Essays on multiple Layers of Uncertainty in Economics

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1 Introduction

Traditionally, economic theory has imposed strong conditions on how well economic agents are informed about various aspects of the model in which they are acting. An ongoing development is to weaken these assumptions. This cumulative dissertation aims to contribute to this trend and consists of four projects: "The (non-)robustness of influential cheap talk equilibria", "The LIBOR Mechanism and Related Games", "Model Uncertainty in Insurance Markets", and "Equilibria with Inertia in Bewley Economies".[1] All four projects come from the field of theoretical microeconomics and are presented in the self-contained chapters 2 to 5. Despite the different models and questions motivating the four chapters, there are deep interrelations along several different dimensions between the projects. By examining these connections in this summary of the dissertation in detail we illustrate the research approach and the research questions that this dissertation considers and answers.[2] We begin with precise descriptions of the four projects, with a particular focus on the research questions and our main contributions. In the second half of this summary we then study the connections of the projects along several different lines, before we give an outlook on future work and developments.

[1]We will refer to them as Non-Robustness, LIBOR, Model, and Bewley, respectively.

[2]I will use "we" throughout the dissertation as a stylistic decision and as chapter 2 is joint work with Christoph Kuzmics.

The (non-)robustness of influential cheap talk equilibria

"The (non-)robustness of influential cheap talk equilibria" is joint work with Christoph Kuzmics. In this project we consider a classical economic situation. An agent has private information which is important for a second agent. The action of the second agent influences the welfare of both agents. In "Persuasion by cheap talk" Chakraborty and Harbaugh (2010) prove the existence of influential cheap talk equilibria despite a large conflict of interest between the two agents. We add uncertainty about the preferences of the first agent in a natural way to their model and show that the slightest amount of uncertainty leads to nonexistence of influential communication in equilibrium.

More precisely, the sender is privately informed about the true state $\theta \in \Theta \subset \mathbb{R}^N$, where $N \geq 2$, and Θ is compact, convex, and nonempty. The sender can send a costless message m from a finite message set M to the receiver. The receiver gets the message and takes an action which affects the welfare of both sender and receiver. The receiver has a full support prior over Θ defined by the distribution function F and will in equilibrium always play his best estimate of the true state, $a = E(\theta|m)$. The sender's utility is a function of the action of the receiver, $u(a) \in \mathbb{R}$. As the message is costless and thereby does not enter directly the utility function of the sender, the model is a cheap talk model.

The paper which started this literature is the seminal Crawford and Sobel (1982). In the latter framework influential communication between one sender and one receiver is only possible if the bias, the conflict of interest, between the expert and the uninformed receiver is not too large. Here influential means that different messages induce different actions with strictly positive probability. If the bias is large, influential communication seemed possible only if messages are costly or verifiable, thus if messages are not cheap. However, Chakraborty and Harbaugh alter the model of Crawford and Sobel in two important points and prove the existence of influential cheap talk equilibria in a situation of a large bias. First, they take the dimension of the state space to be multidimensional, $N \geq 2$. Second, they assume the sender has state-independent preferences, i.e. the state does not enter the utility function of the sender. As the preferences are assumed to be common knowledge ex ante, the state-independence entails that the preferences of the sender are also common knowledge ex interim. Furthermore, the state-independence is interpreted as a large bias as the receiver's preferences in fact depend on the realization of the state. A real world example of state-independent preferences is the following situation between a worker (the expert) and an employer. The true ability of the sender is private knowledge. However, the sender wants, independently of her ability, the maximum wage or the best relation of wage and leisure time.

The state-independence of the expert's preferences at the interim stage requires indifference of the sender between all actions which are induced in equilibrium. The main insight of Chakraborty and Harbaugh is that it is always possible to find an equilibrium with multiple messages and multiple induced actions. They prove this via the Borsuk-Ulam theorem, a result from functional analysis which can be used to prove Brouwer's fixed-point theorem as a corollary.

However, for their result they assume that the preferences of the sender have to be perfectly known to the receiver. Nevertheless, they are able to prove robustness in three particular ways. There exists an influential cheap talk equilibrium if either the dimensionality of the state space is larger than the cardinality of possible sender types or when the receiver has a prior over the type set of the sender which is sufficiently concentrated on one type. Moreover, they present an equivalence result between epsilon-cheap talk equilibria for large biases and cheap talk equilibria in the case of distance preferences. These are interesting robustness checks, but they do not constitute the natural way to test for robustness. Therefore, we introduce uncertainty about the preferences in a different, more standard way which resembles the uncertainty in Harsanyi (1973). In the simplest case of $N = 2$, linear preferences, and uncertainty in one dimension, the utility function becomes $U(a, x) = a_1 + (\rho + x)a_2$. Here $\rho \in \mathbb{R}, \rho \neq 0$, and x is the realization of a continuous random variable X with distribution Φ on support $[-\epsilon, \epsilon]$, where $\epsilon \geq 0$. The realization x of X is private knowledge of the sender. This setup leads to uncertainty about the slope of the indifference curves. The following theorem then holds true.

Theorem 2.2 *If there is uncertainty about the preferences in the linear case, i.e. if $\epsilon > 0$, there does not exist an influential cheap talk equilibrium for any F.*

In the general case we consider not only a set of possible types $t \in T$ of the sender but also require Condition (S) to be satisfied. *Condition (S)* is satisfied if for any two actions a and $a^{'}$, if $U(a, x^*) = U(a^{'}, x^*)$, then $U(a, x) \neq U(a^{'}, x)$ for all $x \neq x^*, x \in T$. This condition has been introduced by Chakraborty and Harbaugh in the appendix of their paper and is basically a generalized single crossing property and for instance satisfied by linear preferences. The second main result of chapter 2 is theorem 2.4.

Theorem 2.4 *If Condition (S) is satisfied and Φ is nonatomic, there does not exist an influential cheap talk equilibrium for any F.*

Thus, the two main results in our paper state that all influential cheap talk equilibria cease to exist as soon as we introduce the slightest amount of uncertainty about the preferences of the sender. In stark contrast, we show via an example that uncertainty about the bias in Crawford and Sobel does not lead to an immediate breakdown of influential communication. Instead uncertainty about the bias plays a surprisingly small role in their framework. Additionally, we explain why a condition such as Condition (S) is needed in the case of general preferences, investigate additive separable preferences which are an intermediate case between the linear and the general case, and consider a set of receivers which differ in their prior.

The LIBOR Mechanism and Related Games

The second project "The LIBOR Mechanism and Related Games" has been motivated by the ongoing LIBOR[3] scandal which started to make headlines throughout the world in summer 2012 and continues to do so. To give the appropriate background to our modeling approach, we start chapter 3 with a historical perspective of the LIBOR, a detailed chronology of the LIBOR scandal, an overview over the administering process and a description of policy responses to answer the problems with the LIBOR mechanism uncovered by the scandal.

We then turn to an analysis of the LIBOR mechanism in a theoretical model. For this end, we consider three strategic games which only differ in the utility function of the players. The players represent the banks. They maximize their expected utility, which depends on three components. First, their financial exposure to an index I which is a statistic of the quotes of

[3]LIBOR is short for London Interbank Offered Rate

the banks. Second, a penalty term for deviating with their quote from their private costs. Third, a term which catches the reputation concerns of the banks quote relative to the index and the quotes of the other banks. All banks quote at the same point in time and without knowing the quotes of the other banks. The three games we investigate are the Trimmed Average Game, the Median Game, and the Average Game which all have their name from the statistic used in the utility functions of the banks. Besides from that, the games are completely identical.

The games are defined as follows. Let $N = \{1, ..., n\}$ be the set of players. All players are informed about the realization of the state variable $\theta \in \Theta = \mathbb{R}^n_+$. This variable represents the private costs of the players. The strategy of a player i is a function $s_i : \Theta_i \to \mathbb{R}_+$. The quotes are denoted by x and defined by the strategy. The utility function is a function of the quotes and the own private value:
$u_i(x_i, x_{-i}, \theta_i) = \nu_i I(x_i, x_{-i}) - (x_i - \theta_i)^2 + \lambda_i(I(x_i, x_{-i}) - x_i)$. Here, $I(x)$ denotes a statistic depending on all quotes, $\nu_i \in \mathbb{R}$ the financial exposure of bank i to the index, and the players weigh their reputational concerns with $\lambda_i \in \mathbb{R}$. In the Median and Trimmed Average Game we set $\lambda_i = 0$ for all i.

We start with the analysis of the Average Game and solve for Nash equilibria which are unique and explain why the Groves mechanism can solve the misrepresentation problem in this game. Also, we explain the conditions under which increasing the number of players is helpful and how this depends on reputation concerns.

After having established the benchmark results in the Average Game, we consider the Median Game. The median is the middle quote if the number of players is odd and the arithmetic mean of the middle two quotes if the number of players is even. In equilibrium, players will quote truthfully or will quote such that their quote directly impacts the median. Moreover, we are able to show a qualitative difference between the equilibria in the even and uneven case. If the number of players is uneven and at least three, in every Nash equilibrium at least one player quotes truthfully, i.e. quotes her private costs. This is not the case in the even case as we show via an example.

In reality, the LIBOR is currently calculated as a trimmed average, i.e. the submitted quotes from the banks are ordered from high to low, the top and bottom 25 percent quantile are deleted, and the remaining quotes are arithmetically averaged. To be able to distinguish between the three different statistics average, median, and trimmed average it is necessary to consider at least five players as for one and two players all notions coincide and for three and four players one has to define the trimmed average either as median or average (except one allows for a different weighting of different quotes). The equilibrium analysis in the Trimmed Average Game leads to the following results. Similarly to the Median Game with even player number there exist equilibria in which all players misrepresent. Moreover, we formulate a sufficient criterion for misrepresentation: If a player quotes such that the quote is not in the top or bottom 25 percent quantile and is not one of the two pivotal quotes which just count for the trimmed average, then the player misrepresents if she has positive exposure $|\nu_i| > 0$ to the index. We show that the bank will quote the pivotal quote if the bank faces sufficiently high exposure to the index. Finally, we are able to prove for the five player game the existence of an equilibrium in which at least one player misrepresents in a maximal way by analyzing all possible equilibrium quote constellations.

In the following paragraph of chapter 3 we compare the three different games and in particular prove theorem 3.16 that states that in all games there exists an equilibrium in which at least one player misrepresents in a maximal way.

Theorem 3.16 *In the Median Game, the Trimmed Average Game and the Average Game there exists for all θ and all ν an equilibrium in which at least one player misrepresents in a maximal way.*

Decisive for the surprisingly easy proof is the notion of maximal equilibrium misrepresentation. We note that this maximal misrepresentation in equilibrium is lowest in the Average Game, followed by the Trimmed Average Game, and highest in the Median Game. This is due to the different marginal influence of individual quotes in the three games.

The appendix provides an equilibrium characterization in the three player Median Game which is interesting from a theoretical point of view. We show that three different kind of equilibria may arise, depending on the parameter constellations of private costs and financial exposure to the index. In the appendix there are also examples of payoff functions and actual trimming rules from reality.

The analysis of the three different games is interesting from a purely theoretical point of view and to understand how the banks were able to manipulate the index. However, the natural question which arises is: What does this imply normatively? What is the right thing to do? Our descriptive study has some important points to make here. Among these are the following recommendations: Increase panel bank sizes whenever possible, make individual quotes temporarily anonymous, base quotes on real transactions whenever possible, decrease the number of currencies and maturities, and be cautious to change from a trimmed average rule to a median or average rule. Several of these suggestions have also been put forward by the Wheatley Review and some are already implemented. However, none of these suggestions really solves the basic underlying incentive problem. Economically speaking one needs an incentive compatible mechanism. In the Average Game the Groves mechanism is able to exactly counterbalance misrepresentation incentives. For this, the mechanism designer has to know the utility functions of the players perfectly well. Unfortunately, in reality this will never be the case. Not only are the private costs of borrowing, the financial exposure to the index, and the penalty function banks may use to connect their quotes to their true costs all private knowledge of the banks, they are also subject to constant change as, for example, banks buy and sell options priced on the LIBOR. We do not see a way of solving the incentive problem besides developing an alternative index to the LIBOR where institutions whose welfare is directly affected by the index are not allowed to influence the index. Thus, there is a need for a qualitatively new interest rate benchmark.

Model Uncertainty in Insurance Markets

"Model Uncertainty in Insurance Markets" revisits the paper "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information" by Rothschild and Stiglitz (1976). We introduce model uncertainty in their framework. First, we examine the benchmark case of a homogenous group of individuals and model uncertainty about the accident probability. Second, we investigate model uncertainty about the fraction of high and low ability types in the heterogeneous population case and in particular provide a new solution to the Rothschild and Stiglitz equilibrium puzzle.

The “Equilibrium in competitive insurance markets” paper has become one of the most influential papers in the theory of economics. In particular, it has been fundamental in the insurance and information literature. The authors consider a simple model of insurers and individuals with two possible states of the world, “accident” and “no accident”. If there happens to be an accident, individuals face accident costs. However, the individuals can buy insurance on a competitive insurance market, i.e. an insurance market without any entry costs for the insurers and where expected profits for the insurers are zero. In the first part of the paper the population of individuals is homogenous. The individuals have the same likelihood of accident and the individuals as well as the insurers know this personal accident probability. Furthermore, insurers are risk neutral and individuals risk averse. The insurers offer contracts which lie on their fair odds line. The contracts on this line give the insurers expected utility zero. The unique equilibrium contract is the full insurance contract which makes the risk averse individuals exactly indifferent between accident and no accident. This contract maximizes expected utility for the individuals while all other contracts, which would be weakly preferred by the individuals make negative expected profits.

The deep insight of Rothschild and Stiglitz is that changing the assumptions on the information distribution slightly can change the results qualitatively. In the second part of their paper, they consider a heterogenous group of individuals, where a known fraction has high ability / low risk while the others have low ability / high risk. Individuals know their personal accident probability. If the insurers knew not only the fraction of high and low risk types but could distinguish high and low types individually, the result from the benchmark case would carry over, with two submarkets. In each submarket a unique equilibrium would exist, the full insurance contract. However, insurers do not know if an individual has high or low risk. In this situation Rothschild and Stiglitz show the following three main findings. Taking information asymmetries into account is important, an equilibrium may not exist, and if an equilibrium exists, it may not be pareto-optimal. The basic force behind these results is that the presence of the low ability individuals constitutes an externality for the high ability types. If insurers offered cheaper contracts for the high ability individuals, low ability individuals would buy them and insurers cannot identify this free riding as they do not know who has high risk and who has low risk.

We revisit their model and introduce model uncertainty into it. We begin by introducing model uncertainty about the accident probability on the side of the insurers in the benchmark case. The insurers know they face a homogenous group of individuals which have an accident with a certain probability in the next period. However, they do not know the precise probability, but only that the probability is an element of a nondegenerate connected interval. Individuals know the accident probability. Suppose the insurers are risk neutral but ambiguity averse concerning the probability interval. This is a sensible assumption as the uncertainty about the precise probability does not vanish with a large number of individuals and independent accident occurrence. Insurers calculate with the highest accident probability for the individuals, hence the insurance contracts get more expensive. Individuals buy less insurance, and for sufficiently high model uncertainty, i.e. a sufficiently high difference between true accident probability and the accident probability upon which the insurers base their calculations, no insurance becomes an equilibrium.

We then inverse the setup and introduce model uncertainty on the side of the individuals. Insurers know the precise probability of accident while the individuals only know that this probability is an element of a nondegenerate connected interval. When individuals are ambiguity averse, they will always buy full insurance. However, in many situations it is sensible to assume ambiguity loving behavior in the sense of overconfidence. Consider driving abilities. In a often repeated study, car drivers are asked about their personal driving abilities relative to driving abilities of the population. More than 70 percent say they are better than average which is mathematically just not possible. Own abilities are overestimated. Going back to the model and thinking of car insurance where insuring companies have very good data on the accident probabilities, overconfidence on the side of the individuals may lead to underinsurance or no insurance at all, depending on the risk aversion and the probability interval.

We also investigate the set of equilibria when individuals have incomplete preferences with inertia. A set of equilibria around the full insurance contract emerges. If this set includes the initial, no insurance allocation, the inertia condition makes the no insurance contract the unique equilibrium. For combinations of a sufficiently large difference between true and maximal accident probability and sufficiently large risk aversion, the latter situation will occur.

The arguably most important and interesting contribution of the "Model" paper is that model uncertainty about the fraction of high and low ability agents may solve the equilibrium puzzle. Consider the heterogenous population model of Rothschild and Stiglitz. They assume the fraction λ of high and low types is perfectly known. It is not only mathematically interesting to allow an interval of possible fractions but also economically more realistic to assume that insurers do not perfectly know the correct fraction λ but rather an interval $\Lambda = [\underline{\lambda}, \overline{\lambda}]$ of possible fractions. Faced with this interval, we assume insurers act in an ambiguity averse way which entails that they reckon with the highest possible fraction of high risk individuals. Two possible types of equilibria may appear in general in the heterogenous population case. The original argument by Rothschild and Stiglitz that there cannot be a pooling equilibrium does not depend on the fractions of high and low types. Consider the separating equilibrium case. If the fraction is perfectly known, then if there are sufficiently few low ability types, there does not exist a separating equilibrium as there exist contracts preferred by both types. However for these equilibrium breaking contracts to exist the number of high risk individuals has to be small as they constitute a negative externality on the low risk ones and make the contract more expensive. In this situation assume the fraction is not precisely known. Insurers thus calculate with the highest fraction of high risk types. For sufficiently large ambiguity in the sense of a sufficiently large fraction of high risk types, there always exists an equilibrium.

Theorem 4.6 *Suppose there is ambiguity about the fraction λ of the high risk individuals. Insurers only know $\lambda \in \Lambda = [\underline{\lambda}, \overline{\lambda}]$. If insurers are ambiguity averse, then there always exists an equilibrium if $\overline{\lambda}$ is sufficiently large.*

We close chapter 4 by giving numerical examples and the figures which illustrate the intuition behind the results.

Equilibria with Inertia in Bewley Economies

In “Equilibria with Inertia in Bewley Economies”, we perform a detailed analysis of the static version of the discrete time case study presented by Dana and Riedel (2013). We investigate the equilibrium set for different amounts of ambiguity in the economy and different ambiguity attitudes. In particular, we are interested in the full insurance and no insurance of uncertainty allocation, how the interplay of risk and Knightian uncertainty affects the set of equilibria, the inertia equilibrium refinement, and robustness considerations.

Dana and Riedel consider two agents which are each endowed with two independent random variables R and U. The endowment $\omega^1 = R + U$ of the first agent is minus the endowment of the second agent. The variable R represents the risk in the economy and its distribution is known to be standard normal. The variable U represents Knightian uncertainty in the sense of Knight (1921). This variable has standard variance but the expectation α is an element of an interval $[-\kappa, \kappa]$, so that $\kappa \geq 0$ becomes a measure for the amount of ambiguity in the economy. Technically, we define the corresponding set of priors via the densities with respect to a reference prior. Both agents have CARA utilities of degree one. We assume agents have incomplete Bewley preferences as introduced by Bewley (2002). Hence, an agent prefers allocation one over allocation two if the agent prefers one over two in all scenarios, i.e. under all elements of the prior set. We call an economy where agents have Bewley preferences B-economy. The decision rule makes the preferences incomplete as soon as there exist two allocations where one is not unanimously better under all priors than the other. In particular, equilibrium allocations tend to be indeterminate when agents have incomplete preferences. To ensure that at least endowment is comparable with an equilibrium, Bewley (2002) suggests the inertia condition. An equilibrium satisfies this property if all agents strictly prefer the equilibrium allocation to the endowment allocation.

In our equilibrium analysis we first consider the equilibrium set without inertia. The following results holds.

Theorem 5.2 *The B-economy has an equilibrium* (c^*, p^*) *with equilibrium consumption* $c^{1*} = -c^{2*} = \alpha U$ *and equilibrium price* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *for* $\alpha \in [-\kappa, \kappa]$. *In particular, the set of equilibria in the B-economy is indeterminate for* $\kappa > 0$.

Thus, the equilibrium set gets thicker the more ambiguity there is in the economy. Without ambiguity, agents know the distribution of the second random variable which is standard normal. Full insurance is the unique equilibrium. As soon as there is ambiguity about the expectation of the second random variable, the equilibrium set becomes indeterminate and for sufficiently large ambiguity, no trade of the uncertain variable becomes an equilibrium. Important to note is that agents always trade as they completely trade away their exposure to the risk representing random variable R. Refining the equilibrium concept with the inertia condition leads to a tension between risk aversion and the status quo bias that stems from the inertia condition.

Theorem 5.4 *The equilibrium* (c^*, p^*) *with* $c^* = \alpha U$ *and* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *satisfies the inertia property if and only if*
$\alpha \in [\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\}]$.
In particular, the set of equilibria with inertia in the B-economy is indeterminate for positive and finite ambiguity ($\kappa \in (0, \infty)$).

For low levels of ambiguity the inertia property has no bite. However, with increasing level of ambiguity the status quo bias becomes more powerful and the inertia condition cuts through the equilibrium set. We precisely calculate for which ambiguity level the equilibrium set is thickest, when full insurance ceases to be an equilibrium, from which points on the inertia cuts through the equilibrium set, and prove uniqueness of equilibrium in the limiting case of infinite ambiguity. Moreover, we highlight that if agents have Gilboa and Schmeidler (1989) preferences, full insurance is always the unique equilibrium, independently of the level of ambiguity.

We then turn to a robustness analysis of our findings. We perform robustness checks in the following different dimensions: Endowment, degree of constant absolute risk aversion, prior sets, and combinations thereof. To take one example, we investigate the equilibrium set if agents are only endowed with uncertainty but not with risk. In this case, the equilibrium with inertia becomes unique as soon as no trade of uncertainty becomes an equilibrium allocation.

Interrelations of the four projects

We now investigate the interrelations of the four projects from several different perspectives.

Financial market regulation, financial crisis and the drying up of markets

The "Bewley" project delivers insights on the financial crisis and the regulation of financial markets. Two prevalent ways of regulation and to measure financial strength are risk measures and stress testing. Artzner et al. (1999) introduce the notion of a coherent risk measure which is an attempt to capture financial risk in a real number. Their starting point is axiomatically. They provide a set of axioms that such a measure of risk is supposed to satisfy. Gilboa and Schmeidler (1989) preferences can be interpreted as coherent risk measures due to the representation result of these risk measures. Exhibiting a stress test means comparing a status quo to a new allocation under a number of scenarios where the new one is only preferred to the current one if it performs better under all scenarios. Thus, there is a correspondence to incomplete preferences plus inertia. Our results point to a possible objectionable consequence of stress testing. Insurance markets may dry up provided sufficiently large ambiguity.

Financial market regulation lies also at the core of the LIBOR project. This set of interest rate benchmarks has been manipulated for years. It is of vital interest for the functioning of financial markets that new rules get established which make such manipulations impossible or at least more difficult. Our descriptive study has some normative implications on this front.

The LIBOR scandal also has an inherent relation to the drying up of a market, the interbank market. After the financial crisis broke lose in 2007, banks were very cautious concerning the financial strength of other banks. Therefore, the unsecured interbank borrowing market, which underlies the theoretical conception of the LIBOR, dried up. The number of credits in this market fell massively. Nevertheless, banks had to come up with quotes which were supposed to give an adequate description of the interest rates on the unsecured market. Despite all the misrepresentation incentives which exist independently of a dry or liquid interbank market, the drying up made the task to find an adequate number for the quote more difficult.

Speaking of drying up of insurance markets and financial crisis our project “Model” provides important insights. Times of financial crisis are in general also times where uncertainty about contract partners grows. We examine the case where insurers face ambiguity about the precise accident probabilities of individuals. In this situation the insurance market shrinks as full or almost full insurance becomes too expensive for the individuals. Moreover, being less fully insured, individuals are also more vulnerable to accidents or, more generally, negative shocks. These accidents may become real threats to the individuals as their income distribution becomes highly dependent on the state. This may signal a generally higher level of fragility of companies and economies in times of high uncertainty.

Information transmission and information extraction

Information transmission and information extraction is a joint topic of the first three projects. In "Non-Robustness" we investigate a sender receiver game. Our main contribution is an impossibility result. As soon as there is some uncertainty about the preferences of the sender, there is no influential communication in equilibrium. It is not possible for sender and receiver to communicate such that the receiver learns something about the true state of the world.

In the "LIBOR" project the receiver which is the financial regulation authority would like to infer from the quotes the true underlying state as well. The regulator would like to extract as much information as possible from the quotes. In the Average Game, full revelation is possible due to the Groves mechanism. In reality, the basic underlying problem is that the true payoff functions of the senders are not known. This makes the problem hard to solve and the LIBOR quotes will give the regulator only an imprecise idea of what the true private costs are.

In the "Model" paper the presence of the high risk individuals poses a negative externality on the low risk individuals and it is not possible for the insurers to separate the groups before the contracts are bought. Also, the low risk individuals cannot communicate credibly that they are low risk. If the insurers believed them, high risk individuals would communicate in the same way.

Equilibria

In all four projects we investigate the equilibrium sets and under which circumstances no equilibrium exists. Moreover, we compare the different equilibria concerning how much information they offer and if they are socially desirable. In the "Non-Robustness" paper we show that only the babbling equilibrium exists, provided there is uncertainty about the preferences. In contrast, we show that influential equilibria continue to exist in Crawford and Sobel (1982) despite bias uncertainty. The "LIBOR" project examines three different but related games in detail. One commonality is that in all three games there exist equilibria in which at least one player misrepresents in a maximal way. Our most important finding in "Model" guarantees the existence of an equilibrium which does not necessarily exist if insurers are better informed about the fraction of high risk individuals in the population. Thus, less information leads to better predictable market behavior in the sense of the existence of a unique equilibrium. The more standard relation in economics is the opposite. In "Bewley" no insurance of uncertainty equilibria arise provided sufficiently large ambiguity. The insurance market dries up, a phenomenon which is economically not desirable as the economy becomes more vulnerable to negative realizations.

Insurance

All four projects have a relation to insurance and insurance markets. The "Model" project investigates the set of equilibrium contracts in competitive insurance markets and the "Bewley" chapter examines when full, partial, and no insurance equilibria exist. From a quite different angle, "Non-Robustness" has an insurance interpretation as well. A situation of state-independent preferences is that of an insurance agent who is privately informed about the quality of a contract but only interested in selling the contract. In our model this expert cannot influence the action of the potential buyer through costless messages as soon as the receiver faces uncertainty about the preferences of the sender. An important part of risk management of companies is to insure against changes of interest rates. The manipulation and manipulability of key interest rates that we discuss in "LIBOR" is therefore an aspect risk managers should be at least aware of.

Robustness

More broadly, a research goal of the thesis is to investigate how robust findings in game theory and economics in general are, if we allow for certain modifications and the introduction of uncertainty. In "Non-Robustness" influential equilibria immediately break down provided uncertainty about preferences; in "LIBOR" the basic incentives to misrepresent are present in all related games, the magnitude of misrepresentation however changes; in "Model" uncertainty may lead to more precise equilibrium predictions, and in "Bewley" the amount of (Knightian) uncertainty in the economy plays a decisive role in the determination of the equilibrium set.

Outlook

Economic theory is a relatively young science and not as matured as physics, for example. Many real world phenomena have not been understood well in economic models so far. Nevertheless, economic theory has been successful in many areas, in particular in microeconomics. To give one example, modern auction theory has helped Germany to make large revenues from the UMTS auction in 2000. Game theory has also been, generally speaking, a success story. While not being able to capture or predict human behavior in many circumstances, it has become an important intellectual tool to think about strategic reasoning and incentives. However, economic theory still has a lot of work to do and a lot of interesting and important questions remain to be answered. With this dissertation we try to support the attempt of economic theory to go more in the direction of robustness and less restrictive assumptions on information about preferences and probabilities.

To be more precise, "Non-Robustness" questions the assumption of perfect information of the receiver about the sender's preferences and comes to the negative conclusion that influential communication ceases to be possible in equilibrium. On the positive side, we show that uncertainty about the bias in "Strategic information transmission" plays a surprisingly small role and does not immediately change communication possibilities. "LIBOR" examines a mechanism from the real world and investigates manipulation possibilities. From our point of view, the language of game theory is natural in this situation as banks are utility maximizing, strategically acting entities. In the "Model" and "Bewley" framework actors do not only face risk but also ambiguity. Thus, we weaken the assumption of the Bayesian paradigm.[4] We

[4] The Bayesian paradigm holds, in the formulation of Gilboa and Marinacci (2011), that any source of uncertainty can and should be quantified probabilistically. Gilboa and

allow the agents to use a set of priors instead of a single probability. In these chapters we apply ambiguity theory and investigate how equilibrium sets change in comparison to models in which agents are better informed in the sense of having a precise probability.

Economic theory and economics in general are, from our perspective, in a transition phase. A lot of models have turned out to be missing essential parts of the real world, in particular in macroeconomics. Some questions may even be unanswerable as the evolution of complex dynamic systems becomes easily chaotic. Nevertheless, it is important to acknowledge three facts. First, economic theory has led already to many deep insights, second, it is a young and difficult science, and third, it is trying to find answers to the many challenges still to be explained and is making progress on this path. We hope to contribute with this dissertation to this ongoing development.

Marinacci state "Since the mid-20th century, economic theory is dominated by the Bayesian paradigm."

2 The (non-)robustness of influential cheap talk equilibria

Abstract

Chakraborty and Harbaugh (2010) prove the existence of influential cheap talk equilibria in sender-receiver games when the state is multidimensional and the preferences of the sender are state-independent. We show that only the babbling equilibrium survives the introduction of any small degree of uncertainty about the sender's preferences in the spirit of Harsanyi (1973). None of the influential equilibria are robust to this kind of uncertainty.

2.1 Introduction

One of the main findings of the cheap talk literature is that influential communication in one sender one receiver games is only possible if the conflict of interest is not too large. This has been shown in the equilibrium characterization by Crawford and Sobel (1982) and expanded by Goltsman et al. (2009). For large biases, credible communication seemed only possible if messages are verifiable or costly (for a survey on this literature, see Sobel (2013)). To the contrary, Chakraborty and Harbaugh (2010) prove the existence of influential cheap talk equilibria in sender-receiver games even if the conflict of interest between the sender and the receiver is large. Thus, their result is in contrast to the intuition that for influential communication preferences cannot be too different. Two things are however key for the result by Chakraborty and Harbaugh which make their framework different from the setting of Crawford and Sobel. First, the preferences of the sender are state-independent and second, the dimensionality N of the state variable θ, whose realization is private knowledge of the sender, is strictly greater than one. The fact that dimensionality of the state can change results qualitatively is known since at least Battaglini (2002)[1], see also Chakraborty and Harbaugh (2007) who show how common interests arise naturally in a multidimensional setting. The additional assumption of state-independence can be seen as an extreme form of conflict of interest. If the sender is a worker with privately known skill level, she may want to receive, regardless of the true state, the maximal wage and therefore has state-independent preferences. In particular, the state-independence implies that if more than one action is induced in equilibrium, the sender has to be indifferent between these actions. This latter fact gives the impression that influential cheap talk equilibria are not very robust and easily break down as soon as the slightest amount of uncertainty about the preferences of the sender is introduced. Chakraborty and Harbaugh themselves show robustness of the equilibrium construction in the presence of limited uncertainty of the receiver about the sender's type in three particular examples. When the number of possible types is lower then the dimensionality N of θ there still exists an influential

[1]He shows that full revelation is generically possible in a multiple sender and multidimensional state world, even for arbitrarily large conflict of interest.

cheap talk equilibrium. Also, if the probability distribution over the set of possible types is sufficiently concentrated on one type, the existence result remains true. Furthermore, Chakraborty and Harbaugh investigate state-dependent Euclidean preferences and present an equivalence result between cheap talk equilibria and epsilon-cheap talk equilibria for large biases.[2]

In this paper we introduce uncertainty in a different, more natural way which resembles the kind of uncertainty in the purification paper by Harsanyi (1973). With this kind of uncertainty which we call Harsanyi-Uncertainty, the intuition that the existence result is not very robust is true. In fact, all influential cheap talk equilibria cease to exist as soon as the slightest amount of uncertainty exists. We begin with the linear case. In this case the way we introduce uncertainty leads to uncertainty about the slope of the indifference curves. The slightest amount of uncertainty suffices to show the nonexistence of influential cheap talk equilibria. For the general case of nonlinear preferences, we suppose additionally that what Chakraborty and Harbaugh call condition (S) is satisfied. This condition basically says that two different actions are not given the same utility by more than one type. In particular, this condition is always satisfied in the linear case. It is necessary to impose a condition like this as otherwise there still may exist an influential cheap talk equilibrium if the indifference curves intersect in a particular way. Given condition (S) there do not exist influential cheap talk equilibria if there exists the slightest amount of uncertainty. To the contrary, there exists an influential equilibrium in Crawford and Sobel despite uncertainty about the bias as we show in the appendix. Moreover, the existence theorem of Chakraborty and Harbaugh is robust to uncertainty about the type of the receiver.

[2] These robustness results appear in the online appendix of the Chakraborty and Harbaugh paper.

The paper is organized as follows. We begin by restating the model and the main finding of Chakraborty and Harbaugh. Then we consider the linear case as a motivating example for the intuition behind our results before we turn to the general case where we generalize the findings of the linear case. Here we also explain why a condition like condition (S) is needed in the general case. In the conclusion we discuss the implications of our results for applications, like the possibility of credible communication of biased lobbyists. In the appendix we show why uncertainty about the bias does not necessarily lead to nonexistence of influential cheap talk equilibria in Crawford and Sobel, we present the proof of the existence theorem of Chakraborty and Harbaugh, consider the case when preferences are additive separable, and show robustness of the existence result when there is a set of possible receivers.

2.2 The Chakraborty and Harbaugh result

A sender (female) is privately informed about the realization of $\theta \in \Theta$, where Θ is a convex and compact subset of $\mathbb{R}^N$ with nonempty interior and $N \geq 2$. The sender can send a costless message m from a finite set of messages M to a receiver (male). The receiver observes the message and takes an action which influences the welfare of both sender and receiver. In equilibrium, the receiver plays his best estimate of the true state, $a = E(\theta|m)$. [3] The prior of the receiver is described by the distribution function F with density f and has full support on Θ. The preferences of both sender and receiver

[3]If the utility function of the receiver has the form $v(a, \theta) = -(a - \theta)^2$, then the best response of the receiver is to play the (conditional) expectation of θ. Here we only consider equilibrium actions of the receiver, in equilibrium he will always play his best estimate of the true state given his prior and the message of the sender

are common knowledge at the interim stage. Thus, the utility function of the sender does not depend on the realization of the state variable θ, it is state-independent. Her utility is a function of the action of the receiver, $u(a) \in \mathbb{R}$. A communication strategy is a mapping from the state space Θ into the set of probability distributions over messages in the message space M. The equilibrium concept is Bayesian Nash. In this setup we define the notions of influential and cheap talk equilibrium. A communication strategy is a *cheap talk equilibrium* if there is no incentive for the sender to deviate to misreporting about the true state of the world. An equilibrium is *influential* if different messages induce different actions with strictly positive probability. The main finding of Chakraborty and Harbaugh (2010) is the existence of influential cheap talk equilibria in addition to the always in cheap talk games existing babbling equilibrium.[4]

Theorem 2.1. *(Chakraborty and Harbaugh) For all u and all F there exists an influential cheap talk equilibrium.*

In the proof, they fix an interior point c and split the state space via a hyperplane h with orientation s where s is an element of the unit sphere. Due to the Borsuk-Ulam theorem[5] there exists a hyperplane h^* with orientation s^* such that if the sender truthfully indicates on which side of the hyperplane the true state is, the sender does not have an incentive to deviate to misreporting as she is exactly indifferent between the induced actions. The receiver does not have an incentive to deviate, as he learns something about

[4] If the receiver takes the same action regardless of the message, the sender is always indifferent between all signals. In particular, it is a best reply for the sender to always send the same message, hence there exists a no communication equilibrium, the babbling equilibrium.

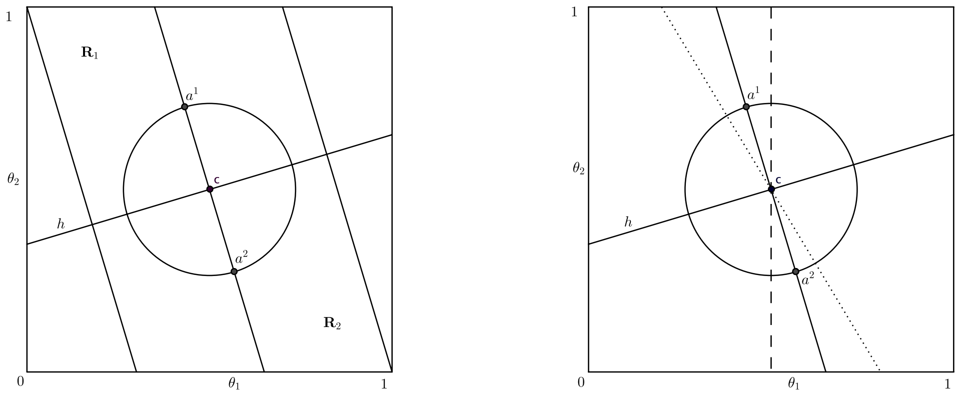
[5] The Borsuk-Ulam theorem states that all continuous odd functions $f : \mathbb{S}^{N-1} \to \mathbb{R}$ have a zero, i.e. there exists s^* such that $f(s^*) = 0$. Here $\mathbb{S}^{N-1}$ is the unit sphere, the set of points which have distance one from the reference point.

the true state and therefore has a more precise updated estimate of the true state. For details of the proof, we refer to the appendix.

2.3 Robustness for linear preferences

Two points are key to the existence proof: the sender's indifference between the two induced actions and the common knowledge of the sender's preferences at the interim stage. Chakraborty and Harbaugh themselves weaken the common knowledge assumption and show robustness of the equilibrium construction in the presence of limited uncertainty of the receiver about the sender's type in three particular examples. When the number of possible types is lower then the dimensionality N of θ there still exists an influential cheap talk equilibrium. Also, if the probability distribution over the set of possible types is sufficiently concentrated on one type, the existence result remains true. Furthermore, they present an equivalence result for distance preferences between epsilon-cheap talk equilibria and cheap talk equilibria when the bias is large.

The three ways in which Chakraborty and Harbaugh allow for uncertainty are however, from our perspective, not the natural ones. We introduce uncertainty in a different, more apparent way and call it Harsanyi-Uncertainty. We start with the simplest case: two dimensions ($N = 2$), linear preferences, and uncertainty in one dimension. Without uncertainty about the preferences of the sender the utility function is $u(a) = a_1 + \rho a_2$, where $\rho \in \mathbb{R}, \rho \neq 0$. We now introduce limited uncertainty about the sender's preferences concerning the second component, $U(a, x) = a_1 + (\rho + x)a_2$, where $x \in [-\epsilon, \epsilon]$ for $\epsilon \geq 0$. If $\epsilon = 0$, we are back in the world without uncertainty. The preferences of the sender cease to be common knowledge as the receiver



(a) No uncertainty (b) Uncertainty

Figure 2.1: Uncertainty vs. no uncertainty in the linear case

does not know x but only $X \sim \Phi[-\epsilon, \epsilon]$. Here X is a continuous random variable distributed according to the continuous distribution function Φ on the interval $[-\epsilon, \epsilon]$. An example is the uniform distribution.[6] The realization x of X is private knowledge of the sender. Therefore, we denote the utility function from the perspective of the sender as U with arguments a and x. For $\epsilon = 0$, there is no uncertainty, thus theorem 2.1 guarantees the existence of an influential cheap talk equilibrium. As soon as $\epsilon > 0$, there is uncertainty about the preferences which induces qualitatively different results concerning the existence of influential cheap talk equilibria. The following theorem 2.2 makes this precise.

Theorem 2.2. *If there is uncertainty about the preferences in the linear case, i.e. if $\epsilon > 0$, there does not exist an influential cheap talk equilibrium for any F.*

[6]Decisive will be the fact that any particular type of a sender will have probability zero due to the continuity of the distribution function. Thus, we could also consider for example a normal distribution with positive variance which then has support $\mathbb{R}$.

Before we give the formal proof, we present the way in which we introduce uncertainty and the intuition behind the contradiction argument in Figure 2.1. In (a), we depict the case without uncertainty about the sender's utility function. For any interior point c there exists a hyperplane h such that if the sender always sends truthfully in which half of the state space the true state lies and the receiver understands the strategy and the message of the sender, the updated estimates of the receiver lie on the same indifference curve of the sender. The updated estimates are a^1 and a^2 and the indifference curve is the straight line through a^1 and a^2. Therefore, an influential equilibrium exists. This is the main insight of Chakraborty and Harbaugh. However, it is crucial for this influential equilibrium to exist that the sender is exactly indifferent between the two updated estimates of the receiver.

Suppose now there is Harsanyi-Uncertainty about the slope of the indifference curve as in (b). The receiver does not know the precise slope, he only knows that the indifference curve of the sender through point c lies somewhere between the dotted and the broken line. The set of possible slopes is a nonempty interval. If the receiver has a nonatomic distribution over this set, with probability zero he will guess the correct one. However, for any two updated estimates of the receiver defined by the hyperplane, there only exists one indifference curve on which both updated estimates lie. Therefore, with probability one, the sender is not indifferent between the two induced actions, which lets her message state-independently. This in turn leads to a contradiction to the assumption that there exists an influential cheap talk equilibrium. We now turn to the formal proof.

Proof. We prove by contradiction. Suppose there exists an influential cheap talk equilibrium. Then there exist $m^1 \neq m^2$ such that $a^1 = E[\theta|m^1] \neq$

$E[\theta|m^2] = a^2$.[7] In equilibrium, a^1 and a^2 have to be best replies. Wlog, $a_2^1 > a_2^2$, else introduce uncertainty in the first dimension of action (if "=") or relabel (if "<"). Suppose there exists $\tilde{x} \in [-\epsilon, \epsilon]$ such that $U(a^1, \tilde{x}) = a_1^1 + (\rho + \tilde{x})a_2^1 = a_1^2 + (\rho + \tilde{x})a_2^2 = U(a^2, \tilde{x})$. This implies $U(a^1, x) > U(a^2, x)$ for all $x > \tilde{x}$ and $U(a^1, x) < U(a^2, x)$ for all $x < \tilde{x}$. If the sender is not indifferent between the two induced actions, it is the unique best response for the sender to make the message sent not state-dependent: For all $\theta, \theta' \in \Theta, \theta \neq \theta'$, it holds $Pr(m^1|x, \theta) = Pr(m^1|x, \theta')$ and $Pr(m^2|x, \theta) = Pr(m^2|x, \theta')$ for all $x \neq \tilde{x}$. As x is unknown to the receiver and X is continuous, $Pr(X = x) = 0$ for all $x \in [-\epsilon, \epsilon]$, in particular $Pr(X = \tilde{x}) = 0$.[8] Therefore, the receiver correctly infers how much the sender gains in the second dimension with probability zero. With probability one, the sender is not indifferent between the two induced actions. Therefore, with probability one the sender will not make the message contingent on the state. Realizing this, the receiver will choose an action independently of the message received, actions a^1 and a^2 are not best responses to the strategy of the sender. Thus, there cannot exist an influential cheap talk equilibrium and the only existing equilibrium is the babbling equilibrium. $\square$

In the particular case of linear preferences with ϵ uncertainty about the sender's preferences concerning the second dimension, the influential cheap

[7] To be precise: Influential only implies that different messages imply different actions with strictly positive probability, not necessarily with probability one. It suffices however to prove the theorem only for the probability one case, as if there exists an influential equilibrium, in particular there exists an influential equilibrium in which different messages induce different actions with probability one.

[8] Here one could precisely quantify in terms of Φ the probability that S prefers a^1 to a^2 due to the ordering of the type space $[-\epsilon, \epsilon]$ and the linearity of the preferences. This is however not important for the contradiction argument and will not be possible in the later, general proof of the nonexistence of influential cheap talk equilibria.

talk equilibria are still robust if we weaken the equilibrium notion to ϵ-equilibrium.[9] To make this precise, consider $\delta = \max_{x\in[-\epsilon,\epsilon]} |[U(a^1, x) - U(a^2, x)]|$. Then the following proposition holds.

Proposition 2.3. For all F there exists an influential cheap talk δ-equilibrium.

2.4 Robustness for general preferences

We now turn to the general case. Wlog, we assume $N = 2$.[10] The sender's utility function has two arguments, action $a \in A \subset \Theta$ and type $x \in T$.[11] In the linear case we have $T = [-\epsilon, \epsilon]$. We assume the type x of the sender is her private knowledge, the receiver only knows $X \sim \Phi(T)$, where Φ is a continuous distribution function with support T.[12] We now formulate condition (S), as stated in the online appendix of Chakraborty and Harbaugh.[13] *Condition* (S) is satisfied if for any two actions a and $a^{'}$, if $U(a, x^*) = U(a^{'}, x^*)$, then $U(a, x) \neq U(a^{'}, x)$ for all $x \neq x^*, x \in T$. For example, linear preferences satisfy this property, as one can see from figure 2.1. More generally condition (S) holds for preferences whose indifference curves satisfy a single crossing property. The following example given in figure 2.2 explains why a condition like condition (S) is needed. Take an interior point c and a hyper-

[9] Let $\epsilon \geq 0$. An equilibrium is an ϵ-equilibrium if it is not possible for a player to gain more than ϵ by unilaterally deviating from the equilibrium strategy.

[10] This is wlog, as the contradiction argument will not depend on the dimensionality of θ.

[11] We choose, in contrast to Chakraborty and Harbaugh, to denote the type by x and not by t, to be able to distinguish between the random variable X and the realization x.

[12] We implicitly assume T has some kind of interval structure to be able to support a continuous distribution function.

[13] They introduce condition (S) for their proposition 6 and use it to split the type set which enables them to apply the implicit function theorem.

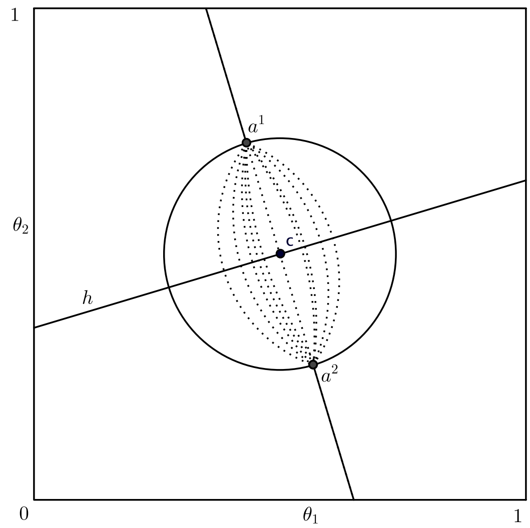


Figure 2.2: Existence of influential cheap talk equilibrium despite Harsanyi-Uncertainty

plane h which splits the state space in two halves. The indifference curves of the different sender types are the dotted lines.[14] If now the updated estimates a^1 and a^2 are exactly such that the indifference curve through a^1 also goes through a^2 for all possible sender types $x \in T$ (violating Condition (S)), then there still exists an influential cheap talk equilibrium. The receiver has no incentive to deviate as he is better informed and the sender has no incentive as the updated estimates lie on the same indifference curve. Condition (S) rules the situation of figure 2.2 out and the following theorem generalizes theorem 2.2 to nonlinear preferences.

Theorem 2.4. *If Condition* (S) *is satisfied and* Φ *is nonatomic, there does not exist an influential cheap talk equilibrium for any* F.

Comment 2.5. Two points are important to note. First, it is straightforward to generalize theorem 2.4 to the case where condition (S) is satisfied

[14]One may think of a continuum of indifference curves between the most left and most right curve

with probability one: Say *Condition (S′)* holds if for any two actions a and a', $\Phi(x \in T | U(a,x) \neq U(a',x)) = 1$. Then theorem 2.4 holds if we replace condition (S) by the weaker condition (S'). The proof is the same.

Second, theorem 2.4 and the condition (S') version of theorem 2.4 give sufficient conditions for the nonexistence of influential cheap talk equilibria, given T is nondegenerate. It might be interesting to investigate necessary conditions for the nonexistence.

Proof. The proof is by contradiction and similar to the proof of the linear case. Suppose there exists an influential cheap talk equilibrium. Hence, there exist messages m^1 and m^2 which induce different actions, $a^1 = E(\theta | m^1) \neq a^2 = E(\theta | m^2)$.[15] Suppose there exists $\tilde{x} \in T$ such that $u(a^1, \tilde{x}) = u(a^2, \tilde{x})$. Condition (S) implies that for all $x \neq \tilde{x}$ it holds $u(a^1, x) \neq u(a^2, x)$. If the sender is not indifferent between the two induced actions, she will send a message which is not state-dependent: For all $\theta, \theta' \in \Theta, \theta \neq \theta'$, it holds $Pr(m^1 | x, \theta) = Pr(m^1 | x, \theta')$ and $Pr(m^2 | x, \theta) = Pr(m^2 | x, \theta')$ for all $x \neq \tilde{x}$. As x is unknown to the receiver and X is a continuous random variable, $Pr(X = x) = 0$ for all $x \in T$, in particular $Pr(X = \tilde{x}) = 0$. Therefore, the receiver can only correctly infer how much the sender gains and loses in the two dimensions with probability zero. Thus, with probability one the sender strictly prefers one induced action over the other induced action which implies that with probability one the sender will send a message which is not state-dependent. Realizing this, the receiver will not make his action contingent on the message as making the action contingent on the message is no longer a best reply. Hence, there cannot exist an influential cheap talk equilibrium. The only equilibrium is the babbling equilibrium. □

[15] As argued before in the footnote in the proof of theorem 2.2, it suffices to consider the probability one case.

Consider an influential cheap talk equilibrium with equilibrium actions $a^1 \neq a^2$ in the case without uncertainty about the preferences. Denote the maximal utility difference of these actions in the uncertainty case by $\delta = \max_{x \in T}[|U(a^1, x) - U(a^2, x)|]$. Then the following proposition holds true.

Proposition 2.6. For all U and all F there exists an influential cheap talk δ-equilibrium.

2.5 Conclusion

The contribution by Chakraborty and Harbaugh provides important insights into how the combination of multidimensionality of the state and state-independent preferences generates influential cheap talk equilibria. The state-independence implies that the sender has to be indifferent between all actions which are induced in equilibrium. This indifference is an unavoidable consequence of the state-independence, but also a drawback of the results as all influential equilibria become fragile in the sense of being not robust. We introduced uncertainty about the preferences of the sender in what we consider the most natural way, first for linear preferences, then for general nonlinear preferences. In the linear case, the receiver knows the slope of the expert's indifference curves precisely up to an ϵ. In the general case the sender's type is an element of a continuous set T. Additionally, we imposed condition (S), i.e. if one type gives the same utility to two different actions, all other types are not indifferent between these two actions. We proved the nonexistence of influential cheap talk equilibria both for the linear and the general case with a contradiction argument. As the sender type does not condition the message on the actual state with probability one, the receiver does not listen to the sender and only the babbling equilibrium continues to exist. These findings follow the intuition. If the sender's utility does not depend on the state, the sender will not disclose information truthfully except the disclosure leads to actions between which the sender is indifferent. Therefore, the results of Chakraborty and Harbaugh do not transfer to more realistic scenarios with some uncertainty about the sender's preferences. In these situations the sender cannot communicate information such that it influences the actions of the receiver.

Our results also let the applications, which Chakraborty and Harbaugh present, appear in a new light. Credible communication is not possible for an expert with state-independent preferences as soon as there is some uncertainty about the type. Consider for example think tanks. They might have a clear political agenda, but try to appear neutral. One might be able to have a pretty good estimation of the precise preferences. As their funding is generally unknown, there nevertheless remains some uncertainty which suffices to break down influential communication, as we have shown. From a different perspective, while the receiver does not learn anything about the true state, he does get some information about the type of the sender. It might be interesting to consider if and how a receiver can assemble information about the true state in a repeated game setting with uncertainty about the sender's preferences.

2.6 Appendix

2.6.1 Uncertainty about the bias in Crawford and Sobel

We have shown that the influential cheap talk equilibria of Chakraborty and Harbaugh (2010) do not survive the introduction of Harsanyi-Uncertainty about the type of the sender. In this section we show that this is not a general problem that all sender-receiver games suffer from. To see this we use a simple example in the spirit of Crawford and Sobel (1982) with a possibly biased sender in which information transmission can still happen despite uncertainty about this bias.

The state space is $\Theta = [-1, 1]$. The prior of the receiver is given by a distribution F (with density f) over Θ that is symmetric around zero.[16] The sender is privately informed about the realization of the state $\theta \in \Theta$ and can send a costless message $m \in M = \{H, L\}$ to the receiver. The receiver observes the message of the sender and takes an action $a \in \mathcal{A} = \Theta = [-1, 1]$. The sender has utility function $u(a, \theta, b) = -(\theta + b - a)^2$ and the receiver utility function $v(a, \theta) = -(\theta - a)^2$. Here, b denotes the sender's bias relative to the receiver, a the action taken by the receiver, and θ the state. Suppose, first, it was common knowledge that the sender's bias is equal to zero. Thus, the game is one of complete common interest. This game has an influential equilibrium in which senders with state below zero send message L and senders with state above zero send message H. The receiver chooses actions which are equal to the conditional expectation of the state conditioning on the observed message and given the sender's strategy. For the case of a uniform prior F, for instance, the receiver chooses action

[16] The assumption of symmetry is not important for the result. It allows us, however, to dramatically simplify the equilibrium calculations.

$a_H = \frac{1}{2}$ upon receiving message H and $a_L = -\frac{1}{2}$ upon receiving message L.

We now introduce Harsanyi-Uncertainty about the bias into this example of a Crawford and Sobel (1982) sender-receiver game.[17] The sender knows her bias precisely, in addition to knowing the state. The receiver does neither know the true state nor the precise bias b. Instead, the receiver only has a prior G (with density g) over an interval $[-\epsilon, +\epsilon]$ of possible biases of the sender for ϵ positive but small. The prior G is assumed to be symmetric around 0 and orthogonal to the prior F over the state space.[18]

We shall now compute an influential equilibrium that is close to the equilibrium without bias uncertainty given above. Suppose that the receiver plays action a_H if he observes message H and action a_L if he observes message L with (without loss of generality) $a_L < a_H$. Then the behavior of the sender must be as follows. If the state θ is below a cut-off of $q(b)$, which depends on the sender's bias, then she sends message L, otherwise she sends message H. The cut-off must be such that the sender with bias b and state equal to this cut-off $q(b)$ is indifferent between the two messages. This consideration leads to $q(b) = \frac{a_H + a_L}{2} - b$. The symmetry in the two distributions implies that $a_L = -a_H$. This in turn implies that the cutoff is $q(b) = -b$ and independent of the two actions.[19] It then remains to calculate the equilibrium action a_H. It is given by the conditional expectation, from the sender's point of view, of the state $q(b)$ given that message H is sent, i.e. given that

[17] Papers with uncertainty about the bias in the cheap-talk literature include Morgan and Stocken (2003) and Li and Madarász (2008). We are not aware of a paper that introduces uncertainty about the bias in a way similar to ours.

[18] In other words the receiver's joint prior about state and bias is the product of the two marginal priors. Bias and state are, in the receiver's view, independently drawn.

[19] Without symmetry in the distributions this would not be true, and calculations would be more cumbersome.

$\theta > q(b)$. For ϵ small enough, this can be expressed as the double-integral

$$2\int_{b=-\epsilon}^{\epsilon}\int_{\theta=-b}^{1}\theta f(\theta)g(b)d\theta db,$$

where the 2 is the reciprocal of the probability that $\theta > q(b)$ (derived from the symmetry in the two distributions). For the special case of two uniform distributions for F and G we obtain $a_H = \frac{1}{2} - \frac{\epsilon^2}{6}$ and $a_L = -a_H$. Thus, except for the receiver's actions being just slightly closer to the center than under the case without bias-uncertainty, the equilibrium has hardly changed. In particular, as ϵ tends to zero the influential equilibrium of the game with bias-uncertainty converges to the original influential equilibrium of the game without bias-uncertainty. To see this not only for the double-uniform prior case, note that, generally, the condition $\theta > q(b) = -b$, as ϵ tends to zero, tends to the condition $\theta > 0$, which is the condition employed in the model without bias uncertainty.

2.6.2 Proof of the Chakraborty and Harbaugh result

Proof. **Theorem 2.1** (Chakraborty and Harbaugh)
Fix $c \in int(\Theta)$ which exists as the interior is nonempty. Let $h_{s,c}$ be the hyperplane through c with orientation $s \in \mathbb{S}^{N-1}$. This hyperplane splits Θ into two nonempty sets $\mathbf{R}^1(h_{s,c})$ and $\mathbf{R}^2(h_{s,c})$. Suppose the receiver actions are $a^1(h_{s,c})$ and $a^2(h_{s,c})$, respectively. As we assume full support of the prior, the two actions are in the interior of the two halfspaces, $a^1(h_{s,c}) \in \mathbf{R}^1(h_{s,c})$ and $a^2(h_{s,c}) \in \mathbf{R}^2(h_{s,c})$. Being interior points, for the two induced actions it holds $\mathbf{R}^1(h_{s,c}) \ni a^1(h_{s,c}) \neq a^2(h_{s,c}) \in \mathbf{R}^2(h_{s,c})$, so that the equilibrium, if it exists, is influential. The expert sends message m^1 if the true state is in $\mathbf{R}^1$ and m^2 if the true state is in $\mathbf{R}^2$. Consider the orientation s of the hyperplane. For a fixed interior point c, the induced estimates

$a^1(h_{s,c})$ and $a^2(h_{s,c})$ are continuous functions of s. For opposite orientations $s, -s \in \mathbb{S}^{N-1}$, we have $\mathbf{R}^1(h_{s,c}) = \mathbf{R}^2(h_{-s,c})$ and $\mathbf{R}^2(h_{s,c}) = \mathbf{R}^1(h_{-s,c})$ implying $a^1(h_{s,c}) = a^2(h_{-s,c})$ and $a^1(h_{-s,c}) = a^2(h_{s,c})$. Consider the difference between the utility in the induced actions: $\Delta(\cdot, c) : \mathbb{S}^{N-1} \to \mathbb{R}$, where $\Delta(s, c) = U(a^1(h_{s,c})) - U(a^2(h_{s,c}))$. This is a continuous odd map in s. According to the Borsuk-Ulam theorem every continuous odd map $\mathbb{S}^{N-1} \to \mathbb{R}$ has a zero. Thus, there exists for every interior c an orientation $s^* \in \mathbb{S}^{N-1}$ such that $U(a^1(h_{s^*,c})) - U(a^2(h_{s^*,c})) = 0$. The expert does not have an incentive to deviate to misreporting, we have found a cheap talk equilibrium. $\square$

2.6.3 Robustness for additive separable preferences

We now turn to the analysis of additive separable preferences. Suppose $N = 2$ and $U(a) = U_1(a_1) + U_2(a_2)$. The additive separability allows to treat the two dimensions separately and it suffices to add uncertainty in one dimension to show that influential cheap talk equilibria are not robust. We introduce uncertainty about the type of the sender as in the linear case: $U(a, x) = U_1(a_1) + U_2(a_2)(1 + x)$, where $x \in [-\epsilon, \epsilon]$ for $\epsilon \geq 0$. If $\epsilon = 0$ there is no uncertainty about the type. If $\epsilon > 0$, the preferences of the sender cease to be common knowledge as the receiver does not know the realization x but only $X \sim \Phi[-\epsilon, \epsilon]$. Here X is a continuous random variable distributed according to the distribution Φ with support $[-\epsilon, \epsilon]$. The realization x is private knowledge of the sender. We introduce the following technical condition. The utility function is said to satisfy *Condition (A)* if the following holds true: If $a \neq a^{'}$, but $U(a, x) = U(a^{'}, x)$, then $U_1(a_1) \neq U_1(a_1^{'})$ and $U_2(a_2) \neq U_2(a_2^{'})$. The following theorem is the analogue to theorem 2.2 for the additive separable case and states that for strictly positive

ϵ all influential cheap talk equilibria cease to exist.

Theorem 2.7. *If condition (A) is satisfied and there is uncertainty about the preferences in the additive separable case, i.e. if $\epsilon > 0$, there does not exist an influential cheap talk equilibrium for any F.*

Proof. The proof is by contradiction and follows the logic of the proofs of theorem 2.2 and theorem 2.4. Suppose there exists an influential cheap talk equilibrium. Thus, there exist messages $m^1 \neq m^2$ which induce different actions $a^1 = E(\theta|m^1) \neq E(\theta|m^2) = a^2$. Suppose there exists $\tilde{x} \in [-\epsilon, \epsilon]$ such that $U(a^1, \tilde{x}) = U_1(a_1^1) + U_2(a_2^1)(1 + \tilde{x}) = U_1(a_1^2) + U_2(a_2^2)(1 + \tilde{x}) = U(a^2, \tilde{x})$. Condition (A) implies $U_1(a_1^1) \neq U_1(a_1^2)$ and $U_2(a_2^1) \neq U_2(a_2^2)$. Thus, $U_2(a_2^1, x) \neq U_2(a_2^2, x)$ for all $x \in [-\epsilon, \epsilon]$ and $U(a^1, x) \neq U(a^2, x)$ for all $x \neq \tilde{x}$. The sender is not indifferent between the two induced actions, she will send a state-independent message: For all $\theta, \theta' \in \Theta, \theta \neq \theta'$, it holds $Pr(m^1|x, \theta) = Pr(m^1|x, \theta')$ and $Pr(m^2|x, \theta) = Pr(m^2|x, \theta')$ for all $x \neq \tilde{x}, x \in [-\epsilon, \epsilon]$. As the realization of X is unknown to the receiver and X a continuous random variable, $Pr(X = \tilde{x}) = 0$. Therefore the receiver can only correctly infer how much the sender gains and loses in the two dimensions with probability zero. As in the linear and general case, this leads to a contradiction and the nonexistence of influential cheap talk equilibria. $\square$

For the additive separable case, one can easily formulate an analogon to propositions 2.3 and 2.6. Suppose for simplicity that $|U_2(a)| \leq 1$ and $\Theta = [0, 1]$. Then the following proposition is true.

Proposition 2.8. For all u and all F there exists an influential cheap talk 2ϵ-equilibrium.

The 2ϵ appears as the length of the type interval is 2ϵ.

2.6.4 Different receiver types

Suppose there is no Harsanyi-Uncertainty about the preferences of the sender. That is, as in Chakraborty and Harbaugh (2010), there is only one type of sender with state-independent utility function $u : \mathcal{A} \to \mathbb{R}$, where $\mathcal{A} = \Theta$ and Θ is a convex and compact subset of $\mathbb{R}^N$ with $N \geq 2$. Instead, there are possibly infinitely many different receiver types in terms of the receiver's subjective belief F over the state space Θ. That is there is a set $\mathcal{F}$ of distributions over the state space. Each receiver privately knows his distribution F. The sender is not informed about the receiver's prior, but holds her own prior H over the set $\mathcal{F}$. This prior H is commonly known and can be a continuous distribution or can have atoms, or can even be a finite distribution.

Theorem 2.9. *Consider a sender-receiver game as defined in Section 2.2 with the information structure as given in this Section. Then this game has an influential equilibrium.*

Proof. The proof follows the existence result of Chakraborty and Harbaugh. Fix an arbitrary $c \in \text{int}(\Theta)$ which exists as Θ is nonempty. Let $h_{s,c}$ be the hyperplane through c with "orientation" $s \in \mathbb{S}^{N-1}$. The orientation is orthogonal to the hyperplane and has (Euclidean) length 1. Thus, $\mathbb{S}^{N-1}$ is the unit sphere in $\mathbb{R}^N$. The hyperplane splits (essentially partitions) the state space into two nonempty regions $\mathbf{R}^1(h_{s,c})$ and $\mathbf{R}^2(h_{s,c})$. The expert sends message m^1 if $\theta \in \mathbf{R}^1$ and m^2 if $\theta \in \mathbf{R}^2$. Receiver type F best responds to the sender's strategy by choosing optimal action $a_i^F(h_{s,c}) \in \mathbf{R}^i(h_{s,c})$ upon receiving message m_i (for $i \in \{1, 2\}$).

The sender, with given fixed prior H, computes, for $i \in \{1, 2\}$, her expected utility $u_i(h_{s,c}) = \mathbb{E}_H \left[a_i^F(h_{s,c})\right]$. For a fixed interior point c, each $u_i(h_{s,c})$ is a continuous function in $s \in \mathbb{S}^{N-1}$. For opposite orientations $s, -s \in \mathbb{S}^{N-1}$, we have $\mathbf{R}^1(h_{s,c}) = \mathbf{R}^2(h_{-s,c})$ and $\mathbf{R}^2(h_{s,c}) = \mathbf{R}^1(h_{-s,c})$ implying $u_1(h_{s,c}) = u_2(h_{-s,c})$ and $u_1(h_{-s,c}) = u_2(h_{s,c})$.

Consider the difference between the two utilities: $\Delta(\cdot, c) : \mathbb{S}^{N-1} \to \mathbb{R}$, where $\Delta(s, c) = u_1(h_{s,c})) - u_2(h_{s,c}))$. The property that $\Delta(s, c) = -\Delta(-s, c)$ makes this a (continuous) odd map in s. The Borsuk-Ulam theorem then implies that there is a $s^* \in \mathbb{S}^{N-1}$ such that $\Delta(s^*) = 0$. Thus, there exists for every interior c an orientation $s^* \in S^{N-1}$ such that $u_1(h_{s^*,c})) - u_2(h_{s^*,c})) = 0$. Thus, we have found an influential cheap talk equilibrium. $\square$

3 The LIBOR Mechanism and Related Games

Abstract

The London InterBank Offered Rate (LIBOR) is the most important set of interest rate benchmarks. Recently there have been reports about systematic manipulation of the LIBOR. We thus investigate incentives and possibilities to rig the LIBOR or related statistics for quote submitting panel banks. Both reputation concerns and financial exposure to the index may lead to misrepresentation of borrowing costs. We show that even in the static model incorrect quoting is the standard and honesty the exception. In particular, we can explain theoretically why the LIBOR quotes were too low during the financial crisis which started in 2007, when increasing panel bank sizes is helpful and why individual quotes should be published with delay. Moreover, we evaluate and compare the performance of different statistics like the median, the trimmed average and the average.

3.1 Introduction and Motivation

"This dwarfs by orders of magnitude any financial scams in the history of markets"
Comment on the LIBOR scandal by Andrew Lo, MIT Professor of Finance, 2012.

The London InterBank Offered Rate (LIBOR) is the most important set of interest rate benchmarks in the financial world. It is estimated that it affects more than $350 trillion worth of derivatives, flexible mortgage rates and student loans. Almost everybody is directly affected by changes of the LIBOR. The BBC has called it "the most important number in the world".[1] The British Bankers' Association writes "The London Interbank Offered Rate (LIBOR) is the primary benchmark for short term interest rates globally."[2] In the last months reports about systematic manipulation of the LIBOR have emerged. These reports call for a systematic investigation of the current LIBOR mechanism in a theoretical model to identify the strengths and weaknesses of the mechanism and the possibilities of banks to manipulate the index.

To this end, we consider several strategic games which are closely related to the actual LIBOR administering process. The LIBOR is currently calculated as the trimmed average of submitted quotes of banks which belong to the LIBOR panel. The top and bottom 25% of the quotes are deleted, the remaining quotes are averaged, the result is the LIBOR. However, all quotes of all banks are published.[3] The quotes of the banks do not have

[1]http://news.bbc.co.uk/2/hi/business/7680552.stm
[2]http://www.bbalibor.com/bbalibor-explained/faqs
[3]With effect from 1st July 2013, the publication of individual bank's submissions to LIBOR will be embargoed for 3 months. Prior to this date, all submissions were

to be based on real transactions, instead banks answer the question: "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11am?" This opens the door to manipulation of the index. We will coin the name "the LIBOR problem" for the problem of building a mechanism and using a statistic such that the resulting index based on the submitted quotes is the same as if the index were calculated using the private values of the banks. Different statistics besides the trimmed average are the median and the average of all quotes. We will formulate games which are identical except for the statistic that is used to aggregate the individual quotes. The statistic enters the utility function of the strategically acting banks which are the players in the game.

Our main findings are the following. Acting strategically, the default quote of a bank is not the true quote, except when the bank cannot influence the statistic. In the Average Game, banks thus always misrepresent their private costs except when reputational concerns and financial incentives exactly balance. In this game, an increase in the panel bank size decreases the magnitude of misrepresentation if reputational concerns are not important for the banks. We find somewhat different results in the Median Game. While it remains true that banks either quote honestly or such that they influence the index, for uneven n there always exists a player who quotes truthfully. This does not hold for even n. The notions of trimmed average, median and average are only distinct if $n \geq 5$. We present a proposition which delivers a comparison between the maximal misrepresentation in equilibrium in the three games. Due to the different marginal impact of the players which influence the statistic, the misrepresentation can be largest in the Median Game, second largest in the Trimmed Average Game and is low-

published immediately.

est in the Average Game. Additionally, we prove the existence of a maximal misrepresentation equilibrium for all considered games. This theorem holds independently of the number of players. In the appendix we characterize the equilibrium set in the three player Median Game. Three different types of equilibria appear, depending on the distribution of the private values and the financial exposure of the banks. A truth-telling equilibrium may arise, an indeterminate equilibrium set where a majority of banks controls the median by quoting the same, and an equilibrium in which only one bank misrepresents while the others quote truthfully.

For the formulation of the different, LIBOR-related games we suggest a specific utility function which captures the incentive structure of the banks. Different factors influence the final quotation. We argue that the utility function has to capture the financial exposure of the bank to the LIBOR or the respective statistic. Moreover, there may be reputational concerns the banks face, in particular, during a financial crisis. As all quotes of all banks are published, high quotes relative to the other quotes or relative to the index may indicate a certain weakness of the financial institution which may entail certain risks like an increased probability of a bank run. Finally, we argue for an anchoring of the bank's quote at the private value. Misquoting is not legal and if detected, punished. Barclays had to pay more than $400 million for the settlement of charges and UBS more than $1 billion for having tried to manipulate the LIBOR. As the reputational concerns only point in the direction of lower quoting and arguably only play an important role in times of financial turmoil and the actual manipulation goes way back before the financial crisis of 2007, we focus in large parts of the paper on financial misrepresentation incentives and the anchoring of the costs in the utility function.

We opt for a linear quadratic structure of the utility function, close to the utility function choices of Chen (2012) and Snider and Youle (2010), the most important contributions in the very small theoretical literature on the LIBOR mechanism. We justify this choice in detail in Section 3.3. We further assume that all players know the private values of all players, while the mechanism designer, the institution that administers the LIBOR, has no information about these private values and only observes what the players actually quote. We do not claim that this strong information asymmetry accurately describes the real world. However, from our perspective it is reasonable to assume that banks are better informed about the actual borrowing costs of other banks than the regulating authority. This may be due to the fact that panel banks are similarly structured and are actually operating on the interbank market and are not only observing it. Nevertheless, this remains a simplification.

The literature which investigates the LIBOR, the quotation patterns, and in particular the possibility of and indications for manipulation, is growing. However, at the moment there are still relatively few contributions. There exist several notable empirical investigations but only two papers which formulate a theoretical model and directly address the LIBOR problem. We start by summarizing the empirical literature.

The first paper which examined LIBOR quotes for suspicious patterns was Abrantes-Metz et al. (2012). They used a screen for collusion and found dubious quote distributions which may indicate manipulation attempts. Abrantes-Metz et al. (2011) use a different approach. Using a test based on Benford's law to find evidence for possible fraud, they find a quote distribution which is highly suspicious. The approach of Kuo et al. (2012) relies on examining different indicators for the borrowing costs of banks like the inferred term borrowing costs derived from FedWire, a data system for interbank borrowing costs based on real transactions. They conclude that the LIBOR quotes are 10-30 basis points lower than the comparison rates in the times of huge financial turmoil after the collapses of Bear Sterns and Lehman Brothers. In other times, the different rates are statistically indistinguishable. In a recent paper by Eisl et al. (2013) EURIBOR and LIBOR fixings are investigated and their performance is compared to alternative rate fixings like the median, the mean, and a randomized procedure. They find that the manipulability of the median is much lower provided the panel of banks is sufficiently large. Therefore, they recommend switching from the trimmed average to the median.

The theoretical literature on the LIBOR mechanism is still small, but due to the ongoing LIBOR scandal a rapid growth of the literature is expected. The first authors who investigated the LIBOR in a theoretical model were Snider and Youle (2010). They sketch a model which predicts quote clustering of the quotes of the banks around the pivotal 25% and 75% quantile. As the LIBOR is calculated on a daily basis and quotes normally do not differ a lot from the previous day, banks have a good knowledge of the next quote of their competitors.[4] Thus, they are able to accurately predict the interval of quotes which are taken for the actual LIBOR calculation and can optimize the position of their own quote inside this interval that entails the quote clustering at the borders. Snider and Youle then look at data and find evidence supporting their theoretical predictions. In particular, they find indications for fraud not only during the times of market turmoil, but also before and after. Interestingly, the first version of their paper has been written in 2009 and thus predates the revelations in the LIBOR scandal.

[4]The new rule of publishing the quotes with delay makes the prediction of future quotes of other banks harder.

A second important contribution is the paper by Chen (2012). She works in a Bayesian framework where the realization of the private value is private knowledge of the respective bank. The private values are independent and identically distributed and known to all players and the mechanism designer. She suggests a utility function similar to ours but only considers the incentive for banks to lower their own quote. In this sense, she is primarily concerned with reputational incentives of the banks. She investigates the best responses of the players and suggests a mechanism to solve the LIBOR problem which is in the spirit of d'Aspremont and Gérard-Varet (1979). She then looks at data and calibrates the parameters of her utility function.

The remainder of the paper is organized as follows. We start by giving background on the LIBOR. We describe the origins of the index, how it became the most important benchmark index in the financial industry, and how it is calculated and administered. We then turn to the LIBOR scandal, the attempted systematic manipulation of the index by the panel members. The scandal really unfurled in June / July 2012, and the magnitude of the scandal is still hard to estimate. However, reports on the manipulation go back to the earliest days of the LIBOR. We go on to describe policy answers to the LIBOR problem, in particular the suggestions of the Wheatley review by Wheatley (2012a), and speculate on the future of the LIBOR as a benchmark index. We present our baseline model of a strategic game in Section 3.3 before we investigate the different LIBOR-related games in Section 3.4. The games only differ in the way the quotes are aggregated. We then point out how our findings add to the understanding of the LIBOR mechanism and what normative implications our results may have on the fixing of the LIBOR problem. In the appendix we present a detailed analysis of the three player Median Game and several examples.

3.2 Background on the LIBOR

In this section we give background on the London Interbank Offered Rate. Starting with a historical perspective, we continue with a description of the current LIBOR mechanism before we present a detailed chronology of the LIBOR scandal and policy approaches in response to the LIBOR scandal. Besides the cited academic articles we draw primarily from newspaper articles and give the references in the footnotes.

3.2.1 Inception and historical development of the LIBOR

The London InterBank Offered Rate was institutionalized in 1986. However, the origins of the rate go back to the late 1960s. In the early days of the LIBOR, the fixing of the rate can best be described as a gentlemen's agreement between likeminded bankers.[5] In that form it was set and governed by a small group of bankers in London between the late 1960s and the early 1980s. The very first LIBOR loan was a $80 million loan by a group of banks to Iran. The banker at Manufacturers Hanover, Minos Zombanakis, who was responsible for the loan, had to come up with a rate fixing. Thus he called several banks in London and asked them to send him their cost of money. He used a trimmed average of the set of rates that were handed in and called it the London InterBank Offered Rate. For the next 15 years the rate was set in a similar way: Banks were asked for their costs of money and a trimmed set of the numbers handed in were arithmetically averaged to fix the LIBOR.

[5]Landon Thomas Jr., Trade Group for Bankers Regulates a Key Rate, July 05, 2012, New York Times

Starting from 1984, the way the LIBOR was governed changed. New financial instruments like interest rate swaps emerged and banks started to trade heavily in these new areas. Banks were worried that nonuniformity and the adhoc structure of the LIBOR calculation would hinder the growth of the new markets for banks. A demand for an institutionalization of the LIBOR arose. Driven by this demand, the British Bankers' Association (BBA) presented the BBA standard for interest rate swaps (BBAIRS), the predecessor of the LIBOR, in 1985. The BBA LIBOR officially commenced on 1.1.1986 and immediately became standard market practice. There have been some minor modifications in the way the LIBOR is governed and calculated since 1986. The composition of the panels has changed frequently and new tenors and currencies have been added. Importantly, the LIBOR question has been modified in 1988. Up to 1988, the question was: "At what rate do you think interbank term deposits will be offered by one prime bank to another prime bank for a reasonable market size today at 11am?" The question was changed to: "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11am?" The new formulation was chosen to enable accountability for the rates, to cite the BBA. Interestingly, the LIBOR-question differs from the EURIBOR-question. The EURo InterBank Offered Rate is also a survey based set of interest rates. The EURIBOR-question is: "Contributing panel banks must quote the required euro rates to the best of their knowledge; these rates are defined as the rates at which euro interbank term deposits are being offered within the EMU zone by one prime bank to another at 11am Brussels time." An analysis of the importance of the two different questions can be found in Eisl et al. (2013). It is important to note that the main idea of asking banks for their borrowing costs and using their answers to calculate a number, the LIBOR, has not been changed since the

1960s.

The significance of the LIBOR has massively increased in the last years and decades. To get an idea of the centrality that the LIBOR plays in the financial world it is best to look at some numbers. It is estimated that contracts with notional value of more than $350 trillion are linked to the LIBOR. More than 50% of all mortgage contracts in the United States have flexible mortgage rates that depend on the LIBOR. Recent documents show that Deutsche Bank has made more than 500 million Euro with bets on the LIBOR only in 2008.[6] The same documents give the information that as of September 30, 2008, Deutsche Bank could "gain or lose as much as about 68 million for each one-hundredth of a percentage point change in the gap between different rates related to LIBOR and the EURIBOR". In other words, the importance of LIBOR is enormous and can hardly be underestimated. A possible manipulation of the LIBOR is of enormous financial interest for the banks. Before we give a chronology of the LIBOR scandal, in which the banks apparently indeed rigged or tried to rig the LIBOR, we first summarize the administering process.

3.2.2 Administering of the LIBOR

In this section we describe how the LIBOR is currently administered. Following the revelations in the LIBOR scandal some changes to the administering process have been implemented recently. The LIBOR used to be published daily in 15 different maturities (overnight, 1 week, 2 weeks, 1 month, 2 months, ..., 12 months) and 10 different currencies (Pound Sterling, US Dollar, Japanese Yen, Swiss Franc, Canadian Dollar, Australian

[6] http://online.wsj.com/article/SB10001424127887324442304578231721272636626.html

Dollar, Euro, Danish Kroner, Swedish Krona, New Zealand Dollar). The last rates for the New Zealand dollar were published end of February 2013, the last for Swedish Krona and Danish Kroner on March 28th, 2013, and the last for Canadian and Australian Dollar at the end of May 2013. Apart from the reduction from 10 to 5 currencies it is also planned to reduce the number of maturities from 15 to 7.

For all currencies and maturities there exist panels which consist of 6-18 members. These members are chosen by the British Bankers' Association (BBA), the organization responsible for the administering process, according to the three criteria scale of market activity, credit rating, and perceived experience in the currency concerned. Panel bank members include the large European, American and Asian banks like Deutsche Bank, UBS, Barclays, Bank of America, Mitsubishi, etc. Some of the panel banks like Barclays and HSBC are themselves members of the BBA, thus had to oversee themselves. Interestingly, Marcus Agius, who was chairman of Barclays till the revelations of the LIBOR scandal of June 2012, was also chairman of the BBA. He stepped down from this position when he stepped down from his chairman position at Barclays.

The most important single LIBOR rate is the three months US Dollar, which is fixed by a panel of currently 18 banks. The actual calculation of the LIBOR is done by the data provider Thomson and Reuters. All panel banks are given a special application which allows the banks to secretly transfer the daily rates for the respective currencies and tenors to Thomson and Reuters. The time window for this transfer is 11.00-11.10am GMT. In this time window, banks submit their set of quotes which is their answer to the question "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11am?" After having received the data packages from the banks, Thomson and Reuters performs a series of statistical tests on the received data sets to ensure no mistakes have been made by the submitting banks. The actual LIBOR rates are then calculated and together with all original quotes of all banks (until June 30th, 2013) published by Thomson and Reuters and other licenced data vendors at precisely 12am GMT. Since July 1st, 2013, individual quotes are published with a delay of 3 month.

3.2.3 The LIBOR scandal

The LIBOR scandal started to unfurl in June / July 2012 when the first major bank, Barclays, admitted that it had submitted quotes for the LIBOR calculation which did not accurately reflect their own perception of the interest rate they would have to pay to borrow on the unsecured interbank market. They agreed to pay a penalty of $450 million for a settlement. However, reports on systematic manipulation of the LIBOR go back much longer. The Wall Street Journal published a series of articles in April and May 2008 which questioned the accurate quoting of banks. On April 16, 2008,[7] the journal printed the article "Bankers cast doubt on key rate amid crisis" by Carrick Mollenkamp where it was stated that "One of the most important barometers of the world's financial health could be sending false signals". The article cited several bankers and the report by Gyntelberg and Wooldridge (2008) which expressed the concern that banks may quote incorrectly. The sources quantified the misrepresentation to be 20-30 basis points below the actual borrowing costs. The article already mentioned the two main reasons for misrepresentation of true borrowing costs. First, "Some banks don't want to report the high rates they're paying for short-term

[7] http://online.wsj.com/article/SB120831164167818299.html

loans because they don't want to tip off the market that they're desperate for cash". In other words, banks submit lower quotes to appear financially stronger. Reputational concerns drive the misrepresentation. The report by Gyntelberg and Wooldridge (2008) suggested that "banks might have an incentive to provide false rates to profit from derivatives transactions." Thus, the second reason for misrepresentation are financial incentives. In a follow-up article published in the Wall Street Journal on May 29, 2008, the reporters Mollenkamp and Whitehouse analyzed in detail the quoting patterns of banks and arrived at the conclusion that some banks may have submitted quotes during the financial crisis which were too low and thus indicated a false financial situation of the banks.[8] In particular, the study compared the LIBOR submissions to credit default swaps, an indicator of financial strength. This article received a lot of attention and triggered research on the question whether the LIBOR is reliable and a credible indicator of the actual borrowing costs of banks. Other financial publications like the Financial Times followed in formulating concerns about the reliability and accuracy of the LIBOR. [9]

The British Bankers' Association responded to the reports. Angela Knight, who at that time was CEO of the BBA, said there is no need to replace or reform LIBOR. She said "I see no reason suddenly to up sticks and change a process that has actually served the financial community world-wide extremely well for a very considerable number of years". Some investigations supported her statement. The already cited Gyntelberg and Wooldridge (2008), a report of the Bank for International Settlements, highlighted incentives for misrepresentation due to financial exposure to the LIBOR. How-

[8]http://online.wsj.com/article/SB121200703762027135.html

[9]Mackenzie, M. and Tett, G. 2008. Libor remarks fail to put unease to rest. Financial Times (June 2)

ever, to quote from the report, “the available data do not support the hypothesis that contributors manipulated their quotes from positions based on fixings.” Moreover, in its global financial stability review of October 2008, the IMF arrived at the conclusion that ”Although the integrity of the U.S. dollar Libor-fixing process has been questioned by some market participants and the financial press, it appears that U.S. dollar LIBOR remains an accurate measure of a typical creditworthy bank’s marginal cost of unsecured U.S. dollar term funding.“[10] Nevertheless, the reports about inaccuracy and problems with the LIBOR continued. Willem Buiter, then professor at the London School of Economics, wrote in August 2008 in Buiter (2008) that ”The unsecured interbank market became illiquid to the point that Libor now is the rate at which banks won’t engage in unsecured lending to each other“. In November 2008, the governor of the Bank of England, Mervyn King, described the LIBOR before parliament in the following way: “It is in many ways the rate at which banks do not lend to each other, and it is not clear that it either should or does have significant operational content.“[11] Documents released in July 2012 prove that the New York Fed and the Bank of England discussed reforms to the LIBOR setting system as far back as June 2008.[12] Mervyn King and Timothy Geithner, then head of the New York Fed and later treasury secretary under President Obama, exchanged a series of emails, in which Geithner gave several suggestions to reform the LIBOR fixing process. These correspondences followed Barclays formulating worries in August 2007 about quotes of other panel banks. In October 2008 there has been an exchange between the deputy governor of the Bank of England, Paul Tucker, and Bob Diamond, the CEO of Barclays.

[10] Global Financial Stability Report, World economic and financial surveys (International Monetary Fund): 76. October 2008

[11] Examination of Mervyn King before Treasure Select Committee, November 25, 2008

[12] http://www.guardian.co.uk/business/2012/jul/13/tim-geithner-mervyn-king-libor

Diamond's account of these exchanges have been interpreted by Barclays employees as suggestions to lower the LIBOR quotes. Tucker however rejects the idea that he meant Barclays to believe that it should quote lower.[13] Barclays quotes were prior to October 2008 usually among the highest of the panel banks. This changed after October 2008, when the executive Jerry Del Missier ordered the head of the money market desk to lower the quotes.[14]

The investigation of financial exposure as a driver of misquoting of banks has been pushed forward by Snider and Youle (2010). The first version of their paper is from autumn 2009. They find evidence for quote bunching around the pivotal quotes, the highest and lowest quotes which are included in the trimmed average. This is a clear indicator for strategic quoting of banks. The comparison between the Eurodollar and the LIBOR is also puzzling. Eurodollar are dollars which are held outside of US territory and thus outside of US jurisdiction. Comparing the Eurodollar bid rate and the LIBOR, which is an offer rate, we find prior to the start of the financial crisis in August 2007 a regular bid-ask spread. The bid rate was 6-12 basis points below the LIBOR quotes. This pattern changed in August 2007 and till summer 2011 the offer rate was below the ask rate. One may call this the Eurodollar-LIBOR paradox. The paradox is a clear indicator that the LIBOR quotes were not accurate but too low.

In May 2011 the Wall Street Journal wrote that regulators are focusing on Bank of America, Citibank and UBS in their investigation.[15] The LIBOR scandal however really broke lose when Barclays admitted misquoting in the LIBOR fixing and agreed to a settlement in June 2012. Barclays not only

[13]http://www.guardian.co.uk/business/2012/jul/13/tim-geithner-mervyn-king-libor
[14]http://www.guardian.co.uk/business/2012/jul/16/libor-barclays-fsa-jerry-del-missier
[15]http://online.wsj.com/article/SB10001424052748703818204576205991698548286.html

agreed to a fine of approximately 450 million but also to the publication of email correspondences. Some of these emails have become famous and can be now found on the web.[16] One email reads: “Hi Guys, We got a big position in 3m libor for the next 3 days. Can we please keep the libor fixing at 5.39 for the next few days. It would really help. We do not want it to fix any higher than that. Tks a lot.” This mail was sent on September 13, 2006, from a senior trader in New York to a LIBOR submitter. Another exchange is summarized in the Telegraph[17] as follows: On 26 October 2006, an external trader made a request for a lower three month US dollar LIBOR submission. The external trader stated in an email to Trader G at Barclays “ If it comes in unchanged I'm a dead man”. Trader G responded that he would “have a chat”. Barclays' submission on that day for the three month US dollar LIBOR was half a basis point lower than the day before, rather than being unchanged. The external trader thanked Trader G for Barclays' LIBOR submission later that day: “Dude. I owe you big time! Come over one day after work and I'm opening a bottle of Bollinger”. A Bollinger is a champagne brand. It is important to note that there has been internal and external communication of the money market desk at Barclays, which is responsible for the LIBOR quote.

Besides Barclays, other banks that have admitted misquoting and settled with the regulators are UBS (approximately 1.5 billion) and RBS (approximately 600 million). Additionally to ongoing investigations by financial authorities against the LIBOR setting banks, LIBOR banks are also sued in civil court. Homeowners in the United States filed a class action against LIBOR banks in October 2012. Flexible mortgage rates which are tied to the LIBOR are mostly reset on the first day of a month.[18] Statistical anal-

[16] Excerpts: http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/9359392
[17]http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/9359392
[18]http://www.forbes.com/sites/halahtouryalai/2012/10/15/

ysis has indicated that on this day quotes have gone up on a regular basis. This is a clear indicator that banks tried via LIBOR manipulation to make mortgage repayments artificially high. In April 2012 a number of municipalities in the United States filed a class action lawsuit against the LIBOR banks as the municipalities earned less on their interest rate swaps due to the too low quotes. The loss to the municipalities is quantified as being at least $6 billion.[19]

These civil court lawsuits and the investigations of the financial authorities are ongoing. More penalties against the banks and more precise estimations of the magnitude of the scandal are expected in the next months and years. The proof of fraud is not easy as the LIBOR quotes are supposed to be best guesses of the borrowing costs. They are not necessarily based on transactions and due to the illiquidity of the interbank market it is often not clear what the true borrowing costs are. Moreover, there does not exist a transaction database for actual interbank lending. The case for such a database is made in Eisl et al. (2013) and Abrantes-Metz and Evans (2012a). The strategy of the prosecutors will mainly be a combination of finding whistleblowers, analyzing internal and external communication of the LIBOR setting employees in the banks, and statistical investigations of the quote patterns. In particular, the statistical investigations will consider quote clustering at the pivotal quantiles, the relation of LIBOR to other measures of borrowing rates, timing of movements of LIBOR quotes, and similarity of LIBOR quotes. There are indications that the manipulation of the LIBOR goes back to the very first days of its existence. Reports say that fraud dates back till at least 1991[20]; other sources speak of fraud already

[19]http://www.bloomberg.com/news/2012-10-09/rigged-libor-hits-states-localities-with-6-billion-muni-credit.html

[20]Mark Gongloff, LIBOR was happening in 1991, July 27, 2012, Huffington Post

in the late 80s.[21] In an updated version of their paper, Snider and Youle (2012) consider data up to June 2012. They are able to find indications that LIBOR manipulation has continued after the financial crisis till 2011, but their tests show no unregular patterns after August 2011.

3.2.4 Policy responses to the LIBOR scandal

The LIBOR scandal has led to an intense and ongoing debate on how to ensure that the LIBOR cannot be manipulated in the future and that it represents the true borrowing costs on the unsecured interbank market. The discussion paper by Wheatley (2012b) and the Wheatley review by Wheatley (2012a) are the most important documents which summarize possible avenues for reforming the LIBOR. They were written by the CEO of the Financial Conduct Authority (FCA), Martin Wheatley. The FCA is the quasi-governmental regulating agency for financial firms in Britain. A summary of the Wheatley review is given by Garcia (2012), and criticism and alternative reform suggestions are put forward by Abrantes-Metz and Evans (2012b) and Abrantes-Metz and Evans (2012a). The following are the main suggestions made in the review. 1. Reduce the number of currencies and maturities. This facilitates the usage of actual transaction data. 2. Make the quotes of individual banks temporarily anonymous, the suggested timeframe is 3 months. This adresses possible reputation concerns and makes collusion of the banks harder. 3. Increase the panel sizes. This decreases the marginal influence of a single bank on the index. 4. Make a clear separation of the trading desk of a panel bank and the bank's LIBOR submitting quotes to give the submitters no indication from which quote patterns the bank would profit. 5. Change from the BBA to a different agency which

[21] http://www.economist.com/node/21558281

administers the LIBOR. Some banks are members of the BBA, thus had to oversee themselves. The responsible agency should be independent. All reform suggestion put forward by Wheatley will be implemented by the Treasury and the implementation process has already started (e.g. point 1 is partially implemented, point 2 fully).[22]

While all these suggestions are sensible and will be supported by the results of our theoretical model in the following sections, it is important to note that the main idea of asking the banks what their perceived borrowing costs are is not changed in the Wheatley review. The reasoning for sticking qualitatively to the existing mechanism is, on the one hand, the lack of alternative benchmarks and, on the other hand, the fact that the LIBOR is currently included in the contracts and the legal way of phasing out the LIBOR is complicated. Thus, according to Wheatley, it is important to continue with the LIBOR for reasons of the stability of the financial markets. These arguments are not wrong. However, and here we begin to speculate about the future of the LIBOR, we expect that the market will move away from using the LIBOR. Alternative benchmarks will be developed and used. The LIBOR is an anachronism from a time when the LIBOR was only used to find out the interbank offered rate but was not used for other purposes such as fixing derivatives. Interesting in this context is also research by psychologists, like Gino et al. (2009). They show that once a culture of fraud has been established, it is very difficult to reestablish a culture of trust.

[22] http://www.moneymarketing.co.uk/regulation/treasury-to-implement-all-wheatleys-libor-recommendations/

3.3 The model

Let $N = \{1, ..., n\}$ be the set of players playing the game, i.e. panel banks. Nature chooses the state $\theta \in \Theta$, where $\Theta = \mathbb{R}^n_+$. All players are informed about the state of the world, i.e. all players know the realization of the n-dimensional random variable θ which represents the vector of private costs of the banks. The strategy of player i is a function $s_i : \Theta_i \to \mathbb{R}_+$, mapping states of nature into the set $\mathbb{R}_+$ of quotes of player i. The actual quotes of the players are denoted by x. To complete the definition of a strategic game, we suggest the following utility function.

$$u_i(x_i, x_{-i}, \theta_i) = \nu_i I(x_i, x_{-i}) - (x_i - \theta_i)^2 + \lambda_i(I(x_i, x_{-i}) - x_i)$$

The particular form of this utility function needs some explanation. $I(x)$ denotes a statistic, which depends on all quotes of all players. The games which we investigate will be completely identical except for the fact that we will consider different kinds of statistics like the average or the median. $\nu_i \in \mathbb{R}$ gives the financial exposure of bank i to the index and the utility is linear in this component. Players face quadratic costs for deviation from their true personal costs. The third component describes reputational concerns of the players of their own quotes relative to the index and play a role as all individual quotes are published in addition to the index. A high quote relative to the index may indicate weakness of the bank. The banks weigh their reputation concerns with $\lambda_i \in \mathbb{R}$.

The linear quadratic structure is a simplification of the true incentive structure that may entail misrepresentation of the banks. However, in the very few other theoretical contributions that investigate the LIBOR problem, namely Chen (2012) and Snider and Youle (2012), similar utility functions are used. A reasoning behind the justification of the structure besides simplicity and tractability goes along the following lines: The more extreme the misrepresentation, the more likely a detection of it and the penalty payment is relative to the magnitude of the misrepresentation. This gives a quadratic penalty term. For small deviations approximations with linear functions are possible, and a huge number of derivatives based on the LIBOR have linear or at least partially linear payoffs. We note moreover that the primary goal of this paper is to arrive at qualitative, not quantitative, conclusions concerning the vulnerability of the LIBOR and related mechanism. We aim at providing ways how to think about the LIBOR problem that may add to the understanding of how to redesign the mechanism in a sensible and more robust way.

In addition to the n players, there is a mechanism designer concerned about the index. This designer does not know the true state θ of the world, but only gets to know the quotes x and is only allowed to use the vector of quotes to determine the index. The objective of the designer is to ensure $I(\theta) = I(x)$, in the ideal case. In general, the designer wants to design the game and the statistic such that the statistic gives, loosely speaking, a good sense of the private values of the players. This is in line with the description of the LIBOR by the BBA: "LIBOR reflects the rates at which contributor banks can borrow money from each other each day, in the world's ten major currencies and for 15 borrowing periods ranging from overnight to 12 months."[23]

[23] http://www.bbalibor.com/bbalibor-explained/faqs

3.4 Equilibrium analysis of the games

We now investigate different games. The games are completely identical except for the statistic in the payoff functions of the players. We begin with the easiest case, the average, before we analyze more complicated statistics like the median or averages which are trimmed in different kind of ways. In the case of the average all quotes directly influence the index. However, in the other two games, the Median Game and the Trimmed Average Game, misquoting may change the index even if the quote does not directly enter the calculation. Misrepresentation may alter the order of the quotes. For a simple example, see figure 3.3 in the appendix.

3.4.1 The benchmark: The Average Game

In this subsection we set $I(x) = A(x) = \sum_{i=1}^{n} x_i/n$, i.e. all quotes influence directly the statistic which is the average of all quotes submitted. We do not impose any restrictions on the λ_i. Thus, the utility function of an individual bank i is

$$u_i(x_i, x_{-i}, \theta_i) = \nu_i A(x_i, x_{-i}) - (x_i - \theta_i)^2 + \lambda_i (A(x_i, x_{-i}) - x_i).$$

We call this game the Average Game which will serve as a benchmark. As all quotes of all banks always enter the calculation of the average, the banks individually optimize without any strategic considerations. The individual optimization is independent of the optimization of all competing players, thus the Average Game is in a sense not really a strategic game. The first order condition (FOC) is

$$\text{FOC}: \frac{\nu_i}{n} - 2x_i + 2\theta_i + \frac{\lambda_i}{n} - \lambda_i = 0.$$

This gives us our first proposition and two corollaries which immediately follow.

Proposition 3.1. The bank's optimal quote is $x_i = \theta_i + \frac{\nu_i}{2n} + \frac{\lambda_i}{2n} - \frac{\lambda_i}{2}$. The deviation in the quotation x_i from the true costs θ_i is $\frac{\nu_i}{2n} + \frac{\lambda_i}{2n} - \frac{\lambda_i}{2}$. In particular, the quotes decrease in λ_i and $A(x) = \sum_{i=1}^{n} \left(\theta_i + \frac{\nu_i}{2n} + \frac{\lambda_i}{2n} - \frac{\lambda_i}{2}\right)/n$.

Corollary 3.2. *Incentive compatibility can be guaranteed via a Groves-scheme with transfers* $t_i = -(\frac{\nu_i}{2n} + \frac{\lambda_i}{2n} - \frac{\lambda_i}{2})$.

Corollary 3.3. *If reputational concerns do not play a role* ($\lambda_i = 0$)*, the misrepresentation decreases in the number* n *of banks in the panel and* $x_i = \theta_i + \frac{\nu_i}{2n} \to \theta_i$ *for* $n \to \infty$.

These findings follow the intuition. The higher the individual concern about the relation of ones own quote to the average, the more important becomes the reputational concern component in the utility function of the banks. This unilaterally leads to lower quotes, as noted in proposition 3.1. As we can calculate the precise misrepresentation of the individual player in the Average Game, we can guarantee truthful quotations of the players with a Groves-scheme like in Groves (1973). Transfer payments are chosen such that misrepresentation incentives are exactly counterbalanced and truthful quoting becomes an equilibrium. The third finding, corollary 3.3, gives information when the often discussed approach of an increase in the panel sizes may help to mitigate the misrepresentation problem. The next two results, proposition 3.4 and corollary 3.5, continue the investigation of this approach.

The society which is represented by the mechanism designer is concerned with the question whether banks quote their true costs and more importantly, if $A(x) = A(\theta)$. The following proposition and the corollary address this problem.

Proposition 3.4. An individual bank quotes honestly if and only if $\nu_i = \lambda_i(n-1)$. The average gives the actual interbank borrowing rate if and only if $\sum_{i=1}^n \nu_i = \sum_{i=1}^n \lambda_i(n-1)$.

The following observation is an immediate consequence of the proposition.

Corollary 3.5. *Increasing the number n of players solves the misrepresentation problem in the limit if and only if $\lambda_i = 0$ for all i.*

For completion, we now present the straightforward proof of proposition 3.4.

Proof.

$$\begin{aligned}
A(x) &= \frac{\sum_{i=1}^n \theta_i + \frac{1}{2n}\sum_{i=1}^n \nu_i + \sum_{i=1}^n \frac{\lambda_i}{2n} - \sum_{i=1}^n \frac{\lambda_i}{2}}{n} = \frac{\sum_{i=1}^n \theta_i}{n} \\
&\Leftrightarrow \frac{1}{2n}\sum_{i=1}^n \nu_i + \sum_{i=1} n\frac{\lambda_i}{2n} - \sum_{i=1}^n \frac{\lambda_i}{2} = 0 \\
&\Leftrightarrow \sum_{i=1}^n \nu_i = \left(\frac{\sum_{i=1}^n \lambda_i}{2} - \frac{\sum_{i=1}^n \lambda_i}{2n}\right) 2n = \sum_{i=1}^n \lambda_i(n-1).
\end{aligned}$$

□

As panel banks also trade with non-panel banks and with other entities like hedge funds, the condition $\sum\limits_{i=1}^n \nu_i = \sum\limits_{i=1}^n \lambda_i(n-1)$ is normally not satisfied even in the case $\lambda_i = 0$ for all i. The default outcome of the average index is such that $A(\theta) \neq A(x)$, thus the average index is an index which normally does not accurately reflect the true borrowing costs on the interbank market. However, as proposition 3.1 shows, increasing the number of banks can attenuate the misrepresentation problem if λ is the null vector. Then there is no systemic misquoting in one direction. This is a difference to the case $\lambda_i > 0$ for some $i \in N$.

In the case of a nonnull vector λ, the more banks there are in the panel, the less likely it is that the average of all quotes is the average of all private costs. The following intuition is behind this. The more banks there are in the panel, the smaller is the marginal impact of a single bank, the more extreme the financial exposure has to be in the positive direction to counterbalance the reputational concerns.

Before we turn to the Median Game in the next subsection, let us briefly evaluate our findings. We have shown a number of interesting results. For example, we can give an explanation why deviations of the LIBOR from market based indicators have been observed since the financial crisis has unfurled. In times of financial crisis, the likelihood of bank runs is higher and reputational concerns of banks become more important. This is reflected in the utility function of the banks by a higher λ_i. In calm markets, which may correspond to the $\lambda_i = 0$ for all i case, banks were primarily concerned with their financial exposure to the LIBOR and not with reputation. Some banks have positive exposure, some negative exposure, the overall effect may cancel out. If there is market turmoil like in a financial crisis, there is an additional incentive for the banks to misrepresent which unanimously points in one direction, that of lower quoting. Exactly this can be observed since the crisis of 2007 / 2008 unfurled. The LIBOR left the traditional close connection to market indicators like credit default swaps (CDS) and has been much lower in the following months. To get these results, we made simplifications. Most importantly, we calculated the index as an average and not as a trimmed average. This leads to the fact that banks do not have to reason about the quoting pattern of the other banks. We now turn to a true game theoretic model and investigate the Median Game, where the statistic in the utility function of the players is the median of all submitted quotes.

3.4.2 The Median Game

The current design of the LIBOR as an average of quotes between two thresholds given by the 25% and 75% quantiles leads us to the formulation and analysis of what we call the Median Game. Consider now the model setup of Section 3.3 with the statistic $I(x) = M(x)$, where $M(x)$ gives the median of the vector of quotes x. We have to distinguish between even and uneven numbers of players and consider the order statistic $(x^{(1)}, ..., x^{(n)})$, i.e. $x^{(1)} \leq ... \leq x^{(n)}$. Suppose n is uneven. Then $M(x) = x^{(\frac{n+1}{2})}$. For even n, the median is the average of the two middle quotes, $M(x) = \frac{1}{2}x^{(\frac{n}{2})} + \frac{1}{2}x^{(\frac{n+2}{2})}$. From now on, we set $\lambda_i = 0$ for all $1 \leq i \leq n$. The utility function in the Median Game has the following form

$$u_i(x_i, x_{-i}, \theta_i) = \nu_i M(x_i, x_{-i}) - (x_i - \theta_i)^2.$$

We investigate the Median Game separately for even and uneven number of players and start with the uneven case. An example for $n = 3$ is shown in figure 3.1 in the appendix. The following two important observations will simplify the further analysis enormously.

Proposition 3.6. Suppose the number of players in the Median Game is uneven. In every Nash equilibrium it holds $x_i = \theta_i$ or $x_i = M(x)$.

Proof. We prove this by showing that all other strategies are strictly dominated. Suppose $x_i \neq \theta_i$ and $x_i \neq M(x)$. In a sufficiently small environment of x_i there exists $\tilde{x}_i$ such that $M(\tilde{x}_i, x_{-i}) = M(x)$ and $(\theta_i - \tilde{x}_i)^2 < (\theta_i - x_i)^2$. Thus the strategy $\tilde{x}_i$ strictly dominates x_i which implies that x_i is not played in any Nash equilibrium. $\square$

Corollary 3.7. *Suppose the number of players in the Median Game is uneven and $n \geq 3$. In every Nash equilibrium at least one player quotes truthfully.*

Proof. The only thing we have to show with reference to proposition 3.6 is that $\theta_i \neq x_i = M(x)$ for all i cannot be a Nash equilibrium. But this is immediate, as any player can profitably deviate to truth-telling. The median does not change but the penalty decreases. □

We now turn to the uneven case, for which an analogue to proposition 3.6 exists while the corollary 3.9 is not true as we show by means of an example.

Proposition 3.8. Suppose the number of players in the Median Game is even. In every Nash equilibrium it holds $x_i = \theta_i$ or $x_i \in \{x^{(\frac{n}{2})}, x^{(\frac{n+2}{2})}\}$

Proof. The proof is almost a copy of the proof of proposition 3.6. We show that all other strategies are strictly dominated. Suppose $x_i \neq \theta_i$ and $x_i \notin \{x^{(\frac{n}{2})}, x^{(\frac{n+2}{2})}\}$. In a sufficiently small environment of x_i there exists $\tilde{x}_i$ such that $M(\tilde{x}_i, x_{-i}) = M(x)$ and $(\theta_i - \tilde{x}_i)^2 < (\theta_i - x_i)^2$. Thus the strategy $\tilde{x}_i$ strictly dominates x_i which implies that x_i is not played in any Nash equilibrium. □

Comment 3.9. Suppose the number of players in the Median Game is even. There exist equilibria in which no player answers truthfully.

Proof. The proof is given by means of an example. For $n = 2$ we are back in the Average Game and players always misrepresent for financial exposure different from 0. Thus consider $n = 4$. The constellation for the example has to be such that two players have negative financial exposure while two others have a positive one. The negative financial exposure players have higher private costs than the positive financial exposure types. Hence, there exist parameter constellations in which the low private value types quote higher than the high private value types such that all misrepresent and no single player can unanimously deviate profitably. One parameter constellation that works is the following: $\theta_1 = \theta_2 = 1.4, \nu_1 = \nu_2 = 1, x_1 = x_2 = 1.6$, and $\theta_3 = \theta_4 = 1.6, \nu_3 = \nu_4 = -1, x_3 = x_4 = 1.4$. For $n > 4$ add identical numbers of identical players to the high and low type, respectively. $\square$

These findings facilitate the further characterization of Nash equilibria in the Median Game. In the appendix we consider the case $n = 3$ and characterize the equilibria for the heterogeneous private value, the heterogeneous financial exposure, and the general case. Here, we continue with the trimmed average game.

3.4.3 The Trimmed Average Game

The LIBOR is presently calculated as a trimmed average and not as a median or average. To discriminate between these three statistics, it is necessary to consider $n \geq 5$. For $n = 1$ and $n = 2$ all notions coincide. For $n = 3$ the trimmed average can be defined either as the average of all quotes or as the middle quote which is the median. Similarly for $n = 4$, the average of the two middle quotes is the median. However, $n = 5$ is the smallest number for which these three statistics are distinct. The statistic in the Trimmed Average Game is defined as $L(x) = \sum_{i=2}^{4} x^{(i)}/3$ in the $n = 5$ player game, where $x^{(i)}$ denotes the order statistic. We call this statistic $L(x)$ as this is closest to the original LIBOR calculation rule. See figure 3.2 in the appendix for the actual rule for $6 \leq n \leq 18$.

We start by presenting general findings for the Trimmed Average Game. They are close to the findings for the Median Game, in particular to the even case. Denote the smallest quote and the highest quote which are averaged in the trimmed average by $x^{(l)}$ and $x^{(u)}$, respectively.

Proposition 3.10. Suppose $n \geq 4$ is the number of players in the Trimmed Average Game. Then in every Nash equilibrium it holds $x_i = \theta_i$ or $x^{(l)} \leq x_i \leq x^{(u)}$.

Proof. With the obvious notation changes, the proof is an exact copy of the proof of proposition 3.8. □

Comment 3.11. Suppose $n \geq 4$. Then there exist equilibria in which all players misrepresent.

Proof. This is shown by means of an example. For $n = 4$, the trimmed average and median coincide and the example of comment 3.9 gives the result. For even n this example for the median also works for the trimmed average if we add identical high and low value players. For uneven $n \geq 5$ consider the example for $n - 1$ and add one identical high or low value player. This completes the proof of comment 3.11. □

The next proposition formulates a sufficient criterion for misrepresentation in equilibrium.

Proposition 3.12. If $\theta_i \in (x^{(l)}, x^{(u)})$, and $\nu_i \neq 0$, then $\theta_i \neq x_i$.

Proof. Assume $\nu_i < 0$. Suppose bank i quotes $x_i = \theta_i \in (x^{(l)}, x^{(u)})$. This quote gives utility $u_i(x_i, x_{-i}, \theta_i) = \nu_i L(\theta_i, x_{-i})$. Consider the quote $\tilde{x}_i = \theta_i - \epsilon$ for $\epsilon > 0$ sufficiently small such that $\tilde{x}_i \in (x^{(l)}, x^{(u)})$. This quote gives utility $u_i(\tilde{x}_i, x_{-i}, \theta_i) = \nu_i L(\tilde{x}_i, x_{-i}) - (\tilde{x}_i - \theta_i)^2$. Comparing the two utilities, it holds $\nu_i L(\tilde{x}_i, x_{-i}) > \nu_i L(\theta_i, x_{-i})$ and $-(\tilde{x}_i - \theta_i)^2 < 0$. As the first inequality depends linearly on the quote and the second quadratically, for sufficiently small ϵ it holds $\nu_i L(\tilde{x}_i, x_{-i}) - \nu_i L(\theta_i, x_{-i}) - (\tilde{x}_i - \theta_i)^2 > 0$. Thus, we have found a strategy which strictly dominates $x_i = \theta_i$. The case $\nu_i > 0$ is proven analogously. □

For sufficiently large exposure to the index a bank will quote $x^{(l)}$ or $x^{(u)}$, depending on the sign of the exposure.

Proposition 3.13. For sufficiently large exposure $|\nu_i|$ to the LIBOR a bank will always quote $x^{(l)}$ if $\nu_i < 0$ and $\theta_i \geq x^{(l)}$. Similarly, if $\nu_i > 0$ and $\theta_i \leq x^{(u)}$, a bank will always quote $x^{(u)}$ provided sufficiently large exposure to the LIBOR.

Proof. Suppose $\nu_i < 0$ and $\theta_i \geq x^{(l)}$. Quoting $x^{(l)}$ gives the utility $u_i(x^{(l)}, x_{-i}, \theta_i) = \nu_i L(x^{(l)}, x_{-i}) - (x^{(l)} - \theta_i)^2$. Thus, whether quoting $x^{(l)}$ strictly dominates any other quote $\hat{x}_i > x^{(l)}$ depends on the relation of the first and the second term in the utility function. For sufficiently large $|\nu_i|$ quoting $x^{(l)}$ strictly dominates any other quote. The case $\nu_i > 0$ and $\theta_i \leq x^{(u)}$ is proven analogously. □

We set $n = 5$ and prove that for all parameter constellations there exists an equilibrium in which at least one player misrepresents in a maximal way.[24] It is important to note that we say a player i who has exposure $\nu_i = 0$ misquotes maximally although i always quotes truthfully having a singleton as the interval of possible quotes.

Proposition 3.14. If $n = 5$, then for all θ and all ν there exists an equilibrium x^* in which at least one player i misquotes maximally, i.e. $x_i^* = \theta_i + \frac{\nu_i}{6}$.

Proof. Suppose there is no maximal misrepresentation in equilibrium. Consider without loss of generality the highest quote that is averaged in the index. If there were exactly one or five players quoting this same quote, players would have a profitable deviation. In case of five players, deviation to truth-telling would be profitable, in case of one player deviation in the

[24]Maximal in the sense of proposition 3.15.

direction of maximal misrepresentation would be profitable. Thus at the highest averaged quote between two and four players quote the same.

Suppose first the highest averaged quote is the highest quote in total. This implies that none of the two to four players quoting this quote has negative financial exposure, all have positive financial exposure. Consider the minimum of the maximal misrepresentation of the players who quote the highest. This becomes an equilibrium if all players who quote the same highest quote deviate to quoting this minimum.

Suppose now that there is a single highest quote. The two to four players who quote the highest averaged quote then all have negative financial exposure. If the quote of two to three players is the highest that is averaged, they have a profitable deviation to a lower quote. If the four players quote the highest averaged quote, this is the lowest quote in total. Consider the maximum of the maximal misrepresentations of these players. This becomes an equilibrium if all players who quote the same lowest quote deviate to quoting this maximum. □

Proposition 3.14 also follows from the general theorem 3.16 in the next subsection in which the existence of a maximal misrepresentation equilibrium in the general Median Game and Trimmed Average Game is shown.

3.4.4 Comparison of the games

The goal of this subsection is to compare the equilibrium sets of the three games with a focus on the magnitude of possible equilibrium misrepresentation. The marginal impact of an individual quote of a player in the Median, the Trimmed Average and the Average Game is different. This fact allows us to prove a result which gives a bound on equilibrium misrepresentations in the distinct games. Denote by $m \leq n$ the number of quotes averaged in the Trimmed Average Game.

Proposition 3.15. The maximal equilibrium misrepresentation of player i is $\frac{\nu_i}{2}$ in the Median Game, $\frac{\nu_i}{2m}$ in the Trimmed Average Game, and $\frac{\nu_i}{2n}$ in the Average Game.

Proof. It is sufficient to note that the equilibrium misrepresentation in the Median, Trimmed Average and Average Game is bounded by the potential marginal influence of a single quote. □

Two things are worth mentioning. First, in the Average Game the equilibrium is unique and players always quote $\theta_i + \nu_i/2n$. Second, for all parameter constellations of private values θ and financial exposure ν there exists not only an equilibrium but in particular an equilibrium in which at least one player misquotes maximally. This is clear for the Average Game and follows for the Median and the Trimmed Average Game from theorem 3.16. The reasoning in the proof depends neither on the number of players n in the game nor the number m of quotes which are averaged.

Theorem 3.16. *In the Median Game, the Trimmed Average Game and the Average Game there exists for all θ and all ν an equilibrium in which at least one player misrepresents in a maximal way.*

Proof. Consider the vector of maximal misrepresentations and order it from high to low. If the maximal misquote x_i enters the calculation of the statistic, quote x_i. If not, change the quote in the direction of θ_i. If the intersection of the interval of possible misrepresentations with the interval of counting quotes is empty, quote θ_i. Else misquote maximally under the constraint that the quote enters the calculation of the statistic. Suppose all players follow this quote strategy. Then this defines a Nash equilibrium in the Average Game, the Trimmed Average Game and the Median Game. □

The median rule increases the marginal impact in comparison with the other two statistics. Moreover, the median rule leads to a more volatile index. By contrast, the average rule has the property of being sensitive to extreme quotes, which may turn out to be a disadvantage in practice. In this respect the trimmed average can be seen as a reasonable compromise between the other, more extreme, statistics. This is in line with the arguments of Wheatley (2012a). However, statistical analysis by Eisl et al. (2013) points out that the median rule would have been less manipulable than the trimmed average between 2006 and 2012. This difference to the results of our model is probably due to two reasons. First, our informational assumption that banks know perfectly well the private values of all banks is not satisfied in reality. Second, the Nash equilibria in the Trimmed Average Game and in the Median Game are difficult to play but the Nash equilibria in the Median Game are relatively more difficult to play. The latter observation calls for a measure of how difficult an equilibrium is to play. Such a measure does not exist so far. An interesting next task could be to design and perform experiments in the lab to examine which equilibria are actually played in the three related games.

3.5 The LIBOR Mechanism from the normative point of view

The LIBOR scandal has made headlines throughout the world in the last months. In this paper we try to contribute to the understanding of how the banks actually manipulated the LIBOR and what modification possibilities of the LIBOR mechanism exist to make the mechanism more robust against manipulation. Thus, we have considered the LIBOR problem from the descriptive point of view. We have modeled the LIBOR fixing process as a strategic game with rational banks that individually maximize their expected utility. We have analyzed several games which are closely related to the actual LIBOR setting process. Concerning our model we emphasize that it is a simplification of the actual LIBOR setting process. We quantified the financial and reputational misreporting incentives and a penalty function for misrepresentation in an adhoc way. Furthermore, our banks are extremely well informed as we assume that the knowledge of θ is common knowledge. In reality, the information distribution between the banks and the regulators is more diffuse and the maximization problem of the individual banks is not common knowledge. This makes the LIBOR problem even harder to solve.

The next step is now to take the normative perspective and ask: "What is the right thing to do?" Our modeling approach and our results not only illuminate how the banks were able to manipulate the LIBOR, they also have normative implications. Our results make the case for several changes of the current process. The LIBOR panels should be increased whenever possible to lower the marginal impact of a single contributing bank. There are however practical problems for increasing panel bank sizes as the number of global player banks who have the necessary information to quote is limited. The LIBOR quotes of the banks should be made temporarily anonymous for two reasons. First, to prevent the quotes from being an indicator of current financial strength of the banks. Second, the immediate publication may serve the banks as a collusion device. Nevertheless, the quotes should be eventually published with delay to enable identification of financial misrepresentation. This change has been implemented in July 2013 and quotes are now published with three months delay. Furthermore, the LIBOR quotes should be based on real transactions whenever possible. To enable this, a database for actual interbank lending has to be established first. Our results do not entail a change from the trimmed mean to the median as the equilibrium misrepresentation in the Median Game can be higher than in the Trimmed Average Game due to the higher marginal influence on the fixing. A switch from a trimmed average to the average is not robust to outliers and thus also problematic to introduce, although the average induces a lower marginal influence of a single bank.

The majority of these modification ideas supported by our model have already been suggested in the Wheatley Review. However, these changes can only attenuate the LIBOR problem but not qualitatively solve it. Maybe the most important difficulty is that the optimization problem of the banks is not known precisely because the utility function of the banks is not known. If it were possible to find out the precise utility function by data analysis, estimations, or reading the balance sheets of the banks, the theory of mechanism design would be able to help solving the LIBOR problem by counterbalancing misrepresentation incentives. In the Average Game the Groves mechanism would solve it. However, the financial exposure to the index, private costs and the penalty function for deviation from the private costs are subject to change and unknown. The main problem remains:

There are huge financial incentives to misrepresent.

We expect the market to move away from this interest rate benchmark. In which directions the markets will evolve and what eventually may take the place of the LIBOR is hard to tell.

However, due to the immense importance of the LIBOR rate for the financial system in general and individuals all over the world in particular it remains an important task of theoretical and practical economists to come up with ideas to guarantee that the LIBOR actually represents what it is supposed to represent or to come up with a sensible alternative.

3.6 Appendix

3.6.1 The three player Median Game

Heterogeneous costs

In this subsection, we assume $n = 3$ and that all players have exactly the same financial exposure $\nu_i = \nu_j = \nu_k = \nu$ to the median index. We set $\nu \geq 0$, the case $\nu \leq 0$ is solved similarly. However, the private values of the players are possibly different. We investigate the four arising cases: $1.\theta_i = \theta_j = \theta_k, 2.\theta_i = \theta_j < \theta_k, 3.\theta_i < \theta_j = \theta_k$, and $4.\theta_i < \theta_j < \theta_k$. We start with the easiest one, that of identical values for all three players.

$1.\theta_i = \theta_j = \theta_k = \theta$:
In the case of identical costs for all agents, exactly two different kinds of equilibria arise: the truth-telling equilibrium and an equilibrium in which one player quotes honestly and the other two quote the same, but not the

truth.

Proposition 3.17. If $\theta_i = \theta_j = \theta_k = \theta$, then for any equilibrium it holds that either $M(x) = x_l = \theta$ for all $l = i, j, k$ or $\exists l \in \{i, j, k\}$ such that $\theta = x_l$ and $M(x) = x_m = x_n \neq \theta$ for $m, n \neq l$. In the second type of equilibrium the dishonest players quote at most $M(x) = x_m = \frac{\nu}{2} + \theta$.

Proof. If all players quote honestly, there is no possibility to unilaterally deviate profitably. Deviation in the other equilibrium is not profitable for the honest player i as the median would not change and the penalty for lying is already minimal. For the other two players j and k a deviation is not profitable if their quote x_j is between θ and the maximizer of $\nu x_j - (x_j - \theta)^2$, thus if $x_j \leq \frac{\nu}{2} + \theta$. If $x_j > \frac{\nu}{2} + \theta$, then the deviation to $x_j = \frac{\nu}{2} + \theta$ will be profitable. Thus both type of quotation patterns constitute Nash equilibria. It remains to show that these are all Nash equilibria. Suppose $x_i < x_j < x_k$. Then either i or k can profitably deviate to quoting x_j. $\square$

$2.\theta_i < \theta_j = \theta_k$:
The same two types of equilibria arise as in the identical costs case.

Proposition 3.18. If $\theta_i < \theta_j = \theta_k$, then in any equilibrium it holds that either $x_i = \theta_i$ and $M(x) = x_j = x_k = \theta_j$ or else there exist players $m, n \in \{i, j, k\}$ such that $M(x) = x_m = x_n > \theta_k$ and $x_l = \theta_l$. In the second type of equilibrium, player i quotes at most $M(x) = x_i = \frac{\nu}{2} + \theta_i$ while the other two quote at most $x_j = \frac{\nu}{2} + \theta_j$. It holds for the nontruthful quote $x_j > \theta_j$. In particular, if $\frac{\nu}{2} + \theta_i < \theta_j$, player i quotes always truthfully.

Proof. The truth equilibrium exists as there is no possibility to unilaterally deviate profitably. In the other equilibrium the two players who quote the same control the median. As the truthful quote is below the quote of the two equal quoting players, one of the two equal quoting players can only change the median by deviating to a lower quote, which is in the specified intervals not profitable. Higher quotation would only induce a higher penalty. □

$3.\theta_i = \theta_j < \theta_k$:

In this situation, the truth equilibrium ceases to exist. Moreover, depending on the true cost structure, new types of equilibria may appear.

Proposition 3.19. If $\frac{\nu}{2} + \theta_i < \theta_k$, there exists only the equilibrium type in which i or j quotes the truth, k quotes the truth and the other i or j sets the median by quoting $M(x) = x_i = \theta_i + \frac{\nu}{2}$. If $\frac{\nu}{2} + \theta_i \geq \theta_k$, there exists only the equilibrium type in which one player quotes the truth while the other two players quote $M(x) = x_l \in [\theta_k, \frac{\nu}{2} + \theta_i]$.

Proof. The truth equilibrium does not exist as i and j could both profitably deviate by quoting higher. If $\frac{\nu}{2} + \theta_i < \theta_k$, the nontruthful player sets the median by its optimal quote $x_i = \theta_i + \frac{\nu}{2}$. If $\frac{\nu}{2} + \theta_i \geq \theta_k$, the deviation of the nontruthful player falls in the domain where the high cost player also can deviate. By quoting the same and thereby controlling the median, no individual player would like to deviate. The low quoter cannot affect the median, and the higher quoters quote in the area where a deviation to a lower quote would not be advantageous. □

$4.\theta_i < \theta_j < \theta_k$:

In the case where all players face different individual costs, the equilibrium structure is similar to the one of case 3.

Proposition 3.20. If $\theta_j + \frac{\nu}{2} \leq \theta_k$, then there only exists the equilibrium in which $\theta_i = x_i, \theta_k = x_k$ and $M(x) = x_j = \theta_j + \frac{\nu}{2}$. If $\theta_j + \frac{\nu}{2} > \theta_k$, then there exists the equilibrium in which $\theta_i = x_i$ and $M(x) = x_j = x_k \in [\theta_k, \theta_j + \frac{\nu}{2}]$.

If additionally $\theta_i + \frac{\nu}{2} \geq \theta_k$ then there exists an equilibrium, in which players i and j or k quote the same $M(x) = x_i \in [\theta_k, \theta_i + \frac{\nu}{2}]$ and the other player j or k quotes the truth.

Proof. The truth equilibrium does not exist as the median cost player can always deviate profitably. The range of possible deviations from the truth depends on the underlying cost structure. With similar reasoning as in the proof of proposition 3.19 one proves that the given quote patterns in fact represent equilibria. □

These four propositions completely characterize the set of pure Nash equilibria in the Median game in the three player cases, where players have identical financial exposure but heterogeneous costs. Depending on the cost constellation, three different types of equilibria arise. The truth-telling equilibrium, an equilibrium where two players control the median and the third quotes truthfully, and an equilibrium in which the top and bottom cost players quote truthfully while the middle cost player deviates in a maximal sense. Interestingly, the second and the third equilibrium type are mutually exclusive. Moreover, the equilibrium set in the second type of equilibrium is indeterminate.

Indeterminacy of equilibrium sets is also a result we get in the next subsection, in which we consider heterogeneous exposure but identical private values.

Heterogeneous exposure

In this subsection, we continue to assume $n = 3$ and $\theta_i = \theta_j = \theta_k = \theta \geq 0$. However, we now allow heterogeneity concerning the financial exposure of

the players. We investigate the four different cases:
1. $0 \leq \nu_i \leq \nu_j \leq \nu_k$, 2. $\nu_i \leq 0 \leq \nu_j \leq \nu_k$, 3. $\nu_i \leq \nu_j \leq 0 \leq \nu_k$, and 4. $\nu_i \leq \nu_j \leq \nu_k \leq 0$. Cases 2 and 3 are the interesting ones, the cases 1 and 4 with exclusively positive or negative financial exposure are similar to the results of the preceding subsection. Basically one has to add indices to the financial exposure. We summarize the findings for the four different cases of financial exposure to the median in the following proposition which characterizes the equilibria. Case 1. corresponds to $-$case 4. and case 2. to $-$case 3.

Proposition 3.21. 1.If $\theta_i = \theta_j = \theta_k = \theta$ and $0 \leq \nu_i \leq \nu_j \leq \nu_k$, the only equilibria are the truth-telling equilibrium and the equilibrium in which one player quotes truthfully and two players lie. In the second type of equilibrium the dishonest players m and n quote $M(x) = x \in \min_{s\in\{m,n\}}[\theta, \frac{\nu_s}{2} + \theta]$.
2. If $\theta_i = \theta_j = \theta_k = \theta$ and $\nu_i \leq 0 \leq \nu_j \leq \nu_k$, the only equilibria are the truth-telling equilibrium and the equilibrium in which i quotes the truth $x_i = \theta$ and j, k quote $M(x) = x_j = x_k \in [\theta, \frac{\nu_j}{2} + \theta]$.
3. If $\theta_i = \theta_j = \theta_k = \theta$ and $\nu_i \leq \nu_j \leq 0 \leq \nu_k$, the only equilibria are the truth-telling equilibrium and the equilibrium in which k quotes the truth $x_k = \theta$ and j, k quote $M(x) = x_j = x_k \in [\frac{\nu_j}{2} + \theta, \theta]$.
4. If $\theta_i = \theta_j = \theta_k = \theta$ and $\nu_i \leq \nu_j \leq \nu_k \leq 0$, the only equilibria are the truth-telling equilibrium and the equilibrium in which one player quotes truthfully and two players lie. In the second type of equilibrium the dishonest players m and n quote $M(x) = x \in \max_{s\in\{m,n\}}[\frac{\nu_s}{2} + \theta, \theta]$.

Proof. As all players face the same costs and an individual cannot unilaterally change the index when all three players quote the same, truth is in all four cases an equilibrium.

1. and 4.: The maximal misrepresentation of an individual agent l is $\nu_l/2$,

if the exposure to the median is positive for all agents. This explains the minimum in 1. The maximal misrepresentation of an individual agent l in the negative exposure case is $\nu_l/2$. This leads to the maximum in 4.

2. and 3.: The direction of possible misrepresentation of two agents is the same if and only if the financial exposure of both agents has the same sign. Thus, only players who both have positive or negative exposure can misrepresent in the same direction in equilibrium. The interval of possible equilibrium misrepresentations is determined by the player with the smaller absolute exposure which is player j. □

Thus, we have, as proposition 3.17 suggested, only two types of equilibria: The truth equilibrium and the indeterminate set of equilibria. We now turn to the general characterization of the equilibria in the Median Game.

General case

In this subsection, we characterize the equilibria in the Median Game in the general case. We thus allow for heterogeneous exposure and heterogeneous costs. A case distinction is now made with respect to the quotes and not with respect to the private values and financial exposure as in the preceding subsections. Decisive in the proof will be to compare the regions of possible misrepresentations of agents. Formally, the *interval of possible misrepresentations* I_i of player i is the region in which player i quotes in equilibrium. Equivalently, one may call it *interval of possible quotes*. It depends on the exposure ν_i and the costs c_i. The following lemma characterizes it.

Lemma 3.22. *The interval of possible misrepresentations* I_i *is given by* $I_i = [\theta_i, \theta_i + \frac{\nu_i}{2}]$

Proof. Suppose $\nu_i \geq 0$. Then player i will always quote at least θ_i and not more than $\theta_i + \frac{\nu_i}{2}$ as for higher quotes the quadratic detection costs always dominate financial exposure benefits by misquoting. The $\nu_i \leq 0$ case is proved similarly. □

The decisive trick for the complete characterization of the equilibria in the Median Game is to investigate all cases of possible overlappings between the intervals of possible misrepresentations.

Proposition 3.23. Suppose $\theta_1 \leq \theta_2 \leq \theta_3$.
(i) If $M(x) = x_1 = x_2 = x_3$, then $x_1 = \theta_1 = \theta_2 = \theta_3$.
(ii) Suppose $x_i < M(x) = x_j < x_k$. Then $\text{int}(I_i) \not\ni x_j \notin \text{int}(I_k)$, $x_j = \theta_j + \frac{\nu_j}{2}$ and $x_i = \theta_i$, $x_k = \theta_k$.
(iii) Suppose $M(x) = x_i = x_j \neq x_k$. Then $x_k = \theta_k$. If $\text{sign}(\nu_i) = \text{sign}(\nu_j)$, then $x_i \in I_i \cap I_j$.
If $\nu_i < 0 < \nu_j$, then i and j both misrepresent in a maximal way.

Proof. (i) Suppose all quotes are the same. Then everybody quotes truthfully as otherwise there would be a profitable deviation to truth-telling for the nontruthful player.

(ii) Suppose all quotes are different in equilibrium. Then the top and bottom quoter are truthful, otherwise they would deviate in the direction of truth-telling. Moreover, the middle quoter misquotes maximally or else an ϵ change in the quote in the direction of financial exposure would be a profitable deviation. The middle quote cannot be in the interior of the maximal misrepresentation intervals of the two honest quoters as otherwise they would have a profitable deviation.

(iii) It holds $x_k = \theta_k$ as k cannot influence the index. If $\nu_i \geq 0 \leq \nu_j$

and $x_k \leq \min\{\theta_i, \theta_j\}$, any element of the intersection $I_i \cap I_j$ can be an equilibrium if both i and j quote it. If $x_k \geq \min\{\theta_i + \frac{\nu_i}{2}, \theta_j + \frac{\nu_j}{2}\}$ but $x_k \leq \theta_i + \frac{\nu_i}{2}$ or $x_k \leq \theta_j + \frac{\nu_j}{2}$, then the respective i or j would deviate to x_k, contradicting the assumption that $x_i = x_j$ as for the other player x_k lies outside of the interval of possible misrepresentations. Moreover, $x_i = \theta_i + \frac{\nu_i}{2} = \theta_j + \frac{\nu_j}{2}$. If $x_k \in I_i \cap I_j$, then $x_i = x_j > x_k$. The case $\nu_i \leq 0 \geq \nu_j$ is proven analogously.

If $\nu_i < 0 < \nu_j$, both i and j misquote maximally. Otherwise, each player had either a profitable deviation in the direction of private costs, if that would not change the median, or in the direction of maximal misrepresentation, if that would change the median. $\square$

It is important to note that we do not get qualitatively new equilibria in comparison to the preceding subsections. We do not explicitly show the existence of the equilibria in proposition 3.23 but we investigate all possible quote patterns of three players and which of these may qualify as equilibria. We again have three possible types of equilibria, the truthful equilibrium, the maximal misrepresentation of the middle quoter and truthful quotes of the top and bottom quoter, and the possibly indeterminate equilibrium type where two players quote the same and the third quotes truthfully. Examples of parameter constellations of the θ and ν vectors that lead to the existence of the respective equilibrium types have been given in the previous subsections of the appendix.

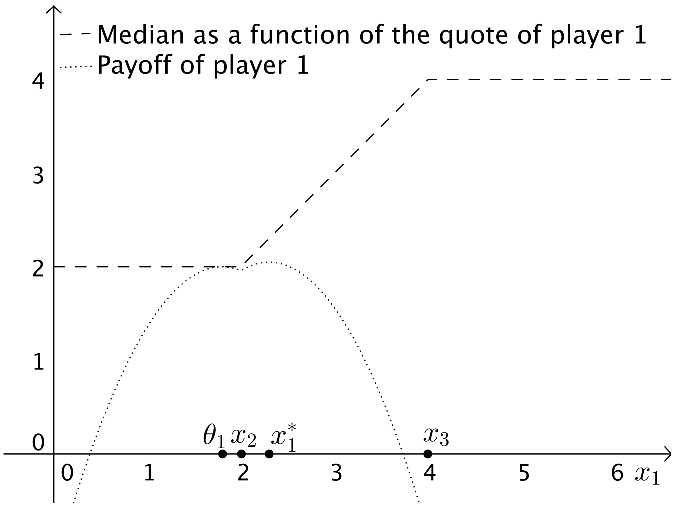


Figure 3.1: Example of payoff for player 1

3.6.2 Examples

On figure 3.1: The first axis is denoted x_1 for the quote of player 1 and the second axis gives the median. θ_1 is the private value of player 1 and x_2, x_3 the quotes of players $2, 3$. The dotted line gives the payoff of player 1 as a function of player 1's quote and the broken line the median as a function of player 1's quote. x_1^* maximizes the payoff of player 1.

No. of Contributors	Methodology	No. of Contributor rates averaged
18 Contributors	top 4 highest rates, tail 4 lowest rates	10
17 Contributors	top 4 highest rates, tail 4 lowest rates	9
16 Contributors	top 4 highest rates, tail 4 lowest rates	8
15 Contributors	top 4 highest rates, tail 4 lowest rates	7
14 Contributors	top 3 highest rates, tail 3 lowest rates	8
13 Contributors	top 3 highest rates, tail 3 lowest rates	7
12 Contributors	top 3 highest rates, tail 3 lowest rates	6
11 Contributors	top 3 highest rates, tail 3 lowest rates	5
10 Contributors	top 2 highest rates, tail 2 lowest rates	6
9 Contributors	top 2 highest rates, tail 2 lowest rates	5
8 Contributors	top 2 highest rates, tail 2 lowest rates	4
7 Contributors	top highest rate, tail lowest rate	5
6 Contributors	top highest rate, tail lowest rate	4

Figure 3.2: Trimming rule of LIBOR

Player	1	2	3	4	5	6	7	Median	Trimmed Average	Average
Private value	1	1.1	1.2	1.3	1.4	1.5	1.6	1.3	1.3	1.3
Quote	1	1.5	1.2	1.3	1.4	1.5	1.6	1.4	1.38	1.357

Figure 3.3: Example for influence of quotes

On figure 3.3: Seven players, the middle five quotes are averaged in the trimmed average. The median changes as the misrepresentation of player 2 changes the order statistic.

4 Model Uncertainty in Insurance Markets

Abstract

We investigate the equilibrium set in competitive insurance markets under model uncertainty. For this end, we consider different modifications and generalizations of the classical model of Rothschild and Stiglitz (1976). The model uncertainty can enter on the individuals' side or on the side of the insurers. Instead of knowing the correct probability that an accident occurs, model uncertainty entails that only an interval of probabilities is known to either insurers or individuals. Depending on the risk aversion, the probability set and the attitude towards model uncertainty, the equilibrium set changes. In particular, a breakdown of the insurance market may occur. Introducing ambiguity about the fraction of high risk individuals paired with ambiguity aversion of the insurers provides a solution to the Rothschild-Stiglitz equilibrium puzzle provided ambiguity is large enough.

4.1 Introduction

The seminal contribution by Rothschild and Stiglitz (1976) is probably one of the most important and influential papers in the theory of economics. It has led to the consensus that the modelling of information structures is important as equilibrium outcomes may depend qualitatively on the distribution of information. Rothschild and Stiglitz investigate a competitive insurance market and compare the equilibrium outcomes when information is symmetric between insurers and individuals but insurance companies cannot discriminate among agents having a high or a low accident probability. They come to three principal conclusions. First, imperfect information makes the competition on markets more complex than in standard models with perfect information. Second, equilibria may not exist and third, competitive equilibria may not be Pareto optimal.

In our paper, we first consider the benchmark model from Rothschild and Stiglitz with a homogeneous group of individuals and introduce model uncertainty. The model uncertainty can either enter on the side of the individuals or on the side of the insurers. This modification induces qualitative changes in the results. We show that, depending on the risk attitude, the model uncertainty, and the model uncertainty attitude, the equilibrium set may change. In particular, underinsurance and a breakdown of the insurance market may happen. We then consider the heterogeneous population case of Rothschild and Stiglitz but introduce model uncertainty about the fraction of high and low risk individuals. If insurers act in an ambiguity averse way as suggested by Gilboa and Schmeidler (1989), then this yields a new solution to the Rothschild-Stiglitz equilibrium puzzle, provided model uncertainty about the fraction is high enough.

The inclusion of model uncertainty stems from ambiguity theory which distinguishes between risk and uncertainty. This distinction goes back to Knight (1921). In his view, risk is characterized by randomness that is precisely measurable, contrary to ambiguity or (Knightian) uncertainty. Ellsberg (1961) differentiated in a more precise way between risk und uncertainty. An event is risky if it has known probability and else it is uncertain. Following Knight and Ellsberg decision theory started in the late 1980's to develop models which allow for this kind of distinction. The axiomatizations by Gilboa and Schmeidler, Bewley (2002) and Schmeidler (1989) were the most influential. With the decision theory at hand, the next task is to apply the theory in different fields of economics like game theory, general equilibrium theory, finance or macroeconomics. For a survey of these applications see Mukerji and Tallon (2004). While interesting findings have been made and some empirical and experimental phenomena are now better understood by using ambiguity theory, from our point of view the understanding of insurance markets under ambiguity is not sufficiently good. In the insurance market context we rather speak of model uncertainty than ambiguity as agents do not precisely know which model is the correct one. For more background on model uncertainty, see also Hansen and Sargent (2008). Two recent contributions which apply Bewley's model and which are related to insurance markets are those of Rigotti and Shannon (2005) and of Dana and Riedel (2013). We consider the introduction of model uncertainty in the insurance market to be natural. Moreover, as we hope our paper shows, it is a fruitful approach with clear results that may shed some light on the mechanisms behind some of the problems on the financial markets in the ongoing financial and economic crisis.

In our variant of the benchmark model of Rothschild and Stiglitz with a homogeneous group of individuals, an individual has a fixed income but will suffer an accident with some probability. She can insure against the accident by buying an insurance on a competitive insurance market. We introduce model uncertainty on either the side of the individuals or the insurers. It is of course a rather bald assumption to assume that model uncertainty exclusively concerns one of the two groups in our model. Nevertheless, this is just an expression of assuming that one of the two groups is better informed than the other and, depending on the example one is considering, it may be natural to assume that insurers or individuals are better informed about the accident probabilities of the individuals. For car insurance, there exists big and extensive data, hence insurers have a very good estimation of the true probabilities. In other circumstances such as insurance of accidents related to nuclear power plants, to give an extreme example, there is hardly any data and insurers are worse informed than the individuals asking for insurance. For a discussion of this point see also for example Jeleva and Villeneuve (2004).

In section 4.3 we start with introducing model uncertainty on the insurer side. This corresponds to a situation where insurers do not have a lot of information and data to assess the riskiness. Individuals are better informed and know their idiosyncratic probability of accident. Insurers only have an idea about the true probability and reckon with an interval of accident probabilities for the individuals. We assume the insurers to be maxmin expected utility maximizers as suggested by Gilboa and Schmeidler (1989), i.e. insurance companies are ambiguity averse. They design their contracts with respect to the prior which would induce the highest costs and the lowest payoff for the insurers in case of accident. However, important to note is that insurers do not have a priori a wrong belief about the accident probability of the individuals. The reaction of ambiguity aversion towards the model uncertainty leads to underinsurance of the individuals and to a drying up of the insurance market. It may even lead to a complete breakdown of the insurance market provided sufficiently low risk aversion of the individuals or sufficiently high costs of insurance contracts.

In section 4.4, the insurers know the average accident probability of the large group of individuals and only care about this average. On the other hand, individuals are not perfectly informed about their idiosyncratic chance of accident, they only know an interval in which the true probability lies. This may correspond to car insurance where insurers have good data to assess the riskiness and calculate the premia of the insurance police. We consider the case where agents are overconfident in own abilities, a phenomenon which has been extensively investigated in economics and psychology, notably by Svenson (1981) and Heath and Tversky (1991). Also Kahneman and Tversky (1996) and Sharot (2011) are important references in this context. In our model agents are assumed to show an extreme kind of overconfidence and take the prior with the lowest accident probability to calculate the demand for insurance. We dwell on the assumption of overconfidence in 4.4.1. For the equilibrium set, the implications of overconfidence lead again to a drying up of the insurance market with possible breakdown for sufficiently strong optimism and sufficiently low risk aversion. However, in this case it may become rational for the individuals to increase their risk. This happens if their overconfidence overcompensates their risk aversion.

In section 4.4 we moreover examine the case where individuals are not overconfident but use incomplete preferences as in Bewley (2002). As the intuition suggests, this leads to indeterminacy of equilibria. Refining the equilibrium concept with the inertia property induces a status quo bias for the individuals. As soon as the uninsured state becomes an equilibrium, the inertia refinement yields uniqueness of this equilibrium.

In section 4.5 we investigate a variant of the Rothschild and Stiglitz model with a heterogeneous group of individuals. In the original Rothschild and Stiglitz paper insurers cannot discriminate between high and low risk types. However, they know the fraction of high risk types in the population. We introduce model uncertainty about this fraction. The insurers are assumed to act in a way towards this model uncertainty as suggested by Gilboa and Schmeidler (1989). Thus, they make a worst case approach. We show that for sufficiently large model uncertainty there always exists an equilibrium. Thus we present a solution for the Rothschild-Stiglitz equilibrium puzzle.

RS is one of the mostly cited papers in economic theory. Many authors have for example attempted to establish conditions that guarantee the existence of equilibrium in the RS model like we do in section 4.5. A recent example for this is Picard (2009). Conditions for underinsurance and breakdown of the insurance market are also well known. If the insurers take prices that are not fair, i.e. if the insurance market is not competitive, rational individuals will only insure partially or not at all. An interesting paper in this context is Jeleva and Villeneuve (2004) who take Yaari (1987) as a starting point to depart from RS. The paper most related to our work is Koufopoulos and Kozhan (2012). They introduce ambiguity about the accident probability on the side of the individuals when there is a heterogeneous population with a known fraction of high and low risk individiduals. They present new existence results under conditions on the probability intervals and examine the efficiency of these equilibria. To the contrary, we investigate ambiguity about the fraction of high risk individuals in the heterogeneous population case.

The structure of this paper is as follows. In the next section, we lay out the benchmark model of Rothschild and Stiglitz (1976). Then we introduce model uncertainty first on the side of the insurers and after that on the side of the individuals. In section 4.5 we suggest a solution to the equilibrium puzzle by assuming ambiguity about the distribution of the population and ambiguity averse insurers. In section 4.6, we calculate a number of examples for individuals that use CARA utility of degree 1. We discuss parametrizations that lead to underinsurance, breakdown of the insurance market, risk increase, indeterminacy of the equilibrium set and uniqueness due to the inertia refinement. Then we conclude by relating to real world phenomena and making an important connection to the ongoing financial crisis. The legend of figures is presented in the appendix.

4.2 The Rothschild-Stiglitz Model

We first take the model of Rothschild and Stiglitz (1976) with a homogeneous population.[1] Thus, we analyze the following simple example. There is an individual whose income in the next period is a random variable. Either she will get W or $W - L$. We interpret W to be the income if there is no accident and $W - L$ the income if there is an accident. Thus, L is the loss for the individual due to the accident. The individual has the opportunity to insure herself with an insurance contract. This contract costs a premium of α^1. In return, the insurance company pays the amount of $\tilde{\alpha}^2$ if there really is an accident. With the contract, the individual alters her pattern of income across the states "no accident" and "accident" such that the income vector becomes $(W-\alpha^1, W-L+\alpha^2)$, with $\alpha^2 = \tilde{\alpha}^2-\alpha^1$. Without insurance the income is $(W, W - L)$ with the first entry for the state "no accident". The vector $\alpha = (\alpha^1, \alpha^2)$ completely describes the insurance contract.

[1] As long as the group of individuals is homogeneous, we sometimes write the individual and not always the individuals as all individuals are equal in all respects.

4.2.1 Demand for Insurance Contracts

Let now W^1 denote the income if there is no accident and W^2 the income if there is an accident. The group of individuals that may buy insurance is completely identical in all respects. We assume the preferences of the agents can be described by the function

$$\tilde{V}(p, W^1, W^2) = (1-p)U(W^1) + pU(W^2), \tag{4.1}$$

where $U(\ )$ represents the utility of income and p the likelihood of an accident. We can express the worth of a contract α for the individuals in the following way

$$V(p, \alpha) = \tilde{V}(p, W - \alpha^1, W - L + \alpha^2).$$

The individual chooses the contract that maximizes her expected utility $V(p, \alpha)$. She will only buy an insurance contract α if it outperforms no insurance, thus the condition $V(p, \alpha) \geq V(p, 0) = \tilde{V}(p, W, W - L)$ has to be satisfied. We assume all individuals to be identical in all respects. In particular, they all know their common accident probability p and are risk averse, $U'' < 0$. We call this model the benchmark model. We will depart from the benchmark model in sections 4.3 and 4.4 by introducing model uncertainty, either on the insurer side or on the side of the individuals. In their paper, Rothschild and Stiglitz then consider a heterogeneous group of agents, one with high accident probability, one with low. While the individual knows her idiosyncratic probability of accident, the insurers cannot discriminate among the agents. In section 4.5 we will consider the heterogeneous group of individuals between which the insurers cannot discriminate and additionally assume ambiguity about the fraction of high risk individuals.

4.2.2 Supply for Insurance Contracts

The worth of a contract α for the insurers is

$$\pi(p,\alpha) = \left[(1-p)\alpha^1 - p\alpha^2\right] = \alpha^1 - p(\alpha^1 + \alpha^2). \tag{4.2}$$

The insurance companies maximize their expected utility. We assume competitiveness of the market. Thus, there is no barrier to entry and $\pi(p,\alpha) = 0$.

4.2.3 Equilibria

We now define the notion of equilibrium that we use.

Definition 4.1. [2] Equilibrium in a competitive insurance market is a set of contracts such that, when customers choose a contract by maximizing their expected utility, the following two conditions are satisfied:
(i) No contract in the equilibrium set makes negative expected profits,
(ii) There does not exist a contract outside the equilibrium set that makes nonnegative expected profits.

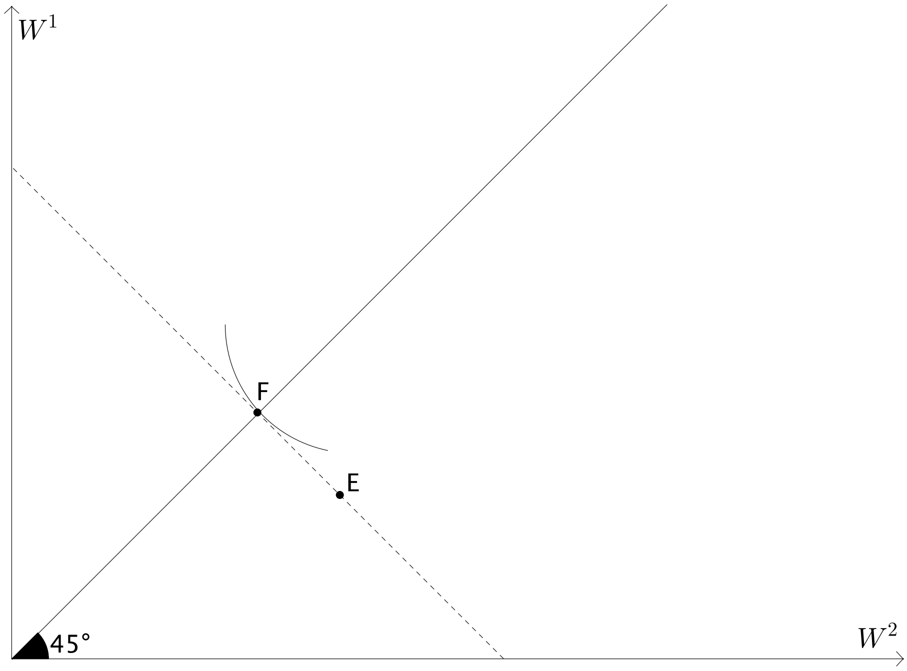
With the definition of equilibrium and the description of the demand and supply functions, we are now ready to analyze which equilibria can possibly arise in our setup.

We analyze the benchmark case mainly graphically through figure 4.1.

The first and the second axis are denoted by W^1 and W^2 for the income in the first state "no accident" and the second state "accident". The individual

[2] Given this definition, an equilibrium is a contract or a set of contracts. As the equilibrium contract fixes the equilibrium income, we sometimes refer also to the income as equilibrium.

Figure 4.1: Benchmark Case



starts in point E with coordinates $E = (\hat{W}^1, \hat{W}^2)$, which is the uninsured state. It lies to the southeast of the 45 degree line as she has higher income if there is no accident. If the individual can buy a contract such that she moves income on the 45 degree line, she has the same income in both states. Indifference curves of the individual are level sets of the equation (4.1). A contract $\alpha = (\alpha^1, \alpha^2)$ moves the individual from E to $(\hat{W}^1 - \alpha^1, \hat{W}^2 + \alpha^2)$. In equilibrium, insurance companies will make expected profit of 0 due to the assumption of competitiveness. This means for a contract α:

$$\pi(p, \alpha) = (1-p)\alpha^1 - p\alpha^2 = 0. \tag{4.3}$$

This equation defines a line EF with slope $\frac{1-p}{p}$. We call the line fair-odds line. We allow the insurers not only to offer contracts with $\alpha^2 \in [0, 1]$, i.e. between E and F, but every contract in the upper right quadrant. Then the insurers offer every point on the line through E and F due to their risk neutrality. Thus, the risk averse individual will in equilibrium choose the contract on this line that maximizes her expected utility. The equilibrium contract is α^*, which moves the individual to the intersection of the fair-odds line and the 45 degree line, the point F. The contract is an equilibrium as it satisfies conditions (i) and (ii) in definition 4.1: It breaks even and any contract which would be preferred by the individual would lead to expected losses for the insurers. α^* alters the income pattern of the individual such that she gets the same income in both states. She is fully insured in equilibrium. The slope of the indifference curve is given by the marginal rate of substitution between the two states $[U'(W^1)(1-p)]/[U'(W^2)p]$. For equal income $W^1 = W^2$, this becomes $(1-p)/p$, which equals the slope of EF, independently of the actual U. The price for full insurance α^{1*} is exactly the expectation of the loss L.

This benchmark case can be seen as a special case of the sections 4.3 and 4.4 when the interval of probability degenerates and becomes a singleton.

4.3 Model Uncertainty on the Side of the Insurers

We begin by introducing model uncertainty on the side of the insurers. Instead of being informed about the true probabilities of the homogeneous group of individuals, insurers only know an interval of probabilities $\mathcal{P}$ in which the true probability p lies. In contrast, individuals know their idiosyncratic probability of accident.

4.3.1 Demand and Supply for Insurance

Individuals calculate their demand as in the benchmark model. However, on the supply side, important differences appear. We assume insurance companies are risk-neutral as in the benchmark model, but in addition assume ambiguity aversion as in Gilboa and Schmeidler (1989).[3] This is an important feature as the companies do not know the accident probabilities of the agents precisely. They only know that the true probability p is an element of the interval $\mathcal{P} = [\underline{p}, \overline{p}]$. Hence, the worth of a contract α for the insurers is

$$\pi(\mathcal{P}, \alpha) = \min_{p \in \mathcal{P}} \left[(1-p)\alpha^1 - p\alpha^2\right] = \min_{p \in \mathcal{P}} \left[\alpha^1 - p(\alpha^1 + \alpha^2)\right] = \alpha^1 - \overline{p}(\alpha^1 + \alpha^2), \tag{4.4}$$

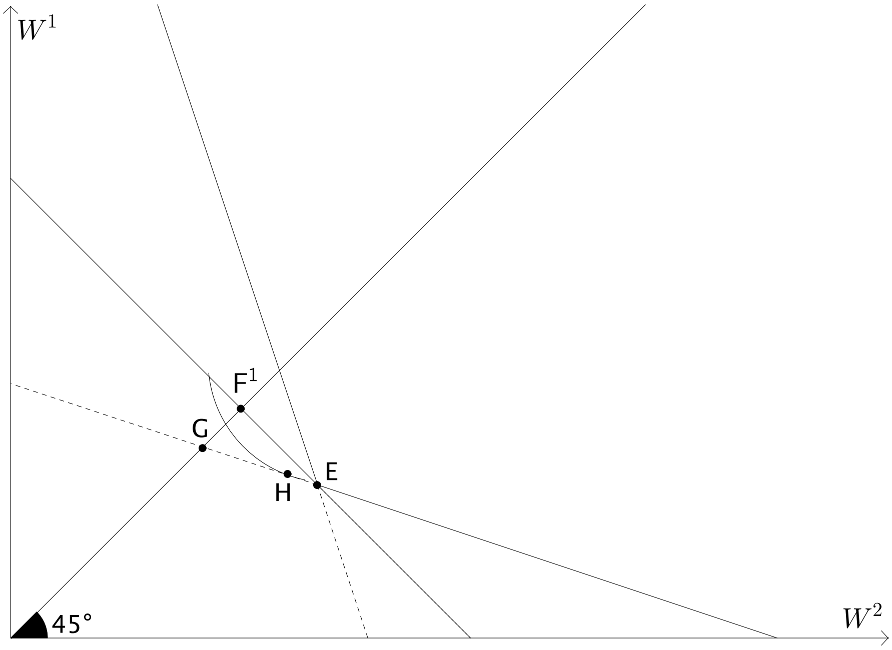
provided $\alpha^1 + \alpha^2 \geq 0$. For $\alpha^1 + \alpha^2 < 0$, the worst case changes and the worth of a contract α for the insurers is

$$\pi(\mathcal{P}, \alpha) = \min_{p \in \mathcal{P}} \left[(1-p)\alpha^1 - p\alpha^2\right] = \min_{p \in \mathcal{P}} \left[\alpha^1 - p(\alpha^1 + \alpha^2)\right] = \alpha^1 - \underline{p}(\alpha^1 + \alpha^2). \tag{4.5}$$

The insurance companies maximize their expected utility. Due to their ambiguity aversion, only the worst scenario with the highest accident probability $\overline{p}$ or the lowest accident probability $\underline{p}$ plays a role. This means that in the considerations of the individual and the companies different probabilities p, $\overline{p}$ and $\underline{p}$ possibly drive the demand and supply respectively.

[3] In our framework it makes perfect sense to suppose these risk and ambiguity attitudes. The risk goes away with a large number of agents and independence of the occurrence of accidents. However, the ambiguity stays also with a large number of individuals.

Figure 4.2: Partial Insurance

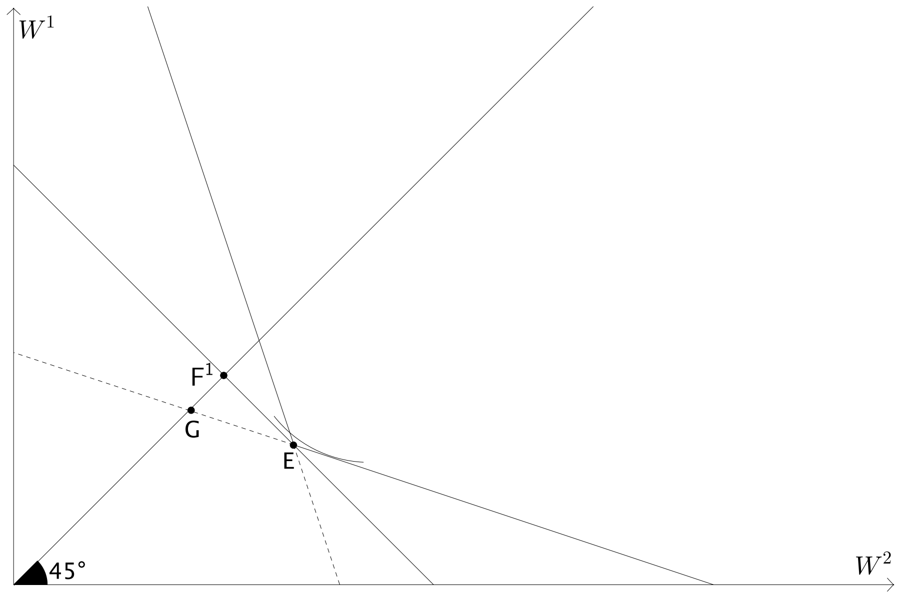


4.3.2 Equilibria

We now identify the equilibria in our model. We analyze this case again mainly graphically through figure 4.2: partial insurance, and figure 4.3: no insurance.

The point E with the coordinates $(\hat{W}_1, \hat{W}_2)$ is the uninsured state of the customer. Due to risk aversion, it lies southeast of the 45 degree line. The point F^1 is the intersection of the 45 degree line and the fair odds line from the benchmark case, i.e. the fair odds line of an insurer who calculates with p. The contract α^* that moves consumption from E to F^1 is the equilibrium in the benchmark case. For the ambiguity averse insurers, the condition $\alpha^1 - \overline{p}(\alpha^1 + \alpha^2) = 0$ has to be satisfied, north west of E. This is due to the assumption of competitiveness of the insurance market which guarantess free entry and perfect competition. If $p = \overline{p}$, the only equilibrium

Figure 4.3: No Insurance



is the full insurance equilibrium F^1, as in the benchmark case. The insurers take the worst scenario for the accident probabilities to design the contract, i.e. they calculate with the highest accident probability. This worst case turns out to be the true case for $p = \overline{p}$. If $p < \overline{p}$, consider the line EG, which has slope $(1-\overline{p})/\overline{p}$. The slope of EF^1 is $(1-p)/p$. For $p < \overline{p}$, we have $(1-\overline{p})/\overline{p} < (1-p)/p$. The insurers only offer contracts which lie on the line EG and to the north west of E. South east of E, the worst case scenario for the insurers change and the fair odds line has a kink in the point E.

One of the following two cases becomes now possible, depending on the degree of risk aversion. First, there is an indifference curve which is tangential to EG and second, there is no such curve.

For the first case, call the tangential point H. The contract α^{**} which moves consumption from E to H is a candidate for an equilibrium. This candidate is the only equilibrium candidate and in fact is an equilibrium as it satisfies the two conditions of equilibrium. First, any contract preferred to

α^{**} would bring the insurance companies expected losses as they calculate the expectation with $\overline{p}$. Second, the contract α^{**} breaks even, i.e. it satisfies (4.5).

In the second case, there is no tangential point of the indifference curve. This case happens for sufficiently low risk aversion and $p < \overline{p}$. Obviously, there can also be no tangential point to the right of the point E due to risk aversion of the individual. In our model, in the point E there is a kink of the fair-odds line. The worst case analysis leads to the kink in E, as the different prior with corresponding fair odds line with slope $(1-\underline{p})/\underline{p}$ becomes decisive. A comparison of the slopes leads to $(1-\overline{p})/\overline{p} < (1-\underline{p})/\underline{p}$. Hence, there is no tangential point of any indifference curve and the breakeven line of the insurers. In this case, the individual holds the initial position and does not buy an insurance contract. Full insurance would cost the individual $\max_{p\in\mathcal{P}} E^p L = E^{\overline{p}} L$.

We summarize the analysis in the following theorem before we interpret these findings economically.

Theorem 4.2. *(a) In comparison to the benchmark model, the insurance becomes more expensive, since the slope of the fair odds line becomes smaller.*
(b) Any point on the line EG between E and G may become an equilibrium, depending on the degree of risk aversion of the individual and on the prior set of the insurers.
(c) (i) For sufficiently low risk aversion, there does not exist a tangential point H on EG, provided $p < \overline{p}$.
(c) (ii) For sufficiently large $\overline{p} > p$, there does not exist a tangential point H on EG, provided risk aversion of the agent.
(d) (i) for fixed $p < \overline{p}$, the equilibrium moves to the east on the line EG with decreasing risk aversion

(d) (ii) for fixed risk aversion, the equilibrium moves to the east on the line EG for increasing $\overline{p}$.

(e) The full insurance price is $\max_{p \in \mathcal{P}} E^p L$.

4.3.3 Economic Interpretation

The fact that the insurers do not know precisely which accident probability the individual has leads to more expensive insurance contracts. Thus, the imprecise knowledge of the insurers about the true probability measure paired with the attitude of ambiguity aversion exerts a negative externality on the individuals. The companies design the contract with respect to the highest accident probability they consider possible for the agent. If for the highest probability $\overline{p}$ it holds $\overline{p} > p$, then the individuals cannot buy the contract which would enable them to alter their pattern of income to the point F^1, because the insurers do not offer this contract. The risk neutral firms only offer contracts that are on the line EG. Thus the individuals choose the point on that line which maximizes their expected utility, i.e. the point H where the indifference curve becomes a tangent. This may be any point on EG, as pointed out in theorem 4.2 (b). For sufficiently low risk aversion, there may be no tangential point on EG as theorem 4.2 (c) states. In the latter case, the individual does not alter her consumption pattern through insuring and stays with her uninsured initial state. Decisive for the location of the equilibrium is the trade off between risk aversion of the individuals and costs of the insurance contract, which depend on $\overline{p}$. Risk increasing behavior is trivially not rational for individuals due to their risk aversion. Finally we do comparative statics in (d). For fixed degree of risk aversion, the higher the difference between $\overline{p}$ and p, the more expensive the insurance contract becomes, the less insurance will the individual buy. This means that the equilibrium moves in the direction of E along the line EG. Similarly, for fixed $\overline{p} > p$, the lower the risk aversion of the individual, the less it pays to insure, the less insurance the individual buys.

4.4 Model Uncertainty on the Side of the Individuals

We now assume that the individuals face model uncertainty. In contrast to the individuals, insurers do not face model uncertainty. They are assumed to be informed about the average accident probability and to care only about this average. We investigate different plausible attitudes of the individuals towards the model uncertainty. The attitude of ambiguity aversion leads trivially to full insurance of individuals. However, we also examine the cases of overconfidence and incompleteness as reactions to model uncertainty. In case of overconfidence partial insurance, no insurance and even negative insurance may become equilibria, depending on the degree of risk aversion and the model uncertainty. If the agents have incomplete preferences, the equilibrium set may become indeterminate.

4.4.1 Overconfidence

We focus in subsection 4.4.1 on overconfidence as reaction of the individuals towards their imprecise knowledge of their idiosyncratic accident probability. Concerning own abilities and skills, individuals show overconfidence in many circumstances. Examples for this have for instance been investigated in the contributions by Svenson (1981) and Heath and Tversky (1991). In Svenson, car drivers are questioned about their driving skills and how safe they are driving. 70-80 percent of the subjects put themselves on the safer half of the distribution. Similar findings have been made in many related studies. Heath and Tversky investigate the competence hypothesis and come to the conclusion, that people prefer betting on their own judgment.

We take these empirical works as motivation for the assumption of overconfidence. In our case, overconfidence takes an extreme form. Individuals use for their choice of insurance the lowest personal accident probability $\underline{p}$. We opt for this extreme overconfidence for two reasons. First, it facilitates calculations and second, qualitatively decisive for our results will only be that agents choose a probability of accident for their calculations which is below the average probability. Our results are robust in this sense.

Demand and Supply for Insurance

The idiosyncratic accident probability $\tilde{p}$ of the individuals is an element of $\mathcal{P} = [\underline{p}, \overline{p}]$. The individuals are informed about $\mathcal{P}$ but not about $\tilde{p}$. Insurers know the average accident probability p. Decisive for the analysis is that all individuals have the same set $\mathcal{P}$. They are allowed to be heterogeneous with respect to their personal accident probability $\tilde{p}$ but they have to be homogeneous concerning $\mathcal{P}$ as the set $\mathcal{P}$ determines their utility maximization problem. Moreover, the individuals are assumed to be homogeneous in their choice of how to react to the imprecise information. Individuals are overconfident in own abilities and pick the most optimistic accident probability when they calculate their insurance demand, i.e. they calculate with $\underline{p}$. Thus, the demand of the individuals is determined by the maximization of

$$V(\underline{p}, \alpha) = (1 - \underline{p})U(W - \alpha^1) + \underline{p}U(W - \alpha^1 - L + \tilde{\alpha}^2).$$

The insurance market is assumed to be competitive. There is free entry and insurers make expected profits of 0. As the insurers take the probability p for the design of the contracts, the worth of a contract for the insurers is

$$(1-p)\alpha^1 - p\alpha^2 = \alpha^1 - p(\alpha^1 + \alpha^2) = 0.$$

The insurers offer all contracts that lie on the line with the slope $(1-p)/p$ that goes through the uninsured state E. We call this line the fair-odds line. The intersection between the fair-odds line and the 45 degree line is the point F. We call this point F *full insurance* point and the corresponding contract *full insurance contract*, as the individual has the same income in this point in both states of the world. In this sense, she is completely insured.

Equilibria

We now turn to the equilibrium analysis. If the prior set $\mathcal{P}$ is a singleton or more generally such that $\underline{p} = p$, we are back in the benchmark case without model uncertainty. For our results we argue mainly graphically with figure 4.4: partial insurance, figure 4.5: risk increase and figure 4.6: no insurance.

E is the typical no insurance state which lies south east of the 45 degree line. F is the full insurance state and lies on the 45 degree line. The line through these two points is the fair odds line of the insurers. They will only offer contracts that lie on this line which has slope $(1-p)/p$. However, the individuals deem a set of probabilities to be possible as idiosyncratic accident chance. This leads to a set of indifference curves through E. Due to the assumption of overconfidence, the indifference curve of the individuals that is decisive for their choice of insurance has slope $(U'(W^1)(1-\underline{p}))/(U'(W^2)\underline{p})$. For equal income in both states $W^1 = W^2$ this becomes $(1-\underline{p})/\underline{p}$ which is

Figure 4.4: Partial Insurance

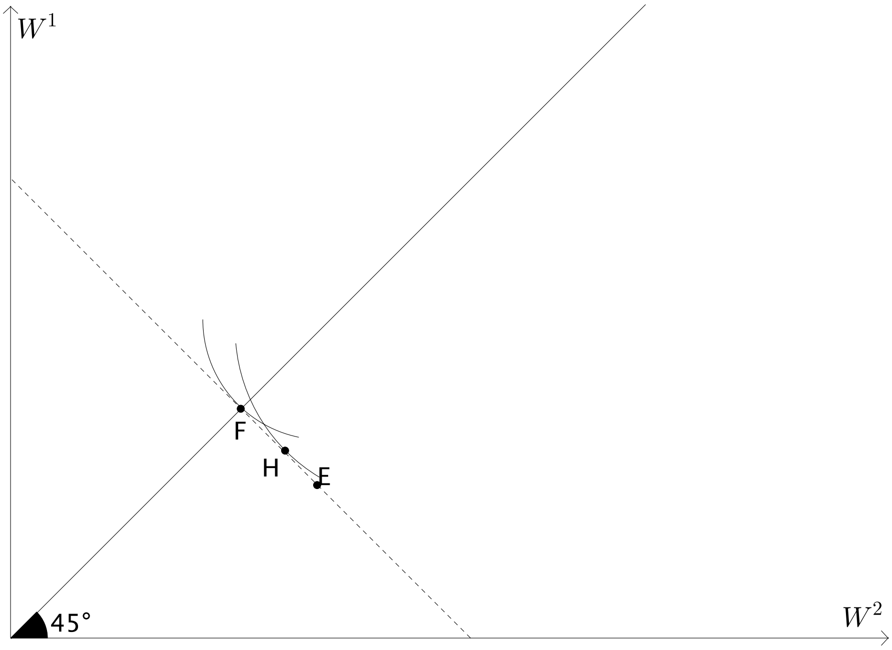


Figure 4.5: Risk Increase

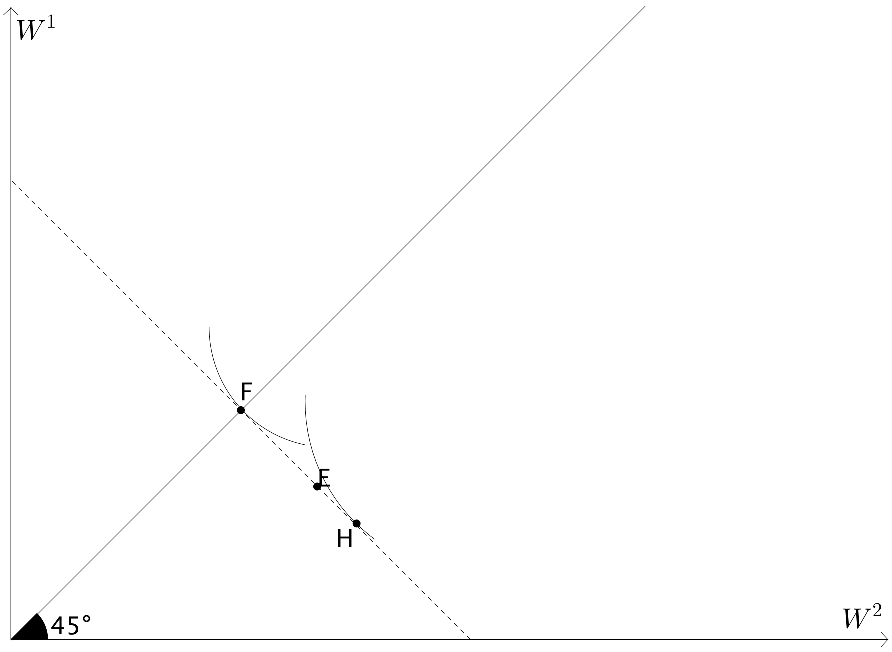
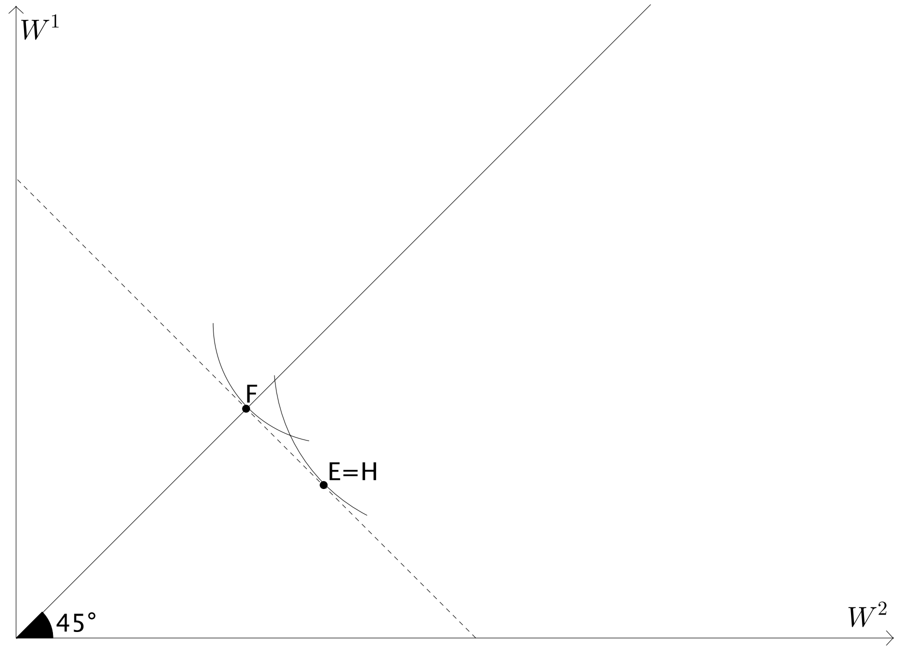


Figure 4.6: No Insurance



larger than the slope of the fair-odds line $(1-p)/p$, if $\underline{p} < p$. To equalize the two slopes, income in the first state, the "no accident" state, has to rise and income in the "accident" state has to fall. This is due to the curvature of U' for which we assume $U'' < 0$. Hence the tangential point of the indifference curve of the agents and the break-even line moves from F in the south-east direction. Depending on the degree of risk aversion described by U'' and the extreme prior $\underline{p}$, the tangential point may lie anywhere on the line EF. For sufficiently low risk aversion and sufficiently high difference $p - \underline{p}$, there does not exist a tangential point in between E and F. Then there may arise an equilibrium to the south east of the no insurance state E, a risk increasing equilibrium, as depicted in figure 4.5. The intuition behind this is as follows. As individuals are overconfident, the contracts the insurers offer are too expensive to insure. Instead, individuals want to insure in a way that may be called "negative". The insurers pay a fixed premium α^1, and if an accident occurs, the individuals pay the insurers α^2. We summarize our findings in the following theorem.

Theorem 4.3. *(a) For $p > \underline{p}$, individuals do not buy full insurance.*
(b) For sufficiently low risk aversion and sufficiently low $\underline{p} < p$, the individuals do not buy insurance at all or even increase their risk.
(c) Any point on the fair-odds line of the insurers may become an equilibrium, depending on the risk aversion of the individuals and on $\underline{p}$.
(d) The lower the risk aversion and the lower $\underline{p}$, the less insurance the individuals buy.

4.4.2 Incomplete Preferences

We now modify our model. The individuals were overconfident in section 4.4.1. Now we assume a different attitude: They have incomplete preferences with inertia as suggested by Bewley (2002). In all other aspects we take the same model as before. Individuals deem a set of probabilities to be possible for themselves, $\mathcal{P} = [\underline{p}, \overline{p}]$. They prefer one point of income over another if and only if it yields higher expected utility under all priors.[4] Additionally, they may have a status quo bias. Individuals only prefer a point of income to remaining uninsured if this point makes them better off in all scenarios. Else they stay uninsured. The behavioral assumption of the status quo bias acts as an equilibrium refinement. The meaningfulness of the status quo bias has been investigated in economics and psychology, we refer to Rigotti and Shannon (2005) for a survey on this.

[4] Bewley preferences and inertia are explained in more detail in chapter 5.

Demand and Supply for Insurance

The supply of the insurance does not change from the previous subsection. The individuals however have now a different attitude towards the set $\mathcal{P} = [\underline{p}, \overline{p}]$. They have incomplete preferences. Thus, their behavior can be described as follows. They pick a probability $q \in \mathcal{P}$ and calculate their demand with respect to this probability

$$V(q, \alpha) = (1-q)U(W - \alpha^1) + qU(W - \alpha^1 - L + \tilde{\alpha}^2).$$

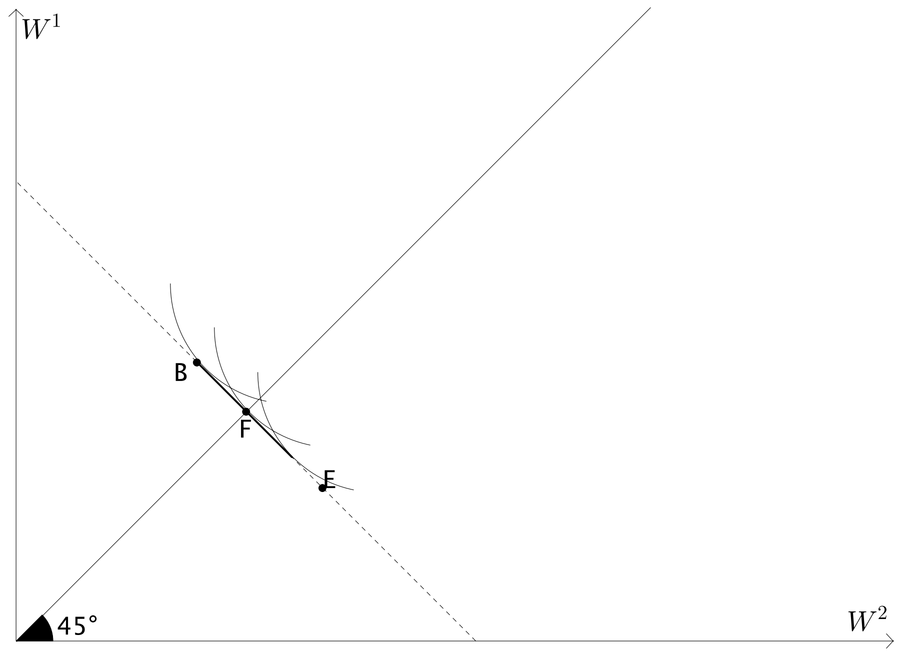
Solving the associated maximization problem leads to an equilibrium income on the line EF and to the acquisition of the corresponding contract. This is an equilibrium as it lies on the line EF and is the best income point the individual can reach with respect to the prior q.

Equilibria

We analyze this case again graphically through figure 4.7: interval of equilibria, and figure 4.8: interval of equilibria containing endowment.

As before, E is the uninsured initial state, F is the full insurance state, and the insurers offer exactly the contracts that alter the consumption pattern of the individual such that it lies on the line through E and F. This line has slope $(1-p)/p$, where p is the accident probability with which the insurers calculate. The individual now has a whole set of indifference curves. The slopes of the indifference curves are $[U'(W^1)(1-\hat{p})/U'(W^2)\hat{p}]$ for $\hat{p} \in \mathcal{P}$. In an equilibrium, the indifference curve is tangential to the contract line which is in this case the fair odds line. For $p = \hat{p}$ the equilibrium is the full insurance state F. However, for $\underline{p} \leq \hat{p} < p$ the equilibrium is on the line EF but to the south east of F. On the other hand, if $p < \hat{p} \leq \overline{p}$, the equilibrium

Figure 4.7: Interval of Equilibria

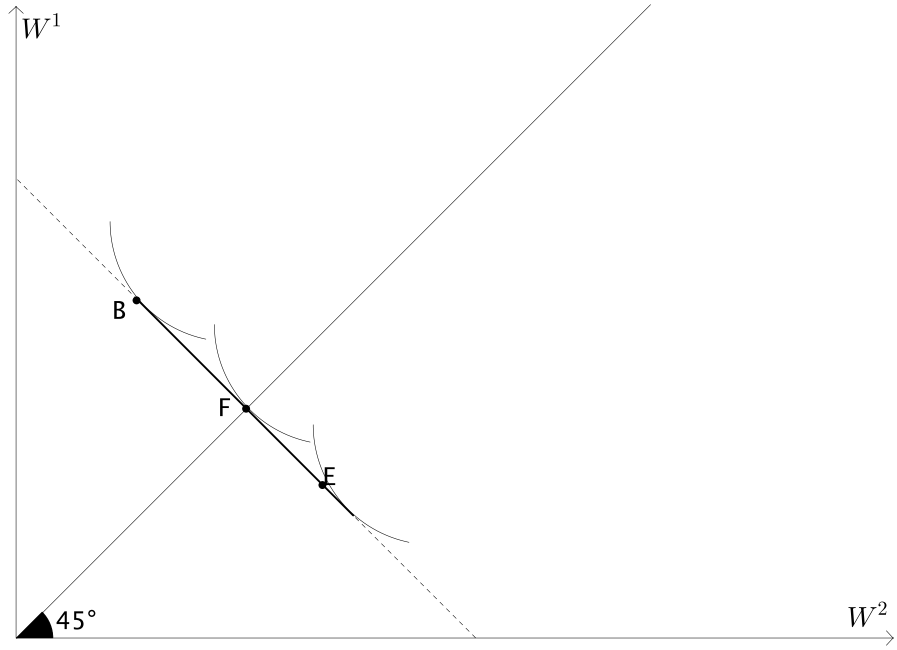


is on the line EF but to the north west of F. Hence, there arises an interval of possible equilibria around the full insurance equilibrium. The length of this equilibrium is increasing in the length of the interval $\mathcal{P}$. In particular, the equilibrium set is indeterminate for positive length of the interval $\mathcal{P}$. Moreover, a bet on the accident by altering the income pattern to a point to the north west of F can become an equilibrium. The indeterminacy and the existence of a "betting" equilibrium B are illustrated in figure 4.7.

For sufficiently large $p - \underline{p}$ and sufficiently low risk aversion, no insurance or risk increase can become an equilibrium, as for example in figure 4.8. If no insurance becomes an equilibrium, the inertia refinement of equilibrium makes this no insurance equilibrium unique. Else the refinement has no bite. The inertia refinement is defined as follows.

Definition 4.4. An equilibrium satisfies the inertia refinement if it is an equilibrium and individuals have higher expected utility of the equilibrium

Figure 4.8: Interval of Equilibria containing Endowment



income in all scenarios $\mathcal{P}$ compared to staying uninsured.

As this definition does not cover the uninsured state, we say the uninsured state always satisfies the inertia refinement, if it is an equilibrium.

We summarize our findings in the following theorem.

Theorem 4.5. *(a) Full insurance is always an equilibrium as we assume $p \in \mathcal{P}$.*
(b) The equilibrium set is an interval on the line EF and includes full insurance. This interval is nondegenerate for nondegenerate $\mathcal{P}$. In particular, this implies indeterminacy of the equilibrium set.
(c) Any point on the line EF may become an equilibrium. In particular, risk increase and bets on the accident may become possible in equilibrium.
(d) If the initial state becomes an equilibrium for sufficiently low risk aversion and sufficiently large $(p-\underline{p})$, the equilibrium refinement of inertia makes

this equilibrium unique. Else the inertia property has no bite.

A remark on the equilibria on the line EF located north-west of F seems to be in order. For these equilibria, a moral hazard problem may arise as individuals profit from the occurrence of accidents. This might motivate individuals to increase their likelihood of accident. Anticipating this, insurers may decide not to offer contracts which correspond to points north-west of F.

4.4.3 Economic Interpretation

In this section we examined the case when model uncertainty is on the side of the individuals but not on the side of the insurers. First, we investigated overconfident individuals. The results in the analysis show that overconfidence in own abilities rationalizes underinsurance or even risk increase. This is a very clear and precise prediction. Due to the generality of the model, these findings can explain underinsurance or deliberate risk increase in many situations. We then turned to the incomplete preferences case. The typical phenomenon of indeterminacy of the equilibrium set when agents have incomplete preferences arises. This is a parallel to the findings of Rigotti and Shannon (2005). The same is true for the uniqueness of the endowment allocation if this is an equilibrium allocation and we refine the equilibrium with the inertia property. In general, the indeterminacy of the equilibrium set is critical. While many findings can be explained due to the flexibility which a thick equilibrium set yields, one can on the other hand say that indeterminacy is not very satisfying as the model does not predict in a precise way where market forces are leading to. It only predicts that market forces will lead into the equilibrium set which is not very informative if the indeterminate equilibrium set is thick. In this context Dana and Riedel (2013) pose the natural question of a refinement possibility of the equilibrium notion in the context of incomplete preferences to reinstall uniqueness, at least locally. They are not optimistic for the existence of such a refinement possibility. Nevertheless, incomplete preferences is a natural way to model individual behavior which has been given increasing attention in the last years.

4.5 Ambiguity about the distribution of the population

In this section we consider the original model of Rothschild and Stiglitz where the population of individuals consists of two different types, a high risk and a low risk type.[5] In the original model of Rothschild and Stiglitz, the insurers cannot discriminate between the two types while the individuals know of which type they are. However, the distribution of high and low risk types inside the population is known to the insurers. The fraction of high risk individuals is λ. One of the main insights of Rothschild and Stiglitz is that in this situation of a heterogeneous group of individuals which the insurers cannot distinguish but where λ is known an equilibrium in general does not exist. For the comfort of the reader we repeat the argument for possible nonexistence in the first part of this section. Then we assume that the insurers do not precisely know the fraction λ but only an interval $\Lambda = [\underline{\lambda}, \overline{\lambda}]$ such that for the true λ it holds $\lambda \in \Lambda$. Moreover, we

[5]High risk and low ability as well as low risk and high ability are synonyms in this section.

assume ambiguity aversion of the insurers.[6] The main result in this section is that for sufficiently large ambiguity, i.e. sufficiently high $\overline{\lambda}$, there always exists an equilibrium if insurers are ambiguity averse. Ambiguity about the distribution of the population paired with the attitude of ambiguity aversion can therefore reestablish the existence of an equilibrium.

4.5.1 Nonexistence of equilibrium in Rothschild-Stiglitz

In a first step we show that there is a unique equilibrium candidate in Rothschild and Stiglitz. We then demonstrate that this candidate is not always an equilibrium.

In our model there are two kinds of individuals which have either high risk or low risk type. Their risk type is described by their accident probability $p^H > p^L$. The fraction of high risk individuals λ is known to the insurers. The average accident probability is thus $\overline{p} = \lambda p^H + (1 - \lambda)p^L$.

Following Rothschild and Stiglitz in their argumentation we begin by showing that there cannot be a pooling equilibrium which is an equilibrium in which both kinds of individuals buy the same contract. Suppose there is a pooling equilibrium α. Then the equilibrium has to lie on the fair odds line through E with slope $\frac{1-\overline{p}}{\overline{p}}$. Otherwise the firms would either lose money or there would be a contract which would be profitable and that would be preferred by all individuals. Consider the indifference curves of the two types of individuals which intersect in the equilibrium α. As they intersect there

[6]As above, the combination of ambiguity aversion and risk neutrality of the insurers makes perfect sense as the risk goes away with an increasing number of individuals provided independence of the occurrence of individual accidents while the ambiguity does not go away with a larger number of individuals.

exists another contract β next to α which is preferred by the low risk individuals to the equilibrium contract but which is worse than α for the high risk individuals. As only the low risk individuals would buy this contract, it makes a profit, thus breaking the equilibrium α. Hence there cannot exist a pooling equilibrium.

Now consider the separating equilibrium case. The contract most liked by the high risk type on the fair odds line of the high risk individual with slope $\frac{1-p^H}{p^H}$ is the full insurance contract. Accordingly, the contract most liked by the low risk individual on the fair odds line of the low risk individual with slope $\frac{1-p^L}{p^L}$ is the full insurance contract as well. However, if the insurers would offer this full insurance contract, the high risk individuals would also buy it which would induce negative expected profits for the insurers. Thus, the insurers cannot offer a contract to the low risk individuals which the high risk individuals prefer to their full insurance contract. In this sense, the presence of the high risk individuals induces a negative externality on the low risk individuals. The contract which gives the high risk individuals full insurance has to be part of any equilibrium. An equilibrium contract for the low risk individuals most not be more attractive to high risk individuals than this full insurance contract. The only remaining equilibrium candidate for low risk is the intersection of the indifference curve of the high risk individuals through the full insurance contract for the high risk individuals with the fair odds line of the low risk individuals. However, this equilibrium candidate does not not always constitute an equilibrium together with the contract that moves consumption to G. Consider a contract γ below the fair odds line of the low risk individual which is preferred to the equilibrium candidate contract by both types. Now the fraction of the population becomes decisive. Suppose the fraction of high risk individuals is sufficiently low such that γ lies below the fair odds line with slope $\frac{1-\overline{p}}{\overline{p}}$. Then the contract γ makes a profit and is preferred by both types of individuals. Hence, it breaks the equilibrium.

Figure 4.9: Rothschild-Stiglitz Equilibrium Puzzle

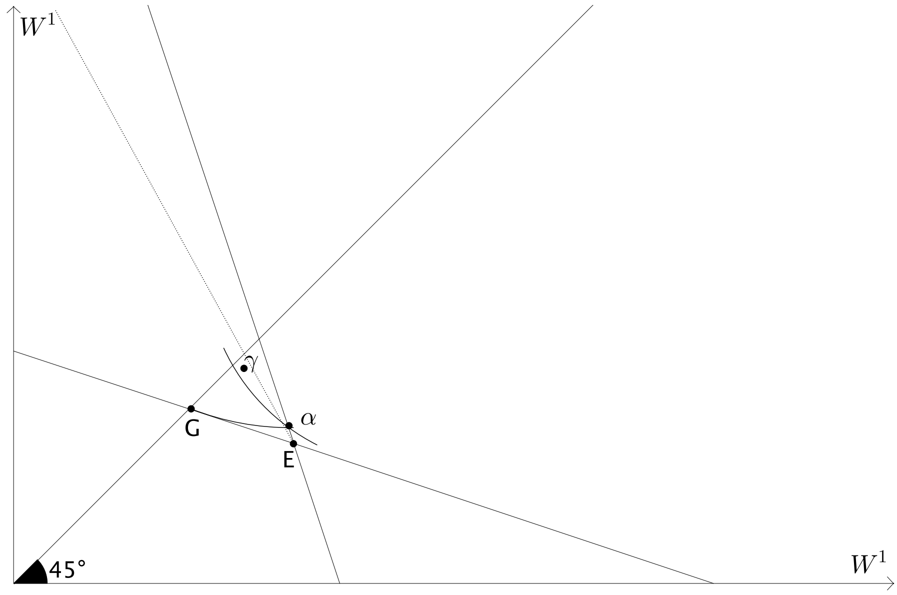


Figure 4.9 illustrates the preceding reasoning. For the equilibrium breaking contract to exist, the population has to consist of sufficiently many low risk / high ability individuals. Our idea to reestablish existence begins exactly at this point. If in the calculation of the insurance contracts the insurers take sufficiently few high ability individuals to exist, the equilibrium exists. We give the details in the next section.

4.5.2 Ambiguity about the distribution of the population

We now assume that the fraction of high risk types is not known to the insurers. Instead, insurers only know that for the true fraction λ it holds true that $\lambda \in \Lambda = [\underline{\lambda}, \overline{\lambda}]$. Additionally, we assume that insurers are ambiguity averse in the sense of Gilboa and Schmeidler (1989), i.e. they calculate with the fraction $\overline{\lambda}$, when designing their insurance contracts. Thus, insurers systematically overestimate the fraction of low ability types among the individuals whenever $\overline{\lambda} - \lambda > 0$. What does this imprecise knowledge about the true distribution of the population paired with the attitude of ambiguity aversion imply? The argument given in the preceding section, that there cannot be a pooling equilibrium in a population with high and low risk types does not depend on the fraction of high and low risk types. Hence the only equilibrium candidate remains the intersection of the fair odds line of the low risk individuals and the indifference curve through the full insurance allocation of the high risk individuals together with the full insurance for high risk types. However, without ambiguity about the fraction of high risk individuals this equilibrium candidate is not an equilibrium for a population consisting of sufficiently many low risk individuals. In reverse, this implies that if there are sufficiently many high risk individuals there exists an equilibrium. Even more holds true. If the insurers calculate with a fraction of high risk individuals that is sufficiently large, the unique equilibrium candidate is in fact an equilibrium. In our modification of Rothschild and Stiglitz insurers know only an interval Λ in which the true fraction lies and act in an ambiguity averse way. This means they use the fraction $\overline{\lambda}$ in their calculation. For sufficiently large $\overline{\lambda}$ the fair odds line with slope $\frac{1-\overline{\lambda}}{\overline{\lambda}}$ does not intersect the indifference curve of the low risk individual through its full insurance contract. Thus, the low risk individual does not deviate to any offered contract which lies on this fair odds line with slope $\frac{1-\overline{\lambda}}{\overline{\lambda}}$. This implies that there does not exist a contract γ which makes a profit and thus would break the equilibrium. We have established the following theorem.

Theorem 4.6. *Suppose there is ambiguity about the fraction λ of the high risk individuals. Insurers only know $\lambda \in \Lambda = [\underline{\lambda}, \overline{\lambda}]$. If insurers are am-*

biguity averse, then there always exists an equilibrium if $\overline{\lambda}$ is sufficiently large.

Figure 4.10: Ambiguity about fraction of high risk types

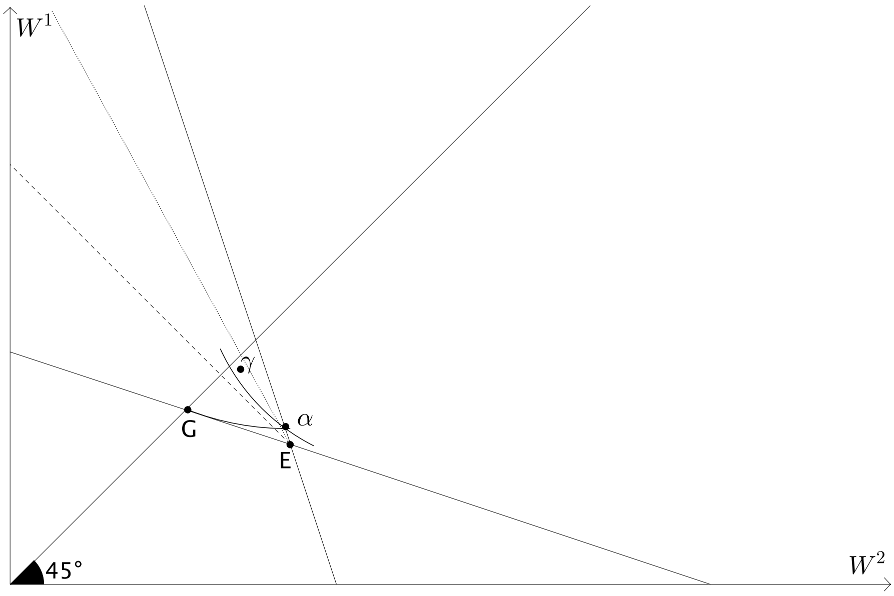


Figure 4.10 demonstrates the theorem and the reasoning in the proof. The unique equilibrium candidate for the low risk type is α, the intersection of the indifference curve of the high risk agents through G and the fair odds line for the low risk types. Between the fair odds lines for the high risk and the low risk types there are the fair odds lines with slope $\frac{1-\tilde{p}}{\tilde{p}}$ where $\tilde{p}$ is the probability that the insurers give the average individual. The line with the fine dots represents the case where the insurers gives the average individual a relatively low risk. The broken line gives the average individual a higher risk. The average individual is determined by the fraction of high and low risk types. The higher λ, i.e. the higher the proportion of the high risk types the smaller the slope of the corresponding fair odds line is. It approaches the fair odds line for the high risk type for increasing λ. As the indifference curve of the low risk type always intersects the 45 degree line above G, for sufficiently large λ in the calculation of the insurers, the

contract γ that might break the equilibrium lies above the fair odds line of the average individual. Thus γ loses money and is not offered by the firm. As γ does not break the equilibrium, the equilibrium candidate consisting of α and the contract which enables the low ability types to insure fully is in fact an equilibrium.

4.6 Examples

In this section we calculate a number of examples for the case of individuals that use CARA utility of degree 1. We begin with the benchmark case. Let $E = (2, 1)$ be the uninsured state. Thus the income is 2 if there is no accident and 1 if there happens to be an accident. The probability p of accident is $\frac{1}{2}$ and insurers and individuals know and calculate with this $p = \frac{1}{2}$. This leads to the slope of the fair odds line which is $\frac{1-p}{p} = 1$, and to the equation of the line through F and E which is $W^2 = 3 - W^1$. The full insurance state $F = (1.5, 1.5)$ yields expected utility of $Eu(1.5) = -\exp(-1.5) \approx -0.2231$. Independently of the utility function and independently of the common p insurers and individuals use for their calculations, risk averse investors insure fully in the benchmark case. This can be seen from the marginal rates of substitution: $\frac{1-p}{p} = \frac{U'(W^1)(1-p)}{U'(W^2)p}$.

Now suppose there is model uncertainty on the insurer side: $p \in \mathcal{P} = [\underline{p}, \overline{p}] = [\frac{1}{4}, \frac{3}{4}]$. To differentiate between the probabilities used by insurers and individuals, we denote them by p^{ins} and p^{ind}, respectively. Then, the costs of insuring one unit north west of E is $p^{ins} = \frac{3}{4}$, due to the ambiguity aversion of the insurers. We denote the number of units that get insured by $i \in \mathbb{R}$. We calculate the marginal utility depending on i:

$$MU(i) = -(1-p^{ind})p^{ins}u'(2-ip^{ins})+p^{ind}(1-p^{ins})u'(2-1+i(1-p^{ins})) = 0,$$

which simplifies to

$$\Rightarrow \frac{3}{8}u^{'}(2-\frac{3}{4}i) = \frac{1}{8}u^{'}(1+\frac{1}{4}i).$$

For CARA utility of degree one, we can solve for i. The optimal i^* is $i^* = -\ln 3 + 1 \approx -0.0986$. This means there is no tangential point of the indifference curve and the fair odds line. For this, i^* would have to be nonnegative. Thus, no insurance is bought by the individuals. We calculate the probability of the insurance company, for which the tangential point would be E, the initial state:

$$p^{ins}_{crit} = \frac{p^{ind}\exp(1)}{p^{ind}\exp(1) + 1 - p^{ind}} = \frac{0.5\exp(1)}{0.5\exp(1) + 1 - 0.5} \approx 0.7311.$$

This implies that if the insurers calculate their fair odds line with respect to an accident probability which is lower than 0.7311, the individual will insure, at least partially. The individual will buy full insurance for $p^{ins} = 0.5$. For $p^{ins} \in (0.5, 0.7311)$ the individual will insure partially.

We now turn to the case where model uncertainty is on the side of the individuals and the individuals are overconfident in own abilities. We again take as interval for the probability of accident $p \in \mathcal{P} = [\underline{p}, \overline{p}] = [\frac{1}{4}, \frac{3}{4}]$. The individuals calculate, due to their overconfidence, with $p = 0.25$. The slope of the fair odds line the insurers offer is 1 as the insurers calculate with $p^{ins} = 0.5$. The equation of the line through E and F is $W^2 = 3 - W^1$. With the same notation as above, we calculate the tangential point of the individuals indifference curves and the fair odds line. The optimal i^* is

$$1 - \ln\left(\frac{0.5 \times 0.75}{0.5 \times 0.25}\right) = 1 - \ln(3) \approx -0.0986,$$

as above. That means, the individuals' overconfidence outperforms their risk aversion and it becomes rational for the individuals to increase their risk. We calculate the critical probability for the individuals, for which they would exactly not change their risk position.

$$0.5 = \frac{p \times \exp(1)}{p\exp(1) + 1 - p} \Rightarrow p^{ind}_{crit} = \frac{1}{2(\exp(1) - 0.5\exp(1) + 0.5)} \approx 0.2694$$

If the individuals give themselves higher probability than p^{ind}_{crit}, they will insure, if they give themselves lower probability, they would be willing to insure "negatively".

Finally, we calculate for the incomplete preferences case, again with $p \in \mathcal{P} = [\underline{p}, \overline{p}] = [\frac{1}{4}, \frac{3}{4}]$. The slope of the fair odds line of the insurers is 1, as they calculate with $p = 0.5$. Of particular interest in this case is the insurance behavior of the individual in the extreme cases. For $p = 0.25$, we already know from the above analysis the corresponding optimal number of insured units $i^*_{p=0.25} \approx -0.0986$. For $p = 0.75$, we get as result $i^*_{p=0.75} \approx 2.67398$. The critical value from which endowment is in the equilibrium set is as above $p^{ind}_{crit} \approx 0.2694$. For $\underline{p} \leq p^{ind}_{crit} \approx 0.2694$, the initial uninsured state is in the equilibrium set. In this case, the inertia refinement would make the initial state equilibrium unique.

Now we give an example how ambiguity about the distribution of the population can reestablish the existence of the equilibrium. Consider again the uninsured initial state $E = (2, 1)$. High risk individuals have accident probability $p^h = 0.6$ and low risk individuals accident probability $p^l = 0.4$. Fair odds lines for these individuals have slope $\frac{1-p^h}{p^h} = \frac{2}{3}$ for the high risk type and $\frac{1-p^l}{p^l} = \frac{3}{2}$ for the low risk type. Every equilibrium in the heterogeneous population case is necessarily a separating equilibrium where high risk individuals buy full insurance. The full insurance allocation for the high risk individuals lies on their fair odds line with slope $\frac{2}{3}$ through E, thus their full insurance allocation is $(1.4, 1.4)$. Full insurance for the low risk would be $(1.6, 1.6)$, however this allocation cannot be offered by the insurers as high risk individuals would buy it. Consider therefore the intersection of the indifference curve of the high risk type with the fair odds line of the low risk individuals. We calculate the location of this intersection point. Expected utility in this point has to be the same for the high risk agent as in the offered full insurance. Thus

$$-\exp(-1.4) = -0.4\exp(-W^1) - 0.6\exp(-4 + \frac{3}{2}W^1),$$

which leads to $W^1 \approx 1.8855$ and $W^2 = 4 - \frac{3}{2}W^1 \approx 1.17175$. The unique equilibrium candidate has the coordinates $(1.8855, 1.17175)$. Consider now the indifference curve of the low risk agent through this equilibrium candidate. It intersects the 45 degree line approximately in the point $(1.5372, 1.5372)$. However, we are interested in finding a particular tangential line of the indifference curve of the low risk agent through the equilibrium candidate, namely the tangential line which passes through the uninsured allocation. This line defines the minimal fraction of high risk individuals for which the equilibrium exists. We calculate the slope of this tangent.

$$\frac{U'(1.8855)(1-0.4)}{U'(W^2)0.4} = \frac{1-\tilde{p}}{\tilde{p}} \Rightarrow \tilde{p} = \frac{1}{\exp(1.17175 - 1.8855)\frac{3}{2} + 1} \approx 0.57647,$$

which defines the critical fraction $\tilde{\lambda} \approx 0.882332$. If the insurers calculate with this fraction or even higher fractions of high risk individuals there always exists an equilibrium.

4.7 Conclusion

We introduced model uncertainty in the model of competitive insurance markets by Rothschild and Stiglitz. We began by assuming that the individuals precisely know their own probability which is the same for all individuals while the insurance companies do not know the precise probability. Instead they only know an interval and then make a worst case analysis for designing the insurance contracts they offer. This makes the insurance costlier. Hence, depending on the risk aversion of the agents and the worst case scenario the insurers consider, any point on the breakeven line of the risk neutral insurers may become an equilibrium. The less insurance is bought the less risk averse the agents are and the higher the costs of the insurance are, which depend on the worst case scenario of the insurers. In particular, the insurance market may break down completely for sufficiently expensive contracts or sufficiently low risk aversion.

The individuals experience a utility loss due to the vague idea of the insurers about the true probability. They would prefer well informed companies that would know the true probability. Moreover, the lack of information on the insurer side also poses a risk for the insurers. If somehow another better informed company would appear on the market, it would be able to offer better conditions for the agents. Therefore it is in the interest of both individuals and companies that insurers have perfect knowledge about the probabilities of accident; for the individuals to increase their expected utility, and for the companies to be able to offer better contracts which hinders opponents to outperform them.

We then introduced model uncertainty on the side of the individuals while insurers were informed about the average probability of accident of the population of individuals. If individuals are ambiguity averse, they obviously insure fully. However, overconfidence in own abilities may induce underinsurance, a breakdown of the insurance markets, or even risk increase. If agents have incomplete preferences as suggested by Bewley (2002), we get indeterminacy of the equilibrium set. Bets on accidents, risk increase, no insurance, and underinsurance may all become equilibria. Refining the equilibrium with the inertia concept makes the equilibrium unique provided no insurance is an equilibrium.

Some of these findings like underinsurance are already explained in the literature in different setups. However, our goal in this note is to describe the role model uncertainty can play in the classical Rothschild and Stiglitz insurance model. In this sense, we hope to add conceptually to the literature.

Moreover, in section 4.5 we present a possibility to guarantee existence in the original Rothschild-Stiglitz model with a heterogeneous population consisting of high and low ability types. Insurers cannot discriminate between the two types while individuals know their idiosyncratic accident probability. However Rothschild and Stiglitz assumed the fraction of high risk individuals to be known. We modified the latter assumption and introduced ambiguity about this fraction. If insurers act in a way that is ambiguity averse in the sense of Gilboa and Schmeidler (1989) we can reestablish existence if the insurers calculate with a scenario that involves sufficiently many high risk individuals. Thus, we give in section 4.5 a solution to the so called Rothschild Stiglitz equilibrium puzzle. A different solution and a short overview over the puzzle can be found in Picard (2009).

This research has in part been inspired by the worldwide economic crisis that has been ongoing since 2007. One particularly important problem that aggravates the turmoils and confuses the issue of finding a way out of the crisis is the drying up of the insurance market. Especially the interbank market has been experiencing this problem. Our simple model may shed some light on the mechanisms that lead to these no trade states. Consider the findings of section 4.3. If the uncertainty about the contract partner becomes large, the costs of insuring increase which leads to lower levels of insurance. The income pattern is no longer altered in a way that the individual (the contract partner that can buy the insurance) is indifferent between the states that may occur. Instead, one state may be much worse than the other, i.e. the accident poses a real threat for the individual. This may also indicate risks for the future of the economies in the current crisis. Companies may be more vulnerable to negative shocks.

4.8 Appendix

Legend of figures

W^1 :	Income in "no accident" state
W^2 :	Income in "accident" state
B :	Betting equilibrium
E :	Initial position
F :	Full insurance
F^1 :	Full insurance in Rothschild and Stiglitz
G :	Full insurance for high risk type
H :	Equilibrium
γ :	Equilibrium breaking contract
α :	Equilibrium candidate
━━━━ :	Equilibrium set (in figure 4.7-4.8)
- - - - - :	Fair odds line (in figure 4.1-4.8)
- - - - - :	High share of high risk type (in figure 4.10)
·············· :	Low share of high risk type (in figure 4.10)
·············· :	Share of high risk type (in figure 4.9)

5 Equilibria with Inertia in Bewley Economies

Abstract

In a case study framework, we investigate the set of equilibria with inertia in a Bewley economy. We identify the equilibria when agents are endowed with two normal random variables where one is standard normal and there is uncertainty about the expectation of the second. We examine the equilibrium with inertia set which enables us to identify the conditions for a breakdown of the insurance market. Furthermore, we prove indeterminacy of the equilibrium set which survives the inertia refinement and highlight the influence of the interplay of risk and uncertainty on the equilibrium with inertia set.

5.1 Introduction

The classic work of Knight (1921) has led to a meaningful and important distinction between risk and uncertainty in the economic literature. Knight saw risk as randomness that can be measured precisely while this is not the case for uncertainty. In his view, the qualitative difference between risk and uncertainty plays an important role in markets that economic theory should take into account.

Ellsberg (1961) rendered the more precise, modern distinction between risk and uncertainty. An event is called risky if it has known probability and uncertain if it has unknown probability. His thought experiments have highlighted important aspects of the difference as in many situations agents tend to prefer gambles with known odds over gambles with unknown odds.

The main motivation of this paper is to understand the market implications better if we allow for a distinction between risk and uncertainty. In particular, we want to investigate three questions. First: What happens to the insurance market if there is risk and uncertainty? Already Knight argued that uncertainty is not insurable. In how far is this true and to what extent does the presence of risk in the market influence the insurance market for uncertainty? Second: Is the set of equilibria indeterminate? In the presence of uncertainty, markets might not be able to find precise prices and the frictions created by uncertainty might lead to indeterminacy. In how far does a status quo bias of the agents change the indeterminacy of the equilibrium set? Third: What influence has the interplay of risk and uncertainty on the equilibrium with inertia set?

To answer these questions, we consider a general equilibrium case study without aggregate uncertainty where agents are endowed with a risky and an uncertain component. This case study has been introduced into the literature by Dana and Riedel (2013). They examine the case study in discrete time and focus on the insurance aspect. They show for which amount of uncertainty in the economy full insurance is an equilibrium with inertia and when the insurance market for uncertainty breaks down. They moreover prove indeterminacy of the equilibrium with inertia set. We investigate the static version of their case study. The static version is easier to handle and thus enables us to describe the equilibrium set sufficiently well for our purposes and to answer the three questions.

In our case study model we rely on Bewley's decision theoretic model. The axiomatization of Bewley's model in Bewley (2002)[1] leads to a set of probability distributions over the state space upon which decision making depends. An allocation is preferred to another if and only if it is unanimously preferred under all probability measures of the prior set. For a singleton prior set, this reduces to standard expected utility. However, if there is no unanimity for two allocations, the preference relation remains incomplete and the two allocations cannot be compared.

In our case study there are two agents which have CARA-utility functions and which are both endowed with two random variables. One random variable models the risky part of the endowment and the other the uncertain part. For this end, we model one random variable as standard normal and the other as normal with standard variance but with expectation in an interval around 0. Thereby, we define the set of priors our Bewley agents use, namely exactly the set of priors such that the first random variable is standard normal under all priors and the second is normal with standard variance and expectation in the interval around 0. The length of the interval gives us a measure for the amount of ambiguity in our economy. The preferences of the agents are incomplete for positive length of the interval. If the interval is a singleton and thus solely consists of 0, the set of priors is a singleton and we are back in the expected utility framework.

In our case study we ask the question which equilibria arise. In particular,

[1]Bewley wrote the paper already in the late 1980's, it was published 2002.

we are interested in the question whether the insurance market breaks down and if the set of equilibria is indeterminate. We thereby follow closely the motivation of Rigotti and Shannon (2005). In their seminal work they show that both a breakdown of the insurance market and indeterminacy of the equilibrium set may happen in their model of a standard Arrow-Debreu economy with agents that have incomplete Bewley preferences.

We confirm their findings by showing that both phenomena may arise in our framework, depending on the amount of ambiguity in the economy. Furthermore, we confirm their finding that if the endowment allocation is an equilibrium with inertia allocation, then it is unique. However, in important points we also depart from Rigotti and Shannon (2005). We investigate explicitely the interplay of risk and uncertainty as agents are endowed with an uncertain and a risky component. The presence of risk gives the agents freedom to choose which amount of uncertainty they want to consume as insuring the risk is already utility increasing. Furthermore, our framework naturally provides us with a measure for the amount of ambiguity in the economy. We take advantage of this measure and describe how the equilibrium set changes for different amounts of ambiguity, i.e. we do comparative statics.

A natural question to ask is how robust our findings are with respect to our choices of the prior sets, the endowments and the utility functions. Our findings are robust with respect to the degree of risk aversion. Different endowments may change the equilibrium with inertia set, we investigate in particular the focal case of only uncertainty as endowment and compare it to the risk and uncertainty case. We also examine the case of different prior sets and can describe the equilibrium set in some cases.

The remainder of this paper is organized as follows. We begin with a precise description of our model, including the explanation of Bewley's decision theoretic model. The analysis which equilibria arise in our economy is performed in section 5.3. In this section, the main theorems 5.2 and 5.4 are directly proven. All other proofs are presented in the appendix. The robustness of our findings with respect to the degree of constant absolute risk aversion, different endowments and different prior sets is subject of section 5.4. We conclude by giving an interpretation which relates our findings to the theory of financial market regulation.

5.2 The Model

We start with a probability space $(\Omega, \mathcal{F}, P_0)$. Let U and R be two random variables on $(\Omega, \mathcal{F}, P_0)$ that are independent and standard normal under P_0, i.e. $U \sim N(0,1)$ and $R \sim N(0,1)$. We define a set of priors $\mathcal{P}$ by defining the densities of the priors with respect to P_0.

$$Q \in \mathcal{P} \Leftrightarrow q = \exp\left(\alpha U - \frac{1}{2}\alpha^2\right),$$

where q is the density of Q w.r.t. P_0, and $\alpha \in [-\kappa, \kappa]$ for some $\kappa \geq 0$. Thus, the prior set consists of all priors under which R is standard normal and U is normal with expectation in the interval $[-\kappa, \kappa]$ and standard variance. R and U are independent under all elements of the prior set. The parameter κ serves as an indicator of the amount of ambiguity / Knightian uncertainty in our economy. The higher κ is, the more uncertainty there is in the economy about the expectation of the random variable U.[2]

[2]The defined prior set is not convex for positive κ. In the continuous time version of this case study, where U and R become predictable processes which are standard Brownian motions under P_0, the prior set is convex. The mathematical background is given by Delbaen (2006). It might then be possible to give a complete characterization

One question that we want to answer in this paper is whether the set of equilibria in the B-economy is indeterminate if we refine the equilibrium definition by adding the inertia property. In the normal framework described in the model above, we will show for our case study that indeterminacy survives the equilibrium refinement of inertia.

We consider two agents with endowment $\omega^1 = U + R = -\omega^2$. Thus, there is no aggregate uncertainty as aggregate endowment is normalized to 0. The agents have incomplete Bewley preferences, i.e. they prefer to consume c to d if c yields higher expected utility than d under all priors in $\mathcal{P}$. Thus, they use the decision criterion

$$c \succsim d \Leftrightarrow E^Q u(c) \geq E^Q u(d) \text{ for all } Q \in \mathcal{P}.$$

For the moment, we assume agents have CARA utilities $u(c) = -\exp(-c)$ of degree 1. We will consider general CARA utilities in section 5.4.

The *full insurance allocation* is the state independent allocation $c^1 = -c^2 = 0$. An allocation c is *feasible* if $\sum c^i = \sum \omega^i$.

An *equilibrium* is a pair (c^*, p^*) of a feasible allocation and a price such that there exists for no agent a budget feasible consumption plan that strictly dominates c^*. Due to the incompleteness of the preferences in the B-economy there may arise the situation that the equilibrium allocation and the endowment allocation are incomparable. We consider this implausible and refine the equilibrium concept by adding the *inertia property.* An equilibrium (c^*, p^*) satisfies the inertia property if for all agents i with $c^{*i} \neq \omega^i$ it holds $c^{*i} \succ \omega^i$. The inertia property induces a status quo bias for the endowment and an equilibrium that satisfies it is called *equilibrium with*

of the equilibrium set with the help of the theory of backward stochastic differential equations.

inertia. The inertia property was suggested by Bewley (2002) as an answer to unmotivated betting. The inertia assumption is a behavioral statement which is well supported by work in economics and psychology. To give an example of an important reference in this context, see Samuelson and Zeckhauser (1988). Rigotti and Shannon (2005) give more details and sources. In many economic situations it is difficult to identify a reasonable status quo. However, in the general equilibrium framework we are using, there is a natural candidate, the initial endowment.

5.3 Equilibria in the B-economy

We investigate the set of equilibria with inertia in our case study. In particular, we are interested in the question whether the set is determinate or indeterminate. We call a set *determinate* if it consists only of isolated points and else *indeterminate*. As a starting point which facilitates the further analysis, we note the following.

Theorem 5.1. *Risk is always fully insured in the B-economy.*

For the proof of theorem 5.1 we refer the reader to the appendix or to Dana and Riedel (2013).

As a benchmark, we consider the set of equilibria in the B-economy without imposing the refinement of inertia.

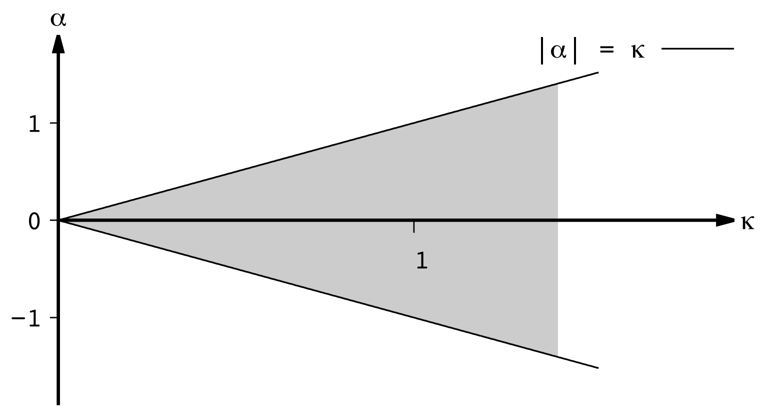
In this case, the following theorem holds.

Theorem 5.2. *The B-economy has an equilibrium* (c^*, p^*) *with equilibrium consumption* $c^{1*} = -c^{2*} = \alpha U$ *and equilibrium price* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *for*

$\alpha \in [-\kappa, \kappa]$. *In particular, the set of equilibria in the B-economy is indeterminate for* $\kappa > 0$.

The indeterminacy follows directly as the equilibrium allocations $c^{1*} = -c^{2*}$ are non-isolated if $\kappa > 0$. Before providing the proof, we sketch the equilibrium set in figure 5.1.

Figure 5.1: Equilibrium consumption set



The first axis denotes the amount of ambiguity κ in the economy, the second the equilibrium consumption α of uncertainty. The equilibrium set grows with κ and is enclosed by the 45 degree lines. The intuition behind this result is that the more ambiguity is present in the economy, the more diverse scenarios are considered possible, the more allocations are undominated. We now continue with the proof of theorem 5.2.

Proof. (Theorem 5.2) We equalize the marginal rates of substitution to calculate equilibrium consumption c^*.

$$\begin{aligned} \exp(-c^{1*})q^1 &= \exp(-c^{2*})q^2 \\ \Rightarrow \exp(-c^{1*})\exp\left(\alpha U - \frac{1}{2}\alpha^2\right) &= \exp(c^{1*})\exp\left(\beta U - \frac{1}{2}\beta^2\right) \\ \Rightarrow c^{1*} &= \frac{1}{2}(\alpha - \beta)U - \frac{1}{4}(\alpha^2 - \beta^2) \end{aligned}$$

Now we calculate the equilibrium price p^* for the first agent.

$$p^{1*} = \exp(-c^{1*})q^1 = \exp\left(-\frac{1}{2}(\alpha-\beta)U + \frac{1}{4}(\alpha^2-\beta^2)\right)\exp\left(\alpha U - \frac{1}{2}\alpha^2\right)$$
$$\Rightarrow p^{1*} = \exp\left(\frac{1}{2}\left(\alpha+\beta\right)U - \frac{1}{4}\left(\alpha^2+\beta^2\right)\right) = p^*$$

Analogously we calculate the equilibrium price for the second agent.

$$p^{2*} = \exp(+c^{1*})q^2 = \exp\left(\frac{1}{2}(\alpha-\beta)U - \frac{1}{4}(\alpha^2-\beta^2)\right)\exp\left(\beta U - \frac{1}{2}\beta^2\right)$$
$$\Rightarrow p^{2*} = \exp\left(\frac{1}{2}\left(\alpha+\beta\right)U - \frac{1}{4}\left(\alpha^2+\beta^2\right)\right) = p^*$$

Markets clear due to the normalization of consumption. For the budget constraint the following equality has to hold for the first agent

$$E^{P_0}p^*(\omega^1 - c^{1*}) = 0.$$

This leads to

$$E^{P_0}\left(\exp\left(\frac{1}{2}\left(\alpha+\beta\right)U - \frac{1}{4}\left(\alpha^2+\beta^2\right)\right)\left(R + U - \frac{1}{2}(\alpha-\beta)U + \frac{1}{4}(\alpha^2+\beta^2)\right)\right) = 0 \tag{5.1}$$

We now assume $\alpha = -\beta$, so that we get

$$E^{P_0}\left(\exp(-\frac{1}{2}\alpha^2)(R + U - \alpha U)\right) = \exp(-\frac{1}{2}\alpha^2)E^{P_0}(R + U - \alpha U) = 0.$$

Walras' law guarantees that the budget constraint is also satisfied for the second agent. This completes the proof of theorem 5.2. □

Comment 5.3. Theorem 5.2 does not give a full characterization of the equilibrium set. The prior set $\mathcal{P}$ is not convex. In the convex hull of $\mathcal{P}$ there may appear additional equilibria with possibly stochastic prices.

We now refine the equilibrium concept and investigate the set of equilibria with inertia in the B-economy. We investigate how the equilibrium set found in theorem 5.2 changes due to the inertia property. Except in the limit case of infinite ambiguity, the indeterminacy of the equilibrium set in the B-economy survives the equilibrium refinement inertia.

Theorem 5.4. *The equilibrium* (c^*, p^*) *with* $c^* = \alpha U$ *and* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *satisfies the inertia property iff* $\alpha \in [\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\}]$. *In particular, the set of equilibria with inertia in the B-economy is indeterminate for positive and finite ambiguity* ($\kappa \in (0, \infty)$).

The indeterminacy is straightforward as equilibrium allocations are non-isolated for $\kappa \in (0, \infty)$, for details consult the appendix.

Concerning full insurance and no trade of uncertainty theorem 5.4 implies the following results.

Corollary 5.5. *For the focal allocations full insurance and no trade of uncertainty we have*
(i) The full insurance allocation is not an equilibrium with inertia allocation for $\kappa > 1$.
(ii) There arises a no trade of uncertainty equilibrium with inertia for $\kappa \geq 1$.

We can describe the equilibrium interval more precisely. In Corollary 5.6 we do comparative statics in κ, investigate which terms are decisive in the interval definition and the thickness of the interval. The proofs can be found in the appendix.

Corollary 5.6. *(i) For the lower bound of the interval, the actual inertia property becomes decisive for* $\kappa \geq -\frac{1}{3} + \frac{\sqrt{7}}{3}$.
(ii) For the upper bound of the interval, the actual inertia property becomes

decisive for $\kappa \geq \frac{1}{3} + \frac{\sqrt{7}}{3}$.

(iii) The thickness of the interval is largest for $\kappa = -\frac{1}{3} + \frac{\sqrt{7}}{3}$.

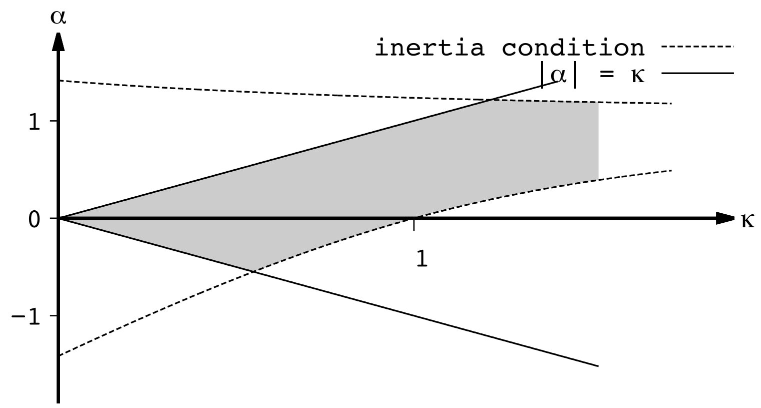
(iv) The thickness of the interval converges to 0 *for limiting* κ.

(v) The thickness of the interval increases monotonically for $\kappa \leq -\frac{1}{3} + \frac{\sqrt{7}}{3}$.

(vi) The thickness of the interval decreases monotonically for $\kappa \geq \frac{1}{3} + \frac{\sqrt{7}}{3}$.

Before we give the proof of theorem 5.4, we illustrate theorem 5.4 graphically via the following figure 5.2.

Figure 5.2: Equilibrium consumption set



In figure 5.2, the equilibrium allocations in a B-economy are indicated for the first agent. The amount of ambiguity κ is plotted against the share α of uncertainty U which the agent consumes in equilibrium. The 45 degree lines enclose the set of equilibrium allocations in B-economies without the refinement of inertia. The two curves are due to the inertia condition, the borders in theorem 5.4. They have for $\kappa = 0$ the value $\sqrt{2}$ and $-\sqrt{2}$ respectively and converge both monotonously against 1. The set of equilibria with inertia is the shaded set. There is a smooth transition from one focal allocation to the other, i.e. from full insurance to no insurance of uncertainty. The equilibrium with inertia is only unique in the limit cases of a risk economy ($\kappa = 0$) and infinite ambiguity. This smooth transition has its origin in the tension between risk aversion and the inertia property. The status quo bias due to the inertia property becomes the more powerful, the higher the ambiguity. In particular, we have for ambiguity $\kappa > 1$ that full insurance is no longer possible in equilibrium. As soon as the inertia property becomes the decisive bound for the equilibrium consumption set, we have with further increasing κ a decrease in the volume of possible trade for insurance in the sense that the thickness of the equilibrium set decreases. Finally, we note that a no trade of uncertainty equilibrium arises for $\kappa \geq 1$, as stated in corollary 5.5.

We turn to the proof of theorem 5.4.

Proof. (Theorem 5.4) We plug equilibrium consumption in the inertia property. Here Q is a probability measure taken from the set of priors $\mathcal{P}$ and γ the expectation of U which corresponds to Q.

$$\begin{aligned} -E^Q \exp(-c^*) &\geq -E^Q \exp(-\omega) \\ \Rightarrow -E^Q \exp\left(-\alpha U\right) &\geq -E^Q \exp\left(-R - U\right) \\ \Rightarrow E^{P_0} \exp\left(-\alpha U + \gamma U - \frac{1}{2}\gamma^2\right) &\leq E^{P_0} \exp\left(-U - R + \gamma U - \frac{1}{2}\gamma^2\right) \end{aligned}$$

Both random variables U and R are standard normal under P_0, so that Laplace Transformation yields

$$\Rightarrow \exp\left(\frac{1}{2}(\gamma-\alpha)^2 - \frac{1}{2}\gamma^2\right) \leq \exp\left(\frac{1}{2} + \frac{1}{2}(\gamma-1)^2 - \frac{1}{2}\gamma^2\right),$$

which simplifies to

$$\frac{1}{2}\alpha^2 - \gamma(\alpha - 1) \leq 1. \tag{5.2}$$

The inequality 5.2 has to hold for all $\gamma \in [-\kappa, \kappa]$. Therefore it suffices to check if 5.2 holds for the most extreme priors, which is either $\gamma = \kappa$ or $\gamma = -\kappa$. We now distinguish the following two cases.

$$\frac{1}{2}\alpha^2 - \kappa(\alpha - 1) \leq 1 \text{ if } \alpha < 1, \tag{5.3}$$

and

$$\frac{1}{2}\alpha^2 + \kappa(\alpha - 1) \leq 1 \text{ if } \alpha \geq 1. \tag{5.4}$$

Solving these equations for α leads to the intervals for α in theorem 5.4. Subject to our restriction $\alpha = -\beta$ there cannot be other equilibria as for all other α the inertia condition fails to hold. The left hand side of either inequality 5.3 or inequality 5.4 would be larger than one. This concludes the proof of theorem 5.4. □

Comment 5.7. There is a stark difference between the equilibrium sets in the B-economy and the Gilboa-Schmeidler (GS) economy. The GS-economy has its name as agents use GS-preferences as axiomatized in Gilboa and Schmeidler (1989). We define the Gilboa-Schmeidler economy in exactly the same framework as the B-economy, we only change the preference relation of the agents. The GS-economy is the economy where agents use the following criterion to compare allocations c and d

$$c \succsim d \Leftrightarrow \min_{Q \in \mathcal{P}} E^Q u(c) \geq \min_{Q \in \mathcal{P}} E^Q u(d).$$

This preference relation is complete and we do not have indeterminacy as the full insurance allocation is the unique equilibrium allocation, independently of the amount of ambiguity κ. To prove the latter, we apply the argument of Billot et al. (2000) as agents always have a common prior and there is no aggregate uncertainty.

Comment 5.8. Equilibrium prices in both theorem 5.2 and theorem 5.4 are deterministic while equilibrium allocations are stochastic. This somewhat surprising result comes from the budget constraint that has to hold in theorem 5.2.

Comment 5.9. With the set of equilibria with and without inertia that we found, we are able to answer the questions when the set of equilibria is indeterminate and when full insurance is an equilibrium allocation. Moreover, the shape of the equilibrium sets enables a clear understanding of the interplay of risk and uncertainty and how they influence the set of equilibria with inertia. The latter aspect is also subject of section 5.4.1, where we change the endowments of the agents.

5.4 Robustness

We investigate the robustness of our findings in the equilibrium analysis. We consider agents with different endowment than before, especially the case, when agents are exclusively endowed with uncertainty and not with risk. Furthermore, we allow the agents to have degree a of constant absolute risk aversion. Finally, we change the prior sets of the agents in several different aspects.

5.4.1 Endowment

In the previous section, we considered agents that were endowed with a risky component and an uncertain component, such that $\omega^1 = R + U = -\omega^2$. In general one can consider agents that have endowment $\omega^1 = aU + bR = -\omega^2$, where $a \geq 0$ and $b \geq 0$. This generalizes the previous case. The results for this case are stated in the following two theorems 5.10 and 5.11. Moreover, we describe the focal case where agents are exclusively endowed with uncertainty $\omega^1 = U = -\omega^2$. In this case, we get the result described in corollary 5.12.

Theorem 5.10. *Suppose the endowment of the agents is $\omega^1 = aR + bU = -\omega^2$. Then the B-economy has an equilibrium (c^*, p^*) with equilibrium con-*

sumption $c^{1*} = -c^{2*} = \alpha U$ *and equilibrium price* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *for* $\alpha \in [-\kappa, \kappa]$. *In particular, the set of equilibria in the B-economy is indeterminate for* $\kappa > 0$.

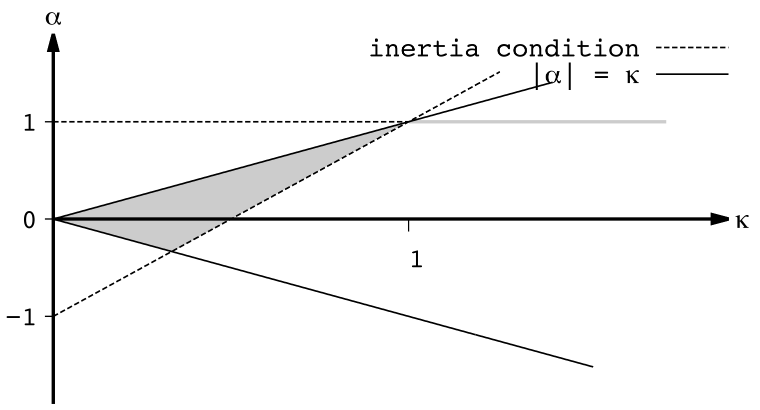
Theorem 5.11. *Suppose the endowment of the agents is* $\omega^1 = aR + bU = -\omega^2$. *The equilibrium* (c^*, p^*) *with* $c^* = \alpha U$ *and* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *satisfies the inertia property iff* $\alpha \in [\max\{-\kappa, \kappa - \sqrt{a^2 + b^2 - 2b\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{a^2 + b^2 + 2b\kappa + \kappa^2}\}]$. *In particular, the set of equilibria with inertia in the B-economy is indeterminate for positive and finite ambiguity* ($\kappa \in (0, \infty)$) *and* $a > 0$.

Corollary 5.12. *Suppose the agents' endowment is* $\omega^1 = U = -\omega^2$. *Then the equilibrium* (c^*, p^*) *with equilibrium consumption* $\omega \neq c^* = \alpha U$ *and equilibrium price* $p^* = \exp(-\frac{1}{2}\alpha^2)$ *satisfies the inertia property iff*

$$\alpha \in [\max\{-\kappa, 2\kappa - 1\}, \min\{\kappa, 1\}]. \tag{5.5}$$

We illustrate the equilibrium set of corollary 5.12 in the following figure 5.3.

Figure 5.3: Equilibrium consumption set



Two points are remarkable concerning this special case. First, the inertia assumption induces borders for α that depend linearly on κ. Second, equation 5.5 can only be satisfied for $\kappa \leq 1$. This second fact is not surprising. When the agents are endowed with risk, they always trade the risk away and thus increase their expected utility. This gives the agents flexibility in their consumption choice of uncertainty. Without risk in their endowment, the agents do not have this flexibility. As soon as the endowment allocation becomes an equilibrium with inertia allocation, this equilibrium becomes unique. We note this in the following corollary.

Corollary 5.13. *Suppose the agents' endowment allocation is an equilibrium with inertia allocation. Then this equilibrium with inertia is unique.*

Equation 5.5 can only be satisfied for $\kappa \leq 1$. The endowment allocation always satisfies the inertia property as this property is only defined for all allocations but the endowment allocation. This implies that for $\kappa \geq 1$ the equilibrium with inertia is uniquely the endowment equilibrium. This fact is a parallel to Rigotti and Shannon (2005) who have the same finding in their paper.

5.4.2 Degree of constant absolute risk aversion

So far, we assumed agents to have CARA utility function of degree 1. Now we consider agents having CARA utility of degree a. This leads to the following generalization of theorem 5.2.

Theorem 5.14. *Suppose agents have CARA utility of degree a. A consumption price pair (c^*, p^*) is an equilibrium in the B-economy if equilibrium consumption is $c^* = \frac{\alpha}{a}U$ and equilibrium price is $p^* = \exp(-\frac{1}{2}\alpha^2)$.*

The theorem is proven as theorem 5.2. Refining the equilibrium set with the inertia property leads to

Theorem 5.15. *Suppose agents have CARA utility of degree a and that agents are endowed with $\omega^1 = R + U = -\omega^2$. The equilibrium (c^*, p^*) with equilibrium consumption $c^* = \frac{\alpha}{a}U$ and equilibrium price $p^* = \exp(-\frac{1}{2}\alpha^2)$ satisfies the inertia property iff*

$$\alpha \in [\max\{-\kappa, \kappa - \sqrt{\kappa^2 - 2a^2 + 2a\kappa}\}, \min\{\kappa, -\kappa + \sqrt{\kappa^2 + 2a^2 + 2a\kappa}\}],$$

which is again proved in the same way as the corresponding theorem 5.4. Thus our findings are robust to changes of the degree of constant absolute risk aversion. For details of the proofs see the appendix.

5.4.3 General degree of constant absolute risk aversion and general endowment

We combine the generalizations of the two preceding subsections.

Theorem 5.16. *Suppose agents have CARA utility of degree z and are endowed with $\omega^1 = -\omega^2 = aR + bU$. A consumption price pair (c^*, p^*) is an equilibrium in the B-economy if equilibrium consumption is $c^* = \frac{\alpha}{z}U$ and equilibrium price is $p^* = \exp(-\frac{1}{2}\alpha^2)$.*

For the equilibrium with inertia set we get

Theorem 5.17. *Suppose agents have CARA utility of degree z and are endowed with $\omega^1 = -\omega^2 = aR+bU$. The equilibrium (c^*, p^*) with equilibrium consumption $c^* = \frac{\alpha}{z}U$ and equilibrium price $p^* = \exp(-\frac{1}{2}\alpha^2)$ satisfies the inertia property iff*

$$\alpha \in [\max\{-\kappa, \kappa - \sqrt{\kappa^2 + (za)^2 + (zb)^2 - 2\kappa zb}\}, \min\{\kappa, -\kappa + \sqrt{\kappa^2 + (za)^2 + (zb)^2 + 2\kappa zb}\}]$$

5.4.4 Prior sets

In the preceding sections, we assumed $\mathcal{P}^1 = \mathcal{P}^2 = \mathcal{P}$. We now consider cases, where $\mathcal{P}^1 \neq \mathcal{P}^2$ becomes possible and where the interval is not symmetric around 0. We only examine the equilibrium set without the inertia refinement. All proofs are straightforward and given in the appendix. In a slight abuse of notation, we define the prior set by specifying the set in which the expectation of U lies.

Theorem 5.18. *Suppose $\mathcal{P}^1 = [-\kappa^1, \kappa^2]$ and $\mathcal{P}^2 = [-\eta^1, \eta^2]$, for $\kappa^1, \kappa^2, \eta^1, \eta^2 \geq 0$. Consider the intersection $\mathcal{P}^1 \cap \mathcal{P}^2 = [-\zeta, \zeta]$ Then the economy has an equilibrium with $c^* = \alpha U$ and price $p^* = \exp(-\frac{1}{2}\alpha^2)$, where $\alpha \in [-\zeta, \zeta]$.*

In particular, this theorem includes the following two special cases which we formulate as corollaries.

Corollary 5.19. *Suppose $\mathcal{P}^1 = [-\kappa, \kappa]$ and $\mathcal{P}^2 = [-\eta, \eta]$, for $\kappa, \eta \geq 0$. Then the economy has an equilibrium with $c^* = \alpha U$ and price $p^* = \exp(-\frac{1}{2}\alpha^2)$, where $\alpha \in \mathcal{P}^1 \cap \mathcal{P}^2$.*

Corollary 5.20. *Suppose $\mathcal{P}^1 = \mathcal{P}^2 = [-\kappa, \eta]$, for $\kappa, \eta \geq 0$. Then the economy has an equilibrium with $c^* = \alpha U$ and price $p^* = \exp(-\frac{1}{2}\alpha^2)$, where $|\alpha| \leq \min\{|\kappa|, |\eta|\}$.*

We now allow for prior sets which may have empty intersection.

Theorem 5.21. *Suppose $\omega^1 = -\omega^2 = R + U$, that agents have CARA-utility of degree 1 and that $\mathcal{P}^1 = [\kappa^1, \kappa^2]$ and $\mathcal{P}^2 = [\eta^1, \eta^2]$ for $\kappa^1, \kappa^2, \eta^1, \eta^2 \in \mathbb{R}$. The allocation αU is an equilibrium allocation with equilibrium price $\exp(-\frac{1}{2}\alpha^2)$ if $\alpha \in \mathcal{P}^1$ and $-\alpha \in \mathcal{P}^2$. In particular, if $0 \notin \mathcal{P}^1 \cap \mathcal{P}^2$, then the full insurance allocation is never an equilibrium allocation.*

One could now continue and formulate the equilibrium with inertia versions of the above theorems. The only new thing in the proofs is that the inertia condition has to be checked individually for both agents as the prior sets of the two agents are different. Moreover, one could combine the general endowment cases and general degree of constant absolute risk aversion cases with the cases where agents have different priors. We do not expect qualitatively new insights from these even more general robustness investigations, thus we stop the robustness analysis at this point.

5.5 Conclusion

We have investigated how agents interacting in a B-economy can share and trade risk and uncertainty. We have examined the set of equilibria in the B-economy. In particular, we have shown that the set of equilibria with inertia in the B-economy is indeterminate except in the limit cases, and a no trade of uncertainty equilibrium arises if the ambiguity κ is high enough. Furthermore, full insurance is not possible in the B-economy for sufficiently large ambiguity if we use the equilibrium notion of equilibrium with inertia. This may in part explain the recent liquidity crisis of the financial markets. Our results are robust to changes of the degree of constant absolute risk aversion. Furthermore, we have investigated the robustness of the assumptions on the prior sets and the endowments of the agents. An important next step is to formulate and analyze the continuous time version of the case study. In continuous time the prior set is convex and a full characterization of the equilibrium sets appears to be possible.

Our results depend on the specific construction of the prior set. The construction provides us with the neat "separation" of risk and Knightian uncertainty, the measure κ for the amount of ambiguity in the economy and allows the comparison of the equilibrium sets in different economies. Investigations with more than two agents or with two different kinds of agents therefore need a different setup.

Following the lines of Dana and Riedel (2013), we finally suggest an interpretation of our results in terms of financial market regulation. Two prevalent ways of regulation are risk measures and stress testing. Exhibiting a stress test means comparing the performance of a new allocation and a status quo allocation under different scenarios. The new allocation is only preferred to the status quo if it is preferable under all scenarios. With the scenarios described by the priors and the endowment serving as the status quo, the usage of B-preferences plus inertia can be interpreted as performing a stress test. For example, the resilience of European banks against different critical economic scenarios was stress tested recently.

A different approach to regulation was taken by Artzner et al. (1999). Their starting point was that a tool to measure risk in financial markets should satisfy certain desirable properties like subadditivity and monotonicity. They introduced coherent risk measures in Artzner et al. (1999). GS-preferences can be interpreted as coherent risk measures due to the representation result for coherent risk measures in Artzner et al. (1999).

Our results show an objectionable consequence of stress testing: The market for insurance dries up provided there is a sufficient amount of ambiguity in the economy and becomes smaller with further increasing ambiguity. Worst case regulation via coherent risk measures never leads to these economically bad no insurance equilibria. In this respect, coherent risk measures may thus be the better way to regulate.

While the latter interpretation is a valid and interesting interpretation of our results, a convincing general theory to evaluate and compare the implications of different forms of financial market regulation has yet to be formulated. In the light of the recent turmoil on financial markets, this remains an important and urgent task of economic theory.

5.6 Appendix

Proof. (Theorem 5.1)
The two agents have the same prior for the distribution of the risk R. If a feasible allocation contained risk for one agent it would contain also risk for the other. By exchanging this risk and thus trading it away completely both agents are better off and have higher expected utility due to their risk aversion. Thus equilibrium allocations do not contain risk components. □

Proof. (Theorem 5.4, indeterminacy part)
Suppose $\kappa \in (0, \infty)$. It suffices to show for indeterminacy that

$$-\kappa + \sqrt{2 + 2\kappa + \kappa^2} - (\kappa - \sqrt{2 - 2\kappa + \kappa^2}) > 0$$

as the interval in theorem 5.4 has positive length in this case. For finite κ it holds

$$-\kappa + \sqrt{2 + 2\kappa + \kappa^2} = -\kappa + \sqrt{(\kappa + 1)^2 + 1} > -\kappa + \kappa + 1 = 1,$$

and

$$\kappa - \sqrt{2 - 2\kappa + \kappa^2} = \kappa - \sqrt{(\kappa - 1)^2 + 1} < \kappa - (\kappa - 1) = 1,$$

which shows nondegeneracy of the interval and thus indeterminacy of the equilibrium with inertia set. □

Proof. (Corollary 5.5)

(i) Suppose $\kappa > 1$. Then

$$\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\} = \kappa - \sqrt{2 - 2\kappa + \kappa^2} = \kappa - \sqrt{(\kappa - 1)^2 + 1} > 0.$$

Thus the full insurance allocation which corresponds to the case $\alpha = 0$ is not an element of the interval $[\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\}]$. This implies that the full insurance allocation does not survive the equilibrium refinement of inertia if $\kappa > 1$.

(ii) Suppose $\kappa \geq 1$. Then

$$\min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\} \geq 1$$

as $\kappa \geq 1$ by assumption and $-\kappa + \sqrt{2 + 2\kappa + \kappa^2} = -\kappa + \sqrt{(\kappa + 1)^2 + 1} \geq 1$. On the other hand,

$$\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\} \leq 1$$

for all κ, as $\kappa - \sqrt{2 - 2\kappa + \kappa^2} = \kappa - \sqrt{(\kappa - 1)^2 + 1} \leq 1$ for all κ. This implies that the no trade of uncertainty allocation which corresponds to $\alpha = 1$ is always an element of the interval $[\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\}]$ if $\kappa \geq 1$. □

Proof. (Corollary 5.6)

(i)

$$-\kappa = \kappa - \sqrt{2 - 2\kappa + \kappa^2}$$
$$\Rightarrow \kappa = -\frac{1}{3} + \frac{\sqrt{7}}{3}.$$

The expression $\kappa - \sqrt{2 - 2\kappa + \kappa^2}$ is strictly monotonically increasing as the derivative is

$1 - \frac{\kappa - 1}{\sqrt{(\kappa - 1)^2 + 1}} > 0$ for finite κ. This shows (i).

(ii) We show (ii) similarly to (i).

$$\kappa = -\kappa + \sqrt{2 + 2\kappa + \kappa^2}$$
$$\Rightarrow \kappa = +\frac{1}{3} + \frac{\sqrt{7}}{3}.$$

The expression $-\kappa + \sqrt{2 + 2\kappa + \kappa^2}$ is strictly monotonically decreasing as the derivative is $-1 + \frac{\kappa+1}{\sqrt{(\kappa+1)^2+1}} < 0$ for finite κ. This proves (ii).

(iii) As the slope of the function $\kappa - \sqrt{\kappa^2 - 2\kappa + 2}$ is larger than one for $\kappa < 1$ and smaller than one for $\kappa > 1$, the only candidates where the equilibrium set may be thickest are $-\frac{1}{3} + \frac{\sqrt{7}}{3}$ and $\frac{1}{3} + \frac{\sqrt{7}}{3}$. The thickness of the interval at these points is $-\frac{2}{3} + \frac{2\sqrt{7}}{3} \approx 1.097$ and $\sqrt{(\frac{1}{3} + \frac{\sqrt{7}}{3} - 1)^2 + 1} \approx 1.023$, respectively. Thus the interval is thickest at the point $\kappa = -\frac{1}{3} + \frac{\sqrt{7}}{3}$.

(iv) The upper bound of the interval converges monotonously from above against 1, $-\kappa + \sqrt{(\kappa+1)^2 + 1} \to 1$ for $\kappa \to \infty$.
The lower bound of the interval converges monotonously from below against 1, $\kappa - \sqrt{(\kappa-1)^2 + 1} \to 1$ for $\kappa \to \infty$. Thus the thickness of the interval converges against 0 for $\kappa \to \infty$.

(v) For $\kappa \leq -\frac{1}{3} + \frac{\sqrt{7}}{3}$ it holds $[\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\}] = [-\kappa, \kappa]$, which implies (v).

(vi) For $\kappa \geq \frac{1}{3} + \frac{\sqrt{7}}{3}$ it holds $[\max\{-\kappa, \kappa - \sqrt{2 - 2\kappa + \kappa^2}\}, \min\{\kappa, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}\}] = [\kappa - \sqrt{2 - 2\kappa + \kappa^2}, -\kappa + \sqrt{2 + 2\kappa + \kappa^2}]$. As $\kappa - \sqrt{2 - 2\kappa + \kappa^2}$ is strictly monotonically increasing in κ and $-\kappa + \sqrt{2 + 2\kappa + \kappa^2}$ is strictly monotonically decreasing in κ, (vi) is implied. □

Proof. (Theorem 5.10) The proof follows the lines of the proof of theorem 5.2. We begin by equalizing the marginal rates of substitution to calculate

equilibrium consumption c^{1*} and then calculate the equilibrium price. As the calculations do not depend on the endowment we get the same results as in theorem 5.2, i.e. $c^{1*} = \frac{1}{2}(\alpha - \beta)U - \frac{1}{4}(\alpha^2 - \beta^2)$ and
$p^{1*} = p^* = \exp\left(\frac{1}{2}(\alpha+\beta)U - \frac{1}{4}(\alpha^2+\beta^2)\right)$.

Markets clear due to the normalization of consumption. For the budget constraint the following equality has to hold for the first agent.

$$E^{P_0} p^*(\omega - c^{1*}) = 0.$$

This leads in the general endowment case to

$$E^{P_0}\left(\exp\left(\frac{1}{2}(\alpha+\beta)U - \frac{1}{4}\left(\alpha^2+\beta^2\right)\right)\left(aR + bU - \frac{1}{2}(\alpha-\beta)U + \frac{1}{4}(\alpha^2+\beta^2)\right)\right) = 0.$$

Suppose now $\alpha = -\beta$. The equation simplifies to

$$E^{P_0}\left(\exp(-\frac{1}{2}\alpha^2)(aR + bU - \alpha U)\right) = \exp(-\frac{1}{2}\alpha^2)E^{P_0}(aR + bU - \alpha U) = 0,$$

as U and R are standard normal distributions under P_0.

Walras' law guarantees that the budget constraint is also satisfied for the second agent. This completes the proof. □

Proof. (Theorem 5.11) The proof follows the lines of the proof of theorem 5.4. We take equilibrium consumption and check when the inertia property is satisfied. Q is a probability measure from the prior set $\mathcal{P}$ and γ the expectation of U which corresponds to Q.

$$\begin{aligned} -E^Q \exp(-c^*) &\geq -E^Q \exp(-\omega) \\ \Rightarrow -E^Q \exp(-\alpha U) &\geq -E^Q \exp(-aR - bU) \\ \Rightarrow E^{P_0} \exp\left(-\alpha U + \gamma U - \frac{1}{2}\gamma^2\right) &\leq E^{P_0} \exp\left(-aR - bU + \gamma U - \frac{1}{2}\gamma^2\right) \\ \Rightarrow E^{P_0} \exp\left((\gamma - \alpha)U - \frac{1}{2}\gamma^2\right) &\leq E^{P_0} \exp\left((\gamma - b)U - aR - \frac{1}{2}\gamma^2\right). \end{aligned}$$

Both random variables U and R are standard normal under P_0, so that Laplace Transformation yields

$$\Rightarrow \exp\left(\frac{1}{2}(\gamma-\alpha)^2 - \frac{1}{2}\gamma^2\right) \leq \exp\left(\frac{1}{2}a^2 + \frac{1}{2}(\gamma-b)^2 - \frac{1}{2}\gamma^2\right),$$

which simplifies to

$$(\gamma-\alpha)^2 \leq a^2 + (\gamma-b)^2.$$

This expression can be written as

$$\alpha^2 - b^2 - 2\gamma(\alpha-b) \leq a^2.$$

This inequality has to hold for all $\gamma \in [-\kappa, \kappa]$. Thus we have to check the most extreme priors $\gamma = -\kappa$ and $\gamma = \kappa$. We distinguish between the following cases.

$$\alpha^2 - b^2 - 2\kappa(\alpha-b) \leq a^2 \text{ if } \alpha \leq b, \tag{5.6}$$

and

$$\alpha^2 - b^2 + 2\kappa(\alpha-b) \leq a^2 \text{ if } \alpha > b. \tag{5.7}$$

We solve these two expressions for α, starting with 5.6

$$\Rightarrow \alpha^2 - 2\kappa\alpha + 2\kappa b - a^2 - b^2 \geq 0$$

$$\Rightarrow \alpha \geq \kappa - \sqrt{\kappa^2 - 2\kappa b + a^2 + b^2}.$$

For 5.7 we get in a similar way

$$\Rightarrow \alpha^2 + 2\kappa(\alpha-b) - b^2 - a^2 \leq 0$$

$$\Rightarrow \alpha \leq -\kappa + \sqrt{\kappa^2 + 2\kappa b + a^2 + b^2}.$$

Thus we have established the borders of the theorem and the condition for α is

$$\alpha \in [\max\{-\kappa, \kappa - \sqrt{\kappa^2 - 2\kappa b + a^2 + b^2}\}, \min\{\kappa, -\kappa + \sqrt{\kappa^2 + 2\kappa b + a^2 + b^2}\}].$$

□

Proof. (Corollary 5.12) The proof follows by setting $b = 0$. □

Proof. (Corollary 5.13) The argument is given in the text before the statement of the corollary. □

Proof. (Theorem 5.14) The proof follows again the lines of the proof of theorem 5.2. Suppose the degree of constant absolute risk aversion is a. The calculation of equilibrium consumption yields

$$-\exp(-ac^{1*})q^1 = -\exp(-ac^{2*})q^2$$

As $c^{1*} = -c^{2*}$, this solves to

$$c^{1*} = \frac{\frac{1}{2}(\alpha - \beta)U - \frac{1}{4}(\alpha^2 - \beta^2)}{a}.$$

For the equilibrium price we then get

$$\begin{aligned} -\exp(-ac^{1*})q^1 &= -\exp(ac^{1*})q^2 \\ \Rightarrow \exp\left(\frac{1}{2}(\alpha + \beta)U - \frac{1}{4}(\alpha^2 + \beta^2)\right) &= p^{1*} = p^{2*} = p^*. \end{aligned}$$

The equilibrium price is independent of the degree of constant absolut risk aversion a. The equilibrium consumption has to be normalized with the CARA degree. We now assume $\alpha = -\beta$ and show that the budget constraint $E^{P_0}p^*(\omega - c^{1*}) = 0$ holds. Plugging in the values for equilibrium consumption and equilibrium price leads to

$$E\left(\exp\left(-\frac{1}{2}\alpha^2\right)\left(R + U - \frac{\alpha U}{a}\right)\right) = \exp\left(-\frac{1}{2}\alpha^2\right)E^{P_0}\left(R + U - \frac{\alpha U}{a}\right) = 0,$$

as R and U are standard normal under P_0. This completes the proof. □

Proof. (Theorem 5.15) We check when the inertia property holds. As before, let Q denote a prior from the prior set $\mathcal{P}$ with corresponding expectation γ.

$$\begin{aligned} &-E^Q\left(\exp\left(-a\frac{\alpha U}{a}\right)\right) \geq -E^Q\left(\exp(-a(R+U))\right) \\ \Rightarrow\ &E^{P_0}\left(\exp\left(-\alpha U\right)\exp(\gamma U - \frac{1}{2}\gamma^2)\right) \leq E^{P_0}\left(\exp(-a(R+U))\exp(\gamma U - \frac{1}{2}\gamma^2)\right). \end{aligned}$$

Laplace transformation yields

$$\Rightarrow \exp\left(\frac{(\gamma-\alpha)^2}{2}-\frac{1}{2}\gamma^2\right) \leq \exp\left(\frac{a^2}{2}+\frac{(\gamma-a)^2}{2}-\frac{1}{2}\gamma^2\right).$$

Taking the logarithm and simplifying leads to

$$\Rightarrow \frac{\alpha^2}{2}+\gamma\,(a-\alpha) \leq a^2.$$

This inequality has to hold for all γ in the prior set, thus we only have to consider the most extreme priors κ and $-\kappa$. We distinguish between the following two cases.

$$\frac{\alpha^2}{2}-\kappa\,(a-\alpha) \leq a^2 \text{ if } \alpha \geq a, \tag{5.8}$$

and

$$\frac{\alpha^2}{2}+\kappa\,(a-\alpha) \leq a^2 \text{ if } \alpha < a. \tag{5.9}$$

We solve these two expressions for α, starting with 5.8.

$$\Rightarrow \alpha^2+2\kappa\alpha-2a^2-2a\kappa^2 \leq 0$$

$$\Rightarrow \alpha \leq -\kappa+\sqrt{\kappa^2+2a^2+2a\kappa}$$

For the second expression 5.7 we get

$$\Rightarrow \alpha^2-2\kappa\alpha+(\kappa-1)2a^2 \leq 0$$

$$\Rightarrow \alpha \geq \kappa-\sqrt{\kappa^2-2a\kappa+2a^2},$$

which concludes the proof. □

Proof. (Theorem 5.16) We follow once again the lines of theorem 5.2. Equalizing the marginal rates of substitution yields the equilibrium consumption.

$$\exp(-zc^{1*})q^1 = \exp(-zc^{2*})q^2$$

$$\Rightarrow \exp(-zc^{1*})\exp\left(\alpha U-\frac{1}{2}\alpha^2\right) = \exp(zc^{1*})\exp\left(\beta U-\frac{1}{2}\beta^2\right)$$

$$\Rightarrow c^{1*} = \frac{\frac{1}{2}(\alpha-\beta)U-\frac{1}{4}(\alpha^2-\beta^2)}{z}$$

The equilibrium price is

$$p^{1*} = \exp(-zc^{1*})q^1 = \exp\left(-\frac{1}{2}(\alpha-\beta)U + \frac{1}{4}(\alpha^2-\beta^2)\right)\exp\left(\alpha U - \frac{1}{2}\alpha^2\right)$$
$$= \exp\left(\frac{1}{2}(\alpha+\beta)U - \frac{1}{4}(\alpha^2+\beta^2)\right)$$

For the second agent

$$p^{2*} = \exp(-zc^{2*})q^2 = \exp\left(\frac{1}{2}(\alpha-\beta)U - \frac{1}{4}(\alpha^2-\beta^2)\right)\exp\left(\beta U - \frac{1}{2}\beta^2\right)$$
$$= \exp\left(\frac{1}{2}(\alpha+\beta)U - \frac{1}{4}(\alpha^2+\beta^2)\right).$$

We now check the budget constraint.

$$E^{P_0}(p^*(\omega^1 - c^{1*}))$$
$$= E^{P_0}\exp\left(\frac{1}{2}(\alpha+\beta)U - \frac{1}{4}(\alpha^2+\beta^2)\right)\left(aR + bU - \frac{\frac{1}{2}(\alpha-\beta)U - \frac{1}{4}(\alpha^2-\beta^2)}{z}\right)$$

For $\alpha = -\beta$ this simplifies to

$$E^{P_0}\exp(-\frac{1}{2}\alpha^2)\left(aR + bU - \frac{\alpha U}{z}\right) = \exp(-\frac{1}{2}\alpha^2)E^{P_0}\left(aR + bU - \frac{\alpha U}{z}\right) = 0$$

which completes the proof. □

Proof. (Theorem 5.17) We follow the lines of theorem 5.4 and check the inertia property.

$$-E^Q\exp(-zc^*) \geq -E^Q\exp(-z(aR+bU))$$
$$\Rightarrow E^Q\exp(-(\alpha U) \leq E^Q\exp(-z(aR+bU))$$
$$\Rightarrow E^{P_0}(\exp(-(\alpha U))\exp(\gamma U - \frac{1}{2}\gamma^2)) \leq E^{P_0}(\exp(-z(aR+bU))\exp(\gamma U - \frac{1}{2}\gamma^2))$$

Laplace transformation yields

$$\Rightarrow \exp(\frac{(\gamma-\alpha)^2}{2} - \frac{\gamma^2}{2}) \leq \exp(\frac{(za)^2}{2} + \frac{(\gamma-bz)^2}{2} - \frac{1}{2}\gamma^2)$$
$$\Rightarrow \frac{\alpha^2}{2} - \frac{(za)^2}{2} - \frac{(zb)^2}{2} + \gamma(zb-\alpha) \leq 0$$

We consider the extreme priors defined by $\gamma = \kappa$ and $\gamma = -\kappa$ and make the following case distinction:

$$\Rightarrow \frac{\alpha^2}{2} - \frac{(za)^2}{2} - \frac{(zb)^2}{2} + \kappa(zb - \alpha) \leq 0 \text{ if } (zb - \alpha) \geq 0, \tag{5.10}$$

and

$$\Rightarrow \frac{\alpha^2}{2} - \frac{(za)^2}{2} - \frac{(zb)^2}{2} - \kappa(zb - \alpha) \leq 0 \text{ if } (zb - \alpha) < 0, \tag{5.11}$$

We solve these expressions for α, beginning with 5.10.

$$\Rightarrow \alpha^2 - 2\kappa\alpha - (za)^2 - (zb)^2 + 2\kappa zb \leq 0$$

$$\Rightarrow \alpha = \kappa \pm \sqrt{\kappa^2 + (za)^2 + (zb)^2 - 2\kappa zb}.$$

The second inequality 5.11 becomes

$$\Rightarrow \alpha^2 + 2\kappa\alpha - (za)^2 - (zb)^2 - 2\kappa zb \leq 0$$

$$\Rightarrow \alpha = -\kappa \pm \sqrt{\kappa^2 + (za)^2 + (zb)^2 + 2\kappa zb}.$$

□

Proof. (Theorem 5.18) Consider the intersection of the prior set $\mathcal{P}^\cap = \mathcal{P}^1 \cap \mathcal{P}^2$. It holds $0 \in \mathcal{P}^\cap$. For the prior set $\mathcal{P}^\cap$ the proof of theorem 5.18 is identical to the proof of theorem 5.2. □

Proof. (Corollary 5.19) Follows from theorem 5.18 by setting $\kappa^1 = \kappa^2 = \kappa$ and $\eta^1 = \eta^2 = \eta$. □

Proof. (Corollary 5.20) Follows from theorem 5.18 by setting $\kappa^1 = \eta^1 = \kappa$ and $\kappa^2 = \eta^2 = \eta$. □

Proof. (Theorem 5.21) We now consider the case where the prior sets of both agents have an empty intersection. Suppose $\alpha \in \mathcal{P}^1$ and $-\alpha \in \mathcal{P}^2$. Then the fact that αU is an equilibrium allocation and $\exp(-\frac{1}{2}\alpha^2)$ is an equilibrium price is proven as in theorem 5.2. The full insurance allocation corresponds to $\alpha = 0$. Thus, if this α is not an element of one of the prior sets, than the full insurance allocation is not an equilibrium allocation.

$\square$

References

Rosa Abrantes-Metz and David Evans. Replacing the libor with a transparent and reliable index of interbank borrowing. *Available at SSRN 2142878*, 2012a.

Rosa Abrantes-Metz and David Evans. Will the wheatley recommendations fix libor? *CPI Antitrust Chronicle November*, 2012b.

Rosa M Abrantes-Metz, Sofia B Villas-Boas, and George Judge. Tracking the libor rate. *Applied Economics Letters*, 18(10):893–899, 2011.

Rosa M Abrantes-Metz, Michael Kraten, Albert D Metz, and Gim S Seow. Libor manipulation? *Journal of Banking & Finance*, 36(1):136–150, 2012.

Philippe Artzner, Freddy Delbaen, Jean-Marc Eber, and David Heath. Coherent measures of risk. *Mathematical finance*, 9(3):203–228, 1999.

Marco Battaglini. Multiple referrals and multidimensional cheap talk. *Econometrica*, 70(4):1379–1401, 2002.

Truman F Bewley. Knightian decision theory. part i. *Decisions in economics and finance*, 25(2):79–110, 2002.

Antoine Billot, Alain Chateauneuf, Itzhak Gilboa, and Jean-Marc Tallon. Sharing beliefs: between agreeing and disagreeing. *Econometrica*, 68(3): 685–694, 2000.

Willem H Buiter. Central banks and financial crises. 2008.

Archishman Chakraborty and Rick Harbaugh. Comparative cheap talk. *Journal of Economic Theory*, 132(1):70–94, 2007.

Archishman Chakraborty and Rick Harbaugh. Persuasion by cheap talk. *The American Economic Review*, 100(5):2361–2382, 2010.

Jiakai Chen. Libor's poker: Discover interbank borrowing costs amid tacit collusion. Technical report, Working Paper, 2012.

Vincent P Crawford and Joel Sobel. Strategic information transmission. *Econometrica*, pages 1431–1451, 1982.

Rose-Anne Dana and Frank Riedel. Intertemporal equilibria with knightian uncertainty. *Journal of Economic Theory*, 2013.

Claude d'Aspremont and Louis-André Gérard-Varet. Incentives and incomplete information. *Journal of Public economics*, 11(1):25–45, 1979.

Freddy Delbaen. The structure of m–stable sets and in particular of the set of risk neutral measures. In *In Memoriam Paul-André Meyer*, pages 215–258. Springer, 2006.

Alexander Eisl, Rainer Jankowitsch, and Marti Subrahmanyam. Are interest rate fixings fixed? an analysis of libor and euribor. *An Analysis of Libor and Euribor (January 15, 2013)*, 2013.

Daniel Ellsberg. Risk, ambiguity, and the savage axioms. *The Quarterly Journal of Economics*, pages 643–669, 1961.

Jun Anthony Garcia. 'fixing the benchmark'-wheatley considers libor overhaul. *Financial Regulation International (September, 2012)*, 2012.

Itzhak Gilboa and Massimo Marinacci. Ambiguity and the bayesian paradigm. In *Advances in economics and econometrics, tenth world congress*, volume 1, 2011.

Itzhak Gilboa and David Schmeidler. Maxmin expected utility with non-unique prior. *Journal of mathematical economics*, 18(2):141–153, 1989.

Francesca Gino, Shahar Ayal, and Dan Ariely. Contagion and differentiation in unethical behavior the effect of one bad apple on the barrel. *Psychological Science*, 20(3):393–398, 2009.

Maria Goltsman, Johannes Hörner, Gregory Pavlov, and Francesco Squintani. Mediation, arbitration and negotiation. *Journal of Economic Theory*, 144(4):1397–1420, 2009.

Theodore Groves. Incentives in teams. *Econometrica: Journal of the Econometric Society*, pages 617–631, 1973.

Jacob Gyntelberg and Philip Wooldridge. Interbank rate fixings during the recent turmoil. *BIS Quarterly Review, March*, 2008.

Lars Peter Hansen and Thomas J Sargent. *Robustness*. Princeton university press, 2008.

John C Harsanyi. Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International Journal of Game Theory*, 2(1):1–23, 1973.

Chip Heath and Amos Tversky. Preference and belief: Ambiguity and competence in choice under uncertainty. *Journal of risk and uncertainty*, 4 (1):5–28, 1991.

Meglena Jeleva and Bertrand Villeneuve. Insurance contracts with imprecise probabilities and adverse selection. *Economic Theory*, 23(4):777–794, 2004.

Daniel Kahneman and Amos Tversky. On the reality of cognitive illusions. 1996.

Frank H Knight. *Risk, Uncertainty and Profits. Boston.* MA: Hart, Schaffner & Marx, 1921.

Kostas Koufopoulos and Roman Kozhan. Optimal insurance under adverse selection and ambiguity aversion. *Available at SSRN 1732268*, 2012.

Dennis Kuo, David Skeie, and James Vickery. A comparison of libor to other measures of bank borrowing costs. *Federal Reserve Bank of New York, Working Paper*, 2012.

Ming Li and Kristóf Madarász. When mandatory disclosure hurts: Expert advice and conflicting interests. *Journal of Economic Theory*, 139(1): 47–74, 2008.

John Morgan and Phillip C Stocken. An analysis of stock recommendations. *RAND Journal of Economics*, pages 183–203, 2003.

Sujoy Mukerji and Jean-Marc Tallon. An overview of economic applications of david schmeidler's models of decision making under uncertainty. *Uncertainty in economic theory*, 63:283, 2004.

Pierre Picard. Participating insurance contracts and the rothschild-stiglitz equilibrium puzzle. 2009.

Luca Rigotti and Chris Shannon. Uncertainty and risk in financial markets. *Econometrica*, 73(1):203–243, 2005.

Michael Rothschild and Joseph Stiglitz. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *The quarterly journal of economics*, pages 629–649, 1976.

W. Samuelson and R. Zeckhauser. Status quo bias in decision making. *Journal of risk and uncertainty*, 1(1):7–59, 1988.

David Schmeidler. Subjective probability and expected utility without additivity. *Econometrica: Journal of the Econometric Society*, pages 571–587, 1989.

Tali Sharot. The optimism bias. *Current Biology*, 21(23):R941–R945, 2011.

Connan Snider and Thomas Youle. Does the libor reflect banks' borrowing costs? *Available at SSRN 1569603*, 2010.

Connan Snider and Thomas Youle. The fix is in: Detecting portfolio driven manipulation of the libor. *Available at SSRN 2189015*, 2012.

Joel Sobel. Giving and receiving advice. In *Advances in Economics and Econometrics*, 2013.

Ola Svenson. Are we all less risky and more skillful than our fellow drivers? *Acta Psychologica*, 47(2):143–148, 1981.

Martin Wheatley. The wheatley review of libor: Initial discussion paper. *HM Treasury*, 1, 2012a.

Martin Wheatley. The wheatley review of libor: Final report. *HM Treasury*, 2012b.

Menahem E Yaari. The dual theory of choice under risk. *Econometrica: Journal of the Econometric Society*, pages 95–115, 1987.

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Summary

This dissertation studies the robustness of influential cheap talk equilibria to uncertainty about the preferences, the LIBOR mechanism from a game theoretic point of view, and applications of ambiguity theory with a particular focus on insurance questions.

The main chapters of this thesis are based on four articles and self-contained. Chapter 2 is joint work with Christoph Kuzmics. We investigate the set of equilibria in the model of Chakraborty and Harbaugh (2010) when the receiver faces uncertainty about the preferences of the sender and prove that there is a qualitative difference to the situation in the model of Crawford and Sobel (1982). Uncertainty about the preferences in the framework of Chakraborty and Harbaugh leads to the nonexistence of all influential equilibria, while in the framework of Crawford and Sobel uncertainty about the bias may still allow for influential communication in equilibrium.

The London Interbank Offered Rate (LIBOR) has been manipulated for many years. We study in chapter 3 the underlying mechanism from the game theoretic perspective and examine how and why different statistics lead to different quote patterns in equilibrium. In particular, we are, on the descriptive side, able to understand the observed quote patterns in the real world, and, on the normative side, able to give advice what kind of changes to the current mechanism may attenuate the problem of the manipulability of the LIBOR.

In chapters 4 and 5 we introduce model uncertainty into the classic insurance market model by Rothschild and Stiglitz (1976) and examine the static version of the case study introduced by Dana and Riedel (2013). Most importantly, we present a new solution to the Rothschild-Stiglitz equilibrium puzzle, and describe precisely the set of equilibria which survives the equilibrium refinement inertia, a refinement introduced by Bewley (2002).

Keywords

Cheap Talk, Influential, Uncertainty, Bias, Preferences, Robust, LIBOR, Game Theory, Mechanism, Manipulation, Model Uncertainty, Ambiguity, Existence, Inertia, Bewley, Incomplete, Risk Aversion, Ambiguity Aversion

Résumé en Français

La présente thèse étudie la robustesse de l'équilibre en communication gratuite qui influe sur l'incertitude concernant les préférences ainsi que le mécanisme du LIBOR du point de vue de la théorie des jeux et les applications de la théorie de l'ambiguïté, l'accent étant particulièrement mis sur les questions d'assurance.

Le principaux chapitres de cette thèse se fondent sur quatre articles et sont indépendants les uns des autres. Le chapitre 2 a été rédigé conjoinement avec Christoph Kuzmics. Nous avons étudié l'ensemble des équilibres du modèle de Chakraborty and Harbaugh (2010) lorsque le récepteur fait face à une incertitude concernant les préférences de l'envoyer et prouvé qu'il y a une différence qualitative par rapport à la situation du modèle de Crawford and Sobel (1982). L'incertitude sur les préférences dans le cadre de Chakraborty and Harbaugh amène à nier l'existence de tout équilibre influencé, alors qu'avec Crawford and Sobel, l'incertitude sur le biais n'empêche pas la communication influente à l'équilibre.

Des manipulation du London Interbank Offered Rate (LIBOR) sont pratiquées depuis des années. Au chapitre 3, nous avons étudié le mécanisme sous-jacent du point de vue de la théorie des jeux et examiné comment et pourquoi des statistiques différentes amènent à différent modèles de cotations à l'équilibre. En particulier, du point de vue déscriptif, nous avons pu suivre les différents modèles de cotation à l'équilibre dans le monde réel et, du point de vue normatif, proposer les types de modifications du mécanisme actuel qui pourraient contribuer à atténuer la manipulabilité du LIBOR.

Aux chapitres 4 et 5, nous avons introduit l'incertitude modélisée dans le modèle classique du marché de l'assurance de Rothschild and Stiglitz (1976) et examiné la version statique de l'étude de cas introduite par Dana and Riedel (2013). Et, surtout, nous présentons une nouvelle solution à l'énigme de l'équilibre de Rothschild and Stiglitz et décrivons avec précision l'ensemble des équilibres qui subsistent à l'inertie du raffinement dans la recherche de l'équilibre, raffinement introduit par Bewley (2002).

Mots clés

Communication gratuite, Influent, Incertitude, Biais, Préférences, Robuste, LIBOR, Théorie des jeux, Mécanisme, Manipulation, Incertitude de modèle, Ambiguïté, Existence, Inertie, Bewley, Aversion du risque, Aversion de l'ambiguïté.