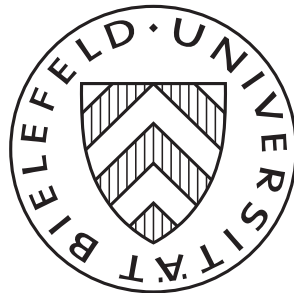# A Case for Standard Theory?

Christoph Kuzmics and Daniel Rodenburger

A case for standard theory?*

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Abstract

Using data from an experiment by Forsythe, Myerson, Rietz, and We-
ber (1993), designed for a different purpose, we test the "standard the-
ory" that players have preferences only over their own monetary pay-
offs and that play will be in (evolutionary stable) equilibrium. In the
experiment each subject is recurrently (24 times) randomly matched
with ever changing opponents to play a 14 player game. We find that
assuming risk-neutrality for all players leads to a predicted evolution-
ary stable equilibrium that, while it can be rejected at the 5% level of
significance, is nevertheless remarkably close to "explaining" the data.
Moreover, when we assume that players are risk-averse and we cal-
ibrate their risk-aversion in one treatment with a simple game, this
theory cannot be rejected at the 5% level of significance for another
treatment with a more complicated game, despite the fact that we
have close to 400 data points.

JEL codes: C57, C72, D72

Keywords: opinion polls, elections, voting, testing, Nash equilibrium, attain-
able equilibrium, symmetries

1 Introduction

We stumbled upon a laboratory experiment that involved randomly chosen human subjects playing a 14 player game.[1] The authors, Forsythe, Myerson, Rietz, and Weber (1993), did not provide a full theoretical analysis of the equilibria (and their properties) of this game. Their objective was different. They wanted to perform an exploratory analysis of how opinion polls may impact voting behavior in an election and were interested in experimentally assessing Duverger's "law", see Duverger (1954), that in any majority-rule election only two parties receive votes. We are simply curious as to what these equilibria are and how the actual play in the laboratory compares to the theoretical equilibrium predictions. We are interested in the data only, in this paper, in as far as it can be seen as a case study of testing "standard theory" in a somewhat complicated game.

With "standard theory" we mean the combination of two things. First, as in "standard economic theory" we assume that players only care about the money they themselves receive in the game. Second, as in "standard game theory" we assume that players play according to some Nash equilibrium.

This 14 player game that subjects play has some special properties that one needs to take into account when analysing the game. While the game is described in detail in Sections 2, 3.1, and 4.1, we need to provide a brief sketch of the game in order to make these special properties understood.

First, each group of 14 players that play the game, is randomly split up into three subgroups. Each group represents a preference over the three alternatives (or candidates) A, B, and C that the subjects are voting on. There are always 4 A, 4 B, and 6 C types, where the type indicates the subject's most preferred alternative. The monetary payoff schedule is chosen in such a way as to put all players of any one type in completely symmetric positions. Moreover, and this is more subtle, also A and B types are in symmetric positions if we relabel their voting strategies from "vote for A" and "vote for B" and so on to "vote for your favorite alternative" and "vote for your second favorite alternative" and so on. All these symmetries suggest one should only consider equilibria that satisfy the appropriate symmetry restrictions, as in (Nash, 1951, Theorem 2), Crawford and Haller (1990), Blume (2000), and Alos-Ferrer and Kuzmics (2013). Following Crawford and Haller (1990) we shall refer to these equilibria as **attainable** in order to

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[1]That is to say, there is no selection bias.

avoid confusion between this concept and what we usually understand by a symmetric equilibrium (in which all players use the same strategies).

The way the experiment was phrased or "framed", however, adds one layer of complication to the game. Forsythe, Myerson, Rietz, and Weber (1993) also wanted to assess the impact of an alternative's ballot position on the outcome of the election. In the game alternative A was always on the top of the list of alternatives as it was presented to the subjects.[2] This means that the "framed" game does allow the distinction between A and B in terms of the language, see Blume (2000), of "the first alternative in the ballot" and so on. This means that the "framed" game, see Alos-Ferrer and Kuzmics (2013), allows, in principle, also all non-attainable strategy-profiles and thus all non-attainable equilibria. For this to be really possible one would need essentially common knowledge that this language can be employed. Whether or not it is employed is then an empirical question.

Second, the game is actually played recurrently (24 times) by an always changing group of 14 subjects, randomly chosen from a group of 28. In fact the 28 subjects are in every session randomly allocated into one of two 14 player groups before they are then allocated a type. Such repetition of playing the game gives hope that play could be in a Nash equilibrium. In one-shot games one cannot usually expect this unless the game has a clear unique focal point, see Schelling (1960). Otherwise even common knowledge of rationality does not lead to a more precise prediction than rationalizability, see Bernheim (1984) and Pearce (1984). However, whenever Nash equilibrium is justified through recurrent play, it is actually the concept of an evolutionary stable strategy of Maynard Smith and Price (1973) and Maynard Smith (1982) or some variation of it that is justified.

There are two experimental treatments in Forsythe, Myerson, Rietz, and Weber (1993). In one treatment subjects only play a voting game. In the other treatment, before playing the voting game, subjects play an opinion poll game, the outcome of which is publicly announced before the voting game.

In this paper we proceed as follows. For each treatment we identify the game played, find all attainable equilibria, identify all evolutionary stable ones among them and then compare those to the empirical findings.

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[2]This could have been done differently if one had wanted to maintain the symmetries by, for instance, employing rotating "pies". See e.g. Blume and Gneezy (2000) and Blume and Gneezy (2010).

After explaining the experimental setup in more detail in Section 2, we find, in Section 3.2, that the voting game has three attainable equilibria and, in Section 3.3, a unique evolutionary stable (ESS) equilibrium among them which is in mixed strategies for A and B type players. In Section 3.4 we find that the null of attainability cannot be rejected. While the data allows us to reject the null that the unique ESS is played, the set of hypothesized strategies that "explains" the data better is not large. We then note, in Section 3.5, that a small change to the "standard theory" can be made so as to perfectly calibrate the data. This is done by changing the "standard economic theory" slightly by allowing subjects to be risk averse.

We then find, in Section 4.2, that the polling game has a theoretically and empirically highly justifiable focal second stage voting behavior. Given this, the polling game has three attainable equilibria and, as we find in Section 4.3, two evolutionary stable (ESS) equilibria among them, both of which are in mixed strategies for A and B type players (albeit very different from the attainable ESS equilibrium in the voting game). In Section 4.4 we find that the null of attainability can again not be rejected, although there is some evidence against it. While the data allows us to reject the null that an ESS is played, the set of hypothesized strategies that "explains" the data better than one of the ESS is again not large. Finally, as we show in Section 4.5, under the risk-aversion version of the "standard economic theory", as calibrated using only the data on the voting games without polls, "explains" the data almost perfectly. The null of the resulting ESS cannot be rejected at the 5% level of significance (despite having close to 400 data points).

A final note on our Propositions in this paper. Their proofs are done partly analytically and partly numerically as we need to solve polynomial equations of degree of up to seven. We employ the multi-dimensional Newton-Raphson method in order to do so. The matlab program with which we do this, is provided as supplementary material. This numerical approach is completely accurate unless we made a mistake in programming.

1.1   Related Literature

There is a large literature on testing "standard theory". Beginning with, within economics, at the latest Güth, Schmittberger, and Schwarze (1982) there is now extensive laboratory research documenting a large variety of behavioral departures from the "standard economic theory" of only self-

interested people. While one can think about the possible effects various theories of fairness concerns or altruism or any other other-regarding preferences have on the equilibria of our game, we understand our paper as not adding much to this particular literature, except perhaps as a case in point that among the many possible theories that could "explain" the data of Forsythe, Myerson, Rietz, and Weber (1993) the "standard economic theory" also suffices.[3]

We are of the opinion that our paper adds something to another strand of the literature on testing "standard theory". The strand that, under certain assumptions about preferences, tests the theories of game theory, specifically the theory of equilibrium play. There is evidence that playing a game only once is not typically sufficient to guarantee equilibrium play, while playing a game often (recurrently with different opponents) does lead to equilibrium play. For instance, VanHuyck, Battalio, and Beil (1990) show that subjects often fail to play any Nash equilibrium in one-shot coordination games, while Cooper, DeJong, Forsythe, and Ross (1990) find evidence of Nash equilibrium play in recurrent coordination games. Similarly O'Neill (1987) finds evidence against laboratory subjects playing minmax (i.e. Nash equilibrium) strategies in zero-sum games, while Walker and Wooders (2001) find mixed evidence that professional tennis players use minmax strategies in their service game, Hsu, Huang, and Tang (2007) find evidence that professional tennis players to use minmax strategies, and Palacios-Huerta (2003) finds strong evidence that professional soccer players (and goalkeepers) use minmax strategies when taking (or defending) penalty kicks.[4] Professionals have played these games often, while laboratory subjects not (or not often enough). Similarly, again, in public good provision games initial play is often not an equilibrium (under material self-interested preferences), but eventual play after repetitions often is. See e.g. Andreoni (1988) and the control group in Fehr and Gächter (2000).

While the experiment of Forsythe, Myerson, Rietz, and Weber (1993) has not been designed specifically to test whether subjects play an evolutionary stable strategy or not, it is, nevertheless, a very suitable case study for such

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[3]One could use this data set also as follows. For any theory of other-regarding preferences one could work out the equilibria under this situation (assuming common knowledge or incomplete information about these preferences, depending on one's theory) and use the actual frequency of play to possibly refute this theory (in this context). Whether or not one would find this interesting depends on what demands one wants to place on the general applicability of the particular theory.

[4]See also Binmore, Swierzbinski, and Proulx (2001) for evidence of minmax play.

an undertaking. Each subject played the game in the lab 24 times with ever changing opponents. The game is not trivial. In fact, one could call it somewhat complex. The game has many equilibria. The game has Nash equilibria that are not evolutionary stable and some that are evolutionary stable. In fact, it turns out, that one of the two treatments in the experiment is a game with two evolutionary stable strategies. The game also allows testing of attainability (i.e. testing of play satisfying some symmetry restrictions) in a game that has symmetries but is presented to subjects in a way that breaks these symmetries.

2 The experimental setup

In this section we describe the experimental setup employed by Forsythe, Myerson, Rietz, and Weber (1993). There are two treatments. In treatment 1 players only play a voting game. In treatment 2 players, before playing the voting game, participate in an opinion poll.

In every game of every treatment exactly 14 players (which are randomly chosen from a pool of 28) are asked to play. The voting game is the same in all treatments and its payoffs are given in Table 1. Four players are randomly assigned the type A, four the type B, and 6 the type C.[5] Each player can choose to vote for one of three candidates, also labelled A, B, and C. The table states the monetary payoffs (in US$) to each type depending on which candidate wins the election. Voters of type A favor candidate A, voters of type B favor candidate B, voters of type C favor candidate C. Voter types are, thus, named in terms of their favorite candidate.

total number	voter type	Election Winner		
		A	B	C
4	A	$ 1.20	$ 0.90	$ 0.20
4	B	$ 0.90	$ 1.20	$ 0.20
6	C	$ 0.40	$ 0.40	$ 1.40

Table 1: Payoff schedule of the voting game

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[5]As there are finite and fixed number of each type, the game here differs somewhat from the voting games studied in e.g. Palfrey (1989), Myerson and Weber (1993), Fey (1997), and Andonie and Kuzmics (2012), where the focus is on large elections and where every player is randomly allocated a type without keeping the fraction of players of any one type constant.

Voters are asked to cast their vote for one of the three candidates or to abstain. If there is a tie between two or three most voted candidates the elected candidate is chosen uniformly randomly.

For treatment 2 players are first asked to state one of the three candidates in an opinion poll (without payoffs), the outcome of which is then publicly announced before players are asked to play the voting game as described above.

3 The voting game without opinion polls

In this section we study the voting game without polls.

3.1 The game

This is a game in which each player of each of the three types of players has four pure strategies: vote for A, vote for B, vote for C, and abstain from voting. There are four players of the A type, four of the B type, and six of the C type. The winning candidate is determined by simple (relative) majority. If there is a candidate with more votes than any other candidate, this candidate is elected with payoff consequences to the various voter types as given in Table 1.[6] If there are two or three candidates with the highest number of votes then one of them is drawn uniformly randomly as the winner with payoff consequences again as given in Table 1.

3.2 Attainable Equilibria

We shall here identify and compute all attainable equilibria of this 14-player voting game that are not weakly dominated. Note that for C types voting for C is a weakly dominant strategy, while for A and B types voting for C and abstaining are both weakly dominated. Thus, we need to concern ourselves only with the choice of A and B types between voting for A and voting for B.

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[6]We here assume that all players have affine preferences of money. This is challenged in Section 3.5.

The game (without considering framing effects induced by ballot position) has inherent symmetries as identified already in Andonie and Kuzmics (2012). If we define an attainable equilibrium (as defined and studied by Crawford and Haller (1990), Blume (2000), and Alos-Ferrer and Kuzmics (2013)) as an equilibrium that respects the inherent symmetries of the game, then we have the following restrictions for the A and B type players. Every A type player must use the same (mixed) strategy, every B type player must use the same (mixed) strategy, and every C type must use the same (mixed) strategy. Let $x_A(A)$ denote the probability that an A type player attaches to pure strategy A. Then, as we assume that A type players do not use strategy C, we must have that $x_A(B) = 1 - x_A(A)$, the probability that an A type player attaches to pure strategy B. A final and perhaps most important restriction of attainability is that $x_A(A) = x_B(B)$ (and, of course also, $x_A(B) = x_B(A)$), where $x_B(B)$ is the probability that a B type attaches to pure strategy B. See Andonie and Kuzmics (2012) for details. Thus, if we restrict attention to undominated attainable strategy profiles, all we need to determine is $x = x_A(A)$.

Proposition 3.1. *The voting game has exactly three undominated attainable equilibria. In all of these, C types play C, and A and B types put zero probability on C and abstaining. The three undominated equilibria are then characterized by the probability an A type attaches to A (same as a B type attaches to B) which in the three equilibria is given by $x = 0$, $x = 1$, and $x = 0.6615$, respectively.*

Proof: We have already argued that C types must play C in an undominated equilibrium and that A and B types will avoid playing C and abstaining. Now consider the supposed equilibrium given by $x = 0$. This implies that all A types play B and all B types play A, which leads to C winning the election by two votes. This implies that no individual can change this outcome by unilaterally deviating from the stated strategy profile. The same argument applies for $x = 1$. So, these are both equilibria.

In order to identify an equilibrium $x \in (0, 1)$ we need to appeal to the indifference principle (i.e. voting for A and B has to be equally good for all A and B types of voters).

Consider w.l.o.g. an A type voter. Let $Y(x)$ denote the random variable that, given probability $x \in (0, 1)$, equals the number of A votes among all 7 other A and B types of voters. Note that for the considered A type voter, a vote between A or B changes the election result if and only if $Y(x) \in \{1, 2, 5, 6\}$.

For instance, if $Y(x) = 1$ an A-vote by the considered A type player will result in a tie between candidates B and C, while a B-vote would result in a win for B.

Let $u_A(A, x)$ denote the expected payoff to the considered A type voter if she votes for A given all others use mixing probability x, and let, similarly, $u_A(B, x)$ denote her expected payoff in this case if she votes for B. The equilibrium condition is then given by $u_A(A, x) = u_A(B, x)$. This equation is given in Appendix B. It is a polynomial in x of degree 7, and the equilibria are given by its zeros. Dividing this polynomial by $x(1-x)$, given that $x = 0$ and $x = 1$ are zeros of this polynomial, leaves a polynomial in x of degree 5. We then apply Newton's method to find all further zeros of this polynomial and obtain only one at $x = 0.6615$.   QED

Note that among all (three) attainable equilibria, in all of which C types vote for C, A and B types clearly as a group and individually prefer the mixed equilibrium with $x = 0.6615$, in which they have at least a positive probability of winning the election.[7]

Note that the voting game has other, non-attainable, equilibria. For instance, it is an equilibrium that all A and B types vote for A, or all A and B types vote for B.

3.3   Evolutionary Stability of Attainable Equilibria

In this section we study the evolutionary stability properties of the three attainable equilibria. We shall adapt the notion of an **evolutionary stable strategy (ESS)** in the sense of Maynard Smith and Price (1973) and Maynard Smith (1982) (see (Weibull, 1995, Definition 2.1) for a textbook treatment) and Palm (1984) (who extended this to general symmetric multi-player games) to the context of attainable strategy profiles. Consider an attainable strategy (profile), characterized by $x \in [0, 1]$, the probability that an A type attaches to playing A (and equivalently that a B type attaches to B). Let $y \in [0, 1]$ denote a mutant strategy that enters with probability $\epsilon > 0$ close to zero.[8] Let $w_\epsilon = (1 - \epsilon)x + \epsilon y$ denote the post-entry mix of strate-

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[7]Indeed of the 48 sessions without polls in the experiment, three are won by candidate A and three are won by candidate B. See Section 3.4.

[8]Throughout the paper we shall only consider mutants separately in each population. That is we do not consider a joint mutation in the population of A and B types and in the population of C types. Of course, any strategy profile that we find to be not evolutionary

gies. Let generally $u_A(x, z)$ denote the payoff to an A type playing strategy $x \in [0, 1]$ when all her (seven) opponents (recall that C types just play C regardless) play strategy $z \in [0, 1]$. Then x is an ESS if for all $y \in [0, 1]$ with $y \neq x$ we have that $u_A(x, w_\epsilon) \geq u_A(y, w_\epsilon)$.

Proposition 3.2. *The only ESS among the three attainable equilibria of the voting game is $x = 0.6615$.*

Proof: The attainability restriction that x is the probability of an A-type playing A and at the same time also the probability of a B-type playing B could also be handled by relabelling all B types strategies from A to B and vice versa with the appropriate transformation in terms of utilities. Then the attainable strategy x is simple a symmetric strategy of the relabelled game, which is now an eight player symmetric game. In Lemma A.1 in the Appendix we provide a simple equivalent condition for a (mixed) strategy $x \in [0, 1]$ to be an ESS for such games, where every player has the same two pure strategies. It implies that, in the present context attainable strategy (profile) x is an ESS if and only if for all $y < x$ close to x A is a best reply to y for an A-type and for all $y > x$ close to x B is a best reply for an A-type.

In order to check the evolutionary stability of the three attainable equilibria, we therefore numerically compute the payoff of an A-type for strategies A and B as a function of x, the probability with which all other A types play A and all B types play B. This is given in Figure 1.

Note that when y exceeds $x = 0.6615$ strategy B is best for an A-type (and analogously strategy A best for a B type). On the other hand, when y is below $x = 0.6615$ strategy A is best for an A-type (and analogously strategy B best for a B type). Thus, by Lemma A.1, the only evolutionary stable strategy is the mixed equilibrium.[9]   QED

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stable is also not evolutionary stable when we allow for more kinds of mutations. Strategy profiles that we find to be evolutionary stable (within each population) may not necessarily be evolutionary stable if we allow joint mutations. We shall, however, not pursue this point.

    [9]This is very similar to the two-player hawk-dove game. Indeed the unframed voting game is in some sense an 8 player (the C types just chose C and can be disregarded) hawk-dove game. The only difference here is that the payoff to an A type for playing A is not a linear function in x (because it is not a two-player game) and is not strictly decreasing.
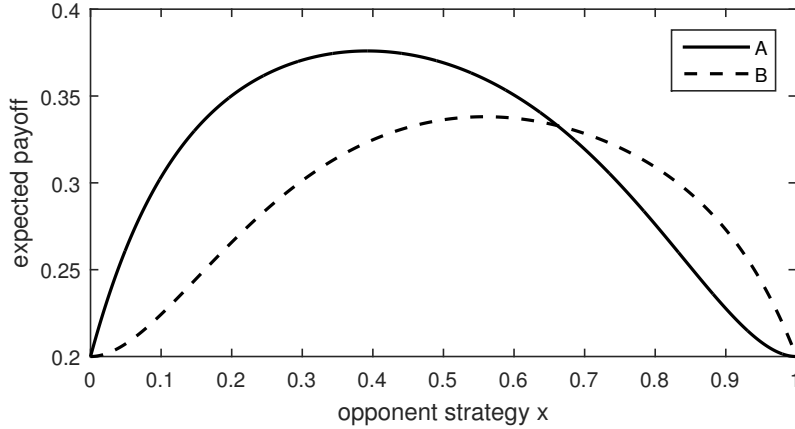
Figure 1: Payoffs to an A-type for playing A (solid line) or B (dashed line) as a function of x

3.4   Empirical Results and Tests

The actual voting behavior in the 48 experimental session without opinion polls is given in Table 2.

		fraction of votes cast by each type for each candidate			
Obs.	Type	A	B	C	Abst.
192	A	0.5625	0.4115	0.0156	0.0104
192	B	0.4271	0.5365	0.0156	0.0208
288	C	0.0139	0.0069	0.9722	0.0069

Table 2: Empirical frequency of votes

Note that roughly 2-3% of subjects choose a weakly dominated strategy in each group of voter types. If we only look at those subjects that do not choose weakly dominated strategies we get the empirical fractions for voter types A and B given in Table 3.

From the empirical results, given in Tables 2 and 3 we can immediately reject the null hypotheses that A and B types play one of the asymmetric pure equilibria, or one of the symmetric pure equilibria. This is even true if we allow for a small enough fraction of "noise" players who just randomize in some arbitrary way. We now turn to testing attainability.

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		fraction of votes cast by each type for each candidate	
Obs.	Type	A	B
187	A	0.5775	0.4225
185	B	0.4432	0.5568

Table 3: Empirical frequency of votes of A and B types among all non-dominated votes

Test 3.1. *The null hypothesis of attainability, i.e. $x_A(A) = x_B(B)$, using a χ^2 test of independence, cannot be rejected at the 5% level of significance. It produces a p-value of 0.8396 ($\chi^2 = 0.8411$).*

Note that in order to perform a χ^2 test of independence we need to first relabel the strategies to take account of the symmetry restrictions. The result is given in Table 4. We can then perform the test and obtain a p-value of 0.8396.

		fraction of votes cast by each type for each candidate			
Obs.	Type	favorite	second favorite	C	Abst.
192	A	0.5625	0.4115	0.0156	0.0104
192	B	0.5365	0.4271	0.0156	0.0208

Table 4: Empirical frequency of relabelled votes for A and B types

Test 3.1 is, thus, also a test of the null of the absence of a ballot position effect. As it cannot be rejected, there is no evidence, in the voting game, that players condition their strategy on the ballot position of the candidates. This is in agreement with the finding of Forsythe, Myerson, Rietz, and Weber (1993).

We can now test the null that A and B types (those that do not play dominated strategies) on average play the evolutionary stable strategy $x = 0.6615$. While the empirical frequency of 0.5672 (average across the two types) is not very far from the hypothesized $x = 0.6615$, the null of $x = 0.6615$ can nevertheless be rejected at the 5% level of significance.

Test 3.2. *The null hypothesis of ESS equilibrium play, i.e. $x_A(A) = x_B(B) = x = 0.6615$, using an exact binomial test must be rejected at the 5% level of significance. It produces a p-value of 0.0002.*

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In this test we are using the pooled data for both A and B types.

Even though the null hypothesis of the evolutionary stable equilibrium probability $x = 0.6615$ is rejected, this theory is nevertheless not so very far from "explaining" the data. In order to see this we identify all possible $x \in [0, 1]$ that would "explain" the data better. That is we identify those values of $x \in [0, 1]$ that produce a higher likelihood of the data than the equilibrium value $x = 0.6615$. One way to attempt this would be to compute the binomial likelihood of 211 "successes" (i.e. an A type choosing A, a B type choosing B) among 372 "trials". This is computationally infeasible. In some sense this is also incorrect. The experiment actually produces data separately for each session of the 8 player game (between A and B types assuming C types play C). However, we do not have this detailed data. Instead we perform a bootstrap, where we simulate the 48 sessions 1000 times and compute the likelihood in each case and compute the average. We find that the set of theories "better than" the evolutionary stable equilibrium theory is approximately the interval (0.4851, 0.6615) for x, which represents approximately 16.66% of the zero-one interval.

3.5   Risk averse players

So far in this Section we considered as the "standard economic theory" that all players have affine preferences over money. In particular for A and B types we postulated payoffs of $u(m) = m$, where $m \in \{1.2, 0.9, 0.2\}$ the three possible distinct payoffs these types can receive (see Table 1). One might call such players risk-neutral.

One could, however, in principle envision that players do not have such affine preferences over money. We could, for example, fix that $u(1.2) = 1.2$ and $u(0.2) = 0.2$ but choose $u(0.9)$ somewhere between 0.2 and 1.2. A person with $u(0.9) > 0.9$ could then be said to be risk-averse and one with $u(0.9) < 0.9$ would be risk-loving.[10] In the present context one could also imagine another reason why a person might have a $u(0.9) > 0.9$. Such a person, in place of an A or B type, might consider the experiment mostly a challenge to achieve coordination for A or B, and not actually care too much about the money

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[10]There is plenty of experimental evidence that people are risk averse even over small gambles (see e.g. Holt and Laury (2002), Barberis, Huang, and Thaler (2006), and Harrison and Rutström (2008)). This is normatively not very appealing as such people would have to be implausibly extremely risk averse over larger gambles (see e.g. Rabin (2000)).

involved.

Proposition 3.3. *The voting game with preferences for A and B types of* $u(0.2) = 0.2$ *and* $u(1.2) = 1.2$ *and* $u(0.9) = 1.0722$ *has a unique undominated attainable ESS with a probability of A types playing A (and B types playing B) of* $x = 0.5672$.

The proof of this proposition is omitted. It follows the same steps as the proofs of Propositions 3.1 and 3.2.

Proposition 3.3, thus, states that the voting game can be calibrated as to generate a unique undominated attainable ESS to perfectly match the observed average frequency $x = 0.5672$ of A types playing A and B types playing B.

4 Opinion Polls before Voting

In this section we study the opinion poll game with the understanding that, after the opinion poll results are publicly announced, the voting game is played.

4.1 The Game

This game is, thus, a two stage game with 14 players, 4 A and 4 B types and 6 C types. There are many possible strategies in this two stage game. Every single player has to choose one of four actions (vote for A, vote for B, vote for C, or abstain) in the first, the opinion poll, stage. Then, for the second stage, the players have to choose a contingent plan of voting for A, B, or C, or choosing to abstain for every possible outcome of the opinion poll stage. Payoffs depend only on which candidate ultimately wins the election and are given as in Table 1.

4.2 Attainable Equilibria

Given that the voting game has multiple equilibria, the two-stage game has many subgame perfect equilibria. In fact even the unframed game has many

attainable subgame perfect equilibria. We here follow the argument in Andonie and Kuzmics (2012) to restrict attention to a small subset of attainable equilibria. This restriction is also justified empirically as we shall see below.

Note first that an attainable equilibrium of the unframed two-stage game requires that after a tie between A and B in the opinion poll an attainable equilibrium of the unframed voting game has to be played. Whenever there is no tie between A and B in the polling stage, there is no such restriction, except that whatever voting behavior follows a certain unequal A-B polling outcome, the voting behavior after the permutation of this polling outcome permuting A and B, has to be the permuted voting outcome. For details see Andonie and Kuzmics (2012).

Of all these possible attainable subgame continuations the "simplest" and perhaps most focal seems to be such that whenever A has more votes than B in the opinion poll, A is chosen by all A and B types in the voting stage, and whenever B has more votes than A in the opinion poll, then B is chosen by all A and B types in the voting stage. This is also the assumed subgame continuation play in Andonie and Kuzmics (2012).

In fact this assumption is also somewhat justified by the experiment as Table 5 illustrates.[11]

| | | Obs. | fraction of elections won by | | |
			A	B	C
Poll Ranking	A leads B	24	0.8750	0	0.1250
	A-B-Tie	11	0.0909	0	0.9091
	B leads A	13	0	0.7692	0.2308
No Polls vs. Polls with Tie	no polls	48	0.0625	0.0625	0.8750
	polls and ties	11	0.0909	0	0.9091

Table 5: Polls determine the election outcome

We now turn to the question what voting equilibrium will be played after a tie in the opinion poll between A and B types. For this subgame, attainability of the unframed two stage game requires an attainable equilibrium of the unframed voting game to be played. There are, however, three such equilibria as we saw in Section 3.2. Unfortunately we do not have the detailed data for the behavior of voters in this case, but we do see from Table 5 that sometimes

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[11]One should also note that what matters to play in the first, the opinion poll, stage, is what players **expect** the continuation play to be in all possible subgames.

(in fact exactly once in eleven cases) the A candidate was elected after a tie between A and B in the polls.[12] This is not possible in the two pure strategy attainable equilibria of the voting game, but is consistent with the mixed strategy equilibrium. This is also the most preferred attainable equilibrium for the group of A and B voters and the only evolutionary stable equilibrium in the voting game (see previous section). We, thus, assume that this is the voting behavior that the players foresee for the second stage when they participate in the opinion poll in the first stage.

We shall term the behavior described in the previous two paragraphs the **focal voting behavior**.

Having solved (or assumed behavior for) the second stage voting game for every subgame, we can write the first stage opinion poll game in reduced form. Note that, under the given assumptions about subgame behavior, choosing to abstain in the opinion poll is equivalent to stating C for all types of players. Thus, each player of each type has three distinct pure strategies: state a preference for A, B, or C. The ultimate payoffs depend only on whether A or B has more votes in the poll or whether there is tie between the two in the poll. These payoffs are summarized in Table 6.

| total number | voter type | Poll Ranking | | |
		A leads B	B leads A	A-B-Tie
4	A	$ 1.20	$ 0.90	$ 0.33
4	B	$ 0.90	$ 1.20	$ 0.33
6	C	$ 0.40	$ 0.40	$ 1.24

Table 6: Payoffs in the polling game as function of the poll outcome

We can, then, finally turn to the equilibrium analysis of the behavior in the polls. Note that, unlike in the voting game, no player of any type has weakly dominated strategies. Consider, for instance, a C type and opponent behavior in the poll such that without her vote A will beat B in the poll by one vote. Then this C type's best strategy is to state a preference for B to create a tie between A and B and, thus, make a C win in the election much more likely. Also for an A or a B type there will be possible opponent strategy profiles that make C a uniquely best strategy.

An attainable strategy profile, again following Andonie and Kuzmics (2012), in this polling game, must be such that all A types use the same (mixed)

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[12]The B candidate never won after an A and B tie in the polls.

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strategy, all B types the same (mixed) strategy, and all C types the same (mixed) strategy. Additional restrictions induced by the symmetries of the unframed game are as follows. Let $x_i(j)$ denote the probability a player of type i attaches to stating a preference for candidate j, for any $i, j \in \{A, B, C\}$. Then we must have that $x_A(A) = x_B(B)$ and $x_A(C) = x_B(C)$ (implying $x_A(B) = x_B(A)$) and $x_C(A) = x_C(B)$. Thus, ultimately, there are three unknowns. These are w.l.o.g. $x_A(A)$, $x_A(B)$, and $x_C(C)$.

Proposition 4.1. *There are exactly three attainable equilibria of the opinion poll game with focal voting behavior and these are given by*

x	x_A	x_B	x_C
x^*	$(0.7955, 0.0723, 0.1322)$	$(0.0723, 0.7955, 0.1322)$	$(0, 0, 1)$
x^{**}	$(0.8872, 0, 0.1128)$	$(0, 0.8872, 0.1128)$	$(0.5, 0.5, 0)$
x^{***}	$(1, 0, 0)$	$(0, 1, 0)$	$(0.3965, 0.3965, 0.207)$

Proof: We need to find all attainable equilibria of the polling game given the assumed focal second stage voting behavior. In principle such equilibria may be in completely or only partially mixed or even pure strategies. We, thus, have to consider all possible support pairs. To give one example, there could be an equilibrium in which A types mix between A and C only (and, by attainability B types mix between B and C) while C types only play C. One would then get one equation from the indifference of A types between A and C and two inequalities as A types must then find playing B worse than (or equal to) playing A and C types must find playing A (and, thus, also B) worse than or equal to playing C.

Altogether there are three possible equilibrium supports to be considered for C types (play the "pure" strategy given by playing A and B with probability $\frac{1}{2}$ each, play pure strategy C, and mix between both). There are seven possible equilibrium supports for A types (B types then follow from attainability). These are pure strategies A, B, and C, mixing between two pure strategies AB, AC, and BC, and mixing between all three pure strategies ABC. Thus, there are twenty-one cases to consider.

For each case we write down the (polynomial of degree up to 7) equalities and inequalities induced by the case and use the Newton-method to find all solutions. The program is available as part of the supplementary material. This procedure is exact and provides us with the stated three attainable equilibria. QED

The expected payoff to all types in these three attainable equilibria are given in Table 7.

	expected payoff of	
Equilibrium	A / B	C
x^*	\$ 0.85	\$ 0.64
x^{**}	\$ 0.93	\$ 0.54
x^{***}	\$ 0.92	\$ 0.55

Table 7: Expected payoffs for every voter type in the three attainable equilibria of the opinion poll.

This reduced polling game also has non-attainable equilibria. For instance, all A and B types playing A and all C types playing C is an equilibrium, in which eventually A is elected. No player can in this case change the outcome by unilaterally deviating to some other strategy. Even six of the A and B type players playing A and the other two B or C and all C types playing C is an equilibrium, in which again no player can change the outcome (of A winning the election) by unilaterally deviating. There are many more non-attainable equilibria.

4.3   Evolutionary Stability of Attainable Equilibria

In this section we study the evolutionary stability properties of the three attainable equilibria.

Proposition 4.2. *Of the three attainable equilibria of the opinion poll game with focal second stage voting behavior, exactly two, x^* and x^{**}, are an ESS.*

Proof: Let us first consider equilibrium x^{***}. We shall look at the population of C types, which in this equilibrium mix between pure strategies A,B and C with probabilities $(0.3965, 0.3965, 0.207)$. Attainability restricts C types to attach the same probability on pure strategies A and B. Thus, attainable strategies for C types can be identified with one number $\alpha \in [0, \frac{1}{2}]$, the probability they attach to pure strategy A (and, thus, also to pure strategy B). They then must attach probability $1-2\alpha$ to pure strategy C. Fixing play of A and B types with their equilibrium play in x^{***} the C types are playing a six player symmetric game with two strategies. We can, thus, appeal to Lemma A.1, given in the Appendix, to check whether or not x^{***} is an ESS.

Figure 2 depicts the payoff to a C type, for playing A (same as B) and C, as a function of α, assuming type A and B players use their prescribed

mixed strategy in equilibrium x^{***}. This Figure, appealing to Lemma A.1, thus shows that the strategy profile x^{***} is not evolutionary stable. This is so because strategy A is the unique best reply to strategies α above the equilibrium value 0.3965 and is strictly worse than strategy C for values α below 0.3965.
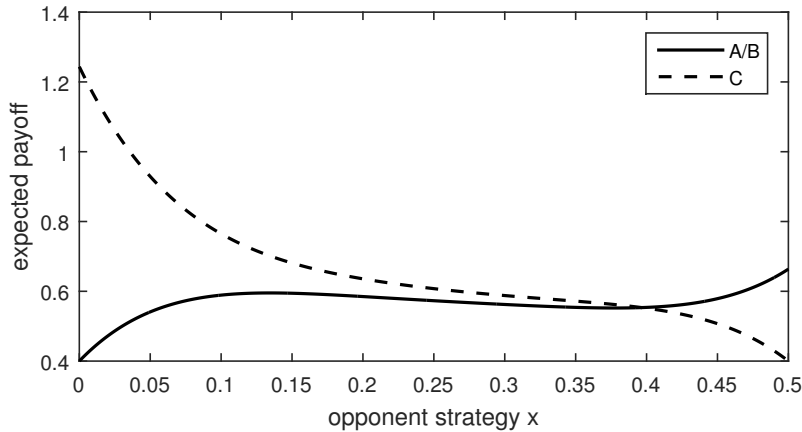

Figure 2: Payoff-difference between playing A or B (solid line) or C (dashed line) for a C-type as a function of α, where $(\alpha, \alpha, 1 - 2\alpha)$ is the assumed mixed strategy of other C types, assuming all A and B types play according to their prescribed mixed strategy in equilibrium x^{***}.

We now turn to equilibrium x^{**}. Figure 3 depicts the payoff to a C type, for playing A (same as B) and C, as a function of α, assuming type A and B players use their prescribed mixed strategy in equilibrium x^{**}. Note that, if, as prescribed in the equilibrium, all other C types play $\alpha = \frac{1}{2}$, then strategy C is strictly worse than strategies A and B. Thus, x^{**} is an ESS from the point of view of C types. We then need to turn to the A and B types in this equilibrium. Figure 4 shows the best response regions for an A type as a function of the mixed strategy assumed by all other A types (and all B types playing accordingly), assuming C types play according to equilibrium x^{**}. One can see from Figure 4, and appealing to Lemma A.1, that no mutant that places probability zero on strategy B can enter the equilibrium x^{**}. This is so because the unique best response to a strategy that puts more probability on A than x^{**} does, while B receives zero probability, is to play C, while the unique best response to a strategy that puts more probability on C than x^{**} does, while B receives zero probability, is to play A. To see that no other mutant can enter x^{**} either consider Figure 5, which shows that the payoff from playing the A types' part of x^{**} against all other A types playing

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y in the simplex (and all B types their corresponding mixed strategy), while C types play as prescribed in x^{**}, is always positive unless $y = x^{**}$.
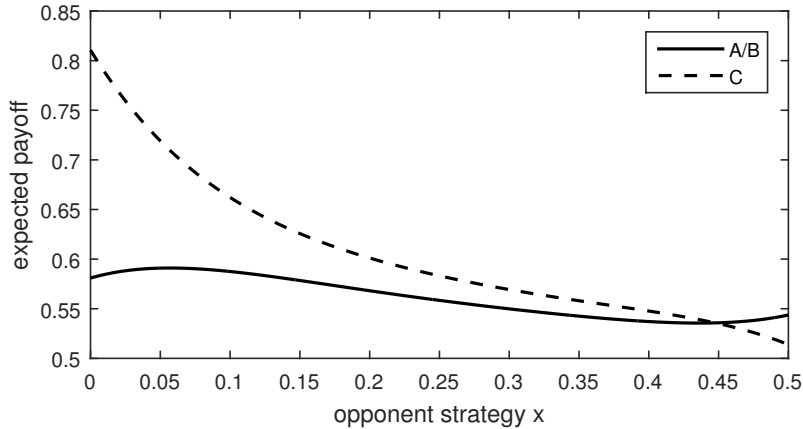

Figure 3: Payoff-difference between playing A or B (solid line) or C (dashed line) for a C-type as a function of α, where $(\alpha, \alpha, 1 - 2\alpha)$ is the assumed mixed strategy of other C types, assuming all A and B types play according to their prescribed mixed strategy in equilibrium x^{**}.

Finally, we explore the evolutionary stability properties of equilibrium x^*. This equilibrium is also an ESS. This can be seen partially from the best response regions given in Figures 6 and 7. Figure 7, in conjunction with Lemma A.1 shows that playing C for C types is evolutionary stable as C is the unique best reply to $\alpha = 0$ (i.e. to all other C types playing C). In Figure 6 we can only partially see that x^* is an ESS. For instance it is clear that a C mutant, who upon entering would lead us to region 1 of the picture would lead to C being the worst strategy. From Figure 6 The effect of mutants entering is, however, not clear for every possible mutant.[13] To fully see that x^* is an ESS we appeal to Lemma A.2 and Figure 8, which shows the payoff difference $u_A(x^*, y^{n-1}) - u(y, y^{n-1})$ which is positive everywhere and equal to zero only for $y = x^*$. \hfill QED

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[13]One can use Figure 6, however, to see that the best-response dynamics (see e.g. Gilboa and Matsui (1991), Matsui (1992), Hofbauer (1995), and Balkenborg, Hofbauer, and Kuzmics (2013)) would always lead to x^*.
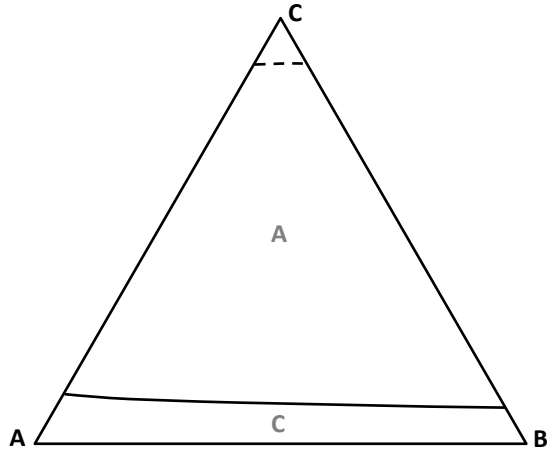
Figure 4: The simplex of mixed strategies for players of type A and the best response regions for an A type player if all other A types (and symmetrically B types) use a mixed strategy in this simplex, while all C types use their prescribed strategy in equilibrium x^{**}. The solid line through the simplex is the indifference line between strategies A and C. The dashed line is the indifference line between strategies B and C. There is no point in the simplex at which an A type is indifferent between A and B.

4.4   Empirical Results and Tests

The actual behavior in the opinion poll in the 48 experimental session with opinion polls is given in Table 8.

		fraction of votes cast by each type for each candidate			
Obs.	Type	A	B	C	Abst.
192	A	0.7552	0.1094	0.0313	0.1042
192	B	0.1771	0.6458	0.0469	0.1302
288	C	0.0833	0.0972	0.7014	0.1181

Table 8:   Empirical frequency of votes in opinion poll

We first test the null of attainability, that is we test independence in the relabelled Table 9.

Test 4.1. *The null hypothesis of attainability, for A and B types, that $x_A(A) = x_B(B)$ and $x_A(C) = x_B(C)$, cannot be rejected at the 5% level of significance. The χ^2 test of independence leads to a p-value of 0.1182 ($\chi^2 = 5.8677$).*
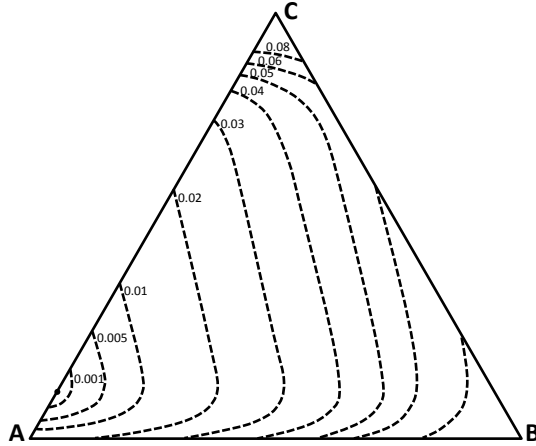
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Figure 5: This picture depicts $u_A(x^{**}, y^{n-1}) - u(y, y^{n-1})$ in terms of iso-utility lines as a function of y.

A few notes are in order here. The null of attainability is also the null of the absence of a ballot position effect. We, thus, cannot reject this null. Thus, at the 5% level of significance, there is no evidence that players condition their strategy on the ballot position. Forsythe, Myerson, Rietz, and Weber (1993) find some evidence of a ballot position effect. This evidence can be reproduced here if we restrict attention to votes cast for A and B only, ignoring those cast for C and abstaining. If we do this test we obtain a p-value of 0.048 ($\chi^2 = 3.91$), which would lead to rejecting the null at the 5% level of significance. This test, however, ignores the randomness in the total number of A and B votes (assuming this total to be exogenously given by the experimenter, which is not the case).[14]

It is clear that subjects are not playing anywhere close to ESS x^{**}. While play is very close to ESS x^* the null of x^* must nevertheless be rejected at the 5% level of significance.

Test 4.2. *The null hypothesis of attainable ESS equilibrium play, i.e. x^* as identified in Proposition 4.2, using a χ^2 goodness of fit test must be rejected at the 5% level of significance. The empirical frequency of play averaged for A*

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[14]Theoretically, one could well expect that subjects who play this game often eventually learn that they could use the ballot position to condition their strategy on. This would then allow the A and B types to be coordinated in more cases if not in all. While there is some suggestion that this is going on here, see also Forsythe, Myerson, Rietz, and Weber (1993) for a discussion, there is no strong evidence of this. It would have been interesting to see what would have happened if the same twenty-eight players had played this game recurrently for more periods.
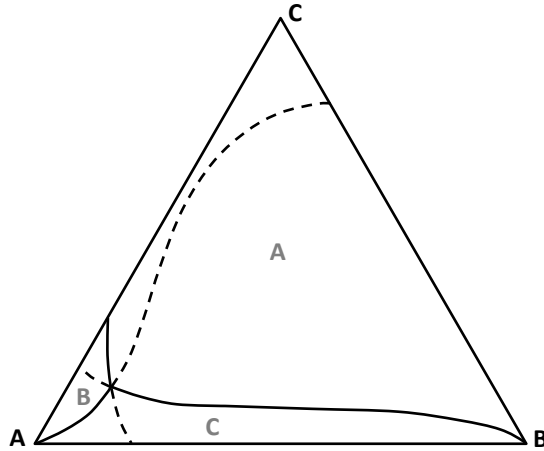
Figure 6: The simplex of mixed strategies for players of type A and the best response regions for an A type player if all other A types (and symmetrically B types) use a mixed strategy in this simplex, while all C types use their prescribed strategy in equilibrium x^*. The three lines in this picture are the indifference lines between each pair of pure strategies. The line emerging from the A corner is the indifference line between strategies B and C. The one emerging from the B corner is the indifference line between A and C. The remaining line is the indifference line between A and B. Suppose we label these six regions clockwise starting with 1 at the top (around the C corner), then in region 1 the A type has preferences $A \succ B \succ C$, in region 2 $A \succ C \succ B$, in region 3 $C \succ A \succ B$, in region 4 $C \succ B \succ A$, in region 5 $B \succ C \succ A$, and in region 6 $B \succ A \succ C$.

and B types is given by (0.7005,0.1432,0.1563) and the p-value is essentially zero ($\chi^2 = 32.7551$).

Aggregating across A and B types the data is "explained better" than with the x^* equilibrium by all $x \in \Delta(\{A, B, C\})$ indicated in Figure 9. Here we use the simulated likelihood approach introduced in chapter 3.4. The area of the grey "spot" is about 1.37% of the area of the simplex.

4.5   Risk averse players

We now return to the possibility of risk-averse players or players who for some other reason value the money amount of 0.9 relatively higher than under the assumption of an affine preference in money. In fact we shall here consider
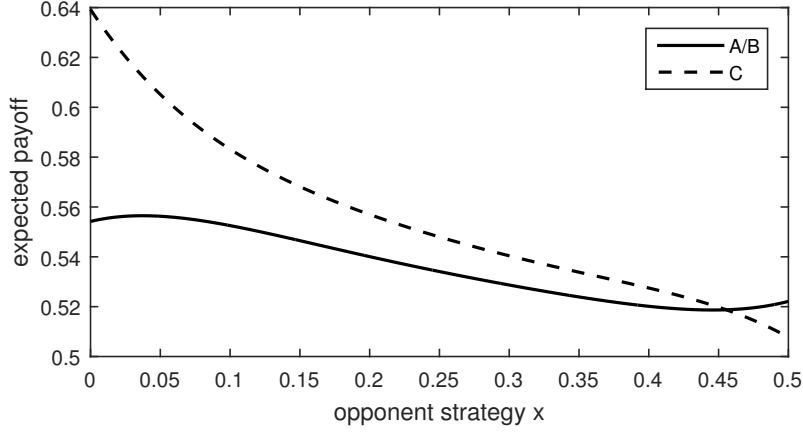
Figure 7: Payoffs for playing A or B (solid line) and C (dashed line) for a C-type as a function of α, where $(\alpha, \alpha, 1 - 2\alpha)$ is the assumed mixed strategy of other C types, assuming all A and B types play according to their prescribed mixed strategy in equilibrium x^*.

Obs.	Type	favorite	second favorite	C	Abst.
		fraction of votes cast by each type for each candidate			
192	A	0.7552	0.1094	0.0313	0.1042
192	B	0.6458	0.1771	0.0469	0.1302

Table 9: Empirical frequency of votes in opinion poll

the case where $u(1.2) = 1.2$, $u(0.2) = 0.2$, and $u(0.9) = 1.0722$, the value that perfectly calibrates or "explains" the outcome of the voting game alone (see Section 3.5).

Proposition 4.3. *There are two evolutionary stable attainable equilibria in the opinion poll under focal voting behavior, when A and B types have utility $u(0.2) = 0.2$ and $u(1.2) = 1.2$ and $u(0.9) = 1.0722$. One is given by $\tilde{\tilde{x}}$ such that $\tilde{\tilde{x}}_A = (0.8272, 0, 0.1728)$, $\tilde{\tilde{x}}_B = (0, 0.8272, 0.1728)$, and $\tilde{\tilde{x}}_C = (0.5, 0.5, 0)$, the other by $\tilde{x}$ such that $\tilde{x}_A = (0.6857, 0.1552, 0.1591)$ and $x_B = (0.1552, 0.6857, 0.1591)$ and $x_C = (0, 0, 1)$. There is a third equilibrium given by (0.9825,0.0175,0), (0.0175,0.9825,0), and (0.3943,0.3943,0.2114). It is not an ESS.*

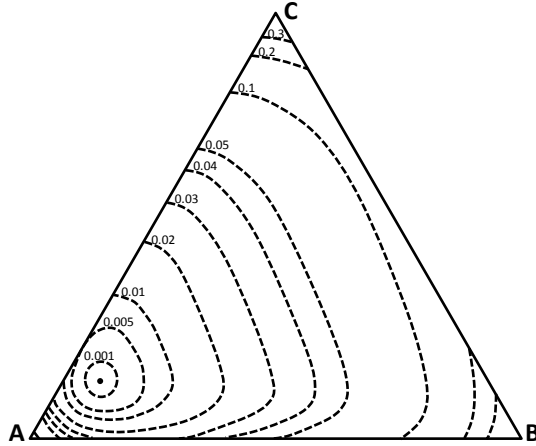The proof is omitted. It follows the steps of the proofs for Propositions 4.1 and 4.2.

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Figure 8: This picture depicts $u_A(x^*, y^{n-1}) - u(y, y^{n-1})$ in terms of iso-utility lines as a function of y.

Again, equilibrium $\tilde{\tilde{x}}$ is not at all consistent with the data. Equilibrium $\tilde{x}$ now is.

Test 4.3. *The null hypothesis of evolutionary stable attainable equilibrium $\tilde{x}$ of the polling game, when A and B types have utility $u(0.2) = 0.2$ and $u(1.2) = 1.2$ and $u(0.9) = 1.0722$, cannot be rejected at the 5% level of significance. The χ^2 goodness of fit test produces a p-value of 0.7799 ($\chi^2 = 0.4972$).*

The null can also not be rejected if we do this test individually for A and B types. The p-values are 0.09889 ($\chi^2 = 4.6276$) for A types and 0.4897 ($\chi^2 = 1.4277$) for B types.

5   Conclusion

What have we learnt from this paper? As the experiment was originally performed for a very different purpose, we first had to carefully identify what "standard theory" would predict in this game. It became clear that one had to take seriously certain symmetry restrictions, termed attainability by Crawford and Haller (1990), even though the game was presented to subjects in a way that also allowed play that violates these symmetry restrictions. As the game was played recurrently in the lab the appropriate equilibrium theory
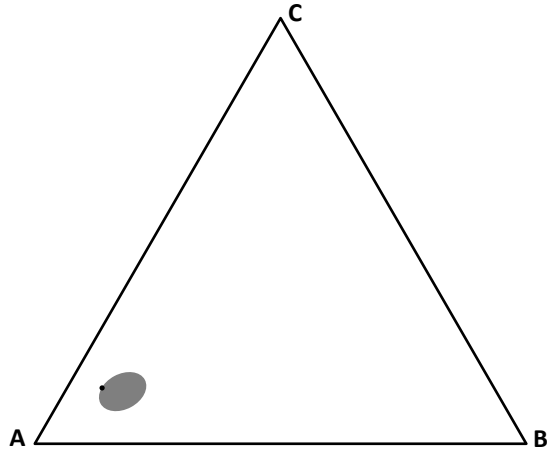
Figure 9: The grey "spot" indicates the set of all hypothesized $x \in \Delta(\{A, B, C\}$ for player types A and B that have a higher likelihood than equilibrium x^*).

is that of evolutionary stability. We, thus, needed to check the evolutionary stability properties of all equilibria we found.

In one treatment, the two-stage game with an opinion poll followed by a voting stage, we needed to identify the focal (perhaps simplest reasonable and empirically founded) attainable subgame continuation play in the voting games for all possible different outcomes of the opinion poll.

We then tested one appropriate "standard theory", in which it is assumed that players care only about the money they receive and do so in a risk-neutral way and play an evolutionary stable equilibrium of the voting game and the opinion poll game under the further assumption of focal voting behavior in the subgames following an opinion poll.

While this theory must be rejected at the 5% level of significance, it is nevertheless quite close to "explaining" the data.

We then reinterpret "standard theory" by only making one change to original notion in that we allow players to be risk-averse. We use one treatment, the voting game alone, to calibrate the risk aversion of the players to give a perfect fit to the data for this treatment. We then take this so calibrated "standard theory" to the treatment with opinion polls followed by a voting stage and test this theory there. It can not be rejected at the 5% level of significance despite the fact that we have close to 400 data points.

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All in all it strikes us that "standard theory" (derived from first principles appropriate to the setting) is not too far off "explaining" the play in this particular laboratory experiment.

A Evolutionary stability in symmetric n-player games

Let $\Gamma = (I, S, u)$ be a symmetric n-player normal form game. That is the set of players is given by $I = \{0, 1, ..., n\}$, where $n \geq 2$. The set of pure strategy profiles is given by $S = \times_{i \in I} S_i$, where S_i is the (finite) set of pure strategies and $S_i = S_j$, for all $i, j \in I$, i.e. every player has the same pure strategies. Finally, $u : S \to \mathbb{R}$ is the payoff function for every player with the understanding that $u(s_1, s_2, ..., s_n)$ is the payoff to a player if she plays pure strategy s_1, while the others play $(s_2, ..., s_n)$. This payoff function has the additional property that it is unaffected by all permutations of $(s_2, ..., s_n)$. Players evaluate mixed strategy-profiles by taking the expected utility. Let Δ denote the set of of mixed strategies, i.e. the set of probability distributions over S_i. For strategies $x, y \in \Delta$ we shall denote by $u(x, y^{n-1})$ the payoff of an x-strategist if all her $n - 1$ opponents play the same strategy y.

We shall use the notion of an **evolutionary stable strategy (ESS)** in the sense of Maynard Smith and Price (1973) and Maynard Smith (1982) (see (Weibull, 1995, Definition 2.1) for a textbook treatment) adapted to symmetric n-player games as in Palm (1984) and Broom, Cannings, and Vickers (1997).

Definition 1. *A (mixed) strategy $x \in \Delta$ of a symmetric n-player normal form game is an evolutionary stable strategy (ESS) if for all mutants $y \in \Delta$ with $y \neq x$ there is an $\bar{\epsilon} > 0$ such that for all $\epsilon \in (0, \bar{\epsilon})$ we have that*

$$u(x, w_\epsilon^{n-1}) > u(y, w_\epsilon^{n-1}),$$

where $w_\epsilon = (1 - \epsilon)x + \epsilon y$ denotes the post-entry mix of strategies.

For symmetric two-strategy n-player normal form game there is a simple equivalent condition for x, now $\in [0, 1]$, to be an ESS. See also (Broom, Cannings, and Vickers, 1997, Section 4) for a discussion of this case.

Lemma A.1. *A (mixed) strategy $x \in [0, 1]$ of a symmetric two-strategy n-player normal form game is an ESS if and only if there is an $\epsilon > 0$ such that*

for all $y < x$ with $x - y < \epsilon$ we have that $u(A, y^{n-1}) > u(B, y^{n-1})$ and for all $y > x$ with $y - x < \epsilon$ we have that $u(A, y^{n-1}) < u(B, y^{n-1})$.

Proof: Consider $x \in [0,1]$ and a mutant y with the property that $y > x$. Then there is an $\alpha \in (0,1)$ such that $y = (1-\alpha)x + \alpha 1$ (i.e. y is a convex combination between playing x and pure strategy A) and the ESS condition reduces to

$$
\begin{aligned}
u_A(x, w_\epsilon^{n-1}) &> u_A(y, w_\epsilon^{n-1}) \\
u_A(x, w_\epsilon^{n-1}) &> (1-\alpha)u_A(x, w_\epsilon^{n-1}) + \alpha u_A(A, w_\epsilon^{n-1}) \\
\alpha u_A(x, w_\epsilon^{n-1}) &> \alpha u_A(A, w_\epsilon^{n-1}) \\
u_A(x, w_\epsilon^{n-1}) &> u_A(A, w_\epsilon^{n-1}) \\
x u_A(A, w_\epsilon^{n-1}) + (1-x)u_A(B, w_\epsilon^{n-1}) &> u_A(A, w_\epsilon^{n-1}) \\
(1-x)u_A(B, w_\epsilon^{n-1}) &> (1-x)u_A(A, w_\epsilon^{n-1}) \\
u_A(B, w_\epsilon^{n-1}) &> u_A(A, w_\epsilon^{n-1}),
\end{aligned}
$$

where $w_\epsilon > x$, given that w_ϵ is a convex combination of x and y and $y > x$. The analogue steps can be made for a mutant $y < x$. $\qquad$ QED

For more than two player games, one can characterize an ESS in terms of first- and higher-order conditions. For two player games see e.g. (Weibull, 1995, Proposition 2.1) for fully characterizing an ESS in terms of a first-order (Nash equilibrium) and a second-order condition. (Broom, Cannings, and Vickers, 1997, p. 935) provide such conditions for n-player games, which one could call first to n-th order conditions. For our purposes a part of their result suffices.

Lemma A.2. (Broom, Cannings, and Vickers, 1997, p. 935) *A (mixed) strategy x of a symmetric n-player normal form game is an ESS if*

1. *$u(x, x^{n-1}) \geq u(y, x^{n-1})$ for all $y \in \Delta$ and*

2. *whenever $u(x, x^{n-1}) = u(y, x^{n-1})$ for some $y \neq x$ then $u(y, y^{n-1}) < u(x, y^{n-1})$.*

Lemma A.2, thus, only provides a sufficient condition for a strategy to be an ESS.

B   Indifference Condition for the Mixed Voting Equilibrium

In the voting game of Section 3.2 the equilibrium condition, $u_A(A, x) - u_A(B, x) = 0$, is given by a polynomial in x of degree 7, which can be written as[15]

$$0.35\mathbb{P}(Y = 1) + 0.35\mathbb{P}(Y = 2) - 0.5\mathbb{P}(Y = 5) - 0.5\mathbb{P}(Y = 6) = 0,$$

where

$$\mathbb{P}(Y = 1) = 4x^3(1 - x)^4 + 3x^5(1 - x)^2$$
$$\mathbb{P}(Y = 2) = 6x^2(1 - x)^5 + 12x^4(1 - x)^3 + 3x^6(1 - x)$$
$$\mathbb{P}(Y = 5) = 3x(1 - x)^6 + 12x^3(1 - x)^4 + 6x^5(1 - x)^2$$
$$\mathbb{P}(Y = 6) = 3x^2(1 - x)^5 + 4x^4(1 - x)^3.$$

As $x = 0$ and $x = 1$ are solutions one can reduce the condition by dividing by $x(1 - x)$ to obtain a polynomial of degree 5, given by

$$0.05(238x^5 - 601x^4 + 692x^3 - 440x^2 + 162x - 30) = 0.$$

The only real root of this polynomial in the interval (0,1) can be determined by Newton's method and is given by $x \approx 0.6615$.

References

ALOS-FERRER, C., AND C. KUZMICS (2013): "Hidden symmetries and focal points," *Journal of Economic Theory*, 148, 226–258.

ANDONIE, C., AND C. KUZMICS (2012): "Pre-election Polls as Strategic Coordination Devices," *Journal of Economic Behavior & Organization*, 84, 681–700.

ANDREONI, J. (1988): "Why free ride?: Strategies and learning in public goods experiments," *Journal of Public Economics*, 37(3), 291–304.

BALKENBORG, D., J. HOFBAUER, AND C. KUZMICS (2013): "Refined best reply correspondence and dynamics," *Theoretical Economics*, 8(1), 165–192.

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[15]The two functions, $u_A(A, x)$ and $u_A(B, x)$, are depicted in Figure 1.

BARBERIS, N., M. HUANG, AND R. H. THALER (2006): "Individual Preferences, Monetary Gambles, and Stock Market Participation: A Case for Narrow Framing," *American Economic Review*, 96(4), 1069–1090.

BERNHEIM, B. D. (1984): "Rationalizable Strategic Behavior," *Econometrica*, 52, 1007–29.

BINMORE, K., J. SWIERZBINSKI, AND C. PROULX (2001): "Does minimax work? An experimental study," *The Economic Journal*, 111(473), 445–464.

BLUME, A. (2000): "Coordination and Learning With a Partial Language," *Journal of Economic Theory*, 95, 1–36.

BLUME, A., AND U. GNEEZY (2000): "An Experimental Investigation of Optimal Learning in Coordination Games," *Journal of Economic Theory*, 90, 161–172.

———— (2010): "Cognitive Forward Induction and Coordination Without Common Knowledge: An Experimental Study," *Games and Economic Behavior*, 68, 488–511.

BROOM, M., C. CANNINGS, AND G. T. VICKERS (1997): "Multi-player matrix games," *Bulletin of Mathematical Biology*, 59(5), 931–952.

COOPER, R. W., D. V. DEJONG, R. FORSYTHE, AND T. W. ROSS (1990): "Selection Criteria in Coordination Games: Some Experimental Results," *American Economic Review*, 80, 218–233.

CRAWFORD, V., AND H. HALLER (1990): "Learning How to Cooperate: Optimal Play in Repeated Coordination Games," *Econometrica*, 58, 571–596.

DUVERGER, M. (1954): *Political Parties: Their Organization and Activity in the Modern State*. Wiley, New York.

FEHR, E., AND S. GÄCHTER (2000): "Cooperation and punishment in public goods experiments," *American Economic Review*, pp. 980–994.

FEY, M. (1997): "Stability And Coordination In Duverger's Law: A Formal Model Of Pre-election Polls And Strategic Voting," *American Political Science Review*, 91, 135–147.

FORSYTHE, R., R. MYERSON, T. A. RIETZ, AND R. J. WEBER (1993): "An Experiment on Coordination in Multicandidate Elections: the Importance of Polls and Election Histories," *Social Choice and Welfare*, 10, 223–247.

GILBOA, I., AND A. MATSUI (1991): "Social stability and equilibrium," *Econometrica*, 59, 859–67.

GÜTH, W., R. SCHMITTBERGER, AND B. SCHWARZE (1982): "An experimental analysis of ultimatum bargaining," *Journal of Economic Behavior & Organization*, 3(4), 367–388.

HARRISON, G. W., AND E. E. RUTSTRÖM (2008): "Risk aversion in the laboratory," in *Research in Experimental Economics*, ed. by J. Cox, and G. W. Harrison, vol. 12, pp. 41–196. Bingley, UK: Emerald Group Publishing Limited.

HOFBAUER, J. (1995): "Stability for the best response dynamics," Unpublished manuscript.

HOLT, C. A., AND S. K. LAURY (2002): "Risk aversion and incentive effects," *American Economic Review*, 92(5), 1644–1655.

HSU, S.-H., C.-Y. HUANG, AND C.-T. TANG (2007): "Minimax play at Wimbledon: comment," *The American Economic Review*, pp. 517–523.

MATSUI, A. (1992): "Best response dynamics and socially stable strategies," *Journal of Economic Theory*, 57, 343–62.

MAYNARD SMITH, J. (1982): *Evolution and the Theory of Games*. Cambridge University Press, Cambridge.

MAYNARD SMITH, J., AND G. R. PRICE (1973): "The logic of animal conflict," *Nature*, 246, 15–18.

MYERSON, R. B., AND R. J. WEBER (1993): "A Theory Of Voting Equilibria," *American Political Science Review*, 87, 102–114.

NASH, J. (1951): "Non-Cooperative Games," *Annals of Mathematics*, 52, 286–295.

O'NEILL, B. (1987): "Nonmetric test of the minimax theory of two-person zerosum games," *Proceedings of the National Academy of Sciences*, 84(7), 2106–2109.

PALACIOS-HUERTA, I. (2003): "Professionals play minimax," *The Review of Economic Studies*, 70(2), 395–415.

PALFREY, T. R. (1989): "A Mathematical Proof Of Duverger's Law," in *Models Of Strategic Choice In Politics*, ed. by P. C. Ordeshook, pp. 69–92. University Of Michigan Press.

PALM, G. (1984): "Evolutionary stable strategies and game dynamics for n-person games," *Journal of Mathematical Biology*, 19(3), 329–334.

PEARCE, D. G. (1984): "Rationalizable Strategic Behavior and the Problem of Perfection," *Econometrica*, 52, 1029–51.

RABIN, M. (2000): "Risk aversion and expected-utility theory: A calibration theorem," *Econometrica*, 68(5), 1281–1292.

SCHELLING, T. C. (1960): *The Strategy of Conflict*. Harvard University Press, Cambridge.

VANHUYCK, J. B., R. C. BATTALIO, AND R. O. BEIL (1990): "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure," *American Economic Review*, 80, 234–248.

WALKER, M., AND J. WOODERS (2001): "Minimax play at Wimbledon," *American Economic Review*, pp. 1521–1538.

WEIBULL, J. W. (1995): *Evolutionary Game Theory*. MIT Press, Cambridge, Mass.