The architecture of modern mathematics: Essays in history and philosophy, edited by José Ferreirós and Jeremy J. Gray, Oxford University Press, Oxford 2006, xii + 442 pp.

In this volume, Ferreirós and Gray have brought together a selection of 14 new essays (only one of them published before) that share a commitment to integrating the philosophical reflection of mathematics with the close historical study of the development of mathematical practices and ideas. In the resulting volume, philosophy and history do not play exactly symmetrical roles: All contributions (with the possible exception of the introduction) present themselves as *historical essays* that highlight some philosophical aspects rather than the other way round. The papers are concerned with mathematical developments between 1800 and 1970, and the majority of them focus on the contributions of one or two great mathematicians within a specific area of mathematics.

The book opens with a long and spirited introduction by the editors, in which they adopt a critical stance towards contemporary mainstream philosophy of mathematics. They see it dominated by attempts to apply "systematic philosophy" to mathematics. The resulting paradigm is criticized for various shortcomings: It tends to ignore the conceptual and methodological *dynamics* of mathematics; it over-emphasizes one element of mathematical practice (viz., proving theorems within the framework of a given axiomatic system) at the expense of other, philosophically no less interesting ones (like conjecturing, explaining, or visualizing); it is obsessed with its pet theories, logic and set theory, and therefore often fails to acknowledge the richness of mathematical concepts and methods; it has suffered from "excessive attention to all-inclusive claims" (p. 18). Such deficiencies, the editors suggest, can be overcome by means of a closer integration of the history and philosophy of mathematics, geared to an ideal of "philosophical reflection" rather than "systematic philosophy". To be sure, similar criticisms have been raised before (for example in Kitcher and Aspray's introduction to their *Minnesota Studies* volume of 1988), and the editors explicitly acknowledge this. Yet one can hardly contest their diagnosis that, by and large, the received picture of the philosophy of mathematics has not changed much during the last 50 years (p. 4 f.). This is despite the many valuable investigations along the lines suggested by Ferreirós and Gray that have of course already been undertaken (among others, by many of the contributors to their volume). Given this, the editors' initiative to present a good selection of recent samples of this work is all the more welcome.

Many of the contributions focus on the philosophical ideas developed by great mathematical practitioners in the context of their work. For example, Jamie Tappenden alerts his readers to the fact that during the time of Frege's mathematical upbringing, which occurred in a context where analysis was widely perceived as a core discipline of mathematics, there was no consensus about what exactly analysis was. In particular, a Weierstrassian tradition with an emphasis on rigor and avoidance of appeal to geometric intuition clashed with a Riemannian one that permitted the application of geometric tools to analysis and pursued conceptual clarity and simplicity as a primary goal. Frege's siding with Riemann's understanding of complex analysis is then shown to be continuous with some of his logico-philosophical insights—e.g., that a function is not an expression and that definitions should be representation-independent.

Riemann himself and his philosophical ideas are subjected to a close investigation in another contribution. José Ferreirós argues that the key to Riemann's philosophy of mathematics is his involvement with physics, which for a while was so serious that he regarded his quest for a unified theory of physical forces as his main occupation. According to Ferreirós, this led Riemann to view mathematics as a part of the natural sciences, which implied the abandonment of its a priori status.

Michael Beaney's paper invites reflections on the historicity of philosophical ideas in mathematics itself, as he reviews the role of historical references in Frege's work and how they relate to his pursuit of the elucidation ("*Erläuterung*") of expressions. The use of historical reference-points like Kant is shown often to be an important part of Fregean elucidation, which is itself an indispensable task as not every expression can be defined in terms of more basic ones.

The volume also contains papers on more unusual and idiosyncratic philosophical ideas of mathematicians. For example, Erhard Scholz portrays Hermann Weyl as an advocate of a "symbolic realism", according to which mathematics acquires meaning from contexts of scientific, technical and social practices, while questions concerning its fundamental ontology should be regarded in an agnostic way. Moritz Epple juxtaposes Felix Hausdorff with his literary alter ego Paul Mongré, under which name he published several pieces of fiction and philosophical reflections in a broadly Nietzschean vein. Epple tries to unearth the common epistemological concerns of the mathematician and the literary writer. Alongside such investigations of individual mathematicians and their philosophical agenda, the volume also presents some interesting and more encompassing efforts to correlate and compare the philosophical reactions of several different mathematicians to certain mathematical and scientific developments. An example is Jeremy Gray's closing essay on the profound changes that transformed mathematics after 1880. He concentrates on geometry and identifies a Leibnizian and a Kantian strategy of coping with the deep philosophical questions about mathematical truth raised by, among other things, the advent of non-Euclidean geometry. How can such historical studies of mathematicians' philosophical ideas (there are several more among the papers within this volume) support the philosophical reflection on mathematics demanded by the editors? I think they can play very helpful roles in this respect, especially if they are concerned with influential, established, or at least coherently defensible philosophical positions. They can serve to demonstrate the contingency of the ideas that dominate the philosophical terrain, in the sense that they show how positions have emerged from changes and tensions within mathematics at certain points in time (as in the case of Frege). They can even highlight alternative paths (as, arguably, in the case of Riemann).

However, if the proposed convergence of history and philosophy of mathematics is to show its whole potential, it must go beyond a mere history *of* philosophy of mathematics. It must also prove its potential to use the history of mathematics in order to lead to convincing, contemporary philosophical insights. The actual mathematical practices of the historical protagonists often offer more important material than their own philosophical reflections. As an illustration of this, Jeremy Avigad has contributed a paper on Dedekind's theory of ideals that exemplifies the great promise of an integrated history *and* philosophy of mathematics. Among the more fascinating aspects of mathematical practice that has received too little attention from traditional philosophy is its considerable freedom: Not only are there myriads of new concepts and structures that could in principle be introduced and investigated, but each of them can also be defined and approached in many very different ways. A case in point is the theory of ideals and its different early treatments, most importantly by Kronecker and Dedekind. Avigad traces the development of Dedekind's several versions of the theory in order to identify the mathematical values that informed his methodological choices. In unexpressed analogy to the so-called "epistemic values" that are familiar from the philosophy of the natural sciences, Avigad gives a precise description of principles of generality, uniformity, and familiarity as they appear in Dedekind's work. The insights enabled by this investigation promise to give rise to many interesting and potentially fruitful historical and philosophical questions—e.g., how did certain mathematical values become established in the mathematical community rather than others, how do they relate to the values of other sciences and even to other ("extra-scientific") values?

Most of the many interesting discussions in this volume offer *impulses* for philosophical reflection rather than their own philosophical conclusions. An exception is the introduction, where Ferreirós and Gray put forward a tentative "hypothetical conception of mathematical practice", which is designed to take the historical richness of the subject into account. It is meant to reflect an insight that most mathematical propositions cannot be taken to be "true in the world we live in", but must be seen as parts of systems based on hypotheses. How exactly this is to be spelled out—and, in particular, how it relates to views that have been discussed at length under the name "deductivism" or "if-thenism"—is not clear. Prima facie, the hard problem for a hypothetical conception is to explain how a whole mathematical theory "is erected on" its hypothetical basis (to use the authors' expression), without loading all the explosive philosophical freight onto an uncritical use of the notion of logical consequence (as deductivism has been criticized of doing). Clearly, systematic philosophizing about mathematics will have to be part and parcel of an integrated enterprise of the history and philosophy of mathematics. Ferreirós and Gray's volume shows that the real convergence has yet to occur, but it also offers enough samples of interesting and insightful work to make the endeavor look promising.

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Harry J. Gensler. ***Historical dictionary of logic.*** Historical Dictionaries of Religions, Philosophies, and Movements, vol. 65. The Scarecrow Press, Lanham (Maryland), Toronto, and Oxford, 2006, xliv + 307 pp.

The book under review is a sort of an encyclopedia of logic. The author declares in the Preface that he understands logic as a discipline analyzing arguments and helping us to see which ones are valid. The aim of the book is to introduce the central concepts of logic in a series of brief, nontechnical articles. The book emphasizes deductive logic but there are also entries on areas like inductive logic, fallacies, definitions as well as key concepts from epistemology, mathematics and set theory.

Logic is nowadays developed by philosophers, mathematicians and computer scientists. In the book under review the philosophical branch of logic is emphasized. Therefore the areas in which philosophers are interested and which they teach are first of all taken into account. The mathematical and computer-science items as more technical and more specialized have not been treated so extensively.

The main part of the book is a dictionary section consisting of 352 entries. They are arranged alphabetically and are devoted to fundamental concepts and problems of logic. The most significant theories and issues arising out of Western philosophy and other traditions, the more common fallacies and argument forms are presented and explained. There are entries on historical periods and figures, including ancient logic, medieval logic, Buddhist logic, Aristotle, Ockham, Boole, Frege, Russell, Gödel, Peano, Quine, etc. One finds there information on propositional logic, modal logic, deontic logic, temporal logic, set theory, many-valued logics, mereology, paraconsistent logic. There are also entries concerning applications of logic to various fields such as philosophy, mathematics or computer science. The presupposed character of the book excluded very technical expositions but the author succeeded to explain technical problems—as, for example, Gödel's incompleteness theorems—in an adequate way not using complicated technical apparatus. The entries vary in length from a sentence or two to several pages.

One should stress that the book under review provides the potential reader not only with knowledge about well established canon of logical notions and concepts but it indicates and explains also many modern, still being developed and studied theories and achievements in logic, in particular those connected with computer science or fuzzy logic as well as with cognitive science.

The front of the book has three important parts: (1) a short notation section, (2) a chronology starting from the ancient Greece and going to 2001, (3) an introduction where an overall view of logic is given (it should provide a broader context for the dictionary entries).