HEAVY QUARKONIA IN QUARK GLUON PLASMA AS OPEN QUANTUM SYSTEMS

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TO MY PARENTS

ABSTRACT

Heavy quarkonia suppression is one of the useful probe for Quark Gluon Plasma formation in heavy ion collisions. It is expected that due to the color screening in QGP, certain quarkonium states will be dissociated at a sufficiently high energy density and temperature. The mechanism predicts a sequential suppression pattern for different quarkonium bound states. Application of this picture to the rapidly evolving medium produced in Heavy ion collision experiments then relies on the fact that quark antiquark pairs created in a given bound or unbound states remain in that same state as the medium evolves. We argue that this scenario implicitly assumes the adiabatic evolution of quarkonium states in the medium. We show quantitatively that this assumption is invalid. The breakdown of adiabatic condition motivates the study of real time dynamics for heavy quarkonium states. Recent developments in this area as well as a noble framework are presented. The framework is applied to harmonic oscillator as a precursor study of realistic heavy quarkonia in QGP. A possible technique to generalize this framework for heavy quarkonium is discussed in this context. These exploratory studies already provide qualitatively novel aspects, which may provide some understanding of recent experimental results. Therefore, it is concluded that the real time dynamics is unavoidable to use heavy quarkonia as a convincing probe for quark gluon plasma in heavy ion collision.

ACKNOWLEDGMENTS

Bielefeld , "the city does not exist" was an internet page appeared to me while searching some photos and facts about the city. The excitement of having a doctoral position in a very new city got threatened by that information. The 'Bielefeld-Conspiracy' wikilink dissolved the matter without any further doubt. Last three years I am living here and now in love with the city. I heartily acknowledge the city which became a home for me from last three years. Three years, of course is not really enough to make a very big progress in research but at the end I feel a satisfactory improvement of my knowledge and ability. For this, I definitely would mention the name of my mentor, my supervisor Prof. Dr. Nicolas Borghini. Progressing and at the same time being careful, having doubts and at the same time keeping faith on own thinking, a very difficult balance I have learnt from him. I have learnt how to make a judicious choice on different thoughts in order to channelize those into different steps of the problem by setting up the priorities. His strong intuition several times showed me the way to crack the problem in certain way even before going to the detail calculation and hence by making it concrete with proper calculation afterwards. During the research on a single topic, I had to go even beyond the specific domain where my learning always got shaped by his expertise on several fields. None of my work would have come to proper conclusion without his master touch to organise the whole issue by putting those in an appropriate platform. Apart from that he always gave me immense mental support by giving example from his own experiences of research. It helped me to keep my motivation into the work even if I have gone through successive failure attempts. No doubt, all these fore sure will help me further in my future to enjoy my scientific career in physics. I have enjoyed a wonderful academic atmosphere in high energy physics floor. All the professors are enthusiastic for every kind of discussions. We were offered several useful courses during these years. My sincere thanks to Prof. Helmut Satz for many fruitful discussion and criticism on my work.

"It does not work" is the regular status of a Ph.D. mind. For me those were tuned into hopes by some very brave and enthusiastic people around me like Florian, Ioan, Sama, Marcel, Markus, Christian and many others. My interaction with Florian, Ioan and Sama was quite different from others. They are full of energy and hope. They became my best friends during this period. Bielefeld high energy physics is full of fun and the 90 percent credit goes to our secretary, Gudrun. She not only laughs loud by herself but makes others

to laugh out loudly no matter how serious they are.
Tamoghna da and Ansu are two important friends in my everyday life since my masters days. I am lucky having such companions. They have guided me, taught me in every situation and spoiled a bit also. The enormous support from such a long distance is only possible by them. They helped me to build my bad habit of asking them about any academic or non academic questions before a self investigation.

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CONTENTS

1 INTRODUCTION

1.1 THE QUARK MATTER

I remember, in high school, just after a lecture on structure of atom, I asked my teacher about the possibility to break the neutron, proton and electron further. I got no answer but a tight slap from my teacher. I realize now what made him so angry. Even if my teacher came up with a positive answer with some new particle as a building block of those, I would have asked the same question again. The slap was not as painful as the thought of making something infinitely divisible which made my teacher restless. The rescue I found by looking back in 55 B. C.[1] is the following
"So there must be an ultimate limit to bodies, beyond
perception by our senses. This limit is without parts, is
the smallest possible thing. It can never exist by itself,
but only as primordial part of a larger body, from which
no force can tear it loose."

Titus Lucretius Carus: De rerum natura,
liber primus 599 – 55 B. C.

The indication (rather the necessity of confinement) was already pointed out in certain philosophical arguments. In modern science, the confinement of quarks, which makes protons and neutrons, is a challenging issue for theoretical high energy and nuclear physicists. It has been understood that pure Yang-Mills theory (on lattice) is confining and physicists are still struggling to understand the mechanism. In a renormalized theory of quantum chromodynamics (QCD), we have seen the role of asymptotic freedom which makes the theory stranger than the abelian one.
The quarks and gluons, which are confined within hadrons, can be liberated in extreme conditions. This is a giant statement in modern era of science. This leads to a careful study of QCD under extreme

1 I saw it in an wondeful lecture series on Introduction to Physics of Quark Matter by Prof. Helmut Satz.

conditions and put forward the experimental dream to observe the constituent degrees of freedom of proton and neutron.
We should start the story from the prediction of asymptotic freedom [1, 2] by Gross, Wilckzek and Politzer in 1973. They came up with the result of how the coupling strength in QCD depends on energy scale.

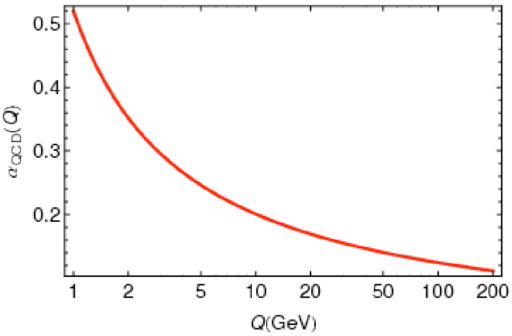
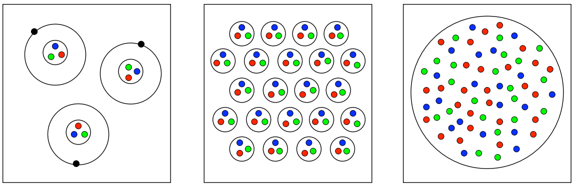


fig-1. *The running coupling constant for QCD [3].*

As we can see in the above figure, with increasing energy Q or with decreasing distance between the quarks, the coupling α_{QCD} becomes small. It happens because the QCD is non abelian theory and the gluons carry color charge (where as photons are chargeless in abelian theory). Asymptotic freedom indicates that inside the proton or neutron, the quarks are not so strongly bound but if we try to tear them apart, they start to attract themselves with a very strong force. In order to make them free, we need infinite amount of energy. There are other ideas around to liberate those fundamental degrees of freedom. We can heat up hadrons to a very high temperature or we can think about a very large hadron density to make them liberated from the specific hadron. Nucleons in general have spatial extension and they are incompressible at the state of hadron. A close packing of nucleons can lead to such a high energy density that the constituent quarks will no longer be associated with a specific hadron. They will have several quarks in their close vicinity which may not belong to the same parent hadron. At this level, a very high energy density of quark matter will be created as it was in the early universe just after the big bang. The nuclear density afterwards decreased and tightly bound quark systems were formed which we see in the normal matter.

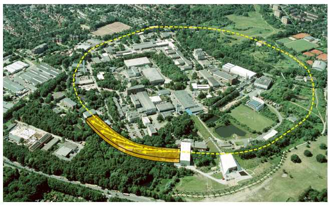


(a) (b) (c)

fig-2. *Increasing density, from atomic (a) to nuclear (b) and to quark matter (c) [4].*

The specific kind of quark matter which we have discussed could be found in the core of neutron stars. Recreating the primordial quark matter in laboratory is a challenge which was chased for the first time around 1984 at Lawrence Barkley National Laboratory (LBNL) and there after in BNL (Brookhaven) and CERN (Geneva). The experimental program already has stepped into recent days with many times more powerful accelerator. The entire program to study extremely dense quark matter is often advertised as Relativistic Heavy Ion Collision. The matter produced in heavy ion collision in laboratory is expected to have certain properties which can be derived from theoretical study of QCD under extreme conditions. Investigation of such QCD matter in very high energy density or temperature demands careful studies of thermodynamics [5, 6] of quantum field theory (mainly quantum chromodynamics). Since last several years a big community is approaching towards this goal to study strongly interacting matter at the extreme conditions and their relevant thermodynamics. Lattice gauge theory is one such promising candidate to investigate theoretically the nature of quark matter in a thermodynamical framework. The phase diagram [7] of QCD was first put forward by Collins and Perry[1] and thereafter many more careful studies have been performed to know the precise behaviour of quark matter in various thermodynamic conditions. Those studies have initiated several important questions. For instance, whether there exists a confinement deconfinement transition for quarks, gluons and what is the nature of the transition. We will discuss the phase diagram in the next chapter with little more detail. Some theoretical predictions have shed light on certain properties of deconfined quark matter which is qualitatively very similar to the electromagnetic plasma state.

1 The very similar idea [8] was put forward almost at the same time by Cabibbo and Parisi.




Geographical view of RHIC, Brookhaven (left) and Large Hadron Collider (right).

In 2001, the Cern relativistic heavy ion collider program announced in a press release that they have found a new state of matter in experiment. The matter produced in the heavy ion collision was very close to the predicted one in theoretical calculation. The fourth state of matter is known as Quark Gluon Plasma. A plasma which is made of nearly free quarks and gluons.

"The combined data coming from the seven experiments on CERN's Heavy Ion programme have given a clear picture of a new state of matter. This result verifies an important prediction of the present theory of fundamental forces between quarks. It is also an important step forward in the understanding of the early evolution of the universe. We now have evidence of a new state of matter where quarks and gluons are not confined. There is still an entirely new territory to be explored concerning the physical properties of quark-gluon matter. The challenge now passes to the Relativistic Heavy Ion Collider at the Brookhaven National Laboratory and later to CERN's Large Hadron Collider."

—In a Cern press release 2001.

In recent years with new advancement in particle accelerators, CERN has launched LHC to carry out several programs. Among them the heavy ion program is of great interest. In CERN, they are trying to create the primordial medium which was supposed to exist in the universe just after the big bang. This little bang creates more possibilities to investigate the deconfined matter made of quarks and gluons.

1.2 PROBES FOR QUARK GLUON PLASMA

The medium produced in ultra relativistic heavy ion collision is very hot and it cools down very rapidly. The persistence of the medium is very short. Experiments show the life time of the medium is of the order of a few fm/c. Therefore, to probe the medium and investigate its properties, external probes are unimaginable. We need to think about certain internal probes for the medium. The probe which we are referring here is specifically important to know whether the medium produced in high energy nucleus nucleus collision is quark gluon plasma or not. The most important thing in this context is the dimension of the internal probes. In order to probe the internal structure of a system of linear size L, the dimension rather the wave length λ of the probe should be less than L. More precisely $\lambda \lesssim a$, when the system has a internal substructure with characteristic size a. In QCD, the fundamental energy scale is Λ_{QCD} which is of the order of inverse hadron size. This fixes the applicable size of the probe less than the hadronic length. For QGP, temperature T is also an important scale to be obeyed as $\lambda < \frac{1}{T}$.

1.2.1 *Electromagnetic probe*

At a very high temperature, the medium could be probed by the radiated virtual or real photons with the wave length much smaller than Λ_{QCD}^{-1}. On the other hand, to obey the scale restricted by the high temperature, one needs to consider high p_T photons or dilepton pairs of very high mass. The temperature scale is really a crucial point as the energy density created in the heavy ion collision brings the temperature equivalence of the order few trillion degree (so as to quote the recent estimate in RHIC and as well as in LHC).

1.2.2 *Jet quenching*

In hadronic collision, high p_T parton showers are being created. Those particles propagate and create further particles. Due to their high momentum (speed), the parton beams become highly collimated and form jets. In high energy collisions one could expect such kind of back to back jets in order to satisfy conservation of momentum. In proton proton collision, we observe such back to back jets. In heavy ion collision, if a medium is being created then the jet should have to travel through the medium. As a result, they should be attenuated due to the strong interaction with the medium.

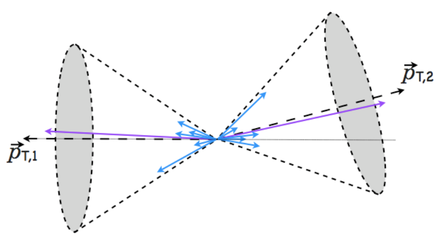


fig-3. *Schematic picture of highly collimated parton beams; Jet.*

In a (non central) collision, two back to back jets can be produced at the edge of the medium and one of them should have to propagate through the medium, while another can leave the medium without travelling much. Hence, one of the jet will be passing through the medium being attenuated.

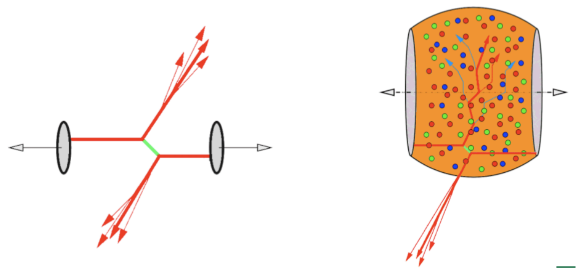


fig-4. *Two back to back jet in the absence of medium (left) and one of the jet is being attenuated in the medium (right).*

The jet which is going through the medium will loose its energy and even may be stopped by the medium. A very high density medium really can stop the jet. To have a quantitative look into this issue, let me describe with little detail.
The loss of energy ΔE of a jet which is propagating through the medium of length L created in a heavy ion collision [9] can be shown to given by,

$$\Delta E = \frac{\alpha_s}{8} C_R \hat{q} L^2 \tag{1}$$

α_s is the interaction strength of the medium constituent and C_R is a constant associated to the specific fast partons. The quantity $\hat{q}$ is known as jet quenching parameter which designates the loss of energy by the jets passing through a medium of certain energy density. Studies have been done to know how the quenching parameter depends on energy density of the medium.

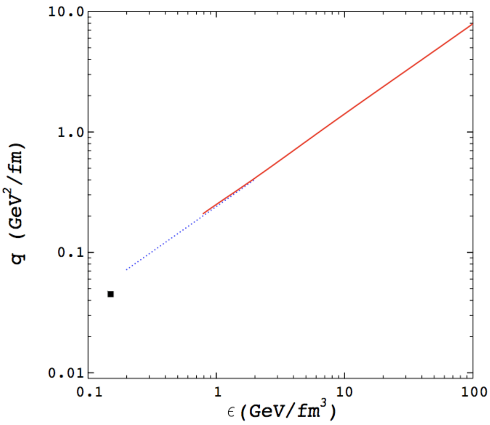


fig-5. *Quenching parameter as a function of energy density for different media (as in Ref. [9]); cold, massless hot pion gas (dotted) and ideal QGP (solid line).*

In the above figure, the dotted line is for a medium made of massless hot pions and the solid line shows what happens in an ideal quark gluon plasma.

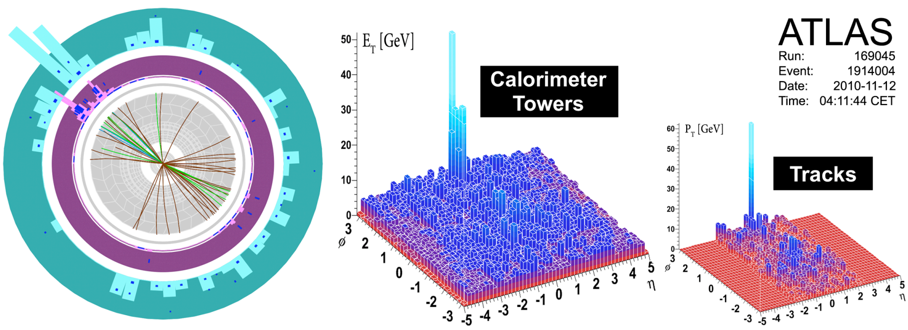


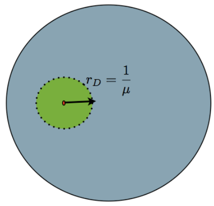
fig-6. *Clear indication of suppression of jet [10]. The recoiling jet is not observed rather the energy deposit in the calorimeter is distributed in a wide azimuthal region.*

Recent measurements [10] on jet produced in heavy ion collisions indicate a strong suppression of parton jets amounting to $\hat{q} \geqslant 1 GeV^2/fm$. In measurement, there is no evidence for two back to back jets which one achieves in case of proton proton collision. This could be a strong evidence for the formation of a medium of high energy density which is unexpected in the medium purely made of hadrons.

1.2.3 *Suppression of quarkonium states*

The deconfined quark gluon plasma is a medium composed of free quarks and gluons. One important property of plasma is the Debye screening. For electromagnetic plasma, the electric field of a source charge is screened due to the freely moving charged particles in the medium. The screening modifies the electric field of the test charge. The modified field is no longer a long range coulomb type rather the field due to the test charge would be short range up to a finite distance.

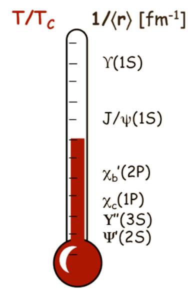
r_D is one characteristic scale of a plasma, known as Debye radius which restrict the the long range interaction within a sphere of influence.



$$V(r,T) = -\frac{\alpha}{r} e^{-\mu(T)r}$$

The medium made of quark and gluon degrees of freedom also carries color charge which, in principle, should provide a color screening characterized by the Debye radius.
Quarkonium is the bound state made of a quark and its antiquark. Due to the color screening, the sphere of attraction about a quark is finite and any bound state which has a bigger size than the Debye radius will no longer be as a bound state in such medium. This could be a good probe to know whether a plasma of color charges has been produced or not.
The very first idea [11] came from Helmut Satz and T. Matsui in 1986 while studying the effect of plasma to such bound states. They explained how color screening could be useful in the context of probing quark gluon plasma. The Debye radius, in principle, depends on the temperature of the medium. Hence, one can infer the possible bound states which can persist at a certain temperature.
The Debye radius decreases with the increase of temperature and therefore by increasing the temperature, one can dissolve lower excited states. A detail description of this will be presented in the next chapter. In order to know the effect of such plasma in quarkonium bound states, one has to investigate properly the Debye screening in quark gluon plasma. Studies have been made in the framework of lattice gauge theory, effective field theory as well as in many different models. Different bound states have different characteristic sizes as well as different binding energies. The higher excited states which have bigger size will be melted when the plasma has the Debye radius just below the size of the specific state. At higher temperature of the plasma, the lower states will start to dissociate as the Debye radius decreases with the temperature. It suggests a pattern of the

dissociation of states with different temperature of the medium. The pattern is sequential and it provides a QGP thermometer [12, 13] . Hence, we have a way to know the plasma temperature from this sequential melting picture.
The studies in this context are mostly driven towards the understanding of heavy quark antiquark potential [14, 15, 16] in the thermal medium of quarks and gluons. This potential is expected to be of Yukawa type. All those careful studies have produced significant amount of data in order to know the dissociation temperature [17] for different bound states of quarkonia. Though all of them [18, 19] do not coincide empirically, the conceptual details are in good agreement. Here, we definitely should mention that the quark antiquark potential is still not well known and there are different arguments available in recent studies. Some of the recent studies pointed out that the effective potential should have an imaginary part in order to describe quarkonium bound states in medium. These issues will be discussed with further detail in chapter 3.



QGP thermometer based on sequential suppression of quarkonia. courtesy: A. Mocsy

Apart from these effective potential studies, an approach through analysing spectral function [20, 21] has been followed in recent lattice studies for QCD. The disappearance of the peaks in the spectral function explains the melting of certain bound states. Studying it at different temperatures, one can give account for dissociation thresholds of different quarkonium states in the deconfined medium. The studies for bottomonia and charmonia have been done carefully in recent days and stands as more accurate description of the probe. Details of those dissociation temperatures for different charmonium and bottomonium states are presented in chapter 2.

1.3 SOME CRITICAL COMMENTS ON QUARKONIA SUPPRESSION AS PROBE IN HEAVY ION COLLISION

In the above section, we have seen how heavy quarkonia could be used as a probe for quark gluon plasma. In experiments, people are trying to observe the signature of charmonium and bottomonium suppressions. Those quarkonium states are formed in heavy ion collision at a very high energy density and thereafter, they evolve in the medium. The created medium then expands very rapidly and cools to freeze out finally. The quarkonium yields come from the medium and we detect them in specific detectors. So, the measurement of different quarkonium states in the detector describes the bound state in the vacuum. Now, association of these states with in medium quarkonia relies on the adiabatic approximation. With the adiabatic approximation, one can neglect the possibilities of energy eigenstate-crossing during their evolution from medium to vacuum states.

Apart from the above criticism, we would like to emphasize that the whole issue of the quarkonia in medium has been treated in thermodynamical ways which applies for a thermalized and static medium. It is worthy to mention that the issue of thermalization in heavy ion collision is not well understood yet. There is even more serious problem when we deal with small systems like heavy quarkonia in a medium which has a size much bigger than that of the small system. There exist two different time scales, namely the time scale of the medium evolution and the time scale of the evolution of quarkonia. In order to be described by thermodynamics, quarkonia should be thermalized with the medium. This requires that the persistence time of quark gluon plasma should be bigger than the time scale of quarkonia. So, before applying the Debye screening picture, one has to know how much time those quarkonia states take in order to be thermalized with the medium.

Even if those states were assured to be thermalized quickly enough with the medium, we can not ignore the issue of rapid cooling of the medium. The sequential suppression pattern which we have discussed earlier relies on the adiabatic evolution of quarkonium states. The adiabatic assumption in this context says that with the evolution of the medium, all the quarkonium states will also evolve smoothly to the same instantaneous energy eigenstates. Before knowing the validity of this adiabatic assumption for the medium produced in heavy ion collisions, one should not adopt the thermodynamical picture in order to use heavy quarkonia as a probe in quark gluon plasma. This issue is discussed in more detail in chapter 3.
In the above section, we have criticised certain issues which are very crucial in the context of making quarkonia suppression a probe for QGP supposed to be created in relativistic heavy ion collisions. Let's list those points.

- The association of vacuum quarkonium states with those in the medium relies on the adiabatic approximation which has not been investigated yet.
- One needs to know the time needed for the quarkonium states to be thermalized with the medium. The thermalization time of quarkonium states should be much smaller than the persistence time of the medium. Otherwise, we can not describe them using thermodynamics.
- We need to investigate whether the rapid cooling of the medium really allows the quarkonia states to evolve adiabatically in the medium or not.

1.4 ORGANISATION OF THE THESIS

We have mentioned certain critical issues related to the suppression of quarkonium states as a probe of quark gluon plasma. Besides, we have discussed two other important probes in brief. The detailed discussion of those are out of the scope of this thesis. We will mainly concentrate on the issues related to the heavy quarkonia. We will investigate those critical points in further detail and will see the necessity of real time dynamics of heavy quarkonia in medium in order to use them as a relevant probe for the medium produced in heavy ion collisions. The static thermodynamical approach is beautiful and explains the suppression in a static quark gluon plasma. It will be pointed out in this thesis that the static approach could not be applied to a medium which evolves so rapidly and persists very short in time.

We have organised the thesis with seven more chapters excluding this short introduction. In Chapter 2, we will discuss non relativistic quarkonium bound states in thermal medium and will give an account of their properties. In the context of the Debye screening mechanism, we will discuss the in medium potential of quark antiquark pair. The sequential suppression picture has been achieved through those thermodynamical calculations. At the end of the chapter, we will present recent theoretical estimations of dissociation temperatures for different charmonium and bottomonium states.

In chapter 3, we will present arguments on the necessity of real time dynamics of heavy quarkonia in order to use them as a probe for QGP. The issues like thermalization and quarkonium time scales are discussed in brief. A comparison with early universe medium is made in order to show differences with the medium created in heavy ion collisions. The adiabatic approximation is scrutinized for quarkonium states in the medium produced in heavy ion collision. The study clearly indicates that one has to think about a dynamical picture rather than static Debye screening.

Chapter 4 is devoted to show recent progress in the context of dynamical evolution of heavy quark bound states. The advantages and disadvantages are also pointed out for those approaches. We will see the justified modelling of quarkonia as an open quantum system which can help us to design a perfect probe of the deconfined quark gluon plasma. Specifically, the time scale of quarkonium in medium will be discussed in a bit quantitative manner with model system like Coulomb bound states in a weakly coupled quark gluon plasma as a thermalized Gaussian bath.

In chapter 5, we will design a new approach to deal with the dynamics of open quantum systems. The new framework is devised for the harmonic oscillator as an open system because that could be a very good precursor study of heavy quarkonium states in medium. The

wave function based approach has an advantage to address real time issues and it is a cost effective technique compared to the approaches based on density matrix. The systematics will also be useful to deal with the dynamics of open systems in many different branches of physics. A connection with coulomb bound state is pointed out also and the technical detail will be discussed in the appendix.
Finally at the end, in chapter 6, we will try to discuss some experimental results till date in order to give an overview of how to use the dynamical evolution of quarkonia as an useful probe. A conclusion for completeness of the thesis is included as the end chapter.

1.5 IMPORTANT FINDINGS

- Sequential suppression pattern of heavy quarkonia is questionable in high energy nucleus nucleus collision.
- Effective potential does not seem to be useful to describe heavy quarkonium bound states in medium in that context.
- Real time dynamics of heavy quarkonia is unavoidable in order to use them as a probe for QGP.
- A systematic approach has been explored to understand the real time dynamics of heavy quarkonia in medium. The framework also could be useful to study dynamics of open quantum system in many other branches of physics.

2

HEAVY QUARKONIA AS PROBE FOR DECONFINED MEDIUM

Red, Green, Blue are three different color charges of quarks (comes also in six different flavors). They always appear as a composite color neutral object. In search of the fundamental constituents of matters, we have learnt from tradition that we have to break the composition in order to have new degrees of freedom. We have successfully broken atoms in order to see the electronic and nuclear degrees of freedom. QCD thermodynamics shows the way to liberate quark and gluon degrees of freedom by going through a phase transition from confined to deconfined regime. The first idea to liberate quark degrees of freedom from hadron came in 1975 by Collins and Perry [7] . The idea was to pack nucleons close together to liberate quark and gluons from the hadronic phase. The first QCD phase digram was given by them and thereafter, lots of careful investigations have been done to achieve more accurate phase diagram.

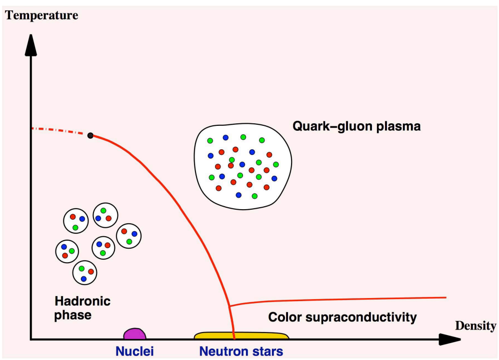


fig-7. *A naive QCD phase diagram shows different states of quark matter with temperature and density*[1].

It is clear from the diagram that by increasing the density at a fixed temperature we can make quarks free from the hadrons and if we increase the temperature further, we can achieve a new state of matter [22] which is known as quark gluon plasma.
By increasing the baryon density high enough and the temperature T_c around 200 MeV, [23] the deconfined plasma state is supposed to be formed. These kind of extreme conditions one can expect either in the core of a neutron star or in the early universe just after the big

1 This diagram has been taken from one of Nicolas Borghini's talk.

bang [24] .
The first attempt made to produce such a high density and temperature in laboratory was around 1984 by colliding two heavy nuclei with each other at very high energy. In recent days, scientists are trying to create that extreme state of matter by colliding two heavy lead nuclei in the Large Hadron Collider and through the gold gold collisions in the RHIC. The programme is known as relativistic heavy ion collision and the created matter, with very high energy density, is expected to be a quark gluon plasma. Now, one has to study this medium in order to know its properties. There are several probes to investigate the medium. We have discussed some of them in brief in the introduction. Until now, several experiments and analysis have confirmed the formation of a medium with very high energy density which behaves more like a perfect fluid, though the issue is not still closed. It needs further studies to confirm the properties and behaviour of the medium.
As the main goal of this thesis is oriented towards one of the probe of QGP, we will focus on that. In 1986, Matsui and Satz showed that in a quark gluon plasma certain quarkonium states will be melted due to the color screening effect of the plasma [11] . So, the dissociation of quarkonia in a deconfined medium could be an excellent probe for that. Before going into details, We would like to give a very short overview of quarkonium bound states in vacuum.

2.1 QUARKONIUM BOUND STATES

Quarkonia are composite particles (mesons) made of a quark and its anti quark. J/ψ was first such kind of particle which was discovered in 1974 in Brookhaven National Laboratory and almost at the same time at Stanford Linear Accelerator Centre. It is one of the bound states of a charm and its antiquark. Bound states of heavy quark antiquark pair are well explained by non relativistic quantum mechanics. Their different properties also have been predicted [25] and measured [26] with moderate success. Their masses $m \geqslant \Lambda_{QCD}$ permit their description in the framework of non relativistic quantum mechanics and therefore, allow us to design their spectroscopic description very well. The quarkonium made of charm and it's antiquark is known as charmonium and for bottom quark pairs it is the bottomonium. They have very stable bound states under strong decay. The Schrödinger equation for these heavy quark pairs reads

$$\left(-\frac{1}{2\mu}\nabla^2 + V(r)\right)\Phi(r) = E\Phi(r), \tag{2}$$

where r is the coordinate of the reduced mass μ corresponding to this two body system. The reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m}{2}. \tag{3}$$

With the appropriate quark antiquark potential one can solve the Schrödinger equation corresponding to such two body system to know different bound states for the quarkonia in vacuum. The potential for heavy quarkonia in first approximation looks like the Cornell potential [27]

$$V(r) = \sigma r - \frac{\alpha}{r}, \tag{4}$$

with the string tension σ and the effective coupling α corresponding to specific quarkonia. We are not considering the spin contribution to the potential for simplicity. Without the spin, we can now find the different bound states designated by three quantum number n, l and l_z.

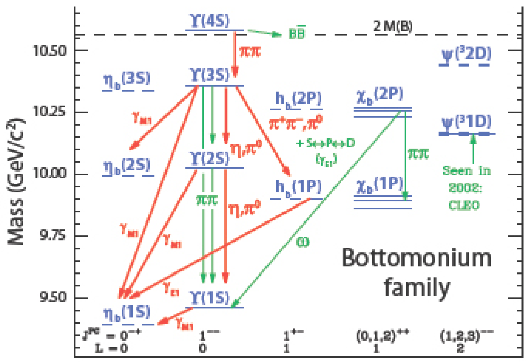


fig-8. *A spectroscopic description of bottomonium family. Source: Pacific Northwest National Laboratories*

We also can calculate the radii of different bound states and can built a spectroscopic description of all those states using these principal (n) and azimuthal (l) and magnetic (l_z) quantum numbers.

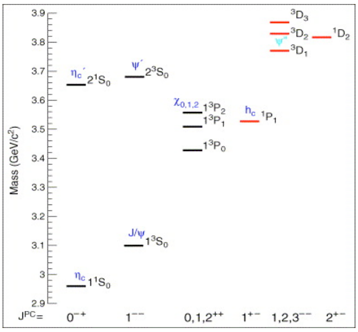


fig-9. *A spectroscopic description of charmonium family [28].*

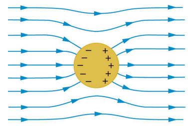
One can give the same description by averaging over l_z. The results are summarised for spin averaged states for different charmonium and bottomonium states [4] in the table below.

States	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
M[GeV]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
ΔE[GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
ΔM[GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
r[fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

Table-1. *Quarkonium properties from non-relativistic quantum mechanics. The table is prepared according to the reference [4].*

The experimentally measured mass M, radius r for different charmonium and bottomonium states are in good agreements with the theoretical predictions by considering the value of the string tension $\sigma \simeq 0.2 GeV^2$ and the effective coupling $\alpha \simeq \frac{\pi}{12}$. The results have been summarised in the table above. This is a very good account of the spectroscopic description for heavy quarkonium states with an error less than 1 percent in the mass determination. The binding energy ΔE shows how tightly different quarkonium states are bound. ΔM is the difference of mass between the experimentally measured values and the same predicted theoretically for different states.

2.2 SCREENING IN PLASMA



The screening prevents electric field to get into the Faraday cage. One can stay there without being electrocuted.

In classical picture of electromagnetic theory (more precisely electrostatic), we have seen that the electric field inside a perfect conductor is zero. Conductors have accumulation of free electrons and if we apply some external electric field, the charge will move to the surface of the conductor in order to cancel the electric field inside it. If we put a test charge inside a conductor, it will also manage its way to the surface. In a sense, the electric field is screened. The electric lines of force can not penetrate the wall of the screen to get into the conductor.
This phenomenon is not obviously going to happen for an insulator. Now, a very opposite phenomenon will be encountered when we heat up normal matter (irrespective of conductor or insulator) up to a very high temperature. Solids form when the thermal energy in the material is low enough to allow the intermolecular bonds to persist. By increasing the thermal energy, liquefaction is possible which still permits the bonds to persist. In gases, the intermolecular bonds are broken due to sufficient thermal energy and molecules are free to move randomly. Raising the temperature further, we can ionise the gas to have freely moving ions and electrons. This state is the new state of matter. It is new because it has some interesting properties which distinguishes it from other states. In this phase, new degrees of freedom come into the play which were previously suppressed. In the plasma state, matter is not electrically neutral and it also conducts electricity. A very interesting phenomenon appears inside the medium when one puts a test charge inside it. The thermalized medium modifies the form of the electric field originated due to the test charge. Now, the potential turns to Yukawa type instead being Coulombic. Therefore, a new scale appears which characterises the shielding of electric field around the test charge. This new quantity is known as Debye screening length. At any point out side Debye radius, the electric field exponentially falls down to zero. Now, the line of forces can not penetrate the screen to go outside. This is exactly opposite to the Faraday cage described in the context of conductor. The Debye screening now has caged the electric field within the sphere of influence.

2.2.1 *Debye Screening radius*

For a test charge Q in medium, the electric field looks

$$V(r) = -\frac{Q}{4\pi\epsilon_0 r} e^{\frac{-r}{r_D}}, \tag{5}$$

where r_D is known as Debye radius which characterizes the plasma. For $r \geqslant r_D$ the potential falls exponentially. We can say that the effect of the test charge is restricted approximately within the Debye sphere

.
Now, consider an electric dipole whose length is bigger than the Debye radius. It will not be any more a dipole rather will dissociate as free charges and will be part of the medium soon.
A similar effect one could expect in quark gluon plasma where the randomly moving charges are color charges and there are three different colors. The QCD phase diagrame has shown us the region where the quark and gluon degrees of freedom are liberated from the hadrons in high density and temperature to form a medium of deconfined quarks and gluons.
Quantum chromodynamics has the confining property which says that at large distances the coupling of the strong interaction become very large. This behaviour is also reflected in the non relativistic potential due to a color charge. The potential has a linear term which shows the confinement.

$$V(r) = -\frac{\alpha}{r} + \sigma r.$$

The constant σ is known as string tension. Studies already have been done to know how these constants α, σ vary with temperature.
Instead of an electric dipole, now we will consider a color dipole in deconfined medium (QGP). Those are strongly bound in vacuum or in a color neutral environment. The bound state of such heavy quark anti-quark pair can be described by the non relativistic potential mentioned above. The prime interest is now to study those bound states in quark gluon plasma. The plasma modifies the potential in a very similar way as it was in electromagnetic plasma. Studies have been done to know this effect in relativistic quark gluon plasma and they show the potential is being modified in the following way[29] ,

$$V(r,T) = -\frac{\alpha}{r} e^{-\mu(T)r} + \frac{\sigma}{\mu(T)}(1 - e^{-\mu(T)r}). \tag{6}$$

Above some critical temperature T_c, the confining part of the potential becomes vanishingly small and we are left with the screened coulomb potential.
The quantity $\mu(T)$ is the inverse Debye radius which is known as the Debye screening mass. The screening mass has been calculated for relativistic plasma of a theory with N_c number of color and N_f flavors with a coupling g using thermal field theory [30] and it shows that the screening mass increases with temperature,

$$\mu^2 = \frac{g^2 T^2}{3} \left(N_c + \frac{N_f}{2}\right) \tag{7}$$

In QED the screening mass is

$$\mu^2 = \frac{1}{3} e^2 T^2. \tag{8}$$

For two flavor QCD, considering $N_c = 3$

$$\mu^2 = \frac{4}{3} g^2 T^2. \tag{9}$$

The $q\bar{q}$ potential (without the string tension part) inside the plasma at temperature T looks

$$V(r, T) = -\frac{\alpha}{r} e^{-\mu(T) r}. \tag{10}$$

This is the effective in medium potential of the quark antiquark pair. The effective potential allows us to forget the medium by considering the effect of the medium in the modified potential. This potential also provides bound states of heavy quarkonia in the medium. It is obvious that none of the bound states will have a size bigger than the Debye sphere. Therefore, the medium does not allow to persist those vacuum bound states which have sizes bigger than r_D. They are missing in the medium and will never be recreated as long as the temperature remain the same. The missing bound states can indicate the formation of quark gluon plasma as well as can predict the plasma temperature. The possibility to use suppression of quarkonium states as a probe in static plasma was first pointed out by H. Satz and T. Matsui [11] and remains one of the strong guidelines to understand the medium.

2.2.2 *Dissociation temperature and sequential melting*

We have already discussed that the Debye radius is a radius of sphere of influence beyond which the field is effectively zero. The temperature plays an important role to it. By increasing temperature, we can reduce the radius of the Debye sphere. Hence, by approaching higher and higher temperature, it is possible to melt shorter and shorter color dipoles in the medium. This sequential melting [31] essentially introduces a QGP thermometer. Just by knowing which states are melted, one can predict the temperature of the medium.
The sequential suppression pattern has been considerably studied for different charmonium [32] and bottomonium [33] states. The threshold temperature for different states are calculated in the framework of lattice QCD and in various phenomenological models. More careful and precise calculation of gauge theory in lattice as well as in other branches to study quarkonium states in deconfined medium can be employed to give more accurate values of the dissociation temperature for different quark antiquark bound states. Till now they all have agreed with a sequential suppression pattern in the dissociation of bound states at least at the qualitative level. Color screening which has been described in the last subsection accounts for this suppression of states.

One can find the properties of various quarkonium states in the table presented in the previous section. If we believe in the in medium potential predicted by lattice gauge theory calculation, the sequential suppression pattern is obvious from the theoretical point of view. We said belief because the definition of in medium potential is still an open issue. Lattice QCD predicts the thermodynamical quantities from which the quark antiquark potenial is used to be extracted. There is no convincing definition for that. Sometimes, it is the free energy which is considered as potential and sometimes the internal energy and other thermodynamical quantities are used to derive the potential. Increasing the energy density of the QGP above deconfinement, first leads to ψ' dissociation, removing those J/ψ which otherwise would have come from ψ' decays. Further increasing the energy density, we can melt χ_c and only for a sufficiently hot medium also J/ψ s disappear. For the bottomonium states, a similar pattern will hold. The pattern for charmonium and bottomonium suppression is shown below.

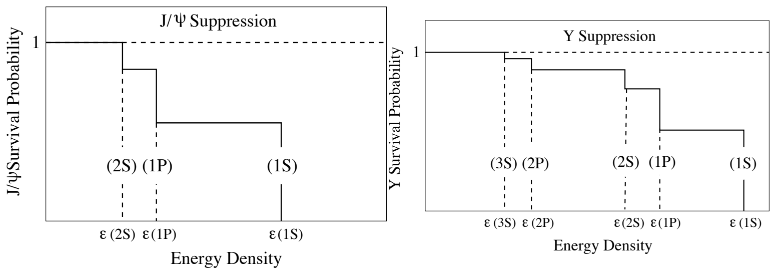


fig-10. *Sequential suppression pattern of* J/ψ *and* Υ *states [4].*

The sequential pattern has been theoretically explored from two different point of views. The recent technique in lattice gauge theory relies on the calculation of the spectral function. The analysis through spectral function can indicate whether a certain bound state can survive at a certain temperature of the plasma. Investigations have been done for several bottomonium and charmonium states. Those studies establish the sequential suppression pattern that can be utilised to design the QGP thermometer.

Another way to look into the same problem is by calculating the quark antiquark potential at a given temperature using effective field theory. The potential calculated previously using thermal effective theory for a quark gluon plasma was in agreement with lattice result at the qualitative level. Both of the disciplines can describe the melting of different quarkonium states in similar fashion by investigating the spectral function or Debye screening mass as a function of temperature. In this context, we should add that, recent studies (both in the effective field theory [16, 34, 35] and calculation on lattice [36]) show that the effective potential may have an imaginary part. This imaginary part indicates a finite life time of the quarkonium bound states in the medium. We will discuss this issue separately in the context of real time dynamics.

2.3 RECENT RESULTS ON DISSOCIATION TEMPERATURE

In the last section, we have seen the sequential melting picture. The quark antiquark potential in medium has been calculated using lattice simulation as well as effective field theory. The Debye screening radius (or mass) gives the radius of influence of colour charges. Investigating this Debye screening radius to know it's dependence on temperature, one can calculate the dissociation temperature for a particular bound state. We have mentioned earlier that by studying spectral function in lattice, one can predict the temperature threshold for a bound state to disappear in the medium. There are several studies available to provide the accurate dissociation temperature for different bound states. Unfortunately they also differ with each other. With the improvement of lattice computation, we are approaching towards much more accurate calculation of the dissociation temperature. For instance in 2001, calculation on lattice by S. Digal, P. Petreczky and H. Satz [17] estimated the dissociation temperatures of different charmonium and bottomonium states in the unit of critical temperature T_c as follows,

state	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
T_d/T_c	1.10	0.74	0.1-0.2	2.31	1.13	1.10	0.83	0.75

In 2006, another chart [37] came out which has different values than the previous one.

state	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
T_d/T_c	2.10	1.16	1.12	> 4.0	1.76	1.60	1.19	1.17

Another prediction by A. Mocsy and P. Petreczky in 2007 was a different estimation once again [38] ,

state	χ_c	ψ'	J/ψ	$\Upsilon(2S)$	$\chi_b(1P)$	$\Upsilon(1S)$
T_{dis}	$\leqslant T_c$	$\leqslant T_c$	$1.2T_c$	$1.2T_c$	$1.3T_c$	$2T_c$

In recent time a calculation by H. T. Ding et. al. shows [39] that both S wave states (J/ψ and η_c) and P wave states (χ_c) disappear at less than 1.5 T_c. For different bottomonium states the recent investigation by G. Aarts et. al. shows that the ground state can survive up to 2 T_c where as 2S state disappears [33] within a range $1.4 \leqslant \frac{T}{T_c} \leqslant 1.68$.

3 NEED FOR REAL TIME DYNAMICS

Debye screening in the quark gluon plasma describes the fate of heavy quarkonia in a static picture where we do not need to know the real time dynamics. We need a QGP thermometer in order to predict the dissociation temperature of different quarkonium states. The sequential melting and screening picture is a beautiful way to understand the in medium behaviour of quark antiquark bound states when the medium is in thermal equilibrium and the temperature is not evolving so rapidly as in the case of relativistic heavy ion collisions. The deconfined quark gluon plasma state is also expected in the early universe when the temperature and energy density was very high. The medium produced in heavy ion collision can not be easily compared to the early universe medium. The process in little bang is much more violent as the fireball evolves very rapidly and the temperature drops down very fast. Therefore, we can not make an easy use of Debye screening mechanism to study quarkonium bound states.
In the previous chapter we have tried to present the suppression of states through Debye screening in plasma which is applicable under certain criteria,

- The plasma should be in thermal equilibrium.
- Quarkonia also should be in thermal equilibrium with the medium.

These two conditions can be assumed for the QGP in the early universe but may not be a valid assumption for the heavy ion collision. It is still an open issue whether the medium produced in heavy ion collision is thermalized or not. There are studies which show that the medium will be thermalized very rapidly by assuming a strongly interacting plasma. Many studies also differ from that. They say that the medium may have a big thermalization time which is even much bigger than that of the persistence of the plasma. There are certain studies which advocate an incomplete thermalization. The time scale of thermalization is still not known. It is worth mentioning that there is another time scale for the interaction of heavy quarkonia with the medium. We have tried to say the same in point no. 2. This is important because the quarkonia in the medium take certain time to feel the temperature of the medium. This issue has not been properly investigated yet. This is really important in order to describe quarkonia even in a thermally equilibrated medium. In order to answer this question, studies have been made recently with simplified models. We will see an overview of those in the next chapter.
The work by Matsui and Satz [11] motivated several theoretical and

experimental studies in last 25 years to employ charmonium and bottomonium suppression as more accurate probe of QGP. The underlying mechanism for the suppression has been thought to be Debye screening. The medium produced in the little bang is not static and it cools very rapidly with its violent expansion in volume. The expansion of medium could be crucial even if we assume the medium is thermalized and evolves being in thermal equilibrium at every instant. This issue has been surprisingly overlooked by the community so far. Nevertheless employing a theory of static medium to explain phenomena in a dynamical medium is also justified under certain criteria. In this chapter I will discuss those implicit assumptions in detail to understand whether Debye screening applies to the rapidly evolving fireball. Before going into that discussion, let's describe the QGP as expected in the early universe. I feel the relevance to do so as it helps to distinguish the little bang from the big bang in the context of deconfined medium. The medium in the laboratory is not exactly the primordial fireball.

3.1 PRIMORDIAL FIREBALL

A very high energy density or temperature one can expect at the very early stage of the universe and therefore that meets the criteria for the formation of quark gluon plasma. If you look back in time to the early stage of the expanding universe, the matter and radiation becomes hotter and dense which results the universe in QGP phase (primordial fireball). The expected time for that is approximately 10^{-5} seconds after the big bang. As the universe expands and cools down with time, a possible QCD phase transition happened within $10^{-5} \sim 10^{-4}$ seconds after the big bang.

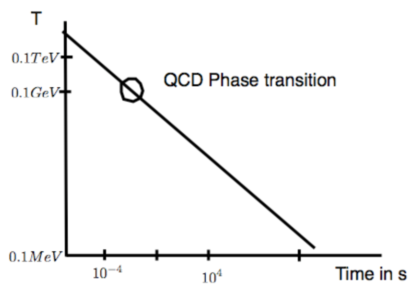


fig-11. *Evolution of temperature in radiation dominant phase of the early universe.*

We can see from the diagram above, the evolution of temperature in the radiation dominant phase of the early universe is rather slow. In such a plasma, thermalization occurs and temperature evolves quasistatically. The time scale of the evolution of the medium is very large compared to the internal time scale of the evolution of heavy quarkonia. A thermal description can more or less describe the fate of quarkonia there. The Hubble time scale t_H for the expansion in the early universe (near the regime of the QCD phase transition) is nearly equal to 10^{-5} second which is very large compared to the relaxation time of the strong interaction (1 fm/c). Therefore quarkonia feel more or less a static medium in thermal equilibrium. The in medium potential description can be validated to describe the possible bound states in the early universe.

$$V(r,T) = -\frac{\alpha}{r}e^{-\mu(T)r} + \frac{\sigma}{\mu(T)}(1 - e^{-\mu(T)r}).$$

Now, the potential changes with time as the temperature falls down. The process is slow enough to support the adiabatic[1] evolution of heavy quarkonium states. It is then justified to use a thermodynamical picture to the early universe QGP as it meets all the criteria properly. Though we can describe the dissociation of quarkonia in early universe using screening picture our goal is not to describe that rather to deal with the same produced in heavy ion collisions. Let us see how far the thermodynamical description holds for the little bang.

3.1.1 *Relativistic heavy ion collision*

In relativistic heavy ion collisions, the fireball is produced by the collision of two heavy nuclei. Then the medium goes through different phases [40] as it has been shown in the figure. For collision energies $\sqrt{s} = 100\text{GeV}$, the nuclei are stopped in the collision to a large extent and a dense and hot expanding fireball with a finite baryon density (finite chemical potential) is formed.

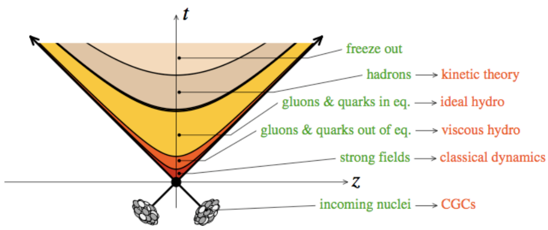


fig-12. *Schematic representation of the various stages of a heavy ion collision as a function of time [40]. Spatial dimensions are denoted by z.*

After the collision approximately at $\tau = 0.2$ fm/c , The partonic constituents form a very dense medium which undergoes a complex evolution without reaching thermal equilibrium. This state is known as

1 The meaning of adiabatic evolution of states is discussed in detail in section 3.3. It should not be confused with adiabatic processes in thermodynamics.

Glasma [41, 42] .

The partons then interact strongly with each other unlike in a proton proton collision for which the partons go through a separate evolution due to negligible interaction. The fragmentation leads to the hadronization. But this is not the case in a heavy ion collision, rather the partons interact and form a new state of matter consists of liberated quarks and gluons. The medium is thought to proceed towards the thermal equilibrium. The themalization in that phase still remains a matter of debate. Considering strongly interacting medium, there are arguments which support a very rapid thermalization which is even less than 1 fm/c. This comprises certain assumptions which are strongly opposed by other trend of thinking suggesting the medium as weakly coupled and undergoes a very slow thermalization process. Even the concept of local thermal equilibrium also faced the ambiguity with data and pointed out an incomplete thermalization throughout the whole process which finally ends at hadronization point. So, this is not really unambiguously understood that in the so called quark gluon plasma phase, the medium is really thermalized or not.

Thereafter the medium starts to cool down rapidly and the hadronic phase appears when the temperature reaches the critical temperature T_c which is around 150 to 200 MeV. In Pb+Pb collision at LHC, it happens approximately at $\tau = 10$ fm/c. We have discussed the suppression of heavy quarkonia state in static medium which shows a sequential suppression pattern. In the high energy nucleus nucleus collision, the produced medium cools down rapidly. The big question is how far one can rely on the results of static thermodynamical picture to describe the fate of quarkonium bound states in such a rapidly evolving medium.

3.2 IMPLICIT ASSUMPTION BEHIND SEQUENTIAL SUPPRESSION IN HEAVY ION COLLISIONS

We have raised two important points in the last section which are important criteria for the use of a thermodynamical approach. For the moment let me assume that these two criteria are satisfied in heavy ion collision. Quarkonia produced in heavy ion collisions start to evolve before they are thermalized in the medium. Let me assume that they have sufficient time to get thermalized with the medium. Still there is the issue of rapid evolution of the medium. The justification behind the expected sequential melting in high energy nucleus nucleus collisions is the following, where for the sake of simplicity we leave aside so-called "initial-state effects".[2] At an early stage after the collision, say some instant t_0, the created deconfined medium reaches high enough energy density so that a given quarkonium state, which we shall refer to as "excited", is suppressed, while another state of the same system, hereafter the "ground state", is bound. The common lore is then that, as the medium expands and cools down ($t > t_0$), the ground state stays unaffected, whereas the depopulated excited state remains suppressed, even when the medium temperature has dropped below its dissociation threshold. The only possibility left to the excited state for being recreated is at the transition to the hadronic phase, through the "recombination" of till then uncorrelated heavy quarks and antiquarks [43, 44]. Justification of this argument relies on two basic ingredients.

There is first the sequential-suppression pattern in the "initial condition" at t_0, whose theoretical foundation is based on Debye screening in static thermalized plasma.

The second element in the scenario is the implicit assumption that "the quarkonium ground state remains the ground state" over the duration of the medium evolution. Recasting this statement more mathematically, a quark antiquark ($q\bar{q}$) pair initially in the eigenstate with lowest energy of the (effective) Hamiltonian describing in-medium quarkonia remains in the lowest-energy eigenstate. More generally, the same will hold for every initially bound state—up to late electroweak decays which take place outside the medium. That is, it is assumed that heavy quarkonia are continuously evolving eigenstates of an adiabatically changing instantaneous Hamiltonian. Accordingly, the scenario for the sequential suppression of quarkonia in the medium created in high-energy nucleus–nucleus collisions relies on the hypothesis that the effective in-medium quark–antiquark potential varies slowly enough that each $q\bar{q}$ pair is at every successive instant in an energy eigenstate. We now wish to investigate the validity of this assumption.

Before going any further and to dispel any confusion, let us note that the adiabaticity we discuss in this context is neither that of the medium evolution related to the production of entropy, nor the adiabatic assumption à la Born–Oppenheimer which allows one to separate gluons from the nonrelativistic heavy quarks when writing down an effective potential for the latter [45].

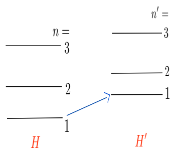
Suppose, at time t_0 the temperature of the medium is T_0. The potential with the Debye screening mass $\mu(T_0)$ provides the energy eigenstates $|\psi_i\rangle, \forall i \leqslant n$. Other energy eigenstates for $i > n$ are already melted at that temperature. In an evolving medium the temperature as well as the quark antiquark effective potential change through the

2 When comparing *relative* yields of different states of a given system, say S-channel charmonia or bottomonia, for a fixed type of nuclear collisions, these effects should play a minor role.

temperature dependent Debye screening mass. The effective Hamiltonian of the quarkonia becomes time dependent. Now, after a while the temperature of the medium becomes T at time t. The Hamiltonian of the system becomes H' with the quark anti-quark potential being

$$V' = -\frac{\alpha}{r} e^{-\mu(T) r}. \tag{11}$$

Sequential melting picture relies on the fact that all the bound states which are still permissible in that temperature will stay in the corresponding eigenstate of H'. For instance, the $|\psi_i\rangle$ corresponding to H will shift as the ith eigenstate of H' and those states which are not permissible will not be recreated from other lower bound states. This prescription runs well if the time scale τ_m of the medium evolution is much bigger than that (τ) of the quarkonia. So, the concept of effective potential (in order to think about the stable bound states) in a evolving medium is fruitful when it provides adiabatic evolution of energy eigenstates. For a slowly evolving medium, the adiabatic approximation is a valid assumption. We should investigate carefully whether the same is applicable for the rapidly evolving fireball produced in heavy ion collisions.



Adiabatic approximation: nth eigenstate remains as nth eigenstate of the new Hamiltonian.

Let us have a quantitative look into the problem. Suppose the system starts with the energy eigenstate $|\psi_i(0)\rangle$ at time t =0 and evolves with the evolution of the time dependent Hamiltonian $H(t)$. The solution of the time dependent Schroedinger equation,

$$|\phi(t)\rangle = \sum_m C_m e^{i\theta_m(t)} |\psi_m\rangle \tag{12}$$

where C_m satisfies the following equation,

$$\dot{C}_m(t) = -C_m \langle \psi_m | \dot{\psi}_m \rangle - \sum_{n \neq m} C_n \frac{\langle \psi_m | \dot{H}(t) | \psi_n \rangle}{(E_n - E_m)} e^{i(\theta_n - \theta_m)}. \tag{13}$$

If we can drop the second term at the right hand side of the above equation, we obtain

$$\dot{C}_m(t) = -C_m \langle \psi_m | \dot{\psi}_m \rangle. \tag{14}$$

In particular when the system starts from the state $|\psi_i\rangle$ initially, the boundary condition $C_i(0) = 1$ and $C_j(0) = i$ for all $j \neq 0$ implies

$$C_i = -C_i(0) e^{i\gamma_i(t)}, \tag{15}$$

where

$$\gamma_i(t) = i \int_0^t \langle \psi_i(t') | \frac{\partial}{\partial t'} \psi_i(t') \rangle dt; \;\; \theta(t) = -\frac{1}{\hbar} \int_0^t dt' E(t').$$

$\gamma_i(t)$ and $\theta(t)$ are known as geometric and dynamical phase, respectively. Hence, the final state can be written as,

$$|\phi(t)\rangle = e^{i\gamma_i(t)} e^{i\theta_i t} |\psi_i(t)\rangle. \tag{16}$$

As we can see the final state at time t is again the ith eigenstate of the Hamiltonian $H(t)$ with a phase factor. So we can say that the concept of instantaneous eigenstates is valid because the quantity

$$\frac{\langle\psi_m|\dot{H}(t)|\psi_n\rangle}{(E_m - E_n)} \ll 1. \tag{17}$$

The precise criterion [46] in terms of a dimensionless ratio would be,

$$\frac{|\langle\psi_m|\dot{H}(t)|\psi_n\rangle|}{(E_m - E_n)^2} \ll 1. \tag{18}$$

The relation also could be realized through the ratio of the time scale τ associated with the evolution of energy eigenstates and that τ_m of the evolution of the Hamiltonian. The time scale (τ) is inverse to the characteristic energy gap between different energy eigenstates of the system where as τ_m is determined from the rate of change of the Hamiltonian. This ratio ($\frac{\tau}{\tau_m}$) is therefore the measure of adiabaticity which should be much less than 1 in order to allow the adiabatic evolution of the energy eigenstates of the quantum mechanical system. For a very rapid evolution of the Hamiltonian, the ratio is very unlikely to be much less than 1, therefore the adiabatic evolution is far from warranted. This ratio for quarkonia in an evolving medium decides the nature of its evolution which will be revealed in the next section.

3.3 VIOLATION OF THE ADIABATIC APPROXIMATION IN HEAVY ION COLLISIONS

For heavy quarkonia, the potential is changing through the temperature of the evolving medium. In this particular context,

$$\langle\psi_m(t)|\dot{H}(t)|\psi_n(t)\rangle = \langle\psi_m(t)|\dot{V}(t)|\psi_n(t)\rangle. \tag{19}$$

The $q\bar{q}$ potential in the evolving medium is changing due to the change of temperature, therefore,

$$\langle\psi_m(t)|\dot{H}(t)|\psi_n(t)\rangle = \langle\psi_m(t)|\dot{T}\frac{dV}{dT}|\psi_n(t)\rangle. \tag{20}$$

Hence the matrix element looks

$$\langle\psi_m(t)|\dot{T}\frac{dV}{dT}|\psi_n(t)\rangle = \dot{T}\int \psi^\star_m(r,t)\frac{dV}{dT}(r)\psi_m(r,t)d^3r. \tag{21}$$

So, the ratio in eq. (18), which determines the adiabaticity, mostly lies in the ratio of the time derivative of temperature (which is outside the integral) as the temperature of the fireball falls down very rapidly. We have mentioned earlier that the in medium potential has not yet been understood properly. Whatever has been investigated in this context, the form of the potentials looks similar apart from the fact that the complex potential introduces an imaginary part in that. Let us investigate the adiabatic condition considering the potential predicted by lattice QCD [15].

$$V(r) \sim \frac{\frac{4}{3}\alpha_s(T)}{r} e^{-A\sqrt{1+N_f/6}\,T g_{2\,\mathrm{loop}}(T)\,r}, \tag{22}$$

$$\frac{dV}{dT} = [\frac{\alpha_s'(T)}{\alpha_s} - A\sqrt{1+N_f/6}(g_{2\,\mathrm{loop}}(T) + T g'_{2\,\mathrm{loop}}(T))r]V(T).$$

We can employ the following relation to simplify above equation,

$$g'_{2\,\mathrm{loop}}(T) \sim -\beta_0 g_{2\,\mathrm{loop}}(T)^3.$$

Now plugging this into above equation we have,

$$\frac{dV}{dT} = (A\sqrt{1+\frac{N_f}{6}}[\beta_0 g_{2\,\mathrm{loop}}(T)^2 - 1]g_{2\,\mathrm{loop}}(T)r)V(T). \tag{23}$$

The contribution from $\frac{\alpha_s'(T)}{\alpha_s}$ is negligibly small compared to the other part. Therefore we have ignored that term.
With $0.5 \leqslant \alpha_s \leqslant 1$,

$$\beta_0 g_{2\,\mathrm{loop}}(T)^2 - 1 = 4\pi\beta_0\alpha_s - 1 \tag{24}$$

where, $\beta_0 = \frac{1}{16\pi^2}(11 - \frac{2N_f}{3})$.

$$4\pi\beta_0 \approx 0.77; \quad \text{for } N_f = 2$$

With that, the value of

$$(A\sqrt{1+\frac{N_f}{6}}[\beta_0 g_{2\,\mathrm{loop}}(T)^2 - 1]g_{2\,\mathrm{loop}}(T)) \text{ is around } 1.4 \text{ to } 2$$

. We see in Eq. (21), the numerator of the Eq. (18) is simply the product of the rate of change of the medium temperature ($\dot{T}$) and the $\langle m(t)|dV/dT|n(t)\rangle$[3]. For the sake of simplicity we have assumed that the medium is (locally) thermalized. For $\dot{T}$, we took the results [47] from a simulation of central Pb–Pb collisions at the LHC within dissipative hydrodynamics considering the evolution of temperature at

3 We have abbreviated ψ_m and ψ_n as m and n respectively.

the center of the fireball: within the first 7 fm/c of the evolution (that is, as long as $T > 200$ MeV), $\dot{T}$ always remains larger than about 30 MeV per fm/c and up to 50 MeV per fm/c in the early stages.

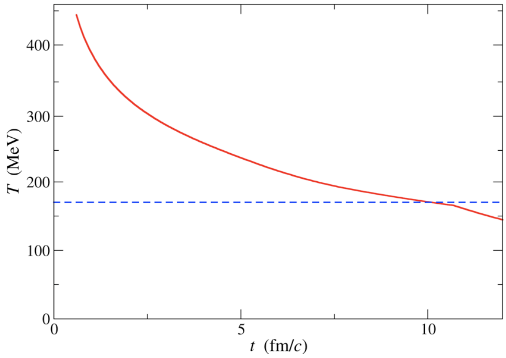


fig-13. *Change of temperature in medium produced in heavy ion collision*[4]

For the $q\bar{q}$ potential, we have calculated the amplitude for a matrix element of dV/dT between eigenstates of the instantaneous Hamiltonian using different energy eigenstates,

$$\left| \left\langle n'(t) \right| \frac{dV}{dT} \left| n(t) \right\rangle \right| \approx 200 - 500 \text{ MeV} \cdot \text{fm}.$$

The numerator in Eq. (18) is thus of the order $(80 - 160 \text{ MeV})^2$. In turn, the denominator is of the order (100-350 MeV)^2 for the excited $b\bar{b}$ states, so that the ratio can be in some cases smaller than 0.1, for other channels larger than 1. Because of those channels, it is far from warranted that the adiabaticity assumption holds. The potential evolves so quickly that a quark–antiquark pair which at some time is in a given instantaneous eigenstate, will, a short while later no longer be in the evolved eigenstate. But it will have components over all the new eigenstates including the new ground state, which shows that even if criterion 18 holds for the latter, yet it is populated by contributions from excited states.

We wish to emphasize here that this "repopulation" mechanism is neither the customary recombination at hadronization, nor the feed-down from late decays, but a natural consequence of the "reshuffling" of $q\bar{q}$ states due to the rapid medium evolution.

A naive picture of the effect of this rapid evolution is provided by dividing the typical size $r_{rms} \approx 0.3$–0.75 fm of a bound bottomonium by the characteristic velocity $v \sim 0.3c$ of the nonrelativistic constituent quark and antiquark, which gives a duration $\tau \approx 1$–2.5 fm/c for an "orbit" of the b quark. On such a time scale, the QGP cools down by

4 This plot has been taken from Prof. Ulrich Heinz in private communication.

30 to 75 MeV, resulting in a significant change in the effective potential (22), which illustrates why the adiabatic evolution of bottomonia is far from being warranted.

As a final argument against using the hypothesis of an adiabatic evolution of $q\bar{q}$ pairs in a QGP, we note that recent studies emphasized the fact that even when criterion (18) is satisfied—i.e., the evolution is slow—, the system with evolving Hamiltonian can be driven from one instantaneous eigenstate to a different one at later times by resonant interactions [48]. The latter leads to Rabi oscillations between eigenstates—that is, they are tailored to induce transitions which violate the adiabatic theorem—on a time scale given by the inverse of the Rabi frequency ω_R.

In the case of a $q\bar{q}$ pair in a quark–gluon plasma at the temperatures found in high-energy nuclear collisions, there are obviously plenty of degrees of freedom around with energies matching possible transition lines. The corresponding Rabi frequencies however depend on the interaction term. Adopting, for the sake of illustration, a dipolar interaction, one finds values of π/ω_R, which in a two-level system is the time after which a transition has occurred with probability 1, of the order 2 to 20 fm/c, depending on the transition Bohr frequency, the medium size and the assumed coupling strength. This means that on such a time scale a $q\bar{q}$ pair certainly does not remain in the same instantaneous eigenstate, which again hints at the invalidity of the adiabatic theorem for heavy quarkonia in a dynamical QGP.

One might be tempted to argue that in an effective-potential approach, the transition-inducing degrees of freedom have been integrated out. Yet the construction of an effective theory ultimately relies on the adiabatic theorem [49], so that it is inconsistent to use the notion blindly here. More precisely, we surmise, although we have not investigated this idea in detail, that the violation of adiabaticity caused by resonant interactions translates into the imaginary part of the effective in medium potential, which physically has the same effect of giving a finite lifetime to the Hamiltonian eigenstates.

3.4 QUARKONIUM BOUND STATE OF COMPLEX POTENTIAL

In section 2.3, we have mentioned that some recent studies already have indicated a complex $q\bar{q}$ static potential to describe heavy quark bound states in thermal medium. The ambiguity of defining the potential is a long time issue and it is still not well understood which quantity should be employed to extract the potential. In perturbative QCD the potential has been derived from thermal Wilson loop. In real time, the potential takes a form which is complex. The key formula behind the extraction of the potential at a fixed temperature in this framework is

$$V(r) = \frac{i\partial_t W}{W} = \text{Re}V(r) + i\text{Im}V(r), \tag{25}$$

where W denotes the expectation value of the thermal Wilson loop at a specific temperature. As temperature varies, the Wilson loop as well as the potential varies. So there is a parametric dependence on temperature for this static potential. A similar case has been studied in lattice QCD by evaluating W in euclidean time and thereafter by extracting the spectral function one can reconstruct the Wilson loop in real time. Hence, by plugging into the above equation, a static potential for heavy quarks can be derived. In both ways (pQCD and Lattice), the imaginary part of the potential has been evaluated. We can see the comparison in the following plot.

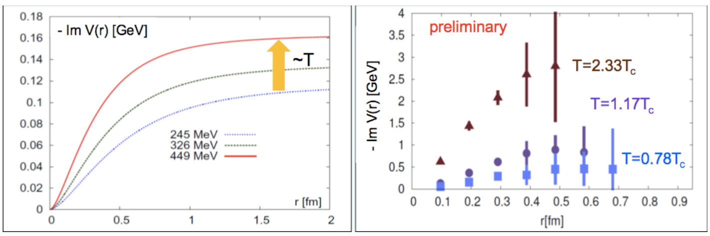


Fig-14.*The left panel has been prepared using perturbative QCD calculation while as the plot at the right side is prepared using lattice QCD.*[5]

Due to this imaginary part, the obvious dynamics of every bound state is dissipative. The bound states then have a finite life time. One relevant issue in this context is to know the final fate of those bound states. Whether they repopulate other states by decaying or they just disappear as free quarks is not easily answered. Nevertheless the idea of complex potential is inspiring the study of dynamical evolution of quarkonia in recent days a lot in the heavy quark community.

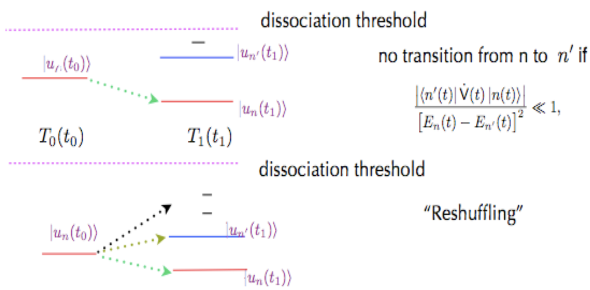
5 We have taken this plot from the talk by T. Hatsuda in Quark Matter 2012.

4 HEAVY QUARKONIA IN MEDIUM AS OPEN QUANTUM SYSTEM

In the last chapter we have already seen that the evolution of quarkonium states in the rapidly evolving medium is not adiabatic. Therefore, one has to consider a new picture apart from the static Debye screening. In thermodynamical description, the distribution of quarkonium state does not change with time. It can not describe the quarkonium states in a transient regime where they are not thermalized with the medium. Breakdown of adiabatic approximation due to the rapid evolution of medium forces us to look back into the problem exploiting their real time dynamics. The dynamical picture then has to be applicable beyond adiabatic approximation. Let us sketch the picture first.

4.1 THE GENERAL PICTURE

Suppose, initially at $t = t_0$, the quarkonium stays in the nth eigenstate $|u_n(t_0)\rangle$ of the effective Hamiltonian $H(t_0)$. Let's say that the corresponding temperature of the medium is T_0. As the temperature changes, the effective Hamiltonian also changes with time. At time t, let the temperature be T and the corresponding Hamiltonian changes to $H(t)$. With the adiabatic approximation, the new state should be $|u_n(t)\rangle$ which is again nth state of the corresponding Hamiltonian $H(t)$. Beyond adiabatic approximation, the initial state will evolve to a superposition of all the energy eigenstates.



We have sketched that schematically in the above picture. At every instant, all the states reshuffle themselves among the spectrum of eigenbasis. This reshuffling is the fate of quarkonium states in a rapidly evolving medium. So, we need to design a formalism which can include this scenario of reshuffling of states. With a proper initial condition, dynamical description leads to a quantitative account for the probability of different states. In the picture we have shown that there are several possible bound states to hold as well as unbound states above the dissociation threshold. Let us phrase the relevant question in a more precise way.

What are the probabilities of having different states of heavy quarkonia at an instant t if it starts from a specific initial state at t_0 and undergoes through an evolution by interacting with the medium?

The interaction with the medium could be time dependent or time independent. For a static medium, the interaction has no explicit time dependence (because the medium is not evolving) but that can offer a dynamics to the bound states. The populations of different bound states evolve even if the medium is static. The initial populations change and finally they reach an equilibrium distribution for a thermalized static medium. Before reaching the equilibrium, those bound state populations go through a transient region of dynamical evolution. This transient regime is important for a medium of finite life time as we mentioned earlier. This is because the medium might disappear before the quarkonium bound states reach the equilibrium. That makes us unable to make use of equilibrium thermodynamics to describe those states. Another case is when the medium evolves rapidly. The interaction between the quarkonia and medium explicitly varies with time and we have the task of considering the evolving interaction into the framework to predict the population of bound states as dynamical quantities.
One way to formulate this problem is to see the small system (heavy quarkonia) as an open quantum system. The open system interacts with the medium hence exchange energy and momentum with the medium. Open quantum system issues have been well studied in several branches of physics like quantum decoherence, quantum optics and many more. Medium which is already being thermalized can be considered as a thermal bath which does not have appreciable changes due to the exchange of energy and momentum with the test system. In heavy ion collision, the medium which is produced at high energy density is not undoubtedly confirmed to be thermalized. We already have given an account how different schools conclude differently regarding this issue.

In this chapter, we will discuss two different approaches made in recent years to investigate real time dynamics of heavy quarkonium states. The first approach which we will describe is based on the evolution of the density matrix using master equation approach. It deals with the evolution of internal degrees of freedom [50, 51]. Another approach [52] employs the stochastic Schroedinger equation and hence by developing a master equation approach describes the evolution of the population for different energy eigenstates.

4.2 MASTER EQUATION APPROACH

A small quantum system coupled to a reservoir is the pedagogic approach to describe open systems. The medium become unimportant when we are interested only in the small system. We then have to integrate out all the medium degrees of freedom to focus on the test system only. Let us first describe the approach for a four level system and then we will see the results coming out of the investigation from simplified model of heavy quarkonia.

4.2.1 *Application to a generic four level system*

The Hamiltonian of system + bath is

$$H = H_S + H_R + V, \tag{26}$$

where H_S and H_R are the Hamiltonians for the system and the reservoir respectively. The interaction is described by 'V'. The medium is assumed to be consists of harmonic oscillators of a continuum frequency span. We are interested to see the evolution of density matrix of the system. Let me write the density matrix of the system plus reservoir as

$$\rho = \rho_S \otimes \rho_R + \rho_{correl} \tag{27}$$

The evolution of this density matrix is described by Heisenberg equation of motion with the total Hamiltonian. In order to focus on the evolution of the open system, one needs to introduce a reduced density matrix just by taking the partial trace over the medium degrees of freedom.

$$\rho^s = Tr_R \rho \tag{28}$$

The density matrix ρ satisfies the Heisenberg equation

$$\frac{d}{dt}\rho = -i[H, \rho]. \tag{29}$$

By introducing partial trace over the medium, dynamics of the reduced density matrix can be derived from Eq. (29). For simplicity, we assume that ρ_{correl} is zero for the case we are interested in. The assumption relies on the fact that there is a time scale, τ_c above which the system loses its correlation with the medium which implies

$$\rho = \rho_S \otimes \rho_R, \quad \forall t \gg \tau_c. \tag{30}$$

Using this assumption, we can derive the master equation for the reduced density matrix which describes the evolution of the population and coherence (off-diagonal) term in the density matrix. For our purpose we do not need to know the evolution of coherence term. We are only interested to see how the population of energy eigenstates evolve.

Let start with a system with the Hamiltonian H_S which has the eigenstate spectrum $|i\rangle$,$|j\rangle$.....corresponding to the energy eigenvalues E_i,E_j... . The reduced density matrix represented in this eigenbasis obeys a set of coupled equations. As we are only interested in the population, we will deal with the diagonal elements ρ^S_{ii} of the matrix. Those linear differential equations read

$$\frac{d\rho^S_{ii}}{dt}(t) = -\sum_{k\neq i} \Gamma_{i\to k}\rho^S_{ii} + \sum_{k\neq i} \Gamma_{k\to i}\rho^S_{kk}, \tag{31}$$

considering master equation with the term up to the second order in interaction. The transition rates are given by Fermi's golden rule. For $E_k \geqslant E_i$ and Bohr frequency $\omega_{ki} \equiv \frac{E_k - E_i}{\hbar}$,

$$\Gamma_{k\to i} = \frac{2\pi}{\hbar^2}\sum_\lambda (\langle n_\lambda\rangle + 1)\left|\langle i; 1_\lambda|V|k; 0\rangle\right|^2 \delta(\omega_\lambda - \omega_{ki}), \tag{32a}$$

$$\Gamma_{i\to k} = \frac{2\pi}{\hbar^2}\sum_\lambda \langle n_\lambda\rangle \left|\langle k; 0|V|i; 1_\lambda\rangle\right|^2 \delta(\omega_\lambda - \omega_{ki}), \tag{32b}$$

where $\langle n_\lambda\rangle$ is the average number of modes λ characterising the medium.

So, one can solve these equations in order to see the dynamical evolution of population. One such example with a four level system has been demonstrated [50] (by populating initially in ground state) in the following plot.

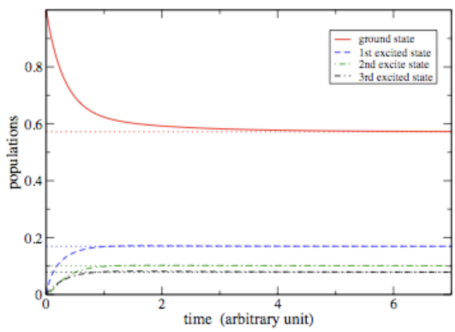


fig-15. *Time dependence of the populations of the states of a 4-level system coupled to a thermal bath. The straight lines correspond to the equilibrium values at the bath temperature [50].*

This plot has been prepared numerically by considering the interaction potential V having a bilinear form

$$V = Xx; \;\; X = \sum_{\lambda} (g_\lambda a_\lambda + g^\star_\lambda a^\dagger_\lambda).$$

x and X characterize the test system and the medium constituents respectively. a_λ and $a^\dagger_\lambda$ are annihilation and creation operator of the mode λ. The coupling term corresponding to the mode is g_λ. We see in the plot that the populations evolve dynamically in a transient regime and then equilibrate to

$$\left(\frac{\rho^{\mathcal{S}}_{kk}}{\rho^{\mathcal{S}}_{ii}}\right)_{\text{eq.}} = \exp\left(-\frac{E_k - E_i}{k_B T}\right).$$

4.2.2 *Master equation approach to quarkonium states*

In the last two sections, we have seen a generic way to deal with the dynamics of an open quantum system through the Master equation approach. In recent years certain attempts have been started to model heavy quarkonia in medium as an open system. The dynamics of the internal degrees of freedom has been calculated at the exploratory level to see the qualitative features. The modelling of quarkonia in pure gluon plasma as an open system interacting with the medium is simple and quite realistic. The medium has been modelled by quantizing chromoelectric field in Weyl gauge. Within the framework of master equation, the only input needed is $\langle n \rangle$ which characterizes the medium. For a thermally equilibrated gluon plasma, it is the Bose-Einstein distribution function. The density matrix of the free medium is given by

$$\rho^{\mathcal{R}} = \frac{e^{-H_{\mathcal{R}}/k_B T}}{\mathrm{Tr}\left(e^{-H_{\mathcal{R}}/k_B T}\right)}. \tag{33}$$

The $q\bar{q}$ states are described as bound states of the quark anti-quark potential. The density matrix for the system then can be constructed by considering all the bound states possible for them. The initial population can be constructed in the similar fashion by populating them in a certain state. The reshuffling picture has been employed to see coupled evolution of the populations of different states. The time scale for the thermalization of quarkonium state also has been investigated as it is very much relevant in the heavy ion collision where the medium persists for a very short time. It has been done with a simplified model of bottomonia by investigating the dynamics of the population of different energy eigen states. The vacuum bottomonium potential can be written as

$$V_{q\bar{q}}(r) = -C_F \frac{\alpha_s \hbar c}{r}, \tag{34}$$

where $C_F = 4/3$ is the usual color factor and the running coupling constant is α_s. The value of α_s has been considered approximately equal to0.25. We should mention here that the eigenstates of Coulomb potential have degeneracies which in this model have been considered explicitly. The degeneracy has been lifted by hand. The calculation has been performed by taking finite number of bottomonium states to see the shuffling among those states even excluding the possible recombinations. Only four eigenstates have been considered in this work and above that an unbound state regime has been designated. Still it is a good preliminary step to see the qualitative behaviour of the dynamics. In the next chapter we will explicitly show a new approach which can avoid these assumptions while dealing the same for harmonic oscillators in a wave function based method.
The interaction of bottomonia with the gluon plasma has been modelled within the dipolar approximation which gives the interaction potential as,

$$V = -\mathbf{d} \cdot \mathbf{E} = -i\sqrt{C_F \alpha_s \hbar c}\, \mathbf{r} \cdot \sum_\lambda \sqrt{\frac{2\pi\hbar\omega_\lambda}{L^3}}\, \epsilon_\lambda (a_\lambda - a_\lambda^\dagger), \qquad (35)$$

A common technique has been employed to quantize the chromoelectric field which is known as box (of size L) quantization. The same also has been utilized for the normalization of bottomonia states. ϵ_λ is the polarization vector of those modes λ which corresponds to the gluons, while **d** and **r** denote the dipole moment and radius operator for the $q\bar{q}$ pair. Now we can plug this in the set of Eq. (31). The equation in vectorial form reads

$$\frac{d\vec{\rho}}{dt}(t) = \mathcal{U}_{\mathcal{R}} \vec{\rho}(t). \qquad (36)$$

We are considering the vacuum quarkonium population to see their evolution. The evolution operator $\mathcal{U}_{\mathcal{R}}$ is diagonal when it is represented in the energy eigenbasis of heavy quarkonium system. The representation is no longer diagonal when medium effect is taken in to account. Still we can diagonalize the matrix by going to another basis sets. This shows a difference between the vacuum bound states and one which is populated in the medium. The physical meaning is that the higher energy $q\bar{q}$ states do not evolve independently from the more bound ones, as in the vacuum. This is because of the medium induced transition. Therefore those states spend some time in a transient regime before they equilibrate in the medium temperature. In this transient regime the populations evolve with the same time scale.

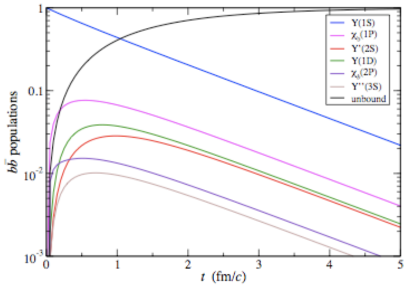


Fig-16. *Dynamics of populations for bottomonia states at* $5T_c$ *[50].*

The above results have been achieved by modelling quarkonia as an open system. The plot at 5 T_c has been prepared by populating the $\gamma(1S)$ initially at $t = 0$. We see,

- all the states evolve together which should be contrasted to the usual sequential picture.

- After the first fm/c or so, one reaches a quasi-equilibrated regime where the populations of all vacuum bound states decay with a characteristic time scale of 1.5 fm/c, while their ratios remain stationary.

Investigation at $2T_c$ shows that the time scale of quarkonium evolution is approximately 8 fm/c. This is quite comparable to the persistence time of the medium produced in heavy ion collisions. I have discussed earlier that even for the thermalized medium, the time scale of quarkonium evolution is important. We see with this study that the equilibrium could not be achieved by quarkonia below the time scale mentioned above. This poses serious caution to the thermodynamic description applied to know the fate of the quarkonium states.

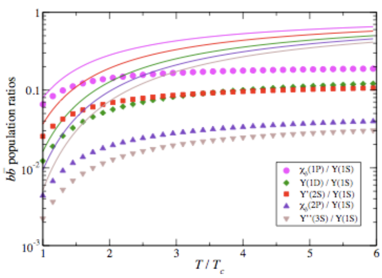


Fig-17.

Temperature dependence of the ratios of bottomonium populations [50]. Symbols: quasi-equilibrium ratios within the master-equation formalism; solid lines: ratios in a thermally equilibrated system.

We can see in the above plot that in the quasi-equilibrium region, population ratios depend on the temperature of the medium. We have seen in this model that the transitions from unbound states to the bound states have not been considered. This amounts to a difference in the prediction of ratios of the populations in thermally equilibrated plasma as predicted by statistical model [43]. The detailed balance condition by considering the back transition

$$\Gamma_{i \to k}\, e^{-E_i/k_B T} = \Gamma_{k \to i}\, e^{-E_k/k_B T} \quad \forall i, k, \tag{37}$$

leads to the equilibrium distribution of the populations which is then proportional to the respective Boltzmann factors. Therefore, we can understand this drawback. Considering a continuum of unbound states and the corresponding back transitions, one can see that the equilibrium is reached infinitely late. Yet that could be avoided by setting some transition probabilities to 0 which is practically equivalent to employ a finite number of states for the purpose.

4.2.3 *Important comments*

We have seen from these studies for a generic 4 level system and also for the exploratory model of heavy quarkonia that dynamical description leads to different conclusion from the static Debye screening picture. The system started initially with it's ground states but the dynamics populates the excited states also. Even for a thermalized static plasma, those excited states which were not initially in the medium can be recreated. This amounts to a difference with sequential melting picture. The sequential melting predicts that the excited states which are already melted can not be recreated and at the same time the ground states will not be affected if the temperature is not sufficient to melt them. We have seen that even at a fixed temperature (we had initially the ground state only), the ground state population has changed by repopulating the excited states.

4.3 EVOLUTION AS A STOCHASTIC PROCESS

In the last section, we have shown one of the attempts to describe quarkonia as open quantum systems in the medium. Instead of trying to design an effective potential, the approach employed the dipolar interaction to describe the dynamics. We should mention that even for a static medium at some temperature, the quarkonium population shows a dynamical behaviour. This could be contrary to the description with an effective potential which is real valued. It has certain similarities with a description governed by a static complex potential. We already have mentioned in the last chapter that the imaginary part provides a finite life time to the bound states of heavy quarkonia.
By approaching through stochastic quantum dynamics, A. Rothkopf

and Y. Akamatsu [52] have tried to show the origin of the imaginary part of the $q\bar{q}$ potential. According to them, the stochastic evolution causes an imaginary part in the potential. Their calculation also applies to a static thermal medium to describe the dynamics of the population. It has been done with one dimensional model by considering the effect of white noise to the dynamics of quarkonium bound states. Let's have a brief look into their idea. The Hamiltonian of the system + medium can be written as,

$$H = H_{med} \otimes I_{sys} + I_{med} \otimes H_{sys} + H_{int}, \quad H^{\dagger} = H,$$
$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] \tag{38}$$

with H_{med} for the medium, H_{sys} for the quark anti-quark pair and H_{int} denotes the interaction between the two. $\rho(t)$ denotes the density matrix corresponding to system +bath. Its evolution is described by the above equation which we have already seen in the master equation approach. We are interested to see the dynamics of the test system, therefore we want to integrate over all the medium degrees of freedom. The density matrix which describes the quark anti quark pair is given by

$$\rho_{q\bar{q}}(t, \mathbf{r}, \mathbf{r}') = \mathrm{Tr}_{med}\Big[\rho(t, \mathbf{r}, \mathbf{r}')\Big] = \langle \Psi_{q\bar{q}}(\mathbf{r}, t)\Psi^{*}_{q\bar{q}}(\mathbf{r}', t)\rangle \tag{39}$$

The wave function $\Psi_{q\bar{q}}$ for the $q\bar{q}$ system has been evaluated stochastically by considering the thermal fluctuation of the medium. Hence, the density matrix has been prepared from an ensemble of wave functions. The stochastic evolution of the wave function is given by

$$\Psi_{q\bar{q}}(\mathbf{r}, t) = \mathcal{T}\exp\left[-i\int_0^t d\tau\left\{-\frac{\nabla^2}{m_q} + 2m_q + V(\mathbf{r}) + \Theta(\mathbf{r}, \tau)\right\}\right]\Psi_{q\bar{q}}(\mathbf{r}, 0), \tag{40}$$

where $V(r)$ is quark anti-quark potential and Markovian noise $\langle\Theta(\mathbf{r}, t)\rangle = 0$ with a spatial correlation $\langle\Theta(\mathbf{r}, t)\Theta(\mathbf{r}', t')\rangle = \frac{1}{\Delta t}\delta_{t,t'}\Gamma(\mathbf{r}, \mathbf{r}')$. The solution of the above equation in first approximation (by expanding the operator in Eq. 40 in Δt)

$$i\frac{d}{dt}\Psi_{q\bar{q}}(\mathbf{r}, t) = \Big(-\frac{\nabla^2}{m_q} + 2m_q + V(\mathbf{r}) + \Theta(\mathbf{r}, t) - i\frac{\Delta t}{2}\Theta^2(\mathbf{r}, t)\Big)\Psi_{q\bar{q}}(\mathbf{r}, t). \tag{41}$$

Here m_q is the heavy quark mass and the interval Δt is a time scale during which the quarkonia states do not evolve considerably though the plasma particles are going through several collisions [52]. We can see though the time evolution is unitary, a complex term emerges from the stochasticity. By taking the thermal average value of the wave function, we obtain

$$i\frac{d}{dt}\langle\Psi_{q\bar{q}}(\mathbf{r},t)\rangle = \Big(-\frac{\nabla^2}{m_q}+2m_q+V(\mathbf{r})-\frac{i}{2}\Gamma(\mathbf{r},\mathbf{r})\Big)\langle\Psi_{q\bar{q}}(\mathbf{r},t)\rangle. \tag{42}$$

One can plug the above equation in to the Eq. (38). This accounts for the evolution of the quarkonium population. The density matrix always can be diagonalized after a finite time t_{dc} with certain choice of basis. This phenomena is called decoherence and the time t_{dc} is known as decoherence time.

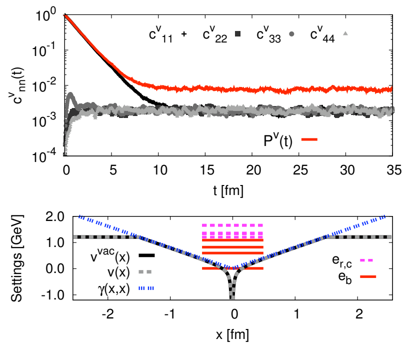


fig-18. *Survival probability* $P^v(t)$ *from the stochastic model [52], based on a lattice QCD inspired parameter set [36] and corresponding values of the initial potential* $v^{vac}(x)$*, the real potential* $v(x)$ *governing the dynamics with the diagonal noise strength* $\gamma(x,x)$[1].

The dynamics of population has been studied in this way considering vacuum $q\bar{q}$ potential as well as by considering a Debye screening in medium potential along with a white noise term. As we can see in the above plot inspired by lattice QCD data set which has been prepared with the initial vacuum $q\bar{q}$ potential $\nu^{\nu ac}$, the real in medium potential governs the dynamics utilising the diagonal noise term $\gamma(x,x)$. The population of quarkonium state decays with time without repopulating other bound states. This is really contrary to the one predicted by the master equation in the last section by Borghini. et. al. Another plot has been prepared by considering perturbative theory data given below.

1 For the other quantities apart from the survival probability, see the ref. [52]. In this discussion we are only interested in $P^v(t)$.

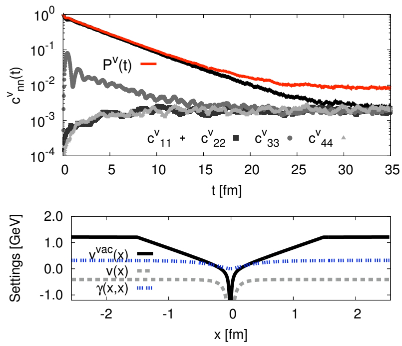


fig-19. *Same as previous plot [52] with a parameter set adapted from a perturbative (PT) study [34] where at* $T = 2.33T_c$ *a Debye screened real potential* $v(x) = \mathrm{Re}[v^{PT}(x)]$ *is accompanied by a small but finite noise term* $\gamma(x, x) = 2\mathrm{Im}[v^{PT}(x)]$.

4.4 CERTAIN REMARKS

In this chapter we have presented recent progress in real time dynamical descriptions of quarkonia in the medium. It is obvious that these studies differ from the usual static picture even at the qualitative level. Here we should mention that the master equation approach by Borghini. et. al. does not employ any in medium effective potential for quarkonia rather makes use of the gluonic interaction to study the dynamics of quarkonium (state) population. But the stochastic description still have not been settled with appropriate potential needed to be considered for $q\bar{q}$ pair. Therefore, different trial inputs have been tested. Of course the model is quite simplified and has dealt only with the one dimensional case. The only thing which confuses a bit is the use of the noise term (due to gluonic interaction) along with an in medium effective potential. By introducing an effective in medium potential, one already has integrated out all the medium degrees of freedom which then can not be used to extract further effect to the quarkonia.
The model utilized in master equation approach is also an exploratory step towards the dynamical issues. It has employed certain assumptions like the time scale of plasma interaction is much smaller than that of the quarkonia. At the same time back transitions from excited states have not been included into the description. Still these two frameworks for the first time step into the domain of real time dynamics. In the next chapter, we will develop a new method to deal the same to overcome existing drawbacks and limitations encounterd in this chapter but that also seems to be possible at the cost of alternative approximation.

5 A WAVE FUNCTION BASED APPROACH

We have discussed the necessity of real time dynamics in Chapter 3 and thereafter have presented some recent efforts to address real time dynamics. We have seen that the adiabatic evolution of quarkonia is not permissible [53] for the rapidly evolving medium produced in heavy ion collision. This is a very strong reason to think about a real time dynamics rather than a thermodynamical description. In this context we also have raised the question about the time scale of heavy quarkonia states to be thermalized with the medium. This question only can be answered in the real time framework. A study has been done by considering quarkonia as open quantum system, hence by modelling it as a four level system. The study has given us a qualitative description and estimation of the thermalization time for quarkonia. In this chapter, we are going to describe a new approach which can overcome those simplified assumptions used in last chapter. The approach, which we are going to demonstrate, is a wave function based approach for an open quantum system. This is important for one more reason as it is cost effective. In general, to deal with open system issues, the usual technique is to use the density matrix based approach. That can be approached through the solution of the master equation or by going through the Feynman-Vernon theory of influence functional [54, 55]. Both techniques rely on the evolution of the reduced density matrix [56]. Wave function based approaches are computationally better because it deals with N number of information instead N^2 number of that as needed in density matrix based frameworks. In this chapter, we will restrict ourselves to the harmonic oscillator model as an exploratory case. In the appendix, we will show a systematic procedure to generalize the approach for Coulomb bound states which can be applicable for heavy quarkonia at first approximation.

5.1 EXPLORATORY MODEL

We are considering the harmonic oscillator as a precursor of heavy quarkonia. The oscillator is interacting with a medium. The model which we will consider is the one which was proposed by Caldeira and Legett [57]. The medium is composed of a large number of harmonic oscillators (ideally infinite number) based on the harmonic approximation [57, 58]. The oscillators in the bath are not interacting among themselves, but all are interacting simultaneously with

the test system. The interaction is considered as bilinear interaction [59, 57] which is based on the assumption that the interaction is not so strong.
The lagrangian for the the whole system (test system and medium) can be written as

$$L = L_0 + L_I + L_B \tag{43}$$

where

$$L_0 = \frac{1}{2}M\dot{x}^2 - \frac{1}{2}M\omega^2 x^2 \tag{44}$$

$$L_B = \frac{1}{2}\sum_j m\left(\dot{X}_j^2 - \omega_j^2 X_j^2\right)$$

$$L_I = \sum_j \lambda x(t) X_j(t)$$

are the Lagrangians for the test oscillator, bath and the interaction part, respectively.
The action of the whole system is given by

$$S = S_0 + \sum_j \Big[\int_{t_i}^{t_f} dt \lambda x(t) X_j(t) + \int_{t_i}^{t_f} dt \frac{1}{2} m\Big(\dot{X}_j^2 - \omega_j^2 X_j^2\Big)\Big], \tag{45}$$

where S_0 is the action for the test system.
We want to investigate the dynamics of internal degrees of freedom like the population of states for the test system. Let the initial state of the test oscillator be $|\psi_0(t_0)\rangle$. The density matrix corresponding to the system is

$$\rho(t_0) = |\psi_0(t_0)\rangle\langle\psi_0(t_0)|. \tag{46}$$

When the density operator is represented in the energy eigenbasis of the system, we have diagonal elements which are known as the population and off diagonal terms are the coherence terms. Here, our interest is to study the population, therefore we will prepare the initial density matrix as diagonal. The ith population is thus given as

$$\rho_{ii}(t_0) = \langle u_i|\psi_0(t_0)\rangle\langle\psi_0(t_0)|u_i\rangle. \tag{47}$$

The system is interacting with the medium whose density matrix will be determined considering the mixed state as we can not access the exact state for the system but may have the knowledge of the probability of different configurations of states. Let us consider that the initial combined density matrix of the system and medium (or say reservoir) is product separable. hence,

$$\rho(t_0) = \rho(t_0)\rho^B(t_0). \tag{48}$$

As we are interested in the test system only, we need to take the partial trace over the medium density matrix. That essentially gives us the reduced density matrix of the test system,

$$\rho^S(t_0) = \mathrm{Tr}_B\,\rho(t_0). \tag{49}$$

We have discussed how this reduced density matrix evolves using the master equation formalism. Another way to understand the same is by going through the Feynman-Vernon theory of influence functional,

$$\rho^S(x,y,t) = \int dx'dy' J(x,y,t;x',y',t_0)\rho^S(x',y',t_0). \tag{50}$$

The density operator is represented in position basis and the quantity $J(x,y,t;x',y',t_0)$ is known as the kernel for the operator in that specific representation,

$$J(x,y,t;x',y',t_0) = \int\int \mathcal{D}x\mathcal{D}y\, \exp\big(iS[x]\big)\exp\big(-iS[y]\big)\mathcal{F}(x,y). \tag{51}$$

The quantity $\mathcal{F}(x,y)$ contains the information of the medium influence and therefore termed as influence functional. It comes through the following path integral,

$$\mathcal{F}(x,y) = \int dY dX dY' dX' \rho^B(X,X')\otimes \int\int \mathcal{D}X\mathcal{D}Y \exp i\big(S[X]-S[Y]-S_I[y,Y]+S_I[x,X]\big). \tag{52}$$

The integral over X, Y and corresponding prime coordinate signifies the partial trace over the medium degrees of freedom. This has been done because we are dealing with the reduced density matrix of the system. At this formal level it may look quite easy to calculate the influence functional by knowing the Lagrangian of the total system and hence we will have the dynamics of the reduced density matrix in coordinate representation. Once this is known, rest is simple to know what will happen to the population if it evolves being in the medium. Appreciable success has been achieved in this framework to describe quantum decoherence and certain phenomena related to quantum dissipation [56, 60]. But this is not all the time easy to go with this procedure especially when one has to describe real time issues and even for description beyond thermal equilibrium. With the intention to achieve a real time description for open systems which could be valid for any time scale and as well as beyond thermal equilibrium,

we are looking into the framework in a different approach. Dealing only with the population provides us certain facilities to make an approach which relies on the initial state of the test system. For coherence term that would be difficult for several reasons.
In this new approach, we will start with the initial state of the test system, let's say $|\psi_0(t_0)\rangle$, which corresponds to the initial population $\rho_{ii}(t_0)$. The system then evolves through the interaction with the medium and finally at time t, we need to know the population

$$\rho_{ii}(t) = \langle u_i|\psi(t)\rangle\langle\psi(t)|u_i\rangle = |\langle u_i|\psi(t)\rangle|^2. \tag{53}$$

This is nothing but the projection of the final state on the energy eigenstate $|u_i\rangle$ in Schroedinger picture. So, the question can be framed as follows,

A system initially in the state $|\psi_0(t_0)\rangle$ evolves by interacting with a medium. What will be the probability of having the system in the state $|u_i\rangle$ at some instant t during the evolution?

To find this answer we need not to go through a density matrix based approach. The initial state of the system and the Lagrangian of the system plus medium is sufficient to find out the answer. The simplest case is to understand when one oscillator (say j*th*) in the medium is going from the $\phi^j_m(X_i)$ to the state $\phi^j_n(X_f)$. the system evolves accordingly and we are interested in taking projection to the required final state we are interested in. let's say we want to see what is the probability of having the test system in its i th energy eigenstate $|u_i\rangle$. The expression is given by

$$\begin{aligned} P_{0\to i} = &\int dx_i dx_f dy_i dy_f \psi_0(x_i)\psi_0^\star(y_i)\otimes \\ &\Big(\int\int \mathcal{D}x\mathcal{D}y \exp\big(iS[x]\big)\exp\big(-iS[y]\big)F^j_{mn}[x,y]\Big)u_i^\star(x_f)u_i(y_f), \end{aligned} \tag{54}$$

$$\begin{aligned} F^j_{mn}[x,y] = &\int \phi^j_m(X_i)\phi^{j\star}_n(X_f)\int\int \mathcal{D}X\mathcal{D}Y \exp i\big(S[X]-S[Y]-S_I[y,Y]+ \\ &S_I[x,X]\big)\phi^j_n(Y_f)\phi^{j\star}_m(Y_i)dX_i dY_i dX_f dY_f. \end{aligned} \tag{55}$$

[

As we are interested in describing the dynamics of the test system, we have to consider all possible initial and final state of the medium constituents. The medium consists of infinite number of non interacting harmonic oscillators and for simplicity we can assume they are dynamically and statistically independent. Therefore,

$$F = \prod_j F^j;\;\; F^j = \sum_{m,n} F^j_{mn}. \tag{56}$$

$$P_{0\to i} = \int dx_i dx_f dy_i dy_f \psi_0(x_i)\psi_0^\star(y_i)$$
$$\Big(\int\int \mathcal{D}x\mathcal{D}y \exp\big(iS[x]\big)\exp\big(-iS[y]\big)F[x,y]\Big)u_i^\star(x_f)u_i(y_f) \tag{57}$$

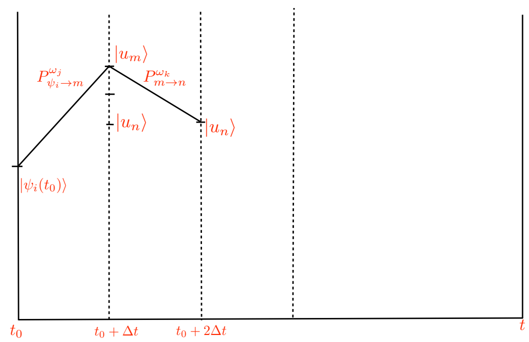
Now, the whole task is to calculate the influence functional offered by the medium and then to solve the modified path integral for the test system. The nice expression gives trouble when going beyond the formal expression to calculate numbers. For thermodynamic calculation it has been employed in lots of literature. A careful investigation made us aware of the fact that many simplifying assumptions have been made to deal with calculation [61] . I am not going to that detail of those simplifying assumptions. For non thermal medium, it still seems to be a nightmare. Having a very close look in to the dynamical description for open systems, we made an attempt to coarse-grain the medium interaction with the test system.

5.2 A COARSE-GRAINED INTERACTION SCENARIO

We already have mentioned in the last section that the medium is made of a large number of non interacting harmonic oscillators. They all are interacting at the same instant with the test system throughout the whole course of time. The problem is then to deal with the propagator of $N+1$ oscillators where N number of harmonic oscillators with various frequencies are interacting with the test system. We want to coarse-grain this problem by considering only two particle interactions. The possible way which we have devised is to divide the time interval t into infinitesimally small intervals Δt. We assume that within this infinitesimal period of time, only one oscillator among the medium constituents interacts with the test system. Other oscillators in the medium are evolving freely during this period. In the next infinitesimal interval again, only one oscillator interacts with the test system. It could be the previous one or a different oscillator. In this way we will have a sequence of two oscillator interactions for the complete span of time t.

The choice of the interacting oscillator from the medium is random, which implies that we can choose any of the oscillators from the medium. For each choice in every infinitesimal time interval leads to a single sequence. In practice we will have to consider all possible such sequences in order to calculate the transition probability from one state to another for the test oscillator. Summing up contributions from all possible sequences, we believe, we can mimic the result of the original problem at first approximation. We still do not have the proof of this statement that $N+1$ particle interaction could be mimicked by

such a two particle coarse-grained interaction. What we have investigated in this respect has convincingly motivated us toward such kind of coarse-graining. We have certain progress towards the proof and justification but we should not discuss those before we undoubtedly figure that out. In order to make a little sense of it, one can go through certain work related to the short time propagators [62, 54].
In the following schematic diagram, I have demonstrated the part of the coarse-graining in a two step process by introducing one possible sequence. The sequence is constructed by considering the fact that the j*th* oscillator with a frequency ω_j is interacting with the test system between t and $t+\Delta t$ where as another oscillator (k*th*) is interacting in the next infinitesimal interval of time.



Schematic picture for course grained interaction scenario.

Corresponding to such different sequences, the probability of (the test oscillator) going from state $\psi_i(t_0)$ to one of its energy eigenstates u_n at $t+2\Delta t$ is given by

$$\begin{aligned} P^{seq.1}_{i\to n} &= \sum_m P^{\omega_j}_{i\to m} P^{\omega_k}_{m\to n}; \\ P^{seq.2}_{i\to n} &= \sum_m P^{\omega_k}_{i\to m} P^{\omega_j}_{m\to n}; \\ P^{seq.3}_{i\to n} &= \sum_m P^{\omega_j}_{i\to m} P^{\omega_j}_{m\to n}. \end{aligned} \tag{58}$$

We have presented three different sequences to span two infinitesimally small time intervals. Many more (infinite number) sequences are possible for this two step progression as there is infinite number of oscillators in the medium. All these sequences are mutually exclusive and therefore the probability $P_{m\to n}$ is the sum of the contribution from all possible sequences,

$$P_{i\to n} = P^{seq.1}_{i\to n} + P^{seq.2}_{i\to n} + ... + P^{seq.p}_{i\to n} + ... \tag{59}$$

Hence, by progressing this way, one can span the full time interval to know the projection to different eigen states finally. In fact due to a distribution of frequency $(f(\omega))$, $P^{\omega_k}_{m\to n}$ becomes a function of frequency $P_{m\to n}(\omega)$. Therefore, Eq.(55) looks

$$P_{i\to n} = \sum_m \int_\omega d\omega d\omega' P_{i\to m}(\omega) f(\omega) P_{m\to n}(\omega') f(\omega'). \tag{60}$$

We see that for two infinitesimal time intervals one has to integrate two times over the frequency ω. For such N number of intervals, we similarly have to integrate N times over the frequency which then effectively introduces a infinite number of such integrals as $N \to \infty$. Finally, the probability of going from $|\psi_i(t_0)\rangle$ at time t_0 to any of the eigenstate $|u_f\rangle$ is given by

$$\begin{aligned} P_{i\to f} = &\int_\omega \int_{\omega'} \int_{\omega''} ....d\omega d\omega' d\omega'' ....P_{i\to m}(\omega) f(\omega) \\ &P_{m\to n}(\omega') f(\omega') P_{n\to q}(\omega'') f(\omega'').... \end{aligned} \tag{61}$$

Before we calculate these ω integral, we need to compute $P^k_{m\to n}$ as basic building block of the above expression.

$$\begin{aligned} P^k_{m\to n} = &\int dx_i dx_f dy_i dy_f u_m(x_i) u^\star_m(y_i) \\ &\Big(\int\int \mathcal{D}x\mathcal{D}y \exp\big(iS[x]\big) \exp\big(-iS[y]\big) F^k[x,y]\Big) u^\star_n(x_f) u_n(y_f) \\ = &\Bigg| \sum_{m',n'} \int dx_i dx_f dX_i dX_f \phi^k_{m'}(X_i) u_m(x_i) \\ &\left(\int\int \mathcal{D}x\mathcal{D}X \exp i\big(S[x] + S[X] + S_I[x,X]\big)\right) \phi^{k\star}_{n'}(X_f) u^\star_n(x_f) \Bigg|^2. \end{aligned} \tag{62}$$

At this stage we can see that the problem boils down to a calculation of two interacting harmonic oscillators with different masses and frequencies.

5.3 PROPAGATOR FOR TWO COUPLED HARMONIC OSCILLATORS

Consider now two coupled harmonic oscillators with masses m_1 resp. m_2 and frequencies ω_1 resp. ω_2, with a bilinear position coupling. The corresponding Lagrangian reads

$$L[x_1, \dot{x}_1, x_2, \dot{x}_2, t] = \sum_{j=1,2} \left[\frac{m_j}{2} \dot{x}_j(t)^2 - \frac{m_j \omega_j^2}{2} x_j(t)^2\right] - \lambda\sqrt{m_1 m_2}\, \omega_1 \omega_2\, x_1(t)\, x_2(t), \tag{63}$$

where the coupling strength λ is dimensionless.

The change of dynamical variables

$$\mathbf{Y}(t) = R\,\mathbf{X}(t) \quad \text{with} \quad \mathbf{X}(t) \equiv \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, \mathbf{Y}(t) \equiv \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$$
$$R = \begin{pmatrix} \sqrt{\frac{M}{m_1}} \cos\varphi & \sqrt{\frac{M}{m_1}} \sin\varphi \\ -\sqrt{\frac{M}{m_2}} \sin\varphi & \sqrt{\frac{M}{m_2}} \cos\varphi \end{pmatrix} \tag{64}$$

in the Lagrangian (63) with an appropriate angle φ allows one to obtain two decoupled "normal" modes with the same masse M and frequencies Ω_1, Ω_2 given by

$$\Omega_{1/2}^2 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\lambda^2\omega_1^2\omega_2^2}\right]. \tag{65}$$

Note that λ should be smaller than 1 to ensure that these roots always take positive values. The angle φ is given by

$$\tan 2\varphi = \frac{2\lambda\omega_1\omega_2}{\omega_2^2 - \omega_1^2} \tag{66}$$

for $\omega_1 \neq \omega_2$, or $\varphi = \frac{\pi}{4}$ when $\omega_1 = \omega_2$.

If the oscillators 1 and 2 play symmetric roles, then one may assign any of the roots to Ω_1^2 (and the other to Ω_2^2). Here, we shall break that symmmetry, by integrating over the states of the harmonic oscillator labelled 2—later, we shall even integrate over different frequencies ω_2. Now, physically one anticipates that, for a fixed frequency ω_1 and a given coupling λ, the effect of oscillator 2 should be maximal when ω_2 is close to ω_1, minimal when ω_2 goes to 0 or ∞. In the latter cases, φ vanishes, so that the matrix $\mathcal{R}$ introduced below [Eq. (68)] becomes diagonal, which signals the decoupling of the oscillators. To ensure that Ω_1 remains of the same order as ω_1 in both limits $\omega_2 \to 0$ and $\omega_2 \to \infty$, one should take for Ω_1 the plus sign in Eq. (65) in the former case, and the minus sign in the latter case. We shall thus take

$$\Omega_1^2 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 + \eta\sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\lambda^2\omega_1^2\omega_2^2}\right],$$
$$\Omega_2^2 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 - \eta\sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\lambda^2\omega_1^2\omega_2^2}\right] \tag{67}$$

with $\eta = +1$ if $\omega_2 < \omega_1$, $\eta = -1$ otherwise. These choices actually lead to $\Omega_1 = \omega_1$, $\Omega_2 = \omega_2$ in the limit $\lambda \to 0$.

Let X, Y denote four-dimensional vectors with

$$X^T \equiv (x_{1,i}, x_{1,f}, x_{2,i}, x_{2,f}), \qquad Y^T \equiv (y_{1,i}, y_{1,f}, y_{2,i}, y_{2,f}),$$

i.e. combining the initial and final values of the position for the coupled resp. decoupled harmonic oscillators together. One easily checks the identity

$$Y = \mathcal{R}\,X \quad \text{with} \quad \mathcal{R} \equiv \begin{pmatrix} \sqrt{\frac{M}{m_1}}\,C & 0 & \sqrt{\frac{M}{m_1}}\,S & 0 \\ 0 & \sqrt{\frac{M}{m_1}}\,C & 0 & \sqrt{\frac{M}{m_1}}\,S \\ -\sqrt{\frac{M}{m_2}}\,S & 0 & \sqrt{\frac{M}{m_2}}\,C & 0 \\ 0 & -\sqrt{\frac{M}{m_2}}\,S & 0 & \sqrt{\frac{M}{m_2}}\,C \end{pmatrix}, \tag{68}$$

with the shorthand notations $C \equiv \cos\varphi$, $S \equiv \sin\varphi$.

Introducing the real symmetric 4×4 matrix

$$\mathcal{S}_{M;\Omega_1,\Omega_2}(\tau) \equiv \begin{pmatrix} A_{M,\Omega_1}(\tau) & 0 \\ 0 & A_{M,\Omega_2}(\tau) \end{pmatrix},$$

the propagator for the two uncoupled harmonic oscillators reads

$$\begin{aligned} \mathcal{K}_{M;\Omega_1,\Omega_2}(\mathbf{Y}_f, t_f; \mathbf{Y}_i, t_i) &= \mathcal{K}_{M,\Omega_1}(y_{1,f}, t_f; y_{1,i}, t_i)\,\mathcal{K}_{M,\Omega_2}(y_{2,f}, t_f; y_{2,i}, t_i) \\ &= \kappa_{M,\Omega_1}(t_f - t_i)\,\kappa_{M,\Omega_2}(t_f - t_i) \\ &\qquad \exp\left[i\,Y^T \mathcal{S}_{M;\Omega_1,\Omega_2}(t_f - t_i)\,Y\right], \end{aligned} \tag{69a}$$

i.e., in terms of the dynamical variables of the coupled oscillators,

$$\mathcal{K}_{M;\Omega_1,\Omega_2}(\mathbf{X}_f, t_f; \mathbf{X}_i, t_i) = \kappa_{M;\Omega_1,\Omega_2}(t_f - t_i)\,\exp\left[i\,X^T \mathcal{A}''_{M;\Omega_1,\Omega_2}(t_f - t_i)\,X\right] \tag{69b}$$

with

$$\kappa_{M;\Omega_1,\Omega_2}(\tau) \equiv \kappa_{M,\Omega_1}(\tau)\,\kappa_{M,\Omega_2}(\tau) \quad , \quad \mathcal{A}''_{M;\Omega_1,\Omega_2}(\tau) \equiv \mathcal{R}^T \mathcal{S}_{M;\Omega_1,\Omega_2}(\tau)\,\mathcal{R}. \tag{69c}$$

Note that $\mathcal{A}''_{M;\Omega_1,\Omega_2}(\tau)$ is again real symmetric, yet with a different determinant from that of $\mathcal{S}_{M;\Omega_1,\Omega_2}(\tau)$. We see that the propagator of two coupled oscillators thus can be written as a product of two oscillators with same mass M and different normal mode frequencies. Therefore the problem even simplifies to the free harmonic oscillators. In the last section, we have seen that due to the coarse-graining, one has to sum over all possible eigenstates of test and medium oscillators. We will choose the energy eigenstates represented in coherent state basis because it helps to simplify the calculation a lot.

5.4 PROPAGATOR AND COHERENT STATES OF A SINGLE HARMONIC OSCILLATOR

Consider first a free one-dimensional harmonic oscillator with mass m and frequency ω, described by the Lagrangian

$$L[x, \dot{x}, t] = \frac{m}{2}\,\dot{x}(t)^2 - \frac{m\omega^2}{2}\,x(t)^2. \tag{70}$$

The corresponding propagator between two space-time points (x_i, t_i) and (x_f, t_f) reads

$$\mathcal{K}_{m,\omega}(x_f, t_f; x, t) = \left[\frac{m\omega}{2i\pi\hbar \sin\omega(t_f - t)}\right]^{1/2} \exp\left\{\frac{im\omega}{2\hbar\sin\omega(t_f - t)}\left[(x_f^2 + x^2)\cos\omega(t_f - t) - 2x_f x\right]\right\}. \tag{71}$$

Let $\tau \equiv t_f - t_i$. This propagator can be recast as

$$\mathcal{K}_{m,\omega}(x_f, t_f; x, t) = \kappa_{m,\omega}(t_f - t)e^{i\mathbf{x}^T A_{m,\omega}(t_f - t_i)\,\mathbf{x}} \tag{72a}$$

where

$$\kappa_{m,\omega}(\tau) \equiv \left[\frac{m\omega}{2i\pi\hbar\sin\omega\tau}\right]^{1/2}, \qquad \mathbf{x} \equiv \begin{pmatrix} x_i \\ x_f \end{pmatrix}$$

$$A_{m,\omega}(\tau) \equiv \frac{m\omega}{2\hbar\sin\omega\tau}\begin{pmatrix} \cos\omega\tau & -1 \\ -1 & \cos\omega\tau \end{pmatrix}, \tag{72b}$$

respectively, denote a coefficient, a real symmetric 2×2 matrix, and a two-component vector.

Let $\alpha \in \mathbb{C}$. The coherent state $|\alpha\rangle$ is the eigenstate of the annihilation operator $\hat{a}$ corresponding to the harmonic oscillator with the eigenvalue α:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle.$$

With the help of the creation operator $\hat{a}^\dagger$ comes

$$|\alpha\rangle = e^{\alpha\hat{a}^\dagger}|0\rangle,$$

where $|0\rangle$ denotes the ground state of the harmonic oscillator.

In Schrödinger coordinate representation, the wavefunction for the coherent state $|\alpha\rangle$ reads

$$\psi_\alpha(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{(\alpha^{*2} - \alpha^2)/4} \exp\left[-\left(\sqrt{\frac{m\omega}{2\hbar}}\,x - \mathrm{Re}\,\alpha\right)^2 + i\,\mathrm{Im}\,\alpha\sqrt{\frac{2m\omega}{\hbar}}\,x\right], \tag{73a}$$

where the exponential prefactor is only a phase factor since $\alpha^{*2} - \alpha^2 = -4i \operatorname{Re}\alpha \operatorname{Im}\alpha$. This expression can also be recast as

$$\psi_\alpha(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-(\operatorname{Im}\alpha)^2 + i\operatorname{Re}\alpha\operatorname{Im}\alpha} \exp\left[-\left(\sqrt{\frac{m\omega}{2\hbar}}\,x - \alpha\right)^2\right]. \quad (73b)$$

The global phase factor $e^{i\operatorname{Re}\alpha\operatorname{Im}\alpha}$ plays no role in the following computations. Yet we shall keep it, since it enters the scalar product

$$\int_{-\infty}^{\infty} \psi_\nu(x)^* \psi_\alpha(x)\, dx = e^{-(|\alpha|^2 + |\nu|^2) + \nu^*\alpha}, \quad (74)$$

which provides a useful check.

5.5 TRANSITION BETWEEN COHERENT STATES OF THE COUPLED OSCILLATORS

The propagator (69) gives access to the transition probability between any pairs of states for the two coupled oscillators. Thus the probability amplitude that oscillator 1 evolving from a coherent state $|\alpha_i\rangle$ to another coherent state $|\alpha_f\rangle$ while oscillator 2 transitions from a coherent state $|\nu_i\rangle$ to the coherent state $|\nu_f\rangle$ is

$$A[|\alpha_i, \nu_i\rangle \to |\alpha_f, \nu_f\rangle] = \int \psi^*_{\alpha_f}(x_{1,f})\, \psi^*_{\nu_f}(x_{2,f})\, \mathcal{K}_{M;\Omega_1,\Omega_2}(\mathbf{X}_f, t_f; \mathbf{X}_i, t_i)$$
$$\psi_{\alpha_i}(x_{1,i})\, \psi_{\nu_i}(x_{2,i})\, dx_{1,f}\, dx_{2,f}\, dx_{1,i}\, dx_{2,i}. \quad (75)$$

Inserting the expressions of the wavefunctions (73a) and the propagator (69) in this equation, one finds

$$A[|\alpha_i, \nu_i\rangle \to |\alpha_f, \nu_f\rangle] = \frac{\sqrt{m_1 m_2 \omega_1 \omega_2}}{\pi\hbar}\, \kappa_{M;\Omega_1,\Omega_2}(t_f - t_i)\, C_{\alpha_i,\alpha_f,\nu_i,\nu_f}$$
$$\int \exp\left[-X^T \mathcal{A}(t_f - t_i)\, X + B^T X\right] d^4X \quad (76a)$$

where $C_{\alpha_i,\alpha_f,\nu_i,\nu_f} \in C$ is a coefficient that only depends on the initial and final coherent states of the oscillators

$$C_{\alpha_i,\alpha_f,\nu_i,\nu_f} \equiv \operatorname{Exp}\big[-\alpha_i^2 - (\alpha_f^*)^2 - \nu_i^2 - (\nu_f^*)^2 - (\operatorname{Im}\alpha_i)^2 - (\operatorname{Im}\alpha_f)^2 - (\operatorname{Im}\nu_i)^2 - (\operatorname{Im}\nu_f)^2 + i[(\operatorname{Re}\alpha_i \operatorname{Im}\alpha_i) - \operatorname{Re}\alpha_f \operatorname{Im}\alpha_f) + (\operatorname{Re}\nu_i \operatorname{Im}\nu_i) - \operatorname{Re}\nu_f \operatorname{Im}\nu_f)]\big].$$

$\mathcal{A}(\tau)$ is a complex symmetric matrix given by

$$\mathcal{A}(\tau) \equiv \mathcal{A}'_{m_1,\omega_1,m_2,\omega_2} - i\,\mathcal{A}''_{M;\Omega_1,\Omega_2}(\tau) \quad \text{with}$$
$$\mathcal{A}'_{m_1,\omega_1,m_2,\omega_2} \equiv \operatorname{diag}\left(\frac{m_1\omega_1}{2\hbar}, \frac{m_1\omega_1}{2\hbar}, \frac{m_2\omega_2}{2\hbar}, \frac{m_2\omega_2}{2\hbar}\right), \quad (76b)$$

a positive definite real symmetric(!) matrix. The vector B is defined by

$$B^T \equiv \left(-\sqrt{\frac{2m_1\omega_1}{\hbar}}\,\alpha_i, -\sqrt{\frac{2m_1\omega_1}{\hbar}}\,\alpha_f^*, -\sqrt{\frac{2m_2\omega_2}{\hbar}}\,\nu_i, -\sqrt{\frac{2m_2\omega_2}{\hbar}}\,\nu_f^* \right). \tag{76c}$$

Eventually, $d^4X \equiv dx_{1,f}\, dx_{2,f}\, dx_{1,i}\, dx_{2,i}$.

The Gaussian integral in Eq. (76a) reads

$$\int \exp\left[-X^T \mathcal{A}(\tau)\, X + B^T X \right] d^4X = \frac{\pi^2}{\sqrt{\det \mathcal{A}(\tau)}}\, e^{\frac{1}{4} B^T \mathcal{A}(\tau)^{-1} B}, \tag{77}$$

which, inserted in Eq. (75), yields

$$A[|\alpha_i, \nu_i\rangle \to |\alpha_f, \nu_f\rangle] = \frac{\pi\sqrt{m_1 m_2 \omega_1 \omega_2}}{\hbar}\, \kappa_{M;\Omega_1,\Omega_2}(t_f - t_i)\, \frac{C_{\alpha_i,\alpha_f,\nu_i,\nu_f}}{\sqrt{\det \mathcal{A}(\tau)}}\, e^{\frac{1}{4} B^T \mathcal{A}(\tau)^{-1} B}. \tag{78}$$

The calculation of the probability amplitude thus amounts to finding the determinant and inverse of the matrix $\mathcal{A}$, Eq. (76b).

One can note that the prefactor of the exponential function in Eq. (78) is actually independent of the coefficients labeling the various coherent states. The latter only enters the exponent as well as the (exponential!) prefactor $C_{\alpha_i,\alpha_f,\nu_i,\nu_f}$ Eq. (76). Moreover, since the vector B is linear in α_i, α_f, ν_i, ν_f, the whole dependence of the probability amplitude on those numbers is simply Gaussian. This then also holds for the corresponding transition probability $\mathcal{P}[|\alpha_i, \nu_i\rangle \to |\alpha_f, \nu_f\rangle] = \left|A[|\alpha_i, \nu_i\rangle \to |\alpha_f, \nu_f\rangle]\right|^2$.

5.6 DISTRIBUTION OF COHERENT STATES

Assume now that the initial state of one of the harmonic oscillators, say that labeled 2, is not exactly known. Instead, one only has the probability $p(\nu_i)$ that it is in the coherent state $|\nu_i\rangle$.

If the average excitation number $\langle n_2 \rangle$ of the oscillator is given, as is for instance the case if the harmonic oscillator is a mode of a field in thermal equilibrium, then this probability reads (see Eq. (10.23) in [63])

$$p(\nu_i) = \frac{1}{\pi\langle n_2 \rangle}\, e^{-|\nu_i|^2/\langle n_2 \rangle}. \tag{79}$$

The probability $\mathcal{P}[|\alpha_i\rangle \to |\alpha_f\rangle]$ that the harmonic oscillator 1 evolves from an initial coherent state $|\alpha_i\rangle$ to another coherent state $|\alpha_f\rangle$, irrespective of the initial and final states of the other oscillator, follows from summing the transition probability over the initial states $|\nu_i\rangle$

weighted by their occurrence probabilities $p(\nu_i)$, and over all final states $|\nu_f\rangle$:

$$\mathcal{P}[|\alpha_i\rangle \to |\alpha_f\rangle] = \int \mathcal{P}[|\alpha_i, \nu_i\rangle \to |\alpha_f, \nu_f\rangle]\, p(\nu_i)\, d^2\nu_i\, d^2\nu_f, \tag{80}$$

where $d^2\nu \equiv d(\mathrm{Re}\,\nu)\, d(\mathrm{Im}\,\nu)$. If the probability $p(\nu_i)$ is given by a Gaussian distribution, as in Eq.(79), then the integrand in Eq. (80) is actually an exponential of a quadratic function of the real and imaginary parts of ν_i, ν_f, so that the computation of $\mathcal{P}[|\alpha_i\rangle \to |\alpha_f\rangle]$ is straightforward.

5.7 DYNAMICS OF POPULATION FOR HARMONIC OSCILLATOR IN A MEDIUM

Up to the last section, we have designed the frame work to deal with the evolution of population of harmonic oscillator bound states in a medium. All the basic ingredient and tools have been discussed. Now, what we need is to apply this wave function based technique to a harmonic oscillator. Our ultimate goal would be to describe heavy quarkonia states in quark gluon plasma. In the appendix, a systematic approach is shown to generalize this precursor study for the realistic quarkonia. Let us give a less mature assurance that the qualitative features will not be drastically different. One can grow the confidence to believe this statement relying on the fact that a Coulomb system is nothing but a "Hydrogenus Oscillator" [64]. Let's see the results which we have obtained in the study of harmonic oscillator.
We have employed the key formula 61 to see the dynamics of different bound states. As a preliminary, we have started with a harmonic oscillator populated initially in it's ground state,

$$\rho(t_0) = \begin{pmatrix} 1 & 0 & . & . \\ 0 & 0 & . & . \\ . & . & . & . \\ . & . & . & . \end{pmatrix} \quad ; \quad \rho_{ii} = |u_i\rangle\langle u_i| \tag{81}$$

As the oscillator initially at t_0 is populated in it's ground state in the medium, we can fix the reduced density matrix $\rho_{00} = 1$ (represented in energy eigenbasis). The corresponding normalized state is $|u_0\rangle$, that is

$$\int_\infty \langle x|u_0\rangle\langle u_0|x\rangle dx = 1. \tag{82}$$

After a while, the state $|u_0(t_0)\rangle$ will evolve to some linear combination of its own energy eigenstates as there is interaction with the medium[1].

1 If it evolves adiabatically, the new state will be again a ground state. We are not interested in the trivial unitary evolution.

Consequently, non zero projection into different eigenstates will populate other diagonal element of the reduced density matrix (81). At any instant 't', the diagonal elements of the matrix can be evaluated from the evolution of the initial wave function,

$$\rho_{jj}(t) = |\langle u_j(t)|u_0(t_0)\rangle|^2; \quad j = 1, 2, ...\infty. \tag{83}$$

The off diagonal terms have not been considered as we are interested to see the dynamics of the population. We can employ Eq. (62) to evaluate the populations ρ_{jj} as a dynamical quantity. we have studied one such case by populating the test harmonic oscillator initially in it's ground state. We can see in eq. (61) that we need to plug the initial and the state on which we intend to take the projection. Plugging ground states for both in the eq. (61) and combining with eq. (62), we have calculated the survival probability of the ground state. This gives the ground state population as a function of time which has been shown in the plot below.

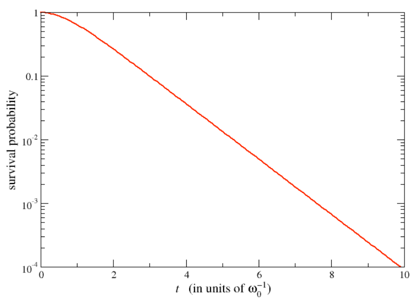


fig-20. *Logarithmic plot of the dynamics of the ground state population. The behaviour is very similar to the quarkonium model discussed in chapter* 4.

As we can see in the above plot, the initial ground state population decreases with time. This amounts to a regeneration of excited states.We have not presented those excited state populations in the plot. It has been shown in Chapter 3 in the context of the master equation approach for heavy quarkonia (as Coulomb system) that the other bound states evolve together. One thing is quite clear from this precursor study through harmonic oscillator that the qualitative feature does not differ from the coulomb system. Therefore, one can even understand the fate of heavy quarkonia in QGP from this generic study at least at the qualitative level. This result also differs from the expectation based on sequential suppression.

5.8 IMPORTANT FEATURES

The previous plot has been prepared by considering those medium modes whose frequencies are comparable to that of the test oscillator. We see in the plot that the decay of initial population is largely due to those comparable modes, whereas modes with very different frequencies leave the test system unaffected. This result meets expected behaviours which qualitatively could be understood for the case of quarkonium bound states also. We expect to find the same for a color dipole model of heavy quarkonia.

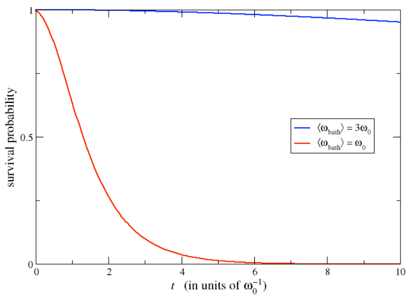


fig-21. *Red and blue lines show the dynamics of population (in linear scale) of the test oscillator due to the comparable modes and the distant modes respectively.*

The result shows that those gluons whose wavelengths are comparable to the size of the dipole, will affect the dipole most. Other high energetic gluons, as they have the de-Broglie wavelength much smaller than the size of the dipole, will see the quarkonia as two different color charges. On the other hand those low energetic gluons, having the corresponding de-Broglie wavelength much bigger than the size of the dipole, will see the dipole as a color neutral obeject. Therefore, they cannot affect the dipole much. This result meets the previously understood hard and soft gluonic effect to the quarkonium states [65].

5.9 WORK IN PROGRESS

In the last couple of sections we have developed the framework for harmonic oscillator as an exploratory study of quarkonia in medium. As we intend to deal with realistic heavy quarkonia, we then have to generalize the scheme for quarkonium states. The $q\bar{q}$ potential in vacuum has two parts, a Coulomb part along with the linear one. For bottomonium, the linear part is vanishingly small at short distances. At a very high temperature, the string tension part becomes negligible which then allows one to treat the $q\bar{q}$ states in the same way as done in Coulomb bound states in Quark Gluon Plasma. The reason behind the study of harmonic oscillator lies in the connection between central potential and 4-D harmonic oscillator. The connection has been well studied using a certain parametrization in the context of Kepler problem. Kustaanheimo-Stiefel transformation [66] was applied to map the Coulomb problem into a problem of harmonic oscillator. The path integral treatment for hydrogen atom or several non gaussian cases remained troublesome for decades. The propagator for hydrogen atom was for the first time calculated by using K-S transformation and by introducing an auxiliary time parameter [67, 64]. A detailed discussion is presented in the appendix.

The interesting thing from the observation of associating Coulomb system with the oscillator lies in the calculability of oscillator system in various cases. We already have presented the framework in this chapter to deal with the harmonic oscillator as an open system in a medium. More precisely, a heavy quarkonia can be thought as color dipole and a deconfined medium of quarks and gluons. Quarkonia will be affected by the chromoelectric field which is very similar to an electric dipole in an electric field. In the quantized form of the chromoelectric field in Wyle gauge the problem boils down to the case where a 4D harmonic oscillator with a constraint interacts with the medium of linear harmonic oscillators. The interaction has a bilinear form which we have seen in chapter 4. A similar case has been studied in the present chapter for a 3D isotropic oscillator. The case of coulomb dipole in QGP is then very similar to that. The only difference is that we have to deal with a four dimensional case. With the progress we have made till now, we can give a reasonable guessing about the results. We are very much convinced with the fact the result will not differ at the qualitative level but definitely, there should difference in the quantitative calculation.

6 CONNECTION WITH THE RECENT EXPERIMENTAL RESULTS

At the very beginning of this thesis, we have mentioned that the study of heavy quarkonia in quark gluon plasma is mostly attributed to design a probe for the medium. From experimental data we can refer observation of different quarkonium states in the vacuum which is obviously the yield coming from the medium. Earlier we have pointed out that the association of those vacuum $q\bar{q}$ states with the same states in the medium relies on the adiabatic approximation throughout their evolution in medium up to the late electroweak decay which takes place outside the medium. In chapter 3, through rigorous calculations, it has been shown that the adiabatic approximation is drastically violated for the rapidly evolving[1] medium produced in heavy ion collisions. Neglecting this issue may lead to a very wrong comparison with the experimental data. This implicit assumption has been employed since decades and the whole meaning of sequential suppression has been compromised. Therefore, the Debye screening is thought to be a sufficient mechanism to describe the observed quarkonium suppression even in the rapidly evolving fireball. We doubt that for several reasons and the role of real time dynamics therefore has been emphasized to understand the suppression in more precise and realistic way. The incomplete suppression of excited states which has been observed in experiment can never be explained through the static picture based on Debye screening. Though, there are other ideas around for sufficient regeneration of those states. Before we present the connection of this argument to the experimental observation, we will discuss what has been measured yet in this context.

6.1 MEASUREMENTS ON CHARMONIUM STATES IN EXPERIMENTS

According to the sequential melting, certain states of heavy quarkonia should not be present in the quark gluon plasma medium. I already have discussed the QGP thermometer in that context. At a very high energy density, excited states melt and if the temperature is sufficiently high, could possibly melt ground states as well. Measurements have been carried out various times both on the charm anticharm boundstates as well as on bottomonia. The earlier expectations from

1 Even for a static medium, the adiabaticity can not be guaranteed due to the resonant interaction of quarkonia with the gluons.

theory was to see the J/ψ particle to be suppressed completely even though other possibilities like cold nuclear effect were also thought to be responsible for the same in an environment different than QGP. In the simplest case without considering any further regeneration, we should not find any J/ψ in the detector if the quark gluon plasma has already been produced. Initial measurements on number $N_{J/\psi}$ in experiments showed suppression but that indicated an incomplete suppression. Those results [68] showed suppression increased with the magnitude of transverse energy. Nevertheless that can not verify the formation of QGP in the way predicted by sequential suppression. It is worth-mentioning that the essence of complete suppression of quarkonium states has never been observed for both the bottomonia and charm pair. The most popular experimental language in this issue deals with terms like single and double ratios. The single ratio is basically comparing different eigenstates of a specific $q\bar{q}$ pair in heavy ion collision with the same in the proton-proton collision.
We should not forget that the Debye screening allows either complete survival or complete suppression of states. The single ratio is then either zero or unity. We really then do not need to define ratios as it signifies only the degree of suppression of quarkonium states.
Let us give a little description on single and double ratio in this context.

6.1.1 *Single and double ratio*

In past 20 years, the J/ψ suppression has been studied extensively in relativistic heavy ion collisions at the Super Proton Synchrotron (SPS) and at RHIC in the energy (center of mass energy ($\sqrt{s_{NN}}$) per nucleon pair) range between 20 to 200GeV. The strong J/ψ suppression was observed in SPS. Observed suppression in Pb-Pb collision was much bigger than to be expected from the cold nuclear matter effect. The excited states like χ_c and $\psi(2S)$ were also measured and significant suppression was observed. In RHIC similar measurement was performed for Au-Au collision. All those experimental observations confirmed suppression of various $c\bar{c}$ states.
In this context, by suppression we mean a part of the produced charmonium states have been observed to be absent in the yield coming out of the medium. This statement may sound a bit tricky as there is no direct way to measure how many states have been created after the heavy ion collision. Nevertheless one can compare the number of states detected in heavy ion collision with the same in p-p collision. By talking in terms of the ratio sounds meaningful in this context as the number of $q\bar{q}$ states in p-p collision can give us a reference of the initially produced states. The single ratio is often known as nuclear modification factor R_{AA} and defined as,

$$R_{AA}^{i} = \frac{Y_{J/\psi}^{i}(\Delta p_t, \Delta y)}{\langle T_{AA}^{i} \rangle \times \sigma_{J/\psi}^{pp}(\Delta p_t, \Delta y)} \tag{84}$$

where $Y_{J/\psi}^{i}$ is defined in the following way,

$$Y_{J/\psi}^{i}(\Delta p_t, \Delta y) = \frac{N_{J/\psi}^{i}}{BR_{J/\psi \to \mu^+\mu^-} N_{events}^{i} A\epsilon^{i}}, \tag{85}$$

for a transverse momentum range Δp_t with rapidity range Δy. $\langle T_{AA}^{i} \rangle$ denotes the average value of nuclear overlap factor and $\sigma_{J/\psi}^{pp}$ is number of J/ψ (in p-p collision) defined in the same way as $Y_{J/\psi}^{i}$. The quantity $N_{J/\psi}^{i}$ is the number of J/ψ normalized with centrality class N_{events}^{i} (i signifies the specific type of nucleus-nucleus collision) where as the quantity BR is branching ratio.

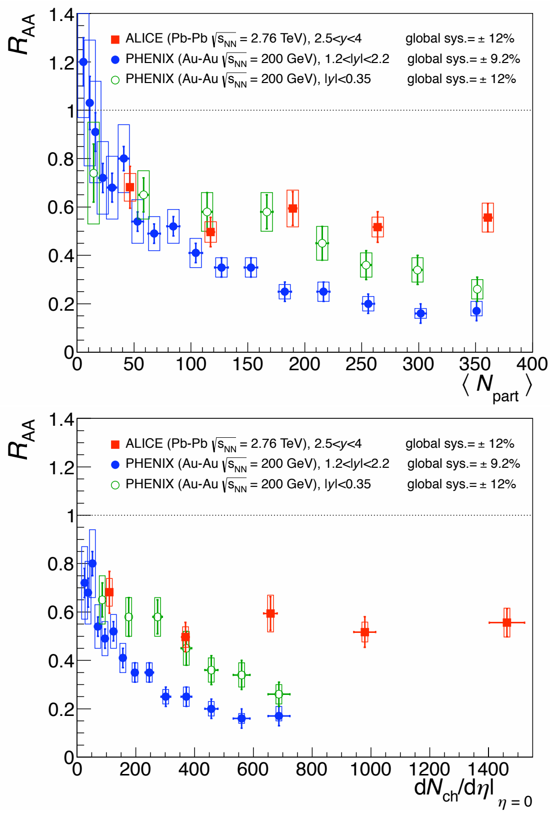


Fig.22.-

(Colour lines) Inclusive J/ψ R_{AA} as a function of the mid-rapidity charged-particle density (top) and the number of participating nucleons (bottom) [69].

We have presented the plot which was prepared from the recent measurement [69] performed at LHC. The measurement was done with a product of acceptance A and efficiency ϵ equal to 0.176 in the rapidity region $2.5 < y < 4$ for $p_t > 0$ at an energy $\sqrt{s_{NN}} = 2.76$ TeV. The nuclear modification factor integrated over most central collision comes out as

$$R^i_{AA} = 0.545 \pm 0.032(\text{stat.}) \pm 0.083(\text{syst.}). \tag{86}$$

6.2 RECENT MEASUREMENTS ON BOTTOMONIUM STATES IN EXPERIMENTS

In last section, we have presented observation made for charmonium states and have given recent results on that. In this section, we will do the same for bottomonium states. Different bottomonium states and their properties has already been discussed in Chapter 2. For charmonium, we speak about a single ratio which can account for the suppression of different states. For bottomonium, very frequently used quantity is known as double ratio though a meaningful description through single ratio is still possible. The measurement at the energy $\sqrt{s_{NN}} = 2.76\text{TeV}$ for $p_T > 4\text{GeV}$ in the rapidity region $|y| < 2.4$ gives the following results [70] ,

$$\begin{aligned}
\Upsilon(2S)/\Upsilon(1S)|_{pp} &= 0.56 \pm 0.13\,(\text{stat.}) \pm 0.02\,(\text{syst.})\,, \\
\Upsilon(2S)/\Upsilon(1S)|_{PbPb} &= 0.12 \pm 0.03\,(\text{stat.}) \pm 0.02\,(\text{syst.})\,, \\
\Upsilon(3S)/\Upsilon(1S)|_{pp} &= 0.41 \pm 0.11\,(\text{stat.}) \pm 0.04\,(\text{syst.})\,, \\
\Upsilon(3S)/\Upsilon(1S)|_{PbPb} &= 0.02 \pm 0.02\,(\text{stat.}) \pm 0.02\,(\text{syst.}) \\
&\quad < 0.07\ (95\%\ \text{confidence level})\,,
\end{aligned} \tag{87}$$

We have seen all those ratios for p-p and Pb-Pb collision which can be employed to compare further by making another quantity which is called double ratio. The double ratio gives a comparison of result in p-p with that in the Pb-Pb collision. Such set of results is given below,

$$\begin{aligned}
\frac{\Upsilon(2S)/\Upsilon(1S)|_{PbPb}}{\Upsilon(2S)/\Upsilon(1S)|_{pp}} &= 0.21 \pm 0.07\,(\text{stat.}) \pm 0.02\,(\text{syst.})\,, \\
\frac{\Upsilon(3S)/\Upsilon(1S)|_{PbPb}}{\Upsilon(3S)/\Upsilon(1S)|_{pp}} &= 0.06 \pm 0.06\,(\text{stat.}) \pm 0.06\,(\text{syst.}) \\
&\quad < 0.17\ (95\%\ \text{CL})\,.
\end{aligned} \tag{88}$$

The nuclear modification factors R_{AA} for bottomonia also have been calculated from those data,

$$\begin{aligned}
R_{AA}(\Upsilon(1S)) &= 0.56 \pm 0.08\,(\text{stat.}) \pm 0.07\,(\text{syst.})\,, \\
R_{AA}(\Upsilon(2S)) &= 0.12 \pm 0.04\,(\text{stat.}) \pm 0.02\,(\text{syst.})\,, \\
R_{AA}(\Upsilon(3S)) &= 0.03 \pm 0.04\,(\text{stat.}) \pm 0.01\,(\text{syst.}) \\
&\quad < 0.10\ (95\%\ \text{CL})\,.
\end{aligned} \tag{89}$$

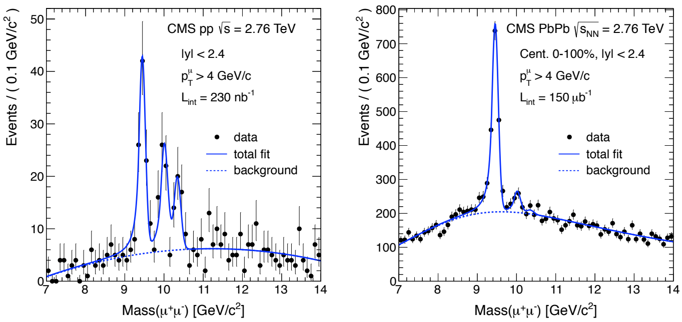


Fig.-23.

Dimuon invariant-mass distributions in PbPb (right) and p-p (left) data at $\sqrt{s_{NN}} =$ *2.76. The solid (signal + background) and dashed (background-only) curves show the results [70] of the simultaneous fit to the two datasets.*

6.3 Discussion on results from sequential suppression vs real time dynamics

We have presented recent results on charmonium and bottomonium measurements in experiments. One thing is noticeable from those results that we have not seen complete suppression of any states. Obviously we have seen that the excited states are being more suppressed than the ground states. In this thesis, we have mainly sketched some of the existing theoretical foundations which have been developed to know the fate of heavy quarkonium states. All those are in agreement on the fact that there will be suppression of quarkonium states in quark gluon plasma but they differ even qualitatively on the nature of suppression. It will be better if we discuss the specific predictions from all those different frameworks.

6.3.1 *Sequential melting picture*

We said earlier that sequential melting picture which is based on the Debye screening mechanism predicts that excited heavy quarkonium states should be melted completely as the energy density is high enough in the medium. The ground state could also be melted if the energy density is high enough to melt them. In this context we should comment that the exact dissociation temperature is still not satisfactorily known. For convenience, if we assume that the medium temperature is sufficiently high to dissociate certain states, the double ratio should be zero for those. But unfortunately we have not seen that yet. The way sequential suppression introduces the QGP thermometer, one should be able to predict the temperature of the medium just by looking at the lowest excited state which has been

melted completely. The experimental results not only show an incomplete suppression of excited states, rather show that the ground states are also partially suppressed. That is confusing in order to refer QGP thermometer to predict the temperature uniquely. The pattern of suppression which still has been realized in the experiment in terms of the number of states is given by
$\Upsilon(3S) < \Upsilon(2S) < \Upsilon(1S)$ or $\psi' < J/\psi$.
The sequential melting picture undoubtedly does not predict this relative suppression of different states.

6.3.2 *From the point of view of complex potential*

In chapter 3, we have presented a very brief discussion on recent developments to describe quark antiquark bound state by a static complex potential. Though the potential has not yet been successfully computed to perform any calculation, still can reflect something at the qualitative level. So far we have seen that the real part of the potential has the Debye screening property and the imaginary part gives account for the decay of different bound states (possibly with different characteristic time). The Debye screening part can determine which bound states can not stay at a specific temperature of the medium. Rest of the bound states will decay with time due to the imaginary part. Thus it predicts that no matter it's ground state or excited one, they decays with time. Therefore apart from the completely suppressed states, one could expect even suppression of ground state in the medium. The complete description then not only depends on the temperature or energy density of the medium but time also plays an important role in this context. Therefore the persistence time of the medium should also be taken care of. This is the distinct feature in it's prediction from the usual Debye screening picture discussed in the context of sequential melting. The suppression of all different states which we see in the experimental results seems accessible in this picture except the incomplete suppression of the excited states which was supposed to be suppressed completely in the medium.

Both of the above pictures do not predict that ground states can repopulate the excited one and vice versa. The latter has no idea about the future of those states which has a finite life time due to the imaginary part of the potential.

6.3.3 *Reshuffling picture*

The reshuffling picture does not describe the fate of heavy quarkonia using effective potential. It shows a dynamical evolution of quarkonium states in medium. We have described in this thesis that it advo-

cates the possibility of incomplete suppression of excited states as well as indicates possible suppression of ground states. According to this picture, we will have a distribution of all possible states in the medium. The population of different states depends on the time they have spent in the medium as well as on the distribution of gluons which interact with heavy quarkonia. Obviously, one needs to know how many different possible states have been created initially after the collision which always can be normalized with same in proton-proton collision. The prediction of this new picture also differs from the description based on complex potential as the former predict the recreation of the excited states. One might think that this picture may give rise to a prediction where excited states are even less suppressed than the ground states. Of course, at the qualitative level we can not throw out the possibility but quantitative description is needed to say anything firmly on that.
A chart on these different predictions from different pictures are presented at the end of this chapter.

6.3.4 *A surprising preliminary in CMS*

In Quark- Matter 2012, some preliminaries were presented[2] on the measurement on charmonium states which shows that excited state ψ' is observed to be less suppressed than J/ψ. The statistics of this outcome is still not very high and therefore we need to wait until they come up with a better statistics. In any case this result is only accessible with the approach through reshuffling picture.

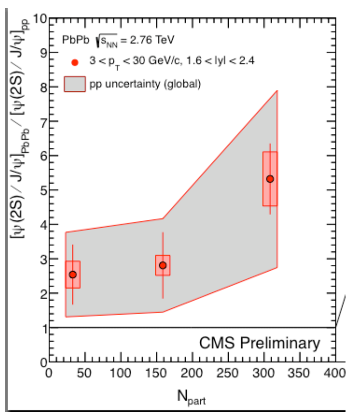
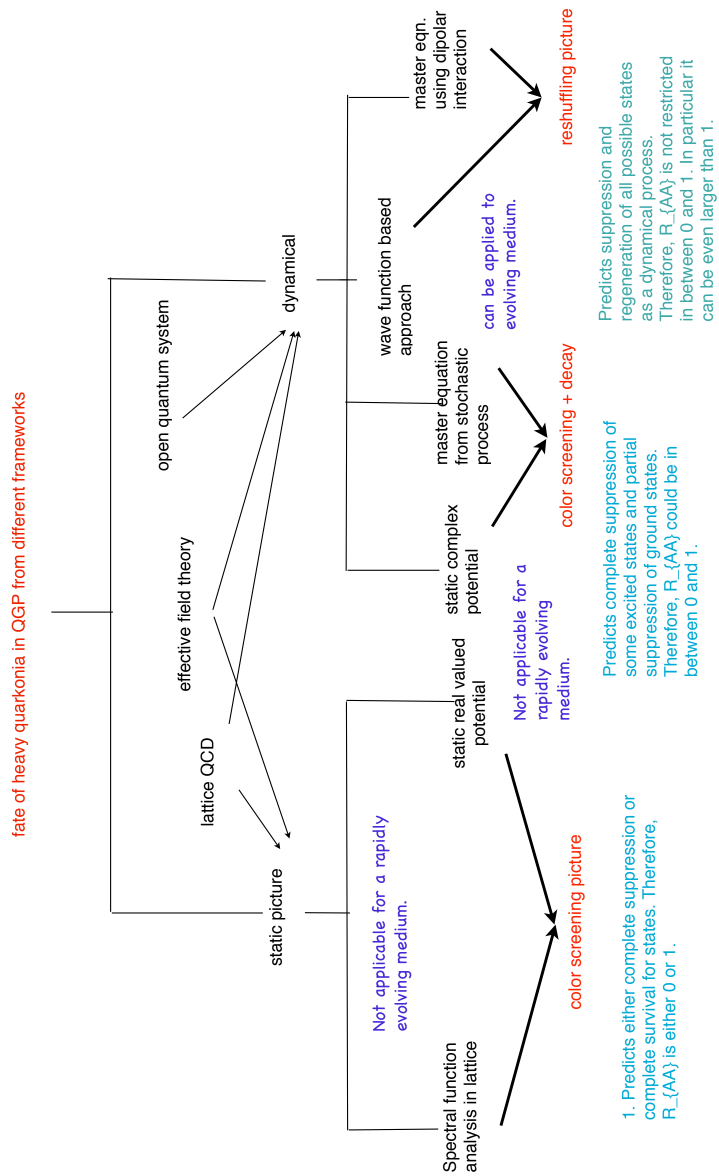


Fig-24 *This plot was presented in quark matter 2012 by CMS collaboration.*

The analysis is now available in Cern's document server (CMS-PAS-HIN-12-007).

2 See the talk by Dong Ho Moon in quark matter 2012.

6.4 A CHART ON THE FATE OF HEAVY QUARKONIA FROM DIFFERENT FRAMEWORKS



7 CONCLUSION

The sole interest of this thesis is to argue why the real time dynamics of heavy quarkoni in medium is un avoidable. Heavy quarkonia are established testing ground for the formation of Quark - Gluon plasma. Statistical QCD has already predicted the melting of quarkonium bound stae in dense quark matter. In thermodynamic picture a quarkonium bound state is either completely dissociated above some threshold temperature or just stays in the specific state below the threshold. The possible transition to other bound states are completely untreated within those frameworks. Apart from that, some recent studies [34, 36, 16] show that a static quark-antiquark potential can be useful to describe the bound state, provided an imaginary part is considered in the potential. This shows a finite life time of the bound state in the medium and initiates the quest for a dynamical picture of the dissociation as well. The question of intermediate bound states becomes relevant within this framework as to show the possible transition to other bound states before the quarkonia melts completely. The dynamical picture becomes more relevant when the medium is expanding and cooling down rapidly, as in the case of heavy ion collision.s

In our recent study [53], we have shown that the sequential suppression pattern of heavy quarkonium states is questionable as the evolution of quarkonium states is not adiabatic in the evolving fireball produced in heavy ion collision. We have clarified how the thermodynamical approaches (describing through an effective potential or approaching through spectral function analysis) relies on the assumption of adiabaticity. The degree of non-adiabaticity has been calculated quantitatively for different quarkonium states. The violation of adiabatic approximation indicates a new picture of quarkonium evolution apart from the static one. According to this picture, there is all the time reshuffling between the bound states. One has to take care of all such transition between different quarkonium states.

The need for a real time dynamics of heavy quarkonia in deconfined medium initiates a search for a picture which is different from the color screening in thermal plasma. A satisfactory justification to formulate the problem in a language of open quantum dissipative system reveals the scope to study the dynamics of quarkonia (with internal degrees of freedom) using the systematics of dissipative dynamics [50, 52, 51]. The problem has been already studied in master equation approach [50] with some limitations. Other approaches with reduced density matrix in path integral face severe problems to draw any conclusion about the dynamics related to the internal degrees of freedom. They are restricted to simple systems and with their macroscopic and semi-classical descriptions. Here we feel the necessity of alternative approaches to deal with quantum dissipative systems to access their microscopic properties. We already have built up a new systematic wave function based approach to quantum dissipative system which will be applicable to shed light on the dynamics of heavy quarkonium states in rapidly evolving fireball produced in heavy ion collision.

7.1 ABOUT THE FRAMEWORK

The new framework for dissipative dynamics is based on wave function of the test system which is interacting with a medium. As it deals with the wave function, the method is less expensive (because we only need to keep track of N number of information instead N^2) compared to other density matrix based approaches like master equation approach or those with the Feynman-Vernon theory of influence functional. To address the real time evolution of different bound state populations, we do not need those off diagonal terms in the density matrix. Furthermore, the evolution equation for the wave function is in practice simpler than the corresponding von Neumann equation for the dynamics of the density matrix. This makes it easier to try and reformulate our findings in terms of an evolution governed by the Schroedinger equation with an effective potential, thereby making contact with other approaches to model quarkonia in a deconfined medium. This general framework has many advantages like,

1. It is applicable to the real time dynamics of open quantum dissipative systems.

2. It is valid for the thermal and non thermal bath, therefore allows us to treat the medium in non equilibrium.

3. It is applicable to arbitrary (physically relevant) small time scale.

4. It can treat the internal degrees of freedom of the open system at the microscopic level which can be employed to achieve macroscopic descriptions as well.

7.2 EXPLORATORY MODEL

With the framework we started to investigate the evolution of harmonic oscillator bound states in a medium as a precursor study for

heavy quarkonia in QGP. We obtained many expected and convincing results. In this picture the harmonic oscillator interacts with a medium of oscillators which can be considered as a good exploratory model to study a color dipole in gluon plasma. The transition between different bound states is triggered by the gluons. In some of the studies we populated the oscillator with it's ground states and investigated the dynamics of the population in the medium. Not surprisingly, we got the exponential decay of the population with time and at the same time the population of other bound states evolves together. This is at first sight somehow in contradiction with the sequential melting picture derived from statistical QCD. The sequential melting breaks down because of the consideration of real time dynamics which could be a key to understand some recent results [69, 70] in heavy ion collision experiments.
We also have noticed that the effect of all gluons are not really important to induce the transitions to different states[65]. The gluons with comparable energies are mostly responsible for the transition to different states. That is obvious because the high energetic gluons will see the color dipole as two different objects where as the low energetic gluons see them as color neutral.
This behaviour we expect also for heavy quarkonia in deconfined medium. It is expected because the one dimensional harmonic oscillator can be generalized[1] for three-dimensional Coulomb potential which is applicable to heavy quarkonia above T_c and even below T_c for bottomonia.

In the harmonic oscillator model there are no unbound states for the system and we only can expect the transition between states and therefore the dissociation is not well modelled. Kustaanheimo-Stieffel transformation [66] allows us to introduce scattering states for short-range Coulomb part of the quark-antiquark potential. With that we can also investigate the dissociation of heavy quarkonia in deconfined medium along with possibilities of transition to other bound states.

7.3 FOR HEAVY QUARKONIA

From exploratory to the realistic heavy quarkonia is just one step generalisation which we are investigating at the moment. We should mention that the quarkonia suppression is still a convincing probe for the formation of QGP in heavy ion collision but with some modifications which come from the real time evolution. Recent experimental results [70] for the population of bottomonium states obtained by the CMS collaboration at the LHC show the ratio of the yield of excited

1 The propagator in coulomb potential is equivalent to that of a 4-D constraint oscillator

states to that of the ground state is neither zero nor equals to the value in proton proton collision. We see a possible explanation of this through the real time dynamics of quarkonia in medium using our framework which takes care the survival of quarkonia in a dynamical way rather than to treat it in a static melting picture. Here I should mention that the approach does not employ any in medium effective potential for quarkonia rather makes use of the gluonic interaction to study the dynamics of quarkonium (state) population. Furthermore the framework might be utilized to shed light on the study of effective potential in a static QGP:
In our work we report that there are justifications to consider the dynamics of quarkonia as that of an open quantum dissipative systems. The dynamics has radical impact to the observables (in the Heavy Ion Collision experiments) which comes along with a very careful microscopic description that scans the problem with more resolution.

7.4 OUTREACH: IMPLEMENTATION THROUGH DYNAMICAL SIMULATION

To allow phenomenological predictons for heavy-ion collision, the formalism needs to be embedded in a dynamical simulation of the evolving medium, which specifies (at each time and position) the density of gluons likely to induce transitions in the quarkonia. Such simulation can either be in transport model or hydrodynamical descriptions.
This implementation is the next necessary step for realistic modelling of quarkonium evolution in heavy-ion collisions, and will be my priority once my present investigations are brought to conclusion.

A

APPENDIX

A.1 PATH INTEGRAL IN AUXILIARY TIME

The general definition of the propagator for a system with a potential $V(x)$ is given by

$$\mathcal{K}(x_f, t_f; x_i, t_i) = \int\int \mathcal{D}x\mathcal{D}p \frac{1}{(2\pi)^3} e^{i\int dt(p\dot{x} - \frac{p^2}{2m} - V(x))}. \tag{90}$$

Using a parametrization of t by an auxiliary time s, we can generalize the above path integral expression. The auxiliary time is defined as s dependent functional

$$t = t^s[x]. \tag{91}$$

Later by choosing a special type of parametrization, we can re-express the propagator in equation (90).

$$\begin{aligned} \frac{dt}{ds} = t'(s) = f\big(x(s)\big); \;\; t_f - t_i = \int_{s_i}^{s_f} dsf\big(x(s)\big) \\ t(s_i) = t_i, t(s_f) = t_f \end{aligned} \tag{92}$$

Obviously, this parametrization will modify the Hamiltonian of the system. The new path integral expression appears

$$\mathcal{K}(x_f, t_f; x_i, t_i) = \int\int \mathcal{D}x(s)\mathcal{D}p(s) \frac{1}{(2\pi)^3} e^{i\int_{si}^{s_f} ds\left(p(s)x'(s) - f\left(x(s)\right)\mathcal{H}\right)}. \tag{93}$$

The auxiliary time puts a constraint which can be incorporated into the expression as

$$\begin{aligned} \mathcal{K}(x_f, t_f; x_i, t_i) = f(x_f) \int_{s_i}^{\infty} ds_f \delta\Bigg(t_f - t_i - \int_{s_i}^{s_f} dsf\big(x(s)\big)\Bigg) \times \\ \int\int \mathcal{D}x(s)\mathcal{D}p(s) \frac{1}{(2\pi)^3} e^{i\int_{si}^{s_f} ds\left(p(s)x'(s) - f\left(x(s)\right)\mathcal{H}\right)}. \end{aligned} \tag{94}$$

Now, we can replace the delta function by introducing a Fourier transformation in the following way,

$$\mathcal{K}(x_f, t_f; x_i, t_i) = f(x_f) \int_{-\infty}^{\infty} dE \frac{1}{\pi} e^{-E(t_f - t_i)} \int_{s_i}^{\infty} ds_f K^E(x_f, s_f; x_i, s_i). \tag{95}$$

Where $K^E(x_f, s_f; x_i, s_i)$ is the propagator in auxiliary time for an E dependent Hamiltonian

$$\mathcal{H}^E = f(x)\big(H(p,x) - E\big)$$
$$K^E(x_f, s_f; x_i, s_i) = \int\int \mathcal{D}x(s)\mathcal{D}p(s)\frac{1}{(2\pi)^3}\, e^{i\int_{si}^{s_f} ds\left(px' - H^E\right)}. \quad (96)$$

This is the final expression for the propagator in auxiliary time quantum description. The corresponding Schrödinger equation one can derive

$$f(x)(H - i\frac{\partial}{\partial t})\psi(x, t; s) = i\frac{\partial}{\partial s}\psi(x; t, s) \quad (97)$$

A.1.1 *Kustaanheimo - Stiefel transformation for 2-D coulomb system*

Suppose, we have a two dimensional coulomb system. The Hamiltonian is given by

$$H = \frac{|p|^2}{2m} - \frac{c}{r};$$
$$|p|^2 = p_1^2 + p_2^2; \;\; r^2 = x_1^2 + x_2^2. \quad (98)$$

p_1 and p_2 are conjugate momenta corresponding to x_1 and x_2. Now, we will introduce a canonical transformation for x_1 and x_2 to u_1 and u_2, known as K-S transformation given below.

$$x_1 = u_1^2 - u_2^2, \quad p_1 = \frac{1}{u^2}(u_1 p_{u_1} + u_2 p_{u_2})$$
$$x_2 = 2u_1 u_2, \quad p_2 = \frac{1}{u^2}(-u_2 p_{u_1} + u_1 p_{u_2}), \quad (99)$$

where p_{u_1} and p_{u_2} are the conjugate momenta corresponding to the transformed coordinates u_1 and u_2 respectively. One can easily check that with this canonical transformation, the Hamiltonian of the coulomb system becomes

$$H = \frac{1}{u^2}(\frac{|p_u|^2}{8m} - c). \quad (100)$$

With the specific choice of $f(x) = |x| = u^2$, the auxiliary Hamiltonian $\mathcal{H}^E$ becomes

$$\mathcal{H}^E = (\frac{|p_u|^2}{8m} - Eu^2 - c). \quad (101)$$

The path integral measure also changes accordingly as

$$d\vec{x} = 2\begin{pmatrix} u_1 & u_2 \\ -u_2 & u_1 \end{pmatrix} d\vec{u} \quad d\vec{p}_x = \frac{1}{2|u|^2}\begin{pmatrix} u_1 & u_2 \\ -u_2 & u_1 \end{pmatrix} d\vec{p}_u\ . \quad (102)$$

$d\vec{x}$, $d\vec{u}$ and $d\vec{p}_x$, $d\vec{p}_u$ present two component position and momentum vectors respectively. The propagator in auxiliary time then looks

$$K^E(u_f, s_f; u_i, s_i) = \frac{1}{4} e^{ic(s_f - s_i)} \int\int \mathcal{D}u(s)\mathcal{D}p(s) \frac{1}{(2\pi)^2} e^{i\int_{s_i}^{s_f} ds \left(p_u u' - \left(\frac{|p_u|^2}{\mu} - Eu^2\right)\right)}. \tag{103}$$

What we see finally is a propagator of harmonic oscillator with mass $\mu = 4m$ and frequency $\sqrt{\frac{-2E}{\mu}}$ when it has been evaluated for auxiliary time . One can easily further return to the usual time for the dynamics of the problem with K^E which should be plugged into (95) after making the reverse transformation to x from u. The integral over s_f is equivalent to that over $(s_f - s_i)$ by setting up the initial reference of auxiliary time s. One more thing should be mentioned in this context about the utility of 2D coulomb system. When one intends to deal with a realistic coulomb system, one has to consider the three dimensional coulomb case. The similar technique will help then also. For the sake of simplicity, we have designed the method for the 2D case first. A generalization for 3D is shown in the next section.

A.2 GENERALIZATION FOR 3D COULOMB SYSTEM

For three dimensional coulomb case, we will again introduce a transformation along with same definition of auxiliary time. The transformation is given by

$$\vec{x} = A\vec{u}; \qquad A = 2\begin{pmatrix} u_3 & u_4 & u_1 & u_2 \\ -u_2 & -u_1 & u_4 & u_3 \\ -u_1 & u_2 & u_3 & -u_4 \\ u_4 & -u_3 & u_2 & -u_1 \end{pmatrix}.$$

The transformation of momentum also given by

$$p = \frac{1}{4r} A p_u. \tag{104}$$

The auxiliary time is defined $dt = u^2 ds$, in the same way as done in the last section. We see that we had to introduce another extra coordinate x_4 to preserve the symmetry. x_4 is defined as

$$x_4 = 2\int dt(u_4\dot{u}_1 - u_3\dot{u}_2 + u_2\dot{u}_3 - u_1\dot{u}_4). \tag{105}$$

One can easily check that the integral measure in this case also be preserved and the propagator will have a very similar expression as last section except a constraint posed by the 4th coordinate x_4.

$$d^4x = \det A d^4u; \quad d^4p = \det A d^4p_u$$
$$d^4xd^4p = d^4ud^4p_u; \quad \det A = 16r^2$$
$$r = \sqrt{\sum_1^3 x_i^2} = \sum_1^4 u_i^2 = u^2. \tag{106}$$

We already have given a sketch to make use of the Kustaanheimo-Stiefel transformation(the canonical transformation)for the generalization of the coulomb propagator in 3D. We do not need to go further in detail of it as there are lots of references dealing with this issue. Let me just write the final expression of the propagator

$$\mathcal{K}(u_f, t_f; u_i, t_i) = \frac{1}{16r_f^2}\int_{-\infty}^{\infty} dE \frac{1}{2\pi} e^{-E(t_f-t_i)} \int_{s_i}^{\infty} ds_f K^E(u_f, s_f; u_i, s_i). \tag{107}$$

Where

$$K^E(u_f, s_f; u_i, s_i) = \int_{-\infty}^{\infty} d(x_4)_f \int_{s_i}^{\infty} ds_f r_f e^{ic(s_f - s_I)}$$
$$\int\int \mathcal{D}u(s)\mathcal{D}p(s)\frac{1}{(2\pi)^4}\, e^{i\int_{s_i}^{s_f} ds\left(p_u u' - (\frac{|p_u|^2}{\mu} - Eu^2)\right)}. \tag{108}$$

Again the inverse transformation will lead to the propagator in x space from u.

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