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The Double Monopoly Model has no Nash Equilibrium

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Abstract

Following Benassy (1988) and Silvestre (1988) a duopoly (double monopoly) model with prices and wages as strategies is defined for the standard two consumer three commodity fix price model. The non-existence result for Nash equilibria in the paper further confirms and generalizes the result by Silvestre (1988). This is in contradiction to the existence proof of Benassy (1988).

Keywords: Objective Demand, Fix Price Model, Nash Equilibrium, Existence

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1 Introduction

Benassy (1988) has developed a framework for studying general equilibrium with price makers, where the nominal price of each good is set by some agent who takes the prices set by the other agents as given, and where the outcome function of the game is determined by a fix–price equilibrium in the sense of Benassy (1978). Assuming uniqueness of the fix–price map and boundedness of all marginal rates of substitution between each good and money, he demonstrates the existence of a Nash equilibrium in prices.

At first sight, this existence result appears to be somewhat at variance with the results obtained by other authors for particular models within this class. In particular, it has been shown by Silvestre (1988) that no Nash equilibrium exists in the Double Monopoly Model of Benassy (1978) for standard parametrizations of utility and production functions. However, these utility and production functions fail to satisfy the above mentioned boundedness assumption. In this paper, we show that under reasonable assumptions on the rationing mechanism, there exists no Nash equilibrium in the Double Monopoly Model which involves positive consumption of goods for the workers.

The paper is organized as follows: In Section 2, we describe the fix–price model which is the basis for the Double Monopoly Model, and in Section 3, we define the strategies and Nash equilibrium of the Double Monopoly game. Section 4 contains the proofs of our results concerning non-existence of Nash equilibrium.

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2 The fix–price model

We consider an economy with 3 agents; a firm, a consumer-shareholder and a consumer-worker, and 3 commodities; labour, a non-produced good called money, and one good that is produced by the firm using labour input. Labour is supplied by the worker alone and the firm is owned by the shareholder alone. Both the worker and the shareholder own some money, which may be consumed directly or used to purchase goods. Money does not enter into the production process of the firm.

2.1 Technology and preferences

Let $U_W(x, m, \ell)$ and $U_S(x, m)$ denote the utility of the worker and the shareholder respectively, where x, m, and ℓ denotes consumption of goods, money and supply of labour respectively. We assume that U_S is twice differentiable, strictly increasing and strictly quasi–concave in x and m and that U_W is twice differentiable, strictly increasing and strictly quasi–concave in x, m and $-\ell$.

The production technology of the firm is given by $x = F(\ell)$, where ℓ is labour input and x is output of goods. We assume that$F$ is twice differentiable, strictly increasing and strictly concave with $F(0) = 0$.

Let p and w denote the money prices of goods and labour. We will also use the following shorthand notation for the agents' subjective prices of goods and labour in terms of money:

$$p_W(x, m, \ell) := \frac{\partial U_W(x, m, \ell)/\partial x}{\partial U_W(x, m, \ell)/\partial m} \tag{1}$$

$$p_S(x, m) := \frac{\partial U_S(x, m, /\partial x}{\partial U_S(x, m)/\partial m} \tag{2}$$

$$w_W(x, m, \ell) := -\frac{\partial U_W(x, m, \ell)/\partial \ell}{\partial U_W(x, m, \ell)/\partial m} \tag{3}$$

$$w_F(p, \ell) := pF'(\ell) \tag{4}$$

The income of the shareholder and the worker is denoted I_S and I_W, respectively, and their budget constraints are given by

$$\begin{aligned} px + m &= I_S := m_S^0 + pF(\ell) - w\ell && (5) \\ px + m &= I_W := m_W^0 + w\ell && (6) \end{aligned}$$

where m_i^0 is the initial money holdings of consumer i for $i \in \{W, S\}$.

2.2 Fix–price equilibrium

For given prices $(p, w) \geq 0$, a fix–price equilibrium for the model will typically involve rationing of some agents. Our main assumption on the rationing scheme is that

Assumption 1 *There exists a unique fix–price equilibrium allocation for every price pair* $(p, w) \geq 0$.

This implies that all relevant aspects of the rationing scheme and its equilibrium outcomes can be represented by a function $X(p, w) = (X_W(p, w), X_S(p, w))$, where for $i \in \{W, S\}$, $X_i(p, w)$ is agent i's consumption of goods. Let $L(p, w) := F^{-1}(X_W + X_S)$ denote the associated labour supply, and let $I_i(p, w)$ denote consumer i's nominal income as given by 5 and 6 for this labour supply. The budget constraints imply that ex post money holdings are given by $M_i(p, w) := I_i(p, w) - pX_i(p, w)$. In order to rule out negative values of consumption, ex post money holdings and labour supply, we must assume that the fix–price map X satisfies $X_i(p, w) \geq 0$ and $M_i(p, w) \geq 0$.

The rationing scheme is assumed to satisfy the usual conditions of no rationing in the money market, and the short–side rule, which states that (i) no agent is forced to buy or sell more than it wants and (ii) that only buyers or only sellers are rationed on each market. This implies that there are three possible fix–price regimes in the model, namely *Classical unemployment* with demand rationing on the goods market and supply rationing on the labour market, *Repressed Inflation* with demand rationing on both markets, and *Keynesian unemployment* with supply rationing on both markets. Formally, the three regimes are defined in terms of market and subjective prices: Given prices (p, w), let $\ell = L(p, w)$, $x_i = X_i(p, w)$ and $m_i = M_i(p, w)$ for $i \in \{W, S\}$, and let $p_W = p_W(x_W, m_W, \ell)$, $p_S = p_S(x_S, m_S)$, $w_W = w_W(x, m, \ell)$, and $w_F = w_F(p, \ell)$. Note that by the short–side rule, we always have $w_F \geq w$, $w_W \leq w$, and $p_i \geq p$ for $i \in \{W, S\}$. A price pair (p,w) is classified as inflationary ($\mathcal{I}$), classical ($\mathcal{C}$), or Keynesian ($\mathcal{K}$) according to the following definitions:

$$\mathcal{I} := \{(p, w) \geq 0 \mid w_F > w, w_W = w \text{ and } (p_W > p \text{ or } p_S > p)\} \tag{7}$$

$$\mathcal{C} := \{(p, w) \geq 0 \mid w_F = w, w_W < w \text{ and } (p_W > p \text{ or } p_S > p)\} \tag{8}$$

$$\mathcal{K} := \{(p, w) \geq 0 \mid w_F > w, w_W < w \text{ and } p_W = p_S = p\} \tag{9}$$

Actually, these definitions exclude the points on the boundaries between regimes. To include the boundary ponts, we define $\bar{\mathcal{I}}$, $\bar{\mathcal{C}}$ and $\bar{\mathcal{K}}$ to be the closures of $\mathcal{I}$, $\mathcal{C}$ and $\mathcal{K}$, respectively. The boundary between e.g. the Keynesian and the inflationary regimes is then given by $\bar{\mathcal{K}} \cap \bar{\mathcal{I}}$. We assume that

Assumption 2 *The function X is Lipschitz continous everywhere and differentiable on $\mathcal{I} \cup \mathcal{C} \cup \mathcal{K}$, but not necessarily on the boundaries.*

The short-side rule completely determines the rationing scheme for both sides of the labour market and the supply side of the goods market, as there is only one agent involved in each of these three cases. On the demand side of the goods market, we will impose the following condition on the rationing scheme: Let

$$R_S(p,w) := F(L(p,w)) - \frac{w}{p}L(p,w) \tag{10}$$

$$R_W(p,w) := \frac{w}{p}L(p,w) \tag{11}$$

denote real profits and real wage income, respectively. We assume that no consumer will get a smaller binding ration if its real income in terms of R_i increases, i.e.

Assumption 3 *For all $i \in \{W,S\}$, if (p,w) and (p',w') are two price pairs with $p_i > p$ and $p_i' > p'$ such that $R_i(p,w) \geq R_i(p',w')$, then $X_i(p,w) \geq X_i(p',w')$.*

3 The Double Monopoly Model

The Double Monopoly Model is a game between the worker and the shareholder, where the strategy spaces are the set of non–negative wages and prices, respectively, and where the outcome function is the fix–price map X. Given a strategy pair (p,w), the utility of the shareholder and the worker are denoted $V_S(p,w)$ and $V_W(p,w)$ and defined by

$$V_S(p,w) := U_S(X_S(p,w), M_S(p,w)) \tag{12}$$

$$V_W(p,w) := U_W(X_W(p,w), M_W(p,w), L(p,w)) \tag{13}$$

A *Nash equlibrium* for the game is a strategy pair (p,w) such that $V_S(p,w) \geq V_S(p',w)$ for all $p' \geq 0$ and $V_W(p,w) \geq V_W(p,w')$ for all $w' \geq 0$. On the interior of regimes where we have sufficient differentiability, a neccessary condition for Nash equilibrium is that $\partial V_W(p,w)/\partial w = 0$ and $\partial V_S(p,w)/\partial p = 0$.

It should be noted that this description of the Double Monopoly Model differs from that which would follow from the general framework of equilibrium with price–making agents in Benassy (1988). In Benassy's framework, the agents choose their consumption plans, production plans and prices simultaneously, while we assume that consumption and production plans are determined in step one by price–taking behaviour under rationing, and that prices are set in step two given the fix–price map. Thus, in Benassy's framework, each agent may coordinate its consumption or production plan with its price strategy, which yields larger strategy spaces and fewer Nash-equlibria than in our version of the model. Hence, if no Nash equilibrium exists in our version of the model, it will a fortiori not exist in the version based on Benassy's framework.

4 Results

We begin by calculating the effects on the utility of the shareholder and the worker from changes in the goods price and the wage rate, respectively. The following goods market equilibrium condition and consumers' budget constraints will be used throughout:

$$\begin{aligned} F(L(p,w)) &= X_S(p,w) + X_W(p,w) && (14)\\ pX_s(p,w) + M_s(p,w) &= m_s^0 + pF(L(p,w)) - wL(p,w) && (15)\\ pX_w(p,w) + M_w(p,w) &= m_w^0 + wL(p,w) && (16) \end{aligned}$$

Consider a price-pair in $\mathcal{I} \cup \mathcal{C} \cup \mathcal{K}$. We differentiate 12 and the shareholder's budget constraint 15 with respect to p, we eliminate $\partial M_S(p,w)/\partial p$ and use 2, 4, 14 and the fact that $\partial U_S/\partial m > 0$ to conclude that $\partial V_S/\partial p$ has the same sign as

$$DV_S := (p_s - p)\frac{\partial X_S}{\partial p} + (w_F - w)\frac{\partial L}{\partial p} + X_W \qquad (17)$$

Proceeding in a similar fashion for the worker, we obtain that $\partial V_W/\partial w$ has the same sign as

$$DV_W := (p_W - p)\frac{\partial X_W}{\partial w} + (w - w_W)\frac{\partial L}{\partial w} + L \qquad (18)$$

Next, we calculate the effects on the real profits and wage income from changes in the goods price and the wage rate, respectively. By differentiating 10 with respect to p and 11 with respect to w and using 4 we obtain

$$\begin{aligned} \frac{\partial R_S}{\partial p} &= \frac{1}{p}\left((w_F - w)\frac{\partial L}{\partial p} + \frac{w}{p}L\right) && (19)\\ \frac{\partial R_W}{\partial w} &= \frac{1}{p}\left(w\frac{\partial L}{\partial p} + L\right) && (20) \end{aligned}$$

The following lemma is an easy consequence of 19, 20, Assumption 3 and the fact that $w_F \geq w$:

Lemma 1 $\partial L/\partial p \geq 0$ *implies* $\partial X_S/\partial p \geq 0$ *and* $\partial L/\partial w \geq 0$ *implies* $\partial X_W/\partial w \geq 0$.

In Propositions 1–3, we show that there is no Nash equilibrium in $\mathcal{C}$, $\mathcal{I}$ or $\mathcal{K}$. Then we show in proposition 4 that there is no Nash equilibrium on the boundaries either. We begin with

Proposition 1 *If $X_W(p,w) > 0$ for $(p,w) \in \mathcal{C}$, then (p,w) is not a Nash equlibrium.*

Proof: Let $(p,w) \in \mathcal{C}$ be given. We first show that $\partial L(p,w)/\partial p > 0$. By definition 8, we have $w_F = w$, thus by 4, the function L is given implicitly by

$$Z(p,w,L) := pF'(L) - w = 0 \tag{21}$$

which since F is strictly concave implies that $\partial L/\partial p > 0$. By Lemma 1, $\partial X_S/\partial p \geq 0$, which since $p_S \geq p$ and $w_F = w$ and $X_W(p,w) > 0$ implies that $DV_S > 0$. Q.E.D.

Proposition 2 *If $X_W(p,w) > 0$ for $(p,w) \in \mathcal{I}$, then (p,w) is not a Nash equilibrium.*

Proof: Let $(p,w) \in \mathcal{I}$ be given. By definition 7, we have $w_W = w$. Suppose first that the worker is not rationed in the goods market. Then $p_W = p$ and $DV_W = L$ which is positive by 14 since $X_W > 0$, hence (p,w) is no Nash equilibrium.

Next suppose that the worker is rationed in the goods market, i.e. that $p_W > p$. The function $L(p,w)$ is determined by the worker's constrained labour supply function $\ell(x_W, w, m_W^0 - px_W)$, which is the solution to the problem

$$\max_{\ell} U_W(x_W, m_W^0 - px_W + w\ell, \ell) \tag{22}$$

where x_W is the worker's binding ration in the goods market. The equilibrium labour supply L(p,w) is given implicitly by

$$Z(p,w,L) := L - \ell(x_W(L), w, m_W^0 - px_W(L)) = 0 \tag{23}$$

where we have indicated the possible dependence of the ration x_W on L. Of course, in fix–price equilibrium we have $x_W(L) = X_W(p,w)$. Since $X_W(p,w) > 0$, it follows by 14 that $L(p,w) > 0$, and since $Z(p,w,0) \leq 0$, the uniqueness assumption 1 implies that $Z(p,w,0) < 0$ and that $\partial Z(p,w,L)/\partial L > 0$ if we exclude the irregular case of $\partial Z(p,w,L)/\partial L = 0$. By the implicit function theorem, we obtain, using the labour supply function $\ell()$:

$$\frac{\partial L}{\partial p} = -\frac{\partial Z/\partial p}{\partial Z/\partial L} = \frac{\partial \ell/\partial p}{\partial Z/\partial L} \tag{24}$$

$$\frac{\partial L}{\partial w} = -\frac{\partial Z/\partial w}{\partial Z/\partial L} = \frac{\partial \ell/\partial w}{\partial Z/\partial L} \tag{25}$$

Note that since the ration x_W is binding for the worker, a change in p has only an income effect on the labour supply ℓ. Consequently, if leisure is a normal good, then $\partial\ell/\partial p \geq 0$ and $\partial L/\partial p \geq 0$ by 24 since $\partial Z/\partial L > 0$. Therefore, by Lemma 1, $\partial X_S/\partial p \geq 0$, which since $p_S \geq p$ and $w_F \geq w$ and $X_W(p,w) > 0$ implies that $DV_S > 0$. Hence (p,w) is not a Nash equilibrium if leisure is a normal good. If leisure is an inferior good, then an increase in w has a positive substitution effect and a positive income effect on the labour supply ℓ. Consequently, $\partial\ell/\partial w \geq 0$ and $\partial L/\partial w \geq 0$ by 25 since $\partial Z/\partial L > 0$. Therefore, by Lemma 1, $\partial X_W/\partial w \geq 0$, which since $p_W \geq p$, $w_W \geq w$ and $L(p,w) > 0$ implies that $DV_W > 0$. Hence (p,w) is not a Nash equilibrium if leisure is an inferior good either. Q.E.D.

Proposition 3 *If $X_W(p,w) > 0$ for $(p,w) \in \mathcal{K}$, then (p,w) is not a Nash equlibrium.*

Proof: Let $(p,w) \in \mathcal{K}$ be given. By definition 9, we have $p_S = p_W = p$ and $w_W < w < w_F$. I.e., the consumers are not rationed in the goods market, but the worker is rationed in the labour market and the firm is rationed in the goods market. Let $x_S(p, I_S)$ and $x_S(p, I_W, L)$ denote the demand functions for goods of the shareholder and the worker, respectively, when L is the binding constraint on the worker's labour supply. In this regime, the equilibrium labour supply $L(p,w)$ is given implicitly by

$$Z(p,w,L) := F(L) - x_S(p, m_W^0 + pF(L) - wL) - x_W(p, m_W^0 + wL, L) = 0 \qquad (26)$$

Since $X_W(p,w) > 0$, it follows by 14 that $L(p,w) > 0$, and since $Z(p,w,0) \leq 0$, the uniqueness assumption 1 implies that $Z(p,w,0) < 0$ and that $\partial Z(p,w)/\partial L > 0$ if we exclude the irregular case of $\partial Z(p,w)/\partial L = 0$. By the implicit function theorem and the Slutsky equation, we obtain from 26 that

$$\frac{\partial L}{\partial p} = \frac{D_p - D_I x_W}{\partial Z/\partial L} \qquad (27)$$

$$\frac{\partial L}{\partial w} = \frac{D_I}{\partial Z/\partial L} \qquad (28)$$

where

$$D_I := \frac{\partial x_W}{\partial I_W} - \frac{\partial x_S}{\partial I_S} \qquad (29)$$

$$D_p := \left.\frac{\partial x_W}{\partial p}\right|_{U_W=const} + \left.\frac{\partial x_S}{\partial p}\right|_{U_S=const} \qquad (30)$$

and where $\partial x_i/\partial p|_{U_i=const}$ are the own price derivatives of the compensated version of consumer i's demand function $x_i()$.

Suppose, by way of contradiction, that (p, w) is a Nash equilibrium. This implies that $DV_S = DV_W = 0$. Substitute for $\partial L/\partial p$ and $\partial L/\partial w$ from 27 and 28 in the expressions for DV_S and DV_W in 17 and 18, respectively, set the result equal to 0, and rearrange, using the fact that $L(p, w) > 0$, $\partial Z/\partial L > 0$ and that $x_W = X_W$ in fix–price equilibrium. This yields:

$$\frac{\partial Z}{\partial L} X_W + (w_F - w)(D_p - D_I X_W) = 0 \tag{31}$$

$$\frac{\partial Z}{\partial L} + (w - w_W) D_I = 0 \tag{32}$$

Note that since $\partial Z/\partial L > 0$, $X_W > 0$, $w_F > w$ and $w > w_W$, 31 and 32 imply that

$$D_I X_W < 0 \tag{33}$$

$$D_p - D_I X_W < 0 \tag{34}$$

By eliminating $\partial Z/\partial L$ from 31 and 32 and cancelling, we obtain

$$\begin{aligned} 0 &= (w_F - w)D_p - (w_F - w_W)D_I X_W && (35)\\ &< (w_F - w)D_p - (w_F - w)D_I X_W && (36)\\ &= (w_F - w)(D_p - D_I X_W) && (37)\\ &< 0 && (38) \end{aligned}$$

which is the desired contradiction. Here, inequality 36 follows by 33 and the fact that $w_W < w$, and inequality 38 by 34 and the fact that $w_F > w$. Q.E.D.

Proposition 4 *If $X_W(p, w) > 0$ for $(p, w) \in (\bar{\mathcal{C}} \cap \bar{\mathcal{I}}) \cup (\bar{\mathcal{C}} \cap \bar{\mathcal{K}}) \cup (\bar{\mathcal{I}} \cap \bar{\mathcal{K}})$, then (p, w) is not a Nash equlibrium.*

Proof: We first note that $L > 0$ since $X_W > 0$. We distinguish between three cases, corresponding to each of the three possible boundaries between the regimes.

Case 1: $(p, w) \in (\bar{\mathcal{C}} \cap \bar{\mathcal{I}}) \setminus \bar{\mathcal{K}}$

By definitions 7 and 8, we have $w_W = w_F = w$. Suppose first that the shareholder is not rationed in the goods market. Then $p_S = p$ and $DV_S > 0$ by 17 and the fact that $L > 0$, hence (p, w) is not a Nash equilibrium in this case.

Next, suppose that $p_S > p$. The smoothness properties of utility and production functions and Assumption 2 imply that p_S is a continous function of p. Since $(p, w) \notin \bar{\mathcal{K}}$, this implies that there is an open set U in R_+ containing p such that $(p', w) \in (\mathcal{C} \cup \mathcal{I}) \setminus \bar{\mathcal{K}}$

with $p'_S > p'$ for all $p' \in U$. Let $\{p^\nu\}$ be a decreasing sequence from U that converges to p and let $L^\nu = L(p^\nu, w)$, $R^\nu = R(p^\nu, w)$ and $X_S^\nu = X_S(p^\nu, w)$, be the labour supply, real income and and shareholder's goods consumption corresponding to each p^ν in the sequence. Also, let $L = L(p,w)$, $R = R(p,w)$ and $X_W = X_W(p,w)$, and define $\Delta R^\nu = R^\nu - R$ and $\Delta p^\nu = p^\nu - p$. From 10 it follows that

$$\Delta R^\nu = \frac{1}{p}\left(pF(L^\nu) - \frac{p}{p^\nu}wL^\nu - pF(L) + wL\right) \tag{39}$$

$$= \frac{1}{p}\left(p(F(L^\nu) - F(L)) - w(L^\nu - L) + wL^\nu(1 - \frac{p}{p^\nu})\right) \tag{40}$$

As $p^\nu \to p$, the continuity of the function $L()$ implies that $L^\nu \to L$. Consequently

$$\frac{\Delta R^\nu}{\Delta p^\nu} \approx \frac{1}{p}\left((pF'(L^\nu) - w)\frac{L^\nu - L}{p^\nu - p} + \frac{wL^\nu}{p^\nu}\right) \tag{41}$$

as $p^\nu \to p$. Since $(p,w) \in \bar{\mathcal{C}}$, the term $(pF'(L^\nu) - w) \to 0$, and by Lipschitz continuity of the function $L()$, the absolute value of $(L^\nu - L)/(p^\nu - p)$ is bounded above by a constant K. This implies that $\Delta R^\nu / \Delta p^\nu > 0$ for p^ν sufficiently close to p, which in turn by Assumption 3 implies that $X_S^\nu \geq X_S$. Since $X_W > 0$, $p_S^\nu > p^\nu$ and $w_F^\nu \to w$ as $p^\nu \to p$, it follows by the discrete version of 17 that an increase in the p increases the utility of the shareholder. Consequently, (p,w) is not a Nash equilibrium.

Case 2: $(p,w) \in \bar{\mathcal{I}} \cap \bar{\mathcal{K}}$

By definitions 7 and 9, we have $w_W = w$ and $p_W = p$. Since $L > 0$, it follows by 18 that $DV_W > 0$, hence (p,w) is not a Nash equilibrium.

Case 3: $(p,w) \in \bar{\mathcal{C}} \cap \bar{\mathcal{K}}$

By definitions 8 and 9, we have $w_F = w$ and $p_S = p$. Since $W_W > 0$, it follows by 17 that $DV_S > 0$, hence (p,w) is not a Nash equilibrium. Q.E.D.

From Propositions 1–4, we now obtain

Theorem 1 *Under Assumptions 1–3, the Double Monopoly Model has no Nash equilibrium with positive consumption for the worker.*

Our assumption of unique fix–price equilibria turned out to be crucial for the non-existence result. Of course, without this assumption, the whole idea of equilibrium with price making agents in fix–price equilibrium would be very weak, and indeed, the same assumption is made by Benassy (1988).

The other crucial assumption is condition 3 which relates rationing of consumers in the goods market to real profits and real wage income. A more general condition would be that rations are non-decreasing if both aggregate production and own nominal income increases. Lemma 1 would then still hold, and the proofs of Propositions 1–3 would go through as before, as would Cases 2 and 3 of Proposition 4. However, there might be Nash equilibria on the boundary between the Classical and the Inflationary regimes if the elasticity of the worker's constrained labour supply with respect to p and the elasticity of the firm's demand for labour with respect to w were both large negative. If leisure were a normal good and the rationing scheme such that either both consumers or none were rationed in the goods market, the labour supply would be a non-decreasing function of p in $\bar{\mathcal{I}}$ whenever the shareholder were rationed in the goods market. This would rule out equilibria on the boundary $\bar{\mathcal{C}} \cap \bar{\mathcal{I}}$ as before.

To conclude, we have shown in this paper that under fairly general conditions on preferences and rationing schemes, there exists no Nash equilibrium in the Double Monopoly Model with positive consumption of goods for the worker. Had we imposed Benassy's boundedness assumption on the subjective goods prices defined in 1–2, we would get a Nash equilibrium in the Keynesian region at a sufficiently high goods price with zero demand for goods and hence zero production. Our assumption of a positive goods consumption for the worker serves the purpose of ruling out this uninteresting case.

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