

Sophistication in Risk Management and Banking Stability: The Short Term*

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Abstract

We explore the impact of sophistication in risk management as required by Basel II on banking stability and market conditions. We compare a competitive banking system in which only average ratings are available with a competitive system in which banks are able to assess the default risk of individual firms. We show that sophistication in banking decreases default probabilities for individual firms and lowers deposit and loan-interest rates. Sophistication decreases the default probability for banks and thus increases banking stability, provided that initial equity is not too low. Otherwise banking stability declines.

Keywords: Financial intermediation, macroeconomic risks, risk management, risk premia, banking regulation, rating.

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1 Introduction

The common assumption underlying banking regulation is that, along with capital requirements, more sophistication in rating and risk management increases the stability of a banking system. This is epitomized in the Basel II regulatory framework. This raises the question of what the aggregate consequences and their impact on stability are when a banking system becomes more sophisticated in its ability to assess the default risk of investing firms. We investigate the case when banks compete for loans and deposits but cannot adjust their initial equity. This situation corresponds to the *short term*. The long term when banks may adjust their initial equity is treated in the twin paper Gersbach & Wenzelburger (2006). There it is shown that sophisticated risk management may decrease banking stability.

We consider a competitive banking system embedded in a macroeconomic environment in which banks offer intermediation services to a population of producing entrepreneurs subject to macroeconomic risk. Entrepreneurs and bank owners have outside options for investing their resources. As in Gersbach & Wenzelburger (2004), risk premia on loans are determined by free exit and free entry conditions for banks. We distinguish between a simple and a sophisticated banking system. Both systems are perfectly competitive and start with the same level of equity, e.g. given by regulatory capital requirements. In a simple banking system, banks are unable to assess the quality of loan applicants individually and charge all entrepreneurs the same loan-interest rate. In a sophisticated banking system, banks are able to assess the default risk of an entrepreneur individually and offer entrepreneur-specific loan rates. The goal of the paper is to investigate the influence of the rating ability of a banking system on market conditions and banking stability.

Our main findings are as follows. First, a sophisticated banking system rewards producing entrepreneurs with low default risks with low loan interest rates. Aggregate repayments of entrepreneurs are therefore lower than in a simple system. As a consequence, the deposit rate of a sophisticated banking system is lower than in a simple banking system, so that its refinancing costs are lower. Second, the sophisticated banking system accumulates more equity only for adverse macroeconomic shocks. The intuition is as follows. Banks earn only the liquidation value of a defaulting entrepreneurs. If a sufficiently large number of bankruptcies occur due to adverse shocks, the simple bank-

ing system's advantage of higher aggregate repayments does not outweigh its higher refinancing costs. Since refinancing costs are unaffected by a macroeconomic shock, the sophisticated system accumulates more equity. Third, we show that the default probability of a sophisticated banking system is only lower if the initial equity levels are sufficiently high to buffer against losses. Otherwise, the simple system is less likely to default, so that sophistication in a banking system decreases bank stability if initial equity is too low. Finally, we develop compact formulas for risk premia, expected losses, as well as value-at-risk and default probabilities for both banking systems embedded in an aggregate equilibrium model.

The approach of this paper is complementary to the work of Gehrig & Stenbacka (2004) who show that uncoordinated screening behavior of competing financial intermediaries creates a financial multiplier and may be responsible for macroeconomic fluctuations. In analyzing the systemic effects of screening activities by firms, this paper contributes to the literature on screening by banks surveyed, for example, in Freixas & Rochet (1997). An interesting question for future research is how even more sophisticated risk management techniques such as the securitization of bank loans including the use of derivative products affect systematic risks as discussed e.g. in Franke & Krahnert (2006).

Our results are related to the literature on banking regulation. Comprehensive surveys with different emphases are given by Bhattacharya & Thakor (1993), Dewatripont & Tirole (1994), Hellwig (1994), Freixas & Rochet (1997), or Bhattacharya, Boot & Thakor (1998). Overall our analysis suggests that the new regulatory policy for banking (Basel II) that requires banks to introduce more sophistication in assessing the default risk of their clients will only be beneficial if banks start with a sufficiently high level of bank capital.

A large body of literature has investigated the consequences of modern risk management techniques in capital markets which have risen dramatically over the last few decades, cf. Carey & Stulz (2006). Although there is a large literature on sophistication in rating techniques¹, its aggregate consequences are unknown. This calls for a more detailed analysis of how sophisticated risk management tools affect the banking system and the macroeconomy.

¹For example, a set of intuitive rating principles have been developed by Krahnert & Weber (2001)

The paper is organized as follows: In the next section we introduce the model and both types of banking systems. In section 3 we examine simple banks, and in section 4 we perform the mirror-image of the analysis for sophisticated banks. In section 5 we compare both systems, leading on from there to our main results. Section 6 concludes.

2 Model

2.1 Households and entrepreneurs

We consider a two-period model with periods $t = 1$ and $t = 2$. The population of agents consists of a continuum indexed by $[0, 1]$. Each agent has individual wealth W in terms of cash in the first period. Agents are divided into two classes. One fraction of agents, indexed by $[0, \eta]$, are potential entrepreneurs. The other fraction, indexed by $(\eta, 1]$, are consumers. Potential entrepreneurs and consumers differ in that only the former have access to investment technologies and will be the owner of banks.

Consumers are endowed with consumption preferences in the two periods of their lives, with c^1, c^2 respectively denoting youthful and elderly consumption in money terms. For simplification, let $u(c^1, c^2) = \ln(c^1) + \delta \ln(c^2)$ be the intertemporal utility function of a consumer, where δ ($0 < \delta < 1$) is the discount factor. Accordingly, a young consumer inelastically saves the amount $s = \frac{\delta}{1+\delta}W$ if he can transfer wealth from period 1 to period 2. We denote the aggregate savings of consumers by $S = (1 - \eta)s$. The assumption that the elasticity of aggregate savings is zero is made for ease of presentation and for the purpose of deriving explicit formulas.²

Potential entrepreneurs are assumed to be risk-neutral and consume only in the second period. Each entrepreneur has to decide whether to invest in a production project that converts period-1 goods into period-2 goods or to channel their funds into an alternative project with return $1 + r_E$ ($r_E > 0$). The alternative investment opportunity may be thought of as an outside option, such as government bonds or investments in other sectors of the economy that are not modeled explicitly.³

²Our model can be qualitatively extended to the case where the interest elasticity of consumer savings is low.

³For tractability we assume that consumers are not allowed to invest in the alternative project. This can be justified by liquidity services of deposits. However, the model could be qualitatively extended to the case where consumers hold a portfolio of deposits and other assets. In this case, the saving function is of the form $S = S(r^d, r_E)$.

The funds required for each investment project are fixed to $W + I$ so that an entrepreneur must borrow I additional units of the good from banks to undertake the investment project. Entrepreneurs are heterogeneous in the quality of their investment projects which depends on their index i . The quality parameter of entrepreneur i is assumed to be private information. If an entrepreneur of type i obtains additional resources I and decides to invest, investment returns in the second period amount to

$$y = q(1 + i)f(W + I),$$

where f denotes a standard atemporal neoclassical production function and $q \in \mathbb{R}_+$ represents an exogenous macroeconomic productivity shock in the economy. Since W and I will remain fixed throughout the paper, we write $f = f(W + I)$. The distribution of q is assumed to be given by a continuous density function $h(q)$ with support on a compact interval $[\underline{q}, \bar{q}]$ with $0 < \underline{q} < \bar{q}$.

Entrepreneurs are price-takers and operate under limited liability. Given a loan interest rate r^c , the expected profit of an investing entrepreneur i is

$$\Pi(i, r^c) := \int_{\underline{q}}^{\bar{q}} \max\{q(1 + i)f - I(1 + r^c), 0\} h(q) dq. \quad (1)$$

Note that $\Pi(i, r^c)$ is monotonically increasing in quality levels i and monotonically decreasing in loan rates r^c . A risk-neutral entrepreneur with the quality parameter $i \in [0, \eta]$ will invest in the production project if the return on the investment project is larger than the return on alternative investments, i.e.,

$$\Pi(i, r^c) \geq W(1 + r_E).$$

We assume that savings are never sufficient to fund all entrepreneurs. Since the interest-rate elasticity of savings is zero, this condition takes the form

$$S := (1 - \eta) s < \eta I.$$

2.2 Simple and sophisticated banking

Depositors cannot observe the quality parameters of entrepreneurs and cannot verify whether or not an entrepreneur invests. The existence of such market frictions necessitates financial intermediation (see e.g. Hellwig 1994). To alleviate there information

problems, we assume that there are n banks, indexed by $j = 1, \dots, n$ ($n > 1$) which are owned by potential entrepreneurs. As these bank owners are risk-neutral and consume only in the second period, the objective of banks is to maximize profits accruing to current shareholders. They monitor loans as delegated monitors in the sense of Diamond (1984) and their monitoring is assumed to be efficient in the sense that they are able to secure both the investment of an entrepreneur and the liquidation value in case of default, cf. Gersbach & Uhlig (2005).

For a comparison based on the same premises, we make the same assumption regarding the competitive environment of the banking sector. First, both banking systems are perfectly competitive with free exit and free entry.⁴ Bank owners have the opportunity to exit the banking industry and to invest their equity alternatively with return r_E ($r_E > 0$). Second, both systems start with the same amount of aggregate equity e_1 which consists of the value physical capital k_1 and cash d_1 . More precisely, each bank in each system starts with the same amount of initial cash holdings $\frac{d_1}{n}$ and the same value of physical capital $\frac{k_1}{n}$ in period 1. The physical capital stock allows banks to perform their intermediation services and consists of branches, IT systems and other components of capital. To simplify the exposition we assume that capital does not depreciate. The central assumption here is that equity is given, e.g. by regulatory capital requirements, so that our short-term perspective is justified. Third, competition among banks determines deposit and loan-interest rates. Each bank j can offer deposit contracts $D(r^d)$, where $1 + r^d$ is the repayment offered for one unit of money. We distinguish between a *simple* and a *sophisticated* banking system which differ only in their ability to rate the quality of entrepreneurs.

1. *Simple Banking System.* The essential feature of the simple banking system is that banks are unable to rate entrepreneurs individually and to adjust loan contracts to the quality parameter i of an entrepreneur. Banks only have an average rating of entrepreneurs and offer all entrepreneurs the same loan contract $C(r^c)$, where $1 + r^c$ is the repayment required from entrepreneurs for one unit of borrowed money.

2. *Sophisticated Banking System.* In a sophisticated banking system, banks are

⁴The free-entry free-exit framework is a standard concept in industrial economics, e.g., see Vives (2004).

able to rate each entrepreneur individually and to offer entrepreneur-specific loan contracts, denoted by $C(r_i^c)$, where r_i^c is the loan interest rate demanded from an entrepreneur of type i .

In both banking systems, banks operate under unlimited liability and loans are only constrained by the amount of equity and deposits. We assume throughout that aggregate uncertainty is canceled out when depositors and entrepreneurs randomly choose banks.⁵ As all banks are identical, they will obtain the same amount of equity and deposits.

With these assumptions, the financial intermediation process in either system is as follows. In the first period banks offer deposit and loan contracts, given by r^d and r^c (simple banking) or by r^d and $\{r_i^c\}_{i=0}^{\eta}$ (sophisticated banking), respectively. Each bank j obtains an amount of d^j in equity and an equal share of deposits from consumers. Entrepreneurs decide which contracts to accept. Money is exchanged. In the second period, entrepreneurs who have chosen the production project produce subject to a macroeconomic shock and pay back loans with limited liability. Banks repay depositors. The vulnerability of a banking system depends on its ability to accumulate equity. This motivates a comparison of equity accumulation in the two banking systems. Of particular interest are the probability distribution of equity and the downside equity risk in the second period. Specifically, we will analyze the default probability for both banking systems, i.e., the probability that equity becomes negative.

3 Competitive Equilibria for Simple Banks

Consider first the case in which banks use simple risk-management tools and are unable to detect the quality parameter i of the production project. Recall that only the amount of equity $\frac{d_1}{n}$ can be used by a bank to finance loans. Physical capital cannot be used to finance loans, but it allows to collect deposits and to grant loans. As all banks are assumed to be identical, we will formulate the equilibrium conditions for the whole banking system.

⁵The exact construction of individual randomness so that this statement holds can be found in Alós-Ferrer (1999). We could also rely on the weaker forms of the strong law of large numbers developed in Al-Najjar (1995) and Uhlig (1996), where independence of individual random variables can be assumed and aggregate stability is the limit of an economy with finite characteristics.

There is an upper boundary for d_1 , denoted by $\bar{d} := \eta I - S$, which, based on the assumption made above, is positive. Since $S + \bar{d} = \eta I$, banks with $d_1 > \bar{d}$ have more equity than needed to finance all entrepreneurs. In this case, excess resources are available at any interest rate. We exclude this uninteresting case from our analysis. For each $d_1 \in [0, \bar{d}]$, there exists a unique *critical entrepreneur* $i_* \in [0, \eta]$, given by

$$i_* = i_*(d_1) := \frac{\bar{d} - d_1}{I}, \quad (2)$$

such that savings are balanced by investments, i.e.,

$$S + d_1 = [\eta - i_*(d_1)] I. \quad (3)$$

3.1 Equilibrium concept

Let $d_1 \in [0, \bar{d}]$ be the amount of equity in the form of cash in the first period. Banks raise funds S that have to be paid back with interest at the end of the second period. In a competitive equilibrium loan demand must equal loan supply, so that (3) holds. Since simple banks are unable to detect the quality parameter i , they charge all investing entrepreneurs the same loan interest rate r^c . Thus, simple banks lend $[\eta - i_*] I$ to firms and will receive payments $P = P(q, i_*, r^c)$ at the end of the second period, given by

$$P(q, i_*, r^c) = \int_{i_*}^{\eta} \min\{q(1+i)f, I(1+r^c)\} di, \quad (4)$$

where $i_* = i_*(d_1)$. Given a pair of interest rates r^d, r^c , the capital of the banking system is given by a function $G(q, \cdot, r^d, r^c) : [0, \bar{d}] \rightarrow \mathbb{R}$, defined by

$$G(q, d_1, r^d, r^c) = P(q, i_*(d_1), r^c) - S(1 + r^d), \quad (5)$$

such that for each shock q and each $r^c, r^d \geq 0$, $d_2 = G(q, d_1, r^d, r^c)$ is the equity level of the banking system at the end of the second period. Note that a priori, physical capital has no influence on second-period equity.

We next define a competitive equilibrium for a simple banking system. Intuitively, a competitive equilibrium is a pair of interest rates (r_*^d, r_*^c) , such that

- (i) no bank exits and no bank enters the market,
- (ii) firms take optimal investment decisions,

(iii) loan demand equals loan supply.

In order to formalize this concept, observe that, given a pair of interest rates r^d and r^c , the expected profits of the banking system are

$$\begin{aligned}\mathbb{E}[G(\cdot, d_1, r^d, r^c)] &= \mathbb{E}[P(\cdot, i_*(d_1), r^c)] - S(1 + r^d) \\ &= \int_{\underline{q}}^{\bar{q}} P(q, i_*(d_1), r^c) h(q) dq - S(1 + r^d).\end{aligned}\tag{6}$$

Formally, a competitive equilibrium for a simple banking system is defined as follows:

Definition 1

Let $d_1 \in [0, \bar{d}]$ denote the capital base and $k_1 > 0$ be the value of physical capital of the banking system operating under unlimited liability. A competitive equilibrium is a pair of interest rates (r_*^d, r_*^c) such that the following conditions hold:

$$\mathbb{E}[G(\cdot, d_1, r_*^d, r_*^c)] = [d_1 + k_1](1 + r_E)\tag{7}$$

$$\Pi(i_*, r_*^c) = W(1 + r_E)\tag{8}$$

$$[\eta - i_*] I = S + d_1\tag{9}$$

$$r_*^d \leq r_E\tag{10}$$

Equation (7) is the free exit and free entry condition. If the expected return on equity were lower than r_E , banks would exit and not offer their intermediation services. If the expected return were higher, new banks would enter until the expected return is again r_E . Condition (7) also rules out that banks will finance entrepreneurs and do not simply invest all of their funds in the risk-less alternative investment opportunity. Indeed if physical capital k_1 is sufficiently high, then

$$\mathbb{E}[G(\cdot, d_1, r_*^d, r_*^c)] \geq d_1(1 + r_E) + S(r_E - r_*^d).\tag{11}$$

Moreover Corollary 1 below shows that (11) also implies $r_*^c \geq r_E$, so that bank owners obtain a non-negative risk premium on equity. Equation (8) is the indifference condition for the critical quality level i_* , which determines the demand for loans. Equation (9) is the equilibrium condition for savings and investments at banks given in (3), showing that the critical entrepreneur $i_* = i_*(d_1)$ defined in (2) is independent of interest rates. In equilibrium all entrepreneurs with sufficiently high quality parameters $i \geq i_*$ invest in

their production projects, while all entrepreneurs with insufficient quality parameters $i < i_*$ invest in the alternative project. The last condition (10) requires that bank equity holders and entrepreneurs who invest in the alternative project are not worse off than by depositing the money at banks.

The banking system operates under *unlimited liability* in the sense that banks (or their bank managers) internalize the default risk that would materialize in losses. This assumption can be justified in various ways. For instance, the non-pecuniary cost of defaults for managers can induce banks to behave as if they were maximizing expected profits. Alternatively, we might consider a banking system that operates under limited liability. Then the l.h.s. of (7) would have to be replaced by

$$\mathbb{E} [\max\{G(\cdot, d_1, r^d, r^c), 0\}].$$

For sufficiently small default probabilities of a bank, both formulations should yield the same qualitative results. Finally, we note how the equilibrium conditions have to be adjusted when there are fixed costs for monitoring loans. Suppose a bank has granted a loan with a face value of I and needs to spend $m \geq 0$ units of resources to secure the liquidation value of defaulting entrepreneurs. Then the free exit and entry condition needs to be reformulated as

$$\mathbb{E} [G(\cdot, d_1, r_*^d, r_*^c)] - [\eta - i_*] I m = [d_1 + k_1] (1 + r_E).$$

3.2 Existence of competitive equilibria

Since savings S are independent of deposit rates, the establishment of the existence and uniqueness of a competitive equilibrium is straightforward. We obtain

Proposition 1

Let $d_1 \in [0, \bar{d}]$ with $i_* = i_*(d_1)$ be arbitrary. Suppose, in addition, that the following conditions hold:

- (i) The ratio between physical capital and savings satisfies $\frac{k_1}{S} \geq \frac{r_E}{1+r_E}$.
- (ii) Entrepreneur i_* invests for a zero loan interest rate, i.e., $\Pi(i_*, 0) > W(1 + r_E)$.
- (iii) Productivity of entrepreneurs is such that

$$\mathbb{E}[q] \int_{i_*}^{\eta} (1 + i) f di \leq [d_1 + k_1 + S](1 + r_E).$$

Then a simple banking system admits a unique competitive equilibrium $\{r_*^d, r_*^c\}$, where $r_*^d = r_*^d(i_*, r_E)$ and $r_*^c = r_*^c(i_*, r_E)$ are given by

$$\Pi(i_*, r_*^c(i_*, r_E)) = W(1 + r_E)$$

and

$$r_*^d(i_*, r_E) = \frac{1}{S} \left\{ \mathbb{E}[P(\cdot, i_*, r_*^c)] - [d_1 + k_1](1 + r_E) \right\} - 1, \quad (12)$$

respectively.

Corollary 1

The loan interest rate satisfies $r_*^c = r_*^c(i_*, r_E) \geq r_E$ and is increasing in r_E .

The proof of Proposition 1 and Corollary 1 is given in the appendix. Condition (i) of Proposition 1 is a sufficient condition that it is attractive for banks to finance entrepreneurs, whereas Condition (ii) ensures that entrepreneurs apply for loans. Condition (iii) states that the largest possible expected aggregate liquidation value is lower than the return on all available funds. It is a sufficient condition to guarantee that the last equilibrium condition (10) holds. Note that the equilibrium loan interest rate r_*^c in Proposition 1 is independent of physical capital k_1 , whereas the equilibrium deposit interest rate is not. Throughout this paper we will suppress this dependency for notational simplicity. Note also that the existence results holds for any value of i_* if $\Pi(0, 0) > W(1 + r_E)$.

To obtain further insight into the nature of equilibrium interest rates, together with their associated risk premia, consider the aggregate losses of the banking system in equilibrium. Using (4), these are formally defined by

$$\begin{aligned} L(q, i_*, r_E) &:= [\eta - i_*]I[1 + r_*^c(i_*, r_E)] - P(q, i_*, r_*^c(i_*, r_E)) \\ &= \int_{i_*}^{\eta} \max \left\{ I[1 + r_*^c(i_*, r_E)] - q(1 + i)f, 0 \right\} di. \end{aligned} \quad (13)$$

Expected aggregate losses in equilibrium are

$$\begin{aligned} \mathcal{L}(i_*, r_E) &:= \mathbb{E}[L(\cdot, i_*, r_E)] \\ &= [\eta - i_*]I[1 + r_*^c(i_*, r_E)] - \mathbb{E}[P(\cdot, i_*, r_*^c(i_*, r_E))]. \end{aligned} \quad (14)$$

Inserting (12) into (6) and using (13) and (14), the bank capital of a simple banking system in the second period is

$$\begin{aligned}
d_2 = G_*(q, d_1, r_E) &:= G(q, d_1, r_*^d, r_*^c) \\
&= P(q, i_*, r_*^c(i_*, r_E)) - \mathbb{E}[P(\cdot, i_*, r_*^c(i_*, r_E))] + e_1(1 + r_E) \\
&= \mathcal{L}(i_*, r_E) - L(q, i_*, r_E) + e_1 + e_1 r_E,
\end{aligned} \tag{15}$$

where $e_1 = d_1 + k_1$ as before. Equation (15) is a compact representation of bank capital at the end of the intermediation process. Future capital is equal to initial equity plus the interest earned on equity plus the difference between expected and realized aggregate losses. On the basis of Proposition 1, the interest-rate margin is given as

$$\Delta = \Delta(i_*, r_E) := r_*^c - r_*^d = \frac{1}{S} [\mathcal{L}(i_*, r_E) + d_1(r_E - r_*^c) + k_1(1 + r_E)]. \tag{16}$$

The interest-rate margin consists of three terms. The first term $\frac{\mathcal{L}(i_*, r_E)}{S}$ represents the premium for macroeconomic risks and is equal to expected losses per unit of deposits. If there is no macroeconomic risk, the term is zero. The second term $\frac{d_1(r_E - r_*^c)}{S}$ represents the additional cost of equity, i.e., the differential between return on equity and equilibrium loan-interest rate. The third term describes the return if the physical capital were liquidated.

3.3 Properties of competitive equilibria

In this section, we derive some intuitive characteristics of competitive equilibria in simple banking systems. Observe first that an entrepreneur with quality level i enters bankruptcy if he is unable to fully pay back his credit, that is, if

$$I(1 + r_*^c(i, r_E)) > q(1 + i)f.$$

The entrepreneur with the lowest quality level who is not bankrupt after encountering the shock q is given by

$$i_B = i_B(q, r_E) := \begin{cases} \frac{I(1 + r_*^c(r_E))}{qf} - 1 & \text{if } q_{\text{TB}}(r_E) \leq q < q_{\text{NB}}(r_E), \\ i_*(r_E) & \text{if } q \geq q_{\text{NB}}(r_E), \end{cases} \tag{17}$$

where

$$q_{\text{NB}}(r_E) := \frac{I[1 + r_*^c(r_E)]}{[1 + i_*(r_E)]f} \quad \text{and} \quad q_{\text{TB}}(r_E) := \frac{I(1 + r_*^c(r_E))}{(1 + \eta)f}. \tag{18}$$

If shocks are sufficiently positive $q \geq q_{\text{NB}}(r_E)$, then no firm goes bankrupt and aggregate losses of banks are zero. For shocks $q_{\text{TB}}(r_E) \leq q < q_{\text{NB}}(r_E)$, all investing entrepreneurs with quality levels $i_* < i < i_B(q, r_E)$ enter bankruptcy, whereas entrepreneurs with quality levels $i \geq i_B(q, r_E)$ pay back their loans fully. On the other hand, all entrepreneurs will enter bankruptcy if $q < q_{\text{TB}}(r_E)$ and losses are maximal.

3.4 Instability

In this section we investigate conditions under which the banking system becomes unstable. In particular, we determine the default probability for the banking system, that is, the probability of negative bank capital d_2 . An individual bank goes bankrupt if $\frac{d_2}{n} < 0$. Due to the assumed symmetry of banks, this is equivalent to the condition $d_2 = G(d_1, q, r_*^d, r_*^c) < 0$, stating that the whole banking system is bankrupt. Using (15), this condition takes the form

$$e_1(1 + r_E) < L(q, i_*, r_E) - \mathcal{L}(i_*, r_E). \quad (19)$$

The banking system will collapse if actual aggregate losses exceed expected aggregate losses by more than the return on equity. Equation (24) implies that a necessary condition for the default of a bank is $q < q_{\text{NB}}(i_*, r_E)$. The default probability for banks can now be determined as follows:

Proposition 2

Let $d_1 \in [0, \bar{d}]$ be arbitrary and assume that

$$e_1(1 + r_E) < L(\underline{q}, i_*, r_E) - \mathcal{L}(i_*, r_E).$$

Then there exists a unique critical level $\underline{q} \leq q_{\text{crit}} \leq q_{\text{NB}}(i_*, r_E)$ for macroeconomic shocks, such that the banking system defaults if and only if $q < q_{\text{crit}}$. The default probability is

$$\Pi_{\text{default}} := \text{Prob}\left(e_1(1 + r_E) < L(q, i_*, r_E) - \mathcal{L}(i_*, r_E)\right) = \int_{\underline{q}}^{q_{\text{crit}}} h(q) dq. \quad (20)$$

The proof of Proposition 2 is given in the appendix. Note that the condition

$$e_1(1 + r_E) < L(\underline{q}, i_*, r_E) - \mathcal{L}(i_*, r_E).$$

depends on the endogenous variable r_*^c . By inserting the equilibrium function $r_*^c(i_*, r_E)$ and by setting $i_* = \frac{\bar{d}-d_1}{I}$, this condition can be expressed solely in terms of exogenous

parameters. Proposition 2 shows that banks will default with positive probability as soon as the buffer $e_1(1 + r_E)$ is too small to insure against adverse macroeconomic shocks.

The equation (20) is a value-at-risk formula for the banking system and for an individual bank. It can also be used to determine the level of capital necessary to limit the default risk of the banking system. Suppose that Π_{default} is predetermined by banking regulation. Then equation (20) determines the required level of bank capital, i.e., the value of e_1 such that the default risk equals Π_{default} .

3.5 Uniformly distributed shocks

To derive more tractable results and to obtain explicit loan-interest rates, we will assume that the macroeconomic productivity shocks are uniformly distributed. The following lemma gives explicit interest rates for this case.

Lemma 1

Under the hypotheses of Proposition 1, assume that the macroeconomic shocks are uniformly distributed such that $h(q) := \frac{1}{\bar{q}-q}$. Then the competitive loan-interest rate takes the form

$$1 + r_*^c(i_*, r_E) = \begin{cases} \frac{(1+i_*)f}{I} \left[\bar{q} - \sqrt{\frac{2(\bar{q}-q)W}{(1+i_*)f}} (1 + r_E) \right] & \text{if } 1 + r_E \leq \frac{(1+i_*)f}{W} \left(\frac{\bar{q}-q}{2} \right), \\ \frac{(1+i_*)f}{I} \left(\frac{\bar{q}+q}{2} \right) - \frac{W}{I} (1 + r_E) & \text{otherwise.} \end{cases} \quad (21)$$

The proof of Lemma 1 is given in the appendix. As seen above, bankruptcies only occur with positive probability if $q_{\text{NB}}(i_*, r_E) > \underline{q}$. Inserting the loan-interest rate (21), we see that this is the case if and only if the alternative project's rate of return $1 + r_E$ satisfies

$$1 + r_E \leq \frac{(1+i_*)f}{2W} (\bar{q} - \underline{q}). \quad (22)$$

Note that condition (22) is expressed solely in terms of exogenous parameters if we replace i_* with $\frac{\bar{d}-d_1}{I}$. A situation where all entrepreneurs go bankrupt does not occur if $q_{\text{TB}}(i_*, r_E) < \underline{q}$. Inserting the loan-interest rate (21), this will be the case if and only if the rate of return $1 + r_E$ of the alternative project satisfies

$$1 + r_E \geq \frac{(1+i_*)f}{2W} \left(\bar{q} - \frac{1+\eta}{1+i_*} \underline{q} \right). \quad (23)$$

To allow for bankruptcies of firms but excluding the extreme case that all firms may go bankrupt, we assume for the remainder of the paper that both conditions (22) and (23) hold.

Solving the integral in (13), aggregate losses in equilibrium $L = L(i_*, q, r_E)$ under the hypothesis of Lemma 1 and the assumption that $q_{\text{TB}}(i_*, r_E) \leq \underline{q}$ take the form

$$L = \begin{cases} 0 & \text{if } q \geq q_{\text{NB}}(i_*, r_E), \\ \frac{(1+i_*)^2 f}{2q} [q_{\text{NB}}(i_*, r_E) - q]^2 & \text{if } \underline{q} \leq q < q_{\text{NB}}(i_*, r_E). \end{cases} \quad (24)$$

Expected losses due to bankruptcies of firms in an equilibrium take the form

$$\begin{aligned} \mathcal{L}(i_*, r_E) &= \int_{\underline{q}}^{q_{\text{NB}}} \frac{(1+i_*)^2 f}{2q} [q_{\text{NB}} - q]^2 \frac{dq}{(\bar{q} - q)} \\ &= \frac{(1+i_*)^2 f}{2(\bar{q} - \underline{q})} \left\{ q_{\text{NB}}^2 \ln \left(\frac{q_{\text{NB}}}{\underline{q}} \right) - 2q_{\text{NB}}(q_{\text{NB}} - \underline{q}) + \frac{1}{2}(q_{\text{NB}}^2 - \underline{q}^2) \right\}, \end{aligned} \quad (25)$$

where $q_{\text{NB}} = q_{\text{NB}}(i_*, r_E)$.

We are now in a position to undertake some comparative statics analyses. First of all, we can clearly see that the equilibrium loan-interest rate (21) is decreasing in r_E . For the interest-rate margin we have the following proposition:

Proposition 3

Under the hypotheses of Lemma 1, let $\{r_^d(i_*, r_E), r_*^c(i_*, r_E)\}$ be a competitive equilibrium of the simple banking system, where*

$$\frac{(1+i_*)f}{2W} \left(\bar{q} - \frac{1+\eta}{1+i_*} \underline{q} \right) \leq 1 + r_E \leq \frac{(1+i_*)f}{2W} (\bar{q} - \underline{q}).$$

If the value of physical capital k_1 is sufficiently small and

$$\frac{q_{\text{NB}}(0, r_E) \ln \left(\frac{q_{\text{NB}}(0, r_E)}{\underline{q}} \right) - [q_{\text{NB}}(0, r_E) - \underline{q}]}{\bar{q} - \underline{q}} < \frac{\bar{d}}{I},$$

then $d_{\text{crit}} \in [0, \bar{d}]$ exists such that

$$\frac{\partial \Delta}{\partial r_E}(i_*, r_E) \leq 0 \text{ if } d_1 \in [0, d_{\text{crit}}] \quad \text{and} \quad \frac{\partial \Delta}{\partial r_E}(i_*, r_E) > 0 \text{ if } d_1 \in (d_{\text{crit}}, \bar{d}].$$

The proof of Proposition 3 is given in the appendix. Proposition 3 implies that higher returns on equity in other investment opportunities influence interest-rate margins

through different channels. On the one hand, a higher value of r_E lowers loan-interest rates (and deposit rates), as entrepreneurs have better alternatives for investing their equity. This lowers banks' expected lending losses, which in turn tends to decrease risk premia. On the other hand, bank owners will demand higher expected returns on bank equity, which in turn requires larger intermediation margins. For a small amount of bank capital the former effect dominates. For a level of bank capital above d_{crit} the relative importance of the effects is reversed.

The default probability for uniformly distributed shocks takes the following explicit form:

Corollary 2

Under the hypotheses of Lemma 1, the default probability is

$$\Pi_{\text{default}} = \frac{q_{\text{crit}} - \underline{q}}{\bar{q} - \underline{q}},$$

where the critical level is given by

$$q_{\text{crit}} = q_{\text{NB}}(i_*, r_E) + A(i_*, r_E) - \sqrt{(q_{\text{NB}}(i_*, r_E) + A(i_*, r_E))^2 - q_{\text{NB}}^2(i_*, r_E)} \quad (26)$$

with $A(i_*, r_E) := \frac{1}{(1+i_*)^{2f}} [\mathcal{L}(i_*, r_E) + e_1(1+r_E)]$.

The proof of Corollary 2 is given in the appendix. Observe that q_{crit} depends essentially on r_E and e_1 .

The preceding results have enabled us to characterize the default probability of the banking system in terms of the underlying exogenous parameters and distributions. In the next section we carry out the same exercise for a sophisticated banking system.

4 Competitive Equilibria for Sophisticated Banks

4.1 Equilibrium concept

We turn to the other polar case in which banks are sophisticated in their rating abilities so that they are able to detect the quality level i of an individual entrepreneur. They can thus determine the firm-specific default probability. The key idea of the equilibrium concept for sophisticated banks is to require that banks charge a *fair risk premium* in

the sense that the average return on each loan is equal to the risk-free return on equity.

Let

$$\mathcal{R}(i, r_i^c) = \int_{\underline{q}}^{\bar{q}} \min \{q(1+i)f, I(1+r_i^c)\} h(q) dq \quad (27)$$

denote the expected repayment by an entrepreneur with quality level i who has received a loan with the face value I at the interest rate r_i^c . In requiring banks to earn the same return on each investing entrepreneur means that an individualized interest rate r_i^c for entrepreneur i has to be such that

$$\mathcal{R}(i, r_i^c) \stackrel{!}{=} \mathcal{R}(i_*, r_{i_*}^c) \quad (28)$$

for all investing entrepreneurs $i \in [i_*, \eta]$. Let d_{i1} and S_{i1} denote the amount of equity and deposits used to finance a loan of size I . As we assume that the debt/equity ratio is the same across loans, we have

$$d_{i1} = \frac{d}{\eta - i^o} \quad \text{and} \quad S_{i1} = \frac{S}{\eta - i^o}$$

for each entrepreneur i who invests into her production project.⁶ As all banks must pay the deposit interest rate r^d on deposits S_{i1} , the individual return of a producing entrepreneur i must be at least

$$\mathcal{R}(i, r_i^c) \geq \frac{1}{[\eta - i^o]} [d_1(1 + r_E) + S(1 + r^d)]$$

so that banks will offer their intermediation services.

The equilibrium concept for a sophisticated banking system is modified as follows. A competitive equilibrium for a sophisticated banking system is a list of deposit- and loan-interest rates $\left\{ r_*^{do}, \{r_{*i}^{co}\}_{i \in [i_*, \eta]} \right\}$ such that

- (i) no banks exit and no banks enter the market to offer intermediation services,
- (ii) firms take optimal investment decisions,
- (iii) loan demand equals loan supply.

More formally, a competitive equilibrium with financial intermediation for a sophisticated banking system is defined as follows:

⁶This corresponds to the capital requirements in the first Basel Accord.

Definition 2

Suppose $d_1 \in [0, \bar{d}]$, $k_1 \geq 0$, and $r_E \geq 0$ be arbitrary. A sophisticated (competitive) equilibrium in a sophisticated banking system is a list $\left\{ r_*^{do}, \{r_*^{co}(i)\}_{i \in [i_*, \eta]} \right\}$ consisting of a deposit-interest rate r_*^{do} and loan-interest rates $r_*^{co} = r_*^{co}(i)$, such that

$$[\eta - i_*] \mathcal{R}(i, r_*^{co}(i)) = [d_1 + k_1](1 + r_E) + S(1 + r_*^{do}), \quad i \in [i_*, \eta], \quad (29)$$

$$\Pi(i_*, r_*^{co}(i_*)) = W(1 + r_E), \quad (30)$$

$$[\eta - i_*] I = S + d_1, \quad (31)$$

$$r_*^{do} \leq r_E. \quad (32)$$

The equilibrium notion derives naturally from the corresponding Definition 1 for a simple banking system. Condition (29) states that banks must earn the same expected return on each loan. Recalling that $\Pi(i, r^c)$ is increasing in quality levels i , condition (30) is the indifference condition for entrepreneurs. As before, (31) is the equilibrium condition for savings and investments at banks determining the critical entrepreneur $i_* = i_*(d_1)$, which is the same as in the simple banking system. In a sophisticated equilibrium all entrepreneurs with sufficiently high quality parameters $i \geq i_*$ invest in their production projects, while all entrepreneurs with insufficient quality parameters $i < i_*$ invest in the alternative project. Hence, both systems finance the same number of production projects. Finally, the last condition precludes that entrepreneurs who invest in the alternative project and bank equity holders do not want to deposit their money at banks.

Condition (29) is equivalent to the free exit and free entry condition, as a bank that offers its intermediation service for entrepreneurs will earn an expected return on equity of $1 + r_E$ if it employs $\frac{d_1 + k_1}{\eta - i_*}$ as equity and $\frac{S}{\eta - i_*}$ as deposits to perform the intermediation services and to fund an individual borrower. To verify that no banks exit or enter a sophisticated banking system in equilibrium, we proceed as follows. The repayments to banks in a sophisticated banking system are

$$P^o(q, i_*, r_E) = \int_{i_*}^{\eta} \min \left\{ q(1 + i)f, I[1 + r_*^{co}(i)] \right\} di. \quad (33)$$

Using equations (27) and (28) which must hold in equilibrium, the expected repayments in a sophisticated equilibrium are given by:

$$\mathbb{E}[P^o(\cdot, i_*, r_E)] = [\eta - i_*] \mathcal{R}(i_*, r_*^{co}(i_*)). \quad (34)$$

Using (29), future bank capital of the sophisticated system in equilibrium is

$$\begin{aligned} d_2^o &= G_*^o(q, d_1, r_E) := P^o(q, i_*, r_E) - S[1 + r_*^{do}(i_*)] \\ &= P^o(q, i_*, r_E) - \mathbb{E}[P^o(\cdot, i_*)] + e_1(1 + r_E), \end{aligned} \quad (35)$$

where $i_* = i_*(d_1)$ as before. Thus

$$\mathbb{E}[G_*^o(\cdot, d_1, r_E)] = e_1(1 + r_E)$$

and the free exit and free entry condition is satisfied in equilibrium.

4.2 Existence of sophisticated equilibria

To establish existence and uniqueness of sophisticated equilibria, observe that i_* is again equal to $\frac{\bar{d}-d_1}{I}$. Condition (30) implies that, in a sophisticated equilibrium, the entrepreneur with the lowest quality level i_* must be indifferent between applying for loans and investing in the alternative project at the rate r_E , i.e., $\Pi(i_*, r_*^{co}) = W(1+r_E)$. This condition coincides with the indifference condition (8) for simple banks. Using Proposition 1, we obtain

Lemma 2

Suppose $\Pi(i_, 0) > W(1 + r_E)$ for some $d_1 \in [0, \bar{d}]$ with $i_* = i_*(d_1)$. Then the interest rate for the lowest-quality entrepreneur applying for loans is given by*

$$r_{*i_*}^{co} = r_*^c(i_*, r_E), \quad (36)$$

where $r_*^c(i_*, r_E)$ is implicitly defined by

$$\Pi(i_*, r_*^c(i_*, r_E)) = W(1 + r_E).$$

Lemma 2 shows that the loan-interest rate for the lowest-quality entrepreneur in the sophisticated system coincides with the loan-interest rate in the simple system. Setting

$$\mathcal{R}_*(i_*, r_E) := \mathcal{R}(i_*, r_*^c(i_*, r_E))$$

for the average repayment of the lowest-quality entrepreneur in equilibrium, we are now ready to establish existence and uniqueness of sophisticated equilibria.

Proposition 4

Let $d_1 \in [0, \bar{d}]$ with $i_* = i_*(d_1)$ be arbitrary and suppose that the following conditions hold.

- (i) The ratio between physical capital and savings satisfies $\frac{k_1}{S} \geq \frac{r_E}{1+r_E}$.
- (ii) Entrepreneur i_* invests for a loan interest rate r_E , i.e., $\Pi(i_*, 0) > W(1 + r_E)$.
- (iii) Productivity of entrepreneurs is such that

$$\mathbb{E}[q] \int_{i_*}^{\eta} (1+i)f \, di \leq [d_1 + k_1 + S](1 + r_E).$$

If

$$\mathcal{R}(\eta, 0) \leq \mathcal{R}_*(i_*, r_E), \quad (37)$$

then a sophisticated banking system admits a unique sophisticated equilibrium

$$\left\{ r_*^{do}, \{r_{*i}^{co}\}_{i \in [i_*, \eta]} \right\},$$

where $r_*^{do} = r_*^{do}(i_*, r_E)$ and $r_{*i}^{co} = r_*^{co}(i, r_E)$, $i \in [i_*, \eta]$ are defined by

$$r_*^{do}(i_*, r_E) = \frac{1}{S} \left\{ [\eta - i_*] \mathcal{R}_*(i_*, r_E) - [d_1 + k_1](1 + r_E) \right\} - 1 \quad (38)$$

and

$$\mathcal{R}(i, r_*^{co}(i, r_E)) = \mathcal{R}_*(i_*, r_E), \quad i \in [i_*, \eta], \quad (39)$$

respectively.

The proof of Proposition 4 is given in the appendix. Conditions (i)-(iii) are completely analogous to those of Proposition 1. The additional Condition (37) requires that a sophisticated banking system is capable of lowering the loan interest such that banks earn the same expected repayment $\mathcal{R}_*(i_*, r_E)$ on loans for all entrepreneurs $i \geq i_*$ who invest in their production projects. Note that the equilibrium loan interest rates $r_*^{co}(i, r_E)$ in Proposition 4 are independent of physical capital k_1 , whereas the equilibrium deposit interest rate r_*^{do} is not. Throughout this paper we will suppress this dependency for notational simplicity.

4.3 Properties of sophisticated equilibria

Let

$$\Delta^o(i, r_E) := r_*^{co}(i, r_E) - r_*^{do}(i_*, r_E)$$

denote the equilibrium intermediation margin associated with the investment project of entrepreneur i . We first obtain the following comparative statics result.

Lemma 3

Let the assumptions of Proposition 4 be satisfied. Then for each $i \in (i_, \eta)$, the following is true:*

$$\begin{aligned} (i) \quad & \frac{\partial r_*^{co}}{\partial i}(i, r_E) \leq 0, & (ii) \quad & \frac{\partial r_*^{co}}{\partial r_E}(i, r_E) < 0, \\ (iii) \quad & \frac{\partial \Delta^o}{\partial r_E}(i, r_E) > 0 & \text{if } & \frac{S}{[\eta - i_*]} < \frac{\partial \mathcal{R}}{\partial r^c}(i, r_*^{co}(i, r_E)) \\ & & & \text{or if } d_1 + k_1 \text{ is sufficiently high.} \end{aligned}$$

The proof of Lemma 3 is given in the appendix. The first property in Lemma 3 is clear. The second property is explained by the feedback of r_E on the loan-interest rates. A higher r_E leads to lower $r_*^{co}(i_*, r_E)$ and thus to lower expected repayments to banks by all entrepreneurs, which implies (ii). Analogously to Proposition 3, a rising r_E has countervailing effects on interest rate margins which depends on parameters. For a sufficiently large level of equity of deposits, interest margins will again rise if the return on equity rises.

An immediate corollary to Lemma 3 is that entrepreneurs whose bankruptcy risk is zero so that their average repayments are equal to their obligations pay the same interest rate. This observation is stated as follows.

Corollary 3

Under the hypotheses of Proposition 4, suppose there exists a quality level $i_{NB} \in [i_, \eta]$ such that*

$$\mathcal{R}(i_{NB}, r_*^{co}(i_{NB}, r_E)) = I(1 + r_*^{co}(i_{NB}, r_E)).$$

Then all entrepreneurs with sufficiently high quality parameters $i \in [i_{NB}, \eta]$ pay the same loan interest rates which is given by

$$1 + r_*^{co}(i, r_E) = \frac{\mathcal{R}_*(i_*, r_E)}{I}.$$

The corollary shows that a sophisticated banking system provides a floor $\frac{\mathcal{R}_*(i_*, r_E)}{I}$ for the loan-interest rates. All entrepreneurs who meet their obligations with certainty will pay the interest rate given by the floor. All other entrepreneurs pay a higher loan-interest rate. For these entrepreneurs the loan-interest rate is monotonically decreasing with the quality of their investment projects.

4.4 Instability

As with simple banking, we are now in a position to derive the default probability of an individual bank, which is equal to the probability of a system-wide collapse of the banking system. Aggregate losses of the sophisticated system are formally defined by

$$L^o(q, i_*, r_E) = \int_{i_*}^{\eta} I[1 + r_*^{co}(i, r_E)] di - P^o(q, i_*, r_E) \quad (40)$$

Using (39), expected aggregate losses are

$$\mathcal{L}^o(i_*, r_E) := \mathbb{E}[L^o(\cdot, i_*, r_E)] = \int_{i_*}^{\eta} I[1 + r_*^{co}(i, r_E)] di - [\eta - i_*] \mathcal{R}_*(i_*, r_E). \quad (41)$$

Inserting (38) and (41) into (35) yields

$$d_2^o = \mathcal{L}^o(i_*, r_E) - L^o(q, i_*, r_E) + e_1(1 + r_E) \quad (42)$$

with $e_1 = d_1 + k_1$. Given the symmetry assumption of this paper, the default condition for an individual bank coincides with the default condition for the whole banking system and is $d_2^o < 0$. Using (42) this condition takes the form

$$e_1(1 + r_E) < L^o(q, i_*, r_E) - \mathcal{L}^o(i_*, r_E). \quad (43)$$

Again, the banking system will collapse if actual aggregate losses are higher than expected aggregate losses plus return on equity. Equation (24) demonstrates that a necessary condition for the default of a bank is $q < q_{NB}(i_*, r_E)$. It follows from (40) that $L^o(q, i_*, r_E)$ is decreasing in q . Therefore, the proof of the following proposition is analogous to that of Proposition 2.

Proposition 5

Let $d_1 \in [0, \bar{d}]$ and $k_1 > 0$ be arbitrary and assume that

$$e_1(1 + r_E) < L^o(q, i_*, r_E) - \mathcal{L}^o(i_*, r_E),$$

where $e_1 = d_1 + k_1$. Then there exists a unique critical level $\underline{q} < q_{\text{crit}}^o < q_{\text{NB}}(i_*, r_E)$ for macroeconomic shocks, such that a sophisticated banking system will default if and only if $q < q_{\text{crit}}^o$. The default probability is

$$\Pi_{\text{default}}^o := \text{Prob}\left(e_1(1 + r_E) < L^o(q, i_*, r_E) - \mathcal{L}^o(i_*, r_E)\right) = \int_{\underline{q}}^{q_{\text{crit}}^o} h(q) dq. \quad (44)$$

Proposition 5 states that banks default with positive probability as soon as the buffer $e_1(1 + r_E)$ is too small to insure against adverse macroeconomic shocks. As in the case of simple banking, equation (44) is a value-at-risk formula. Indeed, if the default probability is stipulated at a certain value Π_{default}^o , equation (44) determines the required equity level $e_1 = d_1 + k_1$ such that an individual bank and hence the banking system defaults with probability Π_{default}^o .

4.5 Uniformly distributed shocks

Assume that the shocks are uniformly distributed on $[\underline{q}, \bar{q}]$. Setting $\underline{r}(i) = (1+i)\underline{q}f/I-1$ and $\bar{r}(i) = (1+i)\bar{q}f/I-1$, the expected repayment of entrepreneur i , given an interest rate r^c , is

$$\mathcal{R}(i, r^c) = \begin{cases} I(1 + r^c) & \text{if } r^c < \underline{r}(i), \\ \frac{1}{\bar{q}-\underline{q}} \left\{ \bar{q}I(1 + r^c) - \frac{I^2(1+r^c)^2}{2(1+i)f} - \frac{1}{2}\underline{q}^2(1+i)f \right\} & \text{if } \underline{r}(i) \leq r^c \leq \bar{r}(i), \\ \frac{\bar{q}+q}{2}(1+i)f & \text{if } \bar{r}(i) < r^c. \end{cases} \quad (45)$$

Inserting the loan interest function given by Lemma 1 into (45) and using the fact that loan-interest rates of both banking systems coincide for the lowest-quality entrepreneur as in Lemma 2, we obtain

$$\mathcal{R}_*(i_*, r_E) = \mathcal{R}(i_*, r_*^c(i_*, r_E)) = \frac{\bar{q}+q}{2}(1+i_*)f - W(1+r_E). \quad (46)$$

From condition (39) we may solve $\mathcal{R}(i, r_{*i}^{co}) = \mathcal{R}_*(i_*, r_E)$ for r_{*i}^{co} for each investing entrepreneurs i , to obtain the following lemma:

Lemma 4

Under the hypotheses of Proposition 4, assume that the macroeconomic shocks are uniformly distributed such that $h(q) := \frac{1}{\bar{q}-\underline{q}}$. Then the deposit-interest rate takes the form

$$1 + r_*^{do}(i_*, r_E) = \frac{1}{S} \left\{ [\eta - i_*] \frac{\bar{q}+q}{2}(1+i_*)f - ([\eta - i_*]W + e_1)(1+r_E) \right\}. \quad (47)$$

The loan-interest rates of sophisticated banking equilibrium are given by⁷

$$1 + r_*^{co}(i, r_E) = \begin{cases} \frac{\mathcal{R}_*(i_*, r_E)}{I} & \text{if } 1 + r_E \geq \frac{(1+i_*)f}{W} \left[\frac{\bar{q}+q}{2} - \left(\frac{1+i}{1+i_*} \right) \underline{q} \right], \\ \frac{(1+i)f}{I} \left[\bar{q} - \sqrt{\frac{i-i_*}{1+i}(\bar{q}^2 - \underline{q}^2) + \frac{2(\bar{q}-q)W}{(1+i)f}(1+r_E)} \right] & \text{otherwise,} \end{cases} \quad (48)$$

where $i \in [i_*, \eta]$.

Lemma 4 implies that loan-interest rates are either independent of the quality of entrepreneurs i [first branch of equation (48)] or decreasing with i (second branch of equation (48)). The first case will occur if the return r_E on outside investments is sufficiently high in relation to the quality of the entrepreneur. The following corollary is analogous to Corollary 2.

Corollary 4

If, in addition, the shocks are uniformly distributed, then the default probability is

$$\Pi_{\text{default}}^o = \frac{q_{\text{crit}} - \underline{q}}{\bar{q} - \underline{q}}.$$

5 Comparison of the Two Systems

5.1 Market conditions

For comparison of the two banking systems, let us first focus on the interest rates. Consider the highly ideal case in which no firm bankruptcies occur in the simple banking system. This will occur if

$$\underline{q}(1 + i_*)f \geq I[1 + r_*^c(i_*, r_E)].$$

Average repayments of entrepreneurs are then

$$\mathcal{R}(i, r_*^c(i_*, r_E)) = I[1 + r_*^c(i_*, r_E)] \quad \text{for all } i \in [i_*, \eta]$$

and by virtue of Proposition 4 we have

$$r_*^{co}(i, r_E) = r_*^c(i_*, r_E) \quad \text{for all } i \in [i_*, \eta],$$

⁷Note that only the smaller solution of the quadratic equation is economically viable.

implying that the simple and the sophisticated banking system have the same loan-interest rates and the same deposit-interest rates.

This situation changes as soon as firm bankruptcies occur in equilibrium. The following proposition shows that loan- and deposit-interest rates in a sophisticated system are lower than loan- and deposit-interest rates in a simple banking system.

Proposition 6

Under the hypotheses of Propositions 1 and 4 assume that

$$\underline{q}(1 + i_*)f < I[1 + r_*^c(i_*, r_E)]$$

and that firm bankruptcies occur with positive probability. Then the following holds:

- (i) $r_*^c(i_*, r_E) > r_*^{co}(i, r_E)$ for all $i > i_*$ with equality holding for $i = i_*$,
- (ii) $r_*^d(i_*, r_E) > r_*^{do}(i_*, r_E)$.

The intuition for Proposition 6 is as follows. In a sophisticated banking system, banks tailor loan-interest rates to the quality level of entrepreneurs. All entrepreneurs with $i > i_*$ obtain lower loan-interest rates in a sophisticated banking system than in a simple banking system. In order to generate equity returns of $1 + r_E$, deposit rates in a sophisticated system must be lower than in a simple banking system, i.e., $r_*^{do} < r_*^d$.

5.2 Default

We next analyze the conditions under which a simple banking system accumulates more capital than a sophisticated system. Recall that the expected bank capital in both banking systems is the same and equal to $e_1(1 + r_E)$. The banking system with lower bank capital will be less likely to collapse if macroeconomic shocks are adverse. An immediate consequence of Proposition 6 is that repayments to simple banks are higher than repayments to sophisticated banks, such that

$$P(q, i_*, r_*^c(i_*, r_E)) \geq P^o(q, i_*, r_E). \tag{49}$$

The critical shock below which entrepreneur i will go bankrupt in the sophisticated system is denoted by $q_{NB}^o(i, r_E)$, such that entrepreneur i is bankrupt for all shocks

$$q \leq q_{NB}^o(i, r_E) := \frac{I[1 + r_*^{co}(i, r_E)]}{(1 + i)f}.$$

Since $r_*^{co}(i, r_E) < r_*^c(i_*, r_E)$ for $i > i_*$, we have

$$q_{\text{NB}}^o(i_*, r_E) < q_{\text{NB}}(i_*, r_E) \quad \text{for all } i > i_*$$

with equality holding for the critical entrepreneur $i = i_*$. Hence the default risk of an individual entrepreneur is lower in a sophisticated system. Observe that (49) holds with a strict inequality for all shocks $q \geq q_{\text{NB}}^o(\eta, r_E)$. This means that repayments to simple banks are higher provided the macroeconomic environment is sufficiently favorable. This scenario occurs with positive probability if $q_{\text{NB}}^o(\eta, r_E) < \bar{q}$.

In order to compare the banking systems' capabilities to accumulate capital, recall that future bank capital of the simple banking system is determined by

$$d_2 = G_*(q, d_1, r_E)$$

with G_* given in (15), while the future bank capital of a sophisticated system is determined by

$$d_2^o = G_*^o(q, d_1, r_E)$$

with G_*^o given in (42). Given the same initial equity level $e_1 = d_1 + k_1$ and the same interest rate r_E , it follows from (15) and (42) that $d_2^o \geq d_2$ if and only if

$$L(q, i_*, r_E) - \mathcal{L}(i_*, r_E) \geq L^o(q, i_*, r_E) - \mathcal{L}^o(i_*, r_E). \quad (50)$$

The following proposition shows that the sophisticated banking system will accumulate more bank capital than the simple system for all shocks below a certain 'break-even' value q_{BE} . As a consequence, a sophisticated system is better able to cope with highly adverse shocks than a simple system. The reverse is true for positive shocks when a simple system will accumulate more bank capital.

Proposition 7

Under the hypotheses of Proposition 6, assume that $\underline{q} < q_{\text{NB}}^o(i_, r_E) < \bar{q}$, such that in both banking systems high-quality entrepreneurs meet their obligations with positive probability while at the same time bankruptcies are possible in both systems. Then there exists a critical shock $q_{\text{BE}} = q_{\text{BE}}(i_*, r_E) \in (\underline{q}, \bar{q})$, such that the sophisticated banking system outperforms the simple system for all shocks $q \leq q_{\text{BE}}$. More precisely,*

$$G_*^o(q, d_1, r_E) \geq G_*(q, d_1, r_E) \quad \text{if and only if } q \leq q_{\text{BE}}.$$

The proof of Proposition 7 is given in the appendix. To illustrate this result, consider an extreme case in which all firms go bankrupt, causing a default of the sophisticated banking system. Such an adverse macroeconomic shock means that all firms will be bankrupt under a simple banking system as well. Since banks earn only the liquidation values, revenues in both banking systems are identical in this case. However, deposit rates are higher in a simple banking system, so their aggregate losses are higher. This explains the lower bank capital for a simple banking system if macroeconomic shocks are below the critical level q_{BE} .

Our main theorem now shows that the default probability of the banking system depends highly on their capital base.

Theorem 1

Let the hypotheses of Proposition 7 be satisfied.

(i) *If*

$$G_*^o(q_{BE}, d_1, r_E) = G_*(q_{BE}, d_1, r_E) > 0, \quad (51)$$

then the default probability of the sophisticated banking system is lower than the default probability of the simple banking system, i.e., $\Pi_{\text{default}}^o < \Pi_{\text{default}}$.

(ii) *If, on the contrary,*

$$G_*^o(q_{BE}, d_1, r_E) = G_*(q_{BE}, d_1, r_E) < 0, \quad (52)$$

then the default probability of the sophisticated banking system is higher than the default probability of the simple banking system, i.e. $\Pi_{\text{default}}^o > \Pi_{\text{default}}$.

In view of (15), Condition (51) is clearly satisfied if initial equity e_1 is sufficiently high. To illustrate Result (i) of Theorem 1, suppose a sufficiently adverse macroeconomic shock occurs. Although loan-interest rates are higher in a simple banking system, revenues do not fully reflect the interest rate differentials with respect to the sophisticated banking system, as under both systems banks will only earn liquidation values for a substantial set of entrepreneurs. However, simple banks face higher refinancing costs $r_*^d > r_*^{do}$ which are unaffected by a macroeconomic shock. Hence, simple banks are more likely to default than sophisticated banks when sufficiently adverse shocks occur.

On the other hand, (51) may be violated for a low level of initial equity e_1 . Then banks may default if moderate adverse macroeconomic shocks and a small number of

firm bankruptcies occur. In this case, a simple banking system has greater benefits from higher average loan rates, which may outweigh the higher refinancing costs in comparison to a sophisticated system. While entrepreneurs benefit from lower loan interest rates in the sophisticated system, the system itself may lack a sufficient amount of repayments. As a consequence, sophistication may decrease bank stability.

6 Conclusion

This paper demonstrated that more sophistication in the assessment of individual default risks of entrepreneurs will only increase banking stability if initial equity is sufficiently high. We showed that sophistication in risk management rewards high-quality entrepreneurs with lower loan rates at the expense of depositors facing lower returns. Sophistication in rating techniques thus has distributional implications for both sides of the market. Our analysis suggests that regulatory policies for banking such as Basel II which require banks to introduce more sophistication in assessing the credit worthiness their clients will only be beneficial if banks are sufficiently healthy in terms of equity.

7 Appendix

Proof of Proposition 1.

It is easy to check that Condition (i) implies that (11) holds. Set $\bar{r}^c := \frac{\bar{q}(1+\eta)f}{I} - 1$ with \bar{q} denoting the highest possible shock. Then the expected profits of any entrepreneur i are zero for loan rates higher than \bar{r}^c , i.e., $\Pi(i, r^c) = 0$ for $r^c \geq \bar{r}^c$ with Π given in (1). Since $\Pi(i, r^c)$ is increasing in quality levels $i \in [0, \eta]$ and strictly decreasing with loan interest rates $r^c \in [0, \bar{r}^c]$, the indifference equation (8) has a unique solution $r_*^c = r_*^c(i_*, r_E) \in [0, \bar{r}^c]$ if Condition (ii) holds, i.e., $\Pi(i_*, r_E) > W(1 + r_E)$. Inserting (6) into the no-entry condition (7) and solving for r_*^d , we can calculate the equilibrium deposit rate r_*^d as a function of i_* and r_E . Since

$$\mathbb{E}[P(\cdot, i_*, r_*^c)] \leq \mathbb{E}[P(\cdot, i_*, \bar{r}^c)] = \mathbb{E}[q] \int_{i_*}^{\eta} (1+i)f \, di$$

Condition (iii) implies (10). This proves the proposition. ■

Proof of Lemma 1.

Notice first that

$$\mathbb{E}[P(\cdot, i_*, r_*^c)] \leq [\eta - i_*]I(1 + r_*^c)$$

Suppose on the contrary that $r_*^c < r_E$. It follows from (11) and (9) that

$$\mathbb{E}[P(\cdot, i_*, r_*^c)] \geq [\eta - i_*]I(1 + r_E),$$

a contradiction. The rest follows from the implicit function theorem. ■

Proof of Lemma 1.

Setting $\underline{r}(i_*) = (1 + i_*)\underline{q}f/I - 1$ and $\bar{r}(i_*) = (1 + i_*)\bar{q}f/I - 1$, the expected profit of entrepreneur i_* given an interest rate r^c is

$$\Pi(i_*, r^c) = \begin{cases} (1 + i_*)f\frac{\bar{q}+q}{2} - I(1 + r^c) & \text{if } r^c < \underline{r}(i_*), \\ \frac{(1+i_*)f}{2(\bar{q}-\underline{q})} \left[\bar{q} - \frac{I(1+r^c)}{(1+i_*)f} \right]^2 & \text{if } \underline{r}(i_*) \leq r^c \leq \bar{r}(i_*), \\ 0 & \text{if } \bar{r}(i_*) < r^c. \end{cases}$$

Then (21) follows from condition (8) when we observe that in the resulting quadratic equation for $r_*^c(i_*, r_E)$ only the smaller solution is economically viable. ■

Proof of Proposition 2.

By assumption, we have $d_2 = G(\underline{q}, d_1, r_*^d, r_*^c) < 0$. Since $G(q, d_1, r_*^d, r_*^c) > 0$ for $q \geq q_{\text{NB}}(i_*, r_*^c)$ and the function G is strictly increasing in q , there exists a unique critical shock $\underline{q} < q_{\text{crit}} < q_{\text{NB}}(i_*, r_E)$ such that $G(q_{\text{crit}}, d_1, r_*^d, r_*^c) = 0$. ■

Proof of Proposition 3.

Let $i_* = i_*(d_1)$, as before, and observe first that (25) is composed of two functions such that $L(i_*, r_E) = \tilde{\mathcal{L}}(i_*, r_*^c(i_*, r_E))$ for a suitably defined function $\tilde{\mathcal{L}}$. Differentiation $\Delta(i_*, r_E)$ with respect to r_E yields

$$\frac{\partial \Delta}{\partial r_E}(i_*, r_E) = \frac{1}{S} \left(\left[\frac{\partial \tilde{\mathcal{L}}}{\partial r^c}(i_*, r_*^c(i_*, r_E)) - d_1 \right] \frac{\partial r_*^c}{\partial r_E}(i_*, r_E) + d_1 + k_1 \right). \quad (53)$$

We have $\frac{\partial r_*^c}{\partial r_E}(i_*, r_E) < 0$ and

$$\frac{\partial \tilde{\mathcal{L}}}{\partial r^c}(i_*, r_*^c) = I(1 + i_*) \frac{q_{\text{NB}}(i_*, r_E) \ln \left(\frac{q_{\text{NB}}(i_*, r_E)}{\underline{q}} \right) - (q_{\text{NB}}(i_*, r_E) - \underline{q})}{\bar{q} - \underline{q}} \geq 0.$$

This implies that the bracket in (53) is positive for d_1 equal or close to zero. Hence, (53) is negative for sufficiently small $d_1 + k_1$.

On the other hand, the bracket in (53) is negative by assumption for $d_1 = \bar{d}$ noting that $0 = i_*(\bar{d})$. Thus, (53) is positive for d_1 sufficiently close to \bar{d} . Provided that k_1 is sufficiently small, the existence of d_{crit} then follows from the intermediate value theorem and by

$$\frac{d}{dd_1} \left(\frac{\partial \tilde{\mathcal{L}}}{\partial r^c}(i_*, r_*^c) \right) < 1 \quad \text{and} \quad \frac{d}{dd_1} \left(\frac{\partial r_*^c}{\partial r_E}(i_*, r_E) \right) > 0.$$

■

Proof of Corollary 2.

Under the hypothesis of Lemma 1, the condition $G(d_1, q_{\text{crit}}, r_*^d, r_*^c) = 0$ is equivalent to

$$\mathcal{L}(i_*, r_E) + e_1(1 + r_E) - \frac{(1 + i_*)^2 f}{2q} [q_{\text{NB}}(i_*, r_E) - q]^2 = 0$$

or, with the help of the notation introduced in the main text,

$$q^2 - 2(q_{\text{NB}}(i_*, r_E) + A(i_*, r_E))q + q_{\text{NB}}^2(i_*, r_E) = 0. \quad (54)$$

The unique solution of (54) lying in $(\underline{q}, q_{\text{NB}}(i_*, r_E))$ is the critical level q_{crit} given in (26).

■

Proof of Proposition 4.

The deposit-interest rate (38) follows from Lemma 2 and Condition (29). The existence of the loan-interest rate function (39) follows from Condition (37), an application of the implicit function theorem, and the fact that $\mathcal{R}(i, r^c)$ is non-decreasing in i and r^c .

■

Proof of Lemma 3.

(i) Follows directly from implicit differentiation of (39).

(ii) Implicit differentiation of (39) yields

$$\frac{\partial r_*^{co}}{\partial r_E}(i, r_E) = \frac{\frac{\partial \mathcal{R}}{\partial r^c}(i_*, r_*^c(i_*, r_E))}{\frac{\partial \mathcal{R}}{\partial r^c}(i, r_*^{co}(i, r_E))} \frac{\partial r_*^c}{\partial r_E}(i, r_E) < 0.$$

(iii) Differentiation gives

$$\frac{\partial \Delta^o}{\partial r_E}(i, r_E) = \left(\frac{1}{\frac{\partial \mathcal{R}}{\partial r^c}(i, r_*^{co}(i, r_E))} - \frac{[\eta - i_*]}{S} \right) \frac{\partial \mathcal{R}}{\partial r^c}(i_*, r_*^{co}(i_*, r_E)) \frac{\partial r_*^{co}}{\partial r_E}(i, r_E) + \frac{d_1 + k_1}{S}$$

Since $\frac{\partial \mathcal{R}}{\partial r^c}(i, r^c) > 0$ for all i and all r^c , the assertion follows from (ii). ■

Proof of Proposition 6.

(i) Since bankruptcies occur with positive probability, the average repayment $\mathcal{R}(i, r^c)$ is increasing in i for sufficiently small $i \geq i_*$. The first assertion then follows from Lemma 2 and Proposition 4.

(ii) Average repayments to simple banks are

$$\mathbb{E}[P(\cdot, i_*, r_*^c)] = \int_{i_*}^{\eta} \int_{\underline{q}}^{\bar{q}} \min\{q(1+i)f, I(1+r_*^c)\} h(q) dq di,$$

while average repayments to sophisticated banks are

$$\mathbb{E}[P^o(\cdot, i_*, r_*^{co}(i_*, r_E))] = [\eta - i_*] \mathcal{R}_*(i_*, r_E).$$

The assumption on bankruptcies implies

$$\mathbb{E}[P(\cdot, i_*, r_*^c)] > [\eta - i_*] \mathcal{R}_*(i_*, r_E)$$

and the assertion follows from a comparison of (12) with (38). ■

Proof of Proposition 7.

We have $d_2^o \geq d_2$ if and only if (50) holds Using (15) and (35), (50) is equivalent to

$$P(q, i_*, r_*^c(i_*, r_E)) - P^o(q, i_*, r_E) \leq \mathbb{E}[P(\cdot, i_*, r_*^c(i_*, r_E))] - [\eta - i_*] \mathcal{R}_*(i_*, r_E). \quad (55)$$

The first term in (55) describes the difference in repayments between the two banking systems, while the second term describes the difference in average repayments. Since $r_*^{co}(i, r_E) < r_*^c(i_*, r_E)$ for all $i > i_*$, the first term of (55) is always non-negative. The assumption regarding $q_{NB}^o(\eta, r_E)$ means that this term is positive for sufficiently high shocks q . Since it is increasing in q , there exists q_{BE} such that (55) hold with equality. This implies that (55) and hence (50) holds if and only if $q \leq q_{BE}$. Since q_{BE} depends on i_* and r_E , this proves the proposition. ■

Proof of Theorem 1.

We need to show that $q_{crit}^o < q_{crit}$ in case (i) and $q_{crit}^o > q_{crit}$ in case (ii). It follows from Proposition 7 and the monotonicity of L and L^o that

$$0 < \mathcal{L}^o(i_*, r_E) - L^o(q, i_*, r_E) < \mathcal{L}(i_*, r_E) - L(q, i_*, r_E) \quad \text{for all } q > q_{BE}$$

and

$$\mathcal{L}^o(i_*, r_E) - L^o(q, i_*, r_E) \geq \mathcal{L}(i_*, r_E) - L(q, i_*, r_E) \quad \text{for all } q \leq q_{BE}.$$

Hence, the critical values must satisfy $q_{crit}^o < q_{crit}$ in case (i) and the opposite inequality must hold in case (ii). This proves the theorem. ■

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