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Double Matching: Social Contacts in a Labour Market with On-the-Job Search

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Abstract

This paper develops a labour market matching model with heterogeneous firms, on-the-job search and referrals. Social capital is endogenous, so that better connected workers bargain higher wages for a given level of productivity. This is a positive effect of referrals on reservation wages. At the same time, employees accept job offers from more productive employers and forward other offers to their unemployed social contacts. Therefore, the average productivity of a referred worker is lower than the average productivity in the market. This is a negative selection effect of referrals on wages. In the equilibrium, wage premiums (penalties) associated with referrals are more likely in labour markets with lower (higher) productivity heterogeneity and lower (higher) worker's bargaining power. Next, the model is extended to allow workers help each other climb a wage ladder. On-the-job search is then intensified and wage inequality is reduced as workers employed in high paid jobs pool their less successful contacts towards the middle range of the productivity distribution.

JEL classification: J23, J31, J64

Keywords: Social networks, referrals, on-the-job search, social capital, wage inequality

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1 Introduction

Careers are not made up of random jumps from one job to another, but rather that individuals rely on contacts acquired at various stages of their work-life, and before.

M. Granovetter (1995, p. 85)

About 30% of job separations in developed countries can be attributed to job-to-job transitions. At the same time there is strong empirical evidence that $30-60\%$ of new hires find jobs through personal contacts. On the one hand, this suggests that workers continue searching on-the-job and climb a wage-ladder by changing employers. On the other hand, employees help their social contacts find a (better paid) job. This paper argues that the two processes are strongly related as individuals simultaneously decide whether to accept an offer and change the job or forward it to a friend. Formally, this study extends the Burdett and Mortensen (1998) model by incorporating referrals into the process of job-to-job transitions. The primary purpose of the paper is to analyze the effect of social contacts on wages and to identify economic factors that lead to premiums or penalties in wages associated with social networks.

This research question, whether the effect of social contacts on wages is positive or negative, remains highly controversial and intensively debated in the literature. For example, in a recent empirical study Pelizzari (2010) shows that in the European Union "... premiums and penalties to finding jobs through personal contacts are equally frequent and are of about the same size." (p. 1). Empirical evidence for the United States is also mixed. Whereas Simon and Warner (1992), Granovetter (1995) and Kugler (2003) estimate a positive effect of referrals on wages, Bentolila, Michelacci and Suarez (2010) provide support for the hypothesis of wage penalties associated with referrals. The same is true for France, where there is contradicting evidence of wage premiums by Margolis and Simonnet (2003) and wage penalties by Delattre and Sabatier (2007). The general conclusion one can draw from these findings, is that labour market conditions and economic environment, as represented by firm and worker heterogeneity, the mechanism of wage-setting, the type of social contacts, the possibility of on-the-job search and other factors may play an important role for the effect of social contacts on wages.

To analyze a number of these factors, I consider a labour market model with on-the-job search and referrals. Social contacts are modelled following a seminal approach by Montgomery (1994), so the population is composed of an endogenous number of two-person groups (dyads) and single workers (monads). Moreover, every connected worker can help his/her dyad partner find a (better paid) job. This is the informal channel of job search. Further, this paper incorporates empirical evidence from sociology that social interaction is organized into foci such as workplaces, clubs, groups, and associations which individuals may belong to. For example, Rivera, Soderstrom and Uzzi (2010) report that the foci of shared activities play an important role in fostering social contacts, however, the loss of shared foci may induce the breaking apart of network ties. In order to incorporate this empirical evidence I assume that unmatched employed individuals randomly form dyad ties with their colleagues ("roommate matching"). At the same time, there is a positive probability of dissolving for the tie if both workers become unemployed. Therefore, the total number of dyads is endogenous in the model and the equilibrium unemployment is decreasing in the speed of contact formation.

Firms are heterogeneous with respect to their productivity and wages are determined by means of Nash bargaining[1]. Thus wages are increasing in the employer's productivity and the reservation wage of the worker. Moreover, on-the-job search implies that workers accept job offers from more productive employers so that job-to-job transitions have a positive effect on worker's earnings. However, job offers from less productive employers are not lost, instead workers forward these offers to their contacts. With respect to the information structure, I assume that connected workers do not observe the flow income of their contacts. Next I distinguish between two modelling regimes. In the first regime, the unemployment status of the worker is explicitly observed by the dyad partner and job information is only transmitted in order to help the unemployed dyad partner find a job. In the second regime, the model is extended to allow connected workers to help each other find a better paid job. However, to simplify the model I assume that unemployment insurance is provided in the form of a public job, thus the state of unemployment is not observed by the dyad partner and job offers are exchanged in an unconditional manner.

The major contribution of this study is to show that job offers transmitted through social contacts are biased towards the left tail of the productivity distribution. This means that the productivity of workers finding jobs through social contacts is below the average in the economy, which has a negative effect on wages. This negative selection effect is an inherent feature of the model in a first information regime, since the distribution of network offers for every unemployed worker is limited from above by the current productivity of the dyad partner. Formally, this means that in a first information regime, the equilibrium productivity distribution with network effects is first order stochastically dominated by the productivity distribution without network effects. Moreover, from an empirical perspective this finding implies that it is essential to control for the (un)observed heterogeneity of employers and hierarchical positions within an occupation in the estimation of the ex-post effect of social contacts on wages. Otherwise, there is risk for the estimator to be biased towards the finding of wage penalties.

Second, this paper identifies a counteracting positive effect of social contacts on wages. This effect arises from the possibility to exchange information within a dyad and leads to endogenous heterogeneity of workers by social capital. For example, in the second information regime, workers are endogenously differentiated into two groups (with high and low social capital) depending on the presence of the dyad partner. This effect is originally investigated in Fontaine (2008) and means that the reservation wage of a connected worker is higher than the reservation wage of a single worker.[2] Therefore, wage bargaining with the same employer will generally lead to a higher negotiated wage if the worker's social capital is high. Further, this paper proves that the value of social capital is increasing in the duration of contacts; that is, a higher speed of contact formation (dissolution) has a positive (negative) effect on differences in the reservation wages between the two groups of workers. This result is supported by the empirical evidence in sociology, for example, Lin (1999) describes it by writing that "the weakest ties are clearly not

[1] To keep the model tractable I assume that the state of unemployment is the only worker's outside option, which means that workers can not hold two jobs at the same time. This simplifying approach was originally applied by Pissarides (1994) and then Gautier (2002). The advantage is that labour contracts are renegotiation-proof.

[2] Burdett and Mortesen (1998) prove that worker's reservation wage is equal to the unemployment benefit if on-the-job search is equally efficient as off-the-job search. Although this requirment is satisfied in the present study, reservation wages are endogenous and strictly below the unemployment benefit. This is because the state of employment is additionally valuable to workers due to the possibility to form and maintain their social contacts.

useful since ties with no strength offer no incentive for exchange".

Next, this paper performs a theoretical Blinder-Oaxaca decomposition of wage differentials between workers finding jobs in a formal and an informal way. For a given level of productivity, jobs obtained through social networks pay higher wages due to the better outside opportunities of connected workers. Nevertheless, workers finding jobs through social contacts are more likely to be employed in low productivity jobs, this is the selection effect. Combining the two effects, this paper shows that wage penalties (premiums) associated with referrals are more likely to be observed in labour markets with stronger (weaker) productivity heterogeneity of employers. From another perspective, wage penalties (premiums) are more likely when the bargaining power of workers is high (low). Intuitively, the sensitivity of wages to outside opportunities is decreasing in the bargaining power, so that workers' reservation wages have a strong impact on earnings if the bargaining power is low. In contrast, the sensitivity of wages to productivity is increasing in the strength of bargaining so that differences in productivities are more likely to be translated into wages when the bargaining power is high.

Finally, and somewhat contradictory to the traditional view (see Montgomery (1991)), this paper illustrates that social networks may reduce wage inequality in the economy. Note that this prediction is obtained in the second information regime when homogeneous workers help each other climb the wage ladder. Intuitively, more successful workers pool their less successful contacts towards better paid jobs in the middle range of the productivity distribution. Therefore, jobs with lowest productivity values are not stable and there are less of them in the stationary steady-state. At the same time, most productive jobs are not frequently transferred through social contacts, so the relative fraction of workers employed in these jobs is decreasing given the overall rise in employment. Together, these two effects imply that social contacts reduce the probability mass in the tails of the distribution by intensifying the process of on-the-job search, so the variance of the equilibrium earnings distribution is reduced.

Related literature

This study is closely related to the literature on social networks in the labour market. The seminal contribution in this field is Montgomery (1991), who shows that employee referrals may serve as a useful screening device if worker's ability is not observed by the potential employer. In the setup of Montgomery workers are more likely to form social links with other workers of the same ability type. As a result firms hire through referrals only if their current employee is of a high-ability type, therefore, referral wages are higher than the average market wage. The key difference of Montgomery (1991) relative to the present study is that firms are ex-ante homogeneous. The advantage of this approach is the possibility of accounting for a positive selection of workers by ability, however, the disadvantage is that the model does not explain selection of referred workers to specific occupations or industries.

In a search theory framework, the positive effect of social contacts on wages is also emphasized by Kugler (2003) and Galenianos (2011). In the former study referrals lower monitoring costs for firms because high-effort referees can exert peer pressure on co-workers, allowing firms to pay lower efficiency wages. Consequently, firms and workers with large networks prefer to use referrals, while others are better off using formal methods. Hence, referrals are used to match well-paid jobs to workers with high social capital and many contacts. However, the critical assumption of the model is that workers use either formal methods or social networks. This is different in the present study where both search methods are used simultaneously.

The negative effect of referrals on wages is investigated in Bentolila, Michelacci and Suarez (2008). Both workers and firms are heterogeneous in their model, as a result social contacts can generate a mismatch between the occupational choice of the worker and his/her productive advantage. Intuitively, if a worker is facing difficulties in finding a job, friends and relatives may help this worker find a job in another sector. This has a negative effect on wages, since the worker loses the productivity advantage by changing the sector. Horvath (2011) extends this result by parameterizing the level of homophily in the society. It is then possible to show that there exists a critical homophily value, such that if the homophily is sufficiently large (small), the presence of social networks decreases (increases) the mismatch compared to a pure market economy. This result is similar to the present study where social contacts can have a positive and a negative effect on wages depending on the characteristics of the labour market. However, the underlying mechanism for penalties and premiums in wages is different. When searching on-the-job workers choose between accepting a job and forwarding it to a friend. This means that referral offers are disproportionately selected from the left tail of the productivity distribution. In contrast, none of the above studies considers the model with on-the-job search.

Further, there are several other studies that can simultaneously generate premiums and penalties in referral wages, they are Sylos-Labini (2004), Pelizzari (2010), Tumen (2012) and Zaharieva (2012). Sylos-Labini (2004) extends the original approach by Montgomery (1991) and distinguishes between family and professional contacts. By assumption there is positive correlation in the ability of professional ties, but there is zero correlation in the ability of family members. As a result, the use of family contacts is likely to have a negative effect on wages, while the opposite is true for professional ties. Family connections are also considered in Zaharieva (2012). In this companion paper firms post wage offers in the regular market, but alternatively they can save on advertising costs and rely on word-of-mouth communication. Wages are then negotiated ex-post between the firm and the applicant and can deviate from the posted market wage depending on the strength of the applicant's bargaining position.

Tumen (2012) considers a population of workers heterogeneous with respect to the cost of maintaining connections. In his model well integrated workers with low costs have higher reservation wages and are able to bargain higher wages. Conversely, workers with higher costs accept wages below the market level. Other related studies in the field of search theory include Fontaine (2004, 2007, 2008). Fontaine (2004) considers the issue of social contacts from the perspective of firms. In his study firms benefit from the social capital of their employees. Bargaining over wages then implies that wages are increasing in the efficiency of networks. Fontaine (2007) applies a similar model to a labour market with two heterogeneous groups of workers. As a result he shows that an economy where more individuals have access to social networks does not necessarily lead to a lower aggregate unemployment rate.

The model by Fontaine (2008) is most closely related to the present study. In that model workers are endogenously differentiated depending on the number of employees in their network. This in turn gives rise to the equilibrium wage dispersion and higher referral wages. Ioannides and Soetevent (2006) support this result by showing that better connected workers experience lower unemployment rates and receive higher wages. My model also has this feature for a given level of productivity. However, in my study endogenous wage dispersion is combined with an exogenous productivity heterogeneity of firms. The advantage of this approach is a possibility to investigate economic factors that lead to penalties or premiums in referral wages.

In a graph theory framework, social networks are analyzed by Calvo-Armengol and Jackson (2004, 2007). The focus of their studies is on differences in the drop-out rates between two exogenous groups of workers (blacks and whites). Calvo-Armengol and Jackson (2007) show that a group of workers with worse initial network connections has higher drop-out rates and persistently lower wages. Further, this study implicitly accounts for the possibility of on-the-job search. In particular, in their model agents are more likely to pass information on if they are more satisfied with their own position. However, the implications of this effect for the steady-state earnings distribution are not considered in their studies, and therefore the negative selection effect of social contacts on wages is not identified.

The plan of the paper is as follows. Section 2 explains notation and the general economic environment. Section 3 presents a reduced intuitive version of the model with endogenous wages, on-the-job search and a binary productivity distribution. In section 4 the model is extended to account for any finite number of productivity values. Section 5 concludes the paper.

2 The theoretical framework

This section describes the setup of the labour market. There is a continuum of infinitely lived risk neutral workers and firms discounting future at a common discount rate r. The total measure of ex-ante identical workers is normalized to one. Firms are heterogeneous with an exogenous discrete productivity distribution $F(y_i) = P\{y \leq y_i\}$, $i = 1, .., n$, where $F(y_n) = 1$. Unemployed workers obtain public unemployment insurance z, such that $z < y_1$.

Job matching Matching between workers and firms is random and employed workers continue searching on-the-job. Both employed and unemployed workers obtain job offers with a Poisson arrival rate λ. Upon the match workers observe the productivity of the job y_i and accept jobs with a positive value gain. Existing jobs are destroyed with a job destruction rate δ.

Social matching Workers employed in the same productivity sector y_i can form social contacts. In this paper I restrict attention to at most one social contact per worker at a given moment of time. So the economy is simultaneously populated by single individuals (monads) and connected two-person groups (dyads). Social matching is modelled using a matching function approach. Let e_0^i denote the measure of monads employed in firms with a productivity level y_i. Then $m(e_0^i) = \phi e_0^i$ denotes the total measure of new dyads with productivity y_i per unit time. This means that $2\phi = 2m(e_0^i)/e_0^i$ is the Poisson intensity parameter of social matching. Note that the special case $\phi = 0$ corresponds to the benchmark economy without dyads.

After the dyad was formed, both workers continue searching on-the-job or may lose their jobs at rate δ. Therefore, all dyads in the economy can be split into three mutually exclusive categories: employed, mixed or unemployed. The total measure of dyads in each category is denoted by d_e, d_m and d_u, respectively. In addition, let e_0 and u_0 denote the total measures of employed and unemployed monads, where $e_0 = \sum_{i=1}^{n} e_0^i$, then it holds:

$$2(d_e + d_m + d_u) + e_0 + u_0 = 1$$

and the overall unemployment rate u in the economy is given by $u = 2d_u + d_m + u_0$.

If both workers in a dyad are unemployed, their contact is subject to the risk of dissolution. Let α denote the separation rate of an individual worker, so that 2α is the dissolution rate of a dyad. Note that a higher value of α is associated with a shorter duration and higher instability of social contacts. In addition, the special case $\alpha = 0$ corresponds to a labour market with permanent social contacts, which can be interpreted as strong family ties. The intermediate situation with $\alpha > 0$ and $\phi > 0$ can then be treated as an economy with weak ties, where social contacts are formed on a temporary basis.

Information structure According to the above description social contacts are formed within the same production sector y_i. However, future productivity and income changes of workers are not observed by their dyad partners. Further, I distinguish between two information regimes. In a first regime, workers observe changes in the employment status of their partners. Moreover, job offers are only forwarded in order to help the unemployed dyad partner find a job. Next, in a second information regime, the model is extended to allow workers to help each other find a better paid job. At the same time, to keep the model tractable, in the second information regime I assume that unemployment insurance is provided in the form of a public job, so that unemployed workers are formally employed in a job paying the flow income z. This simplifies the model since wage and employment status are private information of the worker.

Clearly, personal contacts lead to ex-post endogenous hetegogeneity of workers by social capital, nevertheless in a second regime unobserved information reduces the number of social types to only two groups – with high and low social capital – depending on the existence of a dyad partner. Two workers connected in a dyad may exchange relevant job information and help each other find a (better) job, so these workers' social capital is high, leading to higher endogenous reservation wages. This option is not available to single workers (monads), so their social capital and reservation wages are low.

Wage determination Wages are determined via the concept of Nash-bargaining. This means that wage w_i is increasing in the productivity of the job y_i and the reservation wage of the worker. In addition, labour contracts are continuously renegotiated. On the one hand, it means that changes in the reservation wage of the worker are immediately reflected in the wage. In particular, new social contacts lead to higher reservation wages and higher wages, so that workers with longer tenures are also more likely to receive higher wages. On the other hand, continuous wage renegotiation guarantees that two workers with the same productivity and social capital are paid the same wage independent of their previous employment states.

Optimal strategies Consider a worker with high social capital employed at wage w_i with a corresponding productivity y_i. At a job-finding rate $\lambda\pi_j = \lambda(F_j - F_{j-1})$ this worker obtains a new job offer with a productivity level y_j. If $y_j > y_i$ Nash bargaining with a new employer would lead to a higher wage $w_j > w_i$, so there is a positive value gain for the worker associated with a new job and the job quit is optimal. If $y_j < y_i$ the new job is not valuable for the worker and the offer can be redirected to the dyad partner. If the dyad partner is unemployed or employed with a productivity level below y_j, the job offer will be accepted, otherwise it is lost.

Moreover, in a second information regime it is always optimal for unemployed workers to accept even the least productive job $y_1 > z$. This is due to the fact that reservation wages of unemployed workers are strictly below the unemployment benefit z. Intuitively, the state of employment is additionally valuable for workers due to the possibility to acquire new social contacts. Also the probability of maintaining existing contacts is higher when the worker is employed. This result is an extension of Burdett and Mortensen (1998) who prove that worker's reservation wage is exactly equal to the unemployment benefit if on-the-job search has the same efficiency as off-the-job search. Although this requirement is satisfied in the model, reservation wages of workers are below z due to the presence of network effects.

3 Special case $n = 2$: binary productivity distribution

This section investigates the effect of personal contacts on unemployment and wages in a special case $n = 2$, corresponding to the binary productivity distribution. To ease the notation let $\pi = F(y_1)$ denote the fraction of jobs with a low productivity value y_1, so that:

$$\text{Productivity} = \begin{cases} y_2 & \text{with probability} \quad (1-\pi) \\ y_1 & \text{with probability} \quad \pi \end{cases}$$

Consider the pool of employees. On the one hand, employees always accept job offers from more productive firms y_2, on the other hand, they redirect job offers in the low productivity sector y_1 to their contacts. The reason is that jobs y_1 are not valuable for the employees and can only be strictly gainful for the unemployed workers. On the contrary, job offers in the high productivity sector y_2 are (weakly) valuable for all workers. Note that this setup implies that only the low productivity jobs y_1 are transferred through social contacts.

3.1 Aggregate variables

The structure of the labour market with a binary productivity distribution is illustrated on figure 1. Here black and red arrows indicate the processes of job creation and job destruction. Light blue arrows correspond to the process of social matching: formation and dissolution of social contacts. Blue arrows reflect transitions to better paid jobs, this is the result of on-the-job search, while green arrows indicate job search through personal contacts. Variables $\widetilde{U}$ and U denote the asset values of unemployment for workers with high and low social capital, respectively. In addition, variables $\widetilde{V}_i$ and V_i denote asset values of employment in sector i, $i = 1, 2$, similarly for the two levels of social capital.

Consider changes in the total measure of employed monads $\dot{e}_0$ with the asset values denoted by V_1 and V_2. Employed monads lose jobs at rate δ and form social contacts at rate 2ϕ, so the total outflow of workers from this group is $(\delta + 2\phi)e_0$. At the same time unemployed monads with an asset value denoted by U find jobs at rate λ, so the inflow of workers into this group is λu_0. Similarly, the inflow of workers into the state u_0 is $\delta e_0 + 2\alpha \cdot 2d_u$ stemming from the dissolution of dyads at the instability rate 2α when both dyad partners are unemployed. This reasoning gives rise to the following differential equations for variables $\dot{u}_0$ and $\dot{e}_0$:

$$\dot{u}_0 = 4\alpha d_u - \lambda u_0 + \delta e_0 \qquad \dot{e}_0 = \lambda u_0 - \delta e_0 - 2\phi e_0$$

Next consider changes in the total measure of unemployed dyads d_u where the asset value of each dyad partner is denoted by $\widetilde{U}$. Employed partners in mixed dyads d_m lose jobs at rate δ,

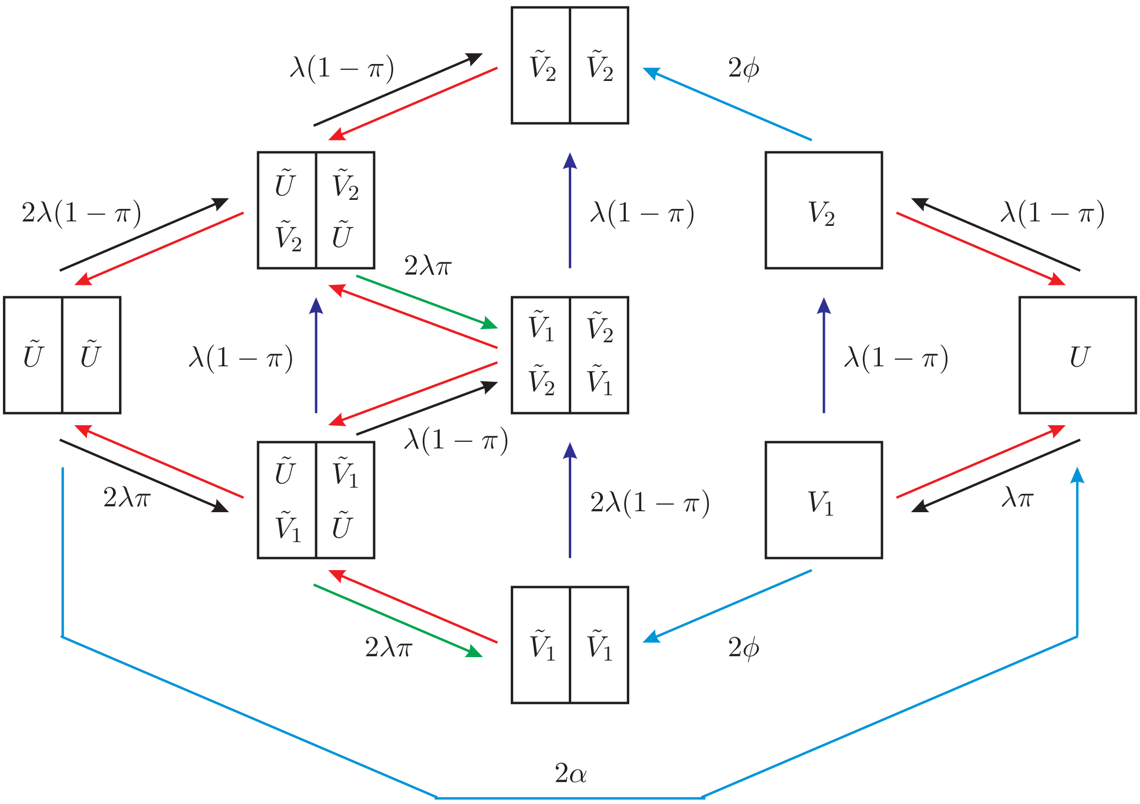

Figure 1: The structure of the labour market

so the inflow of dyads into this state is δd_m. At the same time every partner in the unemployed dyad finds a job at rate λ, so the outflow of dyads from d_u is $2\lambda d_u + 2\alpha d_u$. Similarly unemployed workers in mixed dyads find jobs at rate λ or may obtain job offers from their partners. Since only low paid jobs are transferred through social contacts the rate at which connected unemployed workers find jobs through personal contacts is $\lambda\pi$, so the total inflow of dyads into the state d_e is equal to $\lambda d_m + \lambda\pi d_m + \phi e_0$:

$$\dot{d}_u = \delta d_m - 2\alpha d_u - 2\lambda d_u \qquad \dot{d}_e = \lambda d_m + \lambda\pi d_m - 2\delta d_e + \phi e_0$$

In the steady state it should be true that $\dot{u}_0 = 0$, $\dot{e}_0 = 0$, $\dot{d}_u = 0$ and $\dot{d}_e = 0$, which also implies $\dot{d}_m = 0$ due to the fixed total measure of workers in the economy. The steady state values u_0, e_0, d_u and d_m are described in lemma 1:

Lemma 1: *Denote an additional auxilliary shift variable* $s = \alpha\delta(2\phi + \delta) + (\delta + \lambda + \alpha)\lambda\phi$. *Then the equilibrium measures of unemployed and mixed dyads* d_u, d_m *are given by:*

$$d_u = \frac{0.5\delta^2\phi\lambda}{(\lambda+\delta)s + \lambda^2\pi\phi(\alpha+\lambda)} \qquad d_m = \frac{\delta\lambda(\lambda+\alpha)\phi}{(\lambda+\delta)s + \lambda^2\pi\phi(\alpha+\lambda)}$$

The equilibrium measures of unemployed and employed monads u_0 *and* e_0 *are given by:*

$$u_0 = \frac{\alpha\delta^2(2\phi+\delta)}{(\lambda+\delta)s + \lambda^2\pi\phi(\alpha+\lambda)} \qquad e_0 = \frac{\alpha\delta^2\lambda}{(\lambda+\delta)s + \lambda^2\pi\phi(\alpha+\lambda)}$$

Job search through personal contacts has a negative effect on d_u, d_m, e_0 *and* u_0, *while it has a positive effect on the measure of employed dyads* $d_e = 0.5(1 - u_0 - e_0) - d_m - d_u$.

Proof: Appendix I.

Lemma 1 shows that the second term in the common denominator $\lambda^2\pi\phi(\alpha+\lambda)$ is attributed to network effects. This means that job search through personal contacts has a positive effect on the total measure of dyads in the economy $d = 1 - u_0 - e_0$ and a similar positive effect on the measure of employed dyads d_e. Personal contacts place more workers into jobs so the dissolution of dyads is reduced. In addition consider the two bechmark cases $\alpha = 0$ and $\phi = 0$. In a purely dyadic labour market when $\alpha = 0$ variables d_u and d_m simplify to yield:

$$d_u = \frac{0.5\delta^2}{(\delta+\lambda)^2 + \lambda^2\pi} \qquad d_m = \frac{\delta\lambda}{(\delta+\lambda)^2 + \lambda^2\pi}$$

In the absence of personal contacts these variables are further simplified to $d_u = 0.5\delta^2/(\delta+\lambda)^2$, $d_m = \delta\lambda/(\delta+\lambda)^2$ and $d_e = 0.5\lambda^2/(\delta+\lambda)^2$ which is the case of independent job search, note that the total measure of dyads in a dyadic economy is 0.5. In a monadic labour market when $\phi = 0$ variables u_0 and e_0 simplify to $\delta/(\lambda+\delta)$ and $\lambda/(\lambda+\delta)$ respectively. So the model developed in this paper has a general character and nests a number of benchmark search models.

Informal job search through contacts also has a negative effect on the aggregate unemployment rate in the economy:

$$u = u_0 + 2d_u + d_m = \frac{\delta s}{(\lambda+\delta)s + \lambda^2\pi\phi(\alpha+\lambda)} < \frac{\delta}{\lambda+\delta}$$

The stronger the effect of personal contacts $\lambda^2\pi\phi(\alpha+\lambda)$ the lower is the equilibrium unemployment. In particular, unemployment is decreasing in the proportion of low productivity jobs π since only these jobs are transferred in the informal way. This means that the unemployment rate achieves the minimum for the case $\pi = 1$ – an economy without productivity heterogeneity.

3.2 Equilibrium productivity distribution

Let variables $p_1 = e_0^1/e$ and $p_2 = e_0^2/e$ denote the equilibrium measures of monads employed at productivities y_1 and y_2 respectively and expressed as a proportion of total employment e. Monads are the unconnected workers with a low level of social capital. In addition, let variables $\tilde{p}_1$ and $\tilde{p}_2$ denote the equilibrium proportions of employees with a high level of social capital employed at productivities y_1 and y_2, so that $p_1 + p_2 + \tilde{p}_1 + \tilde{p}_2 = 1$. Table 1 summarises the population structure of the economy reflecting an exogenous heterogeneity of workers by productivity and an endogenous heterogeneity by social capital. The corresponding wages are denoted by w_i and $\tilde{w}_i$, $i = 1, 2$.

	Low productivity y_1, $p(y_1)$	High productivity y_2, $p(y_2)$
Low social capital	w_1, p_1	w_2, p_2
High social capital	$\tilde{w}_1$, $\tilde{p}_1$	$\tilde{w}_2$, $\tilde{p}_2$

Table 1: Wages and productivities by groups of workers

Variables $p(y_1) = p_1 + \tilde{p}_1$ and $p(y_2) = p_2 + \tilde{p}_2$, such that $p(y_1) + p(y_2) = 1$, show the total fractions of workers employed in sectors y_1 and y_2 – this is the equilibrium productivity distribution in the economy. The equilibrium values $p(y_1)$ and $p(y_2)$ are provided by lemma 2:

Lemma 2: *Let variables p_1 and p_2 denote proportions of monads employed at productivities y_1 and y_2. The equilibrium values $p_1 = e_0^1/e$ and $p_2 = e_0^2/e$ are given by:*

$$p_1 = \frac{\pi\alpha\delta^2(2\phi+\delta)}{(2\phi+\delta+\lambda(1-\pi))(s+\lambda\pi\phi(\alpha+\lambda))} \qquad p_2 = \frac{(1-\pi)\alpha\delta^2(2\phi+\delta+\lambda)}{(2\phi+\delta+\lambda(1-\pi))(s+\lambda\pi\phi(\alpha+\lambda))}$$

The equilibrium productivity distribution $\{p(y_1), p(y_2)\}$ is given by:

$$p(y_1) = \frac{\pi\delta}{\delta+\lambda(1-\pi)}\left[1+\frac{\lambda(1-\pi)\phi(\alpha+\lambda)}{s+\lambda\pi\phi(\alpha+\lambda)}\right] \qquad p(y_2) = \frac{(1-\pi)}{\delta+\lambda(1-\pi)}\left[\lambda+\frac{\delta s}{s+\lambda\pi\phi(\alpha+\lambda)}\right]$$

Informal job search has a positive effect on $p(y_1)$ and a negative effect on $p(y_2)$.

Proof: Appendix I.

Lemma 2 proves that network effects shift the equilibrium productivity distribution towards low paid jobs, this is intuitive, since only low paid jobs are transferred through personal contacts. The equilibrium productivity distribution with personal contacts and information exchange is then stochastically dominated by the distribution without contacts:

$$p(y_1) > \frac{\pi\delta}{\delta+\lambda(1-\pi)} \qquad p(y_2) < \frac{(1-\pi)(\lambda+\delta)}{\delta+\lambda(1-\pi)} \quad \text{for} \quad \phi > 0$$

Section 4 further shows that this result persists in the extended model with an arbitrary finite number of productivity values. Indeed, low paid jobs are more likely to be transferred through social contacts which shifts the probability mass of the equilibrium productivity distribution to the left. This is the central explanation of the negative effect of referrals on wages in this paper.

3.3 Asset values and wages

In the second information regime, unobserved information with respect to the flow income of dyad partners implies that connected workers have to form probabilistic beliefs about the employment status and wages of their contacts. Consider an unemployed worker and let variable μ denote an equilibrium probability that the dyad partner of this worker is also unemployed. The measure of unemployed dyads in the economy is d_u, while the measure of mixed dyads where the first partner is unemployed and the second one is employed is $0.5d_m$. For variable μ this means:

$$\mu = \frac{d_u}{d_u+0.5d_m} = \frac{\delta}{\delta+\lambda+\alpha}$$

If the dyad partner of the unemployed worker is also unemployed (with probability μ), the social contact can be destroyed at the dissolution rate 2α. On the contrary, if the dyad partner is employed (with probability $1-\mu$) the social contact persists and there is an additional value for the worker from the possibility of obtaining a new job offer from the partner. Asset values of unemployed workers with high and low social capital are then given by:

$$\begin{aligned} rU &= z + \lambda\pi(V_1 - U) + \lambda(1-\pi)(V_2 - U) \\ r\widetilde{U} &= z + \lambda\pi(2-\mu)(\widetilde{V}_1 - \widetilde{U}) + \lambda(1-\pi)(\widetilde{V}_2 - \widetilde{U}) - 2\mu\alpha(\widetilde{U} - U) \end{aligned}$$

and the asset values of employees searching on-the-job and forming contacts are:

$$
\begin{aligned}
rV_1 &= w_1 + \lambda(1-\pi)(V_2 - V_1) + 2\phi(\widetilde{V}_1 - V_1) - \delta(V_1 - U) \\
r\widetilde{V}_1 &= \tilde{w}_1 + \lambda(1-\pi)(\widetilde{V}_2 - \widetilde{V}_1) - \delta(\widetilde{V}_1 - \widetilde{U}) \\
rV_2 &= w_2 + 2\phi(\widetilde{V}_2 - V_2) - \delta(V_2 - U) \quad \text{and} \quad r\widetilde{V}_2 = \tilde{w}_2 - \delta(\widetilde{V}_2 - \widetilde{U})
\end{aligned}
$$

Asset values for filled jobs are denoted by variables $\widetilde{J}_i$ and J_i, $i = 1, 2$. Wage renegotiation implies that wages are simultaneously updated if the outside option of the worker is changed. This means that firms hiring workers at low wages w_1 and w_2 will have to pay higher wages $\tilde{w}_1$ and $\tilde{w}_2$ respectively at rate 2ϕ when the social capital of the worker is increased. Here note that workers with longer tenures on average earn higher wages than workers with shorter tenures, which is in line with the empirical evidence. Asset values for filled jobs are given by:

$$
\begin{aligned}
rJ_i &= y_i - w_i - \lambda(1-\pi)J_i - 2\phi(J_i - \widetilde{J}_i) - \delta J_i, \qquad i = 1, 2 \\
r\widetilde{J}_i &= y_i - \tilde{w}_i - \lambda(1-\pi)\widetilde{J}_i - \delta \widetilde{J}_i, \qquad i = 1, 2
\end{aligned}
$$

Every job destroyed at rate δ exits the market and is substituted by a new vacant one with the same productivity level, so the total measure of filled and vacant jobs remains unchanged. Worker rents $R_i \equiv V_i - U$ and $\widetilde{R}_i \equiv \widetilde{V}_i - \widetilde{U}$ can be expressed as:

$$
\begin{aligned}
(r + \delta + \lambda(1-\pi) + 2\phi)R_i &= w_i - rU + \lambda(1-\pi)R_2 + 2\phi(\widetilde{R}_i + \Delta U) \\
(r + \delta + \lambda(1-\pi))\widetilde{R}_i &= \tilde{w}_i - r\widetilde{U} + \lambda(1-\pi)\widetilde{R}_2
\end{aligned}
$$

where $\Delta U \equiv \widetilde{U} - U$ is the unemployed worker's value gain from having a dyad partner; this is the endogenous value of social capital in the model. Worker rents R_1 and $\widetilde{R}_1$ associated with jobs in the low productivity sector can be rewritten by inserting variables rU and $r\widetilde{U}$:

$$
\begin{aligned}
(r + \delta + \lambda + 2\phi)R_1 &= w_1 - z + 2\phi(\widetilde{R}_1 + \Delta U) \\
(r + \delta + \lambda + \lambda\pi(1-\mu))\widetilde{R}_1 &= \tilde{w}_1 - z + 2\mu\alpha\Delta U
\end{aligned}
$$

In the following it will be shown that variable ΔU is strictly positive, so that reservation wages of both types of workers are below the unemployment benefit z. This means that workers never reject jobs in the low productivity sector, since $y_1 > z$. There are two distinct reasons for accepting these jobs: unemployed workers without contacts accept a job paying potentially less than z due to the additional probability of forming social contacts when employed. McDonald (2011: 1673) provides empirical evidence and summarizes this idea by writing that "specialization in work can increase a person's social capital as time spent in related occupations increases the opportunity to meet and develop relationships with the kinds of contacts that can provide unsolicited information about future job openings". In addition, connected unemployed workers are willing to accept low productivity jobs in order to eliminate the probability of losing their existing social contacts.

Let $0 < \beta < 1$ denote the bargaining power of workers. Wages are then obtained via the mechanism of Nash bargaining, where the β-fraction of the total job surplus accrues to workers:

$R_i = \beta(R_i + J_i)$ and $\widetilde{R}_i = \beta(\widetilde{R}_i + \widetilde{J}_i)$. This gives the following equilibrium equations for wages:

$$\begin{aligned} w_i &= \beta(y_i + 2\phi\widetilde{J}_i) + (1-\beta)[rU - \lambda(1-\pi)R_2 - 2\phi(\widetilde{R}_i + \Delta U)] \\ &= \beta y_i + (1-\beta)[z + \lambda\pi R_1 - 2\phi\Delta U] \\ \tilde{w}_i &= \beta y_i + (1-\beta)[r\widetilde{U} - \lambda(1-\pi)\widetilde{R}_2] \\ &= \beta y_i + (1-\beta)[z + \lambda\pi(2-\mu)\widetilde{R}_1 - 2\mu\alpha\Delta U] \end{aligned}$$

These equations show that differences in wages can be decomposed into two parts resulting from worker and firm heterogeneity. Inter-sectoral wage differentials are the same for both types of workers and reflect exogenous productivity differences between the sectors: $\tilde{w}_2 - \tilde{w}_1 = w_2 - w_1 = \beta\Delta y$ where $\Delta y \equiv y_2 - y_1$. In addition, wages are heterogeneous within the sector, so that intra-sectoral wage differentials are attributed to differences in the reservation wages of workers:

$$\begin{aligned} \Delta w &\equiv \tilde{w}_1 - w_1 = \tilde{w}_2 - w_2 \\ &= (1-\beta)[\lambda\pi((2-\mu)\widetilde{R}_1 - R_1) - 2\Delta U(\mu\alpha - \phi)] \end{aligned}$$

In the following consider the limiting case $r \to 0$ and denote an auxilliary variable $\sigma = \mu\alpha(\delta + 2\phi + \lambda(1-\pi)(1-\beta)) + \phi\lambda\beta$. Proposition 1 shows that intra-sectoral wage differentials Δw are proportional to the value of social capital ΔU:

Proposition 1: *Wage gap $\Delta w = \tilde{w}_1 - w_1 = \tilde{w}_2 - w_2$ attributed to differences in the social capital of unemployed workers is given by:*

$$\begin{aligned} \Delta w &= \frac{(1-\beta)(\delta + 2\phi + \lambda(1-\pi))\phi}{\delta + 2\phi + \lambda(1-\pi)(1-\beta)} 2\Delta U \\ &= \frac{\lambda\pi\beta(1-\beta)(1-\mu)(\delta + 2\phi + \lambda(1-\pi))\phi}{\sigma(\delta + \lambda(1-\pi) + \lambda\pi\beta) + \lambda\pi\beta(1-\mu)\phi\lambda\beta}(y_1 - z) \end{aligned}$$

Moreover, Δw is a decreasing function of the instability parameter α and an increasing function of the intensity parameter ϕ:

$$\frac{\partial \Delta U}{\partial \alpha} < 0 \quad \frac{\partial \Delta w}{\partial \alpha} < 0 \quad \frac{\partial \Delta w}{\partial \phi} > 0 \quad \frac{\partial \Delta U}{\partial \phi} < 0$$

Proof: Appendix II.

Proposition 1 shows that endogenous wage dispersion Δw disappears if $y_1 = z$, $\beta = 0$ or $\beta = 1$. If $y_1 = z$ low paid jobs are not valuable for workers, so the value of social capital ΔU is equal to zero. This means that reservation wages of both types of workers are the same and equal to z. This is a standard result in a model with on-the-job search. The same is true if $\beta = 0$. On the contrary, if $\beta = 1$ wages are equal to the productivity of jobs: $w_i = \widetilde{w}_i = y_i$, so that differences in the reservation wages are not reflected in wages.

The second group of results from proposition 1 can be explained in the following way. Higher α leads to lower stability and therefore shorter average durations of social contacts, this contributes to a lower value of social capital ΔU. Short-term social contacts are less valuable, so that differences in the reservation wages of workers are reduced. Lin (1999: 482) describes this effect by writing that "the weakest ties are clearly not useful since ties with no strength offer no incentive for exchange". The implications of higher ϕ are similar: social contacts are formed at a higher rate, so the gain from social contacts ΔU is lower. Nevertheless, Δw is higher due to the dominating effect of a higher probability of forming the link.

3.4 Implications of social contacts

What is the average wage paid to workers finding jobs through personal contacts? The answer to this question is $\tilde{w}_1$ – this is the only wage paid to connected workers in the low productivity sector y_1. Further, let w^n denote the average wage paid to workers finding jobs in a formal way, and $\bar{w}$ – the average wage between the two groups. Clearly, $\bar{w}$ is also the average wage in the economy, so that:

$$\bar{w} = w_1 p_1 + w_2 p_2 + \tilde{w}_2 \tilde{p}_2 + (1 - p_1 - p_2 - \tilde{p}_2)\tilde{w}_1$$

This means that the ex-post effect of personal contacts on wages $\tilde{w}_1 - w^n$ is positive (negative) if $\tilde{w}_1 - \bar{w}$ is positive (negative). Therefore, in the following only the term $\tilde{w}_1 - \bar{w}$ is considered and it can be decomposed into a positive and a negative part [3]:

$$\begin{aligned}
\tilde{w}_1 - \bar{w} &= (\tilde{w}_1 - w_1)p_1 + (\tilde{w}_1 - w_2)p_2 + (\tilde{w}_1 - \tilde{w}_2)\tilde{p}_2 \\
&= \Delta w p_1 + (\Delta w + w_1 - w_2)p_2 - \beta \Delta y \tilde{p}_2 \\
&= \Delta w (p_1 + p_2) - \beta \Delta y (p_2 + \tilde{p}_2)
\end{aligned}$$

Proposition 2: *The effect of personal contacts on wages is positive ($\tilde{w}_1 - \bar{w} > 0$) in the absence of productivity heterogeneity meaning that $\Delta y = 0$ or $\pi = 1$, while this effect is negative ($\tilde{w}_1 - \bar{w} < 0$) in a dyadic economy ($\alpha = 0$), if low productivity jobs are not productive ($y_1 = z$) or if workers possess the full bargaining power ($\beta = 1$).*

$$\tilde{w}_1 - \bar{w} = \frac{\alpha \delta^2 \Delta w}{s + \lambda \pi \phi (\alpha + \lambda)} - \frac{(1-\pi)\beta \Delta y}{\delta + \lambda(1-\pi)} \left[\lambda + \frac{\delta s}{s + \lambda \pi \phi(\alpha + \lambda)} \right]$$

Moreover, the effect of personal contacts on wages is negative for $\pi \in (0, \pi^*)$, where the unique threshold value π^* can be obtained from:

$$\frac{\lambda \pi^* \phi(\alpha + \lambda)\alpha \mu \delta}{s(\lambda + \delta) + \lambda^2 \pi^* \phi(\alpha + \lambda)} = \frac{y_2 - y_1}{y_1 - z}(1 - \pi^*)$$

The threshold value π^* is a decreasing function of parameters ϕ, α and y_1 but an increasing function of y_2 and z. **Proof:** Appendix III.

The positive effect of personal contacts is explained by the higher reservation wage of connected workers leading to endogenous wage dispersion Δw. In the absence of productivity heterogeneity ($\Delta y = 0$ or $\pi = 1$) all jobs in the economy are equally productive and there is no distinction between the two sectors. The number of wages in this economy is reduced to only two: $\tilde{w} > w$, so that workers finding jobs through personal contacts are paid higher wages than workers finding jobs in the formal way.

[3] This step is similar to the Blinder-Oaxaca decomposition of differences in wages between two heterogeneous groups of workers. The two parts are called the price and the endownment effects.

The negative effect of personal contacts is stronger with a higher productivity heterogeneity: higher Δy and lower π. This is intuitive since dyad partners only help find jobs in the low productivity sector y_1. Consider the case $\alpha = 0$ corresponding to the dyadic labour market with strong ties. Every worker in this economy is connected to a permanent dyad partner and hence there are no differences in the reservation wages of workers. The number of wages in this economy is again reduced to only two: $w_2 > w_1$, so that workers finding jobs through personal contacts are paid lower wages than workers finding jobs in the formal way.

Moreover, proposition 2 defines a unique threshold value of the probability parameter π^*, such that the negative effect of referrals is dominating for every $\beta < 1$ if the proportion of low paid jobs is sufficiently small: $\pi < \pi^*$. This is illustrated on figure 2. In the opposite case $\pi > \pi^*$ the effect of personal contacts can be both positive and negative depending on the parameter of bargaining power. This is due to the fact that wage dispersion Δw is a non-linear function of β, in particular $\Delta w = 0$ if $\beta = 0$ or $\beta = 1$ reaching maximum at the interior value of β. This gives rise to proposition 3.

Proposition 3: *Denote an auxilliary function $\Delta W(\beta) = \Delta w/\beta$, such that $\Delta W'(\beta) < 0$, and consider the case $\pi^* < \pi < 1$, then there exists a unique threshold value β^* such that the effect of personal contacts on wages $(\tilde{w}_1 - \bar{w})$ is positive if $\beta \in (0, \beta^*)$ and it is negative if $\beta \in (\beta^*, 1]$, the threshold value β^* can be obtained from:*

$$\Delta y \frac{(p_2 + \tilde{p}_2)}{(p_1 + p_2)} = \Delta W(\beta^*)$$

The effect of personal contacts on wages is zero if either $\beta = 0$ or $\beta = \beta^*$.
Proof: Appendix III.

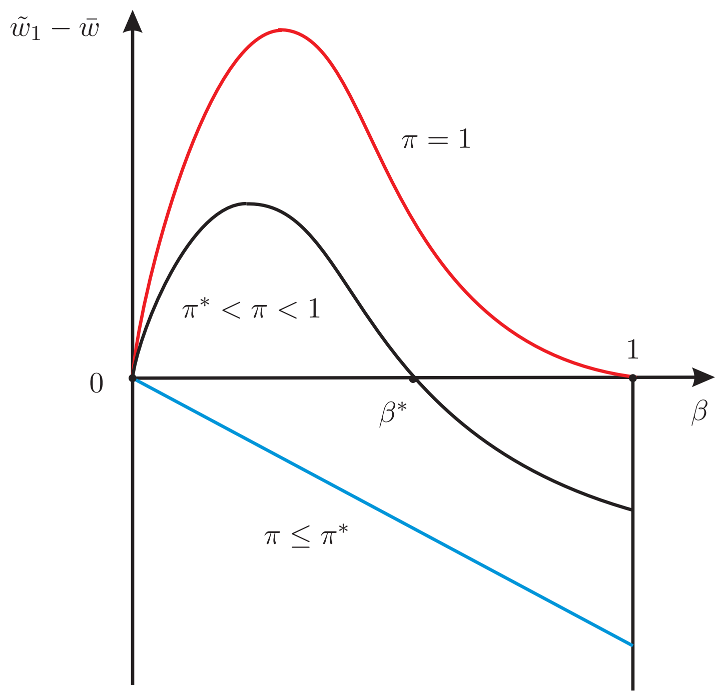

Figure 2: The ex-post effect of referrals on wages

Proposition 3 and figure 2 show that the negative effect of personal contacts on wages is dominating for large values of the bargaining power parameter $\beta > \beta^*$. This is due to the fact that wages are proportional to the productivity, so the negative effect of personal contacts is a linear function of β, reaching the lowest value for $\beta = 1$. In addition, the positive effect of reservation wages Δw is small for $\beta \to 1$, so the negative effect is dominating. On the contrary, the positive effect is large for interior values of β and is dominating for $\beta < \beta^*$.

4 General case: $n > 2$ with exogenous wages

This section explores properties of the labour market with a finite number of productivity values $n > 2$ and shows that the negative effect of personal contacts on wages is an inherent feature of the process of on-the-job search. However, in order to keep the model tractable I impose a simplifying assumption of a binding minimum wage requirement. Without loss of generality I assume that the legal minimum wage in the economy is set to z, this means that wage equations are simplified to yield: $w_i = \beta y_i + (1-\beta)z$. Therefore, the focus of this section is on the negative selection effect of social networks and the traditional (positive) effect of reservation wages on earnings is not considered.

4.1 Aggregate variables

Probability variables $F_i = P\{y \leq y_i\}$ define a cumulative density function of the productivity distribution y_i, $i = 1,..,n$, with a density function $\pi_i = F_i - F_{i-1}$. In addition let h_i define a measure of mixed dyads where the first worker is unemployed and the second is employed at productivity y_i. The corresponding cumulative measure of mixed dyads is denoted by H_i, so that $H_n = 0.5d_m$. Similarly, let g_{ij} denote a measure of employed dyads where workers are employed at productivities y_i and y_j respectively. Finally let G_{ij} denote the corresponding cumulative measure of employed dyads so that $G_{nn} = d_e$.

As a first step in the analysis consider the case when the employment status of the worker is observed by the dyad partner and social connections are only used to help unemployed dyad partners find a job (first information regime). In section 4.3 the model will be extended to allow employed workers to use connections in order to find a better paid job[4]. Changes in the stock of mixed dyads where the second worker is employed at a productivity less or equal to y_k can be described by the following differential equation:

$$\dot{H}_k = \lambda F_k d_u + \delta G_{n,k} - \delta H_k - \lambda(1 - F_k)H_k - \lambda H_k \underbrace{-\lambda \sum_{i=1}^{k} F_i h_i}_{\text{network effects}} \tag{4.1}$$

Unemployed workers find jobs with a productivity less or equal to y_k at rate λF_k. In addition, workers with dyad partners employed at a productivity less or equal to y_k lose jobs at rate δ and the total measure of these dyads is G_{nk}, so the total inflow of dyads into the state H_k is $\lambda F_k d_u + \delta G_{nk}$. Employed workers in mixed dyads lose jobs at rate δ or find a job with a

[4] In a setting when the transmission of job information is conditional on the employment status of the dyad partner, reservation wages of unemployed workers may be situated abobe z, so that low paid jobs y_1 can be rejected if $y_1 - z$ is sufficiently small. However this setting is treated as a first step to the extended model where the transmission of job information is unconditional, in this extended model low productivity jobs $y_1 > z$ are never rejected and so the issue of reservation wages is not discussed in this section.

productivity strictly higher than y_k at rate $\lambda(1-F_k)$, so the first part of the outflow of mixed dyads is $\delta H_k + \lambda(1-F_k)H_k$. Unemployed partners in mixed dyads find jobs in a formal way at rate λ or can obtain an offer in the informal way from their partners. Note that employed workers in mixed dyads h_i forward jobs to their partners with a probability λF_i. This explains the second part of the outflow of mixed dyads $\lambda H_k + \lambda \sum_{i=1}^{k} F_i h_i$.

Next denote Q_k the measure of monads in the economy employed at a productivity less or equal to y_k, formally $Q_k = \sum_{i=1}^{k} e_i^0$, so that $Q_n = e_0$. These monads form social contacts at rate ϕ, so that changes in the stock of employed dyads G_{lk} are given by:

$$\begin{aligned} \dot{G}_{lk} &= \lambda F_l H_k + \lambda F_k H_l - 2\delta G_{lk} - \lambda(1-F_k)G_{lk} - \lambda(1-F_l)G_{lk} + \\ &+ \phi Q_k + \underbrace{2\lambda \sum_{i=1}^{k} F_i h_i + \lambda F_k (H_l - H_k)}_{\text{network effects}} \quad \text{if} \quad l \geq k \end{aligned} \tag{4.2}$$

The total measure of mixed dyads where the first worker is employed in the range between y_k and y_l ($l \geq k$) is equal to $H_l - H_k$. Employed workers in these dyads forward jobs with a productivity below y_k to their dyad partners and these jobs are offered with an arrival rate λF_k. So the additional inflow of dyads into the stock G_{lk} which is attributed to job search through personal contacts is equal to $2\lambda \sum_{i=1}^{k} F_i h_i + \lambda F_k(H_l - H_k)$.

Transform the measures of mixed dyads h_i and H_i into the proper probability distribution $\tilde{h}_i = h_i/(0.5d_m)$ and $\tilde{H}_i = H_i/(0.5d_m)$, so that $\sum_{i=1}^{n} \tilde{h}_i = 1$. In the stationary equilibrium variables $\dot{G}_{nn}$ and $\dot{H}_n$ should be equal to zero, this gives rise to lemma 3.

Lemma 3: *Let $F_i = P\{y \leq y_i\}$, $i = 1,..,n$ denote a cumulative density function of the productivity distribution y_i, $i = 1,..,n$ and let $\bar{F}$ denote an average probability: $\bar{F} = \sum_{i=1}^{n} F_i \tilde{h}_i$. The equilibrium measures of unemployed and mixed dyads d_u, d_m are:*

$$d_u = \frac{0.5\delta^2 \phi \lambda}{(\lambda+\delta)s + \lambda^2 \bar{F} \phi(\alpha+\lambda)} \qquad d_m = \frac{\delta\lambda(\lambda+\alpha)\phi}{(\lambda+\delta)s + \lambda^2 \bar{F} \phi(\alpha+\lambda)}$$

The equilibrium measures of unemployed and employed monads u_0 and e_0 are given by:

$$u_0 = \frac{\alpha\delta^2(2\phi+\delta)}{(\lambda+\delta)s + \lambda^2 \bar{F} \phi(\alpha+\lambda)} \qquad e_0 = \frac{\alpha\delta^2\lambda}{(\lambda+\delta)s + \lambda^2 \bar{F} \phi(\alpha+\lambda)}$$

Job search through personal contacts has a negative effect on d_u, d_m, e_0 and u_0, while it has a positive effect on the number of employed dyads $d_e = 0.5(1-u_0-e_0) - d_m - d_u$.

Lemma 3 is an extended version of lemma 1 for the general case of a finite-state productivity distribution y_i, $i = 1,..,n$. The intuitive interpretation of the results is also similar: personal contacts increase the measure of employed dyads d_e and have a negative effect on the equilibrium unemployment rate.

4.2 Equilibrium productivity distribution

This section analyses properties of the equilibrium productivity distribution in an economy with $n > 2$ production sectors, on-the-job search and personal contacts. First consider the effect of social contacts on the productivity distribution of mixed dyads $\tilde{h}_i$ and $\tilde{H}_i$. Table 2 shows that the new outflow of workers from state h_1 (which is attributed to network effects) is $\lambda F_1 h_1$. This ouflow reflects the fact that connected workers employed at productivity y_1 obtain job offers in the same production sector at rate λF_1 and forward these offers to the unemployed dyad partners. This additional exit rate is however increasing from λF_1 to $\lambda F_n = \lambda$ with a higher productivity level. Indeed, connected workers employed at a maximum productivity y_n will forward *any* new job offer to their unemployed dyad partners.

	e_1	e_2	...	e_{n-1}	e_n	e
h_1	$\lambda\pi_1$	0	...	0	0	λF_1
h_2	$\lambda\pi_1$	$\lambda\pi_2$	...	0	0	λF_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
h_{n-1}	$\lambda\pi_1$	$\lambda\pi_2$	...	$\lambda\pi_{n-1}$	0	λF_{n-1}
h_n	$\lambda\pi_1$	$\lambda\pi_2$	...	$\lambda\pi_{n-1}$	$\lambda\pi_n$	λF_n
H_n	$\lambda\pi_1 H_n$	$\lambda\pi_2(H_n - H_1)$	...	$\lambda\pi_{n-1}(H_n - H_{n-2})$	$\lambda\pi_n(H_n - H_{n-1})$	

Table 2: Outflow of mixed dyads attributed to social contacts

This means that network effects have an asymmetric effect on the productivity distribution of mixed dyads $\tilde{h}_i$: the outflow of dyads is stronger in the right tale of the distribution and it is weaker in the left tale. As a result the productivity distribution $\tilde{H}_i$ with network effects is first order stochastically dominated by the distribution $\tilde{H}_i^*$ without network effects. This is summarized in proposition 3.

Proposition 3: *Let $\tilde{H}_k$ denote a cumulative density function of an equilibrium productivity distribution of mixed dyads. $\tilde{H}_k$ is given by the following recursive equation:*

$$\tilde{H}_k = \frac{\lambda\delta F_k(2(\delta+\alpha+\lambda)+\lambda(1-F_k))}{(\alpha+\lambda)(\delta+\lambda(1-F_k))2(\delta+\lambda)} + \frac{\delta\tilde{Q}_k + \lambda^2(1-F_k)\sum_{i=1}^{k-1}\tilde{h}_i(F_k - F_i)}{(\delta+\lambda(1-F_k))2(\delta+\lambda)}$$

$$\text{where} \quad \tilde{Q}_k = \frac{\phi Q_k}{0.5 d_m} = \frac{2\alpha\delta F_k(\delta+2\phi)}{(\alpha+\lambda)(\delta+2\phi+\lambda(1-F_k))}$$

For $\forall\alpha \geq 0$ the productivity distribution $\tilde{H}_k$ with network effects is first order stochastically dominated by the distribution $\tilde{H}_k^*$ without network effects, where the later is given by:

$$\tilde{H}_k^* = \frac{\lambda\delta F_k(2\delta+\alpha+\lambda+\lambda(1-F_k)) + \delta\tilde{Q}_k(\alpha+\lambda)}{(\alpha+\lambda)(\delta+\lambda(1-F_k))(2\delta+\lambda+\lambda(1-F_k))}$$

Proof: Appendix IV.

Proposition 3 proves that job search through personal contacts shifts the probability mass of the distribution $\tilde{h}_i$ to the left, so the fraction of mixed dyads with workers employed in low productivity sectors is disproportionately increased. In order to isolate this effect consider the simplified case $\alpha = 0$ corresponding to the dyadic labour market with strong ties. Without job information exchange the distribution of mixed dyads is equal to $\tilde{H}_k^* = \delta F_k/(\delta + \lambda(1 - F_k))$ which is a standard productivity distribution in a labour market with on-the-job search. With personal contacts and job information exchange the productivity distribution $\tilde{H}_k$ is first order stochastically dominated by the distribution $\tilde{H}^*_k$:

$$\tilde{H}_k = \frac{\delta F_k}{(\delta + \lambda(1-F_k))}\Big[1 + \frac{\lambda(1-F_k)}{2(\delta+\lambda)}\Big] + \frac{\lambda^2(1-F_k)\sum_{i=1}^{k-1}\tilde{h}_i(F_k - F_i)}{(\delta+\lambda(1-F_k))2(\delta+\lambda)} > \tilde{H}^*_k$$

Next denote $P(y_k)$ a cumulative density function of the equilibrium productivity distribution, which can be calculated as: $P(y_k) = E_k/e$, where $E_k = \sum_{i=1}^{k} e_i$ is the total measure of workers employed at a productivity below or equal to y_k. Category E_k includes (1) employed dyads where either of the two partners is employed at a productivity below or equal to y_k, (2) mixed dyads satisfying the same condition and (3) single monads with a corresponding measure Q_k:

$$E_k = G_{nk} + G_{kn} + 2H_k + Q_k$$

Table 2 shows that the new inflow of workers (attributed to personal contacts) into the employment state e_1 is equal to $\lambda\pi_1\sum_{i=1}^{n} h_i = \lambda\pi_1 H_n$, where $H_n = 0.5d_m$. This is explained by the consideration that *any* employed connected worker will forward a job offer y_1 to the unemployed dyad partner. This inflow is however falling from $\lambda\pi_1 H_n$ to $\lambda\pi_n(H_n - H_{n-1}) = \lambda\pi_n h_n$ with a higher productivity level. Clearly, only workers employed at a maximum productivity level y_n will forward a job offer in the same production sector to their contacts. These results are summarized in proposition 4.

Proposition 4: *Let $F_i = P\{y \leq y_i\}$, $i = 1,..,n$ denote a cumulative density function of the productivity distribution y_i, and let $\bar{F}$ denote an average probability: $\bar{F} = \sum_{i=1}^{n} F_i\tilde{h}_i$. If workers search on-the-job and forward job offers weakly below their current income to unemployed dyad partners, the equilibrium productivity distribution $P(y_k)$ is given by:*

$$P(y_k) \quad = \quad \frac{\delta F_k}{(\delta+\lambda(1-F_k))} + \frac{\lambda\delta\phi(\lambda+\alpha)[F_k(1-\bar{F}) - \sum_{i=1}^{k}(F_k - F_i)\tilde{h}_i]}{(\delta+\lambda(1-F_k))(s+\lambda\phi\bar{F}(\alpha+\lambda))}$$

where

$$F_k(1-\bar{F}) - \sum_{i=1}^{k}(F_k - F_i)\tilde{h}_i = F_k\sum_{i=k+1}^{n}(1-F_i)\tilde{h}_i + (1-F_k)\sum_{i=1}^{k}F_i\tilde{h}_i > 0$$

For $\forall\alpha \geq 0$ the productivity distribution $P(y_k)$ with network effects is first order stochastically dominated by the distribution $P^*(y_k) = \delta F_k/(\delta+\lambda(1-F_k))$ without network effects.

Proof: Appendix IV.

Proposition 4 implies that network effects also have an asymmetric effect on the equilibrium productivity distribution $p(y_k) = e_k/e$: the inflow of workers is stronger in the left tale of the distribution and it is weaker in the right tale. Figure 3 illustrates comparison between the equilibrium productivity distribution $p(y_k)$ (blue line) and the distribution $p^*(y_k)$ without network effects (black line). In addition, the distribution of mixed dyads $\tilde{h}_i$ is illustrated by the black dashed line.

In order to illustrate the effect of social networks I use the exponential distribution $f(y) = 2.5e^{-2.5y}$. So the discretized probability values π_i can be obtained as $\pi_k = f(y_k)/\sum_i f(y_i)$ with a minimum productivity value $y_1 = 0.5$ and a maximim productivity value $y_{20} = 1.45$.

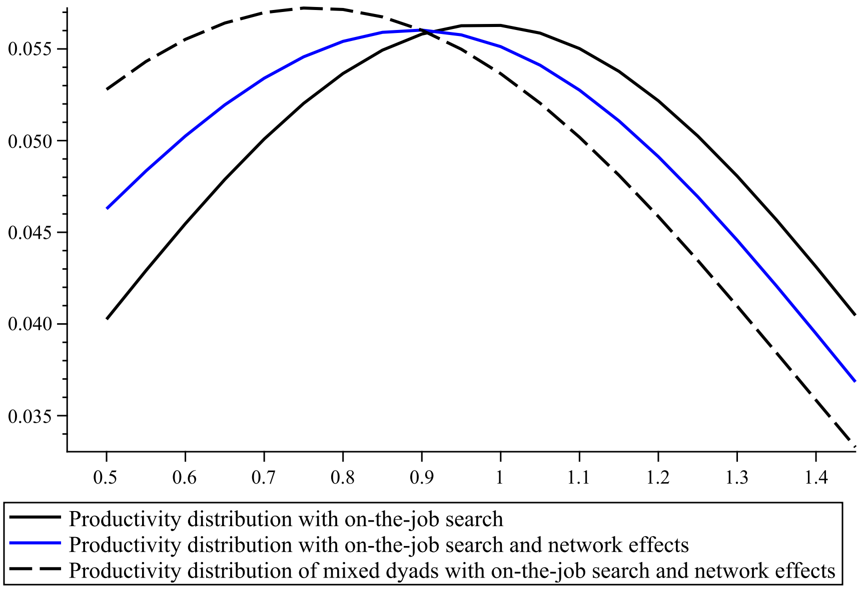

Figure 3: The initial productivity distribution is $f(y) = 2.5e^{-2.5y}$, $\pi_k = f(y_k)/\sum_i f(y_i)$. The parameters used are $\alpha = 0$, $\lambda = 0.5$, $\delta = 0.2$

Burdett and Mortensen (1998) prove that on-the-job search shifts the original productivity distribution towards more productive jobs, so the equilibrium productivity density function $p^*(y_k) = \delta(\delta+\lambda)\pi_k/[(\delta+\lambda(1-F_k))(\delta+\lambda(1-F_{k-1}))]$ is hump-shaped with a unique maximum on the distribution support. This is a joint impact of the downward-sloping exponential density π_k and the effect of on-the-job search. Further, referrals to unemployed workers have a counteracting effect on the productivity density function, so there is more (less) probability mass in the left (right) tail of the equilibrium distributions $\tilde{h}_k$ (dashed line) and $p(y_k)$ (blue line). This also implies that personal contacts have a negative effect on the average wage. Let w^c and w^n denote average wages in economies with and without personal contacts, respectively. These wages can be calculated as follows:

$$\begin{aligned} w^c - w^n &= \beta \sum_{i=1}^{n} y_i p(y_i) + (1-\beta)z - \beta \sum_{i=1}^{n} y_i p^*(y_i) - (1-\beta)z \\ &= \beta \sum_{i=1}^{n} (y_{i+1} - y_i)(P^*(y_i) - P(y_i)) < 0 \end{aligned}$$

which proves that the network effect on wages is negative when connected workers search on-the-job and forward job offers below their current income to unemployed dyad partners.

4.3 On-the-job search with referrals to employees

This section extends the model to the case when employees forward job offers to their employed social contacts. Therefore the strategy of the employee is to accept every offer above the current wage and to forward every other offer to the dyad partner. Thus this strategy is independent of the employment status of the partner. As a result employees in the economy are more likely to change their jobs climbing up the wage ladder. Formally, consider the stock of employed dyads G_{lk}, $l > k$. With a probability $F_i - F_k$ workers employed in jobs with a productivity y_i, $l \geq i > k$ can help their contacts find a better paid job. For every i the total stock of these dyads is $g_{ik} = G_{ik} - G_{i-1k}$. Hence there is an additional outflow of dyads from the stock G_{lk}, $l > k$ which is given by $\sum_{i=k+1}^{l} g_{ik}(F_i - F_k)$. This means:

$$\begin{aligned} \dot{G}_{lk} &= \lambda F_l H_k + \lambda F_k H_l - 2\delta G_{lk} - \lambda(1-F_k)G_{lk} - \lambda(1-F_l)G_{lk} + \\ &+ \underbrace{2\lambda \sum_{i=1}^{k} F_i h_i + \lambda F_k(H_l - H_k)}_{\text{help unemployed}} - \underbrace{\lambda \sum_{i=k+1}^{l} g_{ik}(F_i - F_k)}_{\text{help employed}} \quad \text{if} \quad l > k \end{aligned} \tag{4.3}$$

The equilibrium productivity distribution $P(y_k)$ for the case when on-the-job search is combined with referrals is characterised in proposition 5.

Proposition 5: *Let G_{lk} denote a cumulative density function of employed dyads, so that $g_{lk} = G_{lk} - G_{l-1k}$ if $l > k$. Moreover, let $\tilde{G}_{lk} = G_{lk}/0.5d_m$ and $\tilde{g}_{lk} = g_{lk}/0.5d_m$. If workers search on-the-job and forward job offers to their dyad partners (employed or unemployed) then the cumulative density function of the equilibrium productivity distribution of mixed dyads $\tilde{H}_k$ is given by the following system of linear equations*:

$$\tilde{H}_k = \frac{\delta F_k(2(\delta+\lambda) + \lambda(1-F_k))}{(\delta+\lambda(1-F_k))2(\delta+\lambda)} + \frac{\lambda[\lambda(1-F_k)\sum_{i=1}^{k-1}\tilde{h}_i(F_k - F_i) - \delta\sum_{i=k+1}^{n}\tilde{g}_{ik}(F_i - F_k)]}{(\delta+\lambda(1-F_k))2(\delta+\lambda)}$$

$$\text{where} \quad \tilde{g}_{ik} = \frac{\lambda f_i \tilde{H}_k + 2\lambda F_k \tilde{h}_i + \lambda f_i \tilde{G}_{i-1k}}{2(\delta+\lambda(1-F_k))} \quad i > k \quad \text{and} \quad \tilde{G}_{kk} = \frac{\lambda F_k \tilde{H}_k + \lambda\sum_{i=1}^{k} F_i \tilde{h}_i}{\delta+\lambda(1-F_k)}$$

the equilibrium productivity distribution $P(y_k)$ is given by:

$$P(y_k) = \frac{\delta F_k}{(\delta+\lambda(1-F_k))} + \frac{\lambda\delta[F_k(1-\bar{F}) - \sum_{i=1}^{k}(F_k - F_i)\tilde{h}_i - \sum_{i=k+1}^{n}\tilde{g}_{ik}(F_i - F_k)]}{(\delta+\lambda(1-F_k))(\delta+\lambda(1+\bar{F}))}$$

Proposition 5 shows that there is a negative and a positive effect of referrals on the equilibrium productivity distribution $p(y_k)$. This is illustrated on figure 4. First, employees help their unemployed social contacts find a job. This effect is captured by the term $F_k(1-\bar{F}) - \sum_{i=1}^{k}(F_k - F_i)\tilde{h}_i > 0$. As a result the probability mass of the distribution $p(y_k)$ shifts towards low productivity jobs (solid blue line). Second, employees in more productive jobs (and higher wages) continue helping their social contacts climb the wage ladder. In particular, they forward jobs in the middle range of the distribution to their contacts employed in low paid jobs. This effect is captured by the term $\sum_{i=k+1}^{n}\tilde{g}_{ik}(F_i - F_k)$ and shifts the probability mass of the distribution $p(y_k)$ towards the center (dashed blue line). Overall, figure 4 illustrates that the second shift is dominating and there is less probability mass in the left tail of the equilibrium distribution.

At the same time, note that most productive jobs are rearely transferred through social contacts. This means that a large growth in the overall employment is dominating and the relative proportion of workers employed in most productive jobs is reduced: $P(y_k) = E_k/e$. As a result there is less probability mass in the right tail of the equilibrium distribution. In summary, the final effect of referrals is a larger probability mass in the middle range of the distribution $p(y_k)$, this means that referrals intensify the process of on-the-job search so the variance of the distribution is reduced, but the effect on the average productivity is ambiguous.

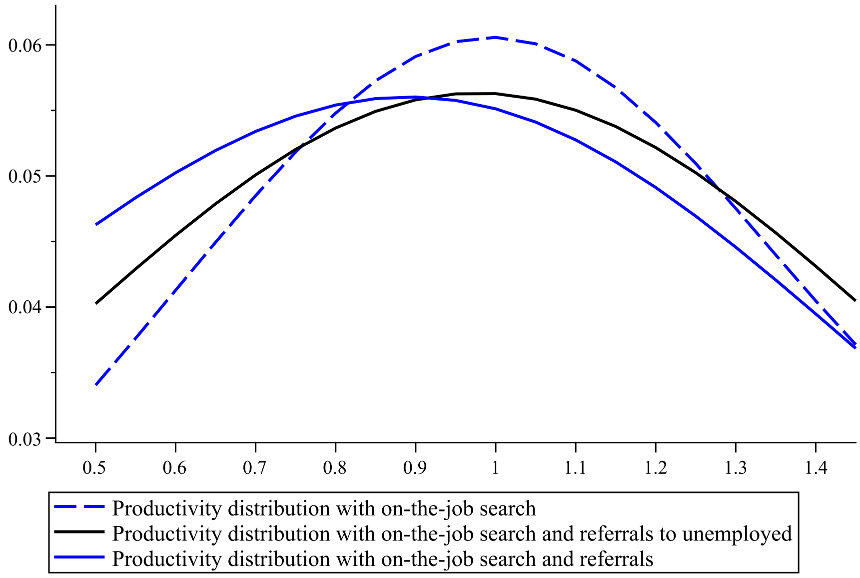

Figure 4: The initial productivity distribution is $\tilde{f}(y_i) = 2.5e^{-2.5y_i}$, $f(y_i) = \tilde{f}(y_i)/\sum_i \tilde{f}(y_i)$. The parameters used are $\alpha = 0$, $\lambda = 0.5$, $\delta = 0.2$

At this point it should be noted that the above result is somewhat in contrast with the traditional view. For example, Montgomery (1991) shows that a higher network density generates greater dispersion of the equilibrium earnings distribution. However, the labour market environment analysed by Montgomery (1991) is different from the present study. First, workers in his study are heterogeneous by ability and there is a problem of asymmetric information. Second, the possibility of on-the-job search and firm heterogeneity are not considered. Consequently, an increase in the network density generates more employee referrals, removing relatively more high-ability workers from the market. As a result, the lemons effect is exacerbated and the market wage falls. In this respect, it is a challenging task for future research to analyse the effect of referrals on wage inequality in a labour market with heterogeneous workers and firms, asymmetric information and on-the-job search.

4.4 Asset values and reservation wages

This section investigates the effect of personal contacts on reservation wages of unemployed workers for the extended case $n > 2$. In addition, the focus of this section is on the second information regime, when employees help their contacts find a (better paid) job. Specifically, this section shows that reservation wages of both types of workers (with high and low social capital) are below the unemployment benefit z, so that no job is rejected by unemployed workers. This is an extesnion of the result by Burdett and Mortensen (1998) who show that in the absence of referrals the reservation wage should be exactly equal to the unemployment benefit if on-the-job search is equally efficient as off-the-job search. Efficiency here is measured in terms of the job-finding rate λ. The situation is different with referrals since workers are more likely to form social contacts when employed and they are more likely to lose their contacts when unemployed.

Due to the problem of unobserved information connected workers have to form probabilistic beliefs about the flow income of their dyad partners. For this reason let $\tilde{p}_j$ denote a probability density function of the productivity distribution considering only connected workers: $\tilde{p}_j = \tilde{P}_j - \tilde{P}_{j-1}$. The cumulative density function $\tilde{P}_j$ can be obtained as: $\tilde{P}_j = (G_{nj}+G_{jn}+2H_j)/(d_m+2d_e)$. In order to simplify the analysis I assume that $\tilde{p}_j$ is a belief distribution for both employed and unemployed dyad partners and there is no learning by means of the information exchange within a dyad. This assumption is clearly restrictive but it allows me to preserve the tractability of the model. Bellman equations for unemployed workers are then given by:

$$
\begin{aligned}
rU &= z + \lambda \sum_{i=1}^{n} (V_i - U)\pi_i \\
r\widetilde{U} &= z + \lambda \sum_{i=1}^{n} (\widetilde{V}_i - \widetilde{U})\pi_i + \underbrace{\lambda(1-\mu)\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{U})\pi_i \tilde{p}_j}_{\text{network effects}} - 2\mu\alpha(\widetilde{U} - U)
\end{aligned}
$$

Suppose the dyad partner of the unemployed worker is employed at productivity y_j. Continuing to search on-the-job this partner will accept job offers with a productivity level above y_j and will forward all jobs weakly below y_j to the connected worker. This means that the attachment value gain of the worker is given by $\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{U})\pi_i$. Neverthless, the productivity variable y_j is not observed by the worker, this gives rise to the expected value gain $\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{U})\pi_i \tilde{p}_j$, reflecting the fact that the dyad partner can be employed at any productivity $y_1, ..., y_n$ with the corresponding probabilities $\tilde{p}_1, ..., \tilde{p}_n$. Similarly, Bellman equations for employed workers are:

$$
\begin{aligned}
rV_k &= w_k + \lambda \sum_{i=k+1}^{n} (V_i - V_k)\pi_i + 2\phi(\widetilde{V}_k - V_k) - \delta(V_k - U) \\
r\widetilde{V}_k &= w_k + \lambda \sum_{i=k+1}^{n} (\widetilde{V}_i - \widetilde{V}_k)\pi_i + \underbrace{\lambda(1-\mu)\sum_{j=k+1}^{n}\sum_{i=k+1}^{j}(\widetilde{V}_i - \widetilde{V}_k)\pi_i \tilde{p}_j}_{\text{network effects}} - \delta(\widetilde{V}_k - \widetilde{U})
\end{aligned}
$$

Consider both types of unemployed workers obtaining a job offer y_1. The net value gains from accepting this job are $R_1 = V_1 - U$ and $\widetilde{R}_1 = \widetilde{V}_1 - \widetilde{U}$ for unemployed workers with low and high social capital, respectively. These value variables can be written as:

$$
\begin{aligned}
(r + \delta + \lambda + 2\phi)R_1 &= w_1 - z + 2\phi(\widetilde{R}_1 + \Delta U) \\
(r + \delta + \lambda(1 + (1-\mu)\widetilde{F}))\widetilde{R}_1 &= w_1 - z + 2\mu\alpha\Delta U \quad \text{where} \quad \widetilde{F} = \sum_{j=1}^{n} F_j \tilde{p}_j
\end{aligned}
$$

Similar to the binary case these equations show that the reservation wages of unemployed workers are below z if $\Delta U > 0$ which is the endogenous value of social capital. Intuitively, unemployed workers with low social capital accept low paid jobs to gain the chance to form social contacts in a working environment, at the same time connected unemployed workers accept these jobs in order to keep the existing contacts. Proposition 6 provides a formal proof of these results.

Proposition 6: *Let $F_i = P\{y \leq y_i\}$, $i = 1, .., n$ denote a cumulative density function of the productivity distribution y_i with a corresponding wage $w_i = \beta y_i + (1-\beta)z$ such that $y_1 \geq z$ where z denotes the unemployment benefit. If connected workers search on-the-job and exchange wage* *offers below their current income, then reservation wages of unemployed workers with high/low social capital are below the unemployment benefit z.* **Proof:** Appendix V.

5 Conclusions

Empirical studies report that referrals can lead to premiums or penalties in wages. Therefore, this paper analyses the role of economic conditions in the labour market in generating a positive or a negative effect of referrals on wages. To address this question I develop a labour market matching model with referrals, heterogeneous firms and on-the-job search. Moreover, social capital is endogenous in the model. When employed, workers can form dyadic ties with their colleagues, however there is a positive probability of dissolving for the tie if both workers are unemployed in the future. Therefore, the total number of dyads is endogenous in the model and the equilibrium unemployment is decreasing in the intensity of contact formation. At the same time, unemployed workers are endogenously differentiated by social capital, since connected workers have better outside opportunities than unemployed workers without contacts. In this setup bargaining over wages leads to endogenous binary wage dispersion in the model, so that connected workers earn higher wages for a given level of productivity (reservation wage effect).

The major contribution of this paper is to show that on-the-job search with referrals leads to a negative selection of referred workers to low productivity jobs. Note that this result obtains in a setup when employees help their unemployed contacts find a job (first information regime). Intuitively, workers accept job offers from firms more productive than their current employer, at the same time job offers from less productive employers are not lost, instead workers forward these jobs to their unemployed acquaintances. This means that the productivity distribution of transferred offers is truncated from above. Therefore, the average productivity of a referred worker is lower than the average productivity in the market (selection effect).

Next, this paper investigates the interaction between the positive reservation wage effect and the negative productivity effect. First, wage premiums are small and the negative effect of referrals is dominating in a labour market with high productivity heterogeneity and strong worker's bargaining power. The latter result is explained by the fact that wages are more sensitive to productivities and less sensitive to reservation wages if the bargaining power of workers is relatively low. This could explain the existing empirical evidence for Italy and Portugal (see Pistaferri (1999) and Addison and Portugal (2002)), where the labour market is characterized by wage penalties associated with referrals. On the contrary, wage penalties are small and the positive effect of referrals is dominating in a labour market with low productivity heterogeneity and weak worker's bargaining power. So the model is also compatible with the empirical evidence for Austria, Belgium and the Netherlands where informal search channels lead to significantly better paying jobs (see Pelizzari (2010)).

Finally, this study demonstrates that social networks may reduce inequality in a homogeneous group of workers. On-the-job search, when workers help each other climb the wage ladder, is a crucial component for this result. On the one hand, workers employed in low productivity jobs pool their dyad partners towards better paid jobs in the middle range of the productivity distribution. This means that low productivity jobs are less stable and there are less of them in the stationary equilibrium. On the other hand, high productivity jobs are rearely transmitted through social contacts, therefore, the relative fraction of workers in this group is reduced given the total growth in employment. As a result in the equilibrium there is more probability mass in the middle range of the productivity distribution, so the variance of the distribution is reduced.

6 Appendix

Appendix I: Proof of lemmas 1-2. From differential equations $\dot{e}_0 = 0$, $\dot{u}_0 = 0$, $\dot{d}_e = 0$ and $\dot{d}_u = 0$ express variables e_0, u_0, d_e and d_m in terms of d_u:

$$\begin{aligned} \delta d_m &= 2(\alpha+\lambda)d_u \qquad \lambda\phi u_0 = 2\alpha d_u(2\phi+\delta) \qquad \phi e_0 = 2\alpha d_u \\ 2\delta d_e &= \lambda(1+\pi)d_m + \phi e_0 = \lambda(1+\pi)\frac{2(\alpha+\lambda)d_u}{\delta} + 2\alpha d_u \\ \delta^2 d_e &= [\lambda(1+\pi)(\alpha+\lambda)+\alpha\delta]d_u \end{aligned}$$

The total labour force is normalized to 1, so that $1 = 2(d_u + d_e + d_m) + e_0 + u_0$, this yields:

$$1 = 2d_u\left[1 + \frac{\lambda(1+\pi)(\alpha+\lambda)+\alpha\delta}{\delta^2} + \frac{2(\alpha+\lambda)}{\delta}\right] + 2\alpha d_u\left[\frac{2\phi+\delta}{\lambda\phi} + \frac{1}{\phi}\right]$$

This means the equilibrium measure of unemployed dyads d_u can be obtained as:

$$d_u = \frac{0.5\delta^2\lambda\phi}{(\lambda+\delta)[(\lambda+\delta+\alpha)\lambda\phi + \alpha\delta(2\phi+\delta)] + \lambda^2\pi\phi(\alpha+\lambda)} = \frac{0.5\delta^2\phi\lambda}{(\lambda+\delta)s + \lambda^2\pi\phi(\alpha+\lambda)}$$

The differential equations for $\dot{d}_m^1$, $\dot{d}_e^1$, $\dot{d}_e^c$ and $\dot{e}_0^1$ are given by:

$$\begin{aligned} \dot{d}_m^1 &= 2\lambda\pi d_u - \delta d_m^1 - \lambda(1-\pi)d_m^1 - \lambda(1-\pi)d_m^1 - 2\lambda\pi d_m^1 + 2\delta d_e^1 + \delta d_e^c \\ \dot{d}_e^1 &= 2\lambda\pi d_m^1 - 2\lambda(1-\pi)d_e^1 + \phi e_0^1 - 2\delta d_e^1 \\ \dot{d}_e^c &= 2\lambda\pi d_m^2 + \lambda(1-\pi)d_m^1 + 2\lambda(1-\pi)d_e^1 - \lambda(1-\pi)d_e^c - 2\delta d_e^c \\ \dot{e}_0^1 &= \lambda\pi u_0 - \delta e_0^1 - \lambda(1-\pi)e_0^1 - 2\phi e_0^1 \end{aligned}$$

This allows to find the equilibrium variables e_0^1 and e_0^2:

$$\begin{aligned} e_0^1 &= \frac{\lambda\pi u_0}{\delta+2\phi+\lambda(1-\pi)} = \frac{\alpha\delta^2(2\phi+\delta)\lambda\pi}{(\delta+2\phi+\lambda(1-\pi))((\lambda+\delta)s+\lambda^2\pi\phi(\alpha+\lambda))} \\ e_0^2 &= e_0 - e_0^1 = \frac{\alpha\delta^2\lambda}{(\lambda+\delta)s+\lambda^2\pi\phi(\alpha+\lambda)}\left[1 - \frac{\pi(2\phi+\delta)}{2\phi+\delta+\lambda(1-\pi)}\right] \end{aligned}$$

Variables p_1 and p_2 are then obtained as: $p_1 = e_0^1/(1-u)$ and $p_2 = e_0^2/(1-u)$. In addition, in the steady state it should be true that $0 = 2\dot{d}_e^1 + \dot{d}_e^c$:

$$\begin{aligned} 0 &= 2[2\lambda\pi d_m^1 - 2\lambda(1-\pi)d_e^1 + \phi e_0^1 - 2\delta d_e^1] + 2\lambda\pi d_m^2 \\ &+ \lambda(1-\pi)d_m^1 + 2\lambda(1-\pi)d_e^1 - \lambda(1-\pi)d_e^c - 2\delta d_e^c \\ &= 2\lambda\pi d_m^1 + 2\lambda\pi d_m - 2\lambda(1-\pi)d_e^1 + \phi e_0^1 + \lambda(1-\pi)d_m^1 - 2\delta(d_e^c + 2\delta_e^1) - \lambda(1-\pi)d_e^c \end{aligned}$$

So that the total measure of workers employed at wage w_1 in employed dyads ($d_e^c + 2d_e^1$) becomes:

$$d_e^c + 2d_e^1 = \frac{\lambda(1+\pi)d_m^1 + 2\lambda\pi d_m + 2\phi e_0^1}{2\delta+\lambda(1-\pi)} \qquad d_m^1 = \frac{2\lambda\pi d_u(2\delta+2(\alpha+\lambda)+\lambda(1-\pi)) + 2\phi\delta e_0^1}{2(\delta+\lambda)(\delta+\lambda(1-\pi))}$$

where the last equation for d_m^1 follows from $\dot{d}_m^1 = 0$ which means $(\delta + 2\lambda)d_m^1 = 2\lambda\pi d_u + \delta(d_e^c + 2d_e^1)$. The equilibrium productivity distribution is then given by $p(y_1) = \tilde{p}_1 + p_1 = (2d_e^1 + d_e^c + d_m^1 + e_0^1)/e$ and $p(y_2) = \tilde{p}_2 + p_2 = 1 - p(y_1)$:

$$2d_e^1 + d_e^c + d_m^1 + e_0^1 = \frac{\lambda\pi d_m(\delta + 2(\alpha+\lambda))}{(\alpha+\lambda)(\delta+\lambda(1-\pi))} + \frac{e_0^1(\delta + 2\phi + \lambda(1-\pi))}{\delta + \lambda(1-\pi)}$$

$$\begin{aligned} p(y_1) &= \frac{\lambda\pi(\delta + 2(\alpha+\lambda))}{(\alpha+\lambda)(\delta+\lambda(1-\pi))} \cdot \frac{\delta(\alpha+\lambda)\phi}{s + \lambda\pi\phi(\alpha+\lambda)} + \frac{\pi\alpha\delta^2(2\phi+\delta)}{(\delta+\lambda(1-\pi))(s+\lambda\pi\phi(\alpha+\lambda))} \\ &= \frac{\pi\delta}{\delta+\lambda(1-\pi)}\left[\frac{s+\lambda\phi(\alpha+\lambda)}{s+\lambda\pi\phi(\alpha+\lambda)}\right] = \frac{\pi\delta}{\delta+\lambda(1-\pi)}\left[1 + \frac{\lambda(1-\pi)\phi(\alpha+\lambda)}{s+\lambda\pi\phi(\alpha+\lambda)}\right] \end{aligned}$$

Appendix II: Proof of proposition 1.

Define the total surplus values $S_i = R_i + J_i$ and $\widetilde{S}_i = \widetilde{R}_i + \widetilde{J}_i$, $i = 1, 2$ so that:

$$\Delta S \equiv \widetilde{S}_1 - S_1 = \widetilde{S}_2 - S_2 = \frac{2\Delta U(\mu\alpha - \phi) - \lambda\pi\beta(1-\mu)\widetilde{S}_1}{\delta + 2\phi + \lambda(1-\pi) + \lambda\pi\beta}$$

where variable ΔU can be obtained from equation:

$$\begin{aligned} 2\mu\alpha\Delta U &= \lambda\pi((2-\mu)\widetilde{R}_1 - R_1) + \lambda(1-\pi)(\widetilde{R}_2 - R_2) \\ &= \lambda\pi\beta(\Delta S + (1-\mu)\widetilde{S}_1) + \lambda(1-\pi)\beta\Delta S = \lambda\beta\Delta S + \lambda\pi\beta(1-\mu)\widetilde{S}_1 \end{aligned}$$

Let $\sigma = \mu\alpha(\delta + 2\phi + \lambda(1-\pi)(1-\beta)) + \phi\lambda\beta$, then

$$\begin{aligned} \sigma 2\Delta U &= \lambda\pi\beta(1-\mu)(\delta + 2\phi + \lambda(1-\pi)(1-\beta))\widetilde{S}_1 \\ 2\Delta U &= \frac{\lambda\pi\beta(1-\mu)(\delta + 2\phi + \lambda(1-\pi)(1-\beta))}{\sigma(\delta + \lambda(1-\pi) + \lambda\pi\beta) + \lambda\pi\beta(1-\mu)\phi\lambda\beta}(y_1 - z) \end{aligned}$$

Further, define a new auxilliary variable $\rho = \delta + 2\phi + \lambda(1-\pi)$, then

$$\Delta S + (1-\mu)\widetilde{S}_1 = \frac{2\Delta U(\mu\alpha - \phi) + (1-\mu)\rho\widetilde{S}_1}{\rho + \lambda\pi\beta}$$

so that the wage difference Δw can be expressed as:

$$\begin{aligned} \frac{\Delta w}{1-\beta} &= \lambda\pi\beta(\Delta S + (1-\mu)\widetilde{S}_1) - 2\Delta U(\mu\alpha - \phi) = \frac{\rho}{\rho + \lambda\pi\beta}[2\Delta U(\phi - \mu\alpha) + \lambda\pi\beta(1-\mu)\widetilde{S}_1] \\ &= \frac{\rho(1-\mu)}{(\rho + \lambda\pi\beta)\sigma}[(\rho - \lambda\beta(1-\pi))(\phi - \alpha\mu) + \sigma]\lambda\pi\beta\widetilde{S}_1 \\ &= \frac{\rho\phi 2\Delta U}{(\rho - \lambda\beta(1-\pi))} = \frac{(\delta + 2\phi + \lambda(1-\pi))\phi}{\delta + 2\phi + \lambda(1-\pi)(1-\beta)}2\Delta U \end{aligned}$$

The sign of $\partial\Delta w/\partial\alpha$ is determined by the sign of $\partial\Delta U/\partial\alpha$ and therefore depends on the change in the ratio $\sigma/(1-\mu)$, which can be obtained as:

$$\begin{aligned} \frac{\sigma}{1-\mu} &= \frac{\delta}{\alpha+\lambda}[\alpha(\rho - \lambda\beta(1-\pi)) + \phi\lambda\beta] + \phi\lambda\beta \\ &= \delta(\rho - \lambda\beta(1-\pi)) + \phi\lambda\beta - \frac{\delta\lambda}{\alpha+\lambda}[\delta + \phi(2-\beta) + \lambda(1-\pi)(1-\beta)] \end{aligned}$$

so that $\sigma/(1-\mu)$ is increasing in α, this means $\partial\Delta U/\partial\alpha < 0$ and $\partial\Delta w/\partial\alpha < 0$. Further, define variable d in the following way: $d = \phi\lambda\beta/[\delta + 2\phi + \lambda(1-\pi)(1-\beta)]$, this means $\partial d/\partial\phi > 0$. Variables ΔU and

Δw can then be expressed in terms of d:

$$
\begin{aligned}
2\Delta U &= \frac{\lambda\pi\beta(1-\mu)(y_1-z)}{(\mu\alpha+d)(\delta+\lambda(1-\pi)+\lambda\pi\beta)+\lambda\pi\beta(1-\mu)d} &\Rightarrow& \quad \frac{\partial\Delta U}{\partial d}<0 \\
\Delta w &= \frac{\pi\rho d(1-\mu)(y_1-z)}{(\mu\alpha+d)(\delta+\lambda(1-\pi)+\lambda\pi\beta)+\lambda\pi\beta(1-\mu)d} &\Rightarrow& \quad \frac{\partial\Delta w}{\partial d}>0
\end{aligned}
$$

Therefore $\partial\Delta U/\partial\phi<0$ but $\partial\Delta w/\partial\phi>0$.

Appendix III: Proof of propositions 2-3. Define an auxilliary function $\Delta W(\beta)=\Delta w/\beta$:

$$
\Delta W(\beta)=\frac{\Delta w}{\beta}=\frac{\lambda\pi(1-\beta)(1-\mu)(\delta+2\phi+\lambda(1-\pi))\phi}{\sigma(\delta+\lambda(1-\pi)+\lambda\pi\beta)+\lambda\pi\beta(1-\mu)\phi\lambda\beta}(y_1-z) \tag{6.4}
$$

The ratio $\sigma/(1-\beta)$ is an increasing function of β, so that $\Delta W'(\beta)<0$ for $\beta<1$.

$$
\begin{aligned}
\tilde{w}_1-\bar{w} &< 0 \quad \forall \quad 0<\beta<1 \quad \Leftrightarrow \\
(p_1+p_2)\Delta W(\beta) &< \Delta y(p_2+\tilde{p}_2) \quad \forall \quad 0<\beta<1 \quad \Leftrightarrow \\
(p_1+p_2)\Delta W(0) &< \Delta y(p_2+\tilde{p}_2) \Leftrightarrow \\
\frac{\lambda\pi(1-\mu)\phi}{(\delta+\lambda(1-\pi))}\frac{\alpha\delta^2(y_1-z)}{s+\lambda\pi\phi(\alpha+\lambda)} &< \frac{(1-\pi)\Delta y}{\delta+\lambda(1-\pi)}\left[\lambda+\frac{\delta s}{s+\lambda\pi\phi(\alpha+\lambda)}\right]
\end{aligned}
$$

This gives rise to the following inequality:

$$
h(\pi)\equiv\frac{\lambda\pi(1-\mu)\phi\alpha\delta^2}{(1-\pi)(s(\lambda+\delta)+\lambda^2\pi\phi(\alpha+\lambda))} \quad < \quad \frac{y_2-y_1}{y_1-z}
$$

Function $h(\pi)$ is such that $h(0)=0$, $h'(\pi)>0$ and it converges to infinity for $\pi\to1$, this means that the last inequality defines a unique value of $0<\pi^*<1$. In addition $(1-\mu)\delta=(\alpha+\lambda)\mu$.

Appendix IV: Proof of propositions 3-4. The distibution H_k can be expressed from equation (4.1):

$$
H_k(\delta+\lambda+\lambda(1-F_k)) = \lambda F_k d_u-\lambda\sum_{i=1}^{k}F_ih_i+\frac{\delta\lambda(1-F_k)H_k}{2\delta+\lambda(1-F_k)}+\delta\frac{\lambda F_kd_m+\phi Q_k+2\lambda\sum_{i=1}^{k}F_ih_i}{2\delta+\lambda(1-F_k)}
$$

$$
\begin{aligned}
& H_k[(\delta+\lambda+\lambda(1-F_k))(2\delta+\lambda(1-F_k))-\delta\lambda(1-F_k)] \\
=\; & (\lambda F_kd_u-\lambda\sum_{i=1}^{k}F_ih_i)(2\delta+\lambda(1-F_k))+\delta\lambda F_kd_m+\delta\phi Q_k+2\lambda\delta\sum_{i=1}^{k}F_ih_i \\
=\; & \lambda F_k[d_u(2\delta+\lambda(1-F_k))+\delta d_m]+\delta\phi Q_k-\lambda^2(1-F_k)\sum_{i=1}^{k}F_ih_i \\
& H_k[(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))+\lambda(\delta+\lambda(1-F_k))+\lambda F_k(\delta+\lambda(1-F_k)] \\
=\; & \lambda F_k[(2\delta+\lambda(1-F_k))+2(\alpha+\lambda)]d_u+\delta 0.5d_m\tilde{Q}_k+\lambda^2(1-F_k)0.5d_m\sum_{i=1}^{k}(F_k-F_i)\tilde{h}_i \\
& \tilde{H}_k(\delta+\lambda(1-F_k))2(\delta+\lambda)=\lambda F_k[2(\delta+\lambda)+2\alpha+\lambda(1-F_k)]\frac{\delta}{\alpha+\lambda} \\
+\; & \delta\tilde{Q}_k+\lambda^2(1-F_k)\sum_{i=1}^{k-1}(F_k-F_i)\tilde{h}_i \quad \text{where} \quad \tilde{H}_i=H_i/0.5d_m \quad \tilde{h}_i=h_i/0.5d_m
\end{aligned}
$$

Further, H_k^* can be obtained from equation (4.1) without network effects:

$$
\begin{aligned}
H_k^*(\delta+\lambda+\lambda(1-F_k)) &= \lambda F_k d_u^* + \delta\frac{\lambda H_k^* + \lambda F_k 0.5 d_m^* + \phi Q_k^*}{2\delta+\lambda(1-F_k)} \\
\tilde{H}_k^*(\delta+\lambda(1-F_k))(2\delta+\lambda+\lambda(1-F_k)) &= \lambda F_k\frac{\delta}{\alpha+\lambda}(2\delta+\lambda(1-F_k)+\alpha+\lambda)+\delta\tilde{Q}_k^* \\
\text{where}\quad \tilde{Q}_k^* = \frac{\phi Q_k^*}{0.5 d_m^*} &= \frac{2\alpha\delta F_k(\delta+2\phi)}{(\alpha+\lambda)(\delta+2\phi+\lambda(1-F_k))} = \tilde{Q}_k
\end{aligned}
$$

$$
\begin{aligned}
\tilde{H}_k^* - \tilde{H}_k &= \frac{\lambda\delta F_k}{(\alpha+\lambda)(\delta+\lambda(1-F_k))}\Big[\frac{\alpha}{2\delta+\lambda+\lambda(1-F_k)} - \frac{2\alpha+\lambda(1-F_k)}{2(\delta+\lambda)}\Big] \\
&+ \frac{\delta\tilde{Q}_k}{\delta+\lambda(1-F_k)}\Big[\frac{1}{2\delta+\lambda+\lambda(1-F_k)} - \frac{1}{2(\delta+\lambda)}\Big] - \frac{\lambda^2(1-F_k)\sum_{i=1}^{k-1}\tilde{h}_i(F_k-F_i)}{(\delta+\lambda(1-F_k))2(\delta+\lambda)} \\
&= -\frac{\lambda\delta F_k[(\delta+\lambda(1-F_k))(2\alpha+\lambda(1-F_k))+(\delta+\lambda)\lambda(1-F_k)]}{(\alpha+\lambda)(\delta+\lambda(1-F_k))(2\delta+\lambda+\lambda(1-F_k))2(\delta+\lambda)} \\
&+ \frac{\delta\lambda F_k\tilde{Q}_k - (2\delta+\lambda+\lambda(1-F_k))\lambda^2(1-F_k)\sum_{i=1}^{k-1}\tilde{h}_i(F_k-F_i)}{(\delta+\lambda(1-F_k))(2\delta+\lambda+\lambda(1-F_k))2(\delta+\lambda)}
\end{aligned}
$$

To complete the proof of proposition 3, it should be shown that this expression is negative:

$$
\begin{aligned}
&-(\delta+\lambda(1-F_k))2\alpha+(\alpha+\lambda)\tilde{Q}_k = -(\delta+\lambda(1-F_k))2\alpha+\frac{2\alpha\delta F_k(\delta+2\phi)}{\delta+2\phi+\lambda(1-F_k)} \\
= \; &-2\alpha(1-F_k)(\lambda+\delta) - \frac{2\alpha\delta F_k\lambda(1-F_k)}{\delta+2\phi+\lambda(1-F_k)} < 0 \quad\Rightarrow\quad \tilde{H}_k^* < \tilde{H}_k
\end{aligned}
$$

$$
\begin{aligned}
E_k &= \frac{2\lambda H_k(1-F_k)+2\lambda F_k d_m + 2\phi Q_k + 4\lambda\sum_{i=1}^{k}F_i h_i]}{2\delta+\lambda(1-F_k)} + 2H_k + Q_k \\
&= \frac{2[H_k(2\delta+2\lambda(1-F_k))+\lambda F_k d_m + 2\lambda\sum_{i=1}^{k}F_i h_i]}{2\delta+\lambda(1-F_k)} + \frac{(2(\phi+\delta)+\lambda(1-F_k))}{2\delta+\lambda(1-F_k)}Q_k \\
&= \frac{\lambda\delta F_k[2(\delta+\alpha+\lambda)+\lambda(1-F_k)]d_m}{(\alpha+\lambda)(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} + \frac{\delta\tilde{Q}_k+\lambda^2(1-F_k)\sum_{i=1}^{k-1}\tilde{h}_i(F_k-F_i)}{(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))}d_m \\
&+ \frac{[2\lambda F_k + 2\lambda\sum_{i=1}^{k}(F_i-F_k)\tilde{h}_i]d_m}{2\delta+\lambda(1-F_k)} + \frac{(2(\phi+\delta)+\lambda(1-F_k))}{2\delta+\lambda(1-F_k)}Q_k \\
&= \lambda F_k d_m\frac{\delta(\delta+\lambda(1-F_k))+\delta(\delta+2(\lambda+\alpha))+2(\alpha+\lambda)(\delta+\lambda(1-F_k))}{(\alpha+\lambda)(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} \\
&+ \lambda d_m\frac{[\lambda(1-F_k)-2\delta-2\lambda(1-F_k)]\sum_{i=1}^{k}(F_k-F_i)\tilde{h}_i}{(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} \\
&+ \frac{2\phi\delta Q_k}{(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} + \frac{(2(\phi+\delta)+\lambda(1-F_k))}{2\delta+\lambda(1-F_k)}Q_k \\
&= \lambda d_m\frac{[F_k(\delta+2(\lambda+\alpha))-(\alpha+\lambda)\sum_{i=1}^{k}(F_k-F_i)\tilde{h}_i]}{(\alpha+\lambda)(\delta+\lambda(1-F_k))} + \frac{2\phi+\delta+\lambda(1-F_k)}{\delta+\lambda(1-F_k)}Q_k \\
&= \lambda d_m\frac{[F_k(\delta+2(\lambda+\alpha))-(\alpha+\lambda)\sum_{i=1}^{k}(F_k-F_i)\tilde{h}_i]}{(\alpha+\lambda)(\delta+\lambda(1-F_k))} + \frac{2\alpha d_u F_k(\delta+2\phi)}{\phi(\delta+\lambda(1-F_k))}
\end{aligned}
$$

This means that with network effects the cumulative density function $P(y_k) = E_k/e$ is given by:

$$\begin{aligned}
\frac{E_k}{e} &= \frac{\lambda[F_k(\delta+2(\lambda+\alpha)) - (\alpha+\lambda)\sum_{i=1}^k (F_k - F_i)\tilde{h}_i]}{(\alpha+\lambda)(\delta+\lambda(1-F_k))} \cdot \frac{\delta\lambda(\lambda+\alpha)\phi}{\lambda(s+\lambda\phi\bar{F}(\alpha+\lambda))} \\
&+ \frac{2\alpha F_k(\delta+2\phi)}{\phi(\delta+\lambda(1-F_k))} \cdot \frac{0.5\delta^2\lambda\phi}{\lambda(s+\lambda\phi\bar{F}(\alpha+\lambda))} \\
&= \frac{\lambda\delta\phi[F_k(\delta+2(\lambda+\alpha)) - (\alpha+\lambda)\sum_{i=1}^k (F_k - F_i)\tilde{h}_i] + \alpha\delta^2 F_k(\delta+2\phi)}{(\delta+\lambda(1-F_k))(s+\lambda\phi\bar{F}(\alpha+\lambda))} \\
&= \frac{\delta F_k[\lambda\phi(\delta+\lambda+\alpha) + \alpha\delta(\delta+2\phi)] + \lambda\delta\phi(\lambda+\alpha)[F_k - \sum_{i=1}^k (F_k - F_i)\tilde{h}_i]}{(\delta+\lambda(1-F_k))(s+\lambda\phi\bar{F}(\alpha+\lambda))} \\
&= \frac{\delta F_k[s+\lambda\phi\bar{F}(\alpha+\lambda)]}{(\delta+\lambda(1-F_k))(s+\lambda\phi\bar{F}(\alpha+\lambda))} + \frac{\lambda\delta\phi(\lambda+\alpha)[F_k(1-\bar{F}) - \sum_{i=1}^k (F_k - F_i)\tilde{h}_i]}{(\delta+\lambda(1-F_k))(s+\lambda\phi\bar{F}(\alpha+\lambda))} \\
&= \frac{\delta F_k}{(\delta+\lambda(1-F_k))} + \frac{\lambda\delta\phi(\lambda+\alpha)[F_k(1-\bar{F}) - \sum_{i=1}^k (F_k - F_i)\tilde{h}_i]}{(\delta+\lambda(1-F_k))(s+\lambda\phi\bar{F}(\alpha+\lambda))}
\end{aligned}$$

Without network effects, the measure of workers E_k^* can be obtained as:

$$\begin{aligned}
E_k^* &= G_{n,k}^* + G_{k,n}^* + 2H_k^* + Q_k^* = \frac{2\lambda H_k^* + \lambda F_k d_m^* + 2\phi Q_k^*}{2\delta+\lambda(1-F_k)} + 2H_k^* + Q_k^* \\
&= \frac{\lambda F_k d_m^*[\delta(\delta+\lambda(1-F_k)) + \delta(\delta+\lambda+\alpha) + (\alpha+\lambda)(\delta+\lambda(1-F_k))]}{(\alpha+\lambda)(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} \\
&+ \frac{2\phi\delta Q_k^*}{(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} + \frac{(2(\phi+\delta)+\lambda(1-F_k))}{2\delta+\lambda(1-F_k)} Q_k^* \\
&= \frac{\lambda F_k(\delta+\alpha+\lambda)(2\delta+\lambda(1-F_k))d_m^*}{(\alpha+\lambda)(\delta+\lambda(1-F_k))(2\delta+\lambda(1-F_k))} + \frac{2\phi+\delta+\lambda(1-F_k)}{\delta+\lambda(1-F_k)} Q_k^* \\
&= \frac{\lambda F_k(\delta+\alpha+\lambda)d_m^*}{(\alpha+\lambda)(\delta+\lambda(1-F_k))} + \frac{2\alpha d_u^* F_k(\delta+2\phi)}{\phi(\delta+\lambda(1-F_k))}
\end{aligned}$$

Therefore, without network effects, the cumulative density function $P^*(y_k) = E_k^*/e^*$ is given by:

$$\frac{E_k^*}{e^*} = \frac{\lambda F_k(\delta+\alpha+\lambda)}{(\alpha+\lambda)(\delta+\lambda(1-F_k))} \cdot \frac{\delta\lambda(\alpha+\lambda)\phi}{\lambda s} + \frac{2\alpha F_k(\delta+2\phi)}{\phi(\delta+\lambda(1-F_k))} \cdot \frac{0.5\delta^2\lambda\phi}{\lambda s} = \frac{\delta F_k s}{(\delta+\lambda(1-F_k))s}$$

Next, it can be shown that $P(y_k) > P^*(y_k)\ \forall k = 1..n-1$, since:

$$\begin{aligned}
F_k(1-\bar{F}) - \sum_{i=1}^k (F_k - F_i)\tilde{h}_i &= F_k - F_k\Big(\sum_{i=1}^k F_i\tilde{h}_i + \sum_{i=k+1}^n F_i\tilde{h}_i\Big) - F_k\sum_{i=1}^k \tilde{h}_i + \sum_{i=1}^k F_i\tilde{h}_i \\
&= F_k\Big(1 - \sum_{i=k+1}^n F_i\tilde{h}_i - \sum_{i=1}^k \tilde{h}_i + \sum_{i=k+1}^n \tilde{h}_i - \sum_{i=k+1}^n \tilde{h}_i\Big) + (1-F_k)\sum_{i=1}^k F_i\tilde{h}_i \\
&= F_k\sum_{i=k+1}^n (1-F_i)\tilde{h}_i + (1-F_k)\sum_{i=1}^k F_i\tilde{h}_i > 0 \quad \text{for} \quad k = 1..n-1
\end{aligned}$$

Appendix V: Proof of proposition 5. The suplus value $R_1 = V_1 - U$ can be expressed as:

$$(r+\delta)(V_1 - U) = w_1 - z - \lambda\pi_1(V_1 - U) - \lambda\sum_{i=2}^n (V_i - U - V_i + V_1)\pi_i + 2\phi(\widetilde{V}_1 - V_1)$$

So that $(r+\delta+\lambda)(V_1-U) = w_1 - z + 2\phi(\widetilde{V}_1 - V_1)$. Similarly, $\widetilde{R}_1 = \widetilde{V}_1 - \widetilde{U}$ is:

$$\begin{aligned}
(r+\delta)(\widetilde{V}_1-\widetilde{U}) &= w_1 - z - \lambda(\widetilde{V}_1-\widetilde{U}) + 2\mu\alpha(\widetilde{U}-U) \\
&+ \lambda(1-\mu)\sum_{j=2}^{n}\sum_{i=2}^{j}(\widetilde{V}_i-\widetilde{V}_1)\pi_i\tilde{p}_j - \lambda(1-\mu)\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_i-\widetilde{U})\pi_i\tilde{p}_j \\
(r+\delta+\lambda)(\widetilde{V}_1-\widetilde{U}) &= w_1 - z + 2\mu\alpha\Delta U + \lambda(1-\mu)\sum_{j=2}^{n}\sum_{i=2}^{j}(\widetilde{V}_i-\widetilde{V}_1)\pi_i\tilde{p}_j \\
&- \lambda(1-\mu)\pi_1(\widetilde{V}_1-\widetilde{U}) - \lambda(1-\mu)\sum_{j=2}^{n}\sum_{i=2}^{j}(\widetilde{V}_i-\widetilde{U})\pi_i\tilde{p}_j = w_1 - z + 2\mu\alpha\Delta U \\
&- \lambda(1-\mu)\pi_1(\widetilde{V}_1-\widetilde{U}) + \lambda(1-\mu)\sum_{j=2}^{n}\sum_{i=2}^{j}(\widetilde{V}_i-\widetilde{V}_1-\widetilde{V}_i+\widetilde{U})\pi_i\tilde{p}_j
\end{aligned}$$

This means $(r+\delta+\lambda(1+(1-\mu)\widetilde{F}))(\widetilde{V}_1-\widetilde{U}) = w_1 - z + 2\mu\alpha\Delta U$ where $\widetilde{F} = \sum_{j=1}^{n} F_j\tilde{p}_j$.

$$rV_{k-1} = w_{k-1} + \lambda\sum_{i=k}^{n}(V_i - V_{k-1})\pi_i + 2\phi(\widetilde{V}_{k-1} - V_{k-1}) - \delta(V_{k-1} - U)$$

Next, consider value gains from on-the-job search $V_k - V_{k-1}$ and $\widetilde{V}_k - \widetilde{V}_{k-1}$:

$$\begin{aligned}
(r+\delta+2\phi)(V_k-V_{k-1}) &= \Delta w_{k-1} + \lambda\sum_{i=k+1}^{n}(V_i-V_k)\pi_i - \lambda\sum_{i=k}^{n}(V_i-V_{k-1})\pi_i + 2\phi(\widetilde{V}_k-\widetilde{V}_{k-1}) \\
&= \Delta w_{k-1} + \lambda\sum_{i=k+1}^{n}(-V_k+V_{k-1})\pi_i - \lambda(V_k-V_{k-1})\pi_k + 2\phi(\widetilde{V}_k-\widetilde{V}_{k-1}) \\
&= \Delta w_{k-1} - \lambda(V_k-V_{k-1})(1-F_{k-1}) + 2\phi(\widetilde{V}_k-\widetilde{V}_{k-1})
\end{aligned}$$

$$\begin{aligned}
r\widetilde{V}_{k-1} &= w_{k-1} + \lambda\sum_{i=k}^{n}(\widetilde{V}_i-\widetilde{V}_{k-1})\pi_i + \lambda(1-\mu)\sum_{j=k}^{n}\sum_{i=k}^{j}(\widetilde{V}_i-\widetilde{V}_{k-1})\pi_i\tilde{p}_j - \delta(\widetilde{V}_k-\widetilde{U}) \\
(r+\delta)(\widetilde{V}_k-\widetilde{V}_{k-1}) &= \Delta w_{k-1} + \lambda\sum_{i=k+1}^{n}(\widetilde{V}_i-\widetilde{V}_k)\pi_i + \lambda(1-\mu)\sum_{j=k+1}^{n}\sum_{i=k+1}^{j}(\widetilde{V}_i-\widetilde{V}_k)\pi_i\tilde{p}_j \\
&- \lambda\sum_{i=k}^{n}(\widetilde{V}_i-\widetilde{V}_{k-1})\pi_i - \lambda(1-\mu)\sum_{j=k}^{n}\sum_{i=k}^{j}(\widetilde{V}_i-\widetilde{V}_{k-1})\pi_i\tilde{p}_j \\
&= \Delta w_{k-1} - \lambda(\widetilde{V}_k-\widetilde{V}_{k-1})(1-F_{k-1}) - \lambda(1-\mu)(\widetilde{V}_k-\widetilde{V}_{k-1})\pi_k\tilde{p}_k \\
&- \lambda(1-\mu)\sum_{j=k+1}^{n}\Big[(\widetilde{V}_k-\widetilde{V}_{k-1})\pi_k + \sum_{i=k+1}^{j}(\widetilde{V}_k-\widetilde{V}_{k-1})\pi_i\Big]\tilde{p}_j
\end{aligned}$$

$$\begin{aligned}
(r+\delta+\lambda(1-F_{k-1}))(\widetilde{V}_k-\widetilde{V}_{k-1}) &= \Delta w_{k-1} - \lambda(1-\mu)(\widetilde{V}_k-\widetilde{V}_{k-1})(F_k-F_{k-1})\tilde{p}_k \\
&- \lambda(1-\mu)(\widetilde{V}_k-\widetilde{V}_{k-1})\sum_{j=k+1}^{n}(F_j-F_{k-1})\tilde{p}_j
\end{aligned}$$

Therefore, one-step surplus gains $V_k - V_{k-1}$ and $\widetilde{V}_k - \widetilde{V}_{k-1}$ can be written as:

$$
\begin{aligned}
(\widetilde{V}_k - \widetilde{V}_{k-1}) &= \frac{\Delta w_{k-1}}{r+\delta+\lambda(1-F_{k-1})(1+\Sigma_{k-1})} > 0, \quad k = 2..n \\
(V_k - V_{k-1}) &= (\widetilde{V}_k - \widetilde{V}_{k-1})\Big[1 + \frac{\lambda(1-F_{k-1})\Sigma_{k-1}}{r+\delta+2\phi+\lambda(1-F_{k-1})}\Big] > 0, \quad k = 2..n
\end{aligned}
$$

$$
\text{where} \quad \Sigma_{k-1} = (1-\mu)\frac{\sum_{j=k}^{n}(F_j - F_{k-1})\tilde{p}_j}{1-F_{k-1}} > 0
$$

Let $r \to 0$ and note that $\tilde{V}_n - V_n = \delta\Delta U/(\delta+2\phi)$, so that:

$$
\begin{aligned}
\widetilde{V}_k - V_k &= \widetilde{V}_{k+1} - V_{k+1} + \frac{\lambda\Delta w_k(1-F_k)\Sigma_k}{(\delta+\lambda(1-F_k)(1+\Sigma_k))(\delta+2\phi+\lambda(1-F_k))} \\
&= \frac{\delta\Delta U}{\delta+2\phi} + \lambda\sum_{i=k}^{n}\frac{\Delta w_i(1-F_i)\Sigma_i}{(\delta+\lambda(1-F_i)(1+\Sigma_i))(\delta+2\phi+\lambda(1-F_i))}, \quad k = 1..n
\end{aligned}
$$

where $\Delta w_n = 0$ by assumption. Denote the second term in the above expression by A_k, so that $\widetilde{V}_k - V_k = \delta\Delta U/(\delta+2\phi) + A_k$, where $A_k > 0$ $\forall k = 1..n-1$ and $A_n = 0$. Finally, variable ΔU can be found as:

$$
\begin{aligned}
(\lambda+2\mu\alpha)\Delta U &= \lambda\sum_{i=1}^{n}(\widetilde{V}_i - V_i)\pi_i + \lambda(1-\mu)\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{U})\pi_i\tilde{p}_j \\
&= \lambda\sum_{i=1}^{n}\Big(\frac{\delta\Delta U}{\delta+2\phi} + A_i\Big)\pi_i + \lambda(1-\mu)\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{V}_1 + \widetilde{V}_1 - \widetilde{U})\pi_i\tilde{p}_j
\end{aligned}
$$

$$
\begin{aligned}
\Big(\frac{\lambda\phi}{\delta+2\phi} + \mu\alpha\Big)2\Delta U &= \lambda\sum_{i=1}^{n}A_i\pi_i + \lambda(1-\mu)\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{V}_1)\pi_i\tilde{p}_j + \lambda(1-\mu)(\widetilde{V}_1 - \widetilde{U})\widetilde{F} \\
&= \lambda\sum_{j=1}^{n}A_j\pi_j + \lambda(1-\mu)\sum_{j=1}^{n}\sum_{i=1}^{j}(\widetilde{V}_{i+1} - \widetilde{V}_i)(F_j - F_i)\tilde{p}_j \\
&+ \lambda(1-\mu)\widetilde{F}\Big[\frac{w_1 - z + 2\mu\alpha\Delta U}{\delta+\lambda(1+(1-\mu)\widetilde{F})}\Big] \quad \text{where} \quad \widetilde{F} = \sum_{j=1}^{n}\sum_{i=1}^{j}\pi_i\tilde{p}_j = \sum_{j=1}^{n}F_j\tilde{p}_j
\end{aligned}
$$

In addition, summation by parts implies $\sum_{i=1}^{j}(\widetilde{V}_i - \widetilde{V}_1)\pi_i = \sum_{i=1}^{j}(\widetilde{V}_{i+1} - \widetilde{V}_i)(F_j - F_i)$ and

$$
\begin{aligned}
2\Delta U\Big[\frac{\lambda\phi}{\delta+2\phi} + \frac{\mu\alpha(\delta+\lambda)}{\delta+\lambda(1+(1-\mu)\widetilde{F})}\Big] &= \frac{\lambda(1-\mu)\widetilde{F}(w_1 - z)}{\delta+\lambda(1+(1-\mu)\widetilde{F})} \\
+ \sum_{j=1}^{n}\sum_{i=j}^{n}\frac{\lambda^2\Delta w_i(1-F_i)\Sigma_i\pi_j}{(\delta+\lambda(1-F_i)(1+\Sigma_i))(\delta+2\phi+\lambda(1-F_i))} &+ \sum_{i=1}^{j}\frac{\lambda(1-\mu)\Delta w_i(F_j - F_i)}{\delta+\lambda(1-F_i)(1+\Sigma_i)}\tilde{p}_j \\
= \sum_{j=1}^{n}\sum_{i=j}^{n}\frac{\lambda^2\Delta w_i(1-F_i)\Sigma_i\pi_j}{(\delta+\lambda(1-F_i)(1+\Sigma_i))(\delta+2\phi+\lambda(1-F_i))} &+ \sum_{i=0}^{j}\frac{\lambda(1-\mu)\Delta w_i(F_j - F_i)}{\delta+\lambda(1-F_i)(1+\Sigma_i)}\tilde{p}_j
\end{aligned}
$$

where $\Delta w_0 = w_1 - z$ and $F_0 = 0$ by assumption, so that $\Sigma_0 = (1-\mu)\sum_{j=1}^{n}F_j\tilde{p}_j = \widetilde{F}$.

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