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Abstract. Scheduling jobs against due dates is one of the most important and best examined objectives in scheduling theory and practice. In this paper the parallel machine version of the well-known total tardiness problem is considered. The objective is to minimize the total tardiness of the jobs, while for all jobs an individual due date is given. The single machine version has been proven to be NP-hard, hence it is unlikely to find polynomially bounded optimization algorithms. Consequently, we concentrate on developing an efficient heuristic. Our extensive computational results confirm that our new heuristic is capable to deliver near optimal results.

Keywords: scheduling, parallel machines, sequencing, tardiness

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1 Introduction and literature review

Scheduling jobs on parallel machines against their due dates is a very common setting from a practical perspective. It can be found, for example, in the beverage industry, where jobs need to be scheduled on parallel bottling machines. Jobs are, for example, beer orders from restaurants, pubs, and wholesalers. Meeting the due dates is important, because you do not want your wholesaler, retailer or local pub run out of beer. The groceries industry is facing a similar situation. Another example is the capital-intensive printing industry, where print jobs (like books) are scheduled on parallel presses and finishing lines. Meeting due dates for orders placed by the different book publishers is extremely important, too, as a publisher will loose revenues if a special title (in the worst case the actual bestseller) is not available on the shelves of the bookstores. The pharmaceutical industry usually runs parallel machines to produce drugs. And they definitely want their blockbusters being available in the drugstores. In this case the customers might be the national sales organization that bundle and forward the orders of their customers. Pretty much every OEM company has parallel lines to assemble the jobs that are placed with him. Even auditors can be seen as parallel machines that work on the auditing jobs acquired by their partners. This are only a very few examples for processing jobs on parallel machines against due dates. Note that in all setting mentioned the jobs are (typically) not split between the parallel machines.

Generally speaking parallel machine problems seem to be harder than single machine problems, since in addition to the sequencing task the jobs need to be assigned to machines. As the total tardiness problem (TTP) is NP-hard for the single machine case (Du and Leung, 1990), its version for parallel machines must be NP-hard, too. This means that finding a polynomially bounded optimization algorithm is very unlikely. Consequently large instances of the problem should be attacked by means of heuristic approaches obtaining near optimal solutions. Some powerful heuristics have been developed for the single machine problem, see Baker and Bertrand (1982), Holsenback and Russell (1992), Panwalkar et al. (1993) and Panneerselvam (2006). For a literature review on the single machine TTP, see Koulamas (1994).

However, the literature on the parallel machine version of the TTP is relatively limited, for an overview see Shim and Kim (2007). We will refer to the parallel TTP as TTP/m in the following. To the best of our knowledge, for the TTP/m only the following heuristics are available. Wilkerson and Irwin (1971), see also Baker (1973), order the jobs according to the earliest due date (EDD) rule and assign them to the machines afterwards. If a job can be finished on time it is assigned to the machine such that it is completed as closely to its due date as possible. If the job will be late, it is assigned to the machine with the lowest workload. Dogramaci and Surkis (1979) apply three different priority rules, namely EDD, shortest processing time (SPT) and minimum slack. For each of these initial sequences they proceed as follows: Among the jobs that have not yet been assigned to a machine take the first job, assign it to the machine with the smallest actual workload and remove it from the list of unscheduled jobs. This step is repeated until all jobs have been assigned to a machine. In a last step, the machines are considered individually and m single machine total tardiness problems are solved. However, as the single machine problem is – as already stated – NP-hard, this procedure is only good for relatively small instances. For the single machine version of the TTP it is well known that the SPT sequence gives relatively good results if the due dates are tight and that the EDD sequence gives relatively good results if the due dates are not that tight. Ho and Chang (1991) make use of this coherence by developing a traffic priority index (TPI) that takes into consideration the congestion of the job shop, i.e. the overall tightness of the due dates. After sequencing the job according to the TPI they assign the jobs to the machines according to the lowest workload rule and apply a greedy-like neighbourhood search afterwards. As mentioned above, the TTP$/m$ consists of a scheduling and a job assignment task, which are interrelated. The three heuristics presented above separate these tasks and thus ignore the parallel nature of the problem. Koulamas (1997) tackles the TTP$/m$ by means of a meta-heuristically approach, namely a hybrid version of simulated annealing. And Bilge et al. (2004) apply a tabu search algorithm to the TTP$/m$.

The number of optimization approaches seems to be limited as well: Lawler (1964) and Root (1965) present optimization algorithms for some special cases, namely when all jobs have the same processing times and due dates, respectively. Gupta and Maykut (1973) develop a dynamic programming based algorithm for the TTP$/m$ that is able to solve problems with up to 10 jobs. Azizoglu and Kirca (1998) are able to solve problems with up to 15 jobs with a branch & bound algorithm. Obviously, limiting enumeration in a branch & bound algorithm by applying different dominance properties is the decisive sleight of this approach. Yalaoui and Chu (2002) are able to solve problems with up to 20 jobs by a very sophisticated branch & bound algorithm. And very recently Shim and Kim (2007) present an even more sophisticated branch & bound algorithm that is capable of solving problems with up to 30 jobs within an hour. However, this means optimal solutions are not available even for mid-sized problems.

The remainder of the paper is organized as follows. In the next section we will introduce the notation needed and present the new heuristic for the TTP/m. The third section will present a problem generator, a mixed-integer programming formulation for the TTP/m and extensive computational results. The last section contains some concluding remarks.

2 A new heuristic for the TTP/m

Notation and Problem Formulation
There are n jobs available at time zero. Each job needs exactly one operation on one of the m identical parallel machines, pre-emption is not allowed. For each job $i = 1, ..., n$ the deterministic and integer processing time p_i and a due date d_i are given. The tardiness $T_i(S)$ of a job i in a schedule S can be calculated by $T_i(S) = \max\{0, C_i(S) - d_i\}$, where $C_i(S)$ denotes the completion time of the job i in the schedule S. Further, let S_j be the sequence of jobs scheduled on machine $j = 1, ..., m$. The objective of the TTP/m is to schedule the jobs on the m machines to minimize the total tardiness $T = \sum_{i=1}^{n} T_i(S)$.

Motivation of the Heuristic
The heuristic we propose is motivated by the parallel nature of the problem, and we try to give consideration to the interdependencies of the scheduling and assignment tasks, respectively. To do so we make use of the so-called NEH heuristic of Nawaz, Enscore and Ham (1983) used for permutation scheduling.[1] See Kalczynski and Kamburowski (2007) for a good analysis of the high quality of the NEH heuristic.

We can limit our search to active schedules as for the TTP/m an optimal schedule without idle times exists. This means that the processing of the m jobs, which are assigned to the first position on each machine, start at the time zero and that the processing of a job immediately starts after its preceding job has been finished.

Our heuristic for the TTP/m consists of two steps: First the m jobs with the smallest

[1]The idea of the NEH heuristic is originally due to Karg and Thompson (1964), who used the same procedure to construct a starting solution for the travelling salesman problem.

due dates are simultaneously assigned to the machines. Generally speaking we use each combination among the m jobs with the smallest due dates as a first assignment. For $m = 2$ we use the starting assignments

- S_1 = (1), S_2 = (2) and
- S_1 = (1,2), S_2 = ().

For $m = 3$ we use

- S_1 = (1), S_2 = (2), S_3 = (3),
- S_1 = (1,2), S_2 = (3), S_3 = ( ),
- S_1 = (1,3), S_2 = (2), S_3 = ( ),
- S_1 = (2,3), S_2 = (1), S_3 = ( ) and
- S_1 = (1,2,3), S_2 = ( ), S_3 = ( ).

And so on. The number of starting assignments, $A(\cdot)$, obviously depends on m:

$$A(m) = 2 + \sum_{j=2}^{m} \sum_{i=0}^{max\{0,m-j-1\}} \binom{m-i}{j}$$

For each of the $A(m)$ starting assignments the following second step is performed, during which the remaining jobs are assigned to the machines one by one. The job with the smallest due date is picked and assigned to the position at which the lowest total tardiness up to this point is incurred, while the predecessor relation of the jobs already scheduled is not altered. For example, if on one of the machines job a is scheduled prior to job b, for a new job i the sequences (a, b, i), (i, a, b) and (a, i, b) are considered.

Heuristic for the TTP/m
Without loss of generality we assume that the jobs are indexed according to the EDD rule, i.e. $d_i \leq d_{i+1}$ for $i = 1, ..., n-1$. If $d_i = d_{i+1}$ then $p_i \geq p_{i+1}$.

Step 0: Generate all $A(m)$ possible starting assignments: sa_1, sa_2, ..., $sa_{A(m)}$. Set $k = 1$.

Step 1: Let $U := \{m+1, ..., n\}$ be the set of unscheduled jobs. Let sa_k be the actual schedule. If one or more machines exist with having no job assigned so far, take the jobs with the smallest index of the set U and assign them to the machines with no job assignments. Remove these jobs from the set U. Now all machines have at least one job assigned to them.

Step 2: Take the job with the smallest index of the set U and insert it optimally into the actual schedule without altering the predecessor and successor relations of the given sequences. The job is first inserted at the last position of machine 1 and then at the first, second, ..., penultimate position of machine 1. The same insertion strategy is applied for machine $2, 3, ..., m$ afterwards. In case of a tie, the algorithm chooses the sequences with the lowest total tardiness which was calculated first. Remove the job from the set U. Repeat Step 2 until U is empty.

Step 3: Store solution (schedule and tardiness) of sa_k. $k := k+1$. If $k \leq A(m)$ go to Step 1.

Step 4: Choose the best of the $A(m)$ solutions.

Let n_j be the number of jobs actually scheduled on machine j. Note that at step 2 each job has to be assigned to $(n_1+1) + (n_2+1) + ... + (n_m+1) = m + \sum_{j=1}^{m} n_j$ positions. Obviously, this step of the heuristic is polynomially bounded, as altogether $(n-m)(m+\sum_{j=1}^{m} n_j)$ schedules have to be evaluated. With $n_j < n$ for $j = 1, ..., m$, the worst case run time of step 2 is bounded by $O(mn^2)$.

The crucial question now is how many starting assignments $A(m)$ exist. Determining the number of starting assignments is similar to allocating identical balls to identical urns. Note that for example S_1 = (1,2), S_2 = ( ) and S_1 = ( ), S_2 = (1,2) are identical allocations from our perspective as the urns (=machines) are identical.

Proposition 1. *The number of starting assignments $A(m)$ is bounded by $(2m)^m/m!$.*

Proof: If the urns are not identical the number of allocations is given by $\binom{2m-1}{m-1}$. Hence

$$A(m) \leq \binom{2m-1}{m-1} = \frac{(2m-1)\cdot(2m-2)\cdot ... \cdot(2m-m)}{m!} \leq \frac{(2m)^m}{m!}.$$

Obviously the upper bound on $A(m)$ is not dependent on the number of jobs. It thus follows that for a given number of machines the heuristic is polynomially bounded by the number of jobs. In fact the growth of $A(m)$ is rather slow: A(2) = 2, A(3) = 5, A(4) = 15, A(5) = 40, ..., A(10) = 1974. Within our computational experiments the number of starting assignments never had a severe impact on the run times.

Example for the TTP/m-Heuristic
Let $m = 2$ and $n = 5$ with

i	1	2	3	4	5
p_i	2	2	9	4	8
d_i	6	8	10	10	13

Step 0: $A(2) = 2$: sa_1: S_1 = (1) and S_2 = (2); sa_2: S_1 = (1,2) and S_2 = ( ). $k = 1$.

Step 1: $U = \{3, 4, 5\}$, sa_1 is the actual schedule.

Step 2: Take the job 3 and assign it to the first and second position of each machine. Note that it does not matter whether job 3 is assigned to machine one or two - in both cases its completion time is 11. Job 3 could have even been assigned to the first position of both machines without causing any tardiness. However, the last mentioned sequences are eliminated by the insertion strategy of step 2: Hence, the new actual schedule is S_1 = (1,3), S_2 = (2) with $T = 1$. As $U = \{4, 5\} \neq \{\}$, repeat step 2:

Take job 4 and assign it to the first, second, and third position of machine one and to the first and second position of machine two. Schedule S_1 = (1,3), S_2 = (2,4) has (unambiguously) the lowest total tardiness with $T = 1$. As $U = \{5\} \neq \{\}$, repeat step 2:

Take job 5 and assign it to the first, second, and third position of both machines. The lowest total tardiness is (unambiguously) obtained by schedule S_1 = (1,3), S_2 = (2,4,5) with $T = 2$. As $U = \{\}$, go to Step 3.

Step 3: Store $S_1 = (1,3)$, $S_2 = (2,4,5)$ with $T = 2$. As $k = 2 \leq 2 = A(2)$ go to Step 1.

Step 1: $U = \{3, 4, 5\}$, sa_2 is the actual schedule. The second machine has no job assignments. Take job 3 and assign it to the first position of machine 2. $U = \{4, 5\}$.

Step 2: Take the job 4 and assign it to the first, second and third position of machine one and to the first and second position of machine two. The lowest total tardiness is obtained by schedule $S_1 = (1,2,4)$, $S_2 = (3)$ with $T = 0$. As $U = \{5\} \neq \{\}$, repeat step 2:

Take job 5 and assign it to the first, second, third, and fourth position of machine one and to the first and second position of machine two. Schedule $S_1 = (1,2,4,5)$, $S_2 = (3)$ has (unambiguously) the lowest total tardiness with $T = 3$. As $U = \{\}$, go to Step 3.

Step 3: Store $S_1 = (1,2,4,5)$, $S_2 = (3)$ with $T = 3$. As $k = 3 > 2 = A(2)$ go to Step 4.

Step 4: The better of the two solutions is $S_1 = (1,3)$, $S_2 = (2,4,5)$ with $T = 2$.

The solution found by the heuristic happens to be optimal.

3 Computational Results

To the best of our knowledge the strongest heuristic for the TTP/m is that of Ho and Chang (1991). To test the quality of our heuristic we decided to compare it to optimal schedules as well as to schedules generates by the Ho and Chang heuristic. For the computational experiments we generated benchmark problems, developed a new integer programming formulation and re-programmed the heuristic of Ho and Chang (1991). This section will describe these steps in detail.

Generation of instances
We will make use of a problem generator to obtain benchmark instances for our computational experiments that are easily reproducible.[2] The problem generator is called

[2]We are gladly willing to distribute the problem generator via e-mail

TTPmGEN. TTPmGEN makes use of the well-known linear congruential random number generator introduced by Lehmer (1951), see Biskup and Feldmann (2001) for a detailed description. Furthermore, TTPmGEN uses two general seeds, one for the generation of the processing times (3,794,612) and one for the due dates (1,794,612). The seeds are altered for each instances: For example the second benchmark problem with fifty jobs uses the seeds 3,794,612 + 2 + 50, and 1,794,612 + 2 + 50, respectively; thus the processing times and due dates are assumed to be stochastically independent. Note that the seeds are not machine dependent. The processing times produced by the generator are of the interval $[1, p_{max}]$, in the following we will use $p_{max} = 10$. The due dates are given by:

$$d_i = p_i + RD \sim \left[1, \left\lceil \frac{\delta}{m} \cdot \sum_{i=1}^{n} p_i \right\rceil\right] \tag{1}$$

where $\lceil x \rceil$ denotes the largest integer greater than or equal to x and the $RD \sim [1, x]$ means that all integers of the interval $[1, x]$ are equally probable. Obviously, the smaller δ is, the tighter the due dates are. We will use values of 1/2, 1/3 and 1/4 for δ. The first benchmark problem with 5 jobs, 2 machines, a processing time range of 10 and a δ of 1/2 is the one that has been used in the example above.

This procedure is generally in line with the due date selection-rules presented by Fisher (1976), Baker and Bertrand (1981) and Baker (1984). However, we decided to avoid generating due dates that are smaller than the processing time of the particular job. Due dates that cannot be met for sure seem to be hard to justify from a practical point of view: Usually due dates are agreed upon by the sales team and a sales person will hardly promise a customer a due date that is smaller than the processing time of the job, especially not if the customer knows what time it would take to process the job. However, a company usually employs a sales team with multiple sales persons. These sales people usually act (at least to a certain degree) independently and negotiate jobs with their particular customers. Especially if customers are pressing for shorter and shorter lead times this can easily lead to a situation where each job has a reasonable large due date but it turns out to be hard or impossible to complete all jobs in time. We feel that this situation is best described by equation (1).

Optimization model for TTP/m

To gain an understanding of the quality of the heuristic, we decided to solve some instances to optimality to compare the optimal and heuristical objective function values. We make use of the following mixed integer programming formulation. Note that the binary variables used do not have a machine index. This is different to the classical formulations of Manne (1960) or Wagner (1959). The following notation is used:

C_j	:=	Completion time of job j $(j = 1,..,n)$
T_j	:=	Tardiness of job j $(j = 1,..,n)$
y_j	:=	Binary variable that takes the value one if job j is the first job on one of the m machines j $(j = 1,..,n)$
x_{ij}	:=	Binary variable that takes the value one if job i is scheduled directly before job j on the same machine $(i, j = 1,...,n, i \neq j)$
$x_{i,n+1}$	:=	Binary variable that takes the value one if job i is the last job on a machine $(i = 1,...,n)$
R	:=	Sufficiently large number

$$\min \quad Z = \sum_{j=1}^{n} T_j \tag{2}$$

s.t.

$$\sum_{j=1}^{n} y_j \leq m \tag{3}$$

At most m jobs may be the first job on a machine.

$$y_j + \sum_{i=1, i\neq j}^{n} x_{ij} = 1 \qquad \forall j = 1,...,n \tag{4}$$

Each of the jobs must either start on one of the machines or precede some other job.

$$\sum_{i=1, i\neq j}^{n+1} x_{ji} = 1 \qquad \forall j = 1,...,n \tag{5}$$

Each of the jobs must either succeed another job or be the last job on one of the machine.

$$C_j \geq p_j \cdot y_j \qquad \forall j = 1, ..., n \tag{6}$$

The completion time for the first job of each machine must be equal to or greater than its processing time.

$$C_j \geq C_i + p_j - R(1 - x_{ij}) \qquad \forall i, j = 1, ..., n, i \neq j \tag{7}$$

For all of the remaining jobs j the completion time C_j must be equal to or greater than its processing time p_j plus the completion time of it direct predecessor C_i. If i is not the direct predecessor of j the subtraction of R makes (7) non-restrictive.

$$T_j \geq C_j - d_j \qquad \forall j = 1, .., n \tag{8}$$

The tardiness is calculated as the difference between completion time and due date.

$$\begin{aligned} &x_{ij} \in \{0, 1\} && \forall i = 1, ..., n, j = 1, ..., n + 1, i \neq j \\ &y_j \in \{0, 1\} && \forall j = 1, ..., n \\ &T_j, C_j \geq 0 && \forall j = 1, ..., n \end{aligned} \tag{9}$$

This model formulation needs $n(n + 1)$ binary variables which is significantly less then using either position-related or sequence-related binary variables. However, for the sake of comparison, we made use of a different model formulation with position-dependent binary variables. Even though this formulation needed significantly more binary variables ($mn(n - m + 1)$ vs. $n(n + 1)$), it turned out to be much faster. For example, the number of iterations needed for $n = 8$ jobs was between 4 million and 40 million with the given model formulation and less than 1 million iterations with position-dependend binary variables. Obviously the inherent structure of a model formulation and the degree of freedom for the binary variables has a higher impact on the solution speed than the pure number of binary variables. Consequently we decided to add more restrictions to our model formulation:

$$x_{ij} + x_{ji} \leq 1 \qquad \forall i, j = 1, ..., n, i \neq j \tag{10}$$

If i is the predecessor of j then j cannot preceed i.

$$\sum_{i=1}^{n} x_{in+1} \leq m \tag{11}$$

At most m jobs can be the last.

$$y_i + \sum_{j=1, i \neq j}^{n+1} x_{ij} \leq 2 \qquad \forall i = 1, ..., n \tag{12}$$

The first job on a machine may have only one successor.

The formulation (2) to (12) turned out to be approximately as efficient as the formulation with position-related binary variables.[3]

Numerical experiments

To evaluate the performance of the proposed heuristic, we generated test problems for $m \in \{2, 3\}, n \in \{5, 8, 10, 50, 100, 250, 500, 1000\}$, and $\delta \in \{0.5, 0.33, 0.25\}$. For each combination we generated 10 instances, i.e. 480 benchmark problems in total. We were able to solve problems with up to 10 jobs to optimality within a reasonable amount of time. We applied our heuristic as well as the heuristic of Ho and Chang (1991) to all 480 test problems. Thus we have data from 1,160 runs in total (180 optimal solution, and 480 runs of our heuristic and that of Ho and Chang, respectively).

To compare the results of the heuristic proposed by us (pro) to the optimal solutions (opt) and to the benchmark solutions of the heuristic of Ho and Chang (ben), we developed three types of relations. The first relation (13) compares the results of our heuristic to the optimal objective function values. It averages its deviation to the optimum over the ten different instances of each combination (m, n and δ). A similar relation is applied to compare the heuristic of Ho and Chang to the optimal objective function values, see (14). The third relation (15) is to compare the two heuristics. The calculation of the relations is as follows:

[3] We are gladly willing to distribute the LINGO codes for both formulations via email.

h	$\in$	{opt,pro,ben}
$TT^{h}_{m,n,\delta,k}$	$:=$	Total tardiness of the kth ($k = 1, ..., K$) instance with the tightness-parameter δ, n jobs and m machines, calculated by the heuristics ($h \in$ {pro, ben}) or the mixed integer program ($h \in$ opt).

$$r_1 = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{TT^{\text{pro}}_{m,n,\delta,k}}{TT^{\text{opt}}_{m,n,\delta,k}} - 1\right) \qquad \forall \delta, n, m \tag{13}$$

$$r_2 = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{TT^{\text{ben}}_{m,n,\delta,k}}{TT^{\text{opt}}_{m,n,\delta,k}} - 1\right) \qquad \forall \delta, n, m \tag{14}$$

$$r_3 = \frac{1}{K}\sum_{k=1}^{K}\frac{TT^{\text{pro}}_{m,n,\delta,k}}{TT^{\text{ben}}_{m,n,\delta,k}} \qquad \forall \delta, n, m \tag{15}$$

For example the results for $m = 2$, $n = 10$ and $\delta = 0.5$ are summarized in table 3.

k	1	2	3	4	5	6	7	8	9	10	$\sum$
$TT^{opt}_{k,2,10,0.5}$	26	39	25	16	15	38	47	31	8	26	271
$TT^{pro}_{k,2,10,0.5}$	26	39	29	17	16	38	49	32	8	27	281
$TT^{ben}_{k,2,10,0.5}$	28	43	31	16	16	38	49	32	8	28	289

Table 3: Results for $m = 2$, $n = 10$ and $\delta = 0.5$

These results lead to the following relations:

$$r_1 = \frac{1}{10}\left(\frac{26}{26} + \frac{39}{39} + \frac{29}{25} + \frac{17}{16} + \frac{16}{16} + \frac{38}{38} + \frac{49}{47} + \frac{32}{31} + \frac{8}{8} + \frac{27}{26} - 10\right) = 0.0402$$

$$r_2 = \frac{1}{10}\left(\frac{28}{26} + \frac{43}{39} + \frac{31}{25} + \frac{16}{16} + \frac{16}{16} + \frac{38}{38} + \frac{49}{47} + \frac{32}{31} + \frac{8}{8} + \frac{28}{26} - 10\right) = 0.0638$$

$$r_3 = \frac{1}{10}\left(\frac{26}{28} + \frac{39}{43} + \frac{29}{31} + \frac{17}{16} + \frac{16}{16} + \frac{38}{38} + \frac{49}{49} + \frac{32}{32} + \frac{8}{8} + \frac{27}{28}\right) = 0.9798$$

For $m = 2$, $n = 10$ and $\delta = 0.5$ our heuristic delivers an average deviation from the optimum of 4% while the heuristic of Ho and Chang delivers an average deviation from the optimum of 6.4%. As r_3 is smaller than one it becomes obvious that our heuristic performs better for this parameter combination than that of Ho and Chang. Table 4 contains the computational results for all parameter combinations:

m	n	δ	r_1 (%)	r_2 (%)	r_3 (%)	m	n	δ	r_1 (%)	r_2 (%)	r_3 (%)
2	5	0.5	0.00	11.43	93.75	3	5	0.5	0.00	30.00	88.89
		0.33	0.00	15.94	90.20			0.33	2.50	12.50	95.00
		0.25	0.00	1.67	98.47			0.25	0.00	10.00	95.00
	8	0.5	2.81	5.62	97.73		8	0.5	18.21	34.56	91.19
		0.33	1.14	1.08	100.09			0.33	2.31	5.49	97.18
		0.25	2.20	2.20	100.00			0.25	0.00	1.42	98.69
	10	0.5	4.02	6.38	97.98		10	0.5	4.03	7.55	97.11
		0.33	4.10	4.12	100.02			0.33	3.46†	7.09†	97.84
		0.25	3.50	2.79	100.24			0.25	1.18†	5.87†	98.47
	50	0.5	-	-	96.06		50	0.5	-	-	94.59
		0.33	-	-	98.77			0.33	-	-	98.67
		0.25	-	-	99.78			0.25	-	-	99.35
	100	0.5	-	-	95.94		100	0.5	-	-	94.57
		0.33	-	-	98.23			0.33	-	-	97.99
		0.25	-	-	98.42			0.25	-	-	98.85
	250	0.5	-	-	97.32		250	0.5	-	-	96.69
		0.33	-	-	99.01			0.33	-	-	98.72
		0.25	-	-	99.07			0.25	-	-	99.01
	500	0.5	-	-	98.18		500	0.5	-	-	97.60
		0.33	-	-	99.42			0.33	-	-	99.27
		0.25	-	-	99.53			0.25	-	-	99.31
	1000	0.5	-	-	98.87		1000	0.5	-	-	98.45
		0.33	-	-	99.44			0.33	-	-	99.55
		0.25	-	-	99.67			0.25	-	-	99.58

Table 4: Results

On first sight our heuristic seems to be superior to that of Ho and Chang. We decided to test this hypothesis by the following two methods:

The first analysis concentrates on the relations r_1 and r_2 by applying the well known U-test from Mann and Whitney (1947) and Wilcoxon (1945) for $m = 2$. This test examines at what level of significance two distributions are different. The first sample $A = a_1, ..., a_{n_A}$ contains the deviations from the optimum of the proposed heuristic with $n_A = 90$ observations. The second sample $B = b_1, ..., b_{n_B}$ comprises the deviations from the benchmark heuristic to the optimum with again $n_B = 90$ observations. The null hypothesis assumes that the distributions are identical, i.e. $A = B$. Each value of A has to be compared with each value of B. The one-tailed test examines if $A < B$. The test

†3 of the 10 instances with $\delta = 0.33$ and 5 of the 10 instances with $\delta = 0.5$ could not be solved to optimality within a reasonable time.

statistic U_A and U_B are calculated by

$$
\begin{aligned}
U_A &= n_A n_B + \frac{n_A(n_A+1)}{2} - T_A = 4586.5 \\
U_B &= n_B n_A + \frac{n_B(n_B+1)}{2} - T_B = 3513.5
\end{aligned}
$$

with T_A and T_B being the sum of the ranks. The sum of ranks is calculated by sorting values from both samples in a single ascending order and assigning each value a rank. In case of a tie the rank of the values is the average of the ranks with the same value. The sum of the ranks of a sample is the total of its ranks.

Obviously in most of the comparisons the value of A is smaller than the value of B ($U_A = 4586.5$ versus $U_B = 3513.5$). We decided to test the null hypothesis by a level of significance of $\alpha = 0.1$. Mann and Whitney show that the distribution of a sample with a size greater of 20 is normal. So we use the following formula to test our hypothesis:

$$
\bar{U} = \frac{n_A n_B}{2} - \underbrace{U(\alpha)}_{1.2815} \cdot \sqrt{\frac{n_A n_B (n_A + n_B + 1)}{12}} = 3602.07
$$

$U(\alpha)$ is taken from the table of the standard normal distribution. As $U_B < \bar{U}$, the null hypothesis must be reject and the contra hypothesis is to be accepted, i.e. $A < B$. This denotes that the probability that the sample B obtains greater values than A is at least 90 percent. The results are similar for the benchmark problems with three machines.

The second analysis describes the deviation between the two heuristics by making use of relation r_3. Let

$$
r_4 = \frac{1}{3} \sum_{\delta \in \{0.5, 0.33, 0.25\}} r_3 \qquad \forall n, m.
$$

For example, $r_4 = 0.9414$ for $n = 5$ and $m = 2$ and thus the proposed heuristic dominates that of Ho and Chang for this parameter combination. Figure 1 summarized this second analysis:

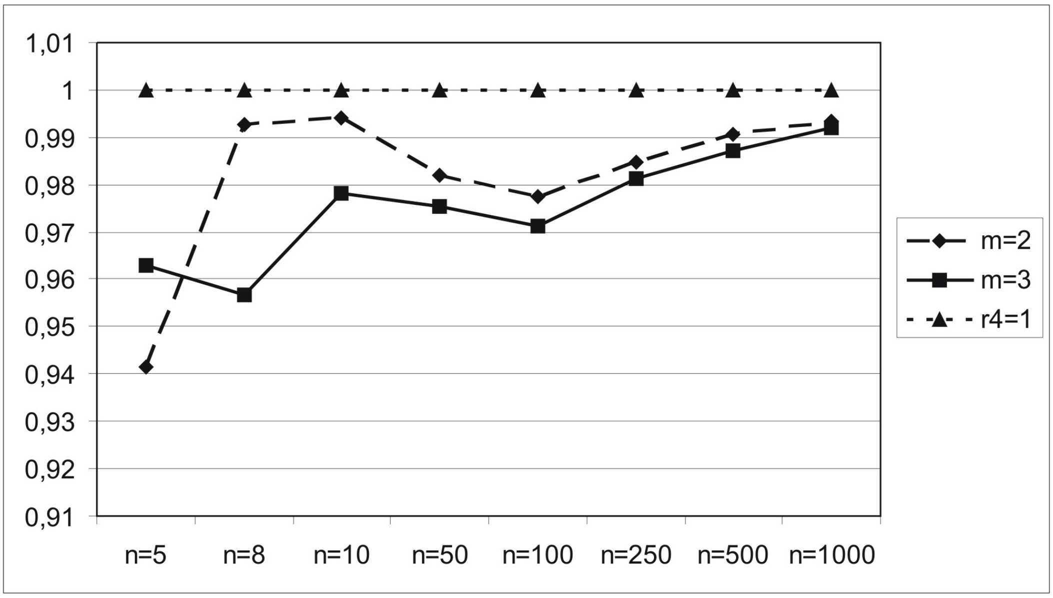

Figure 1: The relation r_4 for $m = 2$ and $m = 3$

From figure 1 it can be easily seen that all values of r_4 are below one, which again supports the above hypothesis. From these two analysis we feel comfortable to claim that our heuristic delivers better results than that of Ho and Chang.

4 Conclusion

In this paper we introduce a new heuristic for the total tardiness problem with parallel machines that is able to deliver near optimal results and that is superior to the existing heuristic approaches. Our goal was to develop a strong heuristic that is at the same time relatively simple to implement. We feel that we achieved this goal.

Generally speaking three approaches to optimization problems exist: heuristics, meta-heuristics and optimization procedures. The existing optimization procedures are limited in their applicability to 30-40 jobs. And because of their interesting and sophisticated dominance properties they are relatively hard to implement. Meta-heuristical approaches are usually capable of delivering near optimal results even for larger problems. However, they are usually not easy to implement, but easier to implement than optimization procedures. And heuristics tend to be the approaches that are easiest to implement and that are fastest even for large problems, but on the flipside they usually deliver worse results than optimization procedures and meta-heuristics. Obviously each approach has its pros and cons. The applicability of the proposed heuristics is now twicefold: it can either be used in practical settings if near optimal schedules are needed with minimum effort. Furthermore the solution obtained by the proposed heuristic can be used as a starting solution for meta-heuristically approaches.

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