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Bertrand, Cournot, Edgeworth, Walras
- A Simple General Equilibrium Analysis

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Abstract:

Bertrand, Cournot, Edgeworth, and Walras equilibria are described and compared for a class of simple convex general equilibrium economies. It is shown that for any finite number of firms, the set of Bertrand equilibrium prices consists of a nondegenerate interval containing the unique Cournot and Walras prices. As the number of firms tends to infinity, Cournot and Walras prices converge to the minimum of average cost. However, the set of Bertrand prices converges to a nondegenerate interval containing the Walrasian and the Cournot prices of almost all economies with finitely many firms.

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1. Introduction

The theory of competition among monopolistic or oligopolistic firms constitutes one of the most extensive research subjects in economics today. It forms the core part of the theory of industrial organisation which has become one of the dominating fields in the past decade. With the development of new concepts and techniques in game theory, oligopoly theory now presents itself in a broader theoretical framework which is much more capable to analyse the complex structure of strategic interaction among economic agents than the traditional one shot full information analysis of static oligopoly theory. In fact it seems that the general dissatisfaction of researchers with the weak descriptive power of static equilibria lead them to analyse, for example, issues of sequential decision making and/or asymmetries in oligopolies. As a consequence some of the fundamental problems of static oligopoly theory were left aside in spite of the fact that their solutions would have an essential influence on more complex strategic models.

Most of the oligopoly models in particular within the literature of industrial organization are presented in a partial equilibrium framework. Until recently no satisfactory general equilibrium approach was available to investigate to what extent the qualitative properties of the partial equilibrium models carry over to a fully closed economy. The lack of a satisfactory general equilibrium oligopoly model is particularly disturbing in the context of normative issues since neither productive efficiency nor Pareto optimality can be examined properly. One other fundamental issue is to what extent economies with large oligopolistic markets approximate competitive economies. The available general equilibrium literature in this area pursues the so called Cournotian approach (for a good account of the literature see the special issue of *Journal of Economic Theory*, Volume 22, Number 2, 1980). One of the prevailing criticisms against this theory centers around the point that it fails to explain how prices are formed, since firms act strategically on quantities while the price setting apparently still requires a Walrasian auctioneer. Bertrand's proposal of strategic price setting is considered to be an acceptable solution to this problem making Cournot's model less convincing. The disturbing and counter intuitive consequence of Bertrand price competition, however, is that competitive equilibrium prices apparently require only two firms which constitutes a paradox to some researchers (Tirole 1988). To resolve it additional restrictions like capacity constraints, increasing marginal cost, and elements of rationing are introduced into the partial equilibrium models to avoid the effect of too much market power of a single firm. But even under these additional restrictions the available limit results (e.g. Allen and Hellwig 1986 a,b) are derived in a partial equilibrium framework

and their relationship to the limit results of the Cournotian general equilibrium approach remain unclear.

The essential difference between the partial and the general equilibrium approach to oligopoly rests in the way 1) how profits earned by oligopolistic firms enter the demand functions faced by oligopolists and 2) how feasibility constraints on markets for input factors required by oligopolists are influenced by the interaction of oligopolistic firms. Typically these markets are taken to be competitive. The partial equilibrium theory of oligopoly typically ignores all income effects as well as the feasibility considerations embodied in 1) and 2). An analysis of these effects is particularly important in models with Bertrand price competition since they influence the feasible strategic possibilities of each firm. If, for example, a firm undercuts the market price of the competitors who loose all of their sales, then the demand function faced by the deviator must have shifted because of an income loss (wages and profits) of consumers. Simultaneously, labor market conditions must change, since zero production by the competitors implies zero factor demand. Thus, for Bertrand equilibria in general rather than in partial equilibrium, a consistent integration of these features is essential. Along similar lines of reasoning one finds that income effects in economies with an increasing number of firms cannot be ignored if firms operate under strictly decreasing returns. In this case aggregate profits imply changes in demand. Simultaneously, the replication of individual technologies generate an increase in aggregate productivity.

In a recent paper (1990) the author suggested a general approach to model Arrow-Debreu economies with oligopolistic firms who take all feasibility and income effects of their demand and supplies fully into account when setting their own prices and quantities. One of the outcomes of this general equilibrium objective demand approach is that in many cases it is not useful to distinguish between price setting or quantity setting oligopolists. Rather, in the general situation, an oligopolist must choose prices and quantities simultaneously given the objective demand set of the rest of the economy which he faces. Within this new general framework this paper describes and compares the different equilibrium concepts of Bertrand, Cournot, Edgeworth, and of Walras. One of the important findings is that Bertrand price competition among producers of homogeneous products in general equilibrium leads to quite different qualitative results than those presented in the partial equilibrium literature. The paper analyses the four equilibrium concepts for a simple class of strictly convex economies with two commodities, one consumer and an arbitrary finite number of identical firms, as

well as for the limiting case when the number of firms tends to infinity.

2. Objective Demand Functions

Consider a private ownership economy with one consumer, $n \geq 1$ identical firms, and two commodities, output $x \geq 0$ and input (labor) $l \geq 0$. The strictly convex technology of each firm is described by a real cost function (input requirement function) $l = c(x)$ for which the following assumption holds:

Assumption A1: $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable with $c(0) = 0$, $c'(x) > 0$, $c''(x) > 0$ for $x > 0$.

The consumer has no endowment of the output commodity and can supply at most $\bar{L} > 0$ units of labor. He receives all profit income and maximizes utility given the price p of the output, the wage rate and total profit. The utility function $u : [0, \bar{L}] \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is quasi-concave, strictly monotonic, and twice continuously differentiable. $u_x(l, x) = \delta u / \delta x(l, x)$ and $u_l(l, x) = \delta u / \delta l(l, x)$ denote the two marginal utility functions. Monotonicity of u implies $u_x > 0$ and $u_l < 0$. The assumptions made so far guarantee that for every $n \geq 1$ there exists a unique Walrasian equilibrium. If the boundary condition $-u_x(0, 0) / u_l(0, 0) > c'(0)$ holds then all equilibrium prices, quantities, and profits are positive.

In order to analyze non-competitive equilibria in this economy the demand behavior of the consumer is best described by the appropriate objective inverse demand function (see Böhm (1990)). For a given wage rate $w_0 > 0$, let

$$p = -w_0 \frac{u_x(l, x)}{u_l(l, x)} = -w_0 R(l, x)$$

denote the nominal price the consumer is willing to pay for the quantity x if he chooses the consumption plan (l, x) . The function R is the marginal rate of substitution, i.e. it defines the relative supporting price for each consumption plan (l, x) . Hence, (l, x) is a utility maximizing supply and demand bundle at prices (w_0, p) and for the total income $I = \text{wages} + \text{profit} = w_0 l + (px - w_0 l) = px$.

Assumption A2: $R_x \leq 0$ and $R_l \leq 0$ with at least one strict inequality.

Assumption A2 stipulates that both commodities are normal and that the marginal rate of substitution is not constant.

For an economy with n firms, called economy n , let

$$X_n = \left\{ (x_1, \dots, x_n) \geq 0 \mid \sum_{i=1}^n c(x_i) \leq L \right\}$$

denote the set of all feasible output plans. Under A1, X_n is a non-empty, compact, convex set. The maximal producible output \bar{y}_n for economy n is given by

$$\bar{y}_n = \max \left\{ \sum_{i=1}^n x_i \mid (x_1, \dots, x_n) \in X_n \right\}.$$

Due to A1, $\{\bar{y}_n\}$ is a strictly increasing sequence in n . $\lim_{n \rightarrow \infty} \bar{y}_n$ is finite if and only if $c'(0) > 0$. For the remainder of the paper set $w_0 = 1$. Then the objective inverse demand function $D^n : X_n \rightarrow \mathbb{R}_+$ is defined by

$$D^n(x_1, \dots, x_n) = R \left(\sum_{i=1}^n c(x_i), \sum_{i=1}^n x_i \right).$$

D^n describes for all feasible output plans those prices at which the bundle $(l, x) = (\sum c(x_i), \sum x_i)$ is utility maximizing given the consumer receives all profits, i.e. D^n incorporates all income effects as well as labor and commodity market feasibility. For most of the analysis in this paper firms' equilibrium decisions will be symmetric since their technologies are identical. In this situation it is convenient to use the "aggregate" inverse demand function

$$\begin{aligned} D_n(x) &= D^n \left(\frac{x}{n}, \dots, \frac{x}{n} \right) \\ &= R \left(n c \left(\frac{x}{n} \right), x \right) \end{aligned}$$

and the "individual" inverse demand function

$$\begin{aligned} d_n(x) &= D^n(x, \dots, x) \\ &= R(nc(x), nx). \end{aligned}$$

It is clear from the definitions that for $n=1$ all of these functions coincide, i. e.

$$D^1(x) = R(c(x), x) = D_1(x) = d_1(x).$$

Given the assumptions A1 and A2 it is now straightforward to verify the properties of the objective demand functions in the following proposition.

Proposition 1: *A1 and A2 imply for all $n \geq 1$ and all $x > 0$:*

- (i) $d_n'(x) < 0$ $D_n'(x) < 0$
- (ii) $d_n(x) > d_{n+1}(x)$
- (iii) $D_{n+1}(x) > D_n(x)$ if $R_1 < 0$.

In other words, both demand functions, "individual" as well as "aggregate", are downward sloping in the conventional price-quantity diagram. Moreover, as the number of firms increases the aggregate demand function shifts to the right and outward due to income effects, while the individual demand function shifts to the left due to efficiency gains in production. Both properties are of course only different aspects of the same effect. The assumption of a single consumer simplifies the analysis substantially. With more than one consumer and/or with weaker assumptions than A2, the objective inverse demand function no longer needs to be downward sloping (see Böhm (1990)). If $R_x = R_1 = 0$, then the objective inverse demand is a constant function independent of the technology. In this case Walras and Cournot equilibria coincide.

3. Bertrand versus Walras

A competitive equilibrium or Walras equilibrium for the economy $n \geq 1$ consists of a price p_n^w and outputs (x_1^w, \dots, x_n^w) such that

$$p_n^w = D^n(x_1^w, \dots, x_n^w)$$

$$p_n^w = c^i(x_i^w) \quad i = 1, \dots, n.$$

Since the cost functions are identical for all firms a Walrasian equilibrium must be symmetric, i.e. $x_i^w = x_j^w$, all $i, j = 1, \dots, n$. Hence, the set of Walrasian equilibria for the economy $n \geq 1$ can be written as

$$WE_n = \{ (x, p) \in \mathbb{R}_+^2 \mid p = d_n(x) = c^i(x) \}.$$

Assumption A3: $\limsup_{x \rightarrow 0} D_1(x) > c'(0)$ and $c'(\bar{y}_1) > D_1(\bar{y}_1)$.

Then one can demonstrate the following proposition.

Proposition 2: *Assumption A1 - A3 imply that for every $n \geq 1$*

Proposition 2: *Assumption A1 - A3 imply that for every $n \geq 1$*

(i) WE_n consists of a unique strictly positive pair (p_n^w, x_n^w) ,

(ii) $\{(p_n^w, x_n^w)\}_{n=1}^{\infty}$ is a strictly decreasing sequence,

(iii) $\lim_{n \rightarrow \infty} p_n^w = \text{Min} \frac{c(x)}{x} = c'(0)$

$\lim_{n \rightarrow \infty} x_n^w = 0$.

Figure 1

Proposition 2 is the general equilibrium analogue of the standard text book partial equilibrium analysis in a market with free entry and increasing marginal costs. The normality of output and leisure which is responsible for the downward sloping demand curve simultaneously drives the limit result, since aggregate demand for

any given price increases with the number of firms and associated higher profits. Figure 1 describes the typical situation for an economy $n > 1$. MC and AC denote the marginal and average cost curves respectively of a single firm.

According to any of the standard presentations of Bertrand price competition in a homogeneous market (see e.g. Tirole (1988)), firms share aggregate demand in some way or another if they all charge the same price. Any firm deviating from such a situation will lose all of its demand if it increases its price above the common level. It will "attract the whole market" if it lowers its price below the common level thereby forcing all other firms to a zero sales position. In a general equilibrium model such a decision to undercut everybody else has two effects, which are ignored in the partial equilibrium setting. First, if all the other firms are forced to produce zero output, they reduce their factor demand to zero. Hence, their profit is zero creating an income loss for consumers. Thus, the demand function faced by the deviator must shift because of a nonnegligible income effect. Second the deviating firm is the only one generating employment. Labor market clearing by one firm then implies a loss in aggregate productivity because of strictly decreasing returns. The general equilibrium analysis here fully incorporates both effects since the objective inverse demand function with one firm alone is well defined. Therefore feasibility considerations and the demand behavior of the consumer imply that the deviating firm faces the objective inverse demand function D_1 , no matter how many other competitors there are in the economy. This leads to the following definition of a Bertrand equilibrium.

Definition: A list (p, x_1, \dots, x_n) is a Bertrand equilibrium for the economy $n \geq 2$, if

$$(i) \quad p = D^n(x_1, \dots, x_n)$$

(ii) for every $i = 1, \dots, n$:

$$(x_i^1, p^1) \text{ such that } p^1 < p \text{ and } p^1 x_i^1 - c(x_i^1) > p x_i - c(x_i)$$

$$\text{implies } p^1 \neq D_1(x_i^1).$$

Condition (i) simply states that firms charge the same price. (ii) excludes the possibility of feasible and profitable downward deviations from the common price. Although there are non-symmetric equilibria in general, only the symmetric ones will be discussed here in detail. The set of symmetric Bertrand equilibria BE_n for economy $n \geq 2$ consists of all pairs $(x, p) \in \mathbb{R}_+^2$ with $p = d_n(x)$ such that (x^1, p^1) with $p^1 < p$ and $p^1 x^1 - c(x^1) > p x - c(x)$ implies $p^1 \neq D_1(x^1)$.

One immediately observes that the price in any Bertrand equilibrium must be less than or equal to p_1^W the Walrasian price in economy 1, since otherwise small price reductions are always beneficial. Moreover the set BE_n is always non-empty. Consider the price \underline{p}_1 which implies zero profit in economy 1, i.e. $\underline{p}_1 = D_1(x) = c(x)/x$. Then, $(x_n, \underline{p}_1) \in BE_n$ if $\underline{p}_1 = d_n(x_n)$, since setting a price lower than \underline{p}_1 implies a loss to the deviator. Continuity of the profit function then suggests that the same argument can be made for a whole interval of prices around \underline{p}_1 for all $n \geq 2$. Hence, one finds that there exists a continuum of symmetric Bertrand equilibria. The precise formulation of these findings is given in Proposition 3.

Proposition 3: *Assume A1 - A3.*

For every economy $n \geq 2$:

- (i) *there exist prices $\bar{p}_n > \underline{p}_n$*
with $p_1^W > \bar{p}_n > \underline{p}_1 > \underline{p}_n > c'(0)$
such that

$$BE_n = \{ (x, p) \in \mathbb{R}_+^2 \mid \bar{p}_n \geq p \geq \underline{p}_n, p = d_n(x) \}$$
- (ii) $\bar{p}_n > p_n^W > \underline{p}_n$,
i.e. $WE_n \in BE_n$,
- (iii) $\bar{p}_n > \bar{p}_{n+1}$ *and* $\lim_{n \rightarrow \infty} \bar{p}_n = \underline{p}_1$
- (iv) $\underline{p}_n > \underline{p}_{n+1}$ *and* $\lim_{n \rightarrow \infty} \underline{p}_n = \lim_{n \rightarrow \infty} p_n^W = c'(0)$.

The proposition substantially modifies the partial equilibrium results and the typical folklore of the effects of price competition in homogeneous markets. Most importantly, (i) states that price competition of the Bertrand type does not lead to a determinate result since a non-degenerate interval of prices is the outcome. (ii) states that Walrasian equilibria are always immune to price cutting for any size of the economy. (iii) and (iv) indicate that the indeterminacy prevailing in any finite market does not disappear as the market grows, but rather increases downward. This shows that, as the number of competitors grows, individual market power forces the maximum Bertrand price down. At the same time, however, more low non-competitive prices become immune to price cutting. Hence with increasing competition any single firm loses its market power at low prices since it cannot maintain the same productivity as several separate firms jointly. Finally, as a

consequence of the Proposition it is immediate that the set of all (symmetric and non-symmetric) Bertrand equilibria has full measure in \mathbb{R}^{n+1} since the inverse demand function D^n and therefore profits are continuous. However, all of the non-symmetric equilibria are inefficient because cost functions are equal.

Proof:

- (i) Consider the regular demand functions $f_n = d_n^{-1}$ and $F = D_1^{-1}$ and define the function

$$H_n : [p_1, p_1^*] \rightarrow \mathbb{R} \text{ by}$$

$$H_n(p) = p f_n(p) - c(f_n(p)) - [p F(p) - c(F(p))].$$

Because of A1 - A3, one easily verifies that

$$H_n'(p) < 0 \text{ and } H_n(p_1) < 0 < H_n(p_1^*).$$

Therefore there exists a unique \bar{p}_n such that $H_n(\bar{p}_n) = 0$ and $p_1^* > \bar{p}_n > p_1$ with the same profit for the firm on d_n and on D_1 . On the other hand, $H_n'(p) < 0$ implies that all $p \in [p_1, \bar{p}_n]$ are Bertrand prices.

Let $p_n = d_n(x) = c(x)/x$. Proposition 1 implies that $p_n < p_1$. Moreover, for all $p \in [p_n, p_1)$ one has

$$p f_n(p) - c(f_n(p)) \geq 0 > p F(p) - c(F(p)) ,$$

which proves (i).

- (ii) is a straightforward consequence of the fact that x_n^w maximizes any firm's profit at p_n^w .

- (iii) Proposition 1 implies that $H_{n+1}(\bar{p}_n) < H_n(\bar{p}_n) = 0$. Therefore, $\bar{p}_{n+1} < \bar{p}_n$ follows from the fact that $H_{n+1}'(p) < 0$. Suppose $\lim_{n \rightarrow \infty} \bar{p}_n = \bar{p} > p_1$. Let

$x_n = f_n(\bar{p}_n)$. Then feasibility and the boundedness of $n c(x_n)$ implies $x_n \rightarrow 0$.

This yields

$$\begin{aligned} 0 &< \tilde{p} F(\tilde{p}) - c(F(\tilde{p})) \\ &= \lim_{n \rightarrow \infty} [\bar{p}_n f_n(\bar{p}_n) - c(f_n(\bar{p}_n))] \\ &= \lim_{n \rightarrow \infty} [\bar{p}_n x_n - c(x_n)] = 0. \end{aligned}$$

Hence, $\lim_{n \rightarrow \infty} \bar{p}_n = p_1$.

(iv) follows directly from Proposition 1.

QED.

Figure 2

Figure 2 provides a geometric characterization for economy $n \geq 2$. In addition to the curves plotted in Figure 1, the critical isoprofit contour for $\bar{Q}_n = \bar{p}_n \bar{x}_n - c(\bar{x}_n) = \bar{p}_n \bar{x}_1 - c(x_1)$ has been inserted with $\bar{p}_n = d_n(\bar{x}_n) = D_1(\bar{x}_1)$. The set of Bertrand equilibria BE_n consists of the whole line segment of d_n for

$p \in [p_n, \bar{p}_n]$. Since d_n shifts to the left as n increases it is clear that $p_n^* \rightarrow MC(0)$ and $\bar{p}_n \rightarrow MC(0)$. For the same reason $\bar{p}_n \rightarrow p_1$, so that

$$BE_n \rightarrow \{0\} \times [MC(0), p_1].$$

The analysis so far has ignored all boundary problems, assuming in particular that feasibility considerations are never binding for all equilibria. With increasing marginal costs it was shown that the size of the market matters for the outcome of price competition, but that there are too many Bertrand equilibria one of which always is the Walrasian one. This contrasts sharply with the results of the partial equilibrium literature claiming that with decreasing returns and/or capacity constraints Walras equilibria are not Bertrand equilibria (Tirole (1988)) and that prices converge to the competitive price in large markets (Allen and Hellwig (1986)). The partial equilibrium literature always uses some form of consumer rationing which is not needed here. Moreover, a consistent analysis with rationing in general equilibrium analysis requires a full consideration of spill over effects between markets, which would lead to a different equilibrium concept with strategic price and quantity setting in a disequilibrium model.

Edgeworth (1897) introduced capacity constraints for firms in order to "solve" the Bertrand paradox. These considerations lead to the so-called Bertrand-Edgeworth equilibria which have been studied extensively in the partial equilibrium literature. As with the situation of decreasing returns, a general equilibrium analysis of Bertrand price competition with capacity constraints does not require any form of consumer rationing. The remaining question then is whether binding capacity constraints for each individual firm change the results of Proposition 3 in a substantial way.

In a general equilibrium model binding output constraints for a firm may have two different causes: 1.) there exist upper bounds for output and/or input quantities because of a capacity choice, or 2.) limited input availability restricts output. In both cases the maximal feasible output K for an economy with one firm constitutes a binding upper bound for the sales of any individual firm, and the lowest feasible price $p_1^1 = D_1(K)$ will be higher in the one firm economy (see Figure 3). This clearly increases the set of potential Bertrand equilibria since for any feasible allocation all prices not greater than p_1^1 but greater than or equal to average cost are Bertrand prices for any economy n . Deviations by any firm below p_1^1 yield no feasible allocation. If the maximal Bertrand price \bar{p}_n calculated for the

case without a capacity constraint is less than the lowest feasible price \underline{p}_1 , then the set of Bertrand equilibria is larger than without the capacity constraint. Since \bar{p}_n decreases monotonically to \underline{p}_1 , this effect appears from some n_0 on. Proposition 4 states these properties. The proof requires only minor modifications from the one of Proposition 3 which is not given.

Figure 3

Proposition 4: *Assume A1- A3 and consider two technological capacity constraints $0 < K_1 < K_2$ with $x_1^w < K_1 < K_2$. If*

$$\frac{c(K_i)}{K_i} < D_1(K_i) < c'(K_i),$$

then the sets of Bertrand equilibria $BE_n(K_i)$ for the constraints K_i , $i = 1,2$ satisfy

(i) $WE_n \in BE_n(K_2) \subset BE_n(K_1)$ for all $n \geq 2$

Moreover, the maximal Bertrand prices satisfy, for all n large enough.

$$(ii) \quad \bar{p}_n(K_1) = \max \{ D_1(K_1), \bar{p}_n \} > \bar{p}_n(K_2) = \max \{ D_1(K_2), \bar{p}_n \} \geq \bar{p}_n$$

and

$$(iii) \quad \lim_{n \rightarrow \infty} \bar{p}_n(K_1) > \lim_{n \rightarrow \infty} \bar{p}_n(K_2) \geq p_1,$$

where \bar{p}_n denotes the maximal Bertrand price for economy n if no capacity constraint exists.

The results show that binding capacity constraints increase the maximal Bertrand price, which is above average cost at full capacity output. Thus the set of Bertrand equilibria, symmetric as well as non-symmetric, increases under capacity constraints. It should be noted that the analysis assumes that all firms charge the same price at equilibrium. This is consistent with the consumer model used here, but different from the typical Bertrand-Edgeworth analysis of partial equilibrium theory which allows for price differentiation. To introduce this in the general equilibrium model here seems to require more than one consumer. It seems that this would not change the general qualitative nature of the results of Proposition 3 and thus the contrast to the partial equilibrium results of the literature remains.

4. Bertrand versus Cournot

Consider now the quantity oligopoly a la Cournot. If each firm $i = 1, \dots, n$ chooses a quantity x_i as its strategy, its profit is given by

$$\Pi_i(x_1, \dots, x_n) = x_i D^n(x_1, \dots, x_n) - c(x_i).$$

Then, a list (x_i^*) , $i = 1, \dots, n$ is a Cournot equilibrium if for all $i = 1, \dots, n$

$$\Pi_i(x_i^*, \dots, x_n^*) \geq x_i D^n(x_i, x_{-i}^*) - c(x_i)$$

for all $x_i \geq 0$ such that $(x_i, x_{-i}^*) \in X_n$.

Since the cost functions are strictly convex and identical for all firms, any Cournot equilibrium must be symmetric. Therefore, for all $n \geq 2$, the set of Cournot equilibria CE_n for economy n can be described as a subset of \mathbb{R}_+^2 .

$$CE_n = \{ (x_n^C, p_n^C) \in \mathbb{R}_+^2 \mid p_n^C = d_n(x_n^C) \}$$

where x_n^C is the best response of each firm and p_n^C is the Cournot equilibrium price. The necessary first order conditions imply

$$R(nc(x_n^C), nx_n^C) + x_n^C [R_1 c'(x_n^C) + R_x] = c'(x_n^C)$$

and A1 - A3 yield $p_n^C > p_n^W$ and $x_n^C < x_n^W$.

Proposition 5: *Assume A1 - A3 and let $\{(x_n^C, p_n^C)\}$ denote a sequence of Cournot equilibria such that p_n^C is bounded above and, for all $n \geq 2$, $nx_n^C \geq \varepsilon$ for some $\varepsilon > 0$. Then*

$$(i) \quad \lim_{n \rightarrow \infty} x_n^C = 0 \quad \lim_{n \rightarrow \infty} p_n^C = c'(0)$$

(ii) *there exists $n_0 \geq 2$ such that for all $n \geq n_0$*

$$CE_n \subset BE_n .$$

Proof: The feasibility constraint $nc(x_n^C) \leq \bar{L}$ and A1 imply $x_n^C \rightarrow 0$. Since $nx_n^C \geq \varepsilon > 0$, the derivatives R_1 and R_x remain bounded below by some constant $M < 0$. Hence the first order condition implies for all n

$$\begin{aligned} p_n^C - w_0 c'(x_n^C) &= R(nc(x_n^C), nx_n^C) - w_0 c'(x_n^C) \\ &= -x_n^C [R_1 c'(x_n^C) + R_x] \leq -M x_n^C (c'(x_n^C) + 1) \\ &\leq -M \bar{x}_n (c'(\bar{x}_n) + 1) \end{aligned}$$

where $\bar{x}_n = c^{-1}(\bar{L}/n)$. Therefore,

$$\lim p_n^C = \lim R(nc(x_n^C), nx_n^C) = w_0 c'(0)$$

and there exists $n_0 \geq 2$ such that $p_n^C \leq p_1$ for all $n \geq n_0$, which proves $CE_n \subset BE_n$ for $n \geq n_0$.

QED

Proposition 5 contains in its first part the general equilibrium analogue of the well known partial equilibrium result, i.e. quantity competition with free entry yields the competitive outcome in the limit with zero profit for each firm and the price equal to the minimum average cost. At the same time with increasing numbers of firms prices will eventually fall below the critical value p_1 . Hence Cournot competition in large markets generates prices which are immune to Bertrand price deviations. It should be noted, however, that the assumptions A1-A3 do not guarantee the existence of Cournot equilibria. Thus, for many cases CE_n may be empty while BE_n was always non-empty.

5. An Example

Consider the following example of a strictly convex economy. Let the consumer's utility function $u : [0, \bar{L}] \times \mathbb{R}_+ \rightarrow \mathbb{R}$ be of the form

$$u(l, x) = \frac{1}{\beta} x^\beta - \frac{1}{\gamma} l^\gamma$$

with $\beta < 0 < 1 \leq \gamma$ and let the real cost function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be given by

$$c(x) = x^\alpha \quad \alpha > 1.$$

If the wage rate w_0 is equal to one, the demand functions D_1, D_n, d_n are

$$D_1(x) = x^\Delta$$

$$D_n(x) = n^{(\alpha-1)(\gamma-1)} x^\Delta$$

$$d_n(x) = n^{\beta-\gamma} x^\Delta,$$

where $\Delta = \beta - \alpha(\gamma-1) - 1 < 0$. One verifies easily that

$$d_n(x) < D_1(x) \leq D_n(x)$$

for all $x > 0$ and for all $n \geq 2$. If $\gamma > 1$, then the second inequality is strict too.

Walrasian equilibria $WE_n = (x_n^W, p_n^W)$ are defined by

$$p_n^W = c'(x_n^W) = d_n(x_n^W).$$

This yields

$$x_n^W = [\alpha n^{\gamma-\beta}]^{1/(\beta-\alpha\gamma)}$$

$$p_n^W = [\alpha^\Delta n^{(\gamma-\beta)(\alpha-1)}]^{1/(\beta-\alpha\gamma)} .$$

$\alpha > 1$ and $\gamma \geq 1 > \beta$ guarantees that

$$\lim_{n \rightarrow \infty} p_n^W = \lim_{n \rightarrow \infty} x_n^W = 0.$$

Moreover, $\beta < 0$ guarantees that

$$\lim_{n \rightarrow \infty} n c(x_n^W) = 0 ,$$

i.e. total labor demand tends to zero. Hence, there exists a finite maximal labor supply $L < 0$, s.t. the equilibria for all economies n are interior feasible allocations.

Cournot equilibria $CE_n = (x_n^C, p_n^C)$ must satisfy

$$D_n(x) \left[1 + \frac{x_i D_n^I}{D_n} \right] = c'(x_i)$$

for every $n \geq 2$ and $x_i = x/n$. One finds that $(x_i D_n^I) / D_n = \Delta/n$, i.e. the elasticity of demand is independent of the quantity x sold in the market, but it decreases with the number of firms. The above condition yields the relationship

$$p_n^C = \frac{1}{1 + \Delta/n} c'(x_n^C)$$

between price and marginal cost, i. e. $-\Delta / (n + \Delta)$ is the percentage markup on marginal cost. The formula shows clearly that the markup goes to zero as the

number of firms grows which yields the limiting result. Solving the above equilibrium condition using d_n yields

$$x_n^C = \left[\frac{1}{1 + \Delta / n} \cdot n^{\gamma - \beta} \right]^{1/(\beta - \alpha\gamma)}$$

and

$$p_n^C = \left[\left[\frac{1}{1 + \Delta / n} \right]^\Delta n^{(\alpha - 1)(\gamma - \beta)} \right]^{1/(\beta - \alpha\gamma)}$$

$\alpha > 1, \gamma \geq 1 > \beta$ and $\Delta < 0$ imply

$$\lim_{n \rightarrow \infty} x_n^C = \lim_{n \rightarrow \infty} p_n^C = \lim_{n \rightarrow \infty} p_n^W = 0.$$

This confirms the result of Proposition 5, that Cournot equilibria converge to Walras equilibria if the number of firms increases without bound. For Bertrand equilibria the interval for the prices has to be described. The lowest Bertrand price p_n is simply that price at which n firms supply at zero profit, i.e. $p_n = c(x_n)/x_n$. Hence

$$n^{\beta - \gamma} x_n^\Delta = x_n^{\alpha - 1}$$

yields

$$p_n = n^{(\gamma - \beta)(\alpha - 1) / (\beta - \alpha\gamma)}$$

The maximal Bertrand price \bar{p}_n guarantees the same profit for each firm whether alone or with $(n-1)$ symmetric competitors in the market. Hence, (x_n, \bar{p}_n) must satisfy

$$\bar{p}_n x_n - x_n^\alpha = \bar{p}_n x_1 - x_1^\alpha$$

with $\bar{p}_n = d_n(x_n)$ and $\bar{p}_n = D_1(x_1)$. Therefore, \bar{p}_n must be a solution of

$$(n^{(\gamma - \beta)/\Delta} - 1) p^{(1 + \Delta)/\Delta} = (n^{\alpha(\gamma - \beta)/\Delta} - 1) p^{\alpha/\Delta}.$$

This yields

$$\bar{p}_n = \left[\frac{n^{\alpha(\gamma-\beta)/\Delta} - 1}{n^{(\gamma-\beta)/\Delta} - 1} \right]^{\Delta/(1+\Delta-\alpha)}$$

$\alpha > 1, \gamma > \beta$ and $\Delta < 0$ implies for all $n \geq 2$

(i) $\bar{p}_n > 1 > p_n$

(ii) $p_1^W > \bar{p}_n > p_n^W > p_n$

(iii) $\lim_{n \rightarrow \infty} \bar{p}_n = 1$

$$\lim_{n \rightarrow \infty} p_n = 0 .$$

Therefore, for all $n \geq 2$, BE_n is a non-degenerate subset of the graph of d_n (see Figure 2) with $\lim BE_n = \{0\} \times [0,1]$. These properties also confirm that for some n_0 the Cournot price $p_{n_0}^C$ must be a Bertrand price, since $p_n^C \rightarrow 0$. Hence $(x_n^C, p_n^C) \in BE_n$ for all $n \geq n_0$. Figure 4 summarizes the results of the price sequences as n grows.

The qualitative features of the example also reveal the fundamental difference of the effect of price competition and of free entry under constant and under decreasing returns. With constant returns i.e. $\alpha = 1$, one finds $\bar{p}_n = p_n = 1$ for all $n \geq 2$. Hence Bertrand competition leads to a determinate result even with two firms. Therefore the number of firms plays no role. With any degree of decreasing returns, i.e. however small the difference $\alpha-1$ may be, price competition among any given number of firms does not yield a determinate outcome. Moreover, free entry does not reduce the indeterminacy in spite of its impact on the aggregate technology which displays constant returns in the limit. Thus the limiting case of free entry does not generate the outcome of the limiting case of the constant returns technology. The fact that, for any fixed $n \geq 2$, $\lim (\bar{p}_n - p_n) = 0$ as $\alpha \rightarrow 1$ implies that price competition among a given set of firms becomes less indeterminate and purely competitive in the limit. But free entry prevents in general that Bertrand price competition implies marginal cost pricing. As a consequence Bertrand equilibria are not Pareto optimal in general, whereas Cournot equilibria become Pareto optimal in the limit.

Figure 4

6. Conclusion

The bulk of the literature treats monopolistic competition in a partial equilibrium setting. This paper has analysed the main noncompetitive equilibrium concepts in a general equilibrium economy. The results were derived for the simple prototype text book model of a strictly convex economy with a unique competitive equilibrium. The primary purpose of the analysis was not to present the most general version of a general equilibrium model with oligopolistic firms, but rather to point out and develop the striking difference of the effects of Bertrand price competition in general equilibrium as compared to the existing partial equilibrium literature. The important results are 1) that for every finite number of firms the set of Bertrand equilibria is a continuum which contains the Cournot as well as the competitive outcome, and 2) that this indeterminacy does not disappear as the number of firms tends to infinity.

With the assumption of only one consumer the general equilibrium description displays an almost partial equilibrium flavor, in spite of the fact that all income and feasibility requirements are considered. Under some generalizations to more than one consumer and more than two commodities, the partial equilibrium flavor is maintained. In such cases the same type of analysis can be carried out leading to similar results. However, in the general situation with more than one consumer or more than two commodities the character of the analysis may change completely. Even with one consumer and weaker assumptions on preferences, the objective inverse demand function may no longer be monotonic (see Böhm(1990)). But it is still true that the general character of equilibria under Bertrand competition remains, i.e. that there exist many equilibria, that Cournot as well as Walras equilibria are outcomes, and that the indeterminacy does not disappear as the number of competitors increases. With a more general structure, however, additional difficulties arise. The possibility of nonsymmetric equilibria has to be taken more seriously. Many additional issues such as the existence of monopolistic equilibria, whether Bertrand, Cournot or Nash in general, may become extremely difficult to deal with. The conceptual extension of equilibria to situations which include rationing may become necessary to obtain equilibria at all, which requires a systematic analysis of spillover effects between markets.

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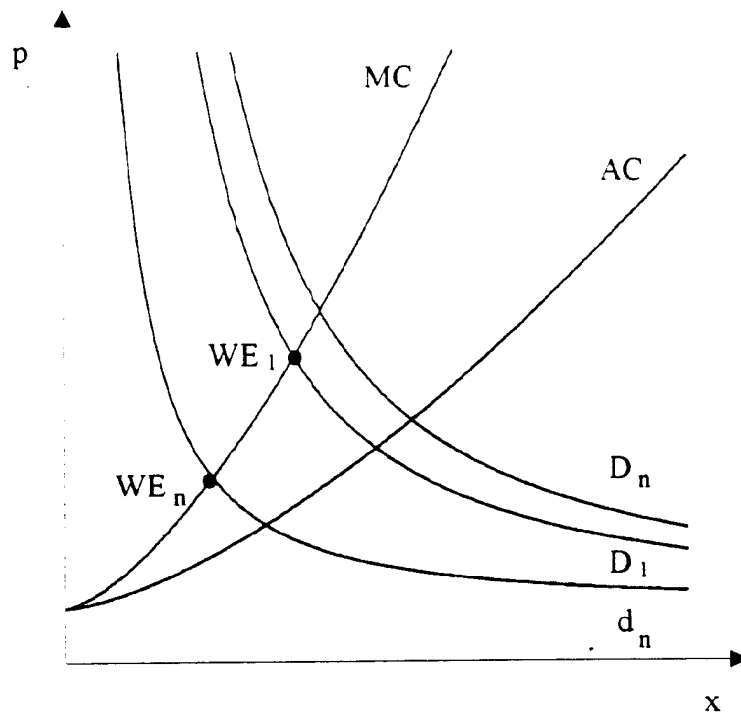


Figure 1

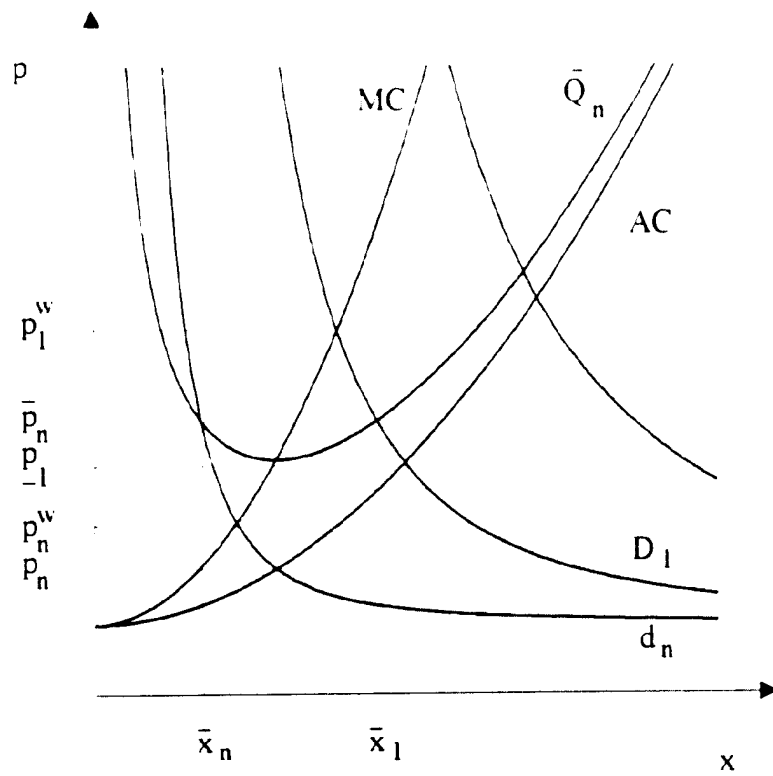


Figure 2

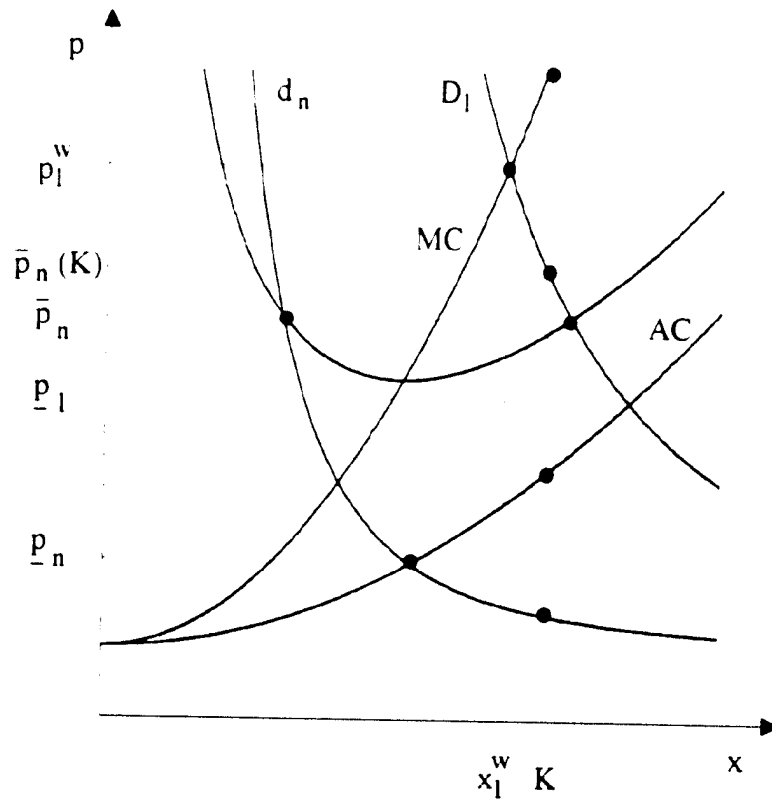


Figure 3

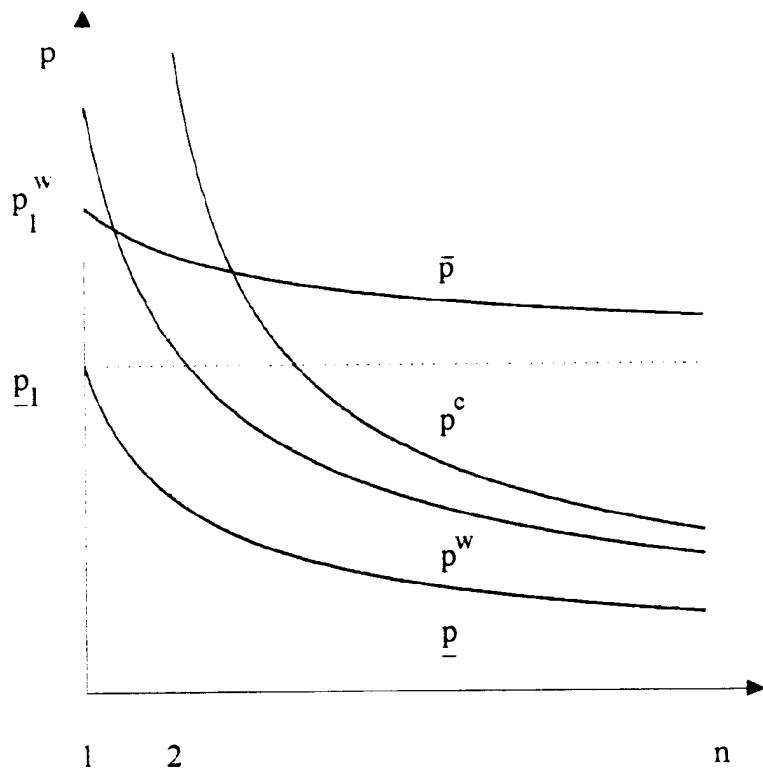


Figure 4

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