On the Role of Asymmetric Bargaining Power in Intermediate Industry for Existence of Spiral Effect

Inauguraldissertation

zur Erlangung des akademischen Grades eines

Doktors der Wirtschaftswissenschaften

an der Fakultät für Wirtschaftswissenschaften

der Universität Bielefeld

vorgelegt von

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Bielefeld, September 2011

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Gedruckt auf alterungsbeständigem Papier nach DIN-ISO 9706

Acknowledgements

First and foremost I would like to express my deep and sincere gratitude to my supervisor, Prof. Dr. Walter Trockel, Institute of Mathematical Economics, University of Bielefeld. I thank him for the patient guidance, encouragement and advice, for his detailed and constructive comments he has provided throughout my work on the thesis. His wide knowledge and logical way of thinking have been of great value for me. It was under his tutelage that I developed a focus and became interested in computational economics. Despite it was a real challenge to satisfy his exacting requirements, I have been not only extremely lucky but it is my honor to have a supervisor who cared so much about my work, and who responded to my questions and queries so promptly. I doubt that I will ever be able to convey my appreciation fully, but I owe him my eternal gratitude. One simply could not wish for a better or friendlier supervisor.

I wish to express my warm and sincere thanks to my second supervisor, Prof. Dr. Herbert Dawid, Dean of the Department of Economics and Business Administration, University of Bielefeld. His understanding, encouraging and personal guidance have provided a good basis for the present thesis and added considerably to my graduate experience. I also wish to express my gratitude for giving me the opportunity to attend conferences and meet so many interesting people.

A very special thanks goes out to Prof. Volker Böhm, Ph.D., University of Bielefeld, who supported me and gave me important instructions during my first steps in Bielefeld Graduate School of Economics and Management. It was under his capable guidance that I completed compulsory and specialization parts of Ph.D. program and was encouraged to start working on my thesis.

Completing this work would have been all the more difficult were it not for the support and friendship provided by the former and present members of the Bielefeld Graduate School of Economics and Management and the Department of Economics and Business Administration. I am indebted to them for their help.

Finally, I would like to thank the board of directors of the Bielefeld Graduate School of Economics and Management for providing the funding which allowed me to undertake this research.

Summary

This thesis proves the existence of the spiral effect in different scenarios with different modeling approaches. With a spiral effect is called the hypothesis that if, due to his bargaining power, one buyer has better procurement conditions than other buyers, he can use them to strengthen his market position in the sales market, which in turn improve his procurement situation, e.g. as he is in a position to negotiate additional quantity discounts.

The analysis is divided into three blocks:

- In "Asymmetric bargaining power in intermediate industry" we provide a model which extends the model of Katz (1987) to the case of the bargaining over the wholesale prices between firms that are Cournot competitors in the final market. We show that the asymmetry in the bargaining weights of downstream firms leads to the asymmetry in their wholesale prices and results in increasing concentration ratio and in increasing profitability of the most efficient firm.
- In "Asymmetric bargaining power in capacity-constrained industry" we extend the model of Kreps and Scheinkman (1983) allowing the costly capacities. We prove that for any capacity pair the capacity-constrained price game (with asymmetric capacity costs) has unique (mixed) equilibrium expected payoffs. In particular, if firms choose Cournot quantities as capacities, the resulting constrained capacity price game has a unique equilibrium outcome, namely Cournot outcome. We also analyze the existing literature on the capacity-constrained price game with asymmetric production costs and check whether this scenario may be incorporated into the prior formal bargaining model. If the asymmetry is not sufficiently high, in equilibrium firms choose Cournot quantities as capacities; if the asymmetry is sufficiently high, the more efficient firm has an incentive to choose the capacity above its Cournot level and price its less efficient rival out of the market.
- In "Dynamic duopoly with sticky prices and asymmetric production costs" we examine the corresponding differential game for different equilibrium concepts: open-loop, feedback - and closed-loop equilibria. We describe the dynamics and the characterization of the particular (stable) fixed points. We show that, similar to the symmetric case, prices in steady state open-loop- and feedback-equilibrium are lower than the prices in the static Cournot game. If the speed adjustment parameter falls then the prices and quantities in all three types of equilibrium converge to the prices and quantities of the static game when both firms act as price-takers. If the speed adjustment parameter grows the steady state price in the open-loop equilibrium converges to the static Cournot equilibrium price, and feedback- and closed-loop equilibrium prices converge to the prices which are lower than the static Cournot price.

Keywords

Spiral effect; Intermediate industry; Bargaining games; Bargaining power; Asymmetric costs; Cournot competition; Bertrand-Edgeworth price competition; Quantity precommitment; Differential games; Sticky prices.

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Chapter 1

Introduction

1.1 Background

The rise to prominence of EDEKA and other big-box retailers has given new life to the debate about whether "buyer power" at intermediate links in a vertically related chain of industries is good or bad for consumers and hence whether it requires the attention of competition authorities. There is no consensus at the present time on the economic and legal meaning of buyer power, although a number of alternative definitions have been put forward. As a result, there has been no clear agreement on the appropriate measurement of buyer power.

Competition Authorities need to distinguish between efficiency enhancing cooperation and cooperation which, on balance, harms competition and consumers. Guidelines on the assessment of horizontal mergers under the Council Regulation on the control of concentrations between undertakings (2004/C 31/03) mention two possible effects of exercising buyer power: Output-Effect and Lock-Effect, including Spiral Effect.

In 2001 buyer power was a central issue in the European Commission's assessment of the merger of the supermarket chains Carrefour and Promodes. European Commission argued, however, that the buyer power which the merger would create could distort competition. It identified two related mechanisms, namely Spiral Effect and Threat Point. The first refers to the self re-enforcing effect of market share and volume discounts, while the second refers to the power that a buyer of even a small percentage of a producer's total output can have in affecting its viability as a business. There are some other theories of harm arising from powerful buyers or buyer groups that are often considered in the literature: Waterbed Effect, Predatory Overbuying, and Dynamic Inefficiency.

On the other hand, buyer power can have a positive impact in a situation where intermediate suppliers have substantial market power. For example, in the food retail sector, an increase in buying power of retailers may countervail (a term coined by Galbraith in 1952) the market power of food manufacturers. Such situation would particularly benefit consumers if retailers behaved competitively in selling, i.e. if there was a low degree of selling power in the retail sector, which would guarantee that the buying price reductions that retailers obtain would be passed on to final consumers.

1.2 Related literature

In the recent literature the main attention has been gained to the negative competitive effect so called "Waterbed Effect". The paper of Inderst (2008) explains the reasons of its occurring by the differentiated products. He assumes two local markets, which are characterized by Hotelling competition. Both local markets are supplied by an incumbent that offers each buyer a limit price so that they do not have incentives to integrate backwards. If two firms from the different local markets merge, the merged unit can spread the fixed cost of backwards integration over both outlets and so obtains a lower input price. The cost advantage obtained in such a way allows the new merged unit to capture some additional demand from its independent rivals and thereby to weak rivals' credible threats to integrate backwards. As a result, rival buyers pay more. Inderst finds that downstream prices fall in all outlets if the costs of switching to the rival source of supply are sufficiently low.

The paper of Majumdar (2005) differs from Inderst in the following aspects. First, it develops more explicit welfare results. In this paper buyer mergers are harmful because the price rises in markets where the chain has already operated. Second, buyer power is modeled by first mover advantage to avoid a potential effect of gatekeeping. Third, he assumes Cournot competition at the downstream market. In the model the supplier serves not only the chain store, but also different independents. His result about the harmfulness of the input price discrimination is applied both to the linear demand and to a class of strictly concave demand functions.

Somewhat similar papers of Matthewson and Winter (1996) and of Gans and King (2002) show how a buyer group with a first-mover advantage can benefit at the expense of smaller buyers, which move second. However, in their models buyers do not compete downstream.

Salop and Scheffman (1983) show the problem of the Waterbed Effect from some different angle, namely that the dominant firm could have an interest on raising its own costs because this would raise rivals' costs sufficiently more. Such behavior harms the competition. However, as Mason (2002) notes, cost raising strategies usually require strong restrictions on parameters if they are to be profitable.

Schiff (2008) analyzes the conditions that lead to the existence of a Waterbed Effect, whereby the prices charged by a multiproduct firm are interdependent. As a result, Waterbed Effect will exist if either the marginal revenue or marginal cost of one good depends on the quantity of the other good.

Katz (1987) develops the model, which is addressed to the price discrimination in input markets. He assumes that a monopoly supplier sells to the downstream firms, which are represented by a chain store and independent local stores. In each local market, the chain store competes with one or more independent stores. The chain store can integrate backwards while independents are assumed not to have this option. Katz compares regimes where price discrimination is allowed with those where it is prohibited. Price discrimination is good for welfare, where it deters integration that would have led to wasteful duplication of upstream fixed costs of production. However, allowing discrimination can reduce welfare by leading to higher prices for downstream firms.

Inderst and Valletti (2006) extend that work of Katz. They consider a monopoly supplier selling to two downstream firms, both of which have the option to integrate backwards at a fixed cost. Inderst and Valletti find that allowing price discrimination enhances pre-existing cost differences when both downstream firms are Cournot competitors. If the more efficient downstream firm obtains additional demand from its competitor who is less efficient, then the rise to a Waterbed Effect occurs.

Buyer mergers are considered in Dobson and Waterson (1997) and von Ungern-Sternberg (1996), where a buyer merger which destroys a retailer increases both downstream market power and upstream bargaining strength with a monopoly supplier. These authors find that buyer mergers harm welfare until downstream competition becomes very intensive, almost perfect.

There are different papers that show that the exercise of buyer power may harm consumers through different channels which are not related to the Waterbed Effect. For example, Chen (2003) analyzes the impact on suppliers' investment incentives, as well as Inderst and Wey (2003) and Vieira-Montez (2004).

The theory of the Waterbed Effect exhibits a major difference compared to the Spiral Effect, another theory of competitive harm from buyer power. According to the theory presented in the paper of Inderst and Valletti (2008), the Waterbed Effect should already work in the short run, as manifested by the coincidence of a decrease in wholesale prices for some buyers and an increase for other buyers. In contrast, the theory of the Spiral Effect is much more prospective: competitive harm should be expected only in the long run, when less powerful rivals have exited the market, thereby probably creating scope for a price increase by the remaining oligopolists.

1.3 Motivation

At the meeting on 18 September 2008 the Working Group on Competition Law gave the following definition of the Spiral Effect: "If, due to his bargaining power, one buyer has better procurement conditions than other buyers, he can use these to strengthen his market position in the sales market. A strengthened position in the sales market can in turn improve his procurement situation, e.g. as he is in a position to negotiate additional quantity discounts. As a result, less efficient (smaller) competitors are squeezed out of the market. In the long term, however, this could lead to price increases if, due to decreasing competitive pressure, the remaining companies are no longer forced to pass on their procurement advantages." The Working Group has also discussed the following questions: Are Spiral and Waterbed Effects only theoretical considerations or possible effects of the exercise of buyer power which should also be taken seriously in practice? What criteria could be applied to assess the risk of their occurrence?

To our knowledge, Spiral Effect has not been modeled formally in the theory. Moreover, it is not clear that such an effect would be necessarily harmful. The Spiral Effect could simply be a process in which lower prices are passed on to end customers allowing a buyer to grow. If such growth allows the buyer to obtain even lower prices, which are then passed on to end customers once again, this would be a virtuous circle that benefits end customers. If the fear is that more efficient firms drive out weaker retailers, this could simply reflect the process of competition.

1.4 Analytical framework

Legal cases involving price discrimination in the sale of intermediate goods typically concern pricing schemes under which customers with large individual purchases or large cumulative purchase volumes receive lower prices than do customers making small purchases. The first analysis of the third-degree price discrimination in intermediate good markets was made by Katz (1987). He shows that models of final good markets are inappropriate for the analysis of intermediate good markets. He sketches two fundamental differences between final and intermediate good markets. First, in an intermediate good market, unlike a typical final good market, the buyers' demands for the product are interdependent. The profits of any given downstream firm and its demand for an input are functions both of the price that firm pays for the input and of the prices that the buyer's product-market rivals pay. A second important difference is that buyers of inputs often have the ability to integrate backward into supply of the good. Many theories why larger buyers, either if they grow organically by becoming more efficient or if they grow by acquisition, obtain a discount compared to smaller buyers are driven by changes in the value of buyers' outside options, one of them is the backward integration.

In our opinion, the analysis of price discrimination is more complex than it is made by Katz. It is important to understand why the large buyers often charge lower prices than the smaller buyers do. Such quantity discounts are typically explained with the argument, that losing a high-volume customer is more costly to the seller than losing a low-volume customer. But the question where the customer goes if it leaves its current supplier is still not answered by such arguments. To make a credible threat to leave, a buyer must have an alternative source of supply. Katz assumes that a large buyer receives lower prices because there are economies of scale in finding an alternative source of supply, and thus the threat of finding an alternative is stronger when it is made by a high-volume buyer. In his model the chain has a stronger threat of backward integration than the local stores because of its larger demand for the input and the economies of scale in production of the intermediate good. This gives rise to the seller's incentives to discriminate. In particular, a buyer may threaten to engage in self-supply using a production technology that exhibits economies of scale or the buyer also could threaten to go to other upstream supplier. In this case, the economies of scale on the buyers' side of the market could arise from a buyer having to incur fixed costs to find and contract with another supplier. Alternatively, the buyer might have to bear fixed costs to modify its production line to utilize an alternative variant of the input. It is also interesting to investigate how the greater bargaining power is related to possibilities of backward integration. Our aim is to show, that even if an explicit threat to integrate backward is not credible, the chain may also receive a discount if it has greater bargaining power.

Chapter 2

Asymmetric bargaining power in intermediate industry

2.1 Introduction

2.1.1 The industry

The upstream industry in our model is represented by a single supplier who produces some output, which he sells to the firms in the intermediary industry. Then these downstream firms produce the same final good and sell it in a local market. The supplier's production takes place after negotiations with downstream firms at a constant, positive marginal cost c. Fixed costs of production do not affect the supplier's behavior because they are sunk.

The price in the local market is described by the inverse demand function $P(\cdot)$, which is a function of the total output sold in the local market, where $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$. Both firms have identical production functions. Assume that they transform one unit of the input into one unit of the output at no additional costs. A natural example where this specification is reasonable is retailing. Firms compete in a horizontally differentiated product market.

Whenever i and j appear in the same expression in the whole work, it means that $i, j \in \{1, 2\}$ and $i \neq j$.

2.1.2 Basic concepts and notations

We present a non-cooperative bargaining process between the supplier and two downstream firms where results from two-person cooperative axiomatic bargaining games are used to define the payoffs of some of the terminal nodes of our extensive game. The definition of the extensive game is quite complex that is why its illustration via game tree is very intuitive.

In our bargaining model the upstream supplier and the downstream firm $i \in \{1, 2\}$ bargain over the wholesale price denoted as w_i. Making the price offer each player binds it to some particular output quantity which he is ready to sell / to buy for the suggested price. These quantities and their dependence on the suggested prices we shall consider in one of the following sections.

We assume that the downstream firms are asymmetric in the sense that firm 1 can integrate backwards (to produce the input by itself, instead of buying it from the supplier) and firm 2 cannot. We shall show that this assumption is crucial for the determination of the firms' behavior depending on the obtained price offers.

Before turning to the formal analysis, let us emphasize that through the whole model we assume that all players possess complete information and they are all rational.[1]

In the model we suggest a stepwise bargaining, because in our case it is a process in which the parties achieve interim settlements step by step, where each settlement is a starting point for further negotiations.

We model the events in the industry in the following steps. We start with the final stage and describe the Cournot competition between both downstream firms in the local market. Then we describe a non-cooperative simultaneous bargaining between each downstream firm $i \in \{1, 2\}$ and the supplier over the wholesale prices w_i for some particular output quantities and finally we show that there is a unique subgame perfect equilibrium wholesale price vector which will be offered and accepted in the first period.

2.2 Equilibrium in the product market

In this section we describe the equilibria in the final subgames that represent the product market. We determine the dependence between the supplier's/firms' price offers and the offered/requested quantities for these prices. We derive the respective equilibrium profits as they play an important role in the determination of the outcome of bargaining.[2]

According to our model both downstream firms are Cournot competitors in the final stage. Therefore let us first describe the respective competition process.

To present the Cournot competition we consider the local market in which two firms "produce" the same product. The aggregate output $\bar{q}$ is the sum of the outputs of both firms. The inverse demand function $P(\bar{q})$ is decreasing so that $P'(\bar{q}) < 0$ for all $\bar{q} \geq 0$. Further we shall derive the equilibrium price $p = P(\bar{q})$ from the aggregate output $\bar{q}$.

$w = (w_1, w_2)$, $q = (q_1, q_2)$, where q_i denotes the output quantity of downstream firm $i \in \{1, 2\}$, which it will have bought by the upstream supplier at unit price w_i. $(w_i, q_i) \longmapsto C_i(w_i, q_i) := w_i q_i$. So $C_1 := C_1(w_1, q_1) = w_1 q_1$ and $C_2 := C_2(w_2, q_2) = w_2 q_2$ are the downstream firms' costs.

A Cournot game for given w_1, w_2 can be modeled as a normal form game.

Definition 2.2.1. *Let* $\Gamma_w := (\bar{Q}^w, \pi)$ *be a game with* $n = 2$ *players, where* $\bar{Q}_i^w$ *is the strategy set for player* i, $\bar{Q}^w = \bar{Q}_1^w \times \bar{Q}_2^w$ *is the set of strategy profiles and* $\pi : \bar{Q}^w \rightarrow \Re_+^2 : q(w) \mapsto \pi^w(q) = (\pi_1^w(q), \pi_2^w(q))$ *is the payoff function.*

[1]With rationality assumption we avoid the possibility of backward integration of particular downstream firm in equilibrium.

[2]Profits of each downstream firm i depend on its own and rival's purchased quantities. The quantities depend on the negotiated wholesale prices, which both rivals pay.

The profit of each downstream firm i depends not only on its own output quantity but also on the output quantity of its rival.

$$\pi_i^w(q) = P(\bar{q})q_i - C_i(w_i, q_i) \tag{2.2.1}$$

Both firms want to choose their outputs in order to maximize their profits. Let us next assume that the first and the second order conditions for the profit maximization in the Cournot game are satisfied everywhere for both firms[3] so that their (reaction) functions are the optimal response for them in the local market.

Downstream firms must buy the output from the upstream supplier, therefore the quantities depend on the wholesale prices w_1 and w_2 for the purchased good.

Notation 1. *For any Cournot game Γ_w based on $w := (w_1, w_2)$ we denote Nash equilibrium quantities by $q^*(w) \equiv (q_i^*(w_1, w_2))_{i=1,2}$.*

Definition 2.2.2. *The output values $[q_1^*(w_1, w_2), q_2^*(w_1, w_2)]$ for any given pair (w_1, w_2) build a Nash equilibrium in the Cournot game Γ_w if the following inequality holds:*

$$\pi_i^w \left[q_i^*(w_i, w_j), q_j^*(w_i, w_j)\right] \geq \pi_i^w \left[\hat{q}_i, q_j^*(w_i, w_j)\right], \ \forall \hat{q}_i, \ i = 1, 2 \tag{2.2.2}$$

In other words, a Nash equilibrium is a set of actions such that no player taking his opponents' actions as given, wishes to change his own action.

Assumption 1. *In any considered two-player Cournot game Γ_w there exists a pure-strategy equilibrium, it is unique and (locally) strictly stable.*[4,5]

The local market is in Cournot equilibrium at price $p^*(w) = P(\bar{q}^*(w))$, where $\bar{q}^*(w) := \sum_{i=1}^{2} q_i^*(w)$. For any $q^*(w)$ the equilibrium profit for each downstream firm i is given below:

$$\pi_i^*(w) := \pi_i^w(q^*(w)) = (p^* - w_i)q_i^*(w), \ i = 1, 2 \tag{2.2.3}$$

Thus, in order to receive a positive profit the purchase price for each downstream firm i must satisfy the following condition: $w_i < p^*$, $i = 1, 2$.

2.3 The game

We consider the following three-player non-cooperative bargaining game G, which lasts $r \geq 1$ rounds. We restrict our attention to subgame perfect equilibria. We assume that the upstream supplier (S) initiates the negotiations with two downstream firms, denoted as firm 1 and firm 2. At the initial time the upstream supplier makes

[3]For these to be true the demand function must be concave (or linear) and the marginal costs increasing in the own output.

[4]Novshek (1985), Bamon and Fraysse (1985) introduce the approach which shows that if a firm's marginal revenue decreases with the other firm's output, a pure-strategy equilibrium exists.

[5]Following Tirole (1988), p. 226 the following sufficient condition for the uniqueness is satisfied: $\left|\frac{\partial R_i(q_j)}{\partial q_j}\right| \left|\frac{\partial R_j(q_i)}{\partial q_i}\right| < 1$, where $R_i(q_j)$ is a continuous reaction function.

price offer to each downstream firm simultaneously and publicly and firms may accept it (A), reject it (R), or withdraw (W) from negotiations. As the illustrated structure of the extensive form of the whole game is very complicated we restrict to the geometric stylized illustrations of the considered problems. The illustration of the initial part of the game, in which the supplier makes price offers is given in Figure 2.1.

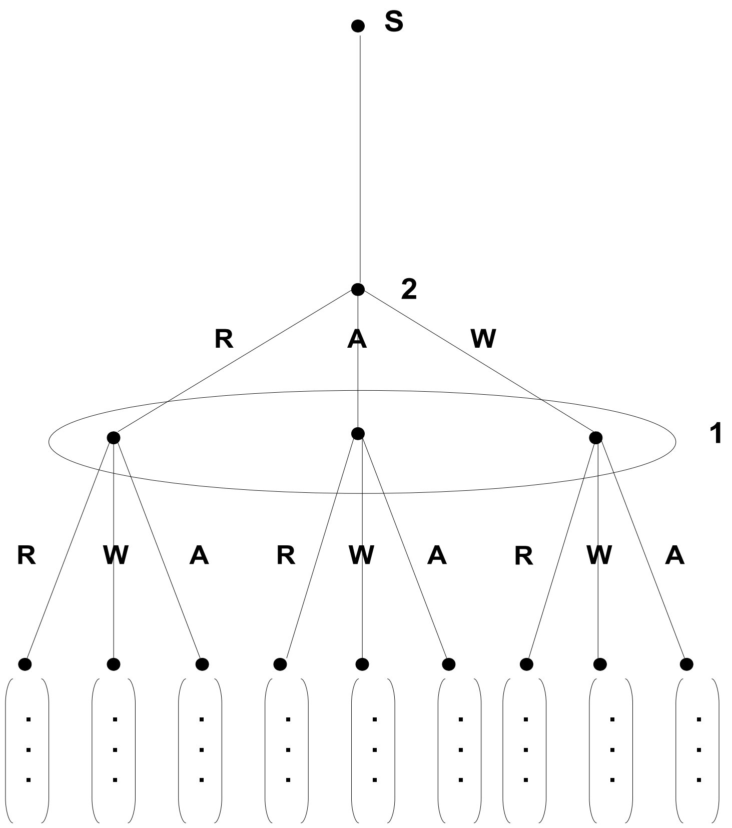

Figure 2.1: The supplier offers the wholesale prices

The circle around the set of nodes (information set) indicates that the decision is made at the same time as the decision at the previous node, and hence downstream firms decide uninformed of each other's choices. The payoffs of all players, depending on their choices of actions will be defined later.

If both firms decide to reject the supplier's offer they will make counteroffers in the next bargaining period, which on its turn the upstream supplier may accept or reject. Geometrically this problem is illustrated in Figure 2.2.

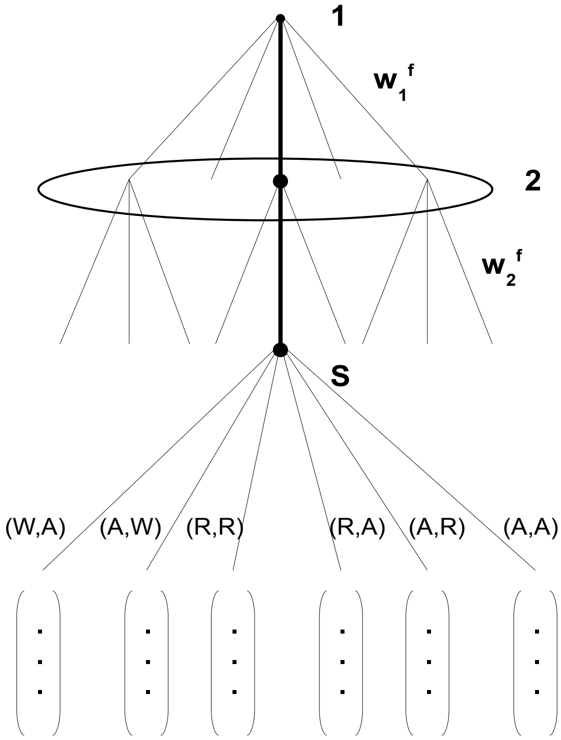

Figure 2.2: Downstream firms' actions constitute the counteroffers

If supplier on his turn rejects both offers, then in the next bargaining period he will make price offers by himself. If in each bargaining period the upstream supplier or both downstream firms reject offers of their opponent, then the branch of the game tree with actions "Reject" becomes infinite as it is illustrated in Figure 2.3.

Players discount stages with a discount factor $\delta \in (0, 1)$. This is a game of perfect information. Each player, when making or responding to an offer, knows all actions taken before his move. The game ends when, and if, an agreement is reached between the upstream supplier and one or both downstream firms. The agreements in such bargaining may appear in different forms, i.e. both downstream firms have come to contract terms, firm 1 has decided to integrate backward, they have decided to withdraw from negotiations, etc.

As it has been shortly mentioned in Section 2.1.1 we assume that the supplier produces the output only when the bargaining is over, namely when the prices and respective quantities are determined and accepted.

The aim of the forthcoming analysis is to prove the existence of the subgame perfect equilibrium and if it exists to show whether it is unique or not. The result depends on the assumptions about the discreteness or continuity of the set of possible alternatives. Such assumption is fundamental for our approach and for our future result and it is also often discussed in the literature on game theory. For example, van Damme, Selten and Winter (1990) show that the result of Rubinstein (1982) who has shown that impatience implies determinateness of the two-person bargaining problem depends also on the assumption of his model that the set of alternatives is a continuum. If the pie can be divided only in finitely many different ways (for example, because the pie is an amount of money and there is the smallest money unit), any partition can be obtained as a result of the subgame perfect equilibrium if the time interval between successive offers is sufficiently small. Rubinstein's theory specifies a unique solution, the uniquely determined subgame perfect equilibrium of his model. Van Damme, Selten and Winter (1990) show that the introduction of the smallest money unit destroys Rubinstein's uniqueness result.

On this stage we make the following statement which is crucial for our future analysis: *As the bargaining in our model takes place over the wholesale prices there exists the smallest possible unit in prices, namely 1 cent.*

This approach also means that there is no lack of credible treats for all players to reject, because there exists the best alternative.

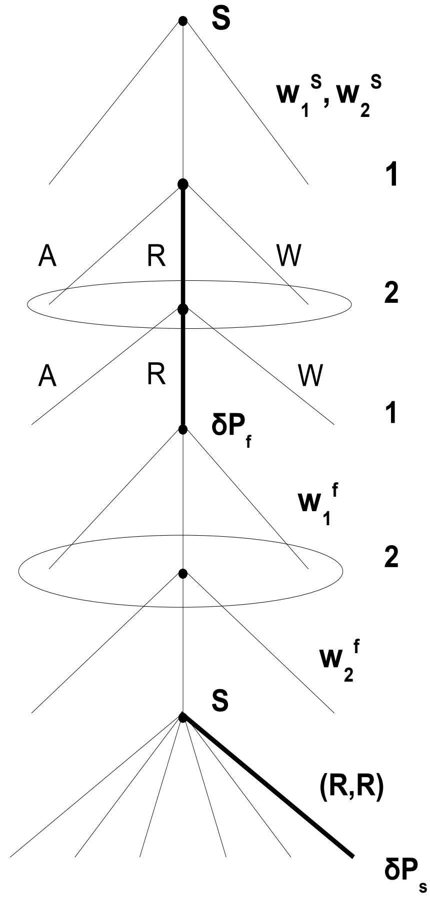

Figure 2.3: All players always reject any offers

2.4 The bargaining model

In order to correctly describe the bargaining process, we must define the aims of all players; their payoffs in case if they reach an agreement; their outside option payoffs, if the agreement is not reached; and the feasible set over which both sides bargain.

2.4.1 The aims of all players

In our model we have three players: the upstream supplier and two downstream firms. The main difference between downstream firms lies in the assumption that only firm 1 possesses the possibility of backward integration.

As we know the bargaining takes place over the wholesale prices. Making some particular price offer each player binds it to some particular quantity of the output, which he wants to sell or to buy at this unit price. The respective output quantities haven't been defined in our model until now, therefore let us consider them more precisely.

In fact we need to determine what quantities are acceptable for the upstream supplier because of his costs by some particular prices w_1 and w_2; and what quantities depending on the situation are acceptable for downstream firms 1 and 2.

For the forthcoming analysis let us now consider the special case of the bargaining between the upstream supplier and a single downstream firm. Earlier we have assumed the common knowledge for all players in our model. Therefore the costs of both players, denoted as C_s for the upstream supplier and C_f for the considered monopolistic downstream firm, as well as the inverse demand function in the local market $P(\cdot)$ are known. Hence it is obvious that we can determine the monopoly output quantity q^m and the monopoly price p^m.

In the initial period when the bargaining starts the upstream supplier offers some particular wholesale price w to the downstream firm, which must decide whether to accept, to reject or to withdraw from negotiations. The price offer w means that the downstream firm should pay this unit price for the output quantity $q^m(w)$.

If the downstream firm accepts this price offer then the bargaining ends and both sides obtain the following payoffs:

The payoff of the upstream supplier: $\pi_s(q^m(w)) = wq^m(w) - C_s(q^m(w))$
The payoff of the downstream firm: $\pi_f(q^m(w)) = (p^m(w) - w)q^m(w)$

If this downstream firm has the possibility of backward integration, then receiving the supplier's offer it will decide whether to integrate or not. To make a decision it will compare its costs without integration $C_f^w(q^m(w)) := wq^m(w)$ with its costs after integration $C_f^{vIB}(q^m(v)) := vq^m(v)$, where v is the price of self-production. Consequently if $w > v$ the upstream supplier earns zero and the integrated downstream firm earns $(p^m(v) - v)q^m(v)$.

Denoting $\pi^m := p^m q^m - C_f^w(q^m)$ we obtain as possible profit distributions without integration for both players all profit distributions from $(0, \pi^m)$ to $(\pi^m, 0)$. Assume that the supplier and the firm negotiate the profits using the Nash bargaining approach. The graphic illustration of the case if the downstream firm does not have an option of backward integration is given on the left hand-side of Figure 2.4.

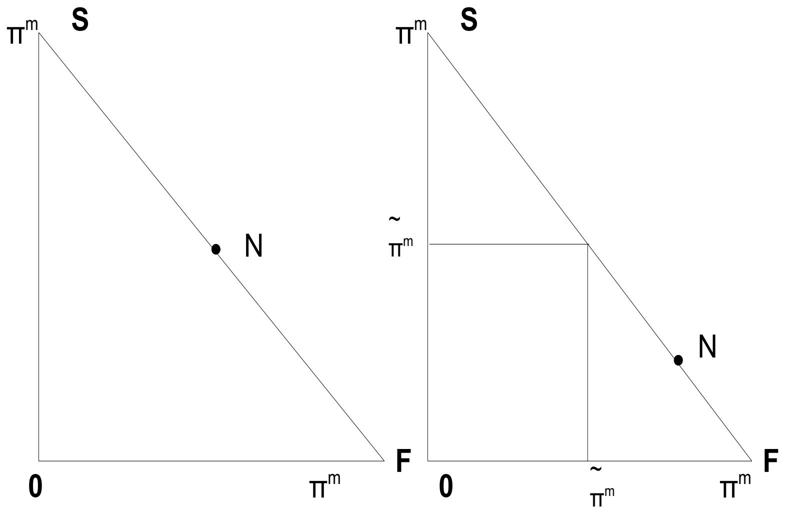

Figure 2.4: Profits of both players via Nash bargaining solution

It is important to mention that even if the downstream firm has the possibility to integrate backwards, it is still interested in negotiations with the upstream supplier, because it cannot supply itself under equal conditions, nevertheless its position in negotiations is stronger comparing with the firm which can not integrate backwards and it is obvious that the distribution of the obtained profits between the downstream firm and the supplier will be different from the case without backward integration option. The bargaining solution for the case when the downstream firm has the backward integration option is illustrated on the right hand-side of Figure 2.4. The monopoly profit is denoted as $\tilde{\pi}^m := (\tilde{p}^m - v)\tilde{q}^m$, where $\tilde{p}^m, \tilde{q}^m$ are the monopolistic price and quantity if $C_1(v, \tilde{q}_1^m) := v\tilde{q}_1^m$.

On this stage let us emphasize that in both cases (with and without backward integration option) the Nash solution (due to linear Pareto frontier) coincides with all other standard solutions: Kalai-Smorodinsky, Perles-Maschler, Raiffa etc. So "Nash" in this specific case does not need a particular justification.

Now let us consider the situation with two downstream firms in the market. The negotiations illustrated on the left hand-side of the Figure 2.4 could take place only between the upstream supplier and the downstream firm without backward integration option. But it is obvious that the downstream firm which has an option of backward integration will enter the local market also in the case if it withdraws from negotiations with the supplier. Hence, there are two downstream firms in the market, which are Cournot competitors in the final stage, and therefore instead of monopoly quantity, - price and - profit appear Cournot quantities, - price and - profits, which we have already considered and defined in Section 2.2. Consequently, the right hand-side illustration on Figure 2.4 will be also impossible.

As firm 1 will enter the local market in all cases, the upstream supplier and the firm 2 may make an attempt to disadvantage firm 1. Hence, let us now consider the underlying extensive game with three players.

We know the cost functions of all three players denoted as $C_s(\cdot)$, $C_1^w(\cdot)$ and $C_2^w(\cdot)$, respectively. As both downstream firms are asymmetric in the existence of the backward integration option there are the following restrictions on their cost functions that must hold. For the downstream firm 2:

$$\min(C_2^w(q^m), C_2^w(q_2^*(w))) > \max_{p \geq 0}(p - w)q^m \tag{2.4.1}$$

For the downstream firm 1:

$$C_s(q^m) < C_1^w(q^m) < p^m q^m \tag{2.4.2}$$

As we have already mentioned in Section 2.3 we restrict our attention on the subgame perfect equilibria, therefore it is important to notice that *the whole following reasoning is based on the assumption of behavior in subgame perfect equilibrium.*

Now let us consider the supplier's behavior when he makes the price offers to both downstream firms. It is obvious that his offers are bound to some particular quantities that he wants to sell at these prices and these quantities from the supplier's point of view are acceptable own strategies. In order to correctly describe the bargaining process we need to define these quantities (the set of supplier's acceptable own strategies).

The supplier has the following options: he can sell Cournot quantities $q_1^*(w)$ and $q_2^*(w)$ to downstream firms; he can offer quantity $q^m(w)$ at some particular price to each firm; or he can force firm 1 to backward integration and as a result to sell the quantity $q_2^*(w)$ to downstream firm 2.[6] Now we consider these three cases in details in order to specify the relation between price offers and quantities.

1. Making the price offer (w_1, w_2) the supplier sells the Cournot quantities $q_1^*(w)$ and $q_2^*(w)$ at unit prices w_1 and w_2 to downstream firms 1 and 2, respectively. Both downstream firms accept these price offers if:

$$C_1^w(q_1^*(w)) < p^*(w)q_1^*(w)$$
$$C_2^w(q_2^*(w)) < p^*(w)q_2^*(w)$$
$$C_1^w(q_1^*(w)) < C_1^{vIB}(q_1^*(w)) \Leftrightarrow w_1 < v$$

2. The offer $(w_1, \bar{w})$ means that the supplier wants to sell the monopoly quantity $q^m(w_1)$ at price w_1 to firm 1, and at price $\bar{w} := 2p^m(w_1)$ to firm 2. Then the

[6]Downstream firm 2 will buy only Cournot quantity because it knows that it will compete with firm 1 in the local market.

following conditions must hold for firm 1 to buy the monopoly quantity:

$$C_s(q^m(w_1)) < w_1 q^m(w_1) < C_1^{vIB}(q^m(w_1)),\ w_1 < p^m(w_1)$$

3. The price offer $(\bar{w}, w_2)$ with $\bar{w} := 2p^m(w_2)$ forces firm 1 to integrate backward and to produce the input by itself. Hence, the upstream supplier aims to sell the Cournot quantity $q_2^*(\bar{w}, w_2)$ at unit price w_2 to downstream firm 2. In such case the following inequality must hold for firm 2 to accept the offer:

$$w_2 q_2^*(\bar{w}, w_2) < p^*(\bar{w}, w_2) q_2^*(\bar{w}, w_2) \ \Leftrightarrow\ w_2 < p^*(\bar{w}, w_2)$$

If both downstream firms decide to reject the supplier's offers they make counteroffers in the next bargaining period. Now let us also consider their price offers and the quantities for which these offers are determined, namely the supplier's acceptable strategies from the firms' point of view.

Let us start with the downstream firm 2 which does not have the backward integration option. Its price offer will be bound to Cournot quantity $q_2^*(w)$, because it knows that it will never be alone in the local market but it will compete with firm 1. Therefore, the offer w_2 means that firm 2 is ready to pay the unit price w_2 for the Cournot quantity $q_2^*(w)$. Thereby the following inequality must hold for the downstream firm 2:

$$w_2 q_2^*(w) < p^*(w) q_2^*(w) \Leftrightarrow\ p^*(w) > w_2$$

The upstream supplier accepts this price offer only if:

$$w_2 q_2^*(w) > C_s(q_2^*(w))$$

Considering price offer of the downstream firm 1, which has the option of backward integration, it is obvious that it has the following choices: firm 1 may buy from the supplier $q^m(w)$, $q_1^*(w)$ or nothing at all and instead produce by itself.

Thus the downstream firms' price offers (w_1, w_2) means that the firm 1 is ready to pay w_1 for $q_1^*(w)$ and the firm 2 the unit price w_2 for $q_2^*(w)$.

As firm 1 will never make an offer which forces it to integrate backwards, we are left with the situation that it offers the supplier the unit price $\tilde{w}$ for the quantity $q^m(\tilde{w})$. Hence, the offer $(\tilde{w}, w_2)$ means that firm 1 is ready to buy $q^m(\tilde{w})$ at price $\tilde{w}$ and firm 2 offers price w_2 for $q_2^*(\tilde{w}, w_2)$. For firm 1 there are following restrictions on the suggested price $\tilde{w}$:

$$\tilde{w} < v,\ \tilde{w} < p^m(\tilde{w})$$

The supplier accepts such a price offer of firm 1 only if:

$$\tilde{w} q^m(\tilde{w}) > C_s(q^m(\tilde{w}))$$

$$\tilde{w} q^m(\tilde{w}) - C_s(q^m(\tilde{w})) > \max_w (p^*(w) q^*(w) - C_s(q^*(w)))$$

The second inequality means that the supplier is better off by accepting the offer of firm 1 selling $q^m(\tilde{w})$ at price $\tilde{w}$, than to choose some other actions, e.g. to sell to both firms Cournot quantities. One of such actions is explained in the next section.

But briefly speaking, if the supplier accepts the offer of firm 2 to sell q_2^* at price w_2, but rejects firm 1's offer, then it will settle some price $\hat{w}_1$ with firm 1 for quantity q_1^* (what the price $\hat{w}_1$ is and how it is determined will be discussed later). Hence, the second inequality means that, whatever $\hat{w}_1$ could be, the downstream firm 1 in order to buy the monopoly quantity q^m should offer such price $\tilde{w}$ that the inequality $\hat{w}_1 q_1^* + w_2 q_2^* < \tilde{w} q^m$ will hold.

Having considered all these issues we have specified the relations between the price offers of both downstream firms and the quantities that they want to buy from the supplier at the suggested prices after they reject the supplier's offers.

2.4.2 Determination of the players' payoffs

2.4.2.1 Preliminaries

For the analysis in this section let us introduce some notations on the payoff vectors of all players by all possible outcomes of negotiations. Hence, we denote the following 3×1 payoff vectors, which will be determined later:

- $P(w_1, w_2)$ the payoffs of all players are determined by agreed w_1 and w_2 (they are proposed and accepted in the same period);
- $P_i(w_j)$ the payoffs are determined by the agreed w_j and w_i, which is a Nash bargaining solution;[7]
- $P_j^w(w_i)$ the payoffs are determined by the agreed w_i (firm j withdraws from bargaining with the supplier in the same period);
- $P_w(w_i)$ the payoffs are determined by the agreed w_i (the supplier withdraws from bargaining with the firm j in the same period);
- P_s indicator s denotes the payoffs when the supplier makes the price offers;
- P_f indicator f denotes the payoffs when the downstream firms make the counteroffers.

As we have already mentioned at the initial time the upstream supplier proposes some wholesale prices to both downstream firms simultaneously and firms may accept or reject them. If both firms decide to reject, they make counteroffers in the next period; if they decide to accept, the negotiation process ends with agreement. If the upstream supplier and firm 1 agree on some price such as firm 1 will buy the quantity q^m, then the negotiation also ends with agreement where firm 2 gets nothing.

Therefore it turns out that in order to find the equilibrium payoffs, we need to determine what happens in the subgames in which one wholesale price has already been accepted and the other - not.

So assume that at some point of the game, at stage t, the upstream supplier and firm j have ended their negotiation by agreeing on price w_j, which becomes known to firm i. Then in such case we assume that we have reached a terminal node of the

[7] The explanation follows below.

game tree where the price for firm i results from the Nash bargaining solution which is just a justified part of the rules of the non-cooperative game. This idea belongs to cooperative game theory. The reasons for choosing the Nash bargaining solution will be discussed in Section 2.5.

As we use the cooperative game approach in our non-cooperative bargaining model let us make some useful notations and definitions concerning the cooperative bargaining game:

Definition 2.4.1. *A bargaining solution is a rule μ that assigns a solution vector $\mu(S, s^0) \in S$ to every bargaining problem (S, s^0) in the class γ.*

As we have shown in the previous section for each pair $w := (w_1, w_2)$ there exist some particular quantities $q_i(w_1, w_2)$, $i = 1, 2$ which are suggested by the supplier to downstream firms or vice versa the downstream firms want to buy from the supplier. For simplicity let us denote the profit of each downstream firm i given in equation (2.2.1) as $\pi_i(w) := \pi_i^w(q_1, q_2)$. The profit of the supplier is $U \equiv U \circ q$ with $U(w) := U^w(q_1, q_2) = \sum_{i=1}^{2} w_i q_i(w) - C_s(\hat{q}(w))$, where $\hat{q}(w)$ is the total output that the supplier produces for both downstream firms.

Definition 2.4.2. *The bargaining game between the upstream supplier and firm i is a pair (S, s^0), where $s^0 \in S \subset \Re_+^2$ and $S \neq \emptyset$ is compact, convex, comprehensive and closed. $S = \{(U, \pi_i) | (U, \pi_i) = (U(w), \pi_i(w)),\ w := (w_1, w_2)\}$; $s^0 = (U^b, \pi_i^b)$, where S is the set of possible utility allocations, s^0 is the threat point, namely the outcome of the event if the bargaining process breaks down.*[8]

The symmetric Nash bargaining solution is determined by the maximization problem $max_{s \in S}(s_1 - s_1^0)(s_2 - s_2^0)$ subject to $s_1 \geq U^b$, $s_2 \geq \pi_i^b$.

Hence, applying the definitions given above the symmetric Nash bargaining solution is taken from the following bargaining problem for given w_j:

$$\max_{w_i} \mu_i(w_1, w_2) = \left[U(w_1, w_2) - U^b\right] \left[\pi_i(w_1, w_2) - \pi_i^b\right] \tag{2.4.3}$$

For the rest of this thesis we make the following additional assumptions that are essential for our future analysis:

Assumption 2. *$U = U(\cdot, \cdot)$ is strictly concave, $U'' < 0$.*

Assumption 3. *$\mu_i(w_1, w_2)$ is strictly concave in w_i, for all $w_i \geq 0$, $i = 1, 2$.*

Turning back to our model when the supplier and firm j agreed on price w_j let us denote the price for firm i as $w_i^{'}(w_j)$. So we assume that for any given fixed w_j the supplier and the downstream firm i agree on the price $w_i^{'}(w_j)$ which is, applying equation (2.4.3), given below:

$$w_i^{'}(w_j) = \arg\max_{w_i} \left[U(w_1, w_2) - U^b\right] \left[\pi_i(w_1, w_2) - \pi_i^b\right], \tag{2.4.4}$$

where U^b and π_i^b are the status quo profits (i.e. the profit obtained if one player

[8]More on the choice of s^0 in Binmore, Rubinstein and Wolinsky (1986).

decides not to bargain with the other player). In the literature U^b and π^b are commonly interpreted as the outside option utility levels of bargaining units. An outside option is often defined as the best alternative that a player can command if he withdraws unilaterally from the bargaining process. If both downstream firms take part in the bargaining then it is obvious that:

$$X := \left\{(w_1, w_2) \in R_+^2 \mid U(w_1, w_2) \geq U^b,\ \pi_i(w_1, w_2) \geq \pi_i^b,\ i = 1, 2\right\}$$

$w_i^{'}(\cdot)$ should not be interpreted as the reaction function of firm i, because it does not represent the best reply of firm i on the behavior of firm j. Instead, it represents the outcome of our non-cooperative game when the particular terminal node of the game tree is reached: w_j has been already accepted and price $w_i^{'}(w_j)$ is an "out of game" part of the rules on the payoff function of the whole underlying non-cooperative game.

To determine the slopes of both functions $w_i^{'}(\cdot)$, $i = 1, 2$ let us consider how the price $w_i^{'}(w_j)$ which results from Nash solution depends on the given w_j. As it is shown in Appendix A.1 the functions $w_i^{'}(\cdot)$ and $w_j^{'}(\cdot)$ are increasing in the wholesale price of the firms' rival. On this stage it is important to notice that function $w_1^{'}(\cdot)$ is increasing until the following inequality holds: $w_1^{'}(w_2)q_1^* + w_2 q_2^* < \tilde{w}q^m$.[9] As we have assumed earlier that functions $w_i^{'}(\cdot)$, $i = 1, 2$ are concave, to ensure that they intersect only once we assume additionally that whenever they intersect $w_1^{'}(\cdot)$ is steeper than $w_2^{'}(\cdot)$.[10] Both functions are illustrated in Figure 2.5.

The analogue of the outside option in our model is the possibility for downstream firms to integrate backwards. As we know in our model only firm 1 can do it. In case of integration, it faces a fixed cost, which must be sunk for production to take place and pays a unit wholesale price v, which is a marginal cost of self-supply.[11] To simplify the following analysis without loss of generality let us assume that fixed costs are zero.

Now let us rewrite the function (2.4.4) for each downstream firm considering the existence of the backward integration option.

$$w_1^{'}(w_2) = \arg\max_{w_1} U(w_1, w_2) \left[\pi_1(w_1, w_2) - \pi_1^{IB}(v, w_2)\right], \tag{2.4.5}$$

where $\pi_1^{IB}(v, w_2)$ is the profit of firm 1 at (v, w_2) if it integrates backward. More precisely it will be described in the next section. But it is obvious that it does not influence the settled price $w_1^{'}(w_2)$.

$$w_2^{'}(w_1) = \arg\max_{w_2} \left[U(w_1, w_2) - U^b\right] \pi_2(w_1, w_2), \tag{2.4.6}$$

Remark 1. *In equation (2.4.5) $U^b = 0$, because if supplier breaks down the negotiation with firm 1, firm 1 will produce the input by itself. In such case firm 2 will still buy from the supplier the same quantity (Cournot) for the same price w_2 because*

[9] More precisely it was explained in Section 2.4.1.

[10] See Tirole (1988), Chapter 5, p. 226; Friedman (1977), pp. 70-74, 168-172.

[11] According to Katz (1987), given the assumption that $v > c$, it is not socially efficient for any downstream firm to integrate; integration will lead to higher industrywide costs of producing a given level of total output.

it will compete with firm 1 afterwards. Hence, there is no sense for the supplier to refuse to deal with firm 1, because he will lose additional profit (from selling to firm 1) and will get nothing substituted from firm 2.

In equation (2.4.6) $\pi_2^b = 0$ *(firm 2 does not have the outside option);* $U^b = \tilde{w}_1 \tilde{q}^m$*, because the upstream supplier will decide not to bargain with firm 2 (to withdraw from negotiations with it) only if he sells to firm 1 the quantity* $\tilde{q}^m$ *for some agreed unit price* $\tilde{w}_1$*, which was explained in Section 2.4.1.*

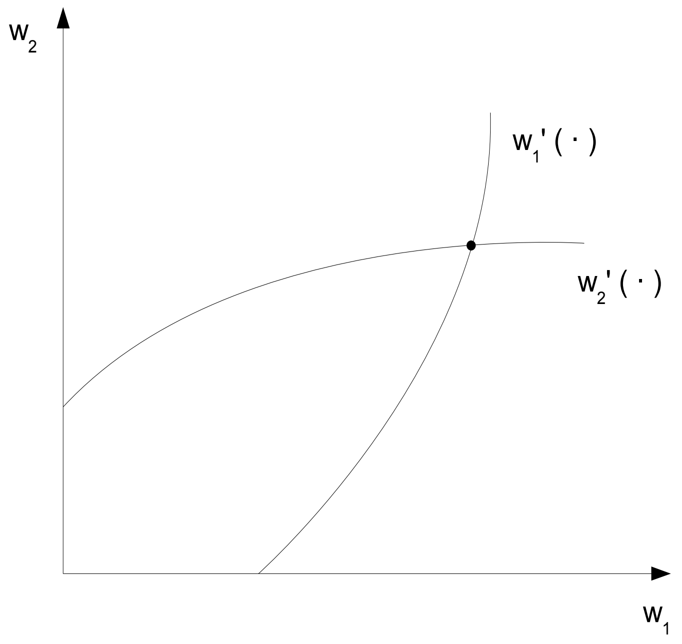

Figure 2.5: Functions $w_i^{'}(\cdot)$ and $w_j^{'}(\cdot)$

2.4.2.2 Payoffs if nobody withdraws

Now let us define the payoff vectors. In each vector described below on the first position the profit of the supplier stays, then of downstream firms 1 and 2, respectively:[12]

$$P(w_1, w_2) = (U(w_1, w_2), \pi_1(w_1, w_2), \pi_2(w_1, w_2)) \tag{2.4.7}$$

$$P_1(w_2) = \left(U(w_1^{'}(w_2), w_2), \pi_1(w_1^{'}(w_2), w_2), \pi_2(w_1^{'}(w_2), w_2)\right), \tag{2.4.8}$$

where w_2 is a price that was negotiated between downstream firm 2 and the supplier and $w_1^{'}(w_2)$ is the price on which the supplier and firm 1 agree (explanation in Section 2.4.2.1 (eq. 2.4.5)).

[12]Important feature of the intermediate good market: the profit of each downstream firm and its demand for an input are functions both of the price that firm pays for the input and of the price that the buyer's product-market rival pays.

2.4.2.3 Payoffs with backward integration

According to our basic assumption that all players are rational utility maximizers, firm 1 decides whether to integrate during negotiations. As the supplier knows firm 1's cost of self-production v he can influence the firm's decision by suggesting some particular price value.

Now let us define the payoff vector if w_2 is accepted, but firm 1 chooses to withdraw, namely to integrate backward.

$$P_1^w(w_2) = \left(U(0, w_2), \pi_1^{IB}(v, w_2), \pi_2(v, w_2)\right), \tag{2.4.9}$$

where:
$U(0, w_2) = w_2 q_2^*(v, w_2) - C_s(q_2^*(v, w_2))$
$\pi_1^{IB}(v, w_2) = (p^*(v, w_2) - v)q_1^*(v, w_2)$
$\pi_2(v, w_2) = (p^*(v, w_2) - w_2)q_2^*(v, w_2)$

Firm 1 makes its integration decision by comparing its expected profit with integration and without it. Hence, firm 1 integrates if and only if:

$$\pi_1^{IB}(v, w_2) - \pi_1(w_1, w_2) \geq 0, \tag{2.4.10}$$

where w_1, w_2 are the wholesale prices, offered by the upstream supplier in the bargaining period in which firm 1 makes a decision. In the future analysis we assume that firm 1 has not integrated backward yet.

Analogously, if price w_1 was agreed upon, but firm 2 chooses to withdraw, then there is the following payoff vector:

$$P_2^w(w_1) = (U(w_1, 0), \pi_1^m(w_1), 0) \tag{2.4.11}$$

As due to our assumption firm 2 does not have an option of backward integration, it will not withdraw from negotiations with the supplier as long as its expected profit isn't negative. Therefore, the illustration in Figure 2.1 of the process when the supplier makes an offer could be simplified as it is shown in Figure 2.6.

$$P_w(w_1) = (U(w_1, 0), \pi_1^m(w_1), 0), \tag{2.4.12}$$

with:
$\pi_1^m(w_1) = (p^m(w_1) - w_1)q^m(w_1)$, where $q^m(w_1)$ is a monopoly output in the local market; $U(w_1, 0) = w_1 q^m(w_1) - C_s(q^m(w_1))$.

$$P_w(w_2) = \left(U(0, w_2), \pi_1^{IB}(v, w_2), \pi_2(v, w_2)\right), \tag{2.4.13}$$

where:
$U(0, w_2) = w_2 q_2^*(v, w_2) - C_s(q_2^*(v, w_2))$;
$\pi_1^{IB}(v, w_2) = (p^*(v, w_2) - v)q_1^*(v, w_2)$;
$\pi_2(v, w_2) = (p^*(v, w_2) - w_2)q_2^*(v, w_2)$.

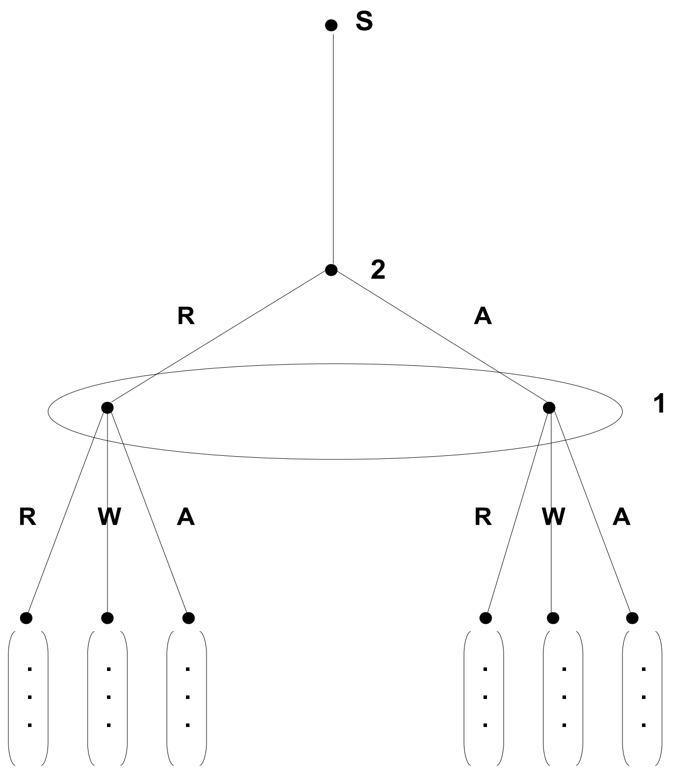

Figure 2.6: The supplier makes a price offer (simplified illustration)

2.4.3 The bargaining procedure

In the previous section we have determined the aims and payoffs of all players. Using these results let us now describe the bargaining procedure.

Remark 2. *The whole following reasoning is based on the assumption of behavior in subgame perfect equilibrium.*

Let us denote the prices that the upstream supplier offers to both downstream firms as w_1^s and w_2^s. In Section 2.4.1 we have already discussed the dependence between the offered prices and suggested to them output quantities. Hence, in the initial period the supplier offers simultaneously (w_1^s, w_2^s). Both downstream firms decide also simultaneously whether to accept or to reject this offer. If both prices are accepted, the payoffs are represented by vector $P(w_1^s, w_2^s)$. If, for example, w_j^s is accepted, but w_i^s - not, then firm i and the supplier adjust the price $w_i^{'}(w_j^s)$, with given price w_j^s, the payoff vector in this case is $P_i(w_j^s)$.

If both initial offers are rejected, then in the next bargaining period $t+1$, where t is the number of the current period, both downstream firms will make a counteroffer to the supplier. The payoff vector in this case is determined by the vector $\delta^{t+1}P_f^{t+1}$.[13]

Let us denote the prices which both downstream firms simultaneously offer to the supplier as w_1^f and w_2^f, respectively. The relation between these price offers and the suggested quantities was also specified in Section 2.4.1. Receiving the offers from downstream firms the supplier has the following choices:

1. he can accept both prices, the payoffs in this case are represented by vector $P(w_1^f, w_2^f)$;
2. he can accept price w_j^f and reject w_i^f, then the payoffs are represented by vector $P_i(w_j^f)$, for $j = 1, 2$;
3. he can reject both prices, the payoffs in this case are represented by vector $\delta^{t+1}P_s^{t+1}$;
4. he can accept price w_i^f and withdraw from negotiations with firm j, the payoffs in this case are denoted by vector $P_w(w_i^f)$, for $j = 1, 2$.

Considering these actions it is important to mention, that only the downstream firm 1 can presumably withdraw from negotiations, as we have already described earlier. As for the downstream firm 2, if it withdraws from the negotiation process with the supplier, it earns zero profit, therefore the action in which it continues to bargain dominates the action to withdraw. The upstream supplier faces a similar situation: we know from the analysis in Section 2.4.1 that the downstream firm 2 will offer such price w_2 for which it is ready to buy the Cournot quantity q_2^*, because it knows that firm 1 will enter the market in all cases, therefore if the supplier plays the action (W, A) (when the downstream firms make a counteroffer), he refuses to sell to firm 1 at least the Cournot quantity q_1^*, so he earns $P_w(w_2^f)$, but if it plays the action (R, A) it earns $P_1(w_2^f)$, which is larger than the payoff by the action (W, A). Hence, the action (R, A) dominates the action (W, A). In subgame perfect equilibrium the downstream firm 2 will never withdraw from the negotiations with the supplier and the supplier will never withdraw from negotiations with the downstream firm 1. He can withdraw from negotiations with firm 2 if firm 1 suggests some particular purchase price w_1 for the monopoly quantity q^m.

Next let us consider more precisely the behavior of all players and how does the non-cooperative bargaining process go on? Let us assume that the supplier has already announced his prices and now both downstream firms must react on these price offers.

2.4.3.1 Supplier offers. Determination of the downstream firms' best actions

The illustration of the considered situation is given in Figure 2.6. Assume that the upstream supplier offers (w_1^s, w_2^s) in order to maximize his utility. Both downstream firms must simultaneously decide whether to accept or to reject this offer. Moreover, firm 1 has an option to withdraw from the negotiations and to produce the output by itself. The reaction of both downstream firms on supplier's offer is illustrated in

[13] See Section 2.4.2.1.

Figure 2.7. As it has been already mentioned, we assume that if the supplier offers the wholesale price $w_1^s > v$, where v is the firm's 1 price of the self-production, the downstream firm 1 withdraws from the negotiation.

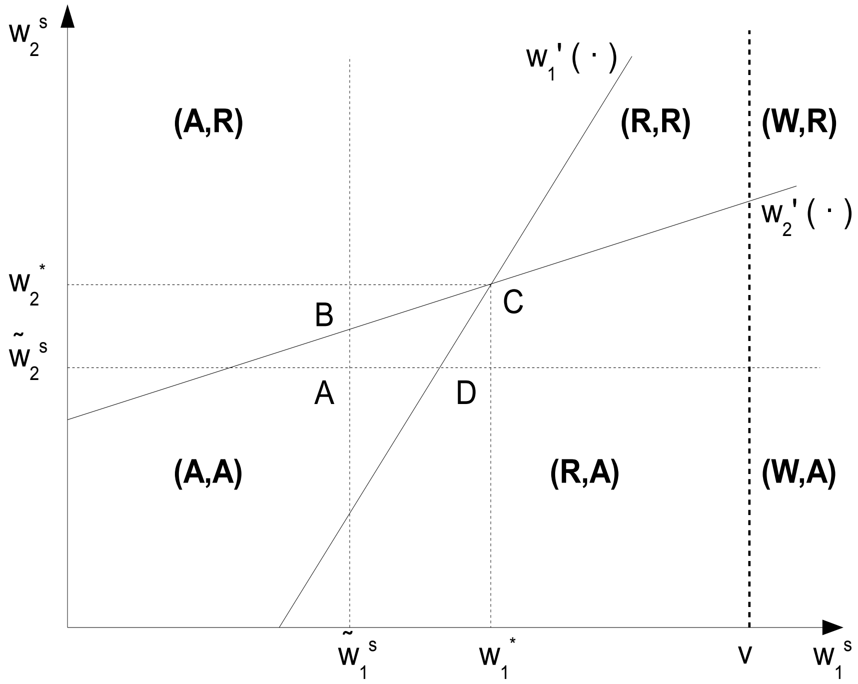

Figure 2.7: Downstream firms' reaction on supplier's price offer

The payoff of each firm depends on the response (action) of its rival as it was shown in the previous section. Let us find the optimal reactions of both downstream firms depending on the behavior of their rival.

To make an optimal decision, each player must generally foresee how his opponent will behave. The first and indisputable basis for such a conjecture is that one's opponent should not play dominated strategies. If an action always gives a lower payoff to a player than another action, whatever the other player does, we may assume that the player will not pick that action.[14] Unfortunately, in many games the elimination of dominated strategies does not go very far toward selecting a unique "reasonable" outcome (or limited set of them). Similarly, in Bertrand or Cournot games of simultaneous choices of prices or quantities, the optimal action for one firm depends on that of the other firm, which means that one already has a lot of undominated strategies.[15]

First let us find the best reaction of firm 1 on the accepted price w_2^s. For simplicity we omit the upper index t in the payoff vectors when we describe the current period. If firm 1 accepts w_1^s, then according to equations (2.4.7) and (2.4.8), firm 1 will get

[14]Tirole (1988), p. 425.
[15]ibid., Chapter 5.

the payoff shown in the payoff vector $P(w_1^s, w_2^s)$, but if it rejects w_1^s its payoff is given in $P(w_1^{'}(w_2^s), w_2^s)$. It is straightforward, that in order to make a decision firm 1 will compare both payoffs. $\pi_1(w_1^s, w_2^s) \geq \pi_1(w_1^{'}(w_2^s), w_2^s)$ if and only if $w_1^s \leq w_1^{'}(w_2^s)$. Thus, firm 1 accepts w_1^s if $w_1^s \leq w_1^{'}(w_2^s)$.

Analogously, the best reaction of firm 2 on the accepted w_1^s is the price w_2^s if $w_2^s \leq w_2^{'}(w_1^s)$. Hence, if $w_1^s \leq w_1^{'}(w_2^s)$ and $w_2^s \leq w_2^{'}(w_1^s)$, in equilibrium both firms may choose the action (A, A).

Next assume that firm 2 rejects the price offer w_2^s. Then if firm 1 accepts w_1^s it receives $\pi_1(w_1^s, w_2^{'}(w_1^s))$. If it rejects w_1^s it receives π_1^f (from the payoff vector P_f^{t+1}). Since $\pi_1(w_1^s, w_2^{'}(w_1^s))$ is a decreasing function in w_1^s, there exists such a unique $\tilde{w}_1^s$ that $\pi_1(\tilde{w}_1^s, w_2^{'}(\tilde{w}_1^s)) = \pi_1^f$. Hence, for all $w_1^s \leq \tilde{w}_1^s$ the best reaction of firm 1 is to accept the price if firm 2 rejects w_2^s; and for all $w_1^s > \tilde{w}_1^s$ the best reaction of firm 1 is to reject the offered price. Therefore, if $w_1^s \leq \tilde{w}_1^s$ and $w_2^s > w_2^{'}(w_1^s)$, then the best action for both downstream firms in equilibrium is to choose (A, R).

Using similar argumentation if the offered prices are so that $w_2^s \leq \tilde{w}_2^s$, $w_1^s \geq w_1^{'}(w_2^s)$ and $w_1^s < v$, then in equilibrium both downstream firms choose (R, A). If $w_2^s \leq \tilde{w}_2^s$, $w_1^s \geq w_1^{'}(w_2^s)$ and $w_1^s \geq v$, then in equilibrium the best reaction is (W, A).

At $w_1^s \geq \tilde{w}_1^s$, $w_2^s \geq \tilde{w}_2^s$ and $w_1^s < v$, in equilibrium both downstream firms may choose (R, R); at $w_1^s \geq \tilde{w}_1^s$, $w_2^s \geq \tilde{w}_2^s$ and $w_1^s \geq v$, they choose (W, R) as the best response.

Result 1. *If the offered prices lie in region ABCD in equilibrium both downstream firms may choose the action (A,A) as well as the action (R,R).*

Summarizing the obtained results let us make the following definition:

$$\varphi := \left\{(w_1^s, w_2^s) \mid w_1^s \leq w_1^{'}(w_2^s),\ w_2^s \leq w_2^{'}(w_1^s)\right\}$$

Hence, for any supplier's price offer (w_1^s, w_2^s) the equilibrium payoffs are given by the following vectors:[16]

$$P_s(w_1, w_2) = \begin{cases} P(w_1^s, w_2^s), & \text{if } \forall (w_1^s, w_2^s) \in \varphi \ \mathbf{(A,A)} \\ P(w_1^s, w_2^{'}(w_1^s)), & \text{if } w_1^s \leq \tilde{w}_1^s,\ w_2^s > w_2^{'}(w_1^s) \ \mathbf{(A,R)} \\ P(w_1^{'}(w_2^s), w_2^s), & \text{if } w_2^s \leq \tilde{w}_2^s,\ w_1^s \geq w_1^{'}(w_2^s),\ w_1^s < v \ \mathbf{(R,A)} \\ P_1^w(w_2^s), & \text{if } w_2^s \leq \tilde{w}_2^s,\ w_1^s > w_1^{'}(w_2^s),\ w_1^s \geq v \ \mathbf{(W,A)} \\ P_1^w(\hat{w}_2), & \text{if } w_1^s \geq \tilde{w}_1^s,\ w_2^s \geq \tilde{w}_2^s,\ w_1^s \geq v \ \mathbf{(W,R)} \end{cases} \tag{2.4.14}$$

If both downstream firms choose action (R,R) to the supplier's price offer (w_1^s, w_2^s) $(w_1^s \geq \tilde{w}_1^s,\ w_2^s \geq \tilde{w}_2^s,\ w_1^s < v)$ then in the next bargaining period they make counteroffers to the supplier and equilibrium payoffs will be determined by the payoff vector $P_f^{t+1}(\acute{w}_1, \acute{w}_2)$. More precisely we shall describe this situation in the next section.

[16] $\hat{w}_2$ is the new negotiated price to which firm 2 and the supplier will come if firm 1 withdraws from negotiations.

2.4.3.2 Counteroffers by downstream firms. Determination of supplier's best actions

Assume that in the previous period both downstream firms rejected the offered prices and now it is their turn to make counteroffers to the upstream supplier. In this section let us find the supplier's best reaction on the downstream firms' price offers. As we have already described the supplier's response "Withdraw" from the negotiations is dominated by the response "Reject". Therefore we do not consider it in this section.

Let us assume that both downstream firms offer the prices w_1^f and w_2^f, respectively. Below we describe these prices more precisely. The supplier's reaction on any pair of (w_1^f, w_2^f) is shown in Figure 2.8. In order to simplify the analysis, let us first consider the case where at least one price offer is accepted.[17] Then we shall consider separately the response (R, R) illustrated in Figure 2.9.

Assumption 4. *We assume that the downstream firm 1 offers to the upstream supplier such price* w_1^f*, that is strictly lower than its own price of self-production* v*. Both Figures 2.8 and 2.9 display this case.*

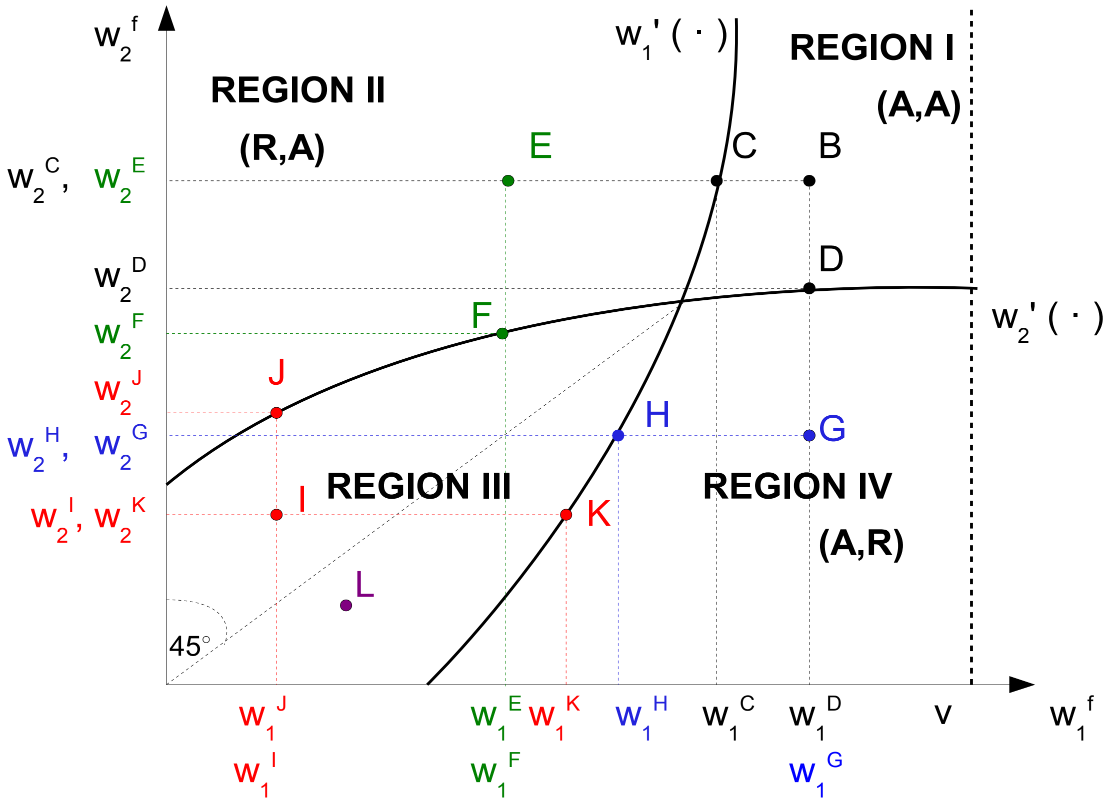

Figure 2.8: Derivation of the supplier's best response

[17] That means that we consider only the actions: (A, R), (R, A), (A, A).

Analyzing the depicted strategies let us first assume that both downstream firms offer such prices, that (w_1^f, w_2^f) lies in Region I, say in some arbitrary point B. If the supplier rejects w_1^B and accepts w_2^B it will lead in the next period to vector C, if he accepts w_1^B and rejects w_2^B it will lead to vector D in the next period. So in C the prices are (w_1^C, w_2^C); in D they are (w_1^D, w_2^D), respectively. The supplier's profit is increasing in the wholesale prices, so he chooses (w_1^B, w_2^B). Hence, in the Region I, the actions (A, R) and (R, A) are dominated by (A, A).

Next let us assume that the price vector lies in Region II, say in some arbitrary point E, then if supplier rejects w_1^E and accepts w_2^E, that will lead to vector C in the next period. If he accepts w_1^E and rejects w_2^E, that will lead to vector F. Comparing the optimal prices in three vectors F, E, C, the best option for supplier is C, because $w_1^C > w_1^E = w_1^F$ and $w_2^C > w_2^F$. Since C dominates E and F, action (R, A) dominates (A, A) and (A, R).

Let us assume that the price vector lies in Region IV, say in some arbitrary point G. Then accepting w_1^G and rejecting w_2^G will lead to the vector D in the next period. Analogously, rejecting w_1^G and accepting w_2^G will lead to the vector H in the next period. As $w_1^D > w_1^H$ and $w_2^D > w_2^H = w_2^G$, D dominates G, and G dominates H, therefore action (A, R) dominates (A, A) and (R, A).

The next interesting case is if prices lie in Region III, let us say in some arbitrary points above or below the 45°-line I or L. Let us consider first point I. If the supplier accepts w_1^I and rejects w_2^I in the next period prices will lie in J. Analogously, if the supplier rejects w_1^I and accepts w_2^I, it will lead to vector K in the next period. Comparing them, as $w_1^K > w_1^I = w_1^J$ and $w_2^J > w_2^I = w_2^K$, both J and K dominate I. The next step is to compare J and K. In J prices are w_1^J and w_2^J, in K prices are w_1^K and w_2^K. As $w_1^K > w_2^J$ and $w_2^K > w_1^J$, the utility of supplier is larger if prices lie in K than in J, and therefore K dominates J, that means that in the region above the 45°-line the action (R, A) dominates the action (A, R). Analogous arguments are necessary to prove that below the 45°-line, in the region where L lies, the action (A, R) dominates (R, A). Along the 45°-line the supplier is indifferent between (A, R) and (R, A).

Result 2. *We have found the supplier's best responses to any possible price offers (w_1^f, w_2^f), assuming that at least one price will be accepted.*

Now let us check when the supplier's optimal response is (R, R). It is clear, that the supplier's response depends on the payoff in the next period, when he will make a counteroffer.

If supplier chooses (R, A) his payoff according to equation (2.4.8) is:[18]

$$U(w_1'(w_2^f), w_2^f) = w_1'(w_2^f)q_1^*(w_1'(w_2^f), w_2^f) + w_2^f q_2^*(w_1'(w_2^f), w_2^f) \tag{2.4.15}$$

$\hat{U}$ with $\hat{U}(w_2^f) := U(w_1'(w_2^f), w_2^f)$ is an increasing function in w_2^f. That means, that there exists some critical price $\tilde{w}_2^f$, at which the payoff, given in equation (2.4.15), is equal to the payoff which will appear in the next bargaining period, when the

[18] As the supplier will sell to both downstream firms we are speaking about Cournot quantities; the relation between price offers and quantities was explained in Section 2.4.1.

supplier makes counteroffers.[19] Therefore it is obvious, that if $w_2^f < \tilde{w}_2^f$, then the action (R, R) dominates; if $w_2^f \geq \tilde{w}_2^f$, then the action (R, A) dominates.

Next let us compare (R, R) with (A, R). The action (A, R) gives the following profit:

$$U(w_1^f, w_2^{'}(w_1^f)) = w_1^f q_1^*(w_1^f, w_2^{'}(w_1^f)) + w_2^{'}(w_1^f) q_2^*(w_1^f, w_2^{'}(w_1^f)) \qquad (2.4.16)$$

$\tilde{U}$ with $\tilde{U}(w_1^f) := U(w_1^f, w_2^{'}(w_1^f))$ is an increasing function in w_1^f and therefore there exists such price $\tilde{w}_1^f$ by which the payoff given in equation (2.4.16) is equal to the supplier's payoff given in vector P_s^{t+1}. Hence, if $w_1^f < \tilde{w}_1^f$, then (R, R) dominates (A, R); if $w_1^f \geq \tilde{w}_1^f$, then the strategy (A, R) dominates.

The supplier's actions are illustrated in Figure 2.9.

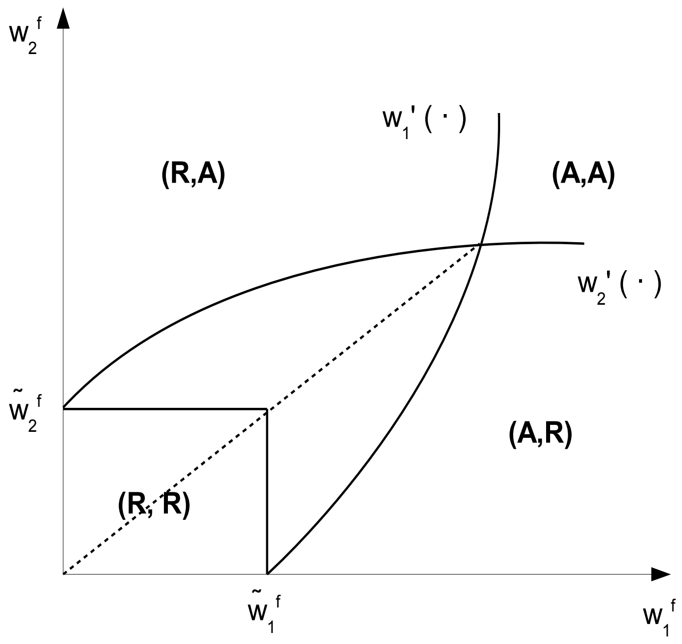

Figure 2.9: The supplier's best response (R,R)

Result 3. *For each downstream firms' price offer (w_1^f, w_2^f) with $w_1^f < \tilde{w}_1^f$, $w_2^f < \tilde{w}_2^f$ in equilibrium the supplier's action (R, R) dominates all other actions.*

To summarize the obtained results let us first make the following definition:

$$\gamma := \left\{ (w_1^f, w_2^f) \mid w_1^f \geq w_1^{'}(w_2^f),\ w_2^f \geq w_2^{'}(w_1^f) \right\}$$

[19]This payoff vector is denoted as $P_s^{t+1}(\check{w}_1, \check{w}_2)$.

For arbitrary counteroffers made by downstream firms the payoff vectors are shown below:[20]

$$P_f(w_1,w_2) = \begin{cases} P(w_1^f, w_2^f), & \text{if } \forall (w_1^f, w_2^f) \in \gamma \ \textbf{(A,A)} \\ P_s^{t+1}(\breve{w}_1, \breve{w}_2), & \text{if } w_1^f < \tilde{w}_1^f, w_2^f < \tilde{w}_2^f \ \textbf{(R,R)} \\ P(w_1^f, w_2'(w_1^f)) \text{ or } P(w_1'(w_2^f), w_2^f) & \text{otherwise.} \ \ \textbf{(A,R) or (R,A)} \end{cases} \tag{2.4.17}$$

Similarly with one of the previous sections, in which we have considered the subgame where the supplier made the price offers, let us determine the optimal counteroffers for both downstream firms if they know the supplier's behavior.

In our model both downstream firms do not cooperate, therefore the choices of the offered prices w_i^f, $i = 1, 2$ depend on the prices of the firm's rival w_j^f. Therefore we must determine the reaction of each downstream firm on the behavior of its rival.

Taking the actions of the supplier as given, the different choices of w_i^f for any given w_j^f are shown in Figure 2.10.

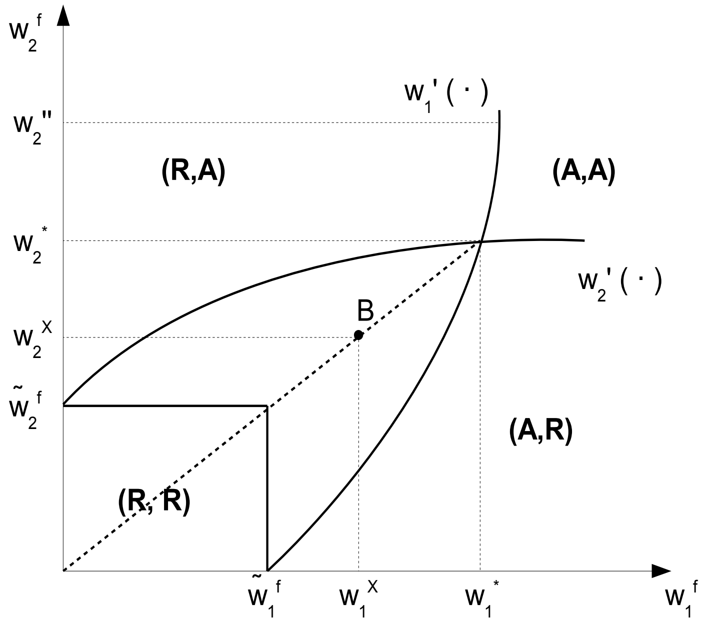

Figure 2.10: Determination of the downstream firms' optimal price offers

- Let us assume that $w_2^f < \tilde{w}_2^f$, then firm 1 will choose price $w_1^f \geq \tilde{w}_1^f$ in order not to end the negotiations by (R, R). But it is to be mentioned that firm 1 will choose the lowest possible value of w_1^f, namely $w_1^f = \tilde{w}_1^f$, because its profit is decreasing in price. So the best response of firm 1 if $w_2^f < \tilde{w}_2^f$ is $w_1^f = \tilde{w}_1^f$.

[20]For simplicity the upper index t which denotes the current period was omitted.

- Let us assume that $w_2^f = w_2^x > \tilde{w}_2^f$. In this case if $w_1^f < w_2^x$, then both prices will lie in region (R, A), so w_1^f will be rejected and w_2^x will be accepted. If $w_1^f = w_2^x$ then the prices lie in B and the supplier is indifferent between strategies (A, R) and (R, A). But if firm 1 suggests the price a little bit larger than w_2^x, it will be better off than by price $w_1^f = w_2^x$. Applying the statement at the beginning of the Section 2.3 about the existence of the smallest possible unit in prices, for all given $w_2^f \in \left(\tilde{w}_2^f, w_2^*\right)$ the best response for firm 1 is $w_1^f = w_2^f + 1$ cent.[21]
- Let us assume that $w_2^f = w_2^{''}$. If firm 1 offers $w_1^f < w_1^{'}(w_2^{''})$, then w_1^f will be rejected and $w_2^{''}$ will be accepted (Region (R, A)). So firm 1 must offer $w_1^f \geq w_1^{'}(w_2^{''})$ in order to get it accepted.

Hence, we can state the following result.

Result 4. *In equilibrium the counteroffer of both downstream firms* (w_1^f, w_2^f) *given the supplier's reaction on the offered prices lies on the intersection of the graphs of the functions* $w_1^{'}(\cdot)$ *and* $w_2^{'}(\cdot)$ *with* $w_1^f = w_1^*$ *and* $w_2^f = w_2^*$.

2.4.3.3 Equilibrium

Due to the analysis made in the previous sections we can finally find equilibrium wholesale prices.

Definition 2.4.3. *A (subgame) perfect equilibrium (Selten 1965) is a set of strategies for each player that in any subgame induces a Nash equilibrium.*

Perfection requires that strategies be in equilibrium whatever the location (subgame) in the game tree, and not only along the equilibrium path.

The basic idea of perfect equilibrium is to select Nash equilibria that do not involve noncredible threats, by (roughly) requiring that the players' behavior be optimal even in situations that are not reached on the equilibrium path.

In order to find a subgame perfect equilibrium in the wholesale prices let us decompose our analysis into several steps. First, let us summarize the results that we obtained in the previous sections concerning the behavior of all players in a subgame perfect equilibrium:

1. Let us start with the upstream supplier. According to our analysis made in Section 2.4.3.2 in the equilibrium he will support all prices lying in Region I (see Figure 2.8), such as $(w_1, w_2) \in \gamma$. The highest prices that both downstream firms accept lie in point C (Figure 2.7). Hence, the supplier knowing the behavior of both downstream firms will offer prices $(w_1^{'}(w_2^*), w_2^{'}(w_1^*)) = (w_1^*, w_2^*)$ in order to get them accepted. If downstream firms accept, then the equilibrium payoff vector $P(w_1^*, w_2^*)$ is realized.

[21]Assuming that the players use Euro as a currency so the smallest unit is 1 cent.

2. Now let us consider the behavior of downstream firms. If the supplier offers prices $w_1'(w_2^*)$ and $w_2'(w_1^*)$ there are two possible ways to act. As we have found out, in the equilibrium in Region ABCD (see Figure 2.7) the downstream firms may choose actions (A, A) and (R, R). If they choose (A, A) the bargaining ends with the equilibrium payoff vector $P(w_1^*, w_2^*)$, where $w_1^* = w_1'(w_2^*)$ and $w_2^* = w_2'(w_1^*)$.

   If the downstream firms choose the action (R, R), then in the next period they will make a counteroffer to the supplier. The profit of each downstream firm is a decreasing function in its wholesale price, so both downstream firms want to offer the lowest possible price. But as they act non-cooperatively, the payoff of each of them depends on the behavior of its rival. According to the analysis which we have made in Section 2.4.3.2 (see Result 4) we have found out that the best price offer for both downstream firms, if they know the behavior of the supplier, are prices that lie on the intersection of the graphs of the functions $w_1'(\cdot)$ and $w_2'(\cdot)$, namely in point C (Figure 2.7). Therefore, both downstream firms will offer prices (w_1^*, w_2^*) to the supplier and he will accept them, so the equilibrium payoff vector $\delta P(w_1^*, w_2^*)$ will be realized.

Hence, it is obvious that neither another proposal nor any other reaction to a proposal can constitute an advantageous unilateral deviation of any player. So (w_1^*, w_2^*) is a subgame perfect equilibrium price vector. Moreover, as $\delta < 1$ the supplier will offer the equilibrium prices and they will be accepted by downstream firms already in the initial period, and the equilibrium payoff vector $P(w_1^*, w_2^*)$ will be realized.

Now it is to prove whether a subgame perfect equilibrium price vector that we have found is unique and there are no other equilibrium price vectors in the considered game.

As we know from our analysis the supplier will offer prices $w_1 \geq w_1'(w_2)$ and $w_2 \geq w_2'(w_1)$. Now without loss of generality let us first assume, that he makes the following offer $(\hat{w}_1, w_2^*)$, with $\hat{w}_1 > w_1'(w_2^*)$ and $w_2^* \geq w_2'(\hat{w}_1)$ and both downstream firms accept. For firm 1 such behavior will lead to the following profit $\pi_1(\hat{w}_1, w_2^*) < \pi_1(w_1^*, w_2^*)$. This cannot possibly be part of a Nash equilibrium, because firm 1 could just reject an offer and ensure itself profit closer to $\pi_1(w_1^*, w_2^*)$, namely $\pi_1(w_1^*, w_2^*) > \pi_1(\hat{w}_1, w_2^*)$ (firm 2 will accept w_2^*). Analogously, there will be also no equilibrium if the supplier offers $(w_1^*, \hat{w}_2)$ with $\hat{w}_2 > w_2'(w_1^*)$.

Second, let us assume that at the initial stage the supplier offers prices (w_1, w_2), so that $w_1 > w_1'(w_2)$ and $w_2 > w_2'(w_1)$. If downstream firms accept, they will obtain profits $\pi_1(w_1, w_2) < \pi_1(w_1^*, w_2^*)$ and $\pi_2(w_1, w_2) < \pi_2(w_1^*, w_2^*)$. These prices (w_1, w_2) are also not equilibrium, because profit of each firm could be easily improved by rejecting its offer. Hence, assume now that downstream firms reject an offer $w_1 > w_1'(w_2)$ and $w_2 > w_2'(w_1)$. They will make a counteroffer in the next bargaining period. As firms' profit functions are decreasing in their own wholesale prices, both firms are better off by offering the lowest possible prices. Now assume that they suggest prices $w_1 < w_1'(w_2)$ and $w_2 < w_2'(w_1)$. If supplier accepts one of them or both, then analogously with previous cases, it is obvious, that there will be also no equilibrium, because the supplier can increase his profit by rejecting one or both offers.

Now assume that the supplier rejects both $w_1 < w_1^{'}(w_2)$ and $w_2 < w_2^{'}(w_1)$. Then in the next bargaining period he will make the price offer. The situation in which all players are better off by rejecting the offers of each other may repeat. Such situation is illustrated in Figure 2.3, where we are in the branch of the game tree when all players reject. Without loss of generality let us assume that permanent rejection of each other offers brings us to some stage t when both downstream firms have already received an offer from the supplier and must decide. After some particular time of regular rejections one or both downstream firms may doubt that they will finally get an offer $w_i^s \leq w_i^{'}(w_j)$, $i = 1, 2$. The player who plays the same game repeatedly may develop a reputation for certain kinds of play. That means that if the player always plays in the same way, his opponents will come to expect him to play that way in the future and will adjust their own play accordingly.[22] Therefore, in order not to end with nothing one particular firm i or both firms will accept an offer, which will end the bargaining process and will lead to the payoff vectors $\delta^t P(w_i^s, w_j^{'}(w_i^s))$, $i = 1, 2$, $i \neq j$ or $\delta^t P(w_1^s, w_2^s)$, respectively. It is obvious that the agreed prices are not the equilibrium prices and the obtained payoff vectors are also not the equilibrium vectors.

On the other hand, as all players are interested in reaching agreement, in each bargaining period they will make more steps to each other in order to find a compromise. That means that with each even period, when it is supplier's turn to make an offer, he will suggest smaller prices to both firms than he has done in the previous period; with each odd period both downstream firms will offer higher prices to the supplier, comparing with the previous period. Hence, assume that at some bargaining period m the upstream supplier offers (w_1^*, w_2^*). As it has been proved earlier both downstream firms will accept this offer, which will lead to the equilibrium payoff vector $\delta^m P(w_1^*, w_2^*)$. As $\delta < 1$ it is straightforward that $\delta^m P(w_1^*, w_2^*) < P(w_1^*, w_2^*)$. That means, that despite the suggested equilibrium prices, $\delta^m P(w_1^*, w_2^*)$ does not result as a subgame perfect equilibrium payoff vector and all players will get smaller payoff than they could get if they made offer and accepted these prices in the initial period.

Summarizing the obtained results we can state the following proposition:

Proposition 2.4.1. *There exists a unique subgame perfect equilibrium wholesale price vector (w_1^*, w_2^*), which lies on the intersection of the graphs of the the functions $w_1^{'}(\cdot)$ and $w_2^{'}(\cdot)$ with $w_1^* = w_1^{'}(w_2^*)$ and $w_2^* = w_2^{'}(w_1^*)$. Moreover this price vector is offered and accepted in the first round and leads to the equilibrium payoff vector $P(w_1^*, w_2^*)$.*

2.5 Concluding remarks

2.5.1 On use of the cooperative game approach

In his paper Nash (1950) introduces the bargaining problem as follows: "A two-person bargaining situation involves two individuals who have the opportunity to

[22] Fudenberg and Tirole (1991), p. 369.

collaborate for mutual benefit in more than one way".[23] Applying such definition almost all human interactions can be seen as bargaining.

According to Binmore, Osborne and Rubinstein (1992) the target of a non-cooperative theory of bargaining is to find theoretical predictions of what agreement, if any, will be reached by the bargainers. One hopes thereby to explain the manner in which the bargaining outcome depends on the parameters of the bargaining problem and to shed light on the meaning of some of the verbal concepts that are used when bargaining is discussed in ordinary language. However, the theory has only peripheral relevance to such questions as: What is a just agreement? How would a reasonable arbiter settle a dispute? What is the socially optimal deal?

In cooperative bargaining theory the bargaining procedure is left unmodeled. Cooperative theory therefore has to operate from a poorer informational base and hence its fundamental assumptions are necessarily abstract in character. As a consequence, cooperative solution concepts are often difficult to evaluate. Sometimes they may have more than one viable interpretation, and this can lead to confusion if distinct interpretations are not clearly separated.

Nash (1953) notices that both cooperative and non-cooperative approaches to the bargaining problem are complementary, namely each of them helps to justify and clarify the other.[24]

As Binmore (2007) observes: "Cooperative game theory sometimes provides simple characterizations of what agreement rational players will reach, but we need non-cooperative game theory to understand why".

Binmore, Osborne and Rubinstein (1992) who introduce the brief and clear discussion about the cooperative and non-cooperative game theoretic shared goals from different approaches do not see cooperative and non-cooperative theories as rivals. They notice that cooperative theory may be seen as "too general"; but equally there is a sense in which non-cooperative theory may be seen as "too special".

We have modeled and analyzed our bargaining problem as a non-cooperative game, but we have used some tools which belong to cooperative game theory. The idea of implementing the cooperative game approach into the model of non-cooperative game is not new. It is widely used in the applied literature for labor negotiations. But, unfortunately, in most papers the formal model of the bargaining process is not presented.

DeMenil (1971) uses the Nash cooperative bargaining solution to model the bargaining between the union and firm. Despite provided empirical support for Nash bargaining solution, the modeling is axiomatic and cannot be generalized or extended.

O'Brien (2002) examines the welfare effects of third degree price discrimination by an intermediate good monopolist selling to downstream firms with bargaining power. He applies a modeling approach that according to a terminology used by Binmore and Dasgupta (1987) belongs to the Nash program. This seeks to justify axiomatic solutions of cooperative games like the Nash bargaining solution from an underlying non-cooperative game. He explicitly models negotiations and motivates the role of outside options, disagreement payoffs and bargaining weights from an

[23]See p. 155.

[24]See p. 129.

underlying non-cooperative bargaining game. Nevertheless, he also does not describe the formal model and the whole bargaining process.

When two players are involved into negotiation process the relationship between cooperative and non-cooperative approaches to bargaining are well understood and described in the literature. But by the existence of more than two players the implementation of the cooperative game approach in underlying non-cooperative games are far less clear. The generalization of the Nash bargaining solution to n players is straightforward, but the extension of its non-cooperative justification seems to be much more difficult problem. For example in Chae and Yang (1994) and Krishna and Serrano (1996) a uniqueness of subgame perfect equilibrium and convergence to the Nash bargaining solution come at the cost of allowing partial agreements, rather than requiring unanimous consent to a comprehensive proposal.

There are many cooperative solution concepts that may be implemented in the models of non-cooperative games such as Raiffa, Nash bargaining, Kalai-Smorodinsky, Perles-Maschler, etc. But which of them are appropriate for the model, and how should they be applied? The answers to these questions are given in the next section.

2.5.2 Critisism

According to Binmore, Osborne and Rubinstein (1992) the ultimate aim of what is now called the "Nash program"[25] is not only to classify the various institutional frameworks within which negotiation takes place but also to provide a suitable "bargaining solution" for each class. As a test of the suitability of a particular solution concept for a given type of institutional framework, Nash proposed that attempts be made to reduce the available negotiation ploys within that framework to move within a formal bargaining game. If the rules of the bargaining game adequately capture the salient features of the relevant bargaining institutions, then a "bargaining solution" proposed for use in the presence of these institutions should appear as an equilibrium outcome of the bargaining game.

In the presented thesis we are concerned with the Nash bargaining solution, which is the leading solution concept for bargaining situations.[26] Nash (1950, 1953) provided a definition as the maximizer of the "Nash product" of players' utility levels and an axiomatic characterization.[27]

The definition and some useful notations to the bargaining solution were shown in Section 2.4.2.1. However, the question of when Nash bargaining solution is appropriate for a two-player bargaining environment involving alternating offers is still open. Binmore, Osborne and Rubinstein (1992) consider the model in which there is a probability p of breakdown after any rejection.[28] They obtain that when a unique subgame-perfect equilibrium exists for each p sufficiently close to one, the bargaining problem (S, s^0), in which S is the set of available utility pairs at time 0 and s^0 is the breakdown utility pair, has a unique Nash bargaining solution. This

[25] See Nash (1953).

[26] See Nash (1950).

[27] To our knowledge this is the first introduced bargaining solution.

[28] Compare with Moulin (1982), Binmore, Rubinstein and Wolinsky (1986) and McLennan (1988).

is the limiting value of the subgame-perfect equilibrium payoff pair as $p \to 0+$.[29] They prove the similar result in the time-based alternating-offers model when the length τ of a bargaining period approaches 0.

In discussion whether other bargaining solutions from cooperative game theory can be implemented the important example is Moulin's (1984) contribution to the Nash program, namely the implementation of the Kalai-Smorodinsky solution for two-person bargaining games in subgame perfect equilibrium. His model begins with an auction to determine who makes the first proposal. The players simultaneously announce probabilities p_1 and p_2. If $p_1 \geq p_2$, then player 1 begins by proposing an outcome a. If player 2 rejects a, then he makes a counterproposal, b. If player 1 rejects b, then the status quo q results. If player 1 accepts b, then the outcome is a lottery that yields b with probability p_1 and q with probability $1 - p_1$. (If $p_2 > p_1$ then it is player 2 who proposes an outcome, and player 1 who responds.) Binmore, Osborne and Rubinstein (1992) criticize his work noting that it is not clear to what extent such an "auctioning of fractions of a dictatorship" qualifies as bargaining in the sense that this is normally understood.

Haake (1998) provides a setup for implementing bargaining solutions and construct a strategic mechanism for n players that implements the Kalai-Smorodinsky bargaining solution in dominant strategies. He shows the uniqueness of dominant strategy equilibria in each of the induced games. From the obtained mechanism he derives an extensive game form so that the final outcome in the unique subgame-perfect equilibrium coincides with the Kalai-Smorodinsky bargaining solution.

Binmore, Rubinstein and Wolinsky (1986) show how some of the data of an economic situation that involves bargaining can be used to apply Nash's bargaining solution to the problem. The main idea is to use the insights of the strategic approach to bargaining in making the modeling judgments involved in the selection of the utility representations and the disagreement point for the application of the Nash's solution.

The axiomatic bargaining theory of Nash presumes that only status quo utilities and the shape of the utility possibilities set are relevant to the bargaining outcome.[30] Chen and Maskin (1999) study a class of economic problems for which bargaining solutions may depend on more than just utility information. For example a solution to the fifty-fifty split of a single good between two bargainers. They show that the requirements of Pareto efficiency, weak symmetry, and technological monotonicity (i.e., bargainers should gain from technological improvement) combine to characterize welfare egalitarianism.

Hendon and Tranass (1991) analyze the bargaining model in a market with one seller and two buyers, which differ only in their reservation price. Differently to our model the seller's output may be sold to one and only one of two buyers, h and l. The smaller reservation price is assumed to be strictly larger than halfway between the reservation prices of the seller and the high buyer, thus making it more profitable for the seller to sell at the low buyer's reservation price than to sell to the high buyer at the two-person bargaining price. They show that no subgame perfect equilibrium exists for stationary strategies and demonstrate the existence of

[29] More in Binmore and Dasgupta (1987).
[30] Welfaristic point of view.

inefficient equilibria in which the low buyer receives the good with large probability, even as friction becomes negligible. Investigating the relationship between the use of Nash and sequential bargaining they notice that the Nash bargaining seems to be applicable only when the sequential approach yields a unique stationary strategy subgame perfect equilibrium.

Gerber and Upmann (2006) provide a general analysis of different solution concepts for the case of a labor market model where negotiations take place between a labor union and an employers' federation. They show that economic policy implications may be very sensitive to the choice of the bargaining solution. They investigate the robustness of the comparative static effects of the bargaining outcome with respect to the different solutions. The obtained comparative static results vary in a significant way across the different solution concepts.[31] Their analysis shows that it is not sufficient to investigate the bargaining outcome in the utility space if one is interested in economic applications. Important information may be lost by considering the utility space only, and remarkable phenomena, such as, for example, seemingly surprising labor market effects, may be veiled and thus overlooked. Despite they make an analysis on the sample of the labor market their results carry over to other economic environments where the equilibrium outcome is determined through multilateral negotiations.

Despite the variety of solution concepts in bargaining theory which may be used to model the outcome of negotiations on economic environments, the Nash bargaining solution is now the most prominent solution concept for bargaining games. Nevertheless whether in a specific economic environment a bargaining outcome is adequately described by the Nash, the Kalai-Smorodinsky, or any other bargaining solution is an open empirical question. As long as this question has not been settled, any policy conclusion derived from a particular bargaining model requires a robustness check with respect to the applied solution concept.[32]

In our model due to the linear Pareto frontier the Nash solution coincides with all other standard solutions such as Kalai-Smorodinsky, Raiffa, etc. Hence, for our model the Nash solution does not need to be justified.

[31]In the model they show that higher reservation utility (or wage) leads to a lower employment level and a higher wage for the Nash solution, while it has an ambiguous employment effect but a positive wage effect for the Kalai-Smorodinsky solution.

[32]More in Gerber and Upmann (2006).

Chapter 3

Extension

3.1 Insertion of buyer power indicator

In the previous sections we have found the unique subgame perfect equilibrium wholesale price vector (w_1^*, w_2^*) with $w_1^* = w_1^{'}(w_2^*)$ and $w_2^* = w_2^{'}(w_1^*)$. We have made our analysis using the assumption, that if at some point of the game the upstream supplier and firm j have ended their negotiation by agreeing on price w_j then we have reached a terminal node of the game tree, where the price for the second player (firm i) is determined using the symmetric Nash solution as a justified part of the rules of our underlying non-cooperative game. Such approach to model negotiations is consistent with what Binmore and Dasgupta (1987) call the "Nash program", which seeks to motivate cooperative approaches to the bargaining problem like the Nash bargaining solution from an underlying non-cooperative game.[1]

In this section we involve asymmetric Nash solution which is justified as reflecting the different "bargaining power" of the players. According to Binmore (1998) "the bargaining power is determined by the strategic advantages conferred on players by the circumstances under which they bargain".[2] Our previous analysis was based on the assumption that firm 1 had the option of integrating backward into the supply of the input and firm 2 did not. This is the crucial source of differences in "strategic advantages" which we justify in the model. It is straightforward that due to the possibility of self-production firm 1 is less risk-averse than firm 2. Hence, the interpretation of the bargaining power in our model is the risk-aversion of downstream firms in negotiations with supplier.

For the future analysis let us introduce α as an indicator of the bargaining power of a downstream firm, $\alpha \in [0, 1]$. We denote the bargaining weight of each downstream firm i as α_i, so that the supplier's bargaining weight is $1 - \alpha_i$, respectively. According to the assumption on the backward integration possibility it is obviously meaningful to assume that $\alpha_1 > \alpha_2$.[3]

[1] See also the Introduction in Binmore, Osborne and Rubinstein's (1992) for a discussion about the cooperative and non-cooperative game theoretic shared goals from different approaches.

[2] See p. 78.

[3] Also intuitively, the more costly it is for a firm to reject the supplier's offer, the less bargaining power the firm has.

3.2 Equilibrium price vector

Turning back to our basic model introduced in the previous chapter when the supplier and firm j agreed on price w_j, but price w_i was not accepted, we assume that we have reached the terminal node of the game tree where the price for firm i is *asymmetric Nash bargaining solution* which is just a justified part of the rules of the non-cooperative game. Hence, the price for firm i is the asymmetric Nash solution to the following problem:[4]

$$\max_{w_i} \mu_i(w_1, w_2) = \left[U(w_i, w_j) - U^b\right]^{1-\alpha_i} \left[\pi_i(w_i, w_j) - \pi_i^b\right]^{\alpha_i} \tag{3.2.1}$$

Rewriting this problem for both downstream firms we obtain:

$$\hat{w}_1'(w_2) = \arg\max_{w_1} U(w_1, w_2)^{1-\alpha_1} \left[\pi_1(w_1, w_2) - \pi_1^{IB}(v, w_2)\right]^{\alpha_1}, \tag{3.2.2}$$

$$\hat{w}_2'(w_1) = \arg\max_{w_2} \left[U(w_1, w_2) - U^b\right]^{1-\alpha_2} \pi_2(w_1, w_2)^{\alpha_2} \tag{3.2.3}$$

The interpretation of the functions $\hat{w}_i'(\cdot)$ is analogous with that of the functions $w_i'(\cdot)$, $i = 1, 2$ given in the previous chapter.[5] They are also increasing in the wholesale price of the firms' rival. They are also concave and intersect only once. Both functions are illustrated in Figure 3.1.

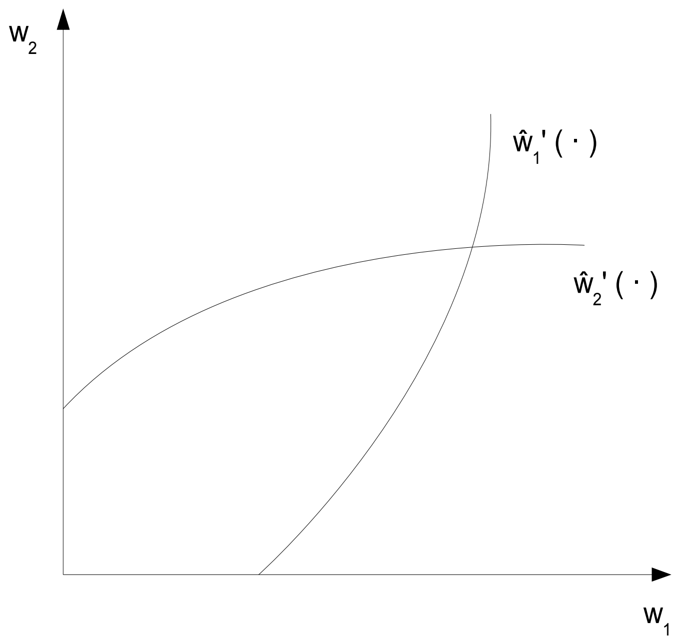

Figure 3.1: Functions $\hat{w}_i'(\cdot)$ and $\hat{w}_j'(\cdot)$

[4]Compare with function (2.4.4).
[5]See p. 17.

Now let us consider how the price $\hat{w}_i^{'}(w_j)$, which results from the asymmetric Nash bargaining solution, on which the upstream supplier and the downstream firm i agree if w_j is accepted and w_i - not, depends on the changes in the bargaining weight of firm i. The first order condition to the problem (3.2.1) after simplification is:

$$(1-\alpha_i)\frac{\partial U(w_i,w_j)}{\partial w_i}\left[\pi_i(w_i,w_j)-\pi_i^b\right]+\alpha_i\frac{\partial \pi_i(w_i,w_j)}{\partial w_i}\left[U(w_i,w_j)-U^b\right]=0 \quad (3.2.4)$$

Next let us rewrite it in the way shown below

$$-\frac{\alpha_i\dfrac{\partial \pi_i(w_i,w_j)}{\partial w_i}}{\pi_i(w_i,w_j)-\pi_i^b}=\frac{(1-\alpha_i)\dfrac{\partial U(w_i,w_j)}{\partial w_i}}{U(w_i,w_j)-U^b} \quad (3.2.5)$$

From equation (3.2.5), using the assumption on the concavity of firms' profit functions, we can state the following result:

Result 5. *An increase in firm i's bargaining weight α_i or in its disagreement profit π_i^b, or a decrease in the supplier's disagreement profit U^b, when all other factors stay equal, requires an increase in firm i's net profit $\left[\pi_i(w_i,w_j)-\pi_i^b\right]$ for equation (3.2.5) to hold, that results in a decrease in firm i's wholesale price for any given w_j.*

Consequently we may state the following proposition about the influence of the bargaining power on the asymmetric Nash bargaining solution presented in this part of the model.

Proposition 3.2.1. *Assume that the upstream supplier and downstream firm j agreed on the wholesale price w_j, but the price w_i was not accepted. After that the supplier and downstream firm i agree on the price which results from the asymmetric Nash bargaining solution to the problem given in equation (3.2.1). Then:*

i) the price $\hat{w}_i^{'}(w_j)$ which results from asymmetric Nash bargaining solution is strictly decreasing in the firm i's bargaining weight α_i;

ii) the difference in both prices, or in other words, the discount for firm i, $(w_j - \hat{w}_i^{'}(w_j))$ is strictly increasing in α_i.

Proof. i) First let us differentiate the equation (3.2.4) with respect to α_i getting:

$$\frac{\partial \hat{w}_i^{'}(w_j)}{\partial \alpha_i}=\frac{\dfrac{\partial^2\mu_i}{\partial w_i^2}\left(\dfrac{\partial U(w_1,w_2)}{\partial w_i}\left[\pi_i(w_1,w_2)-\pi_i^b\right]-\dfrac{\partial \pi_i(w_1,w_2)}{\partial w_i}\left[U(w_1,w_2)-U^b\right]\right)}{\dfrac{\partial^2\mu_1}{\partial w_1^2}\dfrac{\partial^2\mu_2}{\partial w_2^2}-\dfrac{\partial^2\mu_1}{\partial w_1\partial w_2}\dfrac{\partial^2\mu_2}{\partial w_2\partial w_1}} \quad (3.2.6)$$

Given that $\dfrac{\partial U(w_1,w_2)}{\partial w_i}>0$, $\dfrac{\partial \pi_i(w_1,w_2)}{\partial w_i}<0$, $U(w_1,w_2)\geq U^b$, $\pi_i(w_1,w_2)\geq \pi_i^b$, μ_i is strictly concave in w_i, we obtain that the numerator of the fraction is negative. Hence, the sign of the fraction depends only on the sign of the denominator.

Now let us differentiate the equation (3.2.4) with respect to w_i and w_j. Hence we obtain:

$$\frac{\partial^2 \mu_i}{\partial w_i \partial w_j} = -R_i'(w_j)\frac{\partial^2 \mu_i}{\partial w_i^2},$$

where $R_i'(w_j)$ is the reaction function of firm i on the accepted price w_j.

Now we can rewrite the denominator of equation (3.2.6) in the following way:

$$\frac{\partial^2 \mu_1}{\partial w_1^2}\frac{\partial^2 \mu_2}{\partial w_2^2}\left[1 - R_1'(w_2)R_2'(w_1)\right] > 0 \tag{3.2.7}$$

As the bargaining equilibrium is stable due to the assumptions at the beginning of the model description, $[1 - R_1'(w_2)R_2'(w_1)] \geq 0$. Hence, the denominator is positive and therefore the whole fraction is negative.

$\partial \hat{w}_i^{'}(w_j)/\partial \alpha_i < 0$, therefore the wholesale price for each downstream firm i is decreasing in its bargaining weight α_i.

ii) To prove whether the discount of firm i is increasing in its bargaining weight we need to analyze the equation (3.2.8) which is shown below.

$$\frac{\partial(w_j - \hat{w}_i^{'}(w_j))}{\partial \alpha_i} =$$

$$\frac{-\left(\frac{\partial^2 \mu_i}{\partial w_i^2} + \frac{\partial^2 \mu_i}{\partial w_i \partial w_j}\right)\left(\frac{\partial U(w_1,w_2)}{\partial w_i}\left[\pi_i(w_1,w_2) - \pi_i^b\right] - \frac{\partial \pi_i(w_1,w_2)}{\partial w_i}\left[U(w_1,w_2) - U^b\right]\right)}{\frac{\partial^2 \mu_1}{\partial w_1^2}\frac{\partial^2 \mu_2}{\partial w_2^2} - \frac{\partial^2 \mu_1}{\partial w_1 \partial w_2}\frac{\partial^2 \mu_2}{\partial w_2 \partial w_1}} \tag{3.2.8}$$

Analogously to the previous case (i) we obtain that $\partial(w_j - \hat{w}_i^{'}(w_j))/\partial \alpha_i > 0$, which means that the discount for each downstream firm i is increasing in its bargaining weight. □

Omitting the similar analysis of the behavior of all players which was made in the previous chapter and which can be applied here without any restrictions we may summarize the following result:

Proposition 3.2.2. *There exists a unique subgame perfect equilibrium wholesale price vector* $(\hat{w}_1^*, \hat{w}_2^*)$*, which lies on the intersection of the graphs of the the functions* $\hat{w}_1^{'}(\cdot)$ *and* $\hat{w}_2^{'}(\cdot)$ *with* $\hat{w}_1^* = \hat{w}_1^{'}(\hat{w}_2^*)$ *and* $\hat{w}_2^* = \hat{w}_2^{'}(\hat{w}_1^*)$ *shown in Figure 3.2. Moreover this price vector is offered and accepted in the first round and leads to the equilibrium payoff vector* $P(\hat{w}_1^*, \hat{w}_2^*)$*.*

Proof. See proof of Proposition 2.4.1. □

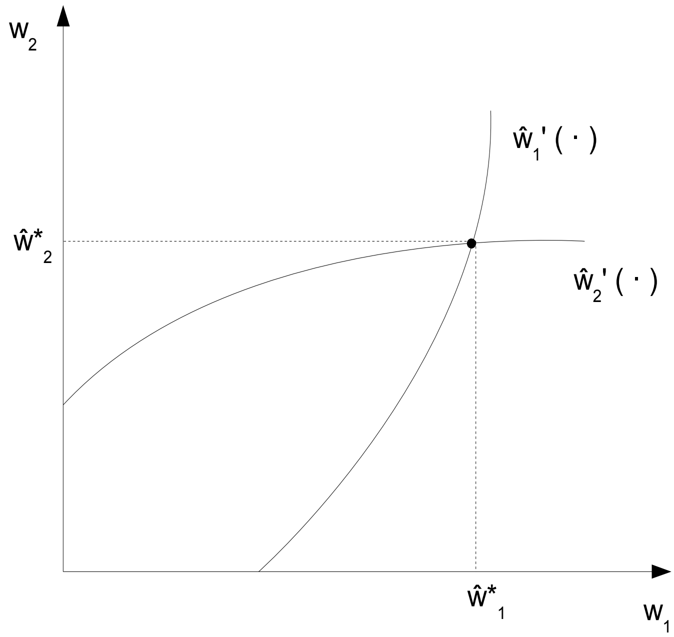

Figure 3.2: Illustration of equilibrium prices

Similarly with the basic model let us graphically illustrate the results of Proposition 3.2.1, namely how the change in the bargaining weight of at least one buyer will change the equilibrium price vector.

Assume that the point A' in Figure 3.3 represents the bargaining equilibrium when downstream firms have symmetric bargaining power $\alpha_1 = \alpha_2$. If firm 1 increases its bargaining power, e.g. by rejecting the costs of self-production, etc. then it will become even more risk-seeking in negotiations with the supplier as it was before, comparing with the downstream firm 2. Graphically the increase in the bargaining power of firm 1 will shift the function $\hat{w}_1'(\cdot)$ to the left, from $\hat{w}_1'(\cdot)$ to $\hat{w}_1''(\cdot)$ as it is illustrated in Figure 3.3. This changes the point of intersection of both functions from point A' to the point A''. Hence, the equilibrium wholesale price of firm 1 falls unambiguously.

The difference in the equilibrium wholesale prices is interpreted as the ability of firm 1 (which has greater bargaining weight) to negotiate discounts. There are the following reasons for that, such as the ability to integrate backward; higher disagreement payoff than by firm 2 and therefore the higher bargaining weight. Summarizing the results of propositions 3.2.1 and 3.2.2 if $\alpha_i > \alpha_j$ then $\hat{w}_i^* < \hat{w}_j^*$. In other words, firm 1 will receive a discount if, other factors equal, it has a greater bargaining weight than firm 2, namely $\alpha_1 > \alpha_2$. So we obtain that $\alpha_1 > \alpha_2$ is a sufficient condition for firm 1 to receive a discount.

In our model we have considered the indicator α as exogenously given. But it is important to determine how to choose the weights α_i and α_j to reflect some possible asymmetries and how to measure the bargaining power.

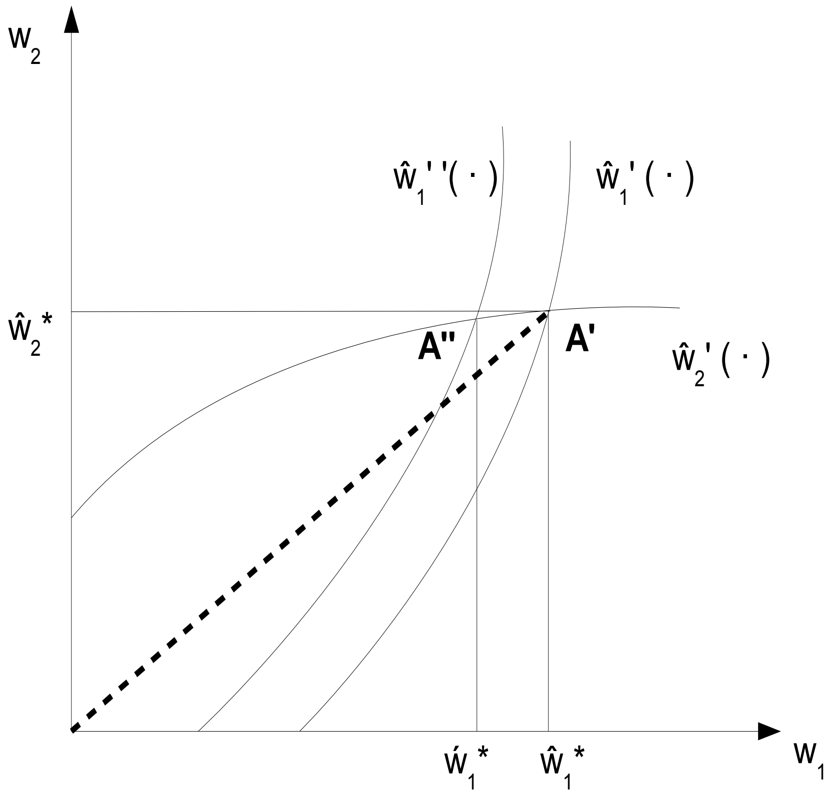

Figure 3.3: Equilibrium prices when α_1 changes

3.3 Measure of the bargaining weight

Binmore et al. (1986) consider two models of alternating offers. The models differ in the source of the incentive of the bargaining parties to reach agreement: the bargainers' time preference and the risk of breakdown of negotiation.

In our bargaining model with a risk of breakdown of negotiations, the time itself is not valuable, but delays are costly because there is an endogenous risk that negotiations might break down after each period in which the firms fail to reach an agreement. Binmore et al. (1986) demonstrate how the power weights α and $1-\alpha$ can be chosen to reflect some possible asymmetries in the procedure and in the parties' beliefs.[6] Hence, assume that Δ_i is the length of the interval between i's reaction to j's proposal and the next point at which i proposes to j and the procedure is symmetric, such as $\Delta_1 = \Delta_2 = \Delta$. But parties differ in their beliefs in the likelihood of a breakdown.[7] Thus, in each bargaining period of length Δ that separates two consecutive bargaining stages there is a positive probability $p = p(\Delta) = 1 - e^{-\lambda\Delta}$ that the process will break down, in which case the outcome will be $b \in X$, where X is the set

[6] More in Binmore et al. (1986), pp. 186-187.

[7] It is assumed that, conditional on the bargaining process' reaching time t and no agreement's being reached before time $t+h$, the probability that the process will break down before time $t+h$ is $\lambda h + o(h)$. That is, the time of the breakdown is exponentially distributed with parameter λ.

of possible agreements, $X = \{(x_1, x_2)|x_1, x_2 \geq 0, x_1, x_2 \leq 1\}$. Let $p_i(\Delta) = 1 - e^{-\lambda_i \Delta}$ be the probability assigned by party i to the event that the process will break down during a single bargaining period. As Δ approaches zero, the unique perfect equilibrium of the model with asymmetric beliefs approaches the solution for $max_{s \in S}(s_1 - s_1^0)^{\alpha}(s_2 - s_2^0)^{1-\alpha}$, where $\alpha_i = \lambda_j/(\lambda_i + \lambda_j)$ and (S, s^0) is the static representation of the exogenous-risk model. Thus, the higher party i's estimate of the probability of breakdown is, the lower its bargaining power is.

In the time preference model, delays are also costly because firms discount the future at positive rates. Assuming that the procedure is asymmetric in the sense that $\Delta_1 \neq \Delta_2$ and computing the unique perfect equilibrium for each of the strategic models, and letting Δ_1 and Δ_2 approach zero while keeping their ratio constant, it is easy to verify that the limiting equilibrium outcomes of both models coincide with the respective asymmetric solutions with poor $\alpha_1 = \Delta_2/(\Delta_1 + \Delta_2)$. The larger Δ_2 is relative to Δ_1, the larger α_1 will be, and hence the "stronger" is party 1. Similar asymmetric solutions arise if the proposer at each time $t\Delta$ is chosen with different probabilities for two players.

Both motivations of avoiding the costs of bargaining delays as well as avoiding the delays because of the risk of breaking down may play a role in negotiations over intermediate good prices, since firms generally discount the future at positive rates and there is often some risk that a profitable opportunity will be exploited by third party.

Muthoo (1999) considers an alternating-offer bargaining model in which both types of delay costs (discounting and the risk of a breakdown) are present. Suppose the one-period discount rates of firm i and the upstream supplier are r_i and r_u, respectively, and there is an exogenous probability p_i that negotiations between the supplier and firm i will break down after any period that one of them rejects the other's offer. His results imply that firm i's bargaining weight in this case is $\alpha_i = (r_i + p_i)/(2p_i + r_i + r_u)$.[8]

3.4 Market share

In Section 3.2, involving the indicator of the bargaining power, we have found the subgame perfect equilibrium wholesale price vector. We have assumed that the source of the higher bargaining power of downstream firm 1 comparing with firm 2 is the existence of backward integration option. We have also shown that if the outside option payoff of firm 1 increases (e.g. firm 1 will find some technology to produce the input for some unit price $\tilde{v} < v$), then its wholesale price will be smaller than by firm 2.

On this stage we finally come to the main aim of this chapter, namely we check how the changes in the buying abilities of one player influence the firms' market shares in the local market, or in other words we want to prove the existence of the Spiral Effect.

[8]Let $\bar{U}_i$ and $\bar{\pi}_i$ be the profits earned by the supplier and firm i, respectively, in each period during their negotiations; let b_{ui} and b_i be their profits (per period) in the event negotiations break down. When both motivations for reaching agreement are present, the results imply that the disagreement profits are $U(b_{ui}) = (\bar{U}_i + p_i b_u)/(r_u + p_i)$ and $\pi_i(b) = (\bar{\pi}_i + p_i b_i)/(r_i + p_i)$.

Usually firms use different instruments in competing for market shares, such as high quality, extra services, low prices, etc. Anyway the cost of production is a very important factor that influences market penetration. Let us denote the market share of each firm i as s_i, where $s_i \equiv q_i(\cdot)/Q(\cdot)$, with $Q(\cdot) = \sum_{i=1}^{2} q_i(\cdot)$, then examining the influence of the bargaining power on the market share we state the following proposition:

Proposition 3.4.1. *Suppose there are two downstream firms with linear demands in the downstream market, which are Cournot competitors. Both firms exercise the bargaining power in negotiations with the supplier. Their bargaining weights are denoted by α_1 and α_2, respectively. Then the increase in the bargaining power of a particular firm leads to the increase of its market share.*

Proof. In Cournot duopoly, under the assumptions on linearity of the demand and cost functions, the firm's equilibrium output is a decreasing function of its own marginal costs and increasing in the marginal costs of its rival. Consequently, the market share of each firm is a decreasing function in the own wholesale price, namely $\partial s_i/\partial w_i < 0$, for $i = 1, 2$. The market share s_i is a composite function of variables w_i and w_j, which on their turn depend on the bargaining weights α_i and α_j. Applying the Chain Rule and using the results of Proposition 3.2.1 that $\partial w_i/\partial \alpha_i < 0$ we obtain that the market share of each downstream firm is increasing in its bargaining weight, $\dfrac{\partial s_i}{\partial \alpha_i} > 0, i = 1, 2$. □

Let us further consider the general case. Therefore we write the FOC for the existence of Cournot equilibrium using elasticity of demand, denoted as ϵ.[9]

$$P(Q)\left[1 - \frac{1}{\epsilon}\frac{q_i}{Q}\right] = w_i, \tag{3.4.1}$$

where $\epsilon = -\dfrac{P(Q)}{Q}\dfrac{\partial Q}{\partial P(Q)}$ and transform it into:

$$\frac{q_i}{Q} = \left[1 - \frac{w_i}{P(Q)}\right]\epsilon \tag{3.4.2}$$

Summarizing (3.4.2) over 2 firms gives us:

$$1 = 2\epsilon - \frac{\epsilon \sum w_i}{P(Q)}, \; i = 1, 2 \tag{3.4.3}$$

Using the definition of the Herfindahl index, which is equal to the sum of squares of the market shares or could be expressed in terms of output quantities

$$H = \frac{\sum q_i^2}{Q^2} \tag{3.4.4}$$

[9] $P(Q) + P'(Q)q_i = C_i'(q_i)$.

and plugging equations (3.4.2) - (3.4.3) into equation (3.4.4) we obtain:

$$H = \frac{\sum q_i^2}{Q^2} = -2\epsilon^2 + 2\epsilon + (1-2\epsilon)^2 \frac{\sum w_i^2}{(\sum w_i)^2} \tag{3.4.5}$$

Cowling and Waterson (1976) suggest the profit-revenue ratio $\frac{\Pi}{Rev}$, where Π and Rev are the profit and the revenue function, respectively, of the whole branch. In case of Cournot oligopoly this profit-revenue ratio is positively related to the Herfindahl index of concentration and inversely related to the elasticity of demand $\frac{\Pi}{Rev} = \frac{H}{\epsilon}$, so one obtains:

$$\frac{\Pi}{Rev} = -2\epsilon + 2 + \frac{(1-2\epsilon)^2 \sum w_i^2}{\epsilon(\sum w_i)^2} \tag{3.4.6}$$

In equation (3.4.6) the concentration ratio is expressed in terms of demand and cost conditions as represented by ϵ and $\frac{\sum w_i^2}{(\sum w_i)^2}$. Equations (3.4.5) and (3.4.6) show that both H and Π/Rev will be larger, the greater are cost or efficiency differential between firms as measured by $\frac{\sum w_i^2}{(\sum w_i)^2}$.[10]

Concluding the above analysis we come to the following result:

Result 6. *Any asymmetry in the bargaining weights leads to an asymmetry in production costs (in our model to an asymmetry in wholesale prices) between firms and results in increasing concentration ratio as well as in increasing profitability of the most efficient firm.*

Demsetz (1973) also indicated the positive correlation between concentration ratio and profitability. The reason of such correlation could be explained in the following way: the asymmetry in firm's costs brings the asymmetry in outputs, so the concentration ratio of the most efficient firm will increase. At the same time the asymmetry brings the additional income for firms with smaller costs that in turn leads to the increase in the average profitability of the branch.

3.5 Conclusion

Chapters 2 and 3 of the presented work is based on the paper of Katz (1987). We have extended his model of quantity competition by involving the bargaining over the wholesale prices between both competitors. We have considered the basic bargaining model and have proved that there exists a unique subgame perfect equilibrium wholesale price vector, which is offered and accepted in the first round and which leads to the equilibrium payoff vector. Then we have extended the obtained model

[10] In the context of our analysis the fraction $\frac{\sum w_i^2}{(\sum w_i)^2}$ may be called Herfindahl index of costs.

assuming that both downstream firms exercise the bargaining power in negotiations with the upstream supplier. We have shown that the asymmetry in the bargaining weights of both downstream firms leads to the asymmetry in their wholesale prices and results in increasing concentration ratio and in increasing profitability of the most efficient firm.

The aim of such analysis is to find the reasons for which the particular firms receive discounts. One of such reasons is the possibility of backward integration. We have shown that the possibility of backward integration that arises in the intermediate good markets can have powerful effects on the equilibrium outcome.

The next reason for the existence of the discounts, presented in our model, is an exercising of the bargaining power. Proposition 3.2.1 shows that even if an explicit threat to backward integration is not credible, firm 1 may also receive a discount if it has greater bargaining power.

According to the definition of the Spiral Effect, if, due to his bargaining power, one buyer has better procurement conditions than other buyers, he can use them to strengthen his market position in the sales market. A strengthened position in the sales market can in turn improve his procurement situation, e.g. as he is in a position to negotiate additional quantity discounts. In support of this opinion Proposition 3.4.1 shows that the increase in the asymmetry between the wholesale prices leads to increase of the market share of the most efficient firm.

As the implications of bargaining for antitrust policy are not well understood, these results may play a useful role for government regulation of a discriminatory pricing. However, bargaining is prevalent in intermediate good markets, where a large share of the antitrust enforcement takes place in developed countries.

Chapter 4

Asymmetric bargaining power in capacity-constrained industry

4.1 Introduction

4.1.1 Motivation

In the previous chapters we have presented a non-cooperative bargaining process between the supplier and two downstream firms where the results from two-person cooperative axiomatic bargaining games were used to define the payoffs of some of the terminal nodes of our extensive game. The upstream supplier had bargained with the downstream firms over the wholesale prices denoted as w for some particular output quantities $q(w)$ which downstream firms then offered in the local market.

In the previous analysis we have assumed that entering the local market both downstream firms were engaged into Cournot competition. With the aim to make the analysis complete it seems to be interesting to investigate the behavior of downstream firms under the Bertrand competition. In order to describe the model it is important to consider the main differences in both kinds of competition, which will influence the future results.

4.1.2 Preliminaries

In Cournot competition producers make their output decisions independently and simultaneously; after that they bring their production to the market. The market price is then set at such level that demand equals the total quantity produced by all firms. In Bertrand competition producers simultaneously and independently set prices. If all firms charge the same price, consumers randomly select among them. If the prices are different, demand is allocated to the low-price producers, who then produce up to the demand they encounter. Any unsatisfied demand goes to the second lowest price producers, and so on. The main difference in these competition models is the price determination, in Cournot it is made by an auctioneer and in Bertrand - by price "competition".

Bertrand competition can be modeled as a normal form game.

Definition 4.1.1. *Let* (P, π) *be a game with* $n \geq 2$ *players, where* P_i *is the strategy set for player* i*,* $P_i = [0, \infty)$*,* $P = P_1 \times P_2 \cdots \times P_n$ *is the set of strategy profiles and* $\pi : P \rightarrow \Re_+ : p \mapsto \pi(p) = (\pi_1(p), \cdots, \pi_n(p))$ *is the payoff function. When each player* $i \in 1, \cdots, n$ *chooses strategy* p_i *resulting in strategy profile* $p = (p_1, \cdots, p_n)$ *then player* i *obtains payoff* $\pi_i(p)$*. Let* p_{-i} *be a strategy profile of all players except for player* i*. Under the assumption of profit maximization, the payoff to each firm* i *is:*

$$\pi_i(p_i, p_{-i}) = p_i D_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})),$$

where $D_i(p_i, p_{-i})$ *represents the total demand for firm* i*'s product at prices* (p_i, p_{-i})*, and* $C_i(D_i(p_i, p_{-i}))$ *is firm* i*'s total minimal cost of producing the output* $D_i(p_i, p_{-i})$*. A Nash equilibrium of this game is sometimes called Bertrand equilibrium.*

The Bertrand model of price competition in which all firms have constant returns to scale technologies with the same cost, $c > 0$, per unit produced, leads to the unique Nash equilibrium in which all firms set their prices equal to marginal costs, $p_i^* = c, \forall i = 1, \cdots, n$. As these prices always equal marginal costs all firms earn zero profit. Many economists have interpreted this result as implying that a market with two identical firms is perfectly competitive or, if costs are similar, approximately competitive. Such situation in which two firms reach Nash equilibrium where both firms charge a price equal to marginal cost is called Bertrand paradox. It is called paradox because it is hard to believe that in industries with few firms, they never succeed in manipulating the market price to make profits. If both firms set the same price, they share the market in some manner. However, if one of them has an absolute cost advantage over its rival that he exploits for setting a lower price, it captures the entire market.

On this stage let us consider the price competition first when both firms have symmetric production costs and then asymmetric.

4.1.2.1 Symmetric production costs

In the "classic" model of Bertrand competition, each of the firms produces an identical product at a constant unit cost of c, that is, $C_i(q_i) = cq_i$. Since their products are perfect substitutes, firms effectively compete for the total demand, $D(p)$, that a monopolist serving the entire market would obtain by pricing at p. The firm setting the lowest price gets all of this demand; in the event of a tie, the firms charging the lowest price share total demand equally. Total demand is sufficiently well-behaved to ensure that the corresponding monopoly profit function, $\Pi(p) \equiv pD(p) - C(D(p))$, is not only continuous, but *(a)* has a unique maximizer, the monopoly price p^m; *(b)* satisfies $\Pi(p) < \Pi(c) = 0 < \Pi(\hat{p}) < \Pi(p^m)$ for $p < c < \hat{p} < p^m$. Despite the continuity of $\Pi(\cdot)$, each firm faces a discontinuous profit function Π_i with

$$\Pi_i(p_i, p_j) = \begin{cases} (p_i - c)D(p_i), & \text{if } p_i < p_j, \text{ for all } i \neq j \\ 0, & \text{otherwise} \end{cases} \tag{4.1.1}$$

4.1.2.2 Asymmetric production costs

Considering the asymmetric duopoly case, where firm 1 has constant unit cost c_1 with $c_1 < c_2$, the results of Bertrand competition that firms price at marginal costs and that they do not make profits do not hold any more for the following reasons: *i*) if both firms charge price $p_1 = p_2 = p = c_2$, then firm 1 could charge an ϵ below c_2 to make sure it has the whole market and *ii*) that firm 1 makes a profit of $(c_2 - c_1)D(c_2)$, and firm 2 makes no profit as long as $c_2 \leq p^m(c_1)$, where $p^m(c_1)$ maximizes $(p - c_1)D(p)$. Thus firm 1 charges above marginal cost and makes a positive profit. But it is a strained conclusion. If c_2 is very close to c_1, then firm 1 makes little profit, and firm 2 makes no profit at all. Typically, oligopoly firms earn positive profits by charging prices above marginal cost. The Bertrand paradox is rare in practice, because among other possible reasons two firms rarely have identical costs. Bertrand paradox also foresees that one producer could satisfy the whole demand if he has the cost advantage over its rival. Edgeworth (1897) found this assumption to be unrealistic and he solved the Bertrand paradox by introducing capacity constraints, by which firms cannot sell more than they are capable to produce.

A variant of Bertrand competition, known as "Bertrand-Edgeworth competition", allows any firm to ration the demand that it faces at given prices by only providing its optimal or competitive supply at its price. Rationing may stem from a physical capacity constraint, $\tilde{q}_i$, that prevents firm i from producing more than $\tilde{q}_i$ units (as in Edgeworth's original formulation), or more generally, from a firm's strategic incentive to refuse to fulfill the quantity demanded of all consumers at a given price. Under Bertrand-Edgeworth competition one must therefore specify how demand is rationed when a firm's quantity demanded at given prices exceeds the amount of product it produces.[1]

4.1.3 Theoretical framework

At this stage let us provide some basic theory that will be useful for the future analysis. Let us start first with a simple case, namely a symmetric duopoly with homogeneous market and two identical firms 1 and 2, which compete in prices. Each firm i, $i \in [1, 2]$, has a capacity $\tilde{q}_i$, so firm i's output must satisfy $q_i \leq \tilde{q}_i$. The marginal cost of unit production (once the capacity is installed) is w_i up to $\tilde{q}_i$ and ∞ after $\tilde{q}_i$. For simplicity we assume an affine linear demand, $P(Q) = 1 - Q$ (or $D(p) = 1 - p$), with $1 > w$.

According to the Bertrand competition model, in equilibrium both firms will sell the goods for price $p_1^b = p_2^b = w$ if they both are able to satisfy the whole demand $D(w)$. Next consider the case when for both firms the following inequality holds $\tilde{q}_i < D(w)$. It is straightforward that by this assumption $p_1^b = p_2^b = w$ will not hold anymore in equilibrium. If one of both firms slightly raises its price, it will not immediately lose its whole demand. Because of the constrained capacities its rival could cover only the part of the demand by price w. In this case the firm with the higher price will still have positive residual demand and still could have

[1] More precisely explained in Baye and Kovenock (2009).

positive profit. The Bertrand-Paradox that both firms make no profit is not valid if production is constrained.

Now let us consider two cases, namely when the prices of both firms differ and then when they are identical:

1. Without loss of generality let us assume that $p_1 < p_2$. Then consumers first try to buy from firm 1, when its supply q_1 is exhausted, consumers turn to firm 2. On this stage it is important to note that we consider identical consumers, by which "income effect" (salary, etc.) does not influence their preferences. Then the aggregate demand at price p_2 depends on the way the constrained supply $\tilde{q_1}$ is allocated among the consumers. As firm 1 can produce only $\tilde{q_1}$, then the demand functions are of the following forms:

$$D_1(p_1, p_2) = \min\left[D(p_1), \tilde{q_1}\right] \tag{4.1.2}$$

$$D_2(p_1, p_2) = \begin{cases} 0, & \text{if } D(p_1) \leq \tilde{q_1} \\ \max\left[0, D(p_2) - \tilde{q_1}\right], & \text{if } D(p_1) > \tilde{q_1} \end{cases} \tag{4.1.3}$$

The first equation of (4.1.3) says that if the capacity of firm 1 is enough to cover the demand $D(p_1)$, no consumer will buy from firm 2 at price p_2. In the second equation, namely when $D(p_1) > \tilde{q_1}$, firm 1 could cover only the part of the whole demand for price p_1. Consumers should then be rationed. It is natural to assume that the rationing scheme is chosen by the low-price firm 1. However, this is not enough to determine what scheme is chosen, since 1's profit is independent of the scheme used.[2] Given that this is so, let us make the following assumption:

Assumption 5. *The low-cost firm (in our case firm* 1*) chooses efficient rationing, the scheme which minimizes the profit of the higher-cost firm (firm* 2*).*[3]

With such rationing, each of the identical consumers is allowed to purchase the same fraction of $\tilde{q_1}$, instead of buying as much as he wants. For identical consumers it is socially efficient, because each of them will receive the same quantity of output and there will be no exchange between them.

Under the assumption about efficient rationing and if $D_2(p_1, p_2) > \tilde{q_1} > 0$ consumers are able to buy some additional quantity of good from firm 2 for price $p_2 > p_1$. So firm 2 sells some quantity q_2 for price:

$$p_2 = P(q_2 + \tilde{q_1}) = 1 - (q_2 + \tilde{q_1}) \tag{4.1.4}$$

From equation (4.1.4) it follows that $q_2 = 1 - p_2 - \tilde{q_1} = D(p_2) - \tilde{q_1}$. As firm's 2 capacity is $\tilde{q_2}$ its sale will be equal to:

$$D_2(p_1, p_2) = \max\left[0, \min\left[D(p_2) - \tilde{q_1}, \tilde{q_1}\right]\right], \text{ if } p_2 \geq p_1 \text{ and } D(p_1) > \tilde{q_1} \tag{4.1.5}$$

[2]Only firm's 2 profit is affected.

[3]This assumption appears to be consistent with the competitive nature of the model.

In case of efficient rationing equation (4.1.5) shows the residual demand that the high-priced firm obtains if the rivals production capacities are constrained.

2. Now let us consider the second case, namely when $p_1 = p_2 = p$. Demand in such case is allocated in proportion to capacities. If firms charge the same price, there should be an assumption about the distribution between consumers. Even if prices of both firms are equal, their costs could be different. Therefore let us consider these two cases separately:

   - If production costs of both firms are equal $w_1 = w_2$ they both face the following demand:

$$\begin{aligned} D_i(p_1, p_2) &= \min\left(\tilde{q}_i, \frac{D(p_i)}{2} + \max\left(0, \frac{D(p_i)}{2} - \tilde{q}_j\right)\right) \\ &= \min\left(\tilde{q}_i, \max\left(\frac{D(p_i)}{2}, D(p_i) - \tilde{q}_j\right)\right), i = 1, 2;\ i \neq j. \end{aligned} \tag{4.1.6}$$

   - At asymmetric production costs, assuming that $w_1 < w_2$, the firms face the following demand:

$$\begin{cases} D_1(p_1, p_2) = \min\left[D(p), \tilde{q}_1\right] \\ D_2(p_1, p_2) = \max\left[0, D(p) - \tilde{q}_1\right] \end{cases} \tag{4.1.7}$$

   We assume that in case of a tie in prices, the low cost firm sells its capacity first.[4]

Analyzing the cases when firms set equal and unequal prices we have specified in equations (4.1.2) - (4.1.7) the resulting demand functions for both firms. Hence it is now possible to define the profit functions for each firm $i = 1, 2$, which are shown below:

$$\Pi_i(p_1, p_2) = (p_i - w_i) D_i(p_1, p_2) \tag{4.1.8}$$

The profit-maximizing prices of each firm $i = 1, 2$ are given by:

$$p_i^* = \arg\max_p \Pi_i(p_1, p_2),\ i = 1, 2 \tag{4.1.9}$$

If the conditions (4.1.10) are satisfied, then (p_1^*, p_2^*) is a Bertrand equilibrium.

$$\begin{cases} \Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*),\ \forall p_1 \\ \Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2),\ \forall p_2 \end{cases} \tag{4.1.10}$$

It is straightforward that the price equilibrium depends on the producers' capacities.

[4]Analogously in Deneckere and Kovenock (1996). They show that equilibrium profits are unaffected by the tie breaking rule, and that equilibrium distributions are altered only in the classical Bertrand region when the low cost firm does not have a drastic cost advantage. In order to break ties when $w_1 = w_2$, they arbitrarily let firm 1 sell its capacity first.

Continuing to follow Bester (2004) the most interesting case is if both producers have relative small capacities, such as:

$$\tilde{q}_i \leq \frac{1 - 2w_i + w_j}{3} = \tilde{q}_i^{\,C} \tag{4.1.11}$$

Inequality (4.1.11) means that the capacity of each firm is not larger than the output that the respective firm would suggest by the quantity (Cournot) competition. Beckmann (1965) obtains that for small capacities the reduced-form profit function has the exact Cournot form in the case of proportional rationing; Levitan and Shubik (1972) extend this result for the case of efficient rationing. They also provide that pure-strategy equilibrium does not exist for larger capacities, unless each firm has enough capacities to supply the whole demand at the competitive price. The equilibrium is thus in mixed strategies, and it is computed in closed form by Beckmann (1965) for proportional rationing and by Levitan and Shubik (1972) for efficient rationing in the special case of symmetric capacities.

The important result is also provided by Kreps and Scheinkman (1983), who characterize the pure-strategy and mixed-strategy equilibrium for efficient rationing for asymmetric capacities. They consider a two-stage game in which both firms simultaneously choose capacities $\tilde{q}_i$ and then, knowing each other's capacity, they simultaneously choose prices p_i, $i = 1, 2$. Their paper shows that if the demand function is concave and if there is an efficient rationing rule then the outcome, namely capacity choices and market price of two-stage game is the same as that of one-stage Cournot game.

The most important results of the paper of Kreps and Scheinkman (1983) are summarized below:

1. The profit function has exact Cournot reduced form. The capacity-constrained price game yields reduced-form profit functions that are identical to Cournot profit functions, in which quantities are to be interpreted as capacities.

2. Cournot outcome in two-stage game. Equilibrium of two-stage (capacity and then the price) game coincides with the Cournot equilibrium, in which quantities are to be interpreted as capacities.

The first result implies the second. These results rest on very strong assumptions, such as efficient-rationing rule, absence of intertemporal price competition and product differentiation. Following Tirole (1988) these features provide some foundations for the Cournot model, in which firms choose quantities and an auctioneer then chooses the price so as to clear the market, as long as quantities are identified with capacities. Thus, Bertrand and Cournot models should not be seen as two rival models giving contradictory predictions of the outcome of competition in a given market. (After all, firms almost always compete in prices). Rather, they are meant to depict markets with different cost structures. Bertrand price competition among even a few firms yields competitive, socially optimal outcome. However it is softened when the firms face sharply rising marginal costs, or when they compete repeatedly. The Bertrand model may be better approximation for industries with fairly flat marginal costs; the Cournot model may be better for those with sharply rising marginal costs. The quantity competition can more generally be seen as a competition in choices of scale, where a firm's choice of scale determines its cost function and thus the conditions of price competition.

Despite a common defense of the Cournot model which was based on the Kreps and Scheinkman's (1983) argument that simultaneous quantity choice followed by simultaneous price-setting can yield a Cournot outcome, the paper of Davidson and Deneckere (1986) shows that this result is sensitive to the choice of the rationing rule.

4.2 Capacity-constrained price games

4.2.1 Basic concepts and notations

In the previous chapter we have developed the model in which both downstream firms compete for the same input which they buy from the upstream supplier. We have also shown that the purchasing input cost of one firm depends on the purchasing strategy of other firm. The important feature is that each firm can overbid its rival and foreclose access to the input supply, or it can make this access more expensive.

In capacity-constrained games that we are going to present in this section, inputs are identified with capacities and each firm can restrict its competitor's capacity by bidding input supplies up.[5]

Our forthcoming research bases on the model which was developed and analyzed in Chapter 2. The events in the industry take place in the following stages: first, the supplier bargains simultaneously with each downstream firm over the wholesale prices for some particular quantities. These quantities are identified with firms' capacities and we denote them as $\tilde{q}_1$ and $\tilde{q}_2$, respectively. After capacities are produced, both downstream firms bring them to the market and engage in Bertrand-like price competition: they simultaneously and independently name prices and demand is allocated in Bertrand fashion, with the assumption that one cannot satisfy more demand than the capacity allows.

It is straightforward that the equilibrium outcome (profit) depends on the choice of capacities by both downstream firms. If capacities of both firms are equal to Cournot quantities it is easy to find the equilibrium in price, which will be the Cournot price.

In the context of our analysis the paper of Kreps and Scheinkman (1983) is of great importance. They show that given capacities for two firms, equilibrium behavior in the second Bertrand stage will not always lead to a price that exhausts capacity, but when those given capacities correspond to the Cournot output levels, in the second stage each firm names the Cournot price. And for the entire game, fixing capacities at the Cournot output levels is the unique equilibrium outcome.

The exploitation of the results of Kreps and Scheinkman for our model is not straightforward, because they based their analysis on the assumptions that two firms had equal capacity costs and no production costs.

[5]Stahl (1985) assumes that the input-supply industry is competitive, he shows that the outcome of the two-stage game in which in the first stage firms bid for inputs (capacities) and in the second stage they choose prices, is competitive. As in Bertrand equilibrium, even two firms producing the final good cannot prevent the price from falling to the level at which the consumers' marginal willingness to pay is equal to the marginal cost of supplying the final good.

In this chapter we base our research on the model which was introduced in Chapter 2. The difference, however, is that the quantities that both downstream firms buy from the upstream supplier are now considered as firms' capacities and we denote them as $\tilde{q}_1(w_1, w_2)$ and $\tilde{q}_2(w_1, w_2)$, respectively. We continue by describing the results of Kreps and Scheinkman more precisely and extend them to the cases of asymmetric capacity costs and asymmetric production costs, which are very important for our whole analysis. Finally, basing on the obtained results we suggest what prices both downstream firms should name in order to reach the equilibrium in profits.

4.2.2 Price competition with capacity costs

4.2.2.1 Case of symmetric capacity costs according to Kreps and Scheinkman (1983)

Kreps and Scheinkman (1983) considered the following two-stage game: capacities are set in the first stage by two producers, who produce a perfectly substitutable final product. Production takes place at zero cost, subject to capacity constraints. Capacity level $\tilde{q}_i$ means that up to $\tilde{q}_i$ units can be produced at zero cost. Both firms have identical production functions. They transform one unit of the input into one unit of the output at no additional costs. In the second stage both firms bring these quantities to the market, where demand is determined by Bertrand-like price competition: they simultaneously and independently name prices p_i from the interval $[0, P(0)]$. The inverse market demand and market demand are denoted by P and $D := P^{-1}$, respectively. Both firms face only the costs for capacity installing, denoted by $b(\tilde{q}_i)$, $b(\tilde{q}_i) \geq 0$, $i = 1, 2$.

The quantities sold by both firms at prices p_1 and p_2, respectively are denoted in equations (4.1.2) and (4.1.3) for the case $p_1 \neq p_2$ and in equation (4.1.6) for $p_1 = p_2$. The net profits of both firms are defined in equation (4.2.1):

$$\pi_i = p_i q_i - b(\tilde{q}_i), \;\; i = 1, 2 \tag{4.2.1}$$

On this stage let us provide some additional assumptions that Kreps and Scheinkman made for their analysis:

Assumption 6. *There is an efficient rationing rule. Customers buy first from the cheapest supplier, and income effects are absent.*

Assumption 7. *Function $P(\cdot)$ is strictly positive on some bounded interval $(0, Q)$, on which it is twice-continuously differentiable, strictly decreasing and concave. For $\tilde{q} \geq Q$, $P(\tilde{q}) = 0$.*[6]

Assumption 8. *The cost to install capacity $\tilde{q}$ is $b(\tilde{q})$, where $b : \Re_+ \to \Re_+$ is twice continuously differentiable and convex on $\Re_+$, and satisfies $0 < b'(0) < P(0)$, and $b(0) = 0$. Without loss of generality the marginal cost of production is assumed to be zero.*[7]

[6] Assumption 1, p. 328.
[7] Assumption 2, p. 328.

Assume that both firms install the capacities $\tilde{q}_1$ and $\tilde{q}_2$, respectively. Using the terminology of Kreps and Scheinkman, beginning from the point where $(\tilde{q}_1, \tilde{q}_2)$ becomes common knowledge there is a proper subgame, namely the "capacity-constrained subgame" with capacities $\tilde{q}_1$ and $\tilde{q}_2$. It is not obvious whether such capacity-constrained subgame has an equilibrium, as payoffs are discontinuous in actions.[8]

The basic fact that Kreps and Scheinkman establish is that for each pair of $(\tilde{q}_1, \tilde{q}_2)$, the associated subgame has unique equilibrium revenues and they give formulas for these revenues. They fix a pair of capacities $(\tilde{q}_1, \tilde{q}_2)$ for the capacity-constrained subgame and, consider the existence of a pure-strategy equilibrium in such subgame, show, that an equilibrium exists if and only if the capacities are not too high.[9] The equilibria in this region are such that both firms charge the prices at which demand is equal to aggregate capacity. Thus, both firms basically dump their quantities in the market, in a manner analogous to Cournot behavior.

Kreps and Scheinkman also characterize the mixed-strategy equilibrium when capacities are high.[10] The profit of a firm with the highest capacity is equal to the Stackelberg follower profit.

Further Kreps and Scheinkman analyze the prior choices of capacities. They show that the Cournot quantities lead to price equilibrium in the pure-strategy region, and that if one firm chooses its Cournot capacity the other firm also is best off choosing its Cournot capacity. They also consider the equilibrium for the entire game and prove, that there is a unique equilibrium outcome, namely the Cournot outcome with $p_1 = p_2 = P(\tilde{q}_1 + \tilde{q}_2)$.

As the results of Kreps and Scheinkman are based on the assumption that both firms have symmetric capacity costs and zero production costs, it is not clear whether these results hold with asymmetry in capacity costs or if firms face production costs. Next we consider these two cases.

4.2.2.2 Case of asymmetric capacity costs

To analyze the case of asymmetric capacity costs let us use the same model and the same assumptions as in the previous section, which are analogous to the model of Kreps and Scheinkman. We also assume that the capacities are costly, but differently from Kreps and Scheinkman, allow them to be asymmetric, so that $b_i(\tilde{q}_i)$ is the cost of firm i of installing the capacity level $\tilde{q}_i$. Assumption 8 holds for both b_i, $i = 1, 2$. Let us make the following additional assumptions on demand:

Assumption 9. *There exists some price $p_0 > 0$, such as:*

$$\begin{cases} D(p) > 0, & \text{if } p < p_0 \\ D(p) = 0, & \text{if } p \geq p_0 \end{cases} \tag{4.2.2}$$

[8] For subgames where $\tilde{q}_1 = \tilde{q}_2$, the existence of a subgame equilibrium is established by Levitan and Shubik (1972) in cases where demand is linear and marginal costs are constant. Also for the case of linear demand and constant marginal costs, Dasgupta and Maskin (1986) establish the existence of subgame equilibria for all pairs of $\tilde{q}_1$ and $\tilde{q}_2$, and their methodology applies to all cases considered here.

[9] They should belong to some region just above the origin in the capacity space.

[10] Lemma 6, p. 332.

Assumption 10. *$D(\cdot)$ is continuous and strictly decreasing on the interval $[0, p_0)$ and twice continuously differentiable on $(0, p_0)$. Furthermore, $p \mapsto pD(p)$ is strictly concave on $[0, p_0)$.*

Analogously to Kreps and Scheinkman let us show the capacity space and divide it into three regions which are of interest in terms of a subgame equilibrium. The results are illustrated in Figure 4.1, where $R_i(\cdot)$ are the corresponding Cournot best-response functions, $i = 1, 2$; $i \neq j$ and $\tilde{q}^*$ is equilibrium quantity of each player in the Cournot game with zero costs.

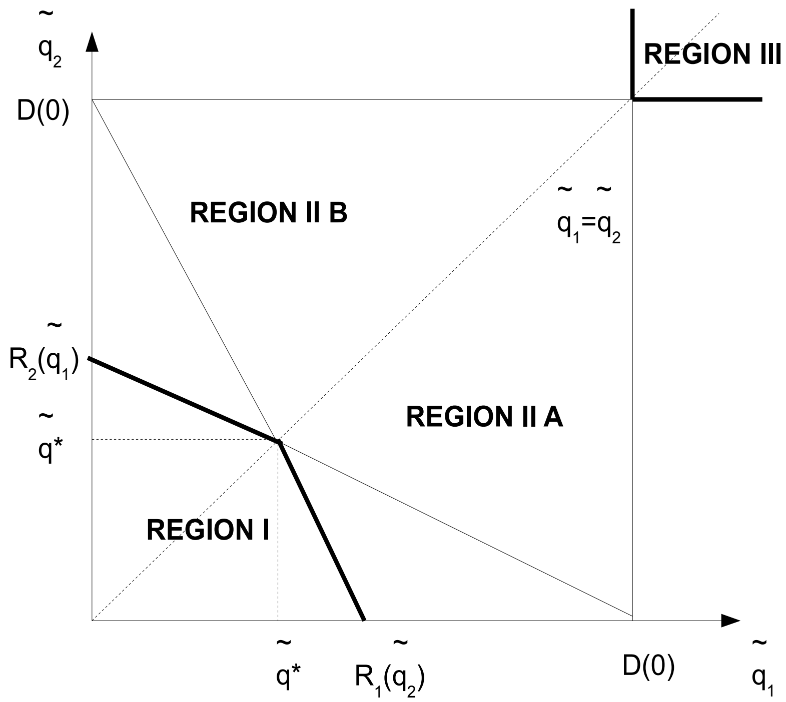

Figure 4.1: Capacity regions for the price subgame with zero costs

Definition 4.2.1. *i) Let Region I* $= \{(\tilde{q}_1, \tilde{q}_2) \in \Re^2_+ : 0 \leq \tilde{q}_1 \leq R_1(\tilde{q}_2),$
$0 \leq \tilde{q}_2 \leq R_2(\tilde{q}_1)\}$;

ii) Let Region IIA $= \left\{(\tilde{q}_1, \tilde{q}_2) \in \Re^2_+,\ (\tilde{q}_1, \tilde{q}_2) \notin III : \tilde{q}_1 > R_1(\tilde{q}_2),\ \tilde{q}_1 \geq \tilde{q}_2\right\}$;
Let Region IIB $= \left\{(\tilde{q}_1, \tilde{q}_2) \in \Re^2_+,\ (\tilde{q}_1, \tilde{q}_2) \notin III : \tilde{q}_2 > R_2(\tilde{q}_1),\ \tilde{q}_1 \leq \tilde{q}_2\right\}$;

iii) Let Region III $= \left\{(\tilde{q}_1, \tilde{q}_2) \in \Re^2_+,\ \tilde{q}_1 \geq D(0),\ \tilde{q}_2 \geq D(0)\right\}$.

Region I: It is the pure-strategy region, where both capacities lie below the best-response functions of firm's rival. As proved by Kreps and Scheinkman, in the equilibrium both firms name price $p_1 = p_2 = P(\tilde{q}_1 + \tilde{q}_2)$.

Region II A and II B: These are the mixed-strategy regions.

In *Region III* the subgame equilibrium is a Bertrand equilibrium with price equals to marginal production cost for both firms, which in the considered case is equal to zero.[11]

For the future analysis let us denote the expected revenue of firm i as $r(\tilde{q}_i)$:

$$r(\tilde{q}_i) = R_i(\tilde{q}_j)P(R_i(\tilde{q}_j) + \tilde{q}_j) \tag{4.2.3}$$

In order to derive equilibrium profits for both firms we use some results of Kreps and Scheinkman, provided by them in Lemmas 5 and 6:[12]

Lemma 5 of Kreps and Scheinkman

Suppose that either $\overline{p}_1 > \overline{p}_2$, or that $\overline{p}_1 = \overline{p}_2$ and $\overline{p}_2$ is not named with positive probability. Then:

- *(a) $\overline{p}_1 = P(R(\tilde{q}_2) + \tilde{q}_2)$ and the equilibrium revenue of firm 1 is $r(\tilde{q}_2)$;*
- *(b) $\tilde{q}_1 > R(\tilde{q}_2)$;*
- *(c) $\underline{p}_1 = \underline{p}_2$, and neither is named with positive probability;*
- *(d) $\tilde{q}_1 \geq \tilde{q}_2$;*
- *(e) The equilibrium revenue of firm 2 is uniquely determined by $(\tilde{q}_1, \tilde{q}_2)$ and is at least $(\tilde{q}_2/\tilde{q}_1)r(\tilde{q}_2)$ and at most $r(\tilde{q}_2)$.*

Lemma 6 of Kreps and Scheinkman

If $\tilde{q}_1 \geq \tilde{q}_2$ and $\tilde{q}_1 > R(\tilde{q}_2)$, there is a (mixed strategy) equilibrium for the subgame in which all the conditions and conclusions of Lemma 5 hold. Moreover, this equilibrium has the following properties. Each firm names prices according to continuous and strictly increasing distribution functions over an (coincident) interval, except that firm 1 names the uppermost price with positive probability whenever $\tilde{q}_1 > \tilde{q}_2$. And if we let $\Psi_i(p)$ be the probability distribution function for the strategy of firm i, then $\Psi_1(p) \leq \Psi_2(p)$: firm 1's strategy stochastically dominates the strategy of firm 2, with strict inequality if $\tilde{q}_1 > \tilde{q}_2$.

Using the results provided in the above Lemmas, in Theorem 4.2.1 we derive the expected equilibrium profits of both firms depending on the regions the capacities are lying in (illustrated in Figure 4.1), but for the case of asymmetric capacity costs. The most interesting case is when both capacities lie in the mixed-strategy Region II.[13]

[11] If $\min_i \tilde{q}_i \geq D(0)$ (Region III), then as in the usual Bertrand game without capacity constraints, $\underline{p}_i = \overline{p}_i = 0$, where $\overline{p}_i$ is the supremum of the support of the prices named by firm i and $\underline{p}_i$ is the infimum of the support. And if $\min_i \tilde{q}_i = 0$, there will be a monopoly case. Thus in future analysis we consider the case where $0 < \min_i \tilde{q}_i < D(0)$.

[12] pp. 331, 332.

[13] The detailed explanation of Theorem 4.2.1 is given in the Appendix A.2.

Theorem 4.2.1. *i) If both capacities lie in the Region I, then:*

$$\pi_i^*(\tilde{q}_1, \tilde{q}_2) = \tilde{q}_i P(\tilde{q}_1 + \tilde{q}_2) - b_i(\tilde{q}_i), \; i = 1,2 \tag{4.2.4}$$

ii)

- *If both capacities lie in the Region II A, then:*

$$\begin{cases} \pi_1^*(\tilde{q}_1, \tilde{q}_2) = R_1(\tilde{q}_2)P(R_1(\tilde{q}_2) + \tilde{q}_2) - b_1(\tilde{q}_1) = r(\tilde{q}_2) - b_1(\tilde{q}_1) \\ \pi_2^*(\tilde{q}_1, \tilde{q}_2) = \underline{p}\tilde{q}_2 - b_2(\tilde{q}_2), \end{cases} \tag{4.2.5}$$

where $\underline{p}$ *is the smallest solution of* $\underline{p} = \dfrac{r(\tilde{q}_2)}{\min\left[\tilde{q}_1, D(\underline{p})\right]}$.

- *If both capacities lie in the Region II B, then:*

$$\begin{cases} \pi_1^*(\tilde{q}_1, \tilde{q}_2) = \underline{p}\tilde{q}_1 - b_1(\tilde{q}_1) \\ \pi_2^*(\tilde{q}_1, \tilde{q}_2) = R_2(\tilde{q}_1)P(R_2(\tilde{q}_1) + \tilde{q}_1) - b_2(\tilde{q}_2) = r(\tilde{q}_1) - b_2(\tilde{q}_2), \end{cases} \tag{4.2.6}$$

where $\underline{p}$ *is the smallest solution of* $\underline{p} = \dfrac{r(\tilde{q}_1)}{\min\left[\tilde{q}_2, D(\underline{p})\right]}$.

iii) If capacities lie in Region III, then:

$$\pi_1^*(\tilde{q}_1, \tilde{q}_2) = \pi_2^*(\tilde{q}_1, \tilde{q}_2) = 0 \tag{4.2.7}$$

Our Assumptions 7 and 8 on the demand and cost functions guarantee that the related Cournot game has a unique equilibrium.

The aim of this section is to check if the results of Kreps and Scheinkman will hold if firms face asymmetric capacity costs. For their results to hold, firms must choose Cournot quantities in Region I and market price should be such that demand is equal to aggregate capacities.

Therefore let us next assume that $\tilde{q_1}^*$ and $\tilde{q_2}^*$ are Cournot equilibrium capacities, with capacity costs b_1 and b_2 by the respective firm. Further it is to prove that if there is a unique subgame perfect equilibrium in this game, then the capacities are equal to Cournot equilibrium quantities and lie in Region I.

Kreps and Scheinkman have found the unique Nash equilibrium in the price competition subgame. In Theorem 4.2.1 we establish the equilibrium profits of both firms which are the functions of the firms' capacities. Hence, applying this theorem it is important to consider the capacity choice of both firms.

1. Let us assume, that both firms choose capacities $\tilde{q}_1, \tilde{q}_2 \in$ *Region I*, then according to Theorem 4.2.1 (*i*), the equilibrium profit of firm i is: $\pi_i^*(\tilde{q}_1, \tilde{q}_2) = \tilde{q}_i P(\tilde{q}_1 + \tilde{q}_2) - b_i(\tilde{q}_i), \; i = 1, 2$.

   For the future analysis we make the following additional assumptions:

   Assumption 11. *Profit functions* π_i^* *are concave in* $\tilde{q}_i$, $i = 1, 2$.

Assumption 12. *We denote $R_i(\tilde{q}_j|b_i) = R_{b_i}(\tilde{q}_j)$ as the Cournot best-response of player i given the capacity cost b_i. It is straightforward, that $R_i(\tilde{q}_j) \geq R_{b_i}(\tilde{q}_j)$ for $\forall \tilde{q}_j$ as illustrated in Figure 4.2.*

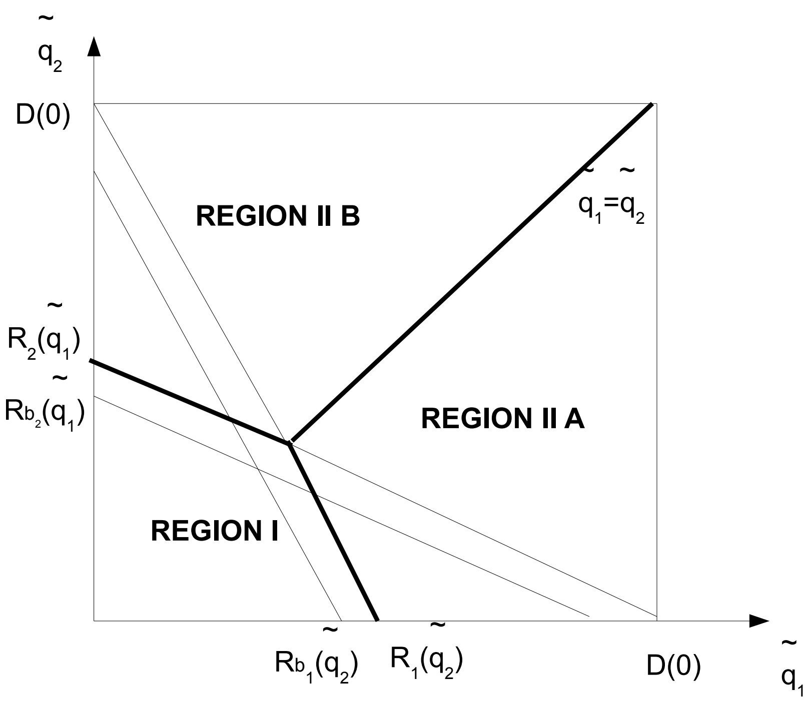

Figure 4.2: Capacity regions for the price subgame with asymmetric capacity costs

If the profit is concave in capacity and both capacities belong to Region I, then using Assumption 12 firm i's profit, given $\tilde{q}_j$, is increasing when capacity $\tilde{q}_i$ gets closer to the $R_{b_i}(\tilde{q}_j)$.

Hence, if $(\tilde{q}_1, \tilde{q}_2)$ is an equilibrium, then the following equalities should hold:

$$\begin{cases} \tilde{q}_1 \stackrel{!}{=} \tilde{q}_1{}^* \\ \tilde{q}_2 \stackrel{!}{=} \tilde{q}_2{}^* \end{cases} \tag{4.2.8}$$

If equations (4.2.8) do not hold, then at least one of both firms can choose other capacity inside of Region I and increase its profit.

Next let us check whether $\tilde{q}_1{}^*$ and $\tilde{q}_2{}^*$ are the unique equilibrium quantities of the considered subgame. According to Figure 4.1, we consider two cases: $\tilde{q}_i{}^* \leq \tilde{q}^*$ and $\tilde{q}_i{}^* > \tilde{q}^*$, $\forall i = 1, 2$.

In the first case if $\tilde{q_1}^* \leq \tilde{q}^*$ and $\tilde{q_2}^* \leq \tilde{q}^*$ it is straightforward that these Cournot capacities $(\tilde{q_1}^*, \tilde{q_2}^*)$ belong to an equilibrium in the pure-strategy Region I. If one firm chooses its Cournot capacity, then the other firm is also best off choosing its Cournot capacity.

Let us consider another case. Without loss of generality we assume that $\tilde{q_1}^* > \tilde{q}^*$, as we consider both capacities lying in Region I (shown in Figure 4.1) it is obvious that $\tilde{q_2}^* < \tilde{q}^*$. If $\tilde{q_1}^* > \tilde{q}^*$ firm 2 has an incentive to change its capacity, then $\tilde{q_2} \notin \mathit{Region\ IIB}$, but $\tilde{q_2} \in \mathit{Region\ IIA}$ is possible. If firm 2 chooses a quantity that belongs to Region II A, then according to equation (4.2.5) in Theorem 4.2.1 and Proposition 1 of Kreps and Scheinkman firm 2 will obtain profit shown in equation (4.2.9):

$$\pi_2^{*IIA}(\tilde{q_1}^*, \tilde{q_2}) = \underline{p}\tilde{q_2} - b_2(\tilde{q_2}) = \frac{r(\tilde{q_2})\tilde{q_2}}{\tilde{q_1}^*} - b_2(\tilde{q_2}) \tag{4.2.9}$$

Since P is concave, the function $p \mapsto pD(p)$ is strictly concave wherever it is positive. As $p \mapsto D(p)$ is decreasing every ray from the origin of the form $p\tilde{q}$ crosses the graph of this function at most once. Using Figure 2 and the proof of Lemma 6 (*iii*) of Kreps and Scheinkman the function $p \mapsto pD(p)$ is maximized at $P(R(0))$, so the demand $R(0)$ at price $P(R(0))$ is higher than $\tilde{q_1}^*$. So the revenue function $r(\cdot)$ is first increasing where it satisfies $r(\tilde{q_2}) = p\tilde{q_1}^*$ and decreasing where it satisfies $r(\tilde{q_2}) = pD(p)$.

Using this argument, the smallest solution of $\underline{p} = \dfrac{r(\tilde{q_2})}{\min\left[\tilde{q_1}^*, D(\underline{p})\right]}$ from Theorem 4.2.1 equation (4.2.5) is also the solution to $r(\tilde{q_2}) = \underline{p}\tilde{q_1}^*$.

Assuming that firm 2 chooses the quantity $\tilde{q_2} \in$ Region II A, we consider how its profit changes with increase of $\tilde{q_2}$ for given $\tilde{q_1}$. Therefore we take the derivative of the profit of firm 2 in Region II A, depicted in equation (4.2.9) with respect to $\tilde{q_2}$:

$$\frac{\partial \pi_2^{*IIA}(\tilde{q_1}^*, \tilde{q_2})}{\partial \tilde{q_2}} = \frac{r(\tilde{q_2}) + \tilde{q_2}r'(\tilde{q_2}) - \tilde{q_1}^* b_2'(\tilde{q_2})}{\tilde{q_1}^*} \tag{4.2.10}$$

As the revenue function of firm i has a form $r(\tilde{q_i}) = R_i(\tilde{q_j})P(R_i(\tilde{q_j}) + \tilde{q_j})$, where $R_i(\cdot)$ is the corresponding Cournot best-response function, its derivative with respect to $\tilde{q_i}$ is:

$$\begin{aligned} r'(\tilde{q_2}) &= R'(\tilde{q_2})P(R(\tilde{q_2}) + \tilde{q_2}) + R(\tilde{q_2})P'(R(\tilde{q_2}) + \tilde{q_2}) + (R'(\tilde{q_2}) + 1) \qquad (4.2.11) \\ &= R'(\tilde{q_2})\left[P(R(\tilde{q_2}) + \tilde{q_2}) + R(\tilde{q_2})P'(R(\tilde{q_2}) + \tilde{q_2})\right] + R(\tilde{q_2})P'(R(\tilde{q_2}) + \tilde{q_2}) \end{aligned}$$

The first term is zero by the definition of $R(\tilde{q_2})$, therefore we have:

$$r'(\tilde{q_2}) = R(\tilde{q_2})P'(R(\tilde{q_2}) + \tilde{q_2}) \tag{4.2.12}$$

Plugging $r'(\tilde{q_2})$ into equation (4.2.10) we obtain:

$$\frac{\partial \pi_2^{*IIA}}{\partial \tilde{q_2}} = \frac{r(\tilde{q_2}) + \tilde{q_2} R(\tilde{q_2}) P'(R(\tilde{q_2}) + \tilde{q_2}) - \tilde{q_1}^* b_2'(\tilde{q_2})}{\tilde{q_1}^*} \tag{4.2.13}$$

$$= \frac{R(\tilde{q_2}) P(R(\tilde{q_2}) + \tilde{q_2}) + \tilde{q_2} R(\tilde{q_2}) P'(R(\tilde{q_2}) + \tilde{q_2}) - \tilde{q_1}^* b_2'(\tilde{q_2})}{\tilde{q_1}^*}$$

According to our previous assumption if $\tilde{q_1}^* \geq R(\tilde{q_2})$ then:

$$\frac{\partial \pi_2^{*IIA}}{\partial \tilde{q_2}}(\tilde{q_1}^*, \tilde{q_2}) \leq \frac{r(\tilde{q_2}) + \tilde{q_2} r'(\tilde{q_2}) - R(\tilde{q_2}) b_2'(\tilde{q_2})}{\tilde{q_1}^*} \tag{4.2.14}$$

Further we plug equation (4.2.12) into (4.2.14) and obtain the following inequality:

$$\frac{\partial \pi_2^{*IIA}}{\partial \tilde{q_2}}(\tilde{q_1}^*, \tilde{q_2}) \leq \frac{R(\tilde{q_2}) P(R(\tilde{q_2}) + \tilde{q_2}) + \tilde{q_2} R(\tilde{q_2}) P'(R(\tilde{q_2}) + \tilde{q_2}) - R(\tilde{q_2}) b_2'(\tilde{q_2})}{\tilde{q_1}^*} \tag{4.2.15}$$

$$\Longleftrightarrow \frac{\partial \pi_2^{*IIA}}{\partial \tilde{q_2}}(\tilde{q_1}^*, \tilde{q_2}) \leq \frac{R(\tilde{q_2}) \left[P(R(\tilde{q_2}) + \tilde{q_2}) + \tilde{q_2} P'(R(\tilde{q_2}) + \tilde{q_2}) - b_2'(\tilde{q_2}) \right]}{\tilde{q_1}^*}$$

In order to understand the sign of the left hand-side of inequality (4.2.15) we analyze the fraction on the right hand-side. The derivative of Cournot profit of firm 2 with respect to $\tilde{q_2}$ is shown below:

$$\frac{\partial \pi_2^{*C}}{\partial \tilde{q_2}} = P(\tilde{q_1} + \tilde{q_2}) + \tilde{q_2} P'(\tilde{q_1} + \tilde{q_2}) - b_2'(\tilde{q_2}) \tag{4.2.16}$$

It is straightforward that equation (4.2.16) is not positive if $\tilde{q_2} \geq R_{b_2}(\tilde{q_1})$. Hence, if $\tilde{q_2} \geq R_{b_2}(R(\tilde{q_2}))$ the right hand-side of inequality (4.2.15) is not positive:

$$\frac{\partial \pi_2^{*IIA}}{\partial \tilde{q_2}}(\tilde{q_1}^*, \tilde{q_2}) \leq 0 \tag{4.2.17}$$

The inequality (4.2.17) means that the profit of firm 2 if its capacity $\tilde{q_2}$ lies in Region II A is increasing if the capacity becomes smaller. As the profit function of firm 2 is continuous in its capacity the best response to $\tilde{q_1}^* \in$ Region I is $\tilde{q_2}^* \in$ Region I.

Summarizing the above made analysis we have obtained the following preliminary result:

Result 7. $(\tilde{q_1}^*, \tilde{q_2}^*) \in$ *Region I are the equilibrium quantities of the considered subgame. Both firms prefer to sell everything and there is a market-clearing price:* $p_1 = p_2 = P(\tilde{q_1}^* + \tilde{q_2}^*)$.

Next it is necessary to check whether these equilibrium capacities are unique in the underlying price subgame. Therefore we consider two cases, first when both capacities belong to Region II A and then when they belong to Region II B.

2. Let us start with the Region II A. Suppose that firm 1 chose capacity $\tilde{q}_1$ and firm 2 chose some capacity $\tilde{q}_2{}'$, so that $(\tilde{q}_1, \tilde{q}_2{}') \in$ *Region II A*. From equation (4.2.5) the profit of firm 1 is:

$$\pi_1^*(\tilde{q}_1, \tilde{q}_2{}') = r(\tilde{q}_2{}') - b_1(\tilde{q}_1) \tag{4.2.18}$$

According to Assumption 8, as the cost function is strictly increasing in $\tilde{q}_1$, the profit of firm 1 in capacity Region II A is strictly decreasing in $\tilde{q}_1$, moreover it is increasing in $\tilde{q}_2{}'$. If in Region II A there exists an equilibrium, it should lie close to or even on the upper frontier of Region II A, namely on the line $\tilde{q}_1 = \tilde{q}_2$. It is obviously possible only if $\tilde{q}_2{}' > \tilde{q}^*$. So if $\tilde{q}_2{}' \leq \tilde{q}^*$ there will be no equilibrium in Region II A.

Applying Proposition 1 of Kreps and Scheinkman, if $\tilde{q}_2{}' > \tilde{q}^*$, on the upper frontier of Region II A, namely on the line $\tilde{q}_1 = \tilde{q}_2$, the highest profit of firm 1 (π_1^{*IIA}) is equal to its lowest profit in Region II B (π_1^{*IIB}):

$$\pi_1^{*IIA}(\tilde{q}_1, \tilde{q}_2{}') = r(\tilde{q}_2{}') - b_1(\tilde{q}_1) = \pi_1^{*IIB}(\tilde{q}_1, \tilde{q}_2{}') = \frac{\tilde{q}_1}{\tilde{q}_2{}'} r(\tilde{q}_1) - b_1(\tilde{q}_1) \tag{4.2.19}$$

Assume that for some small $\epsilon > 0$ firm 1 will choose capacity $\tilde{q}_1 = \tilde{q}_2{}' - \epsilon$, so that $\tilde{q}_2{}' - \epsilon > \tilde{q}^*$ and $(\tilde{q}_2{}' - \epsilon, \tilde{q}_2{}') \in$ Region II B, then according to equation (4.2.6) the profit of firm 1 is:

$$\pi_1^{*IIB}(\tilde{q}_2{}' - \epsilon, \tilde{q}_2{}') = \underline{p}(\tilde{q}_2{}' - \epsilon) - b_1(\tilde{q}_2{}' - \epsilon) \tag{4.2.20}$$

On the lowest frontier of Region II B, namely on the line $\tilde{q}_1 = \tilde{q}_2$ the profit of firm 1 is:

$$\pi_1^{*IIB} = \frac{\tilde{q}_2{}' - \epsilon}{\tilde{q}_2{}'} r(\tilde{q}_2{}' - \epsilon) - b_1(\tilde{q}_2{}' - \epsilon) \tag{4.2.21}$$

Let us consider $\underline{p} = \dfrac{r(\tilde{q}_2)}{\min[\tilde{q}_1, D(\underline{p})]}$. In Region II B, if $\tilde{q}_2 > \tilde{q}_1$ (in our case it is: $\tilde{q}_2{}' > \tilde{q}_2{}' - \epsilon$, where ϵ is positive), then $D(\underline{p}) > \tilde{q}_2{}' - \epsilon$ and firm 1's equilibrium revenue is $\underline{p}(\tilde{q}_2{}' - \epsilon)$, but it is also equal to $r(\tilde{q}_2{}')$.[14] So we obtain: $\underline{p} = \dfrac{r(\tilde{q}_2{}')}{\tilde{q}_2{}' - \epsilon}$. If we plug it into equation (4.2.20), we see that the profit in equation (4.2.20) is higher than in equation (4.2.21):

$$\underline{p}(\tilde{q}_2{}' - \epsilon) - b_1(\tilde{q}_2{}' - \epsilon) > \frac{\tilde{q}_2{}' - \epsilon}{\tilde{q}_2{}'} r(\tilde{q}_2{}' - \epsilon) - b_1(\tilde{q}_2{}' - \epsilon) \tag{4.2.22}$$

This inequality means that firm 1 will rather move from the upper frontier of Region II A, namely from the line $\tilde{q}_1 = \tilde{q}_2$ to the capacity Region II B, where it will increase its profit and will be better off. So we can formulate the second preliminary result of our analysis:

[14] Analogously in Kreps and Scheinkman, proof of Lemma 5 (d), (e), p. 332.

Result 8. *There is no equilibrium in the subgame with* $(\tilde{q}_1, \tilde{q}_2{}') \in$ *Region II A.*

3. Analyzing the case, where capacities of both firms lie in the Region II B and using the analogous arguments as in case of Region II A, we obtain the same result, namely that at least one of both firms could be better off by changing its capacity and moving from the Region II B to the Region II A. That means the following:

   Result 9. *There is no equilibrium for* $(\tilde{q}_1{}', \tilde{q}_2) \in$ *Region II B.*

The results of this section are summarized in the following proposition:

Proposition 4.2.1. *In the capacity-constrained price game (with asymmetric capacity costs) there exists a unique subgame perfect equilibrium outcome (in the two-stage game) with* $(\tilde{q}_1, \tilde{q}_2)$ *the resulting capacities lying in the pure-strategy Region I, described in Definition 4.2.1, and they are equal to the Cournot quantities:* $\tilde{q}_1 = \tilde{q}_1{}^*$ *and* $\tilde{q}_2 = \tilde{q}_2{}^*$*.*

Hence, considering the case of asymmetric capacity costs we can constitute the following general result:

Result 10. *Under the unchanged assumptions on the rationing rule, demand and cost functions, the results of Kreps and Scheinkman hold also for the case of asymmetric capacity costs.*

In the next section we assume that both downstream firms face production costs. The forthcoming analysis is very important in the context of the whole provided work.

4.2.3 Price competition with asymmetric production costs

4.2.3.1 Theoretical framework and basic assumptions

As it has been already mentioned in the previous section, now we complete our analysis assuming that both firms face production costs, moreover we allow them to be asymmetric. As it was presented in the bargaining model in Chapter 2, before entering the local market both firms bargained with the upstream supplier over the wholesale prices. Therefore in this section the production costs are associated with the above mentioned wholesale prices (w_1, w_2). Extending Chapter 2 we have shown that if firm 1 has an option of backward integration it can exercise the bargaining power on the supplier which is larger than the bargaining power of firm 2. We have also shown how this advantage effects the bargaining result, namely that firm 1 can negotiate better buying conditions for itself. This fact explains the asymmetry in production costs. Applying the result obtained in Chapter 2 we make the following statement for the current section: *Allowing the asymmetry in the unit production costs, for any possible pair of negotiated wholesale prices* (w_1, w_2) *the inequality* $w_1 < w_2$ *holds.*

Differently from Kreps and Scheinkman we assume in this section that there are no costs of installing the capacity level, namely $b(\tilde{q}_i) = 0$, $i = 1, 2$. Both firms have identical production functions. The inverse market demand and market demand are denoted by P and $D := P^{-1}$, which are strictly positive and twice continuously differentiable. As in the previous model we assume the efficient rationing rule.

For the forthcoming analysis let us make the following assumption on the cost functions:

Assumption 13. *The cost functions C_i are convex and twice-continuously differentiable, $P(0) > C(0) > 0$ and $C_i(\tilde{q}_i) \neq C_j(\tilde{q}_j)$, $i = 1, 2$.*

$$C_i(q_i) = \begin{cases} w_i q_i, & \text{if } q_i \leq \tilde{q}_i \\ \infty, & \text{if } q_i > \tilde{q}_i \end{cases} \tag{4.2.23}$$

In this section the Assumption 9 on the existence of price level $p_0 > 0$ holds. It determines demand in the same way as it was described in the Assumption 10. Each firm i produces goods at a constant unit cost $0 < w_i < p_0$ up to a capacity level $\tilde{q}_i > 0$. On this stage we present the next additional assumption on demand:

Assumption 14. *$D'(p) + pD''(p) < 0$ holds on the interval $(0, p_0)$.*

Assumption 14 provides that for any cost pairs (w_1, w_2) the Cournot best-response function $R_i(\,\cdot\,|w_i)$, with

$$R_i(q_j|w_i) = \arg\max_{q_i} \left(P(q_i + q_j) - w_i\right) q_i$$

will be well defined and strictly decreasing. Moreover, the related Cournot game has a unique equilibrium $(\tilde{q}_i{}^*(w), \tilde{q}_j{}^*(w))$.

4.2.3.2 Basic notations

Applying equations (4.1.2) - (4.1.7), which determine demand functions, the profit of firm i at price p_i if the rival's price is p_j, is equal to:

$$\pi_i(p_i, p_j|\tilde{q}_i, \tilde{q}_j) = \begin{cases} (p_i - w_i)\min(D(p_i), \tilde{q}_i), & \text{if } p_i < p_j \\ (p_i - w_i)\min\left((\max(0, D(p_i) - \tilde{q}_j)), \tilde{q}_i\right), & \text{if } p_i = p_j;\ w_i > w_j \\ (p_i - w_i)\min\left((\max(0, D(p_i)), \tilde{q}_i\right), & \text{if } p_i = p_j;\ w_i < w_j \\ (p_i - w_i)\min(\tilde{q}_i, \max(0, D(p_i) - \tilde{q}_j)), & \text{if } p_i > p_j \end{cases} \tag{4.2.24}$$

If both prices are equal, we allow the low-cost firm to sell its capacity first.[15]

As $\pi_i(\,\cdot\,|\tilde{q}_i, \tilde{q}_j)|_{\{p|p_i > p_j\}} < \pi_i(\,\cdot\,|\tilde{q}_i, \tilde{q}_j)|_{\{p|p_i < p_j\}}$, which means that the profit of firm i at price $p_i > p_j$ is less than at price $p_i < p_j$, we denote the minmax profit of firm i as:

$$\pi_i^{minmax} = \max_{\{p|p_i > p_j\}} \pi_i(\,\cdot\,|\tilde{q}_i, \tilde{q}_j) \tag{4.2.25}$$

[15]Deneckere and Kovenock (1989) show that equilibrium profits are unaffected by the tie breaking rule, and that equilibrium distributions are altered only in the classical Bertrand region when the low cost firm does not have a drastic cost advantage.

Let $\underline{p}_i$ be the lowest price at which firm i if $p_i < p_j$ can still earn its minmax profit level, so that:

$$\underline{p}_i = \min \left\{ p_i : \pi_i(\, \cdot \, |\tilde{q}_i, \tilde{q}_j)|_{\{p|p_i<p_j\}} \geq \pi_i^{minmax} \right\} \tag{4.2.26}$$

This critical price $\underline{p}_i$ depends on $(\tilde{q}_i, \tilde{q}_j, w_i, w_j)$. Similar to Figure 2 of Kreps and Scheinkman (1983), but for the case of asymmetric production costs, the profit functions from equation (4.2.24) and critical prices from equation (4.2.26) are illustrated in Figure 4.3.

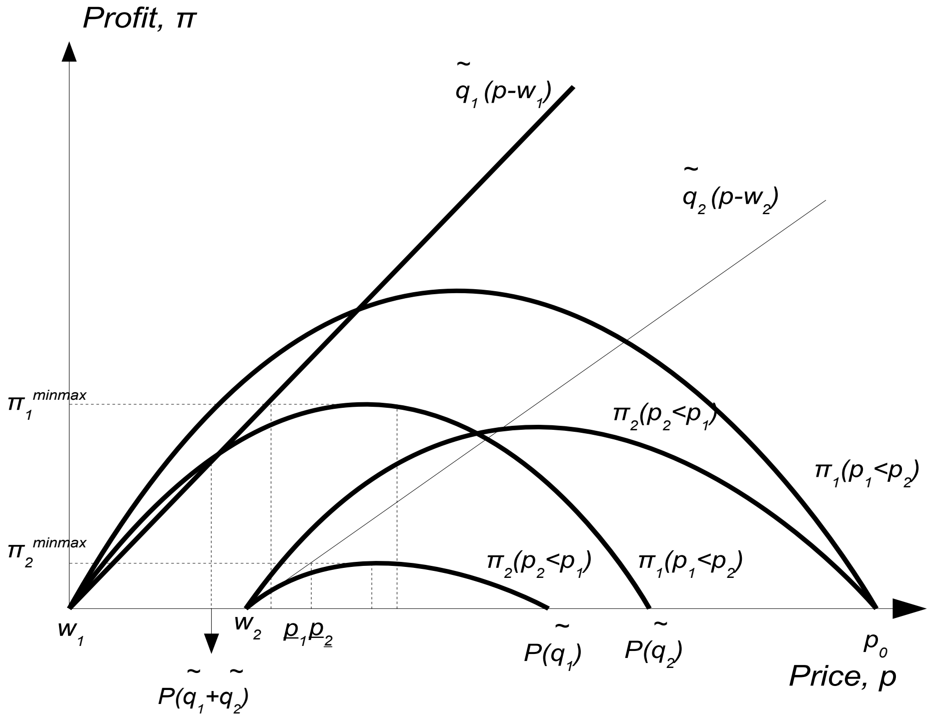

Figure 4.3: Determining the subgame equilibrium

4.2.3.3 Price-setting subgame

Let $S_i = [w_i, p_0]$ be a pure strategy set of firm i and let Σ_i be a corresponding set of mixed strategies (the set of distribution functions F_i on S_i) and π_i is the expected profit function of player i, whose domain can be extended in the natural way to $\Sigma_1 \times \Sigma_2$. According to Osborne and Pitchik (1986) if $F_j \in \Sigma_j$ we have:[16]

$$\pi_i(p, F_j) = \pi_i(p_i, p_j)_{\{p|p_i>p_j\}}(F_j(p) - \alpha_j(p)) + \pi_i(p_i, p_j)_{\{p|p_i=p_j\}}\alpha_j(p) + \\ + \pi_i(p_i, p_j)_{\{p|p_i<p_j\}}(1 - F_j(p)), \tag{4.2.27}$$

[16]See p. 242.

where $\alpha_j(p)$ is the size of atom (if it is present) in F_j at p. We denote $\pi_i(F_i, F_j) = \int_{w_i}^{p_0} \pi_i(p, F_j)dF_i(p)$.

Let $\tilde{q} = (\tilde{q}_1, \tilde{q}_2) \in \Re_+^2$ be a profile of capacities and $w = (w_1, w_2) \in \Re_+^2$ be a profile of possible production costs, then for any quadruple of capacities and costs $(\tilde{q}_1, \tilde{q}_2, w_1, w_2)$ the price-setting subgame $\Gamma(\tilde{q}, w)$ that firms face in the price competition stage is of the normal form $\Gamma(\tilde{q}, w) = \{I, \{\Sigma_i\}, \{\pi_i(F_i, F_j)\}\}$, with set of players $I = [1, 2]$, strategy set Σ_i and $\pi_i(F_i, F_j)$ are payoff functions, $i, j \in I$.

Analogously to Kreps and Scheinkman, who established that for each pair $(\tilde{q}_1, \tilde{q}_2)$ the associated subgame has unique expected equilibrium revenues, let us concentrate on the subgame equilibrium profits of the above described game.

4.2.3.4 Subgame equilibrium profits

Dasgupta and Maskin (1986) proved in Theorem 5 that a (upper semi) continuous-sum game in which individual utility functions satisfy a weak form of lower semi-continuity possesses a mixed-strategy equilibrium.[17] Considering our model $\pi_1(p_1, p_2) + \pi_2(p_1, p_2)$ is upper semi-continuous in (p_1, p_2) because the sum is continuous at all off-diagonal points and the tie breaking rule which we assumed in section 4.1.3 minimizes the total cost of providing the good along the diagonal. The revenues in our model are continuous by assumption, hence, when total profit approaches a point on the diagonal it cannot jump down. Consequentially, we can apply this theorem without any restriction and state the existence of a mixed-strategy equilibrium in the price-setting subgame $\Gamma(\tilde{q}, w)$. Using the results of Dasgupta and Maskin (1986), the next step is to find the Nash equilibrium profits of the subgame $\Gamma(\tilde{q}, w)$ for all quadruples of $(\tilde{q}_1, \tilde{q}_2, w_1, w_2)$.

Similar to the previous section the profits of both firms depend on the region in which their capacities are lying in. To describe the structure of an equilibrium of such subgame, let us divide the capacity space into three regions, as described in Definition 4.2.2 and depicted in Figure 4.4.[18,19]

Definition 4.2.2. *i) Let Region I* $= \{(\tilde{q}_1, \tilde{q}_2) \in \Re_+^2 : \ 0 \le \tilde{q}_1 \le R_1(\tilde{q}_2|w_1),$ $0 \le \tilde{q}_2 \le R_2(\tilde{q}_1|w_2)\}, \ \underline{p} = P(\tilde{q}_1 + \tilde{q}_2) = 1 - \tilde{q}_1 - \tilde{q}_2$.

ii) Let Region II $= \left\{(\tilde{q}_1, \tilde{q}_2) \in \Re_+^2 : \underline{p} = \max\{w_1, w_2\}\right\}$.

iii) Let Region III $= \{\Re_+^2 \backslash (I \cup II), \ (\tilde{q}_1, \tilde{q}_2) \in$ *Region III:* $\tilde{q}_1 \ge 0; \ \tilde{q}_2 \ge 0;$ $\tilde{q}_1 > R_1(\tilde{q}_2|w_1); \ \tilde{q}_2 > R_2(\tilde{q}_1|w_2)\}$.

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[17] Theorem 5, p. 14.

[18] Figure 4.4 is analogous to Figure 2 by Deneckere and Kovenock (1996).

[19] We have also used Theorem 3 by Deneckere and Kovenock in which they proved for the case $c_1 < c_2$ that:

i) If $R_1(0|c_1) < D(c_2)$, there exists a continuous function $\Theta : [0, \infty) \to [0, D(c_1)]$ such that $\underline{p}_2 > \underline{p}_1$, whenever $\tilde{q}_2 > \Theta(\tilde{q}_1)$ and $\underline{p}_2 < \underline{p}_1$, whenever $\tilde{q}_2 < \Theta(\tilde{q}_1)$ and $(\tilde{q}_1, \tilde{q}_2) \notin$ *Region I*. Furthermore, the function Θ satisfies $\Theta(\tilde{q}_1) = R_2(\tilde{q}_1|c_2)$ for $\tilde{q}_1 \in \left[0, \tilde{q}_1{}^C\right]$, $R_2(\tilde{q}_1|c_2) < \Theta(\tilde{q}_1) < \tilde{q}_1$ for $\tilde{q}_1 \in (\tilde{q}_1{}^C, D(c_2)]$.

ii) If $R_1(0|c_1) \ge D(c_2)$, then $\underline{p}_2 \ge \underline{p}_1$ whenever $(\tilde{q}_1, \tilde{q}_1) \notin$ *Region I*.

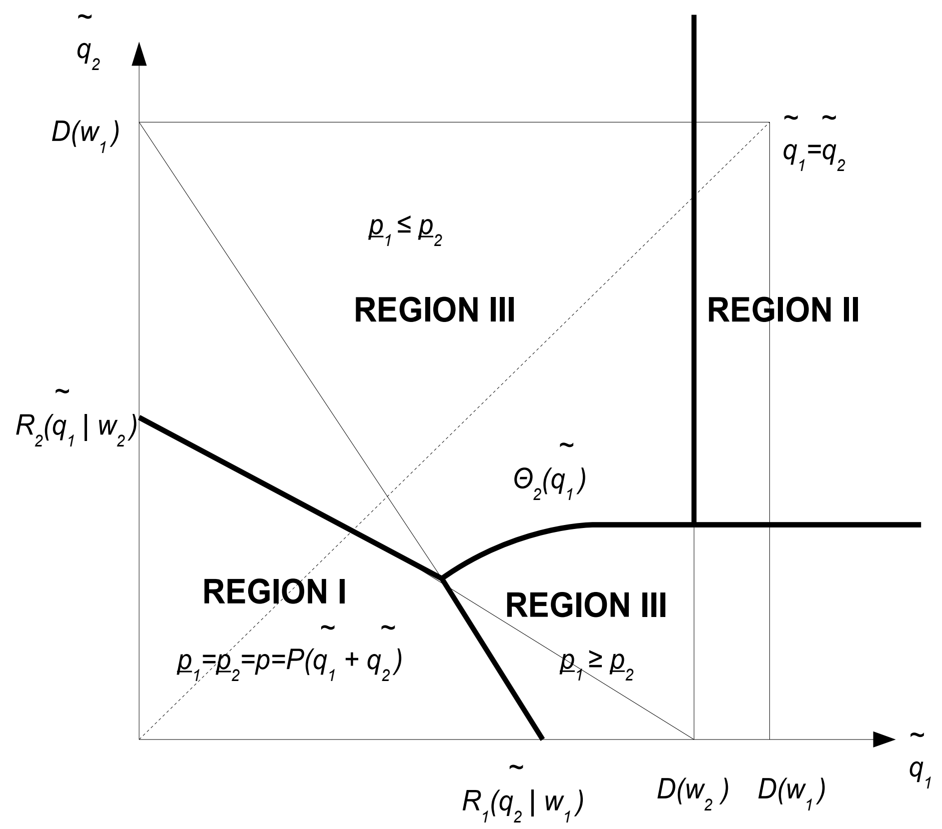

Figure 4.4: Capacity regions for the price subgame, when $w_1 < w_2$

Next we consider the regions illustrated in Figure 4.4. The first capacity region, Region I, lies below the both Cournot best-response functions and it is a pure-strategy region, with each firm setting a price equal to the market clearing price.

Region II is the classical Bertrand region in which the low-cost firm has enough capacity to drive the high-cost firm out of the market, and the high-cost firm is sufficiently large that the low-cost firm finds it profitable to do so.[20]

Region III consists of two regions, namely with $\underline{p}_1 \leq \underline{p}_2$ and $\underline{p}_1 \geq \underline{p}_2$. On this stage we make the following propositions:

Proposition 4.2.2. *At least one firm will earn its minmax profit.*

Proof. Analogously to Kreps and Scheinkman (1983), let us make the following notations: let $s_i \in S_i$ and $s_j \in S_j$. We denote $(F_i(p), F_j(p))$ as a pair of equilibrium price distributions, $\overline{s}_i$ as the supremum of the support of the equilibrium price distributions named by firm i, that is $\overline{s}_i = \inf\{p : F_i(p) = 1\}$ and $\underline{s}_i$ as the infimum

[20] According to Allen et al. (2000).

of the support, that is $\underline{s}_i = \sup\{p : F_i(p) = 0\}$, $i = 1, 2$. Applying Assumption 9 it is obvious that $w_i \leq \underline{s}_i \leq \overline{s}_i \leq p_0$, $i = 1, 2$.

We have $\pi_i(F_i, F_j) \geq \pi_i(p, F_j) \geq \pi_i(p)|_{\{p|p_i>p_j\}} \geq 0$ for all $p \geq 0$ ($p \in S_i$), therefore $\pi_i(F_i, F_j) \geq \pi_i^{minmax}$ for $i = 1, 2$.

If $\overline{s}_i > \overline{s}_j$ or $\overline{s}_i = \overline{s}_j = \overline{s}$ and $\overline{s}_i \notin J(F_j)$, where $J(F_j)$ is the set of atoms of F,[21] or in other words, firm j has no mass point at $\overline{s}$, then $\pi_i(F_i, F_j) = \pi_i(\overline{s}_i, F_j)$. We also have $\pi_i(\overline{s}_i, F_j) = \pi_i(\overline{s}_i)_{\{p|p_i>p_j\}} \leq \pi_i^{minmax}$. Summarizing the above mentioned inequalities we get the following system:

$$\begin{cases} \pi_i(F_i, F_j) \geq \pi_i^{minmax} \\ \pi_i(F_i, F_j) = \pi_i(\overline{s}_i)_{\{p|p_i>p_j\}} \leq \pi_i^{minmax} \end{cases}$$

Hence, from these inequalities it follows that $\pi_i(F_i, F_j) = \pi_i(\overline{s}_i)_{\{p|p_i>p_j\}} = \pi_i^{minmax}$.

Now let us consider the case when $\overline{s}_i = \overline{s}_j = \overline{s}$ and both firms have mass points at $\overline{s}$, then according to Lemma 2 of Osborne and Pitchik (1986) we state that $\pi_i(\overline{s}_i)|_{\{p|p_i<p_j\}} = \pi_i(\overline{s}_i)|_{\{p|p_i=p_j\}} = \pi_i(\overline{s}_i)|_{\{p|p_i>p_j\}}$, $i = 1, 2$ and therefore $\pi_i(F_i, F_j) = \pi_i(\overline{s}_i, F_j) = \pi_i(\overline{s}_i)|_{\{p|p_i>p_j\}}$, so $\pi_i(\overline{s}_i)|_{\{p|p_i>p_j\}} = \pi_i^{minmax}$. □

Using the above obtained results we can state the next proposition, which determines the equilibrium profits of both firms depending on their capacities:

Proposition 4.2.3. *The equilibrium profits of the price-setting subgame $\Gamma(\tilde{q}, w)$ are uniquely determined and given below:*

i) If $(\tilde{q}_1, \tilde{q}_2) \in$ Region I, then:

$$\pi_i^* = (P(\tilde{q}_1 + \tilde{q}_2) - w_i)\tilde{q}_i, \tag{4.2.28}$$

ii) If $(\tilde{q}_1, \tilde{q}_2) \in$ Region II, then:

$$\pi_i^* = \max_{p \leq \underline{p}}(p - w_i)D(p), \tag{4.2.29}$$

iii) If $(\tilde{q}_1, \tilde{q}_2) \in$ Region III and $\underline{p}_1 \leq \underline{p}_2$, then:

$$\begin{cases} \pi_1^* = (\underline{p}_2 - w_1)\min(\tilde{q}_1, D(\underline{p}_2)) \\ \pi_2^* = \max_p(p_2 - w_2)\min(\tilde{q}_2, \max(0, D(p_2) - \tilde{q}_2)) = \pi_2^{minmax} \end{cases} \tag{4.2.30}$$

iv) If $(\tilde{q}_1, \tilde{q}_2) \in$ Region III and $\underline{p}_1 \geq \underline{p}_2$, then:

$$\begin{cases} \pi_1^* = \max_p(p_1 - w_1)\min(\tilde{q}_1, \max(0, D(p_1) - \tilde{q}_1)) = \pi_1^{minmax} \\ \pi_2^* = (\underline{p}_1 - w_2)\min(\tilde{q}_2, D(\underline{p}_1)) \end{cases} \tag{4.2.31}$$

[21] Terminology used by Osborne and Pitchik (1986).

Proof. We make a proof in several steps:

(a) Without loss of generality let us assume that $\underline{p}_1 < \underline{p}_2$ so that we are in the mixed-strategy Region III. As it was proved in Proposition 4.2.2, firm 2, the less efficient firm, lets itself be stochastically undercut and will set such price $\underline{p}_2$ to earn at least its minmax profit level π_2^{minmax}, described in equation (4.2.30).

Let p_1^m be the price that maximizes the profit of firm 1 and assume that $p_1^m > \underline{p}_1$, then using inequality $\underline{p}_2 > \underline{p}_1$ firm 1 can guarantee itself profit higher than its minmax profit: $\pi_1^m > \pi_1^{minmax}$ and will never set price below $\underline{p}_1$.

Meanwhile, the lowest equilibrium distribution extends to $\underline{p}_2$ and the profit of the most efficient firm, firm 1, depends on price $\underline{p}_2$.

(b) If $\pi_2^{minmax} > 0$ and $\underline{p}_2 > \underline{p}_1$, then firm 1 earns at least $\pi_1^*(\underline{p}_2)$.
But if $\pi_2^{minmax} = 0$, then according to equation (4.2.24) the profit of firm 1 is:

$$\pi_1^* = \max_{p \in [w_1, w_2]} [(p_1 - w_1) \min(D(p_1), \tilde{q}_1)]$$

(c) The symmetric results appear in the case with $\underline{p}_1 \geq \underline{p}_2$, so that firm 1, the less efficient firm, earns its minmax profit, described in (4.2.31). The upper level of the equilibrium distribution reaches price $\underline{p}_1$, so that equilibrium profit of firm 2 depends on price $\underline{p}_1$.

□

Proposition 4.2.3 shows that in order to determine the equilibrium profits one must know the values $\underline{p}_1$ and $\underline{p}_2$.

4.2.3.5 Results

Using the propositions of Kreps and Scheinkman (1983) we obtain the following results on the existence of the pure-strategy equilibrium for the case of asymmetric production costs:

Proposition 4.2.4. *Given the asymmetry in the unit production costs, the results of Kreps and Scheinkman, obtained for the pure-strategy Region I, hold. There is a unique Nash equilibrium in the price-setting subgame, with both firms name the market-clearing price $p_1 = p_2 = P(\tilde{q}_1 + \tilde{q}_2)$. Firms sell up to capacity and the profits have also exact Cournot form.*

Proof. In Region I $\tilde{q}_i \leq R_i(\tilde{q}_j)$ for $i = 1, 2$. Let us assume that $p_i < P(\tilde{q}_1 + \tilde{q}_2)$. In this case the profit of firm i is equal to $\pi_i = \tilde{q}_i(p_i - w_i)$. But if firm i charges price $p_i = P(\tilde{q}_1 + \tilde{q}_2)$ its profit is $\pi_i = (P(\tilde{q}_1 + \tilde{q}_2) - w_i)\tilde{q}_i$. So by price $p_i = P(\tilde{q}_1 + \tilde{q}_2)$ firm i is better off than by $p_i < P(\tilde{q}_1 + \tilde{q}_2)$. Hence for each firm i, $\underline{p}_i \geq P(\tilde{q}_1 + \tilde{q}_2)$, which corresponds with the result of Kreps and Scheinkman. In Region I capacities are so low, that both firms sell everything they can produce, so that there is a price that is adopted to clear the market. $p_i < p_j$ is not feasible, because the low-price firm will increase its price as long as it is capacity-constrained. So that

$$\underline{p}_i = \underline{p}_j \geq P(\tilde{q}_1 + \tilde{q}_2) \tag{4.2.32}$$

Let us consider the case when $\overline{p}_i = \overline{p}_j > P(\tilde{q}_1 + \tilde{q}_2)$ and, without loss of generality, assume that $\tilde{q}_1 \geq \tilde{q}_2$. If firm i charges price $\overline{p}_i - \epsilon$, where ϵ is positive and small, then firm i increases its profit and therefore will be better off. If ϵ is small, the loss caused by the price decrease will be also small. That means, that each firm is better off by decreasing its price by some small ϵ. Therefore, there is no equilibrium in such case. Thus

$$\overline{p}_i = \overline{p}_j \leq P(\tilde{q}_1 + \tilde{q}_2) \tag{4.2.33}$$

There are no incentives for both firms to name lower prices, because they will sell its full capacity at price $P(\tilde{q}_1 + \tilde{q}_2)$. From inequalities (4.2.32) and (4.2.33) we obtain $\underline{p}_i = \overline{p}_i = P(\tilde{q}_1 + \tilde{q}_2)$, so $p_1 = p_2 = P(\tilde{q}_1 + \tilde{q}_2)$. Consequently, if capacities of both downstream firms are equal to Cournot capacities that result in Cournot equilibrium price and in Cournot equilibrium profit. □

Proposition 4.2.5. *In the pure-strategy Region I, described in Definition 4.2.2, when firms have asymmetric unit production cost, the price charged by firm i is always $p_i \geq P(R_i(\tilde{q}_j) + \tilde{q}_j)$.*

Proof. As it has been already proved in the previous proposition when both capacities belong to Region I $p_i < p_j$ is not feasible, because in Region I capacities are so low, that both firms sell everything they can produce, so that there is a price that is adopted to clear the market. Assume that $p_i = p_j < P(R_i(\tilde{q}_j) + \tilde{q}_j)$, then the arbitrary firm i, $i = 1, 2$ can raise its price by ϵ. As both firms are capacity constrained, by naming a higher price p firm i would obtain the revenue $(D(p) - \tilde{q}_j)(p - w_i)$, so if $q_i = D(p) - \tilde{q}_j$ the profit of firm i is:

$$\pi_i = q_i \left[P(\tilde{q}_j + q_i) - w_i\right] \tag{4.2.34}$$

As Region I is characterized by $\tilde{q}_i \leq R_i(\tilde{q}_j)$, the following inequality will hold:

$$q_i \left[P(\tilde{q}_j + q_i) - w_i\right] \leq R_i(\tilde{q}_j) \left[P(R_i(\tilde{q}_j) + \tilde{q}_j) - w_i\right]$$

Equation (4.2.34) is maximized at $q_i = R_i(\tilde{q}_j)$. Therefore $p_i = P(R_i(\tilde{q}_j) + \tilde{q}_j)$. □

The next capacity area, Region II, is a Bertrand region, with price equal to marginal production cost for both firms. If one of the firms has a cost advantage, i.e. $w_1 < w_2$, then they both will charge price $p = w_2$. There is no equilibrium if both firms charge $p_1 > w_2$ and $p_2 > w_2$, because by reducing the price by small ϵ each firm will be better off than its rival. If firm 2 charges price $p_2 < w_2$ it will have negative profits. If firm 1 names price $p_1 \in [w_1, w_2)$ and if $D(p) \leq \tilde{q}_1$, it satisfies the whole demand and makes a non-negative profit. By $w_1 < w_2$ the firm 1's highest profit is $(w_2 - w_1)D(w_2)$. The equilibrium over the Region II is unique only if $w_1 = w_2$.

Proposition 4.2.6 summarizes the results on the existence of the pure-strategy equilibrium in Regions I and II.

Proposition 4.2.6. *Given the asymmetry in the unit production costs, $w_1 < w_2$, and Assumptions 9, 10 on demand, a pure-strategy equilibrium in the price-setting subgame exists only in capacity space Regions I and II, moreover:*

i) If both capacities lie in Region I, then in a pure-strategy equilibrium, both firms name price $p_1 = p_2 = P(\tilde{q_1} + \tilde{q_2})$.

ii) If both capacities lie in Region II, then in any equilibrium the prices set by both firms depend on the profit-maximizing price of the most efficient firm (of firm 1), denoted as p_1^{max}, moreover:

- *If the profit-maximizing price of the most efficient firm is equal to or less than the unit production costs of its rival, s.t. ($p_1^{max} \leq w_2$), then in any equilibrium firm 1 gets its highest profit $\pi_1^{max} = (p_1^{max} - w_1)\min(D(p_1^{max}), \tilde{q_1})$ by setting its price at level $p_1 = p_1^{max}$ and profit of firm 2 is $\pi_2 = (p_2 - w_2)D(p_2)$, where $p_2 \geq w_2$.*
- *If the profit-maximizing price of the most efficient firm is larger than the unit production costs of its rival, s.t. ($p_1^{max} > w_2$), then in any equilibrium both firms name prices $p_1 = p_2 = w_2$, with resulting profits $\pi_1 = (w_2 - w_1)\min(D(w_2), \tilde{q_1})$ and $\pi_2 = 0$.*

Proof. i) In Region I there occurs a pure-strategy equilibrium with both firms name prices $p_i = p_j = P(\tilde{q_1} + \tilde{q_2})$, which has been already proved in Proposition 4.2.4.

ii) In Region II, described in Definition 4.2.2, $\underline{p} = \max\{w_1, w_2\}$. As we assumed, that $w_1 < w_2$, then $\underline{p}_2 = w_2$ and $\underline{p}_1 \leq w_2$, and according to equation (4.2.24), the profit of firm 1, if $p_1 \leq w_2 = p_2$, is $\pi_1 = (p_1 - w_1)\min(D(p_1), \tilde{q_1})$.

As p_1^{max} is the price, that maximizes the profit of firm 1, we consider two cases:

a) $w_1 \leq p_1^{max} \leq w_2$. Then firm 1 can achieve the highest possible profit, if it sells first, $\pi_1^* = (w_2 - w_1)D(w_2)$ by setting the price p_1^{max}. If the capacity of firm 1 is not high enough to cover the whole market demand, $\tilde{q_1} < D(w_2)$, it will sell everything up to its capacity and then consumers will buy by firm 2. In such case firm 2 may set a price $p_2 \geq w_2$ to make at least non-negative profit.

b) $p_1^{max} > w_2$. In this case both firms will charge price $p_1 = p_2 = w_2$. There will be no equilibrium, if they both set prices $p_1 > w_2$ and $p_2 > w_2$, because by reducing the price by some small ϵ, each of them will be better off than its rival.

Remark 3.
- *There cannot be an equilibrium in which both firms set p_1 and p_2 strictly above w_2;*
- *Firm 2 does not charge less than w_2 not to make a negative profit;*
- *If firm 1 charges a price $p_1 = w_2 - \epsilon$, where ϵ is small and positive, it could guarantee itself the profit closed to the highest.*

It is also necessary to consider the case whether a pure-strategy equilibrium exists outside of Region $I \cup II$ with $p_1 = p_2$. Assume that there exists a pure-strategy equilibrium outside of Regions I and II, so that $(\tilde{q}_1, \tilde{q}_2) \notin (I \cup II)$. As it has been proved by Kreps and Scheinkman, in the mixed-strategy Region III, where $\tilde{q}_i > R_i(\tilde{q}_j)$ for at least one firm i, the highest capacity firm makes a profit equal to its "Stackelberg follower profit:"[22] $\pi_i = R_i(\tilde{q}_j)P(R_i(\tilde{q}_j) + \tilde{q}_j)$, so that

$$\underline{p}_i > P(\tilde{q}_1 + \tilde{q}_2) \tag{4.2.35}$$

The region, which lies outside of Region II is characterized by $\tilde{q}_1 < D(w_2)$ or by $\tilde{q}_1 \geq D(w_2)$. The first case, where $\tilde{q}_1 < D(w_2)$ means, that firm 1 cannot satisfy the whole demand and so firm 2 will cover the rest, so that it could set the price $\underline{p}_2 \geq w_2$ as we described earlier.

The second case, where $\tilde{q}_1 \geq D(w_2)$ means, that capacity of firm 1 is enough to cover the whole demand and under the previous assumption that the low-cost firm will sell its capacity first, firm 1 could set price $\underline{p}_1 \geq w_2$. Summarizing, we get the following inequalities:

$$\begin{cases} \underline{p}_i > P(\tilde{q}_1 + \tilde{q}_2) \\ \underline{p}_2 \geq w_2 \\ \underline{p}_1 \geq w_2 \end{cases}$$

From these inequalities follows that $p_i \geq \max\{P(\tilde{q}_1 + \tilde{q}_2), w_2\}$, $i = 1, 2$. But in such case there will be also no equilibrium, because either firm has an incentive to decrease its price by some small ϵ and as a result to be better off. $\square$

4.3 Conclusion

In this chapter we have considered the behavior of two price competitors in the downstream market, where they both have simultaneously bargained with an upstream supplier over the wholesale prices for some particular output quantities, which in this chapter we identify with capacities; then firms have brought these capacities to the local market.

The aim of the analysis is to determine the final prices that both downstream firms should offer to the end consumers in order to earn equilibrium profits. Therefore we have turned to the paper of Kreps and Scheinkman (1983) which is the most relevant to the presented work. Discussing the situation when both capacity and production are costly, they assume, that their analysis could be easily modified to show, that the unique outcome will be Cournot outcome, computed by using the sum of two cost functions. Deneckere and Kovenock (1996) consider the case when both capacity and production are costly at the same time and they show that the results of Kreps and Scheinkman do not hold at the price competition stage.

Similar to Kreps and Scheinkman and Deneckere and Kovenock we have examined the model of capacity choice followed by Bertrand-Edgeworth price competition, but separately for the cases of costly capacity costs and production costs and have come to the following results, which differ from the case of identical costs:

[22] The summarizing result of Kreps and Scheinkman for an equilibrium in mixed strategies.

- The results of Kreps and Scheinkman continue to hold in case of asymmetric capacity costs. In such case, in pure-strategy equilibrium both firms choose their Cournot capacities and name the market-clearing price $p_1 = p_2 = P(\tilde{q}_1 + \tilde{q}_2)$.

- In the case when firms have asymmetric production costs, in the price competition stage the results of Kreps and Scheinkman do not hold anymore. If the asymmetry is sufficiently high, the more efficient firm has an incentive to choose the capacity above its Cournot level, as it was shown in the classical Bertrand Region II, even if the less efficient firm chooses its Cournot quantity. The most efficient firm, which has low costs will choose high capacity level and price its less efficient rival out of the market in the subsequent price competition subgame.

Chapter 5

Dynamic duopoly with sticky prices and asymmetric production costs

5.1 Introduction

5.1.1 Motivation

In Chapter 2 we have worked out the theoretical model which represents a non-cooperative bargaining between the supplier and two downstream firms over the wholesale prices, where results from the two-person cooperative axiomatic bargaining games are used to define the payoffs of some of the terminal nodes of the extensive game. Important assumption is that one downstream firm can integrate backwards (to produce the input by itself instead of buying it from the supplier) and other firm cannot. The role of this assumption seems to be crucial for the determination of the firms' actions depending on the obtained price offers.

Extending the model by the assumption that both downstream firms exercise the bargaining power in negotiations with the upstream supplier which is justified by the existence of the backward integration option, it has been shown that the asymmetry in the bargaining weights of both firms leads to the asymmetry in their wholesale prices and finally in concentration ratios in the local market.

In Chapter 4 we have described the behavior of both downstream firms under the price competition. We have determined the final prices that both downstream firms should offer to the end consumers in order to earn equilibrium profits. We have extended the model of Kreps and Scheinkman (1983) to the cases of asymmetric capacity costs and asymmetric production costs.

The current chapter introduces the alternative approach to the already presented work. The outgoing point is the model described in Chapter 2, but we additionally assume that by entering the local market both downstream firms face sticky market prices and we consider the dynamic duopoly model. The aim of the current analysis is to investigate how the changes in the price stickiness influence the equilibrium by open-loop, feedback and closed-loop solutions and to compare the obtained results with the static model of Cournot and perfect competition which were introduced in the previous chapters.

5.1.2 Literature overview and open questions

Nowdays there is an increasing interest in economics to solve problems using a dynamic game theoretical setting; especially in the environmental economics it is a very popular framework, for example, de Zeeuw and van der Ploeg (1991), Kaitala, Pohjola and Tahvonen (1992), Tabellini (2001), Fershtman and Kamien (1987), Petit (1989), Levine and Brociner (1994), van Aarle, Bovenberg and Raith (1995), Douven and Engwerda (1995), van Aarle, Engwerda and Plasmans (2002).

There have been also a number of studies using the differential game theory to analyze dynamic duopoly and oligopoly in a single market. Simaan and Takayama (1978) present an application of differential game theory in the area of microeconomics. They study a dynamic duopoly with sticky prices where two firms are limited by a maximum production capacity, share the same market and try simultaneously but independently to maximize their profits over a certain planning horizon. While the static duopoly theory does not address itself to the question of the process in which changes in the price are brought about, but only compares the prices before and after the change takes place, they use the dynamic market theory, which allows the analysis of how the price changes with time and what trajectory it follows. Simaan and Takayama derive the necessary conditions for a closed-loop Nash equilibrium solution and also more specific results for the special case of linear demand and quadratic cost functions.

The starting point of our research is the paper of Fershtman and Kamien (1987), who study an infinite-horizon model of dynamic duopolistic competition under the assumption that current price does not necessarily adjust instantaneously to its level on a static demand function for that output. They assume that the evolution of price over time is governed by a kinematic equation that specifies, for every given level of output, its change as a function of the gap between its current level and the price indicated by a static demand function. The main objective of their paper is to investigate the relationship between the speed at which the price converges to its value on the static demand function and the resultant stationary subgame perfect Markov equilibrium price. Fershtman and Kamien (1987) study the interactions between symmetric duopolists under open-loop and feedback (subgame perfect) information structures. They demonstrate that the static Cournot equilibrium price is the limit of the open-loop Nash equilibrium price when the speed of adjustment reaches infinity. This is not true for the feedback Nash equilibrium price. In that case the Nash equilibrium price converges to a value below the static Cournot price.

In their next paper Fershtman and Kamien (1990) present a complete analysis of the feedback Nash equilibrium of a finite-horizon linear quadratic differential game with a control constraint. They demonstrate the relationship between the "turnpike properties" of the finite-horizon equilibrium strategies and the infinite-horizon equilibrium strategies. They also analyze a finite-horizon differential game model of duopolistic competition through time under the assumption that prices do not adjust immediately to their level on the demand function for each level of output.

The infinite-horizon dynamic duopoly model of Fershtman and Kamien (1987) was generated by Dockner (1988) as an N-person nonzero-sum differential game. He added to the cases treated in Fershtman and Kamien the one where the oligopolists played under closed-loop but not subgame perfect information structure. His main interest was on the relationship between the static competitive price and the price determined as an asymptotic limit of oligopolistic competition over time when the number of firms went to infinity. It is demonstrated that regardless of the strategy spaces assumed (i.e., open-loop, feedback or closed-loop information structure) the dynamic oligopoly price converges to the long-run competitive price (the one which is equal to the minimum of average costs) as the number of firms goes to infinity. His result allows an interpretation of the long-run competitive equilibrium as the limit of dynamic oligopolistic competition and it is robust with respect to the information structure assumed in the oligopoly case.

In his later paper Dockner (1992) explores the relationship between dynamic oligopolistic competition and static conjectural variations equilibria. The capacity adjustment cost model is used to study dynamic oligopolistic quantity competition over an infinite horizon. It is shown that the stationary closed-loop equilibrium of the dynamic capacity adjustment game coincides with static conjectural variations equilibrium. Under the assumptions that the stationary state closed-loop equilibrium is stable, conjectural variations equilibrium is interpreted as the outcome of dynamic strategic interactions.

In the empirical literature the model specification problem is solved using among others the adoption of the general models nesting a broad set of potential behavioral models. The analysis of the rice export market by Karp and Perloff (1989) is an example of such approach. It nests four models: collusion, price taking, Nash-Cournot open-loop, and Nash-Cournot with feedback. The use of nesting models does not solve the model specification problem, since no framework may nest all possible alternatives. Therefore the researchers must choose among alternative "families" of models. Moreover, the solution of the model may be quite complicated and may require the introduction of simplifying assumptions in order to obtain an explicit form of the structural equations for the econometric model. For example, Karp and Perloff (1989) estimate the degree of competition among rice exporters using a linear-quadratic dynamic oligopoly model in the open-loop and closed-loop strategy spaces. They assume that China, Thailand and Pakistan are either acting as price-takers, collusive, or Cornot-Nash game in their rice export. They consider only some large exporting or importing countries and exclude other small trading countries. However, in the international rice market, a small trading country may produce a large quantity of rice in domestic production. These small trading countries may have a potential to compete with the major trading countries by changing supply and demand flow pattern. Estimating the degree of competition among the exporters the authors find that the rice export market is close to price taking but with some degree of imperfect competition.

Reinganum and Stokey (1985) study oligopolistic resource extraction and demonstrate the importance of the period of commitment in the choice of model strategies, path or decision rule. Applying optimal control theory on a common property aquifer model, they analyze both open-loop and feedback equilibria. They find that feedback equilibria lead to a more rapid extraction level compared to the open-loop equilibria.

Vives and Jun (2004) compare steady states of open-loop and locally stable Markov perfect equilibria (MPE) in a general symmetric differential game duopoly model with costs of adjustment. They show that the strategic incentives at the MPE depend on whether an increase in the state variable of a firm hurts or helps the rival and on whether at the MPE there is intertemporal strategic substitutability or complementarity. They also provide a full characterization in the linear-quadratic case. They ascertain that with price competition and costly production adjustment, static strategic complementarity turns into intertemporal strategic substitutability and the MPE steady-state outcome is more competitive than static Bertrand competition.

Differential games belong to the subclass of dynamic games called state space games, in which modeler introduces a set of state variables to describe the state of a dynamic system at any time during play. The hypothesis is that the payoff-relevant influence of past events is adequately summarized in the state variables. The choice between discrete and continuous time often seems to be quite arbitrary. Discrete time models involve the assumption that no decisions are made between the time instants that define the periods. Differences between discrete-time and continuous time models and open-loop and feedback models are well demonstrated by Karp and Perloff (1993). They develop a feedback, oligopolistic dynamic game model that can be used to estimate the degree of market power in markets with nonlinear adjustment costs in output, investment, or prices. Their model nests various well-known market structures in a larger family. It also provides a simple method for comparing open-loop and feedback equilibrium for a given degree of market power. This model can be used to analyze relatively easy the effects of increasing the number of firms and the costs of adjustment on the equilibrium trajectory and steady state.

All the dynamic game models reviewed above are for a single market and are simplified in order to obtain analytical or closed-form results. For example, demand functions are assumed to be linear and cost functions are assumed to be quadratic and symmetric. While these dynamic models serve to provide general qualitative results, they do not provide the tools to study particular situations where it is too complex to obtain analytical solutions. Moreover, the existing dynamic oligopoly models do not consider the role of transportation costs as well as the different demand functions or the different production costs at various market points. In this regard, our model may be considered as a starting point for a more rigorous extension of the existing dynamic oligopoly models and is an algorithm for studying more complex market situations. In particular, we solve open-loop, feedback (subgame perfect) and closed-loop decision problems.

5.1.3 Modeling strategy

Obviously, many strategic problems in economics are not properly modeled as static games since firms can make decisions more than at one point of time. For example in the Cournot duopoly game, firms need to choose their output levels independently at each point of time T. But which planning horizon should the firm choose is still restrictive in the literature.[1] Apart from the computational point of view that the equilibrium actions in an infinite-planning horizon are much easier to implement and to analyze than those for a finite-planning horizon, there is also at least another reason from economics to consider an infinite-planning horizon. In economic growth theory it is usually difficult to justify the assumption that a firm or government has a finite-planning horizon T, why should it ignore profits earned after T, or utility of generations alive beyond T.

In this chapter we study duopolistic competition between two firms, which produce the homogeneous goods through the time. The main objective is to analyze the intertemporal interaction between both duopolists under the assumption that the price of the product does not adjust instantaneously to the price indicated by its demand function by the given output. For such analysis we use a differential game framework and determine the open-loop, feedback and closed-loop Nash equilibria of the described duopoly game.

In the existing literature on the differential games with sticky prices most authors use the assumption provided first by Fershtman and Kamien (1987) that both firms have identical production costs. Fershtman and Kamien consequently prove that in such case the equilibrium quantities by open-loop and feedback solutions for both firms will be symmetric. Differently from the existing literature we consider the case of asymmetric production costs by both firms and investigate the influence of the speed adjustment parameter as well as the discount rate on the obtained best-response quantities. The main assumptions on the considered model are that the demand is a linear function of price, total cost is a quadratic function of output and there is no uncertainty in the model.

Our analysis starts with a brief review of the differential game approach and follows with the introduction of the model, which will be studied by us, applying open-loop, feedback and closed-loop Nash equilibrium solutions. We add to the cases presented in Fershtman and Kamien the one where the oligopolists play under closed-loop but not subgame perfect information structure. We also give economic interpretations to the use of a particular information concept. At the same time we analyze the impact of the speed adjustment parameter and the discount rate. Finally, we follow the main object of this chapter and compare the obtained results with the static model of Cournot and perfect competition analyzed in the previous chapters and find the conditions under which the obtained equilibrium prices and quantities in both dynamic and static models coincide.

5.2 Basic concepts

5.2.1 On the dynamic game theory

Dynamic game theory brings together four key features in economics; they are: optimizing behavior, the presence of multiple agents/players, enduring consequences of decisions and robustness with respect to variability in the environment. Dealing with problems, which have these four features, the dynamic game theory splits the

[1] Engwerda (2005).

modeling of the problem into three parts.

One part is the modeling of the environment in which the agents act. To obtain a mathematical model of the agents' environment one usually specifies a set of differential or difference equations. These equations are assumed to capture the main dynamical features of the environment. A characteristic property of this specification is that these dynamic equations mostly contain a set of so-called "input" functions. These input functions model the effect of the actions taken by the agents on the environment during the course of the game.[2] A second part is the modeling of the agents' objectives. Usually they are formalized as cost functions which have to be minimized. Since this minimization has to be performed subject to the specified dynamic model of the environment, techniques developed in optimal control theory play an important role in solving dynamic games, the theory of which arose from the merging of static game theory and optimal control theory. The third modeling part describes further reflection of such merging.

Most research is concentrated on the field of static game theory, where all possible sequences of decisions of each player are set out against each other. Characteristic for such game is that it takes place in one moment of time: all players make their choices once and simultaneously, and subject to the choices made, each player receives his payoff. In such formulation the important issues like the order of play in the decision process, information available to the players at the time of their decisions, and the evolution of the game are suppressed, and this is the reason why this branch of game theory is usually classified as "static". In case the agents act in a dynamic environment these issues are, however, crucial and need to be properly specified before one can infer what the outcome of the game will be. This specification is the third modeling part that characterizes the methodology of dynamic game theory.

5.2.2 On the differential game

A differential game is a dynamic game, which is played in continuous time. A linear quadratic differential game studies situations involving two or more decision makers, called the players in the game. These players often have partly conflicting interests and make individual or collective decisions. In a linear differential game the basic assumption is that all players can influence a number of variables which are crucial in realizing their goals and that these variables change over time due to external forces. These variables are called the state variables of the system. It is assumed that the movement over time of these state variables can be described by a set of linear differential equations in which the direct impact of the players' actions is in an additive linear way.

Consequently, the extent to which the players succeed in realizing their goals depends on the actions of other players. Obviously, if one player has information on the action that other player will take, he can incorporate this information into the decision making about his own action. Therefore, information plays a crucial role in the design of optimal actions for the players. So, summarizing, to model the differential game it is necessary to introduce a set of variables to characterize the state of the dynamical system at any instant of time during the game, as well as the

[2] More in Engwerda (2005).

evolution of state variables over time, which are described by the set of differential equations.

5.3 The model

5.3.1 Basic notations and assumptions

Our analysis is based on the model developed in the previous chapters, but we extend it with some additional assumptions which are necessary in differential games' context.

Hence, according to the model mentioned above, two downstream firms have already negotiated their wholesale prices denoted as w_1 and w_2 for Cournot output quantities denoted as q_1^{*C} and q_2^{*C} with the upstream supplier; the upstream supplier had already produced these quantities and both firms bought them.

Now in the context of the dynamic system we consider both firms as players, who play over time t, where $t \in [0, \infty)$. We define $p_i(t)$ as a state variable and $q_i(t)$ as control variable for each player i.

As before whenever i and j appear in the same expression, it means that $i, j \in \{1, 2\}$ and $i \neq j$.

On this stage let us make the following assumptions to describe the simple optimal control problem:[3]

Assumption 15. *There exists set Q such that*

$$q(t) \in Q(p(t), t), \forall t. \tag{5.3.1}$$

Assumption 16. *There exists set P such that $p(t) \in P$, $\forall t$.*

Assumption 17. *The initial value of the state variables is known:*
$p(0) = (p_1(0), p_2(0)) = p_0$

$$p(0) = p_0 \in P \tag{5.3.2}$$

The dynamics of the state variable is described by the following equation:

$$\frac{\partial p_i(t)}{\partial t} \equiv \dot{p_i}(t) = f_i(p(t), q(t), t) \tag{5.3.3}$$

where $p(t)$ is a vector of state variables at time t, $p(t) = (p_1(t), p_2(t))$ and $q(t)$ is a vector of control variables at the same time t (players actions), $q(t) = (q_1(t), q_2(t))$.

The obtained equations (5.3.1) - (5.3.3) are the constraints of the optimal control problem.

The main object of each player is to choose the control path q in an optimal way. In other words to maximize the instantaneous payoff function over time, which

[3]More in Dockner (2000).

depends not only on the choice of the player but also on his rival's choice of action:

$$F_i \equiv F_i(p(t), q(t), t) \tag{5.3.4}$$

Under the above assumptions we derive the first-order necessary conditions for the basic dynamic optimization problem to find a control function $q_i(\cdot)$ that maximizes the payoff function of firm i over time:[4]

$$\max_{q_i(\cdot)} J_i(q_i(\cdot)) \equiv \int_0^\infty e^{-rt} F_i(p(t), q(t), t)dt, \ r \geq 0, \tag{5.3.5}$$

where $r \geq 0$ denotes the discount rate of future profits.
Subject to:
$\dot{p_i}(t) = f_i(p(t), q(t), t)$
$q_i(t) \in Q_i(p(t), t)$
$p_i(0) = p_0 \in P$

Next let us introduce the Hamiltonian function H:

Definition 5.3.1. *The Hamiltonian function of the dynamic optimization problem is defined as*

$$H(p, q, \lambda, t) = F(p, q, t) + \lambda(t)f(p, q, t), \tag{5.3.6}$$

where $\lambda(t)$ is a co-state variable and is often interpreted as a "shadow prices" of states and F is an instantaneous payoff function.

Theorem 5.3.1. *Let (p^*, q^*) which are almost everywhere continuously differentiable be an optimal solution of the dynamic optimization, then there exists a continuous and piecewise continuously differentiable function $\lambda(\cdot) : [0, T] \to \Re^n$ satisfying:*

$$H(p^*(t), q^*(t), \lambda(t), t) = \max_{q \in Q(p^*(t))} H(p^*(t), q, \lambda(t), t), \ \forall \ t \tag{5.3.7}$$

and at each point where $Q(\cdot)$ is continuous:

$$\dot{\lambda}(t) = r\lambda(t) - \frac{\partial H(p^*, q^*, \lambda, t)}{\partial p} \tag{5.3.8}$$

Now let us introduce the main assumption for this chapter:

Assumption 18. *The market price is sticky.*

Sticky is a term used in economics to describe a situation in which a variable is resistant to change. Wages and prices can be sticky. For example, in the absence of competition, firms rarely lower prices, even when production costs decrease (i.e. supply increases) or demand drops. Instead, when production becomes cheaper, firms take the difference as profit, and when demand decreases they are more likely to hold prices constant, while cutting production, than to lower them. Therefore, prices are sometimes observed to be sticky downward. Prices in an oligopoly can

[4] More precisely about control theoretic methods by infinite time horizon in Chapter 3 of Dockner et al. (2000).

often be considered as sticky-upward. The kinked demand curve, resulting in elastic price elasticity of demand above the current market clearing price, and inelasticity below it, requires firms to match price reductions of their competitors to maintain market share.

Economists have tried to model sticky prices in different ways. Models with sticky prices can be classified as either time-dependent, where firms change prices with the passage of time and decide to change prices independently of the economic environment, or state-dependent, where firms decide to change prices in response to changes in the economic environment. The differences can be thought as differences in a two-stage process: in time-dependent models, firms decide to change prices and then evaluate market conditions; in state-dependent models, firms evaluate market conditions and then decide how to respond. Section 5.9 explains more precisely the role of the price stickiness for our whole analysis.

5.3.2 Modeling approach

In the previous section we have introduced all important functions in order to describe the dynamic system. Let us turn to our model and assume that demand is linear in price and the game is played over an infinite interval of time. We denote the output of each firm i as $q_i \geq 0$, $i = 1, 2$.

In the spirit of Fershtman and Kamien (1987)[5] we define the cost functions as quadratic in output, but allow them to be asymmetric for both downstream firms:

$$C(q_i(t)) = w_i q_i(t) + \frac{1}{2} q_i^2(t), \tag{5.3.9}$$

where $w_i \in (0, a)$ is the wholesale price that downstream firm i pays to the supplier, it is a fixed value that was negotiated as shown in Chapter 2 and $q_i(t)$ is the output rate of firm i at time t.

In the analysis of the static model the commodity price $p(t)$ in period t is related to the industry output by continuous and differentiable inverse demand function, which has the following linear form:

$$p(t) = a - b(q_1(t) + q_2(t)) \quad a, b > 0 \tag{5.3.10}$$

Let $\tilde{p}(t)$ denote the price indicated by the inverse demand function for the given level of output:

$$\tilde{p}(t) = a - (q_1(t) + q_2(t)),^{6} \; a > 0 \text{ (const.)} \tag{5.3.11}$$

Additionally we denote $p(t)$ as the current market price. Assuming that there is a price stickiness in the model, the price $\tilde{p}(t)$ will be different from the current price level $p(t)$. The key feature of the price stickiness is that market price does not adjust instantaneously to the price indicated by the demand function.[7] Using the model of Simaan and Takayama (1978) and Fershtman and Kamien (1987) we present the

[5]See also Dockner et al (2000), pp. 267-273.

[6]b=1 is a usual assumption made to simplify the analysis.

[7]This is in contrast to the standard Cournot model where the market price adjusts instantaneously.

change in price by the differential equation shown below, where the evolution of the market price over time is a function of the difference between the current market price and the price specified by the demand function for each level of industry output.

$$\dot{p}(t) = \frac{\partial p(t)}{\partial t} = s[\tilde{p}(t) - p(t)] = s[a - q_1(t) - q_2(t) - p(t)], \qquad (5.3.12)$$
$$p(0) = p_0$$

where $\dot{p}(t)$ is a state variable and $s \in (0, \infty)$ denotes the speed in which price converges to its level on the demand function. For larger values of s the market price adjusts along the demand function more quickly. In the limiting case $s = \infty$ static demand and dynamic demand coincide and yield the same price, i.e, $\lim_{s\to\infty} p(t) = a - q_1(t) - q_2(t)$.

Simaan and Takayama (1978) show that the dynamic demand function shown in (5.3.12) has the same properties (locally) as the static one, i.e., an increase (decrease) in total market supply causes a decrease (increase) in the market price of the commodity.

The instantaneous payoff function of firm i at time t is:

$$F_i(t) = q_i(t)[p(t) - w_i - \frac{1}{2}q_i(t)] \qquad (5.3.13)$$

So under the above assumptions, if $t \in [0, \infty]$, the objective of each firm i is to maximize:

$$J_i(q_i, q_j) = \int_0^\infty e^{-rt}\left[p(t)q_i(t) - w_i q_i(t) - \frac{1}{2}q_i^2(t)\right] dt, \; r > 0, \; i = 1, 2 \qquad (5.3.14)$$

subject to:
$\dot{p}(t) = s[\tilde{p}(t) - p(t)] = s[a - q_1(t) - q_2(t) - p(t)]$
$p(0) = p_0$
$q_i(t) \geq 0$

Hence, each firm i faces the following maximization problem:

$$\max_{q_i(t)} J_i(q_i, q_j) = \int_0^\infty e^{-rt} q_i(t)\left[p(t) - w_i - \frac{1}{2}q_i(t)\right] dt, \qquad (5.3.15)$$

subject to:
$\dot{p}(t) = s[\tilde{p}(t) - p(t)]$
$p(0) = p_0$
$p(t) \geq 0$

In order to investigate the role of the speed of adjustment s, Fershtman and Kamien (1987) solve (5.3.12) for $p(t)$, substitute it into (5.3.14) and finally get the following equation:

$$J_i(q_i, q_j) = \int_0^\infty e^{-rt}\left\{[a - q_1(t) - q_2(t)]\, q_i(t) - \frac{\dot{p}(t)q_i(t)}{s} - w_i q_i(t) - \frac{1}{2}q_i(t)^2\right\} dt \qquad (5.3.16)$$

From equation (5.3.16) it is obvious that firms face a downward sloping linear inverse demand function but the decline in price along it, as a firms' level of output increases, is retarded when s is finite. But if $s \to \infty$, the term $\frac{\dot{p}(t)q_i(t)}{s}$ vanishes and price adjusts instantaneously along the demand function.[8]

There are two major strategies in the literature which are appropriate to solve the problems given in (5.3.15). These are open-loop and feedback strategies, which will be presented in the next sections. As we have assumed the linear quadratic structure at the beginning of our analysis, it will be possible to obtain explicit analytical results for both kinds of equilibrium strategies.

These strategies in the form of selecting the control variables q_1 and q_2 are most commonly employed in the application of the theory of differential games. Open-loop and feedback are terms which are used to distinguish between two different information structures in games. Using the feedback strategies players can run their play at time t relying on the history of play until that date, while open-loop strategies are functions of the calendar time alone.

In order to choose the appropriate strategy one must determine the information structure of the game. If the players never observe any other history than their own, and they are not able to revise their strategies at any subsequent point in time all strategies in such case are open-loop strategies and Nash equilibrium is in open-loop strategy. Equilibrium in open-loop strategy is called open-loop equilibrium. The Nash equilibrium open-loop strategies are relatively easy to determine as they involve a straightforward application of the standard optimal control methods. The choice of the equilibrium strategy will be discussed in Section 5.8.

5.4 Equilibria in static games

5.4.1 Preliminaries

Differently from the problems with a single decision maker there is no unique solution concept for nonzero-sum differential games. One distinguishes, for example, Nash and Stackelberg equilibrium concepts, depending on the players' strategic interactions. The information structure is very important by choosing the solution concept in the oligopoly theory. In the context of our game, the choice of information structure is more related to the concept of strategy spaces and differs from what is known as the economics of information.

In the previous section we have already described the maximization problem which is needed to be solved. We have also briefly discussed two strategies that are mostly used in the literature to solve such problems. In order to analyze and compare the results of the dynamic game which we shall obtain in the next sections with the results of the static game, let us consider the last one more precisely.

On this stage let us turn to the static game of duopolistic competition. As a point of reference we first compute the static Cournot oligopoly price and quantities and then the static equilibrium quantities and price by perfect competition specified for the cost and demand functions given in equations (5.3.9) and (5.3.11). For

[8] More precisely in Fershtman and Kamien (1987), p. 1153.

our forthcoming analysis it is important to distinguish the cases of symmetric and asymmetric production costs.

5.4.2 Symmetric production costs

Now let us consider the case of symmetric production costs $w_i = w_j$.

- By Cournot competition with N firms there are the following equilibrium quantities and price:

$$q^{*C} = \frac{a-w}{N+2}, \quad p^{*C} = \frac{2a+Nw}{N+2} \tag{5.4.1}$$

  For $N = 2$:

$$q^{*C} = \frac{a-w}{4}, \quad p^{*C} = \frac{a+w}{2} \tag{5.4.2}$$

- By perfect competition with N firms there are the following equilibrium price and quantities:

$$p^{*perf} = \frac{a+Nw}{N+1}, \quad q^{*perf} = \frac{a-w}{N+1} \tag{5.4.3}$$

The market price and each firm's profit decrease with the number of firms. Furthermore, since the market price decreases with N, so does the aggregate profit. Indeed, when the number of firms becomes very large ($N \to \infty$), the market price tends to the competitive price w. Thus, Cournot equilibrium with large number of firms is approximately competitive. This is natural, because each firm has only a small influence on the price and thus acts almost like a price-taker.

If $N = 2$ and both firms act like price-takers there are the following equilibrium price and quantities:

$$p^{*perf} = \frac{a+2w}{3}, \quad q^{*perf} = \frac{a-w}{3} \tag{5.4.4}$$

5.4.3 Asymmetric production costs

In the case of asymmetric production costs $w_i \neq w_j$ with $N = 2$ we have the following equilibria

- By Cournot competition:

$$q_i^{*C} = \frac{2a+w_j-3w_i}{8}, \quad p^{*C} = \frac{2a+w_i+w_j}{4} \tag{5.4.5}$$

- By perfect competition:

$$q_i^{*perf} = \frac{a+w_j-2w_i}{3}, \quad p^{*perf} = \frac{a+w_i+w_j}{3} \tag{5.4.6}$$

These results are important for our future analysis, as we shall compare them with the results obtained from the analysis of the dynamic game.

5.5 Strategy spaces in differential game theory

On this stage let us make some useful definitions of the strategy spaces for the forthcoming analysis of the differential game defined below. The definitions are partly taken from Fershtman and Kamien (1987) and Dockner et al (2000). Also Basar and Olsder (1982) provide a detailed discussion of strategy spaces that are frequently employed in economic applications of differential game theory.

For the rest analysis the superscript OL indicates the open-loop Nash equilibrium, F indicates the feedback and CL the closed-loop Nash equilibrium of the relevant variables.

1) **Open-loop strategy space**

Definition 5.5.1. *The open-loop strategy space for player i is given as*

$$S_i^{OL} = \{q_i(t, p_0) | q_i(t, p_0) \textit{ is a piecewise continuous function of time } t \textit{ for all } t \in [0, \infty)\}.$$

The open-loop strategies can be characterized as path strategies. Each player chooses a path of action $q_i(t, p_0)$ to which he commits himself at the outset of the game. Nash equilibrium in such strategies is a pair of paths, such as each player's path is the best response to its rival's path.

Definition 5.5.2. *An open-loop Nash equilibrium for the above game is a pair of open-loop strategies $(q_1^{*OL}, q_2^{*OL}) \in S_1^{OL} \times S_2^{OL}$ such as for every $q_i \in S_i^{OL}$:*

$$J_1(q_1^{*OL}, q_2^{*OL}) \geq J_1(q_1, q_2^{*OL}) \textit{ and } J_2(q_1^{*OL}, q_2^{*OL}) \geq J_2(q_1^{*OL}, q_2) \tag{5.5.1}$$

Each firm i maximizes $J_i(q_i, q_j)$ in equation (5.3.14), given $q_j(t)$. Hence, the equilibrium in the market is a pair of the open-loop strategies, which simultaneously solve two optimization problems - for player i and j.

It is well known that a Cournot-Nash equilibrium in open-loop strategies may be dynamically inconsistent if one or several firms deviate from their equilibrium solution for some time.[9] To avoid the possibility of dynamic inconsistency, a differential game in the open-loop strategy space requires that the firms are obliged to use their original path strategies over the entire planning horizon. However, this behavioral restriction may give rise to an unrealistic representation of oligopolistic competition in some industries.

2) **Feedback and closed-loop strategy spaces**

The class of strategies in which the control depends on the initial condition as well as the state and time is called closed-loop.[10]

In economics a feedback Nash equilibrium is far more interesting and important than an open-loop Nash equilibrium, since the latter requires precommitment, while

[9] More precisely in Reinganum and Stokey (1985).

[10] For a discussion see Basar and Olsder (1982) or Mehlmann (1988).

the former does not. In differential games, assuming the existence and differentiability of value functions, the derivation of a feedback equilibrium is facilitated by solving the Hamilton-Jacobi equation with respect to the value functions. From a practical point of view in many differential games it is very difficult to solve the Hamilton-Jacobi equation, because, in general, it consists of a system of simultaneous partial differential equations. As a result, feedback equilibria are primarily obtained in a special class of differential games so that one can "guess" value functions and feedback strategies that solve the equation.[11]

It is well known that a Cournot-Nash equilibrium in feedback strategies is dynamically consistent even if one or several firms do not use their equilibrium solution for some time. A feedback Cournot-Nash equilibrium is said to be subgame perfect because the decision rule of each firm is the optimal response to the decision rules selected by the other players, when it is viewed from any intermediate time-state pairs. It is, however, difficult to determine the feedback equilibria for general functional forms.[12] The property of being subgame perfect means that after each player actions have caused the state of the system to evolve from its initial state to a new state, the continuation of the game with this new state thought as the initial state may be regarded as a subgame of the original game. A feedback strategy allows the players to do their best in this subgame even if the initial state of the subgame evolved through prior suboptimal actions. Thus, a feedback strategy is optimal not only for the original game as specified by its initial conditions but also for every subgame evolving from it.[13]

The difference between feedback and closed-loop decision rules lies in the fact that closed-loop solutions depend on the initial state (price), while feedback solutions do not. The following definitions of the strategy spaces are taken from Dockner (1988).[14]

Definition 5.5.3. *The feedback strategy space for player i is given as*

$$S_i^F = \{q_i(t,p(t)) \mid q_i(t,p(t)) \text{ is a piecewise continuous function of time } t \text{ and } \\ \text{Lipschitz-continuous with respect to } p(t) \text{ for all } (t,p) \in [0,\infty) \times \Re\} .$$

Definition 5.5.4. *The closed-loop strategy space for player i is given as*

$$S_i^{CL} = \{q_i(t,p(t),p_0) \mid q_i(t,p(t),p_0) \text{ is a piecewise continuous function of time } t \text{ and } \\ \text{Lipschitz-continuous with respect to } p(t) \text{ for all } (t,p) \in [0,\infty) \times \Re\} .$$

Definition 5.5.5. *A closed-loop Nash equilibrium is a pair of closed-loop strategies $(q_i^{*CL}, q_j^{*CL}) \in S_i^{CL} \times S_j^{CL}$ such as for every possible initial condition (p_0, t_0):*

$$J_i(q_i^{*CL}, q_j^{*CL}) \geq J_i(q_i, q_j^{*CL}),$$

for every $q_i \in S_i^{CL}$.

Basar and Olsder (1982) give an explanation about the difference among these equilibrium solutions.[15]

[11] More in Tsutsui and Mino (1990).

[12] More precisely in Reinganum and Stokey (1985).

[13] More in Kamien and Schwartz (1991), p. 275.

[14] p. 50.

[15] Basar and Olsder (1982), pp. 318-327, and Chapter 6.

5.6 Equilibria in dynamic games

5.6.1 Open-loop information structure

5.6.1.1 Preliminaries

Considering our model let us first assume that the players have an open-loop information structure. That means that players must formulate their actions at the moment the system starts to evolve and these actions cannot be changed once the system is running. Therefore, the players have to optimize their performance based on the information that they only know: the differential equation and its initial state.

In Cournot differential game with sticky prices depending on whether the first order condition on controls taken with respect to the output level of any given firm contains the output of its rival or not one obtains either the instantaneous best-response function or best-response at the steady state. An instantaneous reaction function characterizes the optimal behavior of each player at any time during the game, while with best-response at the steady state one only observes player i's best response at the steady state equilibrium.

In the forthcoming section we analyze the underlying model in order to find the best-response functions in the open-loop game. To determine an open-loop Nash equilibrium $(q_1^*(\cdot), q_2^*(\cdot))$ let us define the current value Hamiltonian.

The Hamiltonian function for each player i is:

$$H_i(t) = \left\{ q_i(t) \left[p(t) - w_i - \frac{1}{2} q_i(t) \right] + s\lambda_i(t) \left[a - \sum_{j=1}^{2} q_j(t) - p(t) \right] \right\} \tag{5.6.1}$$

To find the best-response function let us consider the first-order derivative of (5.6.1) with respect to $q_i(t)$ and obtain the first necessary condition for an open-loop equilibrium:

$$\frac{\partial H_i(t)}{\partial q_i(t)} = p(t) - w_i - q_i(t) - \lambda_i(t)s \overset{!}{=} 0 \tag{5.6.2}$$

So the optimal output by the open-loop strategy for each firm i is:[16,17]

$$q_i(t) = \begin{cases} p(t) - w_i - \lambda_i(t)s, & \text{if } p(t) > w_i + \lambda_i(t)s \\ 0, & \text{otherwise} \end{cases} \tag{5.6.3}$$

Equation (5.6.3) states that each player determines his instantaneous production rate according to the rule that marginal revenue equals marginal cost. In this dynamic setting, marginal revenue consists of two terms. The instantaneous marginal revenue is $p(t)$, and from this revenue one subtracts the product of the costate and the adjustment speed parameter $\lambda_i(t)s$. This product represents the long-run effect

[16] $p(t)$ and $q_i(t)$ are assumed to be non-negative.

[17] On the base of equation (5.6.3) it is not possible to create the best-response function of player i at time t, so we need to consider the adjoint condition for the equilibrium, which is given in equation (5.6.5).

of a marginal change in the current production rate since the costate has the interpretation of a shadow price (an imputed value) of the state variable. $w_i + q_i(t)$ in equation (5.6.3) is the instantaneous marginal cost.

The second condition for the equilibrium we obtain from the following equation:

$$\dot{\lambda}(t) = r\lambda(t) - \frac{\partial H(p^*, q^*, \lambda, t)}{\partial p(t)} \tag{5.6.4}$$

So that we obtain the following costate equations

$$\begin{aligned} -\frac{\partial H_i(t)}{\partial p(t)} &= \dot{\lambda}_i(t) - r\lambda_i(t) \\ \Leftrightarrow -q_i(t) + \lambda_i(t)s &= \dot{\lambda}_i(t) - r\lambda_i(t) \\ \Leftrightarrow \dot{\lambda}_i(t) &= -q_i(t) + \lambda_i(t)s + r\lambda_i(t) \\ \Leftrightarrow \dot{\lambda}_i(t) &= \lambda_i(t)(s+r) - q_i(t) \end{aligned} \tag{5.6.5}$$

The transversality condition implies:

$$\lim_{t\to\infty} e^{-rt}\lambda_i(t) = 0 \tag{5.6.6}$$

Differentiating (5.6.2) with respect to t we obtain:

$$\begin{aligned} \left(\frac{\partial H_i(t)}{\partial q_i(t)}\right)\frac{\partial}{\partial t} = \frac{\partial p(t)}{\partial t} - \frac{\partial q_i(t)}{\partial t} - s\frac{\partial \lambda_i(t)}{\partial t} &= 0 \\ \Leftrightarrow \dot{p}(t) - \dot{q}_i(t) - s\dot{\lambda}_i(t) &= 0 \\ \Leftrightarrow \dot{q}_i(t) &= \dot{p}(t) - s\dot{\lambda}_i(t) \end{aligned} \tag{5.6.7}$$

Plugging it into equation (5.6.5) we get:

$$\dot{q}_i(t) = \dot{p}(t) - s\left[(r+s)\lambda_i(t) - q_i(t)\right] \tag{5.6.8}$$

Using the law of motion of the price $\dot{p} = s\left[\tilde{p}(t) - p(t)\right]$ as well as $\tilde{p}(t) = a - q_i(t) - q_j(t)$ and $s\lambda_i(t) = p(t) - w_i - q_i(t)$ which we obtained earlier and plugging these equations into (5.6.8) yields:

$$\dot{q}_i(t) = s\left[a - q_j(t) - p(t)\right] - (r+s)\left[p(t) - w_i - q_i(t)\right] \tag{5.6.9}$$

The first and the second equations in (5.6.10) provide a system of three ordinary differential equations, defined in the feasible region of the (p, q_1, q_2) space. At a steady state market price p_{ss}^{*OL} we must have $\dot{p}(t) = \dot{q}_i(t) = \dot{q}_j(t) = 0$.

$$\begin{cases} \dot{p}(t) = s\left[a - q_i(t) - q_j(t) - p(t)\right] \\ \dot{q}_i(t) = s\underbrace{\left[a - q_j(t) - p(t)\right]}_{q_i(t)} - (s+r)\left[p(t) - w_i - q_i(t)\right] \\ \dot{p}(t) = \dot{q}_i(t) = \dot{q}_j(t) = 0 \end{cases} \tag{5.6.10}$$

Solving this system of equations yields:

$$
\begin{aligned}
2sq_i(t) + rq_i(t) &= (s+r)p(t) - w_i(s+r) \\
\Leftrightarrow q_i(t)(2s+r) &= (s+r)\left[a - q_i(t) - q_j(t) - w_i\right] \\
\Leftrightarrow q_i(t)(3s+2r) &= (s+r)\left[a - q_j(t) - w_i\right] \\
\Leftrightarrow q_i^{BR}(t) &= \frac{(s+r)\left[a - q_j(t) - w_i\right]}{3s+2r} \qquad (5.6.11)
\end{aligned}
$$

$q_i^{BR}(t)$ is a best-response function of firm i at the open-loop equilibrium which has the negative slope:

$$
\frac{\partial q_i^{BR}(t)}{\partial q_j(t)} = \frac{-(s+r)}{3s+2r} < 0, \ \forall s, r \geq 0 \qquad (5.6.12)
$$

From equation (5.6.12) it is obvious that the slope in absolute value is everywhere decreasing in s.

5.6.1.2 Open-loop Nash equilibrium

As we have already mentioned at the beginning we are interested in equilibrium results for cases of symmetric and asymmetric production costs. The first case, namely $w_1 = w_2$ for the analogous game was investigated by Fershtman and Kamien (1987). For our future analysis let us use their Theorem which is presented below:[18]

Theorem 5.6.1. *There is a unique stationary open-loop Nash equilibrium for the above game. The price at this equilibrium is:*

$$
p_{ss}^{*OL} = \frac{3rp^{*perf} + 4sp^{*C}}{4s+3r} \qquad (5.6.13)
$$

and the firms' strategies are given by

$$
q_{ss}^{*OL} = \frac{(a-w)(s+r)}{4s+3r}, \ \textit{if } a \geq w \qquad (5.6.14)
$$

Assuming that both firms have identical cost Fershtman and Kamien (1987) prove that the open-loop equilibrium strategies are symmetric, namely $q_i(t) = q_j(t) = q(t)$.[19]

In order to obtain an equilibrium price trajectory one must differentiate the dynamics $\dot{p}(t) = s(a - [q_1(t) + q_2(t)] - p(t))$ with respect to time and then substitute into this expression the quadruple $(\lambda(t), \dot{\lambda}(t), q(t), \dot{q}(t))$ given by the necessary conditions described above. The result is the following linear second order differential equation for the state $p(t)$, which the equilibrium price trajectory must satisfy

$$
\ddot{p}(t) + (s-r)\dot{p}(t) - (s^2 + 3s(s+r))p(t) = -\left[s^2a + s(2w+a)(s+r)\right] \qquad (5.6.15)
$$

A particular solution of (5.6.15) is given by

$$
p(t) = \frac{-\left[s^2a + s(2w+a)(s+r)\right]}{-(s^2 + 3s(s+r))},
$$

[18] Theorem 1 of Fershtman and Kamien (1987), p. 1155.

[19] ibid., Appendix 1, p. 1162.

after the simple algebraic manipulation it is clear that it is the steady state equilibrium price given in equation (5.6.13).

The characteristic equation associated with the homogeneous part of equation (5.6.15) is given by

$$\mu^2 + (s - r)\mu + (-s^2 - 3s(r + s)) = 0 \tag{5.6.16}$$

It is straightforward that equation (5.6.16) possesses two real roots - one of them is positive and one is negative. Taking the stable solution and using the initial price $p(0) = p_0$, the following price trajectory

$$p^*(t) = p_{ss}^{*OL} + (p_0 - p_{ss}^{*OL})e^{k_1 t}, \tag{5.6.17}$$

with

$$k_1 = -\frac{(s - r) + \sqrt{(s - r)^2 - 4(-s^2 - 3s(r + s))}}{2}$$

is the open-loop Nash equilibrium price trajectory. The price trajectory $p^*(t)$ converges to the steady state level p_{ss}^{*OL} for any value of the initial price p_0. The steady state is globally asymptotically stable.[20]

As we have already mentioned Fershtman and Kamien (1987) consider only the case of symmetric production costs and basing on this statement they prove that the firms' strategies are also symmetric. In our model which was analyzed in Chapter 2 the existence of the asymmetric bargaining power entails the asymmetry in the production costs. In order to make our analysis complete we consider the case of asymmetric production costs also in the dynamic games.

Thus considering the case of asymmetric production costs $w_i \neq w_j$ we use equation (5.6.11). In such situation we cannot use the symmetry in firms' strategies proved by Fershtman and Kamien (1987). Therefore, we state the following proposition:

Proposition 5.6.1. *In the open-loop strategy if both firms have asymmetric production costs there is a unique steady state open-loop Nash equilibrium with strategies and price given below:*

$$q_{i\ ss}^{*OL} = \frac{(s + r)\left[(3s + 2r)(a - w_i) - (s + r)(a - w_j)\right]}{(2s + r)(4s + 3r)} \tag{5.6.18}$$

$$p_{ss}^{*OL} = \frac{4sp^{*C} + 3rp^{*perf}}{4s + 3r} \tag{5.6.19}$$

Proof. The proof follows from the Theorem 1 of Fershtman and Kamien (1987), but for the case of asymmetric production costs which causes the asymmetry in the firms' strategies.

Hence, we obtain that the open-loop steady state equilibrium strategies are

$$q_{i\ ss}^{*OL} = \frac{(s + r)(a - q_j^{*OL} - w_i)}{3s + 2r} \quad i, j \in \{1, 2\},\ i \neq j \tag{5.6.20}$$

[20]More in Fershtman and Kamien (1987) p. 1156; Dockner (2000) pp. 269-270.

After plugging $q^{*OL}_{j\ ss}$ into (5.6.20) we can find that firm i's equilibrium level of output is:

$$q^{*OL}_{i\ ss} = \frac{[(3s+2r)(a-w_i)-(s+r)(a-w_j)]\,(s+r)}{(3s+2r)^2-(s+r)^2} \tag{5.6.21}$$

Given the linear demand, the open-loop equilibrium price is $p^{*OL}_{ss} = a - q^{*OL}_{i\ ss} - q^{*OL}_{j\ ss}$.

Plugging $q^{*OL}_{i\ ss}$ and $q^{*OL}_{j\ ss}$ in the above equation we obtain the steady state equilibrium price in open-loop strategy with asymmetric production costs:

$$p^{*OL}_{ss} = \frac{s(2a+w_i+w_j)+r(a+w_i+w_j)}{4s+3r} = \frac{4sp^{*C}+3rp^{*perf}}{4s+3r} \tag{5.6.22}$$

□

5.6.2 Feedback information structure

5.6.2.1 Preliminaries

In the previous section we applied the open-loop strategy to solve the problem formulated in equation (5.3.15). In the forthcoming sections we consider first the feedback and then present the closed-loop solution. We use the same model as in the previous section but equilibrium strategies (q_i^*, q_j^*) are defined on the state space instead of time domain.

As it has been already mentioned before, the feedback strategies are functions of the current price and they are subgame perfect. Feedback solutions satisfy a kind of "principle of optimality" and are found by using backward induction.[21]

5.6.2.2 Feedback (subgame perfect) Nash equilibrium

In this section we solve our problem using the HJB equations, which are given by

$$rV^i(p) = \max\left\{(p-w_i)q_i - \frac{1}{2}q_i^2 + s\frac{\partial V^i(p)}{\partial p}\left\{a - \sum_{j=1}^{2} q_j(p) - p\right\} \mid q_i, q_j \geq 0\right\}, \tag{5.6.23}$$

where $i, j \in \{1, 2\}$ and $i \neq j$; $V^i(p)$ is the game value of player i associated with the initial price p. Because of the stationarity, the value functions depend on p only. Performing the maximization indicated in the HJB equation yields a unique Markovian output strategy:

$$q_i(p) = \begin{cases} 0, & \text{if } p \leq w_i + sV_p^i(p) \quad \text{case } \mathbf{A} \\ p - w_i - sV_p^i(p), & \text{if } p > w_i + sV_p^i(p) \quad \text{case } \mathbf{B} \end{cases} \tag{5.6.24}$$

Substituting from (5.6.24) into the term in curly brackets on the right-hand side of the HJB equation yields the following differential equations for the value function:

$$\mathbf{A:}\ \ rV^i(p) = sV_p^i(p)(a-p) \tag{5.6.25}$$

[21] Dockner (1988), p. 54.

B: $rV^i(p) = (p-w_i)(p-w_i-sV_p^i)-\frac{1}{2}(p-w_i-sV_p^i)^2+sV_p^i\left[a-3p+w_i+w_j+sV_p^i+sV_p^j\right]$

Starting with case (B) we conjecture that the quadratic value functions of the following form

$$V^i(p) = \frac{1}{2}K_ip^2 - E_ip + g_i \tag{5.6.26}$$

solve the given HJB equations. Here K_i, E_i and g_i are constants to be determined.

$$\begin{cases} V_p^i(p) = K_ip - E_i \\ V_p^j(p) = K_jp - E_j \end{cases} \tag{5.6.27}$$

Substituting the value functions given by (5.6.26), as well as their first order derivatives, into the HJB equations in case (B) provides a set of conditions that must be satisfied by three constants K_i, E_i and g_i.

Substituting (5.6.27) into (5.6.25 case (B)) yields:

$$\frac{1}{2}rK_ip^2 - rpE_i + rq_i = (p - w_i - sK_ip + sE_i)(p - w_i) \tag{5.6.28}$$

$$-\frac{1}{2}(p-w_i-sK_ip+sE_i)^2+(sK_ip-sE_i)(a-3p+w_i+w_j+sK_ip-sE_i+sK_jp-sE_j)$$

After particular algebraic calculations we obtain the following equations:

$$\frac{1}{2}rK_ip^2 - rpE_i + rq_i = (\frac{1}{2} - 3sK_i + s^2K_iK_j + \frac{1}{2}s^2K_i^2)p^2 \tag{5.6.29}$$

$$+[3sE_i - s^2K_iE_i - 2s^2K_iE_j - w_i + sK_i(a + w_i + w_j)]p$$

$$+\frac{1}{2}w_i + \left(\frac{1}{2}sE_i + sE_j - a - w_i - w_j\right)E_i$$

The requirement that these equations are satisfied for all values of p implies that

$$rK_i = 1 - 6sK_i + 2s^2K_iK_j + s^2K_i^2$$

$$\Leftrightarrow s^2K_i^2 + (2s^2K_j - 6s - r)K_i + 1 = 0 \tag{5.6.30}$$

Equating the coefficients of p in equation (5.6.29) we obtain the following expression for E_i

$$-rE_i = 3sE_i - s^2K_iE_i - 2s^2K_iE_j - w_i + sK_ia + sK_iw_i + sK_iw_j, \tag{5.6.31}$$

such that

$$E_i = \frac{sK_i(a + w_i + w_j - 2sE_j) - w_i}{s^2K_i - 3s - r} \tag{5.6.32}$$

Substituting the value of E_j into E_i given in equation (5.6.32) leads to the following expression:

$$E_i = \frac{2s^2K_iw_j - w_i(s^2K_i - 3s - r) - sK_i(a + w_i + w_j)(s^2K_i + 3s + r)}{(3s + r + s^2K_i)(3s + r - 3s^2K_i)} \tag{5.6.33}$$

Now let us turn back to equation (5.6.30). Rewriting it for K_1 and K_2 yields

$$s^2K_1^2 + 2s^2K_1K_2 - 6sK_1 - rK_1 + 1 = 0 \tag{5.6.34}$$
$$s^2K_2^2 + 2s^2K_1K_2 - 6sK_2 - rK_2 + 1 = 0 \tag{5.6.35}$$

Subtracting the second one from the first one yields

$$s^2K_1^2 - s^2K_2^2 - 6sK_1 + 6sK_2 - rK_1 + rK_2 = 0 \tag{5.6.36}$$

such as

$$(K_1 - K_2)\left[s^2(K_1 + K_2) - (6s + r)\right] = 0 \tag{5.6.37}$$

This equation implies that either $K_1 = K_2$, then by substitution into $q_i^* = p - w_i - sV_p^i(p)$ yields to

$$q_i^* = p - w_i - s(Kp - E_i) = (1 - sK)p + sE_i - w_i$$

and equation (5.6.30) is of the following form

$$s^2K^2 + 2s^2K^2 - (6s + r)K + 1 = 0 \quad \Rightarrow \quad 3s^2K^2 - (6s + r)K + 1 = 0 \tag{5.6.38}$$

Then solving this quadratic equation leads to

$$K = \frac{(6s + r) \pm \sqrt{(6s + r)^2 - 12s^2}}{6s^2} \tag{5.6.39}$$

Hence, we constitute a subgame perfect feedback Nash equilibrium for the considered dynamic game with

$$q_i^* = (1 - sK)p + (sE_i - w_i), \tag{5.6.40}$$

where K and E_i are given in equations (5.6.39) and (5.6.33), respectively. In Appendix A.3 we show that with K given in equation (5.6.39) with minus sign the Nash equilibrium is asymptotically stable; we also show that with $K_1 \neq K_2$, $s^2(K_1 + K_2) - (6s + r) = 0$ the obtained equilibrium is not asymptotically stable.

Equation (5.6.40) shows the obtained output strategies of both firms. The results for case (B) are valid only if the output is positive, namely if $(1 - sK)p + (sE_i - w_i) > 0$, which (analogous to Fershtman and Kamien (1987)) means that

$$p \geq \hat{p} := \frac{w_i - sE_i}{1 - sK} \tag{5.6.41}$$

From equations (5.3.12) and (5.6.40) the steady state feedback equilibrium price in case (B) is

$$p_{ss}^{*F} = \frac{a - s(E_i + E_j) + w_i + w_j}{3 - 2sK} \tag{5.6.42}$$

Let us now turn to the case (A) where $p_0 < \hat{p}$ and optimal outputs are zero. Dockner (2000) shows that solving the HJB equation in case (A) shown in equation (5.6.25) yields

$$V(p) = D_0(a - p)^{-r/s}, \tag{5.6.43}$$

where D_0 is constant of integration. Let us define $\hat{V} = V(\hat{p})$. Continuity of the value function implies the boundary condition $\hat{V} = D_0(a - \hat{p})^{-r/s}$. Therefore for case (A) we obtain the value function

$$V(p) = \hat{V}\left(\frac{a-\hat{p}}{a-p}\right)^{r/s} \quad \text{for } p \leq \hat{p} \tag{5.6.44}$$

Recall that we assumed $a > w_i$, $i = 1, 2$. Exploiting this assumption one can show that $a > \hat{p}$ and, since the Nash equilibrium price is increasing over time, this price remains in the interval $[0, \hat{p}]$ only for a finite period of time. At some instant of time, the price will exceed the level $\hat{p}$ and the strategy of zero output is switched to the positive output strategy. This means that the only possible steady state is the one given by (5.6.42). In this steady state production rates are positive.[22]

Summarizing the above obtained results and similar to the paper of Fershtman and Kamien (1987) but with extension to the case of asymmetric production costs we state the following theorem

Theorem 5.6.2.

$$q_i^*(p) = \begin{cases} 0, & \text{if } p \leq \hat{p}, \\ (1 - sK)p + (sE_i - w_i), & \text{if } p > \hat{p} \end{cases} \tag{5.6.45}$$

$$K = \frac{r + 6s - \sqrt{(r+6s)^2 - 12s^2}}{6s^2} \tag{5.6.46}$$

$$E_i = \frac{2s^2Kw_j - w_i(s^2K - 3s - r) - sK(a + w_i + w_j)(s^2K + 3s + r)}{(3s + r + s^2K)(3s + r - 3s^2K)} \tag{5.6.47}$$

$$\hat{p} = \frac{w_i - sE_i}{1 - sK} \tag{5.6.48}$$

Then $(q_i^*(p), q_j^*(p))$ constitutes a global asymptotically stable feedback (subgame perfect) Nash equilibrium for the considered infinite horizon dynamic game.

Theorem 5.6.3. *In the case of asymmetric production costs the steady state feedback equilibrium price converges to a price*

$$p_{ss}^{*F} = \frac{4p^{*C} + 3\sqrt{2/3}\,p^{*perf}}{3\sqrt{2/3} + 4} \tag{5.6.49}$$

Proof. There is a steady state feedback Nash equilibrium price given by

$$p_{ss}^{*F} = \frac{a - s(E_i + E_j) + w_i + w_j}{3 - 2sK} \tag{5.6.50}$$

[22] Precisely in Dockner (2000), Chapter 10.

Analogous to Fershtman and Kamien (1987) let us make the following notations: Let $\beta = \lim_{s\to\infty} sK$, $\alpha_i = \lim_{s\to\infty} sE_i$ and $\alpha_j = \lim_{s\to\infty} sE_j$. Using equation (5.6.46) yields that $\beta = 1 - \sqrt{2/3}$. Similarly, from (5.6.47) we obtain

$$\alpha_i = \frac{2\beta w_j - \beta w_i + 3w_i - \beta(a + w_i + w_j)(\beta + 3)}{(3 + \beta)(3 - 3\beta)} \tag{5.6.51}$$

From equations (5.6.46) and (5.6.47) we obtain

$$\lim_{r\to 0} sK = \beta$$

$$\lim_{r\to 0} sE_i = \alpha_i$$

$$\lim_{r\to 0} sE_j = \alpha_j$$

Thus, as $s \to \infty$ or $r \to 0$ the equilibrium price approaches

$$p_{ss}^{*F} = \frac{a - \alpha_i - \alpha_j + w_i + w_j}{3 - 2\beta} \tag{5.6.52}$$

Substituting α_i and α_j into p_{ss}^{*F} yields

$$p_{ss}^{*F} = \frac{(a + w_i + w_j)(3 - \beta) - (w_i + w_j)}{3(1 - \beta)(3 - 2\beta)}$$

After some transformations we obtain

$$p_{ss}^{*F} = \frac{(2a + w_i + w_j) + (a + w_i + w_j)(1 - \beta)}{3(1 - \beta)(3 - 2\beta)}$$

Substituting the value of β into the above equation yields

$$p_{ss}^{*F} = \frac{4p^{*C} + 3\sqrt{2/3}p^{*perf}}{3\sqrt{2/3} + 4} \tag{5.6.53}$$

□

5.6.3 Closed-loop information structure

The aim of this subsection is to answer the question of how do the results from the previous section change if the firms play closed-loop but do deviate from the subgame perfect equilibrium? To answer this question we consider a particular linear affine closed-loop solution.[23]

The Hamiltonian function of firm i is the same as given by (5.6.1) with the same initial and transversality conditions.

[23] Beside this particular closed-loop solution there exist uncountably many other closed-loop solutions, i.e., Nash equilibria are characterized by informational non-uniqueness (see also Basar and Olsder (1982)).

To solve the optimization problem let us consider the following FOC and hence we obtain the same result (5.6.2) as in the open-loop equilibrium:

$$\frac{\partial H_i}{\partial q_i} = p - w_i - q_i - \lambda_i s \overset{!}{=} 0 \tag{5.6.54}$$

That yields to:

$$q_i^{CL} = \begin{cases} p - w_i - \lambda_i s, & \text{if } p > w_i + \lambda_i s \\ 0, & \text{otherwise} \end{cases} \tag{5.6.55}$$

The kinematic equation for q_i^{CL} is the same as in the case of the open-loop strategy:

$$\dot{q}_i(t) = \dot{p}(t) - s\dot{\lambda}_i(t) \tag{5.6.56}$$

Differently from the open-loop solution now by deriving the adjoint equations we have to take into account the price dependency of the optimal strategies. The adjoint conditions for the optimum, which characterize the interaction between both firms, are shown below:

$$-\frac{\partial H_i}{\partial p} - \frac{\partial H_i}{\partial q_j}\frac{\partial q_j^{CL}}{\partial p} = \dot{\lambda}_i - r\lambda_i, \tag{5.6.57}$$

Taking the derivatives of H_i and q_j^{CL} and substituting them into (5.6.57) we obtain

$$-\frac{\partial H_i}{\partial p} - s\lambda_i = \dot{\lambda}_i - r\lambda_i \tag{5.6.58}$$

Substituting (5.6.58) into (5.6.57) leads to:

$$\dot{\lambda}_i = (2s + r)\lambda_i - q_i \tag{5.6.59}$$

We plug equation (5.6.54) and the definition of $\dot{p}(t)$ given in (5.3.12) into (5.6.56) and obtain:

$$\dot{q}_i = (q_i(t) + w_i)(r + 2s) - p(t)(r + 3s) - sq_j(t) \tag{5.6.60}$$

Applying the stationary point feature $\dot{p}(t) = \dot{q}_i(t) = \dot{q}_j(t) = 0$:

$$q_i(t) = \frac{p(t)(r + 3s) - w_i(r + 2s) + sq_j(t)}{r + 2s} \tag{5.6.61}$$

$$\Leftrightarrow q_i = \frac{(r + 2s)(a - w_i - q_j)}{2r + 5s}$$

After some particular algebraic manipulations for the case of asymmetric production costs we obtain

$$q_{i\ ss}^{*CL} = \frac{(r + 2s)\left[(a - w_i)(2r + 5s) - (r + 2s)(a - w_j)\right]}{(r + 3s)(3r + 7s)} \tag{5.6.62}$$

$$p_{ss}^{*CL} = \frac{a(r + 3s) + (r + 2s)(w_i + w_j)}{3r + 7s} \tag{5.6.63}$$

If the costs are symmetric, $w_i = w_j$, we obtain the following equilibrium quantities and price:

$$q^{*CL} = \frac{(a - w)(r + 2s)}{3r + 7s} \tag{5.6.64}$$

Applying that $p^{*CL} = a - 2q^{*CL}$ we get

$$p^{*CL} = \frac{(r + 2s)(a + 2w) + sa}{3r + 7s} = \frac{sa + 6rsp^{*perf}}{3r + 7s} \tag{5.6.65}$$

5.7 Results

We have based our dynamic game on the assumption that the speed of adjustment s is finite. This is the assumption that makes the game we consider different from a repeated Cournot game in its continuous time version. When s goes to infinity the dynamic structure disappears and the price jumps instantaneously to its level on the demand function for each level of output. Thus, for $s \to \infty$ the game can be viewed as a repeated Cournot game in its continuous time version. On this stage let us examine the limits of the obtained open-loop equilibria.

Hence, using the results obtained in section 5.6.1 we may state the next proposition:

Proposition 5.7.1. *1. By symmetric production costs*

- *If $s \to \infty$ or $r \to 0$ the open-loop steady state equilibrium price p_{ss}^{*OL} and quantities q_{ss}^{*OL} converge to the static Cournot equilibrium price and quantities.*
- *If $s \to 0$ or $r \to \infty$ the open-loop steady state equilibrium price and quantities converge to the static competitive price and quantities.*

2. By asymmetric production costs

- *If $s \to \infty$ or $r \to 0$ the open-loop steady state equilibrium quantities $q_{i\ ss}^{*OL}$ and the static Cournot equilibrium quantities given in (5.4.6) coincide. The steady state open-loop equilibrium price p_{ss}^{*OL} converge to the static Cournot equilibrium price.*
- *If $s \to 0$ or $r \to \infty$ the open-loop steady state equilibrium quantities $q_{i\ ss}^{*OL}$ converge to the static competitive quantities. The steady state open-loop equilibrium price p_{ss}^{*OL} converge to the static competitive price.*

Proof. 1. $w_i = w_j$:

- $$\lim_{s\to\infty} p_{ss}^{*OL} = \lim_{r\to 0} p_{ss}^{*OL} = \frac{a+w}{2} = p^{*C} \tag{5.7.1}$$
- $$\lim_{s\to\infty} q_{ss}^{*OL} = \lim_{r\to 0} q_{ss}^{*OL} = \frac{a-w}{4} = q^{*C} \tag{5.7.2}$$
- $$\lim_{s\to 0} p_{ss}^{*OL} = \lim_{r\to\infty} p_{ss}^{*OL} = \frac{a+2w}{3} = p^{*perf} \tag{5.7.3}$$
- $$\lim_{s\to 0} q_{ss}^{*OL} = \lim_{r\to\infty} q_{ss}^{*OL} = \frac{a-w}{3} = q^{*perf} \tag{5.7.4}$$

2. $w_i \neq w_j$:

- $$\lim_{s\to\infty} q_{i\ ss}^{*OL} = \lim_{r\to 0} q_{i\ ss}^{*OL} = \frac{2a-3w_i+w_j}{8} = q_i^{*C} \tag{5.7.5}$$

- $$\lim_{s\to\infty} p_{ss}^{*OL} = \lim_{r\to 0} p_{ss}^{*OL} = \frac{2a + w_i + w_j}{4} = p^{*C} \tag{5.7.6}$$

- $$\lim_{s\to 0} q_{i\ ss}^{*OL} = \lim_{r\to\infty} q_{i\ ss}^{*OL} = \frac{a - 2w_i + w_j}{3} = q_i^{*perf} \tag{5.7.7}$$

- $$\lim_{s\to 0} p_{ss}^{*OL} = \lim_{r\to\infty} p_{ss}^{*OL} = \frac{a + w_i + w_j}{3} = p^{*perf} \tag{5.7.8}$$

□

Using results obtained in Section 5.6.3 we can state the following proposition examining the limits of the feedback equilibria.

Proposition 5.7.2. *1. If $s \to \infty$ or $r \to 0$ by both symmetric and asymmetric production costs*

a) The feedback (subgame perfect) steady state equilibrium price p_{ss}^{*F} is less than the static Cournot equilibrium price.

b) The feedback equilibrium strategies are

$$q_i^{*F} = \sqrt{2/3}p + (\alpha_i - w_i)$$

with α_i given in equation (5.6.51)

2. If $s \to 0$ or $r \to \infty$ by both symmetric and asymmetric production costs

c) The feedback (subgame perfect) steady state equilibrium price p_{ss}^{*F} converges to the static competitive price p^{*perf}.

d) The feedback equilibrium strategies are

$$q_{i\ ss}^{*F} = p - w_i$$

3. Both firms will decrease their output if price decreases.

Proof. 1. a)

$$\lim_{s\to\infty} p_{ss}^{*F} = \lim_{r\to 0} p_{ss}^{*F} = \frac{p^{*perf} + 2\sqrt{2/3}}{1 + 2\sqrt{2/3}} < p^{*C} \text{ if } w_i = w_j$$

$$\lim_{s\to\infty} p_{ss}^{*F} = \lim_{r\to 0} p_{ss}^{*F} = \frac{4p^{*C} + 3\sqrt{2/3}p^{*perf}}{3\sqrt{2/3} + 4} < p^{*C} \text{ if } w_i \neq w_j$$

The proof follows from the proof of Theorem 5.6.3, where it can be easily seen that the obtained price p_{ss}^{*F} is a convex combination of the static Cournot equilibrium price and the static competitive price and, therefore, is below the static Cournot equilibrium price.

b) From equation (5.6.45) applying that $\lim_{r\to 0} sE_i = \alpha_i$ and $\beta = 1 - \sqrt{2/3}$ the result stated above is straightforward.

2. c) From equation (5.6.42) follows

$$\lim_{s\to 0} p^{*F}_{ss} = \lim_{r\to\infty} p^{*F}_{ss} = \frac{a+2w}{3} = p^{*perf} \text{ if } w_i = w_j$$

$$\lim_{s\to 0} p^{*F}_{ss} = \lim_{r\to\infty} p^{*F}_{ss} = \frac{a+w_i+w_j}{3} = p^{*perf} \text{ if } w_i \neq w_j$$

d)

$$\lim_{s\to 0} q^{*F}_{ss} = \lim_{r\to\infty} q^{*F}_{ss} = p - w \text{ if } w_i = w_j$$

$$\lim_{s\to 0} q^{*F}_{i\ ss} = \lim_{r\to\infty} q^{*F}_{i\ ss} = p - w_i \text{ if } w_i \neq w_j$$

3. The obtained in our analysis feedback equilibrium strategy is an increasing linear function of the state variable, price:

$$q_i^{*F} = \sqrt{2/3}p + (\alpha_i - w_i), \tag{5.7.9}$$

with α_i given in equation (5.6.51).

□

And finally, using the results obtained in equations (5.6.64) and (5.6.65) we may state the following proposition for the case when firms play closed-loop but deviate from the subgame perfect equilibrium:

Proposition 5.7.3. *1. By symmetric production costs*

- *If $s \to 0$ or $r \to \infty$ the closed-loop steady state equilibrium price p^{*CL}_{ss} and quantities q^{*CL}_{ss} coincide with the price and quantities by perfect competition p^{*perf} and q^{*perf}, respectively.*
- *If $s \to \infty$ or $r \to 0$ the closed-loop steady state equilibrium price and quantities are given by*

$$q^{*CL}_{ss} = \frac{2(a-w)}{7}$$

$$p^{*CL}_{ss} = \frac{3a+4w}{7}$$

2. By asymmetric production costs

- *If $s \to 0$ or $r \to \infty$ the closed-loop steady state equilibrium quantities $q^{*CL}_{i\ ss}$ converge to the static competitive quantities. The closed-loop steady state equilibrium price p^{*CL}_{ss} converge to the static competitive price.*
- *If $s \to \infty$ or $r \to 0$ the closed-loop steady state equilibrium quantities $q^{*CL}_{i\ ss}$ and the steady state equilibrium price are given by*

$$q^{*CL}_{i\ ss} = \frac{6a - 10w_i + 4w_j}{21}$$

$$p^{*CL}_{ss} = \frac{3a + 2w_i + 2w_j}{7}$$

Proof. 1. $w_i = w_j$:

- $$\lim_{s\to 0} p_{ss}^{*CL} = \lim_{r\to\infty} p_{ss}^{*CL} = \frac{a+2w}{3} = p^{*perf} \quad (5.7.10)$$

- $$\lim_{s\to 0} q_{ss}^{*CL} = \lim_{r\to\infty} q_{ss}^{*CL} = \frac{a-w}{3} = q^{*perf} \quad (5.7.11)$$

- $$\lim_{s\to\infty} p_{ss}^{*CL} = \lim_{r\to 0} p_{ss}^{*CL} = \frac{3a+4w}{7} \quad (5.7.12)$$

- $$\lim_{s\to\infty} q_{ss}^{*CL} = \lim_{r\to 0} q_{ss}^{*CL} = \frac{2(a-w)}{7} \quad (5.7.13)$$

2. $w_i \neq w_j$:

- $$\lim_{s\to 0} q_{i\ ss}^{*CL} = \lim_{r\to\infty} q_{i\ ss}^{*CL} = \frac{a-2w_i+w_j}{3} = q_i^{*perf} \quad (5.7.14)$$

- $$\lim_{s\to 0} p_{ss}^{*CL} = \lim_{r\to\infty} p_{ss}^{*CL} = \frac{a+w_i+w_j}{3} = p^{*perf} \quad (5.7.15)$$

- $$\lim_{s\to\infty} q_{i\ ss}^{*CL} = \lim_{r\to 0} q_{i\ ss}^{*CL} = \frac{6a-10w_i+4w_j}{21} \quad (5.7.16)$$

- $$\lim_{s\to\infty} p_{ss}^{*CL} = \lim_{r\to 0} p_{ss}^{*CL} = \frac{3a+2w_i+2w_j}{7} \quad (5.7.17)$$

□

5.8 On use of open-loop, feedback and closed-loop strategies

In the preceding analysis we have considered different types of strategies: strategies where players base their actions purely on the initial state of the system and time (open-loop strategies) and strategies where players base their actions on the current state of the system (feedback and closed-loop strategies). The implementation of the second type of strategies requires a full monitoring of the system. To implement these strategies each player has to know the exact state of the system at each point of time. On the other hand, an advantage of these strategies is that as far as the commitment issue is concerned they are much less demanding. If, due to some external causes, the state of the system changes during the game, this has no consequences for the actions taken by the players. They are able to respond to this disturbance in an optimal way, in contrast to the open-loop strategy which implies that the players cannot adapt their actions during the game in order to account for the unforeseen disturbance without breaking their commitment. Since all players are confronted with this commitment promise, one might expect that under such conditions the players will try to renegotiate on the agreed decisions. So open-loop strategies make sense particularly for those situations where the model is quite robust or the players can commit themselves strongly.

Generally, since both rivals cannot react to each other policies, the economic relevance of the open-loop results is mostly rather limited. However, as a benchmark to see how much both parties can gain by playing other strategies, it plays a fundamental role.

A practical advantage of the open-loop strategy is that it is, usually, numerically and analytically more tractable than the feedback strategy.[24]

The feedback solution is strongly time consistent and therefore subgame perfect, while the open-loop solution is only weakly time-consistent, i.e., it is not subgame perfect. Basar and Olsder (1982) give a clear explanation of the difference among these equilibrium solutions.[25]

Mehlmann (1988) and Reinganum (1982) show that there exist classes of games where the closed-loop and the open-loop solutions coincide.[26] But in spite of the circumstances when these two types of strategies coincide, where the actions taken are identical at each point of time, in general they do not.

5.9 Conclusion

The presented chapter is an alternative approach to the previous chapters in which we considered the bargaining between the upstream supplier and two downstream firms in the intermediate market. When the bargaining is over, all players know their wholesale prices and the output quantities were produced, we assume that both downstream firms enter the local market, where in the Chapter 2 and 3 they are involved into Cournot competition and in Chapter 4 they are price competitors. We found the equilibrium prices and quantities for both downstream firms under the considered arts of competition.

Basing on the interpretation of our basic model shown in Chapter 2 it is straightforward to assume that both downstream firms are retailers in the local market. Therefore the consideration of the price stickiness in the market is of great economic importance.

Many recent papers modeling business cycle fluctuations or analyzing monetary policy assume that firms adjust their prices only infrequently.[27] Considering the papers with an empirical work, which measure the price stickiness, a large number of them have shown that certain wholesale and retail prices often go unchanged for many months.[28]

The real world is characterized by sticky prices in the sense that prices do not respond rapidly to innovations in other variables. Intuitively, the absence of compe-

[24]Engwerda (2005), p. 114.

[25]Basar and Olsder (1982), pp. 318-327, and Chapter 6.

[26]Chapter 4 in Mehlmann (1988); see also Fershtman and Kamien (1987); Dockner et al. (2000), Chapter 7.

[27]Goodfriend and King (1997), Rotemberg and Woodford (1997), Clarida, Gali, and Gertler (1999), Chari, Kehoe, and McGrattan (2000), Erceg, Henderson, and Levin (2000), and Dotsey and King (2001) represent only a few examples.

[28]Important references include Carlton (1986), Cecchetti (1986), Kashyap (1995), Levy et al. (1997), Blinder et al. (1998), MacDonald and Aaronson (2001), and Kackmeister (2002).

tition or its lower intensity lets the firms rarely lower prices, even when production costs decrease or demand drops. When production becomes cheaper, firms may take the difference as their profit.

As it has been already mentioned at the beginning we used the paper of Fershtman and Kamien (1987) for our analysis in this chapter. Their results are very important for current research, but unfortunately they cannot be implemented into the work directly. These results were obtained under the basic assumption that both firms had identical production costs. Fershtman and Kamien (1987) consequently prove that in such case the equilibrium quantities by open-loop and feedback solutions for both firms will be symmetric.

With the aim to extend our previous analysis we considered the case of asymmetric production costs and examined the influence of the speed adjustment parameter as well as the discount rate on the obtained output strategies, which to our knowledge were not made in the existing literature.

The main object of this chapter is to investigate whether the equilibrium prices and quantities under symmetric and asymmetric production costs converge in the limits to the same point and to compare the obtained results with the static model of Cournot and perfect competition analyzed in the previous chapters and to find the conditions under which the obtained equilibrium prices and quantities in both dynamic and static models coincide.

We solved our differential game for different information structures using Pontryagins's maximum principle and Bellman's dynamic programming method. We obtained the open-loop, feedback and closed-loop equilibrium solutions. Taking into account, that comparing with the open-loop solution by the feedback and closed-loop solutions player passes the own strategy on the rival's behavior; and since the feedback strategy is independent of the initial price p_0 it is subgame perfect in the sense of Selten (1975) whereas the closed-loop and open-loop solutions do not in general possess this property - the difference between all solutions becomes straightforward.

We have analyzed the underlying differential game of duopolistic competition over time under the assumption that price does not adjust instantaneously to its level on the demand function for each level of output. Then we have considered the difference between the steady state open-loop, feedback and closed-loop Nash equilibria of the game, in the limit, as price adjusts instantaneously.

For each solution we were mostly interested in the relation between the static Cournot price, the competitive price and the steady state open-loop, feedback and closed-loop prices when the speed of adjustment reaches infinity.

Proposition 5.7.2 shows that the feedback equilibrium strategy is an increasing linear function of the state variable, price, and therefore each player will decrease its output when price decreases.

As we know from Fershtman and Kamien (1987) if a firm ignores the reaction of its rival to the change in price and simply makes the Cournot assumption that its rival output will remain at its present level, then it will make its output decision on the basis of the residual demand curve it faces. If, on the other hand, it takes its rival reaction to a price change into account, it will know that as it expands its output and causes prices to fall, its rival will contract his output. Thus, its movement down its residual demand curve will be offset somewhat by an outward shift of the residual demand curve as its rival contracts his output. This, of course, will cause the firm to optimally expand its output beyond the optimal level when its rival reaction to a price change is ignored. Hence, the profits are higher at the steady state open-loop equilibrium than at the steady state closed-loop equilibrium.

If $s \to \infty$ then with both symmetric and asymmetric production costs the steady state open-loop equilibrium price coincides with the static Cournot price and open-loop equilibrium quantities coincide with static Cournot quantities.

Considering the feedback solutions if $s \to \infty$ then by both symmetric and asymmetric production costs the feedback (subgame perfect) steady state equilibrium price p_{ss}^{*F} is lower than the static Cournot equilibrium price. The feedback equilibrium strategies are

$$q_i^{*F} = \sqrt{2/3}p + (\alpha_i - w_i), \tag{5.9.1}$$

with α_i given in equation (5.6.51).

If $s \to \infty$ the closed-loop steady state equilibrium price and quantities for the symmetric and asymmetric production costs are shown in Proposition 5.7.3.

From the proof of Proposition 5.7.1 it is obvious that for the case of symmetric production costs if the price is infinitely sticky, $s \to 0$, the open-loop steady state equilibrium price and equilibrium quantities coincide with the price and quantities of the static game with perfect competition. If production costs of both firms are asymmetric, the open-loop steady state equilibrium price converges to the perfect competitive price of the static game, and equilibrium output strategies also converge to the perfect competitive of the static game, but differently from the previous case they are asymmetric for both firms.

Considering the feedback strategies for symmetric and asymmetric production costs, we have obtained that if $s \to 0$, the feedback steady state equilibrium price converges to the price of the static game with perfect competition (proof of Proposition 5.7.2). The equilibrium strategies are $q_i^{*F} = p - w_i$, $i = 1, 2$.

By closed-loop strategy[29] if $s \to 0$ the closed-loop steady state equilibrium price and quantities under symmetric and asymmetric production costs converge to the price and quantities of the static game with perfect competition.

Hence, if the price is infinitely sticky, under open-loop and closed-loop strategies both firms will sell at least perfect competitive quantities q_i^{*perf}, $i = 1, 2$ and under feedback strategy at least $q_i^{*F} = p - w_i > q_i^{*perf}$, $i = 1, 2$ setting at least the price p^{perf} to such an extent that the resulting equilibrium outputs are higher in the feedback case.

The next result shows that for all positive levels of r and for any finite s, independent on the symmetry or asymmetry in firms' production costs the static Cournot price p^{*C} is higher than the open-loop steady state equilibrium price, concerning the output - under the same conditions on the s and r, the static Cournot output q_i^{*C} is lower than the open-loop equilibrium output.

From our analysis it can be easily seen that the equilibrium output obtained by the feedback solution is larger than under the open-loop solution, which can be explained by the existing possibility for the player to react on the changing behavior of his rival by feedback strategy.

[29]See Proposition 5.7.3.

Hence, for our whole analysis, knowing that the production costs of both downstream firms differ, the results obtained in this chapter are very important. Comparing with the static model, if after the bargaining both firms enter the local market with Cournot quantities as it was modeled in Chapter 2 they both will be better off in the dynamic model with sticky price if it is infinitely sticky, $s \to 0$, playing the feedback strategy, because they will sell higher output quantity than playing open-loop or closed-loop strategies.

If in the local market the price converges instantaneously, $s \to \infty$, then under the open-loop strategy both firms will sell at least Cournot quantities and will obtain profit at least equal to Cournot profit in the static model. In the context of the whole work that means that firm 2 will sell Cournot quantity and firm 1 may produce additional output by itself and sell it in the local market. By the feedback solution the steady state equilibrium price converges to a price below the static Cournot price. Fershtman and Kamien (1987) explain this difference in a following way: *"An intuitive explanation for this difference is that in the feedback strategy, which is subgame perfect, each duopolist knows that the loss in future profits from expansion of current output will be shared by his rival, who will attempt to at least partially offset it by contracting his output. Thus each duopolist will expand his current output beyond the level that he would if he alone bore the full loss in future profits. On the other hand, when the duopolists follow open-loop strategies their ability to shift some of the loss in future profits from expansion of current output on their rival is limited by a commitment to an output path at the outset. This shifting of the loss in future profits on the rival persists when the price adjusts instantaneously for the stationary state feedback strategies but vanishes for the stationary state open-loop strategies."* If $s \to \infty$ the steady state closed-loop equilibrium price is also lower than the static Cournot price, therefore both firms are better off playing the open-loop strategy.

Appendix

A.1 Supplement to section 2.4.2

As we know from Section 2.4.2 the price for downstream firm 1 is a solution to the first order condition of function (2.4.3):

$$\frac{\partial U(w_1,w_2)}{\partial w_1}\pi_1(w_1,w_2)+U(w_1,w_2)\frac{\partial \pi_1(w_1,w_2)}{\partial w_1}=0 \tag{A.1.1}$$

Taking the first order derivative of function $w_1^{'}(\cdot)$ with respect to w_2 we obtain the following equation the sign of which we need to determine:

$$\begin{aligned}\frac{\partial w_1^{'}(w_2)}{\partial w_2} &= \frac{\partial^2 U(w_1,w_2)}{\partial w_1 \partial w_2}\pi_1(w_1,w_2)+\frac{\partial U(w_1,w_2)}{\partial w_1}\frac{\partial \pi_1(w_1,w_2)}{\partial w_2} \\ &+\frac{\partial U(w_1,w_2)}{\partial w_2}\frac{\partial \pi_1(w_1,w_2)}{\partial w_1}+U(w_1,w_2)\frac{\partial^2 \pi_1(w_1,w_2)}{\partial w_1 \partial w_2}\end{aligned} \tag{A.1.2}$$

Rewriting the profit functions of both players (of the upstream supplier and downstream firm 1) using the purchased quantities and the wholesale prices we obtain that the sign of the right hand-side of equation (A.1.2) is positive:
$sign\ \frac{\partial w_1^{'}(w_2)}{\partial w_2} = sign$ (A + B + C +D)

where

$$A=\left[\frac{\partial q_1(w_1,w_2)}{\partial w_2}+w_1\frac{\partial^2 q_1(w_1,w_2)}{\partial w_1\partial w_2}+w_2\frac{\partial^2 q_2(w_1,w_2)}{\partial w_1\partial w_2}+\frac{\partial q_2(w_1,w_2)}{\partial w_1}\right]\pi_1(w_1,w_2)$$

$$B=\frac{\partial U(w_1,w_2)}{\partial w_1}\frac{\partial \pi_1(w_1,w_2)}{\partial w_2}$$

$$C=\frac{\partial U(w_1,w_2)}{\partial w_2}\frac{\partial \pi_1(w_1,w_2)}{\partial w_1}$$

$$D=U(w_1,w_2)\left[(p-w_1)\frac{\partial^2 q_1(w_1,w_2)}{\partial w_1\partial w_2}-\frac{\partial q_1(w_1,w_2)}{\partial w_2}\right]$$

Applying that $\frac{\partial q_i(w_1,w_2)}{\partial w_j} > 0$, $\frac{\partial \pi_i(w_1,w_2)}{\partial w_i} < 0$, $\frac{\partial \pi_i(w_1,w_2)}{\partial w_j} > 0$, $\frac{\partial U(w_1,w_2)}{\partial w_i} > 0$ for $i = 1,2$ and $\frac{\partial^2 q_i(w_1,w_2)}{\partial w_1 \partial w_2} > 0$ (because the higher values of w_j make $q_i(w_1,w_2)$ larger and therefore the increase in w_i has a smaller negative effect on $q_i(w_1,w_2)$ when w_j is high) we obtain that $A > 0$; $B > 0$; $C < 0$; the sign of D is not clear, but we assume that even if it is negative $C + D$ does not dominate $A + B$.

A.2 Proof of Theorem 4.2.1

1. *For Region II A:*

   According to Lemma 5 (a) of Kreps and Scheinkman (1983) $\bar{p_1} = P(R(\tilde{q_2})+\tilde{q_2})$ and the equilibrium revenue of firm 1 is $r(\tilde{q_2})$. From the proof of part (d) and (e) follows:

   The equilibrium revenue of firm i must be $\underline{p} \times D(\underline{p})$. One knows that: $\underline{p} < \bar{p_1} = P(R(\tilde{q_2}) + \tilde{q_2}) \Rightarrow D(\underline{p}) > D(P(\tilde{q_2} + R(\tilde{q_2})) = \tilde{q_2} + R(\tilde{q_2}) \Rightarrow D(\underline{p}) > \tilde{q_2}$.

   So in equilibrium firm 2 certainly gets $\underline{p}\tilde{q_2}$. Firm 1 will not get more than $\underline{p}\tilde{q_1}$. From Lemma 5 (a): $\underline{p}\tilde{q_1} = r(\tilde{q_2})$, so that $\underline{p} = \frac{r(\tilde{q_2})}{\tilde{q_1}}$. Firm 2 will receive $\underline{p}\tilde{q_2} = \frac{r(\tilde{q_2})\tilde{q_2}}{\tilde{q_1}}$.

   As it has been mentioned earlier, the revenue of firm i is $r(\tilde{q_i}) = R_i(\tilde{q_j})P(R_i(\tilde{q_j}) + \tilde{q_j})$, where $R_i(\tilde{q_j})$ is the corresponding Cournot best-response function, but the revenue could also be expressed in the following way:

   $$\underline{p}_i D(\underline{p}_i) = r(\tilde{q_i}) = \max_p p(D(p) - \tilde{q_j}), \tag{A.2.1}$$

   where $\underline{p}_i$ is the smallest solution to equation (A.2.1).

2. *For Region II B:*

   Using analogous arguments as in the proof of Lemma 5 (a) of Kreps and Scheinkman (1983) we obtain that $\bar{p_2} = P(R(\tilde{q_1}) + \tilde{q_1})$ and the equilibrium revenue of firm 2 is $r(\tilde{q_1})$.
   Analogous to the previous part we obtain the following expression: $\underline{p} < \bar{p_2} = P(R(\tilde{q_1}) + \tilde{q_1}) \Rightarrow D(\underline{p}) > D(P(\tilde{q_1} + R(\tilde{q_1})) = \tilde{q_1} + R(\tilde{q_1}) \Rightarrow D(\underline{p}) > \tilde{q_1}$.

   So in the equilibrium firm 1 certainly gets $\underline{p}\tilde{q_1}$. Firm 2 will not get more than $\underline{p}\tilde{q_2}$, but we have already known that it is equal to $r(\tilde{q_1})$, so that $\underline{p}\tilde{q_2} = r(\tilde{q_1}) \Rightarrow \underline{p} = \frac{r(\tilde{q_1})}{\tilde{q_2}}$.

   Firm 1 will then receive: $\frac{r(\tilde{q_1})\tilde{q_1}}{\tilde{q_2}}$.

A.3 Supplement to proof of Theorem 5.6.2

If $K_1 \neq K_2$ in equation (5.6.37) then we have

$$s^2(K_1 + K_2) = r + 6s$$

Substituting from $q_i^*(p) = p - w_i - sV_p^i(p)$ for q_i^* and q_j^* into the differential equation (5.3.12) that shows the change in price in the market with sticky prices we obtain:

$$\dot{p}(t) = s[a - q_1(t) - q_2(t) - p(t)]$$

$$\Rightarrow \dot{p}(t) = s[a - (p - w_i - sV_p^i(p) + p - w_j - sV_p^j(p)) - p(t)]$$

$$\Rightarrow \dot{p}(t) = s[a - 3p + w_i + w_j + s(K_i p - E_i + K_j p - E_j)]$$

$$\dot{p} - sp\,[sK_i + sK_j - 3] = s\,[a + w_i + w_j - sE_i - sE_j] \tag{A.3.1}$$

The particular solution to this first order equation (A.3.1) is

$$\bar{p} = \frac{a + w_i + w_j - s(E_i + E_j)}{3 - s(K_i + K_j)} \tag{A.3.2}$$

The solution to the homogeneous part of (A.3.1) is

$$p(t) = Ce^{s[s(K_i+K_j)-3]t}, \tag{A.3.3}$$

where C is a constant of integration.

Finally, the complete solution of (A.3.1) is

$$p(t) = \bar{p} + (p_0 - \bar{p})e^{s[s(K_i+K_j)-3]t}, \tag{A.3.4}$$

where $p(0) = p_0$ is employed to determine the constant of integration C.

For (A.3.4) to converge to $\bar{p}$ as $t \to \infty$, to be asymptotically stable, we need that $[s(K_i + K_j) - 3] < 0$. Substituting $s^2(K_1 + K_2) = 6s + r$ into this inequality we obtain

$$6 + r/s - 3 < 0 \Rightarrow r/s + 3 < 0,$$

which is not possible because both r and s are nonnegative. That means that asymmetric closed-loop equilibrium $q_1^* \neq q_2^*$ cannot be asymptotically stable.

Considering the case with $K_i = K_j$. The kinematic equation is

$$\dot{p}(t) = s[a - q_i(t) - q_j(t) - p(t)]$$

$$\Rightarrow \dot{p}(t) - sp\,[2sK - 3] = s(a + w_i + w_j - sE_i - sE_j)$$

$$\bar{p} = \frac{a + w_i + w_j - sE_i - sE_j}{3 - 2sK} \tag{A.3.5}$$

$$p(t) = Ce^{s[2sK-3]t} \tag{A.3.6}$$

$$p(t) = \bar{p} + (p_0 - \bar{p})e^{s[2sK-3]t} \tag{A.3.7}$$

For (A.3.7) to converge to $\bar{p}$ as $t \to \infty$, to be asymptotically stable, we need that $s\,[2sK - 3] < 0 \Rightarrow K < \frac{3}{2}s$.

Now let us consider equation (5.6.39) with positive sign

$$K = \frac{(6s + r) + \sqrt{(6s + r)^2 - 12s^2}}{6s^2} \tag{A.3.8}$$

After some algebraic exercises we obtain $K > 3/2s$. Taking the negative sign

$$K = \frac{(6s + r) - \sqrt{(6s + r)^2 - 12s^2}}{6s^2} \tag{A.3.9}$$

implies that $K < 3/2s$, which means that with this K the equilibrium is asymptotically stable.

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