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Efficient Wage Bargaining in a Dynamic Macroeconomic Model

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Abstract

This paper analyzes the implications of bilateral bargaining over wages and employment between a producer and a union representing a finite number of identical workers in a monetary macroeconomic model of the AS–AD type with government activity. Wages and aggregate employment levels are set according to an efficient (Nash) bargaining agreement while the commodity market is cleared in a competitive way. It is shown that, for each level of union power, measured by the share it obtains of the total production surplus, efficient bargaining implies no efficiency loss in production. Depending on the level of union power, temporary equilibria may exhibit voluntary overemployment or underemployment with the competitive equilibrium being a special case.

Due to the price feedback from the commodity market and to income-induced demand effects, all temporary equilibria with a positive labor share are not Nash bargaining-efficient with respect to the set of feasible temporary equilibrium allocations. While higher union power induces a larger share of the surplus and a higher real wage, it always implies lower output and employment. Moreover, the induced nominal equilibrium wage is not always a monotonically increasing function of union power. Therefore, all temporary equilibria with efficient bargaining are only "Second-best" Pareto optimal, i. e. bargaining power and production efficiency do not lead to temporary optimality.

The dynamic evolution of money balances, prices, and wages is analyzed being driven primarily by government budget deficits and expectations by consumers. It is shown that for each fixed level of union power, the features of the dynamics under perfect foresight are structurally identical to those of the same economy under competitive wage and price setting. These are: stationary equilibria with perfect foresight do not exist, except on a set of parameters of measure zero; balanced paths of monetary expansion or contraction are the only possibilities inducing constant allocations; for small levels of government demand, there exist two balanced paths generically, one of which with high employment and production is always unstable, while the other one may be stable or unstable.

Keywords: Efficient Collective Bargaining, Union Power, Monopolistic Wage Determination, Aggregate Demand–Aggregate Supply, Employment, Prices, Wages, Inflation, Expectations, Government Deficits, Monetary Expansion, Perfect Foresight, Dynamics, Stability
JEL Classification: C78, D33, D42, D43, D58, D61, E24, E25, E31, E41, E42, J42, J52

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Contents

1 Introduction

In spite of the fact that in most industrialized countries negotiations between workers unions and syndicates of producers about wage levels and employment conditions occur regularly, their economic significance for the labor market or even more for the evolution of the macroeconomy as a whole is often neglected in the research on labor markets.[1] Taking the number of articles on the subject in the recent *Handbook of Labor Economics* by Ashenfelter & Card (2011a, b) relative to other contributions therein as an indicator, it seems that other theories are considered as more relevant and the motivation to study the impact of bargaining between the two sides on a particular market are not at the forefront of the research in labor economics. Among the many possible macroeconomic models which determine wage and employment levels, those which take a bargaining approach between a producers conglomerate and a workers union are clearly in the minority. This is in contrast to the general empirical observation that such negotiations are observable recurring annual events in most Western economies which induce legally binding agreements which are adhered to in these economies.

Considering the theoretical models of bargaining between groups (as opposed to other wage-employment-determining procedures)[2] from a general microeconomic perspective, the importance of strategic aspects in wage and employment negotiations are well recognized and have been studied extensively. The literature contains several contributions applying game-theoretic notions and concepts (see for example McDonald & Solow 1981; Landmann & Jerger 1999; Gerber & Upmann 2006). However, most of them ignore cross-market effects and carry out the analysis in a partial-equilibrium setting. Thus, any spillovers from other markets or from the income distribution on the general-equilibrium or macroeconomic level are rarely discussed or analyzed, which reduces the validity of their results as contributions to macroeconomics.

One explanation for the lack of more extended game-theoretic considerations in macroeconomic models may lie in the limitations of the game-theoretic approaches and their models themselves. Two essential aspects may explain this absence:

1. the interaction of the labor market with the rest of the economy, and
2. the dynamic aspect of recurring negotiations, of time, and of uncertainty.

With respect to the first point, the existing theories are built primarily on the common principle of bargaining as an allocation device of how to divide a cake of *given* size. If there were strong empirical evidence or a convincing theoretical argument that in fact in most market economies the labor market is a sufficiently independent and isolated unit within the economy, whose rules and allocation principles have little influence on “the size of the cake”, i. e. on GNP, then the underlying premise of a given constant cake would be justified, and the distributive aspects could be separated from the allocative issues on the national level. However, most economists would agree that there are major allocative mechanisms originating from labor market rules to the macroeconomic level. Such spillovers or feedback effects play a role in determining the size of GNP. In addition, most game theorists would also agree that many applications of bargaining theories assume too naively that the negotiations are directed toward *outcomes* to be distributed. In most situations, however, bargaining agreements consist of principles or rules in an allocative environment. Outcomes are the consequences *after* the behavioral

[1] In contrast, the social and legal aspects of wage contracts, of hiring and firing are discussed and analyzed to a large degree.

[2] such as efficiency wages, search theory, matching theory, etc.

response of agents across markets. In other words, outcomes result after the feedbacks between markets take place and the final outcome like GNP and its distributive parts are endogenously determined.[3]

There are always behavioral responses originating from demand and supply behavior, from outside options, and in particular from the feedback effects from other markets and through income effects. Thus, macroeconomic outcomes are the result *after* behavioral consequences in the markets and the spillovers induced, implying that the size of the cake depends on the rules set in the negotiations. Therefore, much of standard bargaining theory may not even be applicable in such cases or has to be reevaluated. It provides essentially a *static* solution concept and framework for negotiations with no consideration for interaction or feedback with an environment or model. Considerations for implications for *outcomes* after induced changes of the environment including the feedback are absent.

For the dynamic implications of repeated negotiations occurring in macroeconomic systems, game theory again does not provide modeling approaches at a satisfactory level to be applied suitably to labor markets. The issues to be solved in a setting of repeated negotiations open a wide range of unsolved problems as to the dynamic setting of the negotiation, the negotiators, the environment, the state variables, and the information, uncertainty, and stochastic shocks. Again, with the cross-market feedbacks playing a qualitative role, the negotiations and their procedures will have an influence on the dynamic evolution of the economy.

The literature on the usage of efficient bargaining taking a macroeconomic perspective is not sizable.[4] McDonald & Solow (1981) study noncompetitive wage setting in partial equilibrium models with capacity-constrained, fully unionized labor markets with one firm and one union. Inter alia, they analyze the cases of the monopolistic union (with the right to manage of the firm) as well as two types of efficient bargaining over wages and employment using the symmetric Nash resp. the Kalai–Smorodinsky bargaining solutions. The agents' objective functions are the profit of the firm resp. the expected excess indirect utility of the representative union member. Indirect utility is measured in nominal wages for a constant reservation wage.[5]

Booth (1996) and Landmann & Jerger (1999) are two prominent presentations addressing and discussing the efficient bargaining solution explicitly in a format which is the closest to the one proposed here. Booth (1996) slightly extends the setting by McDonald & Solow (1981) by applying the generalized Nash bargaining solution while analyzing bargaining over wages alone. This leaves the employment decision to the firm which corresponds to the so called right-to-manage model. Her modeling generalizes the monopolistic-union model and shows that the resulting outcome is not Pareto efficient in a static partial-equilibrium setting.

Landmann & Jerger (1999) present the efficient bargaining model where intertemporal aspects or money plays no role. They present a partial-equilibrium analysis only by assuming fixed

[3]There are many examples from empirical agreements which confirm this fact. For example wage laws for union members, indexed wages rules, minimum wage laws. Trade agreements among countries specify principles of a free trade: no tariffs or duties, no discrimination rules, harmonization of taxes, as in the EU. Financial/monetary principles in a monetary union specify a common currency, mutual free exchange, like IMF, ECB. Cartel agreements specify rules prescribing *dos and don'ts*.

[4]We are not aware of any publications analyzing the role of efficient bargaining and spillovers across markets nor of the dynamics in a closed macromodel.

[5]There are some contributions dealing with specific dynamic or policy issues within models of capital accumulation, as for example Devereux & Lockwood (1991); Kaas & von Thadden (2004); Gerber & Upmann (2006); Koskela & Puhakka (2006) within nonmonetary models. Gertler & Trigari (2009) presents an interesting combination of a market with matching and staggered Nash bargaining in an empirically oriented model.

prices throughout with no analysis of the demand side of the economy or the effects from the income distribution. Moreover, no comparative statics analysis of the role of union power and their implications for allocations is performed.

This paper starts from the general premise that there are significant feedbacks to be studied, which are shown to exist in the standard AS–AD model of a monetary macroeconomy. It analyzes the most innocuous so-called efficient bargaining solution for the labor market as a benchmark model, which assumes the most cooperative structure and solution concept from a strategic point of view. While the literature agrees that this solution concept is empirically the most unlikely, its implications for the macroeconomy must be examined, in particular whether it induces the qualitative properties of efficiency and optimality which the literature seems to assign to it.

The paper derives the structure of the temporary price feedback and discusses the full comparative statics of varying union power, indicating that, in spite of the application of the efficiency criterion used in the labor market separately, the efficiency criterion as well as Pareto optimality fails on the macrolevel. It compares the allocative consequences with other strategic solutions of noncooperative behavior of producers and the union. Finally, the dynamic consequences for allocations and the stability of the evolution under perfect foresight are investigated.

2 The Labor Market with Efficient Bargaining

Consider an economy in discrete time with three markets: a labor market, a commodity market, and a money market, and three sectors: a consumption sector, a production sector, and the public sector consisting of a central government and a central bank.[6]

2.1 The Public Sector

The public sector consists of a government and a central bank. The government demands the produced commodity at a level $g \geq 0$ to produce public goods and services. These are assumed to be pure public goods providing a constant level of utility each period to each type of consumer. In addition, consumer preferences are assumed to be additively separable with respect to the level of the public good so that these do not induce marginal or behavioral effects by consumers. Therefore, the constant level of public services can be and was dropped as an argument in consumer utility functions.

To finance its consumption (the public good's production) the government levies a proportional tax on profits at the rate $0 \leq \tau_\pi \leq 1$ and on wages at the rate $0 \leq \tau_w \leq 1$. Since the government parameters are assumed to be given parametrically in each period, in general, the government budget is not balanced since incomes are endogenously determined. Therefore, the central bank creates/destroys the amount of money according to the need of the government arising from the unbalanced budget. Since money is the only intertemporal store of value held by consumers, any increase (decrease) of the amount of money required to balance the budget of

[6] The model chosen is a standard version of an AS–AD model based on microeconomic principles and embedded in an economy with cohorts of overlapping generations of consumers (see for example Böhm 2010).

the government is equivalent to the amount of savings (changes of the amount of money held by the private sector) in any given period.[7]

2.2 The Production Sector

The nonstorable commodity is produced from labor only by a single profit-maximizing firm.[8] The technology of the single producing firm is described by a differentiable monotonically increasing and concave production $F : \mathbb{R} \to \mathbb{R}$, $L \mapsto F(L)$ satisfying $F(0) = 0$ and the usual Inada conditions which implies that the technical equipment or the stock of capital is constant and does not depreciate.

At a given nominal wage rate $w \geq 0$ for labor and a sales price $p \geq 0$ for the commodity, a production decision L implies current profits $\Pi(p, w, L) := pF(L) - wL$. All profits are paid to consumers, who are the owners or the shareholders of the firm. There is no intertemporal decision making of the producer with no need to retain profits nor to hold money. Therefore, the firm's objective is to maximize profits. Under competitive conditions with prices and wages given, the behavior of the firm in each period in the two markets induces the usual profit-maximizing labor demand function

$$h_{\text{com}}\left(\frac{w}{p}\right) := \arg\max_{L \geq 0}\{pF(L) - wL\} = (F')^{-1}\left(\frac{w}{p}\right) \tag{1}$$

and the commodity supply function $F(h_{\text{com}}(w/p))$.

In noncompetitive situations, in particular under bargaining, pairs (L, w) of employment and wage levels have to guarantee nonnegative profits $\Pi(p, w, L) \geq 0$ for the producer. Therefore, the zero-profit contour implies the participation constraint for the producer

$$w \leq p\frac{F(L)}{L} =: W_{\Pi}(p, L),$$

which defines his reservation wage as a function of the employment level $L > 0$.

2.3 The Consumption Sector

The consumption sector consists of overlapping generations of two types of homogeneous consumers. There are $n_w \geq 1$ workers and $n_s \geq 1$ shareholders in each generation, both of which live for two consecutive periods. The size and composition of the two groups is constant through time implying that at any one time, there are $n_s + n_w$ young resp. old consumers.

Each shareholder consumer receives net profits only in the first period of his life. He spends the proportion $0 < c(\theta^e) < 1$ in the first period and saves the rest in the form of money to be spent on consumption in the second period. Money is the only intertemporal store of value for

[7] To save on notation, we omit, wherever possible, the government parameters g, τ_w, and τ_π in all arguments throughout this paper. When analyzing behavior and markets in any particular period, it is always assumed that money holdings $M \geq 0$ and price expectations $p^e > 0$ are given at the beginning and remain fixed during the period, except when their comparative statics effects are discussed.

[8] This assumption is made for simplicity only, the extension to multiple homogeneous firms organized in a producers association is straightforward.

consumers which carries no interest. Therefore, his consumption/savings decision depends on the expected rate of inflation $\theta^e := p^e/p$.

Each worker supplies labor in the first period of his life to consume in the second period only. His preferences with respect to planned future consumption $c^e \geq 0$ and work $\ell \geq 0$ when young are described by an intertemporal utility function of the form $u(\ell, c^e) = c^e - v(\ell)$ where the function $v : \mathbb{R}_+ \to \mathbb{R}_+$ measures the disutility from labor. The function v is assumed to be continuously differentiable, strictly monotonically increasing, strictly convex, with $v(0) = v'(0) = 0$ and $\lim_{\ell \to \infty} v'(\ell) = \infty$.

Given a wage rate w, an employment level ℓ, and a wage tax τ_w, he saves his total nominal net wage income $(1-\tau_w)w\ell$ in the form of money, to be spent on consumption in the second period of his life. With given price expectations p^e, his planned future consumption satisfies $p^e c^e = (1-\tau_w)w\ell$. Therefore, under competitive conditions and price expectations p^e, his utility-maximizing labor supply is given by

$$\arg\max_{\ell \geq 0} \left\{ u\left(\ell, (1-\tau_w)\frac{w}{p^e}\ell\right)\right\} = (v')^{-1}\left((1-\tau_w)\frac{w}{p^e}\right),$$

which is a continuous, strictly monotonically increasing function of the expected future value of the current nominal wage.

Given the worker's price expectations $p^e > 0$, it is straightforward to define his reservation wage for noncompetitive situations. The labor market participation constraint of a worker for an acceptable employment–wage situation (ℓ, w) must provide a utility at least as high as not working when young. In other words, (ℓ, w) must be a solution of

$$u(0,0) = 0 \leq u(\ell, c^e) = u\left(\ell, (1-\tau_w)\frac{w}{p^e}\ell\right) = (1-\tau_w)\frac{w}{p^e}\ell - v(\ell).$$

This implies the lower bound of the individually acceptable wage rate, i. e. his *reservation wage,* as

$$\frac{w}{p^e} = \frac{1}{1-\tau_w}\frac{v(\ell)}{\ell}, \qquad \ell > 0 \tag{2}$$

which is a strictly increasing function of the employment level. >From these properties one defines directly the aggregate competitive labor supply as

$$N_{\text{com}}\left(\frac{w}{p^e}\right) := n_w \ell = n_w (v')^{-1}\left((1-\tau_w)\frac{w}{p^e}\right)$$

which has a global inverse given by

$$\frac{w}{p^e} = S_{\text{com}}(L) := \frac{1}{1-\tau_w} v'\left(\frac{L}{n_w}\right).$$

With equal treatment of workers one obtains the aggregate reservation wage from equation (2) as

$$\frac{w}{p^e} = S(L) := \frac{n_w}{L(1-\tau_w)} v\left(\frac{L}{n_w}\right),$$

which has an elasticity[9]

$$E_S(L) = E_v(L/n_w) - 1. \tag{3}$$

[9]For any function f we denote its elasticity at x as $E_f(x)$. Thus, $E_v(L/n_w)$ denotes the elasticity of the function v.

This implies a useful relationship between the reservation wage and the competitive inverse labor supply function

$$S_{\text{com}}(L) = E_v(L/n_w)\, S(L) \qquad \text{for all } L. \tag{4}$$

Given the characteristics of each individual young worker, the union is perceived of as an aggregate agent representing the consumer-workers consisting of all homogeneous workers. Since all workers have identical characteristics, the union's bargaining will be concerned with the determination of the wage level w and the aggregate level of employment L, assuming that all workers are treated equally, i. e. each is paid the wage w with individual employment level L/n_w.

2.4 Efficient Wage Bargaining and Employment

It is evident that one of the most challenging questions to investigate concerns the feedback effects or spillover effects between the labor market and the output market since in the closed macroeconomy the impact from wage negotiations on the income distribution will have effects on aggregate demand and therefore on output and income. Moreover, these effects will depend on the market structure chosen on either side.

The framework chosen for the wage bargaining between the union representing the consumer-workers and the producer as a wage determination device consists of an application of a bargaining solution to the *simultaneous* determination of the aggregate employment level L and of the wage rate w under the assumption that the negotiating parties, the union and the producer, are both price takers in the commodity market. With this choice it is possible to discuss best the role of bargaining in general equilibrium and compare the outcomes with the competitive case. Under efficiency considerations, choosing the Nash bargaining solution could be one possibility although in the repeated or dynamic context this may not be the fully convincing.[10] In other words, the producer and the union treat the commodity price as given, implicitly assuming that their bargaining decision has no influence on the induced equilibrium price in the short run. Thus, a *temporary equilibrium with efficient wage bargaining* is defined by a competitive price level p which equalizes aggregate supply and aggregate demand of the commodity market at which the levels of employment and wages induce the desired efficient bargaining solution between the union and the producer.

The result of the bargaining procedure between the union and the producer consists of a joint decision with respect to the employment level L and the wage rate w where the producer's goal is to maximize its net profit while the union tries to maximize the aggregate excess wage bill for the workers. Let $\Pi(p, w, L) = pF(L) - wL$ denote the net profit and $\Omega(p^e, w, L) := wL - p^e S(L)L$ the excess wage bill. Given price expectations and commodity price $(p^e, p) \gg 0$, a bargaining agreement (L, w) is called *individually rational* if Π and Ω are nonnegative. An *efficient bargaining agreement* between the union and the employer is defined in the usual way.

Definition 2.1 *Given $(p^e, p) \gg 0$, a employment–wage pair $(L, w) \in \mathbb{R}^2_+$ is called* efficient *if there exists no other pair (L', w') such that*

$$\Pi(p, w', L') \geq \Pi(p, w, L) \qquad \textit{and} \qquad \Omega(p^e, w', L') \geq \Omega(p^e, w, L)$$

with at least one strict inequality.

[10]From a game-theoretic point of view, the generalized Zeuthen solution for half-space games can be applied which is less specific than Nash; see also the remarks in the introduction and in the conclusion.

To characterize efficient agreements, one may use the associated Lagrangean function

$$\Lambda(w, L, \kappa) = \Omega(p^e, w, L) + \kappa \left(\Pi(p, w, L) - \bar{\Pi}\right)$$

and obtains the first-order conditions of an interior solution $(L, w) \gg 0$ as

$$pF'(L) = p^e(S(L) + S'(L)L), \qquad L > 0. \tag{5}$$

Any positive solution determines the same level of employment for all levels of net profit $\bar{\Pi}$. Moreover, the solution of (5) is identical with that level of employment which would clear the labor market under conditions of perfect competition between the union and the producer for any given pair $(p^e, p) \gg 0$.

This result is well-known from the literature. It occurs in situations of bargaining/cooperative decision making between any two agents who are the only participants trading in the same market, which corresponds to the situation in a vertically integrated industry, a cartel or a bilateral monopoly. In such cases, under efficiency, the two traders internalize all potential net gains and they will decide on a level of trade and price between them which maximizes the sum of their net gains. If they are both facing competitive markets upstream and downstream, the resulting level of activity between them under efficiency is identical to that level of trade which would result under competitive trading, with some mild assumptions. This level guarantees that there are no further joint gains to share. In other words, the level of trade equalizes marginal cost to marginal revenue between the two players and maximizes the cake to share. For the model here between the union and the producer, this implies that the determination of an efficient bargaining solution can be divided into two steps: the choice of the level of employment which depends on the market data upstream and downstream, and the determination of the wage which then turns out to become the central point in the bargaining procedure of sharing the net gains.

Wage Bargaining in the Bilateral Monopoly

As pointed out in the previous paragraph, the employment decision under efficient bargaining turns out to be equivalent to the standard textbook representation when the union and the producer form a bilateral monopoly. For a given price expectations and commodity price $(p^e, p) \gg 0$, the joint net gain is given by

$$\Pi(p, w, L) + \Omega(p^e, w, L) = pF(L) - wL + wL - p^e S(L)L = pF(L) - p^e S(L)L$$

is a function of the employment level alone. Thus, it is necessary that an optimal employment decision maximizes $pF(L) - p^e S(L)L$, independent of the wage decision to be taken. This induces the first-order condition

$$pF'(L) = p^e S(L)\big(E_S(L) + 1\big) \overset{(3)}{=} p^e S(L)\big(E_v(L/n_w) - 1 + 1\big) \overset{(4)}{=} p^e S_{\text{com}}(L), \tag{6}$$

which coincides with (5). Therefore, the employment decision of a bilateral monopoly maximizing joint net gain against the rest of the economy coincides with the one under efficient bargaining. Thus, the employment decision to yield the maximal joint net gain can be separated from the wage decision of how this gain is to be distributed. In this perspective, the labor market has been eliminated, the employment decision L corresponds to an internal decision of a union-producer monopoly, while the decision for the wage rate becomes a "cost allocation issue".

This separability of the employment and the wage decision can be portrayed geometrically in the associated employment–wage space (see Figure 1). For $L > 0$, an acceptable wage must be such that $\Pi \geq 0$ *and* $\Omega \geq 0$, i. e.

$$w \leq p\frac{F(L)}{L} = W_{\Pi}(p, L) \qquad \text{and} \qquad w \geq p^e S(L) =: W_{\Omega}(p^e, L),$$

inducing the two status-quo wage functions W_{Π} and W_{Ω} which correspond to the reservation wage of the producer and of the union respectively. The area between the two functions in Figure 1 defines the set of individually rational employment–wage pairs.

The set of efficient employment–wage choices under bargaining are those on the contract curve shown as the bold red line. Geometrically speaking, each point on the contract curve must

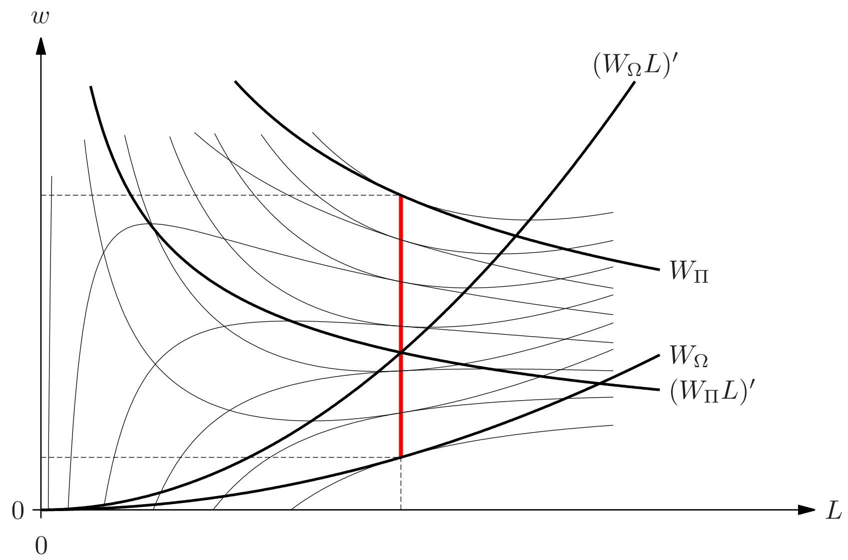


Figure 1: Determining the level of employment

be a tangency point of an iso-utility and of an iso-profit curve (the thin lines). Since all iso-utility/iso-profit curves are of the form

$$W_{\bar{\Pi}}(L) = \frac{pF(L) - \bar{\Pi}}{L} \quad \text{resp.} \quad W_{\bar{\Omega}}(L) = p^e S(L) + \frac{\bar{\Omega}}{L}$$

for all levels $\bar{\Pi}$ and $\bar{\Omega}$, the tangency condition $\partial W(L)/\partial L$ implies

$$\frac{pF'(L)L - W(L)L}{L^2} \stackrel{!}{=} p^e S'(L) - \frac{W(L) - p^e S(L)}{L}.$$

Since $F(L)$ and $-S(L)L$ are strictly concave functions satisfying the Inada conditions, the set of individually rational (L, w) is compact. Moreover, $pF(L) - p^e S(L)L$ is a strictly concave function as well. Therefore, the necessary conditions are also sufficient. Finally, given the strict concavity of both functions, the solution $L > 0$ is unique for any positive given expected inflation rate $\theta^e = p^e/p > 0$. Thus, the solution of equation (5) defines an employment function $h : \mathbb{R}_{++} \to \mathbb{R}_{++}$, $\theta^e \mapsto h(\theta^e)$. Its inverse is given explicitly by

$$\frac{p^e}{p} = \frac{F'(L)}{S(L) + S'(L)L} \stackrel{(6)}{=} \frac{F'(L)}{S_{\text{com}}(L)} := h^{-1}(L), \tag{7}$$

which is differentiable and strictly decreasing since $(h^{-1})'(L) < 0$ holds. Therefore, under efficient bargaining, the level of employment $h(\theta^e)$ is a well-defined, strictly monotonically decreasing, and invertible function of the expected inflation rate θ^e. It is homogeneous of degree zero in price expectations and prices, it is decreasing in expected prices and increasing in the current output price. In addition, the employment level chosen by the two bargaining parties is the same as the one which would result in equilibrium under a perfectly competitive labor market.

Rewriting the condition (7) using the two reservation wage functions, one obtains an intuitive and interesting relationship

$$W_\Omega(p^e, L) = p^e S(L) = \frac{E_F(L)}{E_S(L)+1}\frac{pF(L)}{L} = \frac{E_F(L)}{E_S(L)+1} W_\Pi(p, L). \tag{8}$$

for the relative shares depending on the elasticities of the reservation wage functions, which also characterizes the bargaining level of employment. This stipulates that the ratio between the two status-quo values should correspond to the ratio of their respective elasticities.

The Wage Rate under Bargaining

Given $(p^e, p) \gg 0$ and $L = h(p^e/p) > 0$, the bargaining decision between the two parties concerning the wage rate now constitutes a *bargaining game with constant transfers* since $\Pi + \Omega = pF(L) - p^e S(L)L = W_\Pi(p, L)L - W_\Omega(p^e, L)L$ is a constant sum. Thus, one obtains a special case of a bargaining problem, to which the generalized Zeuthen solution applies (see Rosenmüller 2000). For such games the bargaining power between the two parties is usually measured by a number $0 \leq \lambda \leq 1$, which defines the relative share of the total cake to be alloted to the party having "bargaining power" λ. Thus, for a constant total gain $\Pi + \Omega = W_\Pi(p, L)L - W_\Omega(p^e, L)L$, the weights $(\lambda, 1-\lambda)$ determine a linear redistribution of the total net gain among the two agents.

Therefore, with $L > 0$ and $0 \leq \lambda \leq 1$ given, an application of the generalized Zeuthen solution[11] to the total gain implies choosing the bargaining wage as a convex combination of the two reservation wage levels W_Π (when $\Pi = 0$) and W_Ω (when $\Omega = 0$) with the same weights

$$W(p^e, \lambda, p, L) = \lambda W_\Pi(p, L) + (1-\lambda) W_\Omega(p^e, L), \qquad L = h(\theta^e). \tag{9}$$

Substituting (9) into the utility and into the profit functions yields the payoff vector (Π, Ω) of the bargaining solution

$$\begin{aligned}\begin{pmatrix}\Pi(p^e,\lambda,p,L)\\ \Omega(p^e,\lambda,p,L)\end{pmatrix} &= \begin{pmatrix} pF(L) - W(p^e,\lambda,p,L)L \\ W(p^e,\lambda,p,L)L - p^e S(L)L\end{pmatrix} = \begin{pmatrix} W_\Pi(p,L)L - W(p^e,\lambda,p,L)L \\ W(p^e,\lambda,p,L)L - W_\Omega(p^e,L)L\end{pmatrix} \\ &= \big(W_\Pi(p,L) - W_\Omega(p^e,L)\big)L\begin{pmatrix}1-\lambda\\ \lambda\end{pmatrix} = \big(pF(L) - p^e S(L)L\big)\begin{pmatrix}1-\lambda\\ \lambda\end{pmatrix}.\end{aligned} \tag{10}$$

For given (p^e, p), Figure 2 displays the range of the mapping (10) for different values of the parameter λ, revealing its linear impact on the payoff distribution. A similar linear relationship

[11] Note that the generalized Zeuthen solution (which can only be applied to half-space games) coincides with the generalized Nash solution, yet requiring less properties.

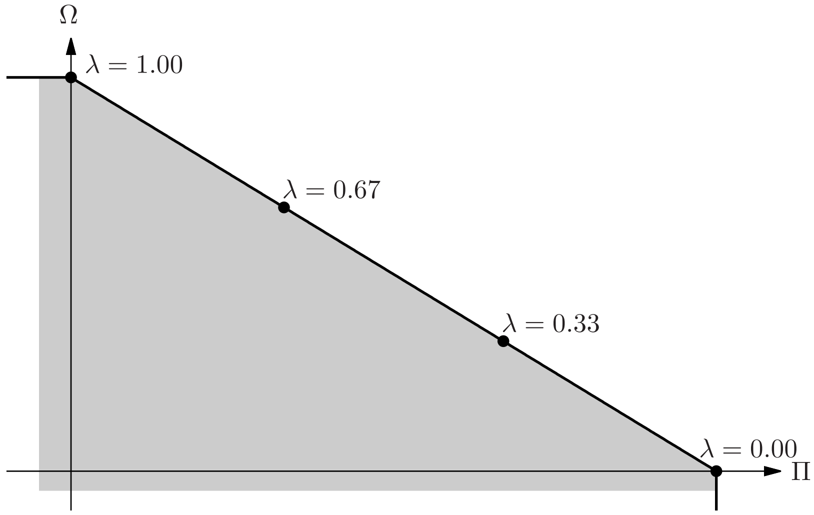


Figure 2: The impact of the bargaining power λ on the equilibrium payoff

holds for the role of λ on the bargaining wage. Finally, substituting (8) into the bargaining wage function (9), one finds that the equilibrium bargaining wage

$$\begin{aligned} W(p^e,\lambda,p,L) &= \left(\lambda + (1-\lambda)\frac{E_F(L)}{E_S(L)+1}\right)\frac{pF(L)}{L} \\ &= \left(\frac{E_F(L)}{E_S(L)+1} + \lambda\frac{E_S(L)+1-E_F(L)}{E_S(L)+1}\right)\frac{pF(L)}{L} \end{aligned}$$

is a multiple of average productivity, and that the equilibrium real wage

$$\frac{W(p^e,\lambda,p,L)}{p} = \frac{1}{E_F(L)}\left(\frac{E_F(L)}{E_S(L)+1} + \lambda\frac{E_S(L)+1-E_F(L)}{E_S(L)+1}\right)F'(L)$$

is a positive multiple of the marginal product of labor (with $L = h(p^e/p)$). Both equations show clearly how the bargaining parameter interacts with the elasticities of the two reservation wage functions

Relative Union Power

As was seen above, an efficient bargaining solution $(L,w) = (h(p^e/p), W(p^e,\lambda,p,h(p^e/p)))$ is defined parametrically for a given $0 \le \lambda \le 1$ measuring the "bargaining power". Thus, the model does not provide a fully endogenous determination of the bargaining power between the union and the producer. However, the efficient level of employment is independent of λ, implying that union–employer negotiations do guarantee productive efficiency. Therefore, the bargaining parameter λ determines exclusively the redistribution of revenue between the two parties, i. e. the share of wages and profits in total revenue.

It is intuitively clear (and also evident from the geometry of Figure 1) that there must be a unique bargaining level for which the parties agree on the competitive wage. This one equalizes marginal cost resp. marginal revenue $((W_\Pi L)'$ resp. $(W_\Omega L)')$. Geometrically speaking, this corresponds to the wage where the respective iso-utility and iso-profit curves are horizontal. Let the unique λ for which this condition holds be denoted by λ_{nat}, the "natural" λ. It is the solution of either

$$W(p^e,\lambda,p,L) \overset{!}{=} \frac{\partial(W_\Pi(p,L)L)}{\partial L} \qquad \text{or} \qquad W(p^e,\lambda,p,L) \overset{!}{=} \frac{\partial(W_\Omega(p^e,L)L)}{\partial L},$$

where $L = h(p^e/p)$. Inserting the definition of $W(p^e,\lambda,p,L)$ into the second equation gives

$$\lambda_{\text{nat}} W_\Pi(p,L) + (1-\lambda_{\text{nat}})W_\Omega(p^e,L) = \frac{\partial(W_\Pi(p,L)L)}{\partial L} = pF'(L) = E_F(L)W_\Pi(p,L).$$

Exploiting (8) then gives

$$\begin{aligned}
E_F(L)W_\Pi(p,L) &= \lambda_{\text{nat}} W_\Pi(p,L) + (1-\lambda_{\text{nat}})W_\Omega(p^e,L) \\
&= \lambda_{\text{nat}} W_\Pi(p,L) + (1-\lambda_{\text{nat}})\frac{E_F(L)}{E_S(L)+1}W_\Pi(p,L) \\
&= \left(\lambda_{\text{nat}} + (1-\lambda_{\text{nat}})\frac{E_F(L)}{E_S(L)+1}\right)W_\Pi(p,L) \\
&= \left(\frac{E_F(L)}{E_S(L)+1} + \lambda_{\text{nat}}\frac{E_S(L)+1-E_F(L)}{E_S(L)+1}\right)W_\Pi(p,L)
\end{aligned}$$

which implies

$$\lambda_{\text{nat}}(L) = \frac{E_F(L)E_S(L)}{E_S(L)+1-E_F(L)}. \tag{11}$$

In other words, $\lambda_{\text{nat}}(L)$ is determined by the elasticities E_S and E_F of the labor supply function and of the production function respectively. Therefore, with isoelastic functions $\lambda_{\text{nat}}(L)$ is constant.

The wage share of total revenue can be computed in a similar manner.

$$\begin{aligned}
\frac{wL}{py} &= \frac{W(p^e,\lambda,p,L)}{W_\Pi(p,L)} = \lambda + (1-\lambda)\frac{W_\Omega(p^e,L)}{W_\Pi(p,L)} \overset{(8)}{=} \lambda + (1-\lambda)\frac{E_F(L)}{E_S(L)+1} \\
&= \frac{E_F(L)}{E_S(L)+1} + \lambda\left(1 - \frac{E_F(L)}{E_S(L)+1}\right) \in \left[\frac{E_F(L)}{E_S(L)+1}, 1\right].
\end{aligned} \tag{12}$$

Therefore, the profit share of total revenue is

$$\frac{\pi}{py} = 1 - \frac{wL}{py} = (1-\lambda)\left(1 - \frac{E_F(L)}{E_S(L)+1}\right). \tag{13}$$

Note that the wage share resp. the profit share for $\lambda_{\text{nat}}(L)$ is $E_F(L)$ resp. $1-E_F(L)$, as expected, since at $\lambda_{\text{nat}}(L)$ the factor shares in total output must be equal to the respective elasticities of the production function F.

Underemployment and Overemployment

Since the bargaining solution $(L,w) = (h(\theta^e), W(p^e,\lambda,p,h(\theta^e)))$ is a joint agreement between the two agents, there can neither be any *involuntary* unemployment nor overemployment. In other words, any difference between $L = h(\theta^e)$ and the desired labor supply $N_{\text{com}}(w/p^e)$ has to be interpreted as a measure of a *voluntary* deviation from the competitive labor supply of the workers, which is a *supply side* measure. Similarly, any difference between L and the desired competitive employment $h_{\text{com}}(w/p)$ by the producer would be a *demand side* measure of *voluntary* deviation relative to the competitive regime.

Here, the voluntary *underemployment rate* will be defined in the usual way as

$$U = \mathcal{U}\left(L, \frac{w}{p^e}\right) := \frac{N_{\text{com}}(w/p^e) - L}{N_{\text{com}}(w/p^e)} = 1 - \frac{L}{N_{\text{com}}(w/p^e)}, \tag{14}$$

which measures the gap between the amount of labor which is actually traded (i. e. worked) and which would be supplied by the workers under competitive conditions at the given wage level. Since the rate of unemployment is defined for all expected real wages and all levels of labor, U defined in (14) can also be negative. This occurs for example if w/p^e is relatively low or L is relatively high. We interpret negative rates of underemployment as overemployment (or overtime).

2.5 Noncompetitive Wage Setting versus Wage Bargaining

It is often conjectured that noncooperative strategic behavior or market power by producers or by unions could be a reason why unemployment in labor markets exists. This section briefly presents the corresponding model with such one-sided deviant behavior on the wage setting and its implication on the level of prices, wages, and on the level of employment[12] at given commodity prices. The comparison between the cooperative and noncooperative temporary equilibria induced for the macroeconomy will be presented in Section 4.

The Monopsonistic Firm and Union Monopoly

Given $(p^e, p) \gg 0$ and the aggregate labor supply function $N_{\text{com}}(w/p^e)$ of workers, the monopsonistic firm choses a wage rate which maximizes

$$pF\left(N_{\text{com}}\left(\frac{w}{p^e}\right)\right) - wN_{\text{com}}\left(\frac{w}{p^e}\right).$$

This implies the first-order condition for an interior solution

$$F'\left(N_{\text{com}}\left(\frac{w}{p^e}\right)\right) = \frac{w}{p}\left(1 + \frac{1}{E_{N_{\text{com}}}(w/p^e)}\right) \quad \left(> \frac{w}{p}\right).$$

Let $\tilde{w} = W_{\text{mon}}(p^e, p) = pW_{\text{mon}}(p^e/p, 1)$ denote the unique solution, and let the induced aggregate employment and aggregate supply be given by

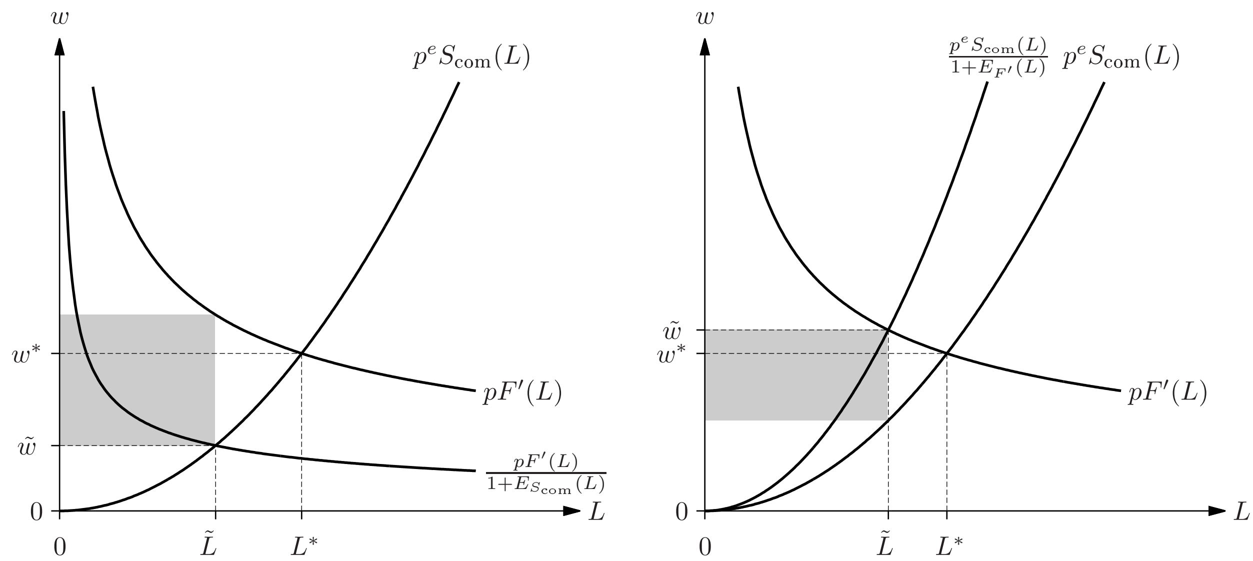
$$\tilde{L} = h_{\text{mon}}\left(\frac{p^e}{p}\right) := N_{\text{com}}\left(\frac{W_{\text{mon}}(p^e/p, 1)}{p^e/p}\right), \qquad AS_{\text{mon}}\left(\frac{p^e}{p}\right) := F\left(h_{\text{mon}}\left(\frac{p^e}{p}\right)\right).$$

The first-order condition implies that for any (p^e, p),

$$h_{\text{mon}}\left(\frac{p^e}{p}\right) < h_{\text{com}}\left(\frac{p^e}{p}\right) \qquad \text{and} \qquad AS_{\text{mon}}\left(\frac{p^e}{p}\right) < AS_{\text{com}}\left(\frac{p^e}{p}\right).$$

[12] see also Böhm (2010)

Therefore, as a consequence, at any given $(p^e, p) \gg 0$, the wage is equal to the marginal reservation wage of workers which is smaller than the marginal value product of labor for the firm. Thus, the firm receives a monopsonistic surplus equal to $pF'(\tilde{L}) - \tilde{w}\tilde{L}$, see Figure 3(a). However, at the same time, the wage is larger than the true reservation wage.



(a) surplus of the monopsonistic firm (b) surplus of the monopolistic union

Figure 3: Wages, employment, and surplus in monopolistic situations; (p^e, p) given

Since the producer accepts the market behavior of the workers as being given by their supply function (which corresponds to their *marginal* reservation wage), it seems as if the firm could exert more power and higher profits in the comparable bargaining situation by lowering the wage to the true reservation wage, which is not an option for the producer to be chosen under market conditions. In other words, the employment–wage decision differs from the efficient bargaining under the most powerful bargaining situation for any given price level p, when $\lambda = 0$.

The situation where a powerful union controls the labor market and sets the wage and the employment level is the symmetric opposite case to the monopsonistic firm and can be treated in a similar fashion. Given $(p^e, p) \gg 0$ and the labor demand function of the producer $h_{\text{com}}(w/p) = (F')^{-1}(w/p)$, the monopolistic union choses a wage rate w which maximizes

$$wh_{\text{com}}\left(\frac{w}{p}\right) - p^e S\left(h_{\text{com}}\left(\frac{w}{p}\right)\right) h_{\text{com}}\left(\frac{w}{p}\right) = wh_{\text{com}}\left(\frac{w}{p}\right) - p^e \frac{n_w}{1-\tau_w} v\left(\frac{h_{\text{com}}(w/p)}{n_w}\right).$$

This implies the first-order condition

$$\frac{w}{p^e}\left(\frac{1}{E_{h_{\text{com}}}(w/p)} + 1\right) = \frac{1}{1-\tau_w} v'\left(\frac{h_{\text{com}}(w/p)}{n_w}\right) = S_{\text{com}}\left(h_{\text{com}}\left(\frac{w}{p}\right)\right)$$

with the solution $\tilde{w} = W_{\text{union}}(p^e, p) = pW_{\text{union}}(p^e/p, 1)$ which induces a level of employment and aggregate supply

$$\tilde{L} = h_{\text{union}}\left(\frac{p^e}{p}\right) := h_{\text{com}}\left(W_{\text{mon}}\left(\frac{p^e}{p}, 1\right)\right), \qquad AS_{\text{union}}\left(\frac{p^e}{p}\right) := F\left(h_{\text{union}}\left(\frac{p^e}{p}\right)\right).$$

For every (p^e, p), this induces a wage equal to the marginal value product which is, however, larger than the competitive wage and larger than the marginal willingness to work of every worker at the associated level of employment. Thus, the workers obtain an aggregate monopolistic surplus equal to $pF'(\tilde{L}) - p^e S_{\text{com}}(\tilde{L})$, see Figure 3(b). As in the case of the monopsonistic firm, the union accepts the labor demand behavior by the producer as being given. Therefore, the wage being equal to the marginal reservation wage of the producer is higher than the true reservation wage, equal to average costs. Thus, at the given price, the powerful union does not obtain access to the full rent from the producer, which it could obtain under bargaining and $\lambda = 1$.

Summarizing the main results of this section, one finds that the employment–wage decision under one-sided strategic behavior in the labor market implies that the powerful side of the market collects an extra rent by exploiting the weaker trader, as is to be expected. Moreover, this induces an inefficient employment allocation since the marginal willingness to work never equals the marginal willingness to hire since only one side of the market is a price taker while the other one is not. This implies a lower level of employment than in the competitive situation at all given prices and price expectations, which is in contrast to the efficient bargaining solution. However, the strategic behavior does not generate unemployment.[13]

3 Temporary Equilibrium with Efficient Wage Bargaining

It is now straightforward to close the model in order to determine the properties of a temporary equilibrium under wage bargaining. The data at the beginning of an arbitrary period are aggregate money balances $M > 0$ held by old consumers, expected prices for the future period $p^e > 0$, and the bargaining parameter $0 \leq \lambda \leq 1$, plus the parameters of the government (g, τ_w, τ_π). Then, a temporary equilibrium with efficient wage bargaining is defined by a pair of prices and wages $(p, w) \gg 0$ such that the price p clears the commodity market competitively while the wage w equals the one set by the union and the producer in the bargaining solution. Associated with the equilibrium is the equilibrium allocation which consists of a pair of feasible employment and output levels $(L, y) = (L, F(L)) \gg 0$.

Since all agents in the economy – consumers, the producer, and the government – are assumed to be price takers in the commodity market, finding a temporary equilibrium is equivalent to finding a price p which equalizes aggregate demand and aggregate supply, where aggregate demand has to be appropriately adjusted to the income distribution induced by the bargaining result.

3.1 The Role of Union Power in Temporary Equilibrium

Aggregate Supply and Aggregate Demand

The bargaining wage $W(p^e, \lambda, p, L)$ and the associated employment level $L = h(p^e/p)$ were derived as a function of price expectations and prices in the previous section where the employment decision turned out to be independent of the bargaining parameter λ. Therefore, given a

[13] for a more detailed discussion see Section 4

pair of price expectations and prices $(p^e, p) \gg 0$, the aggregate commodity supply function is defined by

$$AS : \mathbb{R}_{++} \to \mathbb{R}_{++}, \quad AS(\theta^e) := F(h(\theta^e)).$$

This is a function of the expected inflation rate alone, which is globally invertible and differentiable. Since $h'(\theta^e) < 0$, one finds that $AS'(\theta^e) < 0$ so that, for any given price expectation $p^e > 0$, aggregate supply is a strictly increasing function of temporary commodity prices

$$\frac{\mathrm{d}\, AS(p^e/p)}{\mathrm{d}\, p} > 0.$$

In contrast, the bargaining wage $W(p^e, \lambda, p, h(p^e/p))$ will have an influence on the income distribution and thus on aggregate demand. Since there are four different private consumers plus the government generating aggregate demand, the income distribution between profits and wage income and the total income generated determine aggregate demand.

The assumptions concerning the overlapping-generations structure of consumers imply that all current net wages are saved and a proportion $0 \leq c(\theta^e) \leq 1$ of current net profits is consumed by young shareholders. Therefore, aggregate real demand in any period is the sum of total real money balances $m := M/p$, government demand g, plus the demand by shareholders which is a function of aggregate profits. Thus, given money balances, price expectations, the bargaining weight, and prices (M, p^e, λ, p), the income consistent aggregate demand y must be the solution of

$$\begin{aligned} y &= m + g + c(\theta^e)(1 - \tau_\pi)\frac{\pi}{p} \\ &\overset{(13)}{=} m + g + c(\theta^e)(1 - \tau_\pi)(1 - \lambda)\left(1 - \frac{E_F(L)}{E_S(L) + 1}\right) y \end{aligned}$$

with $y = F(L)$ and $L = h(\theta^e)$. Therefore, one obtains as the income-consistent aggregate demand function

$$\begin{aligned} y = D(m, \theta^e, \lambda) &= \frac{m + g}{1 - c(\theta^e)(1 - \tau_\pi)(1 - \lambda)(1 - \frac{E_F(L)}{E_S(L)+1})} \\ &= \frac{m + g}{1 - c(\theta^e)(1 - \tau_\pi)(1 - \lambda)(1 - \frac{E_F(h(\theta^e))}{E_S(h(\theta^e))+1})}, \end{aligned} \tag{15}$$

which is of the usual multiplier form with respect to money balances and government demand. Observe that aggregate demand is homogeneous of degree zero in (M, p^e, p). Therefore, for given λ, it is a function of real money balances and of the expected rate of inflation. Obviously, $\partial D/\partial m > 0$, i. e. real balances have a positive effect on demand, and $\partial D/\partial \lambda < 0$, i. e. higher bargaining power by the union decreases profits and thus consumption demand by shareholders. In addition, if $\partial D/\partial \theta^e \geq 0$, then the demand is strictly decreasing in the commodity price p, i. e. $\mathrm{d}\, D(M/p, \theta^e, \lambda)/\mathrm{d}\, p < 0$ is negative. This property holds in particular when the savings proportion by shareholders is nondecreasing and when the reservation wage and the production function are isoelastic.

Therefore, given a bargaining weight $0 \leq \lambda \leq 1$ and any pair $(M, p^e) \gg 0$ of money balances and price expectations, the temporary equilibrium is given by a price p which clears the commodity market, i. e.

$$D\left(\frac{M}{p}, \frac{p^e}{p}, \lambda\right) = AS\left(\frac{p^e}{p}\right). \tag{16}$$

Concerning existence and uniqueness, one has the following immediate result.

Lemma 3.1 *Let the aggregate supply function AS be globally invertible with $AS'(\theta^e) < 0$, and assume that $\partial D/\partial\theta^e \geq 0$, $\partial D/\partial m > 0$ hold. Then, for every $(M, p^e) \gg 0$ and $0 \leq \lambda \leq 1$, there exists a unique positive temporary equilibrium price $p > 0$ solving equation* (16)*.*

The uniqueness follows from the fact that the excess demand function is strictly monotonically decreasing. Figure 4 portrays the equilibrium situation in the usual aggregate demand–aggregate supply diagram of the commodity market. As a consequence of Lemma 3.1, one

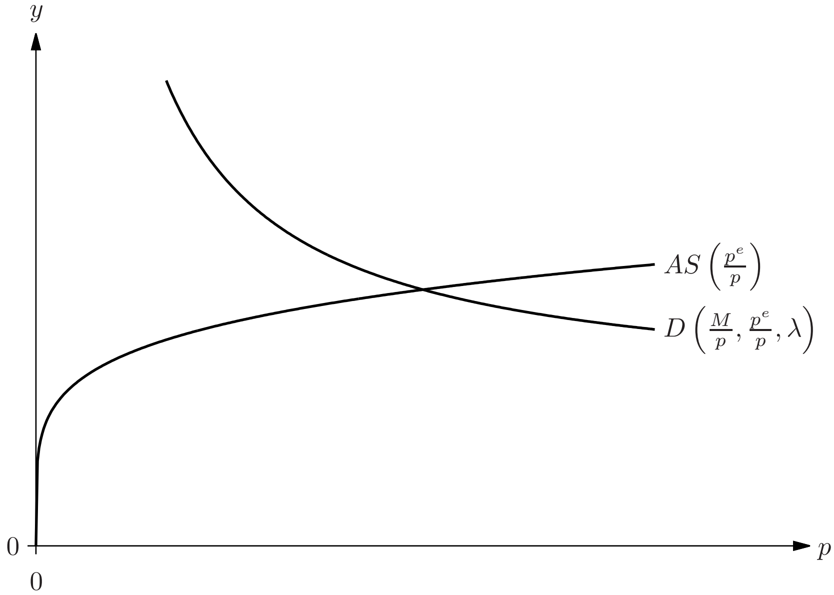


Figure 4: The temporary equilibrium price

obtains the following proposition.

Proposition 3.1 *There exist differentiable mappings $\mathcal{P} : \mathbb{R}^2_{++} \times [0,1] \to \mathbb{R}_{++}$ and $\mathcal{W} : \mathbb{R}^2_{++} \times [0,1] \to \mathbb{R}_{++}$, called the* price law *and the* wage law *respectively, such that*

- *the unique positive temporary equilibrium price is given by*

$$p = \mathcal{P}(M, p^e, \lambda), \tag{17}$$

- *the unique positive temporary equilibrium wage is defined by*

$$w = \mathcal{W}(M, p^e, \lambda) := W\left(p^e, \lambda, \mathcal{P}(M, p^e, \lambda), h\left(\frac{p^e}{\mathcal{P}(M, p^e, \lambda)}\right)\right),$$

  and

- *$\mathcal{P}$ and $\mathcal{W}$ are homogeneous of degree one in (M, p^e), for given λ.*

Properties of the Price Law

Applying the implicit function theorem to (16) with respect to M, one obtains the effect of an increase of money balances

$$\frac{\partial \mathcal{P}}{\partial M} = \frac{\frac{1}{\mathcal{P}}\frac{\partial D}{\partial m}}{-\frac{p^e}{\mathcal{P}^2}F'h' + \frac{M}{\mathcal{P}^2}\frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}^2}\frac{\partial D}{\partial \theta^e}} > 0$$

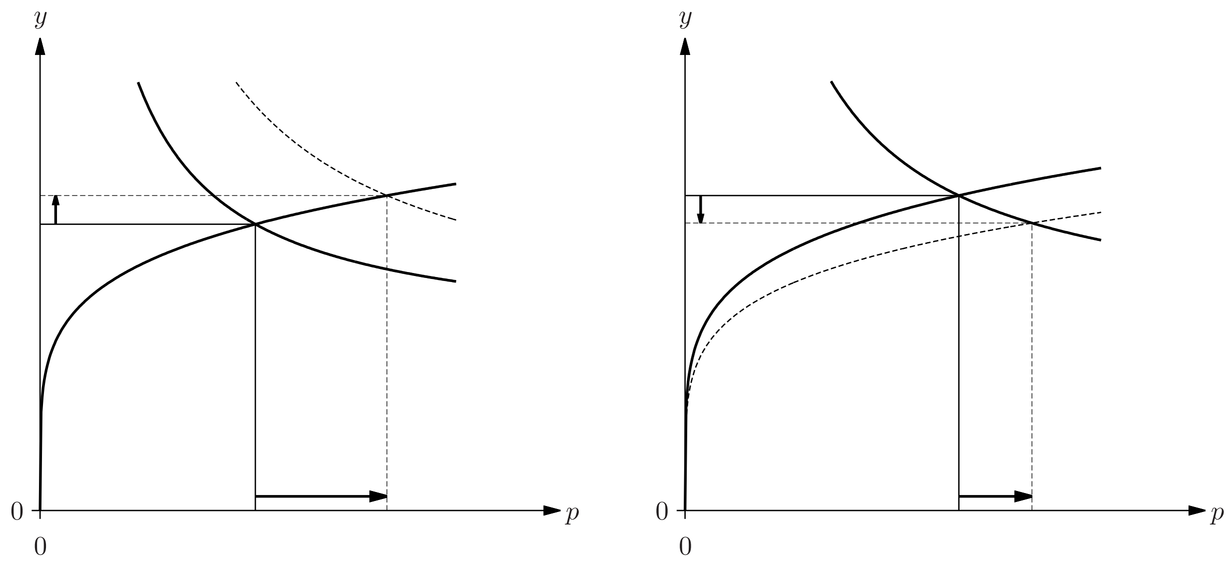


Figure 5: Comparative-statics effects of money balances and price expectations

with an elasticity

$$0 < E_{\mathcal{P}}(M) = \frac{\partial \mathcal{P}}{\partial M}\frac{M}{\mathcal{P}} = \frac{\frac{M}{\mathcal{P}}\frac{\partial D}{\partial m}}{-\frac{p^e}{\mathcal{P}}F'h' + \frac{M}{\mathcal{P}}\frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}}\frac{\partial D}{\partial \theta^e}} < 1. \tag{18}$$

Thus, the temporary equilibrium price is a strictly increasing and strictly concave function of money balances since prices are nonnegative. Applying the implicit function theorem to (16) once more, one obtains a positive expectations effect on prices

$$\frac{\partial \mathcal{P}}{\partial p^e} = -\frac{\frac{1}{\mathcal{P}}F'h'}{-\frac{p^e}{\mathcal{P}^2}F'h' + \frac{M}{\mathcal{P}^2}\frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}^2}\frac{\partial D}{\partial \theta^e}} > 0$$

with an elasticity

$$E_{\mathcal{P}}(p^e) = \frac{\partial \mathcal{P}}{\partial p^e}\frac{p^e}{\mathcal{P}} = \frac{-\frac{p^e}{\mathcal{P}^2}F'h'}{-\frac{p^e}{\mathcal{P}^2}F'h' + \frac{M}{\mathcal{P}^2}\frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}^2}\frac{\partial D}{\partial \theta^e}} < 1, \tag{19}$$

which is also less than one, implying that equilibrium prices are a strictly increasing and strictly concave function in price expectations. Together this implies that the price law $\mathcal{P}$ is strictly concave and homogeneous of degree one in (M, p^e), with a representation of the form $p = p^e\mathcal{P}(M/p^e, 1, \lambda)$ which is strictly increasing and strictly concave in M/p^e.

Output and Employment

Given the price law, one obtains the associated temporary equilibrium allocation consisting of the levels of output and employment as functions of the same data (M, p^e, λ), i. e.

$$\begin{aligned} y = \mathcal{Y}(M, p^e, \lambda) &:= F\left(h\left(\frac{p^e}{\mathcal{P}(M, p^e, \lambda)}\right)\right) \qquad \text{and} \\ L = \mathcal{L}(M, p^e, \lambda) &:= h\left(\frac{p^e}{\mathcal{P}(M, p^e, \lambda)}\right). \end{aligned} \tag{20}$$

which are homogeneous of degree zero in (M, p^e). Using (18) and $0 < E_F(L) < 1$, one obtains the corresponding elasticities of money balances on employment and output as

$$E_{\mathcal{L}}(M) = -E_h(\theta^e)E_{\mathcal{P}}(M) > 0 \qquad \text{and} \qquad E_{\mathcal{L}}(M) > E_F(L)E_{\mathcal{L}}(M) = E_{\mathcal{Y}}(M) > 0. \tag{21}$$

Thus, higher money balances imply higher equilibrium prices but also higher levels of employment and output.

Similarly, applying property (19), $0 < E_F(L) < 1$, and the relationship

$$E_{\mathcal{L}}(p^e) = \underbrace{E_h(\theta^e)}_{<0}\underbrace{(1 - E_{\mathcal{P}}(p^e))}_{\in(0,1)} < 0 \tag{22}$$

yields

$$E_{\mathcal{L}}(p^e) < E_F(L)E_{\mathcal{L}}(p^e) = E_{\mathcal{Y}}(p^e) < 0.$$

Thus, output and employment decline with higher price expectations. Therefore, combined with the zero-homogeneity of the employment law and output law, this confirms the tradeoff between money balances and expectations for a constant level of output and employment. Figure 5 displays the comparative statics results for changes of price expectations and of real money balances.

Properties of the Wage Law

In contrast to the above results, the comparative statics effects of the wage law cannot be signed in general since several diverse effects interact in a nonlinear way. This can be seen partially from the form of the wage law equation

$$w = \mathcal{W}(M, p^e, \lambda) = \lambda W_\Pi\big(\mathcal{P}(M, p^e, \lambda), \mathcal{L}(M, p^e, \lambda)\big) + (1-\lambda)W_\Omega\big(p^e, \mathcal{L}(M, p^e, \lambda)\big), \tag{23}$$

which shows an interaction of the effects of the price law and the employment law in the definition. However, it is possible in some special situations to determine the effects under more restricted conditions. Writing the wage as the associated mark-up over the reservation wage of the workers (or equivalently as a mark-down from the reservation wage of the producer)

$$\begin{aligned} w &= \left(1 + \lambda \frac{E_S(\mathcal{L}(M, p^e, \lambda)) + 1 - E_F(\mathcal{L}(M, p^e, \lambda))}{E_F(\mathcal{L}(M, p^e, \lambda))}\right) W_\Omega(p^e, \mathcal{L}(M, p^e, \lambda)) \\ &= \left(\lambda + (1-\lambda)\frac{E_F(\mathcal{L}(M, p^e, \lambda))}{E_S(\mathcal{L}(M, p^e, \lambda)) + 1}\right) W_\Pi(\mathcal{P}(M, p^e, \lambda), \mathcal{L}(M, p^e, \lambda)), \end{aligned} \tag{24}$$

one observes that the state variables exert their influence on wages via a primary effect through the price and employment laws and a secondary effect through the respective elasticities, which determine the mark-up. Therefore, in situations where the effect of the state variable on the mark-up is small and can be neglected, the wage effect has the same sign as the employment effect, i. e.

$$\begin{aligned} \operatorname{sgn} E_{\mathcal{W}}(M) &= \operatorname{sgn} E_S(L)E_{\mathcal{L}}(M) > 0 \\ \operatorname{sgn} E_{\mathcal{W}}(p^e) &= \operatorname{sgn}\left(E_{\mathcal{P}}(p^e) - (1 - E_F(L))E_{\mathcal{L}}(p^e)\right) > 0 \end{aligned} \tag{25}$$

In this case, wages increase with money balances and with price expectations. This indicates, however, that wages can also fall when employment increases.

The effect of the state variables on the real wage can be determined using the same procedure. Writing the real wage as

$$\begin{aligned}\frac{w}{p} &= \left(\lambda + (1-\lambda)\frac{E_F(\mathcal{L}(M,p^e,\lambda))}{E_S(\mathcal{L}(M,p^e,\lambda))+1}\right)\frac{F(\mathcal{L}(M,p^e,\lambda))}{\mathcal{L}(M,p^e,\lambda)} \\ &= \left(\lambda + (1-\lambda)\frac{E_F(\mathcal{L}(M,p^e,\lambda))}{E_S(\mathcal{L}(M,p^e,\lambda))+1}\right)\frac{F'(\mathcal{L}(M,p^e,\lambda))}{E_F(\mathcal{L}(M,p^e,\lambda))} \\ &= \left(\frac{\lambda}{E_F(\mathcal{L}(M,p^e,\lambda))} + \frac{1-\lambda}{E_S(\mathcal{L}(M,p^e,\lambda))+1}\right)F'(\mathcal{L}(M,p^e,\lambda)),\end{aligned} \tag{26}$$

one finds that it can be written as a positive multiple of average labor productivity or of the marginal product of labor respectively. Therefore, *for given* λ, due to the concavity of the production function with average productivity declining in L, output and employment always move in the opposite direction as the real wage with respect to the state variables (M, p^e), provided that the elasticities do not change too much. Section 4 contains a detailed analysis of the wage law for a specific parametric example.

The Role of Union Power

Since the parameter λ does not influence aggregate supply, the assumption $\partial D/\partial\theta^e \geq 0$ implies that

$$\operatorname{sgn}\frac{\partial \mathcal{P}}{\partial \lambda} = \operatorname{sgn}\frac{\partial D}{\partial \lambda} < 0.$$

Therefore, an increase of union power has a negative effect on the temporary equilibrium price, i. e. the elasticity with respect to union power $E_{\mathcal{P}}(\lambda) < 0$ is negative. Therefore, an increase in union power induces a reduction of prices, output, and employment. Using the properties of the employment law (20) one has

$$E_{\mathcal{L}}(\lambda) = -E_h(\theta^e)E_{\mathcal{P}}(\lambda) < 0 \qquad E_{\mathcal{L}}(\lambda) < E_F(L)E_{\mathcal{L}}(\lambda) = E_{\mathcal{Y}}(\lambda) < 0. \tag{27}$$

Figure 6 portrays the effects of changes of union power on equilibrium prices, showing that there exists a strong nonlinear feedback from the bargaining power on the equilibrium prices, output, and employment. Thus, while the wage bargaining procedure assumes price-taking behavior on behalf of both parties inducing a perceived wage increase under increased union power, the level λ of union power has a negative indirect or spillover effect on the equilibrium price which operates through a negative income effect on aggregate demand.

The bargaining power λ enters in multiple but opposite ways into the wage equation (23), similar to money balances and price expectations (M, p^e). This implies that, in general, the overall effect of union power on the equilibrium wage cannot be signed. However, the effect of λ on the real wage can be determined using the same technique as above. Rewriting the real wage equation (26) as

$$\frac{w}{p} = \left(\frac{E_F(\mathcal{L}(M,p^e,\lambda))}{E_S(\mathcal{L}(M,p^e,\lambda))+1} + \lambda\left(1 - \frac{E_F(\mathcal{L}(M,p^e,\lambda))}{E_S(\mathcal{L}(M,p^e,\lambda))+1}\right)\right)\frac{F(\mathcal{L}(M,p^e,\lambda))}{\mathcal{L}(M,p^e,\lambda)},$$

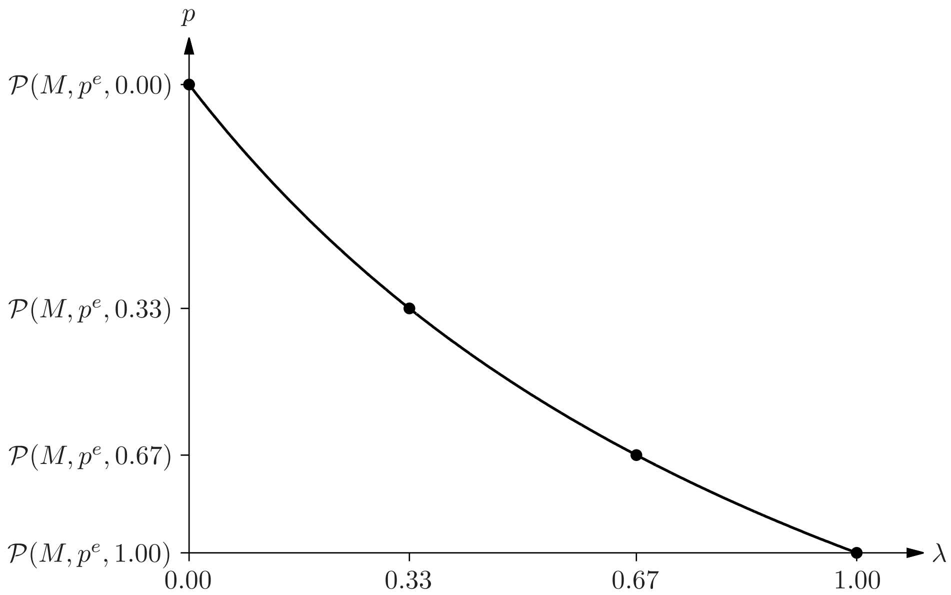


Figure 6: Range of equilibrium prices $\mathcal{P}(M, p^e, \lambda)$ for λ from 0 to 1

one finds that it must increase with union power whenever the wage is nonincreasing or when the effect of λ on the elasticities can be neglected. Section 4 also contains a detailed study of the role of union power for a parametrized version of the model.

3.2 Comparing Bargaining and Competition

The results in the previous section indicate that the level of prices, output, and employment vary inversely with union power λ. It is somewhat surprising that such fairly strong comparative statics properties hold in general. With such clear negative influence on output and employment from powerful but efficient wage bargaining, it is particularly desirable to investigate the role of bargaining in its general relationship to competitive allocations.

To carry out a systematic comparison between temporary equilibria under competition and under efficient wage bargaining, the impact of bargaining on aggregate demand and aggregate supply relative to the competitive case has to be examined. Given the labor demand function of the competitive producer (see equation (1)) $h_{\text{com}}(w/p) = (F')^{-1}(w/p)$, the labor market clearing condition

$$N_{\text{com}}\left(\frac{w}{p}\Big/\frac{p^e}{p}\right) = h_{\text{com}}\left(\frac{w}{p}\right)$$

implies the usual equilibrium relationship between expected inflation and the real wage

$$\frac{p^e}{p} = \theta^e = \frac{w/p}{N_{\text{com}}^{-1}(h_{\text{com}}(w/p))} = \frac{w/p}{S_{\text{com}}(h_{\text{com}}(w/p))} =: W_{\text{com}}^{-1}\left(\frac{w}{p}\right).$$

Using equation (6) with $L = h(\theta^e)$, this induces

$$W_{\text{com}}^{-1}\left(h_{\text{com}}^{-1}(L)\right) = \frac{h_{\text{com}}^{-1}(L)}{S_{\text{com}}\left(h_{\text{com}}\left(h_{\text{com}}^{-1}(L)\right)\right)} = \frac{F'(L)}{S_{\text{com}}(L)} \overset{(7)}{=} \theta^e.$$

Therefore, for all $p^e/p = \theta^e$,

$$h_{\text{com}}\left(W_{\text{com}}(\theta^e)\right) = h(\theta^e), \tag{28}$$

the equilibrium employment decisions in the labor market under bargaining and under competition are identical. This in turn implies that the two aggregate supply functions are the same, i. e. for all θ^e,

$$AS_{\text{com}}(\theta^e) = F\left(h_{\text{com}}\left(W_{\text{com}}(\theta^e)\right)\right) = F(h(\theta^e)) = AS(\theta^e).$$

To define income-consistent aggregate demand under competition, let prices and wages (p, w) be given. The competitive firm chooses its labor input according to the marginal product rule $w = pF'(L)$, implying that the profit share of total revenue is

$$\frac{py - wL}{py} = 1 - \frac{F'(L)L}{F(L)} = 1 - E_F(L).$$

Thus, income-consistent aggregate demand in the competitive case must satisfy

$$y = m + g + c(\theta^e)(1 - \tau_\pi)\left(1 - E_F(L)\right)y,$$

leading to the aggregate demand function under perfect competition in the labor market

$$y = D_{\text{com}}(m, \theta^e) = \frac{m + g}{1 - c(\theta^e)(1 - \tau_\pi)(1 - E_F(L))}, \qquad L = h_{\text{com}}(W_{\text{com}}(\theta^e)) \overset{(28)}{=} h(\theta^e),$$

as compared to the aggregate demand function under bargaining derived from

$$y = m + g + c(\theta^e)(1 - \tau_\pi)(1 - \lambda)\left(1 - \frac{E_F(L)}{E_S(L) + 1}\right)y$$

in (15) as

$$y = D(m, \theta^e, \lambda) = \frac{m + g}{1 - c(\theta^e)(1 - \tau_\pi)(1 - \lambda)(1 - \frac{E_F(L)}{E_S(L)+1})}, \qquad L = h(\theta^e).$$

Thus, the two aggregate demand functions differ essentially only by the size of the multiplier, which depends on λ and on the values of the respective elasticities. Therefore, one finds that, for all (M, p, p^e), aggregate demand under bargaining is strictly decreasing in λ with

$$D(m, \theta^e, 1) < D_{\text{com}}(m, \theta^e) < D(m, \theta^e, 0)$$

and, since aggregate supply is independent of λ and identical in the two cases, that

$$\mathcal{P}(M, p^e, 1) < \mathcal{P}_{\text{com}}(M, p^e) < \mathcal{P}(M, p^e, 0).$$

As a consequence, for given (M, p^e), by the continuity and monotonicity of the price law under bargaining as a function of λ, there must exist a unique value $0 < \lambda_{\text{com}} < 1$, where the temporary equilibrium price at the bargaining equilibrium coincides with that of the competitive equilibrium, i. e. one has

$$\mathcal{P}_{\text{com}}(M, p^e) = \mathcal{P}(M, p^e, \lambda_{\text{com}}).$$

Thus, given the equivalence $\mathcal{P}_{\mathrm{com}}(M, p^e) = p = \mathcal{P}(M, p^e, \lambda_{\mathrm{com}})$ of the equilibrium price under competition and under bargaining for λ_{com}, aggregate supply and aggregate demand at equilibrium must be the same

$$D_{\mathrm{com}}(M/p, p^e/p) = AS_{\mathrm{com}}(p^e/p) = AS(p^e/p) = D(M/p, p^e/p, \lambda_{\mathrm{com}})$$

so that the level of output, employment, and of wages

$$\mathcal{Y}_{\mathrm{com}}(M, p^e) = D_{\mathrm{com}}\left(\frac{M}{p}, \frac{p^e}{p}\right) = D\left(\frac{M}{p}, \frac{p^e}{p}, \lambda_{\mathrm{com}}\right) = \mathcal{Y}(M, p^e, \lambda_{\mathrm{com}}),$$

$$\mathcal{L}_{\mathrm{com}}(M, p^e) = F^{-1}\left(D_{\mathrm{com}}\left(\frac{M}{p}, \frac{p^e}{p}\right)\right) = F^{-1}\left(D\left(\frac{M}{p}, \frac{p^e}{p}\right), \lambda_{\mathrm{com}}\right) = \mathcal{L}(M, p^e, \lambda_{\mathrm{com}}), \text{ and}$$

$$\mathcal{W}_{\mathrm{com}}(M, p^e) = \mathcal{W}(M, p^e, \lambda_{\mathrm{com}})$$

are equalized as well. Therefore, the competitive temporary equilibrium is a special case of the possible equilibria under efficient bargaining for a specific value λ_{com} of union power.

While the coincidence of the two equilibria does not seem surprising at first sight, one should note that this results depends crucially on the fact that the reservation wages for workers and for the firm are defined by the zero-activity level of workers and producers and by the fact that they are common knowledge in the bargaining procedure. These assumptions imply a symmetric no-participation constraint (or threat point) for both sides which induces the specific equilibrium characteristics with no loss in production-effort efficiency, equalizing the real marginal product to the competitive marginal willingness to work. Thus the employment choice corresponds to the competitive one, making the aggregate supply function under bargaining equivalent to the competitive one. Thus, the bargaining equilibrium not only provides an *efficient redistribution* of value added, but it also eliminates inter-party inefficiencies leading to an optimal tradeoff between marginal disutility of effort and marginal productivity of labor. In this sense, the temporary equilibrium with bargaining satisfies conditional Pareto optimality at any level $\lambda > 0$ of bargaining power. Yet, the total value added could always be improved by setting $\lambda = 0$. Combined with a lump-sum redistribution of the surplus, a Pareto improvement could be obtained.[14]

If, however, the reservation wages of either side had been chosen to be the levels of the corresponding competitive inverse demand or supply functions, i. e. their *marginal willingness* to work or hire at given prices and price expectations, conditional Pareto optimality could not be obtained under bargaining since total net value would not have been maximized in equilibrium. In such cases, the bargaining equilibrium would generate allocations with prices and wages, levels of employment and output which are continuous deformations between the two cases of one-sided full market power for the union, i. e. the union monopoly, and the producer monopsony, which were discussed in Section 2. As was shown there, these would suffer from additional inefficiencies and the competitive temporary equilibrium could not be achieved as an equilibrium under efficient bargaining.

[14]Note that this discussion argues only about efficiency in terms of the payoff between the firm and the union and not in welfare terms with respect to the two groups of consumers and their indirect utility. A welfare comparison should use their utility functions. In this case, the effects stemming from underemployment/overemployment would have to be accounted for as well. Moreover, the intertemporal structure of overlapping generations requires additional criteria between old and young consumers and their position in the temporary equilibrium, for which a Pareto criterion is not universally defined.

3.3 Inefficient Redistribution under Efficient Wage Bargaining

The negative feedback of union power on prices, output, and employment derived in (27) indicates that, from a macroeconomic point of view, a strong union under efficient bargaining may not guarantee an overall efficient allocation in temporary equilibrium. In other words, given the data of the economy (M, p^e, λ), output is maximal when $\lambda = 0$ and minimal when $\lambda = 1$. This suggests that the bargaining procedure will never attain the global maximal surplus in the economy unless $\lambda = 0$.

To investigate the role of the bargaining power more closely, consider the payoff vector (Π, Ω) in temporary equilibrium, which is obtained by substituting the price law $\mathcal{P}(M, p^e, \lambda)$ from (17) and the wage law from (20) into the payoff vector (10). This yields

$$\begin{pmatrix}\Pi(M,p^e,\lambda)\\ \Omega(M,p^e,\lambda)\end{pmatrix} = \Big(W_\Pi(\mathcal{P}(M,p^e,\lambda),\mathcal{L}(M,p^e,\lambda)) - W_\Omega(p^e,\mathcal{L}(M,p^e,\lambda)\Big)\mathcal{L}(M,p^e,\lambda)\begin{pmatrix}1-\lambda\\ \lambda\end{pmatrix}$$
$$= \Big(\mathcal{P}(M,p^e,\lambda)F(\mathcal{L}(M,p^e,\lambda)) - p^e S(\mathcal{L}(M,p^e,\lambda))\mathcal{L}(M,p^e,\lambda)\Big)\begin{pmatrix}1-\lambda\\ \lambda\end{pmatrix}.$$

Thus, the efficient bargaining solution at the temporary equilibrium is a linear one-to-one redistribution of the total net surplus

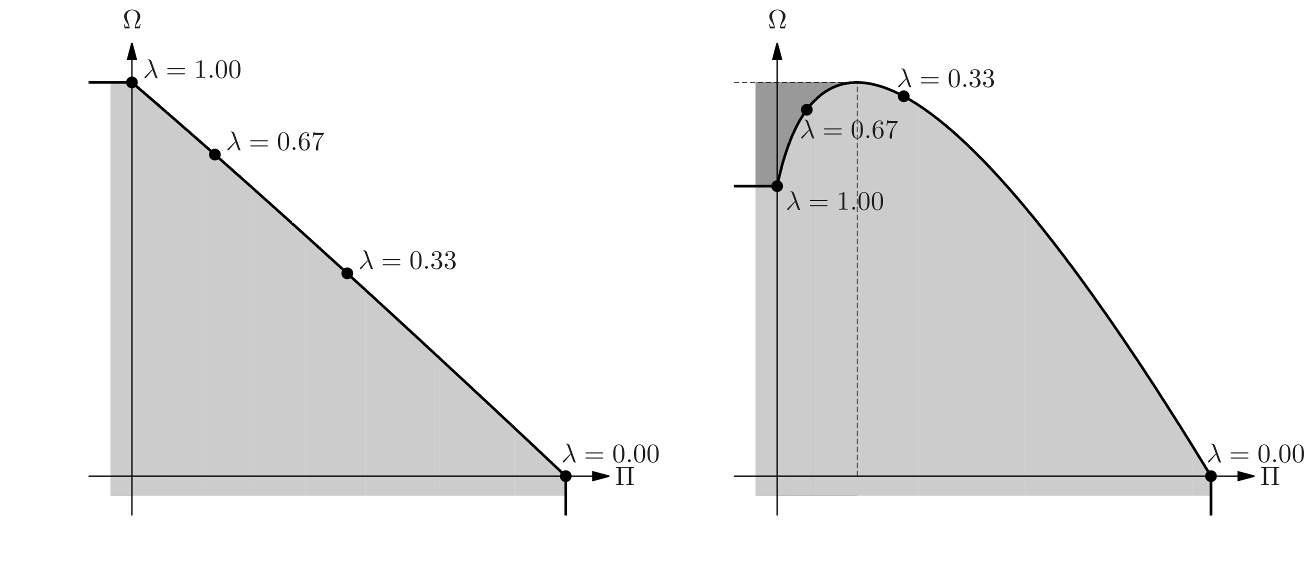
$$\Pi(M,p^e,\lambda) + \Omega(M,p^e,\lambda) = \mathcal{P}(M,p^e,\lambda)F(\mathcal{L}(M,p^e,\lambda)) - p^e S(\mathcal{L}(M,p^e,\lambda))\mathcal{L}(M,p^e,\lambda), \quad (29)$$

implying a marginal rate of substitution between $\Pi(M, p^e, \lambda)$ and $\Omega(M, p^e, \lambda)$ equal to minus one. Taking the derivative of (29) with respect to λ, one finds that

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\,\lambda}\big(\Pi(M,p^e,\lambda)+\Omega(M,p^e,\lambda)\big)\\
=&\frac{\mathrm{d}}{\mathrm{d}\,\lambda}\big(\mathcal{P}(M,p^e,\lambda)F(\mathcal{L}(M,p^e,\lambda)) - p^e S(\mathcal{L}(M,p^e,\lambda))\mathcal{L}(M,p^e,\lambda)\big)\\
=&F(\mathcal{L}(M,p^e,\lambda))\frac{\partial\mathcal{P}(M,p^e,\lambda)}{\partial\lambda} + \underbrace{\frac{\mathrm{d}}{\mathrm{d}\,L}\big(pF(L) - p^e S(L)L\big)}_{\overset{(7)}{=}0}\frac{\partial\mathcal{L}(M,p^e,\lambda)}{\partial\lambda}\\
=&F(\mathcal{L}(M,p^e,\lambda))\frac{\partial\mathcal{P}(M,p^e,\lambda)}{\partial\lambda} < 0
\end{aligned} \quad (30)$$

has a negative sign. Therefore, higher union power λ also induces a lower aggregate equilibrium surplus. Thus, the aggregate surplus is a strictly decreasing function with a global maximum at $\lambda = 0$. Geometrically speaking, this implies that the bargaining possibility frontier for all $0 < \lambda \leq 1$ in temporary equilibrium is strictly below the minus one tradeoff line at $\Pi(M, p^e, 0) - \Omega(M, p^e, 0)$.

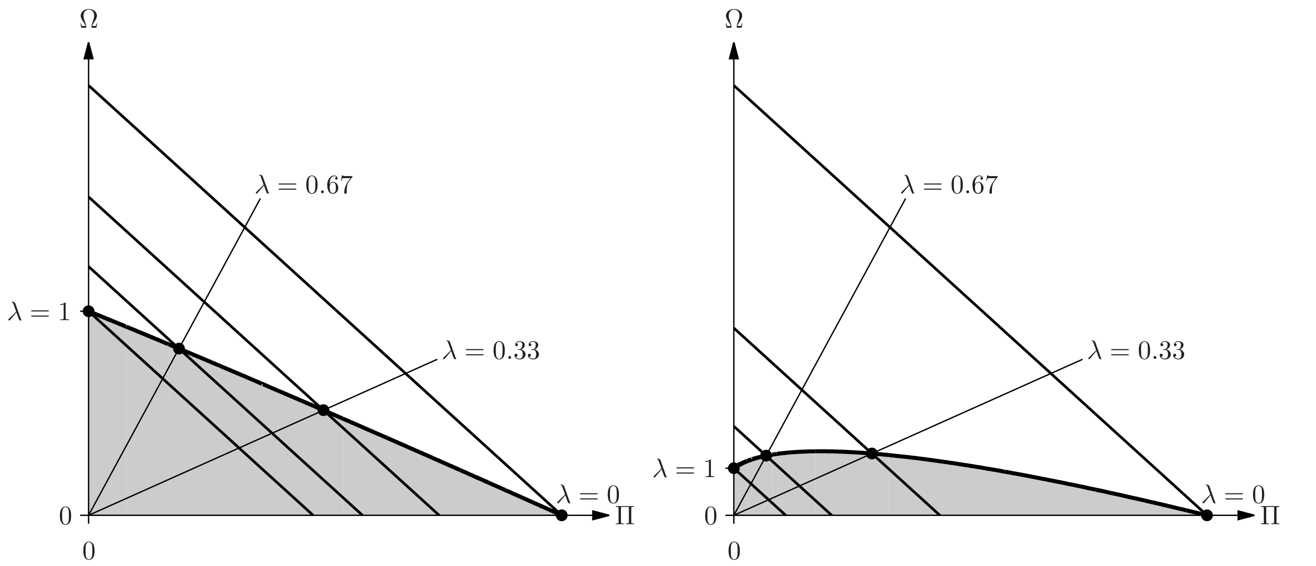
It is obvious that the profit term of the payoff $\Pi(M, p^e, \lambda) - \Omega(M, p^e, \lambda)$ is decreasing in λ while the influence on the wage bill cannot be signed in all cases. In fact, it may be increasing or decreasing depending on the data. Figure 7 displays the payoff frontier in equilibrium for two different levels of government consumption, taking the feedback into account. Both panels show that the distribution of wealth is not linear in λ. While the equilibrium profit always decreases with union power, the right panel clearly shows that even the wage bill may be declining with



(a) high government consumption (b) low government consumption

Figure 7: Net wage bill and profit with price feedback

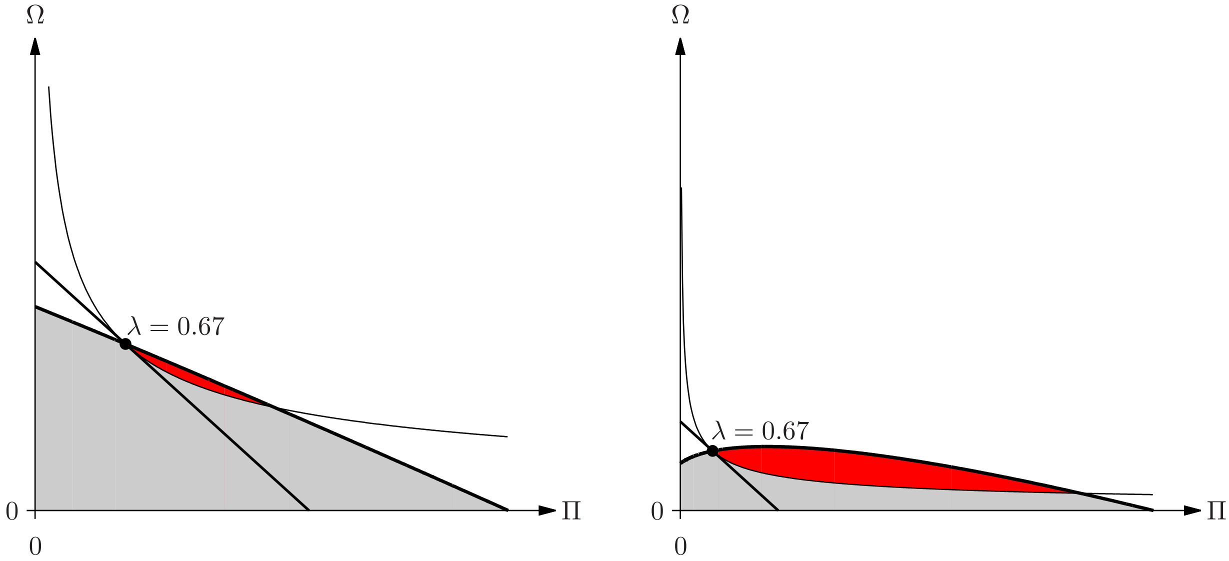
union power in some circumstances. Figure 8 combines Figure 2 and Figure 7 displaying the equilibrium payoffs for four levels of union power ($\lambda = 0.00$, $\lambda = 0.33$, $\lambda = 0.67$, and $\lambda = 1.00$) as intersections of the sharing ratios $\lambda/(1-\lambda)$ and the corresponding associated linear tradeoff frontier (thin downward-sloping lines with prices assumed to be fixed at the respective levels).



(a) high government consumption (b) low government consumption

Figure 8: The role of money balances for $\lambda = 0.00$, $\lambda = 0.33$, $\lambda = 0.67$, and $\lambda = 1.00$

Finally, the two properties of declining aggregate surplus (30) and the linearity of the payoffs *for given* λ imply that the bargaining solution is inefficient at the equilibrium price for all $\lambda > 0$. This follows directly from the fact that the slope of the bargaining frontier must be smaller than one in absolute value at any λ. The argument is given geometrically in Figure 8 and Figure 9.



(a) high government consumption

(b) low government consumption

Figure 9: No efficient Nash bargaining solution under price feedback: the better set (red)

The bargaining frontier is given by the bold downward-sloping curve. To provide the intuition for this result, it is useful to reconsider the bargaining problem. Since both groups are price takers in the commodity market, they assume that its price is given and unaffected by their wage setting for given λ. Thus, the negotiating parties have a perceived payoff frontier with slope minus one while the slope of the bargaining frontier is less in absolute value. In addition to the frontier shown in Figure 8, Figure 9 contains the level curve of the Nash bargaining solution, which must have slope minus one at the equilibrium payoff. Since the slope of the bargaining frontier is flatter, there exists a lower λ and a redistribution at the equilibrium price $p = \mathcal{P}(M, p^e, \lambda)$ which improves the Nash product. The possibility of such improvements is indicated geometrically by the red regions, the feasible upper contour set.

3.4 Summary

For a general discussion of the role of bargaining as a wage determination device, one should note first that temporary equilibria with efficient bargaining exist and they are unique under the same set of assumptions as in other cases of wage setting with price flexibility and market clearing. Thus, temporary equilibria exist so that efficient wage bargaining by itself cannot be the cause for involuntary unemployment.

>From a macroeconomic point of view, however, the most striking result is that higher union power directed toward a desired and successful redistribution from profits to wages in temporary equilibrium always causes lower employment and lower output. This universal negative impact of union power on employment and total output has additional allocative consequences. With constant exogenous demand (government demand plus money balances), an increase of union power implies lower profits and lower effective demand by young shareholders. Production becomes less attractive to producers even if the income distribution (i. e. the profit share in output) stays constant, but the multiplier decreases. In other words, aggregate output to be distributed for private and public consumption declines with higher union power.

Therefore, if total output or aggregate private consumption in temporary equilibrium is considered as a welfare proxy, it would not be desirable to have a strong union imposing a high level of λ. However, the redistribution due to a higher wage bill implies higher savings and demand for money by workers inducing higher expected consumption in the second period. Thus, higher union power also induces an increase of real wealth for workers and higher expected indirect utility. Thus, young shareholders partly pay the bill of high union power through reduced consumption in both periods. Nevertheless, this increase always incurs a macroeconomic cost of lower total.

Finally, it was shown that an efficient bargaining procedure between the participants in the labor market alone does not lead to an efficient outcome with respect to the objective of the bargaining when the remaining market is competitive. Generally speaking, this reconfirms the typical features of results known from Second-best Theory, which say that noncompetitive or deviant behavior in one market alone while all others are competitive does not guarantee Second-best allocations if there are spillovers between markets. Notice that this result equally applies to the competitive temporary equilibrium. In other words, even the fully competitive temporary equilibrium is not efficient with respect to the bargaining criterion, due to the price feedback. Thus, the exogenous parametric setting of the negotiating power of one side of the market induces only an efficient allocation with respect to the *perceived* feasible bargaining set, and which is inefficient with respect to general equilibrium feasibility. Thus, an efficient level of bargaining power would have to be determined endogenously.

$>$From a general welfare perspective, however, it is not clear whether this inefficiency implies also suboptimality and failure to satisfy a Second-best property since both criteria are applied to a comparative-statics analysis of allocations in temporary equilibrium at given money balances and expectations. Therefore, for the *dynamic* macroeconomic perspective taken here with overlapping generations of consumers, the Second-best failure may not seem to be of such primary importance. Moreover, the welfare issue becomes even more complex for sequences of temporary equilibria and requires further criteria and investigations, also with respect to stationary states. What they imply for the dynamic development will be analyzed partly in Section 5. Moreover, arguments will be discussed which would justify an intertemporal adjustment of union power and its consequences, invalidating many arguments of the static comparisons with constant union power.

4 A Parametric Example: the Isoelastic Case

Some further qualitative and quantitative properties of the bargaining model can be obtained when the functional forms of both groups of agents are isoelastic. These features will also prove useful in Section 5 where the dynamic behavior of the model will be discussed. Let the shareholder's utility be given by $\log c_0 + \delta \log c^e$ with $\delta > 0$, which implies a constant propensity $c \equiv \frac{1}{\delta+1}$ to consume out of net profits which is independent of price expectations.

Next, assume that the disutility of effort of the young worker is given by

$$v(\ell) = \frac{C}{C+1}\ell^{1+\frac{1}{C}}, \qquad 0 < C < 1,$$

and let the isoelastic production function be of the form

$$F(L) = \frac{A}{B}L^B, \qquad A > 0, \quad 0 < B < 1.$$

Solving the young worker's first-order condition of optimality $(1-\tau_w)\frac{w}{p^e} = \ell^{1/C}$ yields the individual utility-maximizing labor supply as

$$\ell = \left((1-\tau_w)\frac{w}{p^e}\right)^C,$$

implying an isoelastic competitive aggregate labor supply function

$$N_{\text{com}}\left(\frac{w}{p^e}\right) = n_w\left((1-\tau_w)\frac{w}{p^e}\right)^C.$$

Its inverse is given by

$$S_{\text{com}}(L) = \frac{1}{1-\tau_w}\left(\frac{1}{n_w}L\right)^{1/C}.$$

This is a strictly convex isoelastic function measuring the *aggregate marginal willingness* to work at the aggregate level L when n_w homogeneous workers are employed equally. This is the inverse of the competitive aggregate labor supply function.

The *individual reservation wage* of each worker is the solution of

$$\frac{w}{p^e} = \frac{1}{1-\tau_w}\frac{v(\ell)}{\ell} = \frac{1}{1-\tau_w}\frac{C}{C+1}\ell^{1/C}.$$

Thus, the maximal amount of labor each worker is willing to supply at a given wage w is given by

$$\ell = \left((1-\tau_w)\frac{C+1}{C}\frac{w}{p^e}\right)^C. \tag{31}$$

Therefore, the *aggregate reservation wage* function of the union is given by

$$S(L) = \frac{C}{C+1}\frac{1}{1-\tau_w}\left(\frac{1}{n_w}L\right)^{1/C},$$

which has the same constant elasticity as the aggregate marginal willingness to work of the union. Therefore, one finds that

$$S(L) = \frac{C}{C+1}S_{\text{com}}(L) \qquad \text{and} \qquad N\left(\frac{w}{p^e}\right) = \left(\frac{C+1}{C}\right)^C N_{\text{com}}\left(\frac{w}{p^e}\right).$$

The functions S and S_{com} have the same elasticity $1/C$, which coincides with the elasticity of the individual marginal willingness to work, while N and N_{com} have the same elasticity C.

The inverse of the demand for labor (7) can be computed explicitly

$$\begin{aligned}\theta^e = h^{-1}(L) &= \frac{E_F(L)}{E_S(L)+1}\frac{F(L)}{S(L)L} = \frac{BC}{C+1}\frac{F(L)}{S(L)L}\\ &= A(1-\tau_w)n_w^{1/C}L^{\frac{BC-(C+1)}{C}}.\end{aligned}$$

This yields the labor demand function under bargaining as

$$L = h(\theta^e) = \left(\frac{\theta^e}{A(1-\tau_w)\, n_w^{1/C}} \right)^{\frac{C}{BC-(C+1)}} \tag{32}$$

$$= A^{\frac{C}{C+1-BC}} (1-\tau_w)^{\frac{C}{C+1-BC}} n_w^{\frac{1}{C+1-BC}} (\theta^e)^{\frac{C}{BC-(C+1)}}, \tag{33}$$

which has a constant elasticity satisfying

$$-C < E_h(\theta^e) = \frac{C}{BC-(C+1)} = -\frac{C}{C(1-B)+1} < 0. \tag{34}$$

Therefore, aggregate labor demand under bargaining is an isoelastic, strictly monotonically decreasing function in expected inflation. For a given $p^e > 0$, it is also isoelastic, strictly monotonically increasing, and concave in the price. Substituting labor demand (32) into the production function implies a strictly decreasing isoelastic aggregate supply function in expected inflation given by

$$AS(\theta^e) = \frac{1}{B} A^{\frac{C+1}{C+1-BC}} (1-\tau_w)^{\frac{BC}{C+1-BC}} n_w^{\frac{B}{C+1-BC}} (\theta^e)^{\frac{BC}{BC-(C+1)}}, \tag{35}$$

making it an isoelastic, strictly increasing, and strictly concave function of the commodity price p for any given price expectation p^e.

Regarding the income distribution, equation (13) implies that, for any given union power $0 \leq \lambda \leq 1$, the profit share in output is a given constant

$$\frac{\pi}{py} = (1-\lambda)\left(1 - \frac{BC}{C+1}\right). \tag{36}$$

Thus, with isoelastic production and preferences, the *profit share* under efficient bargaining becomes a linear, decreasing function in λ, independent of the expected inflation rate.

The two properties, an isoelastic utility of shareholders together with an inflation-independent profit distribution (36), imply that there is no inflation feedback into aggregate commodity demand under bargaining. Thus, one obtains from (15) as the income-consistent aggregate demand function

$$D(m,\lambda) = \frac{m+g}{1-c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1})}, \tag{37}$$

which is strictly decreasing in λ and independent of expected prices. Equating aggregate demand (37) and aggregate supply (35), one obtains a unique positive equilibrium price $p = \mathcal{P}(M, p^e, \lambda)$ where the price map $\mathcal{P}$ has the usual properties, i. e. it is increasing and linear homogeneous in (M, p^e). Due to the isoelasticity of aggregate supply given in (35), its inverse with respect to price expectations $\mathcal{P}^e$ is given explicitly by

$$p^e = \mathcal{P}^e(p, M, \lambda) := pAS^{-1}\big(D(M/p,\lambda)\big)$$

$$= pAS^{-1}(1) \left(\frac{M/p+g}{1-c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1})} \right)^{\frac{BC-(C+1)}{BC}}, \tag{38}$$

which is one-to-one, strictly increasing, and strictly convex in p. Notice that the inverse of the price law is an isoelastic function in $(M/p + g)$, which becomes an isoelastic function in p only when exogenous government demand g is equal to zero. Thus, the price law itself is an isoelastic function in M/p^e only when $g = 0$.

In addition to the bounds derived in the general setting of Section 3, one obtains upper and lower bounds for the respective elasticities of the employment function using the isoelasticity of the labor supply function (34).

$$\begin{aligned} 0 < E_{\mathcal{L}}(M) &\overset{(21)}{=} -E_h(\theta^e)E_{\mathcal{P}}(M) = \frac{C}{C(1-B)+1}E_{\mathcal{P}}(M) < E_{\mathcal{P}}(M), \\ -C < E_{\mathcal{L}}(p^e) &\overset{(22)}{=} E_h(\theta^e)\left(1 - E_{\mathcal{P}}(p^e)\right) = -\frac{C}{C(1-B)+1}\left(1 - E_{\mathcal{P}}(p^e)\right) < 0, \\ 0 > E_{\mathcal{L}}(\lambda) &\overset{(27)}{=} -E_h(\theta^e)E_{\mathcal{P}}(\lambda) = \frac{C}{C(1-B)+1}E_{\mathcal{P}}(\lambda) > E_{\mathcal{P}}(\lambda). \end{aligned} \tag{39}$$

Since the output function $\mathcal{Y}(M, p^e, \lambda) = F(\mathcal{L}(M, p^e, \lambda))$ is simply the composition of the production function with the employment function, its elasticities are the same expressions as in (39) each multiplied by B, the elasticity of the production function F. Observe again that all equilibrium maps will be isoelastic functions only if government demand g is equal to zero.

Lower bounds for $E_{\mathcal{W}}(M)$ and $E_{\mathcal{W}}(p^e)$ have been found in (25). In order to establish upper bounds, note that the wage law can be written as a multiple, which neither depends on M nor p^e, of the workers' reservation wage using the constant elasticities of production and labor supply. From (24) one has

$$\mathcal{W}(M, p^e, \lambda) = \left(1 + \lambda\frac{C(1-B)+1}{BC}\right) W_{\Omega}(p^e, \mathcal{L}(M, p^e, \lambda)) \tag{40}$$

which, using (39) and again (34), implies both

$$0 < E_{\mathcal{W}}(M) = E_S(L)E_{\mathcal{L}}(M) = \frac{E_{\mathcal{P}}(M)}{C(1-B)+1} < E_{\mathcal{P}}(M) < 1$$

and

$$0 < E_{\mathcal{W}}(p^e) = 1 - E_S(L)E_{\mathcal{L}}(p^e) = 1 - \frac{1 - E_{\mathcal{P}}(p^e)}{C(1-B)+1} < 1.$$

Therefore, we can conclude that the wage elasticity with respect to money balances and price expectations are positive and less than unit-elastic.

4.1 The Role of Union Power

While union power determines uniquely the relative share $\lambda/(1-\lambda)$ of labor income to profits as a monotonically increasing function in λ, its impact on the other employment–wage related equilibrium values is not necessarily monotonic due to the price feedback. For the wage law

$$\mathcal{W}(M, p^e, \lambda) = W\left(p^e, \lambda, \mathcal{P}(M, p^e, \lambda), \mathcal{L}(M, p^e, \lambda)\right),$$

one finds from (26) that the nominal wage is proportional to the firm's average nominal labor productivity,

$$\mathcal{W}(M, p^e, \lambda) = \left(\frac{BC}{C+1} + \lambda \frac{C+1-BC}{C+1} \right) \frac{pF(L)}{L}. \tag{41}$$

While the term in parenthesis is monotonically increasing in λ and independent of the state variables (M, p^e), the nominal labor productivity itself with $p = \mathcal{P}(M, p^e, \lambda)$ and $L = \mathcal{L}(M, p^e, \lambda)$ is not necessarily increasing in λ. Therefore, due to the price feedback, the nominal wage is not necessarily an increasing function in union power λ. However, from the above equation it follows that the equilibrium real wage

$$\alpha = \frac{w}{p} = \frac{\mathcal{W}(M, p^e, \lambda)}{\mathcal{P}(M, p^e, \lambda)} = \left(\frac{BC}{C+1} + \lambda \frac{C+1-BC}{C+1} \right) \frac{1}{B} F'(\mathcal{L}(M, p^e, \lambda))$$

is a constant multiple of the marginal product of labor, where the constant is an increasing linear function of λ and independent of demand parameters. Thus, in the isoelastic case, the parameter λ determines the *mark-up of the real wage over the marginal product of labor*, which is independent of the state variables M and p^e and of all fiscal and demand parameters. Nevertheless, the latter do affect the temporary equilibrium prices and wages as well as the allocation.

Concerning the nominal payoff, an increase in union power always increases the payoff of the union while decreasing the firm's profit, as shown in Figure 10. There the ranges of the firm's profits, the union's utilities, and the total wage bill (both in nominal and in real terms) are depicted as functions of union power. Notice that the share in total output Π/py is linear in λ

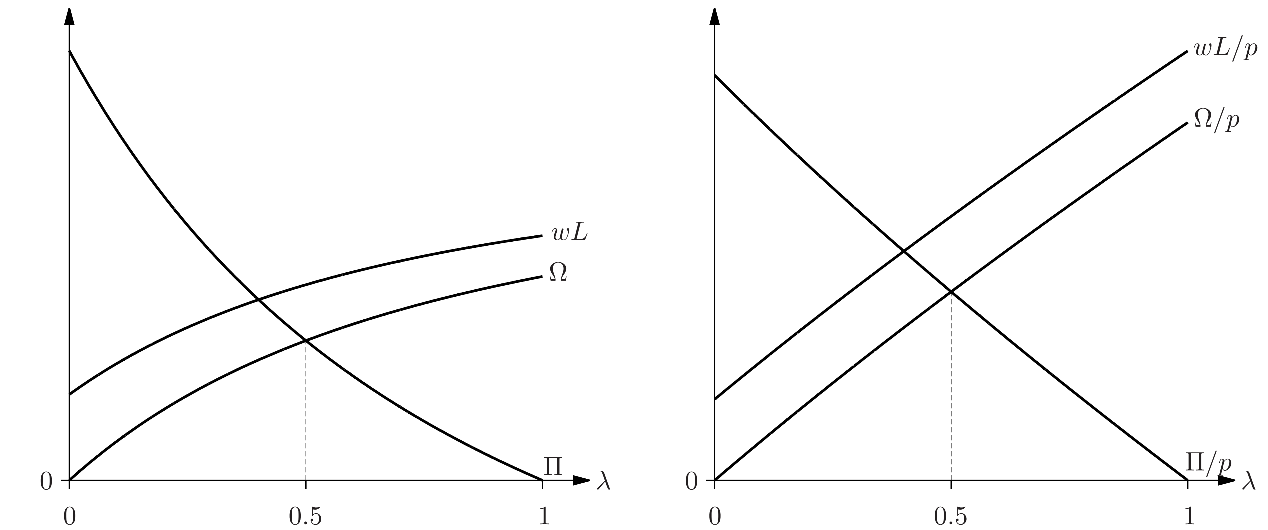


Figure 10: Range of profits, utilities, and wage bill for λ from 0 to 1

while the real profit Π/p is not (panel (b)).

Finally, the rate of underemployment can be calculated explicitly using the wage law and the price law. Because of

$$\frac{w}{p^e} \overset{(24)}{=} \left(1 + \lambda \frac{C(1-B)+1}{BC} \right) S(L) = \left(1 + \lambda \frac{C(1-B)+1}{BC} \right) \frac{C}{C+1} S_{\text{com}}(L), \tag{42}$$

the rate of underemployment can be simplified since N and S are isoelastic. This implies

$$\begin{aligned}\mathcal{U}(M,p^e,\lambda) &= \mathcal{U}\left(L,\frac{w}{p^e}\right) = 1-\frac{L}{N_{\text{com}}(w/p^e)}\\ &= 1-\left(\left(1+\lambda\frac{C(1-B)+1}{BC}\right)\frac{C}{C+1}\right)^{-C}\frac{L}{N_{\text{com}}(S_{\text{com}}(L))}\\ &= 1-\left(\left(1+\lambda\frac{C(1-B)+1}{BC}\right)\frac{C}{C+1}\right)^{-C}.\end{aligned} \tag{43}$$

Thus, with isoelastic production and utility functions, the equilibrium rate of underemployment is a constant determined by union power and by labor market parameters, i.e. by supply side factors only. It is totally independent of the state of the economy (M,p^e) and of fiscal and demand parameters. It is an increasing function of union power. Therefore, high λ imply positive voluntary underemployment and low imply negative voluntary underemployment. Its range is given by the interval

$$\left[1-\left(\frac{C+1}{C}\right)^{C}, 1-B^{C}\right].$$

In addition, one obtains that for the bargaining weight

$$\lambda_{\text{nat}} \equiv \frac{B}{C(1-B)+1},$$

for which the competitive equilibrium is obtained, as the zero of (43), i.e.

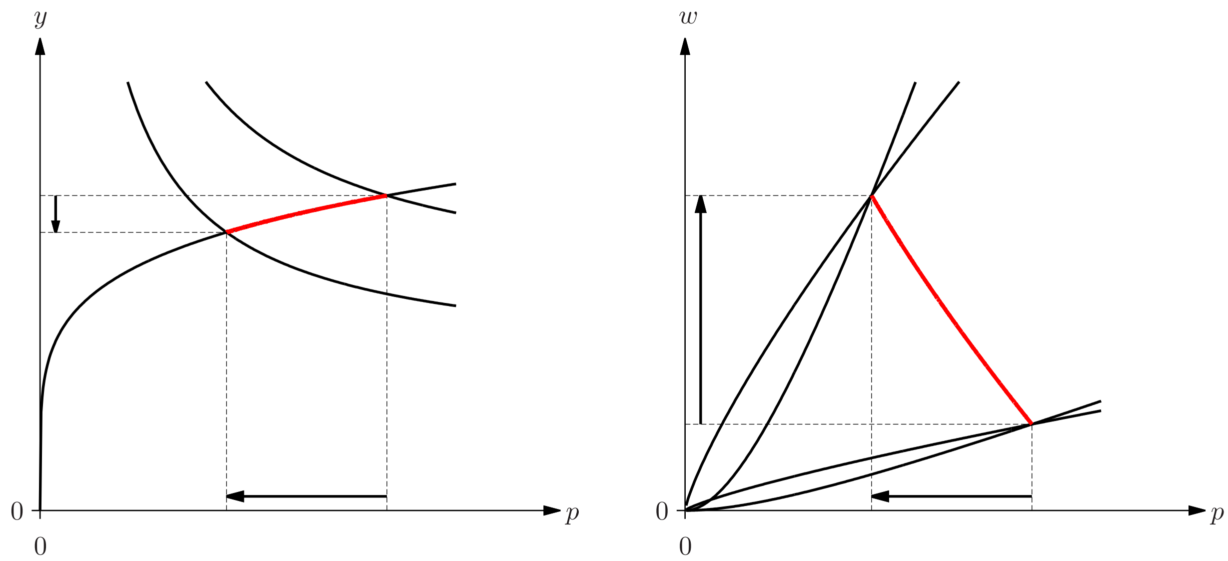
$$\mathcal{U}(M,p^e,\lambda_{\text{nat}}) = 1-\left(\left(1+\frac{B}{C(1-B)+1}\frac{C(1-B)+1}{BC}\right)\frac{C}{C+1}\right)^{-C} = 0.$$

Thus, $\lambda_{\text{com}} \equiv \lambda_{\text{nat}}$ is independent of the state (M,p^e) and of all demand parameters.

Figure 11 portrays the influence of union power on output, prices, and wages for the isoelastic case. Panel (a) depicts the equilibrium situation as the intersection of aggregate demand and aggregate supply, exploiting the fact that the union power has no effect on the aggregate supply curve. Thus, provided that there is no additional expectations feedback in aggregate demand, the influence of higher λ on the temporary equilibrium operates exclusively through the income distribution which causes a negative (downward) shift of the aggregate demand function (see equation (37)). This induces lower prices which then lead to lower employment and lower output.

4.2 Union Power and Wages

To analyze the impact of union power on the nominal wage is more involved than the previous comparisons since, even with isoelastic functions, the wage is not always monotonically increasing in λ. The values of the parameters given in Table 1 were chosen as a benchmark. They are used in Figures 11 and 12(a) for which the wage rate is increasing in λ. The right panel of Figure 11 shows the range of the equilibrium price and of the bargaining wage (the red curve) in temporary equilibrium for λ between zero and one, for values of the parameters where wages



(a) range of prices and output

(b) range of prices and wages

Figure 11: Output, prices, and wages for λ from zero to one

A	B	C	$\tau_\pi = \tau_w$	λ	M	g	p^e	c	n_w
1	0.5	0.5	0.25	0.5	1	1	1	0.5	2

Table 1: Standard parameterization

are monotonically increasing. The diagram has been augmented by the graphs of two functions (the black curves) which represent the market clearing conditions under bargaining for the labor market and the commodity market separately, each parametrized by the commodity price p. To derive their properties, consider first the wage equation (41) in the isoelastic case with *employment consistency* (labor market equilibrium) only, i. e. with $L = h(\theta^e)$. For given p^e, this implies the bargaining wage

$$LE(p, \lambda) := \left(\frac{BC}{C+1} + \lambda \frac{C+1-BC}{C+1} \right) \frac{pF(h(\theta^e))}{h(\theta^e)}, \tag{44}$$

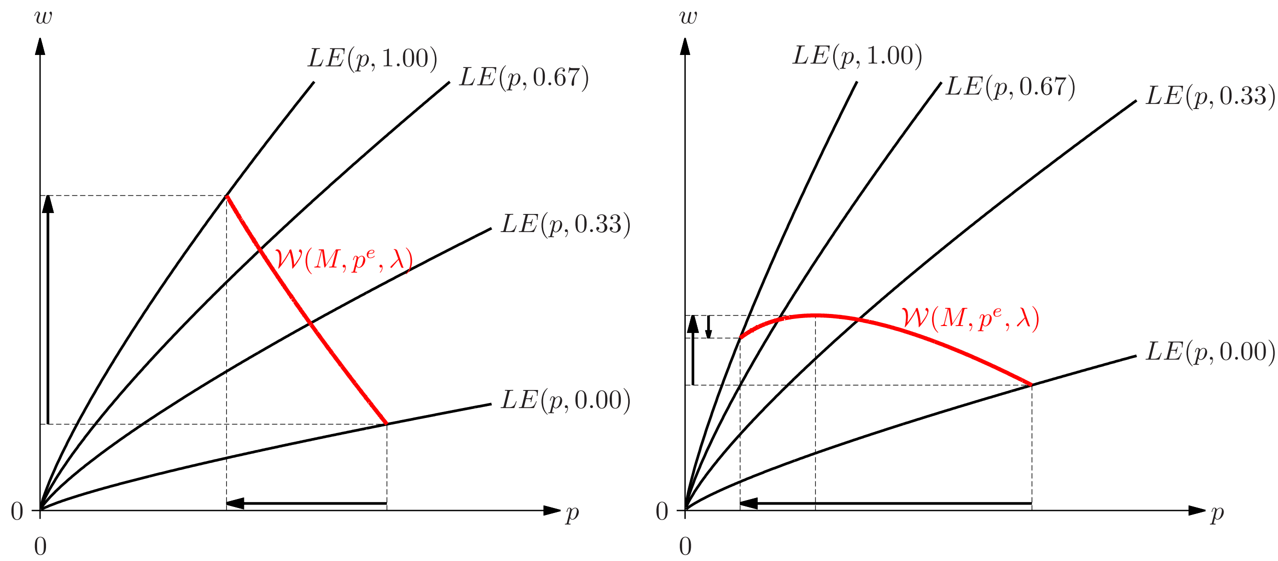
for each commodity price, which is taken as given by workers as well as by the producer. The properties of F and h imply that the function LE is strictly increasing and strictly concave in p. In addition, since h is independent of λ, the employment-consistent bargaining wage LE is strictly increasing in λ as well.

Similarly, for *commodity-market* consistency, $F(L) = D(M/p, \lambda)$ must hold. Therefore, inserting the aggregate demand function for the isoelastic case from (37), one obtains an induced price–wage relation under commodity market equilibrium

$$CE(p, \lambda) := \left(\frac{BC}{C+1} + \lambda \frac{C+1-BC}{C+1} \right) \frac{pD(M/p, \lambda)}{F^{-1}(D(M/p, \lambda))}. \tag{45}$$

With isoelastic functions of consumers and the producer, one finds that the function CE is increasing and convex in p and it is also increasing in λ. Clearly, the intersection of the graphs of the two functions LE and CE defines the temporary equilibrium pair (p, w), which follows also from the equality of aggregate supply and aggregate demand

$$AS(\theta^e) = F(h(\theta^e)) = D(M/p, \lambda),$$



(a) low government demand

(b) high government demand

Figure 12: The role of government demand on prices and wages for λ from zero to one

which is equivalent to equating (44) and (45). As shown above, λ shifts both wage functions upward always decreasing the equilibrium price. However, the impact of union power on the equilibrium bargaining wage may still be ambiguous, depending on whether the demand effect dominates the supply effect. Nevertheless, the associated real wage must always be increasing in λ.

Figure 11(b) portrays a situation of a negatively-sloped price–wage curve, implying a monotonic *increase* in nominal wages as λ changes from zero to one. However, there are situations where the equilibrium bargaining wage is not always monotonically increasing in union power λ. Figure 12 displays the effect of union power for two different levels of money balances with isoelastic functions and given eleasticities. For high levels of money balances (left panel), the wage is globally increasing whereas for low levels, the wage is increasing initially reaching a maximum for some critical level $0 < \lambda < 1$ and then declines with further increases of union power (right panel). The reason for the reverse effect, arises from the fact that the elasticity of the price law cannot be constant as long as government demand is positive *and* that it is a function of money balances. Thus, the level of money balances and of government demand could be potential reasons for the decline in wages.

In order to understand this effect, we investigate the elasticity of the price law and its impact on the wage law. If one computes the elasticity of the wage law (40) with respect to union power

$$E_{\mathcal{W}}(\lambda) = \underbrace{\frac{(C+1-BC)\lambda}{(C+1-BC)\lambda + BC}}_{\text{from the mark-up}} + \underbrace{\frac{E_{\mathcal{P}}(\lambda)}{C(1-B)+1}}_{\text{from } W_{\Omega}},$$

one obtains two distinct effects. The parameter λ affects the workers' reservation wage negatively, but it affects the scaling factor positively. For wages to decrease in union power, the latter needs to be outbalanced by the reservation wage effect. Let us first show that this cannot occur when government demand is equal to zero. Using the explicit form of the inverse of the price law (38), one also obtains an explicit form of the inverse with respect to λ given by

$$\lambda = \Lambda(M, p^e, p) := \frac{1}{\tilde{c}}\left(\left(\frac{p^e}{\tilde{A}}\right)^{\tilde{B}} \frac{M/p+g}{p^{\tilde{B}}} - (1-\tilde{c})\right) \tag{46}$$

with

$$\tilde{A} = AS^{-1}(1), \qquad \tilde{B} := \frac{BC}{C+1-BC}, \qquad \text{and} \qquad \tilde{c} := c(1-\tau_\pi)\left(1 - \frac{BC}{C+1}\right).$$

The function Λ is strictly decreasing in p with elasticity greater than minus one. Therefore, $|E_{\mathcal{P}}(M, p^e, \lambda,)| = |1/E_\Lambda(M, \mathcal{P}(M, p^e, \lambda))| > 1$ in general.

For $g = 0$, one obtains from (46)

$$E_\Lambda(\lambda) = -(1+\tilde{B})\frac{Mp^{-(1+\tilde{B})}}{Mp^{-(1+\tilde{B})} - (1-\tilde{c})(\tilde{A}/p^e)^{\tilde{B}}}.$$

Solving for $Mp^{-(1+\tilde{B})}$ from (46) and substituting implies

$$E_\Lambda(\lambda) = -(1+\tilde{B})\frac{1-\tilde{c}+\lambda\tilde{c}}{\lambda\tilde{c}}$$

and

$$E_{\mathcal{P}}(\lambda) = -\frac{\lambda\tilde{c}}{(1+\tilde{B})(1-\tilde{c}+\lambda\tilde{c})}.$$

Thus, $E_{\mathcal{P}}(\lambda)$ is monotonically decreasing in λ with $E_{\mathcal{P}}(0) = 0$ and

$$-1 < E_{\mathcal{P}}(1) = -\frac{\tilde{c}}{(1+\tilde{B})} < E_{\mathcal{P}}(0) = 0. \tag{47}$$

Therefore, the wage elasticity is positive for all (M, p^e, λ). Moreover,

$$\begin{aligned} E_{\mathcal{W}}(\lambda) &= \frac{\lambda(C(1-B)+1)}{BC+\lambda(C(1-B)+1)} + \frac{E_{\mathcal{P}}(\lambda)}{C(1-B)+1} \\ &= \frac{\lambda(C(1-B)+1)}{BC+\lambda(C(1-B)+1)} - \frac{1}{C+1-BC}\frac{\lambda\tilde{c}}{(1+\tilde{B})(1-\tilde{c}+\lambda\tilde{c})}. \end{aligned}$$

is the difference of two concave and increasing functions in λ with $E_{\mathcal{W}}(0) = 0$ and

$$E_{\mathcal{W}}(1) = \frac{C(1-B)+1}{C+1} - c(1-\tau_\pi)\frac{C+1-BC}{(C+1)^2} > 0.$$

Thus, by continuity, the wage elasticity is also positive for large λ and for all $g > 0$ small.

With this information, we are now able to identify situations numerically were a higher government demand g may lead to a negative elasticity of wages with respect to union power. The properties shown are qualitatively identical in a large neighborhood of the benchmark values. However, for large government demand, one obtains a negative wage effect as displayed in Figure 12(b). The reason for this effect lies primarily in the impact of g on the elasticity of the aggregate demand function. For $g > 0$, one finds that it is an increasing function which becomes less elastic for higher prices such that

$$-1 < E_D(p) := -E_D(M/p) = -\frac{\partial D(M/p,\lambda)}{\partial(M/p)}\frac{M/p}{D(M/p,\lambda)} = -\frac{M/p}{M/p+g} < 0.$$

It seems that this increase of the price elasticity together with the change of the income distribution as λ increases eventually induces the reversal effect for the wage law.

4.3 Comparing Wages, Prices, and Payoffs

The previous section analyzed the allocation and price effects of union power under cooperative bargaining. It was shown that the competitive equilibrium corresponds to a particular value of union power for all states and demand situations. It is an interesting and challenging exercise to carry out an additional comparison of the outcomes under bargaining with those of the two other basic *noncooperative* equilibria, which are often considered in the literature when one-sided wage setting power is discussed for the labor market. These are the situation of a monopolistic union and of a monopsonistic production syndicate or firm, assuming that in all cases the commodity market is cleared competitively and the government behaves identically, taking full account of the general-equilibrium effects of prices and incomes.

Comparing the price–wage pair of a bargaining solution (for a given λ) with the price–wage pairs of the two monopolistic cases (see Section 2.5) and the competitive outcome will yield different answers depending on the given level λ of the bargaining power. Thus, while the price–wage situations for the competitive as well as for the monopolistic situations are uniquely determined, their relative positions to a temporary equilibrium under bargaining will depend on the bargaining power. Therefore, it may be interesting to compare the situation of a *strong union* under bargaining characterized by $\lambda = 1$ with the noncooperative situation of the *monopolistic union*. On the other hand, the price–wage situations and allocations of the noncooperative equilibrium with a *monopsonistic firm* may be compared with those resulting under bargaining induced by a *weak union* under bargaining given by $\lambda = 0$.

In order to understand the influence of the price feedback, which operates in all four cases, it is useful to construct the set of feasible (individually rational) bargaining agreements between the union and the producer *including* the price feedback. Let $(L, w) \gg 0$ denote an arbitrary bargaining agreement. Given the restriction of nonnegativity of the payoffs, (L, w) is called *individually rational* for a given price p if

$$\Pi(p, w, L) = pF(L) - wL \geq 0 \qquad \text{and} \qquad \Omega(p^e, w, L) = wL - p^e S(L)L \geq 0.$$

An agreement (L, w) is called *income/demand-consistent* at p if

$$pF(L) = M + pg + c(1 - \tau_\pi)(pF(L) - wL) \tag{48}$$

which imposes a restriction on feasibility and on the equilibrium price p. Nonnegativity of profit implies that feasible employment levels have to satisfy $F(L) - g \geq 0$. Given the form of the aggregate demand function (48), one can solve for the associated equilibrium price explicitly to obtain

$$p(L, w) := \frac{M - c(1 - \tau_\pi)wL}{F(L)\,(1 - c(1 - \tau_\pi)) - g}, \qquad L \neq L_{\text{crit}} := F^{-1}\left(\frac{g}{1 - c(1 - \tau_\pi)}\right)$$

which must be positive for any $(L, w) \gg 0$. This implies

$$\begin{aligned} \Pi(p(L, w), w, L) =& p(L, w)F(L) - wL = \frac{M - c(1 - \tau_\pi)wL}{F(L)\,(1 - c(1 - \tau_\pi)) - g}F(L) - wL \\ =& \frac{MF(L) - wL(F(L) - g)}{F(L)\,(1 - c(1 - \tau_\pi)) - g}. \end{aligned} \tag{49}$$

The profit function (49) is continuous except at the critical level L_{crit}, where the denominator

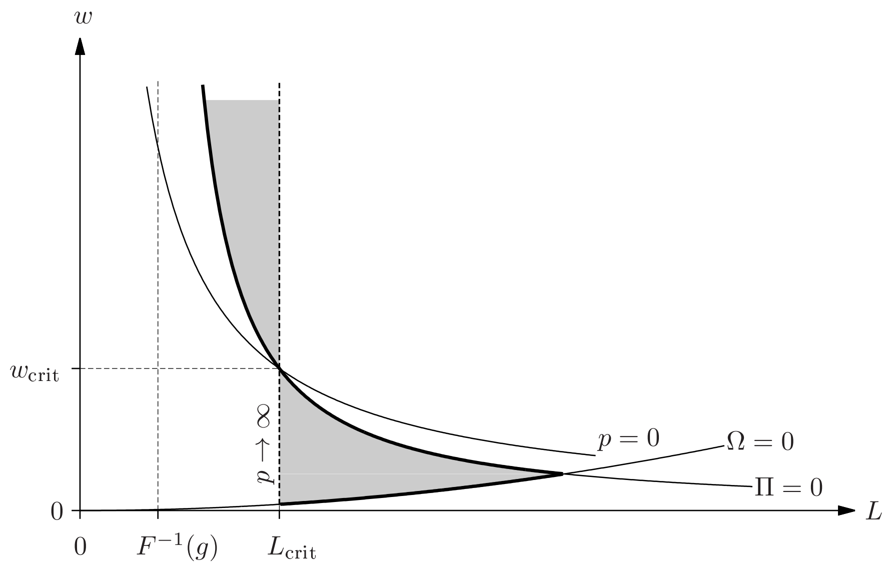


Figure 13: Employment–wage pairs under individual rationality and feasibility

of the price function is zero and changes sign, and where the price and profit become infinite. Thus, the set of bargaining pairs (L, w) with *positive* profit consists of the union of two disjoint open regions allowing unbounded wages for $L < L_{\text{crit}}$ and unbounded employment levels.[15] As a consequence one finds that the set of individually rational and income/demand-consistent employment–wage pairs takes the form of a union of two adjoining sets as depicted in Figure 13. Observe that the two critical employment levels, which are the same for each state of the economy (M, p^e), are determined by demand features and the production function. They are independent of money balances. However, high price expectations may make the lower compact curvilinear triangle empty, implying that all equilibrium allocations must be in the upper region of feasibility. Since unbounded wages with unbounded prices are feasible income/demand-consistent equilibrium allocations for employment levels near the upper critical level, the associated set of payoffs must be unbounded and be equal to all of $\mathbb{R}^2_+$.

By adding the equilibrium points and the λ-efficiency frontier to the above diagrams, one obtains in Figure 14 a comparison of all scenarios in allocation space and in payoff space. For the isoelastic example, all equilibria are in the compact "triangular" region of the employment–wage space. This shows also that the two one-sided strategic monopolistic situations induce inefficient employment levels below the efficiency frontier (left panel of Figure 14). In contrast, the comparison in payoff space confirms the location of the two one-sided monopolistic equilibria *above* the λ-bargaining frontier, see Figure 14(b). In other words, both monopolistic equilibria induce better payoffs which cannot be reached or supported by the cooperative decisions under efficient bargaining. Notice, however, that the union's payoff for $\lambda = 1$ is less than at the noncooperative equilibrium while the producer's profit is higher at the noncooperative situation than under bargaining with $\lambda = 0$. However, these relative positions of the payoffs depend on the price expectations. As Figure 15 shows, the payoffs in both noncooperative equilibria are higher than the maximal payoffs under bargaining when expected prices are high enough. The location in payoff space is surprising and counterintuitive at first. The arguments discussed at the end of the two monopolistic cases show that, for each given price level p in the noncooperative

[15] Strictly speaking, the set also contains the boundary point $(L_{\text{crit}}, w_{\text{crit}})$ since there exists an unbounded interval of positive prices which induce positive profits.

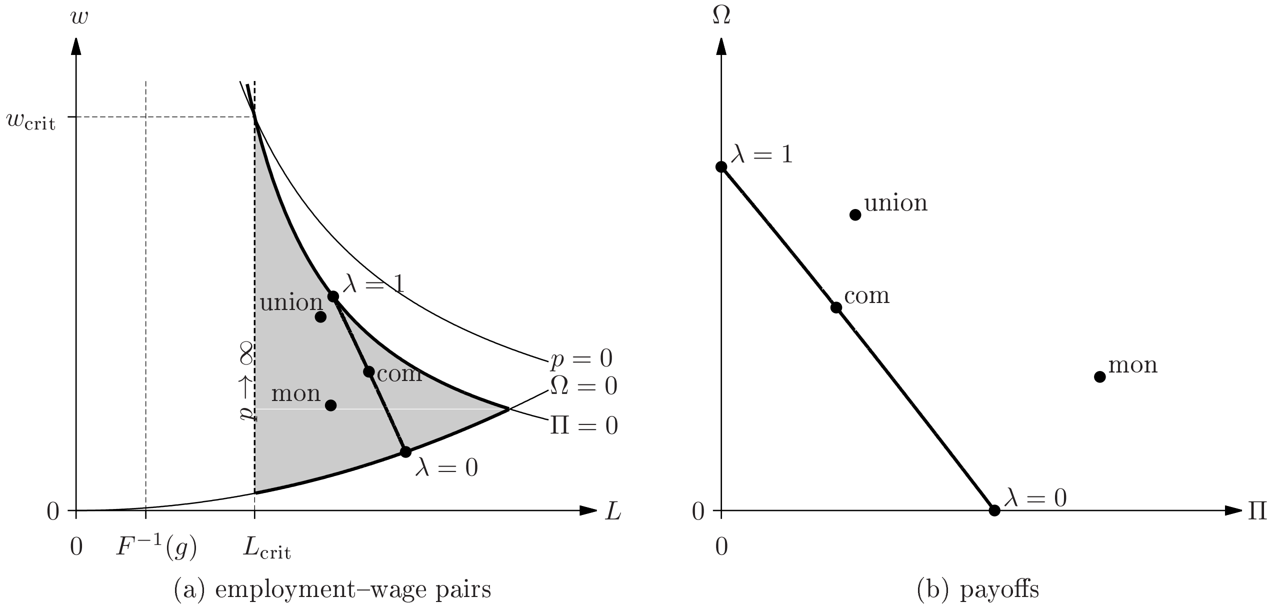


Figure 14: Wages, employment, and payoffs under low price expectations

situation, the monopolist can exert market power to obtain the full rent from the competitive agent, a possibility which neither the union nor the producer can obtain under bargaining. Thus, the price feedback seems to wash out this effect under cooperation.

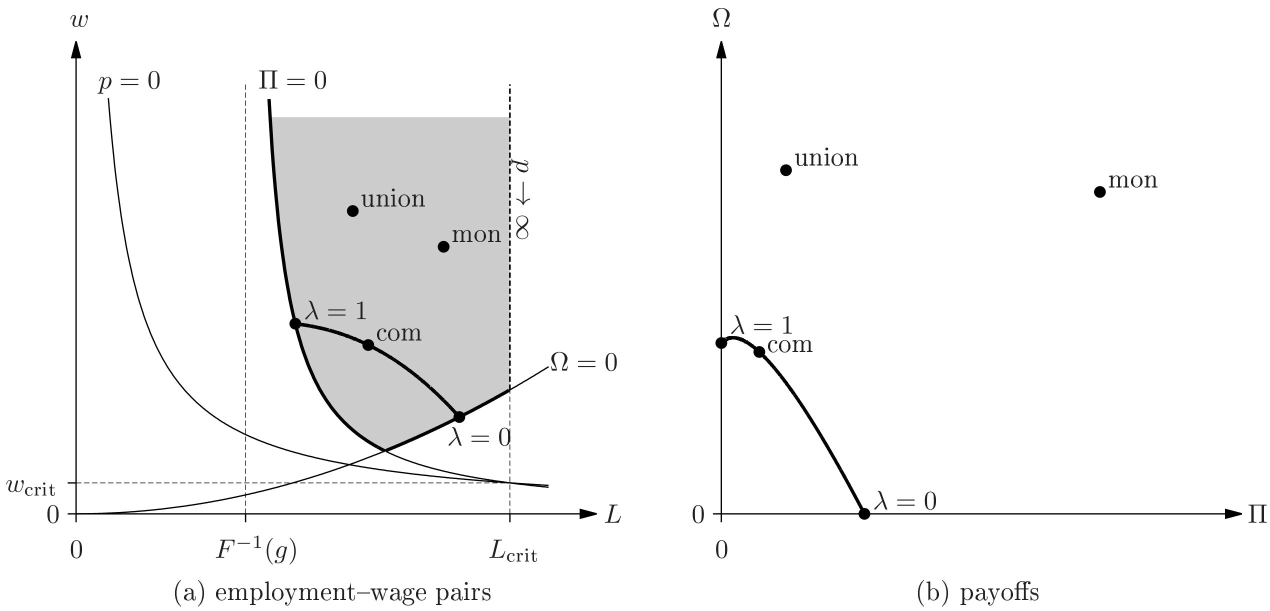


Figure 15: Wages, employment, and payoffs under high price expectations

The diagrams are drawn for the parameters of Table 1 and given values of the government parameters and for given values of the state variables money balances and expectations. Because of continuity, these features are locally robust properties and they will be observed for this isoelastic class of models in different magnitudes and possibly also in different relative orders under different parameters and values of the state variables. However, as some numerical experiments have shown, the basic features are preserved for a wide range of values of the parameters and of state variables. The overall homogeneity of the price law and the wage law does not preclude reversals or opposite effects.

While these result might seem to be counterintuitive at first sight, it is straightforward to discern the two principal reasons why these effects occur. First of all, the maximization of nominal objectives (profit resp. excess wages) creates spillovers between markets even for static general-equilibrium systems, which are primarily due to income effects. Because of these income effects, it is unlikely that the universal comparative-statics results (as often derived in partial-equilibrium models with strategic behavior) will persist in general-equilibrium models. It is known from general equilibrium theory that such effects are due to price normalization, implying different real allocations, relative prices, and nominal values of incomes (profits and wages) under different choices of a numeraire or of price indexes. These results are well documented and have been recognized in many different contexts in particular in welfare economics, international trade, or oligopoly theory whenever income feedbacks are taken into account appropriately with a nonconstant marginal utility of income for consumers.[16] In temporary equilibrium of a monetary economy, these effects clearly do not disappear.

Second, the price feedback, which was shown to be responsible for the inefficiency of the bargaining solution under competitive price taking in temporary monetary equilibrium, operates in each of the three cases endogenously in a different way. There is no structural feature of the model which relates the nominal payoffs, chosen for the bargaining problem neither to the nominal objectives by the monopolist/monopsonist with wage setting and price taking nor to the results induced by the maximization under competitive price and wage taking. Thus, in all three cases, the price feedback and the income feedbacks have a decisive influence on the nominal values chosen for the payoffs in the monetary economy. For these reasons, the four labor market scenarios whose equilibrium characteristics are compared in the price–wage space and in payoff space are in general not comparable with respect to real allocations or nominal payoffs, even under the weak concept of efficiency. Since, in addition, equilibrium prices and allocations depend on the other state variables, an extensive welfare analysis may not lead to conclusive results.

It is worth noting that some properties of the results are specific to the isoelastic model chosen for the numerical analysis since the bargaining parameter λ plays a specific dual role in temporary equilibrium. On the one hand, there is no impact of union power on aggregate supply. Therefore, the interaction of the isoelastic structure between production and labor supply shows that the measure of union power λ exerts a direct influence on the real wage mark-up and on the level of underemployment, making both of them constant in temporary equilibrium. These constants depend on the elasticities of the labor market participants and on union power only. Thus, in a dynamic economy as analyzed in the next section, both of them are constant over time, i. e. independent of (M, p^e), and they are independent of all fiscal and demand parameters in the economy. On the other hand, a powerful union which can choose the parameter λ does not exert absolute control over its seemingly most important endogenous variable the wage rate. Moreover, even for the isoelastic case, it seems unclear whether the wage outcome under bargaining dominates the competitive outcome, in some other sense than the efficiency criterion used above. It remains an open question to what extent the inefficiencies will change or disappear if the bargaining agents chose "real" rather than nominal payoffs as objectives.

[16]see for example Dierker & Grodal (1986); Böhm (1994); Gaube (1997); Roberts & Sonnenschein (1976)

5 Dynamics of Monetary Equilibrium

So far the characteristics of equilibria under bargaining were discussed for an arbitrary given period t with initial money balances M_t held by the private sector, expected prices for the next period by consumers $p^e_{t,t+1}$, and by the union power λ_t. Thus, the triple $(M_t, p^e_{t,t+1}, \lambda_t)$ describes the state of the economy at any given time. Associated with each state are the prices and wages and the levels of output and employment (p_t, w_t, y_t, L_t) in temporary equilibrium which are defined by applying the respective mappings from the previous section.[17]

This section analyzes the dynamic behavior of the economy in equilibrium assuming that union power is constant over time and given exogenously at some level $0 \le \lambda \le 1$. Since $\lambda/(1-\lambda)$ determines the relative share of wages over profits, no other economic variables related to the objectives of the agents are considered. As was shown in the previous section, λ has a significant impact on most important economic variables in every period, like output, incomes, prices, and consumption, which are relevant for welfare. Thus, it would be desirable to relate the specific value chosen for union power to the market data which are induced and to reevaluate the equilibrium outcome with respect to the true objectives of the agents. This leads to an endogenous determination of the measure of bargaining power. For the dynamics, this implies that an adaptive rule or a dynamic mechanism has to be defined based on the data in each period. However, at this stage we examine the dynamics of the monetary economy without providing any justification what level of union power λ would be reasonable to be assumed, leaving such questions to be addressed in future research. Therefore, the dynamic development of the economy will be described completely by characterizing the evolution of the two state variables money balances and expected prices $(M_t, p^e_{t,t+1})$, implying a two-dimensional state space $\mathcal{X} := \mathbb{R}^2_{++}$.

5.1 Perfect Foresight

A sequence $\{p^e_{t,t+1}, p_t\}_{t=t_0}^{\infty}$ of prices and expectations will be said to have the *perfect-foresight property* if $p^e_{t,t+1} = p_{t+1}$ holds for all t. It is one of the main questions of dynamic macroeconomic analysis to find conditions and define the concepts which ensure that perfect-foresight sequences are in fact generated by an associated dynamical system which is globally defined. In other words, a forecasting rule or a predictor has to be defined to ensure perfect foresight along any orbit.[18] In order to guarantee that, for any period t, the actual price p_t coincides with its associated prediction $p^e_{t-1,t}$, the condition

$$p^e_{t-1,t} = \mathcal{P}(M_t, p^e_{t,t+1}, \lambda)$$

must hold for any t. This defines implicitly the functional relationship determining how the forecast in any period for the next one should be chosen as a function of the previous forecast. Therefore, solving (16) for the expected price

$$p^e_{t,t+1} = \psi^*(M_t, p^e_{t-1,t}, \lambda) \equiv \mathcal{P}^e\left(M_t, p^e_{t-1,t}, \lambda\right) := p^e_{t-1,t} AS^{-1}\left(D\left(\frac{M_t}{p^e_{t-1,t}}, \lambda\right)\right)$$

[17] We will assume throughout this section that the aggregate demand function is independent of expected inflation. The general case could be dealt with easily using the result of Lemma 3.1.

[18] see Böhm (2010)

defines the perfect predictor $\psi^*(M_t, \cdot, \lambda)$ since for all (M_t, p^e, λ)

$$\mathcal{P}(M_t, \mathcal{P}^e(M_t, p^e, \lambda), \lambda) = \text{id}_{(M_t,\lambda)}(p^e).$$

Therefore, the two mappings

$$\begin{aligned} M_{t+1} &= \mathcal{M}(M_t, p^e_{t-1,t}, \lambda) := M_t + p_t \left(g - \tilde{\tau} D\left(M_t/p_t, \lambda\right)\right) \\ p^e_{t,t+1} &= \psi^*(M_t, p^e_{t-1,t}, \lambda) \end{aligned} \tag{50}$$

with $p_t = \mathcal{P}(M_t, \psi^*(M_t, p^e_{t-1,t}, \lambda), \lambda)$ and

$$\tilde{\tau} \equiv \tilde{\tau}\left(\frac{p^e_{t,t+1}}{p_t}, \lambda\right) = \tilde{\tau}\left(\frac{\psi^*(M_t, p^e_{t-1,t}, \lambda)}{\mathcal{P}(M_t, \psi^*(M_t, p^e_{t-1,t}, \lambda), \lambda)}, \lambda\right)$$

define the dynamic behavior of money balances and expectations under perfect foresight for any level of bargaining power λ. In addition, $\tilde{\tau}$ denotes the average tax rate which will be derived in (52). Since for all t, one has $p^e_{t-1,t} = p_t$, one can rewrite (50) as

$$\begin{aligned} M_{t+1} = \mathcal{M}(M_t, p_t, \lambda) &= M_t + p_t \left(g - \tilde{\tau} D\left(M_t/p_t, \lambda\right)\right) \\ p_{t+1} = \psi^*(M_t, p_t, \lambda) &= p_t AS^{-1}\left(D\left(\frac{M_t}{p_t}, \lambda\right)\right), \end{aligned} \tag{51}$$

defining equivalent dynamics with perfect foresight in the space of money balances and prices (M, p) for any given level λ of bargaining power.

It is one of the recurring themes of dynamical economies with price expectations that in most cases price dynamics induced under perfect foresight are unstable, a phenomenon which also occurs in the current model. To see this, let $\bar{M} > 0$ denote an arbitrary constant level of money balances and λ be given. Then, (51) reduces to the one-dimensional dynamical system in prices $\mathcal{G} : \mathbb{R}_{++} \to \mathbb{R}_{++}$,

$$p_{t+1} = \psi^*\left(\bar{M}, p_t, \lambda\right) =: \mathcal{G}\left(p_t\right).$$

Rewriting (16) one finds that it has the unique positive fixed point

$$p = \frac{\bar{M}}{D^{-1}\left(AS(\theta^e); \lambda\right)} = \frac{\bar{M}}{D^{-1}\left(AS(1); \lambda\right)},$$

where D^{-1} is the inverse of the aggregate demand function with respect to its first argument M/p. Since the price law is invertible with respect to price expectations with an elasticity strictly between 0 and 1 implies that the unique positive fixed point p is asymptotically unstable since

$$\begin{aligned} \mathcal{G}'(p) &= \frac{\partial \psi^*}{\partial p^e}(\bar{M}, p, \lambda) = \frac{\partial \mathcal{P}^e}{\partial p}\left(\bar{M}, p, \lambda\right) \\ &= \frac{1}{\frac{\partial \mathcal{P}}{\partial p^e}(\bar{M}, \psi^*(\bar{M}, p, \lambda), \lambda)} > \frac{\psi^*(\bar{M}, p, \lambda)}{\mathcal{P}(\bar{M}, \psi^*(\bar{M}, p, \lambda), \lambda)} = 1. \end{aligned}$$

5.2 Dynamics of Money Balances and Prices

One of the main reasons for the price feedback occurring under bargaining originates from the impact of the bargaining power on the income distribution which in turns influences aggregate demand. This has a major influence on the dynamics of savings and money balances which needs to be analyzed in detail to justify the formula suggested in (51) for the demand multiplier.

The aggregate nominal net income of young consumers in any period t is given by

$$(1-\tau_w)w_t L_t + (1-\tau_\pi)\pi_t.$$

Since market clearing implies that the amount of income spent on consumption by the young has to be equal to the amount not spent by the old and by the government, it follows that consumption expenditures by the young are equal to $c(1-\tau_\pi)\pi_t = p_t(y_t - g) - M_t$. Hence, young consumers save

$$\begin{aligned} M_{t+1} &= \underbrace{(1-\tau_w)w_t L_t + (1-c)(1-\tau_\pi)\pi_t}_{\geq 0} \\ &= M_t + p_t y_t \left((1-\tau_w)\frac{w_t L_t}{p_t y_t} + (1-\tau_\pi)\frac{\pi_t}{p_t y_t} \right) + p_t(g - y_t) \\ &= M_t - p_t y_t \left(1 - (1-\tau_w)\frac{w_t L_t}{p_t y_t} - (1-\tau_\pi)\frac{\pi_t}{p_t y_t} \right) + p_t g. \end{aligned}$$

Replacing the wage and the profit share by the elasticities (12) and (13), respectively, and writing $B := E_F(h(\theta^e_{t,t+1}))$ and $C := 1/E_S(h(\theta^e_{t,t+1}))$ for short yields that this term only depends on the expected rate of inflation $\theta^e_{t,t+1}$ and on union power λ,

$$\underbrace{1-(1-\tau_w)\left(\frac{BC}{C+1} + \lambda\left(1-\frac{BC}{C+1}\right)\right) - (1-\tau_\pi)(1-\lambda)\left(1-\frac{BC}{C+1}\right)}_{=:\tilde{\tau}(\theta^e_{t,t+1},\lambda)},$$

which always is between 0 and 1. Therefore aggregate savings are given by

$$\begin{aligned} M_{t+1} &= M_t - \tilde{\tau}\left(\theta^e_{t,t+1},\lambda\right) p_t y_t + p_t g = M_t + p_t\left(g - \tilde{\tau}\left(\theta^e_{t,t+1},\lambda\right) y_t\right) \\ &= M_t + \mathcal{P}(M_t, p^e_{t,t+1},\lambda)\left(g - \tilde{\tau}\left(\frac{p^e_{t,t+1}}{\mathcal{P}(M_t, p^e_{t,t+1},\lambda)},\lambda\right)\mathcal{Y}(M_t, p^e_{t,t+1},\lambda)\right) \\ &=: \mathcal{M}(M_t, p^e_{t,t+1},\lambda) \end{aligned}$$

defining the time-one map of money balances. The function $\tilde{\tau}(\theta^e_{t,t+1},\lambda)$ collects all terms which influence the income distribution. It represents the average tax rate on aggregate income, which can be rewritten as

$$\begin{aligned} \tilde{\tau}(\theta^e_{t,t+1},\lambda) &= 1-(1-\tau_w)\left(\frac{BC}{C+1} + \lambda\left(1-\frac{BC}{C+1}\right)\right) + (1-\tau_\pi)(1-\lambda)\left(1-\frac{BC}{C+1}\right) \\ &= \left(\frac{BC}{C+1} + \lambda\left(1-\frac{BC}{C+1}\right)\right)\tau_w + (1-\lambda)\left(1-\frac{BC}{C+1}\right)\tau_\pi \qquad (52) \\ &= \tau_\pi + \left(\frac{BC}{C+1} + \lambda\left(1-\frac{BC}{C+1}\right)\right)(\tau_w - \tau_\pi), \end{aligned}$$

which shows that it is a constant depending on the parameters of the economy but not on expected inflation. Since the coefficients of τ_w and τ_π add up to unity, $\tilde{\tau}$ is a convex combination of the different tax rates. If a common tax rate τ is imposed on all types of income, $\tilde{\tau}(\theta^e_{t,t+1}, \lambda) = \tau$ and $M_{t+1} = M_t + p_t(g - \tau y_t)$ hold. This implies that union power only affects the short-run tax return if different tax rates are imposed.

Substituting $M_{t+1} = \mathcal{M}(M_t, p^e_{t,t+1}, \lambda)$ into $\psi^* \left(M_{t+1}, p^e_{t,t+1}, \lambda\right)$ gives a perfect predictor depending on the current state only. Therefore the two-dimensional dynamics inducing perfect foresight are

$$\begin{pmatrix} M_{t+1} \\ p^e_{t+1,t+2} \end{pmatrix} = \begin{pmatrix} \mathcal{M}(M_t, p^e_{t,t+1}, \lambda) \\ \psi^* \left(\mathcal{M}(M_t, p^e_{t,t+1}, \lambda), p^e_{t,t+1}, \lambda\right) \end{pmatrix}. \tag{53}$$

5.3 Steady States and Stability

Exploiting the perfect-foresight property

$$\begin{aligned} & \begin{pmatrix} \mathcal{M}(M_t, p^e_{t,t+1}, \lambda) \\ \psi^* \left(\mathcal{M}(M_t, p^e_{t,t+1}, \lambda), p^e_{t,t+1}, \lambda\right) \end{pmatrix} \\ = & \begin{pmatrix} M_t + \mathcal{P}(M_t, p^e_{t,t+1}, \lambda) \left(g - \tilde{\tau}\left(\frac{p^e_{t,t+1}}{\mathcal{P}(M_t, p^e_{t,t+1}, \lambda)}, \lambda\right) D\left(\frac{M_t}{\mathcal{P}(M_t, p^e_{t,t+1}, \lambda)}, \lambda\right)\right) \\ p^e_{t,t+1} AS^{-1}\left(D\left(\frac{\mathcal{M}(M_t, p^e_{t,t+1}, \lambda)}{p^e_{t,t+1}}, \lambda\right)\right) \end{pmatrix} \\ = & \begin{pmatrix} M_t + p_t \left(g - \tilde{\tau}\left(\frac{\psi^*(M_t, p_t, \lambda)}{p_t}, \lambda\right) D\left(\frac{M_t}{p_t}, \lambda\right)\right) \\ p_{t+1} AS^{-1}\left(D\left(\frac{M_{t+1}}{p_{t+1}}, \lambda\right)\right) \end{pmatrix} \end{aligned}$$

and backdating the second equation gives

$$\begin{pmatrix} M_{t+1} \\ p_{t+1} \end{pmatrix} = \begin{pmatrix} M_t + p_t \left(g - \tilde{\tau}\left(\frac{\psi^*(M_t, p_t, \lambda)}{p_t}, \lambda\right) D\left(\frac{M_t}{p_t}, \lambda\right)\right) \\ p_t AS^{-1}\left(D\left(\frac{M_t}{p_t}, \lambda\right)\right) \end{pmatrix} \tag{54}$$

which is an equivalent formulation of the system (53).

Let $(M, p) \in \mathbb{R}^2_+$ be a steady state, for which the two conditions $g = \tilde{\tau}(1, \lambda) D(M/p, \lambda)$ and $1 = AS^{-1}\left(D(M/p, \lambda)\right)$ must hold simultaneously. If $(M, p) \gg 0$, monotonicity, homogeneity, and continuity of aggregate demand in (M, p) imply that there exists a continuum of fixed points since for all $\gamma > 0$, the $(\gamma M, \gamma p)$ are fixed points as well. Geometrically speaking this implies that the set of positive steady states consists of a half-line in the state space $\mathbb{R}^2_+$ with slope $m = M/p$. However, this condition can hold only when $AS(1) = g/\tilde{\tau}(1, \lambda)$. Thus, in the space of parameters of the economy, positive perfect-foresight steady states with a balanced government budget do not exist generically.

To analyze the local stability of any of these fixed points, one obtains as the Jacobian of the system[19]

$$J = \begin{pmatrix} 1 - \tilde{\tau}(1,\lambda)D'(m) & \tilde{\tau}(1,\lambda)D'(m)m \\ \frac{D'(m)}{AS'(1)} & 1 - \frac{D'(m)}{AS'(1)}m \end{pmatrix}$$
$$= \begin{pmatrix} 1 - \tilde{\tau}(1,\lambda)E_D(m)\frac{AS(1)}{m} & \tilde{\tau}(1,\lambda)E_D(m)AS(1) \\ \frac{E_D(m)}{E_{AS}(1)}\frac{1}{m} & 1 - \frac{E_D(m)}{E_{AS}(1)} \end{pmatrix}.$$

The trace and the determinant of J are

$$\operatorname{tr} J = 2 - \tilde{\tau}(1,\lambda)E_D(m)\frac{AS(1)}{m} - \frac{E_D(m)}{E_{AS}(1)}$$

and

$$\det J = \left(1 - \tilde{\tau}(1,\lambda)E_D(m)\frac{AS(1)}{m}\right)\left(1 - \frac{E_D(m)}{E_{AS}(1)}\right) - \frac{E_D(m)}{E_{AS}(1)}\tilde{\tau}(1,\lambda)E_D(m)\frac{AS(1)}{m}$$
$$= 1 - \frac{E_D(m)}{E_{AS}(1)} - \tilde{\tau}(1,\lambda)E_D(m)\frac{AS(1)}{m} = \operatorname{tr} J - 1.$$

The eigenvalues ν_1 and ν_2 are the roots of the characteristic equation $\nu^2 - (\operatorname{tr} J)\nu + \det J$, i. e.

$$\nu_{1,2} = \frac{\operatorname{tr} J \pm \sqrt{(\operatorname{tr} J)^2 - 4\det J}}{2}$$
$$= \frac{\operatorname{tr} J \pm \sqrt{(\operatorname{tr} J)^2 - 4\operatorname{tr} J + 4}}{2}$$
$$= \frac{\operatorname{tr} J \pm \sqrt{(\operatorname{tr} J - 2)^2}}{2}$$
$$= \frac{\operatorname{tr} J \pm (\operatorname{tr} J - 2)}{2}.$$

Thus, one obtains

$$\nu_1 = \operatorname{tr} J - 1 = \det J \qquad \text{and} \qquad \nu_2 = 1.$$

Since

$$\nu_1 = \operatorname{tr} J - 1 = 1 - \underbrace{\frac{\tilde{\tau}(1,\lambda)AS(1)}{m}}_{=g/m} \underbrace{E_D(m)}_{=m/(m+g)} - \frac{E_D(m)}{E_{AS}(1)} = \underbrace{\frac{m}{m+g}}_{\geq 0} - \underbrace{\frac{E_D(m)}{E_{AS}(1)}}_{<0} > 0,$$

both eigenvalues are nonnegative, which excludes the possibility of cycles. To establish an upper bound, note that

$$\nu_1 = E_D(m)\left(1 - \frac{1}{E_{AS}(1)}\right) = \frac{m}{m+g}\left(1 - \frac{BC - (C+1)}{BC}\right) = \frac{m}{m+g}\frac{C+1}{BC} \leq \frac{C+1}{BC},$$

[19]For simplicity, only the case of no inflation feedback on the average tax rate is considered. This is the case for the isoelastic example (Section 4) or under one common tax rate. To improve readability, $D(M/p,\lambda)$ is replaced by $D(m)$. Thus, D' may be written instead of $\partial D/\partial m$.

which is less than unity if both B and C are not "too small". More precisely, because of $m+g=(1-c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1}))AS(1)$ and $g=\tilde{\tau}(1,\lambda)AS(1)$,[20]

$$\nu_1 = \frac{1-\tilde{\tau}(1,\lambda)-c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1})}{1-c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1})}\frac{C+1}{BC}.$$

Consider the case that $B\to 0$ or $C\to 0$. Then $BC/(C+1)\to 0$ so that the first fraction

$$\frac{m}{m+g}\to\frac{1-\lambda\tau_w-(1-\lambda)\tau_\pi-c(1-\tau_\pi)(1-\lambda)}{1-c(1-\tau_\pi)(1-\lambda)}=\frac{(1-c)(1-\tau_\pi)(1-\lambda)+\lambda(1-\tau_w)}{1-c(1-\tau_\pi)(1-\lambda)}$$

is finitely bounded, whereas the second fraction tends to infinity, which implies that ν_1 tends to infinity. Therefore, depending on the parameters of the economy one may find convergence to any fixed point $(M,p)\gg 0$ or divergence.

Note that economically meaningful values for C are in the range $(0,1)$, so $(C+1)/(BC)$ must be greater than 2. In order to compensate this factor in ν_1, the public consumption g must outbalance m, which coincides with "high" tax rates, in particular a "high" wage tax.

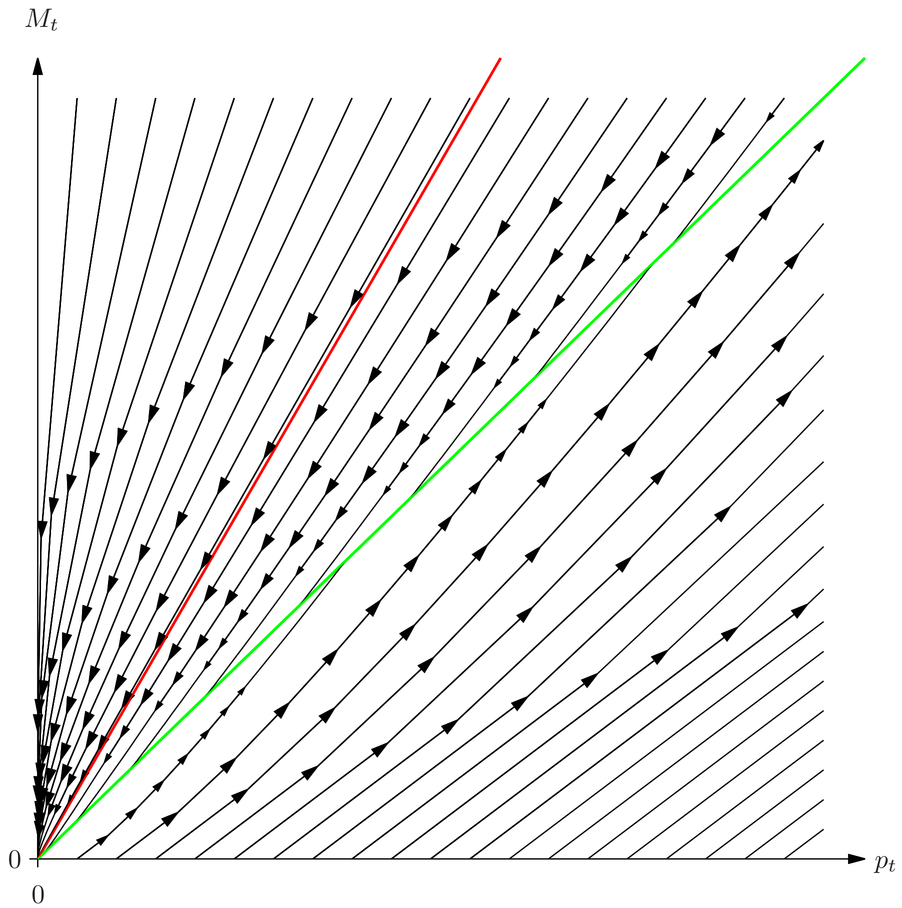


Figure 16: Convergence to continuum of stationary states

Figure 16 displays the situation with a continuum of stationary states under the parameterization given in Table 2. The green half-line is the set of steady states of (54) while the red half-line corresponds to an unstable balanced path (see the next section below). A numerical

A	B	C	g	τ_w	τ_π	λ	c	n_w
1.00	0.95	0.95	$\tilde{\tau}(1,\lambda)AS(1)$	0.50	0.25	0.00	0.99	1

Table 2: Parameterization used in Figure 16.

simulation for the isoelastic case analyzed in Section 4 shows that all orbits starting within the basin of attraction (the area to the lower right of the red line) converge to a positive point on the green line, whereas all paths originating in the triangle to the upper left of the red line converge to zero with prices slower than money balances implying increasing real money balances with unbounded growth of output and employment.

5.4 Dynamics of Real Money Balances under Perfect Foresight

Now consider the generic case with $AS(1) \neq g/\tilde{\tau}(1,\lambda)$. Since fixed points of (53) do not exist, the economically interesting situations are those when money and prices expand or contract at the same rate, implying constant levels of real money balances together with constant allocations.

Definition 5.1 *An orbit* $\{(M_t, p_t)\}_0^\infty$ *is called a balanced path if for all* t *one has* $m_t := M_t/p_t = M_{t+1}/p_{t+1} = m_{t+1}$.

It is clear that balanced paths can be identified with half-lines in the state space $\mathbb{R}^2_+$. Exploiting the homogeneity of the two mappings describing the money dynamics and the price dynamics, (54) induces a one-dimensional system describing the dynamics of real balances, given by

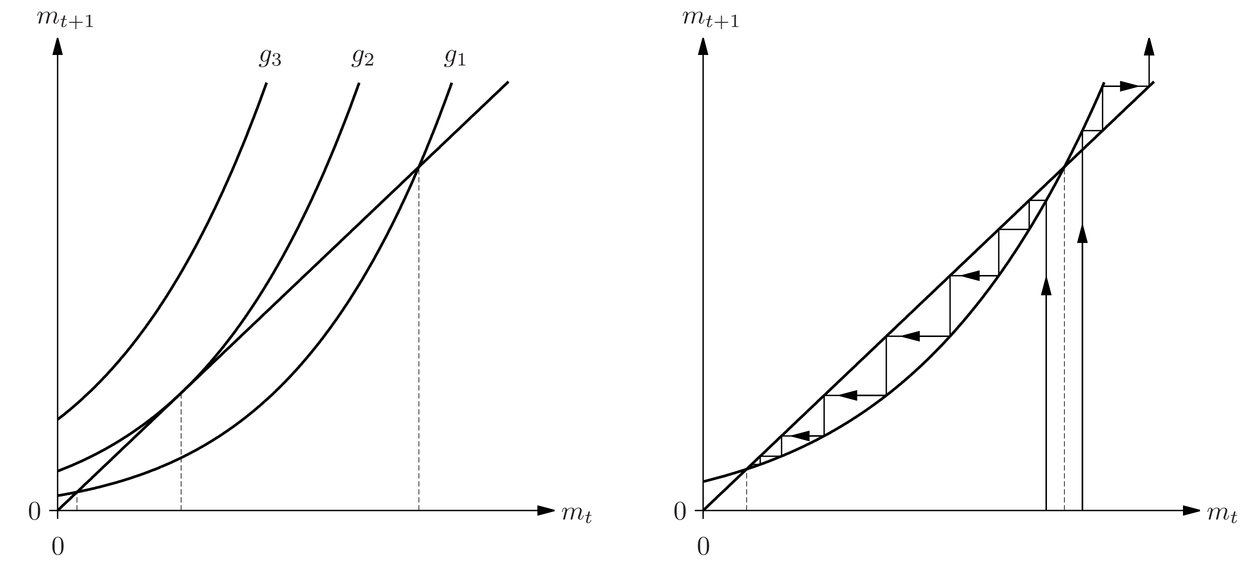
$$
\begin{aligned}
m_{t+1} = \frac{M_{t+1}}{p_{t+1}} &= \frac{\mathcal{M}(M_t, p^e_{t,t+1}, \lambda)}{\psi^*\left(\mathcal{M}(M_t, p^e_{t,t+1}, \lambda), p^e_{t,t+1}, \lambda\right)} \\
&= \frac{p_t\left(\frac{M_t}{p_t} + g - \tilde{\tau}\left(\frac{\psi^*(M_t,p_t,\lambda)}{p_t}, \lambda\right) D\left(\frac{M_t}{p_t}, \lambda\right)\right)}{p_t AS^{-1}\left(D\left(\frac{M_t}{p_t}, \lambda\right)\right)} \\
&= \frac{m_t + g - \tilde{\tau}\left(\psi^*(m_t, 1, \lambda), \lambda\right) D(m_t)}{AS^{-1}\left(D(m_t)\right)} =: \mathcal{F}(m_t).
\end{aligned} \tag{55}
$$

For the isoelastic example, one obtains an explicit isoelastic form of the time-one map

$$
\begin{aligned}
\mathcal{F}(m_t) &= \frac{m_t + g - \tilde{\tau}\left(\psi^*(m_t, 1, \lambda), \lambda\right) D(m_t)}{AS^{-1}(D(m_t))} = \frac{m_t + g - \tilde{\tau}(1,\lambda) D(m_t)}{AS^{-1}(1) (D(m_t))^{\frac{BC-(C+1)}{BC}}} \\
&= \frac{1 - c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1}) - \tilde{\tau}(1,\lambda)}{AS^{-1}(1)} \left(D(m_t)\right)^{\frac{C+1}{BC}} \\
&= \frac{1 - c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1}) - \tilde{\tau}(1,\lambda)}{AS^{-1}(1)\left(1 - c(1-\tau_\pi)(1-\lambda)(1-\frac{BC}{C+1})\right)^{\frac{C+1}{BC}}} \left(m_t + g\right)^{\frac{C+1}{BC}}.
\end{aligned}
$$

Positive fixed points of (55) are associated with positive balanced paths of (54). It is straightforward to show that $\mathcal{F}(m_t)$ is strictly increasing and strictly convex for all m_t which excludes

[20] Note that $\tilde{\tau}(1,\lambda)$ depends on $\frac{BC}{C+1}$, too!

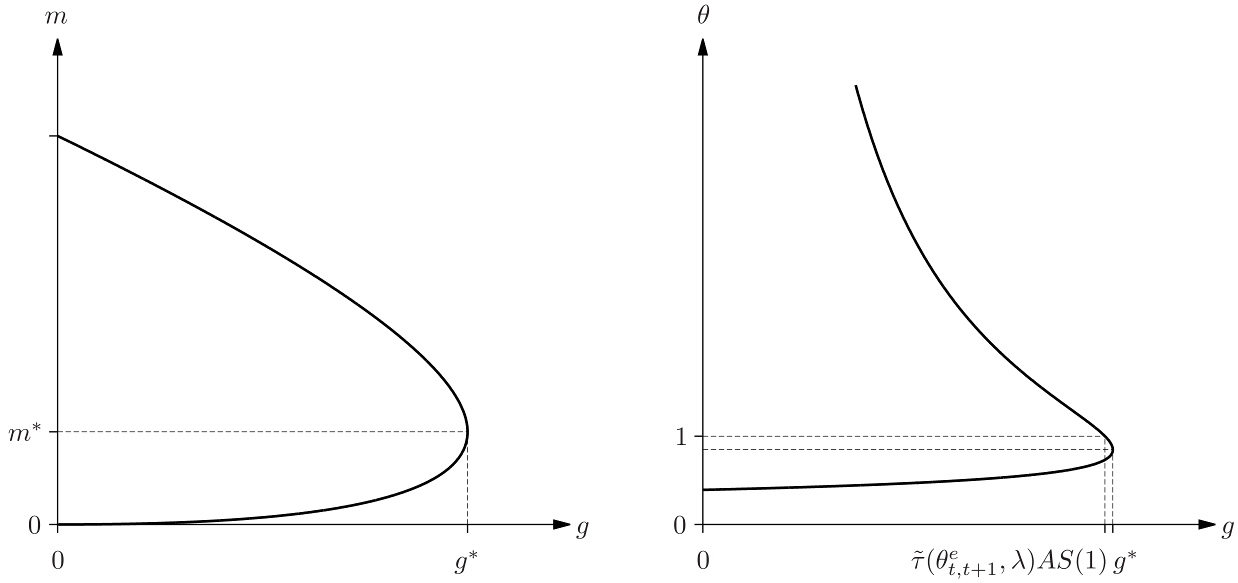


(a) role of public consumption: $0 < g_1 < g_2 < g_3$

(b) convergence/divergence

Figure 17: Existence, uniqueness, and stability

cycles. In addition, for the isoelastic example, $\mathcal{F}$ is isoelastic in $m_t + g$. Moreover, for $g = 0$ one has $\mathcal{F}(0) = 0$, and government consumption $g > 0$ induces a horizontal shift of the mapping. Therefore, there exists a critical level $g^* > 0$ such that $\mathcal{F}$ has no fixed points for $g > g^*$, exactly one fixed point for $g = g^*$, and two positive fixed points for $0 < g < g^*$. The left panel of Figure 17 depicts these three different situations.[21]



(a) stationary real money balances

(b) stationary rates of inflation

Figure 18: Stationary states for parameters as in Table 2

Since the rate of inflation at a fixed point m is a strictly monotonically decreasing function in real money holdings $AS^{-1}(D(m))$, the lower fixed point corresponds to a higher rate of inflation

[21] Since the average tax rate $\tilde{\tau}$ has no expectations effect, these findings correspond one-to-one to the competitive case analyzed in Böhm (2010), which contains a proof of these results.

than the upper one. Therefore, if $g \to 0$, real money balances at the lower fixed point tend to zero, which implies that the equilibrium rate of inflation tends to infinity. However, it is possible that the unstable fixed point induces inflation as well as the stable fixed point could induce deflation (see Figure 18).

Applying the elasticity rules to $\mathcal{F}$ evaluated at a fixed point m yields

$$\mathcal{F}'(m) = E_{\mathcal{F}}(m) = \frac{C+1}{BC}\frac{m}{m+g}$$

which is similar to ν_1 in the two-dimensional case. This shows that the derivative (i. e. the eigenvalue of the one-dimensional system) is bounded from above by $\frac{C+1}{BC}$, which can be arbitrarily large. Therefore, if two fixed points exist, by convexity and monotonicity, the lower one

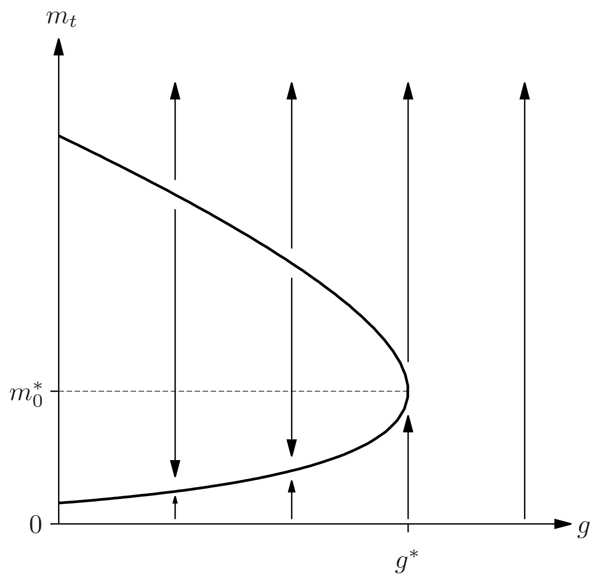


Figure 19: Balanced paths and the role of government consumption

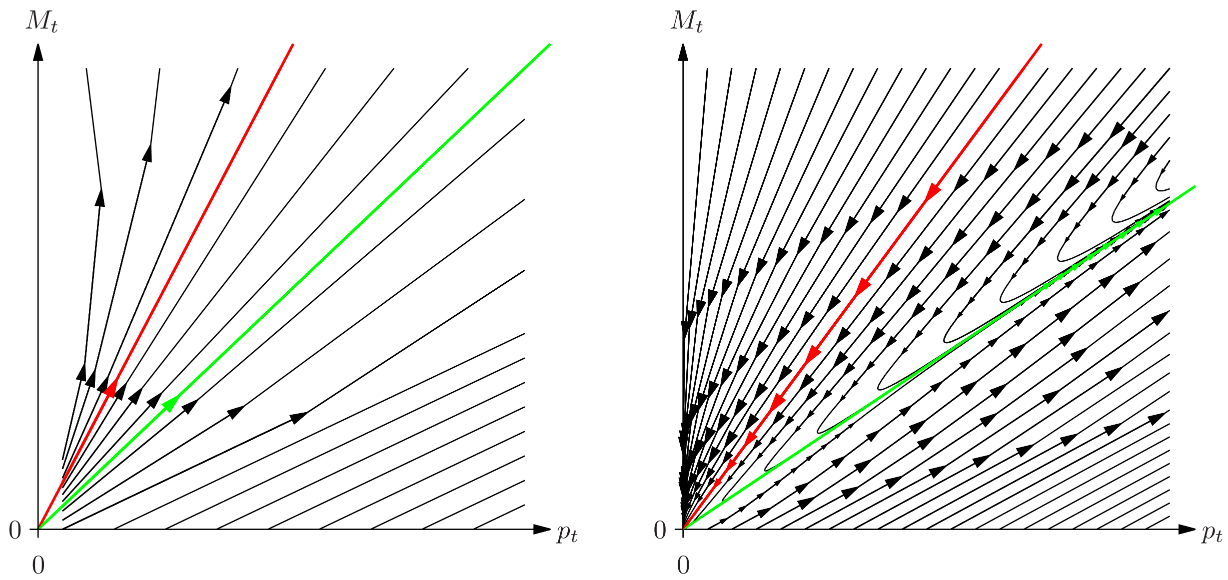
is asymptotically stable with the basin of attraction being the half-open interval between zero (included) and the upper fixed point (excluded). Figure 19 displays the set of steady states in (g, m)-space with their associated stability/instability properties.

5.5 Stable Balanced Paths

It is well-known from models of economic growth that stability and convergence of the ratio of two variables is only a necessary condition for convergence to a balanced path in the two-dimensional state space. In other words, stability in real money balances does not imply convergence to the balanced path.[22] Let $\Delta_t := M_t - mp_t = (m_t - m)p_t$ denote the distance from the balanced path m for any t. Convergence to the balanced path then implies that this distance converges to zero in addition to $\lim_{t\to\infty} m_t = m$. Such a weaker notion of stability in the two-dimensional state space allows for inflation resp. deflation (and thus an unbalanced governmental budget) when there exists a ray or half-line through the origin to which the system (54) converges.

[22] see Deardorff (1970); Böhm (2009); Böhm, Pampel & Wenzelburger (2005); Pampel (2009)

Definition 5.2 (Stable balanced paths) *Let $m = M'_t/p'_t > 0$ denote the level of real money balances associated with a balanced path $\{(M'_t, p'_t)\}_{t=0}^{\infty}$. An orbit $\{(M_t, p_t)\}_{t=0}^{\infty}$ of the dynamical system (54) is said to converge to the balanced path m if $\Delta_t = M_t - mp_t = (m_t - m)p_t$ converges to zero for $t \to \infty$.*



(a) two unstable paths; Table 1, $g \approx 0.9854$ (b) one stable, one unstable path; Table 2, $g \approx 0.21$

Figure 20: Stability of balanced paths in state space

Since, for any balanced path $m > 0$, one has

$$\Delta_{t+1} = (m_{t+1} - m)p_{t+1} = \frac{m_{t+1} - m}{m_t - m}\frac{p_{t+1}}{p_t}(m_t - m)p_t = \frac{m_{t+1} - m}{m_t - m}\frac{p_{t+1}}{p_t}\Delta_t,$$

and since $p_{t+1}/p_t = AS^{-1}(D(m_t))$, the dynamical system (54) induces the two-dimensional dynamical system

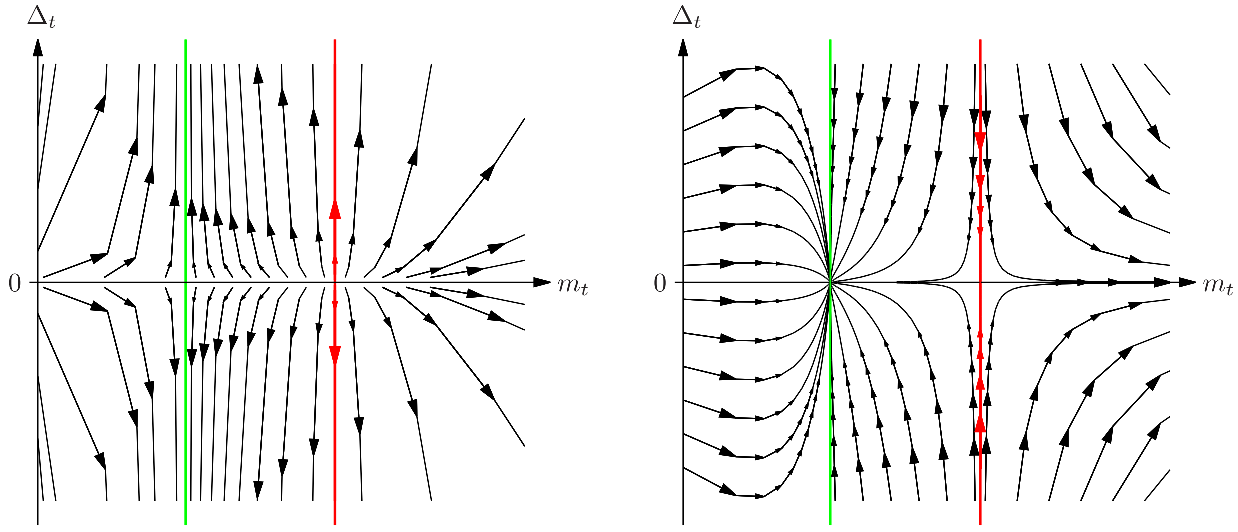
$$\begin{pmatrix} m_{t+1} \\ \Delta_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{F}(m_t) \\ \frac{\mathcal{F}(m_t)-m}{m_t-m} AS^{-1}(D(m_t))\Delta_t \end{pmatrix}. \tag{56}$$

Let $(m, 0)$ be a fixed point of the system (56). The eigenvalues evaluated at $(m, 0)$ are

$$\frac{\partial m_{t+1}}{\partial m_t}(m, 0) = \mathcal{F}'(m) \qquad \text{and} \qquad \frac{\partial \Delta_{t+1}}{\partial \Delta_t}(m, 0) = \mathcal{F}'(m)AS^{-1}(D(m)).$$

Since the upper balanced path has $\mathcal{F}'(m) > 1$, it can never be attracting. For the lower balanced path, one has $\mathcal{F}'(m) < 1$ so that stability occurs whenever $\mathcal{F}'(m)AS^{-1}(D(m)) < 1$. Thus, $(m, 0)$ is asymptotically stable for (56) if one can find values of the parameters such that $\mathcal{F}'(m)AS^{-1}(D(m)) < 1$. Under the parameterization of Table 2, the stable case occurs while the unstable case is associated with the parameterization of Table 1. The results of a numerical analysis of convergence/divergence (with levels of g chosen slightly below the critical level g^*) are given in Figure 20 and Figure 21.

Figure 20 displays several paths under the two different parameterizations. Panel (a) indicates that both balanced paths are unstable under the standard parameterization with $g \approx 0.9854$.



(a) two unstable paths; Table 1, $g \approx 0.9854$

(b) one stable, one unstable path; Table 2, $g \approx 0.21$

Figure 21: Stability of balanced paths in (Δ, m)-space

In contrast, panel (b) shows that for the parametrization given in Table 2 with $g \approx 0.2100$, all paths with initial real money holdings below the level of the unstable steady state of (55) converge to a path with slope stable state of (55). With each path in Figure 20, there is an associated path in Figure 21, showing that in panel (a) the lower steady state is a saddle while it is a sink in panel (b).

6 Summary and Conclusion

There are two main questions which were investigated in this paper. The first one dealt with the allocative consequences of efficient bargaining arrangements between a union and a producer association on the wage rate and on employment as well as on the temporary equilibrium of a macreconomy as compared with the competitive or other noncompetitive equilibria. It was shown that, contrary to common beliefs and results from partial-equilibrium models, an efficient bargaining solution in the labor market combined with a competitive output market induces strong cross-market effects within the macroeconomy which offset the efficiency feature built into the concept at given market prices. In other words, contrary to common understanding and to economic folklore derived from partial-equilibrium models, efficient bargaining between a union and producers in the labor market does not generate the desired efficiency expected for the macroeconomy as a whole. Moreover, it was shown that economic activity, i. e. output and employment, declines with an increase of union power. Thus, high bargaining power leads to low employment and low output in temporary equilibrium at all states, and it may even lead to low wages in certain cases. Thus, a high relative income distribution of wages to profits by a strong union comes at the cost of low real economic activity. Therefore, from a general welfare point of view too much union power may not be desirable.

A comparison of the bargaining outcomes with other one-sided strategic equilibria showed that payoffs under bargaining are often strictly dominated by the one-sided monopolistic equilibria which are inefficient. Thus, the price feedback through the commodity market operates strongly in support of monopolistic behavior, favoring the payoffs of the respective monopolist.

For the second major objective investigating the dynamics of the economy under perfect foresight, it was shown that structurally a macroeconomy with efficient bargaining and constant union power behaves in the same way dynamically as under competition in both markets. Existence and stability of balanced states were shown to depend in the same way on the government parameters and the consequences implied by the budget deficit. For the parametric example with isoelastic functions in both sectors, it was shown that the stability conditions are completely determined by the elasticities in both sectors and by union power. In this case, all orbits are monotonic, and underemployment or overemployment levels are constant. These results imply also that extension to situations with stochastic shocks in production or demand do not change the general conclusion that the properties of rational-expectations equilibria are structurally identical to those of perfect competition.

Finally, it should be noted that the two main underlying assumptions could be contested on several grounds. The assumption of a given constant bargaining power at all times may be questioned since it has no microeconomic justification. It would be desirable to formulate a process which determines the bargaining power endogenously, for example in response to the levels of underemployment or overemployment during the dynamic evolution. A second modification would be of removing the full-efficiency requirement in the bargaining process to one where negotiations are only over wages while the employment levels are determined through the market. This would introduce the right-to-manage principle into the macroeconomy, bringing the model closer to empirically observed negotiations and mechanisms. For both extensions, it would be again more reasonable to examine dynamic adjustments of the measure of union power, increasing the potential for interesting employment and output cycles and tradeoffs.

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