Modelling and Estimation of Dynamic Instability in Complex Economic Systems

JUANXI WANG

This thesis sheds some lights on the ongoing discussions on modelling the economy as a complex evolving system. It introduces a complex systems approach and attempts to unfold the underlying mechanisms of dynamic instability in complex economic system. Moreover, it contributes to the ongoing discussions on early warnings for financial crises and the transmissions of complexity based economic policy. As a complex system, the economic system possesses the characteristics of complex systems. This thesis focuses on three features: critical transitions, catastrophe theory and expectations. The methodologies developed and applied include statistical time series tools and agent-based modelling.

Juanxi Wang received a Research Master degree in Quantitative Economics of Behaviours and Markets from Université Lumière Lyon 2, France (2011) after she earned two Bachelor degrees (BEng and BA) from Sun Yat-sen University in China. In 2011 she was awarded a PhD fellowship from the European Doctorate in Economics - Erasmus Mundus, which allows her to enrol as a PhD student at the University of Amsterdam in the Netherlands and Bielefeld University in Germany. Her main research interests include time series analysis, agent-based modelling, economic dynamics, complex systems and behavioural finance.

JUANXI WANG

Modelling and Estimation of Dynamic Instability in Complex Economic Systems

$$dx = f(x;c)dt + g(x;c)dW$$

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University of Amsterdam Dissertation

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Modelling and Estimation of Dynamic Instability in Complex Economic Systems

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Promotiecommissie:

Promotores: Prof. dr. C.G.H. Diks (Universiteit van Amsterdam)

Prof. dr. H. Dawid (Universität Bielefeld)

Copromotor: Prof. dr. C.H. Hommes (Universiteit van Amsterdam)

Overige leden: Prof. dr. S. Van der Hoog (Universität Bielefeld)

Prof. dr. L.H. Hoogduin (Rijksuniversiteit Groningen)

Dr. R.P.P.P. Grasman (Universiteit van Amsterdam)

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Chapter 1

Introduction and Outline

The instabilities of economic systems are of great concern to governments, central banks, and other regulatory agencies. In particular, in the recent succession of crises, we were faced with a worldwide virtual collapse of financial systems starting with the bursting of the housing bubble. The global impact of these crises has once more stressed the urgency of understanding the unstable nature of economic systems. Questions that often arose were: What are the causes of these extreme events and crises? How can we forecast and prevent them? The response was mainly to assume a universally representative ideal rational agent and to use its rational behaviour as a surrogate for aggregate behaviour. Its theoretical foundation is based upon the dynamic stochastic general equilibrium (DSGE) approach which only allows for equilibrium. In this conventional approach, the economic system is often described in a linear way and is defined as naturally stable. Any extreme events that diverge from calm periods were considered as outliers induced by large exogenous shocks.

Although these conventional approaches successfully explained some phenomena in a linear framework, they fail when it comes to modelling extreme situations in economic systems, such as dramatic changes or crises. Particularly, the global financial crises, which began in 2008, is seen as a stress test for DSGE models or even a challenge to their validity. It seems that something is missing from these conventional economic models that prevents them from represneting the extreme behaviour of economic systems. The then president of the European

central bank, Jean-Claude Trichet addressed the ECB Central Banking conference 2010 stating that

> *When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. Arbitrage broke down in many market segments, as markets froze and market participants were gripped by panic. Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools.*

In order to make major advances in explaining at least parts of the economy, new approaches to economics and finance are required. Jean-Claude Trichet further called for exploring complementary tools from complex systems theory to analyse financial markets and monetary policy transmission:

> *I would very much welcome inspiration from other disciplines: physics, engineering, psychology, biology. Bringing experts from these fields together with economists and central bankers is potentially very creative and valuable. Scientists have developed sophisticated tools for analysing complex dynamic systems in a rigorous way. These models have proved helpful in understanding many important but complex phenomena: epidemics, weather patterns, crowd psychology, magnetic fields. Such tools have been applied by market practitioners to portfolio management decisions, on occasion with some success. I am hopeful that central banks can also benefit from these insights in developing tools to analyse financial markets and monetary policy transmission.*

> *(Speech by Jean-Claude Trichet, ECB Central Banking Conference,*
> *Frankfurt, November 18, 2010)*

Because the application of the theory of complex system presents an important challenge and promising approach in economics, this thesis will build models and perform analyses of the economy as complex system. The analyses in this thesis will show that the complex systems approach helps to gain insights of the underlying nonlinear mechanisms of economic systems.

This thesis serves three important purposes. Firstly, it presents examples to modelling and estimation of complex economic systems. Secondly, it applies complexity quantitative tools, so that complexity mathematical and statistical methodologies are being employed in economic systems. Thirdly, it contributes to the current discussions on early warning systems in economics and the transmissions of economic policy based on complex system approach.

1.1 Complex Economic Systems

The idea to view the economy as a complex system already dates back to Adam Smith. He described the economy as an emergent process based on the self-organised behaviour of independently acting, self-motivated individuals. Anderson *et al.* (1988) described the possibilities associated with the application of complex systems ideas to economics. Some of the successes of early proposed research programs were presented in Arthur *et al.* (1997). Several authors have viewed the economy as a particularly good example of a complex system. For example, Arthur (1999) studied complexity economics and portrayed the economy as a general emergent structure with unfolding of patterns. Stanley *et al.* (2001) suggested that the economy was perhaps one of the most complex of all complex systems. Borland (2005) also wrote that "perhaps one of the most vivid and richest examples of the dynamics of a complex system at work is the behaviour of financial markets".

Conventional economic theory pays rather limited attention to the development of nonlinear patterns with multiple equilibria, but rather to simplify its questions to a static equilibrium in order to seek analytical solutions. On the contrary, complexity economics is a theory at a more general level with multiple equilibria. The dynamic instability of economic systems can

be understood better from a complex systems perspective. For instance, it has led to a better understanding of why stock or housing prices exhibit bubbles and crashes. In particular, the traditional models which are widely used in central banks have been proved to be not efficient in understanding economic crises. Complex system theory would be able to provide appropriate policy tools that could improve early detections. The policy would succeed better by allowing multiple equilibria of economic structures rather than by forcing static outcomes.

As a complex system, economy possesses the characteristics of universal complex systems. One important feature is that complex systems often exhibit large and abrupt changes that appear not to have an outside cause, but instead arise endogenously (Sornette, 2009; Scheffer *et al.*, 2009). These changes are called *critical transitions* or regime shifts in the complex system approach. Examples include epidemics, solar flares, earthquakes, landscape formation, neuronal avalanches in the cortex, and so on. Corresponding phenomena in economics are extreme events such as emergence of bubbles followed by the collapse of stock or housing markets. Once the complex system approach is applied, such extreme events are observed as a naturally integrated phenomena instead of outliers due to exogenous shocks. The mathematical mechanisms behind a critical transition is a *bifurcations*. Chapter 2 of this thesis considers critical transitions in financial markets and attempts to construct early warning signals through identifying characteristics of critical slowing down prior to financial crises.

A complex system can exhibit sudden collapse or behaviour described by *Catastrophe theory*, which is a branch of bifurcation theory. This sheds some lights on "a law of nature". It is particularly useful when qualitatively describing a nonlinear complex system with sudden, discontinuous changes or phase transitions as a result of slow changes in a control parameter. One of the advantages of this approach is that it allows statistical time series analysis. Chapter 3 in the thesis tries to explain housing market crashes using catastrophe theory and estimates a stochastic cusp catastrophe model based on actual housing market data.

Other important features in complex economic system are *expectations feedback* and *adaptive behaviour* between individuals. These are also key ingredients to distinguish complex

economic systems from complex systems in natural sciences. In complex economic systems, agents are boundedly rational and behave merely based on their expectations about the future. The equilibria of the market and the agents' adaptive behaviour co-evolve over time. Even simple examples may generate highly nonlinear complex structures, including bifurcations and even *chaos*. Chapter 4 of this thesis investigates the nonlinear dynamic behaviour of a financial market with boundedly rational mutual fund managers and investors. It aims to find the link between the types of expectations and compensation contractss.

1.2 Bifurcations in Complex Economic System

In a nonlinear dynamical system, when a structural instability leads to a qualitative change in the long-run dynamics of a system, a bifurcation occurs. More specifically, a bifurcation is normally interpreted as a qualitative change in a dynamical system as a control parameter varies. Being a highly nonlinear dynamical system, the economy may exhibit many bifurcations towards instability (Zeeman,1974; Brock and Hommes, 1997, 1998; Scheffer, 2009).

1.2.1 Critical Transitions

There are many different ways in which bifurcations can occur, even in very simple dynamical systems. Take the co-dimension-1 attractor bifurcations for example, Thompson *et al.* (1994) have classified them into three ways: safe, explosive and dangerous bifurcations. This classification is based on the continuity or discontinuity of the attractor path which governs the dynamical outcome that would be observed under a slowly changing control parameter. In safe bifurcations, there is no discontinuous change, merely the continuous growth of a new stable form. It includes the supercritical forms of local bifurcations and a global bifurcation (Thompson and Sieber, 2010b). An explosive bifurcation is a bifurcation with a discontinuously increasing attractor, which grows to a newly enlarged attractor along with itself. These are several kinds of global bifurcations which occupy an intermediate position between the

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safe and dangerous forms. The most commonly discussed bifurcation behaviours in complex systems primarily correspond to the dangerous form, in which the current equilibrium simply disappears, forcing the system to suddenly jump to an entirely different attractor equilibrium. Often on reversal of the control parameter, the response will remain on the path of the new equilibrium, giving rise to hysteresis. It includes local saddle-node bifurcations, subcritcial local bifurcations and some global events.
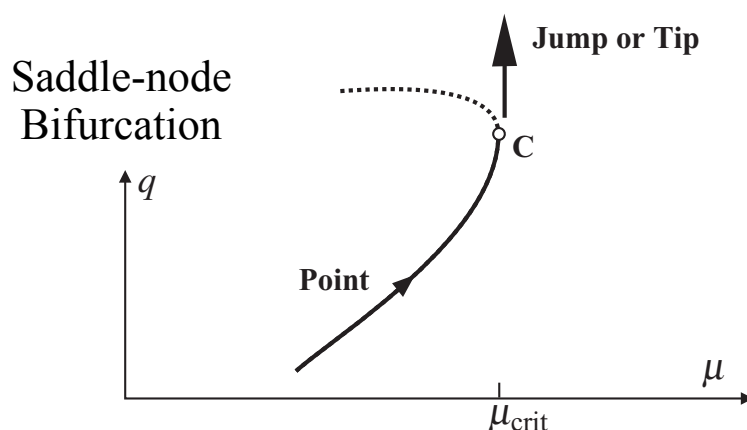
Figure 1.1: Illustration of an example of critical transition. μ represents control parameter. q is the variable. Source: Thompson and Sieber (2010b)

One of the simple examples of dangerous bifurcations, the saddle-node bifurcation is illustrated in Figure 1.1. It is also the most common bifurcation encountered in complex system research in different disciplines. The corresponding phenomenon is normally called a "*tipping point*", "*catastrophic bifurcation*" or "*critical transition*". Applications include abrupt shifts in ocean circulation or climate change, catastrophic shits in fish or wildlife populations, sudden asthma attacks, endogenous market crashes in stock or housing markets, and so on. Two of these evidences are shown in Figure 1.2. Figure 1.2 (a) illustrates the caribbean coral reef collapse around 1984 and indicated by a sudden increase of the percentage of algal cover. Figure 1.2 (b) represents an extreme event in the financial crises in 2008, the values of TED Spread suddenly jumped to an extremely high level.

Recognising them as critical transitions, market crashes in economic systems can now be

(a) Caribbean coral reef collapse   (b) 2008 financial crises
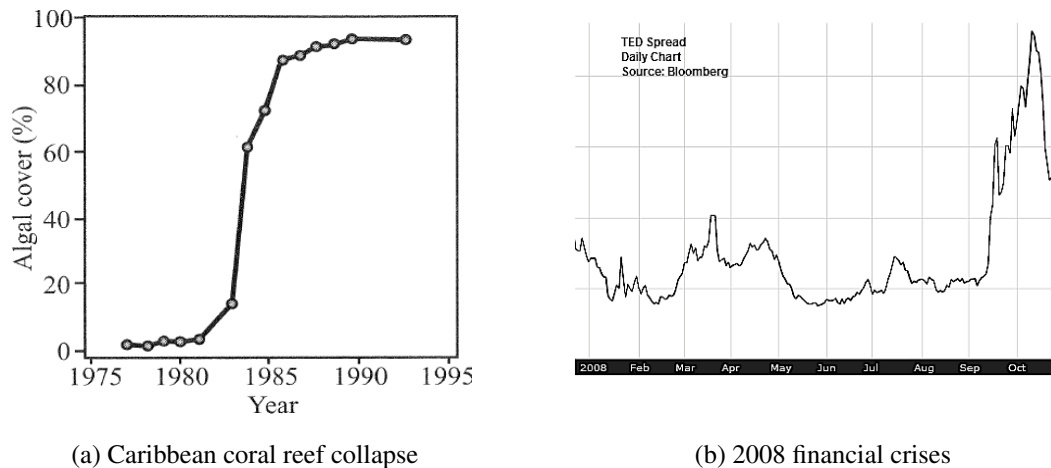
Figure 1.2: Examples of the critical transitions in different complex systems. (a) Caribbean coral reef collapse, indicated by a sudden increase of the percentage of algal cover. Source: Scheffer *et al.* (2009); (b) Financial crises in 2008, indicated by a sudden increase of the values of TED Spread. Source: Bloomberg

understood and analysed by applying the complex systems approach, using concepts from complexity theory to describe and model these phenomena. Foreseeing a critical transition and developing an early warning system is a great challenge in complex systems. Numerous efforts have been devoted to it in different disciplines for many years. Many of them have proved to be successful and have already demonstrated their value. The most important signal that has been suggested as indicator of critical transitions is related to a phenomenon known as "critical slowing down", which is illustrated by the increasingly slow recovering rate from perturbations. By detecting critical slowing down prior to extreme events, we might be able to predict the approaching of critical transitions (Held and Kleinen, 2014; Scheffer, 2009; Dakos *et al.*, 2008; Kefi *et al.*, 2013). Scheffer (2009) suggested the potential of this concept in complex economic systems. In principle, these time series methods have the potential to detect the thresholds statistically without knowing the specific underlying structures. Motivated by this, Chapter 2 of this thesis investigates whether it would have been possible to detect "critical slowing down" prior to several financial market collapses.

1.2.2 Catastrophe Theory

A critical transitions in complex systems is also called a "catastrophe bifurcation". It is described by catastrophe theory which is a branch of bifurcation theory. Catastrophe theory has been proposed to shed some lights on "a law of nature" and to describe phenomena with sudden transformations and divergences (Thom, 1972; Zeeman, 1976). During the last decades, it has proved to be a successful tool to describe the qualitative properties in a wide range of different complex systems, such as physics, engineering, biology, psychology, and sociology. All the observed systems show sudden, discontinuous changes or phase transitions as a result of continuous changes of control parameters. One of the extraordinary findings of catastrophe theory is that it describes the behaviour of nonlinear dynamical systems around the critical points and characterises it by seven simple "universal" forms with less than five control variables and one or two state variables (Thom, 1972; Zeeman, 1976; Gilmore, 1993). The simpler and widely studied form is the so-called cusp catastrophe model which has been applied frequently in social sciences.

Gilmore (1993) has characterised a number of *catastrophe flags* to describe cusp catastrophe quantitatively (see also Van der Maas and Molenaar, 1992; Van der Maas *et al.*, 2003 for comprehensive descriptions). Some of them are closely related to the hypotheses involved within the theory of market crashes in economic systems. The most important one is a *Sudden jump*, which is corresponding to market crashes in economics. It takes place when control parameters are crossing some thresholds. Beyond these thresholds, the state of the system suddenly jumps from one equilibrium to a completely new equilibrium. *Hysteresis* means that the sudden jump occur at different values of the normal variable. For instance, the jump from the first equilibrium to the second one occurs at a different value of the variable than the jump from the second equilibrium to the first one. *Multi-modality*, a multimodal distribution of the state of the system is likely to occur for a range of control parameters. In these regions, the predicted state values are multiple instead of one. *Inaccessibility* is strongly related to multi-modality. It implies that intermediate states between two opposite equilibria are rare. *Critical slowing down* means that

the state of the system takes a certain amount of time before returning to the equilibrium after a perturbation. The rate of decay could be used to detect an approaching catastrophe or a sudden transition.

The recent developments of statistical methods and stochastic forms of catastrophe theory have motivated the application of catastrophe theory to actual data in different complex systems further (Hartelman, 1997; Van der Maas *et al.*, 2003; Grasman *et al.*, 2009; Wilson, 2011; Poston and Stewart, 2014). Motivated by their successes, Chapter 3 studies the cusp model for the housing market. First, we study whether the cusp catastrophe occurs in the housing market. Second, a stochastic cusp catastrophe is fitted to the data. The advantages of this model are: firstly, it captures the general dynamical behaviour of complex system and allows for multiple dimensions; Secondly, it allows application of statistical time series analysis to actual data. Moreover, by understanding the underlying mechanisms of systems, we are able to foresee the future dynamical behaviours.

1.3 Expectations in Complex Economic Systems

Expectations feedback and adaptive behaviours among individuals are important features of complex economic systems. They are also the key ingredients to distinguish complex economic systems from complex systems in the natural sciences. Unlike the traditional neoclassical economic modelling approach which is based on rational expectations hypothesis (REH) and a representative structure with general equilibrium, the complex economic system approach is a behavioural approach using agent-based models in which the economy emerges as a multi-equilibrium complex evolving system with several heterogenous "boundedly rational" agents interacting with each other. In this framework, micro- and macro-behaviour co-evolve over time.

During the last decades, much work has been done on adaptive learning. See Sargent (1993), Evans and Honkapohja (2001) for reviews of the literature. A key underlying assumption is that

agents are not fully rational and do not know the actual "law of motion" of the economy. Instead, they are boundedly rational and use recently observed prices as an anchor to extrapolate the future price (Simon, 1982). Brock and Hommes (1997, 1998) proposed a concept of Adaptive Belief Systems (ABS). Its two important features are *bounded rationality* and *heterogenous expectations*. In these models, equilibrium prices and agents' adaptive expectations co-evolve over time. It shows that even in simple examples, the introduction of adaptive expectations leads to highly nonlinear complex structures. Unstable cycles, bifurcations and chaotic bubble and crash oscillations may arise.

These models have been tested empirically as well as in laboratory experiments with human subjects (Hommes, 2011, 2013; Hommes *et al.*, 2005; Boswijk *et al.*, 2007) and have been proven to be successful tools to analyse the nonlinear dynamical behaviour of economic systems (Hommes and Wagener, 2008; Boswijk *et al.*, 2007). Chapter 4 in this thesis establishes an agent-based model to describe the dynamical behaviour of a financial market with heterogenous agents and studies the influences of some financial factors to its dynamic instability.

1.4 Thesis Outline

This thesis is organised in three self-contained chapters following this introduction. Each has its own introduction and conclusion, and can be read independently from the other chapters. Chapter 2 and Chapter 4 focus on the applications on financial markets, while Chapter 3 focuses on housing markets. Concerning the methodologies applied, Chapters 2 and 3 employ econometrics or time series analysis, which seeks to find underlying structure in the macroeconomic data. Chapter 4 establishes an agent-based model in order to simulate and understand a simple financial market with heterogenous agents and test a financial policy.

In Chapter 2, a new approach to designing early warning signals (EWS) for financial crises is proposed to amend the failing alarm of recent financial crises by traditional financial models. Since the idea of constructing the EWS through identifying characteristics of critical slowing

down on the basis of time series observations is constantly supported by ample empirical and experimental evidence, we observe a number of historical financial crises in order to investigate whether there is indeed evidence for critical slowing down prior to market collapses. The four main events considered are Black Monday 1987, the 1997 Asian Crisis, the 2000 Dot.com bubble burst, and the 2008 Financial Crisis. The analysis in this chapter shows some evidences for critical slowing down before Black Monday 1987, while the results are mixed and insignificant for the other financial crises.

Chapter 3 fits a stochastic cusp catastrophe model to housing market data. Using the deviations of house prices from their fundamental value and quarterly data on long term government interest rates, we fit the stochastic cusp catastrophe model to different countries. The results suggest that cusp catastrophe model can be used to explain the instability of dynamical behaviour of housing market. The cusp catastrophe theory could also guide policy makers to conduct an appropriate interest rate policy.

Chapter 4 establishes an agent-based model to describe the dynamic behaviour of a financial market with mutual fund managers and investors under two types of compensation contracts: asset-based fees and performance-based fees. The fund managers are boundedly rational and with different expectations. They are trend chasers and contrarian traders respectively. The results show that trend chasers always trigger significant fluctuations in the market. Contrarian traders bring along slight up and down oscillations. The market with the performance-based contract is less stable and generates lower returns. We also find out that the inertia parameter decreases the stability of the market. In particular, the heterogenous analysis under different compensation contracts shows that asset maximisers dominate the whole market and produce higher returns. This result explains the current situation that most mutual fund companies favour asset-based compensation schemes.

Chapter 2

Critical Slowing Down as Early Warning Signal for Financial Crises?

2.1 Introduction

The recent financial crisis has intensified theoretical and empirical research on the underlying instabilities of economic systems. Several papers have been concerned with development of an early warning system that could give policymakers and market participants warnings on an upcoming financial crisis. However, most of the available contributions are based on "dynamic stochastic general equilibrium" (DSGE) models. These models seek to understand the whole economy with interacting markets under the assumption that the set of prices will result in an overall equilibrium. Unfortunately, when the crisis came, the serious limitations of such financial models became apparent. They were unable to capture the observed magnitude of stock price fluctuations and failed to provide predictions on several extreme events in financial history. A good understanding of dynamic behaviour of financial system is still lacking. The global financial crisis, which began in 2008, is seen as a stress test for DSGE models or even a challenge to their validity. It may be that something is missing from these conventional economic models that prevents them from describing the behaviour of financial markets. In November 2010, the

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then president of the European central bank, J. Trichet, addressed at the ECB Central Banking Conference stating that "Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner", and he stated "In the face of the crisis, we felt abandoned by conventional tools." (Trichet, 2010). The lessons of the financial crisis for macroeconomic and financial analysis are profound. They lead us to ask questions such as: What are the determinants of crises? Can crises be predicted? Can crises be avoided with sufficient early warnings? We need to develop complementary tools to improve the robustness of our overall framework.

During the last decades, a growing number of researchers have come to recognise that economic systems should be considered as complex systems (Sornette, 2009; Scheffer *et al.*, 2009, 2012; Ball, 2012; Farmer *et al.*, 2012). Unlike traditional economic theories under the assumption of general equilibrium, they describe the economic systems as multi-equilibrium processes, as in ecology or weather systems. They consider market crashes as mainly endogenously driven events. Therefore, these crashes could be predicted by studying the underlying mechanisms. Moreover, inspiration can be obtained from other disciplines where complexity theory has already been applied: ecology, physics, engineering, psychology, biology and so on. Within those fields, sophisticated tools for analysing complex dynamic systems have been developed. Those tools have proven to be helpful in understanding important complex phenomena in global weather forecasting, ecology, epidemiology, crowd psychology and so on.

Several researchers have recently applied complexity tools, and already demonstrated their value in economics and finance (Sornette, 2009; Quax *et al.*, 2013). Applying complexity approaches to financial systems can also help deepen our understanding on the dynamic behaviour of financial markets and (parts of) the economy. In this paper, we will apply concepts from complexity theory to real financial data related to historical crises for the first time. We will also investigate whether the crises considered can be linked to critical transitions of the corresponding financial system. In particular, we will investigate whether in principle it is possible to develop an early warning system based on a number of indicators for different types of

historical crisis.

The exploration of indicators of critical transitions in complex systems has been quite fruit-ful in other disciplines. Important signals that have been suggested in the complexity literature as an early warning indicator are measures of "Critical slowing down". This approach is based on the slowing down of the dynamics of a complex system approaching a critical point. Several authors developed methods to extract signals of critical slowing down from time series data. Kleinen *et al.* (2003) observed that power spectral properties changed as the earth system moved closer to a bifurcation point in a hemispheric thermohaline circulation (THC) model. Held and Kleinen (2004) used a trend in the decay rate of climate sub-systems as an indicator of critical slowing down. The first-order autoregressive coefficient obtained from time series was used as a measure of the decay rate of the system. This method was applied to North Atlantic THC model, providing an early warning signal for the climate system. Livina and Lenton (2007) developed another way of detecting critical slowing down by using detrended fluctuation analysis (DFA). This analysis was originally developed by Peng *et al.* (1994) to detect DNA sequences' long-range correlations. Livina and Lenton (2007) found an early warning signal for an upcoming critical transition in the North Atlantic THC system by investigating model output, as well as Greenland ice core paleotemperature data. The subsequent work by Dakos *et al.* (2008) was the first to study critical slowing down in empirical time series. The results showed increased autocorrelation in eight climate time series prior to critical transitions. Lenton *et al.* (2012) contributed to the early warning signal literature by offering some general guidelines for choosing the parameter settings of the analysis. They improved the robustness of ACF and DFA techniques and gave additional examples showing evidence of critical slowing down in both palaeodata and climate model output. Recently, Kefi *et al.* (2013) expanded the theory of critical slowing down to a broader class of situations where a system becomes increasingly sensitive to perturbations even without catastrophic transitions. They showed that critical slowing down could even be used in a more general sense as an early warning signal.

The possibility of applying early warning tools to financial data was suggested by Scheffer

et al. (2009, 2012). Encouraged by the successes of early warning signals in many complex systems, some efforts have been made to explore the possibility of contructing early warnings from financial time series. For instance, by using information dissipation length (IDL) as indicator, Quax *et al.* (2013) detected early warning signals prior to Lehman Brothers collapse in both USD and EUR interest rate swaps (IRS). They suggested that the IDL may be used as an early warning signal for critical transitions. Tan and Cheong (2014) observed critical slowing down in the U.S. housing market. They detected strong early warning signals associated with a sequence of coupled regime shifts during the period of subprime mortgage loans transition and the supreme crisis. They also found weaker signals during the Asian financial crisis and technology bubble crisis. However, up until now, no evidence of early warning signal has been found in time series data of stock markets. To fill this gap, this paper attempts to apply complexity theory of "critical slowing down" in real financial data for the first time. Four financial crises are analysed: Black Monday 1987, the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. The sample lag-1 autocorrelation and variance are considered as early warning indicators to examine whether financial systems slow down before the critical point is reached.

This paper is outlined as follows. In Section 2.2 we provide some theoretical background of nonlinear dynamical systems. We describe the data and methodology in Section 2.3 and 2.4, respectively. Subsequently, we analyse how well such early warning indicators perform when applied to financial time series. The results of our analyses are presented and discussed in Section 2.5. Section 2.7 provides a summary and conclusions.

2.2 Theory

The proposed mechanism driving critical transitions in complex systems mathematically is a *bifurcation*, which is an abrupt qualitative change in the dynamical systems when one or more control parameters change. Thompson *et al.* (1994) reviewed different types of bifurcations in dissipative dynamical systems. Three categories are classified based on their continuous or dis-

continuous dependence on the control parameters. These categories consist of safe bifurcations, with continuous growth of a new stable attractor, and explosive bifurcations, with a discontinuous growth to a newly enlarged attractor along with itself, and dangerous bifurcations, in which the current attractor simply disappears, forcing the system to jump in a fast dynamic transient to a remote and entirely new attractor. Figure 2.1 illustrates this classification visually. The critical transitions in complex systems are often considered to be dangerous bifurcations (Thompson *et al.*, 1994; Sieber and Thompson, 2012).
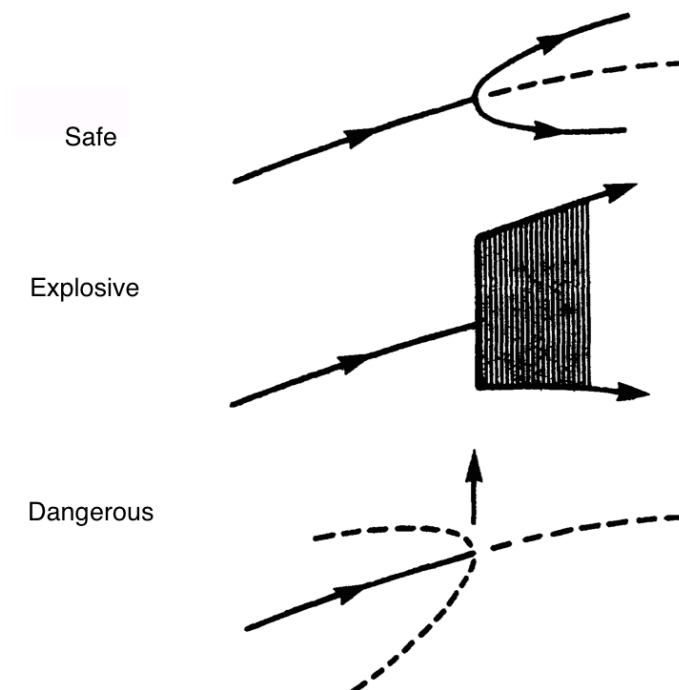
Figure 2.1: Classification of bifurcations according to their outcome (Source: Thompson *et al.* (1994)).

Figure 2.2 provides an example of a saddle-node bifurcation. With the increase of a single control parameter, a critical point is approached. Even a small perturbation would lead to a large qualitative change of the dynamics when the system is very close to this critical point. Once this threshold is exceeded, the whole system transits toward a different attractor. Even if the control parameter is reversed, the response will typically remain close to the new attractor. The

dynamics will not automatically return to the original attractor. This highlights the irreversibility of critical transitions. Scheffer *et al.* (2009, 2012) proposed that this bifurcation can also be used to describe the dynamical behaviour of financial systems, systematic crashes in stock markets for instance. This suggests the possible use of applying complexity theory to financial systems.
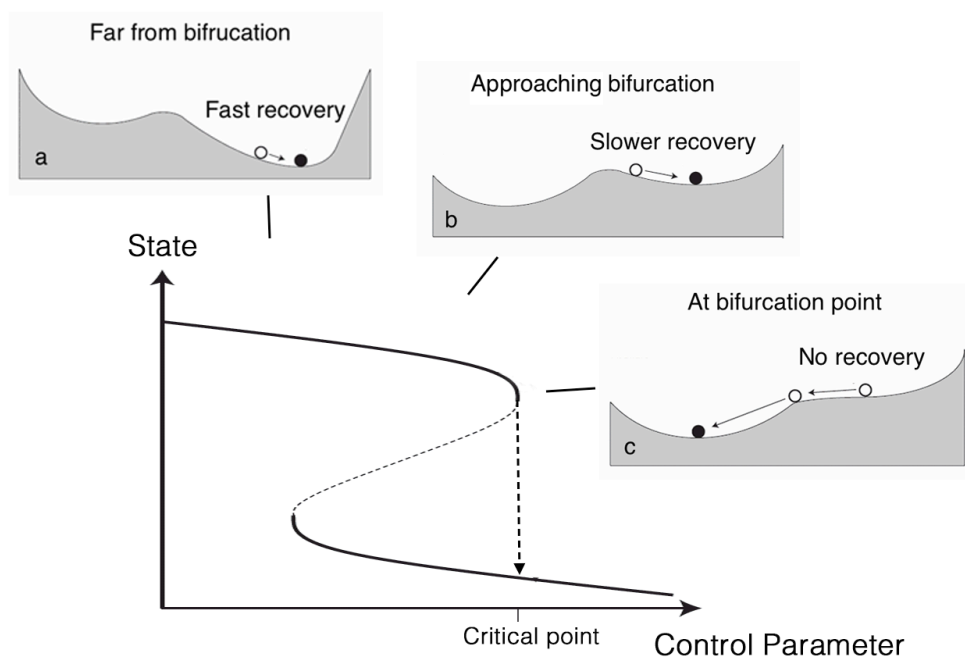
Figure 2.2: A saddle-node fold (Source: Lenton, 2011). Panels *a*, *b* and *c* describe the critical slowing down as an early warning indicator that the system lost resilience on the way to the critical point. Local minima represent stable attractors while the ball shows the present state of the system. (*a*.) Far from bifurcation: small variance and fast fluctuations. (*b*.) Approaching the bifurcation: larger but slower fluctuation with increasing variance; (*c*.) At the bifurcation point: irreversible transition to a new local minimum.

To develop tools to forecast critical transitions based on time series, it is necessary to assume that the observed time series is generated by a rather general nonlinear dynamical system. In the early warning literature it is common to assume that the system is one-dimensional and that the bifurcation of interest can be represented in its normal form, is driven by Gaussian white

noise, and has drift control parameter ρ:

$$dx = f(x, \rho)dt + g(x, \rho)dW, \tag{2.1}$$

where x is the state of the system, $f(x, \rho)$ determines the deterministic part of the system, while $g(x, \rho)dW$ determines the stochastic part. dW is a white noise process. The equilibrium of the undisturbed system is stable in all directions while approaching the critical value. By varying the control parameter ρ, the system reaches a threshold. When a real eigenvalue of the Jacobian matrix $DF_\rho(\bar{x})$ of the steady state finally crosses +1 (with the other real eigenvalue smaller than 1 in absolute value), a saddle-node bifurcation occurs. This bifurcation is corresponding with a critical transition in a time series. This scenario is descried in detail in Rahmstorf (2001), Lenton *et al.* (2008) and Sieber and Thompson (2012). It offers ways to provide early warnings before the critical transition actually happens. As long as we understand the statistical properties of the system approaching a critical transition, we may predict the time of the transition in advance, up to some estimation uncertainty.

Several earlier attempts have been made to develop an early warning system to monitor the risk of financial systems, such as binomial/multinomial logit/probit models (Berg and Patillo, 1999; Kolari *et al.*, 2002; Bussière and Fratzscher, 2006), multivariate probability models (Demyanyk and Hasan, 2010), Markov switching models (Abiad, 2007; Abiad, 2003), binomial tree approaches (Davis and Karim, 2008), and so on. However, their failure to give warnings ahead of the financial crisis in 2008 makes their predictive ability questionable. At the same time, the exploration of indicators of critical transitions in complex systems has been quite successful in other disciplines. Important signals suggested in the literature as early warning indicators are related to critical slowing down; when a dynamical system approaches a critical point, we can expect it to become increasingly slow in recovery from small perturbations. This is characterised by the linear decay rate decreasing to zero. The theory of critical slowing down used to describe this phenomenon, is illustrated in Figure 2.2. Panels *a*, *b* and *c* of Fig. 2.2 show the behaviour of a dynamical system approaching a saddle-node bifurcation. The local minima

of the potential well represent stable attractors and the ball shows the present state of the system. While approaching the bifurcation point, the local minimum on the right becomes shallower, and the recovery of the ball in response to small perturbations is increasingly slowing down. When this local minimum finally disappears, the ball quickly rolls into the minimum on the left. This implies that the system transits into a different steady state. The mechanisms behind this behaviour can be explained in mathematical terms. Approaching a saddle node bifurcation, the maximum real part of the eigenvalue of the Jacobian matrix tends towards 1. It indicates a slower recovery from perturbations (Kuehn, 2011 and Wagener, 2012). Therefore, as long as we are able to detect a signal of slowing down, it would be possible in principle to predict future critical transitions with some accuracy.

In what follows, by exploring critical slowing down prior to some extreme events in financial history, we would be able to assess whether such early warning signals might be possible for financial time series.

2.3   Data

For the analyses time series from subsystems of the economy which clearly indicate critical transitions from one state to another are needed. Stock market prices rather than returns have clear value for helping us forecast these potential transitions. Therefore, time series of daily stock market prices are taken as the dynamical subsystem to be analysed in our work. As long as we find early warning signals in the stock market price system, we are able to detect the approach of critical transitions.

Four financial crises are analysed: Black Monday (October 19, 1987), the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Although the direct causes of these crisis are different, they share the common characteristic that the stock prices for these events displayed similar bubble and burst patterns. Cusp catastrophe theory may possibly be used to describe these critical transitions in financial systems.

A particular time series is studied for each crisis based on its characteristic. For instance, the most popular time series data of stock prices in the literature is the Standard & Poor 500 (S&P 500) index. Therefore, we employ it to detect the early warning signals prior to Black Monday 1987 and the 2008 financial crisis. The Hongkong Hangseng index is the best candidate to describe the Asian events, so we use Hangseng index to analysis the Asian Crisis. The Dot-com Bubble is an information technology crisis which is boosted by the rapid growth of equity values in the internet sector. Thence, the subsystem we choose is NASDAQ Composite. It is related with technology companies and growth companies. The most recent 2008 financial crisis was followed by a credit crisis. In addition with the S&P 500, we analysed the TED spread, since it is an indicator of perceived credit risk. Moreover, as the volatility index of the S&P500, the VIX index is also analysed. Detailed information on data is presented in Table 2.1.

Daily time series data of stock prices, such as the S&P 500 index, the NASDAQ composite and the Hangseng index, were downloaded from Thomson Reuters Datastream for the period from May 1986 until May 2011. The TED spread is derived by calculating the differences between the three-month LIBOR and the three-month T-bill interest rate. These datasets are also available from Datastream. The Volatility index VIX is downloaded from the online Chicago Board Options Exchange (CBOE) database[1] (`http://www.cboe.com/micro/vix/historical.aspx`).

Since we are interested in extracting early warning signals prior to a critical transition, the time series before each transition are carefully selected. The critical transition points are determined by visual inspection according to original records in international news or articles (such as Wikipedia). For the examples that do not have information on timing, we simply choose the points with maximum value in the corresponding period. Since the random growth in stock prices data is in percentage term and not in absolute term, we take logarithms of data. Doing so also linearises the exponential growth in original series and stabilises the variance of the analysed residuals.

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[1]The largest U.S. options exchange and creator of listed options, which offers equity, index and ETF options, including proprietary products, such as S&P 500 options (SPX) and options on the CBOE Volatility Index (VIX)

Crisis	Time	Time Series	Sample Size (N)
Black Monday	1987	S&P500 index	200
Asian Crisis	1997	Hangseng index	500
Dot.com	2000	NASDAQ composite	400
2008 Financial Crisis	2008	S&P500 index	398
		TED spread	280
		VIX index	80

Table 2.1: Summary of financial time series

2.4 Methodology

2.4.1 Detrending

In order to achieve a stationary stochastic process, the first step is to remove the trend pattern from the original time series. The residuals after detrending are further analysed by using linear and nonlinear time series techniques. Subtracting a moving average is the most commonly used technique in detrending. In this paper, we introduce the weighting scheme and use gaussian kernel smoothing. This allows data near the given time point to receive larger weights.

In the analysis of many other complex systems, additional interpolation is needed to have evenly spaced time series. This may produce spurious results (Dakos *et al.*, 2008). Because stock market system can be already considered as a time discrete dynamical system, which is with fixed time-step $\Delta t = 1$ trading days, we skip the interpolation and therefore avoid its possible adverse effects. We detrend time series using a gaussian kernel,

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}, \tag{2.2}$$

based on which the moving average is given by

$$MA_j = \frac{\sum_{i=1}^{N} G(i-j)z_i}{\sum_{i=1}^{N} G(i-j)}, \tag{2.3}$$

where z is the logarithm of original price index with fixed time-step $\Delta t = 1$. The bandwidth is σ.

It is also the standard deviation of the Gaussian kernel distribution. In kernel density estimation, the choice of bandwidth is very important. A large bandwidth would leave out possible details of the trend, underestimate the magnitude of peaks and troughs, and produce an oversmoothed estimate. Therefore when a large bandwidth is used, the estimator may have a large bias. Taking account of this, people may want to choose a bandwidth as small as the data allows. However, there is always a trade-off. When a too small bandwidth is applied, this may result in only a few local data points available to reduce the variance of the estimator, which will reduce the reliability of the estimation results. In this paper, we choose the bandwidth in such a way that we filter out the slower trends visible in the data but still keep the details of the records. Dakos *et al.* (2008) had obtained reasonable results by setting the bandwidths around 10 to 25. In this paper, following Dakos *et al.* (2008), we use bandwidth $\sigma = 10$ and test them with $\sigma = 20$. By subtracting moving average from the logarithm of the original time series, the residuals time series is then given by

$$y_j = z_j - MA_j, \tag{2.4}$$

which fluctuates around 0.

2.4.2 Leading Indicators

An increase in autocorrelation and variance is expected for critical slowing down. Whether this is the case can be estimated using a moving estimation window. Three early warning indicators are considered: AR(1), ACF(1) and Variance.

AR(1) indicator   When approaching a critical transition, consecutive observations in the state of the system become increasingly similar to each other. The lag-1 autocorrelation is considered as the leading indicator to measure critical slowing down. It can be estimated from an autoregressive model of lag-1:

$$f(t_{j+1}) = e^{-\kappa\Delta t} f(t_j) + \theta_\eta \eta_j, \tag{2.5}$$

which is known as an AR(1) model. $f(t_j)$ is the residual y_j in Equation (2.4). $\Delta t = t_{j+1} - t_j$ and η_j is a zero mean innovation. κ indicates the magnitude of the recovery rate. $\lambda = e^{-\kappa\Delta t}$ is the AR(1) coefficient, which is the autoregressive coefficient at lag1. In a saddle-node bifurcation scenario, κ vanishes on the way to the bifurcation point. As κ goes to 0, λ goes to 1.

The random disturbances are assumed to be white noise with zero mean and variance equal to 1. Therefore, the first order autocorrelation coefficient λ is approximated as constant in a local time window of length w. We estimate λ by an ordinary least-square (OLS) fitting method of the regression

$$y_{k+1} = \lambda y_k + u_k, \tag{2.6}$$

with u_k white noise, over the set of indices $k = j - w + 1, \ldots, j$. The local window slides from left to right and traces a series of AR(1) coefficients varying with respect to index. This new series can be interpreted as the time-varying AR(1) coefficient. If it increases, this indicates that the system is driven gradually closer to a bifurcation.

Apart from the bandwidth, the window size is also a very important parameter. A smaller window size allows us to track short term changes in autocorrelation. However, taking a too small window size with very few observations will make the estimation of autocorrelation less reliable. Dakos *et al.* (2008) used half the size of the analysed time series. In this paper, following them, we also choose half the size of analysed time series as the sliding window size.

ACF(1) indicator An alternative and more straightforward way to estimate autocorrelation at lag-1 is by using the first value of the autocorrelation function (ACF)

$$\rho_1 = \frac{E[(y_t - \mu)(y_{t+1} - \mu)]}{\sigma_y^2}, \tag{2.7}$$

where μ is the mean of y_t in the window considered, and σ_y^2 the variance. Like AR(1) indicator, the moving window produces a proxy series of ACF(1). It also serves as an indicator to detect critical slowing down prior to a critical transition.

Variance indicator   An increased slowing down also induces an increased amplitude on the way of approaching threshold. This amplitude corresponds with the variance and is measured by standard deviation:

$$St.Dev. = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(y_t - \mu)^2}. \tag{2.8}$$

This proxy series of variance produced by moving window also serves as one of the early warning indicators preceding a critical transition.

2.4.3   Estabishing Trends

For each indicator observed over time, we test the trends for significance using the noparametric Kendall rank correlation τ between the indicator and time variable (Dakos *et al.*, 2008). It is a statistic tool used to measure the degree of concordance between two pairs of ordinal variables:

$$\tau = \frac{C-D}{N},$$

where C is the number of concordant pairs, D is the number of discordant pairs, $N = n(n-1)/2$. The quantity τ is in the range of $[-1, 1]$. If Kendall's τ is close to 1, the agreement between the two rankings is perfect. A high Kendall's τ suggests a strong trend. In this paper, a strong upward trend which is measured by positive Kendall's τ is expected.

2.5 Results

The evidence for early warning signals is evaluated in two steps. Firstly, we observe the early warning signals before real critical transitions. By using six time series, we examine four well-known extreme financial events in history - Black Monday 1987, the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Secondly, we examine the likelihood of spurious early warnings. The probability of obtaining similar or more extreme early warning signals by chance is estimated using bootstrap time series. In the end, we perform an extensive analysis to examine the robustness of the results with respect to the choice of user-set parameters.

2.5.1 Financial Time Series

First of all, we examine whether there is evidence of critical slowing down in time series data of stock prices. Four financial crisis are investigated.

Black Monday 1987 During a single day, the Dow Jones Industrial Average (DJIA) index lost almost 22%. By the end of that month, most of the major exchanges had dropped by more than 20%. Stock markets around the whole world crashed, beginning in Hong Kong, spreading to Europe, and hitting the United States later after other markets had declined by a significant margin. This event marked the beginning of a global stock market decline, making "Black Monday" one of the most tragic days in recent financial history.

Figure 2.3 shows the analysis of early warning indicators around half a year preceding "Black Monday" by using Standard & Poor 500 index (S&P500) time series. The original time series in Figure 2.3(a) is the logarithm of the daily S&P500 index. It starts from 200 days before the crash and ends to 100 days after it. Stock markets raced upward during the first half of 1987, but experienced a great depreciation in the last few months. The vertical dashed line identifies the critical transition in the time series. Since we are interested in searching for early warning signals before the critical transition, analysis is based on the data before the dashed line. To facilitate explanation, we set the x-axis of the critical transition as 0 to clearly distinguish

the days before and after it. The smoothed central line shows the smoothed time series used for filtering. The dashed arrow shows the width of the moving window. Half the size of the time series is taken as the moving window, following Dakos *et al.* (2008). Figure 2.3(*b*) shows the residuals, that is, the detrended time series used to estimate the early warning indicators.

Figure 2.3(*c*), (*d*) and (*e*) give examples of early warning indicators of AR(1), ACF(1) and Variance. They show that the great crash on "Black Monday" is preceded by an overall upward trends in these indicators. All of these positive trends are confirmed by a positive Kendall rank correlation coefficient τ. Therefore, the examples of early earning indicators showed an increase in the period preceding the critical transition, suggesting that the S&P500 time series indeed slows down before the critical transition.

The Asian Crisis Using the same techniques, we examine the Hangseng time series. Figure 2.4 shows the analysis of early warning indicators around one and a half year before the Asian Crisis. Figure 2.4(*a*) is the logarithm of daily Hangseng index from November 1995 to July 1998. This time series increases in the beginning but collapses around mid-1997, which illustrates the Asian financial crisis in July 1997. The Asian Crisis is a series of currency devaluations along with stock markets declining. The currency market first failed in Thailand because its government no longer pegged their local currency, the Thai Baht, to the U.S. dollar. The currency crisis rapidly caused stock market declines spreading throughout South Asia. Thailand, South Korea and Indonesia were the countries most affected by the crisis. As a result of the crisis, the stock markets in Japan and most of Southeast Asia fluctuated dramatically.

Figure 2.4 has a similar format as Figure 2.3. The smoothed central line in Figure 2.4(*a*) shows the moving average used for filtering. The dashed arrow shows the width of the moving window, which is the half size of the analyzed time series length. Figure 2.4(*b*) shows the remaining residuals used to estimate the early warning indicators.

The examples of upward trends in indicators of AR(1), ACF(1) and Variance shown in Figure 2.4(*c*), (*d*) and (*e*) indicate critical slowing down before critical transitions. All of the trends are significant as measured by Kendall's τ. The increased slowing down before July 1997
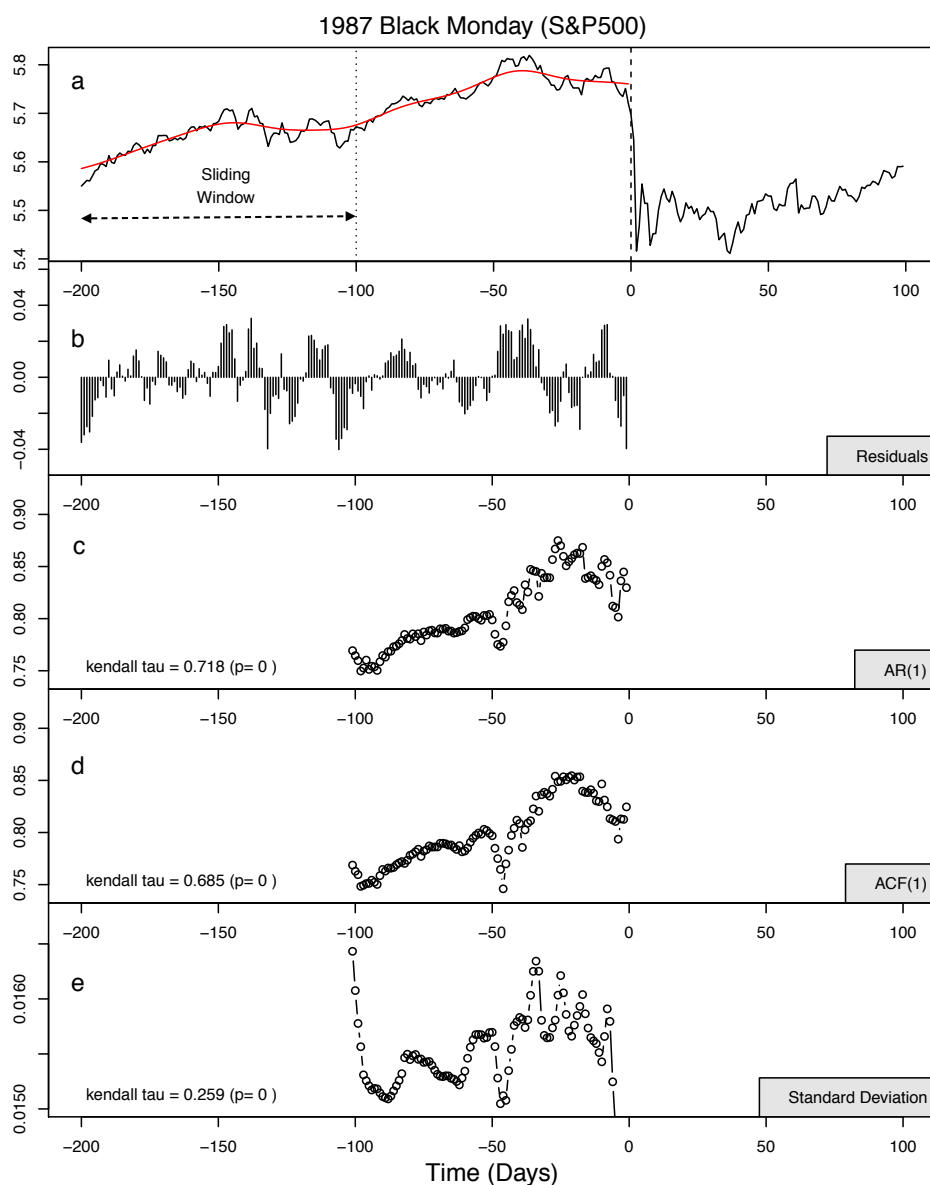
Figure 2.3: Detecting the early warning indicators for "Black Monday" using the S&P500 time series. (*a*) Logarithm of the daily S&P500 time series. (*b*) Residual time series. (*c*) Indicator from AR(1) function. (*d*) Indicator from ACF(1) function. (*e*) Indicator from Standard deviation. The vertical dashed line in (*a*) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (*c*)(*d*) and (*e*). The red line is the smoothed time series used for filtering.

in the Hangseng index provides evidence on early warnings before the Asian Crisis.

The Dot-com Bubble Figure 2.5 presents the analysis of early warning indicators from about one year before the Dot-com bubble collapse. Boosted by the rising of commercial growth of internet, NASDAQ composite index experienced a speculative bubble, as shown in Figure 2.5(a). It peaked around year 2000, the latter part followed a typical boom and bust cycle. When the bubble "bursts", the stock prices of dot-com companies fall dramatically. Some companies went out of business completely, such as Pets.com. Some others survived but their stocks declined by more than 80%, such as Cisco and Amazon.com.

Like the Black Monday and the Asian Crisis, the analysis of early warning indicators give evidence on increased slowing down in NASDAQ time series before Dot-com Bubble. The bubble collapse is preceded by an overall upward trend in the examples of early warning indicators. All the results are robust based on Kendall's τ analysis. This strongly seems to suggest the possibility that critical slowing down could serve as an early warning signal for Dot-com Bubble.

The 2008 Financial Crisis The financial crisis of 2008 is known as the greatest financial crisis since the Great Depression of the 1930s. It was triggered by the bursting of United States housing bubble, which peaked approximately 2005 - 2006. Banks began to give out more loans than ever before to potential home owners. When the housing bubble finally bursted in the latter half of 2007, the secondary mortgage market collapsed. Over 100 mortgage lenders went bankrupt during 2007 and 2008. Several major financial institutions failed, including Lehman Brothers, Merril Lynch, Washington Mutual, Citigroup and so on. The world wide economies experienced a great depression and stock markets around the world went down.

Figure 2.6(a) and (b) show the analysis of early warning indicators around one year before the 2008 financial crisis by using two time series, the S&P500 and the TED spread. Both of the analyses show mixed results. In the analysis of the TED spread time series, the AR(1) and ACF(1) indicators produce strong upward trends with Kendall's tau close to 0.7. This indicates
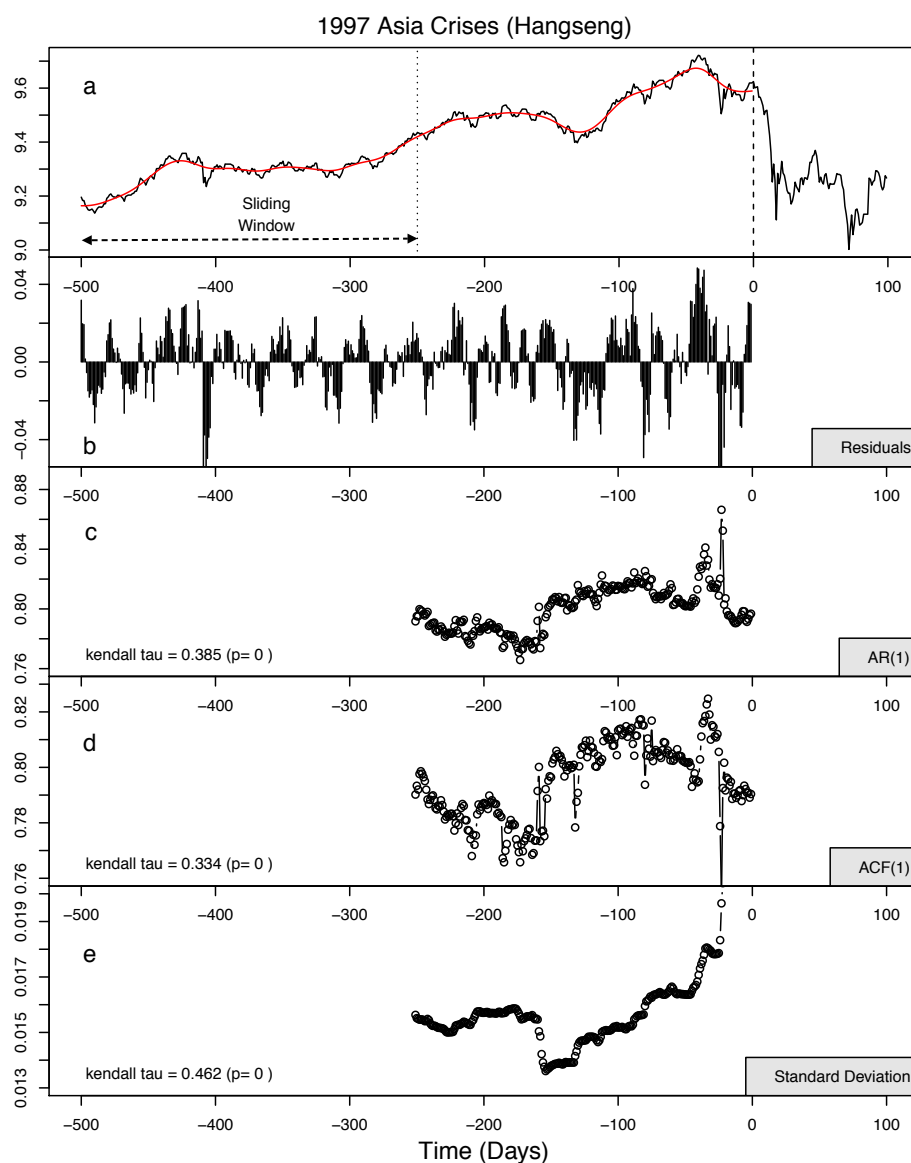
Figure 2.4: Detecting the early warning indicators for the Asian Crisis using time series Hangseng index. (*a*) Logarithm of the daily Hangseng time series. (*b*) Analysis of the residuals time series. (*c*) Indicator from AR(1) function. (*d*) Indicator from ACF(1) function. (*e*) Indicator from Standard deviation. The vertical dashed line in (*a*) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (*c*)(*d*) and (*e*). The red line is the smoothed time series used for filtering.
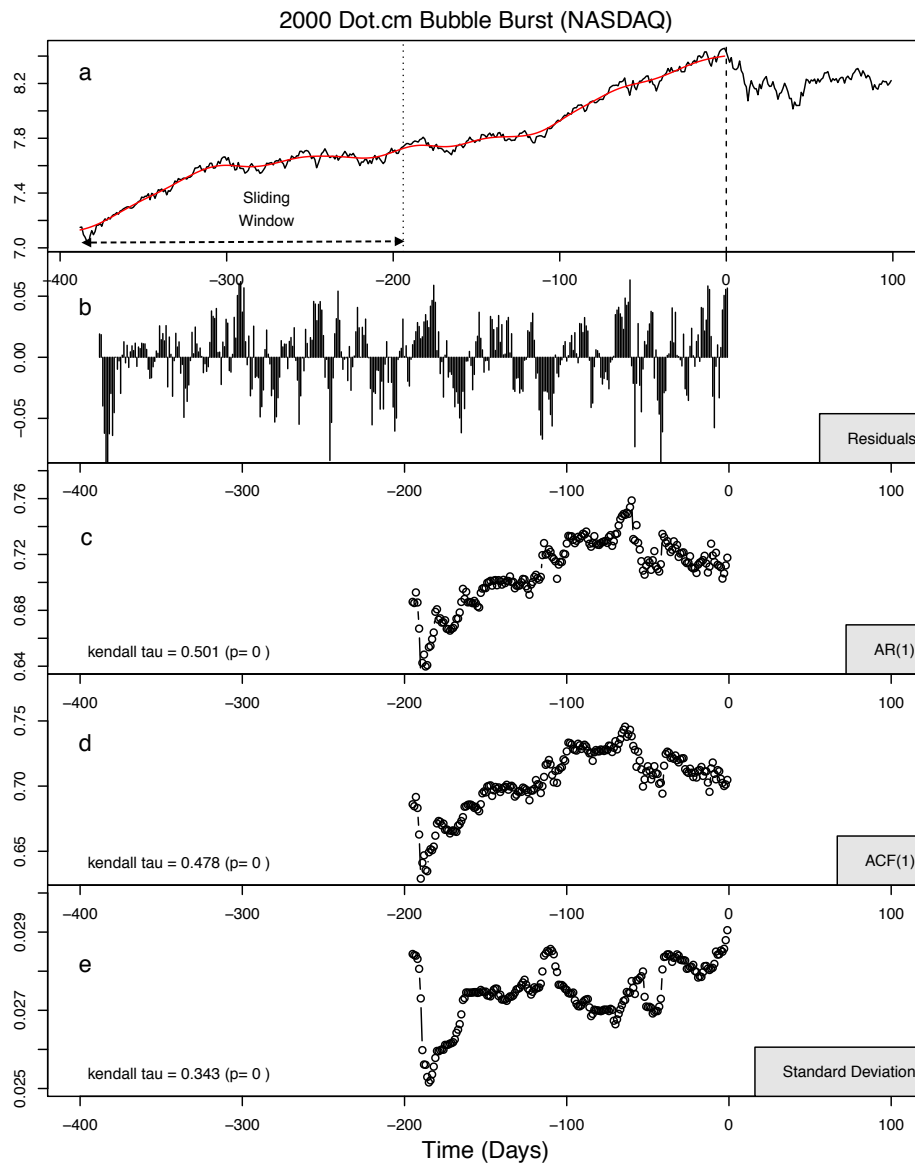
Figure 2.5: Detecting the early warning indicators for the Dot-com Bubbles using NASDAQ composite index. (*a*) Logarithm of the daily NASDAQ time series. (*b*) Analysis of the residuals time series. (*c*) Indicator from AR(1) function. (*d*) Indicator from ACF(1) function. (*e*) Indicator from Standard deviation. The vertical dashed line in (*a*) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (*c*)(*d*) and (*e*). The smoothed line is the smoothed time series used for filtering.

a slowing down preceding the critical transitions in TED spread time series on September 2008. Just around this time Lehman Brothers went bankrupt. However, for the same period, the variance indicator shows a downward trend. In Figure 2.6(b), the analysis of S&P500 time series gives a reversal of the results. AR(1) and ACF(1) indicators show downward trend while the Variance indicator shows an upward trend. The mixed results suggest the importance of applying "composite" indicators. None of the indicators alone would be able to give us accurate predictions. We should also note that Scheffer *et al.* (2009) suggested that the variance need not necessarily increase. Therefore, the promising indicators in our examples are AR(1) and ACF(1), which present increased trend in the analysis of TED spread.

In order to obtain more evidence, we also analyse the volatility index (VIX) in Figure 2.7. What interests us is that the VIX index is an estimated time series itself. It is a popular estimation of the implied volatility of S&P 500 in the next 30 days. Figure 2.7(a) presents the logarithm of the daily S&P500 time series while Figure 2.7(b) shows the logarithm of the volatility of the S&P500 index. Because we are interested in volatility time series, the *x*-axis of the critical transition in volatility index in Figure 2.7(b) is set to 0. As shown in Figure 2.7(a) and (b), this critical transition in VIX is 15 days preceding the real critical transitions in the daily S&P500 time series. In Figure 2.7(b), the time series kept rising after the critical transition for almost a month and decreasing slightly thereafter. However, it did not shift back to the equilibrium before critical transition. It measures the expectations of markets on future stock market volatility. Therefore, our analysis implies that people expected increased volatility of stock market prices already before the financial crisis actually happened. This fear of increased volatility of stock market prices quickly accumulates and lasts for a long time. The whole process coincides with the depression of the economy. It also shows that the early warnings on the critical transitions in volatility index can be considered as the early warnings on the crash in S&P500 index.

The same early warning methodology is applied to VIX time series. The analysis shows significant upward trends in AR(1) and ACF(1) indicators preceding the critical transition in
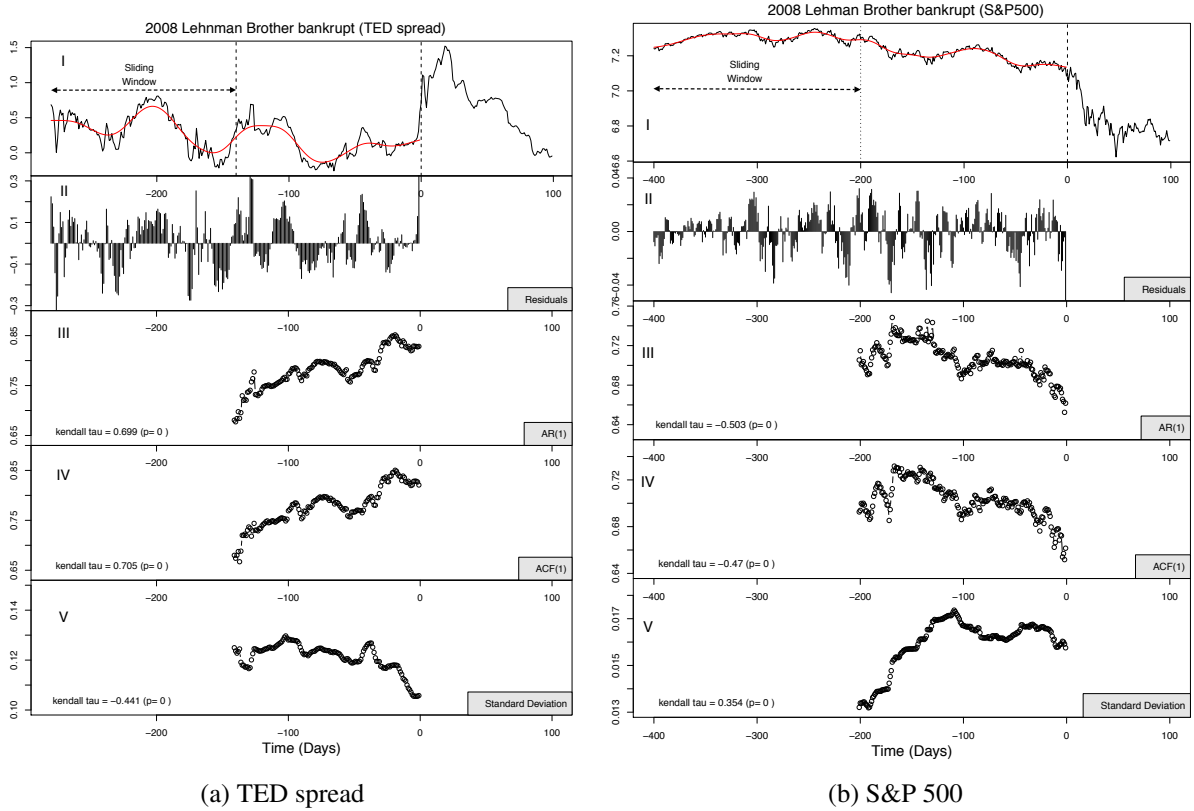
(a) TED spread

(b) S&P 500

Figure 2.6: Detecting the early warning indicators for the 2008 Financial crisis using TED spread and S&P500 index. (*a*) Analysis using TED spread. (*b*) Analysis using S&P500 index. For each of the analysis, (*I*) Logarithm of the daily original time series. (*II*) Analysis of the residuals time series. (*III*) Indicator from AR(1) function. (*IV*) Indicator from ACF(1) function. (*V*) Indicator from Standard deviation. The vertical dashed line in (*I*) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (*III*)(*IV*) and (*V*). The red line is the smoothed time series used for filtering.

VIX time series. It demonstrates increased slowing down and early warnings before the critical transition in the volatility index. However, the variance indicator is just horizontal with a slight downward trend. Moreover, the *p*-value of the estimated Kendall's tau indicates insignificance of the trend. Therefore, no trend is found for the variance indicator.

All the above results and the parameters used in the analysis are summarized in Table 2.2. The symbols "(+)" indicate early warning signals are detected, while the symbols "(-)" indicate that the transitions are not preceded by indicators. As shown in Table 2.2, the window size we choose is half the sample size in each example following Dakos *et al.* (2008). The choice of bandwidth is 10 under the condition that we do not overly smooth the data but still give a stationary time series. We also checked each example using the bandwidth $\sigma = 20$. Although the estimated kendall τ coefficients are slightly different, they give similar trends as in the above examples. This simple test suggests the robustness of the results with respect to the choice of the bandwidth.

Extreme Event	Time Series	N	w	σ	τ		
					AR(1)	ACF(1)	Variance
Black Monay	S&P500	200	100	10	0.718***(+)	0.685***(+)	0.259***(+)
Asian Crisis	Hangseng	500	250	10	0.385***(+)	0.334***(+)	0.462***(+)
Dot-com	NASDAQ	398	194	10	0.501***(+)	0.478***(+)	0.343*** (+)
2008 Crisis	S&P500	400	200	10	-0.503***(-)	-0.470***(-)	0.354***(+)
	TEDspread	280	140	10	0.699***(+)	0.705***(+)	-0.441***(-)
	VIX	80	40	10	0.463***(+)	0.446***(+)	-0.149 (-)

⋆⋆⋆ significant at 1% level, ⋆⋆ significant at 5% level, ⋆ significant at 10% level.

Table 2.2: Studies of early warning indicators for critical transitions in different time series. *N* is sample size. *w* and σ is window size and bandwidth used to do the analysis. τ is estimated kendall's tau coefficient. symbol (+) indicates early warning signals are detected. (-) indicates the transitions are not preceded by indicators.

The summary of the results in Table 2.2 suggests early warnings indicated by the examples of AR(1), ACF(1) and variance preceding the crashes of the stock market prices for Black Monday 1987, the Asian Crisis and the Dot-com Bubble. However, the signals for the 2008 Financial Crisis are mixed. They lead us to the question whether there is increased slowing down
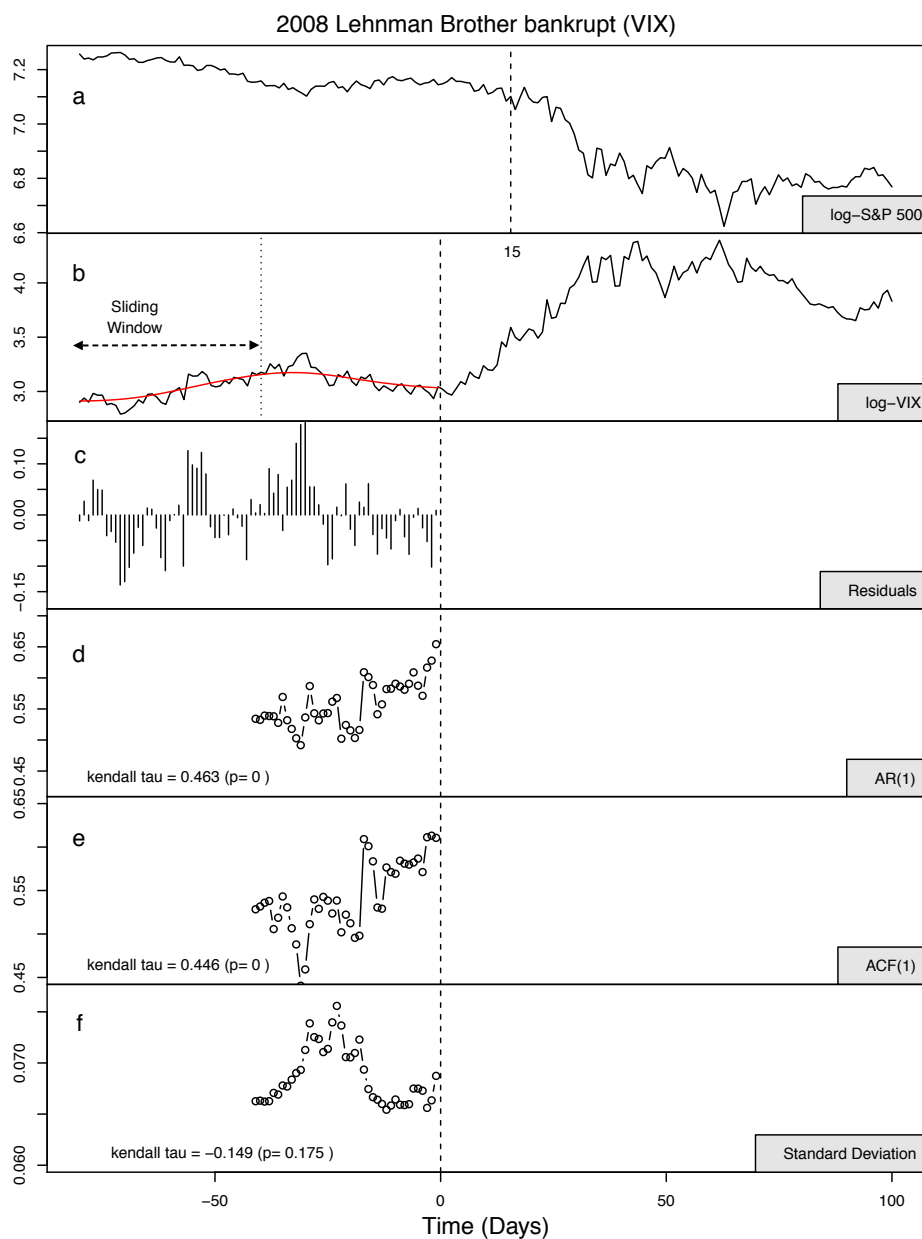
Figure 2.7: Detecting the early warning indicators for the 2008 Financial crisis using volatility index (VIX). (*a*) Logarithm of the daily S&P500 index. (*b*) Logarithm of the volatility of the S&P500 index. (*c*) Analysis of the residuals time series. (*d*) Indicator from AR(1) function. (*e*) Indicator from ACF(1) function. (*e*) Indicator from Standard deviation. The vertical dashed line in (*a*) and (*b*) identifies the critical transitions in the time series of the S&P500 index and the volatility index. The dashed arrow shows the width of the moving window used to compute the indicators shown in (*d*)(*e*) and (*f*). The red line is the smoothed time series used for filtering.

prior to the 2008 Financial Crisis. The AR(1) and ACF(1) methods in TED spread and VIX time series suggest so but they give the opposite results in S&P500 time series. Are these signals just spurious early warnings? Or were there no bifurcations underlying the 2008 financial crisis? To solve these questions, more work is still to be done.

2.5.2   Bootstrapped Time series

In order to test the likelihood of having the trend statistic estimation of kendall's τ by chance, we apply the same early warning methodology to surrogate time series. We also calculate the probability of the trend statistics in surrogate time series being at least as high as for the original records. The surrogate time series are generated by bootstrapping the financial time series of the S&P500 index, the Hangseng index, the NASDAQ composite, TED spread and the VIX index in three different ways.

Bootstrapping Residuals   Firstly, we bootstrap the residuals after detrending following the test in Dakos *et al.* (2008). By resampling the order, we generate surrogate time series with similar means and variances.

Bootstrapping Log-returns   Secondly, instead of residuals, the log-returns of the original time series are bootstrapped. Similar with the first method, we bootstrap the time series by randomly picking data with replacement. Moreover, we take the cumulative sum before detrending.

Parametric GARCH Bootstrap   Thirdly, surrogate time series are generated by fitting a GARCH(1,1) model [2] to log-returns:

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[2]Generalized autoregressive conditional heteroskedasticity, originally introduced by Engle (1982), Bollerslev (1986) represent the dynamic evolution of conditional variances.

$$y_t = \sigma_t \varepsilon_t, \tag{2.9}$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \ t = 1, ..., T,$$

where ε_t is a white noise process with $\varepsilon_t \sim N(0,1)$, σ_t^2 is the volatility. ω, α, β are parameters that satisfy $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ to ensure the positivity of the conditional variance. The process y_t is stationary for $0 < \alpha + \beta < 1$.

Random Segments Bootstrap Spurious early warnings can occur in the fluctuations in a single regime without transiting to a different one. In order to test this possibility, we analyse the early warnings on random segments in financial time series. Figure 2.8 shows an example of the test of the early warning indicators for random segments in Hangseng time series. It follows the same format of figures as in the analysis of financial time series. The analysis is based on a randomly picked segment from a long period of Hangseng time series. However, the sample size is kept the same as it in the case of the Asian Crisis. As shown in Figure 2.8 based on our random segments bootstrap, there is no evidence of early warnings in this case with respect to the early warnings of AR(1), ACF(1) and Variance indicators.

Many other random segments of the financial time series are tested in the same way. However, the trends of the early warning indicators are diverse. Some of them show no early warnings while the others do. The early warning indicators obtained from 1000 random segments are analysed. Figure 2.9(a), (b) and (c) show the histograms of the examples of AR(1), ACF(1) and Variance indicators individually. The dashed lines are Kendall's τ of the indicators of the real critical transitions in the Asian Crisis. The arrows indicate the subsets for which the segments trend statistics are higher than the trend statistics of the true critical transitions. The fractions of these subsets are indicated by percentages in the figure. For all three indicators, the fractions of the subsets are around 20% - 30%. Following the ideas on the analysis of bootstrapped time series in Section 2.5.2, Figure 2.9 implies that the examples of early warning indicators of AR(1),

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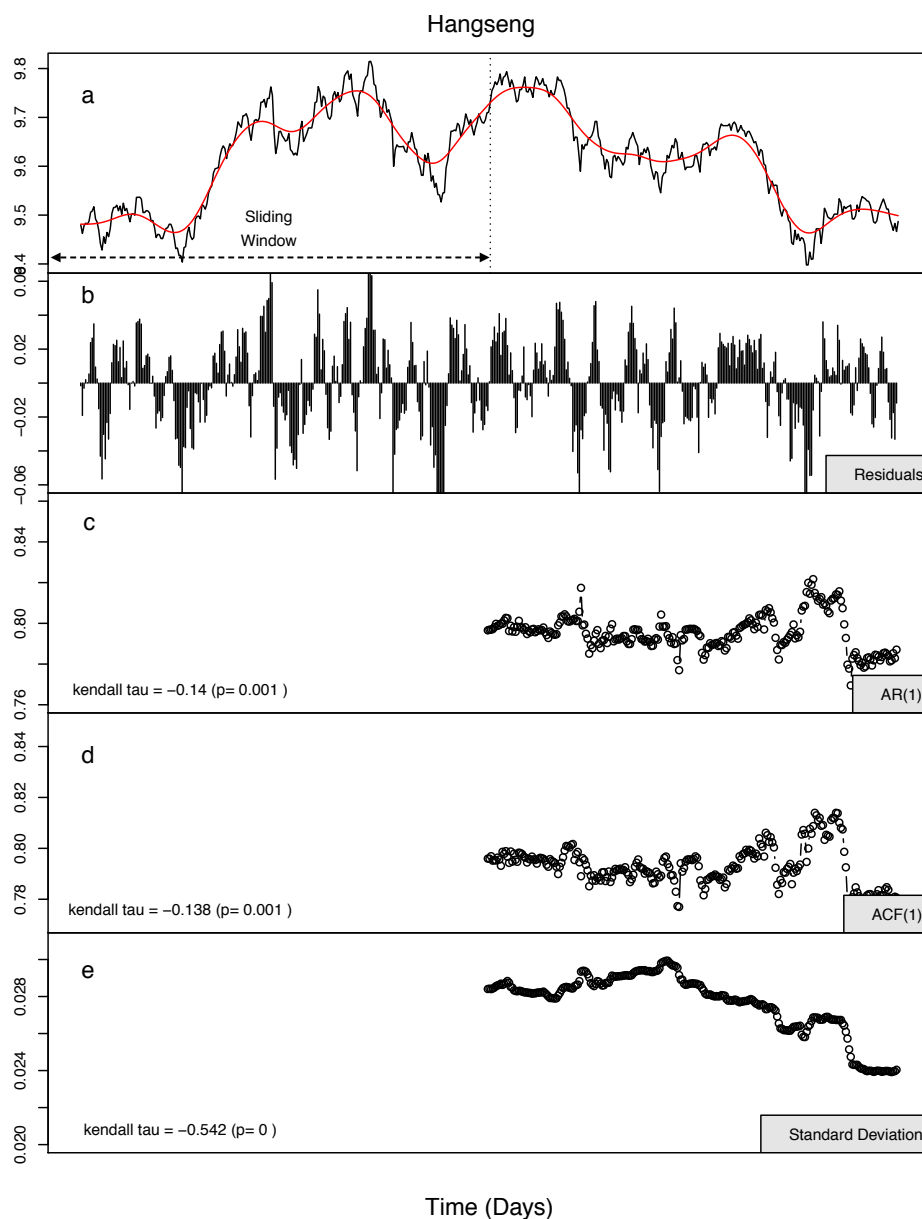
Figure 2.8: Testing the early warning indicators for random segments in Hangseng time series. (*a*) Logarithm of the random segments in Hangseng time series. (*b*) Analysis of the residuals time series. (*c*) Indicator from AR(1) function. (*d*) Indicator from ACF(1) function. (*e*) Indicator from Standard deviation. The dashed arrow shows the width of the moving window used to compute the indicators shown in (*c*)(*d*) and (*e*). The red line is the smoothed time series used for filtering.

ACF(1) and variance give the early warnings in Hangseng index before critical transitions but only at 20% - 30% significance level.
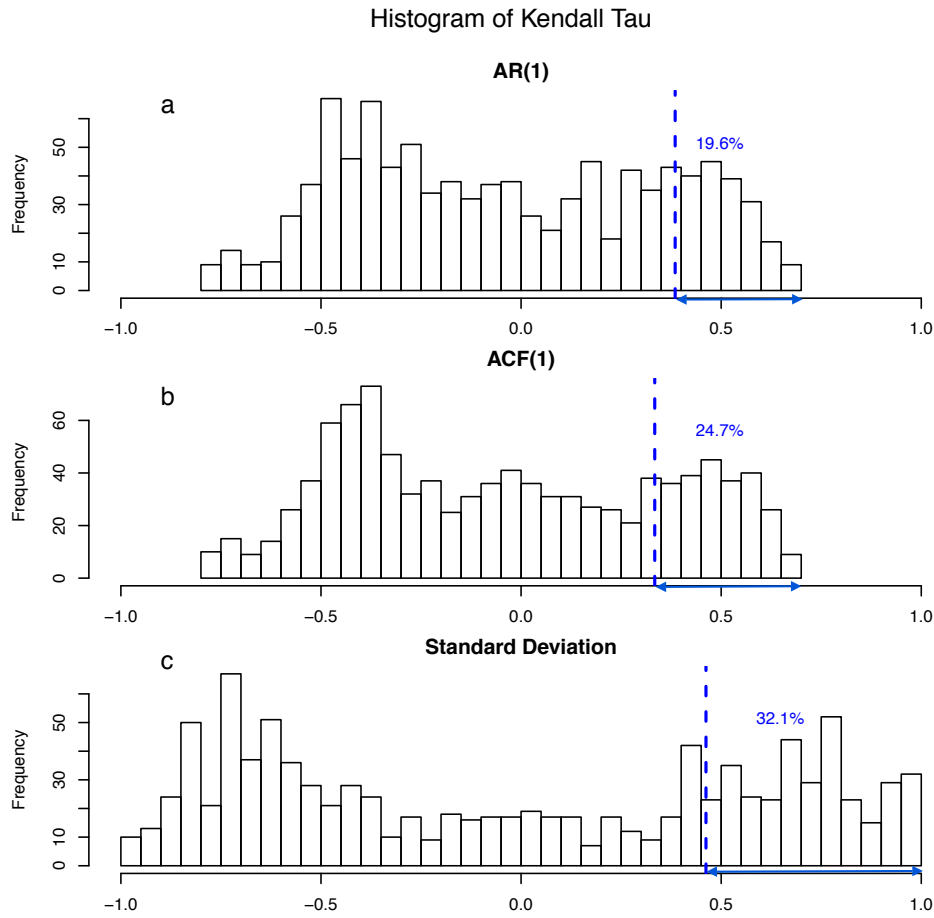
Figure 2.9: Histograms of Kendall's tau by Randomly segments in Hangseng time series from 1988 to 2009. The number of time series is N=1000. a, b and c show the histograms of the examples of AR(1), ACF(1) and Variance indicators individually. The blue dashed lines are the kendall's tau of the indicators of the real critical transitions in Asian Crisis. The blue arrow indicates the subsets which the segments trend statistic is higher than the trend statistic of the true critical transition. The blue percentage numbers indicate the fractions higher than original time series.

Random Walk Bootstrap To compare with financial time series, we perform the analysis on realisations of a random walk process. A simple random walk is presented as:

$$y_t = y_{t-1} + \varepsilon_t, \tag{2.10}$$

where ε_t is white noise.

We generate 1000 random walk process realisations and caculate the early warning indicators based on them. Figure 2.10 shows the histograms of Kendall's τ and p-values of AR(1) and Variance indicators. Due to gaussian detrending in the methodology, $-1 < \tau < 1$ is expected as shown in the figure. Moreover, if it is the methodology which creates spurious trends in Kendall's τ, the distribution would left skewed and clustered near 1. However, Figure 2.10 shows all the Kendall's τ are distributed evenly between -1 and 1, which rejects the hypotheses above.

For the time series data of the S&P500 index, the Hangseng index, the NASDAQ composite, the TED spread and the VIX index, 1000 surrogate time series are estimated under each bootstrap method. The trend statistics of Kendall's τ coefficients and p-values are represented by histograms. Figure 2.11 shows one of the examples of the analysis. It presents the analysis of the Kendall's τ coefficients of the examples of AR(1) indicator in 1000 surrogate S&P500 time series, under the bootstrap method of fitting the GARCH(1,1) model. Figure 2.11(a) presents the probability distributions of the p-values while Figure 2.11(b) shows the histogram of the Kendall's τ coefficients. The dashed line represents the trend statistic of the original τ^* in the Black Monday example.

The likelihood of obtaining trend statistic estimates by chance is estimated by the subsets that the surrogate trend statistics are higher than the trend statistics of the original time series, $p(\tau \geq \tau^*)$. This subset is indicated by the arrow in Figure 2.11(b). It refers to the probability that surrogate kendall's τ lie on the right hand side of the original kendall's τ. The p-values as $p(\tau \geq \tau^*)$ would evaluate the significances of the trend statistic estimations of the early warnings in financial examples. The results for the examples of AR(1) indicators are shown in Table 2.3.

Table 2.3 shows the likelihood of obtaining trend statistic estimates by chance, which is estimated by the probability of the surrogate estimated trend statistic kendall's τ larger than
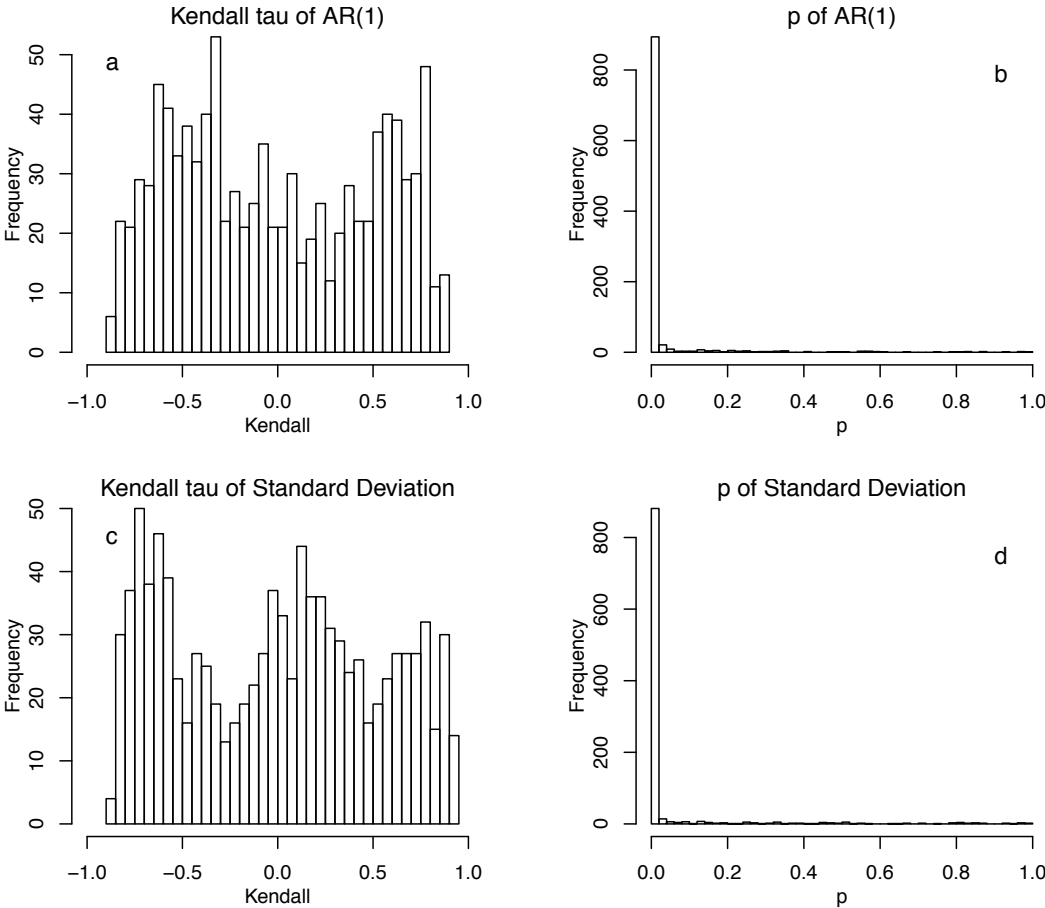
Figure 2.10: Histograms of Kendall's tau and *p*-value in the examples of AR(1) and Variance indicators in random walk process. (a) and (c) show the histograms of the Kendll's tau in the examples of AR(1) and Variance indicators while (b) and (d) show the distributions of *p*-values.

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Histogram of p

Histogram of kendall
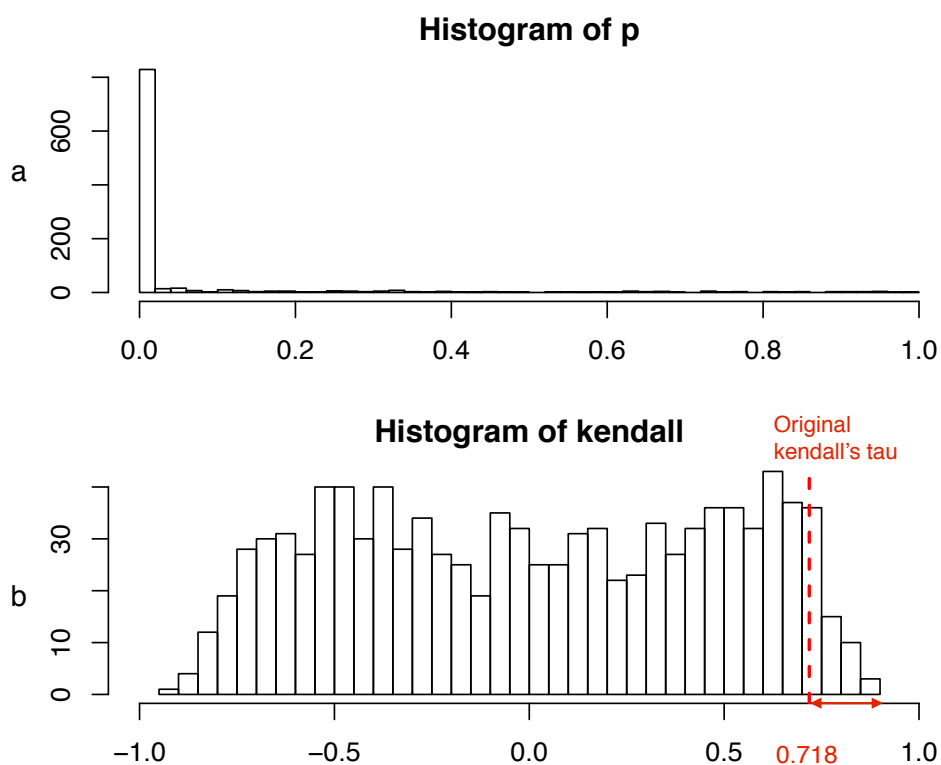
Figure 2.11: Analysis of the Kendall's τ coefficients of the examples of AR(1) indicator in 1000 surrogate S&P500 time series, under the bootstrap method three which is to fit GARCH(1,1) model. (a) presents the bootstrap density of the p-values; (b) shows the histogram of the Kendall's τ coefficients. The Red dashed line represents the trend statistic of the original τ^* in the Black Monday example. The Red arrow indicates the subset which the surrogate trend statistic is larger than the trend statistic of the original residual records. From this subset, only values equal to or higher than the original record are used to estimate the likelihood of acquiring trend statistic estimates this far in the tail by chance.

the trend statistic of the original residual records. Only the results for the examples of AR(1) indicators are shown here. The probabilities vary from case to case, but the differences between the three bootstrap methods are small. This suggests the reliability of the bootstrap results. As shown in Table 2.3, the results are mixed. The probability of having the observed trends by chance are less than 10% in the examples of the Black Monday (S&P500) and the 2008 Financial Crisis (TED spread) in all three bootstrap methods. It implies that the early warning signals we found in both examples are significant, at the 10% significance level. In particular, the performance of the example of the Black Monday (S&P500) is fairly good. Its results are significant at the 5% significance level. It highlights the reliability of the early warning signals preceding Black Monday crash in S&P500 time series. In other cases, the probability of obtaining the trends by chance are around 20% - 30%. The significance levels of the early warnings we found in the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis(VIX) are only around 30%.

Extreme Events	Time Series	Bootstrap $P(\tau > \tau^*)$ [a]					τ^*
		I. Per. Res.	II. Per. Ret.	III. GARCH	IV. Ran. t.s.	V. Ran. wal.	
Black Monday	S&P500	0.055*	0.045**	0.060*	0.025**	0.074*	0.718(+)
Asian Crisis	Hangseng	0.300	0.287	0.309	0.268	0.210	0.385(+)
Dot-com	NASDAQ	0.354	0.335	0.350	0.133	0.174	0.501 (+)
2008 Crisis	S&P500	0.205	0.200	0.183	0.202	0.175	-0.503(-)
	TEDspread	0.072*	0.061*	0.074*	0.098*	0.060*	0.699(+)
	VIX	0.311	0.300	0.288	0.289	0.323	0.463(+)

⋆⋆ 5% level, ⋆ 10% level

Table 2.3: The likelihood of obtaining trend statistic estimates by chance, estimated by the probabilities of the surrogate estimated trend statistic (kendall's τ) larger than the trend statistic of the original residual records. Only the results for the examples of AR(1) indicators are shown here. Symbol (+) indicates early warning signals are detected. (-) indicates the transitions are not preceded by indicators. Surrogate set $N = 1000$.

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[a] $\tau < -\tau^*$ for 2008 Financial Crisis case using S&P500

2.6 Robustness of Parameters

The early warning analysis in this paper is influenced by two key parameters: bandwidth and moving window size. Bandwidth size is a very important parameter when filtering out long term trends from the original time series. There is a trade-off when making the choice. A too narrow bandwidth would not only remove the long run trends but also the short run fluctuations what we intend to study; a too wide bandwidth would not remove enough long run trends. There would be still some slow trends left which may lead to spurious trends of the indicators. A similar trade-off also affects the window size. A smaller window size is good to track short run changes, but a too small window size with too few sample points would make the estimations less reliable.

In order to check the robustness of the parameters in our analysis, we perform an additional analysis by using rolling window and rolling bandwidth. The contour plots in Figure 2.12 and 2.13(a) show the influence of parameters on the observed trends of AR(1) indicators for Black Monday 1987 and the Asian crisis. The black dot indicates the combination of window size and bandwidth size that shows the strongest positive trends. The white dot indicates the parameters used in our early warning analysis. The kendall's tau distributions in Figure 2.12 and 2.13(b) confirm the strong positive trends of kendall's tau in the contour plots. This robustness analysis indicates that the results are quite robust with respect to the parameters chosen. It also shows that even more significant trends could be obtained by moving parameters in the direction of the black dot, which in turn confirms the robustness of the results with respect to changes in the parameters in this paper.

Figure 2.12: Analysis of the robustness of parameters in the example of Black Monday: window size and bandwidth. (a) Contour plots of the rolling window size and bandwidth size. Black dot indicates the assemble of window size and bandwidth size that shows the strongest positive trends. White dot is the parameters used in the early warning analysis (b) Histogram of the kendall's tau in (a)
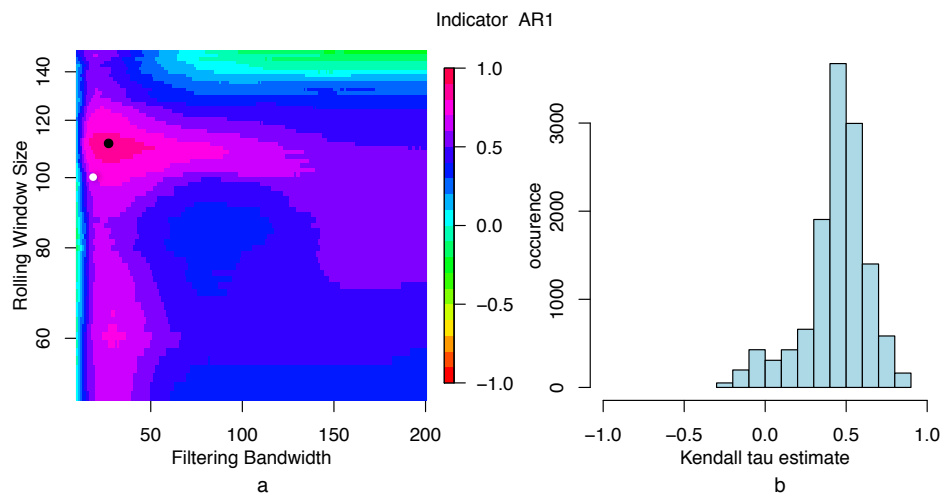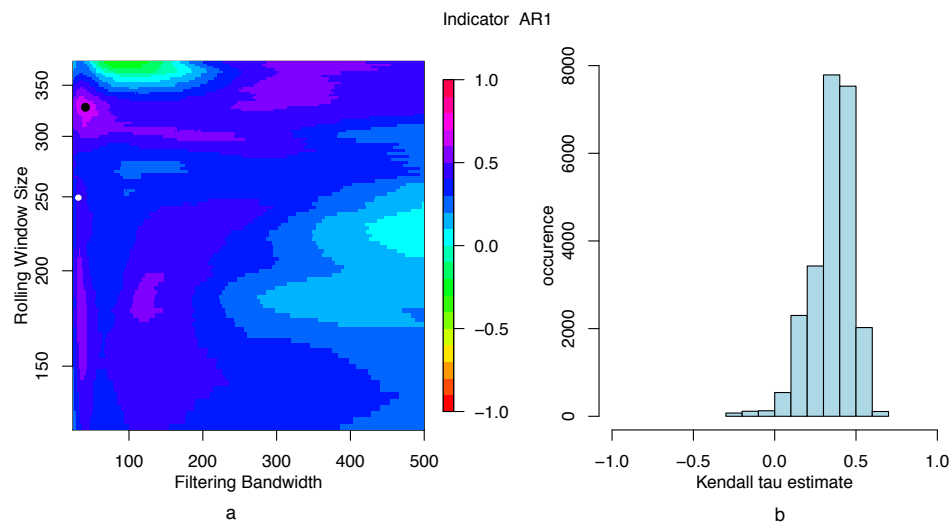

Figure 2.13: Analysis of the robustness of parameters in the Asian Crisis: Window size and bandwidth. (a) Contour plots of the rolling window size and bandwidth size. Black dot indicates the assemble of window size and bandwidth size that shows the strongest positive trends. White dot is the parameters used in our early warning analysis (b) Histogram of the kendall's tau in (a)

2.7 Summary and Conclusions

In the theory of "Critical slowing down", an increase of the first order autocorrelation coefficients towards +1, is considered to be an early warning signal for an upcoming critical transition. This theory is applied to the analysis of time series by Held and Kleinen (2004) and Livina and Lenton (2007). They used the slowing down before critical transitions to identify the bifurcations in climate systems. The method has been applied first to the North Atlantic thermohaline circulation and Greenland ice core paleotemperature using climate model output. Dakos *et al.* (2008) developed this methodology further and applied it to real climate data for the first time. Their analysis provides robust empirical evidence. The first order autocorrelation coefficient indeed increases as the system approaches a critical transition.

Our paper investigates for the first time whether there is evidence of "critical slowing down" in financial systems. Four financial crisis are analysed by using six time series. The results suggest increases in AR(1), ACF(1) and variance indicators precede the crashes of the stock market prices in Black Monday 1987, Asian Crisis and Dot-com Bubble. However, the signals for the 2008 Financial Crisis are mixed. In order to estimate the likelihood of spurious early warnings, the same methodology is applied to various surrogate time series. The analysis of the bootstrapped time series confirms the reliability of the early warning signals preceding the Black Monday crash in the S&P500 time series, but suggests the possibility of spurious early warnings in the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis(VIX) are around 30%. Analysis of random financial time series also shows the real p-value to detect the critical transitions in financial time series are around 20% - 30%. The random walk analysis rejects the hypothesis that the methodology we apply may create increasing trend which gives rise to false early warnings. An additional analysis on the robustness of parameters is performed in the end. It shows that the results are fairly robust with respect to the choices of the parameters.

This paper fills the gap between the theory of "critical slowing down" and its application to financial data. It explores the forecasting ability of the critical slowing down indicators in financial time series. The results detect fairly good early warnings before the crash of Black Monday

1987. The early warnings before the Asian Crisis, the Dot-com Bubble and the 2008 financial crisis are only detected at 20% - 30% significance level. It suggests that critical slowing down can serve as one of the complementary early warning indicators of financial crisis. However, in order to increase the accuracy of the predictions, more sophisticated methods are still desired.

There are a number of possible reasons why the results are mixed. Firstly, so far the tools of detecting critical slowing down are usually linear. With the increase of the complexity of the nonlinear dynamics, the predictions of bifurcations may fail. Sieber and Thompson (2012) tried to extend the techniques to nonlinear features. However, they did not find discernible trends in the nonlinear case. The methodology used in this paper is also based on a first order linear approximation of the dynamics. In this case, due to the complexity of financial dynamic systems, false early warnings can be expected. Secondly, the transitions in complicated financial systems may happen far from local bifurcations and do not have to experience the cusp catastrophe transitions. For instance, early escape due to a noise induced transition (Thompson and Sieber, 2010a). In particular, the emergence of new technology and financial instruments nowadays makes the financial markets even more complex. This could explain the failure to detect the 2008 financial crisis using advance economic and financial approaches. Thirdly, the catastrophe theory approach is based on one dimensional systems with only one control variable, while the situation in financial systems is far more complicated. Perhaps a multivariate approach is required to capture the sophisticated dynamic behaviours in financial systems. Fourthly, asset pricing models are usually based on the assumption that the fundamental price follows geometric random walk process. In particular, the fluctuations around fundamental price are endogenous for heterogenous agents models (HAMs) (Boswijk *et al.*, 2007). They always have the marginally stable eigenvalue 1 already.

Chapter 3

Can A Stochastic Cusp Catastrophe Model Explain Housing Market Crashes?

3.1   Introduction

The collapse of the U.S. housing bubble in 2007 was followed by an enormous worldwide financial crisis. This tragedy has raised great concerns of housing bubbles among financial regulators and researchers. Like stock market bubbles, housing bubbles can be identified through rapid increases in housing prices before they crash. Figure 3.1 illustrates the bust phase of housing price cycles surrounding banking crises from 1899 to 2008 using real housing prices (Reinhart and Rogoff, 2009). The historical average of the declines from peak to trough is 35.5 percent. A number of countries with major housing crashes are included. For instance, Finland, Colombia, the Philippines and Hong kong have experienced the most severe real housing prices crashes in the past 25 years. Their crashes amounted to 50 to 60 percent from peak to trough. Notably, the duration of housing price declines has been quite long lived, averaging roughly 6 years. After the housing market crash of Japan in 1992, real housing prices declined for consecutive 17 years. In particular, housing price declines are even longer lived than equity price declines. The average historical downturn phase in equity prices lasts 3.4 years, still less than half of

the downturn phase in housing prices (Reinhart and Rogoff, 2009). The International Monetary Fund (IMF) recorded that housing price busts lasted nearly twice as long and led to output losses that are twice as large as asset price bursts (IMF World Economic Outlook, 2003). Moreover, financial crises and recessions are often preceded by housing market crashes (Reinhart and Rogoff, 2009). The credit crisis and global financial crisis in 2008 are convincing examples. After housing prices declined in the latter half of 2007, the secondary mortgage market collapsed. A complex chain reaction almost brought down the whole world's financial system. Furthermore, in some papers, housing market bubbles are considered as leading indicators of financial instability and crises (Davis and Heathcote, 2005). For the above reasons, a good understanding of the instability in housing market is crucial.
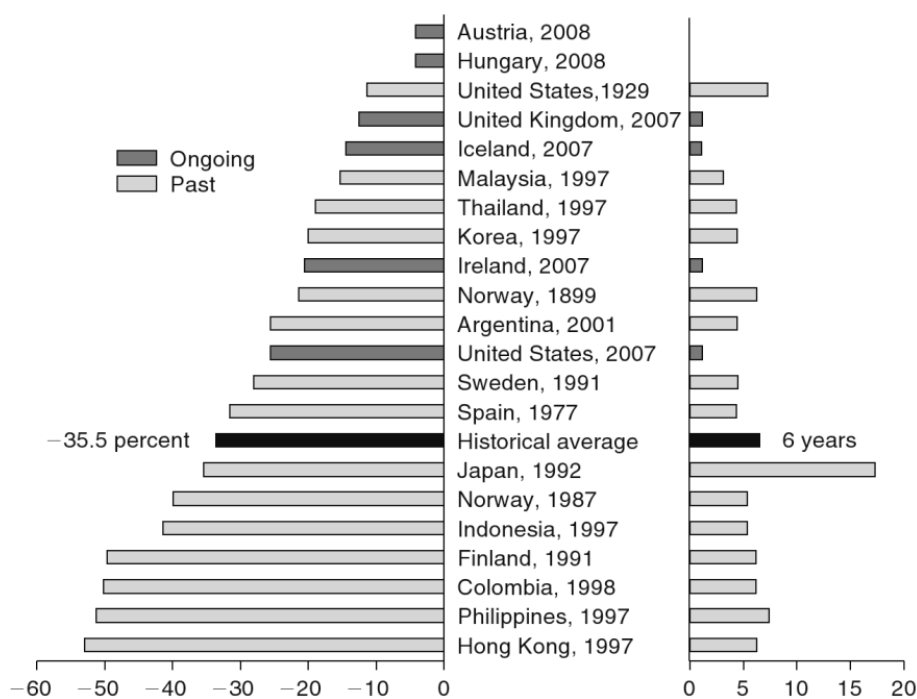
Figure 3.1: The bust phase of housing price cycles surrounding banking crises. Left panel: peak-to-trough price declines; Right panel: years duration of downturn. Source: Reinhart and Rogoff, 2009

Housing market models have been studied extensively in literature. Unfortunately, most of the available research in macroeconmics is mainly based on state-of-the art dynamic stochastic general equilibrium (DSGE) models which are based on fundamentals. However, these tradi-

tional models are insufficient to explain the observed booms and busts in housing prices. A series of papers by Shiller have argued that the changes in economic fundamentals such as population growth, construction costs, interest rates or real rents did not match up with the observed house price fluctuations (Case and Shiller, 2003; Shiller, 2007, 2008, 2012, 2015). Davis and Heathcote (2005) also suggested that DSGE models with housing consumption and production were unable to capture the instability of house price.

During the last decades, a number of theories were proposed that are based on a multiple equilibria approach. Unlike traditional DSGE models under the general equilibrium assumption, they recognise economic systems as complex systems with multiple equilibria. Along these times, a theoretical approach on heterogenous agents models (HAMs) has been introduced to the housing market. It was inspired by the work on heterogenous agent-based financial market models (see Brock and Hommes,1997, 1998 and comprehensive survey in Hommes, 2013). For instance, Kouwenberg and Zwinkels (2011) developed and estimated a HAM model for the U.S. housing market. Their estimated model produced boom and bust price cycles endogenously, which were induced by boundedly rational behaviour of investors. Dieci and Westerhoff (2012, 2013) investigated the speculative behaviour in housing markets using a heterogenous agent approach. Their examples illustrated a variety of situations that can display irregular endogenous dynamics with long lasting, significant price swings around the fundamental price, like in many actual markets. In a recent paper Bolt *et al.* (2014) established and estimated a HAM model for eight different countries. They found evidence of heterogenous expectations from empirical data and identified temporary house price bubbles for different countries.

Although HAMs have proven to be successful theoretical tools to capture temporary deviations from market equilibrium, a method which allows statistical time series analysis is still lacking. In this paper, we fill this gap by using statistical analysis to capture the crash phenomena in real time series. Forecasting market crashes based on previously observed time series is also possible. In this paper, we are attempting to fill this blank. Catastrophe theory has been suggested to be a good candidate (Zeeman, 1974). It captures the instability in many non-

linear dynamical systems and has proven to be an extremely successful tool to investigate the qualitative properties in a wide range of different complex systems ranging from physics and engineering to biology, psychology and sociology. Its applications involve urban and regional systems (Wilson, 1981), quantum morphogenesis (Aerts *et al.*, 2003), the stability of black holes (Tamaki *et al.*, 2003), the size of bee societies (Poston and Stewart, 2012), the cognitive development of children (Van der Maas and Molenaar, 1992), sudden transitions in attitudes (Van der Maas *et al.*, 2003) and so on. In all these applications, behaviour of the observed system shows sudden and discontinuous changes or phase transitions as a result of a small change in control variables. Catastrophe theory offers a mathematical basis for the number and the type of critical points for the classification of nonlinear dynamical systems. Since the economic system has been recognised as a complex system, displaying quick transitions such as market crashes, catastrophe theory might be a good candidate to explain its extreme fluctuations. Zeeman (1974) already proposed that some of the unstable behaviour of stock exchanges could be explained by a model based on catastrophe theory. A similar model can also be applied to currencies, property markets, or any market that admits speculators. The proposed HAM model in financial market was also motivated by some behavioural finance elements in Zeeman's work. Barunik and Vosvrda (2009) and Barunik and Kukacka (2015) tested Zeeman's idea and fitted a stochastic cusp catastrophe model to stock market data. They provided an important shift in application of catastrophe theory to stock markets. Their examples showed that stock market crashes were better explained by cusp catastrophe theory than other models.

In our paper, following the idea of Zeeman (1974), we fit a stochastic cusp catastrophe model to housing market data for the first time. The aim of this paper is twofold. Firstly, we discuss the estimation method of fitting stochastic cusp catastrophe model. Two estimation approaches are studied: Cobb's method and Euler Discretization. Secondly, we apply catastrophe behaviour to the housing markets and study its policy implications further.

There are two main contributions in this paper. Firstly, we study two estimation methods: Cobb's method and Euler Discretization. We show that Euler Discretization gives better fore-

casting ability than Cobb's Method. Secondly, we explain the underlying mechanisms of the instability of housing markets by using a cusp catastrophe model . A critical transition can be distinguished from ordinary fluctuations. We also unfold the underlying link between interest rates and systematic fluctuations in housing markets.

Our analysis sheds some light on the application of catastrophe theory to time series data in social science. We fit a stochastic cusp catastrophe model to housing markets in six different countries. Our results show how the equilibria of the system are changing depending on the interest rate. This scenario can be used to explain several housing price bubbles and crashes in empirical data, such as UK 1978, 1980, 1990, NL 1978, 1990, and the depression of SE after 1990. The policy implication of this paper is that policy makers should commit an interest rate policy which prevents the system from getting too close to the cusp curve that may induce a systemic market crash. To achieve this, the cusp catastrophe fit could provide a reasonable guidebook. This is one of the most appealing contributions of our paper.

This paper is organised as follows. We first introduce, in Section 2, the cusp catastrophe theory and its application to housing markets. Subsequently, we discuss the empirical methods of Cobb and Euler Discretization, estimation variables and empirical data in Sections 3 and 4. The results of Euler Discretization are presented and discussed in Section 5. This paper ends with a summary and conclusion in Section 6.

3.2 Catastrophe Theory

Catastrophe theory has been first proposed by the French mathematician René Thom (1972). Before his work, most models only described phenomena with smooth and continuous changes. However, the world is full of sudden transformations and unpredictable divergences. The proposed catastrophe theory has shed some lights on "a law of nature". Zeeman cooperated with Thom and proposed catastrophe's applications in the fields of economics, psychology, sociology, political studies, and others (Zeeman, 1974, 1977). In particular, he proposed the ap-

plication of cusp catastrophe model to stock markets and qualitatively described the bull and bear markets as a result of interaction between two main types of investors: fundamentalists and chartists (Zeeman, 1974). This work contains a number of important behavioural finance elements, which later led to research on HAM models. However, the biggest difficulty in application of catastrophe theory arises from the fact that it stems from deterministic systems, while most scientific investigations allow for random noise. In order to apply it directly to behavioural science in which random influences are common, we need a bridge to lead catastrophe theory from deterministic to stochastic systems. Loren Cobb was the first to address this challenge. He proposed a stochastic version of catastrophe theory based on Itô stochastic differential equation (Cobb, 1980). Later the development of statistical methods made catastrophe theory very useful and applicable empirically on real data. Unfortunately, this maximum likelihood estimation (MLE) method is not invariant under nonlinear diffeomorphic transformations. Much of the topological generality of catastrophe theory was lost in the statistical portion of this theory (Cobb and Watson, 1980). Hartelman (1997) improved Cobb's method by taking into account the Itô transformation rule. They showed that this model remained invariant under smooth 1-to-1 transformations and can be estimated by a straightforward time series analysis based on level crossings (Wagenmakers *et al.*, 2005). This model has been applied to psychology to model transitions in attitudes successfully (Van der Maas *et al.*, 2003). Barunik and Vosvrda (2009) and Barunik and Kukacka (2015) fitted it to stock market data and showed that stochastic cusp catastrophe model explained the crash of stock market much better than other models. Housing market crashes are often prior to financial crises and recessions. Housing bubbles are considered as leading indicators of financial instability and crises. There are no other examples of understanding housing markets by using catastrophe theory. Therefore, we will use catastrophe theory to explain the dynamical behaviour in housing markets. In what follows, a basic understanding of catastrophe theory is discussed.

Catastrophe theory provides a mathematical basis for systems involving discontinuous and divergent phenomena. In particular, it is effective in those systems where gradually changing

forces lead to abrupt changes in behaviour. The nonlinear dynamics of the system under study follows, in the noise-less case

$$dy_t = \frac{-dV(y_t;c)}{dy_t}dt, \tag{3.1}$$

where y_t represents the state of system. It implies that the studied system changes in response to a change in $V(y_t;c)$; $V(y_t;c)$ is a potential function which is determined by control parameter c, and c determines the specific structure of the system and can consist of one or multiple variables. The system is in equilibrium when the spatial derivative of the potential function equals 0, i.e. $-dV(y_t;c)/dy_t = 0$. The equilibrium corresponds either to a maximum or a minimum of potential function $V(y_t;c)$ with respect to y. When $V(y_t;c)$ takes a minimum, the equilibrium points are stable. The system will always return to it after a small perturbation with respect to system's state; Following the same idea, the equilibrium points are unstable equilibria if the potential function $V(y_t;c)$ takes a maximum. Even a small perturbation will drive the system away from these equilibrium states and move towards a stable equilibrium. The Hessian matrix has eigenvalues equal to 0 in these equilibria, at which a system can give rise to unexpected bifurcations when the control variables are changed. Therefore, catastrophe theory can be employed in systems which can be driven toward an equilibrium state, such as a gradient dynamical systems with critical points.

3.2.1 Cusp Catastrophe

One of the extraordinary findings of catastrophe theory is that it proposed the behaviour of deterministic dynamical systems around the critical points of potential function $V(y_t;c)$. It proposed that this behaviour can be characterised by a set of seven canonical forms with no more than four control variables and one or two canonical state variables (Thom,1972; Zeeman, 1976; Gilmore,1993). In behavioural sciences, the most commonly used canonical form is the so-called cusp catastrophe. In terms of a normalised variable z_t, it describes the sudden, discontinuous transitions in equilibria states as a result of continuous changes in two normal

form control parameters α and β:

$$-V(z_t; \alpha, \beta) = -\frac{1}{4}z_t^4 + \frac{1}{2}\beta z_t^2 + \alpha z_t. \tag{3.2}$$

The equilibria can be obtained by solving the cubic equation:

$$-\frac{\partial V(z; \alpha, \beta)}{\partial z} = -z_t^3 + \beta z_t + \alpha = 0, \tag{3.3}$$

in which the derivative of potential function $V(z; \alpha, \beta)$ equals 0. For descriptive purpose, a statistic so-called Cardan's discriminant is proposed to distinguish the case of three solutions from the case of one solution (Cobb, 1981). This Cardan's discriminant is defined as:

$$\delta = 27\alpha^2 - 4\beta^3. \tag{3.4}$$

When $\delta < 0$, there are three solutions, while when $\delta > 0$, there is only one solution. When δ exactly equals 0, there are three solutions and two of them have the same value. Figure 3.2 gives a visual description of the cusp catastrophe model in an example of a housing market. It illustrates a cusp equilibrium surface living in a three dimensional space. The folded surface with a fold cusp represents the equilibrium surface of system. The floor is a two dimentional control plane which is determined by a set of control parameters, α and β. The height predicts the value of the system's state with respect to control parameters. In the middle of the graph, there are two sheets representing the behaviour of system, and they are connected by a middle sheet making a continuous surface. The difference between the middle sheet and the other two sheets is that the middle sheet represents the least probable state of the system. The curve defining the edges of the fold cusp projected onto the control plane showing an across hatched cusp shaped region. The cusp which marks its boundary is called the bifurcation set (Zeeman, 1974, 1976), for which Cardan's discriminant $\delta = 0$. When $\delta > 0$, the system has only one stable equilibrium state. There is only one predicted state value. However, within the cusp fold,

where $\delta < 0$, the surface predicts two possible stable state values instead of one. This implies that in a system with noise the state variable is bimodal inside the bifurcation area. In addition, this middle surface predicts that certain state values, such as unstable equilibrium states, should not occur frequently. It "anti-predicts" an intermediate value for these values of the control variables (Grasman *et al.*, 2009). Moreover, the system might get into a hysteresis loop by jumping between these two possible state values. The jump from the top sheet to the bottom sheet of the behaviour surface occurs at a different value of the variable than the jump from the bottom to the top sheet does.
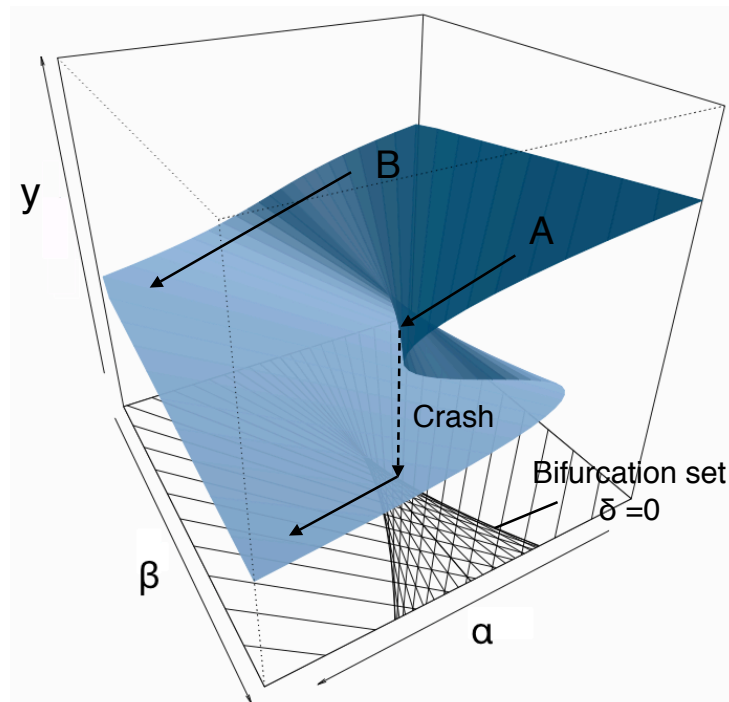
Figure 3.2: Cusp catastrophe model of housing market

3.2.2 Stochastic Cusp Catastrophe

Although Zeeman has proposed that catastrophe theory could be applied to multiple disciplines (Zeeman, 1977), a practical empirical investigation requires a model that allows stochastic shocks. In order to address this issue and to build a bridge between catastrophe theory and real scientific data, several stochastic formations of catastrophe theory which allow empirical

investigations have been proposed (Oliva *et al.*, 1987; Guastello, 1988; Alexander *et al.*, 1992; Lange *et al.*, 2000). Of all the methods, the method of Cobb and Watson (1980) is arguably the most appealing. They proposed to combine the deterministic catastrophe theory with stochastic systems theory by using Itô stochastic differential equation (SDE). It leads to the definitions of stochastic equilibrium state and stochastic bifurcation that are compatible with their deterministic counterparts, in such a way that it establishes a link between the potential functions of deterministic catastrophe systems and the stationary probability density functions of stochastic processes.

Assuming that the (canonical) variable z_t is still governed by the potential function of Eq. (3.2), and that there is a driving noise term with variance σ_z^2 per time unit, the dynamics can be written in terms of the SDE:

$$
\begin{aligned}
\mathrm{d}z_t &= \left. -\frac{\partial V(z;\alpha,\beta)}{\partial z} \right|_{z=z_t} + \sigma_z \mathrm{d}W_t. \\
&= -z_t^3 + \beta z_t + \alpha + \sigma_z \mathrm{d}W_t.
\end{aligned} \tag{3.5}
$$

The deterministic term $-\partial V(z_t;\alpha,\beta)/\partial z_t$ is the drift function, σ_z is the diffusion parameter, and W_t is a Wiener process. z_t is the dependent variable. α and β are the "canonical variates" which are the smooth transformations of actually measured independent control variables $x_1,\ldots,x_n$.

In this stochastic context, on the one hand, if $\alpha = 0$, its density function is symmetric. The sign of α decides whether it is left or right skewed. Thus α is called "asymmetry factor" and determines direction of the skew of the density. On the other hand, as β changes from negative to positive, the density is changing from unimodal to bimodal. β is the "bifurcation factor" and determines the number of modes of the density.

3.2.3 Cusp Catastrophe of Housing Market

The best way to understand the nature of models derived from cusp catastrophe is to illustrate it by examples. Zeeman has considered some popular example applications of catastrophe model to different disciplines, such as ecology, physics and psycology (Zeeman, 1974). His work was the first attempt to explain unstable behaviour of stock market using catastrophe model. Some behavioural finance elements in his work motivated HAM models, in which the instability of market is expectations-driven. The housing market has several common characteristics with the stock market. Their dynamical behaviours are connected in some ways. The booms and burst cycle of housing prices was also proved to be partly driven by heterogenous expectations of agents (Bolt *et al.*, 2014). Several theoretical models have been shown to perform well in the analysis of housing markets and stock markets. For instance, HAMs are successful examples to capture the instability in both the stock market and the housing market (Brock and Hommes, 1997, 1998; Hommes, 2013; Kouwenberg and Zwinkels, 2010; Dieci and Westerhoff, 2012, 2013; Bolt *et al.*, 2014). Since Zeeman has proposed that catastrophe model could be used to explain stock market, it could also be useful in the housing market.

An example of a cusp catastrophe model in housing market is illustrated visually in Figure 3.2. The smooth folded surface with three levels of sheets represents the equilibria of the system. A set of control parameters α and β forms a two-dimensional control plane. The *z*-axis of the 3-dimensional space *y* represents the state variable, such as housing prices.

The catastrophe behaviours are now observable whenever the set of control parameters moves all the way across the cusp equilibrium surface. Each point on the top and bottom sheets of this surface gives equilibrium of system. If the point is on the top sheet and follows the path *A* on the control surface, the corresponding path moves to left on the top sheet until it reaches the fold curve; the top sheet then vanishes, and the path must suddenly jump to the bottom sheet. A small change in control parameters can produce a sudden large change in the state of the system. Alternatively, the path *B* on the control surface outside of the cusp bifurcation exhibits the behaviour of an ordinary market. Its corresponding path moves to the bottom sheet

slowly and smoothly, without catastrophes.

The mechanism of housing market crashes can now be understood. We assume that the equilibrium of a housing market with rising prices is on the top sheet of the behaviour surface. The housing market with falling prices has its equilibrium on the bottom sheet of the equilibrium surface. A crash can then be induced by any event that changes the control parameters enough to push the behaviour point over the fold curve, fall off the "cliff" and jump to the bottom sheet. In particular, if the equilibrium is in the bifurcation set and very close to the cusp curve, even a small perturbation can induce large market collapse. Similar, the "negative" crashes could induced by an upward jump from the bottom sheet to the top sheet. In particular, the equilibrium of the system could also transition from one stable equilibrium to another without passing through the cusp curve. This can be used to explain the slow recovery from a crash in housing markets. A recovery is affected by the slow feedback from the behaviour of housing market on the control parameters. It does not pass through the catastrophe cusp, but slowly and smoothly follow the reversal of the path B. The system is then set for another cycle of boom and bust. Therefore, the duration of housing price declines was quite long lived and the subsequent recovery from a crash was exceptional slow.

3.3 Estimation Methods

In the estimation, α and β are approximated by using a first order Taylor expansion

$$\begin{aligned} \alpha &= \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_v x_v \\ \beta &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_v x_v \end{aligned} \quad (3.6)$$

Then $\alpha_0, \alpha_1 ..., \alpha_v$ and $\beta_0, \beta_1 ..., \beta_v$ are the parameters to be estimated when fitting the catastrophe model to empirical data. $x_1, x_2 ..., x_v$ are the independent variables.

3.3.1 Cobb's Method

The most common estimation method that allows quantitative comparison of catastrophe models with empirical data is that proposed by Cobb (Cobb, 1978, 1980, 1981; Cobb and Watson, 1980; Cobb *et al.*, 1983). He established a stochastic cusp catastrophe by simply introducing a stochastic Gaussian white noise, with the SDE as Eq.(3.5).

In this approach, maximum likelihood estimation is applied to the cusp probability density function (PDF). To simplify the estimation procedure, instead of using the conditional PDF $f(z_t|z_{t-1}, z_{t-2}, ...; \alpha, \beta)$, Cobb considered the invariant PDF which is given by solving the corresponding Fokker-Planck equation. As time passes, $t \to \infty$, the conditional PDF $f(z_t|z_{t-1}, z_{t-2}, ...; \alpha, \beta)$ converges to a stationary and time invariant form $f(z_t|\alpha, \beta)$. With linear transformations of dependent variable $z_t = (y_t - \lambda)/c$, the distribution of system's states on any moment in time is expressed as

$$f(y) = \psi \exp\left[\tilde{\alpha}\left(\frac{y-\lambda}{c}\right) + \frac{1}{2}\tilde{\beta}\left(\frac{y-\lambda}{c}\right)^2 - \frac{1}{4}\left(\frac{y-\lambda}{c}\right)^4\right], \tag{3.7}$$

where $\tilde{\alpha} = \left(\frac{\sigma_z^2}{2}\right)^{-\frac{3}{4}}\alpha$ and $\tilde{\beta} = \left(\frac{\sigma_z^2}{2}\right)^{-\frac{1}{2}}\beta$, $c = r\left(\frac{\sigma_z^2}{2}\right)^{\frac{1}{4}}$, and ψ is normalisation constant so this PDF's integral over the entire range equals 1. For a derivation, see Appendix 3.B.2. The location parameter λ and scale parameter c are kept constant.

Consequently, with the invariant PDF as theoretical PDF in the estimation, and under the assumption that the diffusion function is constant, $\sigma_z = $ constant, the deterministic and stochastic potential function can be linked with each other. The stochastic stable and unstable equilibrium of the potential function are associated with modes and anti-modes of the invariant PDF respectively. The stochastic bifurcations correspond to the changes in the number and type of the modes of the invariant PDF. A qualitative change in the potential function is identical to a qualitative change in the PDF with respect to the change of control parameters. For instance, the PDF changes from unimodal to bimodal as the bifurcation parameter β changes from negative to positive. As a result, by estimating the distribution of invariant PDF, any point on the cusp

equilibrium surface can be estimated according to their closest maximum mode of PDF. The inaccessible middle sheet of equilibrium surface is reflected in the middle of the bimodal that a low probability mode is between two high probability modes.

Based on Cobb's statistical catastrophe theory, a series of works by Hartelman (1997), Hartelman *et al.* (1998), and Grasman *et al.* (2009) implemented and extended Cobb's estimation method. They presented robust and practical computer programs which made it easy to fit cusp catastrophe models on empirical data in a statistical way. In these approaches, Cobb's method is combined with the subspace fitting method of Oliva *et al.* (1987). The state of system is measured as a first approximation of

$$z = w_0 + w_1 y_1 + w_2 y_2 + ... + w_v y_v. \tag{3.8}$$

z is the smooth transformation of the actual state variable of system. $w_0, w_1, ..., w_v$ are the first order coefficient of a polynomial approximation to the smooth transformation. $y_1, y_2 ..., y_v$ is a set of measured dependent variables. Because independent variables α and β are also approximated by the first approximation (see Equation (3.6)), fitting the cusp model to empirical data is then reduced to estimate the parameters of $w_0, w_1, ..., w_v$; $\alpha_0, \alpha_1 ..., \alpha_v$ and $\beta_0, \beta_1 ..., \beta_v$.

Although the Cobb's method has been proved to be a successful tool in several multiple equilibria systems, we should note that it requires a system with dynamics change much more quickly than the feedback reactions of control parameters, due to its nature of time independent. This method gives a good fit of the overall invariant density for the cross-sectional dataset, or for those systems with quickly changing dynamics. For time series dataset or for systems with slow process, particularly when the scale of state of system (z-axis in Figure 3.2) can not be separated from the scale of control parameters (x-axis and y-axis in Figure 3.2), the forecasting ability of Cobb's method may be not sufficient.

3.3.2 Estimations of Euler Discretization

In order to estimate the stochastic differential equation (SDE) of cusp catastrophe with changing time, we consider another numerical method - Euler Discretization, which is the numerical method which seeks to approximate the SDE at discrete times.

Time t is subdivided into intervals of length Δt, so that $t_n = n\Delta t$. Then we could approximate the solution at those times t_n. Because $y = \lambda + rz$ is a scaled and/or translated variable, in terms of $y_t = \lambda + rz_t$ the SDE becomes (see Appendix 3.B.3)

$$\frac{1}{r}dy_t = -\left.\frac{\partial V(z;\alpha,\beta)}{\partial z}\right|_{z=\frac{y_t-\lambda}{r}} + \sigma_z dW_t, \tag{3.9}$$

Euler Discretization gives an approximate equation which predicts a future value of state of system y in terms of past value:

$$\begin{aligned}
y_{t+\Delta_t} &= y_t - \left.\frac{\partial V(\tilde{y}_t;\alpha,\beta)}{\partial \tilde{y}_t}\right|_{\tilde{y}_t=\frac{y_t-\lambda}{r}} r\Delta t + r\sigma_W \varepsilon_t \sqrt{\Delta t} + h.o.t. \\
&\approx y_t + \left(\alpha + \beta\left(\frac{y_t-\lambda}{r}\right) - \left(\frac{y_t-\lambda}{r}\right)^3\right) r\Delta t + r\sigma_z\sqrt{\Delta t}\varepsilon_{t+\Delta_t},
\end{aligned} \tag{3.10}$$

where $\varepsilon_{t+\Delta_t} \sim N(0,1)$.

The distribution of state of system at any moment in time can now be approximated as:

$$f_{y_{t+\Delta_t}}(y|y_t) \approx \psi\exp\left\{\frac{-\left[y - \left(y_t - r\Delta t\left(\left(\frac{y_t-\lambda}{r}\right)^3 - \beta\left(\frac{y_t-\lambda}{r}\right) - \alpha\right)\right)\right]^2}{2\sigma_z^2 r^2 \Delta t}\right\} \tag{3.11}$$

ψ is normalisation constant so the PDF's integral over the entire range equals 1. The location parameter λ and scale parameter c are kept constant. In this paper λ is assumed to be equal to 0 since there is a fundamental equilibrium at 0 (and the mean of the price fluctuations around this

is assumed to be zero). The estimated parameters are $\lambda, c; \alpha_0, \alpha_1, ..., \alpha_v$ and $\beta_0, \beta_1, ..., \beta_v$. The conditional PDF is considered in this approach while invariant PDF is used in Cobb's method. Therefore, by using one-step-ahead forecast, any moment on the equilibrium surface is able to be predicted with changing time. To compare with Cobb's method, the estimation method used here is also Nonlinear Least Squares (NLS). Brillinger (2007) used another approach to estimate a potential function based on a linear model.

We should note that the one-step-ahead forecast is over-parameterised in the sense that the forecasts the model produces are independent of the value we choose for Δt. This might have been expected, since changing the value of Δt simply corresponds to a change of time units, which should not affect the forecasts. We therefore set $\Delta t = 1$ throughout (i.e. we define one time unit to correspond to one quarter, the time interval between consecutive observations in our data set). We do emphasize that the choice of the time scale does affect the estimated numerical values of α, β and r. Doubling Δt can be compensated in the point forecast by multiplying r and β by $\sqrt{2}$ and dividing α by $\sqrt{2}$. The term $\sigma_z^2 r^2 \Delta t$ in the denominator can be considered a single parameter (the forecast variance over a period of one month), which is independent of the choice made for the time unit.

3.3.3   Cobb's Method v.s. Euler Discretization

Cobb's method aims to give a good fit of cusp catastrophe based on the overall invariant density of system. This method requires a system with separated measure scales and quickly changing dynamics. As a matter of fact, it is time independent and would give better fit on cross-sectional dataset rather than time series data. On the contrary, Euler Discretization considers the one-step-ahead forecast by estimating the conditional density of system in time. The assumption of time independent in Cobb's method is relaxed. Intuitively, Euler Discretization should make considerable improvement to Cobb's Method regarding the forecasting ability in a time series framework.

To compare the forecasting ability of these two estimation approaches, the most simple and

straightforward way is to examine their residuals. Figure 3.3 shows the plots of residuals against time in the example of US by using Cobb's Method and Euler Discretization respectively. In Figure 3.3 (a), we observe strongly correlated residuals with clear patterns and obvious deviations from randomness. Moreover, the values of the residuals are big, i.e. bigger than 0.1. This is due to the fact that the predicted values in Cobb's method are estimated based on the closest maximum mode of invariant PDF rather than by past information. Although it may still give a good fit with respect to invariant density distribution, when it comes to the evaluation of the forecasting ability, our analysis shows it is insignificant. Nevertheless, as shown in Figure 3.3 (b), these residuals are randomly distributed and are small, i.e. smaller than 0.04 in absolute value. This suggests that Euler Discretization gives much better predictions than Cobb's method. The residuals in the examples of many other countries are observed under the same patterns, which are shown in Figure 3.13 and Figure 3.14 in Appendix 3.B.1.

(a) Cobb's Method (b) Euler Discretization
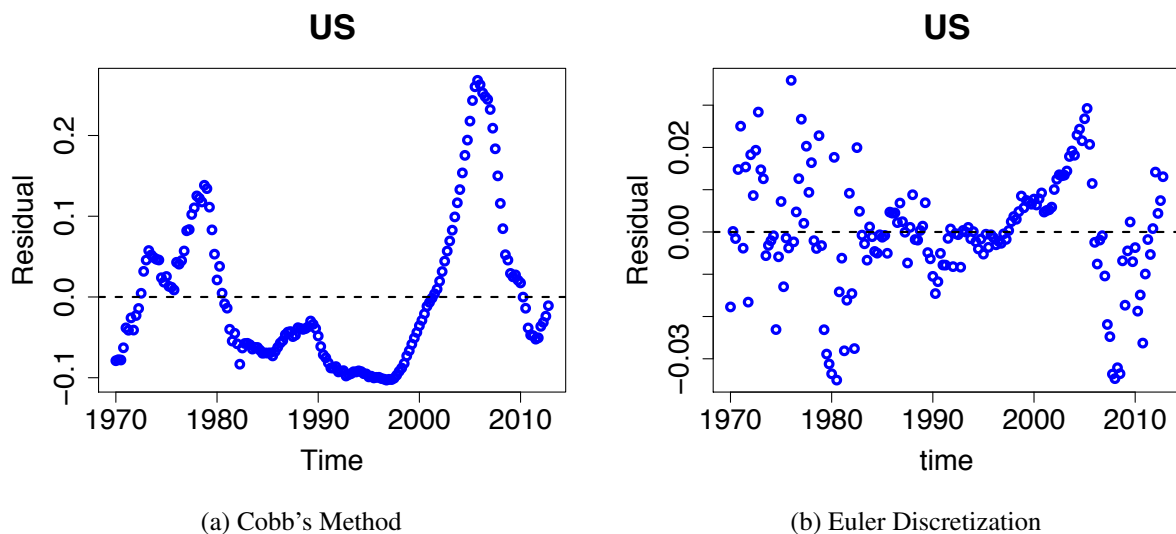
Figure 3.3: Plots of residuals against index by using Cobb's Method and Euler Discretization in the example of US.

Hartelman (1997) and Grasman *et al.* (2009) proposed to use the AIC and BIC to assess the model fit. The AIC and BIC in our examples are presented in Table 3.1. It can be seen that the AIC and BIC by using Euler Discretization are much smaller than using Cobb's method, which suggests a better model fit of Euler Discretization. It further proves that the forecasting ability

of Euler Discretization is more promising than Cobb's Method with respect to housing prices.

		US	JP	UK	NL	SE	BE
Cobb	AIC	497.955	491.587	438.482	357.619	361.697	379.087
	BIC	510.568	504.200	451.096	370.232	373.258	391.049
Euler	AIC	-983.611	-901.021	-715.484	-721.421	-592.250	-782.153
	BIC	-971.021	-888.431	-702.894	-708.831	-580.719	-770.191

Table 3.1: AIC and BIC of Cobb's method and Euler Discretization in different countries.

The possible explanations are following. Firstly, although the estimation method of Cobb has been proved to be a successful tool to fit cusp catastrophe in several areas, most of these examples considered cross-sectional datasets. Rather than observing with changing time, these examples focused on a cross section of time and aimed to approximate the overall stationary density of the system. However, the dataset in our examples are housing price deviations which are time series data. From this point of view, the one-step-ahead forecast in Euler Discretization should be considered as a better candidate than Cobb's method. Secondly, when fitting time series data, Cobb's method presumes separated time scales; it requires that the state of system changes much more quickly compared with the feedback reactions of control parameters. However, housing markets involve very long boom and burst cycles, the noise is big and the speed of the process is slow. Furthermore, because we are going to discuss the forecasting ability of cusp catastrophe model on the housing market, Euler Discretization is considered as a better candidate than Cobb's method regarding forecasting ability. For the above reasons, in what follows, we will analyse the cusp catastrophe behaviour of housing markets using Euler discretisation.

3.4   Estimation Variables

In this paper, we are going to observe the housing markets of six different countries: the United states (US), the United Kingdom (UK), the Netherlands (NL), Japan (JP), Sweden (SE) and

Belgium (BE). The variables in the estimation of the cusp catastrophe model consist of state variables and control variables.

3.4.1 State Variables

The state variable is required to be able to describe the unstable behaviour of housing markets. Bolt *et al.* (2014) estimated a HAM model for housing markets in different countries and observed bifurcations driven by policy parameter out of relative deviations of housing price from fundamental price. Following their success, we also use the relative deviation of housing price from the estimated fundamental price as state variable, which is denoted by

$$y_t = \frac{p_t}{p_t^*} - 1 \approx ln p_t - ln p_t^*. \tag{3.12}$$

The fluctuations of the housing price around the fundamental price (based on expected future rental prices) are described by a model in which agents choose between either buying or renting house. In this model, agents make their decisions at time t based on the expected excess return on investing in housing relative to renting during the period between time t and $t+1$. The fundamental price is assumed as the price that would prevail under rational expectations about the conditional mean of excess return. In equilibrium, the annual cost of home ownership must equal the housing rent adjusted for risk. The supply of the market is the stock of housing. The demand of agents is determined by maximising one-period ahead expected excess returns adjusted for risk. By solving the market clearing condition for price p_t (See Appendix 3.A for detailed calculation), we have the price equation

$$p_t = \frac{1}{1+r+\alpha} \mathbb{E}_{h,t} \left[p_{t+1} + (1+r^{rf})Q_t \right], \tag{3.13}$$

where Q_t denotes the price for renting one unit of housing in the period between times t and $t+1$. Because rents are typically payed up-front at time t, the rent at time t in terms of currency at time $(t+1)$ should be inflated by a factor $(1+r^{rf})$, where r^{rf} is the risk free mortgage rate.

Therefore the cost of renting between t and $t+1$ is given by $(1+r^{rf})Q_t$ in terms of currency at time $(t+1)$. r is the sum of the risk free mortgate rate and the maintenance/tax rate. α is interpreted as a risk premium of buying a house over renting a house. In this model, α is assumed to be constant in order to keep the model tractable. Taking into account the risk premium α in the fundamental price will provide an equilibrium fundamental price from which the market price will deviate by an amount which averages out to 0 in long time series.

The fundamental process underlying the model is assumed to follow a geometric Brownian motion with drift (Boswijk $et\ al.$, 2007):

$$logQ_{t+1} = \mu + logQ_t + \upsilon_{t+1}, \quad \{\upsilon_t\} \overset{i.i.d.}{\sim} N(0,\sigma_\upsilon^2). \tag{3.14}$$

When $g = e^{\mu+1/2\sigma_\upsilon^2} - 1$ and $\varepsilon_{t+1} = e^{\upsilon_{t+1}-1/2\sigma_\upsilon^2}$, one obtains:

$$\frac{Q_{t+1}}{Q_t} = (1+g)\varepsilon_{t+1}, \tag{3.15}$$

such that $\mathbb{E}_t(\varepsilon_{t+1}) = 1$. Therefore, by applying the law of iterated expectations and imposing the transversality condition, the fundamental price at time t is as:

$$p_t^* = \mathbb{E}_t\left[\sum_{i=0}^{\infty} \frac{(1+r^{rf})Q_{t+i}}{(1+r+\alpha)^{i+t}}\right] = \frac{1+r^{rf}}{r+\alpha-g}Q_t, \quad r+\alpha > g. \tag{3.16}$$

It shows that the fundamental price of housing is directly proportional to the actual rent level. Figure 3.4 shows an example of house price and fundamental price in US from 1970 to 2013. Figure 3.4(a) presents the housing price index p_t with the corresponding estimated fundamental values p_t^*. The price deviations of $p_t - p_t^*$ is shown in Figure 3.4(b), which is also the state variable y in our case. The plots of the examples of many other countries are shown in Appendix 3.A in Figure 3.12.

(a) p_t and p_t^* (b) $y_t = p_t - p_t^*$
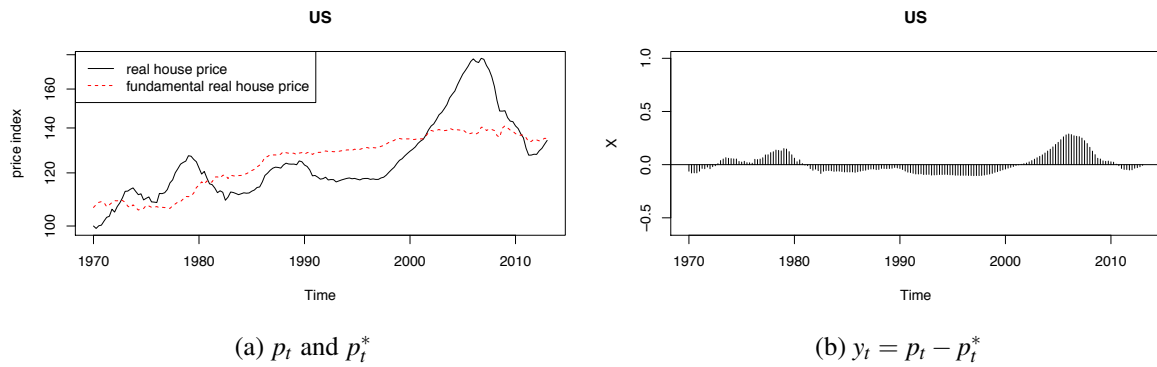
Figure 3.4: The example of housing price and fundamental price in US from 1970 to 2013. (Source: Bolt *et al.*, 2014). (*a*)The housing price indices p_t with the corresponding estimated fundamental values p_t^*; (*b*) The log-difference between the two $p_t - p_t^*$.

3.4.2 Control Variables

We start with a simple model and discuss only one control variable. One of the advantages is that it allows us to observe critical transitions from time series. This could help us to understand the real market crashes in housing system.

One of the parameters which has the greatest influences on the deviations of housing prices from fundamentals is argued to be the mortgage rate. Several papers have pointed out that monetary policy, especially interest rate policy has great impact on housing prices (Bernanke and Gertler, 1995; Shiller, 2006; Muellbauer and Murphy, 2008; Taylor, 2007, 2009; Crowe *et al.*, 2013; Shi *et al.*, 2014). Moreover, empirical evidence of Bolt *et al.* (2014) suggested that several bifurcations of price equilibria may occur driven by interest rates. Therefore, as an important policy parameter, the interest rate is chosen as our control variable.

3.4.3 Data Description

The analysed housing markets are the US, UK, NL, JP, SE and BE. In order to observe the critical transitions from nonlinear dynamics of the housing market, we require time series as long as possible to contain as much information as possible. The investigated time window

ranges from 1970 to 2013 [1]. This contains several well-known housing market crashes, such as those of the United States (2007), Japan (1992), Sweden (1991), the United Kingdom (2007) and the on-going bubbles in many countries.

Quarterly nominal and real house prices for each country are obtained from the housing dataset in the Organisation for Economic Co-operation and Development (OECD). The nominal house price is indexed using 2005 as base year. The real house price index is derived by deflating with the private final consumption expenditure deflator, which is available from the OECD Economic Outlook 89 database. The price-to-rent ratio is defined as the nominal house price index divided by the rent component of the consumer price index, made available by the OECD. The interest rate is indicated by 10-year government bonds yields, downloaded from the OECD iLibrary for US, UK, NL, BE. Because OECD iLibrary does not have the interest rate dating back to 1970, for JP and SE, we downloaded them from Datastream.

3.5   Results and Discussions

The differential equation of cusp catastrophe model is estimated by using Euler Discretization. As described in Section 3.4, the state variable is the relative deviation of the housing price from the fundamental price. For the control parameters, we consider two variants: estimation with constant control parameters, and estimation with the control variable interest rate governing the control parameters α and β.

---

[1]Time window for Sweden (SE) is from 1Q1980 to 1Q2013, and for Belgium is from 2Q1976 to 1Q2013, based on the availability in the datasets.

3.5.1 Constant Control Parameters

As a benchmark, we fit the cusp catastrophe model to housing market data given a constant control variable. Thus the control parameters α and β are constant and are defined by

$$\alpha = \alpha_0 \tag{3.17}$$
$$\beta = \beta_0$$

To describe the stability of system statistically, we investigate Cardan's discriminant $\delta = 27\alpha^2 - 4\beta^3$. $\delta = 0$ indicates the boundary of bifurcation set. When $\delta < 0$, the state of system is in cusp bifurcation region and unstable. The model predicts two possible state values instead of one; if $\delta > 0$, the state of the system is outside of bifurcation region and stable.

Table 3.2 shows the estimated parameters and their corresponding standard errors in all six countries. The parameters λ and σ scale the observed state variable. λ is assumed to be equal to 0 since there is a fundamental equilibrium at 0. Bayesian information criterion (BIC) and Akaike information criterion (AIC) indicate the fitness of model (Hartelman, 1997; Grasman *et al.*, 2009). The standard errors of transformed parameters of Cardan's discriminant δ are obtained by the delta method.

Delta method takes the variance of the Taylor series approximation of a function as standard error. Let G be the transformation function and X be the consistent estimator. X converges in probability to its mean vector U. Let $\bigtriangledown G(X)$ be the gradient of $G(X)$. The first two terms of the Taylor expansion are then an approximation for $G(X)$,

$$G(X) \approx G(U) + \bigtriangledown G(U)^T \cdot (X - U), \tag{3.18}$$

which implies that the variance of $G(X)$ is approximately

$$Var(G(x)) \approx \bigtriangledown G(X)^T \cdot Cov(X) \cdot \bigtriangledown G(X) \tag{3.19}$$

where $Cov(X)$ is the variance-covariance matrix of X. $Var(G(x))$ is thus the standard error of Cardan's discriminant δ.

	US	JP	UK	NL	SE	BE
λ	0.000	0.000	0.000	0.000	0.000	0.000
σ	5.298	3.420 $\cdot$	3.056*	2.982*	2.405 $\cdot$	2.641***
(s.e.)	(2.858)	(2.041)	(1.261)	(1.255)	(1.348e+00)	(0.373)
α	$7.704e-05$	-0.0004	$1.781e-03$	$1.448e-03$	$2.583e-05$	0.003 **
(s.e.)	(5.314e−04)	(0.0005)	(0.001)	(0.001)	(9.484e−04)	(0.0010)
β	$-9.632e-03$	0.014	$1.376e-02$	$1.353e-02$	$1.131e-02$	0.048***
(s.e.)	(1.795e−02)	(0.012)	(0.018)	(0.018)	(2.489e−02)	(0.011)
δ	$3.735e-06$	$-5.528e-06$	$7.526e-05$	$4.673e-05$	$-5.773e-06$	-0.0001
(s.e.)	(1.842e−05)	(2.368e−05)	(0.0001)	(8.298e−05)	(3.799e−05)	(0.0002)
AIC	-983.611	-901.021	-715.484	-721.421	-592.250	-782.153
BIC	-971.021	-888.431	-702.894	-708.831	-580.719	-770.191
Loc.stab.	stable	unstable	stable	stable	unstable	unstable

$***$ significant at 0.1% level, $**$ significant at 1% level, $*$ significant at 5% level, $\cdot$ significant at 10% level.

Table 3.2: Estimation results with respect to constant α and β in different countries. "Stability" shows local stability of the system. The values in the brackets are standard errors.

The local stability of the system, which is indicated by the sign of δ, is also presented in Table 3.2. Using US and SE as examples, Figure 3.5 shows the kernel density of the price deviations in stable and unstable systems. The US shows an unimodal distribution while the SE has a bimodal distribution.

(a) $\delta > 0$ (b) $\delta < 0$
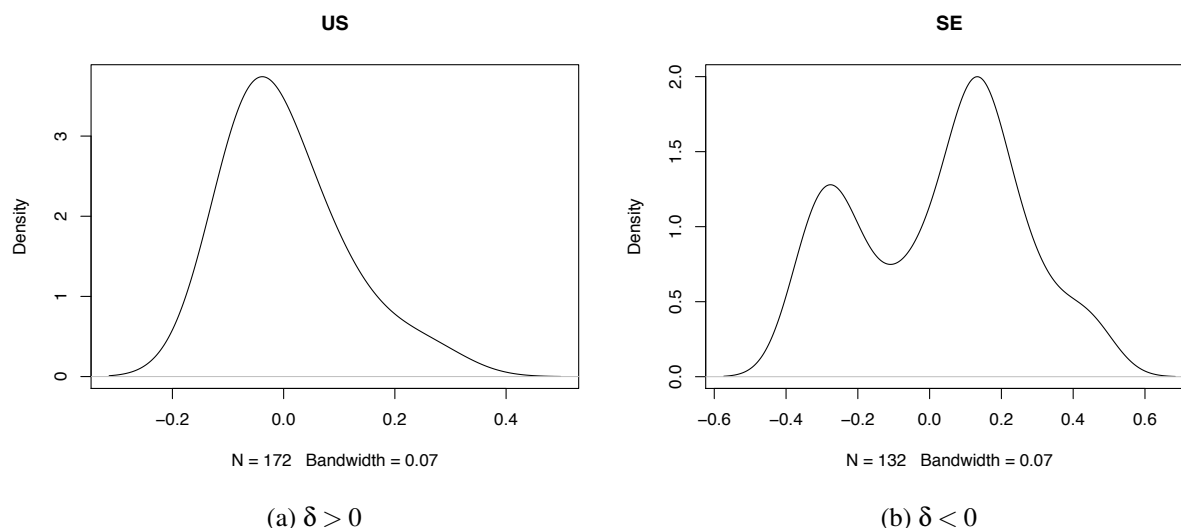
Figure 3.5: The examples of Kernel density estimate on price deviations when $\delta > 0$ and $\delta < 0$.

Figure 3.6 gives a visual illustration of the estimated location of the behaviour points (equilibria) of different countries on the projection of cusp equilibrium surface in Figure 3.2. The cusp shaped shaded area is the control plane which is determined by the sets of control parameters α and β. As shown in the legend, each symbol and colour indicates the estimated parameters of equilibrium of one country. The ellipse around it with the same colour corresponds to its 95% probability region. Consistent with the estimation results in Table 3.2, countries of US, NL and UK are in stable regions, while JP, SE and BE are unstable and inside the grey bifurcation region. With confidence level of 95%, the confidence regions include the points representing the "true" values of behaviour points.
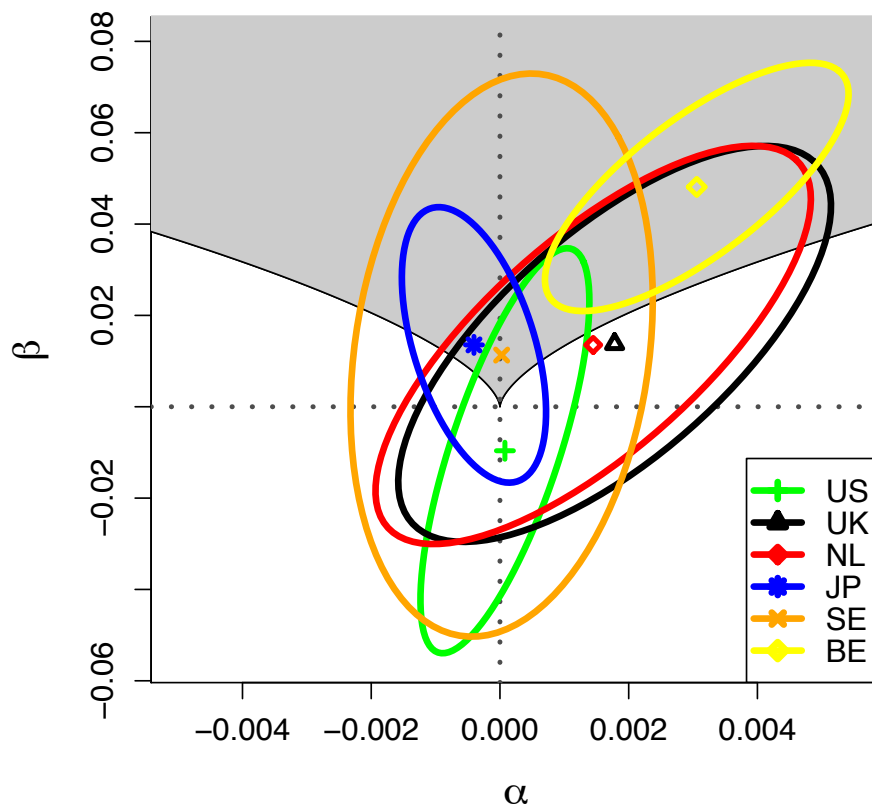
Figure 3.6: The estimated behaviour points of different countries in different regions of the control plane with respect to constant α and β. The cusp shaped shaded area is the control plane. The ellipses correspond to confidence regions with confidence level of 95%.

Notably, by combining Figure 3.6 with the cusp equilibrium surface in Figure 3.2, we are able to forecast the dynamical behaviour of the housing market by observing where its behaviour point is located on the cusp equilibrium surface. For instance, country SE is inside the bifurcation set where the surface predicts two possible state values instead of one. With the change of control parameters, its behaviour point may follow the path A in Figure 3.2 and move close to fold curve. A tiny perturbation on the control variables would induce it to fall off the cusp "cliff" and jump to a different equilibrium suddenly. The US is in the "normal" situation and outside of the cusp bifurcation. If the control variable α changes, its state point would follow the path B in Figure 3.2. Its transition between equilibria is slowly and smoothly without experiencing cusp catastrophes. However, it does not mean that no critical transition could occur. When control variable β increases, it would move into the unstable bifurcation set and experiences a critical transition. We shall observe similar situations for NL and UK. Although they are in stable regions, as long as their control parameters change in certain directions, they may move into bifurcation set and experience possible critical transitions. They are particularly dangerous when they are too close to the bifurcation border. Even a small perturbation may induce a critical transition. Therefore, the changes of corresponding control variables perform an important role in the dynamic behaviour of the systems. By monitoring and controlling these changes, we are able to influence or even prevent the instability of housing markets.

3.5.2 Interest Rate as Control Variable

In what follows, we study the situation when the interest rate is used as a control variable. Following the estimation methods in Section 3.3, the control parameters α and β are now defined by

$$
\begin{aligned}
\alpha &= \alpha_0 + \alpha_1 x, \\
\beta &= \beta_0 + \beta_1 x,
\end{aligned}
\tag{3.20}
$$

where x is the control variable - interest rate. It is indicated by the 10-year government bonds yields. $\sigma, \alpha_0, \alpha_1, \beta_0, \beta_1$ are the parameters to be estimated. Table 3.3 shows the estimation results in different countries. Because we are interested in predicting the changes of equilibria and investigating critical transitions, although some parameters are not very significant, we will later show that the model fits well with respect to the changes of equilibria. Like the example of constant control parameters, we conduct a plot of the control variables with the estimated behaviours for different countries in Figure 3.7 to provide a visual illustration.

	US	JP	UK	NL	SE	BE
λ	0.000	0.000	0.000	0.000	0.000	0.000
σ	2.097	4.482	2.773**	7.495	1.485***	2.533***
(s.e.)	(1.985)	(4.639)	(0.972)	(21.405)	(0.286)	(0.342)
α_0	0.003	−0.002	0.013*	0.005	0.020***	0.015***
(s.e.)	(0.004)	(0.002)	(0.006)	(0.014)	(0.006)	(0.003)
α_1	−0.0005	0.0004	−0.001*	−0.0005	−0.002***	−0.002***
(s.e.)	(0.0006)	(0.0005)	(0.0005)	(0.002)	(0.0007)	(0.0003)
β_0	0.0123	−0.0002	−0.032	−0.094*	0.025	0.051*
(s.e.)	(0.046)	(0.016)	(0.034)	(0.042)	(0.030)	(0.020)
β_1	−0.004	−0.001	0.004	0.013**	0.0009	−0.006*
(s.e.)	(0.006)	(0.003)	(0.004)	(0.005)	(0.003)	(0.002)
AIC	−988.770	−912.718	−732.828	−729.114	−630.110	−831.070
BIC	−969.885	−893.832	−713.943	−710.229	−612.813	−813.128

∗ ∗ ∗ significant at 0.1% level, ∗∗ significant at 1% level, ∗ significant at 5% level, · significant at 10% level.

Table 3.3: Estimated parameters of $\sigma, \alpha_0, \alpha_1, \beta_0, \beta_1$ when using interest rate as the control variable. λ is constant. The values in the brackets are standard errors.

As shown in Figure 3.7, in different countries, their equilibria changing with time trace different paths on the control plane. These tracks are distinguished by different symbols and colours. By comparing with the benchmark in Figure 3.6, we shall see that the interest rate as a control variable has great impact on the instability of housing markets. For the housing markets of NL and UK, although their equilibrium points locate in the stable regions under constant control parameters (Figure 3.6), the changes of interest rate induce them move into the unstable bifurcation region in certain times (Figure 3.7). On the contrary, the equilibrium points of SE, BE and JP are now able to move outside of the bifurcation region. As for the US which remains in stable regions, its equilibrium point also changes with interest rate.
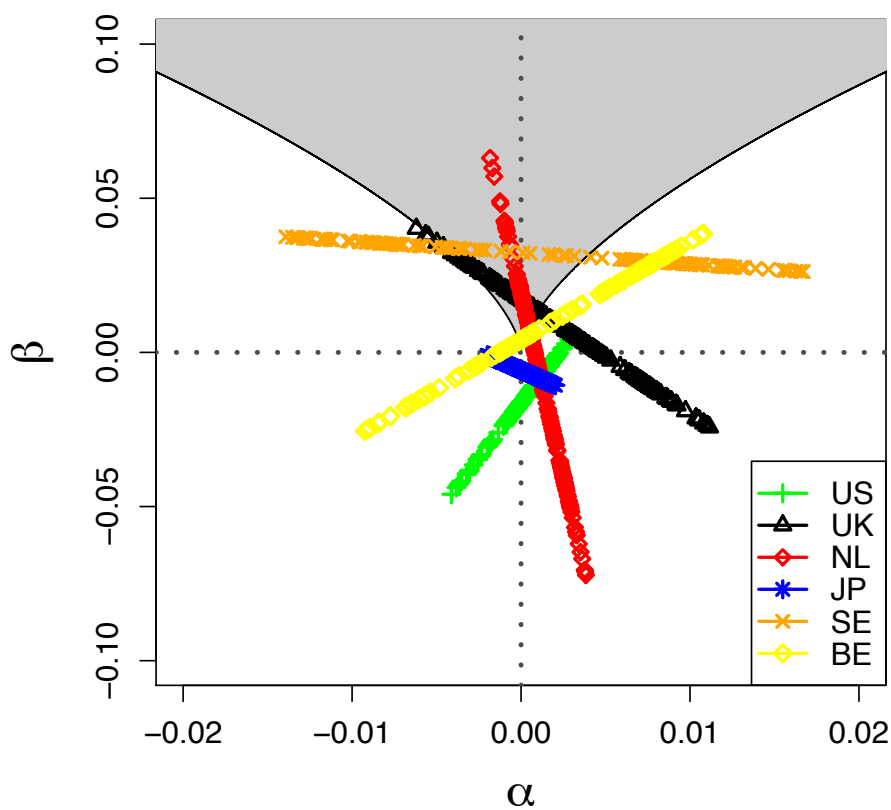
Figure 3.7: The estimated behaviour points of different countries in different regions of the control plane. The control variable is interest rate. The cusp shaped shaded area is the control plane. Each symbol and colour indicate the estimated behaviour of one country.

Equilibrium

Figure 3.8 shows the predicted equilibrium values and the time series of housing price deviations from fundamentals in different countries. It allows us to investigate the equilibria of the system, and also provides information on how the state of system transits from one equilibrium to another.

The black line in Figure 3.8 represents the time series of housing price deviations on fundamentals. The black dotted line indicates the baseline of 0. All six countries exhibit long-lasting periods of fluctuations of price deviations around 0. It can be observed that housing prices have been increasing rapidly since the mid-1990s and have peaked around 2008 in the US and NL.

After that they have dropped considerably for those countries. The JP housing prices peaked earlier around 1990, and subsequently declined to levels below the baseline 0. For the UK, SE and BE, they exhibited peaks around 1990 and 2008. After that, housing prices dropped significantly.

The red and blue lines indicate the estimated equilibria when control parameters are constant (Section 3.5.1), while the scatter plots show equilibria with interest rate as control variables. The red lines or scatters represent stable equilibria on the upper or bottom sheet of cusp equilibrium surface, while the blue lines or scatters imply unstable equilibria on the middle sheet of cusp equilibrium surface.

Although some predicted equilibria do not completely fit the position of real data in the plots, we should note that our objective is to forecast the systematically changes of equilibria. It can be observed that the cusp model forecasts the changes of equilibrium in real housing data well. When the housing price deviation increases or decreases, the cusp model forecasts its equilibrium changes in the same direction. For instance, in the example of SE around mid-1990s, the equilibrium presented a transition from a lower equilibrium to an upper equilibrium. Around the same time, the corresponding housing price data experienced a rapid drop and a gradual increase. Therefore, the cusp model performs well regarding the endogenous changes.

Critical Transition

Figure 3.9 presents the time series of price deviations, interest rates and the cube root of Cardan's discriminant δ in different countries. Under the monitoring of monetary policy, interest rates in all countries exhibited long lasting fluctuations regarding different economic situations. In general, for JP, SE and NL, interest rates were dropping in the observed period. For the US, UK and BE, interest rates peaked around 1980. After that, it was followed by persistent droppings.

Because δ is small and close to 0, transformation to its cube root allows us to visually catch more details on its value around 0. We shall observe its up and down oscillations around 0 in
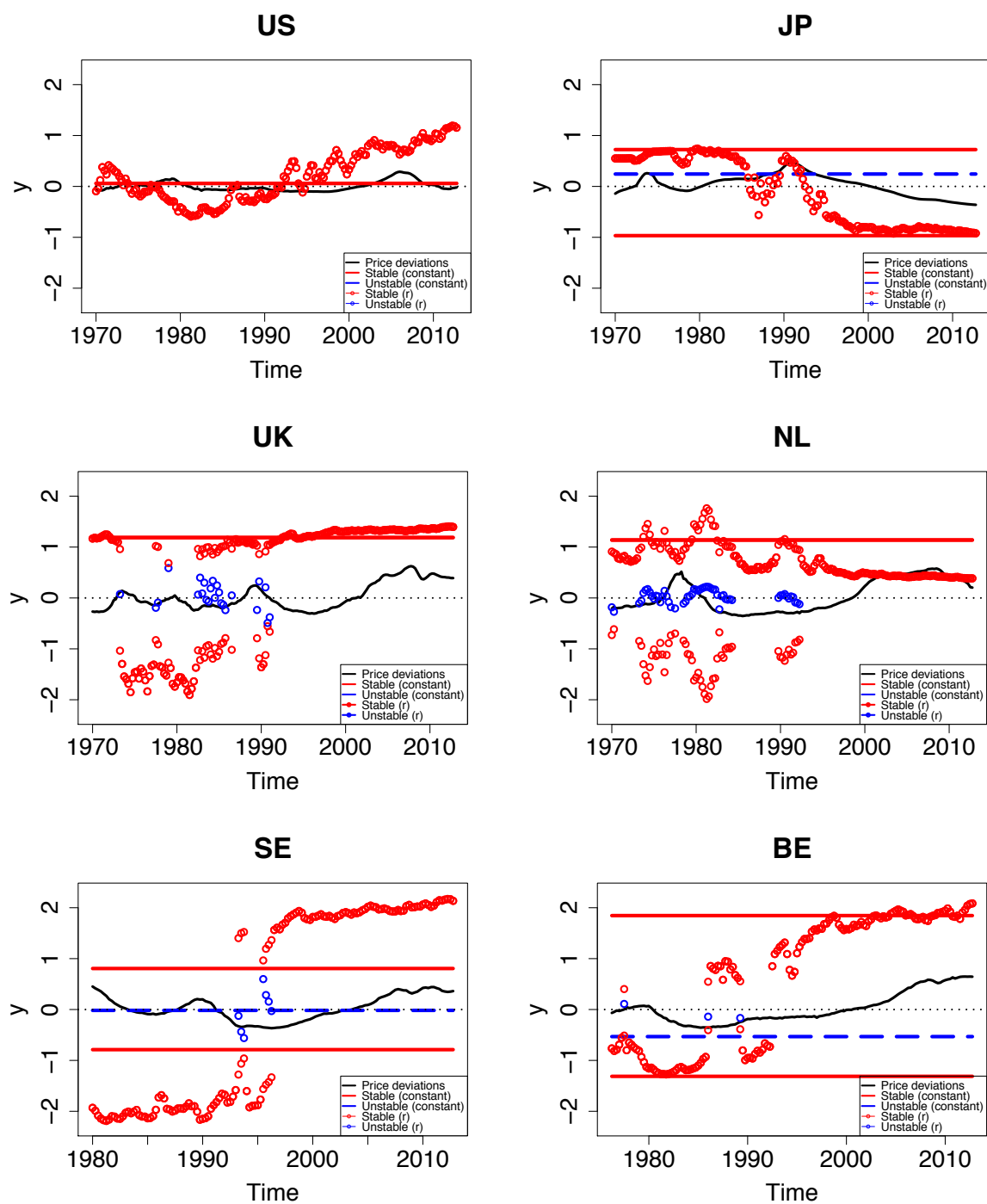
Figure 3.8: Time series of housing price deviations and predicted equilibria in different countries. Black line represents the time series of housing price deviations. Scatter plots indicate estimated equilibria when control variables are interest rate. Red and blue lines indicate the estimated equilibria when control parameters are constant.

different countries. When $\delta > 0$, the system is in a quiet and stable regime; When $\delta < 0$, it is in unstable bifurcation area. As shown in Figure 3.9, the time series of δ is consistent with the tracks of different countries in Figure 3.7. A negative value of δ is observed in the examples of the NL, SE, UK and BE, but not in the US and JP.

We are particularly interested in the corresponding dynamics when δ crosses the 0 baseline from negative to positive, which implies that the state of system moves from the unstable bifurcation region to stable region. This is observed during most of the housing price bubble and burst cycles in the UK, NL, SE and BE. When crossing the bifurcation boundary from inside to outside, a systemic change of system may occur through a "critical transition", with respect to the directions of the changes of the control variable - the interest rate. The corresponding cusp behaviour is that the system's state 'falls' off the cusp curve and jumps from one equilibrium to another.

A fluctuation in time series can be a normal oscillation in a single equilibrium or a systematic change via critical transition between two equilibria. To distinguish between the two, a study of the change of equilibria with respect to the control variables is useful. Figure 3.10 illustrates how the time series of the control variable - the interest rate, corresponded with the bifurcation band in the housing market historically. The grey band indicates the range of interest rate values with respect to $\delta < 0$, which implies the unstable bifurcation region with three equilibria - two stable and one unstable. Because a critical transition happens when the state of system jumps off the cusp curve and ends in a different equilibrium, it can be distinguished when the interest rate cross through the bifurcation band.

By observing the relationship between interest rates and the bifurcation region, displayed across time in Figure 3.10, we are able to study the underlying mechanisms of fluctuations in housing markets. For instance, in the example of SE, the state of housing system has fallen into the bifurcation region twice during 1990 and 2000. This corresponds with the bottom of the downturn of the housing price index. After falling into the bifurcation region for the first time, the state of the system quickly came out of it and went back to the previous stable equilibrium
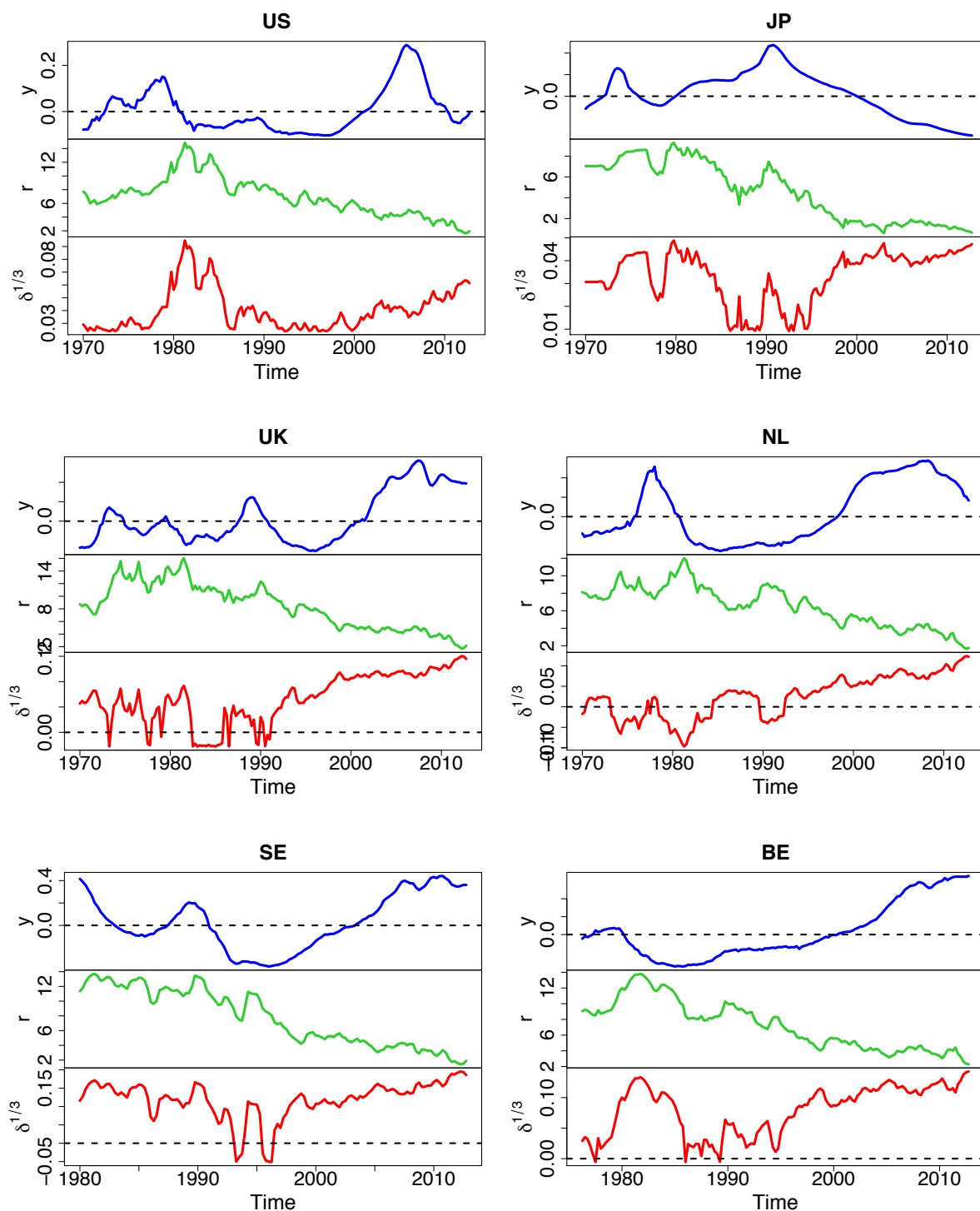
Figure 3.9: Time series of state variable (top panels), interest rate (middle panels) and cube root of Cardan's discriminant δ (bottom panels) in different countries. The control variable is interest rate. Dashed line is baseline of 0.

due to a rise of the interest rate. A year after that, the interest rate dropped and the system was brought down to the multiple equilibria bifurcation region for the second time. For this time, it did not end in the previous equilibrium but transitioned to a new stable equilibrium. This transition induced the retrieval of the housing market. Since then, the SE housing price was continuously increasing. The UK and BE show similar bifurcation bands in their interest rates and exhibited similar systematic fluctuations as SE. They had experienced several critical transitions between two equilibria before mid-1990s, which were consistent with the fluctuations in real housing price. After that, the system of UK came back to its previous stable equilibrium. These behaviours corresponded with the recovery of housing market in these two countries. In particular, the housing system of NL has the widest bifurcation band of all. It remained in a multiple equilibria region for almost a decade between 1973 to 1984, while the corresponding housing price was experiencing a bubble and burst cycle. Nevertheless, its equilibrium never went across the bifurcation band and always came back to its previous stable equilibrium. For the US and JP, there is no bifurcation band. In our analysis, their housing systems remained in a single equilibrium.

To analyse the impact of interest rate on the equilibrium of housing system further, Figure 3.11 presents the bifurcations showing the predicted equilibrium as a function of the interest rate r in different countries. Red scatter represents the stable equilibrium on the upper or bottom sheet of cusp equilibrium surface. Blue scatter represents the unstable equilibrium which lies on the middle sheet.

The results unfold the underlying bifurcations in different housing systems. For the UK, SE and BE, the observed bifurcations are so-called saddle-node bifurcations. With interest rate as a control parameter, the cusp model exhibits three equilibria for a certain range of interest rate. It also shows that as interest rate increases further, the housing system becomes stable again in a new equilibrium. The bifurcation scenario for the example of NL is different. As the interest rate increases, the cusp model exhibits one stable equilibrium. After passing a critical thresholds when interest rate around 7.5, a new saddle-node bifurcation occurs and creates two

new equilibria, one stable and one unstable. The system has been in the multiple equilibria regime from then on. For the US and JP, there are no bifurcations going on during the analysed period.

Policy Implication

The housing bubble and bursts cycles were followed by financial crisis which could create enormous tragedies. They have raised great concerns of the instability of housing price among policy makers. How can a policy maker stabilise the housing price and prevent market instability? As an essential factor in the monetary policy, interest rate has been pointed out to have great influence on the instability of housing market (Bernanke and Gertler, 1995; Shiller, 2006; Muellbauer and Murphy, 2008; Taylor, 2007, 2009; Crowe *et al.*, 2013; Shi *et al.*, 2014). Our study once again shed lights on the importance of interest rates. Moreover, we unfold the underlying link between interest rate and systematic fluctuations in housing market. The dynamic of housing system shows cusp catastrophe behaviour with interest rate as control parameter. It exhibits critical transitions between multiple equilibrium states as a result of the changes of interest rate. It is also dangerous when the system is in the unstable bifurcation region, or gets too close to the cusp curve. Even a small perturbation could induce significant fluctuations. This scenario can be used to explain the majority of housing bubbles and bursts in the data, such as in UK 1978, 1980,1990, NL 1978, BE 1987,1990, and the depression of SE after 1990.

A general lesson for policy makers to be drawn from these examples is that cusp catastrophe may yield important insights on policies that can cause global instability in economics. As we argued in our analysis, policy makers should monitor the instability of economic systems, be alerted when the system approaches a bifurcation, in particular be aware of critical transitions which could lead to significant market bubbles or sudden market collapse. The examples in this paper suggest that interest rate policy plays an important role in keeping the stability of economic system. By performing an appropriate interest rate policy, policy makers are able to prevent endogenous market crashes. There is no empirical evidence to show whether a high

or a low interest rate should be beneficial to the economy. " You can never be too rich or too thin". Taking the SE housing market as an example, as seen in Figure 3.11, if we set the interest rate too high, i.e. between 7 and 9 percent, the market might collapse. However, when the interest rate is set too low, bubbles may arise. Our method gives us an overall picture about the multiple equilibria of the system and how the equilibria change with interest rate policy. It could provide policy makers a reasonable guide to conduct a proper interest rate policy that keeps the economy in a healthy state. It could also help to deal with markets with bubbles and to establish a post-crisis policy on the recession following a housing market collapse.
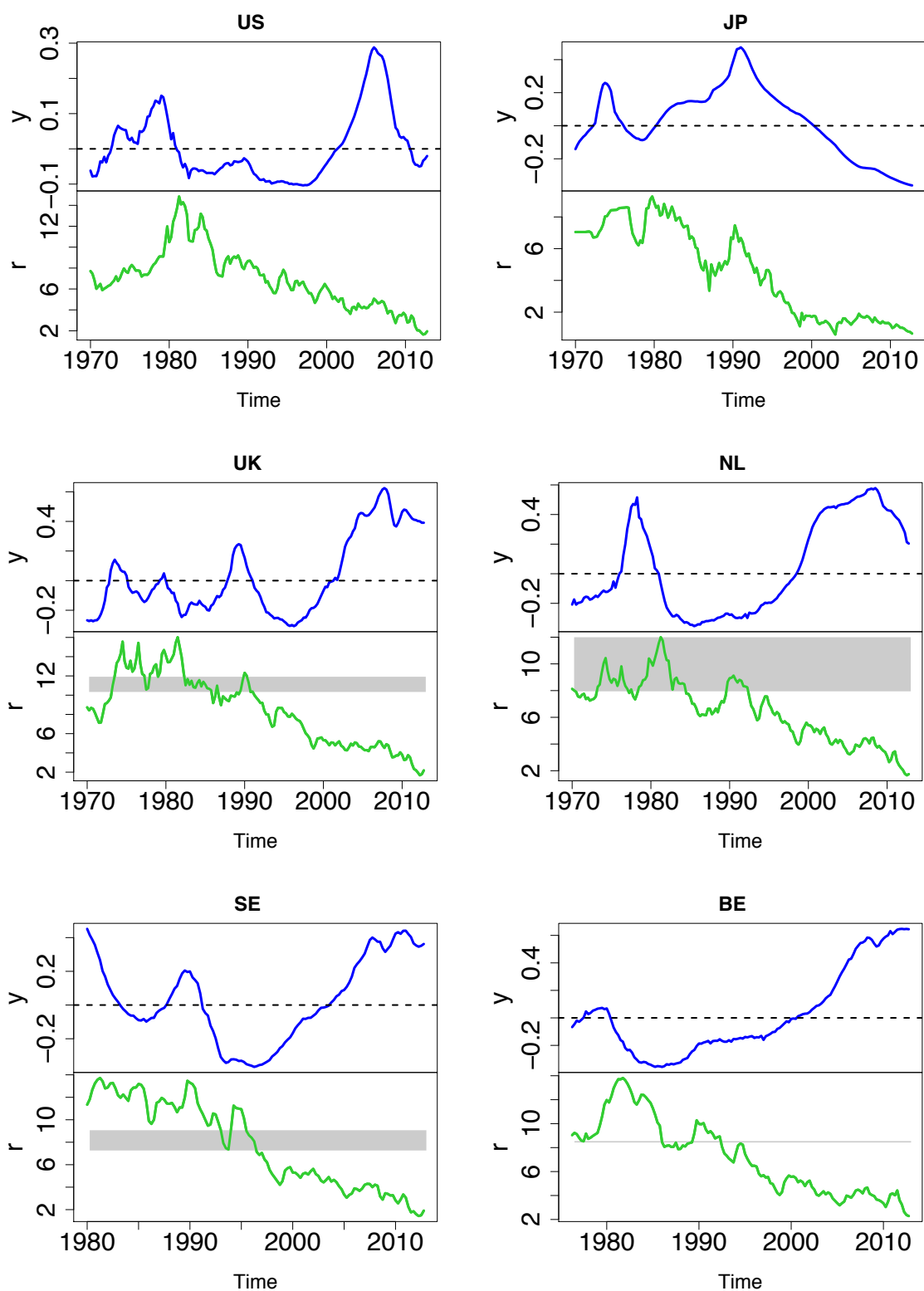
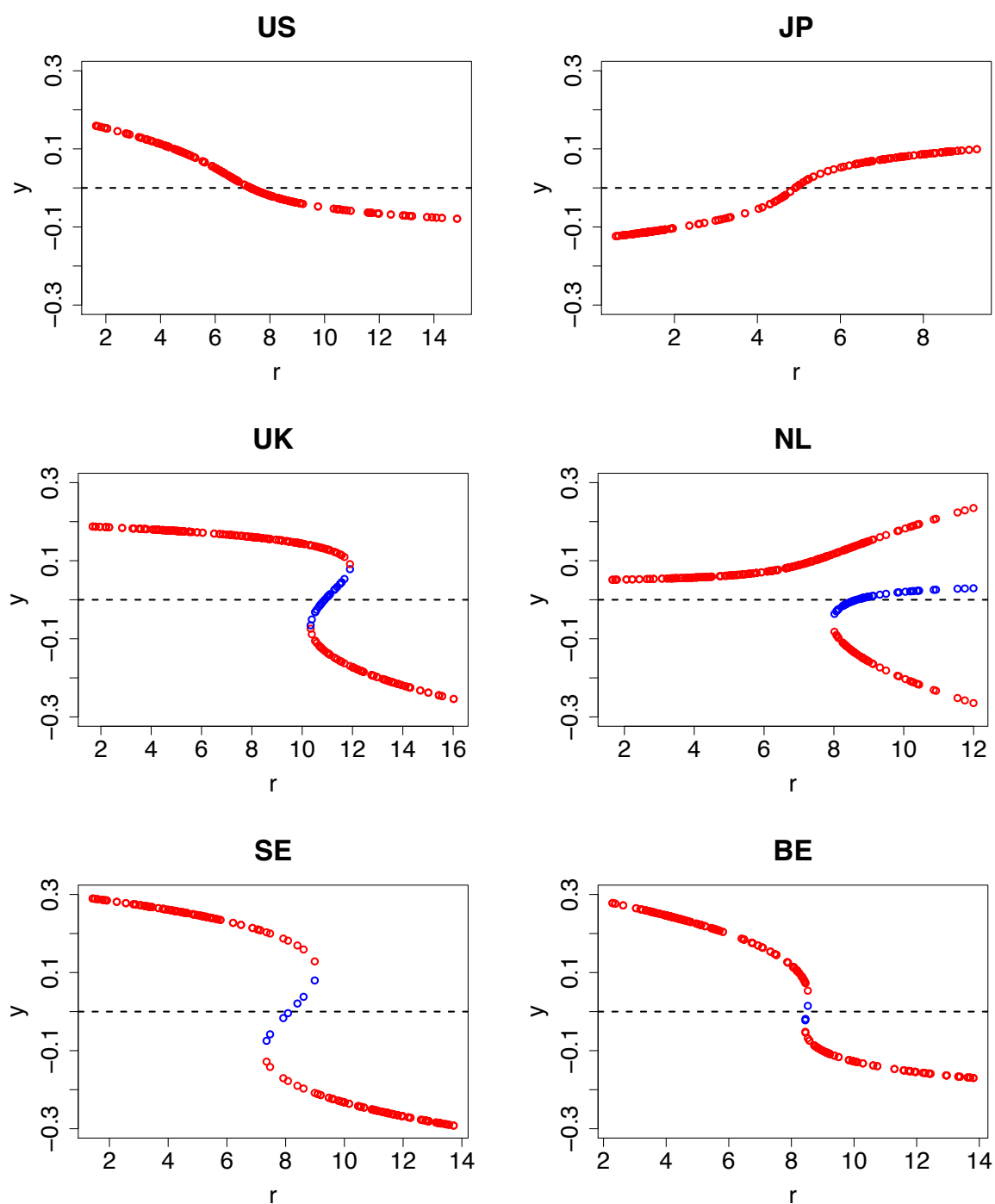Figure 3.10: Time series of housing price deviations, and interest rate with grey bifurcation band.

Figure 3.11: Bifurcations showing the predicted equilibrium as a function of the interest rate *r* in different countries. Red scatter represents the stable equilibrium (up or bottom sheet). Blue scatter represents the unstable equilibrium (middle sheet)

3.6 Concluding Remarks

This paper attempts to find out whether instability of housing market can be explained and predicted by catastrophe theory. A stochastic cusp catastrophe model was fitted to empirical housing market data for the first time. Using housing price deviations and quarterly data on long term government interest rates, we estimated the model for six different countries: United States (US), United Kingdom (UK), Netherlands (NL), Japan (JP), Sweden (SE) and Belgium (BE).

Two estimation approaches are discussed - Cobb's Method and Euler Discretization. The analysis shows that Cobb's Method requires a system for which the state variables change fast compared to the control parameters. It performs well when modeling the overall invariant density of state variables. However, when it comes to forecasting, Euler Discretization always gives better predictions. In this paper, because we are using time series data and our objective is to study the forecasting ability of cusp catastrophe model, Euler Discretization is employed in the later sections.

The estimation results obtained using Euler Discretization are discussed in the later part of the paper. We find that the dynamics of the housing market can be explained by cusp catastrophe behaviour. Under constant control parameters, the housing systems of US, UK and NL are in a normal stable regimes while the housing systems of SE, JP and BE are in unstable bifurcation regime. Nevertheless, when using interest rate as control variable, the interest rate changes the stability of the systems; those systems' equilibria vary with interest rate. The predicted equilibria give us a general picture on the changes of equilibria with time. Time series of cardan's discriminant δ links the changes of system equilibria and the bubbles and bursts cycles in empirical data. Moreover, by observing the relationship between interest rates and bifurcation bands, we are able to study the underlying mechanisms of fluctuations in housing markets. A critical transition can be distinguished when the interest rate cross through its bifurcation band. The underlying bifurcations can be given by the correlation between predicted equilibria and interest rates.

Our results yield important insights into policies that monitor the instability in economics. A change of the main control parameter, interest rate, may move the economic system closer to the unstable region with multiple equilibria. As a control variable, interest rate plays an important role in keeping the stability of economic system. Policy makers should prevent the economic system from moving into the multiple equilibria regions, or from getting too close to the cusp curve that may induce critical transitions. The cusp catastrophe theory could provide policy makers with a reasonable guidebook on interest rate policy.

Appendix 3.A Fundamental Price and Price Deviations

In this model, agents are boundedly rational and have different views about the future values of asset prices. At the same time, agents are allowed to switch from one period to the next between a number of available strategies, based on how well they have performed in the recent past. Agents base their decisions at time t on the expected excess return R_{t+1} on investing in housing relative to renting during the period between time t and $t+1$. Let P_t denote the price of one unit of housing at time t. Let the price for renting one unit of housing in the period between times t and $t+1$ be given by Q_t. Since rents are typically payed up-front (at time t), to express the rent at time t in terms of currency at time $t+1$, it should be inflated by a factor $(1+r^{\mathrm{rf}})$, where r^{rf} denotes the risk free mortgage rate. Therefore, the cost of renting in the period between time t and $t+1$, expressed in terms of currency at time $t+1$, is given by $(1+r^{\mathrm{rf}})Q_t$ rather than Q_t. The ex post excess return R_{t+1} on investing in housing during the period between time t and $t+1$ then is given by the sum of the capital gain minus mortgage/maintenance costs and the saving on rent.

$$R_{t+1} = \frac{(P_{t+1} - (1+r_t)P_t) + (1+r^{\mathrm{rf}})Q_t}{P_t} = \frac{P_{t+1} + (1+r_t^{\mathrm{rf}})Q_t}{P_t} - (1+r_t),$$

where $r_t = r_t^{\mathrm{rf}} + \omega_t$ is the sum of the risk-free (mortgage) rate r_t^{rf} and the maintenance/tax rate ω_t.

The demand, $z_{h,t}$, of agents of belief type h is determined by maximizing one-period ahead expected excess returns adjusted for risk:

$$\mathbb{E}_{h,t}\left(R_{t+1}z_{h,t}\right) - \frac{a}{2}\mathrm{Var}_{h,t}\left(R_{t+1}z_{h,t}\right), \tag{3.21}$$

where a is a measure of risk aversion. For simplicity we assume r_t^{rf} and ω_t to be constant over time: $r_t^{\mathrm{rf}} = r^{\mathrm{rf}}$, $\omega_t = \omega$ (and hence $r_t = r$). Agents are assumed to be homogeneous with respect to their expectations regarding the conditional variance of the excess return, that is,

$\text{Var}_{h,t}\left((P_{t+1}+(1+r^{\text{rf}})Q_t)/P_t-(1+r)\right) = V_t$, while they are heterogeneous concerning their expectations of excess return $\mathbb{E}_{h,t}\left((P_{t+1}+(1+r^{\text{rf}})Q_t)/P_t-(1+r)\right)$. Maximizing Eq. (3.21) leads to the demand for housing:

$$z_{h,t} = \frac{\left(\mathbb{E}_{h,t}\left(P_{t+1}+(1+r^{\text{rf}})Q_t\right)/P_t-(1+r)\right)}{aV_t} = \frac{\mathbb{E}_{h,t}(R_{t+1})}{aV_t},$$

The market clearing condition is:

$$\sum \frac{n_{h,t}\left(\mathbb{E}_{h,t}\left(P_{t+1}+(1+r^{\text{rf}})Q_t\right)/P_t-(1+r)\right)}{aV_t} = S_t, \tag{3.22}$$

where S_t is the stock of housing, and $n_{h,t}$ is the fraction of agents in period t that hold expectations of type h.[2] Solving the market clearing condition for the price P_t leads to the following price equation:

$$P_t = \frac{1}{1+r+\alpha}\sum n_{h,t}\mathbb{E}_{h,t}\left(P_{t+1}+(1+r^{\text{rf}})Q_t\right), \tag{3.23}$$

where $\alpha \equiv aV_t \times S_t$ is assumed to be constant in order to keep the model tractable. Agents require a rate of return on housing equal to $r+\alpha$ rather than $r = r^{\text{rf}}+\omega$. Therefore the parameter α can be interpreted as a risk premium of buying a house over renting a house. Treating α as a constant in the model allows for estimating this extra required rate of return, under the assumption that it is a constant.

We next turn to expectations regarding the fundamental price. Following Boswijk *et al.* (2007), we assume that the fundamental process underlying the model, i.e. Q_t, follows a geometric Brownian motion with drift, i.e.

$$\log Q_{t+1} = \mu + \log Q_t + \upsilon_{t+1}, \quad \{\upsilon_t\} \overset{\text{i.i.d.}}{\sim} N(0,\sigma_\upsilon^2),$$

---

[2]We assume that S is large enough, such that neither type has an incentive to sell short on houses.

with commonly known parameters μ and σ_υ^2, from which one obtains

$$\frac{Q_{t+1}}{Q_t} = (1+g)\varepsilon_{t+1},$$

with $g = e^{\mu + \frac{1}{2}\sigma_\upsilon^2} - 1$ and $\varepsilon_{t+1} = e^{\upsilon_{t+1} - \frac{1}{2}\sigma_\upsilon^2}$, such that $\mathbb{E}_t(\varepsilon_{t+1}) = 1$.

We define the fundamental price as the price that would prevail under rational expectations $\mathbb{E}_t(R_{t+1})$ about the conditional mean of R_t, while taking into account the risk premium α. Taking into account the risk premium in the fundamental price is convenient, as it will provide an equilibrium fundamental price from which the market price will deviate by an amount which averages out to zero in long time series. Under rational expectations on the first conditional moment, we can re-write the price Eq. (3.23) as

$$(1+r+\alpha)P_t = \mathbb{E}_t\left(P_{t+1} + (1+r^{\text{rf}})Q_t\right).$$

By applying the law of iterated expectations and imposing the transversality condition, we obtain the fundamental price at time t, denoted by P_t^*.

$$P_t^* = \mathbb{E}_t\left[\sum_{i=0}^\infty \frac{(1+r^{\text{rf}})Q_{t+i}}{(1+r+\alpha)^{i+1}}\right] = \sum_{i=0}^\infty \frac{(1+g)^i(1+r^{\text{rf}})Q_t}{(1+r+\alpha)^{i+1}} = \frac{1+r^{\text{rf}}}{r+\alpha-g}Q_t, \qquad r+\alpha > g. \quad (3.24)$$

This shows that the fundamental price of housing is directly proportional to the actual rent level. Figure 3.12 shows the examples of house price, fundamental price and price deviations $lnp_t - lnp_t^*$ in all the countries from 1970 to 2013.

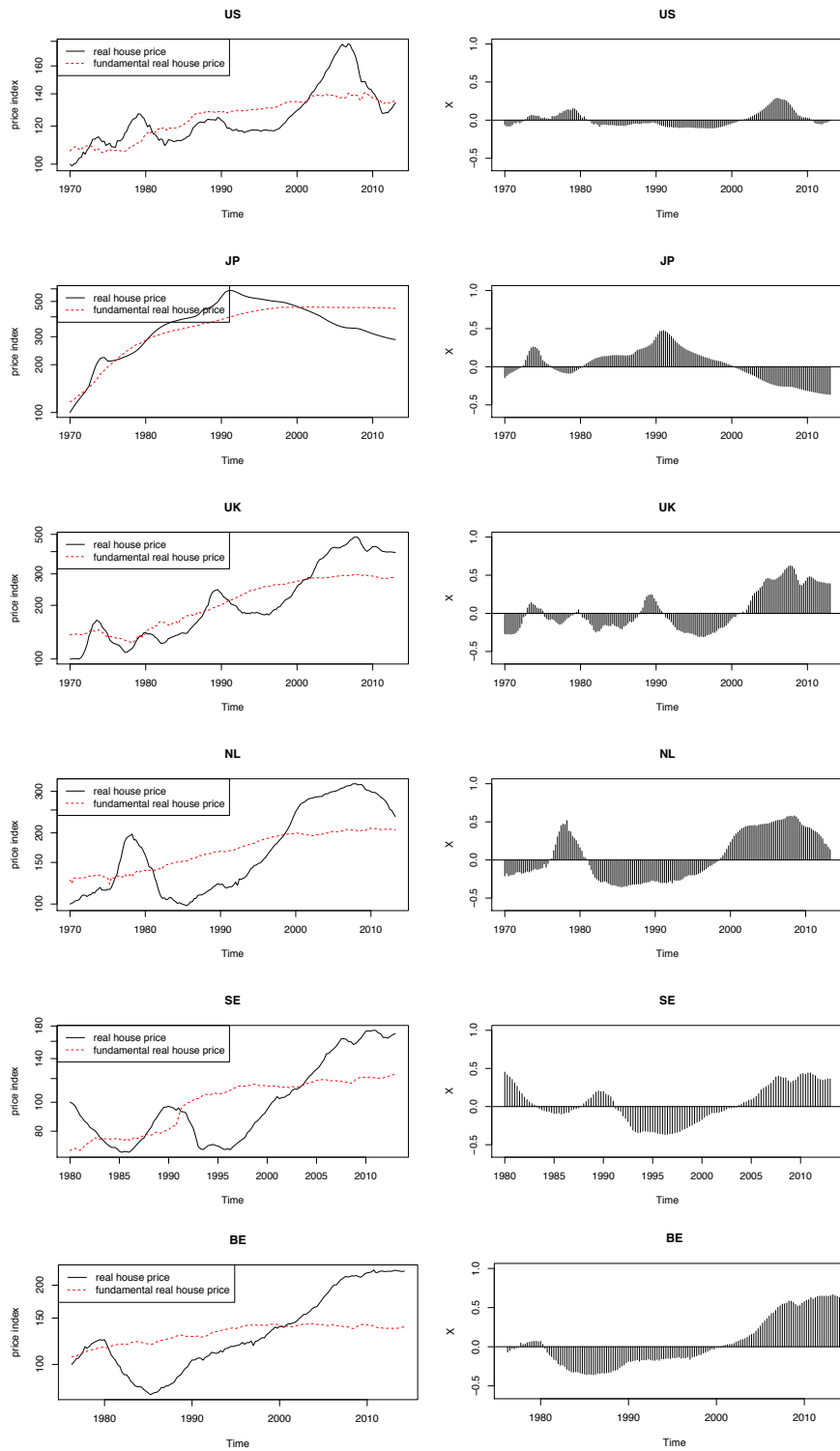Figure 3.12: Housing price indices (left, solid lines, 1970Q1=100), estimated fundamental real housing prices (left, dashed lines) and corresponding price deviations X_t (right).

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Appendix 3.B   Cobb's Method v.s. Euler Discretization

3.B.1   Residuals
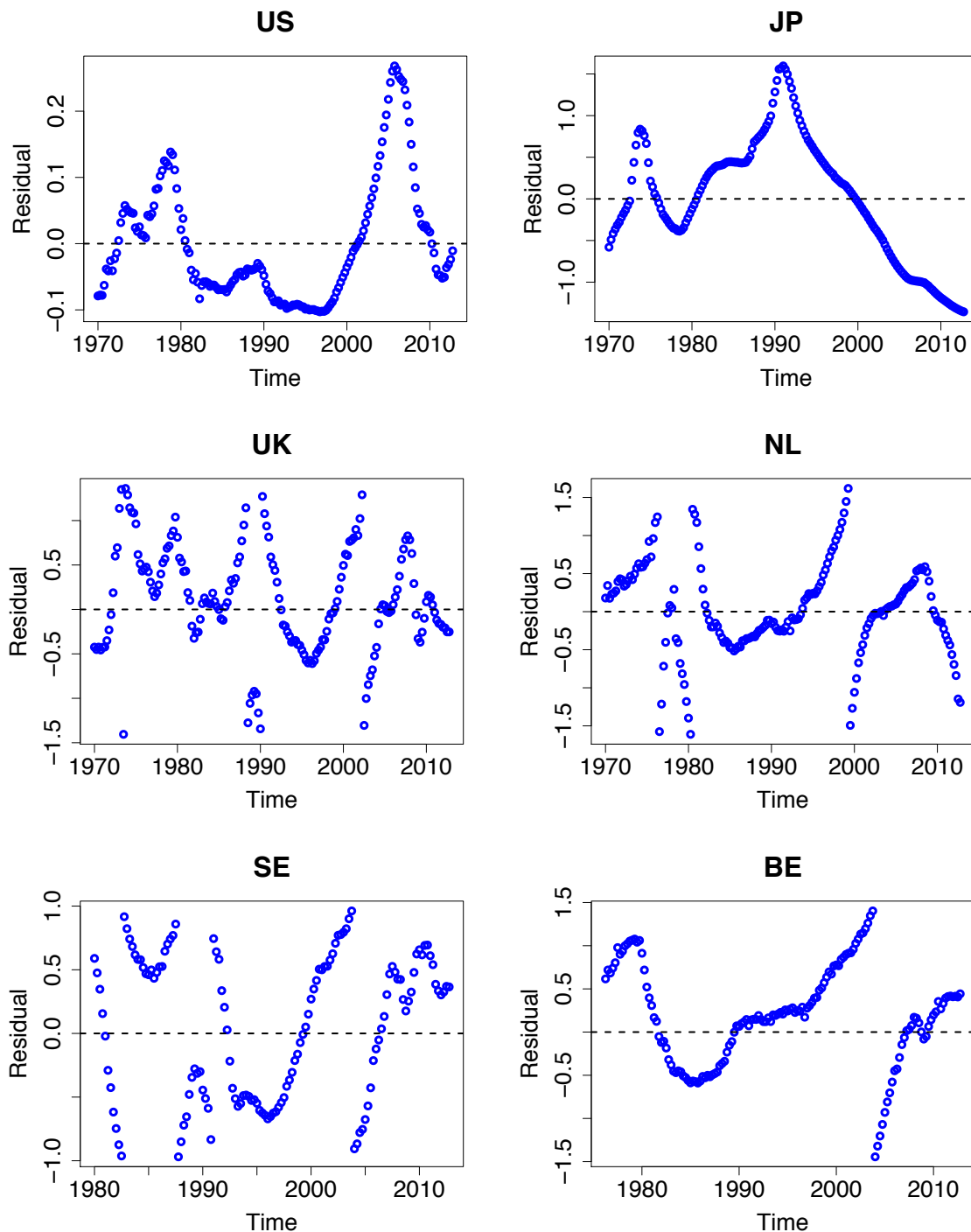
Figure 3.13: Plots of residuals against index by using Cobb's Method.
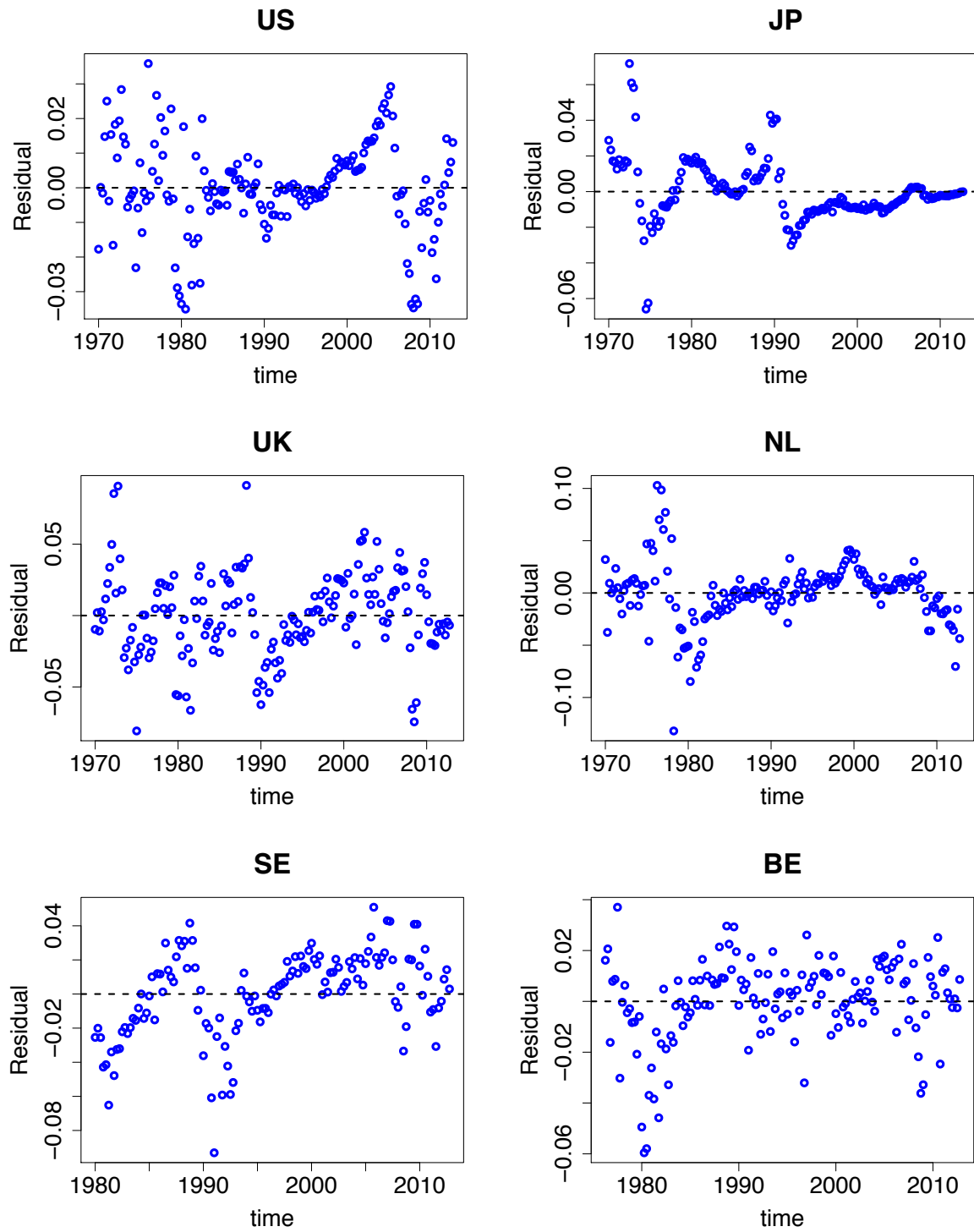
Figure 3.14: Plots of residuals against index by using Euler Discretization.

3.B.2   Cobb's Method

For the stochastic differential equation

$$dy_t = -V'(y_t)dt + \sigma dW_t,$$

with potential $V(y)$, the invariant distribution is the Gibbs distribution with density

$$f(y) = \frac{1}{Z_\sigma} \exp\left(-\frac{2V(y)}{\sigma^2}\right),$$

where Z_σ is a normalization constant (Anderluh and Borovkova, 2008). [With $\varepsilon = \sigma^2/2$ this coincides with Cobb (1978)].

For the canonical CUSP potential $-V(y) = \alpha y + \frac{1}{2}\beta y^2 - \frac{1}{4}y^4$, one finds

$$f(y) = \frac{1}{Z_\sigma} \exp\left(-\frac{\alpha y + \frac{1}{2}\beta y^2 - \frac{1}{4}y^4}{\sigma^2/2}\right).$$

Transforming to

$$\tilde{y} = y / \left(\frac{\sigma^2}{2}\right)^{\frac{1}{4}},$$

(leaving aside trivial translations) we obtain for the density of $\tilde{y}$

$$
\begin{aligned}
f_{\tilde{y}}(\tilde{y}) \quad &\propto \quad \exp\left(\frac{-\alpha\left(\frac{\sigma^2}{2}\right)^{\frac{1}{4}}\tilde{y} - \frac{1}{2}\beta\left(\frac{\sigma^2}{2}\right)^{\frac{1}{2}}\tilde{y}^2 + \frac{1}{4}\left(\frac{\sigma^2}{2}\right)\tilde{y}^4}{\left(\frac{\sigma^2}{2}\right)}\right) \\
&\equiv \quad \exp\left(-\tilde{\alpha}\tilde{y} - \frac{1}{2}\tilde{\beta}\tilde{y}^2 + \frac{1}{4}\tilde{y}^4\right).
\end{aligned}
$$

This suggests $\tilde{\alpha} = \alpha / \left(\frac{\sigma^2}{2}\right)^{\frac{3}{4}}$ and $\tilde{\beta} = \beta / \left(\frac{\sigma^2}{2}\right)^{\frac{1}{2}}$.

3.B.3 Euler Discretization

Suppose that the deterministic part of the (canonical) variable z_t is governed by the potential function

$$V(z; \alpha, \beta) = \alpha z + \frac{1}{2}\beta z^2 - \frac{1}{4}z^4,$$

and that there is a driving noise term with variance σ_Z^2 per time unit, i.e.

$$\mathrm{d}z_t = - \left. \frac{\partial V(z; \alpha, \beta)}{\partial z} \right|_{z=z_t} + \sigma_z \mathrm{d}W_t.$$

The invariant density of z then is proportional to (Gibbs distribution)

$$f_Z(z) \propto \exp\left[-\frac{2V(z)}{\sigma_z^2}\right] = \exp\left[-\frac{\alpha z + \frac{1}{2}\beta z^2 - \frac{1}{4}z^4}{\sigma_z^2/2}\right].$$

If $y = \lambda + rz$ is a scaled and/or translated variable, then $z = (y - \lambda)/r$, and the density of y is proportional to

$$f_Y(y) \propto \exp\left[-\frac{\alpha\left(\frac{y-\lambda}{r}\right) + \frac{1}{2}\beta\left(\frac{y-\lambda}{r}\right)^2 - \frac{1}{4}\left(\frac{y-\lambda}{r}\right)^4}{\sigma_z^2/2}\right]. \tag{3.25}$$

The invariant density fitted by CUSP fit is

$$f_Y(y) = \psi\exp\left[\tilde{\alpha}\left(\frac{y-\lambda}{c}\right) + \frac{1}{2}\tilde{\beta}\left(\frac{y-\lambda}{c}\right)^2 - \frac{1}{4}\left(\frac{y-\lambda}{c}\right)^4\right]. \tag{3.26}$$

Comparing the coefficients of the fourth powers in Eqs (3.25) and (3.26), we see that these coincide only if $c = r\left(\frac{\sigma_z^2}{2}\right)^{\frac{1}{4}}$. It can be readily checked that this implies $\tilde{\alpha} = \left(\frac{\sigma_z^2}{2}\right)^{-\frac{3}{4}}\alpha$ and $\tilde{\beta} = \left(\frac{\sigma_z^2}{2}\right)^{-\frac{1}{2}}\beta$.

In terms of $y_t = \lambda + rz_t$ the SDE is

$$\frac{1}{r}\mathrm{d}y_t = - \left. \frac{\partial V(z; \alpha, \beta)}{\partial z} \right|_{z=\frac{y_t-\lambda}{r}} + \sigma_z \mathrm{d}W_t.$$

Euler discretization gives

$$y_{t+\Delta_t} \approx y_t + \left(\alpha + \beta \left(\frac{y_t - \lambda}{r} \right) - \left(\frac{y_t - \lambda}{r} \right)^3 \right) r\Delta t + r\sigma_z \sqrt{\Delta t} \varepsilon_{t+\delta_t},$$

where $\varepsilon_{t+\Delta t} \sim N(0, 1)$.

Chapter 4

Incentive and Expectations in A Financial Market with Heterogenous Agents

4.1 Introduction

Since the creation of the first fund in 1924, mutual funds have mushroomed over the past ninety years. Rather than managed by private investors, a significant share of financial wealth is delegated to professional mutual fund managers. In 1940, there were approximately 68 funds with \$0.45 billions in assets. By the end of 2013, the number of funds have increased to 7707. They are with \$15,017.68 billions in assets[1], which are approximate the US GDP of the whole year. Nowadays, mutual funds and fund managers are playing important roles in financial industry. They control a large and increasing percentage of the aggregate wealth of investors. However, a potential conflict of interest between mutual fund managers and their investors arises from these considerable controls. Because a mutual fund can be considered as a black box which converts investor's cash into returns, what really happens inside the box remains unclear to investors (Malkiel, 1995).

The absence of costless, complete information and the presence of moral hazard are where these conflicts stem from. Baumol *et al.* (1989) suggested that the primary service supplied by

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[1]2014 Investment Company Fact Book (`http://www.icifactbook.org/index.html`)

mutual fund manager is portfolio management, which includes conducting research and making fund investment decisions. In return, fund managers receive compensation fees under advisory contract for providing those services. On the one hand, the investor assumes that fund manager has common objective with him, which is to maximise expected fund's return. On the other hand, the resources fund manager expends on managing portfolio are in a black box and can not be observable directly. Investors cannot distinguish the effect of a manager's choice from the effect of a randomly determined state of nature. Likewise, exclusively from the observed outcome, investor cannot costlessly distinguish the risk level of a manager's choice. Moreover, there are not many financial regulations to constrain the structure of compensation fees. As Jensen and Meckling (1976) suggested, many of the advisory contracts were only depended on the underlying contracting environment. The question arises whether the portfolio selected is optimal for the investor as well as the fund manager, since the manager may choose a portfolio with a risk level that is not compatible with investor's preference.

Murphy (1999) suggested that it is optimal for contracts to provide a strong link between compensation fees and investor welfare, particularly when investors know little about the production function linking managerial action and investors' objective function. The task is to give fund manager a right incentive to make the best efforts and to achieve the same goal as investors. This contract should detail a risk sharing rule with scheduled monetary rewards. It should also contain a compensation policy for managers. Stracca (2006) reviewed the literature on delegated portfolio management with respects to a principal agent relationship between an investor (the principal) and a fund manager (the agent). He pointed out the fact that it was difficult to design an optimal contract which is compatible with both incentives of investors and managers. The reason is that the manager controls all the efforts and the risks. In industry practice, people tend to favour asset-based compensation schemes. These schemes are usually asset-based and linearly depend on the value of the managed assets. As Ou-Yang (2003) suggested, a portfolio manager should receive a fraction of the total assets under management. Hommes and Milgrom (1987) supported these asset-based compensations by showing that the linear sharing rule is an

optimal contract as it induces an optimal trade-off between risk sharing and effort inducement.

In recent years, performance-based schemes involve in the advisory contracts of many large mutual fund companies, which induces increasing policy discussions and research literature on performance-based incentives. Unlike an asset-based scheme which depends on the amount of assets, a performance-based incentive scheme is based on the performance measures, such as net realised and unrealised gains. One of the claimed advantages of this incentive fee is that better fund's performance is expected from this sort of contract. From the agency literature point of view, Starks (1987) showed that the symmetric type of performance-based compensation contract is aligning the manager's interests with those of investors. Eichberger *et al.* (1999) also suggested that the past relative performance-based reward schemes may arise as optimal contracts by investigating the situation when all fund managers behave exactly as desired by investors.

Moreover, some of the papers have found a significant effect of past performance to the mutual funds and the risk taking behaviour of their fund managers, which support the performance-based incentive contract further. Chevalier and Ellison (1997) provided empirical evidences that the fund's past returns create incentives for fund managers to alter their risk taking behaviours and to change their portfolio choices. There are more papers focused on the strong relationship between flows of new investment into mutual fund and their past performance, such as Ippolito (1992), Sirri and Tufano (1992), Patel *et al.* (1994), Roston (1996) and Goetzmann *et al.* (1997).

There are some arguments regarding whether or not performance-based scheme should be involved in advisory contract. Bines and Thel (2004) suggested that fund manager under performance-based contract might fail to take into account many other components of management, such as portfolio diversification, risk management. Damato (2005) argued that performance-based fees might encourage fund managers to take excessive risks and gamble with their portfolios to obtain higher returns.Moreover, Mendoza and Sedano (2009) suggested that performance-based fees would fail to provide additional incentives to fund managers paid

on increased assets.

Taken into account the rise of performance-based schemes and the popularity of asset-based schemes, comparing these two compensation schemes appears to be an important task. In this paper, in order to understand the implications of incentive contracts, we study a financial market with mutual fund managers and their clients under two compensation schemes: asset-based fees and performance-based fees. An agent-based model is established to describe the dynamic behaviours of a market with investors and mutual fund managers. Investors hire their fund managers according to managers' performance in the past. For managers, they are myopic and make investment decisions by forecasting the market price in the following period. Therefore, Two types of compensation schemes create two types of fund managers: asset maximisers and return maximisers. *Asset Maximisers* are motivated by asset-based compensation schemes and aim to maximise the expected volume of the assets they manage. The managers who are under performance-based compensation schemes aim to maximise the returns of portfolios. This makes them to be *Return Maximisers*.

Our paper is closesly related with the study of Palomino (2005). They evaluated an economy with the combination of fund managers' asset-based compensation schemes and investors' fund picking rules. Investors always use a relative performance rule to evaluate mutual fund managers and allocate money into funds. They found that these relative performance objectives increase the riskiness of the investment strategies. They also suggested that under the relative performance picking rules, asset-based compensation schemes increased investors' expected returns. In our paper, we also establish a model with the combination of fund managers' compensation schemes and investors' fund picking rules. However, instead of exclusively studying one contract, we investigate both asset-based and performance-based schemes. These two compensation schemes are also discussed in Cuoco and Kaniel (2011). They used a dynamic general-equilibrium setting while our paper considers an agent-based model with multiple equilibria. They found that asset-based fees distorted the risk level of managed portfolios. It increases asset prices and has a negative effect on the performance of the investment. They also found

that the effects of performance-based fees fluctuate stochastically over time in response to variations in the excess performance of funds. In our study, we give consistent results with all these papers, but in a complex system approach. Moreover, we are able to observe the coexistence of these two contracts and the dynamic behaviour of the market.

One of the main contributions in this chapter is that we not only observe a single type of fund manager in a homogenous market, but also show the complex behaviours how *heterogenous* types of managers survive in a market. Most of the existing theoretical papers do not address explicitly the issue of multiple funds or heterogenous funds but only focus on a single fund with one manager (e,g. Grinold and Rudd, 1987; Cohen and Starks, 1988). They either investigated single manager and single investor or single manager and multiple investors.

Moreover, this chapter is the first attempt to discuss fund managers by considering an agent-based model. We considered *expectations* and discuss the link between incentive contracts and expectation of agents. A key assumption is that agents do not know the actual "law of motion" of the economy. Instead, they forecast the future based upon time series observations (Simon, 1982). Sargent (1993), Evans and Honkapohja (2001) reviewed the literature on adaptive learning in macroeconomics. In those papers, agents are bounded rational and use the last observed price as an anchor and extrapolate the future price. In a series papers of Brock and Hommes (1997, 1998, 1999), they proposed Adaptive Belief Systems (ABS) which illustrated nonlinear dynamic asset-pricing models with evolutionary strategy switching. In those models, the asset price is driven by evolutionary dynamics of different expectations rule over time. Asset price fluctuations are characterised by irregular switching between a stable phase when fundamentalists dominate the market and an unstable phase when trend chasers dominate and asset prices deviate from rational expectation fundamentals. Boswijk *et al.* (2007) estimated a simple version of heterogeneous agent models on yearly S&P 500 data and provide empirical evidence that stock prices can be characterised by the coexistence of two types of expectation - fundamentals and trend chasers. Moreover, these ABS models have been used to evaluate how likely it is that a stock market bubble will resume. Boswijk *et al.* (2007) suggested that price

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deviations from benchmark fundamentals are triggered by news about economic fundamentals but may be amplified by the strategy of trend chasers.

In our paper, two types of forecasting rules are observed - *Trend Chaser* and *Contrarian*. The manager who behaves as a trend chaser expects prices change to continue in the same direction. He buys assets when prices increase and sells them when prices decrease. On the contrary, a contrarian trader expects a reversal of the latest price change. He acts in an opposite way compared to trend chasers. He sells assets out when prices increase and buys them in when prices decrease. Brock and Hommes (1998) also investigated these two types of trading strategies. They found that both trend chasers and contrarian traders destabilise the market. Contrarians cause a lot of up and down fluctuations around the fundamental, while trend chasers always trigger irregular switching between above and below the rational expectation fundamental. Similar results are also observed in this chapter.

Intuitively, the types of compensation contract should have the same affects to trend chasers and contrarians. However, the results show that due to the significant fluctuations, the influence from compensation contract can barely be seen in the case of trend chasers. Moreover, the coexistence of two types of contracts and two types of expectations fails to stabilise the market. They amplify the small oscillations to chaotic fluctuations. Both contrarian traders and trend chasers favour asset-based contract, but there is always a small fraction of them choose performance-based contract. Neither of the compensation contracts are able to drive the other type out of the market.

This paper is also the first attempt to investigate the nonlinear *dynamic* behaviour in both homogenous and heterogenous market regarding the relationships between investors and fund managers theoreticaly. Most of literature only considered one or two periods.

In a nutshell, the main research questions addressed in the paper are: Will the type of compensation scheme change the manager's risk taking behaviour? If so, what are their impacts to asset price and funds' performance in the long run? Is there any link between incentive contract and expectation? Will this link influence the instability of the financial market? Is there any

type of incentive scheme particularly favouring certain type of expectation rules? Furthermore, what is the policy implications regarding compensation contract?

The model and analysis in this chapter provides answers to these questions in two different aspects. Firstly, the influence of compensation contracts to fund managers under different expectation rules are different. Trend chasers do not care about the motivation contract. Their only concern is to follow the change of price and we do not observe differences between the contracts. However, when it comes to the contrarian traders, the type of compensation schemes plays an important role when they make decisions to choose a portfolio. The different compensation schemes change their risk taking behaviours. They are willing to take more risk under an asset-based contract. Second, this paper shows that when asset maximiser and return maximiser coexist, asset maximisers dominate the market and produce higher returns. Furthermore, this paper investigates the inertia parameter which describes the possibility of an agent sticks to his previous strategy. The results show that it tends to amplify the fluctuations in different types of financial markets.

This chapter is organised as follows. Section 2 presents the model. Section 3 does the maximisation analysis. The simulation results of both homogenous and heterogenous markets are shown in the Section 4. The final section 5 closes this chapter with a summary and conclusion.

4.2 The Model

4.2.1 Basic Setup

There are two kinds of assets in the market, a risk free asset and a risky asset. Agents can choose how much to invest in each of them. Following standard theoretical literature, the risk free asset is referred to insured deposits or government securities. Risky asset stands for the market portfolio or the unit beta portfolio (Friedman and Abraham, 2009).

The risk free asset has a return rate of $r > 0$ per unit per period. The price of the risk free asset is normalised to 1 and the supply of risk free assets is infinite. The risky asset pays an

uncertain dividend per unit at period t denoted by d_t. The assumption is that the dividend follows an exponential process which is commonly used on measuring dividend growth in finance (Asmussen *et al.*, 2000; Albrecher *et al.*, 2005; Avanzi, 2009). Therefore, the dividend d_t is a continuous random variable with the probability density function of an exponential distribution

$$h(x) = \begin{cases} \beta e^{-\beta x} & x > 0 \\ 0 & x \le 0, \end{cases} \tag{4.1}$$

where $\beta > 0$. The supply of the risky asset at time t is constant and is normalized to 1. The price of the risky asset per unit at time t is p_t, which is determined by market clearing.

Agents in this market are mutual fund managers and their investors. Managers make investment decisions and recommend them to their investors. Investors choose a fund manger to follow according to their past performances. Fund managers refer to both the individuals who direct fund management decisions and the companies which provide investment management services. Investors can be private investors, corporations, pension funds, charities, insurance companies and so on.

We assume that the set of managers is m and the set of investors is n. Manger $k \in m(|m| = M)$ is hired by a set of investors $I_{k,t} \subseteq n(|n| = N)$ at period t. $A^I_{i,t}$ is the asset of investor i at period t. To distinguish with manager, we use superscript I to represent investor. Thus, the total assets managed by manager k at period t is

$$A_{k,t} = \sum_{i \in I_{k,t}} A^I_{i,t}. \tag{4.2}$$

We also assume that the overall assets from all the investors are fixed as A.

4.2.2 Market Clearing Price

In period t, on the one hand, fund manager k selects a portfolio $(1 - y_{k,t}, y_{k,t}), y_{k,t} \in [0,1]$ according to his objective. $y_{k,t}$ denotes the fraction of risky assets. This portfolio is recommended

to all the investors who follow him. On the other hand, investors follow the recommendation of their managers in a way of investing a fraction $y_{k,t}$ of assets to the risky assets. The fraction of the risky assets of the portfolio invested by investor i in period t is $y_{i,t}^I$, which is given by

$$y_{i,t}^I = y_{k,t}. \tag{4.3}$$

The individual demand function for risky assets is determined by the fraction of the risky assets invested by investor i and his assets in the previous period,

$$D_{i,t} = \frac{A_{i,t-1}^I y_{i,t}^I}{p_t}, \tag{4.4}$$

where $A_{i,t-1}^I$ is the amount of the assets owned by investor i in previous period $t-1$. Because the supply of risky asset is normalised to 1, market clearing gives the market equilibrium pricing equation as

$$p_t = \sum_{i \in n} y_{i,t}^I A_{i,t-1}^I. \tag{4.5}$$

The computation of the market equilibrium is given in Appendix 4.B.

4.2.3 The Objective of Fund Manager

The objective of mutual fund manager is motivated by his compensation contract with clients. In industry practice, asset-based scheme is commonly used to compensate management. It is usually the fraction of the value of managed assets (Deli 2002, Palomino 2005). We assume that the payment to management is proportional to the amount of managed assets,

$$C_{k,t} = c_{k,t} A_{k,t}, \tag{4.6}$$

where $c_{k,t}$ is trader fee per unit of managed asset. However, over the past few years, a few well-known mutual fund companies have started to use performance-based incentive contract. The

analysis of both asset-based and performance-based fees appears to be an important task. In this paper, we investigated the dynamic behaviour of market under these two types of compensation schemes. We assume that managers are myopic. This makes fund mangers become two types: Asset maximiser and Return maximiser.

Asset Maximiser   Motivated by the asset-based compensation, asset maximiser aims to maximise the expected overall assets he manages, even when his objective is not aligned with investor's welfare. Therefore, mutual fund manager's recommend fraction of risky asset is derived from a maximisation problem of the amount of expected value of the assets he manages in the following period, which is $\max\limits_{y_{k,t}} \hat{A}_{k,t+1}$. The maximisation problem is

$$y_{k,t} = \operatorname*{argmax}_{y_{k,t}} \hat{A}_{k,t+1}. \tag{4.7}$$

Return Maximiser   Return maximiser type of fund manager is motivated by the performance-based compensation contract. He shares the same goal as his clients that he aims to maximise the return of his portfolios. Like asset maximiser, his recommend fraction of risky asset $y_{k,t}$ is derived from the maximisation problem of his objective in the following period, which is to maximise the return of the portfolio, $\max\limits_{y_{k,t}} \hat{R}_{k,t+1}$. The maximisation problem is

$$y_{k,t} = \operatorname*{argmax}_{y_{k,t}} \hat{R}_{k,t+1}. \tag{4.8}$$

4.2.4   Fund Selection by Investors

The Signal

The whole point of fund manger is to leave the investment management function to the professionals. Most investors do not have the knowledge or the time to observe the distribution of portfolio and monitor their fund managers. They always choose their managers by observing the publicly available signals. These signals are mostly attached with fund managers' perfor-

mances. Therefore, managers with better performances always attract more followers. In this model, signal $U_{i,k,t}$ is observed by investors. It is evaluated as the fund managers' performance measurement $P_{k,t}$ subjects to individual investor's error $\delta_{i,k,t}$:

$$U_{i,k,t} = P_{k,t} + \delta_{i,k,t}, \tag{4.9}$$

where $\delta_{i,k,t}$ follows a heavy side distribution.

The managers' performance $P_{k,t}$ is measured as the net realised profits as proposed by Markowitz (1952):

$$P_{k,t} = R_{k,t} - c, c > 0, \tag{4.10}$$

where c represents the trader fee per period incurred by managers, such as the cost of gathering information. More sophisticated strategies require higher costs. $R_{k,t}$ represents the return rate and is defined by using asset pricing model:

$$R_{k,t+1} = (1 - y_{k,t})r + y_{k,t}(\frac{p_{t+1} - p_t}{p_t}) + \frac{y_{k,t}d_{t+1}}{p_t}, \tag{4.11}$$

where $R_{k,t+1}$ denotes the return rate of the asset from period t to period $t+1$. p_t is the price of this asset in period t and d_{t+1} is the dividend paid at the beginning of period t+1. The first and the second terms on the right hand side represent the capital gain of risk free assets and risky assets. The third term is the dividend yield.

Fund Selection

Investors choose their managers by observing publicly available signal $U_{i,k,t}$ which is attached with fund managers' performance measurement $P_{k,t}$. Manager with better performance delivers stronger signal to investors. We assume that the probability for manager k to be chosen by an investor in period $t+1$ is given by:

$$n_{k,t+1} = \frac{exp(\lambda P_{k,t})}{Z_t}, \, Z_t = \sum_{k=1}^{M} exp(\lambda P_{k,t}). \tag{4.12}$$

Z_t is a normalisation factor, so that the fraction $n_{j,t}$ adds up to 1. The rule is that investors tend to choose the manager who has performed well in the most recent past. This picking rule follows a well-known discrete choice model with multinomial logit probabilities. It describes a typical agent chooses an alternative out of a set with a finite number of alternatives. There are several examples in a series of papers as McFadden (1973), McFadden and Reid (1975), etc..

An important parameter λ is the sensitivity of choices. It measures how sensitive the investors are, regarding the choice of optimal strategy. It is inversely related to the variance of the noise terms δ_t. One extreme case $\lambda = 0$, corresponds to infinite variance noise, so that differences in fitness cannot be observed and all fractions will be fixed over time. Investors would distribute themselves evenly across the set of available managers. On the contrary, if λ is infinite, it corresponds to the case without noise, so that the deterministic part of the fitness can be observed perfectly. In each period, all traders choose the optimal forecast. In our case, all investors choose the manager with best performance.

Inertia Parameter κ

Due to the cost of changing manager, sometimes investors may choose to stick to his previous managers or wait for a few periods before switching, even though the public information suggests the optimal strategy is to switch to a new one. An inertia parameter κ describes this behaviour. An investor sticks to the manager he followed in previous period with probability κ. Thus, with probability $1 - \kappa$ he considers to switch and select a manager $j \in m(|m| = M), j \neq k$. Therefore, the probability of an investor follows a manager is

$$
\begin{aligned}
\text{P(stay with manager } k) &= (1 - \kappa)n_{k,t} + \kappa \\
\text{P(switch to manager } j) &= (1 - \kappa)n_{j,t}
\end{aligned}
\tag{4.13}
$$

In the special case of $\kappa = 0$, every investor updates strategy and switches to a new manager; The more general case $0 \leq \kappa \leq 1$, gives some persistence or inertia in the impact of strategies. Reflecting the fact that not all the investors update their choices every period. κ may be interpreted as the average per period fraction of investors who stick to their previous managers. In the extreme case of $\kappa = 1$, they would all stay with previous managers.

4.2.5 The Trading Strategy of Fund Manager

Expectation

In nonlinear economic models, expectations play an important role. People form their expectations about what will happen in the future based on what has happened in the past. For instance, the behaviour of price in the past periods will influence the people's expectation on price in the future. Early paper of Nerlove (1958) first proposed the model of adaptive expectation formation of future prices. Many papers on Learning-to-forecast experiments in Hommes *et al.* (2005, 2011, 2013b) observed the individuals' forecasting behaviour in the laboratory. They all found evidences of bounded rationality and non-fundamental forecasting rules. They also suggested that agents tend to use simple linear rules, in particular trend extrapolating rules.

In this paper, we assume that fund managers are boundedly rational. They are technical traders who use last observed price as an anchor and extrapolate future prices. Two classes of trading strategies are being observed: trend chaser and contrarian. These managers believe that they can forecast market prices p_t. The expectation of fund manager on prices in the next period is

$$\hat{p}_{t+1} = p_t + \eta(p_t - p_{t-1}), \eta \in \mathbb{R}. \tag{4.14}$$

When $\eta > 0$, manager behaves as a trend chaser and expects price changes to continue in the same direction. He buys assets when prices increase and sells them when prices drop. Brock and Hommes (1998) found that trend chasers trigger irregular switching between "optimism" and

"pessimism". Market price fluctuates between temporary growing above efficient-market hypothesis(EMH) fundamental with speculative bubbles and falling below the fundamental. When $\eta < 0$, manager is a contrarian trader who expects a reversal of the latest price change. He sells assets when prices increase and buys them when prices decrease. Some empirical evidences suggested that contrarian strategies generate significant abnormal returns over a long period, such as Fama and French (1998), Capaul *et al.* (1993). Brock and Hommes (1998) showed that contrarians cause a lot of irregular, up and down fluctuations around the fundamental. In particular, when $\eta = 0$, $\hat{p}_{t+1} = p_t$, manager is with naive expectation and simply uses the last observed price. This is initially proposed by Ezekiel (1938) .

Optimial Strategy

In each period, investors are allowed to switch their managers depending on the updated signal of fund managers' performance. In principle, they always switch to the managers with stronger signals. However, we should note that their behaviours are also affected by the inertia parameter κ. In this paper, we consider the optimal portfolio choice of manager j at period t. He assumes that all other managers invest the same portfolio from the previous period with the fraction of risky assets $\hat{y}_{k,t} = y_{k,t-1}$.

From the above notation, the market clearing equation (4.5) is rewritten as

$$p_t = \sum_{k \neq j}^{M} (y_{k,t-1} A_{k,t-1} + y_{j,t} A_{j,t-1}) + \varepsilon. \tag{4.15}$$

To facilitate the computation, we add noises ε to prices. Therefore, the portfolios $(1 - y_{j,t}, y_{j,t})$ recommended by fund manager j is chosen by the way of optimising his expected compensation. His portfolios may vary with different compensation contracts. In this case, the types of compensation contracts play a key role on the fund manager's optimal strategy.

Motivated by their compensation contract, ***Asset Maximiser*** aims to maximise the value of the overall assets he managed. His payoff $y_{j,t}$ is derived from the maximisation problem as

$$y_{j,t} = \underset{y_{j,t}}{\mathrm{argmax}}\, \hat{A}_{j,t+1}. \tag{4.16}$$

According to Equation (4.13), the expected assets are

$$\hat{A}_{j,t+1} = \kappa A_{j,t} + (1-\kappa)A\hat{n}_{j,t+1}. \tag{4.17}$$

$\underset{y_{j,t}}{\mathrm{Max}}\,\hat{A}_{j,t+1}$ is thus achieved by $\underset{y_{j,t}}{\max}\,\hat{n}_{j,t+1}$. This is easily understood by intuition. In order to maximise the managed assets, fund manager has to attract as many investors as possible. According to Equation (4.11), (4.12) and (4.14), the portfolio of asset maximiser in period t is derived from the maximisation equation

$$y_{A,t} = \underset{y_{j,t}}{\mathrm{argmax}} \left\{ \int_0^{\infty} \frac{\beta}{\sum_{k=1}^{M} exp[\lambda(y_{j,t}-y_{k,t-1})(r - \frac{\eta(\hat{p}_t - p_{t-1})+x}{\hat{p}_t}) + \beta x]} dx \right\}. \tag{4.18}$$

Following the same idea, expected **_Return Maximiser_** is motivated by a performance-based contract. The fraction of risky assets in his portfolio $y_{j,t}$ is derived from the maximisation problem of expected return,

$$y_{j,t} = \underset{y_{j,t}}{\mathrm{argmax}}\, \hat{R}_{j,t+1}. \tag{4.19}$$

According to equation (4.11) and (4.14), the portfolio of expected return maximiser is

$$y_{R,t} = \underset{y_{j,t}}{\mathrm{argmax}} \left\{ (1-y_{j,t})r + y_{j,t}\left[\frac{\eta(\hat{p}_t - p_{t-1})}{\hat{p}_t}\right] + \frac{y_{j,t}}{\beta\hat{p}_t} \right\}. \tag{4.20}$$

4.3 Maximisation Analysis

In this model, fund managers are either motivated by performance-based contract or by asset-based contract. The managers who are under performance-based contract are _Return Maximis-_

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ers. They make payoffs by maximising the portfolios' expected returns, which is $\max \hat{R}_{t+1}$. The managers who are motivated by asset-based contract are *Asset Maximisers*. They maximise the whole asset they manage and in turn maximising the probability of investors following them, which is $\max \hat{n}_{t+1}$. Moreover, we assume that these managers are boundedly rational with respect to their price expectation. They are distinguished between *Trend Chasers* and *Contrarian Traders*.
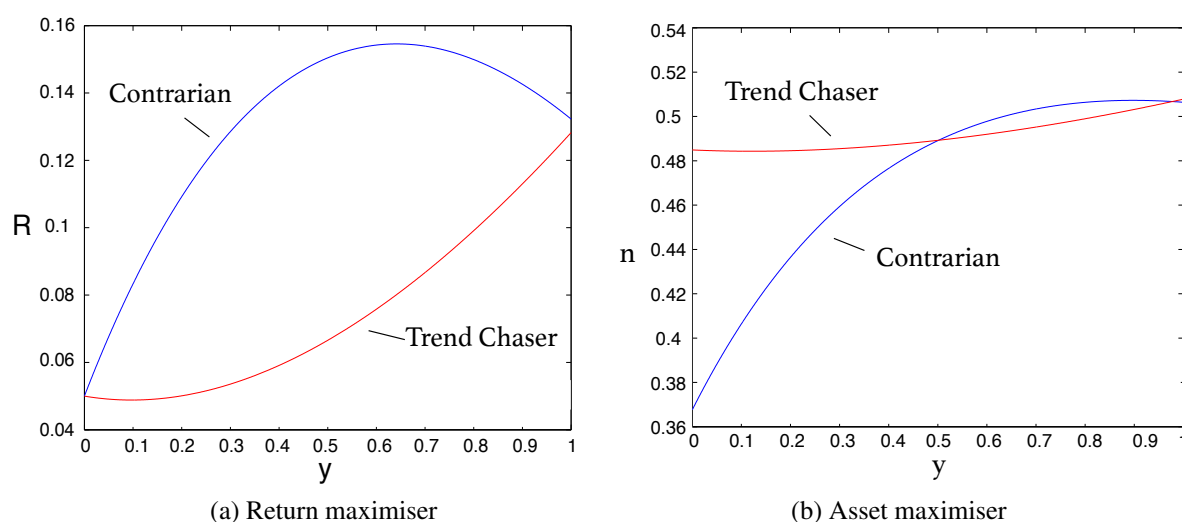

(a) Return maximiser (b) Asset maximiser

Figure 4.1: The relationship between fund manager's payoff and the objective he makes effort to maximise. (a) The relationship between the payoff of return maximiser and the fund's returns; (b) The relationship between the payoff of asset maximiser and the possibility of investors to follow asset maximiser.

Figure 4.1 illustrates the relationship between fund manager's payoff and the objective he maximises. Return maximiser aims to maximise his portfolio's returns, while the objective of asset maximiser is to attract as many investors as possible to follow them. For both return maximiser and asset maximiser, their optimal portfolios present similar patterns with respect to expectations. When the manager is a trend chaser, with $\eta > 0$, the relationship between the choice of risky asset's fraction and fund manager's maximisation objective is convex parabola. For both return maximiser and asset maximiser, they always recommend extreme portfolios to their investors, such as $y = 0$ or $y = 1$, depending on the parameters. For manager with contrarian

expectation, $\eta < 0$, the choice of risky asset's fraction and the objective is in a concave relationship. His objective can be fulfilled with risky asset's fraction anywhere between 0 and 1 by changing the parameters. Therefore, he exhibits a behaviour less extreme compared with trend chaser. To sum up, fund manager does alter his risk taking behaviour in the financial market. The expectations on future price plays a key role. Even under different types of compensation contracts, fund manager always recommends a portfolio in response to his expectations on the trend of future asset prices. Therefore, in the following, we are going to analysis the optimal choices of fund managers' contracts depends on the types of expectations.

4.4 Simulation Results

Unless stated otherwise, we use the following baseline parameter values in the simulations:

- The number of investors and managers are $N = 10$ and $M = 2$. In one type market, all the managers are same type. In two types of market, each manager represents one type.

- In order to avoid wild fluctuations of the market, we keep the parameters of managers' expectation in the range of $-1 < \eta < 1$. They are $\eta = 0.5$ for trend chaser and $\eta = -0.5$ for contrarian trader.

- The average costs per period of managers is $c = 0.1$, the parameter in the density of risky asset's dividend is $\beta = 0.5$, inertia parameter is $\kappa = 0.5$, the sensitivity of investor's choice is $\lambda = 0.9$. A large value of λ allow the system converge quickly.

A summary of the main notations used in the paper is provided in Appendix 4.A. In the dynamic analysis, each simulation runs for $T = 100$ periods.

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4.4.1 One Type Market

Dynamics Under One Type Market

In this section, we analysis a homogenous market with one type of fund manager as a benchmark. Based on the type of expectation and compensation schemes, there are four sorts of managers (see Table 4.1): Trend chaser under asset-based compensation schemes, trend chaser under performance-based compensation schemes, contrarian trader under asset-based compensation schemes, contrarian trader under performance-based compensation schemes. Meantime, investors choose fund managers according to their performance in the past and are also allowed to update their choices over time.

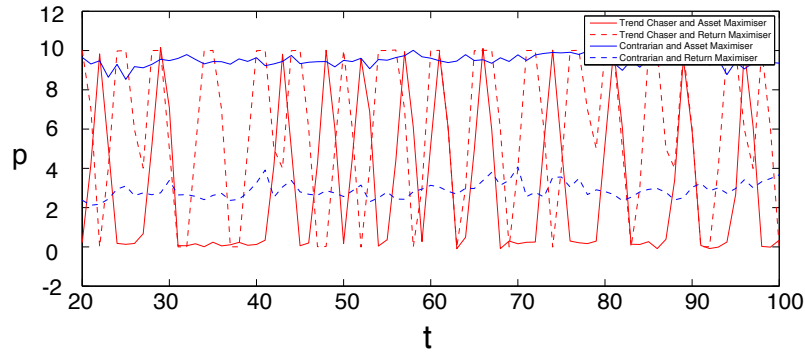|		Compensation schemes	
		Asset-based	Performance-based
Expectations	Trend Chaser	I	II
	Contrarian	III	IV

I. Trend chaser and Asset maximiser; II. Trend chaser and Return maximiser;
III. Contrarian trader and Asset maximiser; IV. Contrarian trader and Return maximiser.
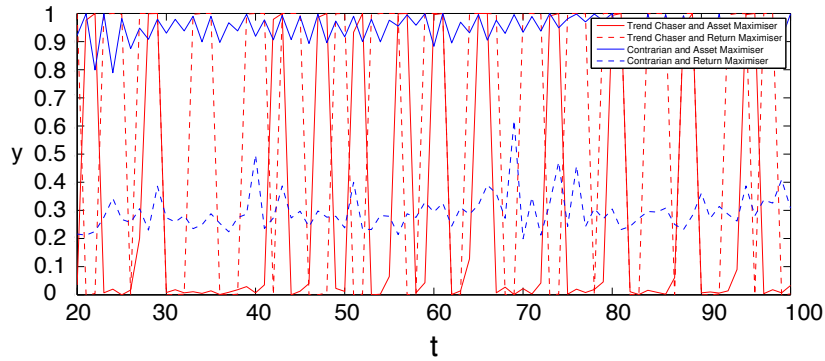
Table 4.1: Types of fund managers

Figure 4.2 presents the dynamic behaviours of the market prices, the fund managers' payoffs and the funds' returns in each market. As shown in Figure 4.2(a), the market prices are driven by the different expectation schemes. When fund managers use the trading strategy of trend chaser, the market exhibits highly irregular switchings of prices; If fund managers use strategy of contrarian, market exhibits small up and down oscillations. This is the case whenever fund managers are under asset-based or performance-based contract. Brock and Hommes (1998) also found that the presence of trend chasers or contrarians may lead to market instability and chaos. In particular, trend chasers trigger irregular and unpredictable switching between the phases of "optimism', with prices following temporary speculative bubbles, or "pessimism", with prices falling bellow the fundamental. Contrarians cause a lot of irregular, up and down oscillations around fundamental. In this homogenous market, investor's choices are largely affected by the

inertia parameter κ. We will discuss the impact of κ in the next section.

(a) Prices

(b) Fraction of risky assets

(c) Returns

Figure 4.2: Dynamics of one type market. (a) Prices *p*; (b) Fund manager's payoffs: the fraction of risky asset *y*; (c) Returns *R*.

Figure 4.2(b) illustrates the dynamic behaviour of the fund managers' choice of the fraction

of risky assets *y*. It also exhibits highly fluctuations in the market with trend chasers and small oscillations in the contrarian case. This is consistent with market prices in Figure 4.2(a). The intuition is that the expectation of trend chasers always drive them to make extreme payoffs every few periods. For instance, if market prices increase in one period, trend chasers would expect the prices keep changing in the same direction in the future. They would raise the fraction of risky assets in their portfolios in the next period. In one type market, all the managers behave in the same way that they are all buying the risky assets. Therefore, the prices keep increasing and managers will raise their risk levels further. In this way, after several periods, managers' payoffs will become fully risky. On the contrary, if market prices dropped, they presume another decline and cut back the fraction of risky assets. The portfolios will have no risk at all after a few periods. The returns of portfolios also exhibit consistent dynamic behaviours with market prices and payoffs (see Figure 4.2(c)). Very high returns can be generated by some portfolios of trend chasers due to highly risk taking behaviour. However, their returns are unstable over time.

We should note that although the types of compensation contracts do not have as much impact as expectations to trend chasers, they do make differences when it comes to the market with contrarians. When these contrarians are asset maximisers and under asset-based contract, they tend to recommend risky portfolios as shown in Figure 4.2(b). Compared with return maximisers, the prices in their markets are higher. Their portfolios give lower returns over time. When contrarians are under performance-based contract, they decrease their risky levels. In the end, return maximisers achieve higher returns than asset maximisers.

In general, our analysis shows that the fund managers' expectations affect the stability in the market. Both of trend chasers and contrarians fail to stabilise the market. Trend chasers tend to trigger fluctuations of the market. This influence is equally effective under both asset-based and performance-based contract. We also observe the influence of different types of compensation contracts and find that they change the risk taking behaviours of contrarians. The asset-based contract induces more risky portfolios, while the performance-based contract

motivates the portfolios with higher returns.

The Impact of Inertia Parameter κ

In the previous section, we investigate the dynamic of one type market with a benchmark inertia parameter $\kappa = 0.5$. It stands for the probability that an investor sticks to his previous manager is 0.5. Because there is only one type of fund manager in the market, inertia parameter plays an important role on the choices of investors. Regarding this, the impact of the changes of inertia parameter κ on the dynamic behaviour of the market is of particular interest. To exclude other influences, such as the impact from expectation parameter η, we analysis different homogenous markets individually. In order to trace the impact of the changes of κ to market dynamic behaviours, we refer to four cases of κ, which are $\kappa = 0, 0.5, 0.8$ and 1.

Contrarians   Figure 4.3 shows the dynamics of prices p (Figure 4.3a), the fund managers' payoffs y (Figure 4.3b) and the returns of portfolios R (Figure 4.3c) in the market with one type of fund managers. These fund managers are with contrarian expectation on the future prices. Moreover, they are under performance-based compensation contract that their objective is to maximise the return of portfolios. To compare, each figure gives simulated time series with respect to κ equals to $0, 0.5, 0.8$ and 1.

In previous section, we showed that the market with contrarian traders was more stable than the market with trend chasers. However, as we shall see in these figures, the slightly up and down oscillations are amplified by raising the inertia parameter κ. When κ is increasing from 0 to 0.5, a manager tends to increase the risk level of his portfolios and raises the prices in the market. In particular, if κ is high, such as over 0.8, we observe significant changes of the portfolios' risk levels in different periods. In some periods the manager would recommend complete riskless portfolios to investors while on the other times he may recommend highly risky ones. This behaviour makes the market prices irregular fluctuation dramatically. We should note that the managers in this market are also return maximisers and aim to optimise the returns of their portfolios. However as show in Figure 4.3(c), with the increasing of inertia

(a) Prices

(b) Fraction of risky assets

(c) Returns

Figure 4.3: Dynamics in one type market with the fund managers with *contrarian expectation*. The fund managers are *return maximisers* and under performance-based compensation schemes. (a) Prices p; (b) Manager's payoffs y; (c) Returns R.

parameter, the returns are decreasing. In this market, when inertia parameter is very high, the returns of the managers' portfolios become very small, even close to 0. The intuition is that the very small returns may motivate the significant change of fund managers' behaviours either by increasing or decreasing their risk levels dramatically, in order to satisfy their objective of maximising returns.

The dynamic behaviours of a market with only one type of fund managers who are asset

maximisers with contrarian expectation are shown in Figure 4.4. Intuitively, the inertia parameter κ should not influence the performance of asset maximiser, since his only concern is to optimise the fraction of investors who follow him. However, the dynamics shows that the slightly up and down oscillations of the market are amplified by raising the inertia parameter κ. Moreover, price are dropping while increasing the level of κ. It suggests the possible price bias and increasing noises given larger κ. When κ increases, the noise is amplified and price is underestimated. However, no matter κ changes or not, returns remain around the same level. In all, κ amplifys fluctuations and make market price to be easily underestimated.
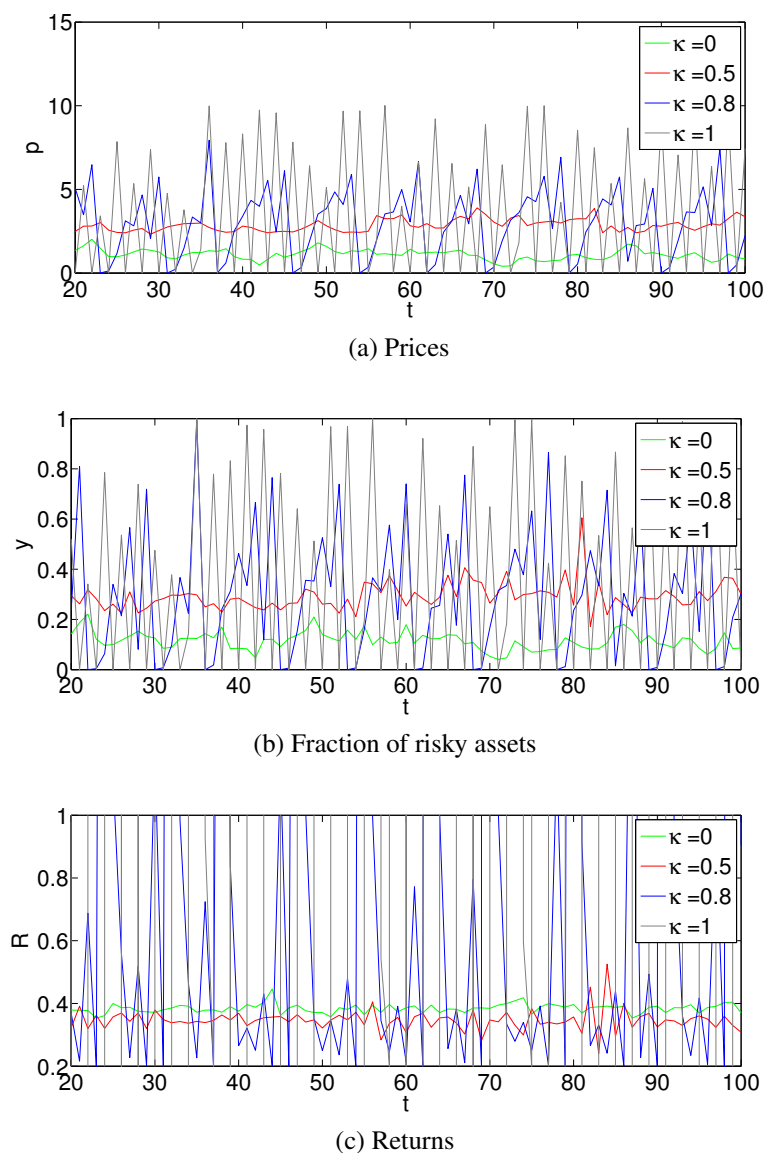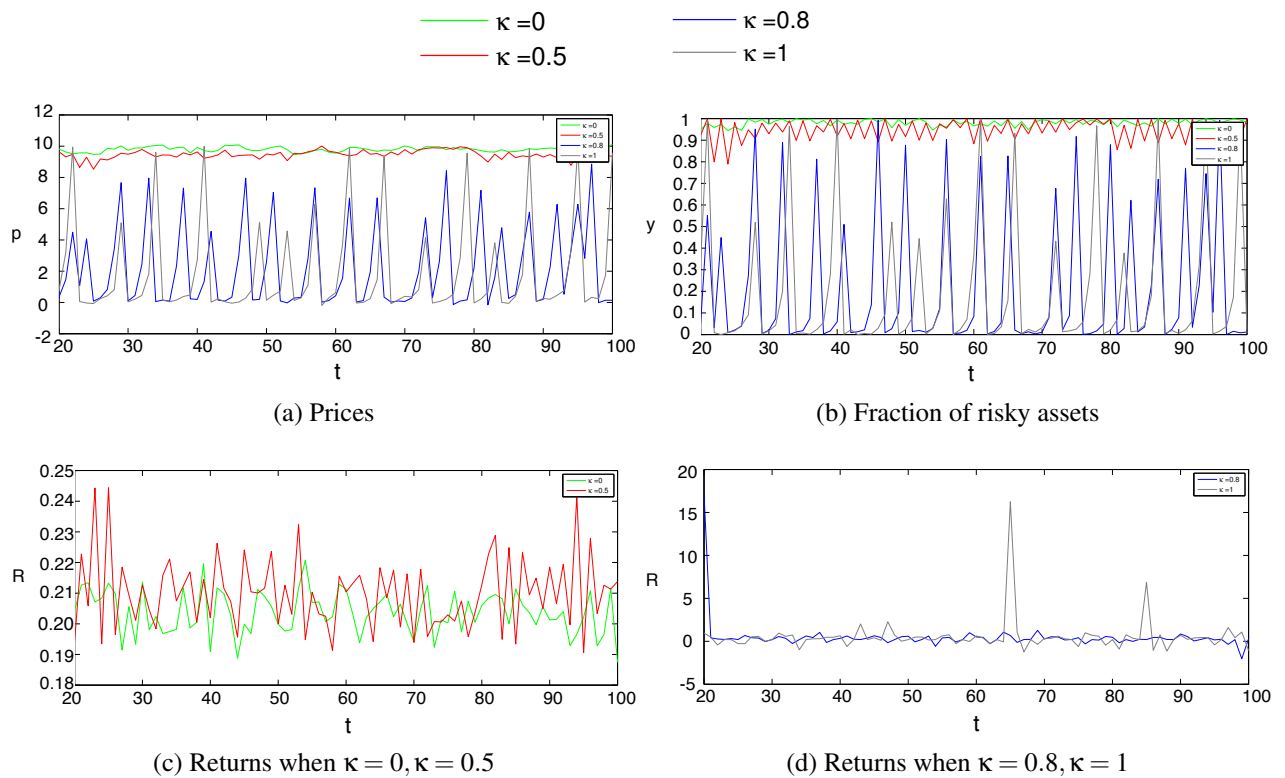

Figure 4.4: Dynamic in one type of market with the fund managers with *contrarian expectation*. These fund managers are *asset maximisers* and under asset-based compensation schemes. (a) Time series of prices; (b) Time series of the fractions of risky assets; (c), (d) Time series of Returns.

Trend Chasers   This section investigates a homogenous market with the fund managers with trend chaser type of expectation. Figure 4.5 shows the analysis of the dynamic behaviour under performance-based compensation schemes. Figure 4.6 is with same format as Figure 4.5 and

presents the analysis under asset-based compensation schemes.

Consistent with the results in Section 4.4.1, trend chasers trigger irregular and significant fluctuations in the market under both types of compensation contracts. They tend to recommend extreme portfolios, fully risky or complete riskless, during all the range of inertia parameter κ. Their risk taking behaviours changes significantly over time. Therefore, the market with trend chasers wildly fluctuates under both asset-based and performance-based contracts. The intuition is that trend chasers always change their risk taking behaviours based on their expectations. They expect the prices change in the same direction as it in the past. When prices increase, they raise the risk level of their portfolios because that they expect another increase of the prices in the future. This speculative behaviour raises the market prices and increases managers' risk levels in turn. After several periods, the trend chaser expectation would drive their risk levels to the extremes. Moreover, the influences of trend chaser expectations are so strong that it dominates the dynamic behaviours of the market over time. Fund managers only concern chasing the change of price. The influence of compensation contract can be neglected. Therefore, the dynamics under performance-based and asset-based compensations exhibit similar patterns.
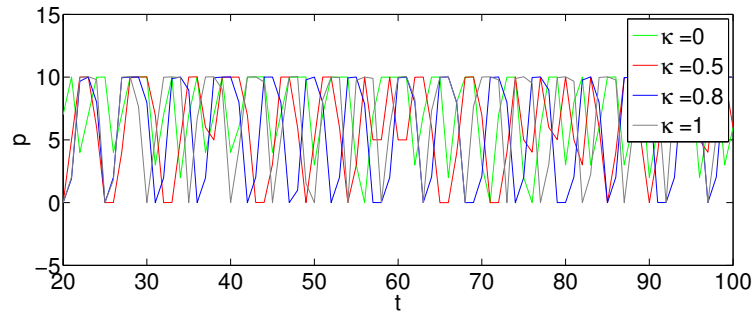
(a) Prices
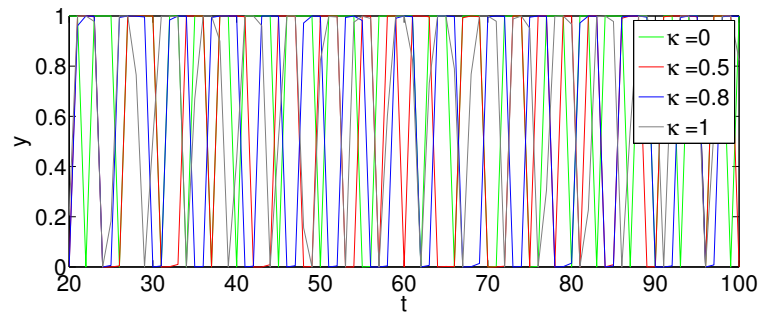
(b) Fraction of risky assets

(c) Returns

Figure 4.5: Dynamic in one type of market with *trend chasers* and *return maximisers*. (a) Prices *p*; (b) Payoffs *y* of managers. (c) Returns of the portfolios

(a) Prices

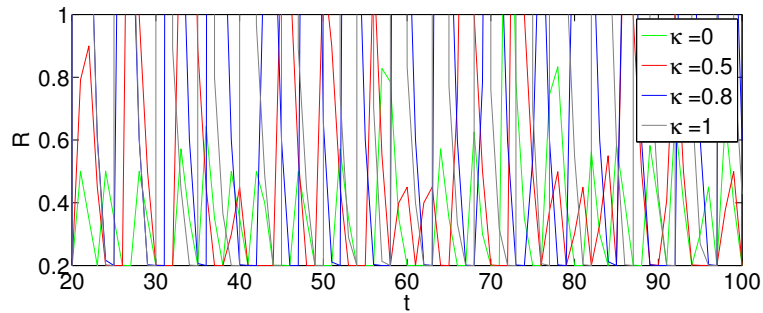(b) Prices

(b) Fraction of risky assets
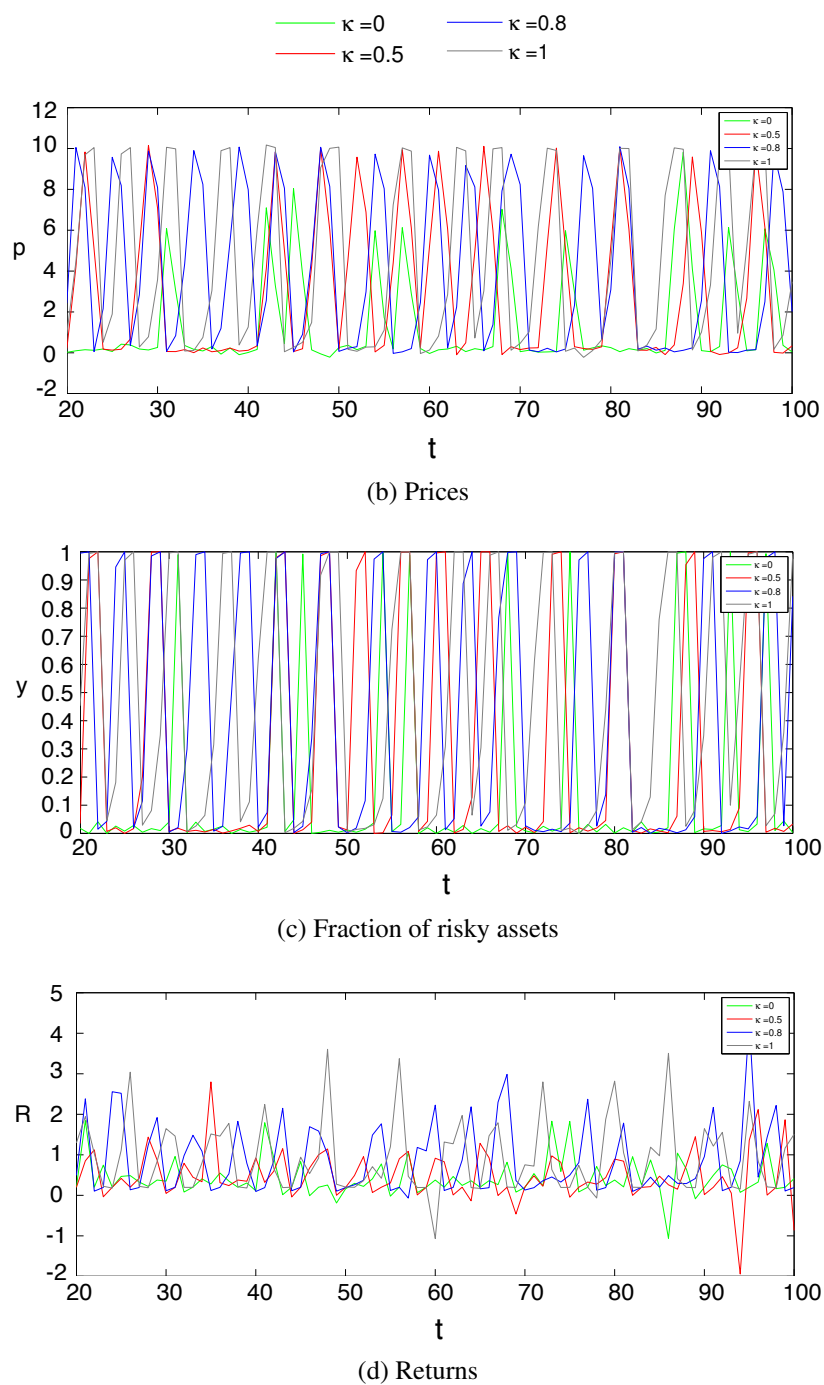
(c) Fraction of risky assets

(c) Returns

(d) Returns

Figure 4.6: Dynamic in one type of market with *trend chasers* and *asset maximisers*. (a) Prices *p*; (b) fund manager's payoffs *y*; (c) returns *R*.

4.4.2 Two Types Market

In previous analysis, we investigated the dynamic behaviours of market with fund managers who are under only one compensation contract. We found some differences between the influence of different compensation contract on different expectations, and also the influence of inertia parameter. However, in the real market those two compensation schemes are coexistence. Although asset-based schemes are the dominant form of compensation types in the advisory contract between mutual fund manager and investors, increasing number of mutual fund companies have been involving performance-based schemes in their contract. In particular, the larger and more complex companies tend to use more explicit performance-based incentives, due to the fact that they usually face higher direct monitoring costs. In this section, we consider heterogenous types of fund manager, asset maximiser and performance maximiser. Investors switch between these two types of fund managers and are allowed to update their choices over time.

Figure 4.7 and 4.8 show the dynamic behaviours of the heterogenous market with the coexistence of return maximiser and asset maximiser types of fund managers under the trading strategies of contrarian and trend chaser. In one type market, fund managers behave differently depend on the two types of expectations. Trend chasers tend to trigger significant fluctuations in the market while contrarians bring small up and down oscillations. However, as we shall see, the coexistence of asset maximisers and return maximisers narrows the differences between these two types of trading strategies. The dynamic behaviours of the market under contrarian expectation exhibits similar patterns as the market under trend chaser expectations. Both of them are highly unstable. The prices and the risk level of portfolios fluctuate dramatically. In other words, the coexistence of two types of compensation schemes increase the risk level of contrarians and motivates them to behave more like trend chaser. Boswijk *et al.* (2007) suggested that stock market bubbles may be amplified by the strategy of trend chasers. There were also several empirical evidences that the relative proportion among forecasting services of trend-following beliefs compared to fundamental mean reverting rules increased in the bubble market (Frankel

and Froot, 1987; Vissing-Jorgensen, 2004). Therefore, asset-based contract may also amplified the stock price bubble by motivating contrarian to behave like trend chaser.

Intuitively, the higher return should be given by return maximiser since he always aims to optimise fund's return. Nevertheless, on the contrary, in this market the portfolios selected by asset maximisers generate higher returns than performance maximiser, and has a significant effect on the average returns of all portfolios in the market. As shown in the results before, inertia parameter κ would make market price to be easily underestimated. Therefore, the negative bias makes the return maximiser give lower optimise returns, while asset maximiser is driven by risk and in turn generating higher returns. The fractions of investors who follow asset maximisers are around 0.9, which leave only around 0.1 of investors to follow return maximisers. Therefore, asset-based schemes become the dominant form of compensation type. However, either types of compensation schemes are able to drive out the other type completely. This is exactly what happens in real financial market. In 1996, 2,190 of 2,351 actively managed equity mutual funds used asset-based management fees whereas only 39 used performance-based fees (Moody, 1996). In recent years, even though performance-based fees have been introduced in a lot of larger mutual fund companies, the dominant compensation scheme in the advisory contracts are still based on asset. For example, in 2004, only 9% of all U.S. mutual funds are under performance-based scheme (Information from Greenwich Associates and the Investment Company Institute).

Our results can be used to explain why a majority of mutual funds favour simple asset-based compensation schemes. This type of contract is able to motivate fund managers use their recourses and expertise knowledges to attract as many assets as possible in the market. In the meantime, they also motivates the portfolios with high returns that they contribute to most of the average return in heterogenous market. This contract is able to meet managers' objective and investors' expectations at the same time. Therefore, asset maximisers dominate the whole market and contribute to most of the average returns in the market.

As a complementary, we also observed the market with heterogenous expectations of trend

chaser and contrarian under different types of compensation contracts. These markets are highly unstable and exhibit wildly fluctuations as the dynamics in Figure 4.8 and 4.7. Both types of traders coexist with fractions varying over time and prices fluctuating chaotically. Either of the type cannot drive out other trader types and fail to stabilizse price fluctuations towards its fundamental value. In all our examples, the influence of trend chaser is very strong turns the oscillations in the market into unpredictable chaotic fluctuations. The influence of motivation contract can be neglected compared with it.
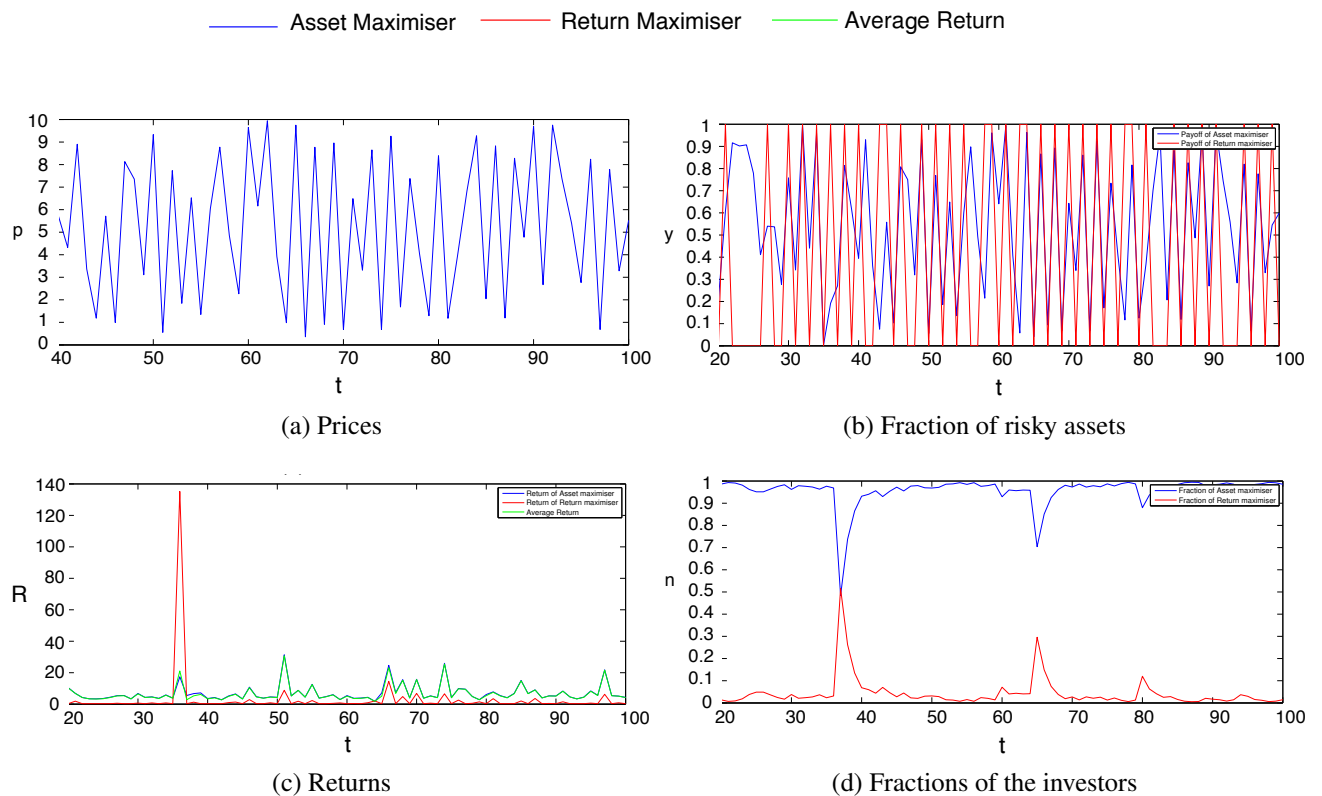

Figure 4.7: Dynamics in a heterogenous market with *return maximisers* and *asset maximisers*. Fund managers are with *contrarian* expectation.(a) Prices p; (b) Payoffs of fund managers y; (c) Returns R; (d) Fractions of the investors who follow different types of fund managers n.
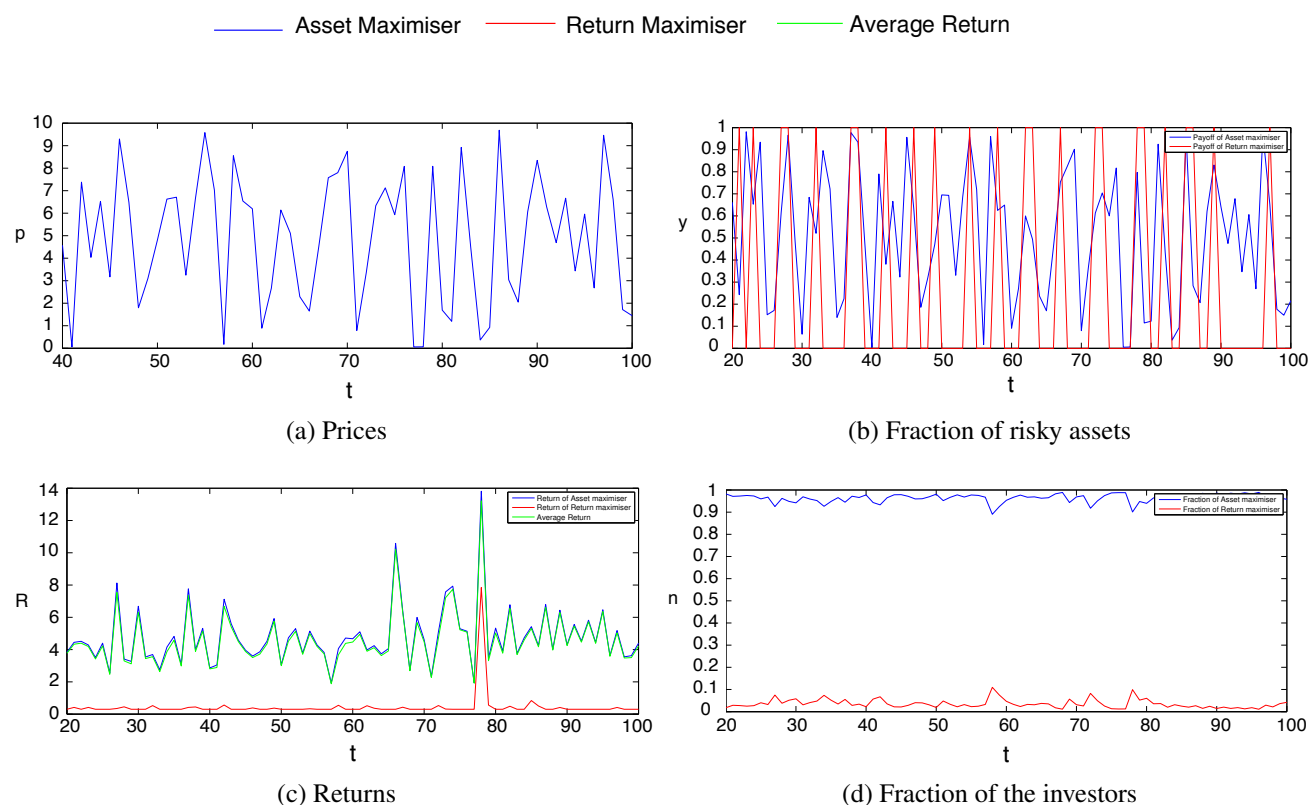
Figure 4.8: Dynamics in a heterogenous market with *return maximiser* and *asset maximiser*. Fund managers are with *trend chaser* expectation. (a) Prices p; (b) Payoffs of fund managers y; (c) Returns R; (d) Fractions of the investors who follow different types of fund managers n.

4.5   Conclusion

In this paper, we established an agent-based model to describe the dynamic behaviours of a financial market with mutual fund managers and investors under two types of compensation contracts: asset-based fees and performance-based fees, using two types of expectations: trend chaser and contrarian. In our frame work, on the one hand, investors choose their fund managers by observing public available performance measures. The manager with better performance attracts more followers. On the other hand, fund managers are myopic and boundedly rational, using last observed price as an anchor to extrapolate future price. They are also motivated by two compensation schemes: asset-based fees and performance-based fees. Fund managers behave differently as asset maximisers and return maximisers. We aim to find the link between the types of compensation contract and expectations.

As a benchmark, we observed homogenous market with only one type of fund managers. The dynamic analysis shows that the expectations have great impact on the stability of market. Trend chasers and contrarian traders make the market less stable. In particular, trend chasers trigger wildly irregular fluctuations, while contrarians brings along small up and down oscillations to the market. This influence is equally effective under both asset-based and performance-based contract. As for the influences from the type of compensation schemes, they do change the risk taking behaviour of contrarian traders. They are willing to take more risk when they are under an asset-based contract. However, for trend chaser, they do not care about motivation contract, but only concern chasing the price.

The inertia parameter κ also plays an important role in the dynamic behaviours of the market. It destabilises the market and induce market price bias. When κ increases, the price is underestimated, which makes asset-based compensation scheme generate higher returns in heterogenous market.

In heterogenous market, the coexistence of two types compensation schemes narrows the differences between trend chaser and contrarian markets. They exhibit similar patterns. Contrarian traders behave very much like trend chasers and amplifies market bubbles. However, in the long run, asset maximisers dominate the market. Most of the average returns in the market are contributed by asset maximisers. In particular, under asset-based contract, fund manager is able to maximise his own compensations and generate a high return portfolio at the same time. These results can be used to explain why a majority of mutual fund companies favour simple asset-based compensations schemes.

Appendix 4.A   Notation

- M - Number of managers

- N - Number of investors

- β - Parameter in the density of risky asset's dividend

- d - Dividend paid to investors at the beginning of each period.

- c - Average costs per period incurred by managers, such as the cost of gathering information.

- y - Fraction of asset which is invested into risky asset

- δ - Random noise, individual investor's error.

- ε - White noise in market price

- R - Return rate

- λ - Sensitivity of investor's choice

- κ - Inertia parameter

- n - Fraction of investor who follows a type of manager

- η - Parameter of expectation of manager on the change of price

Appendix 4.B Market Clearing Price

The individual demand function for the risky asset is determined by the payoff of investor and his asset in the last period,

$$D_{i,t} = \frac{A^I_{i,t-1} y^I_{i,t}}{p_t}, \tag{4.21}$$

where $A_{i,t-1}$ is the asset owned by each investor in the last period $t-1$.

Demand of the market is

$$Demand_t = \sum D_{i,t}. \tag{4.22}$$

Since the supply of risky asset is normalised to 1, market clearing for the risky assets requires

$$Market\,Equilibrium : Demand_t = Supply_t = 1, \tag{4.23}$$

$$\sum D_{i,t} = 1, \tag{4.24}$$

which gives the market equilibrium pricing equation is

$$p_t = \sum_{i \in n} y^I_{i,t} A^I_{i,t-1}. \tag{4.25}$$

Chapter 5

Summary

This thesis sheds some lights on the on-going discussions on modelling the economy as a complex evolving system. It introduces a complex systems approach and attempts to unfold the underlying mechanisms of dynamic instability in complex economic system. Some of them are the first attempts. Moreover, it contributes to the ongoing discussions on early warnings for financial crises and the transmissions of the complexity based economic policy. As a complex system, economic system possesses the characteristics of complex systems. This thesis focuses on three features: *critical transitions, catastrophe theory* and *expectations*. The methodologies developed and applied include statistical time series tools and agent-based modelling.

The main contribution of this thesis is that it considers the economy as a multi-equilibrium complex evolving system with heterogeneous boundedly rational agents interacting with each other, rather than a representative agent model based on rational expectations hypothesis with general equilibrium, as in conventional economic theory. The employment of the complex system approach enhances the understanding of the economy or financial system, and exhibits potential predictive power. This thesis is organised in three self-contained chapters. Chapter 2 employs the notion of critical slowing down to detect critical transitions in financial markets. Chapter 3 discusses catastrophe theory in housing market. Chapter 4 studies expectation formation in a dynamical financial market with heterogenous agents. The detailed outcomes from each chapter are the following.

Chapter 2 explores a complexity approach to designing early warning signals for financial crises, in order to amend the failure to detect the recent financial crisis by using conventional economic models. The appealing idea of constructing early warning signals through identifying characteristics of critical slowing down on the basis of time series observations is supported by the ample empirical and experimental evidence in the complex systems in natural science (Dakos *et al.*, 2008; Scheffer, 2009; Kefi *et al.*, 2013). As a nonlinear system, finance has been repeatedly coined as an important potential application area. This chapter observes a number of historical financial crises to explore the evidence of "critical slowing down" prior to market collapses. The four main events considered are Black Monday 1987, the 1997 Asian Crisis, the 2000 Dot.com bubble burst, and the 2008 Financial Crisis. This chapter shows the sense of evidence for early warnings before the collapse of Black Monday in 1987, while the results are mixed and insignificant for the other financial crises. It suggests that critical slowing down could serve as one of the complementary early warning indicators of financial crises.

Chapter 3 studies the cusp catastrophe model for housing markets. It evaluates whether cusp catastrophe occurs in the housing market, by fitting fits a stochastic cusp catastrophe model to housing market data. This is the first attempt to apply catastrophe theory to housing market data. The results suggest that the instability of dynamical behaviour of housing market can be explained by investigating the behaviour point moves across the cusp equilibrium surface with respect to control parameters. In particular, the predictions of the cusp catastrophe model could also serve as an early warning tool of housing bubbles and crashes, and a guide to interest rate policy.

Chapter 4 considers the importance of expectations in complex economic system. It aims to discuss the link between the types of expectation and compensation contract. It establishes an agent-based model to describe the dynamic behaviour of the financial market with mutual fund managers and investors under two types of compensation contracts: asset-based fees and performance-based fees. The fund managers are boundedly rational with expectations of trend chasers and contrarian traders. The results suggest that expectation formations plays an im-

portant role in the dynamic stability. Trend chasers always trigger significant price fluctuations, possibly leading to bubbles and crashes, while contrarian traders bring about slight up and down oscillations. Heterogenous analysis shows that the fund managers under asset-based contracts dominate the market and generate higher returns. It explains the current situations that asset-based contracts are the main form of compensation schemes in advisory contracts between fund managers and investors.

The general conclusion arising from this thesis is that complexity proves to play an important role in financial market and the economy, which is suggested to be a multi-equilibrium evolving system with several types of heterogenous boundedly rational agents interacting with each other. By adopting the complexity outlook to model and estimate the dynamic instability in the complex economic system, we are better able to understand its underlying mechanisms and to possibly solve the problems involving predictions.

The analysis and results in this thesis have provided successful examples of shifting from conventional economic models to a complex system approach, which is suggested to be a promising direction of future work. A success of the complex system approach to financial markets and the economy could focus on several aspects in the future: firstly, construct a theoretical framework through agent-based modelling, such as heterogenous agents and network models; Secondly, develop empirical and statistical methodologies to analyse the complexity behaviour of economic systems; Thirdly, gain experience and datasets with labotary experiments to match micro and macro through observing the individual and aggregate behaviour of agents. Moreover, providing advanced complexity tools might help policy makers establish more efficient early warning systems for future crises and gain experience of the transmission of complexity-based monetary and macro-economic policy.

References

Abiad, A. (2003). Early warning systems: A survey and a regime switching approach. *IMF Working paper No 03/32, IMF, Washington, DC USA*.

Abiad, A. (2007). *Early Warning Systems for Currency Crises: A Regime-Switching Approach*, volume 104, chapter International Series in Operations Research & Management Science, pp. 155–184. Springer US.

Aerts, D., Czachor, M., Gabora, L., Kuna, M., Posiewnik, A., Pykacz, J. and Syty, M. (2003). Quantum morphogenesis: A variation on thom's catastrophe theory. *Physical Review* E, **67,** number 5, 051926.

Albrecher, H., Hartinger, J. and Tichy, R. F. (2005). On the distribution of dividend payments and the discounted penalty function in a risk model with linear dividend barrier. *Scandinavian Actuarial Journal*, **2005,** number 2, 103–126.

Alexander, R. A., Herbert, G. R., DeShon, R. P. and Hanges, P. J. (1992). An examination of least-squares regression modeling of catastrophe theory. *Psychological Bulletin*, **111,** number 2, 366.

Anderson, P. W., Arrow, K. and Pines, D. (1988). *The economy as an evolving complex system.* Santa Fe Institute Series, Westview Press, US.

Arthur, W. B. (1999). Complexity and the economy. *Science*, **284,** number 5411, 107–109.

Arthur, W. B., Durlauf, N.S. and Lane, D.A. (1997). The economy as an evolving complex system ii. *Proceedings volume XXVII, Santa Fe Institute studies in the science of complexity*.

Asmussen, S., Hojgaard, B. and Taksar, M. (2000). Optimal risk control and dividend distribution policies. example of excess-of loss reinsurance for an insurance corporation. *Finance and Stochastics*, **4,** number 3, 299–324.

Avanzi, B. (2009). Strategies for dividend distribution: A review. *North American Actuarial Journal*, **13,** number 2, 217–251.

Ball, P. (2012). *Why Society is a Complex Matter - Meeting Twenty-first Century Challenges with a New Kind of Science*, 1 edn. Springer-Verlag Berlin Heidelberg.

Barunik, J. and Kukacka, J. (2015). Realizing stock market crashes: stochastic cusp catastrophe model of returns under the time-varying volatility. *Quantitative Finance*, **15,** number 6, 959–973.

Barunik, J. and Vosvrda, M. (2009). Can a stochastic cusp catastrophe model explain stock market crashes? *Journal of Economic Dynamics and Control*, **33,** number 10, 1824–1836.

Baumol, W., Goldfeld, S. M., Gordon, L. A. and Koehn, F.-M. (1989). *The economics of mutual fund markets: Competition versus regulation*, volume 7. Springer.

Berg, A. and Patillo, C. (1999). Predicting currency crises: The indicators approach and an alternative. *Journal of International Money and Finance*, **18,** 561–586.

Bernanke, B. S. and Gertler, M. (1995). Inside the black box: The credit channel of monetary policy transmission. *The Journal of Economic Perspectives*, **9,** number 4, 27–48.

Bines, H. E. and Thel, S. (2004). *Investment management law and regulation*, 2 edn. Panel Publishers.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, **31,** number 3, 307–327.

Bolt, W., Demertzis, M., Diks, C., Hommes, C.H. and van der Leij, M. (2014). Identifying booms and busts in house prices under heterogeneous expectations. *De Nederlandsche Bank Working paper*, **450**.

Borland, L. (2005). Long-range memory and nonextensivity in financial markets. *Europhysics News*, **36,** number 6, 228–231.

Boswijk, H P., Hommes, C. H. and Manzan, S. (2007). Behavioral heterogeneity in stock prices. *Journal of Economic Dynamics and Control*, **31,** number 6, 1938–1970.

Brillinger, D. R. (2007). Learning a potential function from a trajectory. *IEEE Signal Processing Letters*, **14,** number 12, 1.

Brock, W.A. and Hommes, C.H. (1997). A rational route to randomness. *Econometrica*, **65,** number 5, 1059–1095.

Brock, W.A. and Hommes, C.H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, **22,** number 8-9, 1235–1274.

Bussière, M. and Fratzscher, M. (2006). Towards a new early warning system of financial crises. *Journal of International Money and Finance*, **25,** 953–973.

Capaul, C., Rowley, I. and Sharpe, W. F. (1993). International value and growth stock returns. *Financial Analysts Journal*, **49,** number 1, 27–36.

Case, K.E. and Shiller, R.J. (2003). Is there a bubble in the housing market? *Brookings Papers on Economic Activity*, **2003,** 299–342.

Chevalier, J. and Ellison, G. (1997). Risk taking by mutual funds as a response to incentives. *Journal of Political Economy*, **105,** number 6, 1167–1200.

Cobb, L. (1978). Stochastic catastrophe models and multimodal distributions. *Behavioral Science*, **23,** number 4, 360–374.

Cobb, L. (1980). Estimation theory for the cusp catastrophe model. *Proceedings of the Section on Survey Research Mehods*, **26,** 772–776.

Cobb, L. (1981). Parameter estimation for the cusp catastrophe model. *Behavioral Science*, **26,** number 1, 75–78.

Cobb, L., Koppstein, P. and Chen, N. H. (1983). Estimation and moment recursion relations for multimodal distributions of the exponential family. *Journal of the American Statistical Association*, **78,** number 381, 124–130.

Cobb, L. and Watson, B. (1980). Statistical catastrophe theory: An overview. *Mathematical Modelling*, **1,** number 4, 311–317.

Cohen, S. I. and Starks, L. T. (1988). Estimation risk and incentive contracts for portfolio managers. *Management Science*, **34,** number 9, 1067–1079.

Crowe, C., Dell'Ariccia, G., Igan, D. and Rabanal, P. (2013). How to deal with real estate booms: Lessons from country experiences. *Journal of Financial Stability*, **9,** number 3, 300–319.

Cuoco, D. and Kaniel, R. (2011). Equilibrium prices in the presence of delegated portfolio management. *Journal of Financial Economics*, **101,** number 2, 264–296.

Dakos, V., Scheffer, M., van Nes, E.H., Brovkin, V. V.and Petoukho and Held, H. (2008). Slowing down as an early warning signal for abrupt climate change. *Proceedings of the National Academy of Sciences*, **105,** 14308–14312.

Damato, K. (2005). Funds made math mistakes on performance fees. *The Wall Street Journal*, C13.

Davis, E.P. and Karim, D. (2008). Could early warning systems have helped to predict the sub-prime crisis? *National Institute Economic Review*, **206,** 35–47.

Davis, M.A. and Heathcote, J. (2005). Housing and the business cycle. *International Economic Review*, **46,** 751–784.

Deli, D. N. (2002). Mutual fund advisory contracts: An empirical investigation. *The Journal of Finance*, **57,** number 1, 109–133.

Demyanyk, Y. and Hasan, I. (2010). Financial crises and bank failures: A review of prediction methods. *Omega*, **38,** number 5, 315–324.

Dieci, R. and Westerhoff, F. (2012). A simple model of a speculative housing market. *Journal of Evolutionary Economics*, **22,** number 2, 303–329.

Dieci, R. and Westerhoff, F. (2013). Modeling house price dynamics with heterogeneous speculators. In *Global Analysis of Dynamic Models in Economics and Finance*, pp. 35–61. Springer.

Eichberger, J., Grant, S. and King, P. S. (1999). On relative performance contracts and fund and manager's incentives. *European Economic Review*, **43,** 135–161.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica*, **50,** number 4, pp. 987–1007.

Evans, G. W. and Honkapohja, S. (2001). *Learning and expectations in macroeconomics.* Princeton University Press, Princeton, US.

Ezekiel, M. (1938). The cobweb theorem. *The Quarterly Journal of Economics*, **52,** number 2, 255–280.

Fama, E. F. and French, K. R. (1998). Value versus growth: The international evidence. *The Journal of Finance*, **53,** number 6, 1975–1999.

Farmer, J. D., Gallegati, M., Hommes, C., Kirman, A., Ormerod, P., Cincotti, S., Sanchez, A. and Helbing, D. (2012). A complex systems approach to constructing better models for managing financial markets and the economy. *The European Physical Journal Special Topics*, **214,** 361–400.

Frankel, J. A. and Froot, K. A. (1987). Using survey data to test standard propositions regarding exchange rate expectations. *The American Economic Review*, **77,** number 1, 133–153.

Friedman, D. and Abraham, R. (2009). Bubbles and crashes: Gradient dynamics in financial markets. *Journal of Economic Dynamics & Control*, **33,** 922–937.

Gilmore, R. (1993). *Catastrophe theory for scientists and engineers.* Dover, New York, USA.

Goetzmann, W. N., Peles, N., William, P. and Goetzmann, N. (1997). Cognitive dissonance and mutual fund investors. *Journal of Financial Research*, **20,** 145–158.

Grasman, R. P., van der Maas, H. L. J. and Wagenmakers, E. (2009). Fitting the cusp catastrophe in r: A cusp-package primer. *Journal of Statistical Software*, **32,** number 8, 1–28.

Grinold, R. and Rudd, A. (1987). Incentive fees: Who wins? Who loses? *Financial Analysts Journal*, **43,** number 1, 27–38.

Guastello, S. J. (1988). Catastrophe modeling of the accident process: Organizational subunit size. *Psychological Bulletin*, **103,** number 2, 246.

Hartelman, P.A.I. (1997). Stochastic catastrophe theory. Ph.D. Thesis. University of Amsterdam.

Hartelman, P.A.I., van der Maas, H. L. J. and Molenaar, P. C. M. (1998). Detecting and modelling developmental transitions. *British Journal of Developmental Psychology*, **16,** number 1, 97–122.

Held, H and Kleinen, T (2004). Detection of climate system bifurcations by degenerate fingerprinting. *Geophysical Research Letters*, **31,** L23207.

Hommes, C. H. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, **35,** number 1, 1–24.

Hommes, C. H. (2013a). *Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems*. Cambridge University Press.

Hommes, C. H. (2013b). Reflexivity, expectations feedback and almost self-fulfilling equilibria: economic theory, empirical evidence and laboratory experiments. *Journal of Economic Methodology*, **20,** number 4, 406–419.

Hommes, C. H. and Milgrom, P. (1987). Aggregation and linearity in the provision of intertemporal incentives. *Econometrica: Journal of the Econometric Society*, **55,** number 2, 303–328.

Hommes, C. H., Sonnemans, J., Tuinstra, J. and Van de Velden, H. (2005). Coordination of expectations in asset pricing experiments. *Review of Financial Studies*, **18,** number 3, 955–980.

Hommes, C. H. and Wagener, F. (2009). *Handbook of Financial Markets: Dynamics and Evolution*, chapter Complex Evolutionary Systems in Behavioral Finance, pp. 218–271. Elsevier.

Ippolito, R. A. (1992). Consumer reaction to measures of poor quality: Evidence from the mutual fund industry. *Journal of Law and Economics*, **35,** number 1, 45–70.

Jensen, M. C. and Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, **3,** number 4, 305–360.

Kefi, S., Dakos, V., Scheffer, M., Van Nes, E. H. and Rietkerk, M. (2013). Early warning signals also precede non-catastrophic transitions. *Oikos*, **122,** number 5, 641–648.

Kleinen, T., Held, H. and Petschel-Held, G. (2003). The potential role of spectral properties in detecting thresholds in the earth system: application to the thermohaline circulation. *Ocean Dynamics*, **53,** 53–63; 10.1007/s10236-002-0023-6.

Kolari, J., Glennon, D., Shin, H. and Caputo, M. (2002). Predicting large US commercial bank failures. *Journal of Economics and Business*, **54,** number 4, 361 – 387.

Kouwenberg, R. and Zwinkels, R. (2011). Chasing trends in the us housing market. *Available at SSRN http://ssrn.com/abstract=1539475*.

Kuehn, C. (2011). A mathematical framework for critical transitions: Bifurcations, fast-slow systems and stochastic dynamics. *Physica D: Nonlinear Phenomena*, **240,** number 12, 1020–1035.

Lange, R., Oliva, T. A. and McDade, S. R. (2000). An algorithm for estimating multivariate catastrophe models: Gemcat ii. *Studies in Nonlinear Dynamics & Econometrics*, **4,** number 3.

Lenton, T. M. (2011). Early warning of climate tipping points. *Nature Climate Change*, **1,** 201–209.

REFERENCES

Lenton, T. M., Held, H., Kriegler, E., Hall, J. W., Lucht, W., Rahmstorf, S. and Schellnhuber, H. Joachim (2008). Tipping elements in the earth's climate system. *Proceedings of the National Academy of Sciences*, **105,** number 6, 1786–1793.

Lenton, T. M., Livina, V. N., Dakos, V., van Nes, E. H. and Scheffer, M. (2012). Early warning of climate tipping points from critical slowing down: Comparing methods to improve robustness. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **370,** number 1962, 1185–1204.

Livina, V N and Lenton, T M (2007). A modified method for detecting incipient bifurcations in a dynamical system. *Geophysical Research Letters*, **34,** L03712.

Malkiel, B. G. (1995). Returns from investing in equity mutual funds 1971 to 1991. *The Journal of Finance*, **50,** number 2, 549–572.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, **7,** number 1, 77–91.

McFadden, D. (1973). *Conditional logit analysis of qualitative choice behavior*, pp. 105–142. Academic Press, New York.

McFadden, D. and Reid, F. (1975). Aggregate travel demand forecasting from disaggregated behavioral models. *Transportation Research Record*, number 534, 24–37.

Mendoza, A. C. D. and Sedano, M. A. M. (2009). The choice of performance-based fees in the mutual fund industry: The case of spain. *Investment Management and Financial Innovations*, **6,** number 3, 7–17.

Moody (1996). *Moody's Manuals on Banks and Financial Companies*. Moody's Investor Services, New York.

Muellbauer, J. and Murphy, A. (2008). Housing markets and the economy: The assessment. *Oxford Review of Economic Policy*, **24,** number 1, 1–33.

Murphy, K. J. (1999). Executive compensation. *Handbook of labor economics*, **3,** 2485–2563.

Nerlove, M. (1958). Adaptive expectations and cobweb phenomena. *The Quarterly Journal of Economics*, **72,** number 2, 227–240.

Oliva, T. A., Desarbo, W. S., Day, D. L. and Jedidi, K. (1987). Gemcat: A general multivariate methodology for estimating catastrophe models. *Behavioral Science*, **32,** number 2, 121–137.

Ou-Yang, H. (2003). Optimal contracts in a continuous time delegated portfolio management problem. *Review of Financial Studies*, **16,** number 1, 173–208.

Palomino, F. (2005). Relative performance objectives in financial markets. *Journal of Financial Intermediation*, **14,** number 3, 351–375.

Patel, J., Zeckhauser, R. and Hendricks, D. (1994). Investment flows and performance: Evidence from mutual funds, cross-border investments and new issues. *Japan, Europe and the international financial markets: Analytical and empirical perspectives*, 51–72.

Peng, C.-K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E. and Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. *Physical Review* E, **49,** 1685–1689.

Poston, T. and Stewart, I. (2012). *Catastrophe theory and its applications*, 1 edn. US: Dover Publications.

Quax, R., Kandhai, D. and Sloot, P. M. A. (2013). Information dissipation as an early-warning signal for the lehman brothers collapse in financial time series. *Scientific Reports*, **3(2013),** number 1898.

Rahmstorf, S. (2001). *Abrupt cimate change*. UK: Academic Press.

Reinhart, C.M. and Rogoff, K.S. (2009). *This time is different: Eight centuries of financial folly*. Princeton Univ Press, Princeton, US.

Roston, M. (1996). *Mutual Fund Managers and Lifecycle Risk: An Empirical Investigation*. Chicago: University Chicago, Department of Economics,.

Sargent, T. J. (1993). *Bounded rationality in macroeconomics: The Arne Ryde memorial lectures*. Oxford University Press, Oxford, UK.

Scheffer, M. (2009). *Critical transitions in nature and society*. Princeton studies in complexity. Princeton University Press, Princeton, US.

Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, S. R. V.and Carpenter, Dakos, V., Held, H., van Nes, Egbert H., Rietkerk, M. and Sugihara, G. (2009). Early-warning signals for critical transitions. *Nature*, **461,** number 7260, 53–59.

Scheffer, M., Carpenter, S. R., Lenton, T. M., Bascompte, J., Brock, W., Dakos, V., van de Koppel, J., van de Leemput, I. A., Levin, S. A., van Nes, E. H. and et al. (2012). Anticipating critical transitions. *Science*, **338,** number 6105, 344–348.

Shi, S., Jou, J. and Tripe, D. (2014). Can interest rates really control house prices? Effectiveness and implications for macroprudential policy. *Journal of Banking & Finance*, **47,** 15–28.

Shiller, R. J. (2006). Long-term perspectives on the current boom in home prices. *The Economists' Voice*, **3,** number 4.

Shiller, R. J. (2007). Understanding recent trends in house prices and home ownership. *NBER Working Paper*, number w13553.

Shiller, R. J. (2008). Historic turning points in real estate. *Eastern Economic Journal*, **34,** number 1, 1–13.

Shiller, R. J. (2012). *The subprime solution: How today's global financial crisis happened, and what to do about it*. Princeton University Press, Princeton, US.

Shiller, R. J. (2015). *Irrational exuberance*. Princeton University Press, Princeton, US.

REFERENCES

Sieber, J. and Thompson, J. M. T. (2012). Nonlinear softening as a predictive precursor to climate tipping. *Philosophical Transactions of The Royal Society A: Mathematical Physical and Engineering Sciences*, **370,** number 1962, SI, 1205–1227.

Simon, H. A. (1982). *Models of bounded rationality: Empirically grounded economic reason*, volume 3. MIT press, MIT, US.

Sirri, E. R. and Tufano, P. (1992). *Buying and Selling Mutual Funds: Flows, Performance, Fees, and Services*. Division of Research, Graduate School of Business Administration, Harvard University, US.

Sornette, D. (2009). *Why stock markets crash: critical events in complex financial systems*. Princeton University Press, Princeton, US.

Stanley, H. E., Amaral, L. A. N., Gabaix, X., Gopikrishnan, P. and Plerou, V. (2001). Quantifying economic fluctuations. *Physica A: Statistical Mechanics and its Applications*, **302,** number 1, 126–137.

Starks, L. T. (1987). Performance incentive fees: An agency theoretic approach. *The Journal of Financial and Quantitative Analysis*, **22,** number 1, 17–32.

Stracca, L. (2006). Delegated portfolio management: A survey of the theoretical literature. *Journal of Economic Surveys*, **20,** number 5, 823–848.

Tamaki, T., Torii, T. and Maeda, K. (2003). Stability analysis of black holes via a catastrophe theory and black hole thermodynamics in generalized theories of gravity. *Physical Review* D, **68,** number 2, 024028.

Tan, J. P. L. and Cheong, S. A. (2014). Critical slowing down associated with regime shifts in the US housing market. *The European Physical Journal* B, **87:38,** number 2.

Taylor, J. B. (2007). Housing and monetary policy. *Proceedings - Economic Policy Symposium - Jackson Hole, Federal Reserve Bank of Kansas City*, 463–476.

Taylor, J. B. (2009). The financial crisis and the policy responses: An empirical analysis of what went wrong. *Critical Review: A Journal of Politics and Society*, **21,** 341–364.

Thom, R. (1972). *Structural stability and morphogenesis*. Benjamin, New York, US.

Thompson, J. M. T. and Sieber, J. (2010a). Predicting climate tipping as a noisy bifurcation: a review. *International Journal of Bifurcation and Chaos*, **21,** 399–423.

Thompson, J. M. T. and Sieber, J. (2010b). *Predicting climate tipping points*, chapter 3, pp. 50–83. Cambridge University Press, Cambridge, UK.

Thompson, J. M. T., Stewart, H. B. and Ueda, Y. (1994). Safe, explosive, and dangerous bifurcations in dissipative dynamical systems. *Physical Review* E, **49,** 1019–1027.

Trichet, J. C. (2010). Reflections on the nature of monetary policy non-standard measures and finance theory. In *Opening address at the ECB Central Banking Conference*, *Frankfurt*.

Van der Maas, H. L., Kolstein, R. and Van Der Pligt, J. (2003). Sudden transitions in attitudes. *Sociological Methods & Research*, **32,** number 2, 125–152.

Van der Maas, H. L. and Molenaar, P. C. (1992). Stagewise cognitive development: An application of catastrophe theory. *Psychological review*, **99,** number 3, 395.

Vissing-Jorgensen, A. (2003). Perspectives on behavioral finance: Does" irrationality" disappear with wealth? Evidence from expectations and actions. *NBER Macroeconomics Annual*, **18,** 194–200.

Wagener, F. (2012). Regime shifts: Early warnings. *CeNDEF Working paper No. 12-01, University of Amsterdam, Amsterdam, the Netherlands*.

Wagenmakers, E., Molenaar, P., Grasman, R. P., Hartelman, P. A. and van der Maas, H. (2005). Transformation invariant stochastic catastrophe theory. *Physica D: Nonlinear Phenomena*, **211,** number 3, 263–276.

Wilson, A. (1981). *Catastrophe theory and bifurcation: Applications to urban and regional systems*. London : Croom Helm ; Berkeley : University of California Press, c1981.

Zeeman, E. C. (1974). On the unstable behaviour of stock exchanges. *Journal of Mathematical Economics*, **1,** number 1, 39–49.

Zeeman, E. C. (1977). *Catastrophe theory: Selected papers, 1972-1977*. Addison-Wesley, London, Amsterdam, Ontario, Sydney, Tokyo.

Samenvatting (Summary in Dutch)

Dit proefschrift werpt licht op de aanhoudende discussie over de vraag of de economie gemodelleerd moet worden als een complex evoluerend systeem. Het introduceert een complexiteitsbenadering en probeert de onderliggende mechanismes van de dynamische instabiliteit in complexe economische systemen bloot te leggen. Soms zijn dit de eerste pogingen met deze benadering. Daarnaast draagt dit proefschrift bij aan de aanhoudende discussies over de vroegtijdige detectie van financiële crises en over de weerslag van economisch beleid gebaseerd op complexiteit. Als complex systeem beschikt het economische systeem over de eigenschappen van complexe systemen. Dit proefschrift richt zich op drie kenmerken: *kritische transities*, *catastrofetheorie* en *verwachtingen*. De methodologische technieken die worden ontwikkeld en toegepast omvatten statistische tijdreeksanalyse en agent-gebaseerde modellering.

De voornaamste bijdrage van dit proefschrift is dat het de economie beschouwt als een complex evoluerend systeem met meerdere evenwichten en heterogene, begrensd rationele agenten die op elkaar reageren. Dit staat recht tegenover een representatief-agentmodel gebaseerd op de hypothese van rationele verwachtingen en een algemeen evenwicht, zoals in de conventionele economische theorie. Het gebruik van de complexiteitsbenadering verhoogt het begrip van de economie of het financiële systeem, en biedt in potentie een grote voorspellende kracht. Dit proefschrift is onderverdeeld in drie op zichzelf staande hoofdstukken. Hoofdstuk 2 maakt gebruik van de notie genaamd "kritische vertraging" om kritische transities te detecteren in financiële markten. Hoofdstuk 3 behandelt catastrofetheorie in de vastgoedmarkt. Hoofdstuk 4 bestudeert verwachtingsvorming in een dynamische financiële markt met heterogene agenten.

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De gedetailleerde uitkomsten van elk hoofdstuk zijn als volgt.

Hoofdstuk 2 verkent een complexiteitsbenadering voor de vroegtijdige detectie van financiële crises, om de tekortkomingen te herstellen van conventionele economische modellen waarmee de recente financiële crisis niet ontdekt kon worden. Het aansprekende idee van de benadering is om zogeheten "early warning signals" te construeren door het identificeren van kritische vertraging in tijdreeksobservaties. Dit wordt ondersteund door veelomvattend empirisch en experimenteel bewijs uit complexe systemen in de natuurwetenschappen (Dakos et al., 2008; Scheffer, 2009; Kefi et al., 2013). Als niet-lineair systeem is het financiële systeem meermaals aangemerkt als een belangrijk potentieel toepassingsgebied. Dit hoofdstuk beschouwt een aantal historische financiële crises in de zoektocht naar bewijs voor kritische vertraging voorafgaand aan beurskrachs. De vier onderzochte gevallen zijn Zwarte Maandag (1987), de Aziëcrisis (1997), het knappen van de internetzeepbel (2000) en de kredietcrisis (2008). Er wordt bewijs gevonden voor early warning signals voorafgaand aan de crash op Zwarte Maandag in 1987; de resultaten voor de andere financiële crises zijn gemengd en insignifcant. Het hoofdstuk toont aan dat kritische vertraging mogelijk dienst zou kunnen doen als een aanvullende vroegtijdige waarschuwingsindicator van financiële crises.

Hoofdstuk 3 bestudeert een model met een zogeheten "doorncatastrofe" voor de vastgoedmarkt. Onderzocht wordt of een doorncatastrofe plaatsvindt in de vastgoedmarkt door een stochastisch doorncatastrofemodel te fitten op data van de vastgoedmarkt. Dit is de eerste poging om catastrofetheorie toe te passen op data van de vastgoedmarkt. Uit de resultaten blijkt dat de dynamische instabiliteit van de vastgoedmarkt verklaard kan worden door te bestuderen hoe het gedragspunt beweegt over het doornevenwichtsoppervlak ten opzichte van controleparameters. In het bijzonder kunnen de voorspellingen van het doorncatastrofemodel gebruikt worden als een instrument voor vroegtijdige detectie van vastgoedzeepbellen en -crashes, en als een leidraad voor rentebeleid.

Hoofdstuk 4 stelt het belang van verwachtingen in complexe economische systemen aan de orde, en richt zich op het verband tussen verschillende typen verwachtingen en vergoed-

ingsregelingen. Er wordt een agent-gebaseerd model ontwikkeld om het dynamische gedrag te beschrijven van een financiële markt met beleggers en managers van beleggingsfondsen onder twee typen vergoedingsregelingen: gebaseerd op de omvang van het vermogen ("asset-based" contracten) en gebaseerd op de geleverde presentatie (prestatiecontracten). De fondsmanagers zijn begrensd rationeel met trendvolgende en "contrarian" verwachtingen. Uit de resultaten blijkt dat verwachtingsvorming een belangrijke rol speelt in de dynamische stabiliteit. Waar trendvolgers altijd significante prijsfluctuaties tot stand brengen, die mogelijk leiden tot zeepbellen en krachs, brengen contrarian handelaren lichte schommelingen naar boven en beneden teweeg. Heterogene analyse toont aan dat de fondsmanagers met asset-based contracten de markt domineren en hogere opbrengsten genereren. Dit verklaart de huidige situatie waarin asset-based contracten de meest gangbare vorm van vergoedingsregelingen zijn bij adviesovereenkomsten tussen fondsmanagers en beleggers.

De algemene conclusie die uit dit proefschrift naar voren komt is dat complexiteit een belangrijke rol speelt in financiële markten en de economie, die voorgesteld wordt als een evoluerend systeem met meerdere evenwichten en verschillende typen heterogene, begrensd rationele agenten die op elkaar reageren. Door aandacht te hebben voor de complexiteitsdimensie en de dynamische instabiliteit in het complexe economische systeem te modelleren en te schatten, zijn we beter in staat de onderliggende mechanismes van het systeem te begrijpen en mogelijk ook de problemen met betrekking tot voorspellingen op te lossen.

De analyse en resultaten in dit proefschrift bieden succesvolle voorbeelden in het verschuiven van conventionele economische modellen naar een complexiteitsbenadering, die een veelbelovende richting voor toekomstig onderzoek blijkt te zijn. Een succesvolle complexiteitsbenadering voor financiële markten en de economie zou zich in de toekomst op verschillende aspecten kunnen richten: ten eerste, het construeren van een theoretisch kader door agent-gebaseerde modellering, zoals modellen met heterogene agenten en netwerken; ten tweede, het ontwikkelen van empirische en statistische technieken om het complexiteitsgedrag van economische systemen te analyseren; ten derde, het opbouwen van ervaring en datasets van

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laboratoriumexperimenten om micro en macro aan elkaar te koppelen door het observeren van individueel en geagreggeerd gedrag van agenten. Bovendien kan het leveren van geavanceerde complexiteitsinstrumenten beleidsmakers helpen om efficiëntere detectiesystemen op te zetten voor financiële crises en om ervaring op te doen over de weerslag van monetair en macro-economisch beleid gebaseerd op complexiteit.