Pension Systems and their Meaning for Welfare

A Comparison of PAYG and FF Pension Systems

Inauguraldissertation
zur Erlangung des akademischen Grades
eines Doktors der Wirtschaftswissenschaften
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Diplom-Volkswirtin Wencke Böhm
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Recently the news reported that Germany has the lowest birth rates in the world and the balance between young and old citizens will overturn from 2030 on. Therefore, the question whether a pay-as-you-go pension system or a fully funded one provides suitable pension benefits for the retirees becomes even more urgent. During my doctoral studies I had the chance to analyze this question deeply.

I want to thank my professor Bernhard Eckwert for his trust and his professional comments throughout my work. Although I had already found a topic for my thesis, which is not closely related to his own field of research, when I started to work with him he gave me the freedom and his unconditional support to pursue my own research. I benefited a lot from his deep knowledge of economic theory and his intuition for economic mechanism. Further, my supervisor Prof. Itzhak Zilcha was of great importance for me. He gave me the possibility of a research visit to Tel Aviv and supported my work strongly with fruitful discussions. Many ideas for my work were developed during debates with him and via paper recommendations. Also, I want to thank Prof. Böhm for giving me the chance to start my academical career. My colleagues Andreas Szczutkowski, Nikolai Brandt, Bettina Fincke and many more have been of great value for me and for my research. During numerous discussion sessions they helped me to find my way and always provided an open ear.

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Chapter 1

Introduction

History of the Pension System in Germany

Until Otto von Bismarck introduced a law concerning the old-age retirement coverage in Germany in 1889 people were working until their death or they were supported by their children. Some years earlier the government introduced an accident and a disability insurance. During this process it became aware that old people should be prevented from life-long work and poverty as well. The implementation of an old-age pension system went hand in hand with enormous discussions. Finally, parlamentarians agreed on a pension system which pension payments were related to the agents contributions. Pension benefits were not organized in a flat way as the aim was not to augment care against poverty[1]. Determination of the individual pension payments were not made according to the individual average wage income. For simplicity reasons workers were divided into four income groups instead. Workers received an entitlement to pension payments at the age of 70 years (in 1916 this age was reduced to 65 years). In the period 1881/90 life expectancy at birth for men was 37.17 and for women 40.25 years[2]. This makes clear that the number of agents reaching the necessary age for retirement benefits was very low. Workers and employers paid the same amount as contributions and the government granted a so-called "Reichszuschuss" (governmental allowance). Although Bismarck claimed that the state and its institution would live forever and, therefore, pensions would be

[1] For further information see U. Haerendel (2000).

[2] Data taken from G. Wiesner (2001).

safe, the pension system was organized in a fully funded way since a pay-as-you-go system was accepted as unreliable[3].

During the first world war the capital stock of the pension system was used for financing military activities. The hyper-inflation afterwards worsened the problem of missing financial ressources of the system. To fulfill the public pension obligations the government switched to a PAYG system. Although the public saw those risks of a fully funded system, the common opinion still believed in the reliability of this scheme[4]. Fostered by the strong economic growth in the 1920's, Germany returned to a funded pension system. Under the regime of Hitler they changed some components of the social policy. A governmental allowance to pension benefits was no longer given on an individual level but for the general public. The amount of allowance increased significantly. Further, a guarantee for pension benefits was no longer given by the federal states but by the country (Reichsgarantie). At this time, one aim was that pension benefits should not deviate to much from the wage income. In contrast to the habit during the First World War, during the Second World War it was communicated that resources of the pension system were used to finance the acts of war. Hence, the system de facto used current contributions to fulfill its obligations against old-age pensioners, although a PAYG system was not officially implemented yet. In 1957 a great pension reform took place. The government introduced a modified pay-as-you-go system[5]. This means that contribution rates were defined, constant for a ten years coverage period, in such a way that at the end of the period a capital stock existed which was as high as the aggregate pension obligations for one year. At that time pension payments were not an allowance to old-age welfare instead they should more or less substitute wage income. This led to an increase of the contribution rate from 11% to 14% in 1957 and was further increased for the second coverage period to 16.25% and for the third to 18.25%. At the time

[3]Under a pay-as-you-go (PAYG) system pension benefits are payed by current contributions, hence symbolyzes a pure reallocation of wage income from the working generation to the retired generation. Under a fully funded (FF) system agents receive the returns on their contributions as pension benefits.

[4]For further information see P. Manow (2000).

[5]For further information see W. Schmähl (2007).

of the pension reform, the government assumed a constant life-expectancy of the agents, which had already been discussed critically by Georg Heubeck (1956) in his survey. Through this reform, pension benefits got a dynamical component. Benefits were no longer determined at a fixed point of time instead they were determined continuously over the contribution period so that the development of wage income over the work-life was respected. Over the following years mutiple changes of the pension law occured and in 1989/92 and 2001 small reforms of the pension system took place[6]. This has led to today's effective pension law and the structure of the pension system. Pension benefits are still payed according to the own contributions and to the relative position of the own wage income compared to the average wage income. Further, the pension benefits are related to the wage development of the contributors, so that pensioners benefit from the economic development. In 2009 the government decided that pension benefits are bounded from below, meaning that they can only increase or remain constant but never decrease in case of a negative wage development. For sustainability reasons a so-called sustainability factor was introduced in 2002, which relates the number of contributors to the number of pensioners. Owing the demographic change the age at entry was increased from 65 to 67 (adjustable over the next 20 years), the contribution rate raised to 19.9% and due to the decreasing pension level the government decided to boost occupational and private pension insurances. While the occupational pension scheme is boosted only by taxational savings, the private pension scheme, in form of a “Riester-Rente”, is promoted twice. The government gives allowances (if agents save the minimum amount – 4% of the gross wage income) and tax benefits to the private scheme. Both additional precaution instruments are not mandatory and while a high number of agents save in form of a “Riester-Rente”, the occupational scheme is not widely spread.

[6]For further informations see F. Ruland (2007).

Overview of different European Pension Systems

In different European countries old-age retirement welfare is arranged in various ways. A pure fully funded or a pure pay-as-you-go pension system is implemented in none of the European countries. In general, the accommodation in retirement is based on more than one pillar. Three pillars of old-age provisions are well-known: a) the public pension system b) an occupational pension scheme and c) a privately organized old-age provision. The first pillar is typically organized in form of a PAYG system whereas the latter two are funded systems. Countries are divided in two types: a liberal and a conservative welfare state[7]. Countries belonging to the first type are for example Great Britain, Ireland, the USA and to some extend the Netherlands. Germany, France and Austria are examples for a conservative welfare state. Such states want to ensure the living standard of their citizens whereas a liberal welfare state is anxious to prevent its citizens from old-age poverty by granting a minimum level of pension payments. In this context two types of payment schemes are to be mentioned: Bismarck and Beveridge. A Beveridge scheme disburses a flat pension payment to every agent with no regard to his own contributions. This is contrary to a Bismarck scheme where agents receive pension payments according to their own contributions. This scheme is based on the social insurance law introduced by Otto von Bismarck, which we mentioned above. The Beveridge scheme relies on the British national insurance, which was designed mainly by Lord William v. Beveridge. While Bismarck already introduced a social security system in 1881, Beveridge's ideas from 1942 led to the British social legislation in 1946. In liberal welfare states each agent is granted a minimum pension payment, which is not sufficient to ensure one's living standard. Hence, the government of these countries expect their citizens and companies to take measures for an appropriate welfare in retirement. This precaution can be carried out either in the second or the third pillar. Conservative welfare states instead intend to guarantee their citizens an appropriate living standard already by the first pillar, i.e. public pension system. Due to demographic changes it has become harder to fulfill this aim by focussing on the first pillar only. Therefore, conservative welfare states need to enlarge their effort

[7]Facts provided in this paragraph are taken from H. Stöger (2011).

towards the second and third pillar. In the end, this means that agents take more responsibility for their welfare in the retirement period. Often countries grant tax reliefs or fringe benefits to give incentives for an appropriate prevention. The first pillar is often encouraged by governmental grants to indemnify pension payments. The OECD values this practice as critical since it slows down the economic growth and contradicts the principle of fairness regarding future generations. Next, we want to give a small overview how pension systems are structured in different countries.

Great Britain introduced the so-called Basic State Pension (BSP) in 1946, which supplies a flat pension benefit to all agents who contributed to the system for at least the minimal required time span. All employees are covered by this BSP. The pension benefits are low and do not achieve the level of the British social welfare benefits. In the 1970's Britain introduced a mandatory public additional system (SERPS) with the aim to ensure the living standard of agents while retired. This system was replaced by the State Second Pension (S2P). Initially, a contribution-related pension system, the S2P was restructured in 2007 now following the concept of Beveridge, hence disburses flat pension payments. The British government subsidizes the second pillar and allows to compensate the S2P system by an occupational pension scheme. This scheme is not mandatory for companies and only 54% of all employees are covered by such a system. The third pillar is the private pension insurance fund. Further, a so-called Stakeholder Pension (STP) exists. This system was introduced to cover agents who do not have an occupational or a private pension scheme and earn between 9000 and 18500 GBP per year. Altogether the British pension system is hardly able to guarantee its citizens their living standard.

The Netherlands implemented a pension system based on the three pillars as well. In 1956 they introduced a contribution-financed public pension system, the so-called Allgemene Ouderdomswet (AOW). The contribution rate was increased from 6.7% in 1957 to 18% in 2001. Facing the problems of increased costs and a decreased ratio of contributors to beneficiaries, the Dutch goverment established a fund in the late 1990's, financed by tax money, with the aim to guarantee sustainability of the pension system. This system should not ensure living standards but prevent agents from old-age poverty. Pension payments are not granted according to contributions instead according to the timespan an agent has lived in the Netherlands. To achieve the highest possible pension payment, a citizen must live for at least 50 years in the Netherlands. This payment is 70% of the net wage floor for singles and 50% for couples. The second pillar consists of sectoral agreements and is part of the labor pact. Hence, it is mandatory for companies to contribute to an occupational pension scheme. As a result 94% of the employees are covered by such a system. The 6% non-covered employees work for companies which are not liable to sectoral agreements. One distinctive feature of the Dutch second pillar is that contributions are payed although there might be breaks in the occupation. The third pillar, private pension insurance, plays a minor role but is a complementary system for high-wage earner and is promoted by taxational governmental grants.

The aim of the French pension sytem is to ensure the living standard of the citizens. It is divided into two parts: the public pension system – regime de base – which provides 50% of the average income and an additional mandatory occupational pension scheme. In 2003 the government raised the minimal contribution time from 37.5 to 41 years and in 2010 to 41.5 years. After the long contribution period agents receive 50% of their average wage income. The government uses the “best 25 years” to determine the average income. In France, families with more than three children benefit from an income-related pension increase. Hence, large families are financially promoted. Maternity leaves are added to the contribution period up to two years per child. Agents whose income is very low or contribute only for a short period to the pension system are granted with a minimal payment, the so-called minimum vieillesse. These benefits are much lower than the minimal wage income. Agents are differentiated in the public as well as in the occupational pension scheme by their professional category. A big difference to Great Britain or the Netherlands is that benefits out of the occupational pension system are not funded but instead based on a pay-as-you-go system. Their benefits are contribution related according to a Bismarck scheme. Private pension insurance is more or less crowded out as agents already receive a high percentage of their net wage income by combining first and second pillar.

Recent Developments

Nowadays the level of pension benefits approximately lies at 70% of the last net wage income and the contribution rate at 19.9% in Germany. Forecasts say that the level of pension benefits will decrease while the contribution rate will increase up to 22% in 2030 (according to the "Altergrenzenanpassungsgesetz)[8]. There are several reasons for this development. Demographic change is one. In many industrialized western countries the population grows old. Real world data demonstrates a median age of the population in several European countries and Japan above 40 years. In Germany the median age is 44.9[9], in Italy it is 43.5 years. For a comparison, the population of India has a median age of 26.2. The share of "old" agents (65 years and older) in the entire population will increase over time in many European countries. In 1995 the share of "old" agents within the population was for example in Germany 15.4%, in the Netherlands 13,2% and in France 15.0%. The forecast for 2050 predicts the share to be: in Germany 27.7%, in the Netherlands 24.4% and in France 26.8%[10]. A time series visualizes the development of the shares of agents under 20 years and the shares of agents above 65 years in the entire population of Germany in a time period from 1890 to 2000. From this it becomes clear that the share of the young population is decreasing (from 44.9% in 1890 to 21.2% in 2000) and the share of the old population is increasing (from 5.1% in 1890 to 16.2% in 2000). The share of the working generation (age 20 to 65) increased from 50% in 1990 to 62.6% in 2000 and is supposed to stabilize around 60% in the future[11]. In 1871 100 agents of the working population faced 9 retirees while forcecasts guess that in 2040 100 working agents will face 56 retirees, which is a dramatic change. The number of retirees increased from 1960 to 2005 from 7.8 to 19.4 million in the old West German states and if we include the newly-formed East German states to 24.5 million. The length of the retirement period increased from 9.9 years up to 17.2

[8]Information is taken from Ruland (2007).

[9]All data is taken from the *World Factbook 2011 by CIA, USA* and demonstrates estimated values for 2011.

[10]Compare G. Wiesner (2001).

[11]Data taken from the Statistisches Bundesamt in G. Wiesner (2001).

years.

All these numbers visualize that future solvency of the implemented public pension system in Germany is disputable. It is not clear whether a PAYG system will be able to support retirees with suitable pension benefits. Therefore, it is often discussed to switch to a fully funded pension system. Such a system is independent of the demographic development, of the unemployment rate and of the economic progress. Also it is often claimed that an FF system earns higher returns for its retirees than a PAYG system. If an FF system is implemented, agents have to take more responsibility to ensure themselves a sufficient old-age wealth. Many papers claim that an FF system leads to higher economic growth due to higher capital accumulation but often neglect the effects on the wage rate and the interest rate. Nevertheless, these two values have a big impact on the welfare of agents. Before a switch of the systems is implemented it should be clearified whether a PAYG or an FF system leads to higher welfare. This work tries to shed light on the comparison of these two different pension systems in their pure form (no tax benefits or any allowances).

The effects of different pension systems on the economy are manifold. Pension systems do not only influence economic growth through higher or lower physical capital accumulation but have an impact on agents' decisions for children and on investments in education of children. This again are factors which favor economic growth. Hence, an analysis which compares pension systems but disregards the impact on individual decisions besides savings neglects an important aspect. The relation between economic growth and individual welfare (measured in individual life-time utility) is ambiguous. In some economic situations it may be possible to draw conclusions directly from this stylized fact but this is not necessarily the case. Also, it might be that one pension system leads to higher inequality (regarding human capital, i.e. wage income) within the economy. Beside the political implication (instability of political formations, riots, etc.) it is not clear whether this prohibits higher welfare. Although the inequality is higher under one system, it might be that the agent's welfare is superior if each agent earns higher wage income under this system.

The main questions, which we try to answer in this work, are:

- Does higher economic growth go hand in hand with higher individual welfare?
- Do different pension systems have diverse impact on the fertility rate?
- Is the decision on investments in education of children influenced by different pension systems in various ways?
- Can the fertility of agents be increased by transfer payments? And does this lead to higher economic growth?
- Which system leads to higher inequality? Is this necessarily welfare lowering?

While the second chapter mainly addresses the first question, chapters 3 and 4 consider various additional questions. Chapter 2 uses a closed economy and elaborates the crucial relation between the state of development of (various) economies, growth and welfare. It incorporates effects of physical capital accumulation on the rate of return which may reduce welfare. The last two chapters employ a small open economy to focus on additional questions like fertility (chapter 3) and education (chapter 4) without the effects on wage and interest rates.

In all three following chapters we use an OLG-Model. This allows us to analyze intertemporal effects and dynamic processes over time. We consider closed as well as small open economies. Although the small open economy assumption is a strict simplification, it offers the possibility of clear statements on the regarded effects of the pension systems. All agents face a certain non stochastic lifespan, i.e. we abstract from the risks of an early demise.

Model Overview

In the second chapter we analyze economic growth and individual welfare (measured as life-time utility) and show that higher economic growth does not necessarily lead to higher individual welfare. Using the structure of a two-period OLG-Model we present a general equilibrium model of a closed economy populated by homogenous agents. Decisions on education and the number of children are neglected. To accomodate the demographic change, observable in many developed economies, we implement a stochastic process for the development of the population size. In our model the population is shrinking over time. Although agents do not make decisions regarding their schooling, we implement an exogenously given human capital formation process in which human capital is growing by a constant factor over time. Our model coincides with the common result that FF leads to higher physical capital accumulation and, therefore, to higher economic growth. Testing whether higher economic growth goes hand in hand with higher welfare, we find that there is not necessarily a positive link. We show that different economies, endowed with different capital intensity factors, do not all benefit (in terms of welfare) from the same pension system. This means that the pension system, which leads in a highly developed economy to higher welfare, may result in lower welfare in a non-developed economy. A statement on higher welfare under one pension system can, therefore, not be given without knowing the capital intensity factor of the economy. The model considered here concentrates on a comparison of pension systems implemented in two identical economies at the same time. Therefore, we neglect the transition from one system to the other.

The third chapter considers the impact of pension systems and payment schemes on the welfare of heterogenous agents, measured in terms of individual life-time utility. One result is that PAYG as well as FF can be the pension system that ensures the higher possible welfare depending on economic parameters. The two major pension systems are considered, each in combination with two different payment schemes: Beveridge (flat) and Bismarck (contribution related). Heterogeneity of agents is one additional aspect which distinguishes our work from many other papers which often assume homogeneity of agents. For simplicity, we introduce two different types of agents: low-skilled (low individual human capital) and high-skilled (high individual human capital). The agents differ according to their human capital endowment, but are equipped with the same preferences. Depending on their individual types agents prefer different payment schemes regarding their individual welfare. Their birth rates also differ under different schemes. High-skilled agents will have at most the same number of children as the low-skilled agents under all four possible combinations of pension systems and payment schemes. Excluding a learning process, this leads to a decrease of average human capital over time. As this results in a lowering of output per capita, which is often used as a measurement of welfare, we ask whether incentives can be introduced to raise fertility of high-skilled agents. By introducing transfer payments it is possible to raise fertility and through this to increase average human capital. As these incentives work in the intended direction we further analyze whether additional advantages like an increase of individuals welfare go hand in hand.

The fourth chapter presents an equilibrium model of a small open economy with inter-generational altruism. In a benchmark model we analyze how different pension systems influence the human capital formation of agents. Agents decide whether and how much (we exclude credit constraints) to invest in the education process of their children. The aim is to show that agents behave differently (concerning education expenditures) under the two different pension systems. Further, a PAYG pension system leads to higher human capital formation and higher economic output than an FF system under certain conditions. We try to figure out whether this is valid in general or whether certain conditions exist under which a PAYG system leads to higher economic growth. As this model uses a small open economy with mobile physical capital and immobile human capital, the only source for economic growth is aggregate human capital. We introduce two different families of agents who are homogenous within the family but heterogenous concerning their human capital endowment between the families. Regarding their preferences and their ability parameter the agents of the two families do not differ. The influence of the pension systems is different on the two families. It is shown that the family which is initially wealthier (higher human capital leads to higher wage income) will remain the wealthier family over time. We analyze under which pension system the inequality concerning the allocation of human capital within a generation is larger and whether higher economic output goes hand in hand with higher inequality or not. The model shows that under certain conditions a positive link between inequality and economic output exists. In contrast, Galor and Zeira (1993) show that higher inequality goes hand in hand with lower economic output. The main difference to our work is the existence of imperfect credit markets and the existence of two different production functions which either employ unskilled or skilled labor. Our work does not compare the economy under the two different pension systems in a steady state analysis but pointwise in each period.

Small Literature Overview

In a closed economy without pension systems Fanti and Gori (2007) and (2010) analyze the effects of child allowance on endogenous fertility. They claim in both papers that child benefits have an ambiguous effect on the fertility rate since opposing effects exist. They figure out a direct positive effect due to reduced costs of child-rearing but as well a negative effect due to the reduced wage rate. In their paper from 2010 they add schooling expenditures which are financed by all agents in the economy. While child grants are constant over time, expenditures on schooling may vary over time. One result is that an increase of schooling expenditures leads to higher fertility and to higher economic growth. Although we use a small open economy, we see the positive effect of increased human capital on economic growth and as well welfare in chapter 3 and 4.

In the paper by Fenge and Weizsäcker (2009) agents divide their time to work and to have children, assuming that each child reduces the labor time by a certain fraction. Implementing a PAYG pension system, the costs of having children remain private while the benefits become public. They introduce a twofold pension system consisting of a Bismarck pension – benefits are payed proportionally to contributions – and a child pension – benefits according to the number of own children. While a Bismarck pension helps to minimize the labor-leisure (child time) distortion of the PAYG system, a child pension minimizes the fertility distortion. They figure out that child pensions and child benefits (in our work: transfer payments) are perfect substitutes.

Wigger (1999) assumes intergenerational care, meaning that old retired agents receive pension payments and presents from their children. Hence, having children is kind of an insurance of future wealth. Presents depend negatively on the contribution rates as higher contributions imply higher pension benefits. One of his results is that higher contribution rates lead to lower fertility rates since agents know that children make less presents and, therefore, lose their significance as an insurance of old-age wealth.

Soares (2006) as well as Lambrecht et al (2005) explore implications of a learning process on economic growth under PAYG pension systems. Due to altruism as well as egoism (higher pensions) parents finance the learning process of their children and, therefore, improve their human capital which leads to higher economic growth and to higher pension payments.

Kaganovich and Zilcha (2012) present a general equilibrium model in which they implement a human capital formation rule. Public education expenditures are determined by a majoritarian vote. They show that an FF pension leads to higher tax rates for education expenditures than a PAYG system and through this to higher human and physical capital accumulation and to higher economic growth. Kaganovich and Meier (forthcoming) show that the opposite is true in a small open economy. The fourth chapter of this work is similar to those two papers. While using a small open economy we draw our attention to private investments in schooling of children instead of publicly determined education expenditures. Our model uses private schooling as an additional optional part in the human capital formation. It shows that parents invest in education if their preferences for childrens' human capital are strong enough, i.e. if the intergenerational altruism is strong enough.

Glomm and Kaganovich (2008) implement a model of heterogenous agents in which the government funds two programs: public education and social security. Agents contribute a proportional tax to both programs while the benefits which they receive are flat. Parental time and public schooling expenditures are complementary goods in the human capital formation. They show that an increase of the social security tax rate has a reducing effect on the income inequality and on the same time a positive effect on the economic growth. A comparison of the effects under FF and PAYG pension systems is neglected. Differences to our results in chapter 4 can be explained by different human capital formations and the assumption of a

closed economy.
Galor and Zeira (1993) analyze the relation between inequality and growth as well but without the existence of any pension system. One result of their work is that higher inequality goes hand in hand with lower economic output. The main difference to our results in chapter 4 arises through the existence of imperfect credit markets and the existence of two different production functions which either employ unskilled or skilled labor.

Why is a Switch of Pension Systems so difficult?

The difficulty of a transition from a PAYG pension system to an FF one lies in the fact that agents who have already contributed during their whole work-life to the PAYG pension system will not receive any pension payments out of the new implemented FF system. The switch of the pension systems is a clear cut. So far, retirees received pension payments out of the contributions from working agents. After the transition their pension payments will be the return on their own contributions. Therefore, agents, who contributed only to the PAYG system, do not receive anything out of the FF system. Of course, it is a huge social problem how to compensate the generations who contributed (partly) to the PAYG system and face no (or low) public pension payments after the switch. A compensation must assure that these generations are not worse off after the transition.
As a PAYG system is just a pure reallocation of wage-income from the working population to the retired population, there are no financial reserves, which could be used for compensating the transition generation. A switch from one system to the other always includes a structural break and it is not reasonable that this can be done without lowering the welfare of at least some generations. There are two possible szenarios for financing a compensation of the transition generation supposably. The first would be to tax agents by a higher rate until the transition period. The government would save/invest these tax revenue and would finance the compensation payments out of the returns in the crucial period. This would lead to a lowering of the welfare of all agents who would face the higher tax burden. The advantage of this szenario is that after the transition no additional payments are necessary

and only a pure FF pension system exists. The second possibility would be that the government pays the compensation payments financed by a debt and increases the tax rate after the transition for some generations. The compensation payments would have to be high enough so that the transition generation would receive the same welfare as if a PAYG pension system would still be implemented. The tax rate would have to be adjusted in such a way that the generations facing the higher tax burden would not be worse off compared to the case of a PAYG system. This could be achieved if an FF pension system led to higher welfare in general. Then it is possible to compensate the transition generation without setting any generation worse off in terms of life-time utility. In 2000/2001 the German government introduced the so-called “Riester-Rente”. This is a privately financed pension system with the structure of an FF system. Under certain conditions (minimal contributions – 4% of the net wage income) the government advances contributions to this private pension system by granting fringe benefits and tax reliefs. The German government introduced this kind of private pension system because the net-pension-standard was reduced from 70% to 67% during the pension-reform. Since 2005 the German goverment has advanced the “Rürup-Rente” by granting tax benefits as well.

Chapter 2

Pension Systems and their Meaning for Welfare

Abstract: The implementation of a fully funded (FF) pension system in an economy leads to higher economic growth than the implementation of a pay-as-you-go (PAYG) system. But it is not clear how pension systems influence the utility of the population through their effect on wage-rates and rates of return on capital. This chapter shows that PAYG leads to higher welfare under specific conditions. Economies, which vary in their capital intensity factors, generate higher welfare of their agents under different pension systems. Using a closed economy with stochastic population growth we present a general equilibrium model and analyze the relation between economic growth and individual agents' welfare.

2.1 Introduction

As the population of most industrialized countries becomes older and older over time, the question of how to ensure their old-age wealth becomes more important. In these countries we observe a demographic change and the old generation will become the dominant fraction of the population. Real world data demonstrates a median age of the population in several European countries and in Japan above 40 years of age. In Germany the median age is 44.9[1], in Italy it is 43.5 years for instance. For a comparison, the population of India has a median age of 26.2. The *World Factbook 2011* provides time series which indicate that the population in many countries grows old. This may be due to better health care, healthier ways of life, less hard work and many more. In most European countries, except the Netherlands, the old-age security system is organized in the form of a pay-as-you-go (PAYG) pension system. In this case, the young working part of the population pays for the pension payments of the old and retired part of the population. Hence a PAYG system is a pure reallocation of income from young working agents to old retired agents. As the fraction of young working individuals is shrinking, they have to contribute indivually more from their wage income or the retirees will receive a lower pension payment by constant contribution rates. Due to the demographic change, doubts arise whether a PAYG pension system will remain sustainable. Often discussed is a transition from PAYG to a fully funded (FF) pension system to solve the problem of less contributors and more beneficiaries. The PAYG system is not suited to provide an adequate level of retirement payment at acceptable costs for the contributors. Some observers think that the existing pension system has to be changed in a drastic way as they see it already on the "verge to collapse" Börsch-Supan (1998). The health of the pension system is of major interest, as Diamond, Orzag (2005) exposed for the USA that for two-thirds of the elderly beneficiaries the pension payment is the majority of income and for 20 percent of the beneficiaries it is their only income.

[1]All data is taken from the *World Factbook 2011 by CIA, USA* and demonstrates estimated values for 2011

Before taking the difficult task of a transition from a PAYG pension system to an FF one, it should be clear whether an FF system is the favorable option. One aspect is economic growth under both pension systems, which is considered by many papers in the existing literature. A different aspect is individual welfare. In this paper we analyze both and show that higher economic growth does not necessarily lead to higher individual welfare, which is scaled as life-time utility. Using the structure of a two-period OLG-Model we present a general equilibrium model in which we use a closed economy populated by homogeneous agents. Decisions on education and the number of children are neglected. To accomodate the demographic change we implement a stochastic process for the development of the population size. In our model the population is shrinking over time. Although agents do not make decisions regarding their schooling, we implement an exogenously given human capital formation process in which human capital is growing by a constant factor over time. Our model coincides with the common result that FF leads to higher physical capital accumulation and, therefore, to higher economic growth. Analyzing whether higher economic growth goes hand in hand with higher welfare shows not necessarily a positive correlation. Different economies, endowed with different capital intensity factors, do not all benefit (in terms of welfare) from the same pension system. This means, that the pension system which leads in a highly developed economy to higher welfare, may result in lower welfare in a non-developed economy. A statement on the dominance of one pension system can, therefore, not be given without knowing the capital intensity factor of the economy. Our considerations focus on a comparison of pension systems implemented in two identical economies at the same time. Therefore, we neglect the transition from one system to the other.

2.2 The basic model

We consider an overlapping generations economy in which perfectly competitive firms produce a homogeneous good, which can be used for consumption and for investment. They use physical and human capital as inputs in a constant returns neoclassical production technology $F(K, H)$. As we assume a closed economy, physical and human capital are immobile and remain within the economy. The population lives for exactly two periods – adulthood and retirement – and each generation consists of a number of homogeneous agents, who are all endowed with a certain amount of human capital. As we want to study the effects of demographic change and human capital, the size of each generation does not remain constant over time, hence we assume population growth.

The production technology can be described by a classical Cobb-Douglas production function:

$$Y_t = F(K_t, H_t) = A\, K_t^{\delta}\, H_t^{1-\delta} = A\, K_t^{\delta}\, (N_t\, h_t)^{1-\delta} \tag{2.2.1}$$

with $A > 0, 0 < \delta < 1$. Y_t describes total output in period t, K_t is the aggregate stock of physical capital financed by the savings of the previous generation (depending on the type of pension system). H_t denotes the aggregate stock of human capital. All agents supply inelastically one unit of labor in their first period of life. The aggregate stock of human capital $H_t = N_t h_t$ used for production is given by the number of young/adult agents (N_t) times their individual endowment of human capital (h_t). The aggregate stock of physical capital is determined in different ways under the two types of pension systems. Capital is assumed to depreciate completely from one period to the next. Hence aggregate capital never contains old capital. In case of a PAYG pension system the aggregate capital in $t+1$ consists of the aggregate savings s_t of the young agents in period t. Therefore,

$$K_{t+1}^G = N_t\, s_t^G. \tag{2.2.2}$$

Variables belonging to the PAYG system are indexed by G, variables belonging to the fully funded system by F.

The characteristic of a fully funded pension system is that the contributions are invested/saved in the production process and are not only reallocated from young to old agents like under a PAYG system. Hence, aggregate capital consists of private savings and public savings (from the pension system):

$$K_{t+1}^F = N_t\, s_t^F + N_t\, \tau\, w_t^F\, h_t, \tag{2.2.3}$$

where τ denotes the contribution rate. Since the FF pension system is assumed to withold no part of the contributions to cover any costs or to gain proceeds, aggregate contributions are completly invested. Factor markets are competitive so that prices of physical and human capital are given by their marginal products:

$$w_t = \frac{\partial F(K_t, H_t)}{\partial H_t} = (1-\delta)\, A\, K_t^{\delta} H_t^{-\delta} = (1-\delta)\, A\, k_t^{\delta} \tag{2.2.4}$$

$$R_{t+1} = \frac{\partial F(K_{t+1}, H_{t+1})}{\partial K_{t+1}} = \delta\, A\, K_{t+1}^{\delta-1}\, H_{t+1}^{1-\delta} = \delta\, A\, k_{t+1}^{\delta-1} \tag{2.2.5}$$

where $k_t = \frac{K_t}{N_t\, h_t}$ denotes capital in efficiency units. The homogeneous agents live exactly for two periods, i.e. there is no risk of an early demise. In their first period of life, i.e. adulthood, they inelastically supply one unit of labor. In the second period they are retired and do not work, therefore, gain no wage income. Each member of a generation t, adult in period t, is endowed with identical individual human capital denoted by h_t. We assume that the individual human capital grows by an exogenously given factor from one generation to the next. The human capital formation is described by:

$$h_{t+1} = \beta\, h_t \tag{2.2.6}$$

where $\beta > 1$ denotes the exogenously given growth factor of human capital. It directly follows that a generation $t+1$ is more skilled than its preceding generation t. Hence, the quality of labor, offered by the young agents, is higher than the one of their ancestors. If the population size would remain constant over time, it would result in a continuosly growing aggregate stock of human capital. As the population does not remain constant over time but instead decreases, it is not clear whether the aggregate stock of human capital is increasing, decreasing or remains constant over time. The population is growing with a stochastic growth factor, denoted by n_{t+1}. The size of the population is given by:

$$N_{t+1} = (1 + n_{t+1})\, N_t. \tag{2.2.7}$$

n_{t+1} is i.i.d. and uniformly distributed over an intervall $[a, b]$ for all t, with $a < b$. If we want to simulate a decreasing population (demographic change in the western industry economies), at least the parameter a has to be negative and its absolute value has to be greater than that of b. The size of the future generation N_{t+1} is not known by the generation t in period t.

2.3 Individual decisions

In this section our focus lies on the single agent and his decisions regarding savings and consumption. Since he lives for exactly two periods but receives wage income (which can be consumed or invested) only in the first period, he has to ensure consumption in both periods via savings and/or pension payments. As his wage income is non-storable (for later consumption), he invests part of it in the production process. In the second period of his life he receives returns on his investment and pension payments, which he consumes completly as we do not assume any bequest motives.

2.3.1 Decisions under PAYG

The PAYG pension system is the predominant system in most industrialized countries. It is characterized by a reallocation of wage income. Young working agents are forced by the government to contribute a fixed part of their wage income (contributions) to the pension system. The PAYG system takes these contributions but does not invest them in the production process. Instead it gives it directly to the living old-age retirees (beneficiaries).

The agents receive utility out of consumption in both periods. As they have to make their decision regarding today's and tomorrow's consumption (e.g. savings) at the beginning of their first period, we describe their expected utility by the following equation:

$$\mathbb{E}_t\left[U^G(c_{t,t}, c_{t+1,t})\right] = \ln c^G_{t,t} + \mathbb{E}_t[\ln c^G_{t+1,t}] \tag{2.3.1}$$

where $c_{t,t}$ denotes consumption in period t of an agent born/adult in t and $c_{t+1,t}$ is his consumption in $t+1$. As consumption in adulthood depends on common variables only, it is not necessary to regard the expected value of today's consumption. The resulting utility out of future consumption is uncertain in period t because of the uncertain development of the population. All variables of period $t+1$ are random at the point of time t. The young working agents allocate their after-tax (after contribution) wage income W_1 between current consumption and private savings.

Hence they face the following budget constraint in their first period:

$$\begin{aligned} c^G_{t,t} &= W^G_1 - s^G_t \\ &= (1-\tau)\, w^G_t\, h_t - s^G_t \end{aligned} \tag{2.3.2}$$

where w_t is the current competitive wage rate per unit of effective labor. Effective labor means labor multiplied by the individual human capital which symbolizes how productive one unit of labor is. τ is the uniform and flat contribution rate, which agents take as given. Their decision regarding private savings is not limited to positive values. That means, they are allowed to borrow against their future wealth. The interest rate is assumed to be the same for borrowing as well as for lending. Negative savings are equal to credit taking and can be caused by appreciating today's consumption more than tomorrows. Each agent of the same generation is supposed to receive an equal amount of benefits from the pension system while retired. These benefits as well as returns on private savings are no objects of taxation. The total amount of contributions from the new working generation is collected by the pension system and passed to the old generation without any deductions for administration costs. Our agents are assumed to be self-concerned and only value their own consumption, i.e. there are no bequest motives. Therefore, we write the individual budget constraint of an old-age retiree in the following way (which is random at the point of time t)

$$\begin{aligned} c^G_{t+1,t} &= R^G_{t+1}\, s_t + W^G_2 \\ &= R^G_{t+1}\, s^G_t + \frac{N_{t+1}}{N_t}\,\tau\, w^G_{t+1}\, h_{t+1} \\ &= R^G_{t+1}\, s^G_t + (1+n_{t+1})\,\tau\, w^G_{t+1}\,\beta\, h_t, \end{aligned} \tag{2.3.3}$$

where W^G_2 are the benefits an old-age retiree receives from the pension system. The gross rate of return on savings is given by $R^G_{t+1} s^G_t$. w^G_{t+1} denotes the wage rate of the future generation (born in $t+1$) and h_{t+1} is future individual human capital. Each agent derives his decision belonging to his decision variables $s^G_t, c^G_{t,t}, c^G_{t+1,t}$ by maximizing his expected utility of life-time consumption given in (2.3.1) with respect to the budget constraints given by (2.3.2) and (2.3.3).

$$\max_{s_t, c_{t,t}, c_{t+1,t}} \mathbb{E}_t\left[U^G(c_{t,t}, c_{t+1,t})\right] = \max_{s_t, c_{t,t}, c_{t+1,t}} \ln c^G_{t,t} + \mathbb{E}_t[\ln c^G_{t+1,t}]. \tag{2.3.4}$$

Solving the system we obtain from the FOC[2]:

$$s_t^G = \frac{1-\tau}{2+\tau\,(1-\delta)\,\delta^{-1}}\,w_t^G\,h_t. \tag{2.3.5}$$

Due to the logarithmic form of the utility function, the demography factor n_{t+1} is cancelled out and, therefore, the insecurity belonging to the benefit payments from the yet unknown group of contribution payers plays no role in the decision process. If we substitute optimal savings, given by (2.3.5), in equation (2.3.2), we receive consumption in adulthood as:

$$\begin{aligned} c_{t,t}^G &= \frac{1+\tau\,(1-\delta)\,\delta^{-1}}{2+\tau\,(1-\delta)\,\delta^{-1}}\,(1-\tau)\,w_t^G\,h_t \\ &= \frac{1+\tau\,(1-\delta)\,\delta^{-1}}{2+\tau\,(1-\delta)\,\delta^{-1}}\,(1-\tau)\,(1-\delta)\,A\,(k_t^G)^{\delta}\,h_t. \end{aligned} \tag{2.3.6}$$

By plugging (2.3.5) in equation (2.3.3) and then applying formula (2.2.5), we receive consumption in the second period of life as

$$\begin{aligned} c_{t+1,t}^G &= R_{t+1}^G\,\frac{1-\tau}{2+\tau\,(1-\delta)\,\delta^{-1}}\,w_t^G\,h_t + (1+n_{t+1})\,\tau\,w_{t+1}^G\,h_{t+1} \\ &= \delta A(k_{t+1}^G)^{\delta-1}\frac{1-\tau}{2+\tau(1-\delta)\delta^{-1}}w_t^G h_t + (1+n_{t+1})\tau(1-\delta)A(k_{t+1}^G)^{\delta}h_{t+1}. \end{aligned} \tag{2.3.7}$$

Knowing (2.3.5), we now determine the aggregate stock of capital under a PAYG regime, which is given by the aggregate savings of the previous generation:

$$K_{t+1}^G = N_t\,\frac{1-\tau}{2+\tau\,(1-\delta)\,\delta^{-1}}\,w_t^G\,h_t \tag{2.3.8}$$

$$k_{t+1}^G = \frac{1}{1+n_{t+1}}\,\frac{(1-\tau)\,(1-\delta)}{\beta(2+\tau\,(1-\delta)\,\delta^{-1})}\,A\,(k_t^G)^{\delta}. \tag{2.3.9}$$

The determination of K_{t+1}^G and k_{t+1}^G uses a Markovian process. Hence, by knowing today's variables w_t^G, h_t, N_t, k_t^G we can determine tomorrow's capital stock. This relationship shows that tomorrow's aggregate stock of capital, used in the production technology, is uniquely determined. Using this expression for capital in efficiency

[2] To receive s_t in this form, we use the equality $\tilde{k}_{t+1}^G = \frac{N_t\,s_t^G}{(1+\tilde{n}_{t+1})\,N_t\;\beta\;h_t}$ which is the capital in efficiency units in period $t+1$. k_{t+1} is a random variable in period t

units in equation (2.3.7), we calculate tomorrow's consumption by

$$c^G_{t+1,t} = \left(\frac{1}{1+n_{t+1}}\right)^{\delta-1} \left(\frac{(1-\tau)(1-\delta)}{2+\tau(1-\delta)\delta^{-1}}\right)^{\delta} h_t A^{\delta+1}\beta^{1-\delta}\delta \left(1+\tau(1-\delta)\delta^{-1}\right) (k^G_t)^{\delta^2}. \tag{2.3.10}$$

Later we will compare the expected life-time utility of an agent under PAYG and under FF. It is more interesting to compare utilities in the long-run instead of directly after introducing the appropriate pension system, as it is possible to achieve fluctuations of outcome through the introduction process. These fluctuations are supposed to be cancelled out in the long run. Hence we need a conditional equation which determines the capital in efficiency units for an arbitrary period t. The law of motion of k^G_t is given by

$$k^G_t = \frac{1}{1+n_t}\left(\frac{1}{1+n_{t-1}}\right)^{\delta} \cdots \left(\frac{1}{1+n_1}\right)^{\delta^{t-1}} \left(\frac{(1-\delta)(1-\tau)}{\beta(2+\tau\,(1-\delta)\,\delta^{-1})}\,A\right)^{\sum_{i=1}^{t}\delta^{i-1}} k_0^{\delta^t}. \tag{2.3.11}$$

Initial capital in efficiency units is given by $k_0 = \frac{K_0}{H_0}$. We assume that the initial endowment of physical and human capital and the initial size of the population is the same under both pensions systems, hence we drop the superscript G in equation (2.3.11). In the case of a fixed demographic factor n the prefactor would be simplified to $\left(\frac{1}{1+n}\right)^{\sum_{i=1}^{t}\delta^{i-1}}$.

2.3.2 Decisions under Fully Funded

There is a significant dissimilarity in the way the two different types of pension systems work. While the PAYG systems collects contributions from the young agents and transmits them directly to the retirees, the FF system collects the contributions, but then saves them for one period. The savings of the FF pension system are invested in the production technology and earn a return. The single agent is equipped with the same logarithmic and additively separable utility function as under the PAYG system. But as the FF systems works in a varied way, the pension benefit payments are defined differently. So the individual budget constraints differ:

$$
\begin{aligned}
c_{t,t}^F &= (1-\tau)\, w_t^F\, h_t - s_t^F && (2.3.12)\\
c_{t+1,t}^F &= R_{t+1}^F\, s_t^F + W_2^F \\
&= R_{t+1}^F\, s_t^F + R_{t+1}^F \tau\, w_t^F\, h_t && (2.3.13)
\end{aligned}
$$

Equation (2.3.13) demonstrates the significant difference between FF and PAYG. Under a PAYG system the pension benefits depend on the wage income of the succeeding generation. In contrast, now they depend only on current wage income and on the rate of return on savings. In the literature it is often claimed that the aggregate stock of capital is higher under FF than under PAYG. We will later examine the validity of this statement. The decision problem is expressed by

$$
\max_{s_t, c_{t,t}, c_{t+1,t}} \mathbb{E}_t[U^F(c_{t,t}, c_{t+1,t})] = \max_{s_t, c_{t,t}, c_{t+1,t}} \ln c_{t,t}^F + \mathbb{E}_t[\ln c_{t+1,t}^F] \qquad (2.3.14)
$$

From the FOC we receive:

$$
s_t^F = \frac{1}{2}\,(w_t^F\, h_t - 2\,\tau\, w_t^F\, h_t) = \frac{1-2\tau}{2}\, w_t^F\, h_t \qquad (2.3.15)
$$

which will be negative (credit taking) for contribution rates being higher than 50 percent. Due to the logarithmic form of the utility function the rate of return is cancelled out, so that optimal savings do not depend on the stochastic population growth factor. The savings decision is non-random and depends only on well-known variables. Substituting (2.3.15) in equation (2.3.12) yields the current consumption in period t

$$
\begin{aligned}
c_{t,t}^F &= (1-\tau)\, w_t^F\, h_t - \frac{1}{2}\,(w_t^F\, h_t - 2\,\tau\, w_t^F\, h_t) = \frac{1}{2}\, w_t^F\, h_t \\
&= \frac{1}{2}\,(1-\delta)\, A\, (k_t^F)^\delta\, h_t. && (2.3.16)
\end{aligned}
$$

By plugging (2.3.15) into (2.3.13) and applying (2.2.5), consumption in the second period is given by

$$
\begin{aligned}
c_{t+1,t}^F &= R_{t+1}\, (\frac{1}{2}\, w_t^F\, h_t) \\
&= \delta\, A^2\, (k_{t+1}^F)^{\delta-1}\, \frac{1}{2}\,(1-\delta)\,(k_t^F)^\delta\, h_t. && (2.3.17)
\end{aligned}
$$

Hence, the expected life-time utility under FF is independent of the contribution rate τ. This is due to the fact that private savings by agents and investments in the FF pension system are perfect substitutes since the system as well as the agents invest in the same production process. After determining optimal savings in (2.3.15) and knowing that all agents are homogeneous we can determine the aggregate stock of physical capital by using equation (2.2.3)

$$\begin{aligned} K_{t+1}^F &= N_t \, \frac{1-2\tau}{2} \, w_t^F \, h_t + N_t \, \tau \, w_t^F \, h_t \\ &= \frac{1}{2} \, N_t \, (1-\delta) \, A \, (k_t^F)^\delta \, h_t. \end{aligned} \tag{2.3.18}$$

The future capital in efficiency units can be written as

$$\begin{aligned} k_{t+1}^F &= \frac{K_{t+1}^F}{N_{t+1} \, h_{t+1}} \\ &= \frac{1}{1+n_{t+1}} \, \frac{1-\delta}{2\,\beta} \, A \, (k_t^F)^\delta. \end{aligned} \tag{2.3.19}$$

The size of the population is independent of the selected type of pension system, hence, we drop any superscript belonging to the development of the population. Using (2.3.19) for the determination of the future consumption in equation (2.3.17) we receive

$$c_{t+1,t}^F = (1+n_{t+1})^{1-\delta} \, A^{\delta+1} \, \beta^{1-\delta} \, \left(\frac{1-\delta}{2}\right)^\delta \, \delta \, h_t \, (k_t^F)^{\delta^2}. \tag{2.3.20}$$

As we want to express capital in efficiency units in any period $t > 0$ as a function of the initial parameters δ, τ, β and k_0 and the realized values of the stochastic variable n_{t+1}, we receive the following law of motion:

$$k_t^F = \frac{1}{1+n_t} \left(\frac{1}{1+n_{t-1}}\right)^\delta \ldots \left(\frac{1}{1+n_1}\right)^{\delta^{t-1}} \left(\frac{1-\delta}{2\,\beta} \, A\right)^{\sum_{i=1}^t \delta^{i-1}} k_0^{\delta^t}. \tag{2.3.21}$$

2.3.3 Determination of τ - social planner

In this section we will analyze the contribution rate which is chosen by a social planner. The previous section shows that the contribution rate has a direct effect on the optimal savings under a PAYG regime. So it is clear that τ also has an

influence on expected utility. Under an FF regime τ does not have any impact neither on the optimal savings nor on the current and future consumption. This leads to the conclusion that τ has no impact on the expected life-time utility of an agent. We will indeed proof this later.
As all agents in our economy are homogeneous, the optimal contribution rate will be the same for every agent.

Contributions under PAYG

The contribution rate τ lies in the interval $[0,1]$. Contribution rates smaller than 0 are not compatible with the principle of a pension system as this would mean that young agents borrow from their parents. This is not allowed since they cannot repay they debt in the next period as the parents are not alive anymore. Agents are allowed to borrow against their future wealth by choosing a negative savings amount, but not by choosing a negative contribution rate. A rate higher than 1 is also not possible as the agents do not have any initial endowment out of which they could pay the contribution. Hence, the complete amount of their wage income is the maximal contribution to the pension system.
The social planner maximizes the expected life-time utility of all agents with regard to their optimal savings decision. With $d := \tau\,(1-\delta)\,\delta^{-1}$ the objective function is given by:

$$\begin{aligned}
\max_{\tau} \mathbb{E}_t[U^G(\cdot,\cdot)] = &\max_{\tau} \ln\left(\frac{1+d}{2+d}\,(1-\tau)\,(1-\delta)\,A(k_t^G)^{\delta}\,h_t\right) \\
&+ \mathbb{E}_t\left[\ln\left(\left(\frac{1}{1+n_{t+1}}\right)^{\delta-1}\left(\frac{(1-\tau)\ (1-\delta)}{\beta\,(2+d)}\right)^{\delta} h_t\,A^{\delta+1}\,\beta^{1-\delta}\,\delta\ (1+d)\,(k_t^G)^{\delta^2}\right)\right] \\
= &\ln\left(\frac{1+d}{2+d}\,(1-\tau)\,(1-\delta)\,A(k_t^G)^{\delta} h_t\right) \\
&+ \ln\left(\left(\frac{(1-\tau)\ (1-\delta)}{\beta\,(2+d)}\right)^{\delta}\ h_t\,A^{\delta+1}\,\beta^{1-\delta}\,\delta\ (1+d)\,(k_t^G)^{\delta^2}\right) \\
&+ \mathbb{E}_t\left[\ln\left(\frac{1}{1+n_{t+1}}\right)^{\delta-1}\right].
\end{aligned}$$

From the FOC we receive two values for the optimal contribution rate τ.

$$\tau_1(\delta) = \frac{\frac{1}{4}\delta^2 + 2\,\delta - \frac{1}{4} - \frac{1}{4}\,\sqrt{\delta^4 + 8\delta^3 + 14\delta^2 + 8\delta + 1}}{\delta - 1} \tag{2.3.22}$$

$$\tau_2(\delta) = \frac{\frac{1}{4}\delta^2 + 2\,\delta - \frac{1}{4} + \frac{1}{4}\,\sqrt{\delta^4 + 8\delta^3 + 14\delta^2 + 8\delta + 1}}{\delta - 1} \tag{2.3.23}$$

The latter is always negative for all possible values of δ and, therefore, is economically non reasonable. The first $\tau-$value approves positive values for a meaningful interval of $\delta-$values.

Proposition 2.3.1. *The contribution rate, chosen by a social planner, is unique and well-defined for given δ . It is given by*[3]*:*

$$\tau^*(\delta) = \begin{cases} \frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}-\frac{1}{4}\,\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1} & if\ \delta \in [0,\hat{\delta}] = [0, 2\,\sqrt{3} - 3] \\ 0 & else \end{cases}$$

This means that a PAYG pension system is only operating if the capital intensity factor δ takes values in an appropriate interval $\delta \in [0, 2\,\sqrt{3} - 3]$. Otherwise the optimal contribution rate is zero. If the capital intensity is too high, a negative contribution rate will result. This would mean that the retired agents have to hand over commodities to the young generation. As this would contradict the function of a pension system, the contribution rate is limited from below by zero. A zero contribution rate would mean that de facto no pension system exists, hence an economy without a pension system is optimal.

Contributions under FF

As previously claimed, the decision variables of an agent do not depend on the contribution rate τ under an FF pension system. The objective function of the social planner with respect to the agents' optimal savings decision is given by

$$\begin{aligned}\max_{\tau} \mathbb{E}_t[U^F(\cdot,\cdot)] &= \max_{\tau} \ln\left(\frac{1}{2}(1-\delta)A(k_t^F)^\delta h_t\right) + \mathbb{E}_t\left[\ln\left(A^{\delta+1}\beta^{1-\delta}\left(\frac{1-\delta}{2}\right)^\delta \delta\, h_t(k_t^F)^{\delta^2}\right)\right] \\ &= \ln\left(\frac{1}{2}(1-\delta)A(k_t^F)^\delta h_t\right) + \ln\left(A^{\delta+1}\beta^{1-\delta}\left(\frac{1-\delta}{2}\right)^\delta \delta h_t(k_t^F)^{\delta^2}\right) + \mathbb{E}_t\left[(1+n_{t+1})^{1-\delta}\right].\end{aligned}$$

[3] For a proof see the Appendix.

Hence, the social planner can choose any contribution rate $\tau \in [0,1]$ and it will be optimal. The reason for this is that private savings and contributions to the fully funded pension system are perfect substitutes.

2.4 Comparison of the capital in efficiency units

In this section we analyze the development of the physical capital stock (in efficiency units) under the two different pension regimes. The height of the physical capital stock is decisive for the economy's economic perfomance, even more in case of a high capital intensity factor δ. Capital growth stimulates the expansion of the production output, but, nevertheless, it is not clear whether the life-time utility of an agent grows as well. In the first step we will check whether clear statements concerning the comparison of the capital under both systems are possible. The next step will be to prove whether one pension system allows higher growth rates of the capital over time than the other.

Human capital formation and population growth are independent of the pension regime in this model. This means, human capital as well as population size are the same under both systems in all periods $t \geq 0$. This is even more important for the stochastic demography factor n_t. We assume its realizations to be the same, no matter which system is introduced. If a clear statement about the proportion of the capital in efficiency units is possible, the same would be true for the ranking of the aggregate stock of capital and the production output under both pension systems. In most papers, for example Kaganovich and Zilcha (2012), it is claimed that under the FF system a higher aggregate stock of physical capital is received. Define $\pi(n) := \frac{1}{1+n_t}\left(\frac{1}{1+n_{t-1}}\right)^{\delta} \cdots \left(\frac{1}{1+n_1}\right)^{\delta^{t-1}}$ where $n := (n_1, n_2, \cdots, n_{t-1}, n_t)$ is a vector of all realized population growth rates up to period t. A comparison of the capital in efficiency units yields the following equivalence relation:

$$
\begin{aligned}
k_t^G &< k_t^F \\
\Longleftrightarrow \pi(n)\left(\frac{(1-\delta)\,(1-\tau)}{\beta(2+\tau\,(1-\delta)\,\delta^{-1})}\,A\right)^{\sum_{i=1}^{t}\delta^{i-1}} k_0^{\delta^t} &< \pi(n)\left(\frac{1-\delta}{2\,\beta}\,A\right)^{\sum_{i=1}^{t}\delta^{i-1}} k_0^{\delta^t} \\
\Longleftrightarrow \frac{(1-\delta)\,(1-\tau)}{\beta(2+\tau\,(1-\delta)\,\delta^{-1})} &< \frac{1-\delta}{2\,\beta} \\
\Longleftrightarrow \frac{1-\tau}{2+\tau\,(1-\delta)\,\delta^{-1}} &< \frac{1}{2} \qquad (2.4.1)
\end{aligned}
$$

The last inequality is true for all $\tau, \delta \in (0,1)$. From this follows that the capital in efficiency units is always higher under an FF than under a PAYG pension system. This holds true for any arbitrary period t: $k_t^G < k_t^F \ \forall\, t \geq 1$. Hence, the production level is higher under FF than under PAYG. This goes hand in hand with a higher wage income for the agents. The fact that physical capital is always higher under FF than under PAYG in any period t does not necessary mean that the proportion of the growth rates of capital behave in the same way. It could be that the growth rate is higher under PAYG and hence the two aggregate stocks of capital approximate. We will check how the growth rates behave in any arbitrary period t. Define the growth rates under both regimes as:

$$
g^G = \frac{k_{t+1}^G}{k_t^G} = \frac{1}{1+n_{t+1}}\left(\frac{1}{1+n_t}\right)^{\delta-1}\cdots\left(\frac{1}{1+n_1}\right)^{\delta^t-\delta^{t-1}}\left(\frac{(1-\delta)(1-\tau)}{\beta(2+d)}A\right)^{\delta^{t+1}} k_0^{\delta^{t+1}-\delta^t} \qquad (2.4.2)
$$

$$
g^F = \frac{k_{t+1}^F}{k_t^F} = \frac{1}{1+n_{t+1}}\left(\frac{1}{1+n_t}\right)^{\delta-1}\cdots\left(\frac{1}{1+n_1}\right)^{\delta^t-\delta^{t-1}}\left(\frac{1-\delta}{2\beta}A\right)^{\delta^{t+1}} k_0^{\delta^{t+1}-\delta^t} \qquad (2.4.3)
$$

The question is: is the growth rate of capital lower under PAYG than under FF?

$$
\begin{aligned}
g^G &\overset{?}{<} g^F \qquad (2.4.4) \\
\Longleftrightarrow \frac{1-\tau}{2+\tau\,(1-\delta)\,\delta^{-1}} &< \frac{1}{2}
\end{aligned}
$$

The above inequality shows that the answer is: yes. Not only the capital in efficiency units is higher under FF, but also the growth rate is higher. This excludes the possibility of a PAYG system to overhaul its lag concerning the aggregate capital. From this follows that the production outputs will diverge over time.

2.5 Welfare Comparison

So far we have made clear statements about the capital development under the two different pension systems and saw that a fully funded pension system leads to a higher physical capital stock. Nevertheless, we are interested in the individual welfare of a single agent, which is influenced by a pension system. This characteristic should be the basis for the decision, whether to switch from PAYG to FF or not. The aggregate stock of capital has positive effects on wage income, but it has a negative influence on the rate of return on capital. Both effects are decisive for the development of the welfare/expected utility. As agents are homogeneous, aggregate welfare is represented by the welfare of a representative agent.

Society's prevalent opinion is that an FF ensures sustainability of the pension system and higher welfare for agents. In this section we will show whether welfare is higher under FF or under PAYG. We will make pointwise comparisons, hence we analyze the life-time utility of a single agent in any arbitrary period t under both systems. The aggregation of welfare over all periods up to period t is not considered. Hence, the comparison is given by the following equivalence relation:

$$\begin{aligned} & \mathbb{E}_t\left[U^F(c^F_{t,t}, c^F_{t+1,t})\right] > \mathbb{E}_t\left[U^G(c^G_{t,t}, c^G_{t+1,t})\right] \\ \Longleftrightarrow\ & \mathbb{E}_t\left[\ln(c^F_{t,t}) + \ln(c^F_{t+1,t})\right] > \mathbb{E}_t\left[\ln(c^G_{t,t}) + \ln(c^G_{t+1,t})\right] \\ \Longleftrightarrow\ & \mathbb{E}_t\left[\ln\frac{W^F_1 - s^F_t}{W^G_1 - s^G_t}\right] > \mathbb{E}_t\left[\ln\frac{R^G_{t+1}\, s^G_t + W^G_2}{R^F_{t+1}\,(s^F_t + \tau\, w^F_t\, h_t)}\right]. \end{aligned} \tag{2.5.1}$$

By using the expressions for consumption during adulthood and retirement and (in a second step) the laws of motion under both pension systems we can rewrite the above expression in the following way:

$$\begin{aligned} & \frac{\frac{1}{2}(1-\delta)A(k^F_t)^\delta h_t}{\frac{1+d}{2+d}(1-\tau)(1-\delta)A(k^G_t)^\delta h_t} > \frac{\left(\frac{1}{1+n_{t+1}}\right)^{\delta-1}\left(\frac{(1-\tau)(1-\delta)}{2+d}\right)^{\delta} h_t A^{\delta+1}\beta^{1-\delta}\delta(1+d)(k^G_t)^{\delta^2}}{(1+n_{t+1})^{1-\delta}A^{\delta+1}\beta^{1-\delta}\left(\frac{1-\delta}{2}\right)^{\delta}\delta h_t(k^F_t)^{\delta^2}} \\ \Longleftrightarrow\ & \left(\frac{2+\tau(1-\delta)\delta^{-1}}{2(1-\tau)}\right)^{\frac{1-\delta^{t+1}}{1-\delta}} > (1+\tau(1-\delta)\delta^{-1})^{\frac{2}{1+\delta}}. \end{aligned}$$

Decisions on pension systems are made with a long-run horizon. It can be that some generations are worse off directly after the transition from one system to the other.

As long as in the long run all generations are better off, a switch is reasonable. As the capital intensity factor δ is less than one, we know: $\lim_{t\to\infty} \delta^{t+1} = 0$. Therefore, the decisive inequality is given by:

$$\left(\frac{2+\tau\,(1-\delta)\,\delta^{-1}}{2\,(1-\tau)}\right)^{\frac{1}{1-\delta}} \overset{?}{>} \left(1+\tau\,(1-\delta)\,\delta^{-1}\right)^{\frac{2}{1+\delta}} \tag{2.5.2}$$

The previous section showed that a social planner is able to determine an optimal contribution rate $\tau^*(\delta)$ under PAYG and to implement this rate. This optimal $\tau^*(\delta)$ only depends on the exogenous parameter δ. It makes sense to compare the expected utilities under both regimes for an optimally chosen contribution rate. If we plug in $\tau^*(\delta) = \frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}-\frac{1}{4}\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1}$ and assume $0 < \delta < 2\sqrt{3}-3$, we see that the comparison relies only on the capital intensity factor δ. Hence, we are able to determine intervals for δ such that the expected utility is higher under FF than under PAYG. Define the difference of the expected utilities under both pension regimes as:

$$\begin{aligned}
\Delta U(\delta) &= \mathbb{E}_t\left[U^F(c^F_{t,t}, c^F_{t+1,t})\right] - \mathbb{E}_t\left[U^G(c^G_{t,t}, c^G_{t+1,t})\right] \\
&= \left(\frac{2+\left(\frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}-\frac{1}{4}\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1}\right)(1-\delta)\,\delta^{-1}}{2\left(1-\left(\frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}-\frac{1}{4}\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1}\right)\right)}\right)^{\frac{1}{1-\delta}} \\
&\quad - \left(1+\left(\frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}-\frac{1}{4}\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1}\right)(1-\delta)\,\delta^{-1}\right)^{\frac{2}{1+\delta}}
\end{aligned} \tag{2.5.3}$$

Proposition 2.5.1. *The difference $\Delta U(\delta)$ is a function of the parameter δ only. The roots of $\Delta U(\delta)$ are given for at least two values of δ: $\underline{\delta}$ and $\bar{\delta}$ with $0 < \underline{\delta}$.*[4]

For values of δ smaller than $\underline{\delta}$ or larger than $\bar{\delta}$, the difference function $\Delta U(\delta)$ has negative values. This shows that outside the interval $(\underline{\delta}, \bar{\delta})$ a PAYG pension system yields higher welfare for its agents than an FF system. We cannot formally show that no further root of $\Delta U(\delta)$ exists within the interval $(\underline{\delta}, \bar{\delta})$. But numerical simulations

[4]For a proof see the Appendix.

make the suggestion of two unique roots plausible. We are able to appropriate the roots of $\Delta U(\delta)$ by numerical methods as:

$$\underline{\delta} = 0.2654958759$$
$$\bar{\delta} = 0.4641016153 \qquad \approx 2\sqrt{3} - 3.$$

We can show that for $\delta = \delta^{max} \in (\underline{\delta}, \bar{\delta})$ the function $\Delta U(\delta)$ achieves its maximum value, which is positive. Therefore, $\Delta U(\delta)$ receives positive values within the interval $(\underline{\delta}, \bar{\delta})$ and negative values outside the interval. The value of δ^{max} can be determined numerically as $\delta^{max} = 0.340681904 \approx 0.34$ and $\Delta U(\delta^{max}) > 0$. The second derivative[5] of the utility difference function at this point is negative. From this result we can deduce that for capital intensity factors δ lying in the interval $(\underline{\delta}, \bar{\delta})$ the expected utility of a single agent under an FF pension system is strictly higher than under PAYG. This means, an agent can gain higher welfare if the social planner introduces a fully funded pension system. Outside this interval, but within the possible value space of δ, such that an operating PAYG with positive contribution rates exists, the PAYG system is the dominant one regarding life-time utility. Using the boundary behavior of $\Delta U(\delta)$, given in the proof, we can conclude that the fully funded pension system is strongly dominated by a PAYG one for values of δ being small enough.
What we can see from this analysis is that a social planner is able to choose the right pension system in the sense of the highest possible expected utility in the long run. Relying on the prevalent capital intensity factor δ, he will adjust his decision. It is not necessary to consider the height of human capital, the size of the physical capital stock or the technology factor A. Also, the contribution rate is not a decisive parameter since the expected utility under FF is completly independent of τ and under PAYG the social planner determines the optimal contribution rate $\tau^*(\delta)$ by the exogeneously given parameter δ.

To summarize, we cannot claim that the fully funded system is always better in terms of expected welfare than the PAYG one. This stays in contrast to the prediction of which system leads to higher economic growth. We showed numerically that

[5] $\frac{\partial \Delta U(\delta^{max})}{\partial \delta} = 0$ and $\frac{\partial^2 \Delta U(\delta^{max})}{\partial \delta^2} \approx -26.62 < 0$

the question of which pension system leads to higher welfare can be solved uniquely, given the economic parameters. It is clear that the question of which system to introduce, cannot be answered over all economies in the same way. Highly industrialized economies show a much higher fraction of physical capital than non developed enonomies, whose production relies mostly on human capital. We are aware that even in developed economies the capital intensity factor will take values below 0.5[6], hence our determined intervall for δ (so that a PAYG system can be introduced with strictly positive contribution rates) goes quite well in hand with observed real world data.

Proposition 2.5.2. *The PAYG pension system is the correct one in terms of welfare for low developed economies[7] who show up values for the capital intensity factor being small enough, i.e.* $\delta \leq \underline{\delta}$. *For values of* δ *located in the intervall* $(\underline{\delta}, \bar{\delta})$ *we cannot claim the dominance of one pension system unambiguously.*

For a proof see before. □

To demonstrate the dominance of the PAYG system for small δ values, assume $\delta = 0$ (under PAYG the savings decision, current and future consumption are not determined for $\delta = 0$). This leads to:

$$c^F_{t+1,t} = R_{t+1}(s_t + \tau\, w_t\, h_t) = 0 \tag{2.5.4}$$

as $R_{t+1} = \delta\, A\, k^{\delta-1}_{t+1} \overset{\delta=0}{=} 0$. Hence

$$\lim_{\delta \to 0} \ln c^F_{t+1,t} = -\infty \tag{2.5.5}$$

The rate of return on savings for $\delta \to 0$ under PAYG is zero in the limit as well. But under PAYG agents receive a payment from the new young generation in their second period of life. Hence, their consumption in the retirement period is never equal to zero (except for $\tau = 0$).

$$\lim_{\delta \to 0} \ln c^G_{t+1,t} = \ln((1 + \tilde{n}_{t+1})\, \tau\, \tilde{w}_{t+1} \beta\, h_t) > -\infty \tag{2.5.6}$$

[6]Baffes and Shah (1993) showed that the elasticity of output with respect to the private capital is smaller than 0.5 in most OECD and EMENA countries. The values for the rest of the world are considerable below the one for the OECD countries

[7]Low developed economies are in our sense countries with low output elasticities with respect to private capital.

Consumption in the first period does not differ substantially under both pension systems:

$$\lim_{\delta \to 0} \ln c_{t,t}^G = \ln(A\, h_t\, (1-\tau)) \geq \ln\left(\frac{A\, h_t}{2}\right) = \lim_{\delta \to 0} \ln c_{t,t}^F \tag{2.5.7}$$

as $\tau \leq \frac{1}{2}$. Under FF zero consumption in the second period leads to such high disutility that PAYG is the pension system which leads to the higher life-time utility. The second period consumption under FF consists only on the return from the capital market and, therefore, relies on the variability of this market which can lead to zero old-age consumption if the market does not provide a positive return. A PAYG pension system (with positive contribution rates) assures a positive old-age income independent of the capital market. Hence, we can interpret a PAYG system as a type of insurance against low rates of return.

2.5.1 Impact of human capital on welfare

The ratio of young contributors to beneficiaries (under PAYG) is shrinking in many industrialized economies. One way to cope with this problem is to invest in higher human capital. This would mean that not only the quantity of the contributors counts but also the quality. Under a PAYG system higher human capital directly influences the height of the pension payment as it increases the wage income of the contributors. Furthermore, human capital appears through the rate of return on physical capital. There is no direct effect of human capital on the pension payments under an FF pension system, as the pension benefits are not based on the wage income of the successors. But it has an effect on the rate of return like under PAYG.

Using (2.3.6) and (2.3.10), the expected utility of a single agent under a PAYG system is given by

$$\begin{aligned}
\mathbb{E}_t[U^G(\cdot,\cdot)] = &\ln\left(\frac{1+d}{2+d}\,(1-\tau)\,(1-\delta)\,A\,(k_t^G)^{\delta}\,h_t\right) \\
&+\ \ln\left(\left(\frac{(1-\tau)\ (1-\delta)}{2+d}\right)^{\delta}\ h_t\,A^{\delta+1}\,\beta^{1-\delta}\,\delta\ (1+d)\ (k_t^G)^{\delta^2}\right) \\
&+\ \mathbb{E}_t\left[\ln\left(\frac{1}{1+\tilde{n}_{t+1}}\right)^{\delta-1}\right].
\end{aligned}$$

Under a fully funded system the expected utility is given by using (2.3.16) and (2.3.20).

$$\mathbb{E}_t[U^F(\cdot,\cdot)] = \ln\left(\frac{1}{2}(1-\delta)A(k_t^F)^\delta\, h_t\right) + \ln\left(A^{\delta+1}\beta^{1-\delta}\left(\frac{1-\delta}{2}\right)^\delta \delta h_t (k_t^F)^{\delta^2}\right)$$
$$+\,\mathbb{E}_t\left[\ln(1+\tilde{n}_{t+1})^{1-\delta}\right].$$

An increase of the human capital production factor β in period $t = 1$ means that the indivual human capital of an agent born/adult in period $t = 1$ grows stronger than the human capital of their parental generation born/adult in period $t = 0$. Such an increase of the factor β has no direct effect on the structure of savings and consumption decision in the adult period no matter if a PAYG or an FF system exists. This is due to the logarithmic utility function which bears no income effect. Of course, the absolute numbers of savings and consumption will be changed by an increase of β as wage income changes. The factor β affects utility out of consumption in adulthood and in retirement under both pension systems in the same way. We can easily see that the influence of the human capital growth factor β on welfare of the parental generation is positive and equal under both regimes:

$$\frac{\partial \mathbb{E}_t[U^G_{t=0}(\cdot,\cdot)]}{\partial \beta} = \frac{\partial \mathbb{E}_t[U^F_{t=0}(\cdot,\cdot)]}{\partial \beta} = \frac{1-\delta}{\beta} > 0. \tag{2.5.8}$$

There is no effect on the consumption during adulthood of the generation born in $t = 0$. We find only an effect on consumption during retirement. Due to the logarithmic form of the utility function the effect on $c_{t+1,t}$ is the same under both types of pension systems.

We conclude that the generation, whose successors are endowed with higher human capital, gain a higher expected utility. Hence, human capital production has a positive effect on welfare. The first generation (born in period t=1), whose human capital grows stronger than the one of their parents, will benefit and the following generations even stronger (the factor β is not increased further) from an increase of β in $t = 1$. This is due to the fact that the aggregation of k_t is changed. Every generation born after or in $t = 1$ experiences an influence on their consumption in adulthood and retirement. The effect of an increase of β in $t = 1$ on the utility in every succeeding period $t \geq 1$ can be expressed by:

$$\frac{\partial \mathbb{E}_t[U^G(\cdot,\cdot)]}{\partial \beta} = \frac{\partial \mathbb{E}_t[U^F(\cdot,\cdot)]}{\partial \beta} = \frac{2\,t + 1 - \sum_{i=1}^{t} \delta^i - \delta^{t+1}}{\beta}. \tag{2.5.9}$$

This equation shows the effect of an increase in β in period $t = 1$ on the expected utility of a generation born after or in $t = 1$. It demonstrates that the effect of an increased β is for these generations stronger than the effect for the generation born in $t = 0$ as $\frac{2\,t+1-\sum_{i=1}^{t} \delta^i - \delta^{t+1}}{\beta} > \frac{1-\delta}{\beta}$.

Proposition 2.5.3. *An increase in human capital formation in period t, expressed by a higher β, results in a welfare gain of the generation born in period t under a PAYG pension system and under a fully funded one. It is the same under both systems.*

The effect of an increased growth factor on the agent's welfare is the same under both pension regimes due to the logarithmic form of the utility function. As it is positive, a social planner has incentives to boost human capital formation. In our model the human capital formation is given exogeneously. An increase of β could be understood as an improvement of schooling.

2.6 Concluding remarks

In this simple OLG-setting with stochastic population growth and exogenous growth of human capital, we showed that a fully funded system always leads to higher physical capital accumulation. This is consistent with the existing literature. The new result considers the individual utility of agents. We showed that a fully funded system in this respect is not always the best choice. If the government would introduce a pension system with respect to the individual utility of the agents, the main determining parameter would be the capital intensity factor. For countries with a low factor, it is optimal to choose the PAYG system, whereas countries with high capital intensity factors should go for a fully funded one. Hence the dominance of FF is not given in terms of utility for all values of the capital intensity factor. The overall claimed transition to an FF is, therefore, not optimal for all countries.

2.7 Appendix

Proof of Proposition 2.3.1 Define:

$$\begin{aligned} u^G(\tau) &:= \ln\left(\frac{1+d}{2+d}\,(1-\tau)\,(1-\delta)\,A\,(k_t^G)^\delta\,h_t\right) \\ &+\ \ln\left(\left(\frac{(1-\tau)\ (1-\delta)}{2+d}\right)^\delta h_t\,A^{\delta+1}\,\beta^{1-\delta}\,\delta\ (1+d)\ (k_t^G)^{\delta^2}\right) \\ &+\ \mathbb{E}_t\left[\ln\left(\frac{1}{1+n_{t+1}}\right)^{\delta-1}\right] \end{aligned}$$

with $d := \tau(1-\delta)\delta^{-1}$.The first derivative of $u^G(\tau)$ with respect to τ is given by

$$\begin{aligned} \frac{\partial\, u^G(\tau)}{\partial\,\tau} &= \frac{\tau\,(\delta(9-7\delta-\delta^2)-1)+\tau^2\,(2\,(1-\delta\,(4-\delta)))+\delta\,(6\delta-3+\delta^2)}{(\tau\,\delta-\delta-\tau)\,(\tau-1)\,(\tau\,\delta-\tau-2\delta)} \\ &= \frac{g(\tau)}{f(\tau)}. \end{aligned}$$

From the FOC we can determine two optimal values for the contribution rate under PAYG.

$$\begin{aligned} \tau_1(\delta) &= \frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}-\frac{1}{4}\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1} \\ \tau_2(\delta) &= \frac{\frac{1}{4}\delta^2+2\,\delta-\frac{1}{4}+\frac{1}{4}\sqrt{\delta^4+8\delta^3+14\delta^2+8\delta+1}}{\delta-1} \end{aligned}$$

The second value $\tau_2(\delta)$ is negative for all $\delta \in [0,1)$ as the numerator is always positive while the denominator is negative. $\tau_1(\delta)$ presumes positive values for some values of δ. We know that the function $\tau_1(\delta)$ has positive values for δ being small[8] and negative values for δ being large[9]. The function $\tau_1(\delta)$ is continuous and monotonically decreasing[10]. By knowing this we can use the mean value theorem and prove the existence of one unique root of $\tau_1(\delta)$. For $\delta = \hat{\delta} = 2\sqrt{3}-3$ the function is given as $\tau_1(\delta = \hat{\delta}) = 0$. Values of δ larger than $\hat{\delta}$ will result in negative contribution

[8] $\tau_1(\delta = 0) = \frac{1}{2}$

[9] $\tau_1(\delta = 0.9) = -4.77$

[10] $\frac{\partial \tau_1(\delta)}{\partial \delta} = \frac{\sqrt{(\delta^2+6\delta+1)\,(\delta+1)^2}\,(\delta^2-2\delta-7)+5+18\delta+12\delta^2-2\delta^3-\delta^4}{\sqrt{(\delta^2+6\delta+1)\,(\delta+1)^2}\,(\delta-1)^2} < 0$

rates. Therefore, we limit the contribution rate to zero for $\delta > \hat{\delta}$. To prove that $\tau = 0$ is optimal for $\delta > \hat{\delta}$ we have to show that $u^G(\tau)$ decreases in τ. It is sufficient to show that the first derivative of $u^G(\tau)$ is negative for any arbitrary values of $\tau \geq 0$. As we have only two roots of $\frac{\partial\, u^G(\tau)}{\partial\, \tau}$ and $\tau_2(\delta) < \tau_1(\delta)$, it is sufficient to show that $\frac{\partial\, u^G(\tau)}{\partial\, \tau} < 0$ for one value of τ larger than $\tau_1(\delta)$.

$$\left.\frac{\partial\, u^G(\tau)}{\partial\, \tau}\right|_{\tau=0,\delta\geq\hat{\delta}} = \frac{\delta\ (6\,\delta - 3 + \delta^2)}{-2\,\delta^2} < 0$$

as $\delta\ (6\,\delta - 3 + \delta^2) > 0$.

On the interval $\tau \in\ [0,1]$ the function $u^G(\tau)$ has a local maximum for $\tau^*(\delta) = \tau_1(\delta)$ as

$$\begin{aligned}
\left.\frac{\partial u^G(\tau)}{\partial \tau}\right|_{\tau=\tau^*(\delta)} &= 0 \\
\left.\frac{\partial^2 u^G(\tau)}{\partial \tau^2}\right|_{\tau=\tau^*(\delta)} &= \frac{g'(\tau^*(\delta))\, f(\tau^*(\delta)) - f'(\tau^*(\delta))\, g(\tau^*(\delta))}{(f(\tau^*(\delta)))^2} \leq 0 \qquad \forall\, \delta \in\ (0, 2\sqrt{3} - 3]
\end{aligned}$$

and the value of the function at the point $\tau = \tau^*(\delta)$ is positive. Since

$$\begin{aligned}
f(\tau^*(\delta)) &\leq 0 \\
g(\tau^*(\delta)) &\leq 0 \\
f'(\tau^*(\delta)) &\leq 0 \\
g'(\tau^*(\delta)) &> 0,
\end{aligned}$$

we derive $\left.\frac{\partial^2 u^G(\tau)}{\partial \tau^2}\right|_{\tau=\tau^*(\delta)} \leq 0$.

$\square$

Proof of Proposition 2.5.1

The function $\Delta U(\delta)$ depends only on the parameter δ, which is exogenously given and lies within the interval $(0, 1)$. The function is continuous but not monotone. The boundary behavior of this function can be described by:

$$\begin{aligned}\lim_{\delta \to 0} \Delta U(\delta) &= -\infty \\ \lim_{\delta \to 1} \Delta U(\delta) &= -1\end{aligned}$$

For some values[11] of $\delta \in [0, 1]$ the function $\Delta U(\delta)$ presumes positive values. By using the mean value theorem we can prove that the function has at least two roots given by $\underline{\delta}$ and $\bar{\delta}$.

It cannot be shown formally that no further roots exist, i.e. it cannot be proven that $\Delta U(\delta)$ has only one maximum value on the intervall $\delta \in [0, 1]$. This means that $\Delta U(\delta) < 0$ is possible for $\delta \in (\underline{\delta}, \bar{\delta})$. □

[11]For $\delta = 0.35$ the difference function is greater than zero: $\Delta U(\delta = 0.35) \approx 0.22$.

Chapter 3

Pension Systems, Endogenous Fertility and Welfare

Abstract: This chapter is concerned with the influence of different pension systems, i.e. pay-as-you-go (PAYG) or fully funded (FF), and different payment schemes, i.e. Beveridge (flat) or Bismarck (contribution related), on fertility and on welfare of agents. Most existing papers analyze the relation between pension systems and economic growth. For a well-grounded statement on the "quality" of pension systems, welfare should be the crucial criterion. Heterogenous individuals who differ in their human capital endowment are considered. The relation between welfare and/or fertility and pension systems/payment schemes strongly depends on the height of the human capital endowment of agents. This leads to the conclusion that different types of agents prefer different combinations of pension systems and payment schemes. The model shows that all agents have more children under an FF system. Transfer payments to high-skilled agents raise their fertility and, therefore, average human capital implying higher economic growth.

3.1 Introduction

Publically organized pension systems are a main factor of financing consumption and life of old-age pensioners. To visualize numbers, Diamond and Orzag (2005) exposed for the USA that for two-thirds of the elderly beneficiaries, public pension payments are the majority of income and for 20 percent of the beneficiaries it is the only income. Similar numbers can be found for most western countries. Therefore, it is important to ensure sustainability of pension systems. Most western countries organize their public pension system in the form of a pay-as-you-go (PAYG) system. In this system the government collects contributions as a fixed fraction of wage income from the working population and pays them directly to the old age retirees. A PAYG system is, therefore, a pure reallocation of income from young working agents to old retired agents. Children and parents negotiated a so called "generation contract". Such a pension system faces two problems in the real economy: 1. demographic change and 2. low fertility of high-skilled agents. The first leads to a higher fraction of old retired agents to young working agents who contribute to the pension systems. This is due to better health care and better living situations which have led to a higher life span. Furthermore fertility rates decrease throughout the population. This is a common fact in many countries – industrialized as well as developing economies.

A low average human capital leads to low contributions and consequentially to low pension payments. In Germany at least a minimum pension payment is guaranteed. Financing such a public pension system by less (and worse educated) agents could generate a gap in the public budget. A switch from a PAYG to a fully funded (FF) system is often discussed in the public media. Since an FF system does not include a generation contract, i.e. no reallocation of wage income from young working agents to old retired agents, it does not face the problems of a demographic change.

Most existing papers which analyze the advantages of the two pension systems restrain their focus on economic growth. In our opinion welfare of human beings would be a far better criterion to judge on the "quality" of pension systems. We showed in the previous chapter that although an FF system leads to higher economic growth, a PAYG system ensures higher welfare under specific conditions. Within

the framework of an OLG-model and using a small open economy, this chapter considers the impact of the structure of pension systems and payment schemes on the welfare of heterogenous agents, measured in terms of individual life-time utility, in the economy. One result is that PAYG as well as FF can dominate the other one depending on economic parameters. The two major pension systems are considered, each in combination with two different payment schemes: Beveridge (flat) and Bismarck (contribution related). Heterogeneity of agents is one additional aspect which distinguishes our work from many other papers as often homogeneity of agents is assumed. For simplicity, we introduce only two different types of agents: low-skilled (low individual human capital) and high-skilled (high individual human capital). The agents differ according to their human capital endowment but are equipped with the same preferences. Depending on their individual types, agents prefer different payment schemes regarding their individual welfare. Their birth rates also differ under different schemes.
High-skilled agents will have at most the same number of children as the low-skilled agents under all four possible combinations of pension systems and payment schemes. Excluding a learning process, this leads to a decrease of average human capital over time. Since this results in a lowering of output per capita, which is often used as an indicator for welfare, we ask whether incentives can be introduced to raise fertility of high-skilled agents. By introducing transfer payments it is possible to raise fertility and through this to increase average human capital. Incentives, given in the form of transfer payments, are financed as a fraction of the total budget of the pension system meaning that high-skilled as well as low-skilled agents contribute to these transfer payments. As these incentives work in the intended direction we further analyze whether additional advantages like an increase of individual's welfare goes hand in hand.

A learning process is not regarded in this paper, i.e. human capital of each agent is fixed to the amount endowed with at birth. The focus lies on the correlation between pension systems/payment schemes and human capital with reference to fertility. The decision for learning is affected by initial human capital but also by the prevailing pension system. Therefore, the influence of pension systems on the fertility of different types of agents would not clearly be visible if a learning process is included.

3.2 Model

This paper uses an OLG model and distinguishes between two types of agents. Like Kaganovich and Meier (2008) we consider a small open economy where perfectly competitive firms produce a single homogeneous good that can be used for consumption and investment. They use physical and human capital as input factors in a constant returns neoclassical production technology $F(K, H)$. For simplicity the interest rate R is assumed to be constant over time. Using a constant returns technology the wage rate w per human capital is constant as well. In this model physical capital will be mobile whereas human capital is immobile.
Each individual lives exactly for three periods without the risk of an early demise: childhood, working period and retirement. We distinguish between two types of agents who are indexed by their type L or H and by their generation $t = 0, 1, 2 \ldots$. We label the generation whose working period occurs in period t as "generation t". The two types $I = L, H$ of agents differ only in their endowment of human capital, $h^L \neq h^H$, but not in their preferences. Type L agents are low-skilled whereas type H agents are high-skilled so that $h^L < h^H$ holds. Both supply inelastically one unit of labor to the firms in their adult period and receive a wage income which is taxed by a pension contribution rate τ. When agents are adult they decide whether to have children or not and if so how many, $n_t \geq 0$. Since we assume human reproduction to be asexual, fertility is an individual decision of a single agent. Each agent will have children of the same type as himself, i.e. a type I agent will have $n_t^I \geq 0$, $n \in \mathbb{R}_+$, children of the same type I as himself. The total working population in period $t+1$ is, therefore, given by $N_{t+1} = n_t^L N_t^L + n_t^H N_t^H$. Further we assume individual human capital to be constant, i.e. we exclude a learning process. In period $t = 0$ we have the same number of working agents of type L and type H: $N_0^L = N_0^H$, i.e. the initial average human capital is determined as $\bar{h}_0 = \frac{h^L + h^H}{2}$. The average human capital depends only on the human capital of the working population and not on the one

of the retired agents and also not on the one of children.
Since each agent supplies one unit of labor inelastically to the firms, he receives wage income according to his individual human capital $W^I = wh^I$. He has to contribute to the publically financed pension system an amount τwh^I and to release a certain fraction of his wage income to finance child consumption $\rho n_t^I wh^I$. These child costs are mandatory for having children. Each child of an agent I causes the same costs, i.e. there are no positive scale effects. As the costs per child are a fixed fraction of the wage income it is more costly (in absolute terms) to raise children for agents with higher individual human capital. In absolute terms they also contribute a higher amount to the pension system.
The preferences of a single agent are described by a log-utility function which is additively separable in its arguments:

$$U^I(c_{t,t}, c_{t+1,t}, n_t) = \alpha \ln c_{t,t}^I + \beta \ln c_{t+1,t}^I + \theta \ln n_t^I \tag{3.2.1}$$

where $\alpha, \beta, \theta > 0$ are factors displaying preferences for his own consumption when young and respectively when old and for having children. $c_{t,t}^I$ denotes consumption in period t of an adult agent of type I born in $t-1$ and working in t while $c_{t+1,t}^I$ is his consumption in $t+1$. Consumption as a child does not affect utility directly. It is integrated in the utility of parents. Children cannot make any decisions, therefore the analysis of childhood is excluded. The adult working agents allocate their after-tax (after contributions) wage income between current own consumption, savings for future consumption and costs for having children. Hence, agents face the following budget constraint in their working period:

$$c_{t,t}^I = (1 - \tau - \rho n_t^I) wh^I - s_t^I \tag{3.2.2}$$

Consumption budget in the old period, i.e. when retired, is given by returns on savings and by pension payments T_{t+1}^I which depend on the type of the agent. The constraint is therefore:

$$c_{t+1,t}^I = Rs_t^I + T_{t+1}^I \tag{3.2.3}$$

Assuming an utility maximizing agent we can write his objective function as:

$$\max_{s_t, n_t, c_{t,t}, c_{t+1,t}} U^I(c_{t,t}, c_{t+1,t}, n_t) = \max_{s_t, n_t, c_{t,t}, c_{t+1,t}} \alpha \ln c_{t,t}^I + \beta \ln c_{t+1,t}^I + \theta \ln n_t^I \tag{3.2.4}$$

subject to the budget constraints given in (3.2.2) and (3.2.3). Without making further assumptions on pension payments T^I_{t+1} and taking R and w as given, we derive private savings and the number of children born by a single agent I in the following general way:

$$s^I_t = \frac{\beta(1-\tau-\rho n^I_t)wh^I - \alpha\frac{T^I_{t+1}}{R}}{\alpha+\beta} \tag{3.2.5}$$

$$n^I_t = \frac{1-\tau+\frac{T^I_{t+1}}{wh^I R}}{\frac{\rho}{\theta}(\alpha+\beta+\theta)} \qquad \text{for } I = L, H \tag{3.2.6}$$

The decision for savings is not restricted to positive amounts (raising of credit is possible) due to the fact that retirees receive a pension payment out of which they can fulfill their credit obligations. Decisions for children are restricted to positive but not to integer numbers.

Two types of pension systems are considered as well as two different payment schemes – Beveridge and Bismarck – for an analysis of their effects on fertility and welfare. A Beveridge scheme (we use superscript f, short for "flat", for indexation) is a flat system which pays every retiree the same amount no matter how much he contributed to the pension system when young.

$$T^L_{t+1} = T^H_{t+1} = T_{t+1} \tag{3.2.7}$$

A Bismarck scheme (using superscript c, short for "contribution related") is a contribution related system which determines pension payments according to the own contributions:

$$\frac{T^H_{t+1}}{T^L_{t+1}} = \frac{h^H}{h^L}. \tag{3.2.8}$$

A PAYG pension system is indexed by a superscript G whereas a fully funded system is indicated by a superscript F. Whenever it is possible we drop the superscripts.

3.2.1 Decisions under PAYG

The PAYG pension system is the predominant system in most industrialized countries. It is characterized by a reallocation of wage income. Young working consumers

are forced by the government to release a part of their wage income (contributions) to the pension system. The PAYG system collects the contributions but they are not used as physical capital in the production process. Instead, the system gives them directly to the living old-age pensioners (beneficiaries). Hence, we observe a pure reallocation of wage income from young to old adults.
In equilibrium the aggregate amount of pension payments must equal aggregate contributions. As young agents pay for the parental generation, we derive the budget constraint of a PAYG pension system in the following way:

$$N_t^L T_{t+1}^L + N_t^H T_{t+1}^H = \tau w \left(N_{t+1}^L h^L + N_{t+1}^H h^H \right) \tag{3.2.9}$$

with $N_{t+1}^I = n_t^I N_t^I$.

Definition 3.2.1. *Given the initial stock of agents N_0^I with $I = L, H$, individual levels of human capital h^I and the exogenously given and fixed rate of return R, a dynamic competitive equilibrium (DCE) is a sequence of individual agent's decisions $\left\{n_t^I, s_t^I, c_{t,t}^I, c_{t+1,t}^I\right\}_{t=0}^{\infty}$, a sequence of factor prices of labor $\{w_t\}_{t=0}^{\infty}$ and the sequence of pension payments $\left\{T_{t+1}^I\right\}_{t=0}^{\infty}$ so that:*

- *For each $L, H \in I$ and $t = 0, 1, \ldots,$ the collection of $n_t^I, s_t^I, c_{t,t}^I, c_{t+1,t}^I$ solves the individuals' household problem (3.2.2)– (3.2.4) where factor prices w, R and future pension payments T_{t+1}^I are taken as given.*

- *Factor markets clear. Factor markets are competitive, hence, according to the economy's production function the factor prices are determined by their marginal products.*

$$R_t = F_K(\frac{K_t}{H_t}, 1)$$
$$w_t = F_H(\frac{K_t}{H_t}, 1)$$

- *Individual pension payments received by generation t retirees are always defined in such a way that (3.2.9) holds.*

Due to the small open economy assumption R is exogenously given and for simplicity fixed. The constant returns neoclassical production technology in combination with R constant leads to a constant wage rate w.

Plugging (3.2.6), (3.2.7) and (3.2.8) into (3.2.9) we determine equilibrium pension payments under the different payment schemes (e.g. Bismarck and Beveridge) as:

$$\text{Beveridge:} \quad T_{t+1}^{fG} = T_{t+1}^{L^{fG}} = T_{t+1}^{H^{fG}} = \frac{\alpha\tau(1-\tau)w\bar{h}_t^{fG}R}{AR-\alpha\tau} \tag{3.2.10}$$

$$\text{Bismarck:} \quad T_{t+1}^{I^{cG}} = \frac{\alpha\tau(1-\tau)wh^I R}{AR-\alpha\tau} \qquad \text{for } I = L, H \tag{3.2.11}$$

With A defined as $A := \alpha\frac{\varrho}{\theta}(\alpha+\beta+\theta)$. We assume that AR is larger than $\alpha\tau$ so that $AR - \alpha\tau > 0$ and, therefore, all pension payments are positive (negative pension payments are per definition excluded from a system with PAYG and also FF). Therefore, we receive the birth rates in equilibrium for the two payment schemes in the following way:

$$n_t^{I^{fG}} = \frac{\alpha}{A}(1-\tau)\frac{AR + \alpha\tau\left(\frac{\bar{h}_t^{fG}}{h^I} - 1\right)}{AR-\alpha\tau} \tag{3.2.12}$$

$$n_t^{I^{cG}} = \frac{\alpha}{A}(1-\tau)\frac{AR}{AR-\alpha\tau} \qquad \text{for } I = L, H \tag{3.2.13}$$

Equation (3.2.13) shows that individual birth rates under a Bismarck scheme are completely independent of the individual human capital. Both types of agents have the same number of children under this scheme. In contrast birth rates under Beveridge depend on the rate of average human capital in period t to individual human capital. This is due to the fact that every agent receives the same pension payment, i.e. an average one. Under Beveridge, child costs are private whereas benefits from children are in a certain way (flat pension payments) public.

3.2.2 Decisions under FF

There is a significant difference in the way the two different types of pension systems work. While the PAYG system collects contributions from young consumers and transmits them directly to the pensioners, the FF system collects contributions and invests them in the production process for one period. The FF pension system's savings are invested in the production technology and earn a return. The single consumer is equipped with the same logarithmic and additively separable utility function as under the PAYG system. But as the FF system works in a different way, the pension benefit payments are defined differently. The budget constraint of an FF pension system is described by the following equation:

$$N_t^L T_{t+1}^L + N_t^H T_{t+1}^H = R\tau w \left(N_t^L h^L + N_t^H h^H\right) \tag{3.2.14}$$

In equilibrium the two possible pension payments (according to a Bismarck or Beveridge pension scheme) are determined by using (3.2.7) respectively (3.2.8) and plugged into (3.2.14):

$$\text{Beveridge:} \qquad T_{t+1}^{fF} = T_{t+1}^{LfF} = T_{t+1}^{HfF} = \tau w \bar{h}_t R \tag{3.2.15}$$

$$\text{Bismarck:} \qquad T_{t+1}^{IcF} = \tau w h^I R \tag{3.2.16}$$

Plugging (3.2.15) and (3.2.16) in the general formula for birth rates (3.2.6) we derive the equilibrium birth rates under a fully funded pension system with either a Beveridge or a Bismarck scheme:

$$n_t^{IfF} = \frac{\alpha}{A}\left[1 + \tau\left(\frac{\bar{h}_t^{fF}}{h^I} - 1\right)\right] \tag{3.2.17}$$

$$n_t^{IcF} = \frac{\alpha}{A} \qquad \text{for } I = L, H \tag{3.2.18}$$

This shows the same link between individual/average human capital and birth rates as under a PAYG system. Under a Bismarck system birth rates are again independent of individual and average human capital whereas under Beveridge birth rates depend on the ratio of average to individual human capital.

3.2.3 Analysis of birth rates

The previous sections show that birth rates under a Bismarck scheme are independent of human capital, i.e. agents with low human capital decide to have the same number of children as agents with high human capital. This model uses a logarithmic utility function, hence, birth rates and savings are independent of the wage income. Therefore, both types of agents make the same decisions if there is no other distortionary effect. Under Bismarck pension payments are determined according to the relation of individual contributions to contributions by the other agents. From this it follows that an agent with high human capital (contributes more) does not finance partwise old-age consumption of the low human capital agents (contribute less). Therefore, under a Bismarck scheme no distortionary costs influence birth rates. Also, wage income does not determine birth rates but the cost factor ρ of having children plays a major role. The higher the cost factor of children the less children agents decide to have. Under a Beveridge scheme the ratio of average and individual human capital is important for birth rates. If we define the birth rate as a function of human capital $n_t(h)$, we derive the following:

$$\frac{\partial n_t^{fG}}{\partial h} = -\frac{\alpha}{A}(1-\tau)\frac{\alpha\tau\frac{\bar{h}_t}{h}}{AR-\alpha\tau} < 0 \tag{3.2.19}$$

$$\frac{\partial n_t^{fF}}{\partial h} = -\frac{\alpha}{A}\tau\frac{\bar{h}_t}{h} < 0 \tag{3.2.20}$$

The influence of human capital on birth rates under a Beveridge scheme is, therefore, negative. Meaning that under a Beveridge payment scheme (no matter if PAYG or FF) agents with higher individual human capital will have less children, i.e. birth rates decrease in individual human capital. In contrast, individual human capital does not affect birth rates under a Bismarck scheme[1].

Proposition 3.2.2. *Under each pension system and each payment scheme low-skilled agents have at least the same number of children as high-skilled agents.*

If a Beveridge scheme is implemented, agents of type H (with high human capital) suffer a kind of loss through the pension payment. They contribute a significantly higher amount to the pension system than agents with low human capital but as a retiree they receive the same pension payment as agents of type L, i.e. they face higher costs but the same benefits. Knowing this mechanism we can ask whether birth rates are influenced by the type of payment scheme, i.e. whether H-agents (and L-agents) have more children under a Beveridge scheme or under a Bismarck payment scheme.

Proposition 3.2.3. *In an economy with heterogenous agents individuals with low individual human capital (L-type agents) have more children under a Beveridge payment scheme than under a Bismarck scheme. In contrast, high-skilled agents have*

[1] If we would assume a CRRA utility function like $U(c_{t,t}, c_{t+1,t}, n_t) = \frac{1}{1-\sigma}\left(\alpha c_{t,t}^{1-\sigma} + \beta c_{t+1,t}^{1-\sigma} + \theta n_t^{1-\sigma}\right)$ birth rates would depend under all pension systems and payment schemes negatively on human capital as long as $\sigma < 1$.

more children under a Bismarck than under a Beveridge scheme no matter which type of pension system is introduced.

We derive under a PAYG system the following equivalence relation:

$$\begin{aligned} n_t^{LfG} &\geq n_t^{LcG} \\ \Longleftrightarrow \frac{\alpha}{A}(1-\tau)\frac{AR+\alpha\tau\left(\frac{\bar{h}_t^{fG}}{h^L}-1\right)}{AR-\alpha\tau} &\geq \frac{\alpha}{A}(1-\tau)\frac{AR}{AR-\alpha\tau} \\ \Longleftrightarrow \bar{h}_t^{fG} &\geq h^L. \end{aligned} \tag{3.2.21}$$

We know that human capital of high-skilled agents is higher than that of low-skilled agents, i.e. $h^L < h^H$. Therefore we know $h^L \leq \bar{h}_t \leq h^H$ for all periods $t \geq 0$ and for all combinations of pension systems and payment schemes. Only in case where no high-skilled (low-skilled) agents exist, $\bar{h}_t = h^L$ ($\bar{h}_t = h^H$) holds. Hence, L-agents have more (or equal – if no H-type agents exist) children under a Beveridge than under a Bismarck scheme if a PAYG system is implemented. For agents of type H we derive the following equivalence relation:

$$\begin{aligned} n_t^{HfG} &\leq n_t^{HcG} \\ \Longleftrightarrow \frac{\alpha}{A}(1-\tau)\frac{AR+\alpha\tau\left(\frac{\bar{h}_t^{fG}}{h^H}-1\right)}{AR-\alpha\tau} &\leq \frac{\alpha}{A}(1-\tau)\frac{AR}{AR-\alpha\tau} \\ \Longleftrightarrow \bar{h}_t^{fG} &\leq h^H. \end{aligned} \tag{3.2.22}$$

Hence, high-skilled agents have more children under a Bismarck payment scheme. Under a fully funded pension system we derive the same results with an analogous proof.

Comparing birth rates under PAYG and FF we derive a strict ordering of them under the four possible combinations of pension systems and payment schemes.

Proposition 3.2.4. *Both types of agents, low-skilled and high-skilled, have more children under a fully funded pension system than under a PAYG system if the rate of return is sufficiently high. This holds independently of the implemented payment scheme.*

Comparing the birth rates of a low-skilled agent under a Bismarck FF system and under a Beveridge PAYG system leads to the following equivalence relation:

$$
\begin{aligned}
& n_t^{L^{cF}} > n_t^{L^{fG}} \\
\iff & \frac{\alpha}{A} > \frac{\alpha}{A}(1-\tau)\frac{AR + \alpha\tau\left(\frac{\bar{h}_t^{fG}}{h^L} - 1\right)}{AR - \alpha\tau} \\
\iff & R > \frac{(1-\tau)\alpha\frac{\bar{h}_t^{fG}}{h^L} + \alpha\tau}{A} = \frac{(1-\tau)\frac{\bar{h}_t^{fG}}{h^L} + \tau}{\alpha}\frac{\alpha\theta}{\rho(\alpha+\beta+\theta)}.
\end{aligned}
\tag{3.2.23}
$$

Knowing that low-skilled agents have more children under a Beveridge payment scheme we can claim that if $n_t^{L^{cF}} > n_t^{L^{fG}}$ holds, $n_t^{L^{fF}} > n_t^{L^{cG}}$ holds as well. If $R > \frac{(1-\tau)\alpha\frac{\bar{h}_t^{fG}}{h^L} + \alpha\tau}{A}$ holds, we can affirm the following relation regarding the birth rates of low-skilled agents: $n_t^{L^{cG}} < n_t^{L^{fG}} < n_t^{L^{cF}} < n_t^{L^{fF}}$. Hence, a strict ordering of the height of birth rates exists. For high-skilled agents we compare the following:

$$
\begin{aligned}
& n_t^{H^{cG}} < n_t^{H^{fF}} \\
\iff & (1-\tau)\frac{AR}{AR - \alpha\tau} < 1 + \tau\left(\frac{\bar{h}_t^{fF}}{h^H} - 1\right)
\end{aligned}
\tag{3.2.24}
$$

$$
\iff (1-\tau)\alpha\tau < \tau\frac{h^H}{\bar{h}_t^{fF}}(AR - \alpha\tau) \tag{3.2.25}
$$

$$
\frac{(1-\tau)\alpha\frac{h^H}{\bar{h}_t^{fF}} + \alpha\tau}{A} < R \tag{3.2.26}
$$

We know already, high-skilled agents have more children under a Bismarck payment scheme no matter which type of pension system is implemented. Therefore, the relation $n_t^{H^{fG}} < n_t^{H^{cG}} < n_t^{H^{fF}} < n_t^{H^{cF}}$ holds as long as $R > \frac{\alpha}{A}((1-\tau)\frac{h^H}{\bar{h}_t^{fF}} + \tau)$.
If the rate of return is sufficiently high under an FF system, savings used for old-age consumption can be reduced. Lower savings increase free disposal income in adulthood which can be used for raising more children. If the factor expressing "joy" of having children θ is low and the cost factor of children ρ is high, the relation $R > \frac{\alpha}{A}((1-\tau)\frac{h^H}{\bar{h}_t^{fF}} + \tau)$, i.e. $R > \frac{\alpha}{A}((1-\tau)\frac{\bar{h}_t^{fG}}{h^L} + \tau)$, may be fulfilled already for "small" values of R.

We showed above that under an FF pension system both types of agents are willing to have more children than under a PAYG system. Facing the problem of a demographic change, as most industrialized countries are experiencing at the moment, and knowing that most European countries possess a public PAYG pension system, it could be one part of a solution to switch to an FF pension system. This leads to higher fertility of all agents in our model. Such a switch is already often discussed not especially to stimulate fertility but to cope financing gaps due to demographic change.

3.2.4 Welfare Comparison

Many papers dealing with social security systems focus on growth effects of different systems. They conclude that an FF system leads to higher physical capital accumulation and, therefore, to higher economic growth. In our work we can confirm higher physical capital accumulation as well. But our interest lies not on growth effects but on welfare effects since higher economic growth does not necessarily lead to higher welfare. In a closed economy this becomes even more visible as higher capital accumulation leads in equilibrium to higher wages but at the same time to lower rates of return. As we assume a small open economy, rates of return are given exogenously (and fixed for simplicity) and are, therefore, not influenced by higher aggregate capital. Due to the constant returns production function the wage rate is fixed over time as well. Nevertheless, different types of agents can benefit or lose from different pension systems. If we use equilibrium pension payments, birth rates and savings as described above and plug them into (3.2.2) and (3.2.3), we derive equilibrium consumption streams in both periods for all possible combinations of pension systems and payment schemes. Consumption for a type I agent (with

$I = L, H$) in period t is given in the following way:

$$\begin{aligned}
Beveridge\ PAYG: \quad & c_{t,t}^{IfG} = \frac{\alpha}{\alpha+\beta}(1-\tau)(1-\frac{\alpha\rho}{A})\left[h^I + \frac{\alpha\tau}{AR-\alpha\tau}\bar{h}_t^{fG}\right]w \\
Bismarck\ PAYG: \quad & c_{t,t}^{IcG} = \frac{\alpha}{\alpha+\beta}(1-\tau)(1-\frac{\alpha\rho}{A})\frac{AR}{AR-\alpha\tau}wh^I \\
Beveridge\ FF: \quad & c_{t,t}^{IfF} = \frac{\alpha}{\alpha+\beta}(1-\frac{\alpha\rho}{A})[h^I + \tau(\bar{h}_t^{fF} - h^I)]w \\
Bismarck\ FF: \quad & c_{t,t}^{IcF} = \frac{\alpha}{\alpha+\beta}(1-\frac{\alpha\rho}{A})wh^I.
\end{aligned}$$

Consumption in retirement period $t+1$ is given as:

$$\begin{aligned}
Beveridge\ PAYG: \quad & c_{t+1,t}^{IfG} = \frac{\beta}{\alpha+\beta}(1-\tau)(1-\frac{\alpha\rho}{A})\left[h^I + \frac{\alpha\tau}{AR-\alpha\tau}\bar{h}_t^{fG}\right]wR \\
Bismarck\ PAYG: \quad & c_{t+1,t}^{IcG} = \frac{\beta}{\alpha+\beta}(1-\tau)(1-\frac{\alpha\rho}{A})\frac{AR}{AR-\alpha\tau}wh^IR \\
Beveridge\ FF: \quad & c_{t+1,t}^{IfF} = \frac{\beta}{\alpha+\beta}(1-\frac{\alpha\rho}{A})[h^I + \tau(\bar{h}_t^{fF} - h^I)]wR \\
Bismarck\ FF: \quad & c_{t+1,t}^{IcF} = \frac{\beta}{\alpha+\beta}(1-\frac{\alpha\rho}{A})wh^IR.
\end{aligned}$$

The first question we would like to answer is whether different types of agents would prefer different types of payment schemes in terms of individual welfare. We use lifetime utility $U^I(c_{t,t}, c_{t+1,t}, n_t)$ in this section as a measurement for welfare. Since the utility function is monotonically increasing in its three arguments and as all three arguments are larger under one system if one argument is larger, it is sufficient to compare the arguments of the utility functions themselfes. We derive for a type L agent under a PAYG system the following:

$$\begin{pmatrix} c_{t,t}^{LcG} \\ c_{t+1,t}^{LcG} \\ n_t^{LcG} \end{pmatrix} \ll \begin{pmatrix} c_{t,t}^{LfG} \\ c_{t+1,t}^{LfG} \\ n_t^{LfG} \end{pmatrix}$$

and under an FF system:

$$\begin{pmatrix} c_{t,t}^{LcF} \\ c_{t+1,t}^{LcF} \\ n_t^{LcF} \end{pmatrix} \ll \begin{pmatrix} c_{t,t}^{LfF} \\ c_{t+1,t}^{LfF} \\ n_t^{LfF} \end{pmatrix}.$$

Obviously, an agent who has only an endowment of low human capital will gain from a Beveridge payment scheme and suffer from a Bismarck scheme under a PAYG as well as under an FF pension system. This is due to the fact that under Bismarck agents receive only pension payments according to their own contribution but under Beveridge all agents receive the same pension payment no matter how much they contributed themselves. Hence, a low human capital type agent will gain from the higher contributions of type H agents.
For a type H agent the following holds under PAYG:

$$\begin{pmatrix} c_{t,t}^{H^{cG}} \\ c_{t+1,t}^{H^{cG}} \\ n_t^{H^{cG}} \end{pmatrix} \gg \begin{pmatrix} c_{t,t}^{H^{fG}} \\ c_{t+1,t}^{H^{fG}} \\ n_t^{H^{fG}} \end{pmatrix}$$

and under an FF system:

$$\begin{pmatrix} c_{t,t}^{H^{cF}} \\ c_{t+1,t}^{H^{cF}} \\ n_t^{H^{cF}} \end{pmatrix} \gg \begin{pmatrix} c_{t,t}^{H^{fF}} \\ c_{t+1,t}^{H^{fF}} \\ n_t^{H^{fF}} \end{pmatrix}.$$

The discrete arguments, consumption in adulthood $c_{t,t}^{H}$, consumption in retirement $c_{t+1,t}^{H}$ and the number of children n_t^H, are all higher under a Bismarck scheme. Hence, the life-time utility of an H-agent is higher under Bismarck than under Beveridge no matter if a PAYG or an FF system is introduced. This is due to the fact that an H-agent receives less pension payments under a Beveridge scheme as he loses some amount to L-agents and the costs of having children are proportionally higher.

Proposition 3.2.5. *Independently from the pension system prevailing in the economy, low-skilled agents will gain from a Beveridge payment scheme and suffer from a Bismarck scheme, whereas the opposite is true for high-skilled agents.*

If we assume that a Bismarck scheme is implemented, we can investigate whether one type or even both types of agents benefit by introducing either a PAYG or an FF pension system[2]. By comparing the discrete arguments of the life-time utilty

[2] In our comparison we leave the generation which is retired in the period when a pension system is implemented, unattended. In reality this generation is the critical one as we have to solve the problem of financing their pension payments when switching from a PAYG to an FF system.

function of both types of agents we observe the following simultaneously for both agents:

$$\begin{pmatrix} c_{t,t}^{I^{cF}} \\ c_{t+1,t}^{I^{cF}} \\ n_t^{I^{cF}} \end{pmatrix} \gg \begin{pmatrix} c_{t,t}^{I^{cG}} \\ c_{t+1,t}^{I^{cG}} \\ n_t^{I^{cG}} \end{pmatrix} \qquad \text{if } R > \frac{\theta}{\rho(\alpha+\beta+\theta)}.$$

This means that both types of agents derive a higher life-time utility if an FF pension system is introduced as all discrete arguments are higher under FF than under PAYG in case the rate of return is sufficiently high. Under an FF system pension payments depend purely on the rate of return and not on the next generation and, therefore, not on the number of children. Consumption in adulthood, consumption in retirement and having children are substitution goods in our model. If a sufficiently high rate of return leads to high pension payments, agents are in a position to save less and to consume more in adulthood and to have more children. A sufficiently high R leads to a life-time income which is higher under an FF system than life-time income under a PAYG system. The height of the critical value of R depends on the factors α, β, θ and the cost factor of children ρ.

Proposition 3.2.6. *If the rate of return is sufficiently high, i.e.* $R > \frac{\theta}{\rho(\alpha+\beta+\theta)}$*, all types of agents can derive a higher life-time utility under a Bismarck fully funded pension system than under a Bismarck PAYG pension system in all periods* $t \geq 0$.

$R > \frac{\theta}{\rho(\alpha+\beta+\theta)}$ is fulfilled if the cost factor ρ of having children is sufficiently high or the factor θ of "joy" through children is sufficiently low.

3.3 Transfer Payments

In the previous section we were able to show which type of pension system leads to higher welfare in terms of individual life-time-utility. It is quite difficult for a government to observe individual life-time utility. If a government should decide which type of pension system to implement, its decision should, for this reason, rely on a different measurement. One possible measurement is output per capita since aggregate output as well as the number of agents in each generation is easy to determine. The considered economy in this model is a small open one with mobile physical capital but immobile human capital. Due to perfect physical capital mobility, a neoclassic production technology with constant returns and a constant rate of return, physical capital will be supplied in equilibrium in a constant proportion to human capital. This implies that aggregate output grows at the same rate as the human capital accumulation. To determine output per capita we need the average human capital per capita.

In the last section we saw that both types of pension systems and also both types of payment schemes lead to higher birth rates of agents endowed with low human capital. The high-skilled agents, i.e. the ones endowed with high human capital, give birth to less children than the less endowed agents. Therefore, we observe a decreasing human capital per capita/average human capital over time. Also, this means that the economy faces a decreasing output per capita over time. If we interpret output per capita as an indicator for the welfare of agents, welfare decreases over time. The question we would like to address in this section is: is it possible to stimulate fertility of high-skilled agents by giving incentives and does this lead to a higher average human capital? In our simple model individual human capital is assumed to be constant over time. Therefore, average human capital changes by changing the proportions of low- and high-skilled agents in the population. As we want to stimulate only the fertility of high-potential agents, one type of incentives is a transfer payment for agents with high human capital. We assume transfer payments to be payed to type H agents in their retirement period on top of their regular pension payments. Transfer payments are payed out of the pension system's budget, meaning that both types of agents pay for them but only high-skilled agents benefit from them. The aggregate transfer payments are an ϵ-fraction of the pension system's budget. As this budget is changing over time, the absolute value of aggregate transfer payments is changing as well.

Although transfer payments have an impact on pension payments and through this on the budget constraints, they do not structurally change the decision process of low-skilled agents. Therefore, an agent's common savings decision and his decision for children are described by (3.2.5) and (3.2.6). As type H agents receive transfer payments their maximization problem changes slightly:

$$\max U^H = \max(\alpha \ln c^H_{t,t} + \beta \ln c^H_{t+1,t} + \theta \ln n^H_t)$$

s.t.

$$c^H_{t,t} = (1 - \tau - \rho n^H_t) w_t h^H_t - s^H_t \tag{3.3.1}$$

$$c^H_{t+1,t} = R s^H_t + T^H_{t+1} + \frac{B_{t+1}}{N^H_t}. \tag{3.3.2}$$

B_{t+1} (determined later) is the aggregate amount of transfer payments given to all retirees of type H in period $t+1$. Therefore, it has to be divided by the number of type H agents old in $t+1$ to derive the individual transfer payment of a single agent H. Obviously adult consumption is not influenced structurally by a transfer payment. Consumption as a H-type retiree $c^H_{t+1,t}$ now consists not only of a return on savings and pension payments but also of transfer payments. Without making further assumptions on pension payments T^H_{t+1} and transfer payments $\frac{B_{t+1}}{N^H_t}$ the rule for private savings and the number of children is given in the following general way:

$$s^H_t = \frac{\beta(1 - \tau - \rho n^H_t) w h^L - \alpha \frac{T^H_{t+1} + B_{t+1}/N^H_t}{R}}{\alpha + \beta} \tag{3.3.3}$$

$$n^H_t = \alpha \frac{1 - \tau + \frac{T^H_{t+1} + B_{t+1}/N^H_t}{w h^L R}}{\frac{\rho}{\theta}(\alpha + \beta + \theta)}. \tag{3.3.4}$$

3.3.1 Decisions under PAYG

We assume that both payment schemes (Bismarck and Beveridge) are characterized as before. In equilibrium one additional condition has to be fulfilled. The following holds:

$$B_{t+1} = \epsilon \tau w \left(N^L_{t+1} h^L + N^H_{t+1} h^H\right) \tag{3.3.5}$$

$$N^L_t T^L_{t+1} + N^H_t T^H_{t+1} = (1 - \epsilon) \tau w \left(N^L_{t+1} h^L + N^H_{t+1} h^H\right) \tag{3.3.6}$$

Plugging (3.2.6) and (3.3.4) into (3.3.5) and (3.3.6) and using (3.2.7) and (3.2.8) we derive the individual transfer payment under PAYG of a type H agent as:

$$\frac{B^G_{t+1}}{N^H_t} = \frac{\epsilon \tau \alpha w R (1 - \tau) H^G_t / N^H_t}{AR - \tau \alpha} \tag{3.3.7}$$

The transfer payments are determined under both payment schemes in the same way as we assume that high-skilled agents always receive an ϵ-fraction of the pension system's total budget. Meaning that there is no link between the payment scheme and transfer payments. Pension payments under the two types of payment schemes differ and are listed below.

$$\text{Beveridge:} \quad T_{t+1}^{fG} = T_{t+1}^{LfG} = T_{t+1}^{HfG} = \frac{(1-\epsilon)\tau\alpha(1-\tau)w\bar{h}_t^{fG}R}{AR-\tau\alpha} \tag{3.3.8}$$

$$\text{Bismarck:} \quad T_{t+1}^{IcG} = \frac{(1-\epsilon)\tau\alpha(1-\tau)wh^{I}R}{AR-\tau\alpha} \quad \text{for } I = L, H \tag{3.3.9}$$

Obviously pension payments for a type L agent are higher under a Beveridge scheme and for a type H agent they are higher under a Bismarck scheme as $h^L < \bar{h}_t^{fG} < h^H$ (as long as both types of agents exist). This is due to the fact that under a Beveridge payment scheme pension payments are equal for both types of agents. Under such a scheme type L agents benefit from higher contributions of type H agents whereas type H agents suffer due to the same reason. Using these equilibrium pension payments we determine equilibrium birth rates under the two payments schemes for L and H agents in the following way.

Beveridge:

$$n_t^{LfG} = \frac{\alpha}{A}\frac{1-\tau}{AR-\tau\alpha}[(1-\epsilon)\tau\alpha\frac{\bar{h}_t^{fG}}{h^L} + AR - \tau\alpha] \tag{3.3.10}$$

$$n_t^{HfG} = \frac{\alpha}{A}\frac{1-\tau}{AR-\tau\alpha}[(1-\epsilon)\tau\alpha\frac{\bar{h}_t^{fG}}{h^H} + AR - \tau\alpha + \epsilon\tau\alpha\frac{H_t^{fG}}{N_t^{HfG}h^H}] \tag{3.3.11}$$

Bismarck:

$$n_t^{LcG} = \frac{\alpha}{A}\frac{1-\tau}{AR-\tau\alpha}[AR - \epsilon\tau\alpha] \tag{3.3.12}$$

$$n_t^{HcG} = \frac{\alpha}{A}\frac{1-\tau}{AR-\tau\alpha}[AR - \epsilon\tau\alpha + \epsilon\tau\alpha\frac{H_t^{cG}}{N_t^{HcG}h^H}] \tag{3.3.13}$$

With H_t being the aggregate human capital in period t : $H_t = N_t^L h^L + N_t^H h^H$. We can directly see that under a Bismarck scheme birth rates of individuals with high human capital are higher than those of agents with low human capital. This indicates a first success of the politics of incentives. Whether they are higher under a Beveridge scheme has to be proven. We will later take a closer look at this question.

3.3.2 Decision under FF

Under a fully funded pension system the following holds in equilibrium:

$$B_{t+1} = \epsilon R \tau w \left(N_t^L h^L + N_t^H h^H \right) \tag{3.3.14}$$

$$N_t^L T_{t+1}^L + N_t^H T_{t+1}^H = (1-\epsilon) R \tau w \left(N_t^L h^L + N_t^H h^H \right) \tag{3.3.15}$$

To determine the transfer payments received by a single agent of type H in t, we need to divide the aggregate pension payments B_{t+1} by the number of retired type H agents:

$$\frac{B_{t+1}^F}{N_t^H} = \epsilon \tau w \frac{H_t^F}{N_t^H} R \tag{3.3.16}$$

Using the characterizations of payment schemes given by (3.2.7) and (3.2.8) and (3.3.15), we derive equilibrium pension payments under the two payment schemes as:

$$\text{Beveridge:} \quad T_t^{LfF} = T_{t+1}^{HfF} = T_{t+1}^{fF} = (1-\epsilon)\tau w \bar{h}_t^{fF} R \tag{3.3.17}$$

$$\text{Bismarck:} \quad T_{t+1}^{IcF} = (1-\epsilon)\tau w_t h^I R \qquad \text{for } I = L, H. \tag{3.3.18}$$

Obviously pension payments for a type L agent are higher under a Beveridge scheme and for type H agents they are higher under a Bismarck scheme as $h^L < \bar{h}_t^{fG} < h^H$. Plugging these pension payments into (3.2.6) and (3.3.4) we derive equilibrium birth rates under an FF pension system with two different payment schemes:

$$\text{Beveridge:} \quad n_t^{LfF} = \frac{\alpha}{A}[1 - \tau + (1-\epsilon)\tau \frac{\bar{h}_t^{fF}}{h^L}] \tag{3.3.19}$$

$$n_t^{HfF} = \frac{\alpha}{A}[1 - \tau + (1-\epsilon)\tau \frac{\bar{h}_t^{fF}}{h^H} + \epsilon\tau \frac{H_t^{fF}}{N_t^{HfF} h^H}] \tag{3.3.20}$$

$$\text{Bismarck:} \quad n_t^{LcF} = \frac{\alpha}{A}[1 - \epsilon\tau] \tag{3.3.21}$$

$$n_t^{HcF} = \frac{\alpha}{A}[1 - \epsilon\tau + \epsilon\tau \frac{H_t^{cF}}{N_t^{HcF} h^H}]. \tag{3.3.22}$$

Like under a PAYG system we can already see here that transfer payments increase fertility of high-potential agents above the fertility of agents with low human capital at least if a Bismarck scheme is prevailing. Type L agents always have less children through an increase of ϵ whereas type H agents always have more children[3]. How many more children they have depends either on the ratio of average human capital to their own (Beveridge) or on the ratio of aggregate human capital of all agents H_t to aggregate human capital of type H agents $N_t^H h^H$(Beveridge and Bismarck). The less the difference between $\bar{h}_t$ and h^H is the less redistribution through pension payments occur altogether under Beveridge. A single type L agent receives the most redistribution of income if $\bar{h}_t^{fF}$ is close to h^H. The larger the ratio of N_t^L to N_t^H is, the less type H agents contribute to and benefit from the transfer payments and, therefore, a single type H agent receives proportionally more.

3.3.3 Comparison of birth rates

The intention of introducing transfer payments for agents who are endowed with high human capital was to stimulate their endogenous fertility. The aim was to turn around the process of a decreasing average human capital. We now have to prove whether it works or not. A success would be to slow down the process, i.e. to draw the fertility rates of L and H agents nearer or even to increase the fertility rate of type H agents above the one of type L agents. Such a transfer payment is introduced in the initial period $t = 0$ when $N_0^L = N_0^H$. We saw before that under a Bismarck scheme birth rates of both types of agents are equal if no transfer payments exist. In the presence of transfer payments we see directly by comparing (3.3.12) with (3.3.13) and (3.3.21) with (3.3.22) that high-skilled agents have more children than low-skilled agents. Therefore, under a Bismarck payment scheme no matter if a PAYG or an FF pension system prevails the introduction of transfer payments leads to a higher average human capital. Hence, the production per capita increases over time.

Over time a Beveridge scheme leads to a decreasing human capital per capita as long as no transfer payments are active regardless of the pension system implemented. If transfer payments are introduced, the relation of type L and type H agents' birth rates changes.

[3]The first derivative of birth rates with respect to the factor ϵ is given as: $\frac{\partial n_t^{LfF}}{\partial \epsilon} < 0$, $\frac{\partial n_t^{LcF}}{\partial \epsilon} < 0$ and for type H agents: $\frac{\partial n_t^{HfF}}{\partial \epsilon} > 0$, $\frac{\partial n_t^{HcF}}{\partial \epsilon} > 0$

Proposition 3.3.1. *If transfer payments are implemented, birth rates of individuals with high individual human capital (i.e. H-type agents) are higher than the birth rates of individuals with low human capital under all four possible combinations of pension systems and payment schemes if* $\frac{h^H-h^L}{h^H} < \epsilon$. *Under a Bismarck payment scheme this is even true for all* $\epsilon \geq 0$. *Hence, output per capita grows over time.*

Proof. The evidence for a Bismarck payment scheme is proven by a comparison of (3.3.12) with (3.3.13) and (3.3.21) with (3.3.22). To prove the evidence for a Beveridge payment scheme we restrict the analysis to a PAYG pension system as the proof works in the same way for an FF system. High-skilled agents have more children in all periods t if the following equivalence relation holds:

$$n_t^{LfG} < n_t^{HfG} \tag{3.3.23}$$

$$\Longleftrightarrow \frac{N_t^{HfG}(h^H - h^L)}{N_t^{LfG}h^L + N_t^{HfG}h^H} < \epsilon \tag{3.3.24}$$

Knowing that N_t^I can be written as $N_t^I = n_0^I n_1^I \cdots n_{t-1}^I N_0^I$, we can rewrite the LHS expression of (3.3.24) as $\frac{N_0^H(h^H-h^L)}{N_0^H h^H + \frac{n_0^L n_1^L \cdots n_{t-1}^L}{n_0^H n_1^H \cdots n_{t-1}^H} N_0^L h^L}$. As $N_0^L = N_0^H$ under all pension systems and payment schemes we can simplify the former expression to $\frac{h^H-h^L}{h^H + \frac{n_0^L n_1^L \cdots n_{t-1}^L}{n_0^H n_1^H \cdots n_{t-1}^H} h^L}$. This is smaller than $\frac{h^H-h^L}{h^H}$. Therefore, if

$$\frac{h^H - h^L}{h^H + \frac{n_0^L n_1^L \cdots n_{t-1}^L}{n_0^H n_1^H \cdots n_{t-1}^H} h^L} < \frac{h^H - h^L}{h^H} < \epsilon$$

holds, $n_t^{LfG} < n_t^{HfG}$ holds for all t.

□

This means that transfer payments for high-skilled agents lead to higher fertility and, therefore, to higher output per capita. Hence, the government can increase output per capita over time via stimulating fertility of high-skilled agents by giving incentives in the form of transfer payments.

3.3.4 Welfare comparison with/without transfer payments

Transfer payments have been introduced with the aim of stimulating fertility. We note in the previous section that birth rates of high-skilled agents are higher than those of low-skilled agents if the transfer rate is sufficiently high, i.e. $\epsilon > \frac{h^H - h^L}{h^H}$. Therefore, transfer payments suceeded and increased production per capita as well. The remaining open question is whether such a transfer payment will lower or increase individual welfare of the entire population or whether only a part of the population gains and the other part loses through such a policy measurement. For a statement we compare the life-time utility of all agents under the different pension systems and different payment schemes when there exists/does not exist such a transfer payment as described above. We will proceed chronologically which means we will first analyze a Beveridge and a Bismarck PAYG pension system and then continue with Beveridge/Bismarck fully funded pension systems.

Low-skilled agents have to pay for the transfer payments without receiving any compensation. In contrast, the net payment for transfer benefits is for high-skilled agents negative, Hence, they receive a positive net payment. Since there is no direct gain for type L agents, the question is whether they benefit indirectly from higher pension payments or lose at all. For a welfare statement it is sufficient to compare the single arguments of the utility function discretely as the utility function is monotonically increasing as mentioned before.

Given a PAYG pension system with a Beveridge payment scheme we will compare birth rates of type L agents when transfer payments are inactive/active as a first step. In the following we will use the incidental notation: variables $(\tilde{n}_t, \tilde{c}_t, \tilde{c}_{t+1})$ are used under active transfer payments. We know that the average human capital under a Beveridge payment scheme is monotonically decreasing and in the long run tends to h^L as long as no transfer payments are introduced. In case transfer payments are introduced, average human capital monotonically increases over time if $\epsilon > \frac{h^H - h^L}{h^H}$ and tends to h^H. Comparing the birth rates of low-skilled agents in an economy without/with transfer payments we receive the following equivalence relation:

$$\begin{aligned} & n_t^{L^{fG}} < \tilde{n}_t^{L^{fG}} \\ \Longleftrightarrow\ & \bar{h}_t^{FG} < (1-\epsilon)\tilde{\bar{h}}_t^{FG}. \end{aligned}$$

Initially, average human capital is the same, i.e. $\bar{h}_0^{FG} = \tilde{\bar{h}}_0^{FG}$. While $n_t^{L^{fG}}$ is mono-

tonically decreasing, $\tilde{n}_t^{LfG}$ is monotonically increasing if $\epsilon > \frac{h^H - h^L}{h^H}$. For $\tilde{n}_t^{LfG}$ to be larger than n_t^{LfG} in the long run the condition $\epsilon < \frac{h^H - h^L}{h^H}$ has to be fulfilled. But this contradicts the condition for a monotonically increasing average human capital under transfer payments. Following this result, low-skilled agents will in no period have more children when they must pay for transfer payments than without such payments.
The comparison of $c_{t,t}^{LfG} < \tilde{c}_t^{LfG}$ and $c_{t+1,t}^{LfG} < \tilde{c}_{t+1}^{LfG}$ gives the same result. This means that a type L agent cannot gain from transfer payments under a Beveridge PAYG system as the losses through transfer payments are in no period outweighted by higher pension payments due to higher birth rates of type H agents which leads to a raising average human capital. In contrast, a high-skilled agent always gains from transfer payments, which we will show below, as only he receives them.

$$\begin{aligned} n_t^{HfG} &< \tilde{n}_t^{HfG} \\ \Longleftrightarrow \bar{h}_t^{fG} &< \tilde{\bar{h}}_t^{fG}(1 + \epsilon \frac{N_t^L}{N_t^H}) \end{aligned}$$

which is always fulfilled as long as $\epsilon > \frac{(h^H - h^L)}{h^H}$ as this condition leads to increasing average human capital under transfer payments and we know that $\bar{h}_0^{FG} = \tilde{\bar{h}}_0^{FG}$ holds. The same argumentation is true for consumption in working/retirement period. Hence, a type H agent receives higher welfare if transfer payments are active. Using these results we conclude that high-skilled agents benefit from transfer payments whereas low-skilled agents gain a lower welfare if transfer payments are introduced under a Beveridge PAYG system.

Next we consider a Bismarck PAYG pension system. Under such a system pension payments are contribution related, therefore, pension payments of a type L agent depend on how much he contributed to the system. Here he does not benefit from higher contributions of type H agents and, therefore, not from a raising average human capital. Therefore, our conjecture is that under a Bismarck PAYG a type L agent suffers and a type H agent gains from transfer payments. The comparison of birth rates of type L agents leads to:

$$
\begin{aligned}
& n_t^{LcG} > \tilde{n}_t^{LcG} \\
\Longleftrightarrow\ & AR > AR - \epsilon\alpha\tau
\end{aligned}
$$

which holds in all periods $t \geq 0$. The same argument is true for consumption in the periods of working and retirement. Under this system a type H agent receives an additional payment without any net costs for himself. Therefore, it increases his welfare/life-time utility.

$$
\begin{aligned}
& n_t^{HcG} < \tilde{n}_t^{HcG} \\
\Longleftrightarrow\ & 0 < \epsilon\alpha\tau \frac{h^L N_t^L}{h^H N_t^H}
\end{aligned}
$$

which holds true for all periods $t \geq 0$. The same argumentation is used to prove $c_{t,t}^{HcG} < \tilde{c}_t^{HcG}$ and $c_{t+1,t}^{HcG} < \tilde{c}_{t+1}^{HcG}$. We see that under a Bismarck PAYG system not all types of agents gain from transfer payments. More precisely, high-skilled agents benefit in every period whereas low-skilled agents suffer from lower welfare in case transfer payments are introduced.

The same comparison can be done for the two payment schemes under a fully funded pension system and we gain the same result: Low-skilled agents suffer whereas high-skilled agents gain from transfer payments.

Theorem 3.3.2. *Transfer payments introduced with the intention to increase average human capital/output per capita do have opposed side effects. While high-skilled agents benefit from transfer payments in terms of higher life-time utility low skilled agents gain lower welfare compared to an economy whithout such transfer payments.*

Proof. For the proof see above. □

In the long run the fraction of low-skilled agents in the economy tends to zero, meaning that almost no transfer payments are payed to high-skilled agents. Under a Beveridge payment scheme high-skilled agents benefit nevertheless as they receive almost the same pension payments as under a contribution related payment scheme (Bismarck). If a Bismarck payment scheme is prevailing, the positive effects of transfer payments for high-skilled agents vanish in the long run – their pension payments are already contribution related and transfer payments tend to be zero.

3.4 Conclusion

In this paper we analyzed how fertility is influenced by pension systems and by the type of agents. The model shows that agents who are higher skilled, i.e. endowed with higher human capital, have less children than low-skilled agents as long as there are no active transfer payments. This is something we observe in real data as well. The media often reports that people with an academic background have less children than workers with a low level of education. We showed that giving incentives in the form of transfer payments leads to an increase of the fertility of high-skilled agents and that they have even more children than low-skilled agents under certain conditions. This leads to an increasing average human capital and, therefore, to an increasing output per capita over time. Governments are hardly able to observe individual life-time utility. But they can quantify production per capita and use this statistic as an indicator for welfare. In this respect, transfer payments increase welfare. In contrast, measuring the individual life-time utility shows that low-skilled agents suffer and high-skilled agents benefit from transfer payments. Using the results from the comparison of birth rates raises the conjecture that in the long run only high-skilled agents are alive. This leads in our model to the highest possible average human capital and, therefore, output per capita. Leading to the conclusion that an economy whose population is high-skilled and where few inequalities (differences between high- and low-skilled agents) exist, has a high economic output.
A negative proportional transfer payment as it is discussed in Germany in the form of "Kindergeld" is supposed to lower fertility of high-skilled agents and, therefore, aggregate welfare of the economy. This would contradict our aim of introducing transfer payments.

We did not investigate an endogenous decision for learning which would probably change the model. The model was simplified by the assumption of fixed individual human capital and bequest rules of human capital. To introduce a random process of human capital at the time of birth and a learning process is a possible extension of the model. Such a learning process would endogenously change the mixture of low/medium/high skilled agents in the population. This would of course influence our results on welfare.

Chapter 4

How do Pension Systems Influence Decisions on Education?

Abstract: This chapter asks how different pension systems influence the decision by parents for investing in the education of their own children if inter-generational altruism is concidered. In this model human capital is built according to a human capital formation rule. Parents can influence the human capital of their children by investing in education. We show that investments in education are different under the two predominant pension systems (pay-as-you-go and fully funded) and that a PAYG pension system leads to higher human capital under certain conditions. The analysis shows that heterogenous agents (same preferences but different human capital endowment) make different educational investment decisions. This leads to the fact that children of less educated parents do not have the chance to close the gap to the better educated agents. We analyze whether a PAYG system leads to higher inequality concerning the human capital allocation in a small open country and whether higher economic output goes hand in hand with higher inequality.

4.1 Introduction

Pension payments are the major income of individuals when retired. Most western industrialized countries implemented a pay-as-you-go (PAYG) pension system at a time when individuals did not often become much older than the retirement age and birth rates were quite high. A PAYG pension system is based on a contract between generations: children pay pensions for the parental generation. Nowadays life expectancy has dramatically increased and birth rates have decreased. Retirees receive pension payments for a longer period (age at entry constant) which are financed by less contributors. Although France and Germany recently increased the age of entry to retirement, this uprating does not reflect the increase in the life span observable in both countries[1]. Therefore, such a PAYG system faces financing problems in order to guarantee suitable pension payments. It is often discussed to switch from a PAYG to a fully funded (FF) pension system. Besides the technical problems of such a switch (generations who already contribute will not receive pension payments and have to pay double to ensure old-age consumption) it is not clear if an FF system is the dominant one with respect to welfare. Many papers in the existing literature show that an FF system leads to higher economic growth while the effects on welfare are not clear. The intention of this paper is to clearify whether an FF system is "better" in terms of welfare in general or whether a PAYG system is the dominant one under certain conditions. We do not consider how a switch from one system to the other can be solved. This should be done in a next step if an FF system is verified to be the dominant one.

Using an OLG-model with homogenous agents living for three periods we present an equilibrium model of a small open economy with inter-generational altruism. The small-open economy assumption neglects effects arising through human capital formation on the capital market but allows us to clearify the effects of different pension systems on investments in education and welfare in a simple framework. This paper analyzes how different pension systems influence human capital formation of agents.

[1] When Bismarck introduced the public pension system in Germany in 1889 the average expected lifespan was 40 years and in 2010 it was 80 years.

Agents decide whether and how much (we exclude credit constraints) to invest in the education process of their children. The aim is to show that agents behave differently (concerning education expenditures) under the two different pension systems. Further, we show that a PAYG pension system leads to higher human capital formation and higher economic output than an FF system under certain conditions. The common opinion[2] is that an FF pension system leads to higher economic growth (through higher physical capital accumulation). We try to figure out whether this holds in general or whether certain conditions exist under which a PAYG system leads to higher economic growth. We introduce in a next step two different families of agents who are homogenous within the family but heterogenous concerning their human capital endowment between the families. Regarding their preferences and their ability parameter the agents of the two families do not differ. The influence of the pension systems is different on the two families. It is shown that the familiy which is initially wealthier (higher human capital leads to higher wage income) will remain the wealthier familiy over time. We analyze under which pension system the inequality concerning the allocation of human capital within a generation is larger and whether higher economic output goes hand in hand with higher inequality or not. The model shows that under certain conditions there exists a positive link between inequality and economic output. In contrast, Galor and Zeira (1993) show that higher inequality goes hand in hand with lower economic output. The main difference to our paper is the existence of imperfect credit markets, bequests and the existence of two different production functions which either employ unskilled or skilled labor. Agents decide whether to invest in education or not. If they are initially rich (due to positive bequests of parents), they are more likely to invest in education than agents who are initially poor. Poor agents need to take credits for education investments. If these credits are too expensive, they decide to stay unskilled and work in the unskilled production sector. Therefore, rich agents stay rich and poor remain poor while the inequality is raising. Our paper does not compare the two different pension systems in a steady state analysis but pointwise in each period.

[2]See for example Kaganovich and Zilcha (2012)

4.2 The benchmark model

An overlapping generations model (OLG) with a lifespan of three periods for each individual is considered. Like Kaganovich and Meier (2008) we restrict our analysis to a small open economy where perfectly competitive firms (or a single representative firm) produce a single homogenous good. This good can be used for consumption and investment purposes. The production process is given by an aggregate production function $F(K, H)$. Input factors to their constant returns neoclassical production technology are human H and physical capital K. The latter fully depreciates in every period. Due to the small open economy the interest rate r_t is given exogenously. For simplicity, the rate of return $R_t = 1 + r_t$ on physical capital is assumed to be constant over time. Using a constant returns technology with a fixed interest rate the wage rate w per human capital is constant over time as well. This model treats physical capital as a mobile input factor whereas human capital is immobile.
The three periods of each individual are indentified as childhood, adulthood and retirement. In the period of childhood individuals do not make any decisions about consumption, education or savings. Concerning their life-time utility this period can be neglected. The important period is adulthood. In this period each individual supplies inelastically one unit of labor and receives wage income according to his individual human capital $w\,h$. Hence individual human capital h stands for effective labor supply. Decisions on having children, investing in education of children, consumption and savings are made in this period by adult agents. Having children is an asexual process in this model, i.e. each agent has only one parent. In the third period, the retirement period, individuals do not work anymore and, therefore, do not receive any wage income. Thus, old-age consumption has to be financed by returns on savings and pension payments. Bequest motives are completly neglected so that pensioners consume their entire income. Each generation will be indexed with its birth period $t-1$, so an agent of generation $t-1$ was born in $t-1$, will be working in t and will be retired in $t+1$.
In the benchmark model we consider homogenous agents, i.e. they do not differ regarding their preferences and their human capital endowment. Their preferences are described by an additively separable utility function, which is given for an agent

of generation $t-1$ (born in period $t-1$) in the following way:

$$U(c_{t,t-1}, c_{t+1,t-1}, h_t) = \alpha \ln c_{t,t-1} + \beta \ln c_{t+1,t-1} + \theta \ln n_t h_t. \tag{4.2.1}$$

$c_{t,t-1}$ denotes consumption of an adult agent in t who was born in $t-1$. $c_{t+1,t-1}$ denotes his consumption when retired, h_t denotes the human capital of his children and n_t shows how many children he has. In this model the population stays constant over time, i.e. the number of children per agent is set to one $n_t = 1$. Obviously individuals have not only preferences for their own consumption when adult respectively when retired but they also care about the human capital of their children. This expresses parents' altruism towards their offsprings. This is due to the positive correlation between human capital and wage income and through this consumption (of children) is increased by higher human capital. Under a PAYG pension system future human capital has also an indirect effect on the utility as it influences pension payments positively. The human capital formation is given in the following way:

$$h_t = BE_t \tag{4.2.2}$$

where E_t is the amount a single parent spends on the education of its children and B is an ability factor with $B > 0$. In our model human capital formation is purely driven by the parental education investment. This illustrates that parents can influence the human capital of their children by investing more or less in the education of their children. Their total expenditures on education need to be divided by the number of children but, as we assume a constant population, it is redundant in this model. Consumption in the adult period t is limited by the following budget restriction:

$$c_{t,t-1} = (1-\tau)wh_{t-1} - s_t - E_t \tag{4.2.3}$$

where $0 < \tau < 1$ is the contribution rate to the pension system determined by the government and fixed over time. Consumption in the retirement period is determined by

$$c_{t+1,t-1} = Rs_t + T_{t+1} \tag{4.2.4}$$

where T_{t+1} is the pension payment an individual born in period $t-1$ receives when retired in period $t+1$. Agents in this model are assumed to be rational utility maximizer. Hence, their objective function is given by

$$\max_{E_t \geq 0, s_t, c_{t,t-1}, c_{t+1,t-1}} U(\cdot) = \max_{E_t \geq 0, s_t, c_{t,t-1}, c_{t+1,t-1}} (\alpha \ln c_{t,t-1} + \beta \ln c_{t+1,t-1} + \theta \ln h_t) \tag{4.2.5}$$

with respect to the human capital formation (4.2.2) and the two budget constraints (4.2.3) and (4.2.4). In a general setting without distinguishing yet between the two different pension systems and taking R and w as given we derive their decision rules for savings and education investments in the following way:

$$s_t = \frac{1}{P}\left[\frac{1}{\alpha+\beta}(1-\tau)[\theta(1-\tau)wh_{t-1} - (\alpha P) - \theta)\frac{T_{t+1}}{R}] - \frac{\delta}{B}h_{t-1}\right] \tag{4.2.6}$$

$$E_t = \frac{\theta}{P}\left[\frac{T_{t+1}}{R} + (1-\tau)wh_{t-1}\right] \tag{4.2.7}$$

with $P := \alpha + \beta + \theta$. While the savings decision can take negative values, which would indicate credit taking, the education expenditures E_t are restricted to positive numbers, i.e. $E_t \geq 0$ for all periods $t \geq 0$.

4.2.1 Decisions under PAYG

The PAYG pension system is the predominant system in most western economies. It is characterized by a reallocation of wage income between generations. Young working consumers are forced by the government to release a part of their wage income (contributions) to the pension system. The PAYG system collects the contributions but does not invest them in the production function. Instead it gives it directly to the old-age pensioners (beneciaries). Hence, it implements a pure reallocation of wage income from young to old adults. We indicate variables belonging to the two different pension systems by a superscript G for PAYG and by a superscript F for FF pension system. We drop the superscripts whenever it is possible. In equilibrium the aggregate amount of pension payments must equal aggregate contributions. As the adult generation pays for the retired generation, the budget constraint of a PAYG pension system is described by: Aggregate pension payments have to equal the aggregate contributions in the same period.

$$NT_{t+1} = \tau w h_t N \tag{4.2.8}$$

A PAYG pension system is introduced in $t = 0$ in such a way that agents, born in $t = 0$, are the first agents who contribute to the pension system in $t = 1$ while adult. Agents who are adult in $t = 0$ receive benefits in $t = 1$ without having contributed themselfes to the pension system. Agents who are retired in $t = 0$ do not receive any benefits out of the PAYG pension system.

Definition 4.2.1. *Given the stock of agents N and the exogenously given and fixed rate of return R, a dynamic competitive equilibrium (DCE) is a sequence of individual agent's decisions $\{E_t, s_t, c_{t,t-1}, c_{t+1,t-1}\}_{t=0}^{\infty}$, individual levels of human capital $\{h_t\}_{t=0}^{\infty}$, factor prices of labor $\{w_t\}_{t=0}^{\infty}$ and a sequence of pension payments $\{T_{t+1}\}_{t=0}^{\infty}$ so that:*

- *For each period $t = 0, 1, \dots,$ the collection of $E_t, s_t, c_{t,t-1}, c_{t+1,t-1}$ solves the individual household's problem (4.2.2)– (4.2.5) where factor prices w, R and future pension payments T_{t+1} are taken as given.*
- *Factor markets clear. Factor markets are competitive, hence according to the economy's production function the factor prices are determined by their marginal products.*

$$R_t = F_K(\frac{K_t}{H_t}, 1)$$
$$w_t = F_H(\frac{K_t}{H_t}, 1)$$

- *Individual pension payments received by generation t retirees satisfy (4.2.8).*

Due to the small open economy assumption R is exogenously given and for simplicity fixed. The constant returns neoclassical production technology in combination with R constant leads to a constant wage rate w.

If the human capital formation rule (4.2.2) and equation (4.2.7) are considered, equilibrium pension payments are given by

$$T_{t+1} = \tau w h_t \frac{\theta R}{PR - \tau w \theta B} B(1-\tau)w \tag{4.2.9}$$

A PAYG pension system should provide retirees with positive pension payments.

Assumption 4.2.2. $\frac{R}{w} > \frac{\tau B\theta}{\alpha+\beta+\theta}$

This assumption holds troughout the entire model and guarantees positive pension payments. It is fulfilled for τ sufficiently small and the factors revealing preferences for own consumption while adult and while retired are sufficiently large. If the altruism towards children is too large and the contribution rates too high, parents would accept negative pension benefits to ensure their children a higher wealth. Negative pension benefits mean that retired agents pay for the adult agents. Human capital of the children depends positively on the parental human capital. Through this pension payments depend positively on the retiree's own human capital.
If we plug the equilibrium pension payment (4.2.9) under PAYG into (4.2.7), we see:

$$E_t = \frac{\theta}{P}\left[\frac{\theta}{PR - \tau w\theta B}\tau w h_{t-1} B(1-\tau)w + (1-\tau)w h_{t-1}\right]. \tag{4.2.10}$$

The education expenditures are the higher the higher the altruism of parents towards their children is. If parents do not care about children's utility, they do not invest in the education at all. Although this would mean that they do not receive any pension benefits out of the PAYG pension system.

4.2.2 Decision under FF

The mechanism of a fully funded public pension system is completly different than the PAYG system. Contributions to the pension system of working agents are collected and invested in the production technology for one period. Therefore, agents receive the return on their own contributions one period ahead in the form of pension payments. Hence, this system shows no reallocation effects between two consecutive generations and because of the Bismarck payment scheme (pension benefits are payed according to own contributions) also no intra generational redistribution. As there exists only one production technology, private savings and contributions to the pension system are perfect substitutes since they earn the same return[3]. Pension

[3] In this model we neglect any tax benefits or allowances, which a government often provides to FF pension systems.

payments under an FF system have the following structure:

$$T_{t+1} = R\tau w h_{t-1}. \tag{4.2.11}$$

Using (4.2.7) and (4.2.11), the equilibrium education expenditures are:

$$E_t = \frac{\theta}{P} w h_{t-1}. \tag{4.2.12}$$

Although under FF agents do not receive pension benefits which are financed by their children, they invest in the education of their children as long as they care for children's utility. Hence, as long as altruism towards children is active.

4.2.3 Comparison of Welfare under PAYG and FF

In this section we will compare the welfare a single agent achieves under the two possible pension systems. Welfare of a single agent is defined as his life-time utility in this model. The utility function is monotonically increasing in all three arguments $c_{t,t-1}, c_{t+1,t-1}, h_t$, therefore, it is sufficient to compare them separately (we will see that if one argument is larger under one pension system the others are larger under the same system as well).

We set $t = 1$ as the initial period. In this initial period the human capital of adult agents (born in $t - 1 = 0$) is identical under both pension systems. Hence, $h^G_{t-1} = h^G_0 = h^F_0 = h^F_{t-1}$ holds. The human capital of children can already differ under the two system in $t = 1$.

We will now compare the three arguments of the utility function separately to analyze which system provides higher welfare. First we compare the human capital of agents in any period t and question whether it is smaller under an FF pension system.

$$\begin{aligned} &h^G_t > h^F_t \\ \iff &BE^G_t > BE^F_t \end{aligned}$$

Proposition 4.2.3. *In our small open economy a PAYG pension system leads to higher individual human capital than an FF system in any period* $t \geq 1$ *if* $\frac{\tau\theta B}{P} < \frac{R}{w} < \frac{\theta B}{P}$ *holds.*

Proof. For higher individual human capital under PAYG than under FF in every period $t \geq 1$ the following must hold:

$$B\frac{\theta}{P}wh^F_{t-1} < B\frac{\theta}{P}[\frac{\theta}{PR-\tau w\theta B}\tau wh^G_{t-1}B(1-\tau)w + (1-\tau)wh^G_{t-1}]$$
$$\Longleftrightarrow h^F_{t-1} - h^G_{t-1} < \tau h^G_{t-1}[\frac{\theta}{PR-\tau w\theta B}B(1-\tau)w - 1]$$

Start with $t = 1$: we know $h^F_0 = h^G_0$. Hence, for $h^G_1 > h^F_1$ the following must hold:

$$0 < \tau h^G_0[\frac{\theta}{PR-\tau w\theta B}B(1-\tau)w - 1]$$

This holds for $\frac{R}{w} < \frac{\theta B}{P}$. In $t = 2$ human capital is higher under PAYG if

$$h^F_1 - h^G_1 < \tau h^G_1[\frac{\theta}{PR-\tau w\theta B}B(1-\tau)w - 1]$$

holds. The RHS is positive if $\frac{R}{w} < \frac{\theta B}{P}$ and the LHS is negative as shown before. Therefore, $h^G_2 > h^F_2$.

We conclude: Individual human capital is under PAYG higher than under FF for all periods $t \geq 1$ if $\frac{\tau\theta B}{P} < \frac{R}{w} < \frac{\theta B}{P}$. □

Assumption 4.2.4. $\frac{\tau\theta B}{P} < \frac{R}{w} < \frac{\theta B}{P}$

In a small open economy higher individual human capital has no influence on the factor prices R and w. While under a PAYG pension system agents benefit from higher human capital of their children via higher pension benefits, agents do not receive higher pension benefits under an FF system as benefits only depend on their own contributions. Preferences for the consumption of children, i.e. for human capital of children, are the same under both systems, hence, they do not explain the differences of the human capital under different systems. Kaganovich and Zilcha (2012) show that an FF system leads always to a higher human capital in a closed economy. They explain this by the fact that education expenditures have no direct positive effect on the present value of pension benefits under PAYG. Since our model uses a small open economy, physical and human capital accumulation have no effect on the wage rate and the rate of return. Therefore, education expenditures have a positive effect on pension benefits in our framework.

Many papers treating the growth effect of pension systems claim that an FF system leads to higher economic output due to higher physical capital accumulation in a closed economy. The considered economy in this model is a small open one with mobile physical capital but immobile human capital. Due to perfect physical capital mobility, a neoclassic production technology with constant returns and a constant rate of return, physical capital will be supplied in equilibrium in a constant proportion to human capital. This implies that aggregate output grows at the same rate as the human capital accumulation. Since the size of the population stays constant in this model, individual human capital is decisive. Considering a population with homogenous agents, we showed above that a PAYG pension system leads to higher education expenditures under certain conditions, i.e. Assumption 4.2.4. These higher education expenditures lead to higher human capital and, therefore, to higher economic output.
Consumption in adulthood and old-age retirement are given under both pension systems in the following way.

$$\begin{aligned}
c^F_{t,t-1} &= \frac{\alpha}{P} w h^F_{t-1} \\
c^G_{t,t-1} &= \frac{\alpha}{P}[(1-\tau) w h^G_{t-1} + \frac{\theta}{PR - \tau w \theta B} \tau w h^G_{t-1} B(1-\tau) w] \\
c^F_{t+1,t-1} &= \frac{\beta}{P} R w h^F_{t-1} \\
c^G_{t+1,t-1} &= \frac{\beta}{P} R[(1-\tau) w h^G_{t-1} + \frac{\theta}{PR - \tau w \theta B} \tau w h^G_{t-1} B(1-\tau) w]
\end{aligned}$$

Proposition 4.2.5. *In our small open economy, a PAYG pension system leads to higher human capital and to higher welfare of the agents as well under Assumption 4.2.4.*

Proof. Consumption of an adult agent in period t is higher under PAYG than under FF if the following is fulfilled:

$$\frac{\alpha}{P} w h^F_{t-1} < \frac{\alpha}{P}[(1-\tau) w h^G_{t-1} + \frac{\theta}{PR - \tau w \theta B} \tau w h^G_{t-1} B(1-\tau) w] \quad (4.2.13)$$

$$\Longleftrightarrow h^F_{t-1} - h^G_{t-1} < \tau h^G_{t-1}[\frac{\theta}{PR - \tau w \theta B} B(1-\tau) w - 1] \quad (4.2.14)$$

Using the proof of proposition 4.2.3 we can claim that agent's consumption in adulthood is higher under PAYG than under FF under Assumption 4.2.4 The same is true for consumption in the retirement period. □

This comparison does not include agents who are adult or retired in $t = 0$. Agents, adult in $t = 0$ would benefit from the implementation of a PAYG system as they would receive pension benefits without contributing self. Whereas under an FF system they do not receive free pension benefits. Further, we compare only the welfare of one generation and not the aggregate welfare of all agents (adult and retired) under different systems. Under Assumption 4.2.4 agents are endowed with higher human capital under a PAYG pension system and they also receive higher pension payments than under an FF system in every period $t \geq 1$. This enables them to save less for future consumption during their working period and to consume more while adult. Although their return on savings is smaller than under an FF system they consume more since the higher pension benefit is able to overcompensate this "loss".

4.3 Dynasties of heterogenous agents

In this section we distinguish between two dynasties of families. One is indicated by the superscript L and the other by the superscript H. An agent, whose parent belonges to the L family, will belong to this family as well. Therefore, we distinguish between two types of agents: type L and type H where the types only signalize their affiliation to one family. The agents belonging to one familiy are all homogenous and are endowed with the same amount of human capital. We assume that the preferences and the "learning" ability of the agents, indicated by the parameter B, are the same no matter to which family the agents belong. In the initial period $t = 1$ agents, born in $t - 1 = 0$, of the two families differ regarding their human capital endowment. Initially, agents of type L are per assumption endowed with less human capital than agents of type H so that $h_0^L < h_0^H$ holds. The human capital production function is the same as in the benchmark model.

$$h_t^I = BE_t^I \tag{4.3.1}$$

with $I = \{L, H\}$.

In equilibrium the aggregate amount of pension payments for the retirees must equal the total budget of the pension system, i.e. under PAYG the aggregate amount of (new) adult agents' contributions and under FF the return on own aggregate contributions. The number of agents belonging to family L is given by N^L and the number of agents belonging to family H by N^H. The sizes of the families are not necessarily equal but as we exclude population growth in this model they are constant over time. The budget constraints of the two pension systems are given in the following way:

$$\text{PAYG}: \qquad N^L T^L_{t+1} + N^H T^H_{t+1} = \qquad \tau w[N^L h^L_t + N^H h^H_t] \tag{4.3.2}$$

$$\text{FF}: \qquad N^L T^L_{t+1} + N^H T^H_{t+1} = \qquad R\tau w[N^L h^L_{t-1} + N^H h^H_{t-1}] \tag{4.3.3}$$

Both pension systems adopt in this model a Bismarck payment scheme. Under such a scheme, retirees receive pension benefits which are proportional to their own contributions, i.e. $T^H_{t+1} = \frac{h^H_{t-1}}{h^L_{t-1}} T^L_{t+1}$. Using (4.2.7) and (4.3.2), we receive the pension benefits of an agent under PAYG in the following way:

$$T^I_{t+1} = \frac{\theta}{PR - \tau w \theta B} \tau w h^I_{t-1} RB(1-\tau)w \tag{4.3.4}$$

with $I = L, H$ and under an FF system pension payments are determined as:

$$T^I_{t+1} = R\tau w h^I_{t-1}. \tag{4.3.5}$$

Using (4.3.4) and (4.3.5) and plugging them into (4.2.7) we derive the education expenditures under PAYG and under FF.

$$\text{PAYG}: \qquad E^I_t = \qquad \frac{\theta}{P}(1-\tau) w h^I_{t-1} \left[\frac{\theta}{PR - \tau w \theta B} \tau B w + 1 \right] \tag{4.3.6}$$

$$\text{FF}: \qquad E^I_t = \qquad \frac{\theta}{P} w h^I_{t-1} \tag{4.3.7}$$

Equation (4.3.6) shows that educational expenditures are increasing in the individual human capital. From this and from the fact that in the initial period the relation $h^L_0 < h^H_0$ holds, we conclude that agents, belonging to family H, are better educated as type L agents in every period $t \geq 0$ as long as altruism towards children is active.

The same is true under an FF pension system, see (4.3.7). Hence, under both pension systems agents belonging to familiy H are in every period $t \geq 0$ endowed with higher individual human capital than the agents of family L. Equation (4.3.6) and (4.3.7) show as well that the education expenditures are decreasing in the factors α and β which weight the preferences for own consumption when young and old (remember: $P := \alpha + \beta + \theta$).

4.3.1 Inequality and Growth

Galor and Zeira (1993) show that in a small open economy with credit market imperfections inequality regarding human capital allocation harms economic growth. More recently Berg and Ostry (2011) use panel data to show that economies, featuring a more equal allocation of wealth throughout the population, grow faster. This is not necessarily true in our model.

In this section we analyze whether higher inequality (under one pension system) and higher economic growth (under the same system) go hand in hand. The difference in individual human capital endowment is used as an expression for inequality. Inequality occuring in an economy in period t is defined as $IE_t = |h_t^H - h_t^L| = h_t^H - h_t^L$ as we saw that individual human capital is always higher in family H. IE_t measures the inequality between young agents, i.e. children in period t. In period $t-1 = 0$ the inequality among the initial generation is the same under both pension systems as the initial human capital endowment is the same. Comparing the inequality under both pension systems over time leads to the following result.

Theorem 4.3.1. *A PAYG pension system leads to higher inequality if Assumption 4.2.4 holds.*

Proof. Comparing inequality under both systems leads to the following equivalence relation[4]:

[4]Using (4.3.4) and (4.3.6) (respectivally (4.3.5) and(4.3.7)) and plugging them into the human capital formation rule (4.3.1) gives the human capital in every period t. Using this to determine the inequality leads to the fact that inequality in period t depends on human capital of period

$$\begin{aligned} & IE_t^F < IE_t^G \\ \iff & (h_{t-1}^{H^F} - h_{t-1}^{L^F}) < (1-\tau)(h_{t-1}^{H^G} - h_{t-1}^{L^G})[\frac{\theta}{PR - \tau w\theta B}\tau wB + 1] \\ \iff & \frac{(h_{t-1}^{H^F} - h_{t-1}^{L^F})}{(h_{t-1}^{H^G} - h_{t-1}^{L^G})} < (1-\tau)[\frac{\theta}{PR - \tau w\theta B}\tau wB + 1] \end{aligned}$$

Start with $t = 1$: We know $h_0^{H^F} - h_0^{L^F} = h_0^{H^G} - h_0^{L^G}$. Therefore, the comparison reduces to

$$\frac{(h_0^{H^F} - h_0^{L^F})}{(h_0^{H^G} - h_0^{L^G})} = 1 < (1-\tau)[\frac{\theta}{PR - \tau w\theta B}\tau wB + 1]$$

which is fulfilled for $\frac{R}{w} < \frac{\theta B}{P}$. This holds due to Assumption 4.2.4. Therefore, $IE_1^F < IE_1^G$ holds. Using this result we can easily confirm that $IE_t^F < IE_t^G$ holds for every period $t \geq 1$. $\square$

Obviously, the same assumption, which leads to higher human capital under PAYG in the benchmark model leads to higher inequality under PAYG if heterogenous agents exist. Human capital of both types of agents grow with the same rate[5]. As this growth rate is higher under PAYG than under FF, the gap between the human capital endowment of the two families will be higher under PAYG than under FF for all $t \geq 1$.

$t - 1$.

$$\begin{aligned} IE_t^G &= B\frac{\theta}{P}(h_{t-1}^{H^G} - h_{t-1}^{L^G})(1-\tau)w[\frac{\theta}{PR - \tau w\theta B}\tau wB + 1] \\ IE_t^F &= B\frac{\theta}{P}w(h_{t-1}^{H^F} - h_{t-1}^{L^F}) \end{aligned}$$

[5]

Under PAYG: $g^G := \frac{\theta}{P}(1-\tau)w\left[\frac{\theta}{PR - \tau w\theta B}\tau Bw + 1\right]$

and under FF: $g^F := \frac{\theta}{P}w$

In the previous section we mentioned that in equilibrium physical and human capital are used in a fixed relation for production. Therefore, to determine the economic output it is sufficient to know the aggregate human capital. For a comparison of economic output, used as an indicator for economic strength, it is sufficient to compare aggregate human capital.

Proposition 4.3.2. *Under Assumption 4.2.4 a PAYG pension system leads to higher aggregate human capital and through this to higher economic output.*

Proof. Comparing the aggregate human capital leads to the following equivalence relation:

$$\begin{aligned} & H_t^F < H_t^G \\ \iff & N^L h_t^{L^F} + N^H h_t^{H^F} < N^L h_t^{L^G} + N^H h_t^{H^G} \\ \iff & N^L (h_t^{L^F} - h_t^{L^G}) < N^H (h_t^{H^G} - h_t^{H^F}) \end{aligned}$$

If we can show that the LHS is for all t negative and the RHS is for all t positive, it proves that under PAYG the economic output is higher in every period. First, we start with a comparison of $h_t^{L^G}$ and $h_t^{L^F}$ and receive the following equivalence relation.

$$\begin{aligned} & h_t^{L^F} - h_t^{L^G} < 0 \\ \iff & h_{t-1}^{L^F} - h_{t-1}^{L^G} < \tau h_{t-1}^{L^G}[\frac{\theta}{PR - \tau w \theta B} B(1-\tau)w - 1] \end{aligned}$$

Starting with $t = 1$ the comparison reduces to

$$0 < \tau h_0^{L^G}[\frac{\theta}{PR - \tau w \theta B} B(1-\tau)w - 1].$$

This holds for $\frac{R}{w} < \frac{\theta B}{P}$, which is fulfilled due to Assumption 4.2.4. In $t = 2$ the comparison of $h_t^{L^F}$ and $h_t^{L^G}$ is given as

$$h_1^{L^F} - h_1^{L^G} < \tau h_1^{L^G}[\frac{\theta}{PR - \tau w \theta B} B(1-\tau)w - 1]$$

The LHS is negative, as shown before, whereas the RHS is positive. Hence, $h_2^{L^F} < h_2^{L^G}$ holds. From this we can conclude that $h_t^{L^F} < h_t^{L^G}$ holds for all periods $t \geq 1$. The same formalism can be used to prove $h_t^{H^F} < h_t^{H^G}$. Combining these two results it is proven that $H_t^F < H_t^G$ holds for all periods $t \geq 1$. □

This shows that in our framework higher inequality goes hand in hand with higher economic output/strength. Both types of agents have higher human capital under PAYG than under FF as long as Assumption 4.2.4 holds. This is due to higher education expenditures. The stronger growth of human capital leads to higher economic output.

4.3.2 Welfare Comparison

We already saw in the last section that human capital of L and H type agents is higher under a PAYG pension system if $\frac{\tau\theta B}{P} < \frac{R}{w} < \frac{\tau\theta B}{P}$ (Assumption 4.2.4). The welfare comparison in the benchmark model shows that the pension system which leads to higher individual human capital leads to higher consumption when adult and retired. The same is true if two different families are introduced.

Proposition 4.3.3. *A PAYG pension system leads to higher welfare of agents belonging to both families than an FF system if Assumption 4.2.4 holds.*

Proof. For a proof see before. □

4.4 Conclusion

The aim of this paper was to analyze whether an FF pension system is in all aspects (economic growth and welfare) the dominant system in general or whether a PAYG system leads to higher welfare under certain conditions. We were able to prove, that a PAYG system leads to higher welfare and to higher human capital and through this to higher economic output, as we assume a small open economy, if certain conditions regarding the preferences and the contribution rate are fulfilled. This shows, that neither an FF nor a PAYG pension system is in general the dominant system and that the dominance of one system depends on the given parameters. If a government wants to decide which pension system is the right one for a certain economy in terms of welfare and economic growth, the given parameterization has to be taken into account. Heterogenous agents (regarding their human capital endowment) make varying decisions with respect to their investments in the education of their children.

We were able to show that agents with higher human capital invest always more than agents with lower human capital. Meaning that the allocation of human capital will never be homogeneous. If the intention is to close the gap between low and high-skilled agents, education of low-skilled agents must be selectively assisted. This could be done in the form of education vouchers. Nevertheless, experience in Germany shows that they are quite often not demanded. Hence, education has to be made mandatory for the group with low human capital endowment. We analyzed under which system inequality regarding the human capital allocation is higher and showed that, depending on the parameterization, higher inequality goes hand in hand with higher economic output. This is possible in reality as well as the example of China shows. Fast economic growth is possible although inequality within the population is high and is increased through growth even further. In our model, using a small open economy, it is possible to draw conclusions from economic strength to welfare. It is shown, that the pension system which causes higher economic output leads to higher welfare of all types of agents as well.

Chapter 5

Conclusion

This work was concerned with the question whether higher economic growth implicates higher welfare for all agents. Chaper 2 showed that under an FF pension system production output is higher due to a higher physical capital accumulation (aggregate human capital is the same under both systems). In our framework of a closed economy the crucial factor is the capital intensity factor. This factor determines whether welfare is higher under one system or under the other. This suggests that different economies, which vary in their capital intensity, benefit in terms of welfare from distinguished pension systems although an FF system always generates higher economic output. In chapter 3 and 4 we used a small open economy which neglects effects of human and physical capital accumulation on the factor prices. Both chapters showed a positive relation of economic output and welfare but FF led in this framework not necessarily to higher output. Chapter 3 also pointed out that an increase in production output (through an increase in average human capital) does not inevitable enhance welfare of all heterogeneous agents. This makes clear that economic output per capita can be used as an indicator for welfare which is easily evaluable but it gives not necessarily the true impression.

We figured out the relation between implemented pension systems and decisions for children and for investments in their education as well. Depending on the given economic parameters and the preferences of the agents both systems may lead to a larger number of children and to higher investments in the education of children. In the introduction we posed the question whether incentives, given in form of transfer payments, increase the fertility of high-skilled agents and through this the economic output per capita. Such transfer payments worked in our framework (described in chapter 3) in the intended way. They raised fertility and, therefore, average human capital which induced higher economic output per capita. However, only the high-skilled agents experienced a welfare increase while the low-skilled agents suffered from such transfer payments.

Also, we shed light on the question whether higher inequality (in terms of human capital endowment of agents) goes hand in hand with higher economic output. We carved out a positive link between these two measurements, which is confirmed by real world data as well, e.g. the Chinese economy.

Although we analyzed the effects of pension systems on different stylized facts, we are not able to approve the dominance of one system in general. Depending on the given economic parameters the government should decide whether a switch from PAYG to FF is economically reasonable. Nevertheless, the yet open question is how such a switch could be realized. In the introduction we have already pointed out that the transition generation has to be compensated. This is a tough challenge and not solved yet. Hence, further research on this aspect would be reasonable.

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