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Stability of Coalitional Equilibria within Repeated Tax Competition*

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Abstract

This paper analyzes the stability of capital tax harmonization agreements in a stylized model where countries have formed coalitions which set a common tax rate in order to avoid the inefficient fully non-cooperative Nash equilibrium. In particular, for a given coalition structure we study to what extend the stability of tax agreements is affected by the coalitions that have formed. In our set-up, countries are symmetric, but coalitions can be of arbitrary size. We analyze stability by means of a repeated game setting employing simple trigger strategies and we allow a sub-coalition to deviate from the coalitional equilibrium. For a given form of punishment we are able to rank the stability of different coalition structures as long as the size of the largest coalition does not change. Our main results are: (1) singleton regions have the largest incentives to deviate, (2) the stability of cooperation depends on the degree of cooperative behavior ex-ante.

JEL Classification: C71, C72, H71, H77

Keywords: capital tax competition · tax coordination · coalitional equilibria · repeated game

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1 Introduction

This paper studies the stability of capital tax harmonization agreements in a model where countries have formed coalitions to avoid the inefficient fully non-cooperative Nash equilibrium. As incentives for deviations from the cooperative behavior continue to exist, we analyze the stability of any given but arbitrary coalition structure by means of a repeated game setting accounting for deviations by a whole subgroup of countries.

Capital tax competition has been the subject of increasing political and academic interest since the mid-1980s. Next to Wilson (1999) and Wilson and Wildasin (2004) recent surveys of the literature are given by, e.g., Griffith et al. (2008) and Keen and Konrad (2011). It is well established that the structure of payoffs in a standard tax competition model resembles a classical "prisoner's dilemma". In such a static, one-shot model the non-cooperative Nash equilibrium of tax rates is inefficiently low compared to harmonized tax rates. Therefore, a coordination of tax policies can avoid the negative externality that is associated with mobile capital tax bases. For example, the contributions by Zodrow and Mieszkowski (1986), Wildasin (1989), Bucovetsky (1991) and Wilson (1991) analyze if there are Pareto-improving reforms which harmonize capital income taxes.

Given the high costs of tax competition, global tax harmonization is desirable but very unlikely because some countries, e.g., tax havens, prefer lower taxes for commercial reasons.[1] From a political perspective, partial harmonization among a subgroup of countries is therefore easier to achieve (cf. Konrad and Schjelderup, 1999). This is what has been promoted by a variety of policy efforts from several countries, economic unions and international institutions. A very recent example is the announcement of the Council of the European Union to reinforce fiscal stability as a response to the financial crisis by the coordination of a common band of fiscal policy measures, for instance, by the introduction of a common corporate tax base (Council of the European Union, 2011). Other examples include the efforts by the OECD's Center for Tax Policy and Administration, for instance, the list of harmful tax practices. In fact, even if no explicit agreements on the political agenda have been made, there may well be implicit agreements between countries or federations that are linked via policies or institutional arrangements in other fields in order to keep tax competition low (cf. Konrad and Schjelderup, 1999).

[1]Other, well-known factors that add to the reluctance of countries concerning tax harmonization efforts are asymmetries in, e.g., endowments or technologies.

In this paper we abstract from the question how these cooperative agreements have been made, although this is surely a related topic. Rather, we focus on the stability of cooperation taking into account the particular incentives that fiscal spillovers and cooperation among subgroups of countries induce in the long run. In the long run, i.e., if the tax game is played repeatedly, there are strong incentives to raise the tax rates above the inefficient fully non-cooperative Nash equilibrium because deviations from cooperation will be punished. This is what a number of recent studies, e.g., Cardarelli et al. (2002), Catenaro and Vidal (2006) as well as Itaya et al. (2008) have analyzed by applying repeated interactions to the capital tax competition framework. This strand of the literature focuses on the question whether fiscal coordination is sustainable among two asymmetric countries employing grim trigger strategies for the punishment phase of the game.[2] However, these papers deal with the sustainability of overall (global) tax coordination. We analyze the sustainability of tax coordination when there are several tax agreements co-existing, e.g., when there are larger and smaller groups of countries that cooperate.

Konrad and Schjelderup (1999) argue that gains from tax harmonization depend on the response from countries outside the harmonized area and on the size of the tax harmonized area relative to the global economy. Accordingly, they study whether a single subgroup of countries can gain from harmonizing their capital income taxes provided that all other countries do not follow suit by playing non-cooperatively. They show that tax harmonization is Pareto improving for all countries if the tax rates are strategic complements. Itaya et al. (2010) analyze the sustainability of this form of partial tax coordination (within a single subgroup of countries) in a repeated game setting. Also here, all other countries not in the coalition behave non-cooperatively but symmetrically and only singleton regions are allowed to deviate from the cooperative behavior. The main finding of Itaya et al. (2010) is that partial tax coordination is more likely to prevail if the number of regions in the coalition subgroup is smaller and the number of existing regions in the entire economy is larger.[3]

This paper investigates the more general case relaxing two constraints of

[2] Kiss (2011) adds to this literature by analyzing how the introduction of a minimum tax affects the stability of cooperation among N symmetric countries. See also Kessing et al. (2006) who analyze the effect of vertical tax competition on FDI. Here, repeated interaction enables governments and firms to solve the hold-up problem.

[3] Note that, using a numerical analysis with imperfect capital mobility, Rasmussen (2001) finds that the critical mass of countries needed for partial coordination to have a significant impact is a large number of the overall number of economies.

Itaya et al. (2010): First, we allow for any given coalition structure (not only one single coalition). Second, we analyze coalitional deviations in the repeated game, i.e., we analyze the incentives of sub-coalitions to deviate from cooperation.

Let us elaborate on two related papers of the literature strand that analyzes the process or the impact of coalition formation or tax harmonization, respectively.[4]

The process of coalition formation is analyzed by Burbidge et al. (1997). In this fairly general model, different regions may form coalitions to capture efficiency gains by tax rate harmonization. Joining a coalition implies first, choosing a harmonized tax rate such that the coalition's payoff is maximized, and second, committing to a fixed division scheme for the gains from cooperation. Burbidge et al. (1997) study equilibrium coalition structures based on the model of coalition formation from Hart and Kurz (1983) using the concept of a coalition-proof Nash equilibrium (cf. Bernheim et al., 1987). Their main finding is that the grand coalition is not necessarily the equilibrium coalition structure in a setting with more than two regions. This is illustrated by an example with three regions having asymmetric production functions.

Bucovetsky (2009) considers a model of tax competition among regions of different population size. The regions' objective is to maximize the utility of its inhabitants, which depends on the consumption of a private good and the provision of a public good. Bucovetsky (2009) proves that any tax harmonization by a group of jurisdictions benefits the residents of all jurisdictions that are not in the group. He also demonstrates that harmonization increases the average payoff of all regions harmonizing their tax rate. Most remarkably, Bucovetsky (2009) finds that the "biggest threat to the grand coalition" (p. 740) is the coalition structure where $N-1$ regions cooperate and the smallest region remains singleton. Bucovetsky's (2009) work is based on an earlier paper which is quite related to our framework (see Bucovetsky, 2005). For instance, we share the Leviathan type of government[5] and have a similar production function in the one-period game.

In our model, we want to handle explicit solutions so we need to impose specific assumptions: We postulate that (1) the aggregate supply of capital is fixed; (2) each jurisdiction is inhabited by economically identical residents;

[4]Konrad and Schjelderup (1999) offer a brief discussion about the link between tax harmonization and the literature on the profitability of mergers in industrial organization (cf. the references given therein, in particular, Deneckere and Davidson, 1985).

[5]Also Kanbur and Keen (1993) use this kind of objective function.

(3) output in each region is a quadratic function of capital employed. The point of departure is that each region chooses a tax rate to levy on locally employed capital to manipulate its tax base in form of capital movements. Consequently, regions have an incentive to capture the benefits of policy coordination. We allow for any coalition structure to form and derive the equilibrium tax rates and equilibrium tax revenues for a given coalition structure in a first step. In a second step we employ a repeated game setting in order to analyze the stability of cooperation in terms of the related discount factors.
To preview our main finding: We establish that singleton regions have the highest incentive to deviate from the cooperative solution. Furthermore, cooperation is easier to sustain if the environment was acting "more cooperatively" ex-ante.

This paper is organized as follows. In section 2 we set up the basic tax competition model. In section 3, we introduce cooperation into the tax competition model and derive the equilibrium tax rates and equilibrium tax revenues for different coalition structures. In section 4, we introduce the repeated game setting and study the dynamics, in particular, the stability of coalitional equilibria in the tax competition game. Section 4.5 comments on an extension concerning the region's objective, while section 5 illustrates our results by means of a numerical example. We conclude in section 6.

2 The Tax Competition Model

We employ a standard tax competition framework with N identical regions, indexed by $i \in \mathbf{N} = \{1, ..., N\}$. Each region is characterized by a regional government, a representative household and a single firm. The household (labor) is supposed to be immobile, whereas capital is perfectly mobile. Both capital and labor are input factors for the production of a single homogeneous good. The overall capital stock is given by $\overline{K}$ which is equally distributed in the regions. Hence, each region owns $\overline{k} = \overline{K}/N$ units of capital. The production is described by a constant-returns-to-scale type of production function following, e.g., Bucovetsky (1991), Bucovetsky (2009), Grazzini and van Ypersele (2003), Haufler (1997) or Devereux et al. (2008). The production function of region $i \in \mathbf{N}$ is $f(k_i) = (A - k_i)k_i$, where $A > 0$ is the level of productivity, and k_i the per capita amount of capital employed in region i. We assume $A > 2k_i$ for all possible $k_i \leq \bar{K}$. This means that the level of productivity A needs to be sufficiently large such that the equilibrium interest rate is positive.[6] Public goods are financed by a source-based unit tax on capital τ_i for region i.[7] As firms behave perfectly competitively the production factor prices equal their respective marginal productivity

$$r = f'(k_i) - \tau_i = A - 2k_i - \tau_i \tag{1}$$
$$w_i = f(k_i) - k_i f'(k_i) = k_i^2 \tag{2}$$

where r is the net return on capital and w_i is the region-specific wage rate. The no-arbitrage condition in equilibrium for capital is $f'(k_i) - \tau_i = r = f'(k_j) - \tau_j$ for all regions i, j where $i \neq j$. The demand function for capital, depending on the arbitrage-free interest rate r and the regional tax rate τ_i, is then given by $k_i = \frac{A-r-\tau_i}{2}$. To determine the equilibrium interest rate, capital demand need to equal capital supply, $\sum_{i=1}^{N} k_i = N\overline{k}$. Let $\tau = (\tau_1, ..., \tau_N)$ be the vector of tax rates chosen by the regions. We obtain the equilibrium interest rate $r^*(\tau)$ by

$$r^*(\tau) = A - 2\overline{k} - \overline{\tau} \tag{3}$$

where $\overline{\tau} = \frac{\sum_{h=1}^{N} \tau_h}{N}$ is the average capital tax of all regions. Combining (1) and (3) yields the capital demand in equilibrium for region i:

$$k_i^*(\tau) = \overline{k} + \frac{\overline{\tau} - \tau_i}{2} \tag{4}$$

The effects of a changing tax rate on equilibrium capital demand and the equilibrium interest rate are as follows:

$$\frac{\partial r^*(\tau)}{\partial \tau_i} = -\frac{1}{N} < 0 \tag{5}$$
$$\frac{\partial k_i^*(\tau)}{\partial \tau_i} = -\frac{N-1}{2N} = \frac{1}{2N} - \frac{1}{2} < 0 \tag{6}$$
$$\frac{\partial k_j^*(\tau)}{\partial \tau_i} = \frac{1}{2N} > 0 \tag{7}$$

[6] The given level of productivity needs to be sufficient large to ensure capital levels to be strictly smaller than the capital level at which the production function has its maximum.

[7] Lockwood (2004) has shown that in the (standard) tax competition model by Zodrow and Mieszkowski (1986) there are different Nash equilibria in capital taxes depending on the structure of taxes, i.e., ad-valorem or unit taxes. For the sake of readability of our results we employ the unit tax.

for all regions $i, j \in \mathbf{N}$ and $i \neq j$. When a region i augments its own tax rate τ_i, the equilibrium interest rate $r^*(\tau)$ and the capital demand $k_i^*(\tau)$ of this country decreases. However, if another country j increases its tax rate, this has a positive influence on the equilibrium capital demand $k_i^*(\tau)$ of country i. Note that we have the following effect

$$\frac{\partial k_i^*(\tau)}{\partial \tau_i} = -\sum_{j \neq i} \frac{\partial k_j^*(\tau)}{\partial \tau_i} = -\frac{N-1}{2N}.$$

The objective of the regional government is to maximize its tax revenue given by

$$\tau_i k_i^*(\tau). \tag{8}$$

Tax revenues are entirely used to finance public goods. Alternatively, tax revenues could be directly transferred to the representative household. In either case—in contrast to Edwards and Keen (1996)—the Leviathan type of government here does not produce a "waste of resources". A change of the tax rate affects the tax revenue in two respects: First, there is the direct effect of the change in the tax rate itself and second, there is the indirect effect because the equilibrium capital demand responds.

With every region pursuing to maximize its own tax revenue, potential gains of cooperation are ignored. In the next section, we extend the model such that cooperation between the regional governments is allowed for. The standard model will be a special (benchmark) case of this more general setting, namely where regions act as a singleton.

3 Cooperative Behavior

Now, we modify the tax competition framework allowing regions to build any form of coalition structure. For such a given coalition structure, we determine the tax rate, the capital demand and the tax revenues in equilibrium.[8] Before, we have a few words on the concept of a coalition structure and the notion of coalitional equilibrium.

A coalition structure is a partition of the set of players, more precisely a set of coalitions $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ such that their pairwise intersection is empty, $\mathbf{S_m} \cap$

[8] Here, we adopt the same view as Konrad and Schjelderup (1999) who justify the omission of the analysis of the coalition formation process as follows: "the formation of a given coalition may .. be founded on historical, social, political, and economic factors outside the model." (p.160)

$\mathbf{S}_\ell = \emptyset$ for all $m \neq \ell$, and such that their union equals the grand coalition, $\bigcup_{m=1}^{M} \mathbf{S_m} = \mathbf{N}$. For instance, for three regions we have five possible coalition structures, whereas for five regions we already have 52 possible coalitions structures.[9]

As regions are symmetric, the different coalition structures depend on the number of regions in one coalition and on the overall number of coalitions. Thus, if we consider a specific coalition structure, it is enough to know how many regions there are in which coalition. Therefore, our succeeding analysis depends on the sizes of the coalitions. We can associate a coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ to a vector indicating the sizes of the coalitions in the following way: Coalition $\mathbf{S_1}$ consists of regions $1, ..., S_1$, coalition $\mathbf{S_2}$ of regions $S_1 + 1, ..., S_1 + S_2$ and so on. We (usually) denote in non-bold the size of coalition, S_m, and in bold coalition, $\mathbf{S_m}$, containing S_m regions.

The equilibrium concept

The ability of regions to form coalitions implies that we assume regions to behave cooperatively and symmetrically within a coalition but non-cooperatively across coalitions. Our analysis is based on the notion of a "coalitional equilibrium".[10] In our setting, we assume that by forming a coalition the members of this coalition behave symmetrically and agree to set a common tax rate maximizing the coalitional tax revenue.

Definition (symmetric coalitional equilibrium)**.** Given a coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ an action profile of tax rates $(\tau_{\mathbf{S_1}}, ..., \tau_{\mathbf{S_M}})$ is a *symmetric coalitional equilibrium* if for no coalition $\mathbf{S_m}$ in the coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ there is a choice of a common tax rate $\tilde{\tau}_{\mathbf{S_m}}$, symmetric within coalition $\mathbf{S_m}$, that strictly increases the individual tax revenues of all members of the coalition $\mathbf{S_m}$.

Consequently, here, a symmetric coalitional equilibrium is a Nash equilibrium of the game where the different coalitions are interpreted as individual players (maybe differing in a size factor) maximizing joint revenue of the coalition's members. We assume that a coalition sets the tax rate and each

[9] To determine how many coalition structures for a given number of players, N, exist is a combinatorial question. The number of ways a set of N elements can be partitioned into non-empty subsets is the "Bell number". The Bell numbers can be recursively determined by $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_n$ where $B_0 = B_1 = 1$. The first few Bell numbers for $n = 1, 2, 3, 4, 5, 6, 7, 8, ...$ are $1, 2, 5, 15, 52, 203, 877, 4140, ...$.

[10] The formal definition of this idea can be found in Ichiishi (1981), Zhao (1992), Ray and Vohra (1997) or later on Ray (2007). A recent, different application of the coalitional equilibrium can be found in, e.g., Biran and Forges (2011).

region gets an equal share of the tax revenues. This is a reasonable assumption as all regions are symmetric: By agreeing on a common tax rate within the coalition there are no differences in the allocation of capital between the regions in this coalition.

Coalition structures with at least two coalitions

Having defined the equilibrium concept, we analyze a given coalition structure, denoted by $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$, which consists of at least two coalitions, $M \geq 2$. This includes—as a special case—the fully non-cooperative behavior where the number of coalitions is $M = N$. This excludes, however, the grand coalition $\{\mathbf{N}\}$ which is the efficient outcome from an economic perspective for a tax revenue maximization objective. For the grand coalition there are no external effects in terms of capital movements and all available production is absorbed as tax revenues. From a political perspective, however, this scenario is a minor interesting case since an overall (worldwide) harmonization of tax rates is unrealistic.[11]

The regional governments of each coalition maximize the sum of the members' regional tax revenues by choosing a common tax rate within the coalition:

$$\begin{aligned}\sum_{h \in \mathbf{S_m}} \tau_h k_h^*(\tau) &= \sum_{h \in \mathbf{S_m}} \tau_{\mathbf{S_m}} \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right) \\ &= S_m \tau_{\mathbf{S_m}} \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right)\end{aligned}$$

For given tax rates of the other coalitions the first order condition for coalition $\mathbf{S_m}$ is

$$S_m \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right) + S_m \tau_{\mathbf{S_m}} \left(\frac{S_m}{2N} - \frac{1}{2} \right) = 0. \tag{9}$$

The best response function for coalition $\mathbf{S_m}$ reads:

$$\tau_{\mathbf{S_m}} = \frac{N}{N - S_m} \overline{k} + \frac{1}{2} \sum_{\ell \neq m} \frac{S_\ell \tau_{\mathbf{S_\ell}}}{N - S_m} \tag{10}$$

[11] There is an additional technical restriction as the joint tax revenue of all regions have no inner solution for the grand coalition. If all regions cooperate, they will choose a boundary solution for the tax rate so that there is no capital movement across the regions. Thus, the tax rate of the grand coalition is equal to $A - \overline{k}$.

Appendix A.1 shows the computations. Note that the tax rate and with that the capital demand and the tax revenue depend on the given coalition structure. For the ease of notation we omit this dependence in the notation for this section.

The existence of a Nash equilibrium is guaranteed due to the linearity of the best response functions and the fact that their slope is strictly smaller than one:[12]

$$\frac{\partial \tau_{\mathbf{S_m}}}{\partial \tau_{\mathbf{S}_\ell}} = \frac{1}{2}\frac{S_\ell}{N - S_m} < 1.$$

For the ease of notation define

$$\alpha := \sum_{\ell=1}^{M} \frac{S_\ell}{2N - S_\ell} \in \left(\frac{1}{2}, 1\right). \tag{11}$$

We can associate a specific α to every coalition structure depending on the sizes of the coalitions. In Lemma 1, later on, we analyze this factor in more detail. Before, we determine the optimal tax rates:

$$\tau_{\mathbf{S_m}} = \frac{2N\overline{k}}{2N - S_m} + \frac{2N\overline{k}}{(2N - S_m)}\frac{\alpha}{(1-\alpha)} = 2N\overline{k}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{2N - S_m}\right) \tag{12}$$

The computation can be found in Appendix A.2.
The average tax rate is given by

$$\overline{\tau} = \frac{\sum_{\ell=1}^{M} S_\ell \tau_{\mathbf{S}_\ell}}{N} = 2\overline{k}\left(\frac{\alpha}{1-\alpha}\right) \tag{13}$$

and equilibrium capital demand by

$$k^*_{\mathbf{S_m}}(\tau) = \overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} = \frac{\overline{k}}{1-\alpha}\left(\frac{N - S_m}{2N - S_m}\right).$$

Then, the tax revenue is

$$R_{\mathbf{S_m}} = \tau_{\mathbf{S_m}} k^*_{\mathbf{S_m}}(\tau) = 2N\overline{k}^2 \frac{(N - S_m)}{(1-\alpha)^2(2N - S_m)^2}. \tag{14}$$

It is immediately clear:

[12] According to the definition in Konrad and Schjelderup (1999, p.163) equilibrium tax rates are strategic complements, as $\frac{\partial \tau_{\mathbf{S_m}}}{\partial \tau_{\mathbf{S}_\ell}} > 0$.

Proposition 1. *Coalitions of the same size in the same coalition structure set the same tax rate and have the same tax revenue.*

This result is not surprising as all regions are economically identical. Moreover, we obtain:

Proposition 2. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M \leq N$*. The larger a coalition in this coalition structure, the higher its equilibrium tax rate and the smaller its equilibrium tax revenue.*

Proposition 2 shows that cooperation induces higher tax rates. However, taking externalities in form of capital movements into account, the equilibrium tax revenues are lower for larger coalitions. Consider a specific coalition which is relatively large in comparison to the other coalitions. In equilibrium this coalition coordinates on a relatively high tax rate which leads to an outflow of capital given that there are smaller coalitions who coordinate on a relatively low tax rate. This is in line with the findings of Wilson (1991, Proposition 2) for two countries and has been extended by Bucovetsky (2009, Lemma 1) to N countries in a related setting where regions differ in population size.

Proof. For a fixed coalition structure the equilibrium tax rate of the coalitions differ in the factor

$$\frac{N}{2N - S_\ell}$$

for $\ell = 1, ..., M$. This factor increases if the coalition size increases. Hence, the larger the coalition the higher the equilibrium tax rate.
Similarly, for the equilibrium tax revenue we have to look at

$$\frac{N - S_\ell}{(2N - S_\ell)^2}$$

for $\ell = 1, ..., M$. Taking the derivative with respect to S_ℓ gives

$$\frac{\partial \left(\frac{N - S_\ell}{(2N - S_\ell)^2} \right)}{\partial S_\ell} = -\frac{S_\ell}{(2N - S_\ell)^3} < 0.$$

Hence, the larger the coalition the smaller the equilibrium tax revenue. □

Let us re-consider α in detail.

Lemma 1. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M \leq N$*. If two coalitions decide to merge, then* α*, given by* $\sum_{\ell=1}^{M} \frac{S_\ell}{2N - S_\ell}$*, strictly increases.*

The factor α is a measure of concentration for coalition structures. It represents the level of cooperation between regions and therefore reflects the intensity of capital tax competition. Moreover, α is related to the index of capital tax competition as defined in Bucovetsky (2009).

Proof. Assume two coalitions merge. Without loss of generality suppose coalition $\mathbf{S_{M-1}}$ and coalition $\mathbf{S_M}$ decide to form one coalition. We show that α strictly increases. To see this it is sufficient to compare the last two summands of α given by

$$\frac{S_{M-1}}{2N - S_{M-1}} + \frac{S_M}{2N - S_M}$$

and

$$\frac{S_{M-1} + S_M}{2N - S_{M-1} - S_M}.$$

Subtracting the two terms

$$\frac{S_{M-1} + S_M}{2N - S_{M-1} - S_M} - \frac{S_{M-1}}{2N - S_{M-1}} - \frac{S_M}{2N - S_M}$$

yields

$$\frac{(4N - S_{M-1} - S_M)S_{M-1}S_M}{(2N - S_{M-1})(2N - S_M)(2N - S_{M-1} - S_M)} > 0.$$

It follows that

$$\frac{S_{M-1} + S_M}{2N - S_{M-1} - S_M} > \frac{S_{M-1}}{2N - S_{M-1}} + \frac{S_M}{2N - S_M}.$$

So, α strictly increases if two coalitions merge. □

Some further results on the equilibrium tax rate and equilibrium tax revenues can be found in Appendix B, which we illustrate in section 5 in an example.

4 Dynamic Stability of Cooperation

4.1 The setting

In what follows, we analyze under which conditions coalitional equilibria can be sustained as a sub-game perfect equilibrium of the repeated game. Let

$\delta \in [0,1)$ denote the common discount factor. Regions have either implicitly or explicitly agreed to choose their tax rates cooperatively within their coalitions. We assume that the coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ is given. Following the trigger strategies as introduced by Friedman (1971), first of all, each region in each coalition sets the equilibrium tax rate, i.e., all regions act cooperatively within their coalitions if they do not observe any deviation from this behavior. In case a sub-coalition of regions "defects" by breaching the cooperation agreement this will be public information because the equilibrium tax revenues of all regions are affected through capital movements. Define a deviating sub-coalition of regions as follows:

Definition (sub-coalition)**.** Given a coalition $\mathbf{S}_\ell$ of size $S_\ell \geq 2$. We define $\mathbf{S}_\ell^{\mathbf{D}} \subsetneq \mathbf{S}_\ell$ with $1 \leq S_\ell^D < S_\ell$ as a sub-coalition of $\mathbf{S}_\ell$.[13]

The reaction to deviation of all coalitions is to resort to the punishment strategy in the period after the deviation has occurred. This punishment ends up in the fully non-cooperative Nash equilibrium. Here, a word about the punishment strategy is in order.

First of all, the threat which triggers cooperation needs to be sufficiently severe and it is not necessarily restricted to a single political dimension. Within a federation or an economic union, like the EU, there are several ways to punish a defection since countries are linked via (other) common policies and institutional arrangements (cf. Konrad and Schjelderup, 1999). This implies that the threat of punishment can be really high if it also affects other political dimensions.

Second, given that a subset of regions may deviate from its coalition, one could ask why all coalitions adopt the fully non-cooperative strategy although the deviation might come from another coalition. Suppose, only the coalition where the deviation has occurred employs the fully non-cooperative strategy. Then, there still exist substantial incentives to deviate from any other cooperative agreement in all other coalitions (as the succeeding analysis shows). These incentives continue to exist until —ultimately— all regions play fully non-cooperatively.

The chosen punishment strategy that we adopt here constitutes a sub-game perfect Nash equilibrium of the repeated game. It satisfies the condition

[13] This definition of a deviating sub-coalition reflects the idea of "internal blocking" used by Ray and Vohra (2012). In their model, that combines the coalition formation and the blocking approach, they assume that "[...] blocking is internal: only subcoalitions of existing coalitions are permitted to make further 'moves'." (p. 32) Note that our definition also accounts for deviations of singleton regions and they will be of particular importance in the course of our analysis.

that the threat which triggers cooperation must be sufficiently severe and the punishment strategy must be sub-game perfect. We nevertheless provide a discussion of alternative punishment strategies at the end of this section.

Denote by S_ℓ^D the size of a deviating sub-coalition, $\mathbf{S_\ell^D}$, from coalition $\mathbf{S}_\ell$, where the superscript D indicates deviating behavior. Figure 1 summarizes the structure of the repeated game.

$$\{\mathbf{S_1}, ..., \mathbf{S_M}\}$$

All regions play symmetrically and cooperatively within the coalitions and non-cooperatively across coalitions by setting the equilibrium tax rate.

$$\downarrow$$

$$\{\mathbf{S_1}, ..., \mathbf{S}_\ell \setminus \mathbf{S_\ell^D}, \mathbf{S_\ell^D}, ..., \mathbf{S_M}\}$$

Sub-coalition $\mathbf{S_\ell^D}$ (from coalition $\mathbf{S}_\ell$, with $S_\ell \geq 2$, $1 \leq S_\ell^D < S_\ell$) deviates. All other regions continue with cooperative behavior but observe the deviation of sub-coalition $\mathbf{S_\ell^D}$.

$$\downarrow$$

$$\{\,\{1\}\,, ...\,, \{N\}\,\}$$

All other regions play fully non-cooperative "Nash" as a reaction to the deviation of sub-coalition $\mathbf{S_\ell^D}$.

Figure 1: Structure of the Repeated Game.

To judge if a deviation is profitable or not, sub-coalition $\mathbf{S_\ell^D}$ needs to compare the discounted payoffs for deviating vs. for playing cooperatively. Let the subscript of the tax revenue $R_{\mathbf{S}_\ell}^{S_\ell^D}$ indicate the coalition from which the sub-coalition $\mathbf{S_\ell^D}$ has deviated and let the superscript refer to the size of the deviating sub-coalition S_ℓ^D. Deviating implies that each region in sub-coalition $\mathbf{S_\ell^D}$ receives a payoff of $R_{\mathbf{S}_\ell}^{S_\ell^D}$ once. From the next period onwards until infinity the payoff is then

$$R^P_{\mathbf{S}^\mathbf{D}_\ell} = 2N\overline{k}^2 \frac{(N-1)}{(1-\tilde{\alpha})^2(2N-1)^2}$$

with $\tilde{\alpha} = \frac{S^D_\ell}{2N-1} + \frac{N-S^D_\ell}{2N-1} = \frac{N}{2N-1}$. This means the total payoff from deviating is given by

$$R^{S^D_\ell}_{\mathbf{S}_\ell} + \sum_{t=1}^{\infty} \delta^t R^P_{\mathbf{S}^\mathbf{D}_\ell} = R^{S^D_\ell}_{\mathbf{S}_\ell} + \frac{\delta}{1-\delta} R^P_{\mathbf{S}^\mathbf{D}_\ell}.$$

If sub-coalition $\mathbf{S}^\mathbf{D}_\ell$ does not deviate, every region $i \in \mathbf{S}^\mathbf{D}_\ell$ will receive a payoff of $R_{\mathbf{S}_\ell}$ from now, in $t = 0$, until infinity. The total payoff from not-deviating is given by

$$\sum_{t=0}^{\infty} \delta^t R_{\mathbf{S}_\ell} = \frac{1}{1-\delta} R_{\mathbf{S}_\ell}.$$

If the following condition holds, no sub-coalition $\mathbf{S}^\mathbf{D}_\ell$ has an incentive to deviate from the coalitional equilibrium in the infinitely repeated game:

$$\frac{1}{1-\delta} R_{\mathbf{S}_\ell} \geq R^{S^D_\ell}_{\mathbf{S}_\ell} + \frac{\delta}{1-\delta} R^P_{\mathbf{S}^\mathbf{D}_\ell}. \tag{15}$$

In order to sustain a coalitional equilibrium in the dynamic tax competition game we need to find a discount factor that satisfies inequality (15). Such a discount factor $\delta \in (0, 1]$ is "non-trivial" for a payoff structure which satisfies

$$R^{S^D_\ell}_{\mathbf{S}_\ell} \geq R_{\mathbf{S}_\ell} \geq R^P_{\mathbf{S}^\mathbf{D}_\ell}. \tag{16}$$

4.2 Cooperation and punishment tax revenues

First, let us establish the second inequality in (16). By means of Lemma 1 the following proposition establishes that gains from cooperation always exist.

Proposition 3. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M \leq N$. *We have*

$$R_{\mathbf{S}_\ell} \geq R^P_{\mathbf{S}^\mathbf{D}_\ell}.$$

Proposition 3 establishes the well-known inefficiency of the fully non-cooperative Nash equilibrium. When departing from the fully non-cooperative solution by forming coalitions every region is better off.

Proof. Let $\alpha = \sum_{\ell=1}^{M} \frac{S_\ell}{2N-S_\ell} \in \left(\frac{1}{2}, 1\right)$. The equilibrium tax revenue of the coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ with $2 \leq M \leq N-1$ is given by equation (14),

$$R_{\mathbf{S}_\ell} = 2N\bar{k}^2 \frac{(N - S_m)}{(1-\alpha)^2(2N - S_m)^2}$$

for $m = 1, ..., M$.
The equilibrium tax revenue of the punishment is

$$R^P_{\mathbf{S}^\mathbf{D}_\ell} = \frac{2N\bar{k}^2}{N-1}$$

for $i = 1, ..., N$. To prove that gains from cooperation exist it is enough to show

$$(N-1) > \frac{(1-\alpha)^2(2N-S_m)^2}{(N-S_m)}.$$

We know that α strictly increases if two coalitions merge. Moreover, the right-hand side decreases if α increases. Thus, considering the coalition structure with coalition $\mathbf{S_m}$ and the remaining regions as singletons we obtain a lower bound for α given by

$$\alpha \geq \frac{S_m}{2N - S_m} + \frac{N - S_m}{2N-1}.$$

This is equivalent to

$$1 - \alpha \leq 1 - \frac{S_m}{2N - S_m} + \frac{N - S_m}{2N-1} = \frac{(N - S_m)(2N + S_m - 2)}{(2N-1)(2N - S_m)}.$$

The claim follows if we take this upper bound for $1-\alpha$ and establish

$$(N-1) > \frac{(N-S_m)(2N+S_m-2)^2}{(2N-1)^2}.$$

We obtain

$$\begin{aligned}&(2N-1)^2(N-1) - (N-S_m)(2N+S_m-2)^2 \\ &= (S_m - 1)\left((3S_m - 1)(N-1) + S_m^2\right) > 0.\end{aligned}$$

□

4.3 Deviation tax rates and tax revenues

The next step is to compute the revenues from deviation: We allow one sub-coalition $\mathbf{S}_\ell^{\mathbf{D}} \subsetneq \mathbf{S}_\ell$ to change its tax rate while all other regions remain acting cooperatively in the period where the deviation occurs. Assume from now on $S_\ell \geq 2$ and let $\overline{\tau}$ denote the average tax rate for the coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ where each region in each coalition sets the equilibrium tax rate. Sub-coalition $\mathbf{S}_\ell^{\mathbf{D}}$ optimally sets the deviation tax rate

$$\tau_{\mathbf{S}_\ell}^{S_\ell^D} = \frac{N\overline{k}}{N - S_\ell^D} \left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell} \right) \left(\frac{1}{1-\alpha} \right)$$

and obtains a tax revenue of

$$R_{\mathbf{S}_\ell}^{S_\ell^D} = \frac{N\overline{k}^2}{2(N - S_\ell^D)} \left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell} \right)^2 \left(\frac{1}{1-\alpha} \right)^2 .$$

The computation can be found in Appendix A.3.

Proposition 4. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M < N$. *Fix a size of a sub-coalition. The larger the coalition from which a sub-coalition with fixed size deviates, the smaller the deviation tax rate and the smaller the deviation tax revenue.*

This suggests that sub-coalitions which belong to relatively small coalitions have a higher incentive to deviate from cooperation compared to sub-coalitions in relatively large coalitions. From Proposition 2 we know that larger coalitions set higher tax rates but obtain less tax revenues than smaller coalitions. Therefore, in order to make deviating from a larger coalition with relatively low tax revenues profitable, a deviating sub-coalition needs to underbid the remaining regions more than when deviating from a relatively small coalition.

Proof. For a fixed coalition structure the deviation tax rates of the coalitions differ in the factor

$$\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell}$$

for $\ell = 1, ..., M$. The derivative of this expression with respect to S_ℓ is

$$\frac{\partial \left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell} \right)}{\partial S_\ell} = -\frac{S_\ell^D}{(2N - S_\ell)^2} < 0,$$

i.e., the larger the coalition the smaller the deviation tax rate.
Similarly, for the equilibrium tax revenue we have to look at

$$\frac{(2N - S_\ell - S_\ell^D)^2}{(2N - S_\ell)^2}$$

for $\ell = 1, ..., M$. This factor decreases if the size of the coalition increases, i.e., larger the coalition, from which a sub-coalition (of fixed size) deviates, the smaller the equilibrium tax revenue. □

Proposition 5. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M < N$. *The deviation tax rate of sub-coalition* $\mathbf{S_\ell^D} \subsetneq \mathbf{S}_\ell$ *is strictly smaller than the tax rate of regions in coalition* $\mathbf{S_m}$ *if and only if*

$$S_m(2N - S_\ell - S_\ell^D) - S_\ell^D(2N - 2S_\ell) > 0.$$

Moreover, the deviating sub-coalition always strictly underbids its own coalition.

Note that for $S_\ell^D = 1$ the above inequality is always satisfied.[14]

The deviating region acts optimally given the cooperative behavior in the coalitional equilibrium of all other regions. To attract the maximal amount of capital the deviating region certainly needs to underbid the tax rate of its own previous coalition. For all other coalitions it depends—inter alia—on their coalition sizes.

Proof. The tax rate for the deviating sub-coalition $\mathbf{S_\ell^D}$ is given by

$$\tau^D_{\mathbf{S}_\ell} = \frac{N\bar{k}}{N - S_\ell^D} \left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell} \right) \left(\frac{1}{1 - \alpha} \right)$$

and for the remaining regions by

$$\tau_{\mathbf{S_m}} = 2N\bar{k} \left(\frac{1}{2N - S_m} \right) \left(\frac{1}{1 - \alpha} \right).$$

We need to establish that

$$\tau^D_{\mathbf{S}_\ell} < \tau_{\mathbf{S_m}},$$

[14] This can be seen as follows: $S_m(2N - S_\ell - S_\ell^D) - S_\ell^D(2N - 2S_\ell) = (N - S_\ell)(S_m - 2) + S_m(N - 1) > 0$. For $S_m \geq 2$ this is true. For $S_m = 1$ we obtain: $-(N - S_\ell) + (N - 1) = S_\ell - 1$. This expression is strictly greater than 0 as the coalition S_ℓ consists of at least two regions.

which is equivalent to

$$\frac{1}{N - S_\ell^D} \left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell} \right) < 2 \left(\frac{1}{2N - S_m} \right).$$

Manipulating yields

$$\begin{aligned} & (2N - S_\ell - S_\ell^D)(2N - S_m) < 2(N - S_\ell^D)(2N - S_\ell) \\ \Leftrightarrow \quad & 0 < S_m(2N - S_\ell - S_\ell^D) - S_\ell^D(2N - 2S_\ell). \end{aligned}$$

It is easy to find a counter example for which the right-hand side is negative, this is the case for, e.g., $N = 10$, $S_\ell = 5$, $S_\ell^D = 3$ and $S_m = 1$. Therefore, the inequality does not hold in general. Nevertheless, for $S_m = S_\ell$ we get

$$S_\ell(2N - S_\ell - S_\ell^D) - S_\ell^D(2N - 2S_\ell) = (2N - S_\ell)(S_\ell - S_\ell^D) > 0.$$

This proves the claim. □

The following Proposition establishes that the sub-coalition $\mathbf{S_\ell^D}$ indeed has an advantage from deviating from the cooperative behavior.

Proposition 6. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M < N$. *The deviating sub-coalition* $\mathbf{S_\ell^D} \subsetneq \mathbf{S}_\ell$ *realizes a higher one-period deviation revenue than it would obtain from cooperation in* $\mathbf{S}_\ell$. *We get*

$$R_{\mathbf{S}_\ell}^{S_\ell^D} > R_{\mathbf{S}_\ell}.$$

Proof. We consider

$$\frac{N\bar{k}^2}{2(N - S_\ell^D)} \left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell} \right)^2 \left(\frac{1}{1 - \alpha} \right)^2 > 2N\bar{k}^2 \frac{(N - S_\ell)}{(1 - \alpha)^2(2N - S_\ell)^2}$$

which is equivalent to

$$\left(2N - S_\ell - S_\ell^D\right)^2 > 4(N - S_\ell)(N - S_\ell^D).$$

After some algebraic manipulation we obtain

$$(S_\ell - S_\ell^D)^2 > 0$$

which is per assumption on the sizes of the coalitions $\mathbf{S}_\ell$ and $\mathbf{S_\ell^D}$ always true. □

The deviating sub-coalition sets a relatively low tax rate and by that underbids the tax rate from the coalitional equilibrium. The lower tax rate is accompanied by an increase of the tax base. These two effects pointing in opposite directions result in an overall positive effect for the tax revenues, as Proposition 6 shows. Therefore, deviating is profitable in the short run meaning that no coalition structure can be considered as stable, in general. Non-trivial deviations are always profitable and there is no coalition structure that is absorbing in the sense that it leads to high tax revenues and no incentives to deviate.

4.4 The discount factor

In this section we determine the discount factors needed to sustain a coalitional equilibrium. It is clear that the severity of punishment determines the stability of cooperation. Although we study stability with respect to a fixed, jointly committed form of "institutional constraints" concerning the punishment, we can characterize coalition structures according to their degree of stability in the long run by comparing their respective discount factors.
Note that the minimum discount factor is obtained by rewriting equation (15),

$$\begin{aligned}\delta_{\mathbf{S}_\ell}^{S_\ell^D} &= \frac{R_{\mathbf{S}_\ell}^{S_\ell^D} - R_{\mathbf{S}_\ell}}{R_{\mathbf{S}_\ell}^{S_\ell^D} - R_{\mathbf{S}_\ell^{\mathbf{D}}}^{P}} \\ &= \frac{(S_\ell - S_\ell^D)^2(N-1)}{(2N - S_\ell - S_\ell^D)^2(N-1) - 4(2N-S_\ell)^2(1-\alpha)^2(N-S_\ell^D)}. \end{aligned} \tag{17}$$

See Appendix A.4 for the computation.

Let us first study the impact a sub-coalitional deviation on the discount factor.

Proposition 7. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M < N$*. Fix a coalition* $\mathbf{S}_\ell$ *with* $S_\ell \geq 2$*. The larger the deviating sub-coalition* $\mathbf{S}_\ell^{\mathbf{D}} \subsetneq \mathbf{S}_\ell$ *is, the smaller the minimum discount factor* $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$*.*

Proof. We consider the discount factor $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ from equation (17) as a function of the size of a deviating sub-coalition, S_ℓ^D. We take the first derivative with respect to S_ℓ^D to determine extremal points.

$$\frac{\partial \delta_{\mathbf{S}_\ell}^{S_\ell^D}}{\partial S_\ell^D} = -\frac{2\left(S_\ell - S_\ell^D\right)(N-1)}{(N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2}$$
$$- \frac{\left(S_\ell - S_\ell^D\right)^2 (N-1)\left(4(1-\alpha)^2(2N-S_\ell)^2 - 2(N-1)\left(2N - S_\ell - S_\ell^D\right)\right)}{\left((N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2\right)^2}$$

The first order condition is

$$-2\left(S_\ell - S_\ell^D\right)\left((N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2\right)$$
$$- \left(S_\ell - S_\ell^D\right)^2\left(4(1-\alpha)^2(2N-S_\ell)^2 - 2(N-1)\left(2N - S_\ell - S_\ell^D\right)\right) = 0$$

Solving for S_ℓ^D we get two solutions, namely $S_\ell^D = S_\ell$ and $S_\ell^D = 2N - S_\ell$. As we require $S_\ell^D < S_\ell$ we concentrate on $S_\ell^D = S_\ell$. The second derivative of the discount factor $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ with respect to S_ℓ^D is

$$\frac{\partial^2 \delta_{\mathbf{S}_\ell}^{S_\ell^D}}{\partial {S_\ell^D}^2} = \frac{2(N-1)}{(N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2}$$
$$+ \frac{4\left(S_\ell - S_\ell^D\right)(N-1)\left(4(1-\alpha)^2(2N-S_\ell)^2 - 2(N-1)\left(2N - S_\ell - S_\ell^D\right)\right)}{\left((N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2\right)^2}$$
$$- \frac{2\left(S_\ell - S_\ell^D\right)^2 (N-1)^2}{\left((N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2\right)^2}$$
$$+ \frac{2\left(S_\ell - S_\ell^D\right)^2 (N-1)\left(4(1-\alpha)^2(2N-S_\ell)^2 - 2(N-1)\left(2N - S_\ell - S_\ell^D\right)\right)^2}{\left((N-1)\left(2N - S_\ell - S_\ell^D\right)^2 - 4(1-\alpha)^2\left(N - S_\ell^D\right)(2N - S_\ell)^2\right)^3}$$

Evaluating this expression at $S_\ell^{D*} = S_\ell$ we obtain

$$\frac{2(N-1)}{(N-1)(2N - 2S_\ell)^2 - 4(1-\alpha)^2(N - S_\ell)(2N - S_\ell)^2}.$$

To find out whether $S_\ell^{D*} = S_\ell$ is a minimum or a maximum we determine the sign of this expression. It is straightforward to see that the numerator is strictly positive. To show that the denominator is strictly positive, as well, we use that

$$(1-\alpha)^2 \leq \left(1 - \frac{S_\ell}{2N - S_\ell} - \frac{N - S_\ell}{2N-1}\right)^2 = \frac{(N-S_\ell)^2(2N+S_\ell-2)^2}{(2N-1)^2(2N-S_\ell)^2}.$$

Hence we show

$$(N-1)(2N-1)^2 - (N-S_\ell)(2N+S_\ell-2)^2 > 0.$$

This inequality holds true which can be seen by writing the left hand side as

$$(S_\ell - 1)\left[(N+S_\ell)(S_\ell-1) + 2S_\ell(N-1) + 1\right].$$

Therefore, the discount factor $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ as a function of the size of the deviating sub-coalition, S_ℓ^D, attains at $S_\ell^{D*} = S_\ell$ a local minimum. Thus, in the region $S_\ell^D < S_\ell$ the discount factor $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ must be decreasing in S_ℓ^D. This proves the claim. □

Proposition 7 shows that deviations of single regions require a higher minimum discount factor than deviations of sub-coalitions. Hence, it is more attractive for single regions to deviate than for sub-coalitions. The smaller the deviating sub-coalition the higher the minimum discount factor necessary to sustain cooperation. Therefore, we define the minimum discount factor of a coalition to be the one of singleton deviations,

$$\delta_{\mathbf{S}_\ell} := \max\left\{\delta_{\mathbf{S}_\ell}^{S_\ell^D} | S_\ell^D < S_\ell\right\} = \delta_{\mathbf{S}_\ell}^1.$$

Now, in order to sustain a ***coalitional equilibrium***, no region is allowed to have a profitable deviation no matter to which coalition this region belongs. Therefore, given the coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ we need to take the maximal minimum discount factor δ over all coalitions and all possible deviations of sub-coalitions. For all discount factors larger or equal than δ no region has an incentive to deviate from the cooperative behavior. The next Proposition helps us to determine the maximal minimum discount factor δ.

Proposition 8. *Given a coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ with $2 \leq M < N$. The larger the size of a coalition $\mathbf{S}_\ell$ from which a sub-coalition $\mathbf{S}_\ell^{\mathbf{D}}$ (of fixed size) deviates, the larger the minimum discount factor $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$.*

Proof. We consider the minimum discount factor $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ as a function of the coalition size S_ℓ. The numerator of the minimum discount factor $\delta_{\mathbf{S}_\ell}$ given by

$$(S_\ell - S_\ell^D)^2(N-1)$$

is increasing in the coalition size S_ℓ. Thus, if we are able to show that the denominator given by

$$(2N - S_\ell - S_\ell^D)^2(N-1) - 4(2N - S_\ell)^2(1-\alpha)^2(N - S_\ell^D)$$

is a decreasing function in S_ℓ, we are done. To see this we take the derivative of the denominator with respect to S_ℓ and obtain

$$-2(2N - S_\ell - S_\ell^D)(N-1) + 8(1-\alpha)^2(2N - S_\ell)(N - S_\ell^D).$$

Note that we regard the coalition structure hereby as fixed, so the factor α is considered fixed as well. If this derivative is strictly negative, the denominator of $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ is a decreasing function of S_ℓ and hence $\delta_{\mathbf{S}_\ell}^{S_\ell^D}$ is increasing in the coalition size. To see this we show:

$$(2N - S_\ell - S_\ell^D)(N-1) - 4(1-\alpha)^2(2N - S_\ell)(N - S_\ell^D) > 0.$$

By Lemma 1 we have that α increases if two coalitions merge and so $(1-\alpha)^2$ decreases. It is sufficient to take the upper bound for $(1-\alpha)^2$ which is the lower bound for α and corresponds to the case where the regions outside the coalition $\mathbf{S}_\ell$ react non-cooperatively. Thus,

$$(1-\alpha)^2 \leq \left(1 - \frac{S_\ell}{2N - S_\ell} - \frac{N - S_\ell}{2N-1}\right)^2 = \frac{(N - S_\ell)^2(2N + S_\ell - 2)^2}{(2N-1)^2(2N - S_\ell)^2}.$$

Hence, for our claim we require:

$$(2N - S_\ell - S_\ell^D)(N-1)(2N-1)^2(2N - S_\ell) - 4(N - S_\ell)^2(2N + S_\ell - 2)^2(N - S_\ell^D) > 0.$$

Expanding the expression on the left-hand side yields

$$\begin{aligned}
&8S_\ell^D N^4 + 16{S_\ell}^2 N^3 - 12S_\ell^D S_\ell N^3 - 16S_\ell N^3 - 16S_\ell^D N^3 + 4N^3 - 8{S_\ell}^3 N^2 - 12S_\ell^D {S_\ell}^2 N^2 \\
&\quad - 8{S_\ell}^2 N^2 + 40S_\ell^D S_\ell N^2 + 12S_\ell N^2 + 6S_\ell^D N^2 - 4N^2 - 4{S_\ell}^4 N + 8S_\ell^D {S_\ell}^3 N + 16{S_\ell}^3 N \\
&\quad - 11{S_\ell}^2 N - 27S_\ell^D S_\ell N + 4S_\ell N + 2S_\ell^D N + 4S_\ell^D {S_\ell}^4 - 16S_\ell^D {S_\ell}^3 + 16S_\ell^D {S_\ell}^2 - {S_\ell}^2 - S_\ell^D S_\ell.
\end{aligned}$$

This is equal to

$$
\begin{aligned}
&(N-S_\ell)\left[8S_\ell^D N^2(N-2)+5S_\ell^D(N-S_\ell)+11S_\ell(N-S_\ell^D)+S_\ell^D N\right]\\
&\quad+(N-S_\ell^D)(N-S_\ell)\left[8S_\ell(S_\ell N-N-S_\ell)+4S_\ell(S_\ell-2)(N+S_\ell)\right]\\
&\quad+4S_\ell(S_\ell-S_\ell^D)(N-2)N^2+4N\left[(N-1)N-(S_\ell^D-1)S_\ell\right]\\
&\quad+S_\ell\left[N(N-S_\ell^D)-S_\ell\right]+S_\ell^D(N-S_\ell)+S_\ell^D N.
\end{aligned}
$$

This last expression is strictly positive as the assumptions on the coalition structure imply $N \geq 3$ and $S_\ell \geq 2$. This proves the claim. □

Allowing for deviations of sub-coalitions of arbitrary size using Proposition 7 and Proposition 8 we get immediately:

Proposition 9. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M < N$. *The maximal minimum discount factor is given by*

$$\delta = \delta_{\mathbf{S}_{\max}} = \delta^1_{\mathbf{S}_{\max}} = \frac{(S_{\max}-1)^2}{(2N-S_{\max}-1)^2-4(2N-S_{\max})^2(1-\alpha)^2}$$

where S_{max} *is the size of the largest coalition, denoted by* $\mathbf{S_{max}}$*, in the coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$.

Consequently, in order to determine the maximal minimum discount factor for a given coalition structure it suffices to know the size of the largest coalition. All other coalition sizes have no direct impact on the sustainability of the coalitional equilibrium, except for the fact that the discount factor depends on the ex-ante level of cooperation through the factor α.

Let us remark that combining Proposition 7 and Proposition 8 yields another result: For example, if we only allow for deviations of sub-coalitions with a lower bound on the minimal size, then these two Propositions tell us how to determine the discount factor in order to sustain a coalitional equilibrium within a given coalition structure. For example, even if institutional or political reasons (outside our model) require deviations of at least two countries to make defection "effective" we know what the respective discount factor is.

In the following we compare the maximal minimum discount factor between different coalition structures in each case allowing for deviations of sub-coalitions of arbitrary size.

Proposition 10. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $2 \leq M < N$ *let* S_{max} *be the size of the largest coalition, denoted by* $\mathbf{S_{max}}$. *Assume that coalition* $\mathbf{S_m} \neq \mathbf{S}_{\max}$ *splits up into smaller coalitions* $\mathbf{S_{m_1}}, ..., \mathbf{S_{m_k}}$ *with* $k \geq 1$. *Then, the maximal minimum discount factor increases.*

Obviously, the discount factors coincide if the given coalition structure consists of one coalition and singletons. Otherwise the increase of the maximal minimum discount factor is strict as shown in the proof:

Proof. From Proposition 9 we know:

$$\delta = \delta^1_{\mathbf{S}_{\max}}.$$

The general expression for the discount factor is given in (17). It depends in particular on the size of the coalition and the factor α. Per assumption the sizes of the largest coalition in the two coalition structures coincide, which makes α the crucial difference in the discount factor. From Lemma 1 we know that α strictly increases if any two coalitions merge. Therefore if a coalition splits up into smaller coalitions α strictly decreases. In this case $(1-\alpha)^2$ strictly increases and so the denominator of the minimum discount factor in equation (17) strictly decreases, so the discount factor strictly increases. □

Proposition 10 establishes that cooperation is easier to sustain if there is ex-ante more "cooperative behavior" between the regions. For example, compare an arbitrary coalition structure with $\mathbf{S_{max}}$ as the largest coalition with a coalition structure with $\mathbf{S_{max}}$ and the remaining regions act fully non-cooperatively (analyzed by Itaya et al., 2010). Then, the discount factor for the second case (ex-ante less cooperative) is larger than for the first case (ex-ante more cooperative).

Considering the situation in Proposition 10 from the reverse point of view we obtain:

Proposition 11. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $S_2 = ... = S_M = 1$ *and* $2 \leq M < N$. *Suppose, some singleton regions start to form new coalitions. As long as they do not form a coalition with a size strictly larger than* $S_{\max}$*, the maximal minimum discount factor decreases.*

Proof. This can be shown by a similar argument as in Proposition 10. □

Proposition 10 and Proposition 11 depart from two different points of view. In Proposition 10 we study for an arbitrary coalition structure how the maximal minimum discount factor changes if every region outside the maximal size coalition starts to act less cooperatively. In Proposition 11 we start with a coalition structure with one "big" coalition and singleton regions and analyze the influence on the maximal minimum discount factor when some singleton regions form new coalitions, whose size is no larger than the one of the "big" coalition.

Finally, we briefly comment on the punishment, where the assumption, that every region outside the deviating sub-coalition acts non-cooperatively, is relaxed. Let $\hat{\alpha}$ refer to the coalition structure of the punishment. It needs to be shown that the punishment tax revenue,

$$2N\bar{k}\frac{(N - S_\ell^D)}{(2N - S_\ell^D)^2(1-\hat{\alpha})^2},$$

resulting from a less cooperative coalition structure where coalitions outside the deviating sub-coalition or the deviating sub-coalition itself are allowed to continue to act cooperatively, is indeed lower than the tax revenue from cooperation. This implies monotonicity in the tax revenues going from less cooperative coalition structures to more cooperative ones. For this it is necessary to establish the following inequality

$$(N - S_\ell)(2N - S_\ell^D)^2(1-\hat{\alpha})^2 - (N - S_\ell^D)(2N - S_\ell)^2(1-\alpha)^2 \geq 0.$$

It can be shown that all our previous results remain valid, if this inequality is satisfied. Also section 5 indicates by means of a numerical example that this kind of monotonicity in the tax revenues holds. Therefore, we expect that our results can be generalized to other forms of punishment.
By the comparison of the maximal minimum discount factors of two alternative punishment scenarios we immediately observe that changing the punishment to a (maybe more realistic) less harsh scenario with $\tilde{R}^P_{\mathbf{S}^{\mathbf{D}}_\ell} \geq R^P_{\mathbf{S}^{\mathbf{D}}_\ell}$ has of course an impact on the sustainability of cooperation: As the maximal minimum discount factor increases, it becomes more difficult to sustain cooperation.

4.5 Welfare maximization

For the sake of readability of our results we have analyzed a model where the region's objective function is to maximize tax revenues. However, this is not an innocent assumption since the objective function determines the strategic game considerably. A more realistic assumption is to maximize welfare consisting of the region's consumption of a private and a public good, which is financed by tax revenues. In Appendix C, we indicate how this can be done in general. By an numerical example for five regions we find that the equilibrium tax rates and tax revenues for the cooperative behavior seem to follow a similar pattern as with the revenue maximization. The same holds true for the dynamic sustainability of cooperation. Overall, this indicates that our results are likely to be generalized with welfare maximization.

5 Numerical Example

5.1 Equilibrium tax rates and tax revenues for five regions

In order to illustrate our results we introduce in this section an example, where we compare the different coalition structures for five regions, $N = 5$. The number of possible coalition structures is 52 and Table 1 gives an overview. We skipped most of the variations of the coalition structures due to our symmetric setting and the fact that they can easily be obtained by re-naming the players.

	{1}{2}{3}{4}{5}	{1}{2345}	{12}{345}	{12}{34}{5}	{12}{3}{4}{5}	{123}{4}{5}	{12345}
		{2}{1345}	{13}{245}	{13}{23}{5}	{13}{2}{4}{5}	{124}{3}{5}	
		{3}{1345}	{14}{235}	{14}{23}{5}	{14}{2}{3}{5}	{125}{3}{4}	
		⋮	⋮	⋮	⋮	⋮	
			⋮	⋮	⋮		
				⋮			
α	0.55	0.77	0.68	0.61	0.58	0.65	1.00

Table 1: Coalition structures and corresponding α for $N = 5$

For the purpose of this example, it is sufficient to choose one coalition structure from every column of table 1 and compute the corresponding tax rate, capital demand and tax revenue. The results of the equilibrium tax rates, the capital demands and the equilibrium tax revenues are summarized in Tables 2, 3 and 4 below.

Coalition structure	1	2	3	4	5	$\overline{\tau}$
{{1} {2} {3} {4} {5}}	$2.5\overline{k}$	$2.5\overline{k}$	$2.5\overline{k}$	$2.5\overline{k}$	$2.5\overline{k}$	$2.5\overline{k}$
{{1} {2345}}	$5\overline{k}$	$7.5\overline{k}$	$7.5\overline{k}$	$7.5\overline{k}$	$7.5\overline{k}$	$7\overline{k}$
{{12} {345}}	$3.89\overline{k}$	$3.89\overline{k}$	$4.44\overline{k}$	$4.44\overline{k}$	$4.44\overline{k}$	$4.22\overline{k}$
{{12} {34} {5}}	$3.21\overline{k}$	$3.21\overline{k}$	$3.21\overline{k}$	$3.21\overline{k}$	$2.86\overline{k}$	$3.14\overline{k}$
{{12} {3} {4} {5}}	$3\overline{k}$	$3\overline{k}$	$2.67\overline{k}$	$2.67\overline{k}$	$2.67\overline{k}$	$2.8\overline{k}$
{{123} {4} {5}}	$4.09\overline{k}$	$4.09\overline{k}$	$4.09\overline{k}$	$3.18\overline{k}$	$3.18\overline{k}$	$3.73\overline{k}$

Table 2: Equilibrium tax rates for $N = 5$

Coalition structure	1	2	3	4	5
{{1} {2} {3} {4} {5}}	$\overline{k}$	$\overline{k}$	$\overline{k}$	$\overline{k}$	$\overline{k}$
{{1} {2345}}	$2\overline{k}$	$0.75\overline{k}$	$0.75\overline{k}$	$0.75\overline{k}$	$0.75\overline{k}$
{{12} {345}}	$1.17\overline{k}$	$1.17\overline{k}$	$0.89\overline{k}$	$0.89\overline{k}$	$0.89\overline{k}$
{{12} {34} {5}}	$0.96\overline{k}$	$0.96\overline{k}$	$0.96\overline{k}$	$0.96\overline{k}$	$1.14\overline{k}$
{{12} {3} {4} {5}}	$0.9\overline{k}$	$0.9\overline{k}$	$1.07\overline{k}$	$1.07\overline{k}$	$1.07\overline{k}$
{{123} {4} {5}}	$0.82\overline{k}$	$0.82\overline{k}$	$0.82\overline{k}$	$1.27\overline{k}$	$1.27\overline{k}$

Table 3: Equilibrium capital demands for $N = 5$

Coalition structure	1	2	3	4	5	$\sum$
{{1} {2} {3} {4} {5}}	$2.5\overline{k}^2$	$2.5\overline{k}^2$	$2.5\overline{k}^2$	$2.5\overline{k}^2$	$2.5\overline{k}^2$	$12.5\overline{k}^2$
{{1} {2345}}	$10\overline{k}^2$	$5.63\overline{k}^2$	$5.63\overline{k}^2$	$5.63\overline{k}^2$	$5.63\overline{k}^2$	$32.5\overline{k}^2$
{{12} {345}}	$4.54\overline{k}^2$	$4.54\overline{k}^2$	$3.95\overline{k}^2$	$3.95\overline{k}^2$	$3.95\overline{k}^2$	$20.93\overline{k}^2$
{{12} {34} {5}}	$3.1\overline{k}^2$	$3.1\overline{k}^2$	$3.1\overline{k}^2$	$3.1\overline{k}^2$	$3.27\overline{k}^2$	$15.67\overline{k}^2$
{{12} {3} {4} {5}}	$2.7\overline{k}^2$	$2.7\overline{k}^2$	$2.84\overline{k}^2$	$2.84\overline{k}^2$	$2.84\overline{k}^2$	$13.92\overline{k}^2$
{{123} {4} {5}}	$3.35\overline{k}^2$	$3.35\overline{k}^2$	$3.35\overline{k}^2$	$4.05\overline{k}^2$	$4.05\overline{k}^2$	$18.15\overline{k}^2$

Table 4: Equilibrium tax revenues for $N = 5$

Starting from the point of no cooperation, each region receives the lowest values in absolute terms for both equilibrium tax rate and equilibrium tax revenue. This observation refers to the well-known fact that any kind of cooperation is profitable for all the regions. Clearly, this situation is the "classical" tax competition dilemma where tax rates and tax revenues are too low compared to a cooperative solution.
If two regions form a coalition while the remaining regions continue to act non-cooperatively, such that the resulting coalition structure is for example

$\{\{12\}\,\{3\}\,\{4\}\,\{5\}\}$, there is a Pareto-improvement for all regions in terms of tax rates and tax revenues. As suggested by Proposition 2 we observe first, that the coalition of the two cooperating regions sets a higher tax rate, $3\overline{k}$ vs. $2.67\overline{k}$, and becomes a capital exporter where capital demand is $0.9\overline{k}$. Second, the tax revenue of this coalition $\{12\}$ is $2.7\overline{k}^2$ and therefore lower compared to the singleton regions with $2.84\overline{k}^2$, but still strictly higher than the non-cooperative payoff of $2.5\overline{k}^2$.

As cooperation proceeds, for example coalition structure $\{\{123\}\,\{4\}\,\{5\}\}$ forms, we see that every coalition raises its tax rate, compare Proposition 12 in Appendix B. Looking at the capital demands we see that the coalition $\{123\}$ exports more capital than before, which results from an above average increase of their tax rate. In addition, we observe that there is again a Pareto-improvement for all regions. This includes the merging coalitions $\{12\}$ and $\{3\}$ who get a tax revenue of $3.35\overline{k}^2$ compared to $2.7\overline{k}^2$ and $2.84\overline{k}^2$ before, as well as the singletons with $4.05\overline{k}^2$ compared to $2.84\overline{k}^2$. This case exemplifies that even the merging coalitions gain from cooperation which was not clear in general. From Proposition 13 in Appendix B we know that coalitions not merging benefit from the merger for sure.

As already discussed in Bucovetsky (2009) and also obtained here, coalition structure $\{\{1\}\,\{2345\}\}$ offers the highest tax revenues and tax rates which can be received, apart from the grand coalition which would fully absorb production output by taxes.

5.2 A repeated game with five regions

Now, we apply the repeated game to this example.

coalition structure	discount factor		sum of tax revenues
$\{\{1\}\,\{2345\}\}$	$\delta^1_{\{2345\}} = 0.5031$ $\delta^2_{\{2345\}} = 0.3750$ $\delta^3_{\{2345\}} = 0.1837$	$\delta = 0.5031$	$32.5\overline{k}^2$
$\{\{12\}\,\{345\}\}$	$\delta^1_{\{12\}} = 0.0443$ $\delta^1_{\{345\}} = 0.2539$ $\delta^2_{\{345\}} = 0.1019$	$\delta = 0.2539$	$20.93\overline{k}^2$
$\{\{12\}\,\{34\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.0972$ $\delta^1_{\{34\}} = 0.0972$	$\delta = 0.0972$	$15.67\overline{k}^2$
$\{\{12\}\,\{3\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.2195$	$\delta = 0.2195$	$13.92\overline{k}^2$
$\{\{123\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{123\}} = 0.3306$ $\delta^2_{\{123\}} = 0.1414$	$\delta = 0.3306$	$18.15\overline{k}^2$

Table 5: Discount factors for $N = 5$.

This example illustrates one of our main result from Proposition 9, namely that the discount factor associated to a coalition structure is the one for singleton deviations from the largest coalition.

The comparison of the coalition structure $\{\{12\}\,\{345\}\}$ with an ex-ante less cooperative and hence more competitive environment, i.e., there is only coalition $\{123\}$ whereas coalition $\{4\}$ and $\{5\}$ act as a singleton, shows that cooperation is more difficult to sustain. In Proposition 10 we established that this observation holds true in general, so cooperation is easier to sustain in a dynamic setting the more cooperative behavior exists ex-ante under the assumption that the size of the largest coalition is fixed.

Lastly, note that according to Proposition 11 the maximal minimum discount factor decreases from 0.3306 to 0.2539 going from coalition structure $\{\{123\}\,\{4\}\,\{5\}\}$ to coalition structure $\{\{12\}\,\{345\}\}$ where the two singleton regions decide to form a coalition.

Interestingly, a comparison of all possible coalition structures for $N = 5$ shows, that the most stable coalition structure, in the sense of the lowest discount factor, is <u>not</u> the one with the largest sum of tax revenues.

6 Conclusion

This paper has analyzed the stability of coalitional equilibria within repeated tax competition. For any given coalition structure we have determined the equilibrium tax rates and the equilibrium tax revenues. The main contribution of this paper is the analysis of sustainability of cooperative behavior by means of a repeated game setting allowing for deviations of sub-coalitions. We obtain the following results: First, the deviating sub-coalition underbids the tax rates of other regions continuing to act cooperatively and benefits from a one-shot deviation. Nevertheless, for a given form of punishment we have found that there exists a critical minimum discount factor that makes deviations unprofitable and sustains cooperation in the long run. Second, we have established that for an arbitrary coalition structure the discount factor needed to sustain a coalitional equilibrium crucially depends on the size of the largest coalition in this structure and the deviations of singleton regions. Third, the comparison of an arbitrary coalition structure where the largest coalition consists of at least two regions with a coalition structure where there is only the largest coalition and the remaining regions behave fully non-cooperatively, or, in general, with a less cooperative coalition structure but with the same coalition of maximal size, shows: Cooperation is easier to sustain for the first coalition structure with an ex-ante more cooperative behavior. Similarly, for a given coalition considering the behavior of the remaining regions, the discount factor needed to sustain cooperation decreases as long as the remaining regions do not form a coalition larger than the given one. This means, once cooperative behavior has been broadly established it is easier to sustain. Remarkably, a comparison of all possible coalition structures for five regions showed, that the most stable coalition structure, in the sense of the lowest discount factor, is *not* the one with the largest sum of tax revenues.

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References

Bernheim, B. D., Peleg, B., and Whinston, M. D. (1987). Coalition-proof Nash equilibria i. concepts. *Journal of Economic Theory*, 42(1):1–12.

Biran, O. and Forges, F. (2011). Core-stable rings in auctions with independent private values. *Games and Economic Behavior*, 73(1):52–64.

Bucovetsky, S. (1991). Asymmetric tax competition. *Journal of Urban Economics*, 30:167–181.

Bucovetsky, S. (2005). Tax bloc formation among Leviathans. York University.

Bucovetsky, S. (2009). An index of capital tax competition. *International Tax and Public Finance*, 16:727–752.

Burbidge, J. B., DePater, J. A., Myers, G. M., and Sengupta, A. (1997). A coalition-formation approach to equilibrium federations and trading blocs. *The American Economic Review*, 87(5):940–956.

Cardarelli, R., Taugourdeau, E., and Vidal, J.-P. (2002). A repeated interactions model of tax competition. *Journal of Public Economic Theory*, 4(1):19–38.

Catenaro, M. and Vidal, J.-P. (2006). Implicit tax co-ordination under repeated policy interactions. *Recherches économiques de Louvain*, 72:5–18.

Council of the European Union (2011). Conclusions of the Heads of State or Government of the Euro Area of 11 March 2011.

Deneckere, R. and Davidson, C. (1985). Incentives to form coalitions with Bertrand competition. *RAND Journal of Economics*, 16(4):473–486.

Devereux, M. P., Lockwood, B., and Redoano, M. (2008). Do countries compete over corporate tax rates? *Journal of Public Economics*, 92:1210–1235.

Edwards, J. and Keen, M. (1996). Tax competition and Leviathan. *European Economic Review*, 40:113–134.

Friedman, J. (1971). A non-cooperative equilibrium for supergames. *Review of Economic Studies*, 38:1–12.

Grazzini, L. and van Ypersele, T. (2003). Fiscal coordination and tax competition. *Journal of Public Economic Theory*, 5 (2):305–325.

Griffith, R., Hines, J., and Sorensen, P. (2008). International capital taxation, paper prepared for the Mirrlees review. *Reforming the Tax System for the 21st Century (London: Institute for Fiscal Studies).*

Hart, S. and Kurz, M. (1983). Endogenous formation of coalitions. *Econometrica*, 51(4):1047–1064.

Haufler, A. (1997). Factor taxation, income distribution and capital market integration. *The Scandinavian Journal of Economics*, 99(3):pp. 425–446.

Ichiishi, T. (1981). A social coalitional equilibrium existence lemma. *Econometrica*, 49(2):pp. 369–377.

Itaya, J.-I., Okamura, M., and Yamaguchi, C. (2008). Are regional asymmetries detrimental to tax coordination in a repeated game setting? *Journal of Public Economics*, 92(12):2403 – 2411.

Itaya, J.-I., Okamura, M., and Yamaguchi, C. (2010). Partial tax coordination in a repeated game setting. CESifo Working Paper Series 3127, CESifo Group Munich.

Kanbur, R. and Keen, M. (1993). Jeux sans frontieres: Tax competition and tax coordination when countries differ in size. *American Economic Review*, 83(4):877–92.

Keen, M. and Konrad, K. A. (2011). International tax competition. Technical report, Handbook in Public Economics conference, UC Berkeley.

Kessing, S. G., Konrad, K. A., and Kotsogiannis, C. (2006). Federal tax autonomy and the limits of cooperation. *Journal of Urban Economics*, 59(2):317–329.

Kiss, A. (2011). Minimum taxes and repeated tax competition. *International Tax and Public Finance*, forthcoming.

Konrad, K. A. and Schjelderup, G. (1999). Fortress building in global tax competition. *Journal of Urban Economics*, 46(1):156–167.

Lockwood, B. (2004). Competition in unit vs. ad valorem taxes. *Asia-Pacific Financial Markets*, 11(6):763–772.

Rasmussen, B. S. (2001). Partial vs. global coordination of capital income tax policies. Economics Working Papers 2001-3, School of Economics and Management, University of Aarhus.

Ray, D. (2007). *A Game-Theoretic Perspective on Coalition Formation.* Oxford University Press.

Ray, D. and Vohra, R. (1997). Equilibrium binding agreements. *Journal of Economic Theory*, 73(1):30 – 78.

Ray, D. and Vohra, R. (2012). Coalition formation. Presentation at the 17th Coalition Theory Network Workshop, Paris, France.

Wildasin, D. E. (1989). Interjurisdictional capital mobility: Fiscal externality and a corrective subsidy. *Journal of Urban Economics*, 25(2):193–212.

Wilson, J. D. (1991). Tax competition with interregional differences in factor endowments. *Regional Science and Urban Economics*, 21(3):423 – 451.

Wilson, J. D. (1999). Theories of tax competition. *National Tax Journal*, 52(2):269–304.

Wilson, J. D. and Wildasin, D. E. (2004). Capital tax competition: bane or boon. *Journal of Public Economics*, 88(6):1065–1091.

Zhao, J. (1992). The hybrid solutions of an n-person game. *Games and Economic Behavior*, 4(1):145 – 160.

Zodrow, G. R. and Mieszkowski, P. (1986). Pigou, Tiebout, property taxation, and the underprovision of local public goods. *Journal of Urban Economics*, 19(3):356–370.

A Appendix

A.1 Best response functions with coalition structures

We provide the computation of how the best response function for the regional tax rate is determined. The characteristic feature of a coalition is that they agree to set a unique tax rate within the coalition. The coalitional revenue is distributed equally to its members. The objective function of the coalition $\mathbf{S_m}$ is given by

$$\begin{aligned}\sum_{h\in\mathbf{S_m}} \tau_h k_h^*(\tau) &= \sum_{h\in\mathbf{S_m}} \tau_{\mathbf{S_m}} \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2}\right) \\ &= S_m \tau_{\mathbf{S_m}} \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2}\right).\end{aligned}$$

The coalitional tax rate is chosen in such a way that this function is maximized. The derivative with respect to $\tau_{\mathbf{S_m}}$ is:

$$\begin{aligned}&S_m \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2}\right) + S_m \tau_{\mathbf{S_m}} \left(\frac{S_m}{2N} - \frac{1}{2}\right) \\ &= S_m \left(\overline{k} + \frac{1}{2}\sum_{\ell\neq m} \frac{S_\ell \tau_{\mathbf{S}_\ell}}{N} + \left(\frac{S_m}{2N} - \frac{1}{2} + \frac{S_m}{2N} - \frac{1}{2}\right)\tau_{\mathbf{S_m}}\right) \\ &= S_m \left(\overline{k} + \frac{1}{2}\sum_{\ell\neq m} \frac{S_\ell \tau_{\mathbf{S}_\ell}}{N} + \frac{S_m - N}{N}\tau_{\mathbf{S_m}}\right).\end{aligned}$$

Equating this term to 0 leads to

$$\tau_{\mathbf{S_m}} = \frac{N}{N - S_m}\overline{k} + \frac{1}{2}\sum_{\ell\neq m} \frac{S_\ell \tau_{\mathbf{S}_\ell}}{N - S_m}.$$

A.2 Optimal tax rates with coalition structures

We compute the optimal tax rates in line with Bucovetsky (2009) as follows. Given the coalition structure $\{\mathbf{S_1}, \mathbf{S_2}, ..., \mathbf{S_M}\}$, where for $m \geq 2$ the coalitions S_m are singletons. From the first order condition in Appendix A.1 we have

$$S_m \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right) + S_m \tau_{\mathbf{S_m}} \left(\frac{S_m}{2N} - \frac{1}{2} \right) = 0.$$

The optimal tax rates are

$$\tau_{\mathbf{S_m}} = \left(2\overline{k} + \overline{\tau}\right) \left(\frac{N}{2N - S_m} \right).$$

Multiplying both sides with S_m, summing up over all coalitions and dividing by N gives

$$\sum_{m=1}^{M} \frac{S_m \tau_{\mathbf{S_m}}}{N} = \left(2\overline{k} + \overline{\tau}\right) \left(\sum_{m=1}^{M} \frac{S_m}{2N - S_m} \right).$$

On the left-hand side we obtain the average tax rate $\overline{\tau}$. Solving for $\overline{\tau}$ yields:

$$\overline{\tau} = 2\overline{k} \left(\frac{\sum\limits_{m=1}^{M} \frac{S_m}{2N - S_m}}{1 - \sum\limits_{m=1}^{M} \frac{S_m}{2N - S_m}} \right)$$

Defining $\alpha := \sum\limits_{m=1}^{M} \frac{S_m}{2N - S_m}$ the average tax rate can be written as

$$\overline{\tau} = 2\overline{k} \left(\frac{\alpha}{1 - \alpha} \right).$$

Inserting this expression for $\overline{\tau}$ into the optimal tax rates we get

$$\begin{aligned} \tau_{\mathbf{S_m}} &= 2N\overline{k} \left(1 + \frac{\alpha}{1 - \alpha} \right) \left(\frac{1}{2N - S_m} \right) \\ &= 2N\overline{k} \left(\frac{1}{1 - \alpha} \right) \left(\frac{1}{2N - S_m} \right). \end{aligned}$$

A.3 Optimal tax rate and revenue under deviating behavior

Suppose a sub-coalition $\mathbf{S_\ell^D}$ consisting of S_ℓ^D regions (from coalition $\mathbf{S}_\ell$, $S_\ell^D < S_\ell$) deviates while all other regions continue to set the equilibrium tax rate from the coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$. This means there are $S_\ell - S_\ell^D$ regions setting the tax rate $\tau_{\mathbf{S}_\ell}$ and there are S_m regions setting $\tau_{\mathbf{S_m}}$ for $m \neq \ell$.
The regions S_ℓ^D maximize their joint tax revenue given by

$$S_\ell^D \tau_{S_\ell^D} \left(\overline{k} + \frac{\overline{\tau} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{N} + \frac{S_\ell^D \tau_{S_\ell^D}}{N}}{2} - \frac{\tau_{S_\ell^D}}{2} \right)$$

by deviating. The derivative of the joint tax revenue is

$$\begin{aligned} &\overline{k} + \frac{\overline{\tau} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{N} + \frac{S_\ell^D \tau_{S_\ell^D}}{N}}{2} - \frac{\tau_{S_\ell^D}}{2} + \tau_{S_\ell^D} \left(\frac{S_\ell^D - N}{2N} \right) \\ &= \overline{k} + \frac{\overline{\tau}}{2} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{2N} + \tau_{S_\ell^D} \left(\frac{S_\ell^D - N}{N} \right). \end{aligned}$$

Solving the first order condition we get the optimal deviation tax rate

$$\tau_{\mathbf{S}_\ell}^{S_\ell^D} = \frac{N}{N - S_\ell^D} \left(\overline{k} + \frac{\overline{\tau}}{2} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{2N} \right).$$

Capital demand is given by

$$\overline{k} + \frac{\overline{\tau} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{N} + \frac{S_\ell^D \tau_{\mathbf{S}_\ell}^{S_\ell^D}}{N}}{2} - \frac{\tau_{\mathbf{S}_\ell}^{S_\ell^D}}{2}$$

The tax revenue is then obtained by:

$$\begin{aligned} R_{\mathbf{S}_\ell}^{S_\ell^D} &= \frac{N}{N - S_\ell^D} \left(\overline{k} + \frac{\overline{\tau}}{2} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{2N} \right) \left(\overline{k} + \frac{\overline{\tau}}{2} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{2N} - \tau_{S_\ell^D}^D \left(\frac{N - S_\ell^D}{2N} \right) \right) \\ &= \frac{N}{2(N - S_\ell^D)} \left(\overline{k} + \frac{\overline{\tau}}{2} - \frac{S_\ell^D \tau_{\mathbf{S}_\ell}}{2N} \right)^2 \end{aligned}$$

Using the definition of $\tau_{\mathbf{S}_\ell}$ from (12) and $\overline{\tau}$ from (13) the tax rate and the tax revenue can be further simplified to:

$$
\begin{aligned}
\tau_{\mathbf{s}_\ell}^{S_\ell^D} &= \frac{N}{N - S_\ell^D}\left(\overline{k} + \left(\frac{\alpha}{1-\alpha}\right)\overline{k} - \left(\frac{S_\ell^D}{2N - S_\ell}\right)\left(\frac{1}{1-\alpha}\right)\overline{k}\right) \\
&= \frac{N}{N - S_\ell^D}\left(\left(\frac{1}{1-\alpha}\right)\overline{k} - \left(\frac{S_\ell^D}{2N - S_\ell}\right)\left(\frac{1}{1-\alpha}\right)\overline{k}\right) \\
&= \frac{N\overline{k}}{N - S_\ell^D}\left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell}\right)\left(\frac{1}{1-\alpha}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
R_{\mathbf{s}_\ell}^{S_\ell^D} &= \frac{N}{2(N - S_\ell^D)}\left(\overline{k} + \frac{\overline{\tau}}{2} - \frac{S_\ell^D \tau_{\mathbf{s}_\ell}}{2N}\right)^2 \\
&= \frac{N\overline{k}^2}{2(N - S_\ell^D)}\left(\frac{2N - S_\ell - S_\ell^D}{2N - S_\ell}\right)^2\left(\frac{1}{1-\alpha}\right)^2.
\end{aligned}
$$

A.4 Computation of the minimum discount factor

To compute the minimum discount factor we first look at the numerator and obtain

$$
\begin{aligned}
R_{\mathbf{S}_\ell}^{S_\ell^D} - R_{\mathbf{S}_\ell} &= \frac{N\bar{k}^2}{2(N-S_\ell^D)}\left(\frac{2N-S_\ell-S_\ell^D}{2N-S_\ell}\right)^2\left(\frac{1}{1-\alpha}\right)^2 - 2N\bar{k}^2\frac{(N-S_\ell)}{(1-\alpha)^2(2N-S_\ell)^2} \\
&= \frac{N\bar{k}^2}{(1-\alpha)^2(2N-S_\ell)^2}\left(\frac{(2N-S_\ell-S_\ell^D)^2}{2(N-S_\ell^D)} - 2(N-S_\ell)\right) \\
&= \frac{N\bar{k}^2}{(1-\alpha)^2(2N-S_\ell)^2}\left(\frac{(S_\ell-S_\ell^D)^2}{2(N-S_\ell^D)}\right) \\
&= \frac{N\bar{k}^2(S_\ell-S_\ell^D)^2}{(1-\alpha)^2(2N-S_\ell)^2 2(N-S_\ell^D)}.
\end{aligned}
$$

For the denominator we get

$$
\begin{aligned}
R_{\mathbf{S}_\ell}^{S_\ell^D} - R_{\mathbf{S}_\ell^{\mathbf{D}}}^P &= \frac{N\bar{k}^2}{2(N-S_\ell^D)}\left(\frac{2N-S_\ell-S_\ell^D}{2N-S_\ell}\right)^2\left(\frac{1}{1-\alpha}\right)^2 - \frac{2N\bar{k}^2}{N-1} \\
&= N\bar{k}^2\left(\frac{1}{2}\left(\frac{2N-S_\ell-S_\ell^D}{2N-S_\ell}\right)^2\left(\frac{1}{N-S_\ell^D}\right)\left(\frac{1}{1-\alpha}\right)^2 - \left(\frac{2}{N-1}\right)\right) \\
&= N\bar{k}^2\left(\frac{(2N-S_\ell-S_\ell^D)^2}{2(2N-S_\ell)^2(N-S_\ell^D)(1-\alpha)^2} - \frac{2}{(N-1)}\right) \\
&= N\bar{k}^2\left(\frac{(2N-S_\ell-S_\ell^D)^2(N-1) - 4(2N-S_\ell)^2(N-S_\ell^D)(1-\alpha)^2}{(N-S_\ell^D)2(2N-S_\ell)^2(1-\alpha)^2(N-1)}\right) \\
&= \frac{N\bar{k}^2\left((2N-S_\ell-S_\ell^D)^2(N-1) - 4(2N-S_\ell)^2(N-S_\ell^D)(1-\alpha)^2\right)}{(N-S_\ell^D)2(2N-S_\ell)^2(1-\alpha)^2(N-1)}.
\end{aligned}
$$

Hence, the minimum discount factor is given by:

$$
\begin{aligned}
\delta_{\mathbf{S}_\ell}^{S_\ell^D} &= \frac{R_{\mathbf{S}_\ell}^{S_\ell^D} - R_{\mathbf{S}_\ell}}{R_{\mathbf{S}_\ell}^{S_\ell^D} - R_{\mathbf{S}_\ell^{\mathbf{D}}}^P} \\
&= \frac{(S_\ell-S_\ell^D)^2(N-1)}{(2N-S_\ell-S_\ell^D)^2(N-1) - 4(2N-S_\ell)^2(1-\alpha)^2(N-S_\ell^D)}
\end{aligned}
$$

B Further Results

The equilibrium tax rates in equation (12) lead to another result:[15]

Proposition 12. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $3 \leq M \leq N$*. If two coalitions merge, all coalitions raise their equilibrium tax rate. Moreover, the merged coalition raises its equilibrium tax rate above average.*

If only two coalitions merge while all others remain unchanged, this has a positive external effect on the tax rates of all coalitions due to a less competitive environment. This effect also applies to the merging coalitions but for them there is an additional, second effect because of their new, larger size. The first effect, the influence of the competitive environment described by the coalition structure, is given by the factor α, see Lemma 1. The second effect is due to the change in the coalition size which only influences ceteris paribus the merging coalitions. Therefore, the merging coalitions raise their tax rate disproportionally, namely above average.

Proof. The equilibrium tax rates are given by

$$\tau_{\mathbf{S_m}} = 2N\bar{k}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{2N - S_m}\right)$$

for $m = 1, ..., M$ with $\alpha = \sum_{m=1}^{M} \frac{S_m}{2N - S_m}$. The equilibrium tax rate depends the size of the own coalition and the factor α. Without loss of generality suppose coalition $\mathbf{S_{M-1}}$ and coalition $\mathbf{S_M}$ decide to form one coalition. Therefore, the equilibrium tax rates of regions $1, ..., M-2$ are only affected by the change of the factor α. We have already shown that α strictly increases if two coalitions merge (see Lemma 1). Moreover, note that for the merged coalition $\mathbf{S_{M-1}} \cup \mathbf{S_M}$ we have

$$\frac{N}{2N - S_{M-1} - S_M} > \frac{N}{2N - S_M}$$

and

$$\frac{N}{2N - S_{M-1} - S_M} > \frac{N}{2N - S_{M-1}}.$$

Thus the coalition $\mathbf{S_{M-1}} \cup \mathbf{S_M}$ raises its equilibrium tax rate above average. □

[15] This Proposition and the succeeding one are in line with Bucovetsky (2009).

To sum up, the effect of regions forming coalitions is an overall increase of taxes, i.e., the sum of the tax revenues for all regions increases. The next proposition shows: If two coalitions decide to merge, then this has a positive effect on the equilibrium tax revenues of all other regions, that are not involved in the merger.

Proposition 13. *Given a coalition structure* $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ *with* $3 \leq M \leq N$. *If two coalitions merge, the tax revenue of the regions outside the merging coalitions increases.*

Proof. The equilibrium tax revenues are given by

$$2N\bar{k}^2 \frac{(N - S_m)}{(1-\alpha)^2(2N - S_m)^2}$$

for $m = 1, ..., M$ with $\alpha = \sum_{m=1}^{M} \frac{S_m}{2N - S_m}$.
Assume two coalitions merge. Without loss of generality suppose coalition $\mathbf{S_{M-1}}$ and coalition $\mathbf{S_M}$ decide to form one coalition. Similarly to the equilibrium tax rate the equilibrium tax revenue depends the size of the own coalition and the factor α. Hence, the tax rates of regions $1, ..., M-2$ are only affected by the change of the factor α. Lemma 1 already establishes that α increases if two coalitions merge. □

While it is clear from Proposition 13, that all regions increase their tax rates, it is not for sure that the merger of two coalitions ultimately benefits the regions in the merging coalitions. However, all those regions which are not involved in the merger gain from this process. As the sum of the tax revenue increases over all regions, this might produce an incentive to cooperate for the regions in the merging coalitions. At this point of the model it is not yet clear if the members of the merging coalitions gain without receiving a possible transfer from the other regions. This result is in line with Bucovetsky (2009), Proposition 5. Note that he shows in Proposition 6 that, if regions of different size merge, only the smaller of those merging regions definitely gains from cooperation.

C Maximizing Welfare

The governments's objective function determines the strategic game between countries considerably. Therefore, the assumption that regions maximize tax revenues is not innocent. In this section, we consider a numerical example for welfare maximization to get an idea whether our main results hold.

C.1 Cooperative behavior

In line with Cardarelli et al. (2002), Itaya et al. (2008) or Devereux et al. (2008) we assume that the regional government's objective is to maximize a linear utility function which depends on overall (private and public) consumption in society: Private consumption for region i is given by the sum of the labor income and capital income

$$C_i^{private}(\tau) = f(k_i) - k_i f'(k_i) + r\overline{k}.$$

Public consumption is given by the tax revenues

$$C_i^{public}(\tau) = \tau_i k_i.$$

In equilibrium, the welfare in region i is given by overall consumption (private and public) in region i with a marginal cost of public funds $\gamma > 1$:

$$\begin{aligned} W_i(\tau) &= C_i^{private}(\tau) + \gamma C_i^{public}(\tau) \\ &= f(k_i^*(\tau)) - k_i^*(\tau) f'(k_i^*(\tau)) + r^*(\tau)\overline{k} + \gamma \tau_i k_i^*(\tau) \\ &= (k_i^*(\tau))^2 + r^*(\tau)\overline{k} + \gamma \tau_i k_i^*(\tau). \end{aligned}$$

Given a coalition structure $\{\mathbf{S_1}, ..., \mathbf{S_M}\}$ with $2 \leq M \leq N$. For coalition $\mathbf{S_m}$ the objective function is given by

$$\begin{aligned} \sum_{h \in \mathbf{S_m}} W_h(\tau) &= \sum_{h \in \mathbf{S_m}} (k_h^*(\tau))^2 + r^*(\tau)\overline{k} + \gamma \tau_{\mathbf{S_m}} k_h^*(\tau) \\ &= S_m \left(\left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right)^2 + \left(A - 2\overline{k} - \overline{\tau} \right) \overline{k} + \gamma \tau_{\mathbf{S_m}} \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right) \right) \end{aligned}$$

Taking the derivative with respect to $\tau_{\mathbf{S_m}}$ we obtain

$$\begin{aligned} & S_m \left(2 \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right) \left(\frac{S_m}{2N} - \frac{1}{2} \right) - \frac{S_m}{N} \overline{k} + \gamma \left(\overline{k} + \frac{\overline{\tau} - \tau_{\mathbf{S_m}}}{2} \right) + \gamma \tau_{\mathbf{S_m}} \left(\frac{S_m}{2N} - \frac{1}{2} \right) \right) \\ & \quad = S_m \left((\gamma - 1)\overline{k} + \frac{S_m + N(\gamma - 1)}{2N} \overline{\tau} + \frac{S_m(\gamma - 1) + N(1 - 2\gamma)}{2N} \tau_{\mathbf{S_m}} \right) \end{aligned}$$

Equalizing to 0 leads to

$$\tau_{\mathbf{S_m}} = \frac{2N}{N(2\gamma-1) - S_m(\gamma-1)} \left((\gamma-1)\overline{k} + \frac{S_m + N(\gamma-1)}{2N}\overline{\tau} \right)$$
$$= \frac{2N(\gamma-1)\overline{k}}{N(2\gamma-1) - S_m(\gamma-1)} + \frac{S_m + N(\gamma-1)}{N(2\gamma-1) - S_m(\gamma-1)}\overline{\tau}$$

Summing up over all coalitions and dividing by N gives

$$\overline{\tau} = \sum_{m=1}^{M} \frac{S_m 2(\gamma-1)\overline{k}}{N(2\gamma-1) - S_m(\gamma-1)} + \sum_{m=1}^{M} \frac{S_m(S_m + N(\gamma-1))}{N(N(2\gamma-1) - S_m(\gamma-1))}\overline{\tau}$$

$\overline{\tau}$ can be computed as follows:

$$\overline{\tau} = 2\overline{k} \frac{\sum_{m=1}^{M} \frac{S_m(\gamma-1)}{N(2\gamma-1)-S_m(\gamma-1)}}{1 - \sum_{m=1}^{M} \frac{S_m(S_m+N(\gamma-1))}{N(N(2\gamma-1)-S_m(\gamma-1))}}$$
$$= 2\overline{k} \frac{\sum_{m=1}^{M} \frac{S_m(\gamma-1)}{N(2\gamma-1)-S_m(\gamma-1)}}{1 - \sum_{m=1}^{M} \frac{S_m}{N} \frac{S_m}{N(2\gamma-1)-S_m(\gamma-1)} - \sum_{m=1}^{M} \frac{S_m(\gamma-1)}{N(2\gamma-1)-S_m(\gamma-1)}}$$

Define

$$\alpha := \sum_{m=1}^{M} \frac{(\gamma-1)S_m}{N(2\gamma-1) - S_m(\gamma-1)},$$
$$\beta := \sum_{m=1}^{M} \frac{S_m}{N} \frac{S_m}{N(2\gamma-1) - S_m(\gamma-1)}.$$

$\overline{\tau}$ can be then be written in a more simpler way:

$$\overline{\tau} = 2\overline{k}\frac{\alpha}{1-\alpha-\beta}.$$

Computing $\tau_{\mathbf{S_m}}$ we get

$$\tau_{\mathbf{S_m}} = 2\overline{k} \left(\frac{N(\gamma-1)(1-\beta) + \alpha S_m}{N(2\gamma-1) - S_m(\gamma-1)} \right) \left(\frac{1}{1-\alpha-\beta} \right).$$

For the capital demand we get

$$k^*_{\mathbf{S_m}}(\tau) = \overline{k}\frac{1}{1-\alpha-\beta} \left(\frac{N\gamma(1-\beta) - (\alpha + (\gamma-1)(1-\beta))\, S_m}{N(2\gamma-1) - S_m(\gamma-1)} \right).$$

C.2 Repeated interaction

Suppose sub-coalition $\mathbf{S}_\ell^\mathbf{D} \subsetneq \mathbf{S}_\ell$ considers to deviate from the coalitional equilibrium. We determine the optimal deviation tax rate of region j and the respective capital demand.
For the deviation tax rate observe that

$$\frac{\partial\left[(k^*_{\mathbf{S}_\ell^\mathbf{D}}(\tau))^2 + r^*(\tau)\overline{k} + \gamma\tau_{\mathbf{S}_\ell^\mathbf{D}} k^*_{\mathbf{S}_\ell^\mathbf{D}}(\tau)\right]}{\partial\tau_{\mathbf{S}_\ell^\mathbf{D}}}$$

$$= 2\left(\overline{k} + \frac{\overline{\tau} - \frac{S_\ell^D\tau_{\mathbf{S}_\ell}}{N} + \frac{S_\ell^D\tau_{\mathbf{S}_\ell^\mathbf{D}}}{N} - \tau_{\mathbf{S}_\ell^\mathbf{D}}}{2}\right)\left(\frac{S_\ell^D - N}{2N}\right) - \frac{S_\ell^D}{N}\overline{k}$$

$$+ \gamma\left(\overline{k} + \frac{\overline{\tau} - \frac{S_\ell^D\tau_{\mathbf{S}_\ell}}{N} + \frac{S_\ell^D\tau_{\mathbf{S}_\ell^\mathbf{D}}}{N} - \tau_{\mathbf{S}_\ell^\mathbf{D}}}{2}\right) + \gamma\tau_{\mathbf{S}_\ell^\mathbf{D}}\left(\frac{S_\ell^D - N}{2N}\right)$$

$$= (\gamma-1)\overline{k} + \left(\frac{S_\ell^D + N(\gamma-1)}{N}\right)\left(\frac{\overline{\tau}}{2} - \frac{S_\ell^D\tau_{\mathbf{S}_\ell}}{2N}\right) + \left(\frac{S_\ell^D + N(2\gamma-1)}{2N}\right)\left(\frac{S_\ell^D - N}{N}\right)\tau_{\mathbf{S}_\ell^\mathbf{D}}.$$

Solving the first order condition we get the optimal deviation tax rate given by

$$\tau_{\mathbf{S}_\ell^\mathbf{D}} = \left(\frac{2N}{S_\ell^D + N(2\gamma-1)}\right)\left(\frac{N}{N - S_\ell^D}\right)\left((\gamma-1)\overline{k} + \left(\frac{S_\ell^D + N(\gamma-1)}{N}\right)\left(\frac{\overline{\tau}}{2} - \frac{S_\ell^D\tau_{\mathbf{S}_\ell}}{2N}\right)\right)$$

$$= \left(\frac{N}{N - S_\ell^D}\right)\left(\frac{2N(\gamma-1)}{S_\ell^D + N(2\gamma-1)}\overline{k} + \left(\frac{2S_\ell^D + 2N(\gamma-1)}{S_\ell^D + N(2\gamma-1)}\right)\left(\frac{\overline{\tau}}{2} - \frac{S_\ell^D\tau_{\mathbf{S}_\ell}}{2N}\right)\right).$$

Capital demand is given by

$$\overline{k} + \frac{\overline{\tau} - \frac{S_\ell^D\tau_{\mathbf{S}_\ell}}{N} + \frac{S_\ell^D\tau_{\mathbf{S}_\ell^\mathbf{D}}}{N}}{2} - \frac{\tau_{\mathbf{S}_\ell^\mathbf{D}}}{2}$$

$$= \frac{S_\ell^D + N\gamma}{S_\ell^D + N(2\gamma-1)}\overline{k} + \frac{N\gamma}{S_\ell^D + N(2\gamma-1)}\left(\frac{\overline{\tau}}{2} - \frac{\tau_{\mathbf{S}_\ell}}{2N}\right).$$

The welfare can then be computed using the equilibrium tax rate and capital demand. We will stick to a numerical example in what follows.

C.3 A numerical example for five regions

For the repeated game with five regions we employ different levels for the marginal costs of public funds γ. Results are reported in Table 6.

coalition structure	discount factor		overall welfare
$\gamma = 1.1$			
$\{\{1\}\,\{2345\}\}$	$\delta^1_{\{2345\}} = 0.6286$ $\delta^2_{\{2345\}} = 0.6167$ $\delta^3_{\{2345\}} = 0.5652$	$\delta = 0.6286$	$5\bar{k}A - 4.7370\bar{k}^2$
$\{\{12\}\,\{345\}\}$	$\delta^1_{\{12\}} = 0.0729$ $\delta^1_{\{345\}} = 0.3481$ $\delta^2_{\{345\}} = 0.3062$	$\delta = 0.3481$	$5\bar{k}A - 4.8122\bar{k}^2$
$\{\{12\}\,\{34\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.1478$ $\delta^1_{\{34\}} = 0.1478$	$\delta = 0.1478$	$5\bar{k}A - 4.8585\bar{k}^2$
$\{\{12\}\,\{3\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.3713$	$\delta = 0.3713$	$5\bar{k}A - 4.8744\bar{k}^2$
$\{\{123\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{123\}} = 0.4734$ $\delta^2_{\{123\}} = 0.4263$	$\delta = 0.4734$	$5\bar{k}A - 4.8422\bar{k}^2$
$\gamma = 2$			
$\{\{1\}\,\{2345\}\}$	$\delta^1_{\{2345\}} = 0.5648$ $\delta^2_{\{2345\}} = 0.5581$ $\delta^3_{\{2345\}} = 0.5198$	$\delta = 0.5648$	$5\bar{k}A + 10.4000\bar{k}^2$
$\{\{12\}\,\{345\}\}$	$\delta^1_{\{12\}} = 0.0567$ $\delta^1_{\{345\}} = 0.2984$ $\delta^2_{\{345\}} = 0.2680$	$\delta = 0.2984$	$5\bar{k}A + 5.4000\bar{k}^2$
$\{\{12\}\,\{34\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.1197$ $\delta^1_{\{34\}} = 0.1197$	$\delta = 0.1197$	$5\bar{k}A + 2.8099\bar{k}^2$
$\{\{12\}\,\{3\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.2819$	$\delta = 0.2819$	$5\bar{k}A + 1.9401\bar{k}^2$
$\{\{123\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{123\}} = 0.3955$ $\delta^2_{\{123\}} = 0.3604$	$\delta = 0.3955$	$5\bar{k}A + 3.8889\bar{k}^2$

coalition structure	discount factor		overall welfare
$\gamma = 10$			
$\{\{1\}\,\{2345\}\}$	$\delta^1_{\{2345\}} = 0.5143$ $\delta^2_{\{2345\}} = 0.5107$ $\delta^3_{\{2345\}} = 0.4822$	$\delta = 0.5143$	$5\bar{k}A + 255.8181\bar{k}^2$
$\{\{12\}\,\{345\}\}$	$\delta^1_{\{12\}} = 0.0464$ $\delta^1_{\{345\}} = 0.2618$ $\delta^2_{\{345\}} = 0.2387$	$\delta = 0.2618$	$5\bar{k}A + 164.3199\bar{k}^2$
$\{\{12\}\,\{34\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.1010$ $\delta^1_{\{34\}} = 0.1010$	$\delta = 0.1010$	$5\bar{k}A + 121.8140\bar{k}^2$
$\{\{12\}\,\{3\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{12\}} = 0.2296$	$\delta = 0.2296$	$5\bar{k}A + 107.7871\bar{k}^2$
$\{\{123\}\,\{4\}\,\{5\}\}$	$\delta^1_{\{123\}} = 0.3418$ $\delta^2_{\{123\}} = 0.3147$	$\delta = 0.3418$	$5\bar{k}A + 141.4286\bar{k}^2$

Table 6: Discount factors for $N = 5$ with welfare maximization.

There is a good indication that the objective function that we have chosen is not detrimental to our main results. Again, the comparison of all possible coalition structures for $N = 5$ shows that the most stable coalition structure, in the sense of the lowest discount factor, is not the one with the largest overall welfare.