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## EFFICIENCY AND OPTIMALITY OF IMPERFECT COMPETITION IN GENERAL EQUILIBRIUM

by

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## 1. Introduction

Conventional wisdom in microeconomics holds that *imperfect competition in the goods markets* creates suboptimalities in the goods markets but does not effect *efficiency in production*. This is clearly stated, e.g., in the following textbook passage from Binger and Hoffman (1988, p. 384):

*"If all other markets are competitive the resources not employed by the monopolist will be employed in producing other goods and the economy will be moved to a different point on the production possibilities frontier where less of the monopolized good and more of other goods are produced. Thus, from a general equilibrium perspective,* **production** *will be efficient, even if there is some monopoly, as long as other goods are produced competitively and inputs are competitively priced."*

The usual diagram supporting this argument compares a point on the production possibilities frontier where the marginal rate of substitution is unequal to the marginal rate of transformation (the *monopoly* case) with a point where the marginal rate of transformation equals the marginal rate of substitution (the *competitive* case), i.e., points B and A respectively, in the following diagram.

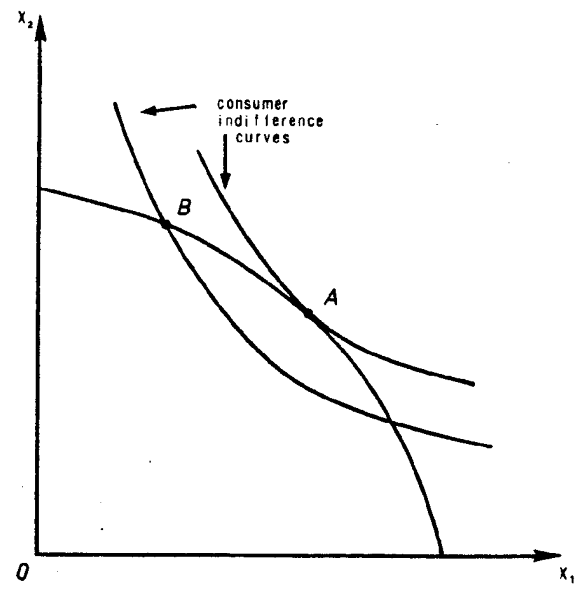


Figure 1

Along similar lines, conventional belief in economic theory was that imperfect competition in commodity markets could not create unemployment in labor markets. This position is strongly supported by Hart (1982) whose results imply that union power may be the primary source for unemployment as a consequence of wage rigidity. The model developed by Hart shows that there is always a market clearing wage rate even if goods markets are not competitive, suggesting essentially that unemployment is a result of imperfections in the labor market alone. However, a recently emerging literature develops quite different results, demonstrating factor unemployment as a consequence of monopolistic competition in commodity markets. These findings are in stark contrast to the traditional view of the effect of monopolistic competition simply driving a wedge between the marginal rates of transformation and the marginal rates of substitution. Factor unemployment as an outcome then implies an additional source of suboptimality in the form of insufficient factor usage generating allocations below the production possibilities frontier. D'Aspremont, Dos Santos Ferreira, and Gerard-Varet (1984, 1989a, b, 1990a, b), Dehez (1985) and Silvestre (1990) show, that Hart's proposition depends on special features of his model and that it does not carry over to a more general framework. In contrast, their results imply unemployment at any positive wage rate as a consequence of commodity market imperfections alone. Hence

*"... perfect competition in the labour market may generate zero wages and unemployment, labour becoming a free good in excess supply. A useful, costless resource is then wasted away, and thus a gross form of inefficiency occurs."* (Silvestre (1990) p. 899).

Sometimes the general equilibrium models used to support such results are not specified in an explicit fashion. However, it seems clear that all authors employ the model proposed by Negishi (1961) in his pioneering work on monopolistic competition in general equilibrium. His work and some of the subsequent publications deal almost exclusively with the question of existence of such an equilibrium. This literature has been reviewed by Negishi himself (1987), but also by Hart (1985) and by Friedman (1982). The techniques developed in Böhm (1990a, b) shed some new light on models of monopolistic competition in general equilibrium. These techniques allow a rigorous formulation of a general equilibrium model with oligopolistic firms in commodity markets when factor markets are competitive. In such a set up it is then possible to investigate the issue to what extent imperfect competition in the output sector of an economy may generate allocations below the production possibilities frontier, which may be of either kind, i.e. inefficient or insufficient usage of input factors.

This paper examines first (sections 2 and 3) the validity of the general efficiency conjecture as e.g. presented by the quotation from Binger and Hoffman (1988). It is shown that the conjecture is correct in a world with full product differentiation among all oligopolists, a fact not recognized in the literature so far. Conversely, efficiency will fail in general, if some firms produce identical products with different technologies. In this case individual cost minimization and competitive factor market clearing do not suffice to obtain efficient allocations. For a class of examples three sources of suboptimality are identified. Section 4 attempts to shed some light on the robustness of the unemployment results of the recent literature. There labor supply is always assumed to be exogenous, whereas sections 2 and 3 treat the case with endogenous labor supply. This makes a comparison of the two types of models difficult. However, by treating the exogenous supply case as the limit of the general endogenous one, it can be shown that the limits of equilibria, if they exist, are typically quite different from the ones with unemployment described in the recent literature.

## 2. Efficiency and Product Differentiation

Consider an economy with a finite set $I = \{1,...,i,...,m\}$ of consumers and a finite set $J = \{1,...,j,...,n\}$ of firms. Firms produce $l \geq 1$ outputs, indexed $h = 1,...,l$, using $k \geq 1$ primary factor inputs, indexed $h = l + 1,...,l + k$, supplied by consumers. Hence, there are $k + l$ commodities in the economy. Firms do not sell to each other, they do not own primary factors, and consumers do not own any endowments of produced goods. Therefore, production sets $Y_j$ of firms $j = 1,...,n$ and consumption sets $X_i$ of consumers $i = 1,...,m$ are subsets of $\mathbb{R}^l_+ \times (-\mathbb{R}^k_+)$. With these conventions a production plan $y_j \in Y_j$ of firm j can be written as $y_j = (x_j,v_j)$ where $v_j \in (-\mathbb{R}^k_+)$ and $x_j \in \mathbb{R}^l_+$, $v_j$ denoting the necessary factor input vector to produce the commodity output vector $x_j$. Given a list $(y_j)$, $j = 1,...,n$ of production plans $y_j \in Y_j$ for all $j \in J$, let $y = \sum_{j \in J} y_j = \sum_{j \in J} (x_j,v_j) = (x,v)$ denote the aggregate production vector.

Definition: *A list of production plans* $y_j = (x_j,v_j)$, $y_j \in Y_j$, $j \in J$ *is efficient if and only if there is no other list of production plans* $y_j' = (x_j',v_j') \in Y_j$, $j \in J$ *such that* $\sum_{j \in J} y_j' \geq \sum_{j \in J} y_j$.

The partition of the commodity space into inputs and outputs makes this standard definition particularly useful to examine the efficiency questions addressed here. For a given aggregate vector $v \in (-\mathbb{R}^k_+)$ of factor inputs, an aggregate output vector $x \in \mathbb{R}^l_+$ is called feasible if there exist production plans $(x_j,v_j) \in Y_j$, $j \in J$ such that $(x,v) \leq \sum_{j \in J} (x_j,v_j)$. Denote by $Q(v)$ the set of aggregate feasible outputs for a given aggregate input vector v. The plan $(x,v)$ is efficient if and only if x belongs to the upper boundary of $Q(v)$, which is the production possibilities frontier for a given v. Let $\partial Q(v)$ denote the production possibilities frontier, and let $\partial^+ Q(v) = \mathbb{R}^n_{++} \cap \partial Q(v)$ denote those points on the frontier where all commodities are produced in positive amounts. The issue of this section is to examine whether any oligopolistic or monopolistic equilibrium with competitive factor markets generates allocations on $\partial^+Q(v)$, thus creating only commodity market distortions and no efficiency loss in factor usage.

Given competitive factor markets, any profit maximizing monopolist or oligopolist minimizes cost at given factor prices and whatever output decision it decides to take. For the general equilibrium model considered here, it turns out that the efficiency of the resulting equilibrium allocation is not automatically guaranteed under cost minimization unless there is complete product differentiation. Let $w \in \mathbb{R}^k_{++}$ denote the price vector for factor inputs and define the cost function for each firm $j = 1,\dots,n$ by

$$C_j(x_j\,, w_j) \;=\; \min\{-\,w\;v_j\;|(x_j,v_j) \in Y_j\}$$

and the conditional factor demand by

$$h_j(x_j\,, w_j) \;=\; \arg\,\min\{-\,w\;v_j\;|(x_j,v_j) \in Y_j\}.$$

Now consider the case of complete product differentiation, i.e. the situation where no two firms are able to produce the same goods. This implies $l \geq n$ and that the set of outputs $L = \{1,\dots,l\}$ can be partitioned into non-empty sets $L_1,\dots,L_n$, such that $h \in L_{j'}$ for any j' implies $y_{jh} = 0$ for all $y_j \in Y_j$ and $j \neq j'$.

**Proposition 1:** *Consider a (noncompetitive) production allocation* $(x_j,v_j)$ , $j \in J$ , *which is cost minimizing for each* $j \in J$ *at factor prices* $w \in \mathbb{R}^k_{++}$, *i.e.* $v_j \in h_j(x_j,w)$, $j \in J$. *Then,* $(x,v) = (\sum x_j\,, \sum v_j)$ *is efficient, if all firms operate under complete product differentiation.*

**Proof:** Suppose the claim were false. Then there exist $(x_j', v_j') \in Y_j$, $j \in J$ such that

$$\sum_{j \in J} (x_j', v_j') \geq \sum_{j \in J} (x_j, v_j).$$

Because of complete product differentiation $x_j' \geq x_j$ must hold for all $j \in J$. This yields $C_j(x_j', w) \geq C_j(x_j, w)$ for all $j \in J$, since cost functions are increasing in output. Then, together with $\sum v_j' \geq \sum v_j$, one obtains

$$\begin{aligned} \sum C_j(x_j', w) &\geq \sum C_j(x_j, w) = -w \sum v_j \\ &\geq -w \sum v_j' \\ &\geq \sum C_j(x_j', w). \end{aligned}$$

Hence, $x_j = x_j'$ for all $j \in J$ and $w (\sum v_j - \sum v_j') = 0$. Since $w \gg 0$ and $\sum v_j \leq \sum v_j' \leq 0$, if follows that $\sum v_j = \sum v_j'$.

QED

The situation with complete product differentiation is clearly very specific. In more general models of imperfect competition analysed in the literature, monopolistically competitive firms very often produce the same commodities. The main prototype model is, of course, the Cournot quantity oligopoly with its Cournot-Nash equilibrium. There and in other non-cooperative equilibrium situations with some quantity competition where not all firms act strategically in the same way (e.g. Bertrand, Chamberlin, Stackelberg), the efficiency question has to be examined. One possible sufficient condition for efficiency is aggregate cost minimization.

**Proposition 2:** *Consider an arbitrary production allocation* $(x_j, v_j)$, $j \in J$. *If* $(x_j, v_j)$, $j \in J$, *minimizes aggregate cost for* $\sum x_j$, *then* $(x_j, v_j)$, $j \in J$, *is efficient.*

**Proof**: Let $(x,v) = (\sum x_j \, , \sum v_j)$ and define the aggregate cost function

$$C(x,w) = \min \{-w \cdot \sum v_j' \mid (x_j' \, , v_j') \in Y_j \, , \sum x_j' = x\} \, .$$

$w \gg 0$ and the usual conditions on $Y_j$ imply that C is strictly increasing in x. Suppose $C(x \, , w) = -w \cdot v$ , but $(x \, , v) = (\sum x_j \, , \sum v_j)$ not efficient. Then, there exists $(\sum x_j' \, , \sum v_j') = (x', v') > (x \, , v) = (\sum x_j \, , \sum v_j)$ .

1) One must have $x' = x$. If not, $-w \cdot \sum v_j' \geq C(x', w) > C(x \, , w) = -w \cdot \sum v_j$ implies

$$w \cdot (\sum v_j - \sum v_j') > 0 \, ,$$

which contradicts $\sum v_j' \geq \sum v_j$ .

2) $v' > v$ implies

$$\begin{aligned} C(x,w) &= -w \cdot v > - w \cdot v' \\ &\geq \sum C_j(x_j',w) \\ &\geq C(x,w) \, . \end{aligned}$$

Therefore, $(\sum x_j \, , \sum v_j)$ is efficient.

QED

It should be noted that aggregate cost minimization is in no way a necessary condition for efficiency, but it is a useful property which can be verified directly from individual cost data. Its failure, however, reveals inefficiency in many cases. In order to demonstrate that efficiency fails in general if some firms produce identical outputs, consider a Cournot duopoly, i.e. an economy with two firms, $n = 2$, and one output, $l = 1$. Assume that the marginal cost functions $MC_1$ and $MC_2$ are strictly increasing in output and that $MC_1(x \, , w) \neq MC_2(x \, , w)$ for all $x > 0$. Let

$p = D(x_1 + x_2)$ denote the inverse demand function. Then, at a Cournot equilibrium $[(x_1,v_1),(x_2,v_2)]$ one must have

$$MC_1(x_1,w) - x_1D'(x_1 + x_2) = D(x_1 + x_2)$$
$$= MC_2(x_2,w) - x_2D'(x_1 + x_2),$$

which is equivalent to

$$MC_1(x_1,w) - MC_2(x_2,w) = (x_1 - x_2)D'(x_1 + x_2).$$

Hence, equality of marginal costs, a necessary condition for efficiency in this case, requires equality of output which is impossible by assumption. Thus, total factor input $v_1 + v_2$ can be rearranged among the duopolists to increase aggregate output $x_1 + x_2$.

It is clear that equality of marginal costs cannot hold in an equilibrium if only some firms behave monopolistically and others behave competitively. Hence such equilibria are typically inefficient. Finally, equality of marginal costs, or an appropriate generalization under non-differentiable cost functions and/or for boundary equilibria, is by itself again only a necessary condition for aggregate cost minimization. There are easy examples of inefficient Cournot equilibria with equality of marginal costs for all firms.

The preceding analysis made an attempt to determine whether imperfectly competitive equilibria are efficient. It was shown that aggregate cost minimization is a reasonable sufficient condition which may be verified if individual cost data are available. Under complete product differentiation this is trivially the case if individual firms minimize costs. However, in general, efficiency will fail, since nothing in the structure of the model or in the non-cooperative equilibrium concepts guarantees the minimizing property nor any other efficiency criterion. Since this reasoning applies to partial as well as to general equilibrium models, one has to conclude that production efficiency is the exception rather than the rule, if quantity competition among firms occurs in the same market.

## 3. Efficiency and Optimality in General Equilibrium - A Geometric Analysis

This section analyses a variety of examples of general Arrow-Debreu economies and develops a simple geometric technique which serves primarily to illustrate the qualitative nature of the failure of efficiency and/or Pareto optimality of non competitive economies. Thus, it supplies a direct tool to exhibit why inefficiencies and suboptimality occur. The method extends the techniques developed in Böhm (1990a).

Consider an economy with two firms and one consumer, with two outputs, and one primary factor (labor), i.e. $l = n = 2$ and $k = m = 1$. The consumer's consumption set is given by

$$X = \{(x_1, x_2, v) \in \mathbb{R}^3 \mid x_h \geq 0 \,,\ h = 1,2,\ 0 \geq v \geq \bar{v}\}$$

where $\bar{v}$ denotes the maximal amount of labor he is able to supply. Preferences are given by a strictly monotonically increasing and strictly quasi-concave, differentiable utility function $U : X \to \mathbb{R}$. Production possibilities are given by two real cost functions $c_j : \mathbb{R}^2_+ \to \mathbb{R}_+$ , $j = 1,2$ , which define for every output pair $(x_{j1}, x_{j2})$ the minimal amount of labor $c_j(x_{j1}, x_{j2}) \geq 0$ necessary to produce $x_j = (x_{j1}, x_{j2})$. Thus, the production possibility sets of the two firms are defined by

$$Y_j = \{(x_1, x_2, v) \mid (x_1, x_2) \geq 0 \,,\ v + c_j(x_1, x_2) \leq 0\} \,,\quad j = 1,2 \,.$$

It will be assumed that there are no fixed cost, i.e. $c(0) = 0$, and that the real cost functions are strictly convex, strictly monotonically increasing, and differentiable. The labor market is competitive with a wage rate normalized to $w = 1$. Cost minimization by both firms implies that labor demand of each firm is equal to $c_j(x_j)$, $j = 1,2$. All profits are distributed to the consumer who is a pricetaker for all three commodities.

**Example 1:** *Product differentiation without separability in utility*

Assume that firm 1 produces commodity 1 and firm 2 commodity 2 only and define $c_j : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $j = 1,2$, in the obvious way. Similarly, for any aggregate level of factor input v, $\bar{v} \leq v \leq 0$, define the set of feasible output combinations as

$$Q(v) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid v + c_1(x_1) + c_2(x_2) \leq 0 \}.$$

Due to the convexity and continuity of the cost functions, Q(v) is a closed convex set. Let $\mathring{Q}(v)$ denote the interior of Q(v) and let

$$\partial Q(v) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid v + c_1(x_1) + c_2(x_2) = 0 \}$$

denote the upper boundary, i.e. the production possibilities frontier for given v.

For the utility function $U(x_1, x_2, v)$, let $U_1 = \partial U/\partial x_1$, $U_2 = \partial U/\partial x_2$, and $U_v = \partial U/\partial v$ denote the functions of the three partial derivatives. Since all income is distributed to the consumer, his demand behavior can best be described by his two objective inverse demand functions

$$\begin{aligned} p_h &= D_h(x_1 , x_2) \\ &= \frac{U_h(x_1 , x_2 , - c_1(x_1) - c_2(x_2))}{U_v(x_1 , x_2 , - c_1(x_1) - c_2(x_2))} \qquad h = 1, 2 . \end{aligned}$$

Notice that the objective inverse demand functions simultaneously reflect labor market clearing and utility maximizing behavior under the appropriate budget constraint, which includes all profits. Feasibility considerations imply that the demand functions are defined on $Q(\bar{v})$, the set of feasible allocations. To avoid boundary problems, only interior allocations will be considered.

The strict convexity of preferences and of technologies implies that there exists a unique Walrasian equilibrium $(x_1^*, x_2^*, v^*)$ which is given by the solution of

$$D_1(x_1^*, x_2^*) = c_1'(x_1^*)$$
$$D_2(x_1^*, x_2^*) = c_2'(x_2^*)$$
$$v^* + c_1(x_1^*) + c_2(x_2^*) = 0 .$$

For the diagrammatic presentation (see Figure 2) it is useful to describe the interior behavior of both firms by the two sets of solutions

$$W_1 = \{(x_1,x_2) \in \mathring{Q}(\bar{v}) \mid D_1(x_1,x_2) = c_1'(x_1)\}$$

and

$$W_2 = \{(x_1,x_2) \in \mathring{Q}(\bar{v}) \mid D_2(x_1,x_2) = c_2'(x_2)\}.$$

$W_1$ and $W_2$ define two lines in $Q(\bar{v})$ whose slopes depend among other things on whether the commodities are substitutes or complements. In either case, their intersection defines the Walrasian equilibrium, i.e. $W_1 \cap W_2 = WE$. Since $v^*$ is assumed to be less than $\bar{v}$, WE is interior to $Q(\bar{v})$. Figure 2 contains the two possibilities frontiers $\partial Q(\bar{v})$ and $\partial Q(v^*)$ and the appropriate indifference curve of the consumer tangent to $\partial Q(v^*)$ at WE.

To describe noncompetitive behavior of both oligopolists, define

$$R_1 = \{(x_1,x_2) \in \mathring{Q}(\bar{v}) \mid \partial D_1(x_1,x_2)/\partial x_1 + x_1 D_1(x_1,x_2) = c_1'(x_1)\}$$

and

$$R_2 = \{(x_1,x_2) \in \mathring{Q}(\bar{v}) \mid \partial D_2(x_1,x_2)/\partial x_2 + x_2 D_2(x_1,x_2) = c_2'(x_2)\}$$

which are the profit maximizing reaction curves (best responses) of the two firms. For $\partial D_j / \partial x_j < 0$, $R_j$ must lie to the left of or below $W_j$ (see Figure 2). It is clear

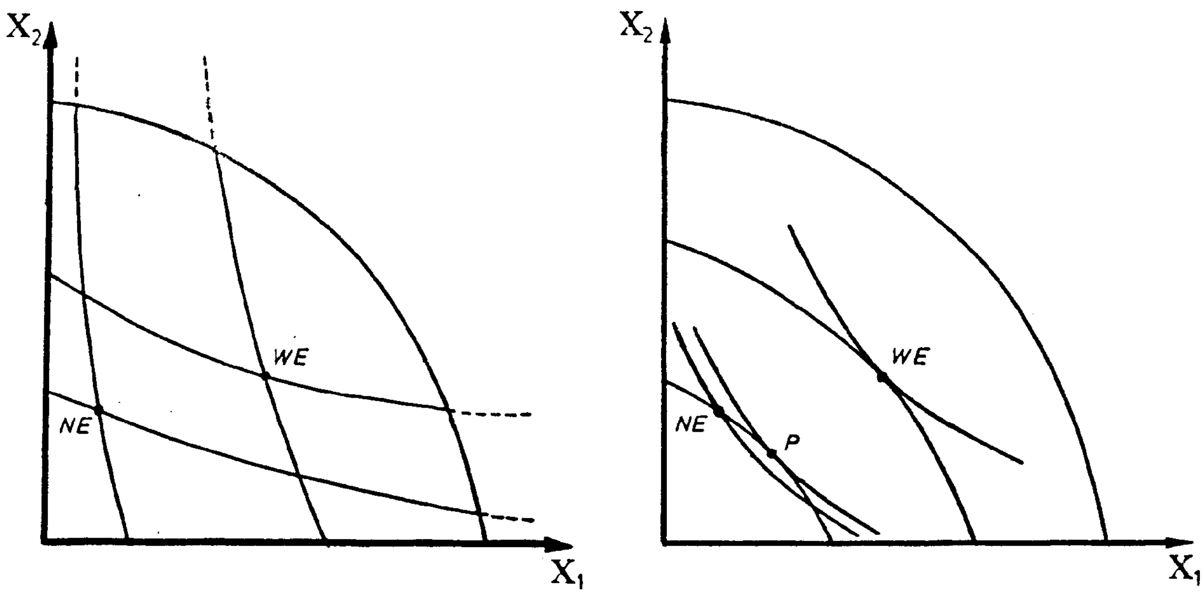


Figure 2

that $R_1 \cap R_2$ = NE defines a Nash equilibrium with output levels $(\tilde{x}_1, \tilde{x}_2)$ and input level $\tilde{v}$. Since the Nash equilibrium is efficient (Proposition 1), NE is on the frontier $\partial Q(\tilde{v})$. However, in spite of efficiency, there are two ways to Pareto improve the Nash allocation. First, given total inputs $\tilde{v}$, one could adjust relative prices $p_1/p_2$ to relative marginal cost, thus eliminating the wedge between the MRS and the MRT for outputs (in Figure 2 move to P). But this still leaves both prices higher than their respective marginal costs. Therefore, as a second improvement, one can move to WE by eliminating the wedge between marginal value product and factor price.

**Example 2:** *Duopoly without product differentiation*

Consider an economy as in the previous example, but with $k = l = m = 1$, and $n = 2$, in other words a homogeneous duopoly. With the appropriate reinterpretation, one may use the same notation and geometric representation as for Example 1. The formal implication of identical commodities now is a utility function of the form $U(x_1 + x_2, v)$ where $x_j$, $j = 1,2$, is the quantity produced by firm j. Figure 3 portrays the four lines $W_j$, $R_j$, $j = 1,2$ and the equilibria WE and NE. Efficiency prevails for a given level v of factor input, if and only if

$$x_1 + x_2 = \text{Max}\left\{x'_1 + x'_2 \mid v + c_1(x'_1) + c_2(x'_2) \leq 0\right\}.$$

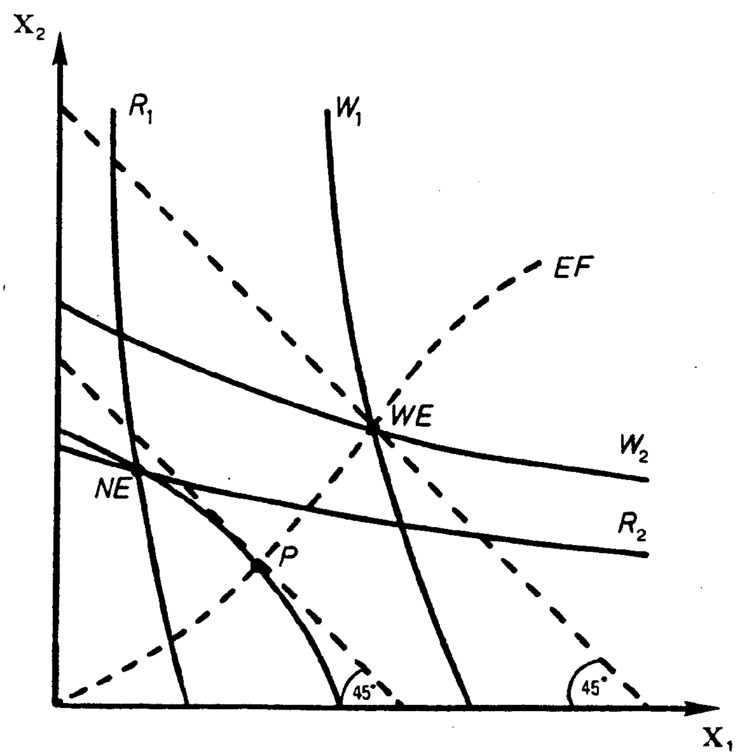


Figure 3

This implies the necessary condition $c_1'(x_1) = c_2'(x_2)$ which is satisfied along an efficiency line EF alone. Thus, all points not on EF are inefficient, and, in particular (Proposition 2), the Nash equilibrium will not be efficient in general because of misallocation of factors between usages for the same output. Hence, moving from NE to P implies an efficiency gain and a welfare gain at a constant aggregate input level. Figure 4 provides another characterization of the inefficiency of the Nash equilibrium NE and of the possible utility gains. NE is below the possibility frontier $\partial Q$, given by the envelop of the two real cost functions $c_1$ and $c_2$. Thus, Pareto improvements are possible through reallocation of factor inputs to reach point P on the frontier by equalizing marginal costs and, in a second step, through setting marginal costs equal to price to obtain the Walrasian equilibrium WE.

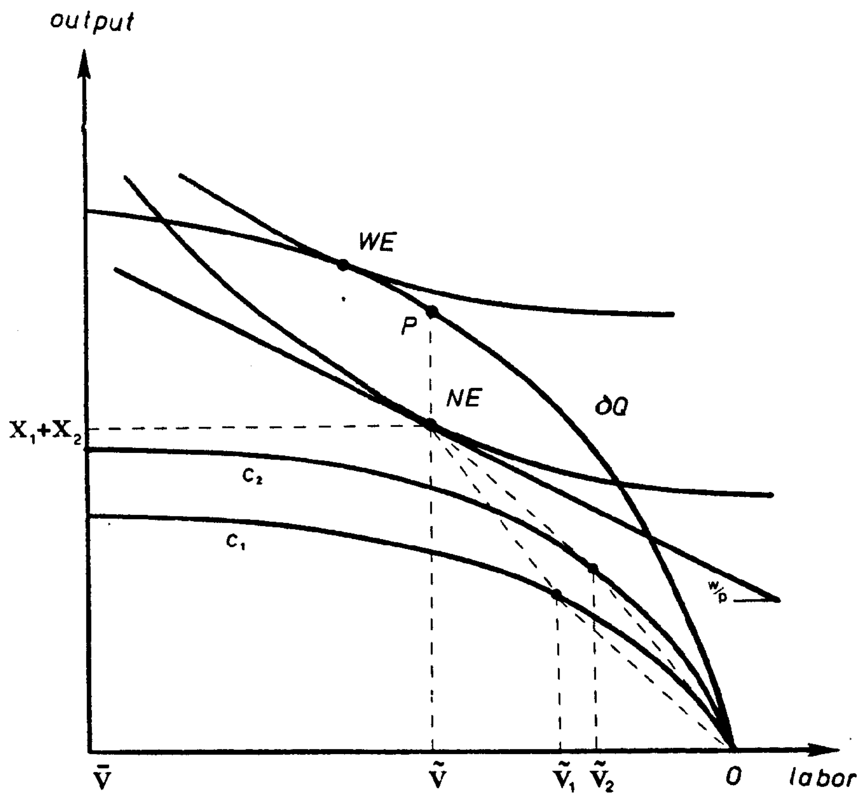


Figure 4

In spite of their simplicity, Figures 2 and 3 capture all essential aspects of the efficiency and welfare losses in noncompetitive economies. Consider again the situation of Example 1 with product differentiation (see Figure 5). Then, the pair

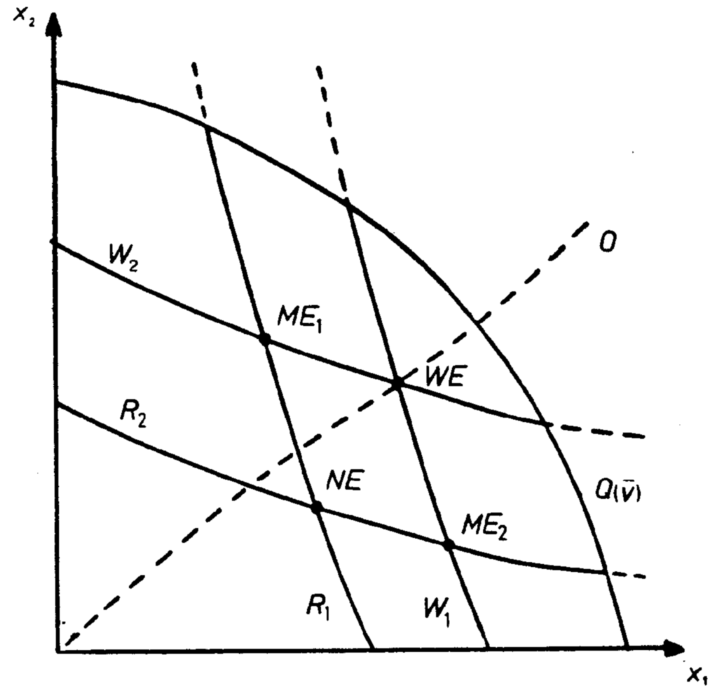


Figure 5

of the two lines $W_i$ and $R_i$ , $i = 1,2$ , also yield the two one-sided monopolies $ME_1 = R_1 \cap W_2$ and $ME_2 = R_2 \cap W_1$ , which must be efficient. However, they suffer from the same optimality failure as the Nash equilibrium. Geometrically this can be seen most easily if one adds the locus of those points where there is no wedge between the MRS in consumption and the MRT in production. Thus, define the optimality line

$$O = \{(x_1, x_2) \in \mathring{Q}(\bar{v}) \mid D_1(x_1, x_2)/D_2(x_1, x_2) = c_1'(x_1)/c_2'(x_2)\}.$$

Then, in general, none of the noncompetitive equilibria whether with or without product differentiation will lie on the optimality line.

Some further arguments now yield a final evaluation of the opening quotation from Binger and Hoffman (1988), or of common economic folklore, as the case may be. Efficiency, they claim, is guaranteed *"... even if there is some monopoly, as long as other goods are produced competitively and inputs are competitively priced"*. Proposition 1 clearly states that efficiency requires product differentiation and that it is independent of whether other goods are competitively priced. The presence of other competitive goods markets, as for example in the two one-sided monopolistic equilibria, does not alter the efficiency property. What matters is the number of firms in an imperfectly competitive market. Example 2 shows that an equilibrium with two Cournot oligopolists is not efficient in general. The same result is true, of course, if only one firm behaves strategically and for the other one price equals marginal cost. Hence, in a strict sense, the statement by Binger and Hoffmann is not wrong, since efficiency prevails always if a noncompetitive firm is alone in a market, i.e. if it is a true monopolist. The qualifying restriction, that there must be other competitive markets, is not necessary. However, quantity competition among several firms will destroy efficiency in general if at least one firm is not a price taker.

Summarizing the findings of Sections 2 and 3 of the paper, one can identify three sources or features of suboptimality in a general equilibrium model with imperfect competition:

1) misallocation of given input factors between different producers for the same output, i.e. inefficiencies in production,
2) misallocation of given inputs between different outputs,
3) misallocation between inputs and outputs.

All three arise in general in spite of cost minimization of all firms and independently of how many commodity markets or firms are noncompetitive. Only feature 1 implies inefficiency and it occurs only, but generally, in imperfectly differentiated markets, i.e. if some output is produced by more than one firm.

Feature 2 essentially stems from the wedge between relative monopolistic prices and the efficient marginal rate of transformation, alluded to by the opening quotation from Binger and Hoffman. Feature 3 arises from the wedge between factor productivity and the equilibrium real wage which coincides with the marginal rate of substitution of a competitive consumer between output and labor. Hence, in spite of the monopolistic behavior in commodity markets, full employment prevails at any given positive wage rate.

## 4. Imperfect Competition and Diminishing Disutility of Labor

The preceding analysis dealt with monopolistic competition in commodity markets, assuming competititve behavior and market clearing in factor markets. Where this assumption was made in the examples, the explicit formulation of the labor market clearing condition for interior consumption allocations required a positive real wage equal to the appropriate marginal rate of substitution between consumption and labor. Thus, labor supply was endogenous with a nonzero disutility for labor. The results derived indicate that monopolistic competition in this setup does not (cannot) generate factor unemployment. In contrast d'Aspremont et al. (1984, 1989a, b), Dehez (1985), and Silvestre (1990) derive distinctly different economic consequences on factor employment from imperfect competition, implying unemployment for all positive wage rates due to monopolistic behavior in the commodity market alone. There are at least two possible explanations for these apparently contradicting results.

One of the explanations could be that most of the contributions do not incorporate fully all income and employment effects generated through price changes, i.e. oligopolists do not completely know the objective demand but only the price component of the notional Walrasian demand. In fact Dehez (1985) attributes the particular results of d'Aspremont et al. (1984) to their taking the true objective demand instead of the restricted one as do Hart and also d'Aspremont et al.

(1990a). However, as d'Aspremont et al. (1990b) show, even with full incorporation of all income effects there may be unemployment at any wage rate. Thus, they claim monopolistic competition in commodity markets instead of wage rigidities can be the sole source of involuntary unemployment.

A second explanation may be that all of the above authors use models with a fixed exogenous labor supply, i.e. without disutility of labor for consumers. In such a case, it is straightforward to define the concept of an equilibrium with unemployment, since spillovers from labor to consumption do not exist. In this case the real wage plays no role for consumption decisions and only income matters. In a model with endogenous labour supply, as presented here, the concept of an equilibrium with involuntary unemployment but with market clearing on imperfectly competitive commodity markets is less clear. Such an analysis would require an integration of monopolistic behavior into an equilibrium theory with quantity constraints, which to our knowledge has not been done yet in a systematic way.

In spite of some other minor structural differences between the model presented in this paper and those in the literature just discussed, the central remaining disparity of the two models consists of the presence or absence of disutility of labor. In both models commodity markets are treated essentially in the same way, i.e. firms maximize against notional demand curves. Since exogenous labor supply can be modeled as a limiting case of endogenous supply, the robustness of the unemployment result could be investigated by a limit argument for a sequence of economies with positive but decreasing disutility of labor. As the following analysis shows, the limits of equilibria with imperfect competition on commodity markets can equally well exhibit unemployment or overemployment, depending on consumer preferences. Whereas overemployment, i.e. excess demand of labor by oligopolists, occurs for all disutility of labor sufficiently small, unemployment appears only as the limit of sequences of Nash equilibria whose employment levels are bounded away from maximal employment. Thus, it seems that, within the class of models which do not fully incorporate spillover effects due to binding quantity constraints, unemployment as a consequence of imperfect competition appears to be a boundary phenomenon for particular preferences.

**Example 3:** *Product differentiation with additively separable utility*

As in Example 1, assume that firm 1 produces commodity 1 and firm 2 commodity 2 only, using the two cost functions $c_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , $i = 1,2$ , where $c_i$ is strictly convex with $c_i(0) = c_i'(0) = 0$, and $c_i'(x) > 0$ for $x > 0$. The utility function is of the form

$$U(x_1,x_2,v) = u_1(x_1) + u_2(x_2) + \delta v$$

with $\delta > 0$ and $\bar{v} \le v \le 0$ and $u_i$ strictly concave, strictly monotonically increasing such that $u_i''' \le 0$ , $i = 1,2$ . Notice that the sign restriction on the third derivative implies that the function $u_i'(x_i) + x_i u_i''(x_i)$ is strictly decreasing in $x_i$ , $i = 1,2$ .

Given these assumptions, consider any positive disutility $\delta$ and define the sets of equilibria discussed in the previous section, i.e.

the Walras equilibrium $WE(\delta)$
the Nash equilibrium $NE(\delta)$
the monopolistic equilibria $ME_i(\delta)$ $i = 1,2$

as subsets of $Q(\bar{v}) \subset \mathbb{R}^2_+$ (see Figure 5). It is straightforward to show that, for all $\delta > 0$, the sets $WE(\delta)$, $NE(\delta)$, and $ME_i(\delta)$, $i = 1,2$ are non-empty for any positive wage rate w. Without loss of generality set w equal to one, as before. Moreover, since there are no crossprice effects in demand, $WE(\delta)$ consists of a unique point $x^*(\delta) = \left(x_1^*(\delta) , x_2^*(\delta)\right) \in \mathbb{R}^2_{++}$ , given by the solution of

$$u_i'(x_i^*) = \delta\, c_i'(x_i^*) \qquad i = 1,2\ ,$$

with an associated equilibrium price vector $p^*(\delta) = \left(c_1'(x_1^*)\ ,\ c_2'(x_2^*)\right)$. The following proposition simply states that Walras equilibria will be on the boundary with maximal employment $\bar{v}$ for all $\delta$ below some positive level $\delta^w$. Hence, the consumer will be at a constant allocation on the boundary of his consumption set. Moreover, the equilibrium real wages will be constant and greater than the marginal rates of substitution between labor and the two goods. In other words, small $\delta$ have neither real nor nominal effects on Walras equilibria at a given wage rate. The proof is a simple exercise and therefore suppressed.

**Proposition 3:** *There exists a unique positive $\delta^w$ such that for every wage rate $w > 0$:*

(i) $\delta > \delta^w$ *implies*

$WE(\delta) \subset \text{int } Q(\bar{v})$ ,

(ii) $\delta \leq \delta^w$ *implies*

$x^*(\delta) = x^*(\delta^w) \in \partial\, Q(\bar{v})$

$p^*(\delta) = p^*(\delta^w)$ .

Consider now the Nash equilibrium. It is clear that $NE(\delta)$ is unique and with quantities less than those at $WE(\delta)$ for all $\delta \geq \delta^w$. But it is also evident that interior Nash equilibria must be increasing as $\delta$ becomes smaller. This follows from the first order conditions

$$u_i'(x_i) + x_i u_i''(x_i) = \delta\, c_i'(x_i)\ ,$$

since the left hand side, i.e. marginal revenue, is assumed to be a strictly decreasing function of $x_i$ , $i = 1,2$. Hence, the behavior of the marginal revenue function determines, whether solutions tend to the boundary of $Q\,(\bar{v})$ or not if $\delta$ tends to zero. The two possible cases, i.e. marginal revenue always positive or negative from some value of $x_i$ on, have distinctly different implications for the type of Nash equilibria, provided by Propositions 4 and 5.

If marginal revenue stays positive, then, for some $\delta$ small, Nash equilibria must be on the boundary of $Q(\bar{v})$ with maximal employment. Then further decreases of $\delta$ imply a factor demand restriction for both duopolists and thus multiple Nash equilibria. Proposition 4 states the formal results.

**Proposition 4:** *Assume that for all* $x_i > 0$ , $i = 1,2$ :

$$u_i'(x_i) + x_i u_i''(x_i) > 0 .$$

*Then, there exists a unique positive* $\delta^N < \delta^W$, *such that for every wage rate* $w > 0$:

(i) $NE(\delta^N)$ *consists of a unique point in* $\partial Q(\bar{v})$,

(ii) *for all* $\delta \leq \delta^N$

$$NE(\delta) \subset \partial Q(\bar{v})$$

*with*

$$NE(\delta) \subset NE(\delta') \quad \textit{for} \quad \delta' < \delta$$

*and*

$$\lim_{\delta \to 0} NE(\delta) = \partial Q(\bar{v}) ,$$

(iii) *for any sequence* $\delta \to 0$ *and* $\left(\tilde{x}_1(\delta) , \tilde{x}_2(\delta)\right) \in NE(\delta)$ *the associated price sequences* $\left(\tilde{p}_1(\delta) , \tilde{p}_2(\delta)\right)$ *become unbounded.*

**Proof:** $(\tilde{x}_1,\tilde{x}_2)$ is a Nash equilibrium for $\delta > 0$ and $w > 0$ if and only if

$$\tilde{x}_i \in \arg\max \left\{ x_i u_i(x_i) - \delta c_i(x_i) \mid c_i(x_i) + c_j(\tilde{x}_j) + \bar{v} \leq 0 \right\}$$

$i \neq j$ , $i,j = 1,2$ . The assumptions on $u_i$ and $c_i$ , $i = 1,2$ guarantee that a solution $(\tilde{x}_1\ \tilde{x}_2)$ of the first order condition

$$(*) \qquad u_i'(\tilde{x}_i) + \tilde{x}_i u_i''(\tilde{x}_i) = \delta\, c_i'(\tilde{x}_i) \qquad i = 1,2$$

increases without bound as $\delta$ tends to zero. Hence there exist $\delta^N > 0$ such that (∗) holds and $c_1(\tilde{x}_1) + c_2(\tilde{x}_2) + \bar{v} = 0$ . The monotonicity of the cost functions implies the uniqueness of $\delta^N$. Moreover, for all $0 < \delta < \delta^N$, and $(x_1,x_2)$ which solve (∗), imply $c_1(x_1) + c_2(x_2) + \bar{v} > 0$ , contradicting feasibility. Therefore, $\delta < \delta^N$ implies that the set of Nash equilibria is given by

$$NE(\delta) = \left\{(\tilde{x}_1,\tilde{x}_2) \in \partial Q(\bar{v}) \mid u_i'(\tilde{x}_i) + \tilde{x}_i\, u_i''(\tilde{x}_i) \geq \delta\, c_i'(\tilde{x}_i) \;,\; i = 1,2\right\} .$$

It is straightforward to see that $\delta' < \delta$ implies $NE(\delta) \subset NE(\delta')$ and $\lim NE(\delta) = \partial Q(\bar{v})$ .

To prove (iii), consider any $(\tilde{x}_1,\tilde{x}_2) \in NE(\delta) \subset \partial Q(\bar{v})$ , i.e. $c_1(\tilde{x}_1) + c_2(\tilde{x}_2) + \bar{v} = 0$. For demand to be satisfied, prices $\tilde{p}_i(\delta)$ , $i = 1,2$ , have to fulfill

$$\tilde{p}_i(\delta) = \frac{u_i(\tilde{x}_i)}{\delta} \qquad i = 1,2 .$$

Therefore $\tilde{p}_i(\delta) \to +\infty$ as $\delta \to 0$ .

QED

Proposition 4 shows quite strikingly that diminishing disutility of labor does not lead to unemployment. On the contrary, for $\delta$ below $\delta^N$, there exists a continuum of Nash equilibria with excess demand on the factor markets and $NE(\delta)$ is a subset of the production possibilities frontier (see Figure 6). For $\delta$ small enough, any point on the frontier becomes a Nash equilibrium, albeit with prices of the commodities arbitrarily large.

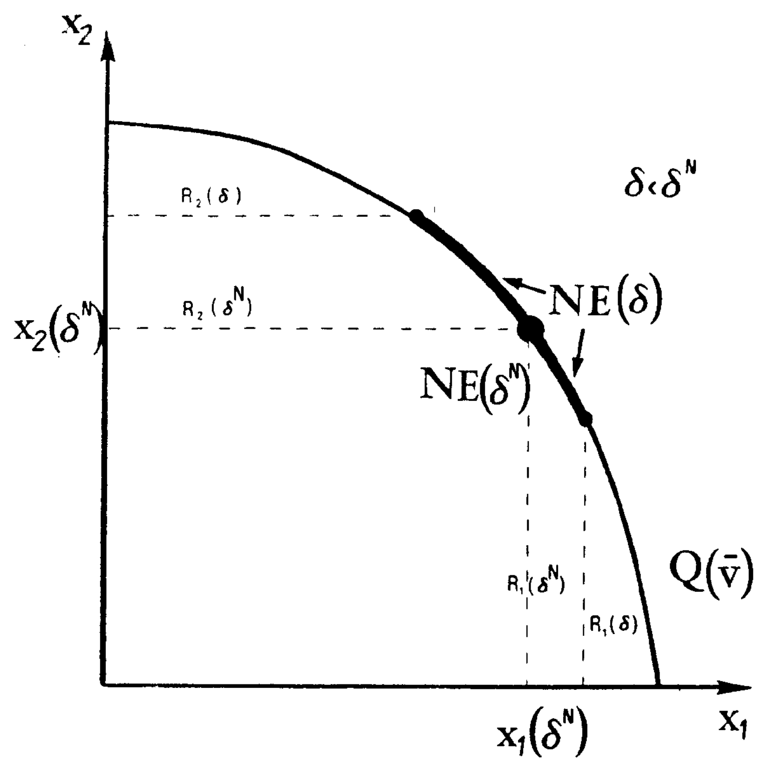


Figure 6

It is clear that the two monopolistic equilibria $ME_i(\delta)$ , i = *1,2* behave in a similar way as the Nash equilibria, but with a restriction generated from the competitor. More precisely, for $\delta$ small, and $i = 1,2$ , one has

$$ME_i(\delta) = \left\{ (\tilde{x}_i , x_j^*) \mid u_i'(\tilde{x}_i) + \tilde{x}_i u_i''(\tilde{x}_i) \geq \delta c_i'(\tilde{x}_i) , u_j'(x_j^*) = \delta c_j'(x_j^*) , c_i(\tilde{x}_i) + c_j(x_j^*) + \bar{v} = 0 \right\} ,$$

which is a subset of $\partial Q(\bar{v})$ . It is straightforward to see that, for each $i = 1,2$ , there exists a unique $\delta_i > 0$ such that $ME_i(\delta_i)$ is unique and on the boundary $\partial Q(\bar{v})$. However, $\delta$ small implies that all allocations on the boundary with larger output than at $ME_i(\delta_i)$ for the competitor and less for the monopolist are also monopolistic equilibria. These results show quite strikingly that imperfect competition on commodity markets does not imply unemployment in general and, on the contrary, that excess demand in factor markets, i.e. overemployment, may occur, if the disutility of labor decreases.

Finally, consider the situation where the marginal revenue functions become negative for some finite output value.

**Proposition 5:** *Let* $\bar{x}_i > 0$ , $i = 1,2$ , *be such that* $u_i'(\bar{x}_i) + \bar{x}_i u_i''(\bar{x}_i) = 0$ *and assume* $\bar{v} + c_1(\bar{x}_1) + c_2(\bar{x}_2) < 0$ . *Then for every positive* $\delta$ *and* $w > 0$, *the associated Nash equilibrium* $NE(\delta)$ *consists of a single point* $\left(\tilde{x}_1(\delta) , \tilde{x}_2(\delta)\right) \ll (\bar{x}_1,\bar{x}_2)$ *in the interior of* $Q(\bar{v})$ . *Moreover,* $\tilde{x}_i(\delta)$ *converges to* $\bar{x}_i$ *as* $\delta$ *converges to zero, and prices* $\tilde{p}_i(\delta)$ *tend to infinity for* $i = 1,2$ .

**Proof:** The assumption on $u_i$ , $i = 1,2$ , guarantees for every $\delta > 0$ a unique feasible solution $\tilde{x}(\delta)$ to

$$u_i'(x_i) + x_i u_i''(x_i) = \delta c_i'(x_i)$$
$$\bar{v} + c_1(x_i) + c_2(x_2) < 0 .$$

$\delta \to 0$ implies $\tilde{x}_i \to \bar{x}_i$ which is interior to $Q(\bar{v})$ by assumption. Moreover,

$$p_i(\delta) = \frac{u_i'\left(\tilde{x}_i(\delta)\right)}{\delta} \qquad i = 1,2 .$$

Hence, $p_i(\delta) \to +\infty$ as $\delta \to 0$.

QED

Proposition 5 indicates that there are situations with monopolistic equilibria where less than maximal employment occurs in the limit as the disutility of labor becomes zero. However, for all $\delta$ small the equilibrium exhibits full employment. In addition, commodity prices as well as profits tend to infinity for any given wage rate. However, relative commodity prices are finite and equal to the appropriate marginal rate of substitution in consumption. At this point it is unclear whether the limiting allocation can also be considered as an equilibrium with imperfect competition. Thus, it seems that the unemployment results of the literature with endogenous labor supply and imperfect competition cannot be considered as limits of monopolistic equilibria with diminishing disutility of labor.

## 5. Conclusions

The results of this paper provide some corrections of the current view on the effects of imperfect competition in general equilibrium. The basic finding is that under market clearing conditions inefficiencies of factor usage arise only under imperfect product differentiation whereas subobtimality has several causes including possibly those of unused factors under excess supply. However, it seems unclear at this point to what extent the excess supply situation is a structurally stable property of equilibria with imperfect competition as suggested by a growing literature. Some further research is needed into the relationship of monopolistic equilibria with and without market clearing before a decisive answer about the consequences of imperfect competition can be given.

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