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Daniel Eckert and Frederik Herzberg

IMW · Bielefeld University
Postfach 100131
33501 Bielefeld · Germany

email: imw@wiwi.uni-bielefeld.de
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It all depends on independence*

Daniel Eckert†
Frederik Herzberg‡§¶

Abstract

Eliaz (2004) has established a "meta-theorem" for preference aggregation which implies both Arrow's Theorem (1963) and the Gibbard-Satterthwaite Theorem (1973, 1975). This theorem shows that the driving force behind impossibility theorems in preference aggregation is the mutual exclusiveness of Pareto optimality, individual responsiveness (preference reversal) and non-dictatorship.

Recent work on judgment aggregation has obtained important generalizations of both Arrow's Theorem (List and Pettit 2003, Dietrich and List 2007a) and the Gibbard-Satterthwaite Theorem (Dietrich and List 2007b).

One might ask, therefore, whether the impossibility results in judgment aggregation can be unified into a single theorem, a meta-theorem which entails the judgment-aggregation analogues of both Arrow's Theorem and the Gibbard-Satterthwaite Theorem. For this purpose, we study strong monotonicity properties (among them non-manipulability) and their mutual logical dependences. It turns out that all of these monotonicity concepts are equivalent for independent judgment aggregators, and the strongest monotonicity concept, individual responsiveness, implies independence. We prove the following meta-theorem: Every systematic non-trivial judgment aggregator is oligarchic in general and even dictatorial if the collective judgment set is complete. However, systematicity is equivalent to independence for blocked agendas. Hence, as a corollary, we obtain that every independent (in particular, every individually responsive) non-trivial judgment aggregator is oligarchic.

This result is a mild generalization of a similar theorem of Dietrich and List (2008), obtained by very different methods. Whilst Eliaz (2004) and Dietrich and List (2008) use sophisticated combinatorial and logical arguments to prove their results, we utilize the filter method (cf. e.g. Dietrich and Mongin, unpublished) and obtain a much simpler and more intuitive derivation of our meta-theorem.

Key words: Judgment aggregation; independence axiom; monotonicity axiom; oligarchy; impossibility results; non-manipulability; partial rationality.

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†Institut für Finanzwissenschaft, Universität Graz, Austria
‡Department of Mathematics, University of California, Berkeley, United States of America
§Institut für Mathematische Wirtschaftsforschung, Universität Bielefeld, Germany

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1 Introduction

The emerging literature on judgment aggregation (for a survey see List and Puppe [14]), which can be seen as an extension of the analysis of aggregation problems from preferences to arbitrary information, has already provided some generalizations of results in classical Arrovian social choice theory (see e.g. Dietrich and List [5]).

Recently, Dietrich and List [4] have established a judgment aggregation analogue to the second-most famous theorem in social choice theory, the Gibbard-Satterthwaite theorem[1].

In the light of recent attempts by Reny [16] and Eliaz [10] at establishing a meta-theorem incorporating both the Arrow and the Gibbard-Satterthwaite theorem[2], their finding that a strong condition of non-manipulability is already equivalent to the conjunction of two properties used in Arrovian type impossibility results, namely independence and monotonicity is suggestive of a common mechanism driving both types of impossibility results.

For the Arrovian framework this property was identified by Eliaz [10] as preference reversal, which essentially means that the collective preference between two alternatives can only be inverted, if there exists an individual whose preference exhibits precisely this inversion.

In order to obtain his meta-theorem, Eliaz defines this property for general social aggregators which encompass both social welfare functions and social choice functions as special cases. This is necessary because, the property of non-manipulability is originally defined for the latter, i.e. for mappings from profiles of preference orderings on a given set of alternatives into this set of alternatives (and not into the set of preferences like in the case of social welfare functions). Essentially a social aggregator is a mapping from the set of all preference profiles into an arbitrary set of binary relations. In case this set is the set of all binary relations with a unique best alternative which is related to any other of the otherwise unrelated alternatives, a social aggregator is equivalent to a social choice function that directly maps into the set of alternatives, whereas in case the co-domain is the set of all preference orderings, we obviously have a social welfare function.

In the framework of judgment aggregation a similar generalization can be obtained by relaxing the condition that the co-domain of a judgment aggregator be identical with the set D of all consistent and complete judgment sets. We shall also relax the condition that the domain of the judgment aggregator be a Cartesian power of N. (For similar generalizations see Dietrich and List [6].) In the following section we formally define judgment aggregators and identify the property, called individual responsiveness, which is shown in the third section to drive both types of impossibility results via a strong variant of independence known as systematicity.

[1] The Gibbard-Satterthwaite theorem essentially establishes that the only non-manipulable social choice function is a dictatorship. See Gibbard [11] and Satterthwaite [17] for the original references; for a survey see Barbera [1].

[2] For an early analysis of this connection see Batteau, Blin, and Monjardet [2].

2 Framework

Let X be a set of formulae of some monotonic logic, e.g. propositional logic or modal logic. X will be called the **agenda**. We assume that X is the union of proposition-negation pairs. In other words, there exists a set Y such that $X = \bigcup_{p \in Y} \{p, \neg p, \}$.

Let D be the set of consistent and complete subsets of X, let D^* denote the set of consistent and deductively closed subsets of X, and let D' be the set of deductively closed subsets of X. Clearly, $D \subseteq D^* \subseteq D'$.

We shall not assume full rationality, neither at the individual, nor at the collective level. While we assume deductive closedness for both individual and collective judgments, consistency is only assumed at the individual level. Consistency of the collective judgment sets in our meta-theorems will be a consequence of the other assumptions imposed.

Definition 2.1 *For any set of individuals N with cardinality n, a **judgment aggregator** is a mapping $f : \mathbb{D}_f \to D'$ from a subset $\mathbb{D}_f$ of the n-fold Cartesian product $(D^*)^n$ of the set of all consistent deductively closed judgment sets into the set of all deductively closed judgment sets.*

For terminological and notational convenience an element $\underline{A} = (A_1, ..., A_n)$ of $\mathbb{D}_f$ will be called a **profile**, and for any profile $\underline{A} \in \mathbb{D}_f$ and any proposition $p \in X$, $\underline{A}(p) = \{i \in N : p \in A_i\}$ will denote the set of individuals who hold proposition p, i.e. whose judgment sets contain p.

3 Rationality axioms for judgment aggregators via individual responsiveness

The following property will be essential for our analysis of judgment aggregators:

Definition 3.1 *A judgment aggregator $f : \mathbb{D}_f \to D'$ satisfies **individual responsiveness** if for every proposition $p \in X$ and all profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$,*
$p \notin f(\underline{A}) \wedge p \in f(\underline{A}') \Rightarrow \exists i \in N : i \notin \underline{A}(p) \wedge i \in \underline{A}'(p)$.

This property can be considered extremely natural, as it only requires that an aggregator should be positively responsive to individual characteristics, i.e. a change in the social outcome should be justified by a corresponding change in an individual's judgment.

It however turns out that this property is quite strong as it can be shown to be closely related to several well-known properties of judgment aggregation rules.

3.1 Arrovian rationality: monotonicity and independence via individual responsiveness

Definition 3.2 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is **independent** if for every proposition $p \in X$, and all profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$,*
$p \in f(\underline{A}) \wedge \underline{A}'(p) = \underline{A}(p) \Rightarrow p \in f(\underline{A}')$.

Lemma 3.3 *Every individually responsive judgment aggregator $f : \mathbb{D}_f \to D'$ is independent.*

Proof. Assume to the contrary that there exist profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ and a proposition $p \in X$ such that $p \notin f(A)$ albeit $p \in f(\underline{A}')$ and $\underline{A}(p) = \underline{A}'(p)$. The latter equality implies plainly that there cannot be an individual i with $i \notin \underline{A}(p)$ and $i \in \underline{A}'(p)$ and therefore contradicts individual responsiveness. (In other words, the change in the collective outcome was not obtained by the change of some individual's judgment with respect to p.) ■

An important class of judgment aggregator attributes are monotonicity properties of various strengths.

Definition 3.4 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is **strongly monotonic** if for all profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ and every proposition $p \in X$,*
$p \in f(\underline{A}) \wedge \underline{A}(p) \subsetneq \underline{A}'(p) \Rightarrow p \in f(\underline{A}')$.

A further strenghtening of strong monotonicity gives the property of isotonicity.[3]

Definition 3.5 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is **isotonic** if and only if for all profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ and every proposition $p \in X$, even*
$p \in f(\underline{A}) \wedge \underline{A}(p) \subseteq \underline{A}'(p) \Rightarrow p \in f(\underline{A}')$.

In fact, the following equivalence is easily seen:

Lemma 3.6 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is isotonic if and only if it is both independent and strongly monotonic.*

Proof. The if-part is clear: In the definition of isotonicity, we may consider profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ with either $\underline{A}(p) = \underline{A}'(p)$ (which leads to independence) or $\underline{A}(p) \subsetneq \underline{A}'(p)$ (which leads to strong monotonicity).
For the only-if part, suppose that f is not isotonic. Then there exist profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ and a proposition p such that

$$\underline{A}'(p) \supseteq \underline{A}(p), \quad p \in f(\underline{A}), \quad p \notin f(\underline{A}').$$

Since f is independent, this means $\underline{A}'(p) \supsetneq \underline{A}(p)$, which, however, contradicts strong monotonicity. ■

For comparison, it is not difficult to see that strong monotonicity actually coincides with Dietrich and List's [4] notion of monotonicity for independent judgment aggregators (see Lemma 3.10 below).

When defining monotonicity, Dietrich and List [4] use the concept of the i-variant of a profile. Two profiles are i-variants if they agree for all individuals but i. Formally:

Definition 3.7 *For every individual $i \in N$ and all profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$, the profile $\underline{A}$ is an i-**variant** of $\underline{A}'$ if and only if $A_i \neq A'_i$ but $A_j = A'_j$ for all $i \neq j$.*

This yields the following variants of monotonicity:

Definition 3.8 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is **monotonic** if for all $p \in X$, all individuals $i \in N$ and all i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$ with $p \notin A_i$ and $p \in A'_i$,*

$$p \in f(\underline{A}) \Rightarrow p \in f(\underline{A}').$$

[3] The notion of isotonicity is, by the way, the direct analogue of the monotonicity concept used in preference aggregation.

Definition 3.9 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is **weakly monotonic** if for all $p \in X$, all individuals $i \in N$ and judgment sets $A_1, \ldots, A_{i-1}, A_{i+1}, A_n$, if there exists a pair of i-variants $\underline{A} = (A_1, \ldots, A_n) \in \mathbb{D}_f$ and $\underline{A}' = (A_1, \ldots, A_{i-1}, A_i', A_{i+1}, \ldots, A_n) \in \mathbb{D}_f$ with $p \notin A_i$ and $p \in A_i'$, then there exists at least one such pair of i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$ such that*

$$p \in f(\underline{A}) \Rightarrow p \in f(\underline{A}').$$

Lemma 3.10 *For an independent judgment aggregator $f : \mathbb{D}_f \to D'$, the following properties are equivalent:*

- *weak monotonicity,*
- *monotonicity,*
- *strong monotonicity, and*
- *isotonicity*

Proof. We have already observed that isotonicity is equivalent to strong monotonicity for independent judgment aggregators. Also, it is clear that strong monotonicity is sufficient for monotonicity and weak monotonocity (even without independence). Dietrich and List [4] proved that monotonicity is both necessary and sufficient for weak monotonocity if f is independent. We will now show that strong monotonicity is necessary for monotonicity if f is independent.
Suppose, for an argument by contraposition, that f is independent, but not strongly monotonic. Then there exist profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ and a proposition p such that

$$\underline{A}'(p) \supsetneq \underline{A}(p), \quad p \in f(\underline{A}), \quad p \notin f(\underline{A}').$$

But n is finite, whence these profiles $\underline{A}, \underline{A}'$ can even be chosen in such a way that $\underline{A}'(p) = \underline{A}(p) \cup \{i\} \supsetneq \underline{A}(p)$ for some individual i. (This can be proven formally through backward induction on the cardinality of $\underline{A}'(p) \setminus \underline{A}(p)$.) The independence of f also implies that $p \notin f(\underline{A}')$ holds for each $\underline{A}'$ satisfying $\underline{A}'(p) = \underline{A}(p) \cup \{i\}$, thus for every i-variant $\underline{A}'$ of $\underline{A}$ with $p \in A_i'$. This contradicts monotonicity. ∎

Corollary 3.11 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is isotonic if and only if it is both independent and weakly monotonic.*

Lemma 3.12 *Every individually responsive judgment aggregator $f : \mathbb{D}_f \to D'$ is strongly monotonic.*

Proof. Assume to the contrary that an individually responsive aggregator f is not strongly monotonic. Due to the independence of f (see Lemma 3.3), we may utilize the characterization of monotonicity in Lemma 3.10 and obtain that f cannot even be monotonic. Then there exists an individual i, a proposition $p \in X$ as well as i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$ such that

$$p \notin A_i, \quad p \in A_i', \quad p \in f(\underline{A}), \quad p \notin f(\underline{A}'),$$

and in particular, the equivalence $j \in \underline{A}(p) \Leftrightarrow j \in \underline{A}'(p)$ holds for every individual $j \neq i$. Hence for all $j \in N$, either $j \in \underline{A}'(p)$ (e.g. ! for $j = i$) or $j \notin \underline{A}(p)$. This contradicts individual responsiveness. ∎

In fact, the following equivalence is easily shown.

Theorem 3.13 *A judgment aggregator* $f : \mathbb{D}_f \to D'$ *is individually responsive if and only if it is both independent and strongly monotonic.*

Proof. The only-if-part is given by Lemma 3.3 and Lemma 3.12.

For the if-part, consider, for a proof by contraposition, a judgment aggregator which is not individually responsive. If this aggregator is not a constant function, then for some proposition p and some profile $\underline{A} \in \mathbb{D}_f$ with $p \in f(\underline{A})$ there exists a profile $\underline{A}' \in \mathbb{D}_f$ with $p \notin f(\underline{A}')$ such that either $\underline{A}(p) = \underline{A}'(p)$ (violating independence) or $\underline{A}(p) \subsetneq \underline{A}'(p)$ (violating strong monotonicity). ■

Corollary 3.14 *For every judgment aggregator* $f : \mathbb{D}_f \to D'$*, the following are equivalent:*

1. f *is individually responsive.*
2. f *is independent and either strongly monotonic or monotonic or weakly monotonic.*
3. f *is isotonic.*

Thus, the natural property of individual responsiveness provides an interesting justification not only for monotonicity[4] but also for the more controversial condition of independence.

3.2 Game-theoretic rationality: non-manipulability via individual responsiveness

It remains to investigate the relation between individual responsiveness and non-manipulability.

Definition 3.15 *A judgment aggregator* $f : \mathbb{D}_f \to D'$ *is* ***non-manipulable*** *if for every individual* $i \in N$*, all* i*-variants* $\underline{A}, \underline{A}' \in \mathbb{D}_f$*, and every proposition* $p \in A_i$*,*
$p \notin f(\underline{A}) \Rightarrow p \notin f(\underline{A}')$*.*
A judgment aggregator is ***manipulable*** *if it is not non-manipulable.*

Dietrich and List [4] employ a stronger notion of non-manipulability, which also rules out that an individual could prevent a collective choice by submitting an inaccurate judgment set:

Definition 3.16 *A judgment aggregator* $f : \mathbb{D}_f \to D'$ *is* ***strongly non-manipulable*** *if for every individual* $i \in N$*, all* i*-variants* $\underline{A}, \underline{A}' \in \mathbb{D}_f$*, and every proposition* $p \in X$*,*
$p \in A_i \wedge p \notin f(\underline{A}) \Rightarrow p \notin f(\underline{A}')$*, as well as*
$p \notin A_i \wedge p \in f(\underline{A}) \Rightarrow p \in f(\underline{A}')$*.*

Obviously, strong non-manipulability implies non-manipulability. The converse holds if the range of the judgment aggregator consists only of complete judgment sets.

[4] Dietrich [3] also studies another weak monotonicity condition, named judgment-set-wise monotonicity, and proves that it is equivalent to monotonicity under the assumption of universal domain (i.e. $\mathbb{D}_f = D$) and independence.

Another aspect of the strength of both non-manipulability conditions consists in the fact that, unlike the usual definition of strategy-proofness based on preferences, non-manipulability excludes any effective manipulation with respect to a particular proposition independently of its potential negative overall effect in terms of individual preference satisfaction with respect to the collective outcome: The opportunity to manipulate by itself does not guarantee that the agent has an incentive to do so. However, Dietrich and List [4] show that strategy-proofness and non-manipulability can coincide.

Lemma 3.17 *1. A strongly non-manipulable judgment aggregator $f : \mathbb{D}_f \to D'$ is non-manipulable.*

2. An independent non-manipulable judgment aggregator $f : \mathbb{D}_f \to D'$ is strongly non-manipulable.

3. If $f : \mathbb{D}_f \to D'$ is a non-manipulable judgment aggregator and $f(\underline{A}) \in D$ for all $\underline{A} \in \mathbb{D}_f$, then f is strongly non-manipulable.

In the proof, we adopt the usual convention that $\neg p$ means q if $p = \neg q$ for some $q \in X$.

Proof.

1. Trivial.

2. Consider, for a proof by contraposition, an individual i, two i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$, and a proposition $p \notin A_i$ such that $p \in f(\underline{A})$, but $p \notin f(\underline{A}')$. The independence of f then entails that $\underline{A}(p) \neq \underline{A}'(p)$ and therefore $p \in A'_i$. Since f is non-manipulable, this readily yields $p \notin f(\underline{A})$, a contradiction.

3. Consider again an individual i, two i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$, and a proposition $p \notin A_i$ such that $p \in f(\underline{A})$. We shall prove that $p \in f(\underline{A}')$. From the completeness of A_i, we obtain $\neg p \in A_i$, and from the consistency of $f(\underline{A})$, we get $\neg p \notin f(\underline{A})$. As f is non-manipulable, this implies $\neg p \notin f(\underline{A}')$. The completeness of $f(\underline{A}')$ finally yields $p \in f(\underline{A}')$.

■

In the rest of this section, we shall address the relation between individual responsiveness and (strong) non-manipulability: We shall prove that individual responsiveness is equivalent to strong non-manipulability, which in turn is equivalent to the combination of non-manipulability and independence.

Lemma 3.18 *Every individually responsive judgment aggregator $f : \mathbb{D}_f \to D'$ is non-manipulable.*

Proof. Consider, for a proof by contraposition, a manipulable judgment aggregator f. Then there exist: an individual $i \in N$, i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$, and a proposition $p \in A_i$ such that $p \notin f(\underline{A})$ but $p \in f(\underline{A}')$. We have to prove that f is not individually responsive. Let $j \in N$. If $j = i$, then $j \in \underline{A}(p)$ as $p \in A_i$. If $j \neq i$, then the equivalence $j \in \underline{A}(p) \Leftrightarrow j \in \underline{A}'(p)$ holds because of $A_j = A'_j$. Hence for all $j \in N$, either $j \in \underline{A}(p)$ or $j \notin \underline{A}'(p)$, contradicting individual responsiveness. ■

Theorem 3.19 *An independent judgment aggregator $f : \mathbb{D}_f \to D'$ is non-manipulable if and only if it is monotonic.*

Proof. Due to Corollary 3.14, Lemma 3.18 already shows that the conjunction of independence and monotonicity is sufficient for non-manipulability.

Consider now, for a proof by contraposition, a judgment aggregator f which is independent, but not monotonic. This gives us a proposition $p \in X$ and some i-variants $\underline{A}, \underline{A}' \in \mathbb{D}_f$ with $p \in A_i$ and $p \notin A'_i$ such that $p \notin f(\underline{A})$, albeit $p \in f(\underline{A}')$. This makes the judgment aggregator f manipulable (by individual i). ■

By Corollary 3.14, we already obtain:

Corollary 3.20 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is individually responsive if and only if it is both non-manipulable and independent.*

The following result, a straightforward generalization of Dietrich and List's [4] findings, states an additional condition on the aggregator's domain under which the strong notion of non-manipulability even entails independence. (Dietrich and List [4] impose the assumption of *universal domain*, i.e. $\mathbb{D}_f = D^n$, instead.)

Lemma 3.21 *Suppose f is a judgment aggregator with $\mathbb{D}_f = E^n$ for some $E \subseteq D^*$. If f is strongly non-manipulable, then f is independent.*

Proof of Lemma 3.21. Consider, for a proof by contraposition, a non-independent judgment aggregator. Then there exists a proposition $p \in X$ and profiles $\underline{A}, \underline{A}' \in \mathbb{D}_f$ such that $\underline{A}(p) = \underline{A}'(p)$ while $p \notin f(\underline{A})$ but $p \in f(\underline{A}')$. Define now, for all $k \leq n$, a profile $\underline{A}^{(k)}$ by $A_i^{(k)} = A'_i$ for all $i \leq k$ and $A_i^{(k)} = A_i$ for all $i > k$. Then $\underline{A}^{(i-1)}$ and $\underline{A}^{(i)}$ are i-variants for all $i > 0$, whilst $\underline{A}^{(0)} = \underline{A}$ and $\underline{A}^{(n)} = \underline{A}'$.

There must be some $i > 0$ such that $p \in f(\underline{A}^{(i)})$ and $p \notin f(\underline{A}^{(i-1)})$ — for, otherwise one could prove by backward induction on i that $p \in f(\underline{A}^{(i)})$ for all $i \leq n$, contradicting $p \notin f(\underline{A}) = f(\underline{A}^{(0)})$.

In other words, there exists an individual $i \in N$ such that for some pair of i-variants $\underline{A}^{(i-1)}, \underline{A}^{(i)} \in \mathbb{D}_f$ (which can be obtained by exchanging some judgment sets of the individuals in $\underline{A}$ by those of the corresponding individuals in $\underline{A}'$) one has $p \notin f(\underline{A}^{(i-1)})$ but $p \in f(\underline{A}^{(i)})$. This individual i can manipulate the collective outcome, in a sense forbidden by strong non-manipulability:

- If $p \in A_i$, then $p \in A_i^{(i-1)}$ by construction of $\underline{A}^{(i-1)}$, although $p \notin f(\underline{A}^{(i-1)})$. In this case, an individual i with judgment set $A_i^{(i-1)} = A_i$ (i.e. i wants p to be socially accepted) can purport $A_i^{(i)} = A'_i$ in order to obtain $f(\underline{A}^{(i)}) \ni p$ as social outcome.
- If $p \notin A_i$, then $p \notin A'_i$ (as $\underline{A}(p) = \underline{A}'(p)$), so $p \notin A_i^{(i)}$ by construction of $\underline{A}^{(i)}$, whilst $p \in f(\underline{A}^{(i)})$. In this case, an individual i with judgment set $A_i^{(i)} = A'_i$ (i.e. i wants to prevent the social acceptance of p) can pretend $A_i^{(i-1)} = A_i$ in order to obtain $f(\underline{A}^{(i-1)}) \not\ni p$ as social outcome.

■

Observe that in this proof, we do not know whether $p \in A_i$, therefore the weak variant of non-manipulability does not suffice to prove independence.

Theorem 3.22 *Suppose f is a judgment aggregator with $\mathbb{D}_f = E^n$ for some $E \subseteq D^*$. Then f is strongly non-manipulable if and only if it is both independent and non-manipulable.*

Proof. If f is strongly non-manipulable, then it is clearly non-manipulable and independent by Lemma 3.21. Conversely, if f is independent and non-manipulable, then it is strongly non-manipulable by Lemma 3.17. ■

Summarizing, we have shown:

Corollary 3.23 *For every judgment aggregator $f : \mathbb{D}_f \to D'$, the following are equivalent:*

1. *f is independent and strongly non-manipulable.*
2. *f is independent and non-manipulable.*
3. *f is individually responsive.*
4. *f is isotonic.*
5. *f is independent and either strongly monotonic or monotonic or weakly monotonic.*

Corollary 3.24 *Suppose $\mathbb{D}_f = E^n$ for some $E \subseteq D^*$. Then even the following are equivalent:*

1. *f is strongly non-manipulable.*
2. *f is independent and non-manipulable.*
3. *f is individually responsive.*
4. *f is isotonic.*
5. *f is independent and either strongly monotonic or monotonic or weakly monotonic.*

4 Oligarchies, systematically

In this section, we adapt the findings of Dietrich and Mongin [7] on the relationship between independence, systematicity, unanimity preservation, and oligarchic (respectively dictatorial) judgment aggregators for the establishment of our meta-theorem.

Our account of Dietrich and Mongin's [7] results is a little more general as we relax the condition $\mathbb{D}_f = D^n$ to $D^n \subseteq \mathbb{D}_f$ in the relevant theorems.

Systematicity is *a priori* a very strong condition as it combines independence and neutrality: Systematicity not only requires that the collective acceptance of any proposition should only depend on the individual judgments on that proposition but also that this pattern of dependence should be the same for every proposition.

Nevertheless, it can be shown to follow from independence as soon as additional assumptions on the logical structure of the agenda are imposed. In the presentation of this argument, we follow Eckert and Klamler [9].

Apart from being a powerful technical tool, the (ultra)filter method illustrates the quasi-duality between the logical structure of the agenda (given by the logical interconnections between the propositions) on the one hand and the social structure of the population (given by the distribution of decision power) on the other hand. It is thus of conceptual interest in its own right.

It is useful to distinguish the concept of a winning coalition in general from the notion of a decisive coalition for a given proposition p in the agenda. The former encompasses all possible coalitions that can force just some collective decision to be made (no matter on what subject), whereas the latter applies to those coalitions which can ensure a favorable collective judgment on some given p.

Through analyzing the set-theoretic closure properties of the collection of winning coalitions, Dietrich and Mongin [7] obtained a necessary and sufficient criterion for a judgment aggregator to be oligarchic. In light of our characterization of individual responsiveness in the first part of the present paper, we shall see that this criterion is, under logical richness assumptions on the agenda, just the conjunction of individual responsiveness and respect for unanimity in a strong sense (see Corollary 4.28).

Given that individual responsiveness is the judgment-aggregation equivalent of preference reversal and that respect for unanimous judgments corresponds to the Pareto property, this Corollary 4.28 is the judgment-aggregation analogue of Eliaz' [10] meta-theorem.

This result has two interesting immediate consequences: First, individual responsiveness and respect for unanimity only entail oligarchic judgment aggregation, whereas in Eliaz' [10] meta-theorem for preference aggregation, the conjunction of preference reversal and the Pareto principle already implies dictatorship. Thus, preference aggregation is more susceptible to dictatorial impossibility results than general judgment aggregation. Second, since individual responsiveness is equivalent to independence and non-manipulability (see Corollary 3.23), we finally obtain an impossibility result featuring non-manipulability as well (Corollary 4.29). Unlike Dietrich and List [6, Theorem 3], we do not need to assume collective rationality. In return, our impossibility result in Corollary 4.29 yields merely an oligarchy, rather than a dictatorship.

4.1 Independence, immediate decisiveness and systematicity

Using a concept well established in social choice theory, one can analyse judgment aggregation rules f by collecting those coalitions which are decisive under f for some proposition in the agenda.

Definition 4.1 *Let $p \in X$. We say that some coalition $U \subset I$ is* ***decisive*** *for p under f if for every $\underline{A}' \in \mathbb{D}_f$, if $U = \underline{A}'(p)$, then $p \in f(\underline{A}')$. The collection of decisive coalitions for p is denoted $\mathcal{W}_p$.*

Extending this concept from coalitions to aggregators, we arrive at the following definition:

Definition 4.2 *A judgment aggregator f satisfies* ***immediate decisiveness*** *if for all $\underline{A} \in \mathbb{D}_f$ and $p \in f(\underline{A})$, the coalition $\underline{A}(p)$ is decisive for p.*

This notion is actually equivalent to independence:

Lemma 4.3 *f is independent if and only if it satisfies immediate decisiveness.*

Proof. First, let f be independent. Let $\underline{A} \in \mathbb{D}_f$ and $p \in f(\underline{A})$. Suppose $\underline{A}' \in \mathbb{D}_f$ is such that $\underline{A}(p) = \underline{A}'(p)$. By independence, $p \in f(\underline{A}')$.

Conversely, let f satisfy immediate decisiveness. Then, for all $\underline{A} \in \mathbb{D}_f$ and every $p \in f(\underline{A})$, the set $\underline{A}(p)$ is decisive — in other words, $p \in f(\underline{A}')$ holds whenever $\underline{A}' \in \mathbb{D}_f$ satisfies $\underline{A}(p) = \underline{A}'(p)$. Thus, we already have

$$\forall \underline{A}, \underline{A}' \in \mathbb{D}_f \left(p \in f(\underline{A}) \Rightarrow \left(\underline{A}(p) = \underline{A}'(p) \Rightarrow p \in f(\underline{A}')\right)\right),$$

which is tantamount to

$$\forall \underline{A}, \underline{A}' \in \mathbb{D}_f \left(p \in f(\underline{A}) \wedge \underline{A}(p) = \underline{A}'(p) \Rightarrow p \in f(\underline{A}')\right).$$

■

Lemma 4.4 *Suppose f is independent. For all $p \in X$ and $\underline{A} \in \mathbb{D}_f$, $p \in f(\underline{A})$ if and only if $\underline{A}(p) \in \mathcal{W}_p$.*

Proof. If $p \in f(\underline{A})$, then $\underline{A}(p) \in \mathcal{W}_p$ by Lemma 4.3. Conversely, if $\underline{A}(p) \in \mathcal{W}_p$, then clearly $p \in f(\underline{A})$ (put $\underline{A} = \underline{A}'$ in the definition of decisiveness). ■

Observe that the equivalence in 4.3 defines what is known in the literature on judgment aggregation as "voting by issues" (cf. Nehring and Puppe [15]).

Obviously, the aggregation problem crucially depends on the properties of the agenda, essentially the logical connections between the propositions in the agenda.

Following Dokow and Holzman [8], the logical connections between the propositions in any agenda X are captured by a binary relation $\vdash^* \subset X \times X$ of conditional entailment.

Definition 4.5 *For any distinct propositions $p, q \in X$, we write $p \vdash^* q$ if there exists a minimally inconsistent superset S of $\{p, \neg q\}$ (i.e. there exists some set $P \subset X \backslash \{p, \neg q\}$ such that $S := P \cup \{p, \neg q\}$ is inconsistent while every proper subset of S is a consistent set of propositions).*

Thus for any two distinct propositions $p, q \in X$, $(p, q) \in \vdash^*$ means that there exists a set of propositions $P \subset X \backslash \{p, \neg q\}$ conditional on which holding proposition p entails holding proposition q.

Example 4.6 *Consider an agenda $X = \{p, \neg p, q, \neg q, p \vee q, \neg(p \vee q), p \wedge q, \neg(p \wedge q)\}$. Then the set $S = \{\neg p, p \vee q, \neg q\}$ is a minimally inconsistent set which establishes the conditional entailment relation between e.g. $\neg p$ and q.*

Definition 4.7 *An agenda X will be called* ***totally blocked*** *if the relation of conditional entailment has full transitive closure, i.e. if $T(\vdash^*) = X \times X$.*

Total blockedness means that any proposition in the agenda is related to any other proposition by a sequence of conditional entailments.

Example 4.8 *Verify that the above agenda* $X = \{p, \neg p, q, \neg q, p \vee q, \neg(p \vee q), p \wedge q, \neg(p \wedge q)\}$ *is totally blocked, while neither the agenda* $Y = \{p, \neg p, q, \neg q, p \wedge q, \neg(p \wedge q)\}$ *nor* $Z = \{p, \neg p, q, \neg q, p \vee q, \neg(p \vee q)\}$ *is totally blocked.*

While this property might appear quite strong, Dietrich and Mongin [7] rightly observe that it is compatible with highly indirect logical connections.

In social choice theory the Pareto principle — enshrining respect for unanimous preferences — is firmly established. In the framework of judgment aggregation, two ways to express this principle present themselves:

Definition 4.9 $f : \mathbb{D}_f \to D'$ *is said to be* ***unanimity-preserving*** *if and only if for all* $p \in X$ *and for every* $\underline{A} \in \mathbb{D}_f$ *such that* $\underline{A}(p) = N$*, one has* $p \in f(\underline{A})$*.*

Definition 4.10 $f : \mathbb{D}_f \to D'$ *is called* ***strictly unanimity-respecting*** *if and only if for all* $p \in X$ *and* $\underline{A} \in \mathbb{D}_f$*, one has* $p \in f(\underline{A})$ *if* $\underline{A}(p) = N$ *and* $p \notin f(\underline{A})$ *if* $\underline{A}(p) = \emptyset$*.*

Obviously, if $f : D^n \to D$, then f is already strictly unanimity-respecting if it is just unanimity-preserving.

Lemma 4.11 *Suppose* $D^n \subseteq \mathbb{D}_f$ *and* f *is independent as well as unanimity-preserving. For all* $p, q \in X$*, if* $p \vdash^* q$*, then* $\mathcal{W}_p \subseteq \mathcal{W}_q$*.*

Proof. Let $P \subseteq X \setminus \{p, \neg q\}$ be a non-empty set such that $P \cup \{p, \neg q\}$ is minimally inconsistent. Consider any $U \in \mathcal{W}_p$. Since every consistent set can be extended to a consistent and complete set, there exists a profile $\underline{A} \in D^n \subseteq \mathbb{D}_f$ such that $\underline{A}(p) = U \in \mathcal{W}_p$, $\underline{A}(q) = U$ and $\underline{A}(r) = N$ for all $r \in P$. By Lemma 4.3, $p \in f(\underline{A})$, and since f is unanimity-preserving, also $P \subseteq f(\underline{A})$. However, $P \cup \{p\} \vdash (\neg q \to \bot)$ by modus ponens (as $P \cup \{p, \neg q\} \vdash \bot$), so $P \cup \{p\} \vdash q$ by modus tollens.

Therefore, $f(\underline{A}) \vdash q$, and since $f(\underline{A}) \in D'$, finally $q \in f(\underline{A})$. Again by Lemma 4.3, we arrive at $U = \mathcal{W}_q \in \underline{A}(q)$. ■

In fact, the logical structure of the agenda can induce a strong neutrality condition known as systematicity in the judgment aggregation literature:

Definition 4.12 *A judgment aggregator* $f : \mathbb{D}_f \to D'$ *is* ***systematic*** *if for any propositions* $p, q \in X$*, and any profiles* $\underline{A}, \underline{A}' \in \mathbb{D}_f$*,*
$p \in f(\underline{A}) \wedge \underline{A}'(q) = \underline{A}(p) \Rightarrow q \in f(\underline{A}')$*.*

Lemma 4.13 *Assume that* X *is a totally blocked agenda. Suppose* $D^n \subseteq \mathbb{D}_f$ *and* f *is independent as well as unanimity-preserving. Then,* f *is systematic.*

Proof. An iterated application of Lemma 4.11 yields $\mathcal{W}_p = \mathcal{W}_q$ for all $p, q \in X$. Therefore, for any $p, q \in X$ and $\underline{A}, \underline{A}' \in \mathbb{D}_f$ with $\underline{A}(p) = \underline{A}'(q)$, we have the equivalence of $\underline{A}(p) \in \mathcal{W}_p$ and $\underline{A}'(q) \in \mathcal{W}_q$. By Lemma 4.4, this means that $p \in f(\underline{A})$ holds if and only if $q \in f(\underline{A}')$. ■

4.2 Impossibility results

For the following reasoning, we collect all coalitions which win the collective outcome for some proposition into a single set:

Definition 4.14 *If f is a judgment aggregator, the family $\mathcal{F}_f := \{\underline{A}(p) : \underline{A} \in \mathbb{D}_f, \quad p \in f(\underline{A})\}$ is called the collection of* ***winning coalitions****.*

In order to use $\mathcal{F}_f$ as a technical tool, we need the following result:

Lemma 4.15 *Suppose f is systematic. For all $\underline{A} \in \mathbb{D}_f$ and $p \in X$, one has $\underline{A}(p) \in \mathcal{F}_f$ if and only if $p \in f(\underline{A})$.*

Proof. If $\underline{A}(p) \in \mathcal{F}_f$, then there is some $\underline{A}' \in \mathbb{D}_f$ and some $q \in f(\underline{A}')$ such that $\underline{A}(p) = \underline{A}'(q)$. But as f is systematic, this can only be true if already $p \in f(\underline{A})$.
The converse implication is trivial. ■

Definition 4.16 *X is called* ***rich*** *if and only if there are propositions $p, q \in X$ such that each of the propositions $p \wedge q, \neg p \wedge q, p \wedge \neg q$ is consistent and an element of X.*

Definition 4.17 *f is called* ***trivial*** *if either $X \subseteq f(\underline{A})$ for all $\underline{A} \in \mathbb{D}_f$ or $f(\underline{A}) \cap X = \emptyset$ for all $\underline{A} \in \mathbb{D}_f$.*

Remark 4.18 *If f is strictly unanimity-respecting, then it is not trivial.*

Remark 4.19 *If f is unanimity-preserving and $f : \mathbb{D}_f \to D$, then f is strictly unanimity-respecting (and hence non-trivial).*

The next lemma is the basis for our impossibility results. It must be noted that it does not assume unanimity preservation or strict respect for unanimity (i.e. the Pareto principle) as an extra axiom. This result is a mild generalization of a result by Herzberg [12] who obtained it as part of an algebraic characterization of systematic non-trivial judgment aggregators; we shall give a direct proof.

Lemma 4.20 *Assume X is rich. If $D^n \subseteq \mathbb{D}_f$ and $f : \mathbb{D}_f \to D'$ is systematic as well as non-trivial, then $\mathcal{F}_f$ is a filter.*

Herein, we have imposed another moderate assumption about the richness of the agenda, which mirrors Lauwers and Van Liedekerke's [13] aggregator domain condition (A1') and is related to a similar agenda condition imposed by Dietrich and List [6, agenda condition (i)] (which in turn was first introduced as non-median space condition in abstract aggregation theory by Nehring and Puppe [15]).
Proof. We write $\mathcal{F}$ for $\mathcal{F}_f$.

1. *Superset closedness.* Suppose $U' \supseteq U \in \mathcal{F}$. Since every consistent set can be extended to a consistent and complete set, there exists a profile $\underline{A} \in D^n \subseteq \mathbb{D}_f$ such that

   $$\forall i \in U \quad p \wedge q \in A_i, \quad \forall i \in U' \backslash U \quad p \wedge \neg q \in A_i, \quad \forall i \in N \backslash U' \quad \neg p \wedge q \in A_i.$$

   Since $\underline{A}(p \wedge q) = U \in \mathcal{F}$, we have $p \wedge q \in f(\underline{A})$ by Lemma 4.15 and thus $p \in f(\underline{A})$ as $f(\underline{A})$ is deductively closed. Hence $\underline{A}(p) = U' \in \mathcal{F}$.

2. *Intersection closedness.* Suppose $U, U' \in \mathcal{F}$. Since every consistent set can be extended to a consistent and complete set, there exists a profile $\underline{A} \in D^n \subseteq \mathbb{D}_f$ such that

$$\forall i \in U \cap U' \quad p \wedge q \in A_i, \quad \forall i \in U \setminus U' \quad p \wedge \neg q \in A_i, \quad \forall i \in N \setminus U \quad \neg p \wedge q \in A_i.$$

Then

$$\mathcal{F} \ni (U \cap U') \cup (U \setminus U') = \underline{A}(p),$$

so $\underline{A}(p) \in \mathcal{F}$. Likewise

$$\mathcal{F} \ni U' \subseteq (U \cap U') \cup (N \setminus U) = \underline{A}(q),$$

so $\underline{A}(q) \in \mathcal{F}$ since we have already established that $\mathcal{F}$ is closed under supersets. By Lemma 4.15, $p, q \in f(\underline{A})$ so $p \wedge q \in f(\underline{A})$ as $f(\underline{A}) \in D'$. Therefore $U \cap U' = \underline{A}(p \wedge q) \in \mathcal{F}$.

3. *Non-triviality.* We have already seen that $\mathcal{F}_f$ is superset-closed. If it were the case that $\emptyset \in \mathcal{F}_f$, then $\underline{A}(p) \in \mathcal{F}_f$ for every $p \in X$ and every $\underline{A} \in \mathbb{D}_f$, therefore $p \in f(\underline{A})$ for every $p \in X$ and every $\underline{A} \in \mathbb{D}_f$. This would contradict the non-triviality of f.

   Similarly, if it were the case that $\mathcal{F}_f = \emptyset$ then $p \notin f(\underline{A})$ for every $p \in X$ and every $\underline{A} \in \mathbb{D}_f$, hence $f(\underline{A}) \cap X = \emptyset$ for every $\underline{A} \in \mathbb{D}_f$, which again would contradict the non-triviality of f.

■

Definition 4.21 *A judgment aggregator $f : \mathbb{D}_f \to D'$ is said to be* ***oligarchic*** *if there exists some $M \subseteq N$ such that for all $p \in X$ and $\underline{A} \in \mathbb{D}_f$,*

$$p \in f(\underline{A}) \Leftrightarrow \forall i \in M \quad p \in A_i.$$

Lemma 4.22 *Assume f is systematic. Then f is oligarchic if and only if $\mathcal{F}_f$ is a filter.*

Proof.

1. By the classification of filters on finite sets: $\mathcal{F}_f$ is a filter if and only if there is a set $M_f \subseteq N$ such that

$$\mathcal{F}_f = \bigcap_{i \in M_f} \{U \subseteq N \ : \ i \in U\},$$

2. Combining this with Lemma 4.15: $\mathcal{F}_f$ is a filter if and only if there is a set $M_f \subseteq N$ such that for all $p \in X$ and $\underline{A} \in \mathbb{D}_f$,

$$\begin{aligned} p \in f(\underline{A}) \Leftrightarrow \underline{A}(p) \in \mathcal{F}_f \Leftrightarrow \forall i \in M_f \quad i \in \underline{A}(p) \\ \Leftrightarrow \forall i \in M_f \quad p \in A_i \Leftrightarrow p \in \bigcap_{i \in M_f} A_i, \end{aligned}$$

   thus $f(\underline{A}) = \bigcap_{i \in M_f} A_i$ for all $\underline{A} \in \mathbb{D}_f$.

3. Therefore $\mathcal{F}_f$ is a filter if and only if there is a set $M_f \subseteq N$ such that $f(\underline{A}) = \bigcap_{i \in M_f} A_i$ for all $\underline{A} \in \mathbb{D}_f$.

■

Another mild assumption on the agenda involves the existence of contingent sentences. (A sentence p is **contingent** if both $\{p\}$ and $\{\neg p\}$ are consistent.)

Lemma 4.23 *Suppose X contains a contingent sentence. If f is oligarchic and $f : \mathbb{D}_f \to D$, then f is dictatorial.*

Proof. Let M_f be the set of oligarchs. Note that $M_f \neq \emptyset$, for otherwise we would have $f(\underline{A}) = \mathcal{P}(X) \notin D$ for all $\underline{A} \in \mathbb{D}_f$. Suppose, for a contradiction that M_f has more than one element. Then there are individuals $i, i' \in N$ such that $f(\underline{A}) \subseteq A_i \cap A_{i'}$ for all $\underline{A} \in \mathbb{D}_f$. Consider a contingent sentence p, and let $\underline{A}$ be a profile with $p \in A_i$ and $p \notin A_{i'}$. By consistency of A_i, $\neg p \notin A_i$. Therefore, $p, \neg p \notin A_i \cap A_{i'}$, so $p, \neg p \notin f(\underline{A})$, whence $f(\underline{A}) \in D$, contradiction. ■

Theorem 4.24 *Assume X is rich. If $D^n \subseteq \mathbb{D}_f$ and $f : \mathbb{D}_f \to D'$ is systematic as well as non-trivial, then f is oligarchic.*

Proof. Combine Lemma 4.22 with Lemma 4.20. ■

The technical agenda richness condition specified in Lemma 4.20 and Theorem 4.24 follows, for example, from total blockedness.[5]

Lemma 4.25 *Suppose X is totally blocked and there exists a contingent sentence in X. Then there are $p, q \in X$ such that the sets $\{p, q\}, \{\neg p, q\}, \{p, \neg q\}$ are each consistent.*

Proof. Dietrich and List [6, Lemma 8] have shown that there exists a minimal inconsistent $Y \subseteq X$ of cardinality ≥ 3. By the compactness of propositional logic, Y must be finite. Let $Y = \{p, q, r_1, \ldots, r_m\}$ for some $m \geq 1$ and sentences $p, q, r_1, \ldots, r_m$ in X. Since $Y = \{p, q, r_1, \ldots, r_m\} \vdash \perp$, modus ponens yields $\{p, r_1, \ldots, r_m\} \vdash (q \to \perp)$, therefore $\{p, r_1, \ldots, r_m\} \vdash \neg q$ by modus tollens. But $\{p, r_1, \ldots, r_m\}$ is consistent as Y is minimal inconsistent, therefore $\{p, \neg q\}$ is consistent, too. Interchanging the rôles of p and q in this argument shows that $\{q, \neg p\}$ is consistent as well. Finally $Y = \{p, q, r_1, \ldots, r_m\}$ is minimal inconsistent, so $\{p, q\}$ must also be consistent. ■

Corollary 4.26 *Assume X is a totally blocked agenda which is closed under $\wedge$ and contains a contingent sentence. Then X is rich.*

Corollary 4.27 *Assume X is a rich, totally blocked agenda. Suppose $D^n \subseteq \mathbb{D}_f$. If $f : \mathbb{D}_f \to D'$ is independent as well as strictly unanimity-respecting, then f is oligarchic and hence monotonic as well.*

Proof. Since X is totally blocked, Lemma 4.13 ensures that f is systematic. Therefore, Lemma 4.25 allows us to apply Theorem 4.24. ■

Thus, we finally arrive at the meta-theorem announced earlier in this section:

Corollary 4.28 *Assume X is a rich, totally blocked agenda. Suppose $D^n \subseteq \mathbb{D}_f$. The aggregator $f : \mathbb{D}_f \to D'$ is both individually responsive and strictly unanimity-respecting if and only if it is oligarchic.*

[5]This was pointed out to the authors by Dr. Franz Dietrich (personal communication).

Proof. Let f be individually responsive and strictly unanimity-respecting. By Lemma 3.3. f is independent. Therefore, Corollary 4.27 shows that f is oligarchic.

Conversely, every oligarchic aggregator is not only strictly unanimity-respecting, but also independent and monotonic and hence individually responsive by Theorem 3.13. ■

An important consequence, Corollary 4.28 and Corollary 3.23 provide the following impossibility result:

Corollary 4.29 *Assume X is a rich, totally blocked agenda. Suppose $D^n \subseteq \mathbb{D}_f$. The aggregator $f : \mathbb{D}_f \to D'$ is independent, non-manipulable and strictly unanimity-respecting if and only if it is oligarchic.*

As shown by Dietrich and List [6, Theorem 3], total blockedness is not only sufficient, but even necessary for the oligarchy results in Corollaries 4.27, 4.28, 4.29, provided the population has at least three members.

If we impose the additional assumption that the range of f is contained in D, that the collective judgment set is complete, then Lemma 4.23 allows us to replace the attribute "oligarchic" by "dictatorial" in Theorem 4.24 and Corollaries 4.27, 4.28, and 4.29.

5 Discussion

Assuming a totally blocked agenda, we have shown that strictly unanimity-respecting judgment aggregators are oligarchic if they are either individually responsive or both indepedent and (in a weak sense) non-manipulable. However, individual responsiveness is also equivalent to the conjunction of independence and monotonicity. Thus, the independence property is an important ingredient in our meta-theorems for (oligarchic) impossibility results in judgment aggregation.

The concept of individual responsiveness can be seen as a justification for assuming judgment aggregation rules to be independent. An alternative approach in defending the independence property would be to invoke Dietrich and List's [4] strong concept of non-manipulability, which is equivalent to the conjunction of monotonocity and independence. However, this strong notion of non-manipulability is already equivalent to the conjunction of independence and a weaker, but simpler notion of non-manipulability. For this reason, independence should be vindicated via individual responsiveness rather than non-manipulability.

Summarizing, we have found three reasons why independence is a pivotal axiom in judgment aggregation:

1. Independence follows from individual responsiveness, the analogue of the meta-theorem condition of preference reversal.
2. For independent judgment aggregators, all non-manipulability and monotonicity properties are equivalent.
3. In combination with the Pareto principle (unanimity preservation), independence yields oligarchies under certain agenda conditions.

Mathematically, the independence axiom derives its importance from its intricate relation to systematicity, as systematicity is a key to the application of the filter method.

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