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The Economics of Cultural Formation of Preferences

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Abstract

This paper introduces a generalized representation of the formation of continuous preferences (which can reflect different intensities). The preference intensity that a child adopts is formed as the collective outcome of all role models for preference intensities — which are derived from the socioeconomic actions of adults — that it socially learns from. We then show how the adopted preference intensities induce preferences over socioeconomic choices. Finally, this cultural formation of preferences process is endogenized as resulting out of optimal parental socialization decisions. This framework thus endogenously determines the intergenerational evolution of preference intensities and the induced preferences.

Keywords: Socialization; Preference Evolution; Endogenous Preferences; Cultural transmission
JEL–Classification numbers: C72, J13, Z13

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Michael Pichler

1 Introduction

The concept of preferences is one of the most important cornerstones of economic theory, since preferences provide economic agents with the necessary means to choose between different possible socio–economic actions. The question of how preferences are being formed is thus of central interest to economic theory. The aim of the present paper is to contribute to the resolution of this question by providing a general framework that represents the *formation of continuous preferences*.

With the latter, we mean those types of preferences that can reflect different intensities (or magnitudes, valuations, strengths, importances...), located in a convex subset of the real line. Notably, this characterization is not very restrictive since, assumingly, most types of preferences can be (re–)interpreted in a continuous way (e.g. instead of asking whether a person has a 'status preference', one can ask *how important* status is for the person). Specifically, it contains preference types that are in standard use in economic theory, like the degree of altruism, the intensity of preferences for leisure or for social status, the patience (intensity), etc.; but notably, it also contains continuous cultural traits and concepts like the values, attitudes, (strength of) norms and 'continuous opinions' that a person adopts.

A natural question that arises in the context of this characterization of continuous preferences is then which of the possible intensities a person adopts, and how a process that determines this can be described in formal terms. Our approach will be to let the preference intensities be formed in the socialization period of a person, out of social learning from *role models* for preference intensities[1]. This latter concept has substance, since we derive it from the observable socio–economic actions of the adults. Given the preference intensity that a person has adopted at the beginning of its adult period, we show how this can be interpreted such as to induce preferences over the choices over the role models for preference intensities, thus the underlying socio–economic actions. The central importance of this step is that it closes the circle between the socio–economic actions taken by one adult generation and the preferences over these actions by the succeeding adult generation. We thus obtain a fully consistent and closed representation of the *evolution* (!) of the preference intensities and the induced preferences of a sequence of generations.

Related Literature By basing the formation of preferences process on the children's social learning, the approach of the present paper stands in a natural relation to the literature on the economics of cultural transmis-

[1] Our viewpoint will be primarily that of an economist, with references to findings in the socio–psychological literature on child socialization whenever needed. A thorough placement of the present paper within this literature is though far beyond scope. See e.g. Grusec and Hastings [29] and Grusec and Kuczynski [30] for related book long treatments.

sion[2]. This literature has been established by Bisin and Verdier [7, 8, 9] and Bisin et al. [6], and is based on the work of Cavalli-Sforza and Feldman [15, 16] and Boyd and Richerson [12] in evolutionary anthropology. It studies the population dynamics of the distribution of a discrete set of preferences (respectively cultural traits) under an endogenous intergenerational cultural transmission mechanism. The endogeneity stems from the purposeful parental choice of socialization intensity, which effectively determines the probability that the child will directly adopt the preferences of the parents. Parents engage into the cost of purposeful socialization in order to avoid (decrease the probability) that their child will not adopt their preferences — in which case parents encounter subjective utility losses[3].

However (as the name reveals), this theory considers the probabilistic *transmission* of preferences and does not approach the formation of the latter, restricting its applicability mainly to *discrete preferences* (respectively cultural traits). So far, little has been contributed to resolve the question of the cultural formation of continuous preferences. Important early treatments of the topic are Cavalli-Sforza and Feldman [16] in a theoretical, and Otto et al. [35] in an empirical context. More recently, both Bisin and Topa [5] and Panebianco [36] proposed representations of the formation of a preference intensity (respectively value of a cultural trait) as a weighted average between a role model that is taken by the family and the (weighted) average of the preference intensity in the population.

In this respect, the major limitation of both contributions is, however, that they do not explicitly consider the family's choice of role models (and also not the construction of role models themselves), but do (implicitly) assume that the family always chooses their 'target value', i.e. the optimal preference intensity, as a role model (Bisin and Topa [5]), respectively that the parent chooses a role model that is exactly in accordance with its preference intensity (Panebianco [36]). Given this degenerate view on the family's behavioral choices, the family's socialization decision is then restricted to choosing its weight in the formation of the preference intensity of their child.

[2] As Bisin and Verdier [7, p. 299] point out, this approach is thus distinct from models that are based on evolutionary selection mechanisms (where preferences/traits are either genetically inherited or imitated, with the reproductive/'imitative' success being increasing in the material payoff of the different preferences/traits), like Rogers [41], Bester and Güth [4], Fershtman and Weiss [21], Kockesen et al. [32], [24], and from models that deal with the agents' introspective self selection of preferences, as in e.g. Becker [2] and Becker and Mulligan [3].

[3] The properties of the model framework have been applied in several different contexts, such as e.g. preferences for social status (Bisin and Verdier [7]), voting and political ideology (Bisin and Verdier [8]), corruption (Hauk and Sáez-Martí [31]), hold up problems (Olcina and Penarrubia [34]), gender discrimination (Escriche et al. [20]), etc. For an overview over the literature on cultural transmission see Bisin and Verdier [10].

Contributions The present paper adds to this literature in a substantial way. In a first step, we introduce a suitable conceptualization of role models for preference intensities. We derive these from the choices of socio–economic actions of adults. Specifically, we assume that any feasible socio–economic action is characteristic for the display of exactly one intensity of the preferences. Thus, the *role models* for the children's social learning of preference intensities are the *displayed preference intensities* of the observed socio–economic actions of adults.

In a second step we then show how the preference intensity that a child adopts through the socialization process results as a weighted average between the displayed preference intensity that is *chosen* by its family[4] and the average displayed preference intensity that the child observes in its general adult social environment; and further, how the adopted preference intensities can then be interpreted to induce preferences over displayed preference intensities, respectively the underlying socio–economic actions.

In a third step we introduce one possible framework to endogenize this cultural formation of preferences process, which will be based on purposeful socialization decisions of parents[5]. That parents are willing to engage into associated costs of active socialization stems from the fact that they obtain an inter–generational utility component. Specifically, we let this utility be negatively related to the distance between the adopted preference intensity of their adult children and a parentally perceived *optimal* preference intensity.

The parental decision problem is it then to choose their weight in the child's socialization process (as in previous contributions) and their displayed preference intensity, given their perceived optimal preference intensity and given the average displayed preference intensity of the general social environment. Since the latter results of the individual parents' choices, this introduces strategic interaction. We characterize the corresponding parental best reply choices, and introduce conditions under which a Nash equilibrium in pure strategies exists. These equilibrium choices determine the inter–generational *evolution* of the preference intensities (and with it the preferences) of the society.

Outline The further outline of this paper is as follows. Section 2 introduces the general representation of the cultural formation of preferences process, while as section 3 delivers a framework for its endogeneization. The

[4]The family's choice of a displayed preference intensity can be interpreted as the generalized and continuous equivalent to the 'preference shaping demonstration effect' of Cox and Stark [17]; see also Stark [48].

[5]Alternative, and not elsewhere cited, approaches that deal with preference endogeneity in 'non–purposeful–socialization' frameworks are based on e.g. 'bandwagon' or 'snob' effects (Leibenstein [33]), 'keeping up with the Joneses' (Duesenberry [19]), 'emulation effects' (Veblen [49]) or 'interdependent preferences' (Pollak [38]).

proofs of the propositions in the latter section can be found in Appendix A. Section 4 discusses additional aspects that show routes how to apply the model, and section 5 concludes. Finally, Appendix B characterizes the dynamic evolution of the preference intensities if all parents have 'imperfect empathy' (this concept is due to Bisin and Verdier [7] and is shortly discussed in section 3.1).

2 Cultural Formation of Preferences

... or: We are all the sum total of our experiences.

In this section, we will show how children adopt intensities of any type of continuous preferences (e.g. 'patience (intensity)') through social learning from role models for preference intensities, and how the adopted preference intensities induce preference relations over choices of the role models in the adult life period. This kind of closed circle is the motivation to label the representation of the socialization process that this paper proposes as *cultural formation of preferences*.

Consider an overlapping generations society populated by a continuum of adults[6], $a \in A = [0, 1]$ endowed with Lebesgue measure λ, and their children. For ease of exposition, we will assume that reproduction is asexual and every adult has one offspring, so that we can denote with $\tilde{a} \in \tilde{A}$ the children of the parents $a \in A$.

Let us assume that all adults have available the same feasible set of socio–economic actions, $X \subseteq \mathbb{R}^n$. The structure of the latter is such that any typical element $x \in X$ is the characteristic role model for exactly one preference intensity (PI). We will call this the *displayed preference intensity* (DPI) of a choice of socio–economic actions x, $\phi^d(x) \in \mathbb{R}$.[7] Thus, there exists a displayed preference intensity function

$$\phi^d : X \mapsto \mathbb{R}$$

where $\phi^d(X)$ then corresponds to the set of possible DPIs[8]. Subsequently, it will be convenient to denote the DPI of the socio–economic actions of adult $a \in A$, $x_a \in X$, as $\phi^d_a := \phi^d(x_a)$.

Example 1 (Patience Preferences)**.** *Consider the case of 'patience preferences', and assume that there is only one socio–economic action category*

[6]The logic of the cultural formation of preferences process that is presented in the present paper would be preserved in the case where the set of adults is finite.

[7]This can be interpreted in the way that any adult who observes another adult $a \in A$ taking socio–economic actions $x \in X$ could reflect upon this observation by the statement that 'adult a behaves as if she would have a PI of $\phi^d(x)$'.

[8]The function ϕ^d assigns to any element of X a *relative* position in $\phi^d(X)$. Thus, any affine transformation of ϕ^d, $b + d\phi^d$, where $b \in \mathbb{R}$ and $d \in \mathbb{R}_{++}$, would represent the same DPIs, since it assigns the same relative positions in $b + d\phi^d(X)$.

that serves as a role model for the social learning of patience (intensity). Let this be the share of adult period income that is saved for pension period consumption. Denoting as $y_a \in \mathbb{R}_{++}$ *the adult period income, and as* $s_a \in [0, y_a]$ *the savings of adult* $a \in A$ *(there is no lending), we thus have that* $x_a \equiv \frac{s_a}{y_a} \in [0,1] \equiv X$*. Naturally, we want* ϕ^d *to be strictly increasing in the present case, so that we can simply choose* $\phi^d(x) = x$ *and then* $\phi^d(X) = [0,1]$.

We will now introduce the representation of the socialization process that this paper proposes. This will be established on grounds of the *tabula rasa* assumption, which means in the present context that children are born with undefined PI, and equally, with undefined preferences (a corresponding assumption is also taken in the literature on the economics of cultural transmission, see e.g. Bisin and Verdier [9]). This assumption implies that we restrict the analysis of the determination, respectively formation, of preferences to cultural factors ('nurture'), while as the issue of the contribution of genetic inheritance ('nature') is left aside[9].

On this basis, we then let the formation of the PI that a child adopts result out of social learning from the DPIs of adults (only) that it is confronted with. Specifically, this is being embedded in a framework of socialization inside the family and by the general adult social environment, or 'direct vertical and oblique socialization'. This terminology stems from Cavalli-Sforza and Feldman [16], and is distinguished from 'horizontal socialization', viz. the socialization influence of members of the same generation (which we leave unconsidered in the present paper). In this context, we will let the PI that a child $\tilde{a} \in \tilde{A}$ adopts be formed according to a weighted average between the DPI of its family, i.e. its single parent $a \in A$, $\phi_a^d \in \phi^d(X)$, and the *average* DPI of the (child's) general social environment[10], $A \backslash \{a\}$,

$$\int_{A \backslash \{a\}} \phi_{a'}^d \, d\lambda\left(a'\right) = \int_A \phi_{a'}^d \, d\lambda\left(a'\right) =: \phi_A^d \in \text{con } \phi^d(X).^{11}$$

The weight that the DPI of the parent of a child $\tilde{a} \in \tilde{A}$ has in the socialization process of the child will be called the *parental socialization success*

[9] An introduction to the cross–disciplinary 'nature–nurture' debate can be found in Rogers [41]; Sacerdote [42, 43, 44] provides for empirical investigations of the relative importances of both influences.

[10] Indeed, to require that the child's social learning from the general social environment is in terms of the *average* DPI of its members constitutes a strong and restrictive assumption. It means that all DPIs of the unrelated adults are assumed to have an identical social learning impact on the child. Both Sáez-Martí and Sjögren [45] (in the cultural transmission of preferences context) and Panebianco [36] (in the cultural formation of preferences context) introduce possible alternatives to this assumption.

[11] To see that the average choice of a continuum of players endowed with Lebesgue measure and with identical choice set (a subset of $\mathbb{R}^n$) is indeed located in the convex hull of the choice set, confer e.g. Rath [40, p. 430].

share, $\hat{\sigma}_a \in [0,1]$. This corresponds to the cognitive impact of the parental DPI relative to the cognitive impact of the general social environment's average DPI. Factors that would determine this relative cognitive impact would include the social(ization) interaction time of the parent with its child, as well as the effort and devotion that the parent spends to socialize its child to the chosen DPI[12].

We now obtain the formation of the PI that a child $\tilde{a} \in \tilde{A}$ adopts through the 'direct vertical and oblique socialization' process, $\phi_{\tilde{a}}$, as

$$\phi_{\tilde{a}} = \hat{\sigma}_a \phi_a^d + (1 - \hat{\sigma}_a)\phi_A^d. \tag{1}$$

We will call this the *parental socialization technique*[13]. It embodies the view that the parents set a PI benchmark, ϕ_a^d, and can invest into their parental socialization success share, $\hat{\sigma}_a$, to countervail the socialization influence that the child is exposed to in its general social environment, ϕ_A^d. Since the final adopted PI of a child is by construction a convex combination of all DPIs that it observes, the set of possible PIs (that a child can adopt), then coincides with the convex hull of the set of possible DPIs, $\operatorname{con} \phi^d(X) \subseteq \mathbb{R}$ (a convex subset of the real line).

Example 2 (Discrete Choice Sets)**.** *To illustrate the last point consider any discrete choice set of socio–economic actions, and let us take the simplest (non–degenerate) example where $X = \{0,1\}$, e.g. not buying or buying a status good. Let again $\phi^d(x) = x$, so that $\phi^d(X) = \{0,1\}$. However, under the formation of PIs (1), we have that the set of possible PIs is* $\operatorname{con} \phi^d(X) = [0,1]$. *Thus, although adults can only display through their socio–economic actions that they either disfavor/not have ($x = 0$) or favor/have ($x = 1$) a certain preference (e.g. 'status'), the children can adopt also any intermediate PI through the socialization process.*

We will assume that the PI that a child adopts through the socialization process is being internalized and kept in its adult life–period. Notably, the concept of an adopted PI of an adult corresponds to a *cognitive element* in the cognitive dissonance theory of Festinger [22] — and so does the concept of a DPI. According to the cognitive dissonance theory, people dislike dissonance between cognitive elements, the strength of which depends on the degree of the dissonance. In the present context, it is immediate that this degree of dissonance is being determined by the (Euclidean) distance between a DPI and the adopted PI. Thus, adults can compare and rank different DPIs based on their distance to the adopted PI. Obviously then, since socio–economic actions are pre–images of DPIs, the adopted PI of an

[12] See e.g. Grusec [27] for an introductory overview of theories on determinants of parental socialization success.

[13] Equation (1) is a generalization of the representation of the formation of continuous preferences (traits) in Bisin and Topa [5] and (with certain respects) Panebianco [36].

adult does also constitute a 'filter' under which adults can compare and rank different choices of socio–economic actions.

Assumption 1 (Preferences). *$\forall a \in A$,*

(a) the adopted PI, $\phi_a \in \text{con } \phi^d(X)$, induces a complete and transitive preference relation $\succ^{\phi_a}$ over DPIs $\phi^d_a \in \text{con } \phi^d(X)$,[14] *and*

(b) the preferences $\succ^{\phi_a}$ are single–peaked with peak ϕ_a. This means that $\forall \phi^d_a, {\phi'}^d_a \in \text{con } \phi^d(X)$, $\phi^d_a \succ^{\phi_a} {\phi'}^d_a \Leftarrow {\phi'}^d_a <> \phi^d_a \leq\geq \phi_a$.

Given their basic properties, we will represent the preferences $\succ^{\phi_a}$ by single–peaked utility functions with peak ϕ_a

$$u^{\phi_a} : \text{con } \phi^d(X) \mapsto \mathbb{R}$$

which are strictly increasing/decreasing at all $\phi^d_a \in \text{con } \phi^d(X)$ such that $\phi^d_a < / > \phi_a$.

Example 3 ('Displayed Patience' Utility). *Continuing the first example, assume that adults earn interest on their savings and, thus, their pension period consumption is $(1+r)s_a$, $r \in \mathbb{R}_+$ (prices are constant and there is no other pension period income and also no bequests).*

Assuming Cobb–Douglas utility, the life–time utility out of the adult savings decision can be represented as $u^{\phi_a}(s_a) = (y_a - s_a)^{1-\phi_a}((1+r)s_a)^{\phi_a}$, i.e. consumptions in the first and second life period are weighted according to the 'impatience' and 'patience' (intensities). Dividing and multiplying the right hand side of the latter by y_a, we obtain $u^{\phi_a}\left(\phi^d_a\right) = \left(1-\phi^d_a\right)^{1-\phi_a}\left(\phi^d_a\right)^{\phi_a} \cdot \left(y_a(1+r)^{\phi_a}\right)$. Thus, we have transformed utility out of a socio–economic choice into utility out of the choice of 'displayed patience (intensity)', ϕ^d_a. It is immediate that $\frac{\partial u^{\phi_a}\left(\phi^d_a\right)}{\partial \phi^d_a} >=< 0 \ \forall \phi^d_a \in [0,1]$ such that $\phi^d_a <=> \phi_a$ so that the single peak property is satisfied naturally (furthermore, u^{ϕ_a} is strictly concave).

3 Endogenous Cultural Formation of Preferences

. . . or: How far does the apple fall from the tree?

In the previous section, we have introduced a representation of the inter–generational formation of continuous preferences. One major innovation that this approach embodies is that it interconnects the socio–economic (respectively DPI) choices of the adult generation with the preferences over

[14] Equally, thus, $\phi_a \in \text{con } \phi^d(X)$, induces a complete and transitive preference relation $\succ^{\phi_a}$ over socio–economic actions $x_a \in X$, where $\forall x_a, x'_a \in X$, $x_a \succ^{\phi_a} x'_a \Leftrightarrow \phi^d(x_a) \succ^{\phi_a} \phi^d(x'_a)$.

the available choices that the next generation adults adopt. Thus, any model framework that determines the adult socio–economic (respectively DPI) choices, together with the parental socialization success shares, equally endogenizes the cultural formation of preferences process (see section 4 for a more detailed discussion). In the present section, we will lay down one specific way of achieving this endogeneization based on purposeful socialization decisions of parents.

3.1 Motivation for Purposeful Socialization

In a first step, we have to clarify what motivation parents have to actively engage in their children's socialization process, i.e. what induces them to purposefully employ their socialization technique (the functioning of which we assume here the parents to be fully aware of). Basically, we let this motivation stem from the fact that parents also obtain an inter–generational utility component that is either related to the adopted PI of their adult children and/or to the DPI (respectively the underlying socio–economic actions) that they expect their adult children to take.

As far as the latter expectations are concerned, we make here an assumption on a specific form of parental myopia: Although parents obtain an inter–generational utility component, which eventually induces them to choose a DPI that does not coincide with their adopted PI (see below), we assume that they do not realize that this form of behavior changing impact will also be present in their adult children's decision problems. Thus, any parent $a \in A$ expects its adult child to choose a DPI that is in the set of maximizers of its 'own' utility function, $\arg\max_{\phi^d_{\tilde{a}} \in \phi^d(X)} u^{\phi_{\tilde{a}}}\left(\phi^d_{\tilde{a}}\right)$. Under the following assumption, $\phi^d(X)$ is convex (and compact, which will be needed in the propositions below), and thus $\phi^d(X) = \text{con}\ \phi^d(X)$. This then guarantees by the single–peakedness of the utility functions that $\arg\max_{\phi^d_{\tilde{a}} \in \phi^d(X)} u^{\phi_{\tilde{a}}}\left(\phi^d_{\tilde{a}}\right) = \phi_{\tilde{a}}$, $\forall a \in A$, so that the parental expectations of their adult children's DPIs are uniquely determined[15].

Assumption 2 (Convexity and Compactness)**.** *$X \subseteq \mathbb{R}^n$ is non–empty, convex and compact, and ϕ^d is continuous. If $n > 1$, then ϕ^d is additionally concave.*

Given this form of myopic expectations, it is independent of whether the inter–generational utility component of a parent is related to the adopted PI or expected DPI of its adult child, since the latter coincides with the first. Under this property, we will now assume that any parent perceives an *optimal preference intensity*, such that if the adult child adopts this optimal

[15] That parents are not aware of the inter–generational utility of their children does also have the simplifying consequence that they do not care about their whole dynasty (this point has already been made by Bisin and Verdier [9, p. 305] in the context of cultural transmission of preferences).

PI, then this is considered by the parent to be 'inter–generational utility maximal'. These parent–specific optimal PIs are subject to what we call *construction rules*. Thereby, for any parent a construction rule is determined by two 'ingredients'. The first one specifies a (set of) subset(s) of adults, respectively reference group(s); and the second one specifies the construction of the optimal PI that a parent perceives out of characteristics of the adults in these reference group(s) that are either observable (notably the DPIs of adults) or known to an individual parent.

Definition 1 (Construction Rule)**.** *Let $\mathcal{A}$ be a σ–algebra of subsets of A. Then, the* construction rule *for the optimal PI perceived by parent $a \in A$ is a pair $\left(R_a, \hat{\phi}_{\tilde{a}}\right)$, where $\emptyset \neq R_a \subseteq \{a\} \cup \mathcal{A}$ and where $\hat{\phi}_{\tilde{a}} : \{a\} \cup \mathcal{A} \mapsto \text{con } \phi^d(X)$, $\hat{\phi}_{\tilde{a}}\left(R_a\right) \in \text{con } \phi^d(X)$.*

To ease the interpretation of this conceptualization, we will briefly introduce three sensible types of construction rules for optimal PIs (this list is not meant to be exhaustive — and notably, one could consider combinations of the three types mentioned).

CR 1 The optimal PI of a parent $a \in A$ is identical to its adopted PI, $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}\left(\{a\}\right) = \phi_a \in \text{con } \phi^d(X)$.

One justification to consider this construction rule is based on a special form of parental altruism called 'imperfect empathy'. This concept has been introduced into the economics literature by Bisin and Verdier [7]. Parents are altruistic and fully internalize the utility of their adult child's socio–economic actions (respectively DPI). Nevertheless, parents can not perfectly empathize with their child and can only evaluate their adult child's utility under their own (not the child's) utility function — which attains its maximum at the adopted PI of the parent.

CR 2 The optimal PI of a parent $a \in A$ is identical to a parent–specific (model–exogenous) PI, $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}\left(\{a\}\right) = e_a \in \text{con } \phi^d(X)$.

One motivation for this construction rule could be that the preference under scrutiny is a 'good preference' where parents thus want to maximize the PI of their adult children. This also would concern certain characteristics (preferences) that are e.g. favorable on the labor market (and thus increase the future expected income of the adult child, which the parents would aim to maximize if they are altruistic — and if their own utility function is increasing in monetary payoff).

CR 3 The optimal PI of a parent $a \in A$ is identical to the average DPI of a subset (with strictly positive measure) of the adults, $R_a \subseteq A$, and $\hat{\phi}_{\tilde{a}}\left(R_a\right) = \frac{1}{\lambda(R_a)} \int_{R_a} \phi^d_{a'} \, d\lambda\left(a'\right) \in \text{con } \phi^d(X)$.

> One potential justification for this construction rule is the case of *endogenous* behavioral norms that equate to the average DPI of the respective subset of the adults. Norms are typically maintained by members of a group (a subset of the adults) through a system of social rewards and punishments (see e.g. Arnett [1]). In the present context, these could be related to the parents' success or failure to guarantee that the child will behave according to the behavioral norm.

Given the construction rules and the resulting optimal PIs, we assume further that parents perceive utility losses for deviations of the adopted PI of their children from these optimal PIs (note the structural analogy to the before introduced preferences and utility that are induced by adopted PIs). Specifically, for any parent $a \in A$, we introduce the parameter $i_a \in \mathbb{R}_+$ that shall capture the strength of the perceived inter–generational utility losses. We will call this the parent's *inter–generational preference intensity*[16]. Notably, this latter type of PI can also be interpreted as being subject to a cultural formation of preferences process. Nevertheless, we choose here for simplicity a degenerate representation of this process and assume that the inter–generational PIs are invariably passed over from an adult to its child, $i_{\tilde{a}} = i_a$, $\forall a \in A$.

Assumption 3 (Inter–generational Utility). *$\forall a \in A$,*

(a) there is an inter–generational utility function $v^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\cdot\,|i_a\right) : \operatorname{con}\phi^d(X) \mapsto \mathbb{R}$, $v^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\phi_{\tilde{a}}\,|i_a\right) \in \mathbb{R}$, where

(b) $\forall i_a \in \mathbb{R}_{++}$, $v^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\cdot\,|i_a\right)$ is single–peaked with peak $\hat{\phi}_{\tilde{a}}\left(R_a\right)$, thus strictly increasing/decreasing at all $\phi_{\tilde{a}} \in \operatorname{con}\phi^d(X)$ such that $\phi_{\tilde{a}} < / > \hat{\phi}_{\tilde{a}}$.[17]

3.2 Optimization Problems and Best Replies

In the last step toward the construction of the parental optimization problems, let us finally discuss the cost associated with investments into controlling the parental socialization success share. These would concern e.g. the opportunity cost of the time parents spend for the active socialization of a child, as well as the (psychological) cost of the effort and devotion invested. We will represent these cost by an indirect cost function of choices of socialization success shares. Notably, we also allow for the dependence of

[16] In the case of the motivations given for the first and second type of construction rule, the inter–generational PI could be interpreted as the 'intensity of parental altruism'. In the case of behavioral norms (the motivation for the third type of CR), it could be interpreted as the (perceived) strength of the norm, i.e. how important conformism to the behavioral norm is for the members of the group. As a consequence, it then also reflects the intensity of the social rewards and punishments within that group.

[17] Under imperfect empathy of adult $a \in A$, one could straightforwardly specify $v^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\phi_{\tilde{a}}\,|i_a\right) = i_a u^{\phi_a}\left(\phi_{\tilde{a}}\right)$.

the cost of any such choice on the 'credibility' that children would assign to their parents' implicit claims that their proposed PIs (their choices of DPIs) are the optimal ones for the children to adopt. In the present context, it seems reasonable to let this 'credibility' depend on the level of satisfaction, i.e. utility, that the parents could generate out of their choices of DPIs[18]. For any $a \in A$, we thus propose a parental socialization success share cost function $c : [0,1] \times \mathbb{R} \mapsto \mathbb{R}_+$, $c\left(\hat{\sigma}_a, u^{\phi_a}\left(\phi_a^d\right)\right) \in \mathbb{R}_+$.[19]

The parental optimization problem is it then to choose a DPI and its socialization success share such as to maximize the life–time utility net of the cost of achieving the chosen socialization success share. Assuming additive separability of the (inter–generational) utility and cost functions (which significantly simplifies the subsequent analysis), we obtain, $\forall a \in A$,

$$\max_{\left(\phi_a^d, \hat{\sigma}_a\right) \in \phi^d(X) \times [0,1]} u^{\phi_a}\left(\phi_a^d\right) + v^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\phi_{\tilde{a}} \left| i_a\right.\right) - c\left(\hat{\sigma}_a, u^{\phi_a}\left(\phi_a^d\right)\right) \tag{2}$$
$$\text{s.t. } \phi_{\tilde{a}} = \hat{\sigma}_a \phi_a^d + (1 - \hat{\sigma}_a)\phi_A^d.$$

Thus, the optimization problems of the parents basically consist of trading off 'own' utility losses that they experience when choosing a DPI that deviates from their adopted PI together with the cost of a choice of their socialization success share against resulting inter–generational utility gains that they experience through improvements of the location of their children's adopted PI.

The first important issue to note with respect to these optimization problems is that they induce individual sets of pairs of best reply choices against the average DPI of the general social environment, and subject to the optimal PI, the adopted PI and the inter–generational PI. For any $a \in A$, we will therefore denote any of the pairs of best reply choices as $\left(\phi_a^d\left(\phi_A^d, \hat{\phi}_{\tilde{a}}\left(R_a\right), \phi_a, i_a\right), \hat{\sigma}_a\left(\phi_A^d, \hat{\phi}_{\tilde{a}}\left(R_a\right), \phi_a, i_a\right)\right)$, which will subsequently be abbreviated as $\left(\phi_a^d\left(\cdot\right), \hat{\sigma}_a\left(\cdot\right)\right)$. Furthermore, together with the average DPI of the general social environment, any of the parental best replies also determines a best reply location of the adult child's adopted PI (through the formation of PIs (1)), $\phi_{\tilde{a}}\left(\phi_a^d\left(\cdot\right), \hat{\sigma}_a\left(\cdot\right), \phi_A^d\right)$.

[18] We find (indirect) support of this hypothesis in Sears et al. [47] (the child's desire to imitate positive features of the parent), and in Grusec and Goodnow [28] (in the context of factors that determine the child's acceptance of parental messages).

[19] That parents can choose their socialization success shares within the whole unit interval is a non–trivial assumption (which is though also taken in Bisin and Topa [5] and Panebianco [36]). It means that both an exclusive parental socialization of a child, i.e. $\hat{\sigma}_a = 1$, as well as an exclusive socialization of a child by the general social environment, i.e. $\hat{\sigma}_a = 0$, are possible (as well as all intermediate cases).

Notably, the assumption that every adult $a \in A$ has available *all* choices $\left(\phi_a^d, \hat{\sigma}_a\right) \in$ con $\phi^d(X) \times [0,1]$ implies that through their socialization technique, parents can, irrespective of $\phi_A^d \in$ con $\phi^d(X)$, achieve *any* $\phi_{\tilde{a}} \in$ con $\phi^d(X)$ (which differs from previous contributions).

The following assumption specifies additional properties of the (inter–generational) utility and cost functions. These will allow for a significant characterization of the individual pairs of best reply choices, as well as of the resulting best reply locations of the adopted PIs of the adult children.

Assumption 4 (Slope). $\forall a \in A$,

(a) u^{ϕ_a} *and* $v^{\hat{\phi}_{\tilde{a}}(R_a)}(\cdot\,|i_a)$ *are continuous, and differentiable at their peaks,*

(b) c *is continuous, and differentiable with respect to the first argument at the point* $\hat{\sigma}_a = 0$*, where* $\frac{\partial c(0,\cdot)}{\partial \hat{\sigma}_a} = 0$*, strictly increasing in the first argument* $\forall \hat{\sigma}_a \in (0,1]$*, and decreasing in the second argument.*

Note that since both the utility and inter–generational utility function are single peaked, it follows under Assumption 4 (a) that $\forall a \in A$, $\frac{\partial\, u^{\phi_a}(\phi_a)}{\partial\, \phi_a^d} = 0$, as well as $\frac{\partial\, v^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\hat{\phi}_{\tilde{a}}(R_a), i_a\right)}{\partial\, \phi_{\tilde{a}}} = 0$. Thus, parents perceive zero (inter–generational) utility losses for marginal deviations of their chosen DPI from their adopted PI, respectively of their adult child's adopted PI from the optimal PI.

In the following two propositions, we assume that the construction rules for the optimal PIs of all parents are as such that the individual parents' decisions have (at most) a negligible impact on the location of their own optimal PI.

Proposition 1 (Characterization of Best Replies). *Let Assumptions 1–4 hold. Then, if*

(a) $\phi_A^d \neq \hat{\phi}_{\tilde{a}}(R_a)$*, generically*[20] $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$ *and* $\hat{\sigma}_a(\cdot) > 0$*, while always* $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$.

(b) $\phi_A^d = \hat{\phi}_{\tilde{a}}(R_a)$*, it holds that* $\phi_a^d(\cdot) - \phi_a = 0$ *and* $\hat{\sigma}_a(\cdot) = 0$*, hence* $\phi_{\tilde{a}}\left(\phi_a, 0, \hat{\phi}_{\tilde{a}}(R_a)\right) - \hat{\phi}_{\tilde{a}}(R_a) = 0$.

Proof. In Appendix A.

[20] There are two kinds of exceptions to the generic characterization. The first is that if the deviation of the best reply DPI from the adopted PI into the characterized direction is not possible, i.e. if the adopted PI of a parent coincides with (the relevant) one of the boundaries of $\phi^d(X)$, then the best reply DPI will coincide with that boundary (while as still generically $\hat{\sigma}_a(\cdot) > 0$). The second is that in the cases where $\hat{\phi}_{\tilde{a}}(R_a) > \phi_a$ and $\phi_A^d \in \left(\phi_a, \hat{\phi}_{\tilde{a}}(R_a)\right)$, respectively where $\hat{\phi}_{\tilde{a}}(R_a) < \phi_a$ and $\phi_A^d \in \left(\hat{\phi}_{\tilde{a}}(R_a), \phi_a\right)$, it can also hold that $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = 0$ and $\hat{\sigma}_a(\cdot) = 0$, hence $\phi_{\tilde{a}}\left(\phi_a, 0, \phi_A^d\right) = \phi_A^d$.

The (generic) results of this proposition are illustrated in Figure 1. The left pair of graphs stylizes case (a) of Proposition 1, and the right pair the case (b). In both pairs of graphs, in the left interval (all intervals correspond to the set of possible DPIs) the context of the adult's decision problem is depicted, while as in the right interval a corresponding best reply choice is stylized. As can be seen both from Proposition 1 directly, as well as from the graphical illustration, the results feature two dominant characteristics.

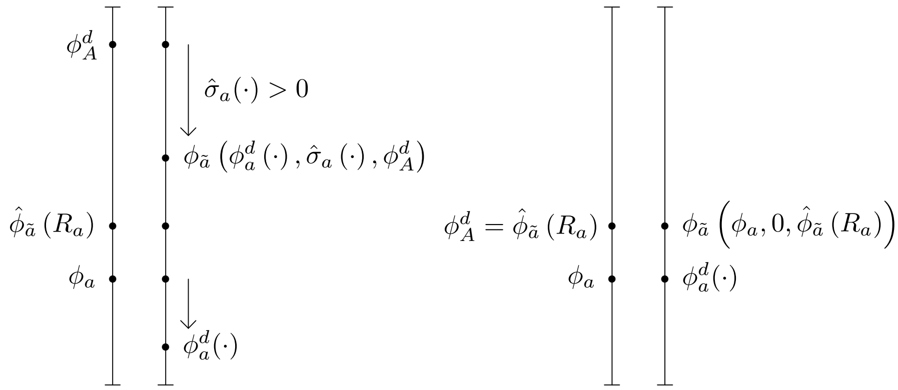

Figure 1: Characterization of Best Replies

The first concerns the generic location of the best reply choices. If the average DPI does not coincide with the optimal PI, then parents countervail the respective socialization influence on their children by choosing strictly positive *socialization instruments*. This means first that they choose a DPI that deviates from their adopted PI — and this deviation is into the opposite direction as the deviation of the average DPI from the optimal PI (if such a choice is available); and second, this behavioral countervailing is coupled with a strictly positive choice of their parental socialization success share (since otherwise, their chosen DPI would be fully ineffective in the child's socialization process).

That parents would generically choose strictly positive socialization instruments even for very small deviations of the average DPI from the optimal PI is due to the fact that marginal investments into the socialization instruments are (utility) costless (while as the resulting strictly positive decrease in the distance of the adult child's adopted PI from the optimal PI yields a strictly positive inter–generational utility gain). Obviously, if the average DPI exactly coincides with the optimal PI, then parents have no incentives to actively employ their socialization technique and choose zero investments into their socialization instruments.

The second dominant characteristic concerns the location of the adult children's adopted PIs that would result out of the parental best reply choices. Despite the parental countervailing in the case of suboptimal socialization influences of the general social environment, the investments into their socialization instruments would never be intense enough such as to guarantee that their adult children's adopted PIs would exactly coincide with the optimal PIs. Hence, there is always a strictly positive deviation of the adopted PI of an adult child from the parentally perceived optimal PI, the direction of which accords with the direction of deviation of the average DPI from the optimal DPI.

That the latter result holds for even very small deviations of the average DPI from the optimal DPI stems from the fact that parents do not perceive inter–generational utility losses for an only marginal deviation of the adult child's adopted PI from the optimal PI (while at any already strictly positive choice of the socialization instruments, the marginal cost of additional investments to further reduce the distance between the adult child's adopted PI and the optimal PI would be strictly positive). Again obviously, in the case where the average DPI is optimal, the adopted PI of an adult child will also coincide with the optimal PI.

The following list of assumptions will be prerequisite for a further characterization of the parental best reply choices in terms of comparative statics.

Assumption 5 (Curvature)**.** *$\forall a \in A$,*

(a) u^{ϕ_a} and $v^{\hat{\phi}_{\tilde{a}}(R_a)}(\cdot\,|i_a)$ are $\mathcal{C}^2$ and strictly concave, c is $\mathcal{C}^2$ and convex, and

(b) $\operatorname{sign}\left(\hat{\phi}_{\tilde{a}}(R_a) - \phi_{\tilde{a}}\right) \frac{\partial^2 v^{\hat{\phi}_{\tilde{a}}(R_a)}(\phi_{\tilde{a}}|i_a)}{\partial \phi_{\tilde{a}}\, \partial i_a} > 0$, i.e. the marginal cost of a deviation of the adopted PI of the adult child from the optimal PI is strictly increasing in the inter–generational PI.

Note that Assumption 5 (b) is only necessary for the results related to the second column of the comparative statics matrix below to hold.

Proposition 2 (Comparative Statics of Best Replies)**.** *Let Assumptions 1–5 be satisfied. Then, if $\phi^d_A \neq \hat{\phi}_{\tilde{a}}(R_a)$ and the optimization problem of parent $a \in A$ is strictly concave at its best reply choice, and if the two* socialization instruments $\left|\phi^d_a(\cdot) - \phi_a\right|$ *and $\hat{\sigma}_a(\cdot)$ are 'not too strong substitutes', then*[21]

$$\begin{pmatrix} \frac{\partial\left|\phi^d_a\left(\phi^d_A,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)-\phi_a\right|}{\partial\left|\phi^d_A-\hat{\phi}_{\tilde{a}}(R_a)\right|} & \frac{\partial\left|\phi^d_a\left(\phi^d_A,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)-\phi_a\right|}{\partial\, i_a} \\ \frac{\partial\,\hat{\sigma}_a\left(\phi^d_A,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)}{\partial\left|\phi^d_A-\hat{\phi}_{\tilde{a}}(R_a)\right|} & \frac{\partial\,\hat{\sigma}_a\left(\phi^d_A,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)}{\partial\, i_a} \end{pmatrix} \gg 0.$$

[21] A technical version of the latter condition can be found in the proof of this proposition. Note that these comparative statics are subject to a fixed location of the parental PI. Furthermore, we assume here that none of the constraints of the decision variables is binding at the best reply choices. This assumption rules out both kinds of 'non–generic' cases in Proposition 1 (in the second kind, the lower bound for the parental socialization success shares would be binding).

Proof. In Appendix A.

The first column of the comparative statics matrix shows that (under the relevant conditions), parents use their investments into their socialization instruments and the average DPI of the general social environment as *cultural substitutes*. This means that if the average DPI becomes more favorable (i.e. its distance to the optimal PI becomes smaller), then parents would reduce investments into both socialization instruments.

The second column sheds light on the role that the inter–generational PI plays in determining the parental socialization decisions. Under the conditions of Proposition 2, parents with a higher inter–generational PI would choose more intense investments into their socialization instruments for any given strictly positive distance between the average DPI and the optimal PI. This follows since the socialization PI basically determines the weight that parents put on their inter–generational utility. Thus, given a higher inter–generational PI, parents are willing to engage more 'own' utility losses and socialization success share cost such as to reduce their comparatively larger inter–generational utility losses.

3.3 Equilibrium and Evolution

After having characterized the individual best reply choices of a DPI and a parental socialization success share, the next step is to discuss the existence of a (pure strategy) Nash equilibrium of the game that is induced by the strategic interdependence of the individual parental choices. To do this, it will be important to carefully clarify the nature of the possible forms of the strategic interdependences, given the general model structure of the present section.

First of all, as has already been discussed, the net life–time utility of an individual parent, i.e. the object of its optimization problem (2), depends on the location of the average DPI of the general social environment (while as it is independent of the other parents' choices of their socialization success shares). Second, the decisions of the other adults could influence the net life–time utility of an individual parent via the construction rule for its optimal PI (as e.g. in the third type of construction rule introduced in section 3.1). In this respect, for the existence proposition below to hold, we will require the additional normalization that if the construction rule of a parent is based on the DPIs and/or socialization success shares of a subset of the adults, then this is only only in terms of the respective *average(s)*.

The general structure of the strategic interdependence of the individual parental decisions is then such that there is a finite partition of the adult set, $\{A_j\}_{j=1}^{k}$ (and $\mathcal{A}$ is the σ–algebra induced by this partition), where the net expected life–time utility of all individual parents depends on the tuple of pairs of average DPIs and average parental socialization success shares, $\left\{\phi^d_{A_j}, \hat{\sigma}_{A_j}\right\}_{j=1}^{k}$, where $\forall j = 1, \ldots, k$, $\phi^d_{A_j} := \frac{1}{\lambda(A_j)} \int_{A_j} \phi^d_{a'}\, d\lambda\left(a'\right)$ and $\hat{\sigma}_{A_j} := \frac{1}{\lambda(A_j)} \int_{A_j} \hat{\sigma}_{a'}\, d\lambda\left(a'\right)$.[22]

The payoff, i.e. the net life–time utility, that any parent gains out of its own decision pair and any given profile of pairs of average decisions of the subsets of adults is determined by the parent's adopted PI and inter–generational PI, as well as on the construction rule for its optimal PI. These triples thus fully characterize the adults, and we will denote the payoff function of an individual adult $a \in A$ as $\mathcal{P}\left(\cdot, \cdot \,\middle|\, \phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right) : \left(\phi^d(X) \times [0,1]\right)^{k+1} \mapsto \mathbb{R}$, $\mathcal{P}\left(\left(\phi^d_a, \hat{\sigma}_a\right), \left\{\phi^d_{A_j}, \hat{\sigma}_{A_j}\right\}_{j=1}^{k} \,\middle|\, \phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right) \in \mathbb{R}$.[23] We hence obtain a family of games, parametrized by the tuple of adult profile triples,

$$\Gamma\left(\left\{\phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right\}_{a \in A}\right) = \left(A, \left(\phi^d(X) \times [0,1]\right)^{A}, \left\{\mathcal{P}\left(\cdot, \cdot \,\middle|\, \phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right)\right\}_{a \in A}\right).$$

Definition 2 (Nash Equilibrium[24])**.** *Call a tuple* $\left\{\phi^{d^*}_a, \hat{\sigma}^*_a\right\}_{a \in A}$ *a* Nash equilibrium *of the game* $\Gamma\left(\left\{\phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right\}_{a \in A}\right)$*, if for almost all* $a \in A$, $\forall \left(\phi^d_a, \hat{\sigma}_a\right) \in \phi^d(X) \times [0,1]$, $\mathcal{P}\left(\left(\phi^{d^*}_a, \hat{\sigma}^*_a\right), \left\{\phi^{d^*}_{A_j}, \hat{\sigma}^*_{A_j}\right\}_{j=1}^{k} \,\middle|\, \phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right) \geq \mathcal{P}\left(\left(\phi^d_a, \hat{\sigma}_a\right), \left\{\phi^{d^*}_{A_j}, \hat{\sigma}^*_{A_j}\right\}_{j=1}^{k} \,\middle|\, \phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right)\right)$.

Proposition 3 (Nash Equilibrium Existence)**.** *If Assumptions 1—3 hold, and if the functions* $\hat{\phi}_{\tilde{a}}$ *are continuous for every* $a \in A$*, then a Nash equilibrium exists for any parametrized game.*

Proof. This proof is a straightforward generalization of the proof of Theorem 2 in Rath [40], and can be obtained from the author as a separate note.

[22]Note that we do explicitly *neither* require here that the construction rules for the optimal PIs of all parents are based on all (or even any) of the subsets of the partition of the adult set, *nor* that they depend both on the average DPI and the average parental socialization success share of a subset. In the first case, the optimal PIs would then simply be constant for all values that the average DPIs and average parental socialization success shares take on all 'irrelevant' subsets; and in the second case, this would be true for all values of either the average DPI or the average parental socialization success share of a subset concerned.

[23]The representation of the individual payoff functions does also incorporate the dependence of the parental payoffs on the average DPI of the general social environment, since $\phi^d_A = \sum_{j=1}^{k} \lambda\left(A_j\right) \phi^d_{A_j}$. Also note for completeness that since the parental payoffs are independent of the other parents' choices of socialization success shares, they are simply constant $\forall \hat{\sigma}_A = \sum_{j=1}^{k} \lambda\left(A_j\right) \hat{\sigma}_{A_j} \in [0,1]$.

[24]This definition follows Schmeidler [46] and Rath [40].

The existence result above means that in any given period, we can use (a selection of) the Nash equilibrium choices for substitution in the formation of PIs equation (1). By doing so, we obtain an endogenous representation of the inter–generational formation of PIs, i.e. we have endogenized the cultural formation of preferences process.

In a dynamic context, the model framework does hence endogenously determine the evolution of the DPIs and the underlying socio–economic choices (and the parental socialization success shares), as well as the evolution of the PIs and the induced preferences of a society. Notably, these dynamics will be subject to a specification of the (initial) tuple of adult profile triples. This means to specify (a) the initial tuple of PIs (which are the state variables of the model and evolve endogenously), (b) the inter–temporarily fixed tuple of inter–generational PIs, and (c) the tuple of construction rules for optimal PIs. Lacking a theory of the formation of the latter, it is sensible to assume for simplicity that they are (like the inter–generational PIs) invariantly passed over from a parent to its child, hence inter–temporarily fixed. Furthermore, to impose a minimum level of structure on the analysis, it would in any case be sensible to consider only assignments of equal types of construction rules to all parents (e.g. one of the three types of construction rules introduced in section 3.1).

Notably, among the three types of (initial) adult profile tuples, it is the specification of the tuple of construction rules that can be supposed to most centrally govern the qualitative properties of the dynamics of any specified model (within the framework of the present section). Roughly spoken, the reasoning for this is that in any given period, the optimal PIs determine the direction of the purposeful socialization efforts of the parents, independent of the contextual effects that are induced by their adopted PIs (and the fixed inter–generational PIs govern the relative strength of the two effects).

This 'power' of the tuple of construction rules can be illustrated by means of a particularly simple example. Consider the case where all parents have the first (imperfect empathy) type of construction rules. Then, it is easy to show (and we did in Appendix B) that the following results hold for every possible tuple of pairs of initial PIs and inter–generational PIs: (a) Between any two succeeding periods, the PIs of the adults assimilate (almost surely), which means that the minimum PI that any of the adults has adopted is strictly increasing while the maximum PI that any of the adults has adopted is strictly decreasing (but the PI dispersion stays strictly positive) and thus (b) the tuple of PIs of the adults converges to a point where the PIs of all adults are identical, and (c) any such point is a steady state.

Of course, not all possible specifications of the tuple of construction rules will yield as simple characteristics of the resulting dynamics. In any case, it shall have become clear from the above discussion that any significant qualitative characterization of dynamic properties will have to be based on a sensible specification of the model framework of the present section.

4 Applications

In the preceding two sections, we have laid down a general framework to determine the inter–generational formation of continuous preferences. Given its generality, this framework can be specified for applications in a large variety of different settings and socio–economic questions. In what follows, we will briefly outline four different dimensions along the lines of which any application, respectively specification, of the model could be oriented.

Level of the Analysis Any analysis of the properties of a specified model can be pursued on two different levels. The first, 'meta–level analysis', takes place at the level of the intensities of the preference under scrutiny, and concerns the evolution of the PIs and DPIs, as discussed already above. Interesting issues in this context would then typically be to characterize the dynamics of the model under different specifications of the tuple of (initial) adult triples. Specifically, it would be of interest to identify specifications of tuples of construction rules under which (stable) heterogeneous and/or homogeneous steady state distributions of the PIs exist (for the first type of construction rule as presented in section 3.1, we could show that only homogeneous steady state distributions can exist; see Appendix B).

The second, 'empirical analysis', would take place at the level of the observable socio–economic choices of the adults. For this end, it would be necessary to clarify (a) which socio–economic choices are supposed to serve as the role models for the social learning of the intensities of the preference under scrutiny, and (b) how the relationship between the socio–economic choices and the DPIs can be represented in terms of the DPI function. Given this, the 'meta–level analysis' would additionally answer the question of the evolution of the underlying socio–economic choices.

Complexity of the Adult Problem The *purposeful socialization* framework of section 3 embeds parents that are endowed with inter–generational concern in a strategic socialization interaction environment, in which they choose optimal DPIs and socialization success shares. This structure entails a certain degree of complexity, which could, however, be decreased by employing alternative (less 'rich') designs of the parental optimization problems. These would either feature a lower dimensionality and/or would eliminate the strategic socialization interaction — and it depends indeed on the specific application, which of these alternatives would eventually be suitable.

One alternative that reduces the dimensionality of the parental optimization problem would be to assign (strictly positive) exogenous socialization success shares[25], but to leave endogenous the choices of DPIs. Even, by

[25] This would notably have the consequence that the 'power' of the parental socialization

setting the socialization success shares equal to one so that the children are exclusively socialized by their parents, one could additionally eliminate the strategic socialization interaction in the choices of DPIs (while as other forms of strategic interaction could then still be introduced into the model). Another alternative would obviously be to exogenously fix the chosen DPIs of the parents while as the decision of their socialization success shares is left endogenous (as in Bisin and Topa [5] and Panebianco [36]). This approach would also additionally eliminate the strategic socialization interaction[26].

The double effect of reducing the dimensionality of the parents' decision problems as well as doing away with the strategic socialization interaction could furthermore be achieved by considering a *naive socialization framework* where the adults (parents) fully neglect the children's preference formation process or are not aware of it[27] — while this process is still taking place. In such a setting, one would again have to assign (exogenous) parental socialization success shares[28]. Notably, in the competitive socio–economy version of such a model, all adults would always choose to behave exactly in accordance with their adopted PI. This follows since the parents would lack the behavior shifting incentives that would be created by the presence of a (non–constantly zero) inter–generational utility component. Thus, one would typically aim at giving additional substance to such a framework, e.g. by introducing alternative forms of strategic interaction, or by considering a social planner problem (as discussed below).

Finally, one could eliminate the strategic interaction in the decision problems by basing these on the parents' *expectations* of the average DPI of the general social environment (which would sensibly be based on the average DPI that the adults have observed in their child period). The drawback of this approach would be that one could not allow for the alteration of the parents' decisions upon observations of average DPIs that do deviate from the expectations. Thus, on the transitory path, parents would generically

technique is being reduced in that the parents can no longer control the adopted PIs of their children within the whole set of possible PIs. Rather, the sets of PIs that parents can achieve for their children given their fixed (and non–equal one) socialization success shares depend in this case on the location of the average DPI of the general social environment.

[26] A remark analogous to the one in the previous footnote applies here.

Also note that the latter approach would do away with the possibility of *consistently* introducing into the model other forms of strategic interaction that depend on the DPIs, respectively the underlying socio–economic choices of the adults.

[27] Parents would fully neglect the process if their inter–generational PIs would be zero. In this case their inter–generational utility would be constantly zero (for all choices that they have available). In the case where parents have an additional form of myopia and are not aware of the PI formation process, they simply lack the inter–generational utility component at all.

[28] In the simplest possible way, one could even assign to the parental socialization success shares the value zero so that effectively, there is oblique socialization only. Thus, all children of a society would adopt the same PI, which coincides with the average DPI of the general social environment.

not choose best reply choices against the true realized average DPI of the general social environment.

Social Planner Problem Given the closed circle between the adopted PIs of the adults, their chosen DPIs (and underlying socio–economic actions) and the induced adopted PIs and preferences of the next adult generation, the cultural formation of preferences frameworks opens routes toward new kinds of social planner problems.

To discuss these, let us first of all clarify possible ways how a social planner could intervene in the cultural formation of preferences process. The first way would be targeted directly at the 'meta–level' of the PIs, and would primarily concern the social planner serving for an additional source of child socialization, next to the family and the general social environment. This could e.g. be in the form of the influence that the designs of the legal system and the institutions (including schools and media) of a society have in the socialization process of a child; see Bowles [11] for an overview of related issues. Within the terminology of the present paper, the social planner could thus effectively set a DPI coupled with (investments into) its socialization success *relative* to the socialization successes of the family and the general social environment[29].

The second possible way of social planner intervention is only indirectly targeted at the level of the PIs, and would concern 'standard' socio–economic incentive shifting policies, like e.g. a consumption tax or pension schemes in the context of first and third example in section 2. Since these measures are designed such to influence the adults' socio–economic decisions, the same is being achieved in terms of the corresponding adults' choices of DPIs — which in turn influences the formation of the PIs of the children.

Let us now discuss the possible motivations of a social planner to actively employ its socialization technique. The first motivation can result out of the usual form of a benevolent social planner's aim of maximizing the weighted sum of the life–time utilities of a sequence of generations. Notably, since the social planner is assumed to be aware of the inter–temporal externalities that are inherent in the cultural formation of preferences process, she has, via her two ways of intervention, access to a new level of efficiency: She can inter–connect the question of the optimal inter–generational distribution of utilities with the question of the optimal inter–generational distribution of utility functions, since they are determined by the cultural formation of

[29]Let us shortly discuss the most simple ad–hoc formal representation of the 'socialization technique of the social planner'. Assume that for every $\tilde{a} \in \tilde{A}$ the social planner, g, can choose (eventually child–specific) DPIs, $\phi^d_{g\tilde{a}}$, within a choice set of possible DPIs (which we leave unspecified here and which can, but does not have to, coincide with the set of possible DPIs of the adults), and can determine its (eventually child–specific) socialization success shares, $\hat{\sigma}_{g\tilde{a}}$, within an also unspecified subset of the unit interval. Then, its socialization technique for child $\tilde{a} \in \tilde{A}$ could be $\phi_{\tilde{a}} = \hat{\sigma}_{g\tilde{a}}\phi^d_{g\tilde{a}} + (1 - \hat{\sigma}_{g\tilde{a}})\left(\hat{\sigma}_a\phi^d_a + (1 - \hat{\sigma}_a)\phi^d_A\right)$.

preferences process.

The second motivation can be in terms of the social planner perceiving, respectively having information about, a socially optimal (distribution of) the PIs and/or DPIs within the society, which it aims at instilling in a paternalistic way; see e.g. Qizilbash [39] for a discussion of related issues. The typical question would then be whether the social planner can design a transitory policy regime such as to achieve this form of social optimum in the steady state.

Structure of the (initial) Adult Profile Triples Additionally to what has already been said above, it could be of interest to characterize the properties of a specified model for different degrees of symmetry embodied in the distribution of the (initial) adult profile triples on the adult set. Obviously, the maximum symmetry would be achieved in the case of a representative agent model, while as the minimum symmetry would correspond to assigning any arbitrary distribution of (initial) adult profile triples on the adult set.

As an intermediate step, one could partition the adult set into subsets of adults that have identical (initial) adult profile triples. Thus, one would obtain a discrete set of adult types, which could be interpreted as *cultural groups*. Under suitable conditions that guarantee the inter–temporal PI symmetry of the members of the groups, one could then answer the question of behavioral (DPI) and cultural (PI) assimilation of the groups[30]. Within the present continuous preferences framework, this would constitute the analogue to the analysis on the dynamics of the population distribution of discrete preferences in the economics of cultural transmission of preferences literature.

5 Conclusions

This paper has introduced a general representation of the formation of continuous preferences. We showed in the first main part of this paper (section 2) how children adopt preference intensities through social learning from role models for preference intensities that they observe in their social environment. Thereby, we derived these role models, which we call *displayed preference intensities*, from the socio–economic actions of adults. We then showed how to interpret the preference intensities that adults have adopted such as to construct and characterize preferences over displayed preference intensities, respectively the underlying socio–economic actions. The representation of the socialization process that this paper proposes thus constitutes a consistent and closed circle between the socio–economic actions

[30]The author is currently working on such an analysis in a two cultural groups setting. The work in progress of this paper, Pichler [37], is available upon request.

taken by one adult generation and the preferences over these actions by the succeeding adult generation.

In the second main part of the paper (section 3), we proposed one possible way to endogenize the cultural formation of preference process as resulting out of purposeful parental socialization decisions. On the one hand, these consist of the choice of a displayed preference intensity; and on the other hand they consist on investments into the weight that this role model has in the socialization process of the child relative to the weight that the observed average displayed preference intensity of the general social environment has. Thus, basically, the parents decision problem is to choose best replies against this representative role model of the general social environment, subject to the location of the optimal preference intensity that they would like their children to adopt.

We could show that, generically, whenever the average displayed preference intensity of the general social environment does not coincide with the optimal preference intensity, then parents countervail this suboptimal socialization influence on their children by choosing strictly positive *socialization instruments*. This means first that they choose a displayed preference intensity that deviates from their (utility maximal) adopted preference intensity — and this deviation is into the opposite direction as the deviation of the average displayed preference intensity from the optimal preference intensity; and second, this behavioral countervailing is coupled with a strictly positive choice of their relative socialization weight.

Furthermore, we could show that under certain conditions, parents use their investments into their socialization instruments and the representative role model of the general social environment as *cultural substitutes*. This means that if the representative role model of the general social environment becomes more favorable (i.e. its distance to the optimal preference intensity becomes smaller), then parents would reduce investments into both socialization instruments.

Finally, we showed conditions under which a pure strategy Nash equilibrium of the 'strategic socialization interaction game' of the parents exists. These equilibrium choices determine the inter–generational *evolution* of the preference intensities and the preferences of the society.

The power of the model that is presented in the present paper arguably lies in its generality which allows for a large number of possible forms of adoptions and specifications such as to apply it to an accordingly large variety of different socio–economic questions. In section 4, we also outlined lines along which any such application could be oriented. But notably, these lines were embedded within the basic structure of the present model, which is the formation of continuous preferences in the *socialization period* of a person. However, the very logic of the processes described stands in a natural relation to models that are concerned with the formation/evolution/adaption/learning of any kind of preferences/attributes/opinions with continuous structure (in the sense of the present paper) in the *adult period* of a person. Among others, Friedkin and Johnson [23], Demarzo et al. [18], Brueckner and Smirnov [13, 14] and Golub and Jackson [25, 26] provide for related analyses within a social network structure.

Despite the generality of the model presented, there is however still considerable room for further generalizations. Among other possible directions, this would concern (a) considering an n–dimensional representation of the formation of continuous preferences with an optional endogeneization of the formation of the inter–generational preference intensities (although the latter extension would substantially increase the analytical complexity of the model), (b) endogenously determining the formation of the construction rules of parents, (c) consistently introducing 'horizontal socialization' and the socialization influence of institutions (like the legal system, schools, media, etc.), (d) dropping the assumption of asexual reproduction and potentially endogenizing the reproduction decision, (e) allowing for a pro–active role of the children in the formation process of their preferences, and (f) considering a representation of displayed preference intensities subject to heterogeneous choice sets of socio–economic actions. Finally, instead of considering a continuum of adult decision makers, one could embed the endogenous cultural formation of preferences framework in a finite population setting.

A Proofs

Proof of Proposition 1 First note that since by Assumption 4, the target functions of the parental optimization problems (2) are continuous and since the choice sets are compact (Assumption 2), a non–empty set of maximizers, i.e. parental best reply choices, must exist. Consider below any $a \in A$.

Case $\phi_A^d \neq \hat{\phi}_{\tilde{a}}(R_a)$: It will be sensible to start the proof of this case by showing the second part first. Assume, by way of contradiction, that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$. For this to hold, it would necessarily have to hold that $\operatorname{sign}\left(\phi_a^d(\cdot) - \hat{\phi}_{\tilde{a}}(R_a)\right) = -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$ together with $\hat{\sigma}_a(\cdot) > 0$. But this can never be subject to a best reply choice, since e.g. the choice of (the same) $\phi_a^d = \phi_a^d(\cdot)$ together with a $\hat{\sigma}_a < \hat{\sigma}_a(\cdot)$ such that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a, \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = 0$ would yield the same 'own' utility, but strictly larger inter–generational utility as well as strictly lower socialization success share cost. Now assume that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = 0$, for which to hold it would be necessary that $\operatorname{sign}\left(\phi_a^d(\cdot) - \hat{\phi}_{\tilde{a}}(R_a)\right) \in \left\{0, -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)\right\}$ together with $\hat{\sigma}_a(\cdot) > 0$. In this case, the slope of the inter–generational utility function is zero, while the slope of the socialization success share cost function is strictly positive. From this, it follows that there is always an alternative choice pair where $\phi_a^d = \phi_a^d(\cdot)$ and $\hat{\sigma}_a < \hat{\sigma}_a(\cdot)$, thus $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a, \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$, but for which it holds that the resulting reduction in the socialization success share cost strictly dominates the inter–generational utility loss. It thus must hold that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$.

We will now show the first part of the proof for the present case. Assume, again by way of contradiction, that $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = \operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$ and $\hat{\sigma}_a(\cdot) \in [0,1]$. From above, we know that under the present assumption $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = \operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$. It then follows that there always exists an alternative choice pair where $\hat{\sigma}_a = \hat{\sigma}_a(\cdot)$, and where $\operatorname{sign}\left(\phi_a^d - \phi_a\right) = \operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right)$ but $\left|\phi_a^d - \phi_a\right| < \left|\phi_a^d(\cdot) - \phi_a\right|$, and $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$ but $\left|\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right| \leq \left|\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right|$. Such a choice yields (a) strictly larger 'own' utility, (b) larger inter–generational utility and (c) less cost of achieving $\hat{\sigma}_a(\cdot)$ given (a). Thus, the best replies must satisfy $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) \in \left\{0, -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)\right\}$.

Assume next that $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$ and $\hat{\sigma}_a(\cdot) = 0$. But this can not be a best reply since the choice $\phi_a^d = \phi_a$ and $\hat{\sigma}_a = \hat{\sigma}_a(\cdot) = 0$ would yield (a) strictly larger 'own' utility and (b) identical inter–generational utility and identical socialization success share cost. Hence $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) \in \left\{(0,0), (0,+1), \left(-\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right), +1\right)\right\}$.

Let us from now on consider the case where a choice pair that satisfies the third sign combination of above is available, i.e. the adopted PI does not coincide with the relevant boundary of $\phi^d(X)$.[31] We first rule out that nevertheless $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) = (0,+1)$. To see that this can never be a best reply note that at such a choice, the slope of the 'own' utility function is zero. It then follows that there always exists a choice pair where $\hat{\sigma}_a = \hat{\sigma}_a(\cdot)$, and where $\operatorname{sign}\left(\phi_a^d - \phi_a\right) = -\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$, $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$ but $\left|\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right| < \left|\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right|$, such that the resulting strictly positive gain in inter–generational utility strictly

[31] In the other case, then the best replies satisfy $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) \in \{(0,0), (0,+1)\}$. To see that if $\hat{\phi}_{\tilde{a}}(R_a) \geq \phi_a$ and $\phi_A^d \notin \left(\phi_a, \hat{\phi}_{\tilde{a}}(R_a)\right)$, or $\hat{\phi}_{\tilde{a}}(R_a) \leq \phi_a$ and $\phi_A^d \notin \left(\hat{\phi}_{\tilde{a}}(R_a), \phi_a\right)$, then the best replies must satisfy the second sign combination follows basically the same line of argumentation as in the rest of the proof below.

dominates the combined loss in 'own' utility and the increase in the socialization success share cost.

Finally, consider the cases where $\hat{\phi}_{\tilde{a}}\left(R_a\right) \geq \phi_a$ and $\phi_A^d \notin \left(\phi_a, \hat{\phi}_{\tilde{a}}\left(R_a\right)\right)$, or $\hat{\phi}_{\tilde{a}}\left(R_a\right) \leq \phi_a$ and $\phi_A^d \notin \left(\hat{\phi}_{\tilde{a}}\left(R_a\right), \phi_a\right)$.[32] It rests to show that in these cases $\operatorname{sign}\left(\hat{\sigma}_a\left(\cdot\right), \phi_a^d\left(\cdot\right) - \phi_a\right) = (0,0)$ can not be subject to a best reply[33]. To see this, note that at such a choice, both the slope of the socialization success share cost function and the slope of the 'own' utility function are zero. But this then again implies that there always exists an alternative choice where $\operatorname{sign}\left(\phi_a^d - \phi_a, \hat{\sigma}_a\right) = \left(-\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right), +1\right)$, $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a, \phi_A^d\right) - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right) = \operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d\left(\cdot\right), \hat{\sigma}_a\left(\cdot\right), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right)$, but $\left|\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a, \phi_A^d\right) - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right| < \left|\phi_{\tilde{a}}\left(\phi_a^d\left(\cdot\right), \hat{\sigma}_a\left(\cdot\right), \phi_A^d\right) - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right|$, and such that the resulting strictly positive gain in inter–generational utility strictly dominates the combined loss in 'own' utility and the increase in the socialization success share cost.

Case $\phi_A^d = \hat{\phi}_{\tilde{a}}\left(R_a\right)$: These best reply choices yield the maximum possible net life–time utility.

Proof of Proposition 2 Denote the Lagrangean of the optimization problem (2) of an adult $a \in \mathcal{A}$ as $\mathcal{L}\left(\phi_a^d, \hat{\sigma}_a \middle| \phi_A^d, \hat{\phi}_{\tilde{a}}\left(R_a\right), \phi_a, i_a\right)$, which we will abbreviate subsequently as $\mathcal{L}\left(\phi_a^d, \hat{\sigma}_a \left|\cdot\right.\right)$. Any pair of best replies, $\left(\phi_a^d\left(\cdot\right), \hat{\sigma}_a\left(\cdot\right)\right)$ must satisfy the first order conditions. Further, since we assume that the optimization problem is strictly concave at this best reply choice (so that the determinant of the Hessian matrix is strictly positive), all conditions for the Implicit Function Theorem are satisfied.

We will now show that $\exists \left|b_a\right| \in \mathbb{R}_{++}$, such that if $\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot) \left|\cdot\right.\right)}{\partial \left|\phi_a^d - \phi_a\right| \partial \hat{\sigma}_a} > -\left|b_a\right|$ i.e. the two socialization instruments are 'not too strong substitutes' at the parental best reply choice, then the desired signs of Proposition 2 hold.

To do this, we will transform the representation of the comparative statics matrix of Proposition 2 into a representation that involves only the sensitivities of the best reply choices to the relevant parameters. For this, it will be convenient to distinguish the cases where $\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right) = +1/-1$, so that by Proposition 1, it generically holds that $\operatorname{sign}\left(\phi_a^d\left(\cdot\right) - \phi_a\right) = -1/+1$ (the other, 'non–generic', cases are disregarded in Proposition 2). Thus, for the results in the first row of the matrix in Proposition 2 to hold, we require that

$$\operatorname{sign}\begin{pmatrix} \dfrac{\partial \phi_a^d\left(\cdot\right)}{\partial \left|\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right|} & \dfrac{\partial \phi_a^d\left(\cdot\right)}{\partial i_a} \end{pmatrix} = \left(-1/+1 \quad -1/+1\right). \tag{A.1}$$

[32] In the other cases, no further restriction of the signs is possible, so that we have that $\operatorname{sign}\left(\phi_a^d\left(\cdot\right) - \phi_a, \hat{\sigma}_a\left(\cdot\right)\right) \in \left\{\left(-\operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right), +1\right), (0,0)\right\}$.

[33] Except for the special case $\phi_A^d = \hat{\phi}_{\tilde{a}}\left(R_a\right) = \phi_a$, see below.

Next, note that $\left|\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right| = \operatorname{sign}\left(\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right)\left(\phi_A^d - \hat{\phi}_{\tilde{a}}\left(R_a\right)\right)$, so that the entries of the first column of the matrix of Proposition 2 could be decomposed accordingly. It is straightforward to show (by the Implicit Function Theorem) that

$$\operatorname{sign}\left(\frac{\partial\,\phi_a^d\left(\cdot\right)}{\partial\,\hat{\phi}_{\tilde{a}}\left(R_a\right)}, \frac{\partial\,\hat{\sigma}_a\left(\cdot\right)}{\partial\,\hat{\phi}_{\tilde{a}}\left(R_a\right)}\right)' = -\operatorname{sign}\left(\frac{\partial\,\phi_a^d\left(\cdot\right)}{\partial\,\phi_A^d}, \frac{\partial\,\hat{\sigma}_a\left(\cdot\right)}{\partial\,\phi_A^d}\right)'$$

and, thus, as far as the signs of the comparative statics are concerned, it is irrelevant, how a marginal change in the absolute distance between ϕ_A^d and $\hat{\phi}_{\tilde{a}}\left(R_a\right)$ is 'composed', and we can restrict our attention to marginal changes of ϕ_A^d only. Thus, for (A.1) to hold, it is necessary that

$$\operatorname{sign}\begin{pmatrix} \frac{\partial\,\phi_a^d(\cdot)}{\partial\,\phi_A^d} & \frac{\partial\,\phi_a^d(\cdot)}{\partial\,i_a} \\ \frac{\partial\,\hat{\sigma}_a(\cdot)}{\partial\,\phi_A^d} & \frac{\partial\,\hat{\sigma}_a(\cdot)}{\partial\,i_a} \end{pmatrix} = \begin{pmatrix} -1/-1 & -1/+1 \\ +1/-1 & +1/+1 \end{pmatrix}. \tag{A.2}$$

We can now use the Implicit Function Theorem to derive a necessary condition for these signs to hold. First note that since the Lagrangean is strictly concave at the best reply choice, the second partial derivatives with respect to the two decision variables are strictly negative, while as the cross second partial derivative

$$\frac{\partial^2\,\mathcal{L}\left(\phi_a^d\left(\cdot\right), \hat{\sigma}_a(\cdot)\,|\cdot\right)}{\partial\,\phi_a^d\,\partial\,\hat{\sigma}_a} = \frac{\partial^2\,i^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\phi_{\tilde{a}}\left(\cdot\right)\right)}{\partial\,\phi_{\tilde{a}}^2}\hat{\sigma}_a(\cdot)\left(\phi_a^d\left(\cdot\right) - \phi_A^d\right) + \frac{\partial\,i^{\hat{\phi}_{\tilde{a}}(R_a)}\left(\phi_{\tilde{a}}\left(\cdot\right)\right)}{\partial\,\phi_{\tilde{a}}} - \\ - \frac{\partial\,u^{\phi_a}\left(\phi_a^d\left(\cdot\right)\right)}{\partial\,\phi_a^d}\frac{\partial^2\,c\left(\hat{\sigma}_a(\cdot), \partial\,u^{\phi_a}\left(\phi_a^d\left(\cdot\right)\right)\right)}{\partial\,u^{\phi_a}\left(\phi_a^d\right)\partial\,\hat{\sigma}_a}$$

is ambiguous in sign. It is furthermore straightforward to show that

$$\operatorname{sign}\begin{pmatrix} \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,\phi_A^d} & \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,i_a} \\ \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a\,\partial\,\phi_A^d} & \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a\,\partial\,i_a} \end{pmatrix} = \begin{pmatrix} -1/-1 & -1/+1 \\ +1/-1 & +1/+1 \end{pmatrix}.$$

Given these signs, it follows from the Implicit Function Theorem that (A.2) is true if $\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,\hat{\sigma}_a} < / > b_a \in \mathbb{R}_{++} / \mathbb{R}_{--}$ where

$$b_a = \min / \max \\ \begin{pmatrix} \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a^2}\frac{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,\phi_A^d}}{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a\,\partial\,\phi_A^d}} & \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a^2}\frac{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,i_a}}{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a\,\partial\,i_a}} \\ \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,{\phi_a^d}^2}\frac{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a\,\partial\,\phi_A^d}}{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,\phi_A^d}} & \frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,{\phi_a^d}^2}\frac{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\hat{\sigma}_a\,\partial\,i_a}}{\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\,\phi_a^d\,\partial\,i_a}} \end{pmatrix}$$

Remembering that $\operatorname{sign}\left(\phi_a^d\left(\cdot\right) - \phi_a\right) = -1/+1$, this condition is equivalent to requiring that $\frac{\partial^2\,\mathcal{L}\left(\phi_a^d(\cdot),\hat{\sigma}_a(\cdot)|\cdot\right)}{\partial\left|\phi_a^d - \phi_a\right|\partial\,\hat{\sigma}_a} > -\left|b_a\right|$. $\square$

B Dynamics under Imperfect Empathy

In this section, we will derive the qualitative properties of the dynamic evolution of the PIs under the first (imperfect empathy) type of construction rules of section 3.1. To do so, it will be useful to first introduce the following terminology.

Definition 3 (PI Assimilation)**.** *Consider any two succeeding periods and let* $\phi^m := \max_{a\in A}\phi_a$*,* $\phi_m := \min_{a\in A}\phi_a$*, and* $\tilde{\phi}^m := \max_{\tilde{a}\in\tilde{A}}\phi_{\tilde{a}}$*,* $\tilde{\phi}_m := \min_{\tilde{a}\in\tilde{A}}\phi_{\tilde{a}}$*. Then, we speak of* PI assimilation *if* $\phi_m < \tilde{\phi}_m < \tilde{\phi}^m < \phi^m$*.*

Definition 4 (Symmetric PI Point)**.** *Call a tuple* $\{\phi_a\}_{a\in A}$ *a* symmetric PI point *if* $\forall\,(a,a')\in A^2\ \phi_a=\phi_{a'}$*.*

Definition 5 (Steady State)**.** *Call a tuple* $\{\phi_a,\phi_{\tilde{a}}\}_{a\in A}$ *a* steady state *if for almost all* $a\in A$ $\phi_{\tilde{a}}=\phi_a$*.*

Finally, let $\{\phi_a^0\}_{a\in A}$ denote the tuple of initial PIs of the adults.

Proposition 4 (Dynamics under Imperfect Empathy)**.** *Let Assumptions 1—3 hold, let* $\hat{\phi}_{\tilde{a}}$ *be continuous for every* $a\in A$*, and let* $R_a=\{a\}$ *and* $\hat{\phi}_{\tilde{a}}(\{a\})=\phi_a$ *hold in any period and for every* $a\in A$*. Then,* $\forall\left\{\phi_a^0,i_a\right\}_{a\in A}\in\left(\text{con }\phi^d(X)\times\mathbb{R}_+\right)^A$*, it holds that (a) for every two succeeding periods, the PIs are assimilating almost surely, thus (b) the PIs converge to a symmetric PI point, and (c) any symmetric PI point is a steady state.*

Proof. Consider any period and any $\{\phi_a,i_a\}_{a\in A}\in\left(\text{con }\phi^d(X)\times\mathbb{R}_+\right)^A$. Let $a^m:=\{a\in A\,|\phi_a=\phi^m\}$ and $a_m:=\{a\in A\,|\phi_a=\phi_m\}$. Assume that $\phi^m-\phi_m>0$ and further that $\lambda\left(A\backslash a^m\right)>0$ and $\lambda\left(A\backslash a_m\right)>0$.

First, we will show that in Nash equilibrium $\phi_A^{d^*}\in(\phi_m,\phi^m)$. To see this consider the parental best replies to $\phi_A^d\geq\phi^m$. From Proposition 1 (a), it follows that in this case $\forall a\in a^m$, $\phi_a^d(\cdot)\leq\phi^m$ and $\forall a'\in A\backslash a^m$, $\phi_{a'}^d(\cdot)<\phi^m$. Since in any Nash equilibrium, almost all adults choose best reply strategies (see Definition 2), and since $\lambda\left(A\backslash a^m\right)>0$, it then follows that $\phi_A^{d^*}<\phi^m$ must hold. By the same logic, $\phi_A^{d^*}>\phi_m$.

For the next step, let us denote with A^N the set of adults that choose best reply strategies in the Nash equilibrium of a given period (where $\lambda\left(A^N\right)=1$). Again by Proposition 1 (a), it follows that for every $a\in A^N$ such that $\phi_a\in\left(\phi_A^{d^*},\phi^m\right]$ it must hold that $\phi_{\tilde{a}}\left(\phi_a^{d^*},\hat{\sigma}_a^*,\phi_A^{d^*}\right)\in\left(\phi_A^{d^*},\phi_a\right)$, and for every $a\in A^N$ such that $\phi_a\in\left[\phi_m,\phi_A^{d^*}\right)$, we have $\phi_{\tilde{a}}\left(\phi_a^{d^*},\hat{\sigma}_a^*,\phi_A^{d^*}\right)\in\left(\phi_a,\phi_A^{d^*}\right)$. It follows that $\phi_m<\min_{a\in A^N}\phi_{\tilde{a}}\left(\phi_a^{d^*},\hat{\sigma}_a^*,\phi_A^{d^*}\right)<\max_{a\in A^N}\phi_{\tilde{a}}\left(\phi_a^{d^*},\hat{\sigma}_a^*,\phi_A^{d^*}\right)<\phi^m$. Thus, $\phi_m<\tilde{\phi}_m<\tilde{\phi}^m<\phi^m$ almost surely.

Since for any two succeeding periods the PIs assimilate almost surely for any tuple of pairs of (first period) PIs and inter–generational PIs, it follows that for any tuple of initial PIs coupled with any tuple of inter–generational PIs, the PIs converge to a symmetric PI point.

We will finally show that indeed any symmetric PI point is a steady state. Consider any symmetric PI point and denote the identical PI of the adults as $\phi \in \text{con}\ \phi^d(X)$. We will show first that $\phi_A^{d^*} = \phi$. To see this, simply note that by Proposition 1 (a) the best replies to the cases where $\phi_A^d <> \phi$ must satisfy that $\forall a \in A$, $\phi_a^d(\cdot) >< \phi_a = \phi$. Thus, only the case $\phi_A^d = \phi$ can be supported by best replies (of the adults of A^N). Given $\phi_A^{d^*} = \phi$ it then follows from Proposition 1 (b) that $\forall a \in A^N$, $\left(\phi_a^{d^*}, \hat{\sigma}_a^*\right) = (\phi, 0)$ and $\phi_{\tilde{a}}\left(\phi, 0, \phi\right) = \phi$. $\square$

References

[1] Arnett, J. J. (1995). Broad and narrow socialization: The family in the context of a cultural theory. *J. Marriage Fam.*, 57:617–628.

[2] Becker, G. (1996). *Accounting for Taste.* Harvard University Press, Cambridge MA.

[3] Becker, G. and Mulligan, C. B. (1993). The endogenous determination of time preference. *Quart. J. Econ.*, pages 729–758.

[4] Bester, H. and Güth, W. (1998). Is altruism evolutionarily stable? *J. Econ. Behav. Organ.*, 34(2):193–209.

[5] Bisin, A. and Topa, G. (2003). Empirical models of cultural transmission. *J. Europ. Econ. Ass.*, 1(2-3):363–375.

[6] Bisin, A., Topa, G., and Verdier, T. (2009). Cultural transmission, socialization and the population dynamics of multiple-trait distributions. *Int. J. Econ. Theory*, 5(1):139–154.

[7] Bisin, A. and Verdier, T. (1998). On the cultural transmission of preferences for social status. *J. Pub. Econ.*, 70(1):75–97.

[8] Bisin, A. and Verdier, T. (2000). A model of cultural transmission, voting and political ideology. *Europ. J. Polit. Economy*, 16(1):5–29.

[9] Bisin, A. and Verdier, T. (2001). The economics of cultural transmission and the dynamics of preferences. *J. Econ. Theory*, 97(2):298–319.

[10] Bisin, A. and Verdier, T. (2008). Cultural transmission. In Durlauf, S. N. and Blume, L. E., editors, *The New Palgrave Dictionary of Economics.* Palgrave Macmillan, Basingstoke.

[11] Bowles, S. (1998). Endogenous preferences: The cultural consequences of markets and other economic institutions. *J. Econ. Lit.*, 36(1):75–111.

[12] Boyd, R. and Richerson, P. J. (1985). *Culture and the Evolutionary Process.* University of Chicago Press.

[13] Brueckner, J. K. and Smirnov, O. (2007). Workings of the melting pot: Social networks and the evolution of population attributes. *J. Reg. Sci.*, 47(2):209–228.

[14] Brueckner, J. K. and Smirnov, O. (2008). Social networks and the convergence of population attributes: A generalization. *J. Reg. Sci.*, 48(2):359–365.

[15] Cavalli-Sforza, L. L. and Feldman, M. (1973). Cultural versus biological inheritance: phenotypic transmission from parents to children. *Am. J. Hum. Genet.*, 25:618–637.

[16] Cavalli-Sforza, L. L. and Feldman, M. W. (1981). *Cultural Transmission and Evolution: A Quantitative Approach.* Princeton University Press, Princeton.

[17] Cox, D. and Stark, O. (1996). Intergenerational transfers and the demonstration effect. Boston College Working Papers in Economics 329., Boston College Department of Economics.

[18] Demarzo, P. M., Vayanos, D., and Zwiebel, J. (2003). Persuasion bias, social influence, and unidimensional opinions. *Quart. J. Econ.*, 118(3):909–968.

[19] Duesenberry, J. (1949). *Income, Saving and the Theory of Consumer Behavior.* Cambridge, MA: Harvard University Press.

[20] Escriche, L., Olcina, G., and Sanchez, R. (2004). Gender discrimination and intergenerational transmission of preferences. *Oxford Econ. Pap.*, 56(3):485–511.

[21] Fershtman, C. and Weiss, Y. (1998). Social rewards, externalities and stable preferences. *J. Pub. Econ.*, 70(1):53–73.

[22] Festinger, L. (1957). *A theory of cognitive dissonance.* Evanston, IL: Row, Peterson.

[23] Friedkin, N. E. and Johnson, E. C. (1997). Social positions in influence networks. *Social Networks*, 19:209–222.

[24] Galor, O. and Moav, O. (2002). Natural selection and the origin of economic growth. *Quart. J. Econ.*, 117(4):1133–1191.

[25] Golub, B. and Jackson, M. O. (2009). How homophily affects learning and diffusion in networks. Working Papers 2009.35, Fondazione Eni Enrico Mattei.

[26] Golub, B. and Jackson, M. O. (2010). Naïve learning in social networks and the wisdom of crowds. *Amer. Econ. J.: Microecon.*, 2(1):112–49.

[27] Grusec, J. E. (2002). *Handbook of Parenting: Practical Issues in Parenting*, chapter Parental Socialization and Children's Acquisition of Values, pages 153–167. Lawrence Erlbaum Associates.

[28] Grusec, J. E. and Goodnow, J. J. (1994). Impact of parental discipline methods on the child's internalization of values: A reconceptualization of current points of view. *Dev. Psychol.*, 30:4–19.

[29] Grusec, J. E. and Hastings, P. D., editors (2007). *Handbook of Socialization: Theory and Research*. New York: The Guilford Press.

[30] Grusec, J. E. and Kuczynski, L., editors (1997). *Parenting and Children's Internalization of Values*. New York: John Wiley and Sons.

[31] Hauk, E. and Sáez-Martí, M. (2002). On the cultural transmission of corruption. *J. Econ. Theory*, 107(2):311–335.

[32] Kockesen, L., Ok, E. A., and Sethi, R. (2000). The strategic advantage of negatively interdependent preferences. *J. Econ. Theory*, 92(2):274–299.

[33] Leibenstein, H. (1950). Bandwagon, snot, and veblen effects in the theory of consumers' demand. *Quart. J. Econ.*, 64:183–207.

[34] Olcina, G. and Penarrubia, C. (2004). Hold up and intergenerational transmission of preferences. *J. Econ. Behav. Organ.*, 54(1):111–132.

[35] Otto, S. P., Christiansen, F. B., and Feldman, M. W. (1994). Genetic and cultural inheritance of continuous traits. Morrison institute for population and resource studies working paper no. 0064, Stanford University.

[36] Panebianco, F. (2010). "driving while black": A theory for interethnic integration and evolution of prejudice. Eurodiv Paper 70.2010.

[37] Pichler, M. M. (2010). Cultural formation of preferences and assimilation of cultural groups.

[38] Pollak, R. A. (1976). Interdependent preferences. *Amer. Econ. Rev.*, 66(3):309–20.

[39] Qizilbash, M. (2009). Well-being, preference formation and the danger of paternalism. Papers on Economics and Evolution 2009-18, Max Planck Institute of Economics, Evolutionary Economics Group.

[40] Rath, K. P. (1992). A direct proof of the existence of pure strategy equilibria in games with a continuum of players. *Econ. Theory*, 2(3):427–33.

[41] Rogers, A. R. (1988). Does biology constrain culture? *Am. Anthropol.*, 90(4):819–831.

[42] Sacerdote, B. (2002). The nature and nurture of economic outcomes. *Amer. Econ. Rev.*, 92(2):344–348.

[43] Sacerdote, B. (2007). How large are the effects from changes in family environment? a study of korean american adoptees. *Quart. J. Econ.*, 122(1):119–157.

[44] Sacerdote, B. (2008). Nature and nurture effects on children's outcomes: What have we learned from studies of twins and adoptees? In *Handbook of Social Economics* (forthcoming). Amsterdam: North Holland.

[45] Sáez-Martí, M. and Sjögren, A. (2007). Peers and culture. IEW - Working Papers iewwp349, Institute for Empirical Research in Economics - IEW.

[46] Schmeidler, D. (1973). Equilibrium points in nonatomic games. *J. Stat. Phys.*, 7(4).

[47] Sears, R. R., Maccoby, E. E., and Levin, H. (1957). *Patterns of child–rearing.* Evanstan, IL: Row, Peterson.

[48] Stark, O. (1995). *Altruism and Beyond.* Cambridge University Press.

[49] Veblen, T. (1934). *The theory of the leisure class.* New York: Modern Library.