Sophistication in Risk Management and Banking Stability: The Long Term*

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Abstract

We investigate the question of whether sophistication in risk management fosters banking stability. We compare a simple banking system in which an average rating is used with a sophisticated banking system in which banks are able to assess the default risk of entrepreneurs individually. Both banking systems compete for deposits, loans, and equity. While a sophisticated system rewards entrepreneurs with low default risks by low loan interest rates, a simple system acquires more equity and finances more entrepreneurs. Expected repayments in a simple system are always higher and its default risk is lower if productivity is sufficiently high.

Keywords: Financial intermediation, macroeconomic risks, risk management, risk premia, banking regulation, rating.

JEL Classification: D40, E44, G21

1 Introduction

The popular view of banking regulation is that more sophistication in rating and risk management increases the stability of banking systems. This view has motivated the introduction of modern risk management techniques during the last decade. While such techniques which cause the transformation of risk are well established in capital markets[1], the application of sophisticated rating tools by commercial banks is a more recent development. From the perspective of a single institution, it is clear that costless sophistication of risk management techniques is always beneficial. However, whether or not this holds for the economy as a whole is a largely unresolved issue.

In this paper we argue that a systemic perspective may lead to quite different conclusions from those derived from the perspective of a single institution. We investigate the issue of to what extent the investment behavior of banks and the stability of the banking system will change with the introduction of more sophistication in rating and risk management. We explore the case when banks which compete for equity, loans, and deposits may adjust their capital base and hence their balance sheets. This situation corresponds to the long term. The short term when banks do not adjust their equity is treated in Gersbach & Wenzelburger (2006). There it is shown that sophisticated risk management is only beneficial if initial bank capital is sufficiently high.

We consider a competitive banking system in which banks offer their intermediation services to a population of entrepreneurs who have three investment opportunities. They may either invest their initial wealth into a production project which is subject to macroeconomic risks, into equity of banks, or into an alternative investment project with an exogenously given return. Banks compete for equity and deposits and offer loans as delegated monitors. Risk premia on loans are determined by the equity market as banks need to offer sufficient returns in order to attract equity. We compare two polar cases, a simple and a sophisticated banking system. In the simple system banks are unable to rate the risk of an investment project individually. They attribute the same default probability to each borrower and thus use the same rating. In the sophisticated system banks use an infinitely fine rating system in which each borrower is individually rated. Loan interest rates are tailored according to the default probability of an entrepreneur.

Our main findings are as follows. First, relative to the simple banking system in which

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[1] A large body of literature investigates the consequences of modern risk management techniques for capital markets which have risen dramatically over the last decades, cf. Carey & Stulz (2006).

all entrepreneurs are offered the same credit terms, a sophisticated system offers high-quality entrepreneurs low interest rates and low-quality entrepreneurs high interest rates. Second, loan demand in a simple banking system is higher than in a sophisticated system because credit terms for low-quality entrepreneurs are more favorable. This allows simple banks to attract more equity while in a sophisticated system more resources are invested into the alternative project. Hence, concerns are justified that sophisticated rating as imposed by the Basel II regulatory framework will make it difficult for middle-sized firms to obtain loans. Third, aggregate repayments in the simple banking system are on average higher than in the sophisticated system in macroeconomic environments with low productivity. As simple banks acquire more equity, their capital buffer against adverse macroeconomic shocks is larger. Hence, although sophisticated banks have higher average quality projects, the default risk of the sophisticated system is generally higher.

The approach of this paper is complementary to the work of Gehrig & Stenbacka (2004) who show that uncoordinated screening behavior of competing financial intermediaries creates a financial multiplier and may be an independent source of fluctuations. In analyzing the systemic effects of screening activities by firms, this paper contributes to the literature on screening by banks surveyed, for example, in Freixas & Rochet (1997). The focus of this paper is more on the consequences for market conditions and systemic defaults when banks introduce more sophisticated rating tools. An interesting question for future research is how even more sophisticated risk management techniques such as the securitization of bank loans including the use of derivative products affect systematic risks as discussed in Franke & Krahnen (2006).

Our results are related to the literature on banking regulation. Comprehensive surveys with different emphases are given by Bhattacharya & Thakor (1993), Dewatripont & Tirole (1994), Hellwig (1994), Freixas & Rochet (1997), or Bhattacharya, Boot & Thakor (1998). Overall, we suggest that increased sophistication in rating[2] as advocated in the new banking regulatory framework (Basel II) may produce unintended negative side effects. Indeed, the analysis of our paper indicates that increased sophistication in banking may create more instability in the long run.

The paper is organized as follows: In the next section we introduce the model and both types of banking systems. In Section 3 we examine simple banks, and in Section 4 we perform the mirror-image of the analysis for sophisticated banks. In Section 5 both systems are compared and our main results are presented. Section 6 concludes.

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[2]Krahnen & Weber (2001) develop a comprehensive set of intuitive rating principles.

2 Model

2.1 Households and entrepreneurs

We consider a two-period model with periods $t = 1$ and $t = 2$. The population of agents consists of a continuum indexed by $[0, 1]$. Each agent has individual wealth W in the first period. Agents are divided into two classes. One fraction of agents, indexed by $[0, \eta]$ $(0 < \eta < 1)$, are potential entrepreneurs. The other fraction, indexed by $(\eta, 1]$, are consumers. Potential entrepreneurs and consumers differ in that only the former have access to investment technologies.

Consumers are endowed with consumption preferences in the two periods of their lives, with c^1, c^2 respectively denoting youthful and elderly consumption in money terms. For simplification, let $u(c^1, c^2) = \ln(c^1) + \delta \ln(c^2)$ be the intertemporal utility function of a consumer, where δ $(0 < \delta < 1)$ is the discount factor. Accordingly, a young consumer inelastically saves the amount $s = \frac{\delta}{1+\delta}W$ if he can transfer wealth from one period to the next at a certain interest rate. We denote the aggregate savings of consumers by $S = (1 - \eta)s$.

Potential entrepreneurs are assumed to be risk-neutral and to consume only in the second period. Each entrepreneur has to decide whether to invest in a production project that converts period-1 goods into period-2 goods, to provide equity for banks, or to invest her funds in an alternative project with return r_E $(r_E > 0)$. The alternative investment opportunity may be thought of as an outside option, such as government bonds or investments in other sectors of the economy that are not modeled explicitly.[3]

The funds required for each production project are fixed at $W + I$, so that an entrepreneur must borrow I additional units of the good from banks to undertake the production project. Entrepreneurs are heterogeneous in the quality of their production projects which depends on their index i. The quality parameter of entrepreneur i is assumed to be private information. If an entrepreneur of type i obtains additional resources I and decides to invest, investment returns in the second period amount to

$$y = q(1 + i)f(W + I),$$

where f denotes a standard atemporal neoclassical production function and $q \in \mathbb{R}_+$ represents an exogenous macroeconomic productivity shock in the economy. Since W

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[3]For simplicity we assume that consumers are not allowed to invest in the alternative project. This can be justified by liquidity services of deposits. However, the results carry over to the case in which consumers hold a portfolio of deposits and other assets. In this case, the saving function is of the form $S = S(r^d, r_E)$ where r^d is the deposit rate.

and I will remain fixed throughout the paper, we write $f = f(W+I)$. The distribution of shocks q is assumed to be given by a continuous density function $h(q)$ with support on a compact interval $[\underline{q}, \overline{q}]$ with $0 < \underline{q} < \overline{q}$.

Entrepreneurs are price-takers and operate under limited liability. Given a loan interest rate r^c, the expected profit of an investing entrepreneur i is

$$\Pi(i, r^c) := \int_{\underline{q}}^{\overline{q}} \max\{q(1+i)f - I(1+r^c), 0\}\, h(q)dq. \tag{1}$$

Note that $\Pi(i, r^c)$ is monotonically increasing in quality levels i and monotonically decreasing in loan rates r^c. A risk-neutral entrepreneur with the quality parameter $i \in [0, \eta]$ will prefer to invest in the production project rather than into the alternative investment project if the return on the production project is expected to be larger than the return on the alternative project, i.e., if

$$\Pi(i, r^c) \geq W(1 + r_E).$$

We assume that savings are never sufficient to fund all entrepreneurs. Since the interest-rate elasticity of savings is zero, this condition takes the form

$$S := (1 - \eta)\, s < \eta\, I. \tag{2}$$

2.2   Banking sector

Following Gersbach & Wenzelburger (2004), we assume that depositors cannot observe the quality parameters of entrepreneurs and cannot verify whether or not an entrepreneur invests. The existence of such market frictions necessitates financial intermediation (see e.g. Hellwig 1994). To alleviate there information problems, we assume that there are n (commercial) banks, indexed by $j = 1, \ldots, n$ $(n > 1)$, which are owned by entrepreneurs so that banks are risk-neutral. Banks finance production projects and maximize profits accruing to shareholders. They monitor loans as delegated monitors in the sense of Diamond (1984) and their monitoring is assumed to be efficient in the sense that they are able to secure both the investment of an entrepreneur and the liquidation value in case of default, cf. Gersbach & Uhlig (2005).

To avoid that properties other than the ability to rate entrepreneurs determine the results, we make the same assumption regarding the competitive structure of the banking sector. First, both banking systems compete for equity and loans while facing a given deposit interest rate $\overline{r}^d$. Each bank j can offer deposit contracts $D(\overline{r}^d)$, where $1 + \overline{r}^d$

is the repayment offered for one unit of money. Second, banks raise equity by issuing equity contracts. An equity contract specifies that the holder is entitled to obtain a share of dividends in proportion to the resources he has given to a particular bank. By providing equity, entrepreneurs become owners of a bank. A bank becomes a legal entity and can only operate if it obtains a positive amount of equity.[4] Third, both banking systems are perfectly competitive. Bank owners have the opportunity to invest in the alternative project with return $1 + r_E$ $(r_E > 0)$.

Competition among banks determines the level of equity and loan interest rates. We distinguish between a simple and a sophisticated banking system which differ only in their ability to rate the quality of an entrepreneur's production project.

1. *Simple Banking System.* The essential feature of the simple banking system is that banks are unable to rate entrepreneurs individually and to adjust loan contracts to the quality parameter i of an entrepreneur. Banks only have an average rating of entrepreneurs and offer all entrepreneurs the same loan contract $C(r^c)$, where $1 + r^c$ is the repayment required from entrepreneurs for one unit of borrowed money.

2. *Sophisticated Banking System.* In a sophisticated banking system, banks are able to rate each entrepreneur individually and to offer entrepreneur-specific loan contracts, denoted by $C(r_i^c)$, where r_i^c is the loan interest rate demanded from an entrepreneur of type i.

In both banking systems, banks operate under unlimited liability and loans are only constrained by the amount of equity and deposits. We assume throughout that aggregate uncertainty is canceled out when depositors and entrepreneurs randomly choose banks.[5] As all banks are identical, they will obtain the same amount of equity and deposits in a particular banking system.

With these assumptions, the financial intermediation process in either system is as follows. Given $\overline{r}^d$, banks in the first period offer equity contracts and loan contracts r^c (simple banking) or loan contracts $\{r_i^c\}_{i=0}^{\eta}$ (sophisticated banking), respectively. Each bank j obtains an amount of d^j in equity and an equal share of deposits from consumers.

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[4]Otherwise the legal entity is not founded as there are no owners. As banks take all equity capital they can obtain, the equilibrium value will be determined by the supply of equity.

[5]The exact construction of individual randomness so that this statement holds can be found in Alós-Ferrer (1999). We could also rely on the weaker forms of the strong law of large numbers developed in Al-Najjar (1995) and Uhlig (1996), where independence of individual random variables can be assumed and aggregate stability is the limit of an economy with finite characteristics.

Entrepreneurs decide which contracts to accept. Money is exchanged. In the second period, funded entrepreneurs produce subject to a macroeconomic shock and pay back loans with limited liability. Banks repay depositors and equity holders.

Some remarks regarding the relationship between $\overline{r}^d$ and r_E are in order. We assume $\overline{r}^d \leq r_E$ and hence entrepreneurs have no incentive to bring their wealth to banks as deposits. Recall that we assumed that households can only provide deposits. If, on the contrary, households could invest their funds at no costs in the alternative project, we would have to assume $\overline{r}^d = r_E$. This is a special case of our model.

We are now ready to investigate to what extent the ability of a competitive banking system to rate firms reduces its vulnerability to firm bankruptcies. To this end, we will compare how equity develops in both types of banking systems. We are particularly interested in the distribution and downside risk of equity in the second period. It is intuitively clear that the lower accumulated equity is in period 2, the more the stability of the banking sector is endangered.

3 Competitive Equilibria for Simple Banks

We develop the equilibrium concept for a simple banking system in which banks are unable to rate entrepreneurs individually. Recall that each entrepreneur owns W units of funds that she can invest either into a production project, into equity of the banking system or into the alternative project. If the total investment of entrepreneurs into the banking system is d, the amount of equity of an individual bank is $\frac{d}{n}$. As all banks are assumed to be identical, the equilibrium conditions will be formulated for the whole banking system.

We will assume throughout that the amount of equity which economic agents are able to supply at expected gross returns $1 + r_E$ suffices to balance loan demand and supply. Let $i_* \in [0, \eta]$ be the critical quality level of an entrepreneur such that all entrepreneurs $i \in [i_*, \eta]$ invest in their own production project and all entrepreneurs $i \in [0, i_*)$ either provide bank equity or invest in the alternative project. Since each entrepreneur is endowed with W units of funds, the available total funds $S + i_* W$ required for financing the entrepreneurs must be larger than the credit volume $[\eta - i_*]I$, that is,

$$S + i_* W \geq [\eta - i_*]I. \tag{3}$$

Given the condition on available deposits (2), condition (3) holds for all large enough

critical quality levels

$$i_* \geq \underline{i} := \frac{\eta I - S}{I + W}. \tag{4}$$

In other words, at least $\underline{i}$ entrepreneurs must provide their endowments for equity in order to meet (3). Given $i_* \geq \underline{i}$, banks' equity d_* must satisfy

$$S + d_* = [\eta - i_*]I. \tag{5}$$

In a competitive equilibrium, (5) states that loan supply must equal loan demand. The remaining resources $i_* W - d_*$ may then be invested into the alternative project.

3.1   Equilibrium concept

Let $\bar{r}^d \geq 0$ be the deposit interest rate of the banking system which is the same for all banks. Banks receive funds S from consumers that have to be paid back with interest at the end of the second period. In a simple banking system, banks lend $[\eta - i_*]I$ to firms and charge the same loan interest rate r^c to all investing entrepreneurs. Given the loan interest rate r^c and some critical quality level i_*, banks' payments $P = P(q, i_*, r^c)$ at the end of the second period are

$$P(q, i_*, r^c) = \int_{i_*}^{\eta} \min\left\{q(1 + i)f,\, I(1 + r^c)\right\}di. \tag{6}$$

The equity level of the banking system in the second period is given by

$$G(q, i_*, r^c) = P(q, i_*, r^c) - S(1 + \bar{r}^d). \tag{7}$$

We next define a competitive equilibrium for a simple banking system. Intuitively, a competitive equilibrium is an equity level and a loan interest rate (d_*, r_*^c) such that

   (i) the equity market clears,

  (ii) firms take optimal investment decisions,

 (iii) the market for loans is balanced.

In order to formalize this concept, observe that given a critical quality level i_* and a loan interest rate r^c, the expected profits of the banking system are

$$\mathbb{E}\left[G(\cdot, i_*, r^c)\right] = \mathbb{E}\left[P(\cdot, i_*, r^c)\right] - S(1 + \bar{r}^d) \tag{8}$$

$$= \int_{\underline{q}}^{\bar{q}} P(q, i_*, r^c)\, h(q)dq - S(1 + \bar{r}^d).$$

Formally, a competitive equilibrium for a simple banking system is defined as follows.

Definition 1

Let $\bar{r}^d \leq r_E$ be given. A competitive equilibrium (with positive investment in the alternative project) of a simple banking system which operates under unlimited liability is a triplet (i_*, d_*, r_*^c) such that

$$\mathbb{E}\left[G(\cdot, i_*, r_*^c)\right] = d_*(1 + r_E), \tag{9}$$

$$\Pi(i_*, r_*^c) = W(1 + r_E), \tag{10}$$

$$\left[\eta - i_*\right]I = S + d_*, \tag{11}$$

$$i_* W > d_*. \tag{12}$$

Equation (9) is the equilibrium condition in the equity market.[6] If the return were lower than r_E, no equity would be supplied and hence no bank could operate. If the expected return were higher, more equity would be supplied by entrepreneurs with $i \leq i_*$ and hence this cannot be an equilibrium either. The equilibrium condition (9) forces a spread $r_*^c - \bar{r}^d$ that accounts for the risk of losses on loans and possible differences in capital costs $\bar{r}^d$ (deposits) and r_E (equity). Note that for $r_E = \bar{r}^d$, the spread $r_*^c - \bar{r}^d$ is the risk premium banks must obtain in order to generate the return on equity $1 + r_E$. Equation (10) is the indifference condition for the critical quality level i_*, which determines the demand for loans. Equation (11) is the equilibrium condition for savings and investments at banks already given in (5), showing that the critical entrepreneur i_* is independent of the loan interest rate. The last condition (12) ensures that there are enough entrepreneurs who invest into equity so that the banking system has enough funds to finance production projects. Throughout this paper, we focus on equilibria with positive investment in the alternative project as economically these are the more plausible ones.

3.2 Existence of competitive equilibria

Since savings S are independent of deposit rates, the existence and uniqueness of a competitive equilibrium is straightforward to establish.

Proposition 1

Let $r_E \geq 0$ be given and suppose that the following conditions are satisfied:

(i) $\Pi(\underline{i}, 0) > W(1 + r_E) > \Pi(\underline{i}, r_E)$.

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[6]Alternatively, this equation may be interpreted as a free-entry and free-exit condition for banks, cf. Gersbach & Wenzelburger (2006).

(ii) *The average repayments of the entrepreneur with the highest quality level η satisfies*

$$\mathcal{R}(\eta, r_\eta^c) := \int_{\underline{q}}^{\overline{q}} \min\left\{q(1+\eta)f, I(1+r_\eta^c)\right\}h(q)dq > I(1+r_E),$$

where $r_\eta^c \geq 0$ is given by $\Pi(\eta, r_\eta^c) = W(1+r_E)$.

Then for $\overline{r}^d \leq r_E$ sufficiently close to r_E, a simple banking system admits a unique competitive equilibrium (i_*, d_*, r_*^c), where $i_* = i_*(r_E)$, $r_*^c = r_*^c(r_E)$, and $d_* = d_*(r_E)$.

Corollary 1

The loan interest rate satisfies $r_*^c = r_*^c(r_E) > r_E$ for $r_E = \overline{r}^d$.

The proofs of Proposition 1 and Corollary 1 are given in Appendix A. Condition (i) in Proposition 1 requires that entrepreneur $\underline{i}$ is willing to invest in her production project for zero loan interest rates while she will invest into equity or into the alternative project if the loan interest rate is higher than r_E. The second condition (ii) states that the average return of the highest quality entrepreneur η at the loan interest rate at which she is indifferent between investing in the two projects is higher than the return on I at the interest rate of the alternative project r_E. Hence it is attractive for banks to finance at least high-quality production projects.

3.3 Instability

We are now in a position to calculate the default probability of a simple banking system. Writing

$$P_*(q, r_E) := P\big(q, i_*(r_E), r_*^c(r_E)\big)$$

for repayments in a competitive equilibrium, average repayments of entrepreneurs are

$$\mathbb{E}\big[P_*(\cdot, r_E)\big] = d_*(1 + r_E) + S(1 + \overline{r}^d) \tag{13}$$

and hence are positive. To obtain further insight into the nature of equilibrium interest rates, consider aggregate losses of the banking system in equilibrium. Using (6), these are formally defined as

$$
\begin{aligned}
L(q, r_E) \quad &:= \quad [\eta - i_*(r_E)]I\big[1 + r_*^c(r_E)\big] - P_*(q, r_E) \tag{14} \\
&= \quad \int_{i_*(r_E)}^{\eta} \max\left\{I\big[1 + r_*^c(r_E)\big] - q(1+i)f, 0\right\}di.
\end{aligned}
$$

Expected aggregate losses in equilibrium are

$$
\begin{aligned}
\mathcal{L}(r_E) &:= \mathbb{E}\big[L(\cdot, r_E)\big] \\
&= [\eta - i_*(r_E)]I\big[1 + r_*^c(r_E)\big] - \mathbb{E}\big[P_*(\cdot, r_E)\big].
\end{aligned}
\tag{15}
$$

Inserting (13), (14), and (15) into (7), future bank capital of a simple banking system in the second period is

$$
\begin{aligned}
G_*(q, r_E) &:= G\big(q, i_*(r_E), r_*^c(r_E)\big) \\
&= P_*(q, r_E) - \mathbb{E}\big[P_*(\cdot, r_E)\big] + d_*(1 + r_E) \\
&= \mathcal{L}(r_E) - L(q, r_E) + d_*(1 + r_E).
\end{aligned}
\tag{16}
$$

An entrepreneur with quality level i goes bankrupt if she is unable to fully pay back her credit, that is, if

$$
I\big(1 + r_*^c(r_E)\big) > q(1 + i)f.
$$

The entrepreneur with the lowest quality level who is not bankrupt after encountering the shock q is given by

$$
i_B = i_B(q, r_E) :=
\begin{cases}
\dfrac{I\big(1 + r_*^c(r_E)\big)}{qf} - 1 & \text{if } q_{\text{TB}}(r_E) \le q < q_{\text{NB}}(r_E), \\
i_*(r_E) & \text{if } q \ge q_{\text{NB}}(r_E),
\end{cases}
\tag{17}
$$

where

$$
q_{\text{NB}}(r_E) := \frac{I\big[1 + r_*^c(r_E)\big]}{\big[1 + i_*(r_E)\big]f} \quad \text{and} \quad q_{\text{TB}}(r_E) := \frac{I\big(1 + r_*^c(r_E)\big)}{(1 + \eta)f}.
\tag{18}
$$

If shocks are sufficiently positive $q \ge q_{\text{NB}}(r_E)$, then no firm goes bankrupt and aggregate losses of banks are zero. For shocks $q_{\text{TB}}(r_E) \le q < q_{\text{NB}}(r_E)$, all investing entrepreneurs with quality levels $i_* < i < i_B(q, r_E)$ enter bankruptcy, whereas entrepreneurs with quality levels $i \ge i_B(q, r_E)$ pay back their loans fully. On the other hand, all entrepreneurs will enter bankruptcy if $q < q_{\text{TB}}(r_E)$ and losses are maximal.

It follows directly from (16) that the future bank capital is on average positive, so that a simple banking system will not default on average. The probability of a system-wide default by banks can now be calculated as follows. An individual bank is bankrupt if second period equity is negative. Due to the assumed symmetry of banks, this is equivalent to the condition $G_*(q, r_E) < 0$ stating that the whole banking system is bankrupt. Using (16), this condition takes the form

$$
d_*(1 + r_E) < L(q, r_E) - \mathcal{L}(r_E).
\tag{19}
$$

By (17) a necessary condition for the default of a bank is $q < q_{\text{NB}}(r_E)$. The default probability for banks can now be determined as follows.

Proposition 2

Under the hypotheses of Proposition 1, assume that

$$d_*(1 + r_E) < L(\underline{q}, r_E) - \mathcal{L}(r_E),$$

so that banks may default. Then there exists a unique critical level $\underline{q} < q_{\text{crit}} < q_{\text{NB}}(r_E)$ for macroeconomic shocks, such that the banking system defaults if and only if $q < q_{\text{crit}}$. The default probability is

$$\Pi_{\text{default}} := Prob\Big(d_*(1 + r_E) < L(q, r_E) - \mathcal{L}(r_E)\Big) = \int_{\underline{q}}^{q_{\text{crit}}} h(q)dq. \qquad (20)$$

Proposition 2, the proof of which is given in Appendix A, shows that banks default with positive probability as soon as the buffer $d_*(1 + r_E)$ is too small to insure against negative macroeconomic shocks. If the macroeconomic shocks are uniformly distributed, the default probability takes the following explicit form.

Corollary 2

If the shocks are uniformly distributed, the default probability is

$$\Pi_{\text{default}} = \frac{q_{\text{crit}} - \underline{q}}{\overline{q} - \underline{q}}.$$

Observe that q_{crit} depends on r_E. The preceding results allows us to characterize the default probability of the banking system in terms of the underlying exogenous parameters and distributions. The equation (20) is a value-at-risk formula for the banking system and for an individual bank. Suppose that Π_{default} is predetermined by banking regulation. Equation (20) determines then the required level for bank capital d_* such that the default risk is equal to Π_{default}.

In the next section we carry out the same exercise for a sophisticated banking system.

4   Competitive Equilibria for Sophisticated Banks

4.1   Equilibrium concept

We turn to the other polar case in which banks are sophisticated in their rating abilities so that they are able to detect the quality level i of an individual entrepreneur. They can thus determine the firm-specific default probability. The key idea of the equilibrium concept for sophisticated banks is to require that banks charge a *fair risk premium* for

each loan in the sense that the average return on each loan is equal to the risk-free return on equity. Let

$$\mathcal{R}(i, r_i^c) = \int_{\underline{q}}^{\overline{q}} \min \left\{ q(1+i)f, \ I(1+r_i^c) \right\} h(q)dq$$

denote the expected repayment from an entrepreneur with quality level i who has received a loan size I at the interest rate r_i^c. In requiring banks to earn the same return on each investing entrepreneur, an individualized interest rate r_i^c for entrepreneur i has to be such that

$$\mathcal{R}\big(i, r_i^c\big) \overset{!}{=} \mathcal{R}\big(i^o, r_{i^o}^c\big)$$

for all investing entrepreneurs $i \in [i^o, \eta]$. Let d_i and S_i denote the amount of equity and deposits used to finance the loan of quality I. We assume that the debt/equity ratio is the same across loans[7]. Hence, we have

$$d_i = \frac{d}{\eta - i^o} \quad \text{and} \quad S_i = \frac{S}{\eta - i^o}.$$

As all banks must pay a fixed deposit interest rate $\overline{r}^d$ on deposits S_i, the individual return of an investing entrepreneur i must be at least

$$\mathcal{R}\big(i, r_i^c\big) \geq \frac{1}{[\eta - i^o]} \big[d(1 + r_E) + S(1 + \overline{r}^d)\big]$$

so that entrepreneurs with low-quality projects are willing to supply equity.

Intuitively, a competitive equilibrium for a sophisticated banking system is a list consisting of a critical entrepreneur, an equity level, and loan interest rates

$$\left\{ i_*^o, d_*^o, \left\{ r_*^{co}(i) \right\}_{i=i_*^o}^{\eta} \right\}$$

such that

  (i) the equity market clears,

 (ii) firms take optimal investment decisions,

(iii) the market for loans is balanced.

More formally, a competitive equilibrium with financial intermediation for a sophisticated banking system is defined as follows:

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[7]This corresponds to the capital requirements in the first Basel Accord.

Definition 2

Let $\overline{r}^d \leq r_E$ be given. A sophisticated (competitive) equilibrium (with positive investment in the alternative project) of a sophisticated banking system is a list

$$\left\{ i_*^o, d_*^o, \left\{ r_*^{co}(i) \right\}_{i=i_*^o}^{\eta} \right\}$$

consisting of a critical entrepreneur i_*, an equity level d_*^o and loan interest rates $r_*^{co}(i)$, $i \in [i_*^o, \eta]$, such that

$$[\eta - i_*^o]\, \mathcal{R}\big(i, r_*^{co}(i)\big) \;=\; d_*^o(1 + r_E) \;+\; S(1 + \overline{r}^d), \quad i \in [i_*^o, \eta], \tag{21}$$

$$\Pi\big(i_*^o,\, r_*^{co}(i_*^o)\big) \;=\; W(1 + r_E), \tag{22}$$

$$\big[\eta - i_*^o\big] I \;=\; S + d_*^o, \tag{23}$$

$$i_*^o W \;>\; d_*^o. \tag{24}$$

Condition (21) states that on average banks must earn the same return on each loan. In particular, on average banks must earn $1 + r_E$ on equity on each individual loan. In the next section we will show in more detail that (21) is equivalent to market clearing in the equity market. Condition (22) is the indifference condition for entrepreneurs, recalling that $\Pi(i, r^c)$ is increasing in quality levels i. As before, (23) is the equilibrium condition for savings and investments at banks determining the critical entrepreneur i_*^o, whereas Condition (24) guarantees that the required equity for banks is available.

4.2 Existence of sophisticated equilibria

The existence of sophisticated equilibria can be established as follows. Recall for this purpose that according to (4) at least $\underline{i}$ entrepreneurs are required to provide banks with equity.

Proposition 3

Let $r_E \geq 0$ be arbitrary and suppose that the following holds:

(i) $\Pi(\underline{i}, 0) > W(1 + r_E) > \Pi(\underline{i}, r_E)$,

(ii) $\mathcal{R}(\underline{i}, r_o^c) > \mathcal{R}(\eta, 0)$ for r_0^c which satisfies $\Pi(\underline{i}, r_o^c) = W(1 + r_E)$.

Then for $\overline{r}^d \leq r_E$ sufficiently close to r_E, a sophisticated banking system admits a unique sophisticated equilibrium

$$\left\{ i_*^o, d_*^o, \left\{ r_*^{co}(i) \right\}_{i=i_*^o}^{\eta} \right\},$$

where $i_*^o = i_*^o(r_E)$, $r_*^{co}(i) = r_*^{co}(i, r_E)$ and $d_*^o = d_*^o(r_E)$.

Corollary 3

Loan interest rates $r_*^{co}(i, r_E)$, $i \in [i_*^o, \eta]$ are non-increasing in quality levels. If there exists an entrepreneur $i_{\mathrm{NB}} \in [i_*^o, \eta]$ such that

$$\underline{q}(1 + i_{\mathrm{NB}})f \geq I\big[1 + r_*^{co}(i_{\mathrm{NB}}, r_E)\big],$$

so that all entrepreneurs $i \geq i_{NB}$ meet their obligations, then loan interest rates are given by

$$1 + r_*^{co}(i, r_E) = \frac{\mathcal{R}\big(i_*^o, r_*^{co}(i_*^o, r_E)\big)}{I}, \quad \text{for all } i \in [i_{\mathrm{NB}}, \eta].$$

The proofs of Proposition 3 and Corollary 3 are given in Appendix A. Condition (i) in Proposition 3 requires that entrepreneur $\underline{i}$ is willing to invest in her production project for zero loan interest rates but will either invest into equity or the alternative project for loan interest rates above r_E. Condition (ii) guarantees that the banking system is capable of tailoring loan interest rates according to the quality of the production project so that risk premia are fair and entrepreneurs are still willing to invest.

Corollary 3 shows that a sophisticated banking system provides a floor $\frac{\mathcal{R}\big(i_*^o, r_*^{co}(i_*^o, r_E)\big)}{I}$ for the loan-interest rates. All entrepreneurs who meet their obligations with certainty will pay the same interest rate which is given by the floor. All other entrepreneurs pay a higher loan-interest rate. For these entrepreneurs the loan-interest rate is monotonically decreasing with the quality of their production projects.

We show in detail that the equilibrium condition (21) is equivalent to market clearing in the equity market. According to the definition of a sophisticated equilibrium, it follows from (21) that equilibrium loan interest rates must satisfy

$$\mathcal{R}\big(i, r_*^{co}(i, r_E)\big) = \mathcal{R}\big(i_*^o, r_*^{co}(i_*^o, r_E)\big), \quad i \in [i_*^o, \eta]. \tag{25}$$

The repayments to banks in a sophisticated banking system are

$$P_*^o(q, r_E) = \int_{i_*^o(r_E)}^{\eta} \min\Big\{ q(1 + i)f, \, I\big[1 + r_*^{co}(i, r_E)\big] \Big\} di. \tag{26}$$

Taking expectations and using (25), the expected repayments in a sophisticated equilibrium are

$$
\begin{aligned}
\mathbb{E}\big[P_*^o(\cdot, r_E)\big] &= \big[\eta - i_*^o\big]\, \mathcal{R}\big(i_*^o, r_*^{co}(i_*^o, r_E)\big) \\
&= d_*^o(1 + r_E) + S[1 + \overline{r}^d]. \tag{27}
\end{aligned}
$$

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In view of (21), the future bank capital of the sophisticated system in equilibrium is

$$G_*^o(q, r_E) \; := \; P_*^o(q, r_E) \; - \; S[1 + \overline{r}^d]$$

$$= \; P_*^o(q, r_E) - \mathbb{E}\big[P_*^o(\cdot, r_E)\big] \; + \; d_*^o(1 + r_E). \tag{28}$$

Thus

$$\mathbb{E}\left[G_*^o(\cdot, r_E)\right] = \; d_*^o(1 + r_E)$$

which is the equilibrium condition in the equity market.

4.3 Instability

Similar to the case of simple banking, we can derive the default probability of an individual bank, which is equal to the probability of a system-wide collapse of the banking system. Aggregate losses of the sophisticated system are formally defined by

$$L^o(q, r_E) = \int_{i_*^o(r_E)}^{\eta} I\big[1 + r_*^{co}(i, r_E)\big] di \; - \; P_*^o(q, r_E). \tag{29}$$

Using (25), expected aggregate losses are

$$\mathcal{L}^o(r_E) := \mathbb{E}\big[L^o(\cdot, r_E)\big] = \int_{i_*^o(r_E)}^{\eta} I\big[1 + r_*^{co}(i, r_E)\big] di \; - \; \mathbb{E}\big[P_*^o(\cdot, r_E)\big]. \tag{30}$$

Inserting (29) and (30) into (28) yields

$$G_*^o(q, r_E) = \mathcal{L}^o(r_E) - L^o(q, r_E) + d_*^o(1 + r_E). \tag{31}$$

The default condition for an individual bank and for the banking system is $G_*^o(q, r_E) < 0$ which, using (31), takes the form

$$d_*^o(1 + r_E) < L^o(q, r_E) - \mathcal{L}^o(r_E). \tag{32}$$

It can readily be seen from (31) that the future equity of the banking system is positive for sufficiently high shocks q.

It follows from (29) that $L^o(q, r_E)$ is decreasing in q. Therefore, the following proposition is proven analogously to Proposition 2.

Proposition 4
Under the hypotheses of Proposition 3, assume that

$$d_*^o(1 + r_E) < L^o(\underline{q}, r_E) - \mathcal{L}^o(r_E),$$

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so that banks may default. Then there exists a unique critical level $\underline{q} < q^o_{\mathrm{crit}} \leq q_{\mathrm{NB}}(r_E)$ for macroeconomic shocks, such that a sophisticated banking system defaults if, and only if, $q < q^o_{\mathrm{crit}}$. The default probability is

$$\Pi^o_{\mathrm{default}} := Prob\Big(d^o_*(1 + r_E) < L^o(q, r_E) - \mathcal{L}^o(r_E)\Big) = \int_{\underline{q}}^{q^o_{\mathrm{crit}}} h(q)dq. \qquad (33)$$

Corollary 4

If, in addition, the shocks are uniformly distributed, the default probability is then

$$\Pi^o_{\mathrm{default}} = \frac{q^o_{\mathrm{crit}} - \underline{q}}{\overline{q} - \underline{q}}.$$

Similar to the case of a simple banking system, (33) is a value-at-risk formula for a sophisticated banking system. Proposition 4 states that banks default with positive probability as soon as the buffer $d^o_*(1 + r_E)$ is too small to insure against negative macroeconomic shocks. If losses exceed average losses, this case will occur for a banking system whose capital base is too small.

5 Comparison of the two Systems

For a comparison of the two banking systems, let us first consider the special case in which no firm bankruptcies occur in the simple banking system. This will occur if

$$\underline{q}(1 + i_*)f \geq I\big[1 + r^c_*(r_E)\big],$$

where $i_* = i_*(r_E)$. The expected repayment of banks from entrepreneurs are then

$$\mathcal{R}\big(i, r^c_*(r_E)\big) = I\big[1 + r^c_*(r_E)\big] \quad \text{for all } i \in [i_*, \eta]$$

and by virtue of Proposition 3 we have

$$i^o_* = i_* \quad \text{and} \quad r^{co}_*(i, r_E) = r^c_*(r_E) \quad \text{for all } i \in [i_*, \eta],$$

implying that the simple and the sophisticated banking system charge the same loan-interest rates and finance the same number of entrepreneurs.

This situation changes as soon as firm bankruptcies are possible. In the next theorem we compare the loan interest rate in the simple banking system with the schedule of loan interest rates in a sophisticated banking system.

Theorem 1

Let the hypotheses of Propositions 1 and 3 be satisfied and assume that firm bankruptcies occur with positive probability. Then there exists $i_{\mathrm{ER}} \in [i_*^o, \eta)$ with

$$
\begin{aligned}
(i) & \quad r_*^c(r_E) < r_*^{co}(i, r_E), \quad i \in [i_*^o, i_{\mathrm{ER}}), \\
(ii) & \quad r_*^c(r_E) = r_*^{co}(i_{\mathrm{ER}}, r_E), \\
(iii) & \quad r_*^c(r_E) > r_*^{co}(i, r_E), \quad i \in (i_{\mathrm{ER}}, \eta].
\end{aligned}
$$

The proof of Theorem 1 are given in the appendix. Theorem 1 shows that loan interest rates for high-quality borrowers fall below the loan interest rate that is obtained in a simple banking system. This is a result of competition which enforces the same return on equity in both systems. Sophisticated banks change higher interest rates to intermediate quality borrowers in order to compensate for higher default risk and reward high-quality borrowers by low loan interest rates. An immediate consequence of Theorem 1 is that sophisticated banks expect higher repayments from low-quality entrepreneurs than simple banks. In this sense sophisticated banks eliminate cross-subsidization between their borrowers.

The ability of the two systems to attract equity is compared next.

Theorem 2

Let hypotheses of Propositions 1 and 3 be satisfied and assume that bankruptcies occur with positive probability. Then the following properties hold:

$$
\begin{aligned}
(i) & \quad i_*(r_E) < i_*^o(r_E), \qquad (ii)\ d_*(r_E) > d_*^o(r_E), \\
(iii) & \quad \left[i_*(r_E)W - d_*(r_E) \right] < \left[i_*^o(r_E)W - d_*^o(r_E) \right].
\end{aligned}
$$

The proof of Theorem 2 is given in the appendix. The intuition for Theorem 2 may be described as follows. In order to adjust loan interest rates to the quality level of entrepreneurs and to generate equity returns of $1 + r_E$ at the same time, sophisticated banks charge lower quality entrepreneurs higher loan interest rates, making it less attractive for them to invest into their production projects. Hence, simple banks invite more entrepreneurs to invest into their production projects than sophisticated banks, i.e., $i_* < i_*^o$. Market clearing in the loan and equity market enables a simple banking system to attract the necessary equity so that more production projects are financed, i.e., $\eta - i_*^o < \eta - i_*$. As a consequence, in a sophisticated banking system more resources are invested in the alternative project, while at the same time high-quality entrepreneurs pay lower loan interest rates.

We next address the question of which of the two banking systems will accumulate more second-period equity. From the market clearing conditions

$$\mathbb{E}\big[G_*(\cdot, r_E)\big] = d_*(1 + r_E) \quad \text{and} \quad \mathbb{E}\big[G_*^o(\cdot, r_E)\big] = d_*^o(1 + r_E),$$

we infer from Theorem 2 that expected future equity of the simple banking system is higher than in the sophisticated system. Using (13) and (27), we obtain the following result.

Proposition 5

Under the hypotheses of Propositions 1 and 3, expected repayment of the simple banking system is higher than expected repayment of the sophisticated system, that is,

$$\mathbb{E}\big[P_*(\cdot, r_E)\big] - \mathbb{E}\big[P_*^o(\cdot, r_E)\big] = [d_* - d_*^o](1 + r_E) > 0. \tag{34}$$

To further illustrate the consequences of Theorem 2, consider a worst-case scenario in which in both systems *all* entrepreneurs enter bankruptcy, such that banks in both systems will receive only the respective liquidation values. For such a shock denoted by q_{low}, we have

$$P_*(q_{\text{low}}, r_E) = q_{\text{low}} f[\eta - i_*]\Big[1 + \frac{\eta + i_*}{2}\Big] = q_{\text{low}} f\Big[[\eta - i_*] + \tfrac{1}{2}[\eta^2 - i_*^2]\Big]$$

for the simple banking system and

$$P_*^o(q_{\text{low}}, r_E) = q_{\text{low}} f[\eta - i_*^o]\Big[1 + \frac{\eta + i_*^o}{2}\Big] = q_{\text{low}} f\Big[[\eta - i_*^o] + \tfrac{1}{2}[\eta^2 - i_*^{o2}]\Big]$$

for the sophisticated banking system. Since by Theorem 2 (i), $i_* < i_*^o$, we have

$$P_*(q_{\text{low}}, r_E) > P_*^o(q_{\text{low}}, r_E),$$

so that in a worst-case scenario the simple banking system receives higher repayments than the sophisticated banking system. This argument carries over to the case with adverse shocks in which not necessarily all entrepreneurs enter bankruptcy. We have

$$P_*(q, r_E) - P_*^o(q, r_E) = \int_{i_*(r_E)}^{i_*^o(r_E)} \min\big\{q(1 + i)f, I\big[1 + r_*^c(r_E)\big]\big\} di \tag{35}$$

$$+ \int_{i_*^o(r_E)}^{\eta} g(i, q) di,$$

where

$$g(i, q) := \min\big\{q(1 + i)f, I\big[1 + r_*^c(r_E)\big]\big\} - \min\big\{q(1 + i)f, I\big[1 + r_*^{co}(i, r_E)\big]\big\}.$$

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The first term on the r.h.s. of (35) is a positive volume effect and reflects the fact that the simple system finances more production projects. Using Theorem 1, there exists a quality level i_{ER} and a critical shock $q_{\text{ER}} > \underline{q}$ such that

$$q_{\text{ER}}(1 + i_{\text{ER}})f = I\big[1 + r_*^c(r_E)\big] = I\big[1 + r_*^{co}(i_{\text{ER}}, r_E)\big].$$

For the shock q_{ER} entrepreneurs with quality levels $i \geq i_{\text{ER}}$ fully meet their repayment obligations in both systems, implying that $g(i, q_{\text{ER}}) \geq 0$ for $i \geq i_{\text{ER}}$. On the other hand, all entrepreneurs $i < i_{\text{ER}}$ default at the shock q_{ER}, so that $g(i, q_{\text{ER}}) = 0$ for $i < i_{\text{ER}}$. This implies that for all shocks $q \leq q_{\text{ER}}$,

$$G_*(q, r_E) - G_*^o(q, r_E) = P_*(q, r_E) - P_*^o(q, r_E) > 0.$$

Hence the simple banking system outperforms the sophisticated system for adverse shocks. This result is illustrated in Figure 1. It is summarized in the following proposition.

Proposition 6

Let the hypotheses of Propositions 1 and 3 be satisfied. Then there exists a critical shock $q_{\text{BE}} = q_{\text{BE}}(r_E) \geq q_{\text{ER}}$, so that the simple banking system outperforms the sophisticated system for all shocks $q < q_{\text{BE}}$. More precisely,

$$G_*(q, r_E) > G_*^o(q, r_E) \quad \text{for all} \quad q < q_{\text{BE}}.$$

We conclude this section by comparing the default probabilities of the two banking systems. Observe that the critical value q_{BE} given in Proposition 6 depends significantly on the quality level i_{ER} which in turn is determined by the distribution of the macroeconomic shocks. The lower i_{ER} is, that is, the more entrepreneurs pay lower loan interest rates in the sophisticated system, the higher q_{BE} is. A priori, it cannot be ruled out that $q_{\text{BE}} \geq \overline{q}$. In this case the simple banking system outperforms the sophisticated system for all shocks and its default probability Π_{default} is lower than the default probability of the sophisticated system Π_{default}^o.

Matters are different if $q_{\text{BE}} < \overline{q}$ and repayments in the sophisticated system are higher for sufficiently high shocks, i.e., $P_*^o(q, r_E) > P_*(q, r_E)$ for $q > q_{\text{BE}}$. A sufficient condition for the case where no firm bankruptcies occur in both systems is

$$\int_{i_*^o}^{\eta} I\big[1 + r_*^{co}(i, r_E)\big]\, \mathrm{d}i > \big[\eta - i_*\big] I\big[1 + r_*^c(r_E)\big].$$

Our last theorem states conditions under which the default probability of a simple banking system is lower than the default probability of the sophisticated system.
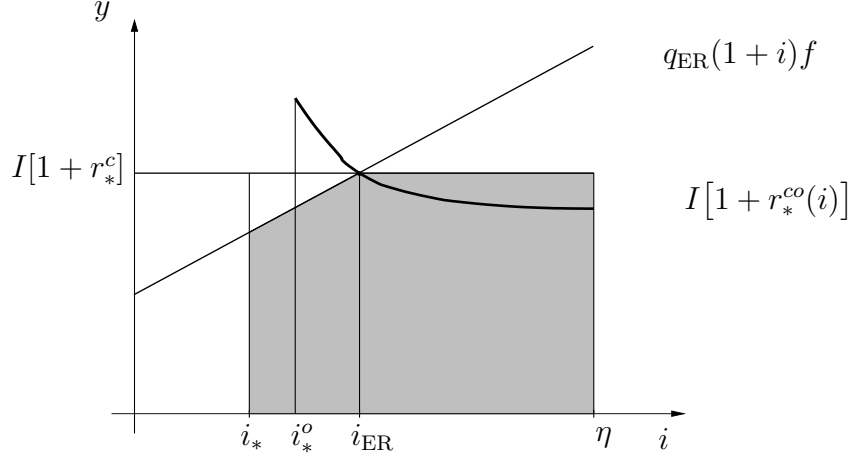
Figure 1: Repayments of entrepreneurs.

Theorem 3

Let the hypotheses of Propositions 1 and 3 be satisfied and assume that the probability density h of the macroeconomic shocks is strictly positive. Let $q_{BE} = q_{BE}(r_E)$ be given by Proposition 6. Suppose that the productivity of entrepreneurs is high enough, i.e.,

$$P_*^o(q_{BE}, r_E) > S(1 + \overline{r}^d)$$

so that the sophisticated system will not default in response to shocks $q \geq q_{BE}$. Then the default probability of the simple banking system is lower than the default probability of the sophisticated banking system, i.e.,

$$\Pi_{\text{default}} < \Pi_{\text{default}}^o.$$

6 Conclusions

We have shown that sophistication in risk management benefits production projects of high-quality entrepreneurs by lowering loan rates at the expense of production projects of lower quality entrepreneurs. Sophisticated banks reduce the credit access of entrepreneurs with intermediate quality levels and attract less equity than simple banks. As a consequence, expected repayments of the simple system are always higher and its default risk is lower for sufficiently low deposit rates. These results suggests that more sophistication in the assessment of individual entrepreneurs' default risk may decrease banking stability if entrepreneurs' productivity is sufficiently high. This may be a serious concern for the impact of banking regulation.

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A Appendix

Proof of Proposition 1.

Step 1. Set

$$\overline{r}^c := \frac{\overline{q}(1+\eta)f}{I} - 1,$$

with $\overline{q}$ denoting the highest possible shock. Then profits of entrepreneur i are on average zero, i.e., $\Pi(i, r^c) = 0$ for $r^c \geq \overline{r}^c$ with Π given in (1). Condition (10) takes the form

$$\Pi(i, r^c) - W(1 + r_E) \stackrel{!}{=} 0, \quad i \in [\underline{i}, \eta]. \tag{36}$$

Since for all $i \in [\underline{i}, \eta]$ and each $r_E \geq \overline{r}^d$ sufficiently close to $\overline{r}^d$,

$$\Pi(i, 0) \geq \Pi(\underline{i}, 0) > W(1 + r_E) > \Pi(i, \overline{r}^c) = 0,$$

for each $i \in [\underline{i}, \eta]$, equation (36) has a solution $r^c = \varphi(i)$ such that

$$\Pi(i, \varphi(i)) - W(1 + r_E) = 0 \quad \text{for all } i \in [\underline{i}, \eta]. \tag{37}$$

Since $\Pi(i, r^c)$ is strictly increasing in i, this solution is uniquely determined. Since $\Pi(i, r^c)$ is decreasing in $r^c \in [0, \overline{r}^c]$, φ is increasing in i.

Step 2. Consider the function $F : [\underline{i}, \eta] \to \mathbb{R}$, defined by

$$F(i) := \frac{\mathbb{E}[P(\cdot, i, \varphi(i))]}{[\eta - i]I} + \frac{S(r_E - \overline{r}^d)}{[\eta - i]I} - (1 + r_E). \tag{38}$$

Using (8) and (11), condition (9) takes the form

$$F(i_*) = \frac{\mathbb{E}[P(\cdot, i_*, r^c_*)]}{[\eta - i_*]I} + \frac{S(r_E - \overline{r}^d)}{[\eta - i_*]I} - (1 + r_E) \stackrel{!}{=} 0. \tag{39}$$

By Assumption (i), $W(1 + r_E) > \Pi(\underline{i}, r_E)$ so that $\varphi(\underline{i}) < r_E$. Hence,

$$\mathbb{E}[P(\cdot, \underline{i}, \varphi(\underline{i}))] - [\eta - \underline{i}]I(1 + r_E) + S(r_E - \overline{r}^d) \tag{40}$$

$$\leq [\eta - \underline{i}]I[\varphi(\underline{i}) - r_E] + S(r_E - \overline{r}^d).$$

$$< 0$$

as long as r_E is sufficiently close to $\overline{r}^d$. This implies $F(\underline{i}) < 0$. On the other hand, it follows from

$$[\eta - i]\mathcal{R}(i, r^c) \leq \mathbb{E}[P(\cdot, i, r^c)], \quad i \in [\underline{i}, \eta)$$

that $F(i) > 0$ for sufficiently large i because

$$F(i) \geq \frac{1}{I}\mathcal{R}\big(i, \varphi(i)\big) + \frac{S(r_E - \overline{r}^d)}{[\eta - i]I} - (1 + r_E) > 0, \tag{41}$$

noticing that $r_\eta^c = \varphi(\eta)$ as defined in Assumption (ii). We infer from Step 1, that $F(i)$ is increasing in i. It then follows from (40) and (41) that for each r_E sufficiently close to $\overline{r}^d$ there exists a unique $i_* = i_*(r_E)$ so that $F(i_*) = 0$. Clearly, $r_*^c = \varphi(i_*)$ and since by construction $i_* > \underline{i}$, Condition (11) is satisfied. This completes the proof.

∎

Proof of Corollary 1.
It follows from (39) that

$$0 = F(i_*) < 1 + r_*^c(r_E) + \frac{S(r_E - \overline{r}^d)}{[\eta - i_*]I} - (1 + r_E).$$

This proves the corollary.

∎

Proof of Proposition 2.
By assumption, we have $G_*(\underline{q}, r_E) < 0$. Since $G_*(q, r_E) > 0$ for $q \geq q_{\mathrm{NB}}(r_E)$ and the function G_* is strictly increasing in q, there exists a unique critical shock $\underline{q} < q_{\mathrm{crit}} < q_{\mathrm{NB}}(r_E)$ so that $G_*(q_{\mathrm{crit}}, r_E) = 0$.

∎

Proof of Proposition 3.
Step 1. Analogous to the first step in the proof of Proposition 1, we can establish the existence of a function $\varphi : [\underline{i}, \eta] \to \mathbb{R}$ with $r^c = \varphi(i)$ such that

$$\Pi\big(i, \varphi(i)\big) - W(1 + r_E) = 0 \quad \text{for all } i \in [\underline{i}, \eta],$$

provided that $r_E \geq \overline{r}^d$ is sufficiently close to $\overline{r}^d$. Since $\Pi(i, r^c)$ is strictly increasing in i, this solution is uniquely determined. Since $\Pi(i, r^c)$ is decreasing in $r^c \in [0, \overline{r}^c]$, φ is increasing in i.

Step 2. Consider the function $H : [\underline{i}, \eta] \to \mathbb{R}$, defined by

$$H(i) := \frac{1}{I}\mathcal{R}\big(i, \varphi(i)\big) + \frac{S(r_E - \overline{r}^d)}{[\eta - i]I} - (1 + r_E). \tag{42}$$

Inserting (23), Condition (21) for i_*^o then takes the form

$$H(i_*^o) = \frac{1}{I}\mathcal{R}\big(i_*^o, \varphi(i_*^o)\big) + \frac{S(r_E - \overline{r}^d)}{[\eta - i_*^o]I} - (1 + r_E) \overset{!}{=} 0.$$

Since

$$[\eta - i]\,\mathcal{R}(i, r^c) < \mathbb{E}\big[P\big(\cdot, i, r^c\big)\big], \quad i \in [\underline{i}, \eta),$$

we have $H(i) < F(i)$ for all $i \in [\underline{i}, \eta)$, where the function F has been defined in (38). Hence based on (40), $H(\underline{i}) < 0$, provided that r_E is sufficiently close to $\overline{r}^d$. On the other hand, clearly

$$\mathcal{R}\big(\eta, \varphi(\eta)\big) + S(r_E - \overline{r}^d) > 0, \tag{43}$$

such that $H(i) > 0$ for sufficiently large i. Since $\varphi(i)$ is increasing in i and $\mathcal{R}(i, r^c)$ is increasing in both arguments, $H(i)$ is increasing in i. It now follows from (43) that for each r_E sufficiently close to $\overline{r}^d$, there exists a unique $i_*^o = i_*(r_E)$ such that $H(i_*^o) = 0$. Since by construction $i_*^o > \underline{i}$, Condition (24) is satisfied.

Step 3. By assumption $\mathcal{R}\big(\underline{i}, \varphi(\underline{i})\big) > \mathcal{R}(\eta, 0)$ and $\varphi(\underline{i}) = r_o^c$. It follows from the monotonicity of $\mathcal{R}(i, r^c)$ and $\varphi(i)$ that

$$\mathcal{R}\big(i_*^o, \varphi(i_*^o)\big) > \mathcal{R}(\eta, 0).$$

Hence there exists a function $r_*^{co}(i, r_E)$ which satisfies

$$\mathcal{R}\big(i, r_*^{co}(i, r_E)\big) = \mathcal{R}\big(i_*^o, \varphi(i_*^o)\big), \quad i \in [i_*^o, \eta].$$

■

Proof of Corollary 3.

The proof follows directly from the implicit function theorem and the fact that $\mathcal{R}(i, r^c)$ is non-decreasing in i and r^c.

■

Proof of Theorem 1.

In the two proofs of Propositions 1 and 3 that $r_*^c = \varphi(i_*)$ and $r_*^{co}(i_*^o, r_E) = \varphi(i_*^o)$, respectively, where φ was defined in (37). Moreover, it was shown that φ is monotonically increasing. Since by Theorem 2 $i_*^o(r_E) > i_*(r_E)$, we have $r_*^{co}(i, r_E) > r_*^c(r_E)$ in a neighborhood of $i \geq i_*^o(r_E)$.

Assume, on the contrary, that

$$r_*^{co}(i, r_E) > r_*^c(r_E) \quad \text{for all} \quad i \in [i_*^o, \eta]. \tag{44}$$

It follows from (13) that average repayments of entrepreneurs in a competitive equilibrium are

$$\mathbb{E}\big[P_*(\cdot, r_E)\big] = \big[\eta - i_*(r_E)\big] I(1 + r_E) - S(r_E - \overline{r}^d) \tag{45}$$

and similar for the sophisticated system

$$\mathbb{E}\big[P_*^o(\cdot, r_E)\big] = \big[\eta - i_*^o(r_E)\big] I(1 + r_E) - S(r_E - \overline{r}^d). \tag{46}$$

Hence

$$\mathbb{E}\big[P_*(\cdot, r_E)\big] - \mathbb{E}\big[P_*^o(\cdot, r_E)\big] = \big[i_*^o(r_E) - i_*(r_E)\big] I(1 + r_E). \tag{47}$$

It follows from (45) that

$$\mathbb{E}\big[P_*(\cdot, r_E)\big] \le \big[\eta - i_*(r_E)\big] I(1 + r_E)$$

and because of the monotonicity of the integrand

$$\int_{i_*(r_E)}^{i_*^o(r_E)} \int_{\underline{q}}^{\overline{q}} \min\big\{q(1 + i)f, I\big[1 + r_*^c(r_E)\big]\big\} h(q) dq \, di \le \big[i_*^o(r_E) - i_*(r_E)\big] I(1 + r_E). \tag{48}$$

Setting

$$g(i, q) := \min\big\{q(1 + i)f, \ I\big[1 + r_*^c(r_E)\big]\big\} - \min\big\{q(1 + i)f, \ I\big[1 + r_*^{co}(i, r_E)\big]\big\},$$

we have

$$\mathbb{E}\big[P_*(\cdot, r_E)\big] - \mathbb{E}\big[P_*^o(\cdot, r_E)\big] \le \big[i_*^o(r_E) - i_*(r_E)\big] I(1 + r_E) \tag{49}$$
$$+ \int_{i_*(r_E)}^{i_*^o(r_E)} \int_{\underline{q}}^{\overline{q}} g(i, q) \, h(q) dq \, di.$$

Equations (47) and (49) imply that

$$\int_{i_*(r_E)}^{i_*^o(r_E)} \int_{\underline{q}}^{\overline{q}} g(i, q) \, h(q) dq \, di \ge 0. \tag{50}$$

The initial hypothesis (44) implies that

$$g(i, q) \le 0 \quad \text{for all} \quad i \in [i_*^o, \eta], \ q \in [\underline{q}, \overline{q}]$$

with the strict inequality holding for i sufficiently close to i_*^o. Then (50) implies that there exist i_0 and q_0 such that

$$g(i_0, q_0) > 0.$$

The assertion then follows from the continuity of g and the monotonicity of $r_*^{co}(i, r_E)$.

∎

Proof of Theorem 2.

It follows from the proofs of Propositions 1 and 3 that the critical entrepreneurs i_* and i_*^o are given by the conditions

$$F(i_*) = 0 \quad \text{and} \quad H(i_*^o) = 0,$$

where F and H were defined in (38) and (42), respectively. Again, since

$$[\eta - i]\, \mathcal{R}(i, r^c) \le \mathbb{E}\big[P\big(\cdot, i, r^c\big)\big], \quad i \in [\underline{i}, \eta),$$

we have $H(i) < F(i)$ for all $i \in [\underline{i}, \eta)$, such that $i_* < i_*^o$. This proves the first claim. The second claim follows directly from the balance equation, so that $d_*^o < d_*$. The third claim is an immediate consequence of the first two assertions.

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Proof of Theorem 3.

Since both systems face a positive default risk,

$$S(1 + \overline{r}^d) > P_*(\underline{q}, r_E) > P_*^o(\underline{q}, r_E).$$

On the other hand

$$P_*(q_{\mathrm{BE}}, r_E) > P_*^o(q_{\mathrm{BE}}, r_E) > S(1 + \overline{r}^d).$$

Since P_* and P_*^o are increasing in q, it follows from Propositions 5 and 6 that $q_{\mathrm{crit}} < q_{\mathrm{crit}}^o$, where the critical values q_{crit} and q_{crit}^o are given by

$$P_*(q_{\mathrm{crit}}, r_E) = P_*^o(q_{\mathrm{crit}}^o, r_E) = S(1 + \overline{r}^d).$$

Recalling the definitions of the default probabilities (20) and (33), this completes the proof.

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References

AL-NAJJAR, N. I. (1995): "Decomposition and Characterization of Risk with a Continuum of Random Variables", *Econometrica*, 63, 1195–1224.

ALÓS-FERRER, C. (1999): "Individual Randomness in Economic Models with a Continuum of Agents", Working paper no. 9807, University of Vienna.

BHATTACHARYA, S., A. BOOT & A. V. THAKOR (1998): "The Economics of Bank Regulation", *Journal of Money Credit and Banking*, 30(4), 745–770.

BHATTACHARYA, S. & A. V. THAKOR (1993): "Contemporary Banking Theory", *Journal of Financial Intermediation*, 3, 2–50.

CAREY, M. & R. STULZ (eds.) (2006): *The Risks of Financial Institutions*. University of Chicago Press, forthcoming.

DEWATRIPONT, M. & J. TIROLE (1994): *The Prudential Regulation of Banks*. MIT Press.

DIAMOND, D. (1984): "Financial Intermediation and Delegated Monitoring", *Review of Economic Studies*, 51, 393–414.

ELSINGER, X., X. LEHAR & M. SUMER (2005): "Using Market Information for Banking System Risk Assessment", Forthcoming *International Journal of Central Banking*.

FRANKE, G. & J. KRAHNEN (2006): "Default Risk Sharing Between Banks and Markets: The Contribution of Collateralized Debt Obligations", in *The Risks of Financial Institutions*, ed. by M. Carey & R. Stulz. University of Chicago Press, forthcoming.

FREIXAS, X. & J.-C. ROCHET (1997): *Microeconomics of Banking*. MIT Press, Cambridge, MA.

GEHRIG, T. & R. STENBACKA (2004): "Screening Cycles", Working paper, University of Freiburg.

GERSBACH, H. & H. UHLIG (2005): "Debt Contracts and Collapse as Competition Phenomena", Forthcoming *Journal of Financial Intermediation*.

GERSBACH, H. & J. WENZELBURGER (2004): "Do risk premia protect from banking crises?", Discussion Paper No. 519, Bielefeld University and CEPR Discussion Paper No. 4935.

———— (2006): "Sophistication in Risk Management and Banking Stability: The Short Term", Discussion Paper 550, Department of Economics, University of Bielefeld.

HELLWIG, M. F. (1994): "Banking and Finance at the End of the Twentieth Century", WWZ-Discussion paper no. 9426, University of Basel.

KRAHNEN, J. & M. WEBER (2001): "'Generally Accepted Rating Principles: A Primer", *Journal of Banking & Finance*, 25, 3–24.

UHLIG, H. (1996): "A Law of Large Numbers for Large Economies", *Economic Theory*, 8, 41–50.

VIVES, X. (2004): "Innovation and Competitive Pressure", CEPR Discussion Paper No. 4369.