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On Maskin monotonicity of solution based social choice rules*

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Abstract

Howard (1992) argues that the Nash bargaining solution is not Nash implementable, as it does not satisfy Maskin monotonicity. His arguments can be extended to other bargaining solutions as well. However, by defining a social choice correspondence that is based on the solution rather than on its realizations, one can overcome this shortcoming. We even show that such correspondences satisfy a stronger version of monotonicity that is even sufficient for Nash implementability.

JEL Classification: C71; C78; D61

Keywords: Maskin monotonicity, social choice rule, bargaining games, Nash program, mechanism, implementation

1 Introduction

Hurwicz (1994) in contrast to large parts of the literature stresses the fundamental difference between games and mechanisms (game forms).

The concept of a game form, that allows it to formally separate the rules of a game from players' individual evaluations of the outcomes, is a cardinal tool for applications of

*Dedicated to Leo Hurwicz whose conceptual rigor and clarity has become a treasured benchmark for the profession.

game theory. The possibility to choose the outcome space of a mechanism (or game form) according to the specific needs of the problem to be modelled makes implementation theory a powerful instrument. A key role in that theory is played by the property called Maskin monotonicity, that is a necessary property for a social choice rule to be implementable in Nash equilibrium (see Maskin (1999)).

While many specific applications almost naturally distinguish "the" suitable mechanism, thus outcome function, this is not the case when cooperative solutions are to be Nash implemented. The aim to relate cooperative solutions of coalitional games to Nash equilibria of non-cooperative games in strategic or extensive form goes back to Nash (1951, 1953) and is now commonly referred to as the "Nash program". The exact relation between the Nash program and implementation theory has been addressed explicitly in the literature only in the last decade. Serrano (1997) states: "The Nash program and the abstract theory of implementation are often regarded as unrelated research agendas", and Bergin and Duggan (1999) write: "... because the implementation-theoretic and traditional approaches both involve the construction of games and game forms whose equilibria have specific features, considerable confusion surrounds the relationship between them."

Several articles have recently tried to dispose of this confusion: Dagan and Serrano (1998), Serrano (1997, 2005a, b), Bergin and Duggan (1999), Trockel (2002a, 2003). At the heart of the problem lies the fact that a cooperative solution as a technical concept is distinct from a social choice rule. Consequently, Nash implementation of a cooperative solution is literally impossible, as it is not well defined. A crucial step in making solutions implementable is therefore the interpretation of a solution as a social choice rule. Formally, this means the suitable definition of a solution based social choice rule that carries the characteristic features of the underlying solution.

While social choice rules are mappings associating certain outcomes to profiles of preferences or utility functions, solutions associate feasible (monetary or utility) payoffs of players to certain coalitional games. A basic task is it therefore to understand the relation between utility profiles and coalitional games. In their seminal paper, Bergin and Duggan (1999) explain this problem by use of the notions of "effectivity" and "supportability".

Supportability associates with a coalitional game an underlying profile of utility functions supporting it. Effectivity associates with any utility profile a coalitional form to describe the potential strategic effects on coalitional worths. While here and likewise in Trockel (2002a, 2003) the relation between social choice rules and solutions is formally analyzed, it is ignored in large parts of the literature, a fact that contributes to the "confusion" mentioned above.

Nash in his non-cooperative foundation of the Nash bargaining solution left the supportability problem unsettled. Implementation in the sense of mechanism theory was not yet an issue for him.

Howard (1992) and Moulin (1984) provided early implementations in subgame perfect equilibria of the Nash and the Kalai-Smorodinsky solutions, respectively. They both ignored, or better avoided, the effectivity problem by introducing solutions directly as social choice functions defined on a space of utility profiles.

In order to implement bargaining solutions like those of Nash or Kalai-Smorodinsky one has to generate an outcome space and to define solution based social choice rules. This corresponds to solve in that context the supportability-effectivity problem.

There are obviously several possibilities to factorize a payoff vector function into an outcome function and a vector of utility functions. The two extreme cases are to take

a) the outcome space as identical to the strategy space, choosing the outcome function as the identity map and the utility functions as the payoff functions;

b) the outcome space to be the space of payoff vectors, choosing the utility functions as projections to payoffs and the outcome function as the payoff vector function.

For different choices of outcome space and preferences on the outcome space one clearly gets different solution based social choice rules. And Maskin monotonicity may very well depend on the actually selected solution based social choice rule.

Howard (1992) argues that, due to a lack of Maskin monotonicity, the Nash bargaining solution fails to be Nash implementable. That a suitably defined Nash bargaining social choice rule **is** in fact Nash implementable has been demonstrated by von Damme (1986), Naeve (1999), and Trockel (2000, 2002b).

In the next section we shall revisit the example by Howard and show that by choosing a different outcome function we can define a Nash social choice rule that is Maskin monotonic. We shall extend this discrete context to its convexification where our reasoning remains true. In section 3 we provide an alternative approach to Howard's example that allows it to avoid the violation of Maskin monotonicity. Section 4 briefly sketches that the situation with some other Pareto efficient solutions is similar. We particularly focus on the Kalai-Smorodinsky solution. Again, the examples are discrete and chosen in such a way that bargaining solutions are well defined and unique but allow for straightforward extensions to the convexified bargaining sets. The key property for this conclusion is some symmetry property of the considered bargaining solutions. The concluding section 5 considers essential monotonicity that for more than two players is sufficient for Nash implementability. For any Pareto efficient bargaining solution we establish essential monotonicity, hence Maskin monotonicity, of the induced solution based social choice rule in our setup.

2 Howard's Example

We consider a bargaining problem, in which two agents negotiate over the alternatives Q, S, V. If they do not come to an agreement the outcome is the status quo alternative Q.

The set of admissible utility profiles on $A := \{Q, S, V\}$ is $U := \{u, u'\} \equiv \{(u_1, u_2), (u_1', u_2')\}$ where $u_i, u_i',\ i = 1, 2$ are real valued (von Neumann-Morgenstern) utility functions that are defined on A as follows:

$$\begin{aligned} u(Q) &= u'(Q) = (q_1, q_2) \equiv (q_1', q_2') = (0, 0) \\ u(S) &= u'(S) = (s_1, s_2) \equiv (s_1', s_2') = (1, 1) \\ u(V) &= (v_1, v_2) = (0, 2) \qquad u'(V) = (v_1', v_2') = (3/4, 2) \end{aligned}$$

A bargaining solution in this framework is a mapping $\hat{\lambda} : \{u(A), u(B)\} \longrightarrow u(A) \cup u'(A)$ with $\hat{\lambda}(u(A)) \in u(A)$ and $\hat{\lambda}(u'(A)) \in u'(A)$. The Nash solution is the bargaining solution $\hat{\nu}$ that solves $\max_{\hat{\lambda}} \hat{\lambda}_1(u(A))\ \hat{\lambda}_2(u(A))$ and $\max_{\hat{\lambda}} \hat{\lambda}_1(u'(A))\ \hat{\lambda}_2(u'(A))$.

The Nash social choice rule in this model is given by the correspondence $\hat{\varphi}^{\hat{\nu}} : U \Longrightarrow A$ with $\hat{\varphi}^{\hat{\nu}}(w) := \operatorname{argmax}_{a \in A}\ w_1(a)\ w_2(a)$.

As depicted in Figure 1, for the profile u, we obtain $\hat{\varphi}^{\hat{\nu}}(u) = \{S\}$ because $u_1(S)\, u_2(S) = s_1\, s_2$ maximizes the Nash product $u_1(a)\, u_2(a)$ on A. For the profile u' we get $\hat{\varphi}^{\hat{\nu}}(u') = \{V\}$. Indeed, now $u_1'(V)\, u_2'(V) = v_1'\, v_2' = 3/2 > 1 = u_1'(S)\, u_2'(S)$.

Hence, the switch from profile u to profile u' results in a different social optimum in A. In particular, outcome S drops out of the Nash correspondence. However, we see no preference reversal involving S that is induced by that switch: S remains the best outcome for player 1 and the second ranked outcome for player 2. Therefore, Maskin monotonicity **is violated**.

The arguments do not change when we replace A by the mixture set generated by A. For instance, let Q, S, V be defined as $(1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$, respectively, and let $\Delta A := \textit{convex hull}(A)$. As any point in ΔA is a convex combination of (i.e., a probability distribution over) Q, S, V, its utility is simply the expected value of u_i or u_i' $(i = 1, 2)$,

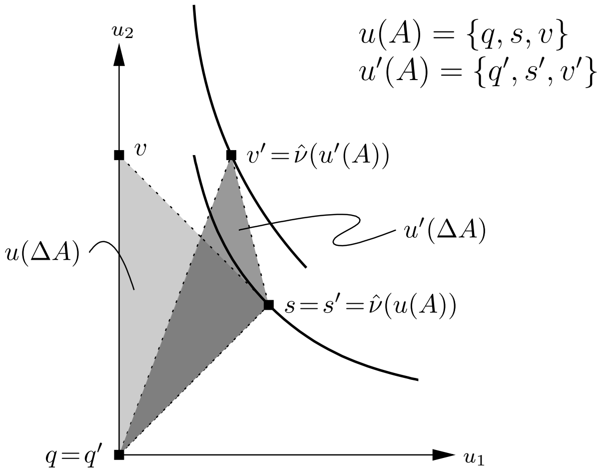

Figure 1: Howard's example for the Nash solution

respectively. Now, u and u' –for convenience we denote their extensions to ΔA again by u, u'– map ΔA onto different compact convex sets.

Next, we present an alternative to Howard's model in which Maskin monotonicity is satisfied.

3 Alternative model for Howard's Example

Let $B := u(A) = \{(0,0), (1,1), (0,2)\}$ and $B' := u'(A) = \{(0,0), (1,1), (3/4, 2)\}$ two bargaining games with status quo point $(0,0)$ and $\mathcal{B} := \{B, B'\}$ be the set of feasible bargaining games. Let $\lambda, \nu : \mathcal{B} \longrightarrow \mathbb{R}^2$ be bargaining solutions defined by

$$\lambda(B) = \lambda(B') = (1,1), \qquad \nu(B) = (1,1), \quad \nu(B') = (3/4,\ 2).$$

Obviously, ν is the Nash solution on B as it maximizes the Nash product on B and B'. Observe that B and B' are exactly the two bargaining problems considered in the previous section.

Now, define the outcome space $\tilde{A}$ to be the set of all bargaining solutions on $\mathcal{B}$, i.e., $\tilde{A} := \{\alpha : \mathcal{B} \longrightarrow B \cup B' \mid \alpha(B) \in B, \quad \alpha(B') \in B'\}$.

On $\tilde{A}$ we define profiles of utility functions $\tilde{u}, \tilde{u}'$ by setting for any $\alpha \in \tilde{A}$,

$$\tilde{u}(\alpha) := \alpha(B) \qquad \text{and} \qquad \tilde{u}'(\alpha) = \alpha(B').$$

Let $\tilde{U} := \{\tilde{u}, \tilde{u}'\}$. The bijection between $\tilde{U}$ and $\mathcal{B}$ associating $\tilde{u}$ with B and $\tilde{u}'$ with B' provides the effectivity/supportability of Bergin and Duggan (1999) in our specific context!

Next, we define our Nash social choice rule $\varphi^\nu : \tilde{U} \Longrightarrow \tilde{A}$ by

$$\varphi^\nu(\tilde{u}) := \underset{\alpha \in \tilde{A}}{\operatorname{argmax}}\ \tilde{u}_1(\alpha)\, \tilde{u}_2(\alpha) \supseteq \{\nu, \lambda\}$$

$$\varphi^\nu(\tilde{u}') := \underset{\alpha \in \tilde{A}}{\operatorname{argmax}}\ \tilde{u}'_1(\alpha)\, \tilde{u}'_2(\alpha) \supseteq \{\nu\}$$

Note that in particular $\lambda \notin \varphi^\nu(\tilde{u}')$. Thus, when switching from $\tilde{u}$ to $\tilde{u}'$ the former social optimum λ is no longer one at preferences $\tilde{u}'$ (see Figure 2). But, now a preference reversal involving social optima is involved. Indeed, while at $\tilde{u}$ we have $\tilde{u}(\lambda) = \tilde{u}(\nu) = (1, 1)$, we get $\tilde{u}'_1(\nu) = 3/4 < 1 = \tilde{u}'_1(\lambda)$ and $\tilde{u}'_2(\nu) = 2 > 1 = \tilde{u}'_2(\lambda)$ at profile $\tilde{u}'$. Thus, ν is strictly better than λ for player 2. Hence, Maskin monotonicity is **not violated**.

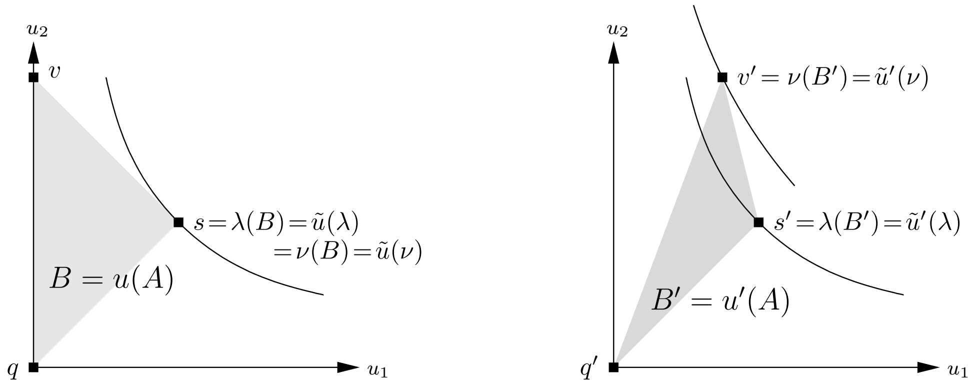

Figure 2: Alternative approach to Howard's example

Note that the two examples capture the same situation; a socially desired outcome is no longer desirable after a switch of utility profiles. But due to a different choice of the outcome space, and hence, of the social choice rule, Maskin monotonicity may or may not be satisfied. As we demonstrate in Section 5, the solution based choice correspondence φ^ν does satisfy Maskin monotonicity. In fact, weak and full Nash implementation of the Nash bargaining solution based on social choice rules have been established in Trockel (2000, 2002b).

4 Further examples

Howard's observation that the specific Nash social choice rule is not Maskin monotonic is not limited to the Nash bargaining solution. Figure 3 illustrates an example with five physical outcomes $A = \{Q, S, V, W^1, W^2\}$ and two profiles of utility functions u, u' given by

$$u(S) = u'(S) = (5/4, 3/4), \qquad u(V) = u'(V) = (1, 1), \qquad u(W^1) = u'(W^1) = (2, 0),$$
$$u(W^2) = (0, 6/5), \qquad u'(W^2) = (0, 2), \qquad u(Q) = u'(Q) = (0, 0)$$

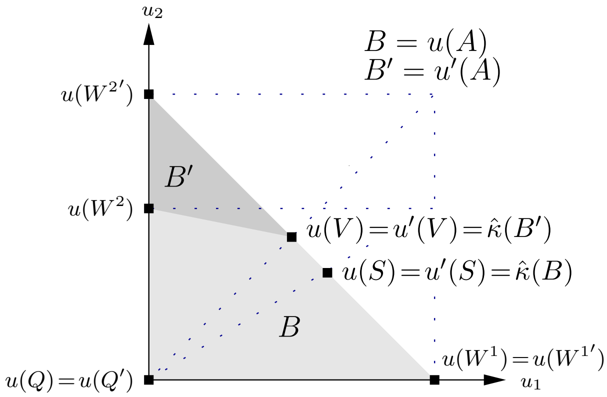

Figure 3: The Kalai-Smorodinsky solution

Again, we consider the two bargaining problems $B = u(A)$ and $B' = u'(A)$. Analogously to the definition of the Nash social choice rule in Section 2, we define the Kalai-Smorodinsky social choice rule $\hat{\varphi}^{\hat{\kappa}} : \{u, u'\} \Longrightarrow A$ by $\hat{\varphi}^{\hat{\kappa}}(w) := \text{argmax}_{a \in A} \min_{i=1,2} \frac{w_i(a)}{\max_{a' \in A} w_i(a')}$. Immediate calculations reveal $\hat{\varphi}^{\hat{\kappa}}(u) = \{S\}$ and $\hat{\varphi}^{\hat{\kappa}}(u') = \{V\}$. Again, physical outcome S is no longer desirable, when moving from u to u', but the ranking of outcomes in A are identical in u and u'. So, $\hat{\varphi}^{\hat{\kappa}}$ is not Maskin monotonic. As in Section 2, nothing is altered, when considering ΔA and extensions of u and u'.

By a closer inspection of Figure 3, it is straightforward that the lack of Maskin monotonicity can be replicated for any Pareto efficient and symmetric bargaining solution.[1] However, for positive implementation results of the Kalai-Smorodinsky solution, we refer to van Damme (1987), Haake (2000), or Trockel (1999). In the next section, we show in general that any solution based social choice correspondence that stems from a Pareto efficient bargaining solution is Maskin monotonic.

5 Monotonicity

Trockel (2002a) shows that any solution based social choice rule stemming from a Pareto efficient bargaining solution does satisfy Maskin monotonicity – a necessary condition for Nash implementability. As we demonstrate in this section a solution based social choice correspondence in fact satisfies a stronger version of monotonicity: *essential monotonicity*. Yamato (1992, Theorem 2) shows that this version is sufficient for Nash implementation,

[1]Roughly, one has to define u, u' such that $u'(A)$ is obtained from $u(A)$ by exchanging coordinates, but without reversing preferences over A. With an appropriate choice of utilities of S and V the solution switches between these physical outcomes.

when there are at least three players.[2]

We consider a population $I := \{1, \ldots, n\}$ of n players. An *n-person bargaining game* B consists of a closed and convex subset of $\mathbb{R}^n$ – the utility possibility set – and an interior point – the status quo point – such that the set of status quo dominating points is bounded. Let $\mathcal{B}$ be a nonempty set of (admitted) bargaining games for n persons.

We define the *outcome space* $\tilde{A}$ to be the set of all bargaining solutions on $\mathcal{B}$, i.e., $\tilde{A} := \{\alpha : \mathcal{B} \longrightarrow \mathbb{R}^n \,|\, \alpha(B) \in B,\, B \in \mathcal{B}\}$. By $\tilde{U}$ we denote the set of all (admitted) profiles of utility functions on $\tilde{A}$ such that there is a well defined one-to-one correspondence between $\tilde{U}$ and $\mathcal{B}$ along the effectivity/supportability results in Bergin and Duggan (1999). To be precise, $\tilde{u} = (\tilde{u}_1, \ldots, \tilde{u}_n) \in \tilde{U}$ if and only if there is $B \in \mathcal{B}$ such that for all $\alpha \in \tilde{A}$ we have $\tilde{u}(\alpha) = \alpha(B)$, meaning that player i evaluates bargaining solutions by the utility they assign to him in bargaining problem B. Therefore, we henceforth identify utility functions profile $\tilde{u}$ with bargaining problem B or $\tilde{u}'$ with B'.

Let $\eta \in \tilde{A}$ be a prespecified bargaining solution. Define a (solution based) social choice correspondence $\varphi^\eta : \tilde{U} \Longrightarrow \tilde{A}$ by $\varphi^\eta(\tilde{u}) := \left\{\alpha \in \tilde{A} \,|\, \alpha(B) = \eta(B)\right\} = \left\{\alpha \in \tilde{A} \,|\, \tilde{u}(\alpha) = \tilde{u}(\eta)\right\}$. That means, φ^η assigns to $\tilde{u} \in \tilde{U}$ all bargaining solutions in $\tilde{A}$ that coincide with η on $\tilde{u}$ (i.e., on B). Put differently, when defining $\varphi^\eta(\tilde{u})$, the corresponding bargaining problem B is the only relevant one. Therefore, if η is supposed to be a desirable bargaining solution, then all solutions that coincide with η on B should be equally desirable and are therefore collected in $\varphi^\eta(\tilde{u})$ as well.

For $i = I$, $\tilde{u} \in \tilde{U}$ and $\alpha \in \tilde{A}$ define i's lower contour set of α at $\tilde{u}$ by $\mathbb{L}_i(\tilde{u}, \alpha) := \left\{\alpha' \in \tilde{A} \,|\, \tilde{u}_i(\alpha') \leq \tilde{u}_i(\alpha)\right\}$. A social choice correspondence $F : \tilde{U} \Longrightarrow \tilde{A}$ is *Maskin monotonic*, if for all $i \in I$, $\tilde{u}, \tilde{u}' \in \tilde{U}$, $\alpha \in F(\tilde{u})$, $\mathbb{L}_i(\tilde{u}, \alpha) \subseteq \mathbb{L}_i(\tilde{u}', \alpha)$ implies $\alpha \in F(\tilde{u}')$.

Let $\tilde{M}$ be a subset of $\tilde{A}$ and $F : \tilde{U} \Longrightarrow \tilde{A}$. An outcome $\alpha \in \tilde{M}$ is *F-essential* for $i \in I$ in $\tilde{M}$, if there exists $\bar{u} \in \tilde{U}$ with $\alpha \in F(\bar{u})$ and $\mathbb{L}_i(\bar{u}, \alpha) \subseteq \tilde{M}$. Denote by $Ess_i(\tilde{M}, F)$ the set of F-essential outcomes for i in $\tilde{M}$. F satisfies *essential monotonicity*, if for all $i \in I$, $\tilde{u}, \tilde{u}' \in \tilde{U}$, and all $\alpha \in F(\tilde{u})$, $Ess_i(\mathbb{L}_i(\tilde{u}, \alpha), F) \subseteq \mathbb{L}_i(\tilde{u}', \alpha)$ implies $\alpha \in F(\tilde{u}')$.

Theorem (Yamato(1992), Theorem 2) *Suppose $n \geq 3$. If F satisfies essential monotonicity, then F is Nash implementable.*

Proposition *Let $\eta \in \tilde{A}$ be a Pareto efficient bargaining solution. Then φ^η is essentially monotonic. Hence φ^η is Nash implementable, if there are three or more players.*

[2]See also Danilov (1992). In Yamato's work, this condition was originally termed *strong monotonicity*, but is now more frequently, and more appropriately, found under the term we use.

Proof. We start with two immediate observations.

1. For all $\tilde{u} \in \tilde{U}$ we have $\eta \in \varphi^\eta(\tilde{u})$.
2. For all $\tilde{u} \in \tilde{U}$, $i \in I$ and $\beta \in \varphi^\eta(\tilde{u})$, $\mathbb{L}_i(\tilde{u}, \beta) = \mathbb{L}_i(\tilde{u}, \eta)$.

Now, let $\tilde{u}, \tilde{u}' \in \tilde{U}$ and $\alpha \in \varphi^\eta(\tilde{u})$ be such that $Ess_i(\mathbb{L}_i(\tilde{u}, \alpha), \varphi^\eta) \subseteq \mathbb{L}_i(\tilde{u}', \alpha)$ for all $i \in I$. We need to show $\alpha \in \varphi^\eta(\tilde{u}')$.

First, for all $i \in I$, any $\beta \in \varphi^\eta(\tilde{u})$ is φ^η-essential in $\mathbb{L}_i(\tilde{u}, \alpha)$. To see this, take $\tilde{u}$ as utility profile $\bar{u}$ in the definition of essential outcomes. Then, clearly, $\beta \in \varphi^\eta(\tilde{u})$ and by the second observation $\mathbb{L}_i(\tilde{u}, \beta) \subseteq \mathbb{L}_i(\tilde{u}, \alpha)$. Hence, for all $i \in I$ we have

$$\varphi^\eta(\tilde{u}) \subseteq Ess_i(\mathbb{L}_i(\tilde{u}, \alpha), \varphi^\eta) \subseteq \mathbb{L}_i(\tilde{u}', \alpha).$$

With the first observation, $\eta \in \mathbb{L}_i(\tilde{u}', \alpha)$, and therefore $\tilde{u}'_i(\eta) \leq \tilde{u}'_i(\alpha)$ $(i \in I)$, which is equivalent to $\eta(B') \leq a(B')$, where B' is the bargaining problem identified with $\tilde{u}'$. Since η is Pareto efficient, $\eta(B') = a(B')$, i.e., $\tilde{u}'_i(\eta) = \tilde{u}'_i(\alpha)$ $(i \in I)$, implying $\alpha \in \varphi^\eta(\tilde{u}')$. □

It is easy to see that essential monotonicity implies Maskin monotonicity. We can therefore confirm the following result in Trockel (2002a), as it is a direct corollary of the Proposition.

Corollary *Any solution based social choice correspondence with underlying Pareto efficient bargaining solution is Maskin monotonic.*

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