

# **A normative approach to the stability of interbank and banks-firms systems by means of a multi-agent model**

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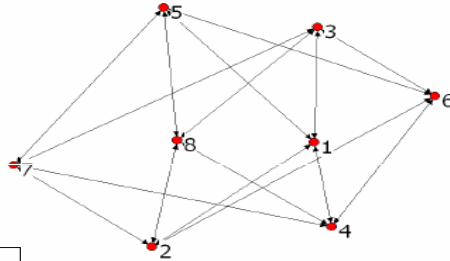
**Zweitgutachter**

Jr. Prof. Dr. Sander van der Hoog, Universität Bielefeld

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# Interbank Network

If we consider a system with 25 banks and also the direction of the links we have:



#  $N = 2^{600}$  possible networks

Which one is the best for the stability of the financial market?

*“The policy predictions of the models that are in use aren’t wrong, they are simply non-existent.”*

J.D. and D. Foley. The economy needs agent-based modeling. Nature.

*“Economic theory failed to envisage even the possibility of a financial crisis like the present one. A new foundation is needed that takes into account the interplay between heterogeneous agents.”*

Lux, T. and F. Westerhoff. Economics crisis. Nature Physic.



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*Dedicato a mia madre, mio padre, mia sorella e a tutto il resto della famiglia.*



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# 1 Introduction to the Topic

The stability of the interbank system is a very important topic in economics. In the last century the capitalistic society has suffered several times from financial instability such as the Wall Street crash of 1929 and the financial crisis of 2008.

The economic failure of Lehman Brother and Washington Mutual in 2008, which created panic and contagion in the interbank market, was caused by sub-prime mortgage foreclosures. The origin of such a problem dates back to 2006 and was caused by the US real estate bubble which “deflated” because of an increase of interest rate and, hence, the insolvency of several owners of sub-prime mortgages. Although the problem originated in US financial services, it quickly spreads to other countries and other sectors. In Europe one of the first country to feel the effect of recession was Denmark, followed by Iceland, Ireland, Portugal, Greece and Italy. Thus we can conclude that the contagions between the elements of a system (for example, bank-bank) or among different systems (for example, banks-countries) is an economic topic that must be analysed as well as studied to understand and, if possible, to prevent the negative consequences of such contagions.

The influences exerted by entities can have on the rest of a system constitutes a topic that has received a lot of attention from several researchers during the past few years. Many interlinked elements constitute a network, and social network analysis is the field of science that studies these relations. Jackson (2008) summarizes the contribution of these studies regarding connections among general entities for example, among people. Modern financial systems show a high degree of interdependence: banks can exchange money with other banks, firms can be linked with other firms to buy or sell products. The network analysis is a mathematical tool useful to understand these complex structures. Allen et al. (2008), in their "Networks in Finance", summarize the capacity of networks to describe complex structures in an economy. They analysed papers that worked on the network theory, in order to describe the contagion effect among banks as well as the effects of social networks by means of exchange of information on investment decisions. Haldane (2009) in his “Rethinking the financial network”, provides an introduction to different aspects such as topology, connectivity and stability. These topic are discussed in comparison with other studies in which the same topics are studied in different fields (eco-systems, wealth, among others). Figure 1 reports the banking network of Austria in 2003 as an example of the interbank structure.

This work deals with the relationships between two networks, banks and firms ones, and also with the interbank market. This work adopts a constructive approach

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and thus analyses the properties of these structures while proposing rules aimed at decreasing the financial instability of firms and banks. To test such rules we need an instrument that can simulate a stylized society comprising banks, interbank markets and firms. To handle such complex system we use a multi-agent approach (Wooldridge (2002)). This method models interacting agents, in our case banks and firms, that evolve over the time by means of discrete dynamical equations determining the behaviour and status of each element.

If social network analysis is a means to describe the property of a complex system in which simple components interact with each other and create a complex behaviour overall, a multi-agent system acts as the means to set up such systems. Taken together, they are an ideal mathematical tool to propose new rules with the aim to increase the stability of an economic system, overcoming the limit of classic hypothesis-bases economic analysis (rational agents, representative agent, among others) that does not show satisfactory results describing real life (Stiglitz (2009), Lux et al. (2009), Farmer et al. (2009)).

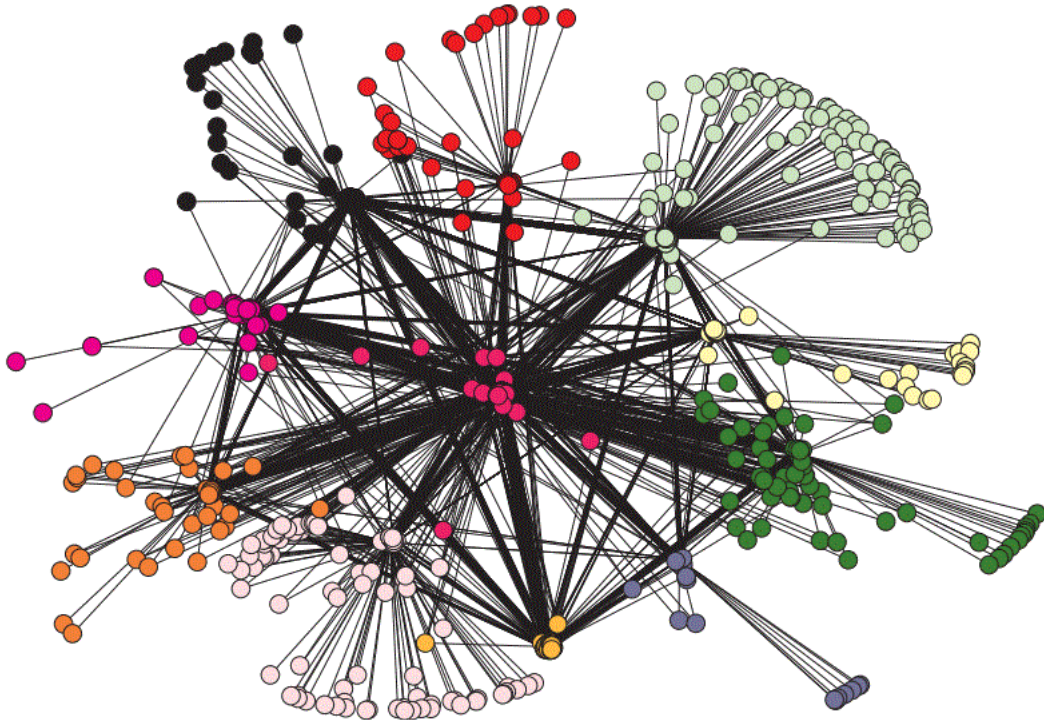


Figure 1: The banking network of Austria in 2003. Boss et al. (2003). The network topology of the interbank market.

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**Core Literature** The field of computer agent model has several applications including disaster response, online trading, and modelling of social structures. The last topic has assumed, after the economic crisis hit in the US banks and insurances sector and spread to other countries an important role as it enables one to understand and control systems which cannot be fully comprehended with traditional economic methods. Farmer et al. (2009) criticize the fact that most economic models used by the US government are based either on empirical statistical models, which are fitted to past data, or on ‘dynamic stochastic general equilibrium’, pointing out that the former method can fail in the case of great changes and the latter is based on unrealistic hypotheses. They propose to use the multi-agent system that allows us to analyse complex systems and provides results of simple interactions between the elements. This perspective is shared by Gallegati et al. (2005) who provide a model of society comprising banks and firms. They point out that a simple multi-agent model can describe phenomena describing real life events and this cannot be represented by the models based on representative agents. Ashraf et al. (2010) provide a work in which a society with firms and banks is simulated and the role of banks, which can facilitate the entry of trading firms and influence their exit decisions in the self-organized network, is studied. They analyse the conflicts between macroeconomic stability and microprudential bank regulations in scenarios where different returns from the entire economy are considered. The theory proposed by L. Arciero et al. (2008) is more oriented to economic crisis. Here a multi-agent based model of crisis simulation is presented. The purpose is to use this kind of simulation to test the stability and resilience of the financial systems. The authors note that a multi-agent system can perform better than normal stress testing methodologies by gathering the effect of complex, non linear response: a model based on multi-agent is a valid instrument for the Central Bank to calibrate interventions.

If the multi-agent system can simulate other systems, such as an interbank market, the social network analysis is an ideal instrument to study the properties of such complex systems. Soramaki et al. (2007) analyse the interbank structure of commercial banks, both before and after the impact of 11 September, over USA Fedwire system by using concepts of social network such as the number of links, connectivity, reciprocity, average degree, average path length, average eccentricity and clustering coefficient. Boss et al. (2003), starting from the Austrian bank balance sheet database, and also by using local entropy maximization (see Sheldon and Maurer (1998)) to fulfil the missing data in the interbank network, analyse the interbank system with degree distributions, clustering coefficient, and average shortest path length parameters. Müller (2006) presents an analogous study for the Swiss interbank market. Upper et al. (2002) analyse the German interbank market by using real data and local entropy maximization. The contagion among the banks is studied: the authors simply assumed that a bank fails if a prefixed percentage of loan is given to a previously failed bank is bigger than the bank’s book capital. They conclude that the failure of a single bank could lead to the breakdown of up to 15 % of the banking system in terms of assets even if there are the Central Bank

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regulation like limiting the exposure of each borrower bank to the group of debtors. Similar analysis are computed in Upper (2007). In Furfine (1999), an analysis of the Federal Reserve's large-value transfer system during February and March 1998 is carried out. The robustness of interbank relations is tested by forcing the failure of the most significant bank, the failure of the second most significant bank, the failure of the 10th such bank, and the joint failure of the two most significant banks. The result of this analysis shows that a wide spreading of the contagion of failed assets with respect to the percentage of the total amount is unlikely, but not impossible to happen. In Elsinger et al. (2006), the network analysis for the interbank market is repeated. Starting from the unique data-set provided by the Austrian Central Bank they run simulations to study the possibility of contagion among banks. The results is a low probability of contagion as well as the existence of some scenarios in which contagion accounts for up to 75% of all bank defaults. Another approach to study bank defaults and propagation of contagion in the interbank structure is to build an artificial interbank network as in Nier et al. (2007). In this paper a model is built and the study of default is analysed with regard to several banking parameters such as bank capitalization, probability of interbank linkages and size of shocks. Small increase in connectivity, the number of links among banks with respect to the total potential links, increases the contagion effect; however beyond a certain threshold value, connectivity improves the ability of a banking system to absorb shocks.

The multi-agent model proposed in this work is inspired mainly by papers describing the behaviour of firms, banks and the Central Bank activities. Iori et al. (2006) create and analyse a model with homogeneous and heterogeneous kinds of banks. Each bank receives a stochastic shock on its deposits, returns on investments and pays dividends. The simulation makes it evidence that, when banks are homogeneous, the insurance role of interbank lending prevails. In this situation, higher reserve requirements can lead to a higher incidence of bank failures. When banks are heterogeneous in average liquidity or average size, contagion effects may arise. The model of Delli Gatti et al. (2008) provides a simulation of a society comprising credit networks among firms and firms-banks. There are two kinds of firms: one that produces a single product, using intermediate goods, and the other that produces the intermediate product. The two layers of firms are connected by credit relations that can change step by step. Each firm can ask banks for money, if it needs liquidity to start the production; therefore there is also a credit network among firms and banks, even if an interbank market is not present. Di Guilmi et al. (2011) propose a model, analogous to the previous one, to analytically solve the problem by means of a master equation (ME), a tool used in mechanical statistics. It is a first-order differential equation, which quantifies the evolution through the time of the probability of observing a given number of agents in a certain state. A complete model with banks, interbank and firm sectors is proposed by Georg and Poschmann (2010). The key feature of this multi-agent model is the presence of the Central Bank that can provide loan money to banks by accepting only a percentage of the bank assets as securities. Here the Central Bank's presence always increases the

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stability of the system.

The interbank market and the connections between firms and banks are regularized by institutions like the Central Bank and the Government that impose rules (Basel Committee (2010), “Basel III, a global regulatory framework for more resilient banks and banking systems”, Basel Committee on Banking Supervision, Basel) and can intervene in the system to adapt to these rules, if necessary, to help banks and firms. Freixas et al. (2009) analyse the appropriate response of the Central Bank’s interest rate policy to banking crises. When using an analytical model with three dates, a continuum of competitive banks and a unit continuum of consumers, the results display how, during crisis periods, an intervention by the Central Bank to decrease the interbank interest rate facilitates the reallocation of liquid assets among banks. The results also show that in a period of no crisis, the Central Bank must grant that interbank rates are high enough to provide incentives for banks to hold enough liquid assets ex ante. For example, Gertler M. and N. Kiyotaki (2009), analyse direct Central Bank lending as a means to mitigate the impact of the crisis. In Allen et al. (2009) a study about the role of this institution in interbank market is provided, with suggestions to design policies aimed at preventing such crisis or mitigating its effect.

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## 1.1 Outline Of The Work

The questions that drive this thesis are:

- < How can the Government or other institutions like the Central Bank create some rules to determine interbank relationships in such a manner that we have an improvement on the problem of the stability of the interbank system? >
- < Has this regularized interbank system got properties similar to real life interbank markets?>
- < How can the Government or other institutions like the Central Bank create some rules to determine interbank and banks-firms relationships so as to get improvements in the stability of the entire economic system? >

To answer these questions we present a work organized in four chapters.

In Chapter 2 we introduce a multi-agent model that describes a society comprising banks and firms. This is the means that we use to propose, test, and calibrate normative approaches aimed at increasing the stability of the system. Section 2.1 presents the firms world is presented. We have two layers of firms: Upper-firms (U) that produce intermediate products and Down-firms (D) that produce the final output. Production process, workers required and prices of both products are analysed. Section 2.2 describes the relations that occur among banks and firms when they need loan to produce taking into account fixed and stochastic costs. Section 2.3 describes the method that D firms choose to select U firms and how the firms select banks in case a loan is necessary. In Sections 2.4, 2.5 and 2.6, the equations that update the net value of firms and banks considering the presence of potential loans, purchased/sold product, costs, credit/debts and oscillations in deposits for banks are developed. Section 2.6.1 deals with the description of the interbank connections. In Section 2.7 the values of the parameters of the model are reported. Section 2.7.1 describes the steps of the algorithm that is used to understand the time priority between the parts, firms and banks, presented in the model. Section 2.7.2 lists the difference between the proposed model and the two papers by which it is inspired. Section 2.8 describes the evolution of society, presenting the description of the code necessary to simulate it.

In Chapter 3 we propose a first normative approach to establish which connections among banks are allowed and which ones are denied in the interbank markets. The results are reported in Section 3.4. In subsection 3.4.1 a network analysis of the simulated interbank system is reported; while in the subsection 3.4.2 we compare the parameters characterizing this optimized market with the real interbanks' properties, in the USA and EU market.

In Chapter 4 we want to improve the result got in Chapter 3 by proposing further rules with the aim to decrease the instability of the system comprising banks and firms. In Section 4.1 we introduce an index, the difference of monthly returns between banks and firms systems. By this the Central Bank can change parameters,

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such as the interbank interest rate, in an appropriate way to increase the stability of the entire society presented in Chapter 2 and previously influenced by the rule proposed in Chapter 3. The results are then reported in Section 4.1.1. A stress analysis of the bank system is presented in Section 4.2.

In Chapter 5 we propose a statistical and sensitivity analysis of the model. This part will conclude with comments on the results, answers to the proposed questions, and proposals of future developments of the research.

Appendix. In this section an appendix regarding social network analysis and dealing with the introduced concepts that are used in chapter 3 to analyze the interbank market is presented.

In Chapter 5, we propose a statistical and sensitivity analysis of the model. This part will conclude with comments on the results, answers to the proposed questions, and proposals for future research. In the Appendix section, we prepare an appendix regarding the social network analysis and deal with the already introduced concepts used in Chapter 3 to analyse the presented interbank market.

The work presented in this thesis can be classified in the macroprudential framework for financial supervision and regulation:

“... in order to improve the safeguards against financial instability, it may be desirable to strengthen further the macroprudential orientation of current prudential frameworks, a process that is already under way.”  
(Borio (2003)).





## 2 The Model

In this chapter we create a model that describes a credit network characterized by credit relationships connecting downstream firms and upstream firms as well as credit relationships connecting these firms with a banking system. The model is inspired by Delli Gatti et al. (2008). In this paper a multi-agent system with banks and two layers of firms are modelled. The network, which is the connection among firms and among firms-banks, is not static (Delli Gatti et al. (2006)), but evolves step by step. The model has several restrictions on the conditions and thus this chapter implements the possibility of an interbank market playing the role of a safety network, inspired by Iori et al. (2006). The economy consists of three sectors: downstream firms D, an upstream sector U, and a banking system of Z banks. The D firms produce the final product, while the U firms produce the intermediate product that the D firms need for their production. In every period each D firm looks for the U firm with the lowest price of intermediate goods. At the same time every firm searches for the bank with the lowest interest rate. Each bank has the possibility to lend/borrow money by using interbank lending contracts. We assume that the D firms sell all the output they produce at a stochastic price. The number of firms and banks is exogenous, and the U firms do not hold inventories of intermediate goods because they produce them ‘on demand’ of the D firms.

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## 2.1 Firms

We assume the production function of each D firm  $i$

$$Y_i = \min \left( \frac{1}{\delta_d} N_i, \frac{1}{\gamma} Q_i \right) \quad (2.1)$$

where  $N$  is employment,  $Q$  is the intermediate product necessary to produce,  $\delta_d > 0, \gamma > 0$  are parameters. We assume Leontief production function. The level of production  $Y$  is then constrained at time  $t$

$$Y_{i,t} = \phi A_{i,t}^\beta \quad (2.2)$$

where  $\phi > 1, 0 < \beta < 1$  are parameters.  $A_{i,t}$  is the net value of D firm  $i$  at time  $t$ . This hypothesis, financially constrained output function, implies that for every D firm with a high net value, an increasing of its value allows the firm to have a small increase in the produced output.

Eqn. (2.2) has an economic meaning. Delli Gatti et al. (2008) point out that this hypothesis is consistent with the solution of a maximization problem: maximizing the expected profits net of bankruptcy costs weighted by the probability of bankruptcy (see Greenwald and Stiglitz (1993)).

The discrete-time equation that allows the system to update the net value of each D firms is described in Eqn. (2.18). From Eqn. (2.1) and Eqn. (2.2), we get

$$N_{i,t} = \delta_d \phi A_{i,t}^\beta \quad Q_{i,t} = \gamma \phi A_{i,t}^\beta \quad (2.3)$$

We assume that D firms sell all output, the final good, at each step of the algorithm and thus there are no stocks in the model.

The Upstream firms produce the intermediate good by means of a linear technology, which employs only labour

$$Q_{j,t} = \frac{1}{\delta_u} N_{j,t} \quad (2.4)$$

where  $\delta_u > 0$  and  $N_{j,t}$  is the work force for the U firm  $j$  at step  $t$ .

Many D firms can be linked to a single U firm but each D firm has only one supplier of intermediate goods. By assumption each failed U or D firm is replaced by a new

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one and each U firm does not have any reserve: it produces and sell the requested output. In the paragraph ‘Partner’ we will discuss the way in which the links between the two layers of firms are formed. This is done using the preferred-partner choice rule.

In each period the supplier  $j$  receives orders from a set of D customers which will be denoted by  $\Phi_j$ . The request of the intermediate good of the  $j$ -th U firms will be

$$Q_{j,t} = \gamma \sum_{i \in \phi_j} Y_{i,t} = \gamma \phi \sum_{i \in \phi_j} A_{i,t}^\beta \quad (2.5)$$

U firms produce exactly these amount of intermediate product. Hence the production of U firms is demand-constrained. The demand for labour is obtained by using eqn. (2.4):

$$N_{j,t} = \delta_u Q_{j,t} \quad (2.6)$$

The price the supplier is charging is defined as

$$P_{j,t} = \alpha A_{j,t}^{-\alpha} + p_f \quad (2.7)$$

where  $a > 0$  and  $p_f > 0$  is a constant minimum price.

That is: the price charged to each and every D firm belonging to  $\Phi_j$  is decreasing with the net value of the U firms. At the beginning of the simulation every U firm is, on average value, identical. Hence if a firm can sell product, its net value will increase. This decreasing shape implies a monopolistic behaviour of U firms: if a U firm has sold products, it increases its net value, then it will keep the price low to increase its number of consumers in spite of other U concurrent firms.

Every D firm produces the same kind of output. We assume that the cost  $u_t$  of this final product is stochastic. It is a random variable with uniform density.

$$u_t \sim U(u_{min,t}, u_{max,t})$$

so the price is the same for every D firm in a given step of the algorithm. We allow the expected value of the price to change with respect to the time-step by keeping the variance fixed: we assume a supply and demand law (Besanko, and Braeutigam (2002)) for the price; thus the more quantities of the final product is available at step  $t$ , the more the average cost of it decreases at step  $t + 1$ . Also the opposite is true: if the quantity of the final product is scarce, its average cost will be relatively high. We assume

$$\bar{u}_{t+1} = \alpha_{pr} + \beta_{pr} Y_t$$

$$Var(u) = v_{pr}$$

with  $\alpha_{pr} > 0$ ,  $\beta_{pr} \leq 0$ ,  $v_{pr} > 0$  and  $Y_t$  the total output produced by the D firms at step  $t$ .

A high realization of the random variable output price can be interpreted as a high demand regime that will increase the net value of the D firms in successive steps. Vice-versa, a relative low output price is a demand regime in which a low quantity of product is requested. Generic consumers buy in every step the entire amount of the available final product. In Figure 2, we report the schema of the principal components and relations in the model.

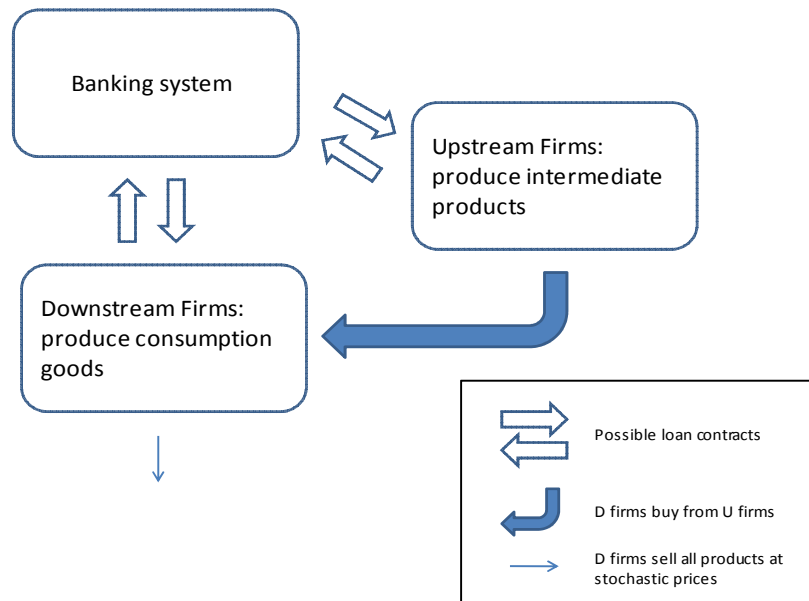


Figure 2: Principal components and relations in the model.

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## 2.2 Bank-Firm Exchange of Money

The financing gap, the difference between the firm's expenditures and net value of itself, is filled by bank credits.

For U firms, the financing gap is the difference between the wage bill, a deterministic cost, plus stochastic cost, and net value plus the value of the sold intermediate product. This demand for credit is

$$B_{j,t} = \max(0, W_{j,t} + \bar{\eta}\lambda_u\mu_u Q_{j,t} - E_{j,t} - A_{j,t}) \quad (2.8)$$

$$W_{j,t} = wN_{j,t} \quad E_{j,t} = P_{j,t}Q_{j,t} \quad (2.9)$$

where  $W_{j,t}$  is the wage bill, and  $w$  is the cost for one worker.  $E_{j,t}$  is the value of the intermediate output that firm  $j$  creates during the step of the algorithm  $t$ . In the demand for credit, we have a fixed cost due to workers as well as a stochastic cost due, for example, to maintenance of machines for production and cost of energy. It is proportional to the quantity of product. The variable  $\eta_{j,t}$  is supposed to be uniformly distributed in  $(0,1)$  and  $\bar{\eta}$  is the expected value equal to 0.5. The rule of the parameter  $\lambda_u$  is, exactly like  $w$  which is the cost for one worker for the fixed charge, the unitary cost of the stochastic charge.

When each U firm needs money, it cannot know exactly the stochastic cost a priori and thus it uses the expected value of the random variable  $\eta_{j,t}$  times a factor  $\mu_u$  that can guarantee enough money also in worse situations. In our case we simply impose  $\mu_u = 2$ , equivalent to a worst-case management of stochastic cost. We assume that each U firm will try to avoid complete risk: it will allocate, during each step of the algorithm, a quantity of money for the stochastic cost that is bigger or equal to the realization of it. If a firm allocates lower amount of money, it could fail if the realization of cost is bigger than the available quantity of money necessary to face the entire production.

According to these hypotheses, the U firm fails if it cannot get enough requested money from the banking system to have non-negative net value at the end of each month.

For D firms, we do not assume that the financing gap is the difference between the wage bill plus stochastic cost and net worth. We assume that the financing gap for D firms is the difference between the wage bill plus a random cost plus cost of intermediate product and net worth. It means that the acquisition of intermediate

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goods cannot be financed by means of trade credit: intermediate products must be paid at the beginning of each period simultaneously to the purchase of that material from U firms. Consequently, the cost of intermediate product for D firms is then a part of the financing gap, because D firms cannot have enough money to pay the provider.

The expenditures comprises of wages, the expected stochastic cost, and the cost of intermediate products.

$$B_{i,t} = \max(0, W_{i,t} + E_{i,t} + \bar{\eta}\lambda_d\mu_d Q_{i,t} - A_{i,t}) \quad (2.10)$$

$$W_{i,t} = wN_{i,t} \quad (2.11)$$

where  $E_{i,t} = p_{j,t}Q_{i,t}$  is the cost of the purchased intermediate products from U firms  $j$ .

The concept of stochastic cost is analogous to the cost of the U firms. We impose furthermore  $\mu_d = 2$ , equivalent to a worst-case management of the stochastic cost.  $\lambda_d$  is the unitary cost of the stochastic charge.

Following Delli Gatti et al. (2008), we assume out of simplicity that the required loan does not refer to a term relating to the income due to the selling of the final product.

We define self-financing at step  $t$  a firm that does not need bank loans, equivalent to have for U firm  $j$  and D firm  $i$ :

$$B_{x,t} = 0 \quad x = i, j \quad (2.12)$$

For example, a U firm  $j$  is self-financed at step  $t$  if

$$(w\delta_u + \bar{\eta}\lambda_u\mu_u - \alpha A_{j,t}^{-\alpha} - p_f) \gamma \phi \sum_{i \in \phi_j} A_{i,t}^\beta < A_{j,t}$$

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The terms outside the brackets are always non negative. This inequality is true if for example

$$w\delta_u + \bar{\eta}\lambda_u\mu_u < \alpha A_{j,t}^{-\alpha} + p_f$$

the price of its produced intermediate product (Eqn. 2.7) proposed by a U firm  $j$  in a step  $t$  is bigger than a critical value depending on cost of workers and stochastic cost, and thus it will be self financed during this step.

For a D firm  $i$  the self-financing condition is true if

$$W_{i,t} + E_{i,t} + \bar{\eta}\lambda_d\mu_d Q_{i,t} - A_{i,t} \geq 0$$

equivalent to:

$$\left( w\delta_d + \left( \alpha A_{j,t}^{-\alpha} + p_f \right) \gamma + \bar{\eta}\lambda_d\mu_d\gamma \right) \phi \geq A_{i,t}^{1-\beta}$$

Each firm looks for the bank with the lowest interest rate, which is computed, relative to the  $x$  firm, for the  $z$  bank:

$$r_{z,t}^x = \nu + \sigma A_{z,t}^\sigma + \theta (l_{x,t})^\theta \quad x = i, j \quad (2.13)$$

$$l_{x,t} = \frac{B_{x,t}}{A_{x,t}} \quad x = i, j \quad (2.14)$$

$l_{x,t}$  is the leverage ratio of the  $x$  firm,  $\sigma > 0, \theta > 0$  are constant parameters for bank world.  $i$  is D firm index,  $j$  is U firm index.

$\nu$  is the official discount rate set by the Central Bank (see Dawid et al. (2011)). It is an exogenous constant parameter in this chapter and indicates a minimum requested interest rate. In our model the Central Bank does not have the possibility to lend money, it has the possibility to make monetary policy (Chapter 4) and impose an interbank structure (Chapter 3). One aim of this work is to find a rule that will allow the Central Bank to change this parameter to decrease the instability in the system.

Interest rate proportional to the net value of the banks allows financial institutions, which have small net values, to propose lower interest rate, thereby increasing the

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possibility to be selected by firms that will ask for loans. The third part of the definition of the interest rate depends on the leverage ratio of the firm. Leverage is an index of the firm's bankruptcy probability, a relatively big value of it increases the requested risk premium (Bernanke and Gertler (1989)).

## 2.3 Partners

Each D firm has a relationship with one U firm. Initially, the network of firms is random, and thus the links among D and U firms are established at random. In the successive steps this network changes:

- The  $i$  D firm chooses a partner looking at the prices of a randomly selected number  $M$  of U firms. If the minimum observed price, say the price of  $j_t$  firm, is lower than the price offered by the selected U firm in previous step,  $j_{t-1}$ , then  $i$  will switch to  $j_t$ , if it is not the case,  $i$  will continue to deal with  $j_{t-1}$ . The consequences of these rules are that the total number of nodes is constant, but the branches of the networks can evolve step by step.
- This preferred-partner choice rule is applied to the relationship between D firms and banks as well as to U firms and banks. The D and U firms choose a bank partner looking at the interest rate of a randomly selected number  $N$  of banks. If the minimum observed interest rate  $r_{z_t}^x$  offered by bank  $z_t$  at step  $t$  to generic firm  $x = j, i$ , is lower than the interest rate proposed by the previous-step selected bank  $r_{z_{t-1}}^x$ , then  $j$  ( $i$ ) will switch to bank  $z_t$ . If it is not the case, firms will continue to be linked with  $z_{t-1}$ . The loan requested by a firm could not be completely provided by a single bank. Banks will give the minimum between their availability and the requested loan. If the first selected bank cannot offer the total amount of the requested money, the firm will search for the rest of it by asking other banks. The firm will select the bank with the second lowest proposed interest rate, and so on. If all banks in the  $j$  ( $i$ ) firm list of contacts do not provide the total amount of money, we assume, for the sake of simplicity, that the firm gets in any case these partial loans.

The procedure to choose the partner is activated in every period. In Figures 3, 4 and 5 the schema of U firms - banks and inter bank relations, D firms-banks and inter bank relations and D firms - U firms relations, are reported.



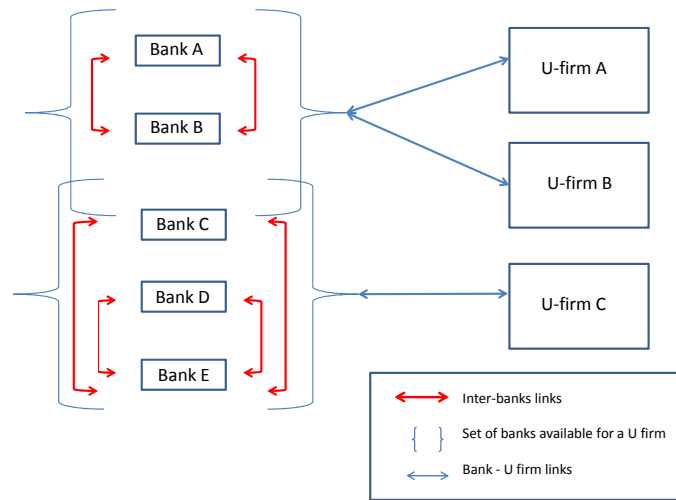


Figure 3: U firms - Banks and inter bank relations.

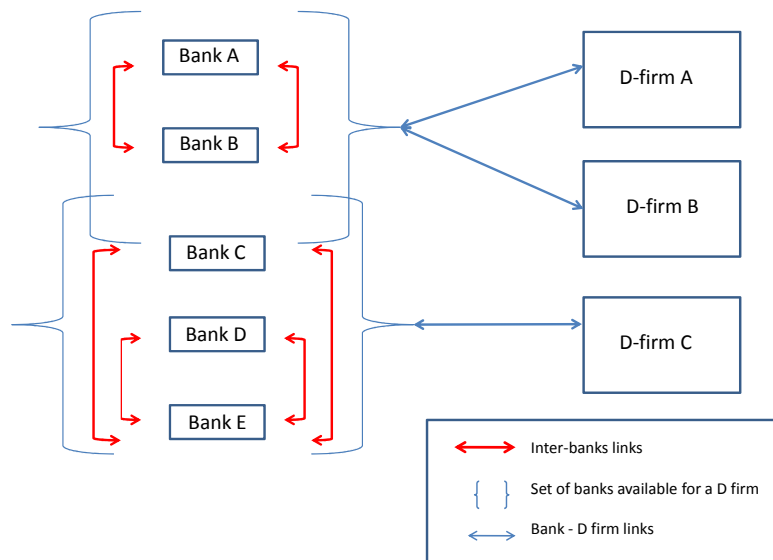


Figure 4: D firms - Banks and inter bank relations.

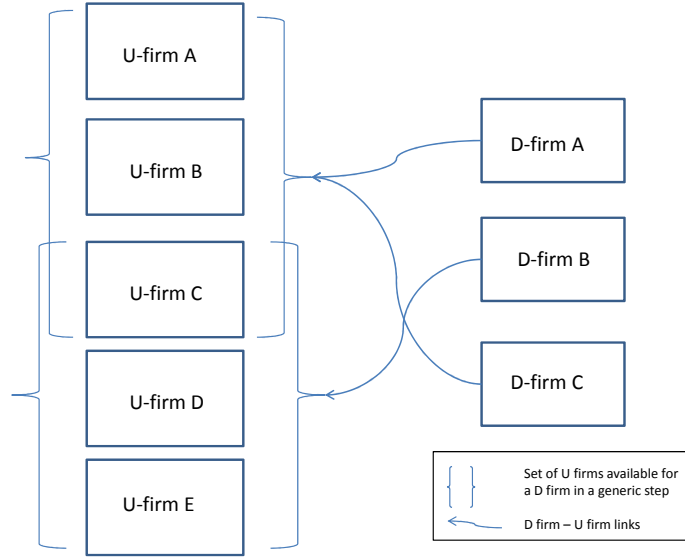


Figure 5: D firms - U firms relations.

## 2.4 Theoretic Cash Flow at the End of Month for Banks and Firms

We assume that the eventual loan contracts among banks and firms as well as the selling of intermediate outputs are stipulated at the beginning of each time. D firms buy the intermediate product from U firms not by means of credit contracts, like in Delli Gatti et al. (2008), but purchasing it at the beginning of the periods. At the end of each period firms must pay the loan contracts with banks, if they exist, and D firms sell the final product and get money from it, pay workers and the stochastic cost.

We define the theoretic cash flow at the end of the month of the  $i$  firm of the D set, the flow of money at end of each period  $t$ :

$$\pi_{i,t} = u_t Y_{i,t} - (1 - r_{z,t}^i) B_{i,t} - W_{i,t} - \eta_{i,t} \lambda_d Q_{i,t} \quad (2.15)$$

with  $W_{i,t} = wN_{i,t}$  the cost of workers,  $u_t$  the stochastic price of final product,  $Y_{i,t}$  the output of  $i$  firm,  $r_{z,t}^i$  the interest rate charged by bank  $z$  to firm  $i$ , and  $\eta_{i,t} \lambda_d Q_{i,t}$  is the stochastic cost. Assuming  $B_{i,t} = 0$ , we get

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$$\pi_{i,t} = \phi A_{i,t}^\beta (u_t - w\delta_d - \eta_{i,t}\lambda_d\gamma)$$

so the theoretical cash flow for a self-financed D firm is proportional to its net value and it is positive only if the stochastic price of the output satisfies this condition:

$$u_t > w\delta_d + \eta_{i,t}\lambda_d\gamma$$

the price of the sold output must be bigger than a quantity proportional to the costs allocated for the workers and the stochastic cost.

Similarly the theoretical cash flow at the end of the month  $\pi_{j,t}$ , the flow of money at the end of each period  $t$  for U firm  $j$ , is

$$\pi_{j,t} = - \left(1 - r_{z,t}^j\right) B_{j,t} - W_{j,t} - \eta_{j,t}\lambda_u Q_{j,t} \quad (2.16)$$

with  $W_{j,t} = wN_{j,t}$  the cost of workers,  $r_{z,t}^j$  the interest rate charged by bank  $z$  to firm  $j$ , and  $\eta_{j,t}\lambda_u Q_{j,t}$  is the stochastic cost.

If a U firm  $j$  is self-financed the cash flow is

$$\pi_{j,t} = - (w\delta_u + \eta_{j,t}\lambda_u) \gamma \phi \sum_{i \in \phi_j} A_{i,t}^\beta$$

It must pay the workers and the stochastic cost, both proportional to the quantity of the produced intermediate output that is proportional to the net value of the set of D firms that buys from it (Eqn 2.5).

The theoretical cash flow of the  $z$  bank at the end of the month, without considering interbank lending, is

$$\pi_{z,t} = \sum_{i \in I_{z,t}} \left(1 + r_{z,t}^i\right) B_{i,t} + \sum_{j \in J_{z,t}} \left(1 + r_{z,t}^j\right) B_{j,t} \quad (2.17)$$

with  $I_{z,t}$  the set of D firms interacting with bank  $z$  at time  $t$  and  $J_{z,t}$  the set of U firms interacting with bank  $z$  at time  $t$ .

---

## 2.5 Net Value of Firms

The net worth of a generic firm is the sum of the last-period net-worth, the theoretical cash flow at the end of the month (the flow of money at the end of each period  $t$ ) and the rest of money flow that is carried out at the beginning of each time, that is credit from the bank world (Eqn. 2.8 and 2.10) and cost (earn) of the intermediate product for D (U) firms.

The net worth of the  $i$  D firm is defined as follows:

$$A_{i,t+1} = A_{i,t} + B_{i,t} - E_{i,t} + \pi_{i,t} \quad (2.18)$$

with  $A_{i,0} > 0$ , the initial net value of firm  $i$ , realization of a random variable of a constant density function (Section 2.7).

The net worth of  $j$  U firm is similarly

$$A_{j,t+1} = A_{j,t} + B_{j,t} + E_{j,t} + \pi_{j,t} \quad (2.19)$$

with  $A_{j,0} > 0$ , the initial net value of firm  $j$ , realization of random variable of a constant density function (section 2.7).

U Firm (D firm)  $j$  ( $i$ ) goes bankrupt if at the end of time  $t$  the net value (Eqn. 2.19) (Eqn. 2.18 for D firm) is negative:

$$A_{j,t} + B_{j,t} + E_{j,t} + \pi_{j,t} < 0$$

$$A_{i,t} + B_{i,t} - E_{i,t} + \pi_{i,t} < 0$$

If it happens in the next step, a failed U firm (D firm) is replaced with a new one. The dimension of the net value of the new one is a realization of the random variable of the density distribution that we have used to create the set of firms at time  $t = 1$ .

From this formulation of the problem, we can see that if the net worth of a firm is bigger than the cost that it must face, it does not contact any banks. If, instead, the cost is bigger than the of net-worth, the firm looks for money in the world of banks to get enough resource to face that production level.

We can summarize the effect of an increase in net value of D firms:

- 
- increase of production (Eqn. 2.2) and consequently an increase in earning (Eqn. 2.15);
  - increase in the number of required workers (Eqn. 2.3);
  - increase in the required intermediate product (Eqn. 2.10), which implies an increasing financial gap (Eqn. 2.10);
  - decreasing of financial gap (Eqn. 2.10) and consequently a decrease in the theoretical cash flow for the bank (Eqn. 2.17);
  - increase in the production of U firms, which is a direct consequence of the more required intermediate product necessary for the creation of the final product (Eqn. 2.5);
  - increase in the stochastic cost (Eqn. 2.15), which implies an increase in the computation of financial gap relative to its expected value (Eqn. 2.10).

We can summarize the effect of an increase in the net value of U firms:

- a decreasing offer price for the intermediate product (Eqn. 2.7);
- increase in the number of required workers (Eqn. 2.9);
- decreasing financial gap (Eqn. 2.8) and consequently a decrease in the theoretical cash flow for the bank (Eqn. 2.17);
- increase in the stochastic cost (Eqn. 2.16), which implies an increase in the computation of the financial gap relative to its expected value (Eqn. 2.8).

## 2.6 Net Value of Banks

We define the net value of a bank as the available amount of money of a bank. To create its dimension at time  $t = 1$  we use a density distribution that fits the dimension of real banks. In empirical papers such as Berger et al. (1995), Jones and Critchfield (2005), it was identified a skewed density function for banks' dimension. Janicki et al (2006) study the possibility of fitting this empirical distribution with log-normal density and Zipf's law. The former does not fit the upper right tail of the size distribution in an appropriate way, while the latter fits better this part of the distribution, but the quality of the fit is poor outside the tail. The purpose of this work is to study the situation of bankruptcies of banks, such as banks with small net value: we use a log-normal density to create the initial value of the net values of banks. We assume that at the beginning each bank  $z$  receives a deposit  $R_z$  and that its net value is proportional to it:

$$A_{z,0} = \alpha_R R_{z,0} \tag{2.20}$$

---

with  $\alpha_R > 1$ . The difference between  $A_{z,0}$  and  $R_{z,0}$  is the equity of the bank. We, therefore, assume that the density distribution of the variable  $R$  is

$$f(R) = \frac{\exp\left(-\frac{(\log(R)-\mu_R)^2}{2\sigma_R^2}\right)}{\sqrt{2\pi\sigma_R^2 R}}$$

with  $\mu_R$  and  $\sigma_R$  parameters.

We start the description of the dynamics of the banking world, excluding interbank lending.

Each bank  $z$  receives stochastic shocks to its liquid reserves  $R_z$ . These shocks can be interpreted as the results of fluctuations generated from cash deposits, electronic transfers and other operations that, like the stochastic cost for firms, include every flow of money that is not fixed. Each bank must also pay the interest to the deposits. We assume that in this step each bank allocates money, proportional to its net value, to pay fixed costs, for example for workers. At the start of each period, each bank inherits an amount  $A_{z,t}$  from the previous period.

The net worth of  $z$  bank at step  $t$ , before the possible loans to the firms' world, is

$$A_{z,t}^* = A_{z,t} + sc_{z,t} - r_R R_{z,t} - \psi A_{z,t} \quad (2.21)$$

$A_{z,0}$ , and the reserve  $R_{z,0}$ , are defined in Eqn. (2.20).  $r_R \geq 0$  is the interest rate that the bank must pay for deposits. Parameter  $\psi \geq 0$  summarizes the fixed cost for bank, for example, taxes and salary of workers. We assume, out of simplicity, that they are proportional to the net value.

The term  $sc_t$  is the shock to the reserve  $R_{z,t}$ . It is defined (Iori et al. (2006), heterogeneous case):

$$sc_{z,t} = |R_{z,0} + \varepsilon_{z,t}^s R_{z,0} - R_{z,t}| \quad (2.22)$$

where  $\varepsilon_{z,t}^s$  is a random variable. With Eqn. (2.22) we assume the independence of the deposits' variations among banks excluding the possibility of "panic event" during which the owners of deposits withdraw large amount of money from the bank system (Diamond and Dybvig (1983), Jacklin and Bhattacharya (1988), Calomiris and Kahn (1996)).

The dynamical equation that updates the reserve for each bank is then

$$R_{z,t+1} = R_{z,t} + sc_{z,t} \quad (2.23)$$

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We assume a minimum reserve  $res_{z,t}$  kept by each bank defined

$$res_{z,t} = \chi(sc_{z,t} + R_{z,t}) \quad (2.24)$$

with  $0 \leq \chi \leq 1$  a parameter imposed by the Central Bank. Each bank must not use this amount of money for any investments. In this chapter this parameter is constant. A low level of this parameter allows banks to have relative high quantity of money to be invested in the firm's world. It also increases at the same time the possibility for the banks to have a bad debt because, loans to firms are deemed risky investments.

The amount available is therefore:

$$L_{z,t}^* = A_{z,t}^* - res_{z,t} \quad (2.25)$$

At this step, banks have the possibility to make investments. We assume that each bank  $z$  can receive, at the beginning of each step  $t$ , a set of requested loans  $w_{z,t}$  from D and U firms asking for credit. This opportunity defines the maximum possible loan that a bank can give. The actual investment in the firms' world then satisfies:

$$L_{z,t} = \min \left[ \max \left[ 0, L_{z,t}^* \right], w_{z,t} \right] \quad (2.26)$$

After loans, bank receives, at the end of step, the return on these plus principal, less the ones relative to those firms that fail in this step. If a borrower firm linked with a bank goes bankrupt, it cannot pay back money. We define  $bd_t$  as Bad-Debt, the amount of money that is not received by banks due to bankruptcies of the failed borrower firms. We have not Bad-Debt for U firms relative to the possibility of the failure of D firms that purchase from U, because D firm buys and pays intermediate products at the beginning of each period. If a D firm has not enough money and it cannot get enough liquidity from the banking system to purchase intermediate products, then it will not get the product from the U firms. If a D firm has not enough money to pay the ordered intermediate product from the U firm, and it has not got money from the bank to do it, then the entire order from the U firm is deleted and the status of the U firm is updated. In this case we assume that the D firm must pay the workers all the same at the end of period for the planned, but not executed, production.

The net value of  $z$  bank at the end of step  $t$ , is:

$$A_{z,t+1} = A_{z,t}^* - L_{z,t} + \pi_{z,t} - bd_{z,t} \quad (2.27)$$

where  $\pi_{z,t}$  is the theoretical cash flow at the end of the month of the  $z$  bank (Eqn. 2.17),  $L_{z,t}$  is the actual investments, defined in Eqn. (2.28) with the constraint Eqn (2.26) and  $bd_{z,t}$  is the bad-debt for bank  $z$  (Eqn. 2.29)

$$L_{z,t} = \sum_{i \in I_{z,t}} B_{i,t} + \sum_{j \in J_{z,t}} B_{j,t} \quad (2.28)$$

$$bd_{z,t} = \sum_{i \in I_{z,t}^f} (1 + r_{z,t}^i) B_{i,t} + \sum_{j \in J_{z,t}^f} (1 + r_{z,t}^j) B_{j,t} \quad (2.29)$$

where  $B_{i,t}$  and  $B_{j,t}$  are defined in Eqn. (2.10) and Eqn. (2.8),  $bd_{z,t}$  is the value of the missed return of money due to bankruptcy of firms linked with  $z$ . At the step  $t$ ,  $I_{z,t}$  is the set of D firms interacting with bank  $z$ ,  $J_{z,t}$  is the set of U firms interacting with bank  $z$ ,  $I_{z,t}^f$  is the set of D firms that fail and that has a credit relation with bank  $z$ ,  $J_{z,t}^f$  is the set of U firms that fail and that has a credit relation with bank  $z$ . From these definitions we get  $I_{z,t}^f \subseteq I_{z,t}$ ,  $J_{z,t}^f \subseteq J_{z,t}$ .

$r_{z,t}^j > 0$  is the interest rate charged by bank  $z$  to U firm  $j$  (Eqn. 2.13),  $r_{z,t}^i > 0$  is the interest rate charged by bank  $z$  to D firm  $i$  (Eqn. 2.13).

Without the interbank system, a bank that ends a month with negative net value (Eqn. 2.27) will fail and be replaced by a new one.

We can summarize the effect of an increase in the net value of a bank:

- As increase in the interest rate proposed to firms (Eqn. 2.13).
- An increase in its fixed cost (Eqn. 2.21).
- An increased quantity of money available for firms (Eqn. 2.26).

## 2.6.1 Interbank System

At this stage, we can introduce the interbank market with a safety network role. Credit linkages among banks are defined by a connectivity matrix  $Z_{ab}$  whose elements are either one or zero; a value of one indicates that a credit linkage exists between the banks  $a$  and  $b$ , while zero indicates no relationship. In this chapter  $Z_{ab}$  is randomly chosen at the beginning of the simulation. We define the parameter  $Pc$  the probability that the generic element  $z$  of the matrix  $Z_{ab}$  is one. With  $Pc = 1$ ,



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we have a fully connected interbank market; while, in the opposite case, we have no interbank market. Different from firms' preferences, we assume that this matrix remains constant over time. The interbank interest rate  $r_b$  is, in this chapter, an exogenous parameter.

We note that matrix  $Z_{ab}$  is a priori non symmetric. Element  $z(a, b)$  equal to one implies the possibility for bank  $b$  to ask bank  $a$  for a loan. It is not true if  $z(b, a)$  is zero.

From this definition of the interbank market, we argue that every element on the diagonal of the matrix are always zero because one bank cannot exchange money with itself.

In the next chapters we allow the Central Bank to control the interbank connections and the interbank interest rate to stabilize the market.

Allowing the interbank market, the generic and complete evolutionary behaviour for a bank is:

$$A_{z,t+1} = A_{z,t}^* - L_{z,t} + \pi_{z,t} - bd_{z,t} + ci_{z,t} + c_{z,t} - bc_{z,t} \quad (2.30)$$

With interbank market a borrower bank tries to get money to have non negative net value. It asks lender banks to get the necessary loans, provided that the borrowers and lenders are linked in an interbank relationship. A bank is now a potential lender if its net value (Eqn. 2.27) minus its reserve (Eqn. 2.24) is positive.

With respect to Eqn. (2.27), we have now variables that describe the interbank relations among banks:

$$ci_{z,t} = \left[ - \sum_{z \in \alpha_{z,t}} b_{z,t} + \sum_{z \in \beta_{z,t}} b_{z,t} \right] \quad (2.31)$$

is the total lending and borrowing contracts  $b$  that bank  $z$  pays to and receives from other banks during step  $t$

$$c_{z,t} = (1 + r_b) \left[ \sum_{z \in \alpha_{z,t-1}} b_{z,t-1} - \sum_{z \in \beta_{z,t-1}} b_{z,t-1} \right] \quad (2.32)$$

is the last step lending and borrowing contracts  $b$  that bank  $z$  handles with other banks, increased of interest, at step  $t$

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$$bc_{z,t} = (1 + r_b) \left[ \sum_{z \in \alpha_{z,t-1}^f} b_{z,t-1} \right] \quad (2.33)$$

is missed flows of money due to bankruptcy of other banks with a contract relationship with  $z$ .

$\alpha_{z,t}$  is the set of banks that have received money from bank  $z$  in step  $t$ ,

$\beta_{z,t}$  is the set of banks that have lent money to bank  $z$  in step  $t$ ,

$\alpha_{z,t-1}^f$  is the set of banks that have received money from bank  $z$  in step  $t - 1$  and fails, so they cannot give back the credit at step  $t$ . We have  $\alpha_{z,t-1}^f \subseteq \alpha_{z,t-1}$ ,

$r_b > 0$  is the interest rate of inter-bank contracts.

We note that, in each step  $t$ , a bank  $z$ , after receiving returns from investments in the firms' world, can have a negative net value or not. If it has negative net value it will search for loans in interbank markets and will be a borrower ( $\alpha_{z,t} = \emptyset$ ). If it has a positive net value, it can give loans in interbank markets, and it will be a lender ( $\beta_{z,t} = \emptyset$ ).

At the beginning of every step, the value of each bank changes due to payments of interest to deposits, stochastic shocks to them and payments of fixed costs (Eqn. 2.21). After this step, there is the possibility to invest in the firms' world by means of loan (Eqn. 2.26).

At the end of every step, the status of the bank is updated by means of Eqn (2.27). Afterwards each bank starts to pay back the debts, if they exist, with the interest it has with other banks. If for a bank there is at least one debt that it cannot pay back, it will be a borrower bank: it will search in interbank market enough money to pay back the not yet paid debt.

Now, two types of banks can be distinguished, those classified as lender, if they have positive net value, and those classified as borrower. Borrowing banks ask for loan to get a non negative net value, that is equivalent to ask for money to pay back the not yet paid back debt. It is assumed that each borrowing bank contacts lending banks in a random order, subject to the condition that the borrower and the lender are linked in a interbank relationship (unitary element in the matrix  $Z_{ab}$ ). The debt contract is stipulated at the end of the period, and it must be paid at the end of the next period. The amount of money exchange between two banks is the minimum between the request and the bid. If a bank has not got enough money from the first contacted lending bank, it continues to search for money in other available interbank markets. A borrowing bank does not receive the requested money from

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the contacted lending banks until it has got enough credits to pay back the entire amount of money it has searched for in the interbank system. Banks that in the previous steps were lender receive the amount of money from the previous steps' borrower banks having needed access to the interbank market and have got enough money to pay the debt with interest.

The entire process is repeated till there is exchange of money in the interbank system.

If a bank has at least one debt with a bank that, after requesting for money in the interbank system, cannot pay back, its net value will be negative: the bank will be declared as a failed one. If, after the interbank market's exchange of money, a bank has a positive amount of money, but not enough to pay the debt, the quantity is given back to the creditors and it is declared as failed.

The number of banks is constant and, thus the failing banks would be replaced by new ones generated from the same log-normal density distribution by which the algorithm has created the initial population of banks.

### **2.6.1.1 Consideration about the Bankings and Firms' System**

The interbank network is a matrix created at the beginning of the simulation and does not change anymore; instead, the firms-banks network evolves during time. The idea is that firms can search, similar to the real world, for other banks; while banks have only fixed bank-partners which for example, have some agreements of collaborations. In the next chapter, we develop the idea that the Central Bank, or the Government, can regularize the interbank relationships.

Another difference is that firms can ask for money at the beginning of the period and must give it back at end of the period. Banks can contact other banks for inter-bank loans at the end of the period and must give back money at the end of the next period. Firms fail if at the end of the period it has not enough money to repay the debt-credit contract with banks; they can ask for further loans from those banks with which they have agreements of collaborations. If the lender banks cannot offer the borrower bank enough credit, the latter simply fails.

Both for firms and banks, the number of elements is constant. So, the failed firms are replaced by new firms and failed banks are replaced by new banks. The basic idea beyond this assumption is that the purpose of this work is to create rules that can decrease the instability of the entire system by avoiding or replacing the failed firms or banks. Thus, we can get a system in which there could be too few banks to handle the entire financial market and/or a system in which there are not enough possibilities of investment for banks in the firms' world.

In the next chapter, we study rules which, imposed by the Central Bank or the Government, reduce the degree of freedom of banks to make the system more stable. At this step, the unique and notable reduction of freedom for lender banks is that they must give loans to borrower banks, if they are asked to.

## 2.7 Parameters

The constant number of D firms, U firms and banks, respectively  $I$ ,  $J$  and  $Z$ , are  $I = 90$ ,  $J = 60$  and  $Z = 25$ . The number of the steps of algorithm  $T = 600$  and repetition of itself  $Rep = 30$ . We can identify a step with a month and so we have 50 years' simulation. The values of the parameters is common with Delli Gatti et al. (2008) and Iori et al. (2006); they are selected from these papers. These values and those of the other parameters are adjusted accordingly to ensure the presence of the failing firms and banks and also to ensure no divergence in the time series (Figures 38, 40, 42, 44).

We report the proposed values of parameters:

PARAMETERS FIRM'S WORLD		
Constrained output of D firms	$\phi = 1.2$	$\beta = 0.8$
Labor requirement of D and U firms	$\delta_d = 0.4$	$\delta_u = 1.04$
Intermediate goods requirement of D firms	$\gamma = 0.3$	
Price on intermediate product	$\alpha = 0.1$	
Real wage	$\omega = 1$	
Number of potential partners	$M = 5\%$ of firms*	$N = 10\%$ of banks **
Fixed cost intermediate product		$P_f = 0.9$
Parameter random cost	$\lambda_d = 0.08$	$\lambda_u = 0.025$
Parameter expected value stochastic cost	$\bar{\eta}_d = 0.5$	$\bar{\eta}_u = 0.5$
Parameter "worst stochastic cost"	$\mu_d = 2$	$\mu_u = 2$

\* U firms for D firms

\*\* Banks for U and D firms

The cost of final product is a uniform random variable with expected value  $\bar{u}_{t+1}$  (section 2.1):

$$\bar{u}_{t+1} = \alpha_{pr} + \beta_{pr} Y_t$$

$$Var(u) = v_{pr}$$

$$\text{with } \alpha_{pr} = 0.8, \beta_{pr} = -0.003, v_{pr} = \frac{1}{12} (0.4)^2$$

The initial net value of D firm  $i$  and U firm  $j$  is extracted from

$$A_{i,0} = |1 + \sigma_{\Omega} \varepsilon_x|$$

$$A_{j,0} = 0.015 |1 + \sigma_{\Omega} \varepsilon_x|$$

with  $\sigma_\Omega = 0.5$  and  $\varepsilon_x = N(0, 1)$ . The same density distribution is used to replace failed U and D firms.

PARAMETERS OF BANK'S WORLD		
Percentage cost of bank's workers	$\psi = 0.01$	
Minimum reserve factor	$\chi = 0.01$	
Interest rate on inter-banks loans	$r_b = 0.0025$	
Interest rate on bank loans	$\sigma = 0.002$	$\theta = 0.002$
Interest rate on deposits	$r_R = 0.01$	
Probability to have a inter-banks link	$Pc = 0.5$	
Bank rate	$v = 0.015$	

The parameter for log-normal density function for the definition of deposits and net value in  $t = 1$  and for new banks, inserted replacing the failed ones, are:

$$\mu_R = -0.0945$$

$$\sigma_R = 0.2$$

The net values and deposits are proportional (eqn. 2.20). Value of the parameter:

$$\alpha_R = 1.2$$

The random variable for the shock of deposits is

$$\varepsilon_{z,t}^s \sim U(-0.25, 0.25)$$

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## 2.7.1 Step of the Algorithm

We report the actions of banks and firms in a generic step of the simulation.

### D FIRMS

1. At the beginning of each month, each D firm searches for a U firm from which it can buy intermediate products. The searching method is the ‘preferred partner choice rule’. To start production, firms must have enough net value to cover the cost of workers, the cost of the expected stochastic cost, and the cost of intermediate products that they are buying. If a D firm has not enough money for this operation, it searches for a loan in the banking system. The searching method is the ‘preferred partner choice rule’.
2. At the end of the month, D firms sell all output and pay the debt, with interest, to the bank, if there is one. If, at the end of these operations, the net worth of some firms is negative, they will be in default.

### U FIRMS

1. At the beginning of each month, each U firm can receive requests of intermediate products from a set of D firms. To start production, a firm must have enough net value to cover the cost of workers and the cost of the expected stochastic cost. If a firm has not enough money for this operation, it searches for a loan in the banking system.
2. At end of the month, U firms pay the debt, with interest, to the bank, if there is one. If, at the end of these operations, the net value of some firms is negative, they will be in default.

### BANKS

1. At beginning of each month, each bank receives stochastic shocks to its liquid reserves, must pay interest to depositors, and also must pay fixed costs. After this fluctuation, if the net value is positive, a bank can make loans in the U and D firms’ world.
2. At the end of the month, banks get returns of the firms credits’ contracts, if there are some. Each bank that has a debt (credit) contract in the interbank market pays (receives) it with interest. If, at the end of these operations, the net value is negative, banks can ask for debt contracts which they will pay at the end of the next month. If, after asking partner-banks, a bank has negative net value, it fails.

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## 2.7.2 Assumptions

Here we list the assumptions of Delli Gatti et al. (2008), Iori et al. (2006) and their differences with this model.

- Assumptions of Delli Gatti et al. (2008):

- Ass.A1) The number of banks and number of firms are exogenous.
- Ass.A2) Firms sell all output they produce, so there is an infinite demand for products.
- Ass.A3) D firms buy intermediate products from U firms by means of commercial credit contracts.
- Ass.A4) Selection of partner for D firms relative to U firms. Selection of partner for D and U firms relative to banks: ‘preferred partner choice rule’.
- Ass.A5) D firms: financial constrained production proportional to net value. Unconstrained availability of workers.
- Ass.A6) Price of final goods: stochastic price.
- Ass.A7) Many D firms can be linked to a single U firm but each D firm may only have one supplier of intermediate goods.
- Ass.A8) For U firms financial gap is the difference between wage bill and net value. The loan request by a U firm is either completely provided by a bank, or the firm receives no loan.
- Ass.A9) For D firms financial gap is the difference between wage bill and net value. The loan request by a D firm is either completely provided by a bank, or the firm receives no loan.
- Ass.A10) Many firms can be linked to a single bank but each firm will not have more than one bank that supplies credit.
- Ass.A11) Failed firms/banks are replaced with new entrants.
- Ass.A12) With a small probability  $\epsilon > 0$ , each D firm chooses a U firm at random, otherwise it selects a U firm by only comparing the proposed price in a set of U firms and selects the one which proposes the lowest price.
- Ass.A13) With a small probability  $\epsilon > 0$ , each D and U firms chooses a bank at random; otherwise they select a by bank only comparing the proposed interest rate of a set of banks and selecting the one which proposes the lowest interest rate.
- Ass.A14) The interest rate proposed by the banks to firms are inversely proportional to the net value of banks and proportional to the leverage ratio of the firm.

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Differences between the assumptions of Delli Gatti et al. (2008) and this model:

- Ass.A3)\* D firms buy the intermediate product from U firms not by means of credit contracts, but they purchase it at the beginning of the periods. If a D firm has not enough money to pay the ordered intermediate product because it did not get its request of loan from the banking system, the order from the U firm is deleted and the status of the U firm is updated. With no credit contracts, D firm searches for money in the banking system at the beginning of each step, if it has not enough money to buy the intermediate product. We make this assumption to have only exchange of money in the interbank system at the end of each step.
- Ass.A6)\* The price of final goods produced by D firms: stochastic price, with the average value following a supply and demand law. (Besanko, and Braeutigam (2002))
- Ass.A8)\* For U firms, the financing gap is the wage bill, a deterministic cost, plus stochastic cost minus the net value minus the value of the sold intermediate product. The loan request by U firms could not be completely provided by a single bank. The banks will give the minimum between their availability and the requested loan. If a bank cannot offer the entire requested money, U firms will search for the rest of it by addressing other banks. If all banks in the U firm list of contacts do not provide the total amount of money, we assume that the firm gets these partial loans anyway, which have already been stipulated with the contacted banks. The stochastic cost is assumed to consider the potential cost that production (fixed cost) cannot cover.
- Ass.A9)\* For D firms, the financial gap is the wage bill, a deterministic cost, plus a stochastic cost plus cost of intermediate product minus the net worth. The loan requested by D firms could not be completely provided by a single bank. Banks will give the minimum between their availability and the requested loan. If a bank cannot offer the entire requested money, D firms will search for the rest of it by asking other banks. If all banks in the D firm list of contacts do not provide the total amount of money, we assume that the firm gets these partial loans anyway, which have already been stipulated with the contacted banks. If the firm does not get the total requested loan, it cannot produce; but it must pay the monthly salary to the workers engaged in producing a quantity of product that it has planned to produce. The stochastic cost is assumed to consider the potential cost that production (fixed cost) cannot cover.
- Ass.A10)\* Many firms can be linked to a single bank and each firm can have more than one bank that supplies credit. This assumption is made to avoid



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the impossibility for a set of banks to be exposed in the same risky investment. We assume that each bank, which gives loan to D firms/U firms that cannot cover the total amount of requests, does not know if this firm will get the rest of money from the rest of the banking system.

Ass.A12)\* There is not the possibility that with a probability  $\epsilon > 0$ , the D firm chooses a firm of type U at random: it selects a U firm only comparing the proposed prices of a set of those firms. It selects the firm that offers the lowest price. This assumption is made to avoid the possibility for D firms to make mistakes of selecting a U firm randomly.

Ass.A13)\* There is not any possibility that with a probability  $\epsilon > 0$  D and U firms would choose a bank at random. Each firm selects a bank only by comparing the interest rate proposed by a set of banks and selects the one with the lowest interest rate. If that bank cannot offer the entire quantity of money, the firm will ask the other contacted banks for the rest of the necessary loan. The firm selects the bank with the second lowest proposed interest rate and so on. This assumption is made to avoid the possibility for firms to make mistakes in selecting banks randomly.

Ass.A14)\* The interest rate proposed by the banks to firms is proportional to the net value of banks, to the leverage ratio of the firms, and to the official discount rate set by the Central Bank. The interest rate proportional to the net value of the banks allows those banks which have got interbank credit, and then having small net value, to propose a lower interest rate. This will increase the possibility to have firms that will ask these banks for loans. The interest rate proportional to official discount rate set by the Central Bank allows this institution to control the financial relations between banks and firms.

- Assumptions of Iori et al. (2006).

Ass.B1) The demand for credits are assumed to be stochastic. Each bank can make multi-periodical investments, getting returns from them in the future steps of algorithm.

Ass.B2) For the banks the investments (loans) are risk-free.

Ass.B3) The matrix of inter-bank lending relations is chosen at the beginning.

Ass.B4) Each bank receives stochastic shocks to its liquidity reserves, generated by the deposit withdraws as well as electronic transfers.

Ass.B5) A bank can undertake dividend payments to shareholders.

Ass.B6) Banks can receive return from a loan at the beginning of a period and make loans at the end of the period.

Ass.B7) It is assumed that a bank pays all its debts, if it can. A bank cannot refuse to give loan if it can give it.

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- Ass.B8) Surplus banks are assumed to give priority to dividends and loans for firms and then to loans to banks.
  - Ass.B9) A borrowing bank does not receive actual funds in the interbank market until it has lined up enough credit to ensure that it is not going to fail during the current period.
  - Ass.B10) Bankrupt banks are not replaced with new entrants.

Differences between the assumptions of Iori et al. (2006) and this model:

- Ass.B1)\* Request for loans to the banks are not assumed to be stochastic, but are endogenous, created by the firms' sector. Firms always receive, from each contacted bank, the minimum between the requested money and the availability of the bank. Even if credit is given from one or more banks, the requested loan could not be always completely fulfilled. The debt repayment period is fixed and set to one period: each bank cannot make multi-periodical investments. Banks will have the returns on investments at the end of each month.
- Ass.B2)\* Loans for banks are not risk-free: firms that ask for money can fail.
- Ass.B5)\* A bank can not undertake dividend payments to shareholders. We make this assumption to avoid to insert in the model properties that do not directly influence the behaviour of the algorithm with respect to the stability of the firms' and banking world.
- Ass.B6)\* Banks make loans to firms at the beginning of a period, and they receive interest and principal at the end of it. Inter-bank loans are stipulated also at the end of the period and they must be repaid, in this case, at end of the next period. We make this assumption to synchronize the moment in which banks have the possibility of risky investments in the firms' system with the moment during which firms ask for loan contracts at the beginning of each step.
- Ass.B10)\* Bankrupt banks are replaced with new entrants. We make this assumption to let the system to find normative approaches that can regularize an interbank market by focusing only on the links among banks and avoiding the possibility to have a more complex system in which their number can decrease. The density function by means of which we create the initial banking system and replace the failed banks is the log-normal density.

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## 2.8 Algorithm Implementation

In the following steps the functions used to implement the multi-agent system are described. Step\_1 is identified with the file name A2, Step\_2 with A3 and so on, while Step\_10 is identified with the file name A10a, Step\_11 with A11, Step\_12 with A11a, Step\_13 with A12, and so on.

Step\_1) We create the matrix  $Z_{ab}$ , which is the fixed matrix of the inter-bank relationships; the parameters and the variables are initialized (see Section 2.7); and the algorithm starts. The algorithm iterates itself from Step\_2 till Step\_18 until the maximum number of iterations is reached. The algorithm is repeated more times and the analysed indexes are the average value with respect to these repetitions.

### BEGINNING OF THE STEP

Step\_2) By means of Eqn. 2.3, we compute the quantity of product and number of required workers for each D firm.

Step\_3) By means of Eqn. 2.7, we compute the offered price proposed by each U firm.

Step\_4) Each D firm uses preferred-partner choice rule to select U firm for trading.

Step\_5) By means of Eqn. 2.5 and Eqn. 2.6, we can now compute the number of the workers needed and the planned output for U firms. There is no rationing in the labour market and thus each firm automatically gets the quantity of workers that it needs.

Step\_6) With Eqn. 2.10 and Eqn. 2.11, we get the financing gap for D firms.

Step\_7) With Eqn. 2.8 and Eqn. 2.9, we get the financing gap for U firms.

Step\_8) By using Eqn. 2.21, Eqn. 2.22, Eqn. 2.23 and Eqn. 2.24 we get the net value of each bank before potential loans. If it is positive, the bank can finance firms. Interest on deposits are computed with respect to the last time deposit. At the beginning of every period, each bank's liquid resource changes due to payment of interest to depositors, which is equivalent to a stochastic shock to them in this model. Now each bank allocates money to pay its fixed cost. Each bank allocates a percentage of net value for the minimum reserve ration imposed by the Central Bank.

Step\_9) Each D firm uses the preferred-partner choice rule to select banks; in particular, the interest rate offered by the consulted banks is created by means of Eqn. 2.13 and Eqn. 2.14. The offered interest rate depends on the D firms' leverage ratio Eqn. 2.14. Each D firm asks, if it needs, one bank for money to get the entire loan. If a bank cannot give all requested money to a firm, this firm asks other banks to get the rest of requested

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money. A firm will ask another bank for a loan with the hypothesis that they have firm-bank relations. If banks grant the requested loan, we update the net worth of banks and firm. Firms compare the interest rate offered by the best previous-step selected bank with  $N$  new banks. A bank has lending capacity given in Eqn. 2.26. We assume that D firms ask banks for money before U firms, and that, both for D firms and U firms, the order with which each firm contacts banks is random.

Step\_10) Before going to the ‘end of period’ step of the algorithm, we update the net worth of each D firm by means of Eqn. 2.18 and Eqn. 2.15, and relatively only to the ‘at the beginning of period’ flow of money. If a D firm has not enough money to pay the ordered intermediate product from U firm, as it did not get its request loan from the banking system, the order from the U firm is deleted and the status of the U firm is updated by means of equations present in (Step\_5) and (Step\_7). By hypothesis, we assume that D firms pay workers in any case.

Step\_11) Each U firm uses the preferred-partner choice rule to select banks, in particular the interest rate offered by the consulted banks is created by means of Eqn. 2.13 and Eqn. 2.14. The offered interest rate depends on the U firms leverage ratio Eqn. 2.14. Each U firm asks, if it needs, for money from one bank to get the entire loan. If a bank cannot give all the requested money to a firm, this firm asks other banks to get the rest of it. The firm will ask another bank for a loan with the hypothesis that they have firm-bank relations. If banks grant the requested loan, we update the net value of the banks and the firm. Firms compare the interest rate offered by the best previous-step selected bank with  $N$  new banks. A bank has the lending capacity given in Eqn. 2.26. We assume that D firms ask banks for money before U firms (see Step 9), and that, both for D firms and U firms, the order with which each firm contacts banks is random.

Step\_12) Before going to the ‘end of period’ step of the algorithm, we update the net worth of each U firms by means of Eqn. 2.19 relatively only to the ‘at the beginning of period’ flow of money. If a U firm has not got not enough money from banks, we assume that it will produce in any case and pay the workers and also that it will fail at the end of the period.

END OF THE STEP

Step\_13) We can now update the net worth of each D firm by means of Eqn. 2.18 and Eqn. 2.15, excluding the part that we have already updated in Step\_10 regarding the flow of money at the beginning of the step. We also compute the bad-debt between D firms and banks.

Step\_14) We can now update the net worth of each U firms by means of Eqn. 2.19 and Eqn. 2.16, excluding the part that we have already updated in

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Step\_12 regarding the flow of money at the beginning of the step. We also compute the bad-debt between U firms and banks.

- Step\_15) Using Eqn. 2.27 we update the net value of each bank. The part relative to the loan given to the firms, at the beginning of the step, is already updated in Step 9 and Step 11. Some banks can not have a positive net value. These banks will be the potential borrowers in Step\_16. The bad-debt part, relative to the financial relations between banks and firms, are updated in Step\_13 and Step\_14.
- Step\_16) Banks paid the previous step debts with other banks, if they exist, and receive money from other banks if some loans during the previous step have been taken out. Now, two types of banks can be distinguished: those classified as potential lenders, if they have cash, and potential borrowers. Banks with at least yet to be paid debt, try to get credit in the interbank market to pay it. They are classified as borrowing banks. It is assumed that each borrowing bank contacts lending banks in a random order, provided that the borrower and lender are linked in an interbank relationship. Once a borrowing bank contacts a lending bank, an agreement is reached between the two about how much credit will be exchanged. This is the minimum demand and supply of the two banks. A borrowing bank that is left with a negative net value contacts other banks and tries to make further agreements. A borrowing bank gets the requested credits from the contacted lender banks only if it can guarantee that it will not fail: the total requested credit is enough to pay back the rest of the debt. If it is the case, then the bank can receive the credit and payback the rest of debts. The entire procedure is repeated till no exchange of money happens. If a bank cannot pay back the entire debt with interest, the amount of money not enough to pay it is given back to the creditors in any case. The credit that a borrowing bank has got now will be the next step bank debt. Equations that describe these procedure are: Eqn. 2.30, Eqn. 2.31, Eqn. 2.32 and Eqn. 2.33.
- Step\_17) All banks that now are left with negative net value are deemed to be in default. We know now which banks are failed; we record the status of these banks and replace them with new ones by using the log-normal density function (Section 2.6).
- Step\_18) The bankrupt firms are replaced with new entrants by using density functions of Section 2.7.



# 3 Maximization of the Stability of the Interbank System

‘We [believed] the problem would come from the failure of an individual institution. That was the big mistake. We didn’t understand just how entangled things were.’ (Gordon Brown, former British prime minister at the Institute for New Economic Thinking’s. Bretton Woods Conference, 9 April 2011).

In this third chapter we want to find out rules that increase interbank stability. We use the model created in the second chapter to give an answer to these questions:

- < How can the Government or other institutions like the Central Bank create some rules to determine interbank relationships in such a manner that we have an improvement on the problem of the stability of the interbank system? >
- < Has this regularized interbank system got properties similar to real life interbank markets?>

We will create a set of rules that can maximize the stability of the interbank system.

## 3.1 Financial Stability

With the multi-agent system presented in Chapter 2 we can now propose a normative approach for the stability of the interbank and banks-firms systems. In Schinasi’s (2004) ‘Defining financial stability’, a survey and comments on financial stability definitions are proposed. With the comparison in section 3.4.2 with data on the USA and EU interbank markets, we have decided to quote these definitions:

Roger Ferguson (Board of Governors of the U.S. Federal Reserve System) ‘It seems useful...to define financial stability...by defining its opposite: financial instability. In my view, the most useful concept of financial instability for the Central Banks and other authorities involves some notion of market failure or externalities that can potentially impinge on real economic activity. “Thus, for the purposes of this paper, I’ll define financial instability as a situation characterized by these three basic criteria: (i) some important set of financial asset prices seem to have diverged sharply from fundamentals; and/or (ii) market functioning and

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credit availability, domestically and perhaps internationally, have been significantly distorted; with the result that (iii) aggregate spending deviates (or is likely to deviate) significantly, either above or below, from the economy's ability to produce.'

Tommaso Padoa-Schioppa (the European Central Bank) '... [financial stability is] a condition where the financial system is able to withstand shocks without giving way to cumulative processes, which impair the allocation of savings to investment opportunities and the processing of payments in the economy. The definition immediately raises the related question of defining the financial system... [which] consists of all financial intermediaries, organized and informal markets, payments and settlement circuits, technical infrastructures supporting financial activity, legal and regulatory provisions, and supervisory agencies. This definition permits a complete view of the ways in which savings are channeled towards investment opportunities, information is disseminated and processed, risk is shared among economic agents, and payments are facilitated across the economy.'

The paper also reports a proposition which encompasses the properties that the definition of financial stability should have:

*A financial system is in a range of stability whenever it is capable of facilitating (rather than impeding) the performance of an economy, and of dissipating financial imbalances that arise endogenously or as a result of significant adverse and unanticipated events.*

In Section 3.2, we propose a rule that creates an interbank market structure with the purpose of increasing stability of a part of the economy system, the bank system itself. In Section 4.1 we propose a rule that increases the stability, starting from the differences in the performance between firms and bank returns, and thus a rule based on 'financial imbalances that arise endogenously'. In Section 4.2. we check the robustness of the proposed rules by testing it with 'significant adverse and unanticipated events': imposing exogenous negative shocks in the interbank system.

Following Roger Ferguson's suggestion, we will define an index in Section 3.3 that defines the range of instability of the system. To find the best rules for the system, we will solve an optimization problem and check the performance of the economy with these imposed rules.



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## 3.2 Interbank Network Formation

‘The implications of our findings are that regulators seeking to address systemic risk should pay particular attention to the network structure of financial relationships between banks that determine the extent of any banking crisis. (Krause and Giansante (2010)).

In the model developed in Chapter 2, we have a fixed structure of this market described by a matrix,  $Z_{ab}$ , that is a random matrix in which each element has a probability  $P_c$  to be equal to one. The purpose of this chapter is to impose rules in the structure of the interbank market to increase the stability of the system.

Iyer et al. (2010) study the contagion effect through links among banks. The results of the paper let the authors assert that a central institution should regularize the interbank market imposing, for example, a limited exposure in a single institution, to change the capital requirement of each bank in dependence of its kind of exposure in the interbank market and limiting the degree of freedom of banks in searching for funding in the interbank market. We follow this last proposal.

The idea in this step is to create a matrix that describes the interbank network by means of criteria which allow the system to have a lower instability. The element of this network, namely banks, evolves during the steps and so the rule that creates the financial connections among banks, should also update them.

A bank that needs money, asks the Central Bank to get access to the interbank system. The Central Bank gives a set of allowed potential lenders. The borrower bank keeps the degree of freedom to choose the order of the selected lender banks, if more than one. This operation is repeated for each borrower. Banks that in previous steps were lenders, receive the amount of money from previous step borrowers banks that have needed access in interbank market and have got enough money to avoid to fail. At this point, the procedure of creating new allowed links in the interbank market is repeated till no exchange of money among the banks happens.

A lender bank  $a$  will have money to invest in the interbank system if:  $A_{a,t} - A_{res,t} > 0$ ; where  $A_{a,t} > 0$  and  $A_{a_{res},t}$  are the net value and the reserve (Eqn. 2.24) of bank  $a$  at step  $t$  before the interbank market starts. A borrower bank  $b$  at step  $t$  will have  $A_{b,t} < 0$ . Each borrower bank will enter the interbank market to contact lender banks to get enough money to reach a non negative net value. The variable that we consider is:

$$x_{a,b;t} = A_{a,t} - A_{a_{res},t} + A_{b,t} \quad (3.1)$$

The choice of an indicator based on the dimension of the banks is justified in Drehmann and Tarashev (2011), where the authors investigate whether simple indicators can approximate more complex measures of a bank’s systemic importance.

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The variable  $x_{a,b;t}$  indicates the difference between the maximum quantity of money that the lender bank  $a$  can offer in the interbank market and the request of money from the borrower bank  $b$  at step  $t$ . It is an indicator of the excess supply in a interbank connection. If  $x_{a,b;t} \geq 0$  the lender  $a$  can entirely cover the request of borrower  $b$ ; otherwise, the borrower would have excess demand for liquidity and would be forced to ask at least two banks to have the right quantity of money necessary to avoid the bankruptcy.

Now we focus on the creation of the interbank network by using this variable.

To build a network we can divide the domain of  $x_{a,b}$  into  $N$  sub-sets  $X_i, i = 1, 2, \dots, N$ :

$$[x_0, x_1), [x_1, x_2), \dots, [x_{N-1}, x_N]$$

that create a partition of the domain. If in each subset we define a binary function

$$g_i(x_{a,b;t}) : X_i \rightarrow \{0, 1\} \quad x_{a,b;t} \in X_i, i = 1, 2, \dots, N$$

With respect to the entire interval we have functions  $g_i(x_{a,b;t})$  that gives us  $N$ -dimensional binary values. To divide the interval into subsets, we can use  $N$  equal intervals. A more efficient system is to use the simulated density function  $f(x)$  of the variable  $x_{a,b;t}$  got from 20 repetitions of the simulation, each composed by 600 steps. We can define a partition of intervals such that

$$\int_{x_0}^{x_1} f(x) dx = \int_{x_1}^{x_2} f(x) dx = \dots = \int_{x_{N-1}}^{x_N} f(x) dx$$

so as to find the  $N$  quantiles.

We define the binary vector  $G = (g_1(x), g_2(x), \dots, g_N(x))$  where  $g_i(x) \in \{0, 1\}$  as defined above.

$$G = \{(g_1(x), g_2(x), \dots, g_N(x)) : g_i(x) \in \{0, 1\}\} \quad i = 1, 2, \dots, N,$$

The vector in which we record the values of the  $N$  functions is  $g_i(x_{a,b;t}), i = 1, 2, \dots, N$ . Figure 6 reports an example of function  $G = [g_1(x), g_2(x), \dots, g_N(x)]$ , with  $N = 5$  and the domain of density function being  $f(x), [-1, 1]$ .

By means of this function the Central Bank can allow a financial connection from lenders to the generic borrower  $b$  every time it looks into the interbank system for money. If  $a$  is a lender and  $g_i(x_{a,b;t})$  assumes equal value of one, then a financial connection is allowed; otherwise not.

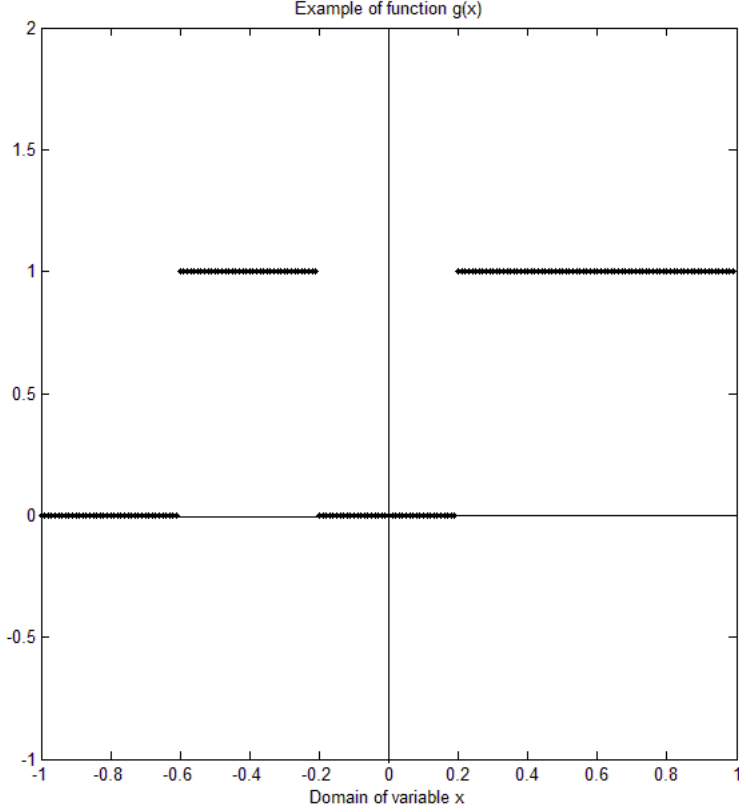


Figure 6: Example of function  $G = [g_1(x), g_2(x), \dots, g_N(x)]$  with  $N = 5$  and domain of density function  $f(x)$ ,  $[-1, 1]$ .

### 3.3 Instability Index of the System

The instability index of bank  $z$  with respect to a rule  $G$  is given by :

$$inst_G(A_{z,t}) = \begin{cases} |A_{z,t_{end}}| & \text{if } A_{z,t_{end}} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.2)$$

where  $A_{z,t_{end}}$  indicates the net value of bank  $z$  at the end of the step  $t$ . If  $A_{z,t_{end}} < 0$ , we have that the bank  $z$  has not got enough credit in the interbank system and so it fails in the step  $t$ . We point out that the instability for the bank  $z$  is positive when it fails and it identifies the loss of money consequences of its bankruptcy.

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It is important to note that our purpose is to identify the vector  $G$ , constant with respect to the steps, by minimizing the average instability of the bank system with respect to all banks, all steps of the algorithm and fixed all the other parameters. We repeat the algorithm several times and compute the average instability of the interbank system also with respect to all repetitions of the algorithm itself.

The instability index of the entire banking system is given by:

$$Inst\left(\{A_{z,t,r}\}_{z \in Z, t \in T}, Rep, T, Z; G\right) = \frac{100}{Rep} \sum_{r=1}^{Rep} \frac{1}{T} \sum_{t=1}^T \frac{1}{\sum_{z \in Z} |A_{z,t_{end};r}|} \sum_{z \in Z} inst_G(A_{z,t;r}) \quad (3.3)$$

where  $Rep$  is the number of repetitions of the algorithm,  $T$  is the number of steps,  $Z$  is the set of banks in the system. This function give us the index of instability of the system. We note that  $Inst(A_{z,t}, Rep, T, Z; G) \in [0, 100]$ . This index assumes a value equal to 100, if in all repetitions and steps of the algorithm every bank fails at the end of each month, is 0 if in all repetitions and steps no banks fail at end of each month.

The optimization problem to create the best interbank structure is:

$$G^* = \arg \min_G Inst\left(\{A_{z,t,r}\}_{z \in Z, t \in T}, Rep, T, Z; G\right) \quad (3.4)$$

With six sub-sets that composed the partition of the domain of  $x$ , we have 64 different vectors  $G$ . To have the guarantee that  $G^*$  is the global optimum of the optimization problem, we solve it by enumeration: we will try every possible solution.

The idea is to find a rule that minimizes the instability by decreasing the degree of freedom of banks in searching for credit in the interbank system. The Central Bank can check that this kind of rule is respected by creating a document of exchange of money among banks similar to the propose of De Larosiere Report (2009) for a global initiative to create an international register of claims between financial institutions.

We introduce another index:

$$Inst_2\left(\{A_{z,t,r}\}_{z \in Z, t \in T}, Rep, T, Z; G\right) = \frac{100}{Rep} \sum_{r=1}^{Rep} \frac{1}{T} \sum_{t=1}^T \frac{1}{|Z|} \sum_{z \in Z} inst_{2,G}(A_{z,t;r}) \quad (3.5)$$

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with  $|Z|$  the cardinality of the set of banks.

$$inst_{2,G}(A_{z,t}) = \begin{cases} 1 & \text{if } A_{z,t_{end}} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.6)$$

This index gives us the average number of failed banks (Iori et al. (2006)).

During the optimization (3.4), solved by enumeration, we record this index for each possible tested solution  $G$ . The aim is to check if the solution  $G^*$  is also the solution that minimizes Eqn. (3.5). We check

$$\arg \min_G Inst_2(\{A_{z,t}\}_{z \in Z, t \in T}, Rep, T, Z; G) = \arg \min_G Inst(\{A_{z,t}\}_{z \in Z, t \in T}, Rep, T, Z; G) \quad (3.7)$$

if this identity is true, minimizing the average number of bankruptcies in a bank system is equivalent to minimize the average loss of money in that system.

### 3.4 Results

The domain of the function  $G$  is divided into the quantiles of order 6.

$$[-0.6063, -0.1044, -0.0396, 0, 0.0323, 0.1136, 1.2972]$$

The solution of problem (3.4) is  $G^* = [0, 1, 1, 0, 0, 0]$ , it means that if the variable  $x_{a,b}$  defined in Eqn. (3.1) assumes values  $-0.1044 \leq x_{a,b} < 0$ , each time that bank  $b$  needs access to the interbank market, then bank  $a$  can lend bank  $b$  money:  $Z_{ab}(a, b) = 1$ . The value of instability index (Eqn. 3.3) is 0.062.

In Figure 7, we report the index of instability for the banking system (Eqn. 3.3) for each of the 64 rules. The values are ordered from the highest to the lowest values, and each of them is the average value with respect to 30 repetitions of the simulation with a specific rule.

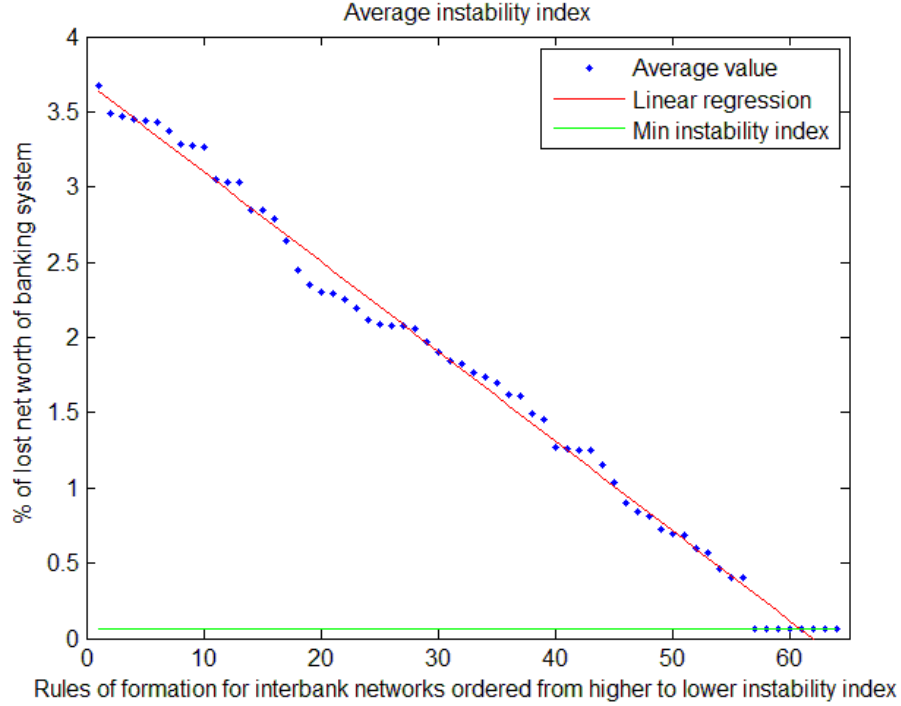


Figure 7: Instability index (Eqn. 3.3) with respect to 25 banks, 600 steps and 30 repetitions of the algorithm.

In Figure 7 we have a discontinuity between the 8th and 9th best rules. The best eight rules are

1. [0, 1, 1, 0, 0, 0]
2. [1, 1, 1, 0, 0, 0]
3. [0, 0, 1, 0, 0, 0]
4. [1, 0, 1, 0, 0, 0]
5. [1, 1, 0, 0, 0, 0]
6. [0, 1, 0, 0, 0, 0]
7. [0, 0, 0, 0, 0, 0]
8. [1, 0, 0, 0, 0, 0]

We note that the eight best rules are the ones that allow no exchange of money from  $a$  to  $b$ , if the value of  $x_{a,b}$  belongs to a part of the domain in which it assumes a non negative value. Only eight out of 64 rules have this property. It means that the non negative value of the variable  $x_{a,b}$  must not be associated with a link in the interbank system from  $a$  to  $b$ . This property implies that only values of  $x_{a,b}$  which are negative are efficient to create links in the interbank system:

$$A_{a,t} - A_{a_{res},t} < -A_{b,t}$$

---

By definition, if the bank  $b$  is requesting for money, we have  $A_{b,t} < 0$ , so it implies that an interbank link between the lender  $a$  and the borrower  $b$  is allowed only if the request of money of bank  $b$  is bigger than the quantity of money that the bank  $a$  can loan. The bank  $b$  must ask more than one bank to get enough quantity of money that is necessary to avoid to the failure: an interbank system is efficient regarding the instability index, if the risk of credit of a borrower bank is diversified among several lender banks. A interbank market in which a lender bank can offer the entire request of money is therefore not efficient and a disconnected interbank system (empty-network) outperforms this case. The empty-network is outperformed by others created by the best six rules in which the diversification of the credit risk among several lender banks is the unique allowed system of loan. The best rule creates an average index of instability 7.33% lower than the one generated by the absence of the interbank network.

To answer the question (3.7), we report the rule that minimize the number of bankruptcies in the system:  $[0, 0, 0, 0, 1, 1]$ . Minimizing the average number of bankruptcies in a banking system is not equivalent to minimizing the average loss of money in that system. The best eight rules that minimize the bankruptcies are:

1.  $[0, 0, 0, 0, 1, 1]$
2.  $[0, 0, 0, 1, 1, 1]$
3.  $[0, 0, 1, 0, 1, 1]$
4.  $[0, 0, 1, 1, 1, 1]$
5.  $[0, 1, 1, 1, 1, 1]$
6.  $[0, 0, 1, 1, 0, 1]$
7.  $[0, 0, 0, 1, 0, 1]$
8.  $[0, 1, 1, 1, 0, 1]$

We note that the difference with respect to the rules that minimize the instability index is the uniquely allowed presence of lenders with a relative high amount of money to lend. The robustness, with respect to the number of bankruptcies in the system, of a configuration in which big banks can be connected with other banks is confirmed in Nier et al. (2007). In Figure 8, we report the bankruptcy index for the banking system (Eqn. 3.5) for each of the 64 rules. The values are ordered from the highest to lowest values, each of them is the average value with respect to 30 repetitions of the simulation with a specific rule. We report in Figure 9 the same bankruptcy index (Eqn. 3.5) with respect to the order of the rules that minimizes the index of instability (Eqn. 3.3). Both figures also reported the rule that minimizes this instability index (Eqn. 3.3).

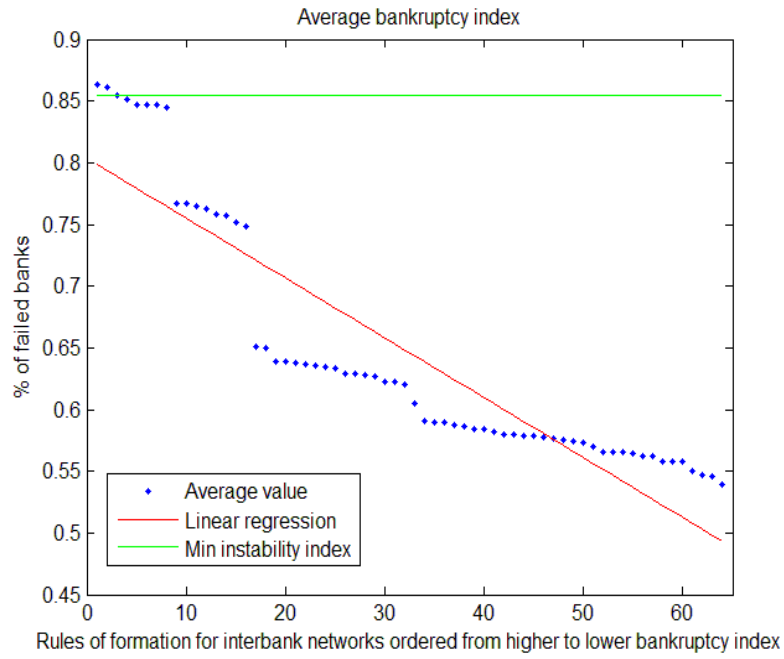


Figure 8: Bankruptcy index (Eqn. 3.5) with respect to 25 banks, 600 steps and 30 repetitions of the algorithm.

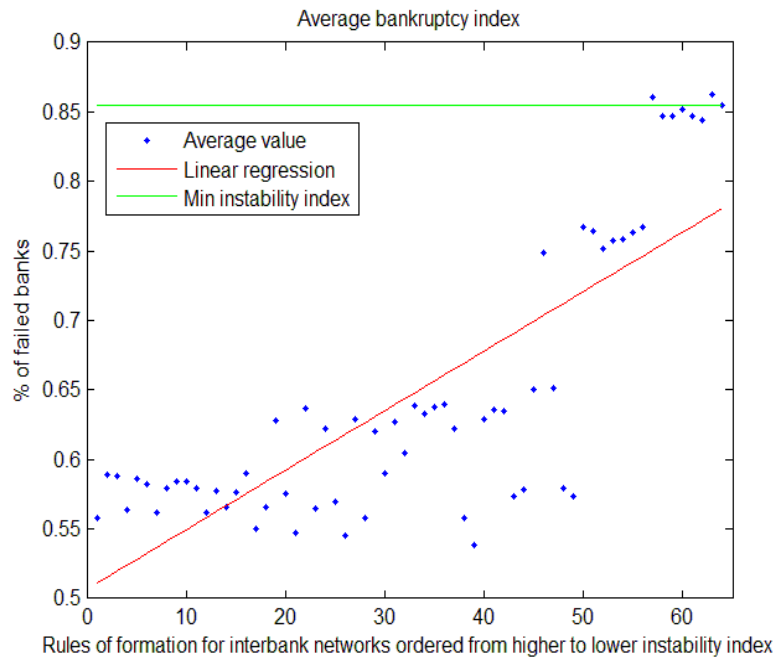


Figure 9: Bankruptcy index (Eqn 3.5) with respect to 25 banks, 600 steps and 30 repetitions of the algorithm ordered with respect to the rules that minimize the index of instability.



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The best eight solutions for the problem of minimization of instability index (3.3) have the highest bankruptcy index. From these figures we can argue that the presence of only small lenders, forcing the borrower banks to diversify its request of money in the interbank system, decreases the instability index (Eqn 3.3) of the system but not the number of bankruptcies (Eqn 3.5) which is minimized with the presence of large lenders (Nier et al. (2007)): the highest average loss of money in the banking system is associated with a low average number of failed banks and the lowest average loss of money is associated with the highest average number of failed banks.

### 3.4.1 Network Analysis of Simulated Interbank System

We compute the properties of the interbank network with respect to the links that banks in the proposed multi-agent system effectively used and not with respect to the theoretically allowed links among the banks imposed by the Central Bank. Banks use the interbank market in the step  $t$  to pay back the debt stipulated in  $t - 1$  and to get new credit. We identify this interbank network as the ‘simulated interbank network’.

Figure 10 shows the density (see Appendix) of the simulated interbank networks, created with the 64 proposed rules, expressed in percentage. The density of a network is the quantity of existing links with respect to the total possible number. With this definition, we have a unitary value when all links are used, while we get zero if the interbank market does not exist (see Appendix). The x axis, the 64 rules, is ordered with respect to the generated instability index (Eqn. 3.3): from left, the highest instability index, to right with the lowest one and solution of problem (3.4). The average density with respect to this ordination of x axis is decreasing. The eight best rules that create a low index of instability, also create low density in the interbank market: in a relatively stable interbank market the banks need access to the interbank market, that has a role of safety network (Section 2.6.1), with relative lower frequency. The value of density with the best rule is 0.018% .

In Figure 11, the reciprocity expressed in percentage is reported. The reciprocity of a network gives a measure of the ratio of the number of links pointing in both directions to the total number of links. With this definition, a unitary value is for a purely indirect network, while we get zero for a purely direct one (see Appendix). An interbank system has low reciprocity if a generic borrower bank, which is borrower at steps  $t - 1$  and  $t$ , chooses different lenders in these two steps. The borrower bank  $a$  in the step  $t - 1$  will ask the set of lenders  $B$  and in the step  $t$  a set of lenders  $C$ . In the step  $t$ , it will give the banks  $B$  money back and receive money from banks the  $C$ . If  $B = C$  we have links in the simulated interbank market, pointing in both directions so the reciprocity should be, relatively to the other proposed rules, high. In Figure 11, we get a low level of reciprocity for the best eight rules that minimize the instability index : the best rule generates a reciprocity of 0.005%, and thus the we have on average  $B \neq C$ .

If banks  $B$  and  $C$  have exchange of money among themselves, we will have ‘triangles’ in the interbank market:  $a$ ,  $B$  and  $C$  are linked. In Figure 12, we report, expressed in percentage, the value of cluster coefficient in the interbank market. This parameter is a quantity proportional to the number of complete triads (triangles) divided by the number of connected triples of vertexes in the indirect network representing the simulated interbank (see Appendix). The best rules present a relatively low level of this coefficient (0.016%), so we can conclude that in a stable interbank market  $B$  and  $C$ , having financial relations with  $a$ , have not exchange of money between themselves. A representation of these relations is plotted in Figure 13. A low level of clustering implies also that if a generic lender lends two or more borrowers money, these borrowers have not financial relationship during that step.

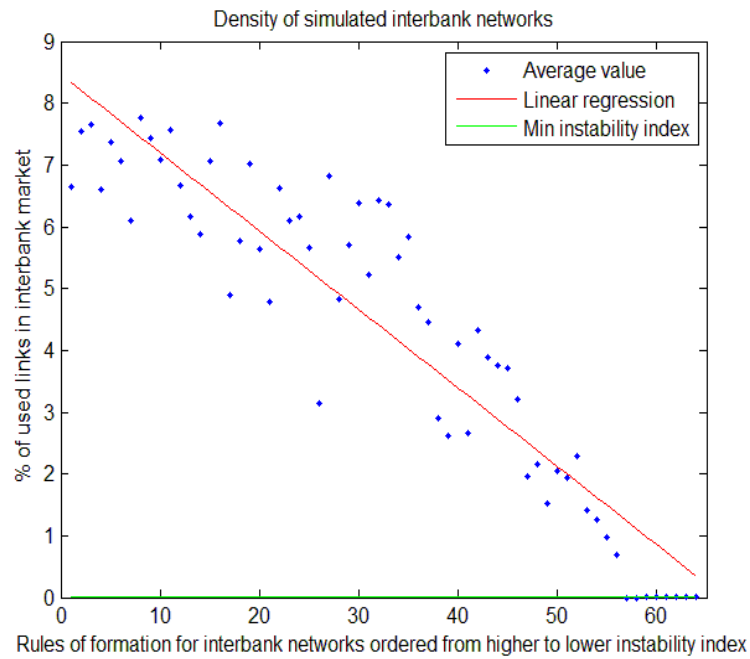


Figure 10: Density of simulated interbank networks with respect to 25 banks, 600 steps and 30 repetitions of the algorithm.

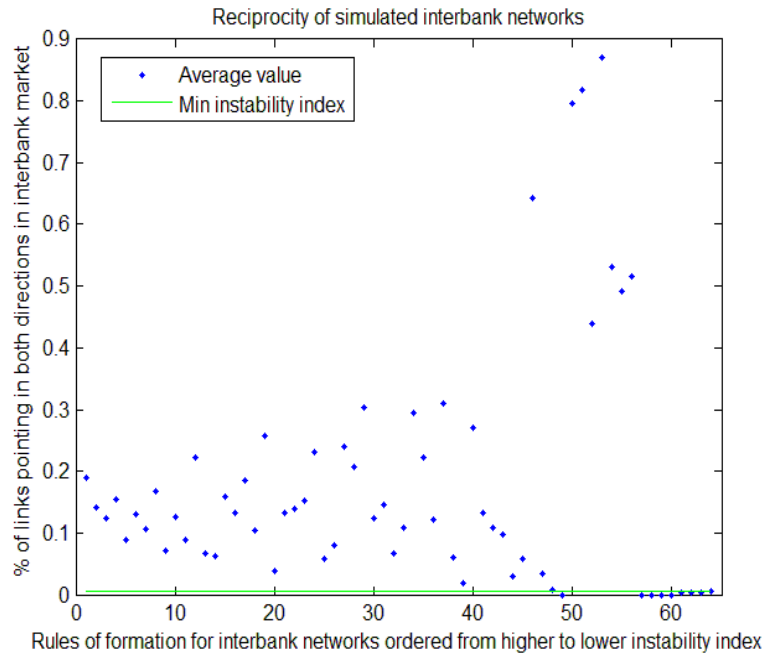


Figure 11: Reciprocity of simulated interbank networks with respect to 25 banks, 600 steps and 30 repetitions of the algorithm.

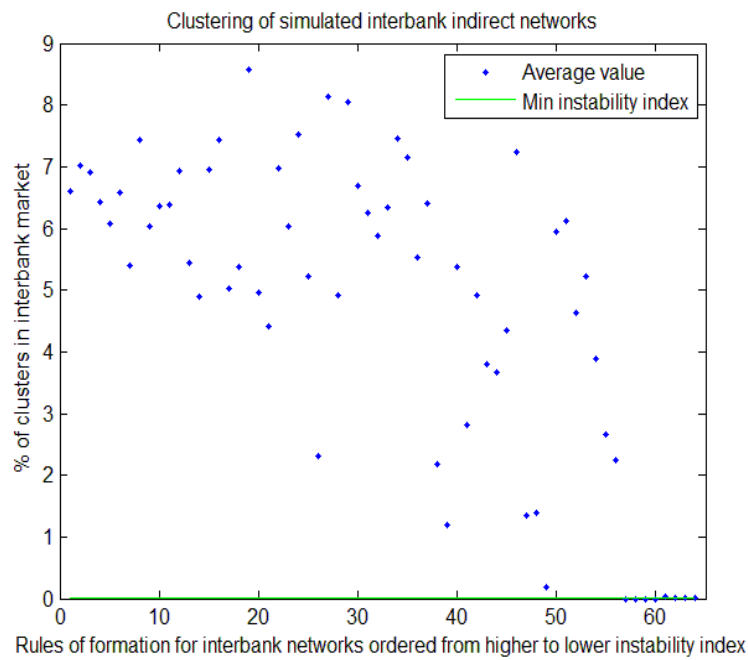


Figure 12: Clustering of simulated interbank networks with respect to 25 banks, 600 steps and 30 repetitions of the algorithm.

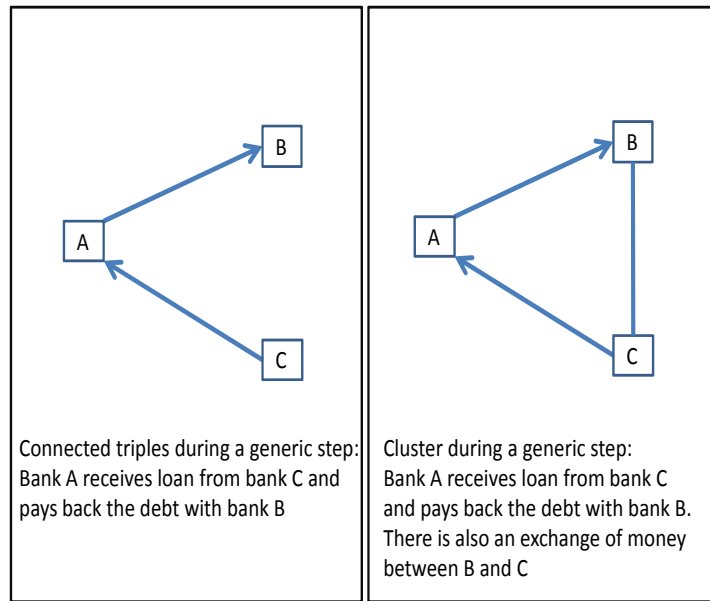


Figure 13: Example of connected triples and cluster in an interbank network.

### 3.4.2 Comparison with Empirical Data

We analyse the created interbank networks and compare the results of the proposed normative approach for the interbank system with data received from the empirical literature, relative to the real life interbank system. In Soramaki et al. (2007), for example, the authors explore the network topology of the interbank payments transferred among commercial banks over the Fedwire Funds Service on 11 September, 2001. In Gabrieli (2011), it is proposed that a detailed network analysis of high-frequency data on unsecured loans traded in e-MID from January 2006 until November 2008 during which the global financial crisis started, two months later the bankruptcy of Lehman Brothers (15.09.2008). Saltoglu B. and Yenilmez T. E. (2010), perform a study of the Istanbul Stock Exchange overnight money market data in the year 2000 and in which there is a period of crisis at the end of that year. The authors argue that the average reciprocity value, which below 1% is a significant character of the Turkish overnight money market, has to be considered as an indicator of systemic risks such as that density.

We report average density and average reciprocity of the USA on 11-09-01 and the EU during November 2008 interbank markets got from Soramaki et al. (2007) and Gabrieli (2011) :

---

Parameters\Markets	USA 11-09-2001	EU 11-2008
Density	0.26%	2.88%
Reciprocity	22.3%	0.1%

The value of density and reciprocity for the USA market on 11-09-2001 is comparable with the value of other periods: the value of these parameters during the fourth quarter of 2004 is 0.3% and 21.5% (Soramaki et al. (2007)).

Reciprocity of a network gives a measure of the ratio of the number of links pointing in both directions to the total number of links. With this definition, a unitary value is for a purely indirect network while we get zero for a purely direct one (see Appendix). Density of a network is the quantity of existing links with respect to the total possible number. With this definition, we have a unitary value when all links are used while we get zero if the interbank market does not exist (see Appendix). We use these data to compute the Euclidean distance between the vectors of the properties of the USA and EU interbank network (each composed of two elements: density and reciprocity parameters) with that one generated from each rule  $G$  (see Eqn. 3.4) in the multi-agent model. With this analysis we check if the rule that in the previous study (section 3.4) has generated the more stable interbank structure is also the rule that generates the interbank market, analysed with respect to the effectively used links among banks, more similar to those in the USA. The same analysis is repeated for EU market.

In Figures 14 and 15, we report the Euclidean distances of the two dimensional vectors, composed of density and reciprocity and generated by the 64 different rules, with respect to the vectors of the interbank market in USA and in EU.

The interbank system proposed in this multi-agent system has a role of safety network. If we assume that the model can capture the principal properties of a real-life interbank market, we can argue that a real-life interbank system, with characteristics not similar to the ones of the model generated by the best rule (solution of Eqn. 3.4), can be improved by adopting similar rules. Critical times, like 11 September 2001 for the USA and November 2008 for EU, are examples to test these similarities and to understand if the real life interbank system can act as safety networks in an efficient way.

In Figure 14, the Euclidean distance between USA interbank market and the simulated one decreases from left to right: the more the proposed rule decreases the instability in the model, the more the difference with the USA interbank market on average decreases. The best rule that decrease the instability index has a Euclidean distance of 0.223 and the one with lowest Euclidean distance of 0.219 . This USA market has the property to have a high reciprocity (22.3%) and low density (0.26%) and that rule generates the highest reciprocity index (Figure 11) and low density (Figure 10): 0.87% and 1.41%.

In Figure 15, the Euclidean distance between the EU interbank market and the simulated one decreases from left to right: the more the proposed rules decrease the

instability in the model, the more the differences with the EU interbank market on average decreases. The best rule that decreases the instability index has a Euclidean distance of 0.0287 and the one with the lowest Euclidean distance of 0.0079. This EU market has the property to have a relative low reciprocity (0.1%) and high density (2.88%), and that rule generates a reciprocity index (Figure 11) and a density of 0.0189% and 2.61% (Figure 10).

In Figure 16, the density and reciprocity for the USA and EU markets are reported together with the relative network properties of simulated interbank networks, with respect to different rules. From the analysis of the comparisons we can conclude that a normative approach tends also to decrease on average the dissimilarity from real-life markets, and also from a lower level of reciprocity in the USA market and a lower level of density in EU one could increase their stability if the proposed model can adequately describe the real-life interbank markets.

In this multi-agent model, the principal limit is that the banks can only stipulate one month-financial contract: create debt at step  $t$  to repay it at  $t + 1$ . A more realistic model, calibrated with respect to the real world, and with a more detailed interbank system, could be used for future analyses based on this approach to investigate how stabilized are the real-life interbank markets.

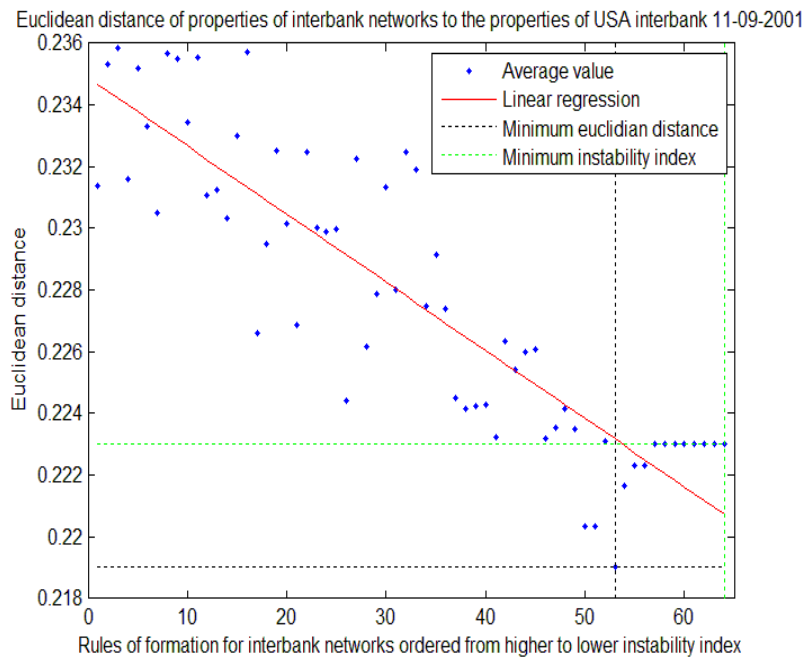


Figure 14: Euclidean distance of properties of interbank networks to the properties of the USA interbank 11-09-2001. 25 banks, 600 steps and 30 repetitions of the algorithm.

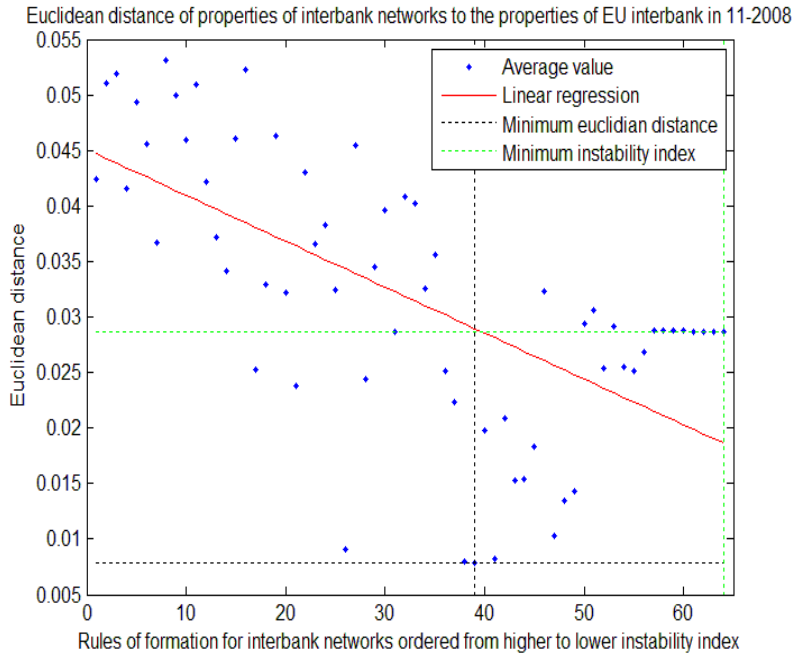


Figure 15: Euclidean distance of properties of interbank networks to the properties of EU interbank in 11-2008. 25 banks, 600 steps and 30 repetitions of the algorithm.

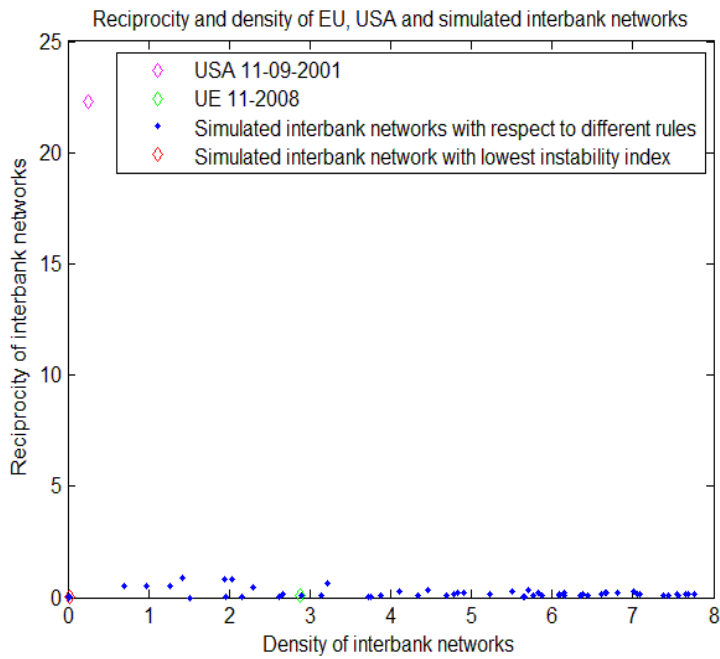


Figure 16: Reciprocity and density of the EU, USA and simulated interbank networks with respect to different rules. 25 banks, 600 steps and 30 repetitions of the algorithm.





# 4 Maximization of the Stability of the Bank-Firm System

In this fourth chapter we want to find out some rules that allow the system to increase the stability of the system comprising banks and firms. In this part, we use the model created in the Chapter 2 and the interbank market developed in Chapter 3 to create further normative approaches to answer the questions of the thesis:

- < How can the Government or other institutions like the Central Bank create some rules to determine interbank and banks-firms relationships so as to get improvements in the stability of the entire economic system? >

## 4.1 Improvement of the Stability of Banks-Firms System

In this step, the institution can change the same parameters of the simulated society, which, till now, were assumed constant, to improve the stability of the interbank and bank-firms system. We assume that the Government or other institutions can have the data relative to the single step returns of the banks system and the firms system.

We define  $K = J \cup I \cup Z$  the set of U firms, D firms and banks,  $A_{k,t_{end}}$  net value at the end of the period  $t$ , before the failed banks and firms are replaced. The replacing is the last operation in each step of the algorithm. After it we have the beginning of the next step.  $A_{k,t-\tau+1}$  is the net value at the beginning of the step  $t - \tau + 1$ . The return of the banks, U firms, D firms system, computed with respect to each interval  $\tau \geq 0$  arbitrarily chosen is:

$$\varsigma_K(t; \tau) = \frac{1 + \sum_{k \in K} A_{k,t_{end}}}{1 + \sum_{k \in K} A_{k,t-\tau+1}}$$

By definition, the net value of each element of this simulated society at the beginning of each step is not negative, because if the net worth of a bank, U firm, D firm at the end of previous month is negative, it fails, and so at the beginning of the new period we replace it with a new insitution that has a not negative net worth.

---

We assume that the Central Bank can impose and change during months these variables:  $\nu$ ,  $\chi$  and  $r_b$ .

- $\nu$  : the official discount rate set by the Central Bank.

This variable influences the return of banks and the economy of firms because it changes directly the interest rate proposed by banks to firms. We report the definition of it (Eqn. 2.13)

$$r_{z,t}^x = \nu + \sigma A_{z,t}^\sigma + \theta (l_{x,t})^\theta \quad x = i, j$$

$l_{x,t}$  is the leverage ratio of the  $x$  firm (Eqn. 2.14),  $\sigma > 0, \theta > 0$  are constant parameters for bank world.  $i$  is D firm index,  $j$  is U firm index.

- $\chi$ : parameter that identifies the minimum reserve ratio that must be kept by each bank.

As in the Bank for International Settlements (BIS) (1988) and Blenck et al. (2001), this is related to the management of the risk imposed by international standards: in this model, it is the part of deposits that each bank cannot invest. We propose a dynamic change of this parameter with the aim to offer better performance of a static parameter. We report the equation that defines the deposits not allocable in the risky assets (Eqn 2.24)

$$res_{z,t} = \chi (sc_{z,t} + R_{z,t})$$

- $r_b$ : is the interbank interest rate.

In previous chapters we have imposed it to be an exogenous constant parameter. Now we allow it to change. The interbank interest rate is the rate that every bank must pay in case of debt contracts to a lender bank. An example of its average value is the London Interbank Offered Rate, LIBOR, that is defined for one day, one month, two months, six months, one year etc ('Behaviour of Libor in the Current Financial Crisis', (2009)). A low interbank interest rate can help banks that need liquidity to avoid failing, but also decrease the return of lender banks.

We define the difference between the returns of the bank and firms' system, which is the imbalances that arises endogenously at step  $t$ :

$$\varsigma_{Diff}(t; \tau) = \zeta_Z(t; \tau) - \frac{1}{2} (\zeta_J(t; \tau) + \zeta_I(t; \tau)) \quad (4.1)$$

---

with  $J, I, Z$  the set of U firms, D firms and banks. The firms' world is composed of two layers and thus, we use the average value of them.  $\zeta_{Diff}(t)$  is the controlling variable: the parameter that the institution uses to change the controlled variables  $\nu, \chi, r_b$ .

A lower level of official discount rate can improve the economy of the firms' world, decreasing the probability of bankruptcies among firms that can easily repay the debt. A decreasing probability of the failed firms can decrease the instability index of firms and can increase also their average return  $\zeta(t; \tau)$ , because the number of failed firms, which have negative net values at the end of the step, will decrease. If, instead, the value of  $\nu$  does not decrease, the firms, with hypothesis to have returns lower than banks, can need, step by step, more credit from the bank to avoid failing. The request of credit with relatively high interest rate (Eqn. 2.13) will increase the probability of bankruptcies in firms and, then, increase the probability of bankruptcies for banks, due to not repaid credits. An opposite reasoning is true if the performance of firms are bigger than the banks. See Diamond and Rajan (2009) for an analysis of the interest rate during crisis.

The dynamic change of  $\chi$ , parameter that identifies the minimum reserve kept by each bank, is similar to  $\nu$  ones. A decrease in  $\chi$  will allow the firms to have the possibility to get more credit from the banks. On the other hand, it exposes these banks to risky assets with the possibility to go bankrupt if the firms will not pay back the debt. If the economic situation of the banks, measured by the returns, is better than that of the firms, the first effect, more credit for firms, will give the entire system more benefits than the possible negative impact due to the second effect. An opposite reasoning is true if the performance of firms is bigger than that of the banks.

$r_b$ , the interbank interest rate, is a parameter that concerns only the banking system. In Freixas et al. (2009), the authors show that interbank rates should be low during a financial crisis and high in normal times. Following the events of the financial crisis in 2008, starting with financial troubles in the banking system and propagate to the firms, we can argue that if not positive periods for the bank system starts, the return of it will be lower than firms' system. This difference of performance is then an index that can suggest the Central Bank to change the interbank interest rate.

We assume that the relations between the set of the variables are

$$\nu(t+1) = \begin{cases} \nu(t) (1 + \Delta_\nu \cdot \zeta_{Diff}(t)) & \text{if } t = n\tau; n \in \mathbb{N} \\ \nu(t) & \text{otherwise} \end{cases} \quad (4.2)$$

$$\chi(t+1) = \begin{cases} \chi(t) (1 + \Delta_\chi \cdot \zeta_{Diff}(t)) & \text{if } t = n\tau; n \in \mathbb{N} \\ \chi(t) & \text{otherwise} \end{cases} \quad (4.3)$$

$$r_b(t+1) = \begin{cases} r_b(t) (1 + \Delta_{r_b} \cdot \zeta_{Diff}(t)) & \text{if } t = n\tau; n \in \mathbb{N} \\ r_b(t) & \text{otherwise} \end{cases} \quad (4.4)$$

we insert constraints to the maximum and minimum value for the controlled variables:

$$\nu(t) \in [0.1\nu(0), 10\nu(0)] \quad \forall t \in \{1, 2, \dots, T\}$$

$$\chi(t) \in [0.1\chi(0), 10\chi(0)] \quad \forall t \in \{1, 2, \dots, T\}$$

$$r_b(t) \in [0.1r_b(0), 10r_b(0)] \quad \forall t \in \{1, 2, \dots, T\}$$

The constraints are imposed to keep economic meaning of them. If a variable will exceed the maximum in a step, it will be forced to have the maximum value for that step. The analogous is true for value below the minimum.

The values of the variable  $\nu(t)$ ,  $\chi(t)$ ,  $r_b(t)$  are computed by means of equations (4.2), (4.3) and (4.4) at the beginning of step  $t$ .

$\nu(0) = 0.015$ ,  $\chi(0) = 0.01$ ,  $r_b(0) = 0.0025$  are the set of parameters proposed in paragraph 2.7 and used in chapter 3 for the optimization of interbank market. These values are the benchmark, equivalent to impose  $\Delta_\nu = \Delta_\chi = \Delta_{r_b} = 0$  in Eqns (4.2), (4.3) and (4.4).

These equations represent the normative approaches by means of them we try to decrease the instability of the system composed of banks, U firms and D firms. The parameters to fix are then  $\Delta = \{\Delta_\nu, \Delta_\chi, \Delta_v\}$ ,  $\tau$ . We impose  $\tau$  and we use an optimization system, analogous to that described in chapter 3, to establish the remaining. We introduce these definitions of instability index:

$$inst_{G^*}(A_{k,t}) = \begin{cases} |A_{k,t_{end}}| & \text{if } A_{k,t_{end}} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.5)$$

$$Inst(\{A_{k,t}\}_{k \in K, t \in T}, Rep, T, K; G^*, \Delta) =$$

$$\frac{100}{Rep} \sum_{r=1}^{Rep} \frac{1}{T} \sum_{t=1}^T \frac{1}{\sum_{k \in K} |A_{k,t;r}|} \sum_{k \in K} inst_{G^*, \Delta}(A_{k,t;r}) \quad (4.6)$$

with  $k = j, i, z$  U firm, D firm and bank,  $K = J, I, Z$  set of U firms, D firms and banks. The system is influenced by the values of the parameters  $\Delta$ , and by the interbank structure characterized by vector  $G^*$  solution of the Eqn (3.4).

The instability index of the entire economic system is the average value of this index computed for bank and firms' sector. The index for the firms' sector is the average value of the ones computed for U and D firms.

$$\begin{aligned}
& INST \left( \{A_{k,t}\}_{k \in Z \cup IUJ, t \in T}, Rep, T, Z, J, I; G^*, \Delta \right) = \\
& \frac{1}{2} Inst \left( \{A_{k,t}\}_{k \in Z, t \in T}, Rep, T, Z; G^*, \Delta \right) + \\
& + \frac{1}{4} \left( Inst \left( \{A_{k,t}\}_{k \in J, t \in T}, Rep, T, J; G^*, \Delta \right) + Inst \left( \{A_{k,t}\}_{k \in I, t \in T}, Rep, T, I; G^*, \Delta \right) \right)
\end{aligned} \tag{4.7}$$

This index assumes values equal to 100 if in all repetitions and steps of the algorithm every bank, U firm and D firm fail at the end of each month, it is 0 if no element fails. To find the optimal set of parameters  $\Delta^* = \{\Delta_\nu^*, \Delta_\chi^*, \Delta_{rb}^*\}$  to have more stability in the system, we solve this optimization problem

$$\Delta^* = \arg \min_{\Delta} INST \left( \{A_{k,t}\}_{k \in Z \cup IUJ, t \in T}, Rep, T, Z, J, I; G^*, \Delta \right) \tag{4.8}$$

To solve it we assume that the Central Bank can check in each time interval  $\tau$  of the algorithm the variable  $\varsigma_{Diff}(t; \tau)$ . To have the guarantee to get the global optimum among possible solutions, we solve it by enumeration. We impose the search for the best parameters in this proposed set:

$$\Delta_\nu^*, \Delta_\chi^*, \Delta_{rb}^* \in \{-0.4, -0.2, 0, 0.2, 0.4\}.$$

We introduce the average return for the system

$$\begin{aligned}
& RETURN \left( \{A_{k,t}\}_{k \in Z \cup IUJ, t \in T}, Rep, T, Z, J, I; G^*, \Delta \right) = \\
& = \frac{1}{2} \frac{1}{Rep} \sum_{r=1}^{Rep} \frac{1}{T} \sum_{t=1}^T \frac{1 + \sum_{z \in Z} A_{z, t_{end}; r, G^*, \Delta}}{1 + \sum_{z \in Z} A_{z, t; r, G^*, \Delta}} +
\end{aligned}$$

$$+ \frac{1}{4} \left( \frac{1}{Rep} \sum_{r=1}^{Rep} \frac{1}{T} \sum_{t=1}^T \frac{1 + \sum_{i \in I} A_{i,t_{end};r,G^*,\Delta}}{1 + \sum_{i \in I} A_{i,t;r,G^*,\Delta}} + \frac{1}{Rep} \sum_{r=1}^{Rep} \frac{1}{T} \sum_{t=1}^T \frac{1 + \sum_{j \in J} A_{j,t_{end};r,G^*,\Delta}}{1 + \sum_{j \in J} A_{j,t;r,G^*,\Delta}} \right) \quad (4.9)$$

with  $k = j, i, z$  U firm, D firm and bank,  $K = J, I, Z$  set of U firms, D firms and banks.

In the study of the solution of the optimization problem (4.8) we analyse the possibility to have minimization of the instability index and maximization of Eqn. (4.9) with respect to the tested set of parameters  $\Delta$ . We test the hypothesis that the set of parameters that minimize the instability index of the entire economy also maximize its return.

$$\begin{aligned} & \arg \min_{\Delta} INST \left( \{A_{k,t}\}_{k \in Z \cup I \cup J, t \in T}, Rep, T, Z, J, I; G^*, \Delta \right) = \\ & = \arg \max_{\Delta} RETURN \left( \{A_{k,t}\}_{k \in Z \cup I \cup J, t \in T}, Rep, T, Z, J, I; G^*, \Delta \right) \end{aligned} \quad (4.10)$$

To analyse the role of the bank system we repeat the normative process only considering the instability index of bank: the optimization process (4.8) will be computed with respect to the minimization of instability of bank system:

$$\arg \min_{\Delta} Inst \left( \{A_{k,t}\}_{k \in Z, t \in T}, Rep, T, Z; G^*, \Delta \right) \quad (4.11)$$

### 4.1.1 Results

To solve the problem (4.8) we select  $\tau = 6$  and  $Rep = 30$ . In Figure 17, we report the average instability for the entire economic system. The x axis, the 125 tested rules, is ordered with respect to the generate instability index (Eqn. 4.7) from left, highest instability index, to right with the lowest one and solution of (4.8). The same x axis definition is kept in the rest of the figures.

The minimum instability index for the optimized system is 0.476. In the graphics is reported the benchmark, the index of instability if in the system is non applied any rules, equivalent to impose  $\Delta_{\nu} = \Delta_{\chi} = \Delta_{r_b} = 0$ , and present also in the rest of the figures. For the benchmark, we have  $\nu(0) = 0.015$ ,  $\chi(0) = 0.01$ ,  $r_b(0) = 0.0025$  with instability index 0.494: the proposed normative approach decreases the instability of 3.643%.

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The set of parameters that minimize the instability index is  $\{-0.4, -0.4, 0.4\}$ : a negative linear relation between the official discount rate and difference of returns between banks and firms (Eqn. 4.2), a negative linear relation between the minimum reserve and it (Eqn. 4.3), and a positive linear relation between interbank interest rate and it (Eqn. 4.4). If the return of bank system is bigger than that of firms, a decreasing official discount rate and minimum reserve factor allow firms to get more loan from banks with lower interest rate, with the possibility of increasing their performance. The interbank interest rate is directly proportional: if the return of the bank system is low the interbank rate will decrease, and the banks can easily exchange money among themselves with the possibility to increase their financial stability.

In Figure 18, we report the average instability index of the firms' world, average value of the one for U firms and D firms. The minimum value is 0.867, the original one is 0.926. We get a decrease of 6.371%.

In Figure 19, we report the average instability index for bank. The original value of it, got solving the problem (3.4), is 0.062, with normative approach (Eqn. 4.8) it is 0.085, with increase of 37.096%. The increase of instability index of bank is the consequence of the fact that the normative approach tends to increase the overall index of the system, comprising bank and firms' worlds: a penalization in a sector, the bank's one, can increase the stability of the entire system, result confirmed also if we analyze the average number of banks and firms that fail.

Figure 20 represents the bankruptcy index for firms, average value of the indexes (3.5) computed for U firms and D firms: the original value is 1.869, with the optimal rule we get 1.805, with a decrease of 3.424%.

Figure 21 represents the bankruptcy index (3.5) for banks: the original value is 0.857, while the optimal rule we get 1.154, with an increase of 34.655%.

An analogous result is obtained by focusing on the return (4.9). Figure 22 represents the return of firms' world: the original value is 0.951, with the optimal rule we get 0.952, with an increase of 0.105%. Figure 23 represents the return of the banking world: the original value is 0.987, while the optimal rule we get 0.982, with a decrease of 0.506%.

To check the identity (4.10), we report in Figure 24 the average value of the return for the entire system. The original value is 0.969, while the optimal rule we get 0.967, with a decrease of 0.206%. The identity (4.10) is not true: the rule penalize the bank's ones, impeding the possibility to have a maximization of the return of entire system when we minimize its instability.

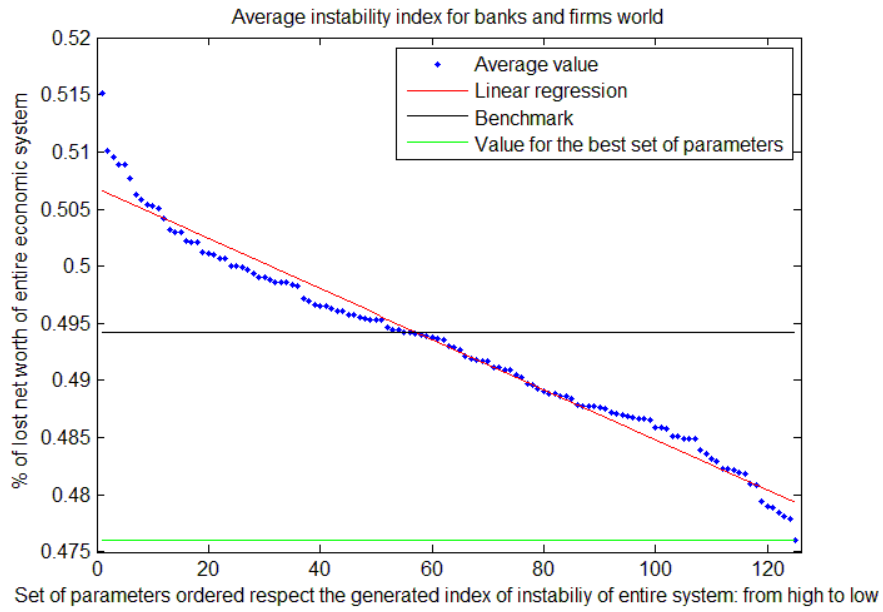


Figure 17: Average instability index for entire economy. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

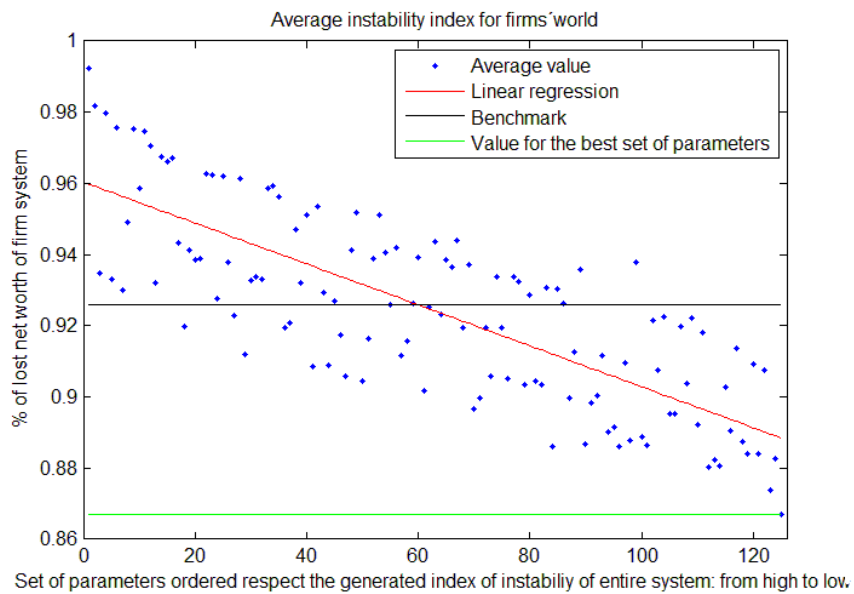


Figure 18: Average instability index for firms. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.



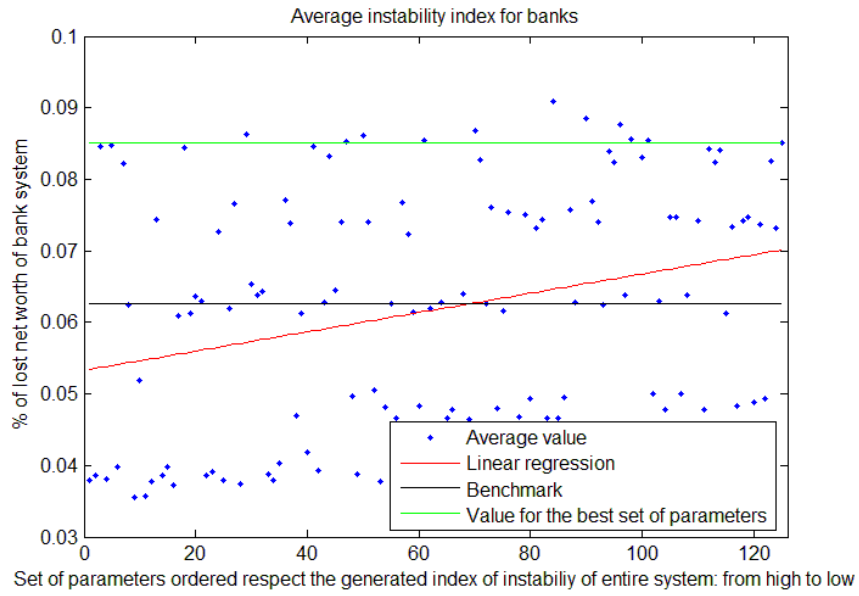


Figure 19: Average instability index for banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

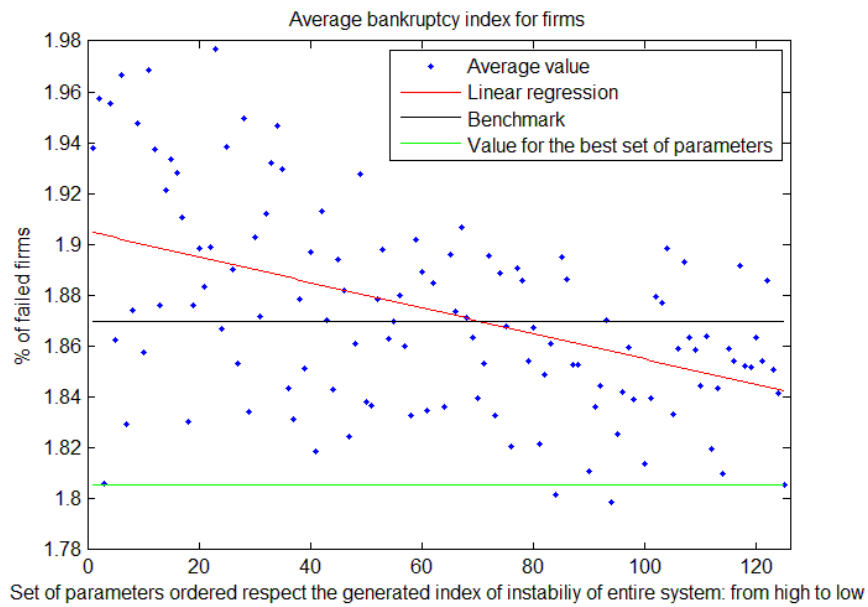


Figure 20: Average bankruptcy index for firms. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

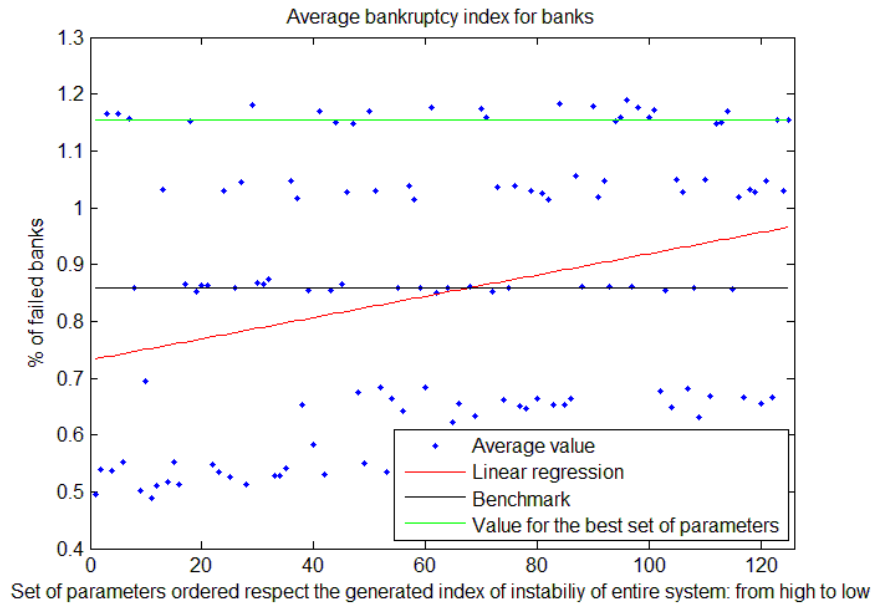


Figure 21: Average instability index for banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

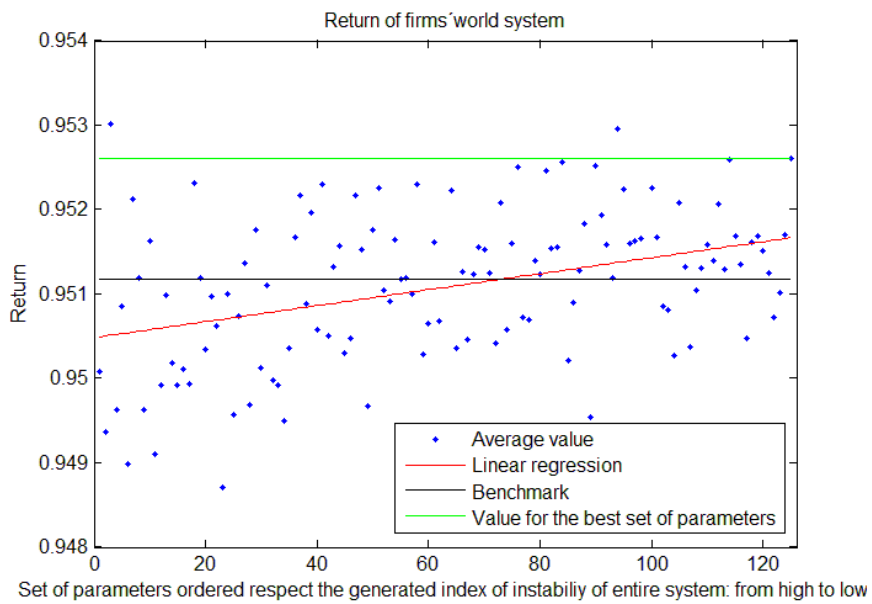


Figure 22: Average return for firms. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

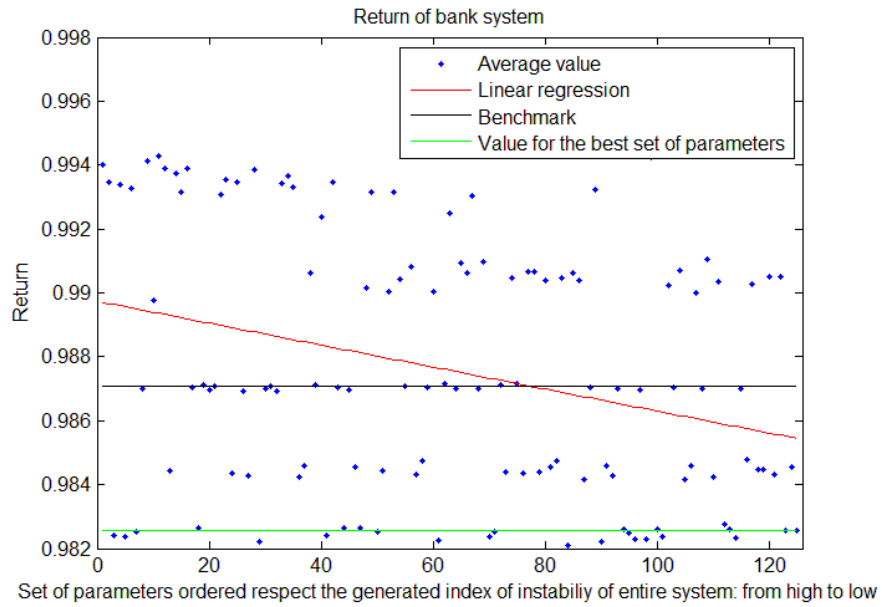


Figure 23: Average return for banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

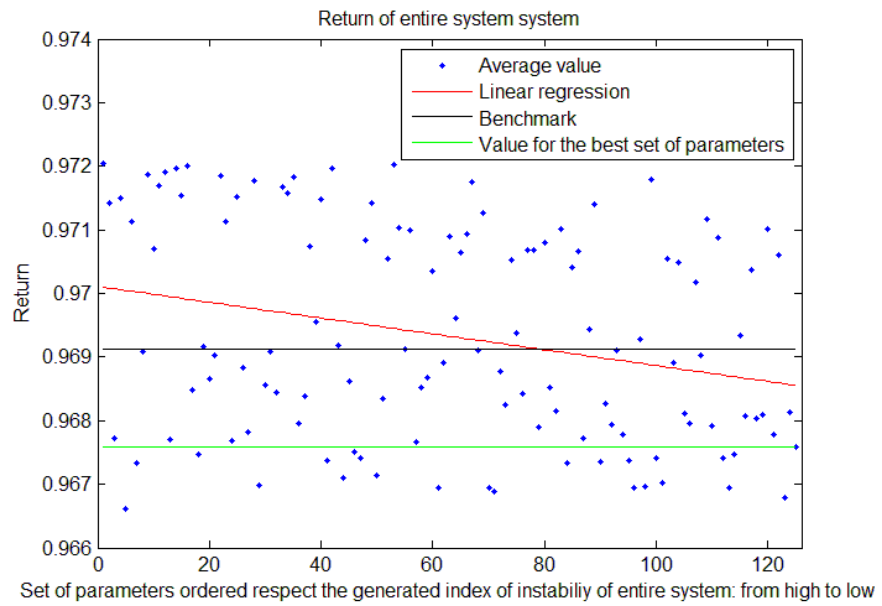


Figure 24: Average return for entire economy. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

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To analyse the role of the banking system we repeat the normative process only considering the instability index of bank: the optimization process (4.8) will be computed with respect to the minimization of instability of the banking system (see Eqn. 4.11).

The minimum instability index for the banking system, plotted in Figure 25, is 0.035 for the best rule. Comparing the result got in Figure 19, where the normative approach is computed with respect to the banking and firms system (Eqn. 4.7) and the best rule generates an instability index for banks of 0.085, we have a decrease of instability of 58.821%. The optimization is with respect to the banking system: the found best rule penalizes the firm system to advantage the bank one. In Figure 26 the best rule increases the instability index of the firm system of 1.296% with respect to the benchmark. The penalization in the firm system to advantage the bank one is confirmed also analysing the bankruptcy index for firms (Figure 29), and banks (Figure 28), the return for firms (Figure 31), and banks (Figure 30). Considering bank and firms (Eqn. 4.7) we get a decrease in the instability of 1.578% with respect to the benchmark. The result is plotted in Figure 27.

The rule that minimizes the instability of the bank system is  $\{0.4, -0.2, -0.4\}$ . A positive linear relation between the official discount rate with the difference of returns between bank and firms (Eqn. 4.2), negative relation between the minimum reserve for banks with it (Eqn. 4.3) and negative relation between interbank interest rate (Eqn. 4.4) with it.

If the return of bank system is bigger than the one of firms, an increase in the official discount rate can increase the return on investment for banks and also the possibility of loss of it due to the bankruptcy of firms. Minimizing the index of instability for banks, we get a positive linear relation (+0.4): the increase of the return of investment get over the increased possibility of loss of the investment. Also, negative linear relations (-0.2) between the reserve factor for banks and the difference of the return between banks and firms confirms that an increase of the quantity of money of the bank available in investment in firms world can increase the stability of banks even if these investments are risky. A negative linear relation (-0.4) between the interbank interest rate and the difference of the return between banks and firms is the consequence of the other previous two relations: if the return of the bank is bigger than that of the firm, the official discount rate increases and when it increases the probability of firms to fail. The reserve factor decreases, exposing banks more to risky investments. In this context, a decrease in interbank interest rates can facilitate the flow of money among lender banks and borrowers, that have faced loss due to the failure of firms in which they have invested.

Figures 25, 28 and 30 report five clusters in the graphics, respectively, for the instability index, the bankruptcy index, and the return for banks. Each cluster is associated with a value of the parameter  $\Delta_r$  that links the difference of return between banks and firms and the official discount rate (Eqn. 4.2). This parameter is then the most important among the three in the minimization of instability index of banks.

From left in the figures to right, equivalent to the highest generated instability index for bank to the lowest one, we have respectively:  $\Delta_\nu = -0.4, -0.2, 0, 0.2, 0.4$ . The positive relations between the discount factor and the difference of the returns always increase the stability of the bank system allowing it to have higher interest rate from the investments in the firm system, if the bank system's return is bigger than the firms' one. These rule advantages the bank system to the the detriment of firms' system (see Figures 26, 29 and 31).

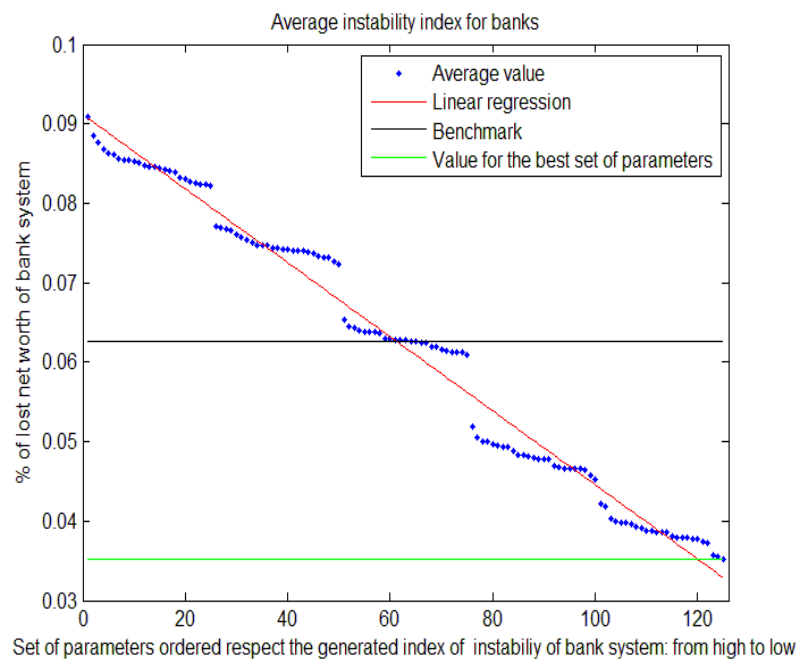


Figure 25: Average instability index for banks minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

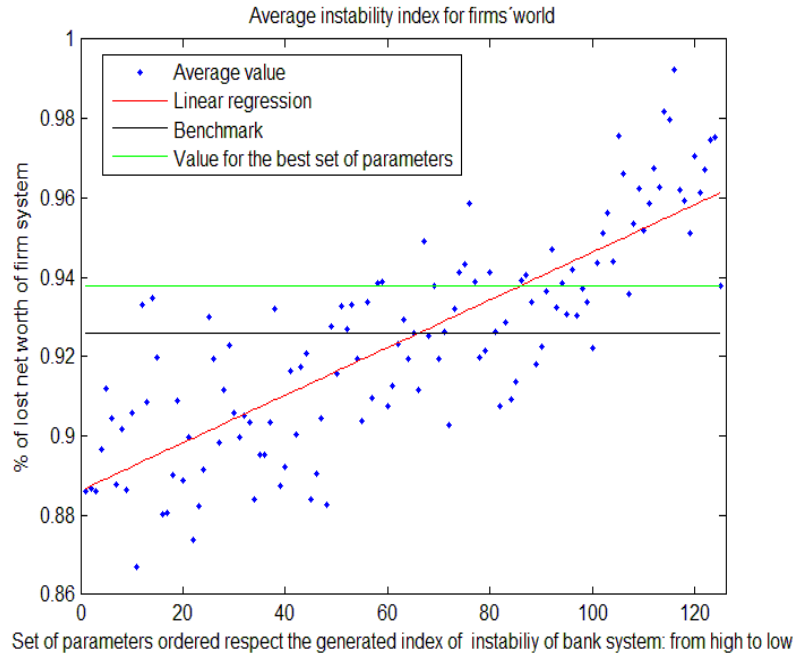


Figure 26: Average instability index for firms minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

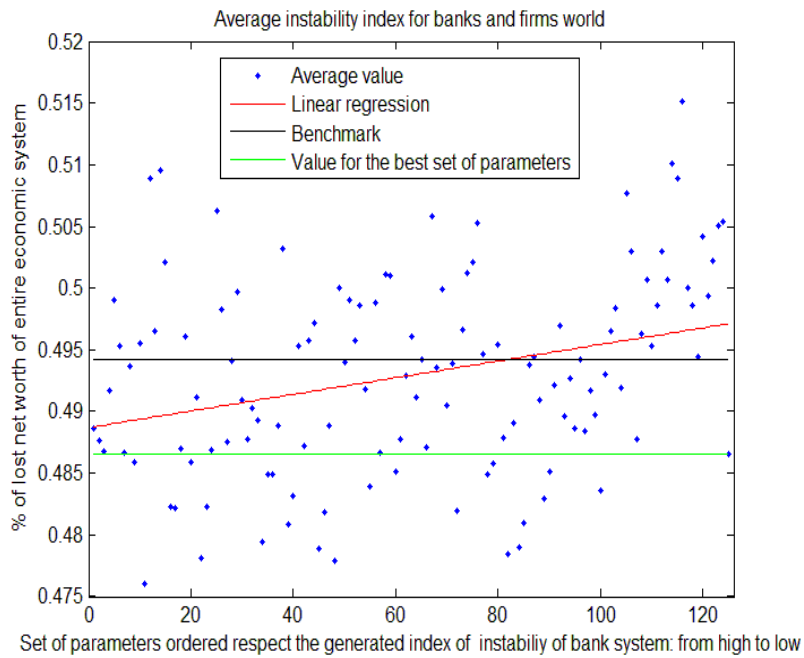


Figure 27: Average instability index for entire economy minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

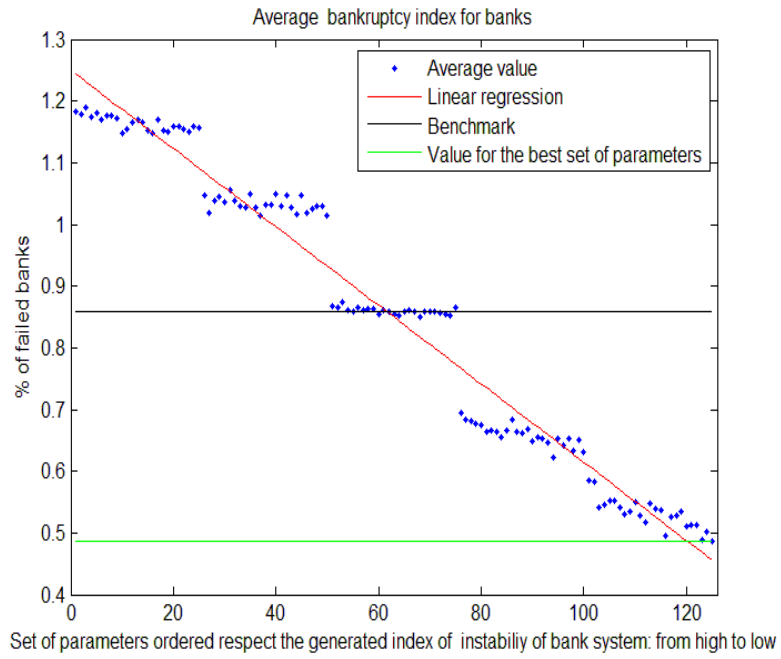


Figure 28: Average bankruptcy index for banks minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

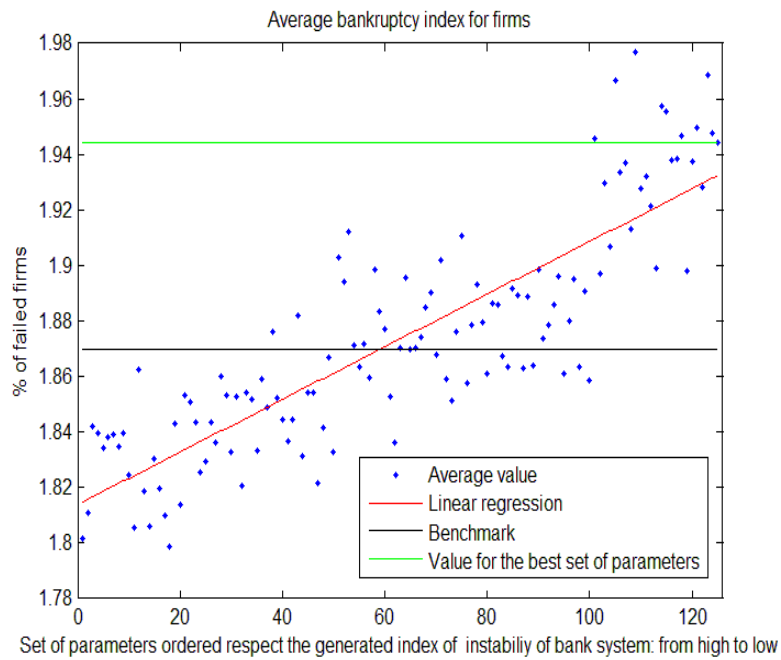


Figure 29: Average bankruptcy index for firms minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

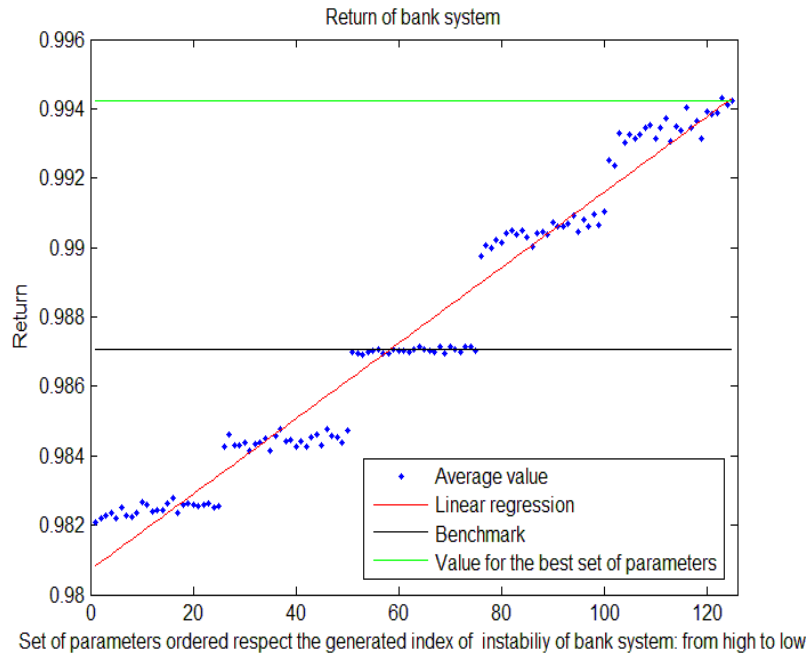


Figure 30: Average return for banks minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

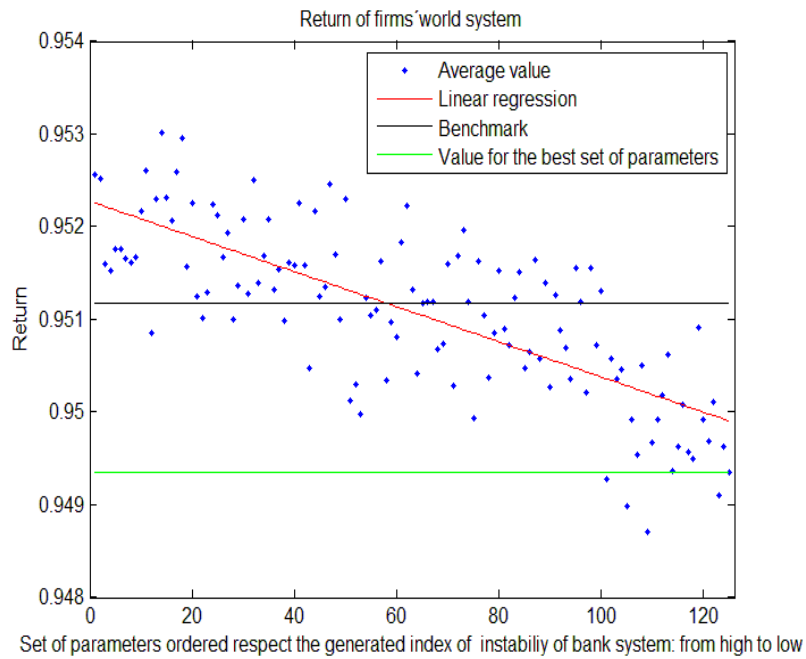


Figure 31: Average return for firms minimizing the instability index of banks. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.



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## 4.2 Stress Test for Interbank System

To study the stability of the inter-bank structure. we insert exogenous shocks in the system. These shocks can be interpreted like assets that are shared by banks and negatively influence their net values (Radelet and Sachs (1998); Edison et al. (2000)). An exogenous shock can distress only banks with positive net worth (see Upper (2006), (2007)). The aim of this analysis is to check how a negative impact on some banks can influence the entire interbank network and economy. An exogenous shock does not automatically imply a bankruptcy of the affected banks: a bank can save itself asking for money in the interbank system. In Gai and Kapadia (2010), the authors provide a study on the banking network with entirely arbitrary structure and in which they randomly choose a bank and force it to go bankrupt. The result is that, while the probability of contagion may be low, the effects can be extremely widespread when problems occur. The result is confirmed in Gai and Kapadia (2008) where the network among banks is enriched with financial instruments that can increase the interconnections. A definition of stress test for the financial system is reported in Jones et al. (2004).

The proposed stress analysis randomly choose a set of  $n$  banks with positive net values, to reduce them to 1% of their original values and invert the sign of them. It happens after the banks receive the returns on loans in the firms' world and before the possibility to exchange money in the interbank market. With this shock, the selected potential lender banks<sup>1</sup> are now potential borrowers: they search for money in the interbank system to avoid failing. We impose the shock for different value of  $n$ , maximum quantity of potential lender banks transformed into potential borrowers, in each step. This parameter spans from 1 to 25, which is the entire bank system. We examine the instability index for the system with the normative approaches, described in Chapters 3 and 4, and without. In this case the interbank system is not imposed so we compute the index of instability for random interbank network with parameter  $Pc$ , the probability that the generic element  $z(a, b)$  of matrix  $Z_{ab}$  is one, equal to 0.25, 0.5, 0.75, 1 (see Section 2.6.1).

We repeat the algorithm 30 times and we report the average value of instability index (Eqn. 3.3) for banks in Figure 32 and Eqn. (4.7), the average value of the instability index for banks and firms in Figure 33. In both figures, the normative approach guarantees that the indexes of instability are not bigger than the non-regularized markets. These ones reach a plateau of instability index for the maximum quantity of potential lender banks transformed into potential borrowers around 20. In this range, we have in some steps not enough potential lenders to be transformed in potential borrowers. When it happens, all potential lenders are transformed in potential borrowers and the entire bank system fail. For  $n$  equal to 25 we get in every step the failure of the entire bank system and, by definition, its instability

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<sup>1</sup>During a step we could have a quantity of potential lenders present in the market and that can receive the shock lower than  $n$ . This parameter indicates then the maximum quantity of potential lender banks transformed into potential borrowers.

index is 100 for banks and 50, plus a part proportional to that of firms, for the entire economy (Figure 33).

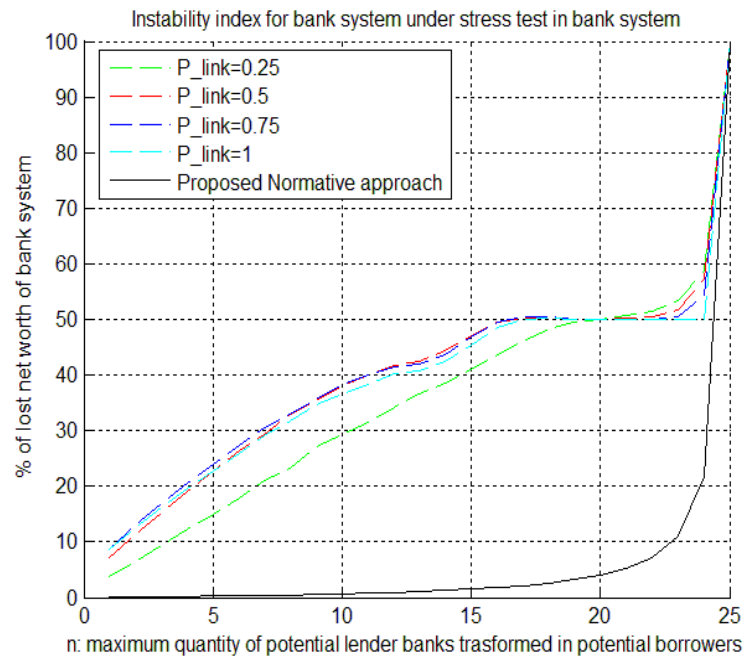


Figure 32: Instability index for the banking system under stress test in banking system. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

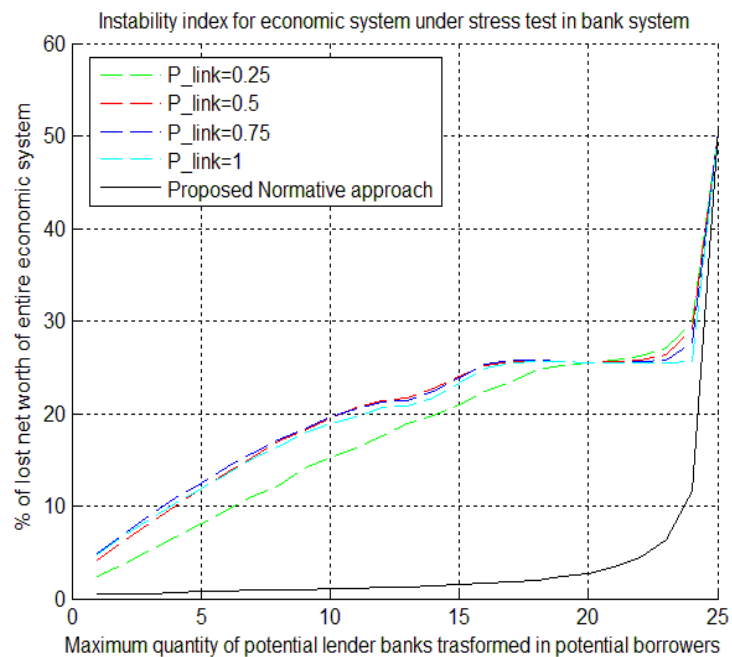


Figure 33: Instability index for economic system under stress test in bank system. 25 banks, 150 firms, 600 steps and 30 repetitions of the algorithm.

# 5 Analysis of results, Conclusion and Future Development

This chapter summarizes and interprets the results that answer the questions proposed in Section 1.1. A future development of the topic is discussed. Statistical indexes describe the properties of the multi-agent system presented in Chapter 2. The same information is exposed for the simulated societies in Chapters 3 and 4 in which normative rules are imposed. The purpose is to point out how the rules change the simulated society.

## 5.1 Analysis of Results and Sensitivity Analysis

In Figure 34 and Figure 35, we report the density of the instability index (Eqn. 3.3) for banks for the model proposed in Chapter 2 and the same after applying the normative rules proposed in Chapters 3 and 4. In Figure 36 and Figure 37, we report the density of the instability index for the entire economic system (Eqn. 4.7) for the model proposed in Chapter 2 and the same after applying normative rule proposed in Chapters 3 and 4 (the parameters of the multi-agent system are reported in Section 2.7). The effect of the normative approach is a reduction of the value of instability indexes for banks and for the entire economy.

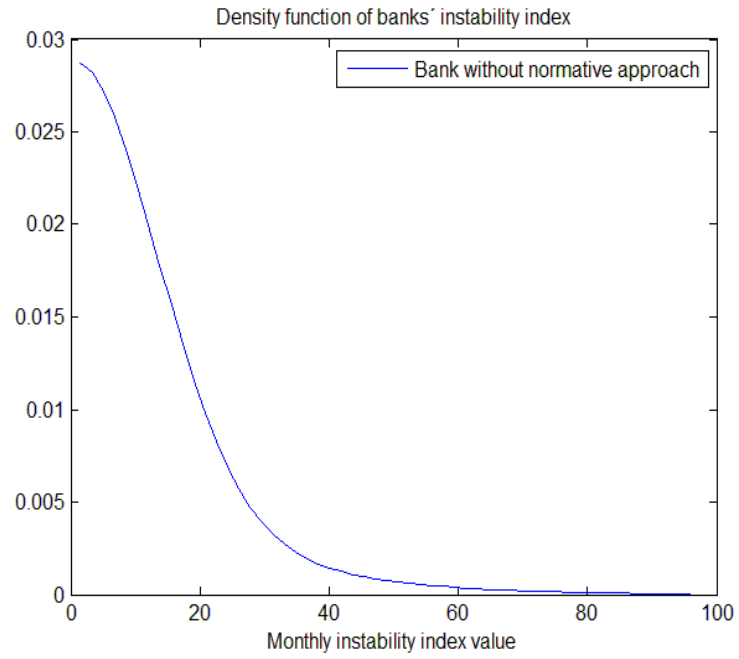


Figure 34: Density function of instability index for banks without normative approach.

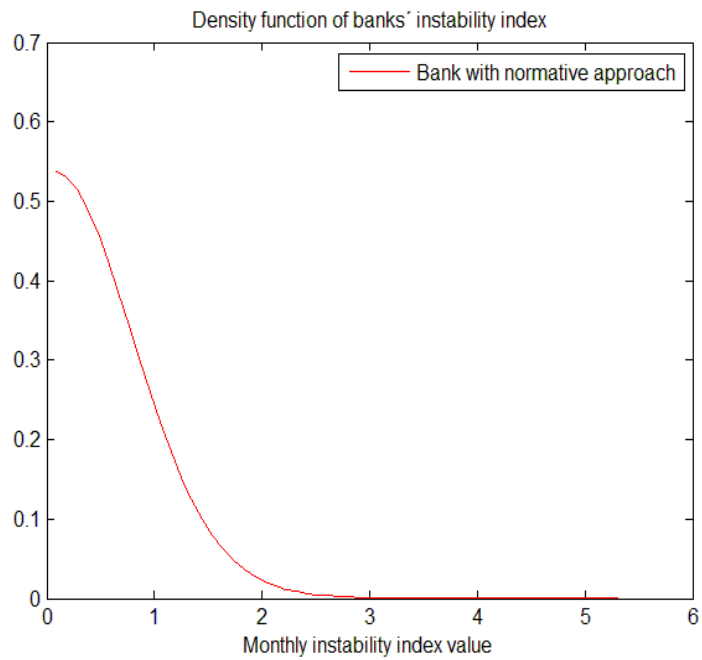


Figure 35: Density function of instability index for banks with normative approach.

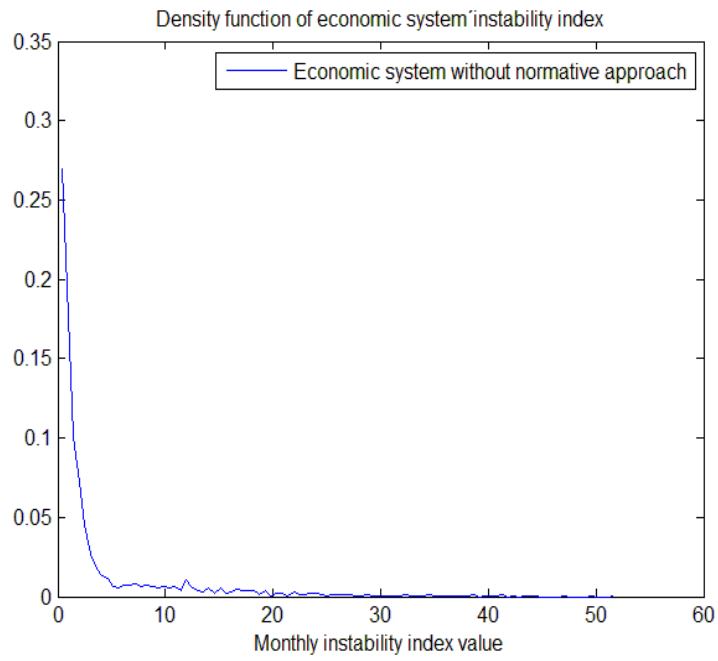


Figure 36: Density function of instability index for the entire economic system without normative approach.

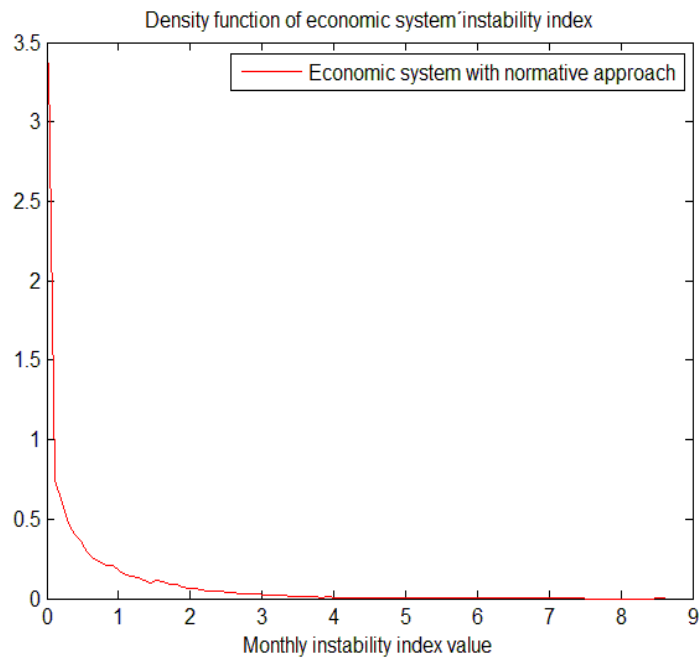


Figure 37: Density function of instability index for the entire economic system with normative approach.

In the following tables 1, 2 and 3, we report statistical properties of models presented in chapter 2, 3 and 4: index of instability (I.I.), bankruptcy index (B.) and return (R.) for banks, for firms and for the entire economy. The instability index for banks is Eqn. (3.3), for the entire system is Eqn. (4.7). The bankruptcy index for bank system is Eqn. (3.5). An analogous definition is used for instability index of firms. The return for the entire system is Eqn. (4.9) in which the return of banks and firms are part of the definition. The indexes for firms system is the average value of the ones for U firms and for D firms. The entire economy is composed by the banking and firms' system. Each index is computed with respect to a simulation of 600 months. The simulation is repeated 30 times and the average value with respect to the repetitions is reported. Model A is the one without rules (Chapter 2), model B is the model A with rules proposed in Chapter 3, and model C is the model A with rules proposed in Chapters 3 and 4. In model A, the probability to have connections among banks in the interbank market is  $Pc = 0.5$  (paragraph 2.7).

From table 1, 2 and 3 we can argue that the banking system decreases its instability index with the rule proposed in Chapter 3 (model B) and increases with the rules proposed in Chapter 4: it is penalized to improve the stability of firms' world and the stability of the entire system. Furthermore, a lower level of the standard deviation of the instability index is a positive index for banks and firms: they do not face many months of zero level of instability with months with high level of it. If we associate the maximum of the instability index and bankruptcy index with financial crisis, we can argue that, with the proposed normative approach, they assume a lower average intensity for banking system and for the entire economy. These results assume more importance if we compare also the average return and its standard deviation: the proposed rules can increase the stability of financial system decreasing the fluctuation of the return and keeping its average value comparable with the non-regularized economy.

Banks system	A	B	C
Average I.I.	2.5639	0.0622	0.0851
Standard deviation of I.I.	8.8429	0.1872	0.2230
Max of I.I.	73.4010	1.7461	2.0172
Average B.	0.5871	0.8572	1.1544
Standard deviation of B.	1.9371	1.8989	2.2107
Max of B.	19.2000	11.4667	13.4667
Average R.	0.9701	0.9872	0.9828
Standard deviation of R.	0.1399	0.0554	0.0570
Max of R.	1.4398	1.1776	1.1812

Table 1.

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Firms system	A	B	C
Average I.I.	1.0550	0.9263	0.8672
Standard deviation of I.I.	1.7705	1.6274	1.6093
Max of I.I.	11.5882	10.7912	11.2229
Average B.	2.1863	1.8693	1.8056
Standard deviation of B.	1.6099	1.6620	1.5936
Max of B.	10.3981	12.3519	11.3889
Average R.	0.9870	0.9512	0.9521
Standard deviation of R.	0.0788	0.0789	0.0792
Max of R.	1.1195	1.1244	1.1281

Table 2.

Entire system	A	B	C
Average I.I.	<u>1.8094</u>	0.4941	<u>0.4765</u>
Standard deviation of I.I.	4.5297	0.8197	0.8231
Max of I.I.	<u>37.6685</u>	5.4238	<u>5.6711</u>
Average B.	1.3867	1.3632	1.48
Standard deviation of B.	1.2805	1.2714	1.3702
Max of B.	11.2028	7.7537	8.4556
Average R.	0.9785	0.9692	0.9671
Standard deviation of R.	0.0818	0.0418	0.0425
Max of R.	1.2204	1.0973	1.1102

Table 3.

In the following set of figures we report, with respect to the steps of the algorithm, the effect of the normative approach proposed in Chapters 3 and 4. The instability index, bankruptcy index and return are represented for banks and for the entire economy, that is the average value between banks and firms system. The simulation is repeated 30 times and the average value, with respect to the repetitions, of max, min and average value are reported for each step.

The figures confirm the reduction of the maximum instability index for banks with the normative approach (Figure 39) in comparison with the one without it (Figure 38). We have the same results for the entire economy (Figure 41 and 40). In Figure 43 and 42, we get the monthly bankruptcy index for banks with and without the normative approach. If the average value of this index increases with the proposed rule (Table 1), the maximum value assumes lower values. This effect is also represented, comparing the Figures 45 and 44, the monthly bankruptcy index for the entire economy with and without the normative approach.

Figures 47 and 46 report the monthly returns of the banking system with and without the normative approach. The proposed rules stabilize the returns of that system reducing the amplitudes of the oscillations around its average value. We have the same result considering the entire economy in Figures 49 and 48.

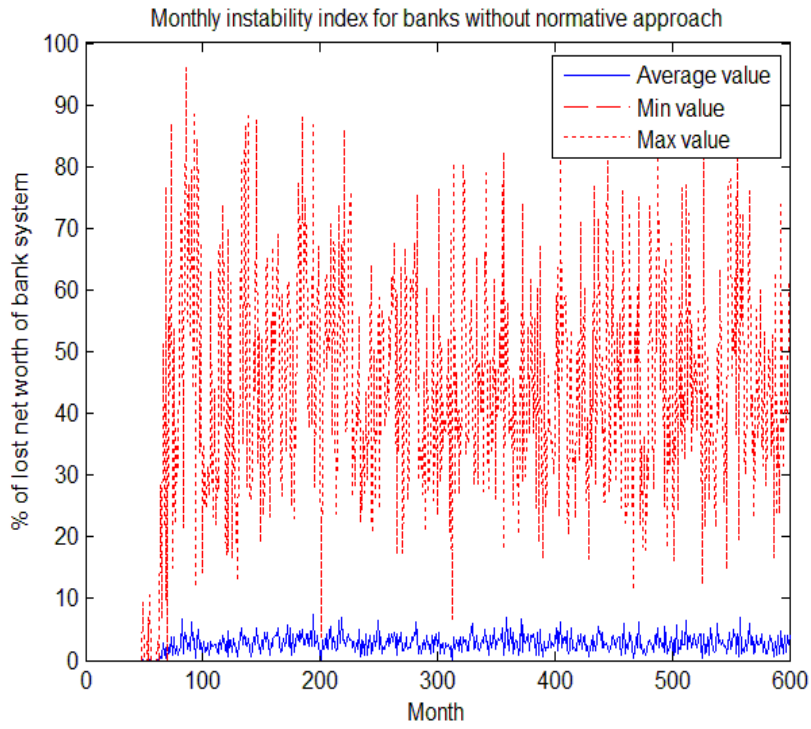


Figure 38: Monthly instability index for banks without normative approach.

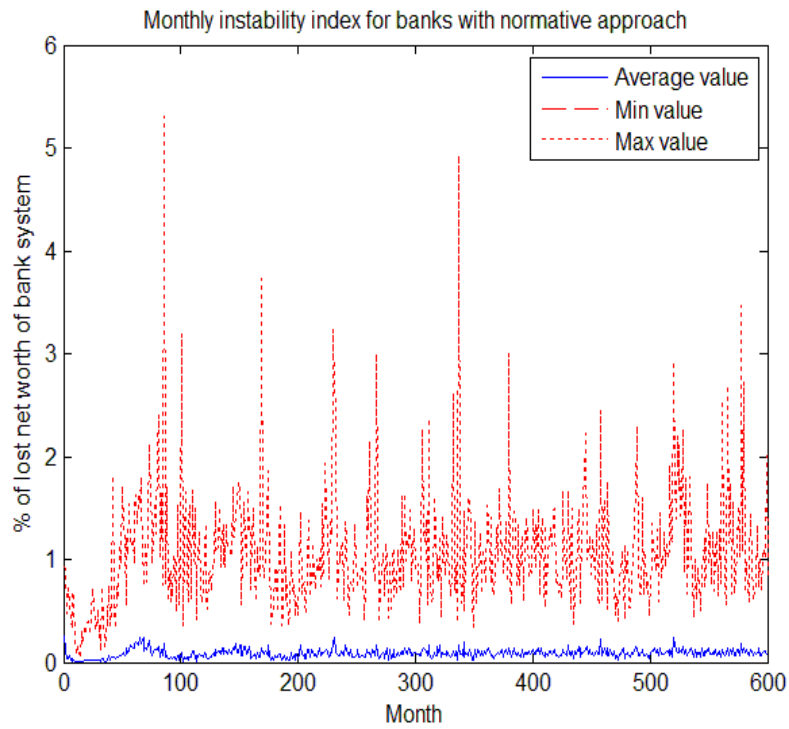


Figure 39: Monthly instability index for banks with normative approach.



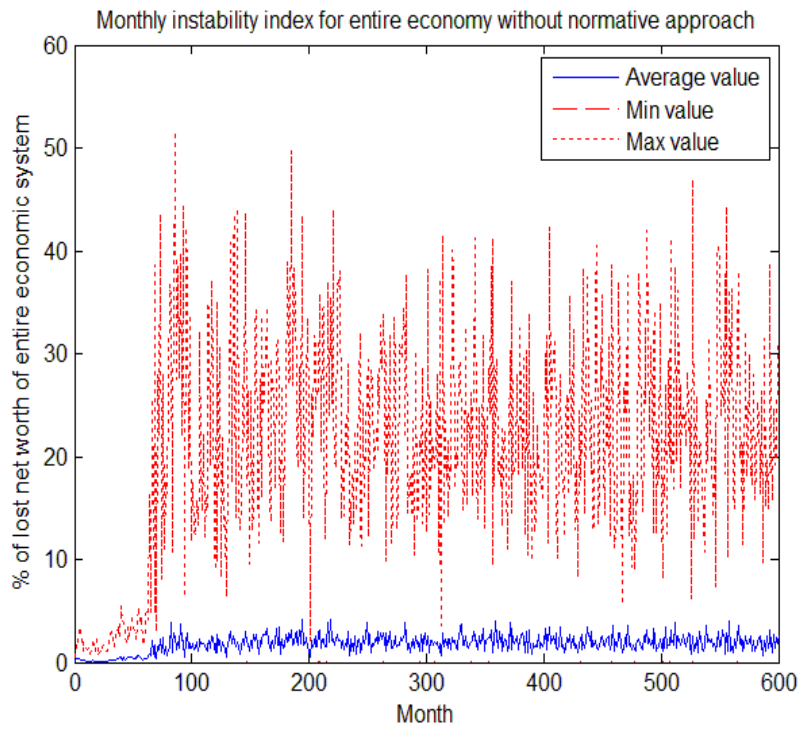


Figure 40: Monthly instability index for entire economy without normative approach.

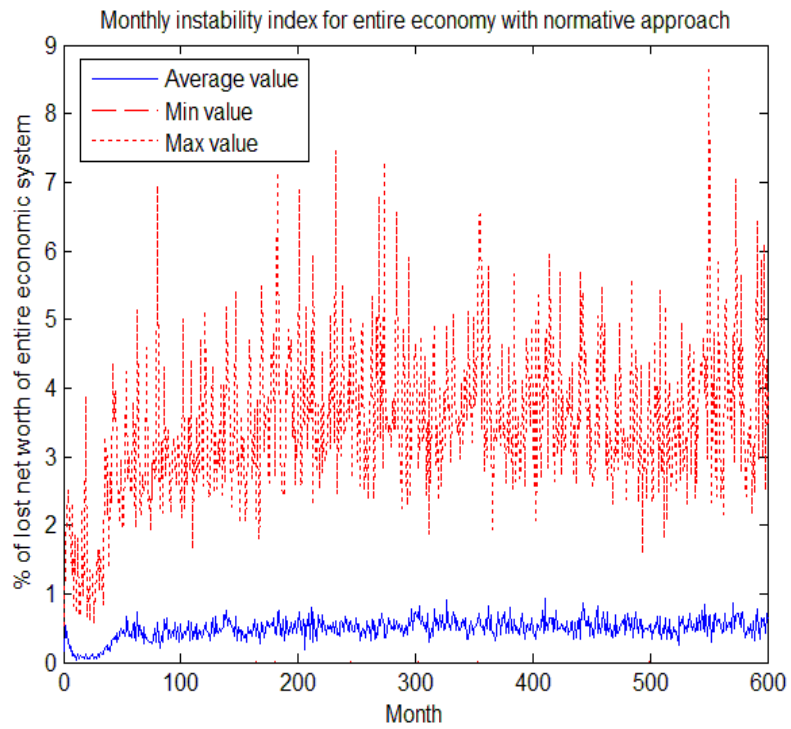


Figure 41: Monthly instability index for entire economy with normative approach.

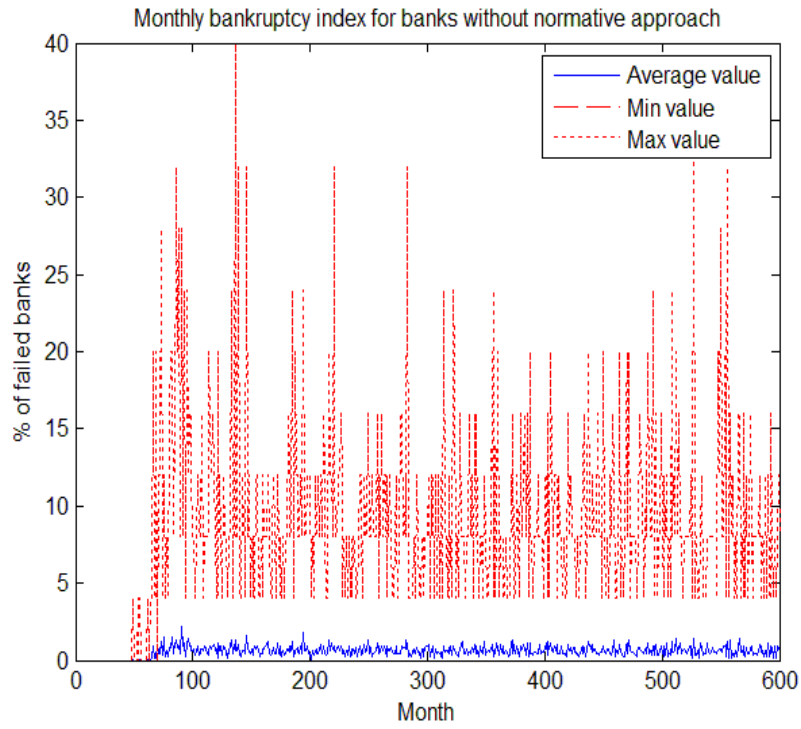


Figure 42: Monthly bankruptcy index for banks without normative approach.

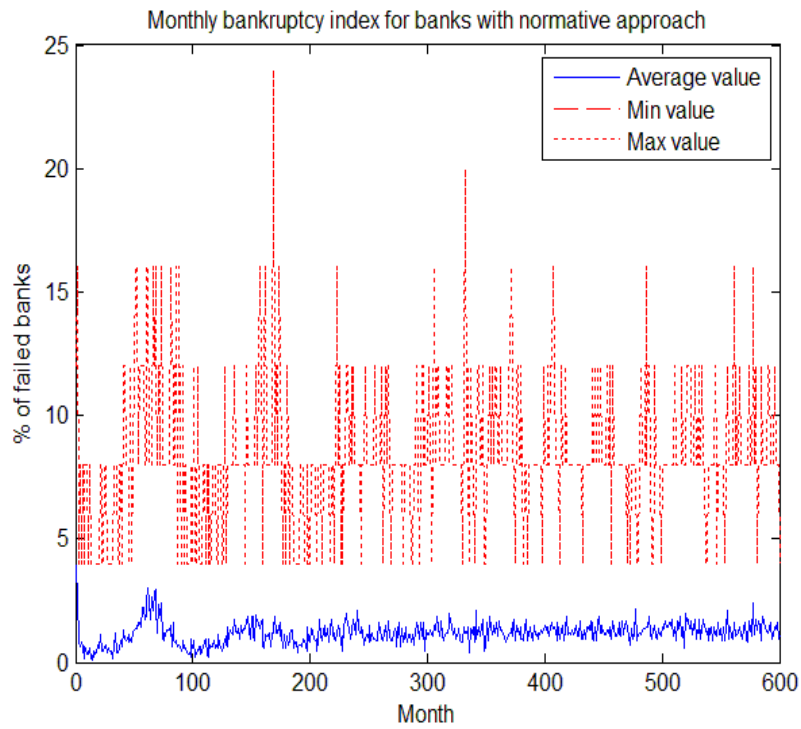


Figure 43: Monthly bankruptcy index for banks with normative approach.

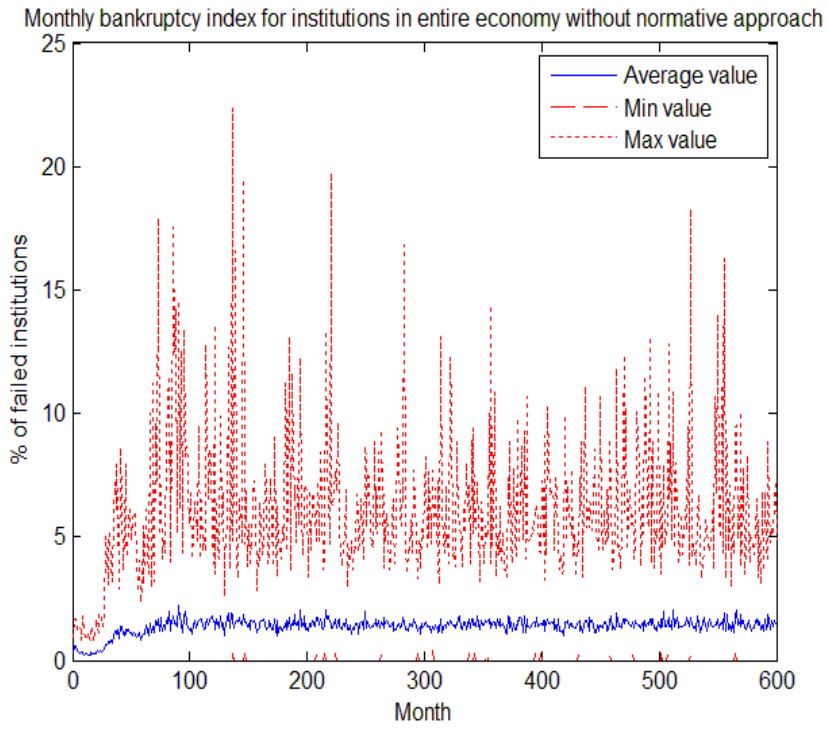


Figure 44: Monthly bankruptcy index for entire economy without normative approach.

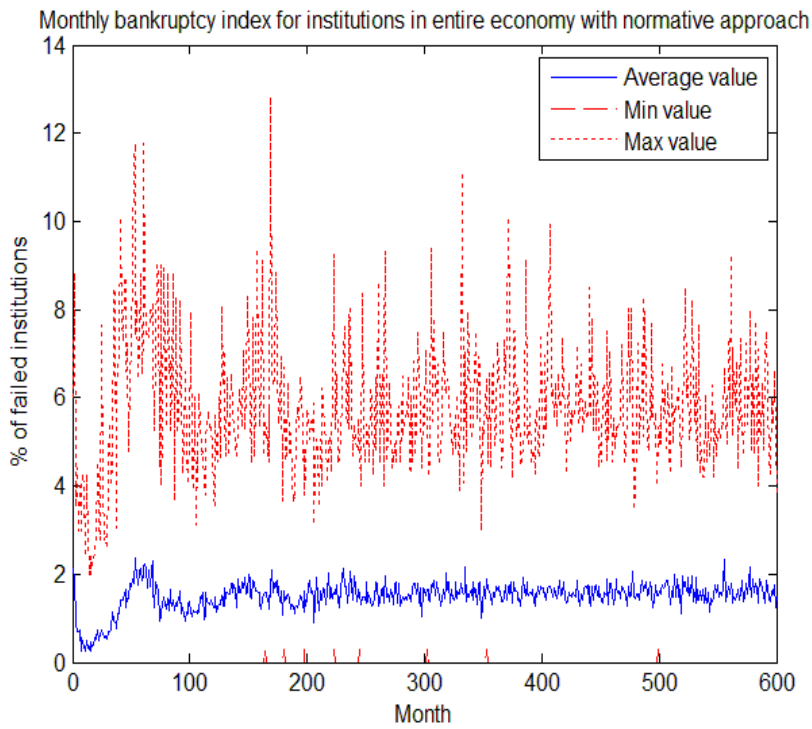


Figure 45: Monthly bankruptcy index for entire economy with normative approach.

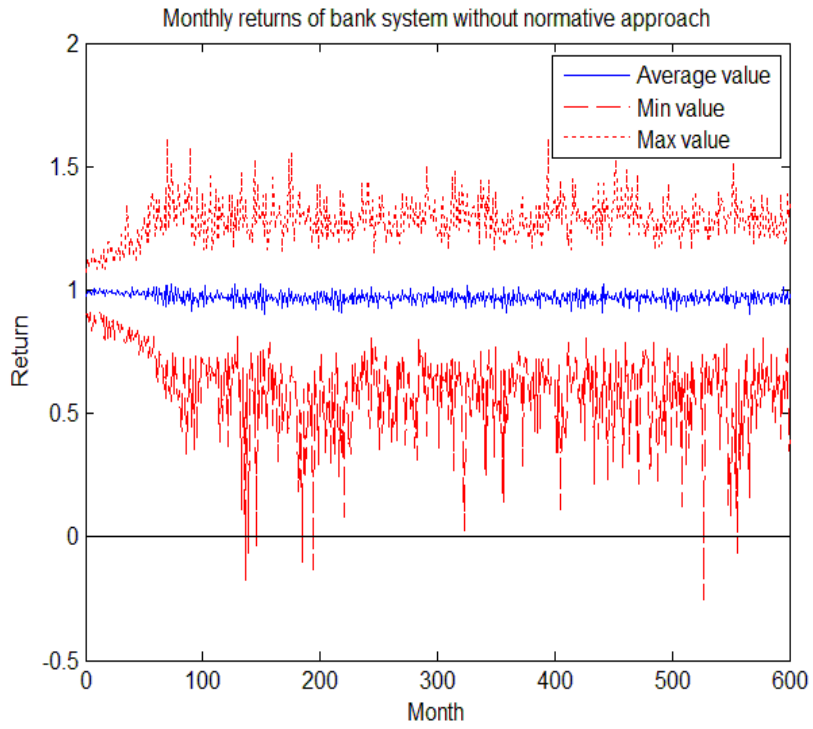


Figure 46: Monthly return of banks without normative approach.

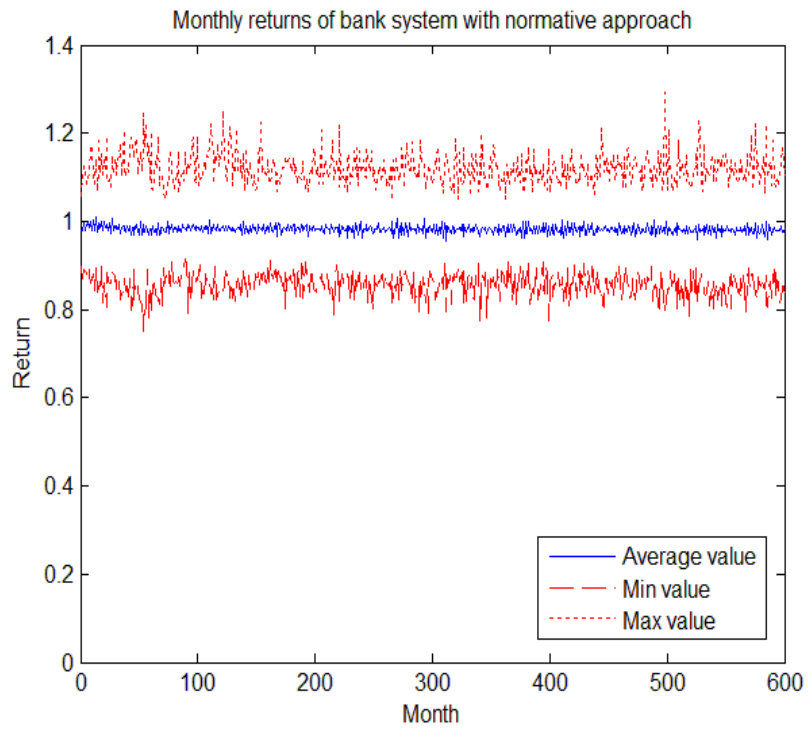


Figure 47: Monthly return of banks with normative approach.

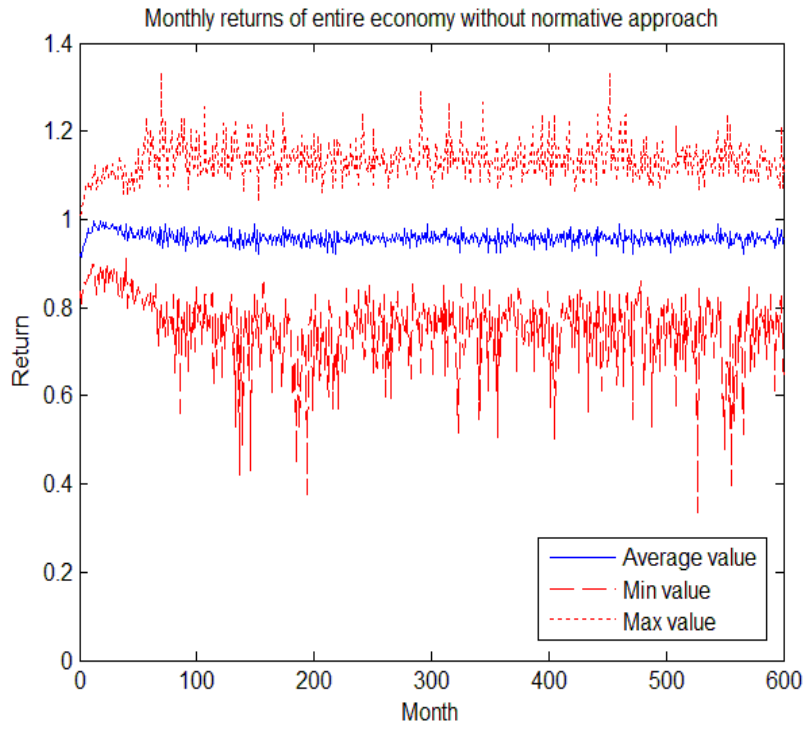


Figure 48: Monthly return of entire economy without normative approach.

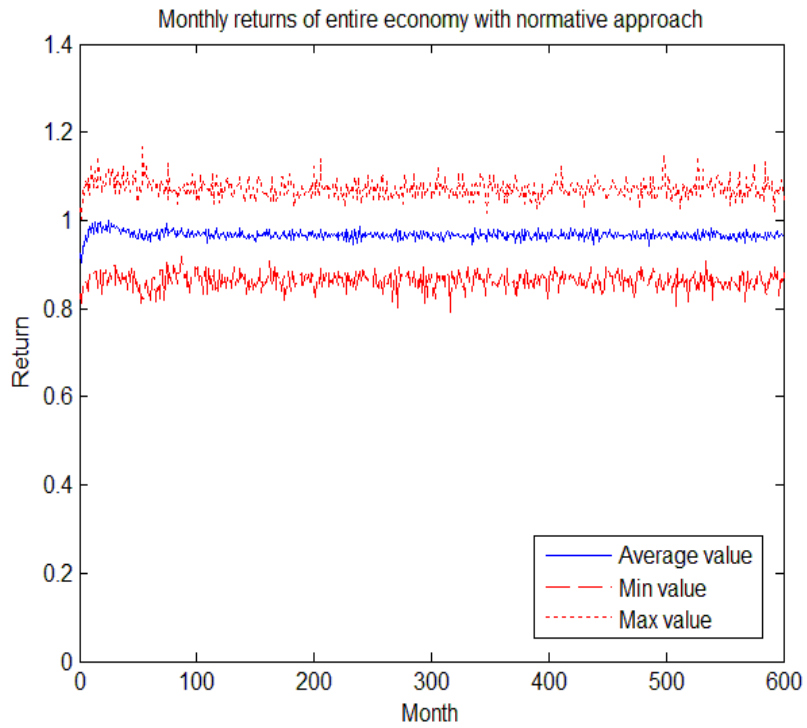


Figure 49: Monthly return of entire economy with normative approach.

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We report the Table 1, 2 and 3 for different value of parameters. Index of instability (I.I.), bankruptcy index (B.) and return (R.) for banks, for firm systems and for the entire economy. The index of instability for banks is Eqn. (3.3), for the entire system is Eqn. (4.7). The bankruptcy index for banks is Eqn. (3.5). An analogous definition is used for the instability index of firms. The return for the entire system is Eqn. (4.9) in which the return of banks and firms are part of the definition. The indexes for the firms' system is the average value of the ones for U firms and for D firms. The entire economy is composed by banks and firms system. Each index is computed with respect to a simulation of 600 months. The simulation is repeated 30 times and the average value with respect to the repetitions is reported. Model A is the one without rules (Chapter 2), model B is the model A with rules proposed in Chapter 3 and model C is the model A with rules proposed in chapter 3 and 4.

The best solutions of problem (Eqn. 3.4) do not change. In particular, the eight best rules are all the ones that don't allow exchange of money from the lender  $a$  to the borrower  $b$  if the value of  $x_{a,b}$  belongs to a part of the domain in which its values assumes non-negative values (see Section 3.4) . It implies that the non-negative value of that variable must not be associated with a link in the interbank system from  $a$  to  $b$ . The bank  $b$  must ask more than one bank to get the right quantity of money necessary to avoid failing: an interbank system is efficient regarding the instability index, if the risk of credit of a borrower bank is diversified among several lender banks.

For the banking system, the normative approach proposed in Chapter 3 to decrease the instability index also increases the bankruptcy index and the average return. The results confirm the difference between instability index and bankruptcy index to examine the behaviour of the banking system. By applying also the normative approach proposed in Chapter 4, we decrease further the instability index and the bankruptcy index in all cases, except for the model with increased interbank interest rate  $r_b = 0.01$  (+300%) in which both indexes increase.

In the firms' system, the normative approach, proposed in Chapters 3 and 4, always produces a decrease in the average bankruptcy index, in the average instability index and in its maximum value. The normative approach always reduces the maximum of the instability index, but increases the maximum of bankruptcy. This result implies the decrease of standard deviation for the instability index, but an increase in the same for the bankruptcy index.

For the entire economy, we always get a decrease in its instability index and an increase in the average return. Its average bankruptcy decrease, except in the Table 15 and 18 in which the high average bankruptcy for banks increases this value. The values of maximum and standard deviation always decrease for the instability index and return. The proposed set of rules stabilizes the economy in the sense that the society faces a decrease of the oscillation of the instability index and return and a decrease of the average value of the instability index itself.

Banks system	A	B	C
Average I.I.	2.6483	0.0622	0.0465
Standard deviation of I.I.	9.2475	0.1900	0.1891
Max of I.I.	73.3643	1.7689	2.0850
Average B.	0.5798	0.8569	0.6422
Standard deviation of B.	1.9320	1.8755	1.7212
Max of B.	18.9333	11.0667	13.2000
Average R.	0.9702	0.9870	0.9906
Standard deviation of R.	0.1431	0.0551	0.0524
Max of R.	1.4515	1.1786	1.1794

Firms system	A	B	C
Average I.I.	1.0681	0.9093	0.8788
Standard deviation of I.I.	1.7541	1.6366	1.6466
Max of I.I.	12.2274	10.9888	11.4218
Average B.	2.1929	1.8916	1.8611
Standard deviation of B.	1.5839	1.6727	1.7828
Max of B.	9.8889	11.8981	11.9352
Average R.	0.9454	0.9504	0.9511
Standard deviation of R.	0.0793	0.0788	0.0788
Max of R.	1.1210	1.1214	1.1278

Entire system	A	B	C
Average I.I.	1.8582	0.4858	0.4626
Standard deviation of I.I.	4.7151	0.8241	0.8284
Max of I.I.	37.2801	5.5627	5.7281
Average B.	1.3863	1.3743	1.2517
Standard deviation of B.	1.2681	1.2627	1.2427
Max of B.	11.1824	7.6361	8.4241
Average R.	0.9578	0.9687	0.9709
Standard deviation of R.	0.0821	0.0485	0.0478
Max of R.	1.2300	1.1068	1.1044

Table 4,5,6. Model with minimum reserve factor  $\chi = 0.001$  (-90%)

Banks system	A	B	C
Average I.I.	2.4771	0.0645	0.0464
Standard deviation of I.I.	8.3649	0.1826	0.1708
Max of I.I.	67.9885	1.6003	1.6933
Average B.	0.5949	0.8731	0.6502
Standard deviation of B.	1.9138	1.8759	1.7175
Max of B.	18.2667	11.6000	11.4667
Average R.	0.9704	0.9869	0.9908
Standard deviation of R.	0.1358	0.0544	0.0519
Max of R.	1.4226	1.1715	1.1696

Firms system	A	B	C
Average I.I.	1.0610	0.9252	0.9134
Standard deviation of I.I.	1.7573	1.6398	1.6253
Max of I.I.	11.4917	10.9520	10.5541
Average B.	2.2024	1.8852	1.8831
Standard deviation of B.	1.5957	1.6709	1.7309
Max of B.	10.4259	13.2963	12.3056
Average R.	0.9451	0.9507	0.9509
Standard deviation of R.	0.0791	0.0787	0.0785
Max of R.	1.1205	1.1301	1.1246

Entire system	A	B	C
Average I.I.	1.7690	0.4949	0.4799
Standard deviation of I.I.	4.2888	0.8255	0.8177
Max of I.I.	34.3214	5.5657	5.3049
Average B.	1.3986	1.3791	1.2667
Standard deviation of B.	1.2652	1.2618	1.2238
Max of B.	10.5472	8.0833	7.9037
Average R.	0.9577	0.9688	0.9708
Standard deviation of R.	0.0787	0.0483	0.0475
Max of R.	1.2138	1.1023	1.1050

Table 7,8,9. Model with minimum reserve factor  $\chi = 0.02$  (+100%)



Banks system	A	B	C
Average I.I.	2.4665	0.0616	0.0457
Standard deviation of I.I.	8.7768	0.1915	0.1737
Max of I.I.	75.8458	1.8783	1.8691
Average B.	0.5813	0.8480	0.6331
Standard deviation of B.	1.9692	1.8992	1.6770
Max of B.	20.1333	11.8667	10.8000
Average R.	0.9710	0.9873	0.9909
Standard deviation of R.	0.1399	0.0551	0.0516
Max of R.	1.4420	1.1778	1.1636

Firms system	A	B	C
Average I.I.	1.0613	0.9296	0.8910
Standard deviation of I.I.	1.7522	1.6216	1.6727
Max of I.I.	11.4119	11.0801	11.0311
Average B.	2.1690	1.8767	1.8696
Standard deviation of B.	1.5979	1.6724	1.7678
Max of B.	10.0000	12.0278	12.1389
Average R.	0.9462	0.9512	0.9509
Standard deviation of R.	0.0794	0.0786	0.0786
Max of R.	1.1247	1.1235	1.1225

Entire system	A	B	C
Average I.I.	1.7639	0.4956	0.4683
Standard deviation of I.I.	4.4887	0.8176	0.8408
Max of I.I.	38.6150	5.5988	5.5592
Average B.	1.3752	1.3624	1.2514
Standard deviation of B.	1.2984	1.2691	1.2234
Max of B.	11.6917	7.8843	7.4537
Average R.	0.9586	0.9692	0.9709
Standard deviation of R.	0.0808	0.0484	0.0475
Max of R.	1.2202	1.1123	1.1048

Table 10,11,12. Model with interest rate on inter-bank loans  $r_b = 0.0005$  (-80%)

Banks system	A	B	C
Average I.I.	2.8472	0.0656	0.0851
Standard deviation of I.I.	9.4624	0.1956	0.2261
Max of I.I.	69.7670	2.1508	2.1270
Average B.	0.5878	0.8609	1.1613
Standard deviation of B.	1.9300	1.8786	2.2251
Max of B.	19.4667	11.2000	13.0667
Average R.	0.9675	0.9869	0.9825
Standard deviation of R.	0.1422	0.0545	0.0570
Max of R.	1.4181	1.1711	1.1846

Firms system	A	B	C
Average I.I.	1.0768	0.9075	0.8614
Standard deviation of I.I.	1.7658	1.6318	1.5588
Max of I.I.	11.6958	11.2332	10.6308
Average B.	2.1739	1.8735	1.7984
Standard deviation of B.	1.5672	1.6690	1.6115
Max of B.	9.7407	13.2315	12.0000
Average R.	0.9459	0.9512	0.9529
Standard deviation of R.	0.0789	0.0786	0.0789
Max of R.	1.1217	1.1238	1.1267

Entire system	A	B	C
Average I.I.	1.9620	0.4865	0.4732
Standard deviation of I.I.	4.8193	0.8237	0.7892
Max of I.I.	35.4500	5.6387	5.4049
Average B.	1.3808	1.3672	1.4799
Standard deviation of B.	1.2712	1.2698	1.3846
Max of B.	11.5796	7.9870	8.3843
Average R.	0.9567	0.9691	0.9677
Standard deviation of R.	0.0812	0.0487	0.0492
Max of R.	1.2101	1.1035	1.1108

Table 13,14,15. Model with interest rate on inter-bank loans  $r_b = 0.01$  (+300%)

Banks system	A	B	C
Average I.I.	3.2270	0.1023	0.0953
Standard deviation of I.I.	10.0991	0.2467	0.2427
Max of I.I.	77.4704	2.2786	2.3040
Average B.	0.7460	1.3396	1.2898
Standard deviation of B.	2.2686	2.3648	2.3464
Max of B.	22.9333	14.4000	13.7333
Average R.	0.9617	0.9799	0.9806
Standard deviation of R.	0.1473	0.0586	0.0578
Max of R.	1.4671	1.1882	1.1817

Firms system	A	B	C
Average I.I.	1.1099	0.9146	0.8549
Standard deviation of I.I.	1.8080	1.5908	1.6246
Max of I.I.	11.8528	10.5881	10.5661
Average B.	2.1506	1.7983	1.8115
Standard deviation of B.	1.5799	1.5993	1.6057
Max of B.	8.8426	11.6574	11.7222
Average R.	0.9464	0.9526	0.9521
Standard deviation of R.	0.0792	0.0790	0.0793
Max of R.	1.1196	1.1254	1.1244

Entire system	A	B	C
Average I.I.	2.1684	0.5084	0.4751
Standard deviation of I.I.	5.1461	0.8055	0.8238
Max of I.I.	39.5141	5.3182	5.3610
Average B.	1.4483	1.5690	1.5506
Standard deviation of B.	1.4109	1.4463	1.4394
Max of B.	12.9028	8.9102	8.9343
Average R.	0.9541	0.9663	0.9664
Standard deviation of R.	0.0834	0.0497	0.0495
Max of R.	1.2301	1.1172	1.1067

Table 16,17,18. Model with official discount rate  $v = 0.005$  (-66.66%)

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Banks system	A	B	C
Average I.I.	1.0901	0.0177	0.0172
Standard deviation of I.I.	5.2305	0.1006	0.1022
Max of I.I.	49.1154	1.3804	1.2996
Average B.	0.2360	0.2604	0.2549
Standard deviation of B.	1.0470	1.0044	1.0327
Max of B.	9.4667	6.9333	7.6000
Average R.	0.9886	0.9970	0.9970
Standard deviation of R.	0.1069	0.0455	0.0463
Max of R.	1.3554	1.1537	1.1567

Firms system	A	B	C
Average I.I.	1.0676	0.9402	0.9079
Standard deviation of I.I.	1.7534	1.6285	1.5990
Max of I.I.	10.8412	10.8282	10.4622
Average B.	2.1958	1.8860	1.8901
Standard deviation of B.	1.5680	2.0466	1.9847
Max of B.	11.0833	13.4259	13.1759
Average R.	0.9451	0.9510	0.9506
Standard deviation of R.	0.0788	0.0781	0.0785
Max of R.	1.1205	1.1205	1.1239

Entire system	A	B	C
Average I.I.	1.0789	0.4790	0.4626
Standard deviation of I.I.	2.7933	0.8164	0.8018
Max of I.I.	25.4736	5.4318	5.2371
Average B.	1.2159	1.0732	1.0725
Standard deviation of B.	0.9517	1.1343	1.1226
Max of B.	6.7852	6.9361	6.9833
Average R.	0.9669	0.9740	0.9738
Standard deviation of R.	0.0670	0.0460	0.0460
Max of R.	1.1801	1.0992	1.0980

Table 19,20,21. Model with official discount rate  $v = 0.03$  (+100%)

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## 5.2 Conclusion and Future Development

In this part we conclude this work by answering the questions framed in Section 1.1:

- < How can the Government or other institutions like the Central Bank create some rules to determine interbank relationships in such a manner that we have an improvement on the problem of the stability of the interbank system? >
- < Has this regularized interbank system got properties similar to real life interbank markets?>
- < How can the Government or other institutions like the Central Bank create some rules to determine interbank and banks-firms relationships so as to get improvements in the stability of the entire economic system? >

With results got in Chapters 3 and 4, we can assert that:

1. It is possible to create a set of rules that could select the possible lenders for a bank that needs access to the interbank market. Such rules increase the stability of the banking system with respect to a non-regularized interbank structure. These rules force banks, which search for money in interbank system, to diversify their request of loan among more than one lender-bank: an interbank system has low instability if the risk of credit of a borrower bank is diversified among several lender banks.
2. No, it has not. The best possible solution with respect to the problem of the stability of interbank system has not properties similar to real-life interbank markets. If the simulated model can capture the behaviour of the real-life interbank system, this normative approach could increase the stability of real market.
3. It is possible to improve the stability of the entire economic system by changing the discount rate, minimum reserve for banks and interbank interest rate in a proportional way with respect to the index composed of the difference between the return of the banking system and the return of firms' system.

The results 1 and 3 are robust also in case of exogenous shocks in the banking system (Section 4.2).

Another result, emerged in Chapter 3, is that identity (3.7) is not true: the set of rules minimizing the instability index (the percentage of loss of money in the considered system) for the banking system does not minimize its number of bankruptcies. Future research should avoid using the number of failed institutions as the principal index of instability of a system.

For future developments of this model, possible improvements could be:

- Active role of the Central Bank in lending money (Georg C.P. and Poschmann J. (2010)) and, for example, lender of last resort.
- To insert the possibility for lender banks to refuse a request of money from borrower banks.

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- To insert the presence of other typologies of credit contract, like government bond.

The principal limit of this model is the absence of multi-step events like the possibility for banks to make loans to firms with returns in successive steps of the algorithm (see Iori et al. (2006)). An analogous concept could be the presence of interbank exchange of money with different interbank interest rates, depending on the temporal windows after which the debt must be paid back.

## 6 Appendix: Network Analysis

In this appendix we introduce concepts of network analysis that allow us to study the property of interbank structure. Reference: Jackson (2008).

Matrix  $Z_{ab}$ , that defines the interbank structure, is a direct unweighted network: it describes the presence/absence and directions of links among the elements of the network.

Let the set of the banks be

$$Z = \{1, 2, \dots, z\} \quad 3 \leq z < \infty$$

If  $i \in Z$  and  $j \in Z$ , we indicate with  $ij$  the link that start from  $i$  and goes in  $j$ . Directed link  $ij$  indicates the possibility for bank  $j$  to ask  $i$  for a loan. The network  $Z_{ab}$  is represented with a squared matrix in which each element is

$$Z_{ab}(i, j) = \{0, 1\} \quad \forall i, j \in Z$$

A bank cannot lend/borrow money to/from itself, so it is always true

$$Z_{ab}(i, i) = 0 \quad \forall i \in Z$$

From now on we indicate with matrix  $g$ , directed unweighted network, the matrix  $Z_{ab}$  defined in chapter 2, and with  $z$  the cardinality of set  $Z$ .

Let  $g^N$  be the set of all subsets of  $Z$  and  $G$  be the set of all networks

$$G = \{g : g \subseteq g^N\}$$

We define the set of links of a bank  $i$  that start from it self

$$L_{i \rightarrow}(g) = \{ij \mid ij \in g\} \quad i, j \in Z$$

It is the set of banks to which banks  $i$  can lend money. We define the set of links of a bank  $i$  that arrives in it

$$L_{i \leftarrow}(g) = \{ji \mid ji \in g\} \quad j, i \in Z$$

is the set of banks from which bank  $i$  can borrow money.

We define the associated indirect unweighted network of  $g$ , the network  $g^U$ . It is possible to get  $g^U$  from  $g$  noting that for a indirect network the direction of the links are not considered, so

$$ji \equiv ij$$

We define the set of links of an element  $i$  for the network  $g^U$

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$$L_i(g^U) = \{ji \mid ji \in g^U\} \quad i, j \in Z$$

Matrix that represents the network  $g^U$  starting from  $g$ , is got by means of the relations

$$g_{i,j}^U = \begin{cases} 1 & \text{if } g_{i,j} = 1 \\ 1 & \text{if } g_{j,i} = 1 \\ 0 & \text{otherwise} \end{cases} \quad i, j \in Z$$

We define ‘‘in-degree’’ of bank  $i$  the number of banks from which it can borrow money

$$l_{i\leftarrow}(g) = |L_{i\leftarrow}(g)| \quad i \in Z$$

We define ‘‘out-degree’’ of bank  $i$  the number of banks to which it can lend money

$$l_{i\rightarrow}(g) = |L_{i\rightarrow}(g)| \quad i \in Z$$

We define ‘‘degree’’ of a bank  $i$  the number of banks to which it can lend money and from which it can borrow money

$$l_i(g) = |L_{i\leftarrow}(g)| + |L_{i\rightarrow}(g)| \quad i \in Z$$

## DENSITY

We define the density of network  $g$  of interbank system the quantity  $D(g) \in [0, 1]$  of existing links with respect to the total possible number.

$$D(g) = \frac{\sum_{i=1}^z l_i(g)}{z(z-1)} \quad i \in Z$$

## RECIPROCITY

Reciprocity  $r(g)$  of a network  $g$  gives a measure of the ratio of the number of links pointing in both directions to the total number of links. With this definition,  $r(g) = 1$  is for a purely indirect network while  $r(g) = 0$  for a purely direct one.

$$r(g) = \frac{\sum_{i=1}^z \sum_{i=1, i \neq j}^z g_{i,j}^*}{\sum_{i=1}^z \sum_{i=1, i \neq j}^z g_{i,j}} \quad i, j \in Z$$

with

$$g_{i,j}^* = \begin{cases} 1 & \text{if } (g_{i,j} = 1) \wedge (g_{j,i} = 1) \\ 0 & \text{otherwise} \end{cases}$$

For a completely disconnected network  $g^{Dis}$ , we define  $r(g^{Dis}) = 0$ .

## CLUSTERING COEFFICIENT

We define, following (Boss et al. (2003)), the clustering of a network  $g$  a quantity proportional to the number of complete triads (triangles) divided the number of connected triples of vertexes



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$$\text{cluster coefficient } (g) = 3 \frac{\text{number of triangles on the graph representing } g^U}{\text{number of connected triples of vertices representing } g^U}$$

If connected triples of vertexes is zero and, then, the number of triangles on the graph is zero, we use impose this parameter equal to zero.

This coefficient is computed with respect to the associated indirect network matrix, the network  $g^U$ .



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