A Note on Symmetry Breaking in a World Economy with an International Financial Market*

Tomoo Kikuchi †

October 2003

Discussion Paper No. 509

UNIVERSITY OF BIELEFELD
Department of Economics

Abstract

This paper analyzes the world economy model presented in Matsuyama (2002) with the help of numerical methods. We exhibit that his necessary and sufficient conditions for the asymmetric steady states do not cover all possible cases when the world interest rate is determined endogenously in the international financial market. Additional asymptotically stable asymmetric steady states emerge. Borrowing constraints can be binding for all countries in symmetric as well as asymmetric steady states. All countries converge to symmetric steady states if they are not credit rationed initially, if their initial conditions are sufficiently similar or if the sum of their initial conditions sufficiently high, so that the productivity of investment project allows the capital accumulation of countries to adjust to the same level via the international financial market. In other words, the international financial market can function as an equalizing force as well as magnifying inequality.

Keywords: Convergence, Symmetry-breaking, Financial market imperfections, World economy
JEL classification: E44, F43, O11

*The research for this paper is part of the project "Endogene stochastische Konjunkturtheorie von Realgüter– und Finanzmärkten" supported by the Deutsche Forschungsgemeinschaft under contract number Bo. 635/9–1,3. I am indebted to V. Böhm, T. Pampel and J. Wenzelburger for useful discussions.

†University of Bielefeld, Department of Economics, P.O.Box 100131, D-33501 Bielefeld, Germany; tkikuchi@wiwi.uni-bielefeld.de

1 Introduction

The aim of this note is to analyze the world economy model presented in Matsuyama (2002) with the help of numerical methods. Numerical results reveal new features of symmetry breaking when the world interest rate is determined endogenously. The model tries to explain the symmetry breaking of groups of countries in the world economy motivated by the empirical literature on polarization of the world economy (See Quah 1997). In section 2 the world economy model is presented where the implicit condition for the world interest rate and the capital accumulation functions are derived. In section 3 we specify the functional form of the two dimensional dynamic system. Section 4 analyzes the stability property of symmetric steady states. Section 5 delivers the numerical results and examines the propositions in Matsuyama (2002). Section 6 concludes. We use the notation of Matsuyama (2002) for this paper unless defined otherwise. For notational simplicity k_t^i should refer to $k_t^{(i)}$ the capital of countries in group i at time t when it does not cause confusions.

2 The World Economy Model

Aggregate output Y_t in each period is produced from the total amount of labor L_t and physical capital K_t by use of a concave linear homogenous production function $F(L_t, K_t)$. Let us denote $k_t = \frac{K_t}{L_t}$ the capital intensity and $y_t = \frac{Y_t}{L_t}$ the output per capita in period t, then

$$y_t = f(k_t)$$

where we assume a continuum of homogenous agents with unit mass, i.e. $K_t = k_t$ or $L_t = 1$ and $f(k)$ is C^2 and satisfies $f(0) = 0$ and $f''(0) = \infty$. We also assume a competitive factor markets i.e. $W(k_t) := f(k_t) - k_t f'(k_t)$ where $W'(0) = \infty$ and $W''(k) < 0$.

There are overlapping generations of two-period lived agents, who supply one unit of labor inelastically in the first period and consume only in the second period. The homogenous agents in a group of countries $i = 1, 2$ with income $W(k_t^i)$ have opportunities to become either a lender or a borrower in international financial market in period t to transfer income to the next period. The lender receives $r_{t+1}W(k_t^i)$ in $t+1$. The borrower can start only one investment project, for which 1 unit of final goods is required implying a necessary borrowing of $1-W(k_t^i)$. There exist a homogenous linear technology across countries to transform the 1 unit of final goods into R units of physical capital, which is assumed to be not too productive so that $W(R) < 1$. These R units of physical capital are used as an input in the aggregate production function. Then the investor's profit will be $Rf'(k_{t+1}^i) - r_{t+1}(1-W(k_t^i))$. The non-arbitrage condition in the international financial market is therefore $R\rho_{t,t+1}^{e,i} = r_{t,t+1}^e$ where $\rho_{t,t+1}^{e,i}$ is the expected marginal productivity of aggregate capital $f'(k_{t+1}^i)$. Additionally, there exist a non-default condition, which limits the borrowing to an amount which is defined to be a fraction of expected revenue of the investment project. Thus the non-default condition is written as $\lambda R\rho^{e,i}_{t,t+1} = r^e_{t+1}(1 - W(k^i_t))$. Summarizing the two conditions, we obtain

$$r^e_{t+1} = \mathcal{R}^e(k^i_t, \rho^{e,i}_{t,t+1}) \quad := \begin{cases} \dfrac{\lambda R\rho^{e,i}_{t,t+1}}{(1 - W(k^i_t))} & \text{if } k^i_t < K(\lambda) \\ \\ R\rho^{e,i}_{t,t+1} & \text{if } k^i_t \geq K(\lambda) \end{cases} \tag{1}$$

where $\lambda = 1 - W(K(\lambda))$.

If we assume perfect foresight of agents for the aggregate marginal production of the economy, $\rho^{e,i}_{t,t+1} = f'(k^i_{t+1})$. Solving the equation for k^i_{t+1}, the capital accumulation function of each country in group i is defined as

$$k^i_{t+1} = \Psi(k^i_t, r^e_{t,t+1}) \quad := \begin{cases} \Phi\left[\dfrac{r^e_{t,t+1}(1 - W(k^i_t))}{\lambda R}\right] & \text{if } k^i_t < K(\lambda) \\ \\ \Phi\left(\dfrac{r^e_{t,t+1}}{R}\right) & \text{if } k^i_t \geq K(\lambda) \end{cases} \tag{2}$$

where $\Phi(x) := (f')^{-1}(x)$.

The credit demand of each group i in the international financial market is therefore

$$b^i_t = \mathcal{B}(\Psi(k^i_t, r^e_{t,t+1})) \quad := \begin{cases} \Phi\left[\dfrac{r^e_{t,t+1}(1 - W(k^i_t))}{\lambda R}\right]\dfrac{1}{R} & \text{if } k^i_t < K(\lambda) \\ \\ \Phi\left(\dfrac{r^e_{t,t+1}}{R}\right)\dfrac{1}{R} & \text{if } k^i_t \geq K(\lambda). \end{cases} \tag{3}$$

Let X be a fraction of the countries with k^1_t and $(1 - X)$ be a fraction of the countries with k^2_t at time t. If we assume perfect foresight of the agents for the world interest rate, $r^e_{t,t+1} = r_{t+1}$. Then, the equilibrium interest rate $r_{t+1} = \mathcal{R}(k^1_t, k^2_t)$ in the international financial market is implicitly determined by equating the credit demand of each group to the credit supply of each group

$$X\mathcal{B}(\Psi(k^1_t, r_{t+1})) + (1 - X)\mathcal{B}(\Psi(k^2_t, r_{t+1})) = XW(k^1_t) + (1 - X)W(k^2_t). \tag{4}$$

Substituting $r^e_{t,t+1} = r_{t+1} = \mathcal{R}(k^1_t, k^2_t)$ into (2), a two dimensional dynamical system is defined by

$$k^i_{t+1} = \Psi(k^i_t, \mathcal{R}(k^1_t, k^2_t)), \qquad i = 1, 2. \tag{5}$$

3 Functional Form

Since it is not possible to solve for $r_{t+1} = \mathcal{R}(k^1_t, k^2_t)$ explicitly in general, we use the Cobb-Douglas production function $y = f(k) = A(k)^\alpha$ with $0 < \alpha < 1$. Hence, $f'(k) = \alpha Ak^{\alpha-1} = x$ and $(f')^{-1}(x) = \Phi(x) = (\frac{x}{\alpha A})^{\frac{1}{\alpha-1}}$ and $W(k) = (1 - \alpha)A(k)^\alpha$.

$r_{t+1} = \mathcal{R}(k_t^1, k_t^2) :=$

$$
\begin{cases}
\dfrac{\left[[XW(k_t^1)+(1-X)W(k_t^2)]R\right]^{\alpha-1}\lambda R\alpha A}{\left[X(1-W(k_t^1))^{\frac{1}{\alpha-1}}+(1-X)(1-W(k_t^2))^{\frac{1}{\alpha-1}}\right]^{\alpha-1}} & \text{if } k_t^1 < K(\lambda), k_t^2 < K(\lambda) \\[2em]
\dfrac{[[XW(k_t^1)+(1-X)W(k_t^2)]R]^{\alpha-1}\alpha A\lambda R}{\left[X[1-W(k_t^1)]^{\frac{1}{\alpha-1}}+(1-X)\lambda^{\frac{1}{\alpha-1}}\right]^{\alpha-1}} & \text{if } k_t^1 < K(\lambda), k_t^2 \geq K(\lambda) \\[2em]
\dfrac{[[XW(k_t^1)+(1-X)W(k_t^2)]R]^{\alpha-1}\alpha A\lambda R}{\left[X\lambda^{\frac{1}{\alpha-1}}+(1-X)[1-W(k_t^2)]^{\frac{1}{\alpha-1}}\right]^{\alpha-1}} & \text{if } k_t^1 \geq K(\lambda), k_t^2 < K(\lambda) \\[2em]
[[XW(k_t^1)+(1-X)W(k_t^2)]R]^{\alpha-1}\alpha AR & \text{if } k_t^1 \geq K(\lambda), k_t^2 \geq K(\lambda).
\end{cases}
\tag{6}
$$

Substituting (6) into (2) we obtain a two dimensional dynamical system.

$$k_{t+1}^1 = \Psi(k_t^1, \mathcal{R}(k_t^1, k_t^2)) = \tilde{\Psi}^1(k_t^1, k_t^2) :=$$

$$
\begin{cases}
\dfrac{(1-\alpha)AR[X(k_t^1)^\alpha+(1-X)(k_t^2)^\alpha]}{X+(1-X)\left[\frac{1-W(k_t^2)}{1-W(k_t^1)}\right]^{\frac{1}{\alpha-1}}} & \text{if } k_t^1 < K(\lambda), k_t^2 < K(\lambda) \\[2em]
\dfrac{(1-\alpha)AR[X(k_t^1)^\alpha+(1-X)(k_t^2)^\alpha]}{X+(1-X)\left[\frac{\lambda}{1-W(k_t^1)}\right]^{\frac{1}{\alpha-1}}} & \text{if } k_t^1 < K(\lambda), k_t^2 \geq K(\lambda) \\[2em]
\dfrac{(1-\alpha)AR[X(k_t^1)^\alpha+(1-X)(k_t^2)^\alpha]}{X+(1-X)\left[\frac{1-W(k_t^2)}{\lambda}\right]^{\frac{1}{1-\alpha}}} & \text{if } k_t^1 \geq K(\lambda), k_t^2 < K(\lambda) \\[2em]
(1-\alpha)AR[X(k_t^1)^\alpha+(1-X)(k_t^2)^\alpha] & \text{if } k_t^1 \geq K(\lambda), k_t^2 \geq K(\lambda).
\end{cases}
\tag{7}
$$

$k^2_{t+1} = \Psi^2(k^2_t, \mathcal{R}(k^1_t, k^2_t)) = \tilde{\Psi}^2(k^1_t, k^2_t) :=$

$$\begin{cases} \dfrac{(1-\alpha)AR[X(k^1_t)^\alpha + (1-X)(k^2_t)^\alpha]}{X\left[\frac{1-W(k^1_t)}{1-W(k^2_t)}\right]^{\frac{1}{\alpha-1}} + (1-X)} & \text{if } k^1_t < K(\lambda), k^2_t < K(\lambda) \\ \dfrac{(1-\alpha)AR[X(k^1_t)^\alpha + (1-X)(k^2_t)^\alpha]}{X\left[\frac{1-W(k^1_t)}{\lambda}\right]^{\frac{1}{\alpha-1}} + (1-X)} & \text{if } k^1_t < K(\lambda), k^2_t \geq K(\lambda) \\ \dfrac{(1-\alpha)AR[X(k^1_t)^\alpha + (1-X)(k^2_t)^\alpha]}{X\left[\frac{\lambda}{1-W(k^2_t)}\right]^{\frac{1}{1-\alpha}} + (1-X)} & \text{if } k^1_t \geq K(\lambda), k^2_t < K(\lambda) \\ (1-\alpha)AR[X(k^1_t)^\alpha + (1-X)(k^2_t)^\alpha] & \text{if } k^1_t \geq K(\lambda), k^2_t \geq K(\lambda). \end{cases} \tag{8}$$

4 Stability of Symmetric Steady States

Proposition 1
The symmetric steady state is stable, if $\bar{k} = \bar{k}^1 = \bar{k}^2 > K(\lambda)$.

Proof.
For $k^1_t \geq K(\lambda), k^2_t \geq K(\lambda)$,

$$\begin{aligned} k^i_{t+1} &= \tilde{\Psi}^i(k^1_t, k^2_t), \qquad i = 1, 2 \\ &= (1-\alpha)AR(X(k^1_t)^\alpha + (1-X)(k^2_t)^\alpha). \end{aligned}$$

Let

$$J(\bar{k}^1, \bar{k}^2) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \dfrac{\partial k^1_{t+1}}{\partial k^1_t} & \dfrac{\partial k^1_{t+1}}{\partial k^2_t} \\ \dfrac{\partial k^2_{t+1}}{\partial k^1_t} & \dfrac{\partial k^2_{t+1}}{\partial k^2_t} \end{pmatrix}.$$

Since $\tilde{\Psi}^1(k^1_t, k^2_t) = \tilde{\Psi}^2(k^1_t, k^2_t)$ in this case, we obtain $a = c, b = d$ and the characteristic polynominal can be written as

$$\begin{aligned} p(\nu) &= \nu^2 - (a+d)\nu + ad - bc \\ &= \nu^2 - (a+d)\nu = 0 \end{aligned}$$

Thus, the eigenvalues of the system are

$$\begin{aligned} \nu_1 &= 0 \\ \nu_2 &= a + d = \alpha. \end{aligned}$$

where $\bar{k} = \left(\frac{1}{(1-\alpha)AR}\right)^{\frac{1}{\alpha-1}}$.
It follows that $-1 < \nu_1, \nu_2 < 1$. Q.E.D.

Proposition 2
If $\bar{k} = \bar{k}^1 = \bar{k}^2 < K(\lambda)$ and if we assume the same fraction of countries for each group, i.e. $X = \frac{1}{2}$, the symmetric steady state is stable if $\frac{f(\bar{k})W'(\bar{k})}{1 - W(\bar{k})} < 1$.

Proof.
For $k_t^1 < K(\lambda), k_t^2 < K(\lambda)$ and $X = \frac{1}{2}$,

$$k_{t+1}^1 = \tilde{\Psi}^1(k_t^1, k_t^2) = \frac{(1-\alpha)AR\frac{1}{2}\left[(k_t^1)^\alpha + (k_t^2)^\alpha\right]}{\frac{1}{2} + \frac{1}{2}\left[\frac{1 - W(k_t^2)^\alpha}{1 - W(k_t^1)^\alpha}\right]^{\frac{1}{\alpha-1}}}$$

$$k_{t+1}^2 = \tilde{\Psi}^2(k_t^2, k_t^2) = \frac{(1-\alpha)AR\frac{1}{2}\left[(k_t^1)^\alpha + (k_t^2)^\alpha\right]}{\frac{1}{2} + \frac{1}{2}\left[\frac{1 - W(k_t^1)^\alpha}{1 - W(k_t^2)^\alpha}\right]^{\frac{1}{\alpha-1}}}.$$

We can observe that $a = d, b = c$. Then,

$$\begin{aligned} p(\mu) &= \mu^2 - 2a\mu + a^2 - b^2 \\ (\mu - a)^2 &= b^2 \\ \mu_1 &= a + b \\ &= \frac{1}{2}\alpha(1-\alpha)AR\bar{k}^{\alpha-1}\left(1 + \frac{A\bar{k}^\alpha}{1 - (1-\alpha)A\bar{k}^\alpha}\right) \\ &\quad + \frac{1}{2}\alpha(1-\alpha)AR\bar{k}^{\alpha-1}\left(1 - \frac{A\bar{k}^\alpha}{1 - (1-\alpha)A\bar{k}^\alpha}\right) \\ &= \alpha \\ \mu_2 &= a - b \\ &= \alpha(1-\alpha)AR\bar{k}^{\alpha-1}\left(\frac{A\bar{k}^\alpha}{1 - (1-\alpha)A\bar{k}^\alpha}\right) = \frac{f(\bar{k})W'(\bar{k})}{1 - W(\bar{k})}. \end{aligned}$$

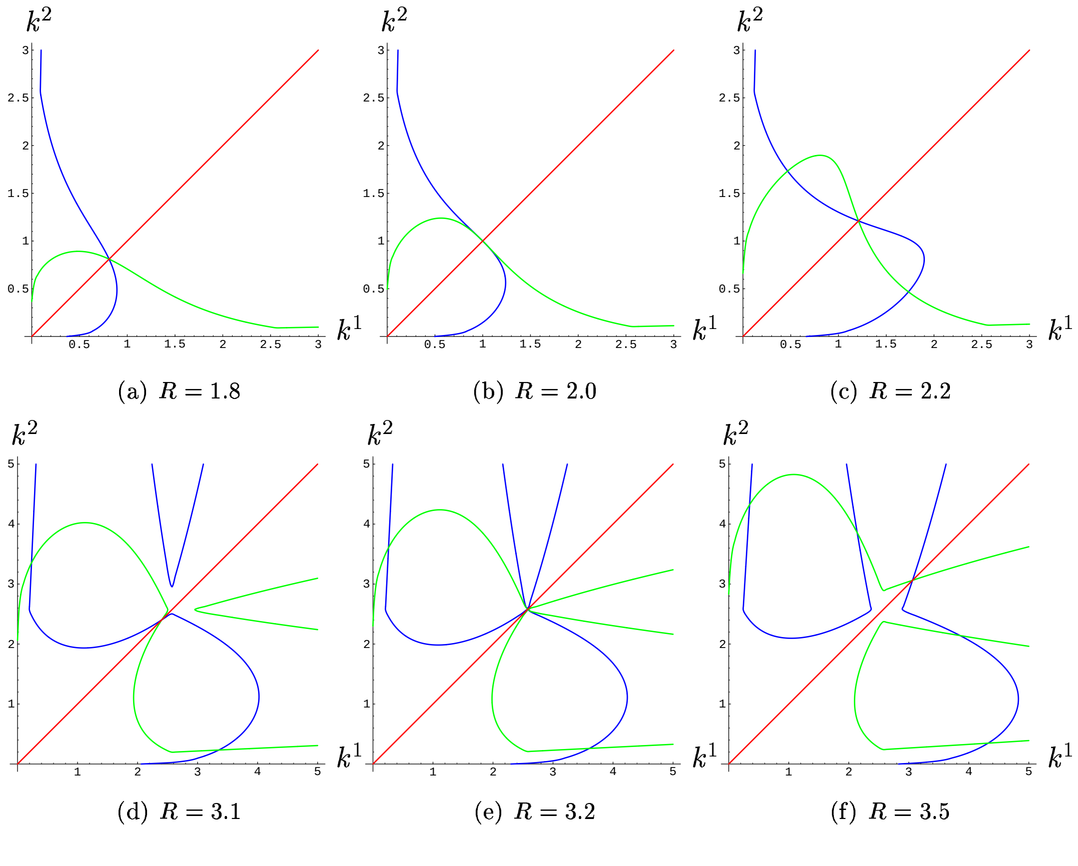
where $\bar{k} = \left(\frac{1}{(1-\alpha)AR}\right)^{\frac{1}{\alpha-1}}$.

It follows that $0 < \mu_1 < 1$ and $0 < \mu_2 < 1$ iff $\dfrac{f(\bar{k})W'(\bar{k})}{1 - W(\bar{k})} < 1$. Q.E.D.

5 Numerical Investigations on Stability and Existence of Steady States

In our numerical investigation we used $A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$ as our standard parameter set varying $R \in (0, 4)$ to analyze the behavior of the system. Different values did not yield quantitatively different results. They induce similar effects on the behavior of the system through their influence on the productivity of the economy.

Figure 1 shows null contours of the functions $\Delta k^1 := k^1 - \tilde{\Psi}^1(k^1, k^2)$ and $\Delta k^2 := k^2 - \tilde{\Psi}^2(k^1, k^2)$ in blue and in green respectively for different values of R. The symmetric steady state value is increasing in R. The system has one fixed point for $0 < R \leq 2$, three fixed points for $2 < R \leq 3.2$ where $\bar{k} = K(\lambda) = 2.56$ and five fixed points for $K(\lambda) < R < 4$.

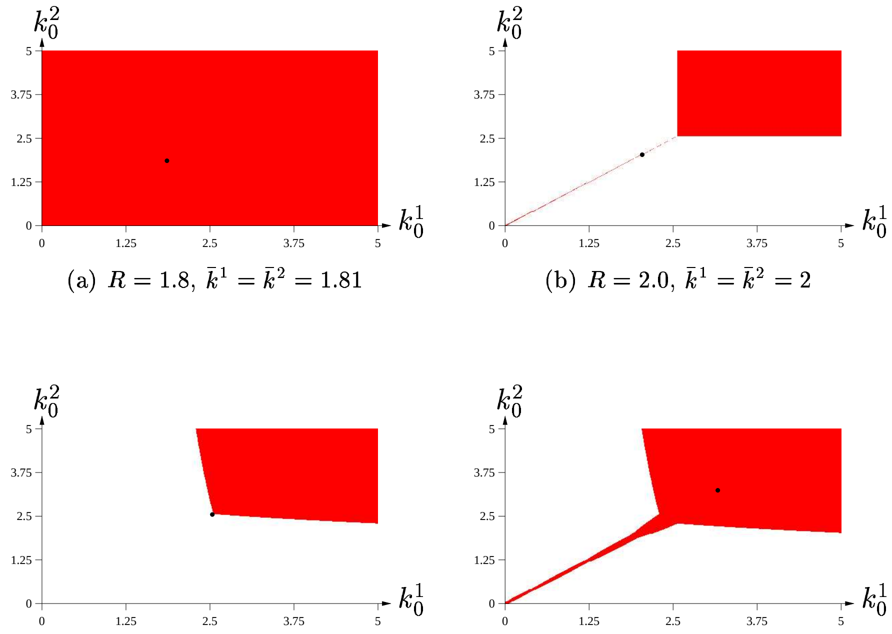


(a) $R = 1.8$ (b) $R = 2.0$ (c) $R = 2.2$

(d) $R = 3.1$ (e) $R = 3.2$ (f) $R = 3.5$

Figure 1: Null Contour Plot: $A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$

Figure 2 shows the basins of attraction for the symmetric steady states for different values of R. We can observe that the symmetric steady state loses its global asymptotic stability at $R = 2$, i.e. when $\mu_2 = 1$ and regains its asymptotically stability for $R > 3.2$, i.e when $\bar{k} = \bar{k}^1 = \bar{k}^2 > K(\lambda)$. In general, after the loss of global asymptotic stability the basin of attraction for the symmetric steady state enlarges while the corner moves down along the diagonal expanding its angle over 90 degree for higher R. The basin of attraction emerges along the diagonal for $R > 3.2$, i.e. for $\bar{k} > K(\lambda) = 2.56$. In principle a symmetric initial condition $k_0^1 = k_0^2$ always converges to a symmetric steady state.



(a) $R = 1.8$, $\bar{k}^1 = \bar{k}^2 = 1.81$

(b) $R = 2.0$, $\bar{k}^1 = \bar{k}^2 = 2$

(c) $R = 3.2$, $\bar{k}^1 = \bar{k}^2 = 2.56$

(d) $R = 3.5$, $\bar{k}^1 = \bar{k}^2 = 3.0625$

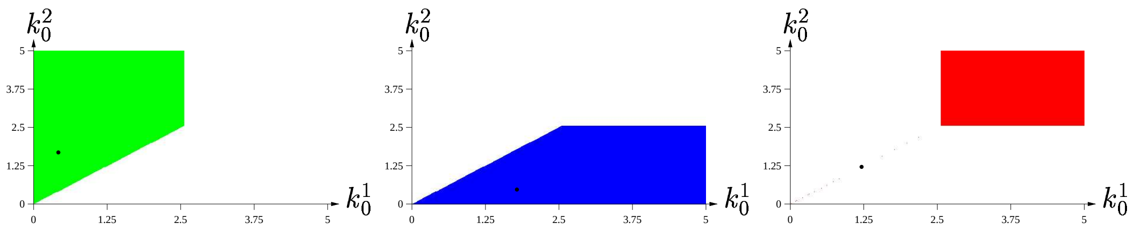
Figure 2: Basin of attraction: $A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$

These numerical observations suggest that, after losing its global asymptotic stability, the symmetric steady states obtain for $k_0^1, k_0^2 \geq K(\lambda)$, where k_0^1, k_0^2 jump on the stable manifold, i.e. the capital accumulation of countries adjust to the same level, within one period. For k_0^1 or $k_0^2 < K(\lambda)$ the symmetric steady state still obtains if the initial values are sufficiently similar to each other or the sum of initial values sufficiently high, so that the productivity of investment project R allows the adjustment of capital accumulation to the same level to take place.

parameter	description	restriction
α	elasticity of production	$0 < \alpha < 1$
A	scaling parameter	$0 < A < \infty$
R	investment productivity	$R < [\frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}$
X	a fraction of countries with k_0^1	$0 < X < 1$
λ	degree of the efficiency in the financial market	$0 < \lambda \leq 1$
k_0^1	initial capital of each country in group 1	$0 < A < \infty$
k_0^2	initial capital of each country in group 2	$0 < A < \infty$

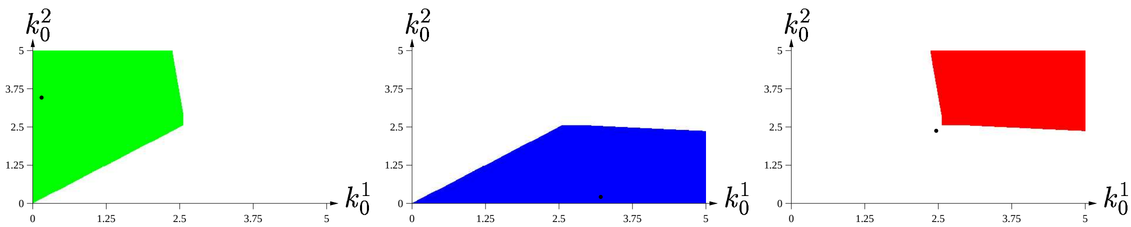
Table 1: Parameter set

The basins of attraction for all steady states are shown for $R = 2.2, 3.1$ and 3.5 in Figures 3, 4 and 5. As it can be seen from these figures, after the symmetric steady state loses the global asymptotic stability, asymptotically stable asymmetric steady states emerge. In Figure 3, both groups of countries 1 and 2 are rationed in the international financial market. In Figures 4 and 5, either the group of countries 1 or 2 are rationed in asymmetric steady states while both groups are rationed in symmetric steady states in Figure 4 but not in 5. There are only three basins of attraction for $R = 3.5$ implying that the two inner asymmetric steady states observed in Figure 1(f) are unstable.



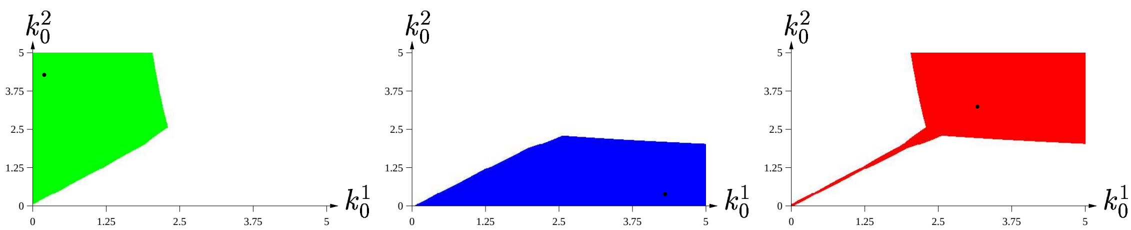
(a) $\bar{k}^1 = 0.467544$, $\bar{k}^2 = 1.73246$ (b) $\bar{k}^1 = 1.73246$, $\bar{k}^2 = 0.467544$ (c) $\bar{k}^1 = \bar{k}^2 = 1.21$

Figure 3: Basin of attraction: $R = 2.2, A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$



(a) $\bar{k}^1 = 0.233132$, $\bar{k}^2 = 3.35387$ (b) $\bar{k}^1 = 3.35387$, $\bar{k}^2 = 0.233132$ (c) $\bar{k}^1 = \bar{k}^2 = 2.4025$

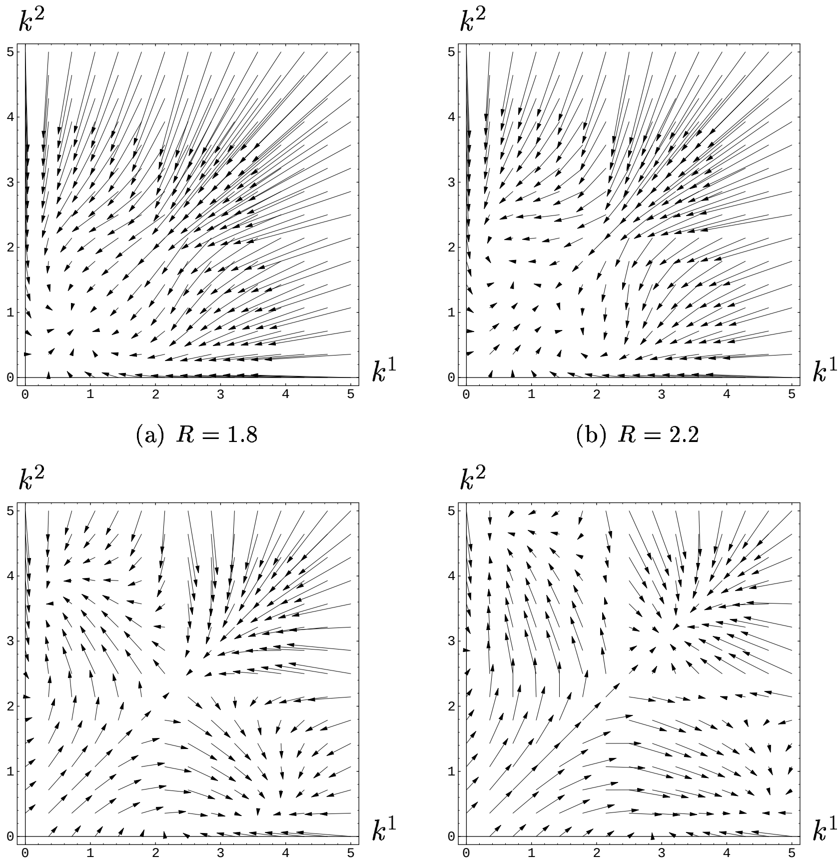
Figure 4: Basin of attraction: $R = 3.1, A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$



(a) $\bar{k}^1 = 0.347723$, $\bar{k}^2 = 4.32263$ (b) $\bar{k}^1 = 4.32263$, $\bar{k}^2 = 0.347723$ (c) $\bar{k}^1 = \bar{k}^2 = 3.0625$

Figure 5: Basin of attraction: $R = 3.5, A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$

Since the eigenvalues for the asymmetric steady state could not be obtained analytically, we illustrate the dynamic behavior of the system by means of vector fields in Figure 6. The direction of the arrow is equal to the direction of the vector field at its base point. The length of the arrow is proportional to the magnitude of the vector field. The figures show how stable asymmetric steady states emerge for $R > 2$.



(a) $R = 1.8$ (b) $R = 2.2$ (c) $R = 3.1$ (d) $R = 3.5$

Figure 6: Vector field $(\Delta k^1, \Delta k^2)$: $A = 1, \alpha = 0.5, \lambda = 0.2, X = 0.5$

Matsuyama treats each country in the world economy as a small open economy and derives the conditions for the asymptotic stability of symmetric steady states and the conditions for the existence of asymptotically stable asymmetric steady states in his Propositions 3 and 4 respectively.[1] We have not obtained any analytical results for the asymmetric steady states. However, our numerical results suggest that additional asymmetric steady states to those under small open economy emerge when the world interest rate is determined endogenously. This observation implies some modifications of his Proposition 4.

Proposition 3 in Matsuyama (2002)

Let $R_c \in (0, R^+)$ be defined by $f(K^*(R_c)) = 1$. Then,

a) If $K^*(R) < K(\lambda)$ and $R < R_c$, the state in which all the countries have $k^* = K^*(R)$, is a stable steady state of the world economy.

b) If $K^*(R) < K(\lambda)$ and $R > R_c$, there exists no stable steady state in which all countries have the same level of capital stock.

[1] See Matsuyama (2002) for the proof of Propositions 3 and 4.

c) If $K^*(R) > K(\lambda)$, the state in which all the countries have $k^* = K^*(R)$, is a stable steady state of the world economy.

Proposition 4 in Matsuyama (2002)

Let $R_c \in (0, R^+)$ and $\lambda_c \in (0,1)$ be defined by $f(K^*(R_c)) = f(K(\lambda_c)) = 1$. The world economy has a continuum of stable steady states, in which a fraction $X \in (X^-, X^+) \subset (0,1)$ of the countries have the capital stock, $k_L < K(\lambda)$, and a fraction $1 - X$ of the countries have the capital stock equal to $k_H > K(\lambda)$, if and only if $\lambda < \lambda_c$, $f'(K(\lambda)) > \lambda f'(K^*(R))/[1 - W(K^*(R))]$ where $R < R_c$, and $\lambda < f'(K(R))K(\lambda_c)$. Furthermore, $X^- > 0$ if $R > R_c$ and $X^+ < 1$ if $K^*(R) < K(\lambda)$.[2]

The additional variables in Matsuyama's Propositions 3 and 4 are defined for our specification of the functional form in Appendix. R_c is obtained by solving $\mu_2 = 1$ for R. Hence, Matsuyama's Proposition 3 for the stability of symmetric steady states is identical with our Propositions 1 and 2. This coincidence is not surprising as there is no interaction in international financial market in symmetric steady states.

For the asymmetric steady states Matsuyama's Proposition 4 implies that there exists no stable asymmetric steady states with $X^+ < 1$ when $K^*(R) \geq K(\lambda)$. However, Figure 5 (a), (b) illustrate a situation where $K^*(R) > K(\lambda)$ and contrary to his Proposition 4 there exist asymptotically stable steady states. Here asymptotically stable asymmetric steady states and an asymptotically stable symmetric steady state coexist. Furthermore, Figure 3 (a), (b) illustrate a situation where $\bar{k}^1, \bar{k}^2 < K(\lambda)$, i.e. both group of countries are credit rationed in asymptotically stable asymmetric steady states. This reveals additional asymmetric stable steady states to those defined in his Proposition 4.

6 Concluding Remarks

We have analyzed the world economy model presented in Matsuyama (2002) using numerical methods. Our results coincide with the analytical conditions by Matsuyama for the symmetric steady states. However, for the asymmetric case we find asymptotically stable steady states which do not fulfill the necessary and sufficient conditions in his paper. In addition we found asymptotically stable asymmetric steady states in which both groups of countries are credit rationed in the international market. These new numerical findings suggest that new steady states emerge when the world interest rate is determined endogenously in the international financial market in addition to those found for a small open economy. However, the economic implication still remains the same: relative positions in initial capital accumulation determine the long run positions of countries. The symmetric steady state prevails only if countries are not credit rationed initially, if the initial conditions are sufficiently similar or if the sum of initial values are sufficiently high, so that the productivity of investment project allows the

[2] The condition $f'(K(\lambda)) > \lambda f'(K^*(R))/[1 - W(K^*(R))]$ where $R < R_c$ seems to be redundant as a positive fraction of both group of countries $X^- > 0$ requires $R > R_c$.

capital accumulations of countries to adjust to the same level via the international financial market. In other words, the international financial market can function as an equalizing force as well as magnifying inequality.

Appendix

With our specification of the functional form in Section 3, the value of the symmetric steady states is defined by

$$\begin{aligned} k^* &:= RW(k^*) = R(1-\alpha)A(k^*)^{\alpha} \\ K^*(R) &= (AR(1-\alpha))^{\frac{1}{1-\alpha}}, \end{aligned}$$

the critical value of capital accumulation above which the countries are no longer credit constrained is defined by

$$\begin{aligned} W(K(\lambda)) &:= 1-\lambda \\ K(\lambda) &= W^{-1}(1-\lambda) = \left(\frac{1-\lambda}{(1-\alpha)A}\right)^{\frac{1}{\alpha}}, \end{aligned}$$

the critical value of productivity of investment project above which the symmetric steady states lose their global asymptotic stability is

$$\begin{aligned} f(K^*(R_c)) &= A(R_c(1-\alpha)A)^{\frac{\alpha}{1-\alpha}} := 1 \\ R_c &= \left(\frac{1}{A}\right)^{\frac{1-\alpha}{\alpha}} \frac{1}{(1-\alpha)A}, \end{aligned}$$

and the critical value of degree of the efficiency in the financial market above which the symmetric steady states lose their global asymptotic stability is defined by

$$\begin{aligned} f(K(\lambda_c)) &= A\left(\frac{1-\lambda_c}{(1-\alpha)A}\right) := 1 \\ \lambda_c &= 1-(1-\alpha)A. \end{aligned}$$

References

MATSUYAMA, KIMINORI (2002): "Financial Market Globalization, Symmetry-Breaking, and Endogenous Inequality of Nations", *Discussion Paper, Northwestern University.*

QUAH, DANNY T. (1997): "Empirics for Growth and Distribution: Polarization, Stratification, and Convergence Clubs,", *Journal of Economic Growth*, 27–59.