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Information and Discrimination: Foundations and Applications to Credit and Labor Markets

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Contents

Chapter 1

General Introduction

One thing that is ubiquitous in our economy is information imperfection, by which we refer to a situation in which different parties of a transaction have access to different information. One common example happens in the labor market. Workers know better about their own skills, industriousness, productivities, etc., while employers can hardly know the true competence of prospective workers, although they are able to observe certain information on the applicants such as education levels or certificates. Such imperfections are also shared on the capital market: the relationships between banks and borrowers act similarly to an employment relationship between employers and job seekers. Borrowers are usually more familiar with their projects and possess more accurate information about the quality of their projects, while lenders such as banks have limited access to such information and have difficulties knowing whether it is likely for borrowers to default. More examples are trading goods between a seller and a buyer, signing an insurance contract between an insurer and his client, and so on.

The impact of information depends on the market structure and to what extent market participants can access to the information. The problems caused by the information imperfections can be alleviated when there are signals which can reveal the hidden information, at least to some extent. Thus, it is common for the party with informational disadvantage to resort to such signals even when it is costly. For example, employers ask for certificates or design tests and interviews to infer the true productivities of candidate workers; banks carry out investment analysis or ask for collaterals or other requirements from the borrowers. Yet on the other hand, the party with more information may also have the incentive to signal or reveal their private information when they can expect to be treated differently. The signal, or the information derived from the signals, plays an important role in the decision-making process.

1.1 Imperfect Information and Information criteria

In a decision problem with information asymmetry, it is possible and common for the party with information disadvantage to collect information before the decision is made. We could say that information is signaled no matter this information collection is active or passive. Decision makers observe signals and update their beliefs, and the updated beliefs - the posterior beliefs - are taken use of to make their decisions. Yet the information contained in the signals is not always true and precise, since signals and the unknown characteristics are generally not perfectly correlated. It is natural to think that more reliable signals with more accurate information could contribute to a more efficient performance. In order to formulate this evaluation of signals, an information system or an information structure is thus used to describe the relationship between signals and underlying states, various information criteria are proposed based on the information system. By virtue of this, we are allowed to consider which way of collecting information can provide less noisy information and are therefore more preferable.

One classical criterion was developed by Blackwell (1951, 1953), who considers the comparison of two experiments from a statistical perspective. That is, a more informative system should be statistically sufficient for a less informative one, which also means that a less informative system can be duplicated from a more informative one by adding some random transmission errors, or "garbling" as is called. Large literature follows Blackwell's informativeness criterion, yet alternative criteria have also been introduced (see Lehmann (1988), Kim (1995), Persico (2000), Jewitt (2007), Quah and Strulovici (2009)).

On the other hand, there are information criteria which are defined on the expectation conditional on signal realizations. Since decisions, after signals being observed, are made according to the posterior distributions, the expectation conditional on signal realizations seems to be crucial. A more reliable signal is supposed to have larger impact on the posteriors, and thus the conditional expectations should be more disperse (see Ganuza and Penalva (2010)). This kind of criteria is very intuitive and therefore of interest to us. It comes the question that which kind of information criteria could be adopted in a certain decision-making problem with asymmetric information. Therefore, the relationship between classical information criteria and the dispersion of conditional expectations are examined in the first chapter, which also provides a theoretic foundation of how to formalize the revelation of information.

1.2 Information imperfection on the credit market and credit crisis

In the credit market, it has long been understood that asymmetric information plays a central role in determining the market equilibria (since Stiglitz and Weiss (1981)). Large literature has investigated the significant influence of the asymmetric information on the credit market, showing how the strategies and the interactions of lenders and borrowers are determined in the circumstances with asymmetric information. Lenders' lack of information on the relevant characteristics of the borrowers may result in the outcome of underinvestment. Credit is said to be rationed in this sense. However, the possibility for the flip side of this story is also extensively studied: the investment level may turn out in excess of the social efficient level (see De Meza and Webb (1987) and Alberto and Filippo (2013)). With asymmetric information lenders have difficulties knowing whether it is likely for borrowers to default, and this leads to moral-hazard and adverse-selection problems in the financial market. Therefore, financial intermediaries such as banks may choose either screening or monitoring, or both technologies to alleviate these problems. We pursue our study along their lines, but proceed from a different perspective: when borrowers can choose to disclose the quality of projects or to keep silent, to which extent may the revelation of information impact the amount of the projects financed, and thus, the type of the equilibrium?

From another viewpoint, by taking look at the financial crisis that we experienced, especially the credit crisis during 2007-2008, we saw that it was primarily driven by the investment and activities concerning the opaque securities such as mortgage-backed securities (MBS), credit default swaps (CDS). Complicated structured financial vehicles and the securitization were booming. Credit was offered even when the investors were lack of relevant information on the projects. However, as the turmoil burst, lending standards tightened and credit is hoarded. A credit crunch was spread throughout the financial market, and the tightening of credit led to contractions in the real economy (see Brunnermeier (2009), Acharya and Skeie (2011) and so on). The lending with insufficient information and the investment in the opaque structured products naturally lead us to considering a possible explanation of credit crisis from the perspective of information asymmetry.

Therefore, in Chapter 3 we consider a signaling model in a competitive credit market where borrowers have the option of revealing information or keeping opaque, discuss the opacity of the market in the equilibrium, and attempt to explain the credit crunch and the instability of the financial market.

1.3 Imperfections of labor market and discrimination

Labor market is another example where imperfect information prevails. The asymmetric information affects employers' hiring decision, as well as workers' decisions on skill or education investment. A prominent example is statistical discrimination. Lang and Manove (2011) show that under asymmetric information, the blacks overinvest in observable characteristics (e.g. education) to overcome the disadvantage in job hunting as employers find it more difficult to evaluate the blacks compared to the whites. Empirical work with similar facts can also be seen in Rivkin (1995), and Cameron and Heckman (2001).

Yet Lang and Manove (2011) are silent about labor market friction captured by employment rate which becomes another central measure of discrimination in nowadays literature. The purpose of the fourth chapter is therefore to propose a tractable theory based on a relatively new branch of equilibrium search model to explain why when discrimination is potentially present in the market and why the discriminated group may invest more in skills compared to the favored one. Different from the random search model, Moen (1997) along with Shimer (1996) suggest that if individual firms are able to post wage to maximize the expected profit, then the socially optimal allocation of resources results. The critical difference between directed search (with wage posting) and random search (with wage bargaining) is whether the information on the wages is available for the workers. If yes, then workers' search strategy depends on these wages, or their search is directed by the wages. Moreover, workers take into account the trade-off between wages and the induced matching probability to maximize their expected utility from search; firms perceive this relationship that is induced by their posted wages, subject to which the profit is maximized. The search externality can be internalized and th efficiency can result.

The role of asymmetric information is absent in both Moen (1997) and Shimer (1996). While Guerrieri *et al.* (2010) consider adverse selection problem in competitive search context. The private information of the workers exists ex ante (before search) and persists ex post (after matching). In our model, wages can not be conditioned on the group identity, it is as if there is asymmetric information ex ante (before search); what makes our context differ from the information system in Guerrieri *et al.* (2010) is that once a worker-firm match gets formed, the information is revealed in the sense that firms can do what they want - to discriminate at the hiring stage. Another related work is by Grout (1984), which gives rigorous theoretical investigation on the holdup problem in a context without search friction. If the gains from investment have to be shared ex post with the trading partner, the social incentive of investment is dampened. In our model, we argue that holdup is related to our discrimination problem: when workers sink skill investment cost before entering into labor market, firms are able to extract larger surplus by creating some discriminatory hierarchy among workers. This is a re-interpretation of the result from Lang, Manove and Dickens (2005).

1.4 Outline of the thesis

The rest of this thesis is organized with three self-contained chapters. Each can be read independently from one another. It starts from a theoretical perspective by looking into information criteria, and then considers two problems regarding credit market and labor market, respectively.

In Chapter 2, I review some information criteria and pay special attention on the precision criteria which are defined on the conditional expectations by Ganuza and Penalva (2010). More specifically, the study seeks to find links between the dispersion of conditional expectations and Blackwell's informativeness criterion. The result shows that Blackwell's informativeness criterion does not necessarily imply or be implied by the dispersion of conditional expectations in general discrete cases, although Blackwell's informativeness can imply the dispersion of conditional expectations when the signal is binary. Besides, Persico's accuracy criterion is also analyzed in this chapter. Following Persico's accuracy criterion, a similar criterion is constructed, which can imply the dispersion of conditional expectations under some, though strict, conditions.

In Chapter 3, which is coauthored with Prof. Bertrand Wigniolle, we consider a lender-borrower relationship where borrowers have better information on their own projects than lenders and can choose to disclose this information or not. Signaling is costly and is borne by the borrowers. The decision of information revelation is endogenized, and so is the market opacity. We characterize the equilibrium with respect to the risk-free interest rate and show that the existence and the characteristics of the equilibrium depend on the level of the risk-free interest rate: there only exists an opaque equilibrium, in which all borrowers do not reveal information and are funded, when the interest rate is low; there only exists transparent equilibrium, in which only borrowers who own good projects reveal information and are financed, when the interest rate is high enough; and there are multiple equilibria where both opaque and transparent equilibria can be possible when the interest rate is in an intermediate range. The existence of multiple equilibria admits possible jump from an opaque equilibrium to a transparent one, which occurs with a decrease of interest rate and a reduction of credit granting at the same time. Moreover, we extend the model to an OLG context and examine the long run convergence of the equilibrium. Under different parameters, the market is likely to converge either to an opaque or a transparent stationary state, and for some configurations of parameters there is no convergence in the long run - there may be permanent oscillations between opaque and transparent equilibria. Both the static and the dynamic frameworks provide possible explanations of the instability of the credit market and indicate a possible way of explaining the phenomenon of credit crunch during the financial crisis.

In Chapter 4, which is coauthored with Sheng Bi, we study how the hiring discrimination may affect the skill investment decision of workers in the directed search model. We see that a holdup problem on workers' skill investment arises when employers can adopt discriminatory hiring norms to extract higher than socially optimal profit. When hiring priority is determined both by productivity-dependent (skill level) and -independent characteristics (discrimination), the decision of skill investment becomes strategic between the discriminated and favored groups. In frictional markets with posted wage, the skill investment in equilibrium depends on market tightness. The discriminated group, compared to the favored one, tends to underinvest in skills when the market gets tighter, although there also exists an equilibrium where the favored group underinvests in skills while the discriminated one does not. The payoffs of both workers and firms are also analyzed here. Besides, we further discuss on the problem in a wage bargaining context. With bargained wage, similar equilibrium in which the favored group underinvests exists, and firms incur cost for an intermediate range of bargaining power when they discriminate.

Chapter 2

Informativeness and the dispersion of posterior distributions

Abstract

In a decision problem with uncertainty the decision maker can receive signals which reveal some of the information about the unknown true state. In order to evaluate the informativeness of different signals, or different information systems in which signals are generated, various criteria have been developed over the past few decades. Since decisions, after signals being observed, are made according to the posterior distributions, the expectation conditional on signal realizations is likely to be crucial in deciding the informativeness of signals. Therefore, the relationship between some classical information criteria and the dispersion of conditional expectations (Ganuza and Penalva's supermodular precision criterion) is examined here. The study shows that Blackwell's informativeness criterion does not necessarily imply or be implied by the dispersion of conditional expectations in general discrete cases, although binary cases could somehow enlighten us. Besides, Persico's accuracy criterion is also analyzed in this paper. Following the accuracy criterion, a similar criterion is constructed, which can imply the dispersion of conditional expectations under some conditions.

2.1 Introduction

Uncertainty exists nearly everywhere in the economy, such as the future price of a stock, the uncertain return of a project, or the unobservable skill of a worker in the labor market. The recognition of imperfect information has had a profound influence and has provided a remarkable method for explaining economic and social phenomena. Although they may not be sure about the true state, agents can, in most cases, overcome the uncertainty by obtaining related information that is conveyed in some signals, which may be derived from personal investigation, or suggested by an expert, or purchased from some institutions, or even stolen. Their decision will then be based on the revised knowledge about the true state, i.e. the posterior belief of the state.

But how could we tell the quality of the information? How will distinctive signals influence the decision process? Consider an extreme case where the signals could precisely reveal the future state, then the revised belief will depend completely on the realized value of the signal rather than the prior belief. On the other hand, if the informational content of the signal is relatively low, the posterior belief will be similar to the prior belief and different signal realizations will not form much different posterior beliefs. That is to say, the posterior distribution is less disperse under less informative signals. This could also be extended to the expectation of one's payoff conditional on signal realizations. So more information should lead to more disperse conditional expectations. In fact, following this intuition, Ganuza and Penalva (2010) propose a new kind of criteria for evaluating different information systems, which they refer to as *precision criteria*.

This type of precision criteria are practical since the decisions are often made on the basis of conditional expectations, and they are therefore useful for a large range of economic decision problems, such as auctions, investments on education, etc., and the formalization of precision criteria provides an easier way to interpret the informativeness of signals. Nevertheless, it is unconventional to evaluate informativeness based on conditional expectations rather than directly on the underlining information systems.

Therefore, I try to bridge the gap between the precision criteria and the traditional informativeness criteria, such as the informativeness criterion (Blackwell, 1951 and 1953) and the accuracy criterion (Persico, 2000), which are considered in this paper. As for the precision criteria, we focus here only on Ganuza and Penalva's supermodular criterion, which is defined with the dispersive order. By examining the properties of the dispersive order, we see that Blackwell's informativeness does not generally imply or be implied by the supermodular dispersion, although there can be some link between the two criteria in a binary case in the sense that there are only binary signal realizations. In addition, we provide a necessary condition of the dispersive order for discrete random variables as well as a characterization for the case with binary random variables. Moreover, we consider Persico's accuracy criterion, with which we construct a similar criterion. And the result shows that the kind of accuracy criteria is possible to be connected with the dispersion of conditional expectations (in term of Ganuza and Penalva's supermodular criterion).

As for the literature on information criteria, it dates back to Blackwell (1951, 1953), where he introduces the informativeness criterion by considering two statistical experiments. It follows the intuition that the statistic from a more informative experiment should be more sufficient. Therefore, Blackwell's informativeness criterion is also referred to as the sufficiency criterion. In a more economic context, it means that more informativeness always provides a higher value to decision makers regardless of the decision problems in question or prior beliefs that they share. That is, all the utility maximizers prefer a more informative system.

Although large literature follows the Blackwells informativeness criterion[1], alternative criteria have also been introduced. For example, Lehmann (1988) considers information structures with monotone likelihood ratio property (MLRP), which indicates that higher signals are more favorable. He proposed the so-called effectiveness criterion from the intuition that better information should be more correlated with the true state. Following the same intuition, Persico (2000) formalizes Lehmanns criterion and call it accuracy criterion. Moreover, Kim (1995) points out the Blackwell's criterion is too restrictive and provides a so-called efficiency criterion with the consideration of a agency model.

More recently, Jewitt (2007) relates the Blackwell's criterion with Lehmann's and extends Lehmann's results in a general single crossing preference (SCP) case, and Quah and Strulovici (2009) even generalize the results to the cases where SCP does not hold. Even closer to the purpose of this paper, Brandts *et al.* (2014) provide different information criteria based on the joint density of signals and underlying states, which imply the dispersion properties of the conditional expectations.

This paper focuses exclusively on the supermodular precision. In fact, due to the intrinsic property of the dispersive order, which the supermodular precision bases on, the connection between this criterion and traditional criteria can hardly

[1] See Le Cam (1964), Ponssard (1975), Crémer (1982), Schlee (2001), Eckwert and Zilcha (2003), etc.

be built up. We see that the dispersive order, and thus the supermodular precision criterion, require highly on the prior distributions, which other criteria usually do not need. Therefore, a less demanding criterion by losing the harsh restrictions on the priors may provide more possibility of linking the dispersion of posteriors and other information criteria.

In this paper, we start with reviewing the most essential concepts in section 2.2. Then, in section 2.3, we examine the relationship between Blackwell's informativeness criterion and the supermodular precision criterion for discrete cases. In section 2.4, we provide conditions which can link Perciso's accuracy criterion with the supermodular precision criterion. Finally, section 2.5 concludes this chapter.

2.2 Informativeness and dispersive orders

2.2.1 Information systems

In the setting of statistical decision theory, uncertainty can be characterized by a random state of nature. As stated before, a decision maker takes actions before knowing the true state but after observing a signal which is correlated to the true state. We further assume that the agent has a prior probability distribution of the state of nature, then he can infer additional information from the signal and revise his belief about the true state via Bayes' Rule. In this context, an information system is used to formalize how the signals are generated by the true state.

Definition 2.1. *The triplet (Ω, Y, F) is defined as an information system, where Ω is the set of states, Y is the set of signals, and F is a stochastic transformation from Ω to Y, represented by the conditional density function $f(y|\omega)$, for any $y \in Y$ and $\omega \in \Omega$.*

In the finite discrete case, suppose that there are N possible states, i.e. $\Omega = \{\omega_1, ..., \omega_N\}$, and M different signal values, i.e. $Y = \{y_1, ..., y_M\}$. Then, the transformation F can be represented by a $N \times M$ matrix with the entry f_{ij} showing the probability of generating a signal y_j under the state ω_i, where $i \in \{1, 2, ..., N\}$ and $j \in \{1, 2, ..., M\}$. Note that $\sum_j f_{ij} = 1$ for any i (and similarly, we also have $\int_Y f(y|\omega)dy = 1$).

In fact, for a certain decision problem, an information system is sometimes described with another term - the information structure, which mainly refers to the joint distributions of the signals and the states, or more specifically, the family of probability density functions (or distributions) of the signals conditional on the state of nature. So we may also use simply the transformation F to indicate the information system (Ω, Y^F, F) when there is no ambiguity.

Then, for a given prior $\pi(\cdot)$ defined on Ω, agents can update their beliefs; the posterior belief is given by:

$$\nu(\omega|y) = \frac{f(y|\omega)\pi(\omega)}{\mu(y)}, \quad \forall \omega \in \Omega, \forall y \in Y, \tag{2.1}$$

where $\mu(y) = \int_\Omega f(y|\omega')\pi(\omega')d\omega'$. Decisions then will be made according to the posterior belief $\nu(\omega|y)$.

The availability of additional information allows agents to better react to the risky environment, which leads naturally to the consideration of the value of information conveyed in the signals, or of a certain information system (the way how signals are related to the true states). Consider an expected utility maximizer with a utility function $u(a, \omega)$, where $a \in A$ is the action the agent takes. After receiving a signal y, the agent chooses the optimal action $a^*(y)$ which solves the maximization problem $max_{a \in A} \int_{\omega \in \Omega} u(a, \omega)\nu(\omega|y)d\omega$. And the value of the information is

$$V := \int_{y \in Y} \mu(y)\Big(\int_{\omega \in \Omega} u(a^*(y), \omega)\nu(\omega|y)d\omega\Big)dy. \tag{2.2}$$

That is, the value of information is the agent's ex ante expected utility from an optimal chosen decision rule. Then, it is feasible to compare two different information systems for a given decision problem.

2.2.2 The preference over different signals

In the decision problem with uncertainty, the decision is made based on the posterior distribution after a certain signal is obtained. Thus, we may ask a question - which signal does a decision maker prefer? It is natural to assume that a decision maker will infer a higher expected state or a higher expected utility from the observation of a signal with higher value, and thus prefer a higher signal. Actually, this intuitive assumption can be ensured by the following property.

Definition 2.2. *The densities $\{f(y|\omega)\}_{y \in Y, \omega \in \Omega}$ have the monotone likelihood ratio property (MLRP)*[2]*, if for every $y' > y$ and $\omega' > \omega$, we have*

$$f(y'|\omega')f(y|\omega) - f(y'|\omega)f(y|\omega') \geq 0. \tag{2.3}$$

[2]In our case, we may also directly say that an information system satisfies MLRP.

Note that $\frac{f(y'|\omega)}{f(y|\omega)}$ is the ratio of the densities between receiving a higher signal realization and a low one in a certain state. And the definition of MLRP states that this ratio is greater in a larger state ω' compared to that in a lower state ω. According to Milgrom (1981), the family of densities $\{f(y|\omega)\}_{y\in Y,\omega\in\Omega}$ has the MLRP if, and only if, $y' > y$ implies that the posterior distribution $\nu(\cdot|y')$ dominates the posterior distribution $\nu(\cdot|y)$ in the sense of first-order stochastic dominance for every non-degenerate prior distribution $\pi(\cdot)$, which is also called that the signal y' is *more favorable* than the signal y. By the definition of first-order stochastic dominance, we can easily get that a higher signal is thus related to a higher underlying state.

The MLRP seems natural and it is not difficult to be satisfied by relabeling the states and signals while keeping the correlations between them unchanged. In the rest of this paper, we consider information systems which have the MLRP if without any other specification.

2.2.3 Blackwell's informativeness

When there is more than one way of obtaining information, i.e. more than two information systems available, we would consider the problem of how to choose a better one. For this purpose, various criteria have been proposed during the past few decades in order to evaluate two information systems in terms of the informativeness of the generated signals. One classical criterion is developed by Blackwell (1953) which follows the intuition that a more informative system should have a higher value regardless of the decision problems in question or prior beliefs that the decision makers share. That is, all the utility maximizers prefer a more informative system. On the other hand, a more informative system should be statistically sufficient for a less informative one, which means that a less informative system can be duplicated from a more informative one by adding some random transmission error. Actually, Blackwell shows that the two ideas coincide with each other. Following the second idea, more informativeness can be formally defined as below[3].

Definition 2.3. *Let* (Ω, Y^F, F) *and* (Ω, Y^G, G) *be two information systems. Then* (Ω, Y^F, F) *is said to be more informative than* (Ω, Y^G, G)*, denoted by* $F \succ_i G$*, if there exists a stochastic transformation* Γ *from* Y^F *to* Y^G *represented by a stochastic*

[3]See Marschak and Miyasawa (1968), where the term "garbling" is used for this stochastic signal transformation process.

density kernel γ *such that for all* $\omega \in \Omega$ *and* $y^G \in Y^G$*, it holds true that*

$$g(y^G|\omega) = \int_{y^F \in Y^F} \gamma(y^G, y^F) f(y^F|\omega) dy^F. \tag{2.4}$$

For the finite case, information systems can be represented with matrices, thus Definition 2.3 can be modified by replacing equation (2.4) with the following condition:

$$G = F\ \Gamma \tag{2.5}$$

where Γ is a stochastic matrix.

As a matter of fact, a more informative system, as defined above, is indeed preferred by every expected utility maximizer, i.e. $V^F \geq V^G$ for all utility functions u, as stated in the well-known Blackwell's Theorem.

2.2.4 Orders of dispersion

Intuitively, signals containing more useful information have a stronger impact on posterior distributions, which may in turn lead to more dispersive conditional expectations. Thus, Ganuza and Penalva (2010) introduce a new kind of criteria for evaluating different information systems by applying different stochastic orders to conditional expectations of the states, which they call the precision criteria. Two criteria - the supermodular precision and the integral precision - based on two different orders are discussed in their study. For instance, the supermodular precision is defined on the basis of the dispersive order, which is also one of most important dispersion criterion we will focus on later.

Definition 2.4. $\tilde{x}$ *and* $\tilde{z}$ *are two real-valued random variables with distributions* F_x *and* G_z*, respectively. Then,* $\tilde{x}$ *is said to be greater than* $\tilde{z}$ *in the dispersive order (denoted by* $\tilde{x} \geq_{disp} \tilde{z}$*), if for any* $0 < p \leq q < 1$*,*

$$F_x^{-1}(q) - F_x^{-1}(p) \geq G_z^{-1}(q) - G_z^{-1}(p) \tag{2.6}$$

where F_x^{-1} *and* G_z^{-1} *are quantile functions of* $\tilde{x}$ *and* $\tilde{z}$*. That is,* $F_x^{-1}(p) = inf\{x|F_x(x) \geq p\}$ *and* $G_z^{-1}(p) = inf\{z|G_z(z) \geq p\}$ *for any* $p \in [0, 1]$*.*

Rearranging equation (2.6), we can directly get the following characterization of the dispersive order.

Lemma 2.1. $\tilde{x} \geq_{disp} \tilde{z}$ *if, and only if,* $F_x^{-1}(p) - G_z^{-1}(p)$ *is increasing in* $p \in (0, 1)$*.*

From the definition, the dispersive order requires that the jump of the quantile function of the two random variables coincide with each other in the discrete case. Formally, we have

Lemma 2.2. *Let $\tilde{x}$ and $\tilde{z}$ be two discrete random variables with the support $\{x_1, ..., x_n\}$ and $\{z_1, ..., z_n\}$, respectively, where $x_1 < ... < x_n$ and $z_1 < ... < z_n$. Then, $\tilde{x} \geq_{disp} \tilde{z}$ implies that $Pr(\tilde{x} = x_i) = Pr(\tilde{z} = z_i)$ for any $i \in \{1, ..., n\}$.*

Proof. See appendix.

From the above lemma, we see that the dispersive order exposes relatively strict restrictions on the probability distributions of the two random variables, if they are discrete and have the same number of possible realizations. Note that if the two random variables have the same support, then they follow the same distribution. We may refer to this necessary condition of the dispersive order as the *equal probability condition*.

Now we can introduce Ganuza and Penalva's criterion based on the dispersive order to describe the dispersion property of the conditional expectations. The intuition is that a more informative signal should provide more effective revisions for updating the belief, i.e., having more influence on the posteriors and thus the conditional expectations. When the signal is fully uninformative, the expectation conditional on signal realizations will equal to the unconditional expectation for any signal realization; while when signals can effectively revise posterior belief, the conditional expectation varies a lot with different signal realizations. Hence, it is natural to link the precision of signals with the dispersion of the expectation conditional on signals.

Let $\tilde{\omega}$ be the random variable of the state and $\tilde{y}^F$ and $\tilde{y}^G$ be random variables of the two signals generated from information system (Ω, Y^F, F) and (Ω, Y^G, G), respectively[4]. Then, as in Ganuza and Penalva (2010), we can define the so-called supermodular precision criterion as follows.

Definition 2.5. *An information system (Ω, Y^F, F) is said to be more supermodular precise than another system (Ω, Y^G, G), denoted by $F \succ_{sm} G$, if $E[\tilde{\omega}|\tilde{y}^F] \geq_{disp} E[\tilde{\omega}|\tilde{y}^G]$.*

Roughly speaking, the dispersive order requires necessarily that the more dispersive random variable has a broader support. Thus, in order to compare any

[4]Without any other specification, we will use in the paper the tilde sign to represent the corresponding random variables.

realizations of different signals, an alternative characterization of supermodular precision can be formalized by introducing a transformed signal which is defined as $\tilde{x}^k = F^k(\tilde{y}^k)$, $k \in \{F, G\}$, where F^k is the cumulative distribution function of $\tilde{y}^k$. According to the probability integral transform [5], the new signal is uniformly distributed on the interval [0,1] when F^k is continuous and strict increasing, regardless of the original distribution of signal $\tilde{y}$[6]. With this transformation of variables, we now restate Lemma 1 of Ganuza and Penalva (2010) in the following, which provides another characterization of the supermodular precision criterion.

Lemma 2.3. (Ω, Y^F, F) *is more supermodular precise than* (Ω, Y^G, G) *if*

$$E^F[\tilde{\omega}|x'] - E^F[\tilde{\omega}|x] \geq E^G[\tilde{\omega}|x'] - E^G[\tilde{\omega}|x] \tag{2.7}$$

for any $x, x' \in (0, 1)$ *such that* $x' > x$.

Lemma 2.3 describes the relation between signal precision and the conditional expectations in a more explicit way. It is obvious that the difference between the two conditional expectations, denoted by $\Delta E(x) := E^F[\tilde{\omega}|x] - E^G[\tilde{\omega}|x]$, is monotonically increasing in x. Moreover, the conditional expectation for the more precise information system, $E^F[\tilde{\omega}|x]$, is always steeper than that in the less precise one, $E^G[\tilde{\omega}|x]$, which shows that the conditional expectation is more sensitive to the changes in signal realizations if the system is more precise.

Similar as the supermodular precision criterion, Ganuza and Penalva (2010) also introduce the integral precision which is also defined on conditional expectations but with another stochastic order - the convex order. But in the following of this paper we just focus on the supermodular precision criterion and try to find links between this precision criterion with traditional criteria.

[5]The probability integral transform theorem states that if X is a random variable with continuous cumulative distribution function $F_X(x)$, then $U = F_X(X)$ is uniformly distributed over the interval [0,1].

[6]The transformed signal is uniformly distributed only when the distribution function F is continuous and strictly increasing, yet Lehmann (1988) provides a construction of an equivalent signal, which can solve the problem of the discontinuity of the distribution function.

2.3 Relationship between informativeness and supermodular precision

2.3.1 Binary examples

Example 1. Each project requires an initial investment of 1 and generates a payoff $\tilde{q}$ in the next period. There are two types of projects, namely, low- and high-revenue projects, with payoffs of q_L and q_H, respectively, where $q_L < q_H$. Assume that a fraction π of the projects are of low-revenue, while the high-revenue projects comprise the remaining fraction $1 - \pi$. Assume that $0 < \pi < 1$. The investment decision is made after observing a payoff-related signal $\tilde{y} \in \{y_L, y_H\}$. Let p_t be the probability of receiving signal y_t when the project is of type s, where $t, s \in \{L, H\}$ and $t \neq s$, i.e., $p_t := Pr(y_t|q_s)$. Then, p_t can be interpreted as the "error probability". Furthermore, we assume that $p_L = p_H = p \in [0, \frac{1}{2})$. Intuitively, the larger the error probability p is, the less information the system can provide. Specifically, $p = 0$ indicates the full informativeness of the information system, while $p = \frac{1}{2}$ shows that the system is fully uninformative. Therefore, the information system is characterized by $(\{q_L, q_H\}, \{y_L, y_H, F\})$, where

$$F = \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}$$

.

Lemma 2.4. *For the binary case stated as above, the larger the error probability p is, the smaller the difference between the conditional expected payoffs is. That is, $E[\tilde{q}|y_H] - E[\tilde{q}|y_L]$ is decreasing in p.*

Proof. See appendix.

In another word, Lemma 2.4 shows that

$$E^F[\tilde{q}|y_H] - E^F[\tilde{q}|y_L] > E^{F'}[\tilde{q}|y_H] - E^{F'}[\tilde{q}|y_L], \text{ for any } 0 < p < p' < 1/2, \quad (2.8)$$

where E^F and $E^{F'}$ represent the expectation calculated under the information systems with parameters p and p', respectively. Equation (2.8) exhibits a property of the dispersive order, although, by Lemma 2.2, the necessary condition of $E^F[\tilde{q}|\tilde{y}] \geq_{disp} E^{F'}[\tilde{q}|\tilde{y}]$ requires that the probability of $E^F[\tilde{q}|\tilde{y} = y_L]$ and $E^{F'}[\tilde{q}|\tilde{y} = y_L]$ are equal, which is also equivalent to $Pr^F(\tilde{y} = y_L) = Pr^{F'}(\tilde{y} = y_L)$. Therefore, to draw the conclusion of the dispersion of the conditional expectations we need in addition that

$$\pi(1-p) + (1-\pi)p = \pi(1-p') + (1-\pi)p'.$$

In fact, the equal probability condition together with equation (2.8) is also sufficient to show the dispersive orderings. More generally, we have the following characterization of the dispersive order in a binary case.

Lemma 2.5. *Let $\tilde{x}$ and $\tilde{z}$ be two discrete random variables, with the supports $supp(X) = \{l_x, h_x\}$ and $supp(Z) = \{l_z, h_z\}$, respectively. Then, $\tilde{x} \geq_{disp} \tilde{z}$ if, and only if, (1) $h_x - l_x \geq h_z - l_z$; and (2) $Pr(\tilde{x} = l_x) = Pr(\tilde{z} = l_z)$.*

Proof. See appendix.

Hence, we know that for some prior distribution $p < p'$ implies that $E^F[\tilde{q}|\tilde{y}] \geq_{disp} E^{F'}[\tilde{q}|\tilde{y}]$[7]. That is, the information system F is more supermodular precise than the information system F'. Nevertheless, in order to ecsure condition *(2)* in Lemma 2.5, the only possible prior distribution for this example is $\pi = 1/2$.

Now we consider some general binary information systems.

Example 2 There are two information systems which are characterized by $(\{q_L, q_H\}, \{y_L, y_H\}, F)$ and $(\{q_L, q_H\}, \{y_L, y_H\}, G)$, respectively, where

$$F = \begin{pmatrix} 1-p_1 & p_1 \\ p_2 & 1-p_2 \end{pmatrix} \quad and \quad G = \begin{pmatrix} 1-q_1 & q_1 \\ q_2 & 1-q_2 \end{pmatrix}$$

with $p_1, p_2, q_1, q_2 \in (0,1)$, $p_1 + p_2 < 1$ and $q_1 + q_2 < 1$.

Proposition 2.1. *F and G are two information systems as stated above, with the prior distribution $(\pi, 1-\pi)$. Then, $F \succ_i G$ implies $F \succ_{sm} G$ for the prior $0 < \pi < 1$ such that $Pr^F(\tilde{y} = y_L) = Pr^G(\tilde{y} = y_L)$, i.e. $\pi(1-p_1) + (1-\pi)p_2 = \pi(1-q_1) + (1-\pi)q_2$.*

Proof. (See the appendix.)

Although the informativeness criterion implies the supermodular precision criterion which is defined by the dispersion of the conditional expectations, the implication also depends on the prior, which ensures one of the necessary conditions of the dispersive order.

[7]Moreover, if the investment decision is made on the basis of the expected utilities of the possible payoffs, we can also have the same conclusion that a higher error probability results in a smaller dispersion of conditional expected utilities as long as the utility function is increasing.

2.3.2 General discrete cases

Now we consider a more general case where $\sharp Q = n$ and $\sharp Y = m$, the result becomes unclear. Let $Q = \{q_1, ..., q_n\}$, sorted in increasing order, be a finite set of possible payoffs, with the prior probability $\{\pi_1, ..., \pi_n\}$; $Y = \{y_1, ..., y_m\}$ is the set of signals, which is also sorted in increasing order. Two information systems are identified by the following matrices, F and G, respectively.

$$F = \begin{pmatrix} f_{11} & \cdots & f_{1m} \\ \vdots & & \vdots \\ f_{n1} & \cdots & f_{nm} \end{pmatrix} \quad and \quad G = \begin{pmatrix} g_{11} & \cdots & g_{1m} \\ \vdots & & \vdots \\ g_{n1} & \cdots & g_{nm} \end{pmatrix}.$$

Suppose that F is more informative than G in Blackwell's sense, denoted as $F \succ_i G$. That is, there exists a stochastic matrix $\Gamma = (\gamma_{ij})_{m\times m}$ such that

$$G = F\Gamma \tag{2.9}$$

Actually, the result in Lemma 2.5 can be easily generalized to the finite discrete case. However, the implication between informativeness and the supermodular precision does not hold any more, even for those priors which can ensure the equal probability condition to hold. An counter example is show in the following.

Let

$$F = \begin{pmatrix} 0.9195 & 0.0517 & 0.0288 \\ 0.1053 & 0.2399 & 0.6548 \end{pmatrix}, \; G = \begin{pmatrix} 0.7141 & 0.1974 & 0.0885 \\ 0.2489 & 0.1381 & 0.6130 \end{pmatrix}.$$

We can check that both F and G satisfy MLRP. And for such information systems, there exists a stochastic matrix

$$\Gamma = \begin{pmatrix} 0.7638 & 0.1902 & 0.0459 \\ 0.1043 & 0.4208 & 0.4750 \\ 0.2191 & 0.0261 & 0.7549 \end{pmatrix},$$

such that $G = F\Gamma$. That is, $F \succ_i G$.

However, even for the prior $(0.4113, 0.5887)$, which ensures the same probability of each signal realization, i.e. $Pr^F(y_i) = Pr^G(y_i)$, for any $i \in \{1, 2, 3\}$, we have

$$E^F[\tilde{q}|y_3] - E^F[\tilde{q}|y_2] = 0.1010 < E^G[\tilde{q}|y_3] - E^G[\tilde{q}|y_2] = 0.4082.$$

In fact, the property at the extremes can be kept, while the monotonicity of

$E^F[\tilde{q}|y_s] - E^G[\tilde{q}|y_s]$ with respect to y_s can be possibly violated in the intermediate values when there are more than two signal realization values. The MLRP guarantees the monotone increasing of the conditional expectations in both information system, yet the speed of the increasing can differ, which possibly makes the difference of the conditional expectations fluctuate.

Although the result cannot be duplicated for general information systems, it is true as long as the signal space contains only 2 elements, i.e. $m = 2$. Formally,

Proposition 2.2. *F and G are two information systems with finite states and 2 signals, and the prior distribution is* $(\pi_1, ..., \pi_n)$*. Then,* $F \succ_i G$ *implies* $F \succ_{sm} G$ *for the prior* $(\pi_1, ..., \pi_n)$ *such that* $Pr^F(\tilde{y} = y_i) = Pr^G(\tilde{y} = y_i)$ *for all* $i \in \{1, ..., n\}$.

Proof. See appendix.

That is, as long as there are binary signal realizations, the expected payoffs can still be ranked by the dispersive order even when the set of possible payoffs contains more than two elements. Moreover, all the results can be reproduced for the expected utility of the payoffs, as long as the utility function is strictly increasing.

From the above analysis we see that the equal probability requirement of the dispersive order makes the connection between supermodular precision and Blackwell's informativeness very difficult. However, even when we ignore the requirement on the priors, when the signals have more than realizations, the dispersive property of the conditional expectations may not be obtained. In fact, the precision criteria, which are defined on the conditional expectations instead of on information systems, is already out of the convention. As shown in Brandt *et al.* (2014), these precision criteria do not possess the property that the ordering is invariant to the relabeling of the unknown states.

Still, it is natural for us to think about discarding the equal probability requirement when we consider the discrete cases, yet still keeping the core idea of Ganuza and Penalva (2010).

2.4 Other information criteria and the dispersion of posteriors

Alternative criteria for comparing two different information systems have also been introduced in the literature. For example, Lehmann (1988) considers information systems with MLRP, which is also known as the property of affiliation; for this certain class of decision problems, Lehmann proposed the so-called effectiveness criterion. With MLRP, we have the property that the higher a signal realization is, the higher the underlying state we can infer. As the information system describes the correlation between the signals and the true state, a signal from a better information system should be more correlated with the true state. Persico (2000) formalizes Lehmann's criterion, where the informativeness of an information system is defined in the following way, where the term "accurate" is used.

Adopting Persico's notations, in the following we denote the signal as $\tilde{x}$ and the underlying state as $\tilde{v}$. More specifically, let $\tilde{v}$ be a random variable representing the unknown state with support $\mathcal{V}$. For any $v \in \mathcal{V}$, let $H(v)$ and $h(v)$ be the prior distribution and density functions of V, respectively. Decisions are made after observing a realization of signal $\tilde{x}$ that is related to the unknown state, and the conditional distribution and density of $\tilde{x}$ are denoted by $F(\cdot|v)$ and $f(\cdot|v)$, respectively. Assumed that the signal $\tilde{x}$ is uniformly distributed on the interval $[0, 1]$. That is, the marginal density of $\tilde{x}$, denoted by $\mu(x)$, equals unity for any $x \in [0, 1]$. According to Persico,

Definition 2.6. *Two information structures with signals $\tilde{x}^\theta$ and $\tilde{x}^\eta$ are denoted by I^θ and I^η, respectively, which admit MLRP. Then, information structure I^θ is said to be more accurate than the other I^η, if*

$$T_v(x) := (F^\theta)^{-1}\big(F^\eta(x|v)|v\big) \tag{2.10}$$

is non-decreasing in v, for every x.

The function $T_v(\cdot)$ can be seen as a transformation of the signals, and the increasing property of T_v means that the transformation adds more correlation on the less informative signal $\tilde{x}^\eta$ and yields a more accurate signal $\tilde{x}^\theta$.

Clearly, equation (2.10) is equivalent to

$$F^\theta(T_v(x)|v) = F^\eta(x|v). \tag{2.11}$$

Inspired by Persico's criterion, we now consider two information structures with signals $\tilde{x}^\theta$ and $\tilde{x}^\eta$, respectively, which satisfy the following property:

$$f^\theta(T_v(x)|\tilde{v} \le v) = f^\eta(x|\tilde{v} \le v), \ \forall v \in \mathcal{V}, \tag{2.12}$$

In the following part of this section, we show the informativeness captured by such a criterion could have some relationship with the dispersion of the expectations conditional on signals. In order to proceed the analysis, it is useful to observe and show that the conditional density of the signal given the state is lower than some value v, $f(\cdot|\tilde{v} \leq v)$, is decreasing for any given v.

Lemma 2.6. *For any given $v \in \mathcal{V}$, $f(x|\tilde{v} \leq v) := \dfrac{\int_{\underline{v}}^{v} f(x, v')dv'}{H(v)}$ is decreasing in x, where $f(x, v)$ is the joint density of signals and states.*

Proof. Directly from Lemma 1 in Brandt *et al.* (2014).

Before turning to investigating the links between the accuracy of information and the dispersion of conditional expectations, we first show a necessary lemma as follows.

Lemma 2.7. *For any given $v \in \mathcal{V}$, there exists $\bar{x}_v \in (0, 1)$ such that $T_v(\bar{x}_v) = \bar{x}_v$, i.e. $f^{\theta}(\bar{x}_v|\tilde{v} \leq v) = f^{\eta}(\bar{x}_v|\tilde{v} \leq v)$. Moreover, if $T_v'(x) < 1$, we have $T_v(x) > x$, $\forall\ 0 \leq x < \bar{x}_v$ and $T_v(x) < x$, $\forall\ \bar{x}_v < x \leq 1$.*

Proof. For any $v \in \mathcal{V}$, we have

$$\int_0^1 f^{\theta}(x'|\tilde{v} \leq v)dx' = \int_0^1 f^{\eta}(x'|\tilde{v} \leq v)dx' = 1 \tag{2.13}$$

Since $f^{\theta}(\cdot|\tilde{v} \leq v)$ and $f^{\eta}(\cdot|\tilde{v} \leq v)$ are continuous, by the mean value theorem for integration, there must exist an $\bar{x}_v \in (0, 1)$ such that $f^{\theta}(\bar{x}_v|\tilde{v} \leq v) = f^{\eta}(\bar{x}_v|\tilde{v} \leq v)$. That is, $T_v(\bar{x}_v) = \bar{x}_v$.

Let $\tilde{T}_v(x) := T_v(x) - x$, then $\tilde{T}_v'(x) = T_v'(x) - 1 < 0$ since $T_v'(x) < 1$. That is, $\tilde{T}_v(x)$ is decreasing in x. Note that $\tilde{T}_v(\bar{x}_v) = T_v(\bar{x}_v) - \bar{x}_v = 0$, thus we have

$$\tilde{T}_v(x) > \tilde{T}_v(\bar{x}_v) = 0, \qquad \text{for any } 0 \leq x < \bar{x}_v. \tag{2.14}$$

$$\tilde{T}_v(x) < \tilde{T}_v(\bar{x}_v) = 0, \qquad \text{for any } \bar{x}_v < x \leq 1. \tag{2.15}$$

■

Moreover, taking derivative of equation (2.12) with respect to x yields

$$\frac{\partial}{\partial x} f^{\theta}(T_v(x)|\tilde{v} \leq v)\ T_v'(x) = \frac{\partial}{\partial x} f^{\eta}(x|\tilde{v} \leq v). \tag{2.16}$$

From Lemma 2.6 we know both $\frac{\partial}{\partial x} f^{\theta}(\cdot|\tilde{v} \leq v))$ and $\frac{\partial}{\partial x} f^{\eta}(\cdot|\tilde{v} \leq v))$ are negative, which implies $T_v'(x) > 0$.

Now we can get the link between condition (2.12) and the dispersion of the conditional expectations.

Proposition 2.3. *Suppose that for any given $v \in \mathcal{V}$, $\frac{\partial^2 f^\theta(x|\tilde{v}\le v)}{\partial x^2} > 0$ for any $0 \le x < \bar{x}_v$ and $\frac{\partial^2 f^\theta(x|\tilde{v}\le v)}{\partial x^2} < 0$ for any $\bar{x}_v < x \le 1$. Then, condition (2.12) with $T'_v(x) < 1$ implies that $E^\theta[\tilde{v}|\tilde{x}^\theta]$ is more dispersive than $E^\eta[\tilde{v}|\tilde{x}^\eta]$.*

Proof. Note that $0 < T'_v(x) < 1$ for any $v \in \mathcal{V}$, then from equation (2.16) we have

$$\frac{\partial}{\partial x} f^\theta(T_v(x)|\tilde{v} \le v) < \frac{\partial}{\partial x} f^\eta(x|\tilde{v} \le v). \tag{2.17}$$

Now consider the case where $0 \le x < \bar{x}_v$. By Lemma 2.7, we have $T_v(x) > x$. Therefore,

$$\frac{\partial}{\partial x} f^\theta(T_v(x)|\tilde{v} \le v) > \frac{\partial}{\partial x} f^\theta(x|\tilde{v} \le v), \tag{2.18}$$

since $\frac{\partial^2 f^\theta(x|\tilde{v}\le v)}{\partial x^2} > 0$.

When $\bar{x}_v < x \le 1$, $T_v(x) < x$, and the property of the second order derivative of $f^\theta(x|\tilde{v} \le v)$ implies the same as equation (2.18). Therefore, by equations (2.17) and (2.18), we have

$$\frac{\partial}{\partial x} f^\theta(x|\tilde{v} \le v) < \frac{\partial}{\partial x} f^\eta(x|\tilde{v} \le v), \quad \text{for any } x \in [0,1]. \tag{2.19}$$

Equivalently,

$$\left|\frac{\partial}{\partial x} f^\theta(x|\tilde{v} \le v)\right| > \left|\frac{\partial}{\partial x} f^\eta(x|\tilde{v} \le v)\right|, \quad \text{for any } x \in [0,1], \tag{2.20}$$

which shows that the graph of $f^\theta(x|\tilde{v} \le v)$ is steeper than that of $f^\eta(x|\tilde{v} \le v)$ on the whole interval of $[0,1]$.

For any $x \in [0,1]$, the expectation of $\tilde{v}$ conditional on signal $\tilde{x}^k = x$ is given by

$$\begin{aligned} E^k[\tilde{v}|\tilde{x}^k = x] &= \int_{\mathcal{V}} v f^k(v|x)dv = \int_{\mathcal{V}} v dF^k(v|x) = vF^k(v|x)\Big|_{\underline{v}}^{\bar{v}} - \int_{\mathcal{V}} F^k(v|x)dv \\ &= \bar{v} - \int_{\mathcal{V}}\int_{\underline{v}}^{v} f^k(v'|x)dv'dv = \bar{v} - \int_{\mathcal{V}}\int_{\underline{v}}^{v} f^k(x,v')dv'dv. \end{aligned}$$

where $k \in \{\theta, \eta\}$.

Denote $\Delta(x) := E^\theta[\tilde{v}|\tilde{x}^\theta = x] - E^\eta[\tilde{v}|\tilde{x}^\eta = x]$, then we get

$$\begin{aligned}\Delta(x) &= \int_{\mathcal{V}} \int_{\underline{v}}^{v} \big(f^\eta(x,v') - f^\theta(x,v')\big) dv' dv \\ &= \int_{\mathcal{V}} H(v) \ \Big(\ \frac{\int_{\underline{v}}^{v} f^\eta(x,v')dv'}{H(v)} - \frac{\int_{\underline{v}}^{v} f^\theta(x,v')dv'}{H(v)} \ \Big) \ dv \\ &= \int_{\mathcal{V}} H(v) \ \big(\ f^\eta(x|\tilde{v} \leq v) - f^\theta(x|\tilde{v} \leq v) \ \big) \ dv \qquad (2.21)\end{aligned}$$

Thus, we can see that equation (2.19) provides a sufficient condition for $\Delta'(x) > 0$, which means $E^\theta[\tilde{v}|x] - E^\eta[\tilde{v}|x]$ is increasing in x. Therefore, we have shown that $E^\theta[\tilde{v}|\tilde{x}^\theta]$ is more dispersive than $E^\eta[\tilde{v}|\tilde{x}^\eta]$. ■

2.5 Conclusion

With looking into the intrinsic property of the dispersive order, this paper focuses on the supermodular precision criterion, which is defined on the dispersion of the expectation conditional on signal realizations. We attempt to link this precision criterion with other information criterion in this chapter. Yet in fact, the dispersive order requires require highly on the prior distributions in the discrete cases. For example, when the random variables have the same number of realizations, the dispersive order requires what we call the equal probability condition. This requirement on priors is usually not needed for other criteria, although it could be solved by considering the normalization of signals. Even so, the connection between the supermodular precision criterion and Blackwell's informativeness can hardly be built up except for the binary case. As long as there are more than two signal realizations, the monotonicity of the difference between the two conditional expectations can not be kept. And in the last section of this paper, another criterion which is based on Persico's criterion, with the normalization of signals in a continuous case, is taken into account, yet it is still difficult to see a clear and interpretable link to the dispersion of conditional expectations. For the same purpose, some new criteria are proposed and thoroughly discussed in Brandt *et al.* (2014), where more formulations of dispersion are considered and more links are presented.

In this paper we see that Blackwell's informativeness criterion does not necessarily imply or be implied by the dispersion of conditional expectations in general cases, while in the monotone decision problems the relationship between dispersion of conditional expectations and some informativeness criterion, which is defined similar as accuracy criterion, could be built up under some conditions. This could provide theoretical basis for applying the dispersive orders directly to decision analysis. Actually, this is just a preliminary study in this topic, and more informativeness and dispersion criteria are under investigation.

Appendix 2.A Proofs of propositions

Proof of Lemma 2.2. Let F_x and G_z be the distribution functions of $\tilde{x}$ and $\tilde{z}$, respectively. Let $P_i := F_x(x_i)$ and $Q_i := G_z(z_i)$ for any $i \in \{1, 2, ..., n\}$.

By contradiction, we suppose there exists some $k \in \{1, 2, ..., n\}$ such that $Pr(\tilde{x} = x_k) \neq Pr(\tilde{z} = z_k)$ and $Pr(\tilde{x} = x_i) = Pr(\tilde{x} = z_i)$ for any $i < k$. That is, k is the smallest index where there are unequal probabilities. Hence, we have $P_i = Q_i$ for any $i < k$ and $P_k \neq Q_k$.

If $P_k > Q_k$, let $\alpha = Q_k$ and $\beta = P_k$, thus $\alpha, \beta \in (0, 1)$ with $\alpha < \beta$. And we have

$$F_x^{-1}(\beta) - F_x^{-1}(\alpha) = x_k - x_k = 0$$

and we know that

$$G_z^{-1}(\beta) - G_z^{-1}(\alpha) \geq z_{k+1} - z_k > 0,$$

Note that $F_x^{-1}(\alpha) = x_k$, since $P_{k-1} = Q_{k-1} < Q_k$, i.e. $F_x(x_{k-1}) < \alpha$. And the inequality comes from $G_z^{-1}(\beta) \geq z_{k+1}$. Therefore, we have $F_x^{-1}(\beta) - F_x^{-1}(\alpha) < G_z^{-1}(\beta) - G_z^{-1}(\alpha)$, which contradicts $\tilde{x} \geq_{disp} \tilde{z}$ by Definition 2.4.

If $P_k < Q_k$, we first consider the case $P_k < Q_k < P_{k+1}$. Take $\alpha = Q_k$ and $\beta = P_{k+1}$, then we have $F_x^{-1}(\alpha) = F_x^{-1}(\beta) = x_{k+1}$, while $G_z^{-1}(\alpha) = z_k$ and $G_z^{-1}(\beta) = z_{k+1}$. Therefore, we have $F_x^{-1}(\beta) - F_x^{-1}(\alpha) = 0 < G_z^{-1}(\beta) - G_z^{-1}(\alpha)$ which forms the same contradiction as above.

When $P_k < P_{k+1} \leq Q_k$, there must exist some $l > k + 1$ such that $P_{l-1} < Q_{l-1} < P_l$ since $P_n = Q_n = 1$. Take $\alpha = Q_{l-1}$ and $\beta = P_l$, then we have $F_x^{-1}(\alpha) = F_x^{-1}(\beta) = x_l$, while $G_z^{-1}(\alpha) = z_{l-1}$ and $G_z^{-1}(\beta) = z_l$. We get the same contradiction as before. ■

Proof of Lemma 2.4. Note that the probability of receiving a low signal is given by

$$Pr(y_L) = Pr(q_L)Pr(y_L|q_L) + Pr(q_H)Pr(y_L|q_H) = \pi(1 - p) + (1 - \pi)p,$$

then we have

$$Pr(q_L|y_L) = \frac{Pr(q_L)Pr(y_L|q_L)}{Pr(y_L)} = \frac{\pi(1-p)}{\pi(1-p)+(1-\pi)p}$$

and

$$Pr(q_H|y_L) = \frac{Pr(q_H)Pr(y_L|q_H)}{Pr(y_L)} = \frac{(1-\pi)p}{\pi(1-p)+(1-\pi)p}.$$

Thus, the conditional expectation of the payoff is

$$\begin{aligned} E[\tilde{q}|y_L] &= Pr(q_L|y_L)q_L + Pr(q_H|y_L)q_H \\ &= \frac{\pi(1-p)}{\pi(1-p)+(1-\pi)p}q_L + \frac{(1-\pi)p}{\pi(1-p)+(1-\pi)p}q_H. \end{aligned}$$

Similarly, the expected utility of the payoff conditional on the high signal is given by

$$E[\tilde{q}|y_H] = \frac{\pi p}{\pi p+(1-\pi)(1-p)}q_L + \frac{(1-\pi)(1-p)}{\pi p+(1-\pi)(1-p)}q_H.$$

(It is obvious that $E[\tilde{q}|y_L] = q_L$ and $E[\tilde{q}|y_H] = q_H$ when the error probability $p = 0$ - the information system is fully informative, while when $p = \frac{1}{2}$ the two conditional expectations are the same and equal to the unconditional expectation, i.e. $E[\tilde{q}|y_L] = E[\tilde{q}|y_H] = \pi q_L + (1-\pi)q_H$.)

Taking derivatives of the conditional expected utilities of payoffs with the error probability p yields that

$$\begin{aligned} \frac{dE[u(\tilde{q})|y_L]}{dp} &= \frac{\pi(\pi-1)u(q_L)+\pi(1-\pi)u(q_H)}{[\pi(1-p)+(1-\pi)p]^2} \\ &> \frac{\pi(\pi-1)u(q_L)+\pi(1-\pi)u(q_L)}{[\pi(1-p)+(1-\pi)p]^2} = 0 \end{aligned}$$

where the inequality follows the fact that u is increasing and $q_H > q_L$. Similarly, we have

$$\frac{dE[u(\tilde{q})|y_H]}{dp} = \frac{\pi(1-\pi)u(q_L)+\pi(\pi-1)u(q_H)}{[\pi p+(1-\pi)(1-p)]^2} < 0$$

Denote $\Delta E := E[u(\tilde{q})|y_H] - E[u(\tilde{q})|y_L]$, then we get

$$\frac{d\Delta E}{dp} = \frac{dE[u(\tilde{q})|y_H]}{dp} - \frac{dE[u(\tilde{q})|y_L]}{dp} < 0.$$

Therefore, we show that $E[\tilde{q}|y_H] - E[\tilde{q}|y_L]$ is decreasing in p. ■

Proof of Lemma 2.5. Let F_x and G_z be the distribution functions of $\tilde{x}$ and $\tilde{z}$, respectively. First, suppose $\tilde{x} \geq_{disp} \tilde{z}$. (2) is a direct result of Lemma 2.2. Let $Pr(\tilde{x} = l_x) = Pr(\tilde{z} = l_z) = \bar{p}$, we have $\bar{p} \in (0, 1)$. Let $p = \bar{p}$ and $p' = \bar{p} + \varepsilon$, where $\varepsilon \in (0, 1-\bar{p})$. Then, by Definition 2.4, we have $F_x^{-1}(p') - F_x^{-1}(p) \geq G_z^{-1}(p') - G_z^{-1}(p)$. That is, $h_x - l_x \geq h_z - l_z$, and thus (1) is true.

Now suppose (1) $h_x - l_x \geq h_z - l_z$, and (2) $Pr(\tilde{x} = l_x) = Pr(\tilde{z} = l_z)$. We show that condition (2.6) holds true for any $0 < p \leq p' < 1$. For any $\bar{p} < p \leq p' < 1$, we have $F_x^{-1}(p') - F_x^{-1}(p) = h_x - h_x = 0$ and $G_z^{-1}(p') - G_z^{-1}(p) = h_z - h_z = 0$. For any $0 < p \leq p' \leq \bar{p}$, we have $F_x^{-1}(p') - F_x^{-1}(p) = l_x - l_x = 0$ and $G_z^{-1}(p') - G_z^{-1}(p) = l_z - l_z = 0$. For $0 < p \leq \bar{p} < p' < 1$, we have $F_x^{-1}(p') - F_x^{-1}(p) = h_x - l_x$ and $G_z^{-1}(p') - G_z^{-1}(p) = h_z - l_z$. According to (1), we have $F_x^{-1}(p') - F_x^{-1}(p) \geq G_z^{-1}(p') - G_z^{-1}(p)$. Thus, condition (2.6) holds true for any $0 < p \leq p' < 1$, and $\tilde{x} \geq_{disp} \tilde{z}$.

■

Lemma 2.8.

$$F \succeq^i G \iff p_1(1-q_2) \leq (1-p_2)q_1 \quad \textit{and} \quad p_2(1-q_1) \leq (1-p_1)q_2.$$

The proof of the above lemma follows directly from the definition of Blackwell's informativeness, and the proof of Proposition 2.1 will need this above lemma.

Proof of Proposition 2.1. As before, it is sufficient to show that the result holds true for the conditional expected utilities of the payoffs when the utility function is increasing. The conditional expectations of the utilities in the more informative information system are given by

$$E[u(\tilde{q})|y_L] = \frac{\pi(1-p_1)u(q_L) + (1-\pi)p_2u(q_H)}{\pi(1-p_1) + (1-\pi)p_2}$$

$$E[u(\tilde{q})|y_H] = \frac{\pi p_1 u(q_L) + (1-\pi)(1-p_2)u(q_H)}{\pi p_1 + (1-\pi)(1-p_2)}$$

For the less informative one, we have

$$\bar{E}[u(\tilde{q})|y_L] = \frac{\pi(1-q_1)u(q_L) + (1-\pi)q_2u(q_H)}{\pi(1-q_1) + (1-\pi)q_2}$$

$$\bar{E}[u(\tilde{q})|y_H] = \frac{\pi q_1 u(q_L) + (1-\pi)(1-q_2)u(q_H)}{\pi q_1 + (1-\pi)(1-q_2)}$$

Thus, we have the difference of the expected utilities conditional on signal y_L in the two different information systems

$$
\begin{aligned}
&E[u(\tilde{q})|y_L] - \bar{E}[u(\tilde{q})|y_L] \\
&\qquad = \frac{\pi(1-p_1)u(q_L) + (1-\pi)p_2 u(q_H)}{\pi(1-p_1) + (1-\pi)p_2} - \frac{\pi(1-q_1)u(q_L) + (1-\pi)q_2 u(q_H)}{\pi(1-q_1) + (1-\pi)q_2} \\
&\qquad = \frac{\pi(1-\pi)[u(q_H) - u(q_L)][p_2(1-q_1) - (1-p_1)q_2]}{[\pi(1-p_1) + (1-\pi)p_2][\pi(1-q_1) + (1-\pi)q_2]} \leq 0
\end{aligned}
$$

since $p_2(1-q_1) \leq (1-p_1)q_2$, which, according to Lemma 2.8, derives from the assumption that $F \succeq^i \bar{F}$. Thus, we have $E[u(\tilde{q})|y_L] \leq \bar{E}[u(\tilde{q})|y_L]$. Similarly, the first inequality of the equivalence condition in Lemma 2.8 yields that

$$
E[u(\tilde{q})|y_H] - \bar{E}[u(\tilde{q})|y_H] = \frac{\pi(1-\pi)[u(q_H) - u(q_L)][(1-p_2)q_1 - p_1(1-q_2)]}{[\pi p_1 + (1-\pi)(1-p_2)][\pi q_1 + (1-\pi)(1-q_2)]} \geq 0
$$

That is, $E[u(\tilde{q})|y_H] \geq \bar{E}[u(\tilde{q})|y_H]$. In addition, we get

$$
\bar{E}[u(\tilde{q})|y_L] - \bar{E}[u(\tilde{q})|y_H] = \frac{\pi(1-\pi)[u(q_H) - u(q_L)](q_1 + q_2 - 1)}{[\pi(1-q_1) + (1-\pi)q_2][\pi q_1 + (1-\pi)(1-q_2)]} \leq 0
$$

since $q_1 + q_2 \leq 1$. Therefore, we can conclude that $E[u(\tilde{q})|y_L] \leq \bar{E}[u(\tilde{q})|y_L] \leq \bar{E}[u(\tilde{q})|y_H] \leq E[u(\tilde{q})|y_H]$, i.e. the conditional expected utilities in the more informative system are more dispersive compared to the less informative one. ■

Proof of Proposition 2.2. In order to shed some light on the more general discrete cases, we start the proof with the $N \times M$ case. Suppose that $F \succ_i G$. That is, there exists a stochastic matrix $\Gamma = (\gamma_{ij})_{m \times m}$ such that $G = F\Gamma$. Let $u_i = u(q_i)$ be the utility of the payoff for any $i = 1, ..., n$, where u is strictly increasing. Thus, for any signal y_s, $s = 1, 2, ..., m$, the conditional expected utility under the more informative information system F is

$$
E^F[u(\tilde{q})|y_s] = \sum_{i=1}^{n} \frac{\pi_i P(y_s|q_i)}{P^F(y_s)} u(q_i) = \frac{\sum_{i=1}^{n} f_{is}\pi_i u_i}{P^F(y_s)}
$$

where $P^F(y_s) = \sum_{j=1}^{n} \pi_j P^F(y_s|q_j) = \sum_{j=1}^{n} \pi_j f_{js}$.

Similarly, in the less informative information system G, we have

$$
E^G[u(\tilde{q})|y_s] = \frac{\sum_{i=1}^{n} g_{is}\pi_i u_i}{P^G(y_s)}
$$

where $P^G(y_s) = \sum_{j=1}^{n} \pi_j g_{js}$.

Denote $\Delta(y_s) \equiv E^F[u(\tilde{q})|y_s] - E^G[u(\tilde{q})|y_s]$, then we have

$$\begin{aligned}
\Delta(y_s) &= \frac{\sum_{i=1}^{n} f_{is}\pi_i u_i}{P^F(y_s)} - \frac{\sum_{i=1}^{n} g_{is}\pi_i u_i}{P^G(y_s)} \\
&= \frac{\sum_{i=1}^{n} f_{is}\pi_i u_i \sum_{j=1}^{n} \pi_j g_{js} - \sum_{i=1}^{n} g_{is}\pi_i u_i \sum_{j=1}^{n} \pi_j f_{js}}{P^F(y_s)P^G(y_s)} \\
&= \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} f_{is}g_{js}\pi_i\pi_j u_i - \sum_{i=1}^{n}\sum_{j=1}^{n} f_{js}g_{is}\pi_i\pi_j u_i}{P^F(y_s)P^G(y_s)} \\
&= \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} f_{is}g_{js}\pi_i\pi_j (u_i - u_j)}{P^F(y_s)P^G(y_s)} \\
&= \frac{\sum_{i<j} \pi_i\pi_j (u_i - u_j)(f_{is}g_{js} - f_{js}g_{is})}{P^F(y_s)P^G(y_s)} \qquad (2.22)
\end{aligned}$$

The fourth equation is obtained by swapping the indexes i and j and exchanging the summations. Since $\pi_i > 0$, $\pi_j > 0$, $P^F(y_s) > 0$, $P^G(y_s) > 0$ and $u_i - u_j < 0$, the sign of (2.22) depends on the term $f_{is}g_{js} - f_{js}g_{is}$. Since we know $G = F\Gamma$, we can have

$$f_{is}g_{js} - f_{js}g_{is} = f_{is}\sum_{k=1}^{m} f_{jk}\gamma_{ks} - f_{js}\sum_{k=1}^{m} f_{ik}\gamma_{ks} = \sum_{k\neq s}(f_{is}f_{jk} - f_{ik}f_{js})\gamma_{ks}.$$

The MLRP of F shows that $f_{is}f_{jk} - f_{ik}f_{js}$ could be either positive, if $k > s$, or negative, if $k < s$. Yet in the cases where $m = 2$, we can confirm that $\Delta(y_1) < 0$ and $\Delta(y_2) > 0$. That is, the expected utility of payoff is more disperse under a more informative system even when the set of possible payoffs contains more than two elements. However, the result seems unclear when the signal space expands ($m > 2$). In fact, the property at the extremes can be kept, while the monotonicity of $E^F[u(\tilde{q})|y_s] - E^G[u(\tilde{q})|y_s]$ with respect to y_s can be possibly violated.

■

Chapter 3

Endogenous information revelation in a competitive credit market and credit crunch

Coauthored with Professor Bertrand Wigniolle

Abstract

This paper focuses on the role of dissipative signal in a competitive credit market with asymmetric information. Borrowers can resort to costly certification, which signals the quality of their projects; signaling is costly and borne by the borrowers. This has the effect of easing the information disadvantage on the side of lenders for their making decision on loan issuance. Besides the finding of equilibria in a one-shot economy, we spot a close relationship between the opacity of credit market and the fundamental funding cost, i.e. the risk-free interest rate. To further examine its dynamic interaction with the market degree of opacity, the interest rate is endogenized by extending the model to an OLG context. We show that the market is likely to converge to either an opaque or a transparent stationary state, and more interestingly, for some configurations of parameters there exist permanent oscillations between two different regimes, which provides us a theoretical support of the (in)stability of the credit market and indicates a possible way of explaining the credit crunch during the financial crisis.

3.1 Introduction

We had gone through the vast financial instability during the global financial crisis of 2007-2008, which was primarily driven by mortgage-backed securities. The foundations laid in the pre-crisis period are the low interest rate environment and a decline in lending standards. Structured investment vehicles were booming and the extent of securitization brought excessive opacity into the financial market. Yet as the turmoil burst, the capital of financial institutions eroded and, at the same time, lending standards tightened. Even with injections of liquidity from central banks, banks started hoarding funds and became reluctant even to lend to each other[1]. A credit crunch was spread through economic agents, and the tightening of credit led to contractions in the real economy - asset prices dropped, unemployment increased, and growth of outputs bogged down[2].

The information asymmetry between borrowers and lenders could be one reason to explain the inappropriate lending. Having the information advantage, borrowers may resort to a costly way of revealing their information in order to increase the chance of being financed. With this key assumption of costly information disclosure, we are able to represent the existence of different equilibria, and even the existence of multiple equilibria. Moreover, we are able to observe from the model that, unqualified borrowers may also be financed if they keep opaque and when the credit market is relatively loose. Thus, the information asymmetry and the costly information revealing could be a possible way of explaining the credit crunch we experienced and the volatility of the credit market.

In this paper, we consider a signaling problem in competitive credit market. In our model, borrowers, who seek for funding to finance their projects, have private information about the return on their own projects, and they are offered with the option of costly disclosing the information or remaining opaque. Banks, who collect deposits and serve as a lender, charge different interest rates according to the type of the borrowers (opaque or transparent). Endogenizing the interest rate by considering imperfectly elastic credit supply, we show that, according to different values of the safe interest rate, there may exist three types of equilibria - the opaque equilibrium in which all borrowers do not reveal information and are funded, the transparent equilibrium in which only borrowers who own good projects reveal information and are financed, and the multiple equilibria where

[1]Interbank spreads in Europe soared to nearly 200 basis points at the peak in September 2008, while up to 500 basis points in the US, compared to the level around 10 basis points before the crisis (Heider *et al.*, 2009).

[2]See also Brunnermeier (2009), Acharya and Skeie (2011) and so on.

both opaque and transparent equilibria can be possible. Besides, we extend the context to a dynamic framework by endogenizing the interest rate through an OLG model, where savings at period t constitute the credit supply on the credit market at period $t+1$. At each period, the economy can be either in an opaque or a transparent equilibrium. We provide the dynamics of the credit market which is governed by the evolution of two variables - the interest rate and the type of the state. Both opaque and transparent steady states can possibly occur in the long run.

The main result in the static model is that the level of risk-free interest rate determines the type of the equilibrium in the credit market, and thus the opacity of the market. We show that there only exists transparent equilibrium when the safe interest rate is high, while low interest rate leads to opaque equilibrium. More interestingly, when the interest rate is in an intermediate range, there exists possibility of multiple equilibria, which shed some light on the indeterminacy of the credit market. For example, a possible case is that there is an abrupt jump from an opaque equilibrium to a transparent equilibrium, which indicates a situation of credit crunch - a decrease of credit supply together with a decrease of the interest rate. By integrating the basic static model into an OLG framework, we form the dynamics of the safe interest rate, showing that there may exist the convergence to different types of equilibria in the long run; and with some parameter settings, there may even exist cyclical oscillations between opaque and transparent equilibria. The indeterminacy in the static context is spread out in the long run, and we may even have systematically permanent fluctuations in the financial market and as well as in the real economy.

Since the pioneer work of Stiglitz and Weiss (1981), large literature has studied the role of asymmetric information played on determining the credit market equilibria. Lenders lack of information on the relevant characteristics of the borrowers may result in the outcome of underinvestment. Credit is said to be rationed in this sense. Yet, the possibility for the flip side of this story is also extensively studied: the investment level may turn out in excess of the social efficient level. For example, De Meza and Webb (1987) shows that if expected returns on the project could differ, overinvestment may occur under some plausible assumptions on the distribution function of the project return. A recent paper by Alberto and Filippo (2013) reinforces this direction in a dynamic context how the adverse selection fosters a strong boost on investment, capital accumulation and capital inflows. We pursue our study along their lines, but proceed from a different perspective: how are the equilibria impacted by taking into account the borrowers' option of costly revealing the information on their project returns.

As for signaling problems in credit market, studies can be dated back to Leland and Pyle (1977), where they consider a similar problem and assume that signaling through the choice of one's financial structure. And classical treatment on adverse selection problem in the credit market can, for example, be found in Besanko and Thakor (1987), where the principal designs contracts to induce self-selection. More studies can be found in Milde and Riley (1988), Cremer and Khalil (1992), and Tirole (2006). Yet in our model, we simplify the principal (banks)'s decision-making problem without losing the key insights in such literature.

Regarding the study on the macroeconomic dynamics in credit market, similar results are found as in Azariadis and Smith (1998), where they establish the results by considering the production sector and focus on how the capital stock changes and therefore the switches among different equilibria. In our paper we focus more on the credit market and start from the decision of typical borrowers and lenders, which provides more microeconomic foundations.

One feature of our model is to incorporate the decision of dissipative signaling. In the static model with endogenous interest rate, despite the simple setup we choose, the decision of information revelation is endogenized, and so is the market opacity. Unlike the literature with asymmetric information in credit market, borrowers in our model can choose to be transparent or opaque depending on which financial environment they are experiencing - namely, the level of the safe interest rate and the willingness of banks' lending. Thus, we are able to characterize the equilibrium with respect to a safe exogenous market interest rate, and therefore, to address the link between fundamental funding cost and the type of the equilibrium, as well as the market opacity and the aggregate output.

Another feature is that we address the long run fluctuations in addition to the short-term indeterminacy. Therefore, we are able to demonstrate the switches in the type of equilibrium and credit crunch may explain economic fluctuations.

The rest of the paper is organized as follows. Section 3.2 considers the signaling problem in a static model and decision making of the two type agents is analyzed; full characterization of the equilibrium is given in section 3.3; and in section 3.4 we consider endogenized interest rate and show how credit supply affects the market equilibrium and the opacity. Then, in section 3.5 we consider the problem in an OLG model and show the possible stationary states and the transition between different states in the long run. And the final section concludes this chapter.

3.2 The model

3.2.1 Assumption

Consider a credit market populated by two kinds of agents: entrepreneurs and investors. The entrepreneurs, who are also termed as borrowers, need to raise capital for launching their projects, and the investors, also termed as lenders, seek meanwhile to provide loans for them. Both borrowers and lenders are risk-neutral. Each borrower needs to raise 1 unit of fund to proceed with his project, which yields a random return of V. However, borrowers are heterogeneous; the return to the project varies across borrowers and is private information of the owner of the project himself. The borrower knows the true value of the return v that his project will realize[3], while lenders only know the cross-sectional distribution $H(v)$ of the project return V. The associated density function $h(v)$ is positive and continuous for any $v \in [\underline{v}, \bar{v}]$ and zero elsewhere.

Unlike the models in most of the literature, where borrowers who have the private information are all required to report or signal the information (whether truthfully or falsely), borrowers in our model, when facing a certain loan contract proposed by a lender, have the option to choose either to publish information to attract investment or keep silent. Publishing information is costly; the cost is $c > 0$ and borne solely by the borrower. This cost can be regarded as, for example, a cost due to being committed to obtaining certain certificate, being exposed to harsher auditing or monitoring terms, which the borrow has to pay during or at the end of the implement of the project. We call the borrowers who reveal information *transparent* borrowers and those who do not reveal information *opaque* borrowers. The borrowers who choose to be transparent reveal full information on the return to their projects[4] and the cost is paid at the end of the period when the project has been accomplished. Lenders know the exact realization of the return v once the information is released; otherwise, they have no more information on the return other than the distribution of V, $H(v)$.

Lenders (Banks) offer the loans which must be repaid with interest at the end of the period. The repayments required by lenders are different for transparent and opaque borrowers. So lenders propose a loan contract with a pair of repayment

[3] A weaker assumption could be that borrowers have better information than lenders, but not full information, which could be described by using some information criteria. Yet strong assumptions here could also capture the properties that we would like to show in this model.

[4] We could consider the cost as a certification cost, so we could focus on the situation where only truthful revelation is possible.

requirements (R_1, R_2), in which R_1 is the required repayment for opaque borrowers, and R_2 for transparent borrowers[5] [6]. The timeline of the model is summarized in Figure 3.1.

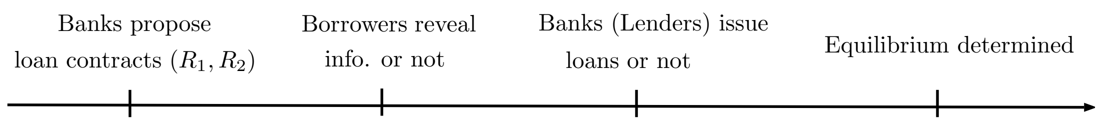

Figure 3.1: Timing of the Decisions.

The repayment R_i is fulfilled only when the realization of the return on the project v, net of the certification cost c, exceeds the corresponding amount. That is, when a bank issues loan to an opaque borrower to finance his project, R_1 can be fulfilled only if $v \geq R_1$; when a bank invests in a transparent project, R_2 can be fully repaid if $v - c \geq R_2$. Otherwise lenders could only collect whatever the projects realize, i.e. v and $v - c$ for opaque or transparent financing, respectively. Borrowers have no initial endowment to be pledged.

Thus, the payoff of a transparent borrower who owns a project with the return v, given that his loan application is approved, is

$$\pi_B^T = max\{v - c - R_2, 0\}. \tag{3.1}$$

And the payoff of an opaque borrower with a project of the return v, if he can obtain the loan, is

$$\pi_B^O = max\{v - R_1, 0\}. \tag{3.2}$$

If the application of loans is rejected, the payoff of either type of the borrowers is zero, i.e. $\pi_B^T = \pi_B^O = 0$.

If $v < R_1$, the borrower obtains a null gain no matter whether he undertakes the project or not. It is assumed in this case that the borrower always undertakes the project if he can get a loan. This assumption can be explained by the fact that the entrepreneur can also get non-market benefits or private outcomes from leading

[5] Assume that the credit is not constrained; a lender will provide 1 unit of loan once she decides to invest - the loan size of all contract is 1, so R_1 also equals to $1 + r_1$, where r_1 is the required interest rate on the loan.

[6] The interests charged by banks (R_1, R_2) are assumed to be independent of the return on the projects, since we consider a competitive credit market in the model and the competition on the market will finally drive the interest rate down to the level with which zero-profit condition is satisfied.

a project, such as gaining experience, extending social networks, and building reputations. As a consequence of this assumption, when there exist opaque projects being financed in an equilibrium, all opaque borrowers get loans in this equilibrium.

Consider the funds are supplied by depositors at a safe interest rate r_0, and denote $R_0 = 1 + r_0$. Then, the profit of a bank, if it offers the loan to a transparent borrower with a project of return v, is

$$\pi_L^T = min\{R_2, v - c\} - R_0. \tag{3.3}$$

The expected profit of a bank, if it provides the loan to an opaque borrower, is

$$\pi_L^O = E[min\{R_1, V\}| \text{ V is opaque }^{7}] - R_0. \tag{3.4}$$

To summarize, we have the expected payoff of both borrowers and lenders shown below in Figure 3.2.

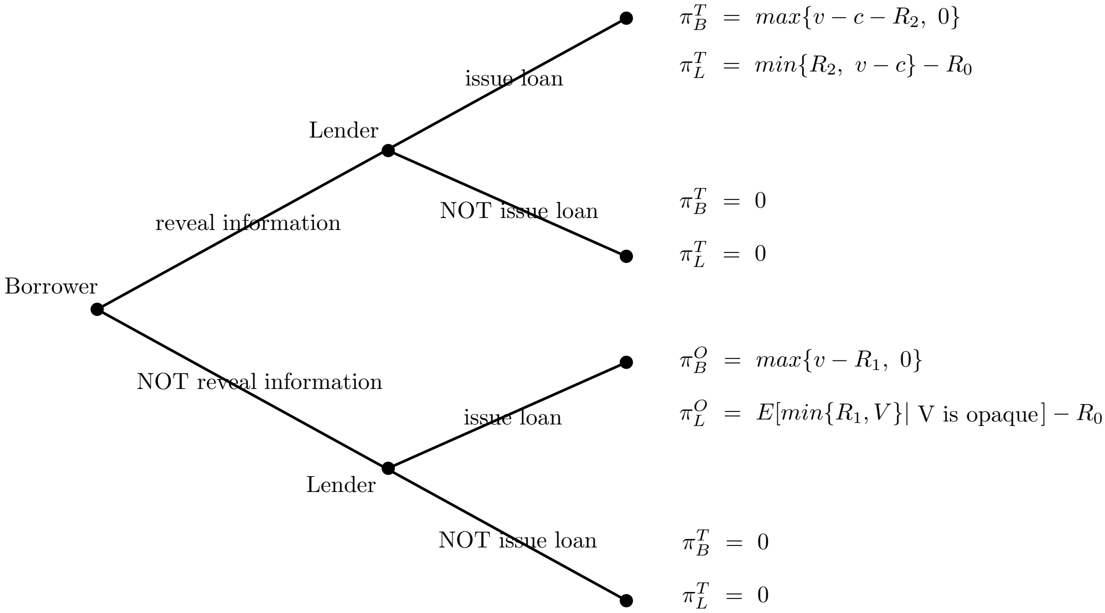

Figure 3.2: Payoffs of Borrowers and Lenders.

[7] "V is opaque" refers to the borrowers who choose not to reveal information.

3.2.2 Optimal decisions

Now we consider the decisions of some typical lender and borrower. The decision made by the borrower of whether or not to reveal information depends on the strategy taken by the lender. Thus, we first take account of the lender's decision. The lender choose to invest in a transparent borrower if and only if $\pi_L^T \geq 0$. That is,

$$min\{R_2, v - c\} - R_0 \geq 0 \quad \Leftrightarrow \quad R_2 \geq R_0 \text{ and } v \geq R_0 + c. \tag{3.5}$$

As for the decision of investing in an opaque project, we first introduce two notations: define h_O and h_T as the seeming probability density functions corresponding respectively to opaque and transparent projects. That is, the density function of V, $h(v)$, is divided into two parts according to the types of the borrowers. Thus, we have $h(v) = h_O(v) + h_T(v)$ for all $v \in [\underline{v}, \bar{v}]$, where h_O and h_T are the parts of the density function which refer to opaque and transparent borrowers, respectively. Both h_O and h_T are non-negative and defined on $[\underline{v}, \bar{v}]$. The corresponding seeming distribution functions are denoted by $H_O(v) = \int_{\underline{v}}^{v} h_O(v)dv$ and $H_T(v) = \int_{\underline{v}}^{v} h_T(v)dv$.

The lender issues loans to opaque borrowers if and only if the expected payoff of her investing in opaque projects is greater than or equal to that of not investing, i.e. $\pi_L^O \geq 0$. Note that $H_O(\bar{v})$ is the fraction of opaque borrowers, equation(3.4) can be written as

$$\pi_L^O = \frac{\int_{\underline{v}}^{R_1} vh_O(v)dv + R_1\big(H_O(\bar{v}) - H_O(R_1)\big)}{H_O(\bar{v})} - R_0. \tag{3.6}$$

Therefore, if opaque projects can be financed in an equilibrium, R_1 must satisfy

$$Z_O(R_1) := \frac{\int_{\underline{v}}^{R_1} vh_O(v)dv + R_1\big(H_O(\bar{v}) - H_O(R_1)\big)}{H_O(\bar{v})} \geq R_0. \tag{3.7}$$

Note that the function Z_O is non-decreasing in R_1, and $Z_O' < 1$ for any $R_1 \in (\underline{v}, \bar{v})$, with $\min Z_O(R_1) = Z_O(\underline{v}) = \underline{v}$ and $\max Z_O(R_1) = Z_O(\bar{v}) = E[V|\text{V is opaque}]$.

Then, we can consider the borrower's decision of whether or not to release information about the return on his project. There are two cases in which borrowers have the incentive to be transparent.

Case 1: A borrower can be willing to reveal information if he cannot get loans as an opaque applicant. So firstly, the condition of borrowers' choosing to be transparent in this case is that banks issue loans to transparent borrowers ($\pi_L^T \geq 0$)

but not to opaque ones ($\pi_L^O < 0$) . Secondly, the payoff of being transparent for the borrower must be non-negative, i.e. $\pi_B^T = max\{v - c - R_2, 0\} \geq 0$, which is always true. Therefore, borrowers choose, in such a case, to reveal information under the following conditions,

$$\begin{cases} min\{R_2, v - c\} \geq R_0 \\ Z_O(R_1) < R_0 \end{cases} \tag{3.8}$$

which are equivalent to

$$\begin{cases} R_0 \leq R_2 \leq v - c \\ R_0 \leq v - c \\ Z_O(R_1) < R_0 \end{cases} \tag{3.9}$$

Case 2: The second case in which borrowers have the incentive to reveal information is, if: firstly, they can get loans if they reveal information; secondly, in contrast to Case 1, they can also get loans if they do not reveal information; thirdly, the payoff of being transparent is however higher than that of being opaque, which makes being transparent more attractive. Thus, borrowers reveal information and become transparent if

$$\begin{cases} min\{R_2, v - c\} \geq R_0 \\ Z_O(R_1) \geq R_0 \\ max\{v - c - R_2, 0\} - c \geq max\{v - R_1, 0\} \end{cases} \tag{3.10}$$

which can be simplified as follows,

$$\begin{cases} R_0 \leq R_2 \\ R_0 \leq v - c \\ Z_O(R_1) \geq R_0 \end{cases} \quad \text{and} \quad \begin{cases} R_1 \geq R_2 + c, \text{ if } v \geq R_2 + c. \\ R_1 \geq R_0 + c, \text{ if } v < R_2 + c. \end{cases} \tag{3.11}$$

From Case 1 and 2, we can deduce that a borrower does not reveal information if: either $v < R_0 + c$, or $v \geq R_0 + c$ but with $Z_O(R_1) \geq R_0$. In the first situation, the return on the projects net of the signaling cost is too low to assure lenders of any positive profit, and revealing information about the low-quality of one's project is simply ruling out any possibility of getting financed - no bank invests in the projects if she foresees the default. In the latter situation, projects are good enough to attract investment from lenders, but they can also be financed if they remain opaque. Then it can be more profitable for borrowers to remain opaque. One of the possible circumstances is when $v \geq R_2 + c$ and $R_1 < R_2 + c$, where being opaque provides the borrower with a payoff $\pi_B^O = v - R_1$, which is larger than that of being transparent, $\pi_B^T = v - c - R_2$. The other case is when $v < R_2 + c$ and $R_1 < R_0 + c$, where we have $\pi_B^T = 0$ and $\pi_B^O = v - R_1 > 0$ since we are in the case where $v \geq R_0 + c > R_1$.

In any of these cases the conditions ((3.9) and (3.11)) are easier to be satisfied for a larger certification cost c. That is, borrowers tend to be less willing to be transparent when the cost of information disclosure is relatively high, which is consistent with our intuition. Note also that if $v = R_2 + c$ or if $R_1 - c = R_2$, it may be a limit case in which the borrower is indifferent between revealing information or not.[8]

To sum up, we have the following lemmas.

Lemma 3.1. *No borrower with $v < R_0 + c$ chooses to be transparent in any circumstance.*

Lemma 3.2. *Regardless of the decision of the lenders, if:*

1. *$R_1 < R_2 + c$, no borrower with $v \geq R_2 + c$ reveals information; thus, all borrowers are opaque.*
2. *$R_1 > R_2 + c$, all borrowers with $v \geq R_2 + c$ choose to be transparent (and the rest remains opaque);*
3. *$R_1 = R_2 + c$, borrowers with $v \geq R_2 + c$ are indifferent between revealing information or not.*

As stated in Lemma 3.1, bad borrowers have unanimous decisions under any circumstance, while good borrowers do not. However, good borrowers' decision depends more on the relative values of the two different repayment requirements rather than the absolute values. What really matters is the difference between the two types of repayments, compared with the cost of information disclosure. According to the optimal decisions of borrowers, we can get the aggregate loan demand with respect to the repayments R_1 and R_2, on the opaque and transparent markets respectively; similarly, the aggregate loan supply can be derived by virtue of the optimal decision of lenders. The aggregate demand and supply of loans are depicted in Figure 3.3 and 3.4, for transparent and opaque markets respectively.

[8] It is also indifferent for borrowers to reveal information or not when $R_0 + c < v < R_1 < R_2 + c$ and $Z_O(R_1) \geq R_0$, in which case $\pi_B^O = \pi_B^T = 0$. Yet, the interval $(R_0 + c, R_2 + c)$ is empty in equilibrium where $R_2 = R_0$.

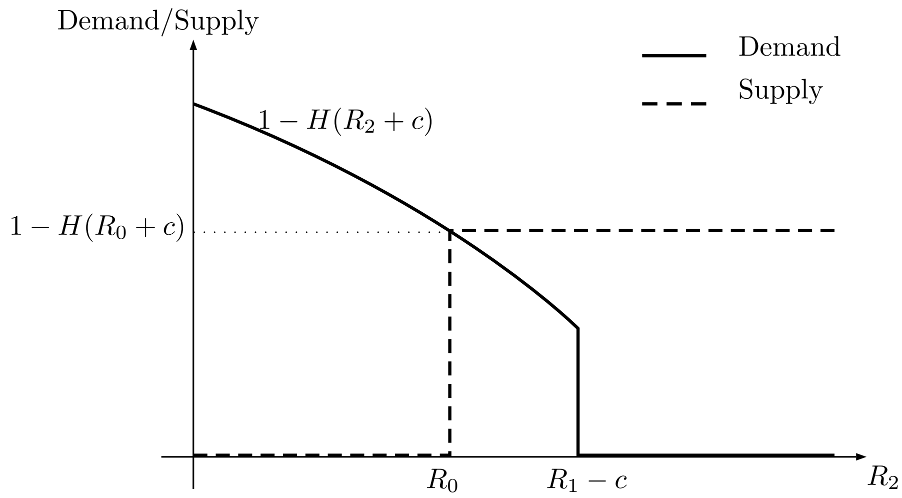

Figure 3.3: Aggregate demand and supply of loans on the transparent market.

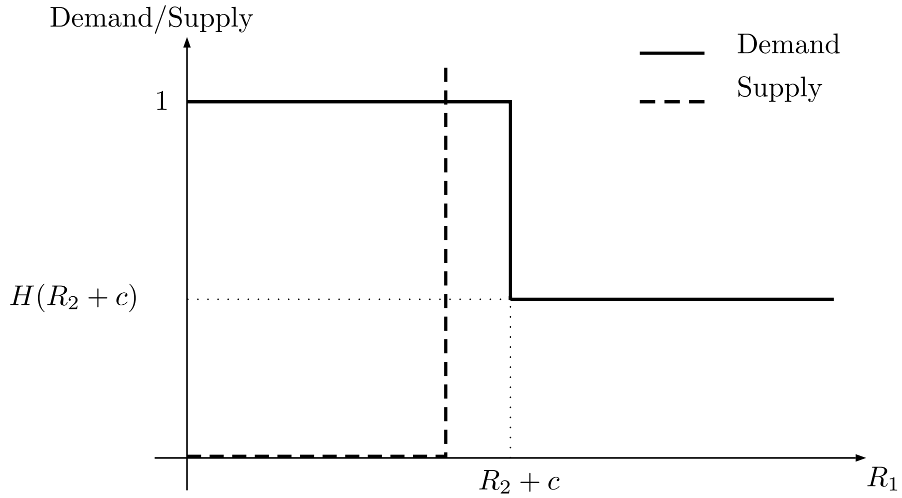

Figure 3.4: Aggregate Demand and supply of Loans on the opaque market.

3.3 Market equilibrium

3.3.1 Definition

In a competitive credit market with free entry, one necessary condition for the market equilibrium is the zero-profit for the banks. The equilibrium can be captured by the seeming density functions h_O and h_T and a loan contract (R_1^*, R_2^*). In the equilibrium, banks maximize their profits by deciding whether to grant loans or not;

borrowers, with the consideration of lenders' decision, maximize their payoffs by choosing between revealing information or being opaque, captured by the seeming density functions h_O and h_T. Furthermore, due the competition in the credit market, banks earn zero-profit in the equilibrium, by which the level of equilibrium interests (R_1^*, R_2^*) can be pinned down. The existence and the type of equilibria are determined by the parameters in the model.

The equilibrium of the loan market is solved under the assumption that borrowers always prefer to implement their project rather than doing nothing when the profit in both cases is zero. As a consequence, when there is a market for opaque projects, all projects (both opaque and transparent projects) are financed. Indeed, an opaque project that is financed can never lead to a negative profit for the borrower. Therefore, he prefers to implement it rather than inaction. Under this assumption, the equilibrium can be defined as follows:

Definition 3.1. *An equilibrium of the credit market is characterized by density functions for opaque and transparent projects h_O and h_T, loan repayments for opaque and transparent projects R_1^* and R_2^* such that:*

1. $h = h_O + h_T$.
2. *if Supp*$(h_T) \neq \emptyset$, *then* $R_2^* = R_0$ *and Supp*$(h_T) \subset [R_0 + c, \bar{v}]$. *Moreover,*
   a *either* $\forall R_1 \geq R_{0,}\ Z_O(R_1) < R_0$ *;*
   b *or* $\exists R_1^* \geq R_0 + c$ *such that* $Z_O(R_1^*) = R_0$.
3. *if Supp*$(h_T) = \emptyset$, *then an equilibrium corresponds to a value* $R_1^* \leq R_0 + c$ *such that* $\int_{\underline{v}}^{R_1^*} vh(v)dv + R_1^*\big(1 - H(R_1^*)\big) = R_0$.

The definition corresponds to different possible cases. If *Supp*$(h_T) \neq \emptyset$, there may exist a market for transparent projects, where the loan repayment R_2^* must equal to R_0 due to the zero-profit condition in equilibrium. *2*.a of Definition 3.1 corresponds to Case 1 in section 3.2, where there is no market for opaque projects. A borrower can obtain a loan only if he is transparent (and qualified). *2*.b corresponds to Case 2, where a market for opaque projects exists with a loan repayment R_1^*. But since the required interest from opaque borrowers is relatively high, $R_1^* \geq R_0 + c$,, revealing information can make qualified borrowers benefit from the low interest for transparent financing. Finally, *3* of Definition 3.1 refers to the case where there is no market for transparent projects. No borrower reveals information if they are able to be financed as opaque borrowers with a lower financing cost, $R_1^* \leq R_0 + c$.

3.3.2 Characterization

As we have discussed above, no borrower with project $v < R_0+c$ reveals information under any circumstance, since to disclose the insufficient quality of the project would disable them from being financed. What matters more is the decisions of those borrowers with $v \geq R_0 + c$. We may refer to the borrowers with $v \geq R_0 + c$ as good borrowers, and those with $v < R_0 + c$ as bad ones. To avoid the triviality, we assume that

Assumption 3.1. $\bar{v} > R_0 + c$.

That is, at least some borrowers are good. Besides, it also shows that the cost of information disclosure is relatively small compared to the maximum value of the possible return on a risky project. By this assumption, we also have $H(R_0+c) < 1$.

Before providing a full characterization of the equilibrium, we first look more into two extreme cases: borrowers with $v \geq R_0 + c$ are all opaque, or all of them are transparent. If there exists an equilibrium where all borrowers are opaque, i.e., $h_O = h$, the highest repayment that a lender can set is $R_1 = R_0 + c$; otherwise, good borrowers would choose to be transparent. Note that zero-profit condition requires $Z_O(R_1) = R_0$, and Z_O is increasing in R_1, thus to ensure the existence of an equilibrium with the repayment $R_1^* \leq R_0 + c$ such that $Z_O(R_1^*) = R_0$, it is necessary to have $Z_O(R_0 + c) \geq R_0$ with $h_O = h$. That is,

$$\psi(R_0) := \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)\big(1 - H(R_0 + c)\big) \geq R_0. \tag{3.12}$$

The inequality $\psi(R_0) \geq R_0$ has a simple interpretation. It means that, if no borrower reveals information, the expected gain of a lender, with requiring loan repayment R_0+c, is strictly higher than the funding cost R_0. Therefore, there may exist a repayment loan $R_1^* < R_0 + c$ that ensures the equilibrium of the opaque market. We consider this case, in which all (financed) projects are opaque, as an *opaque* equilibrium.

At the other extreme, if all good borrowers, those with project return $v \geq R_0+c$, choose to be transparent (and get funded) in an equilibrium, the repayment of transparent loans, R_2^*, must be equal to R_0. The reason why they choose so could be that there is no market for opaque projects when the repayment of opaque loans R_1^* is smaller than $R_0 + c$ and therefore attractive to good borrowers. That is,

$$Z_O(R_1) < R_0, \text{ for any } R_1 \leq R_0 + c. \tag{3.13}$$

Note that Z_O is increasing, the necessary condition for the non-existence of opaque market becomes

$$Z_O(R_0 + c) = \frac{\int_{\underline{v}}^{R_0+c} vh(v)dv}{H(R_0 + c)} \; < \; R_0,$$

since $h_O = h$ for any $v < R_0 + c$ and $h_O = 0$ for any $v \geq R_0 + c$. Rearranging the above condition yields,

$$\phi(R_0) := \int_{\underline{v}}^{R_0+c} vh(v)dv + R_0\big(1 - H(R_0 + c)\big) < R_0. \tag{3.14}$$

The first term of $\phi(R_0)$ can be seen as the maximum of possible expected gain on opaque market (though it does not exist under condition (3.14)), while the second term, $R_0(1 - H(R_0 + c))$, is the gain from issuing loans to transparent borrowers. The sum of the two terms is smaller than R_0 so that it is unprofitable if, given that all good borrowers are transparent, lenders invest also in bad projects. Therefore, we can have the case in which only good borrowers reveal information and have the loans approved. This is referred to as a *transparent* equilibrium.

The two functions above play important roles in depending the existence and the type of the equilibrium, and we see that both function $\phi(R)$ and $\psi(R)$ are increasing, and that $\phi(R) < \psi(R)$ for any R. Moreover, we have

- If $\phi(R_0) > R_0$, then $\psi(R_0) > \phi(R_0) > R_0$;
- If $\psi(R_0) < R_0$, then $\phi(R_0) < \psi(R_0) < R_0$.

Now we provide a complete characterization of the different types of equilibria that may exist in this model. First, we consider that the opaque equilibrium in which all borrowers choose to be opaque.

Proposition 3.1 (Opaque Equilibrium)**.** *Assume that the following inequality holds:*

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + R_0\big(1 - H(R_0 + c)\big) > R_0 \tag{3.15}$$

Then, there exists a unique market equilibrium that is opaque: no borrower reveals information and all projects are financed with a loan repayment $R_1^* < R_0 + c$ which is determined by

$$\int_{\underline{v}}^{R_1^*} vh(v)dv + R_1^*\big(1 - H(R_1^*)\big) = R_0.$$

Under the above condition, the whole market is opaque, while another case that we have discussed above is the transparent equilibrium, in which only borrowers who reveal information can be financed. And the transparent equilibrium is characterized in the following proposition.

Proposition 3.2 (Transparent Equilibrium)**.** *Assume that:*

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)\big(1 - H(R_0+c)\big) < R_0. \tag{3.16}$$

Then, there exists a unique market equilibrium that is transparent: all borrowers with $v \geq R_0 + c$ reveal information and are financed with repayment R_0. The borrowers with projects $v < R_0 + c$ choose to be opaque and are not financed.

Under this condition, only a mass $1 - H(R_0 + c)$ of borrowers can get loans and proceed with their projects. Although the credit market is completely transparent and therefore there is no risk due to the lack of information, the credit is rationed and production is hampered.

Proposition 3.3 (Multiple Equilibria)**.** *Assume that the two following inequalities hold:*

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + R_0\big(1 - H(R_0+c)\big) \;<\; R_0; \tag{3.17}$$

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)\left(1 - H(R_0+c)\right) \;>\; R_0. \tag{3.18}$$

Then there exist 3 types of equilibria:

1. *a transparent equilibrium in which borrowers with $v \geq R_0 + c$ reveal information and are financed with loan repayment R_0, whereas borrowers with $v < R_0 + c$ remain opaque and are not financed.*

2. *an opaque equilibrium in which no borrower reveals information and all projects are financed by loans with repayment $R_1^* < R_0 + c$ such that*

$$\int_{\underline{v}}^{R_1^*} vh(v)dv + R_1^*\big(1 - H(R_1^*)\big) = R_0.$$

3. *a multiplicity of (unstable) equilibria such that $R_1^* = R_0 + c$. All borrowers with $v < R_0 + c$ remain opaque. Borrowers with $v \geq R_0 + c$ are split into two parts - either opaque or transparent. Transparent projects are financed*

by loans with repayment R_0 and opaque borrowers by loans with repayment $R_0 + c$. h_O is such that

$$\int_{\underline{v}}^{R_0+c} \frac{vh(v)}{H_O(\bar{v})} dv + (R_0 + c)\,\frac{(H_O(\bar{v}) - H_O(R_0 + c))}{H_O(\bar{v})} = R_0.$$

We have seen the first two possibilities in the previous propositions, and the market opacity is the same as stated above, i.e. $\alpha_O = 1$ and $\alpha_T = 0$. In the third case of Proposition 3.3, all borrowers are financed, and at least a mass $H(R_0 + c)$ of borrowers are opaque (bad/unqualified). Besides, since good borrowers are indifferent between revealing information or not, the equilibrium no longer exists once there is some deviation of any good borrower, and is therefore unstable.

The three propositions are based on the inequalities, which rely on the two functions ϕ and ψ. Proposition 3.1 corresponds to the case $\phi(R_0) > R_0$ (and thus $\psi(R_0) > R_0$). Proposition 3.2 is obtained when $\psi(R_0) < R_0$ (and thus $\phi(R_0) < R_0$). Finally, Proposition 3.3 corresponds to the intermediate case $\phi(R_0) < R_0 < \psi(R_0)$. Furthermore, if we consider the two functions ϕ and ψ at some threshold values, i.e. when $\phi(R_0) = R_0$ or $\psi(R_0) = R_0$, we can also have multiple equilibria (see Proposition 3.6 and 3.7 in the appendix).

The existence of multiple equilibria provides us a possible way of understanding the instability of the credit market, while we will first characterize the equilibrium with respect to the safe interest rate, which helps to link the type of equilibrium with the funding cost, and leave more detailed explanations on the multiple equilibria in the following subsections.

3.3.3 Characterization with respect to the safe interest factor R_0

As shown in the previous propositions, the existence and the type of the equilibrium depend on the signs of $\phi(R_0) - R_0$ and $\psi(R_0) - R_0$, and thus on the value of R_0 and the shape of the two functions. We denote $\hat{R}$ and $\check{R}$ as the threshold values such that $\phi(\hat{R}) = \hat{R}$ and $\psi(\check{R}) = \check{R}$, respectively. That is, we have

$$\begin{aligned}
\int_{\underline{v}}^{\hat{R}+c} vh(v)dv + \hat{R}\big(1 - H(\hat{R} + c)\big) &= \hat{R} \\
\int_{\underline{v}}^{\check{R}+c} vh(v)dv + (\check{R} + c)\big(1 - H(\check{R} + c)\big) &= \check{R}
\end{aligned}$$

Besides, we know that

$$\begin{aligned}\phi'(R) &= c\,h(R+c) + \big(1 - H(R+c)\big) > 0\\ \psi'(R) &= 1 - H(R+c) > 0, \quad \psi''(R) = -h(R+c) < 0.\end{aligned}$$

Both functions are strictly increasing, and moreover, ψ is strictly concave, with $min\ \phi(R) = \phi(\underline{v} - c) = \underline{v} - c$, $min\ \psi(R) = \psi(\underline{v} - c) = \underline{v}$, and $max\ \phi(R) = \phi(\bar{v} - c) = max\ \psi(R) = \psi(\bar{v} - c) = E[V]$. Due to the monotonicity and the concavity of the function ψ, it is sufficient to have $\psi(\bar{v} - c) = E[V] < \bar{v} - c$ to ensure the uniqueness of the threshold $\check{R}$. This sufficient condition imply that the cost of disclosing information is relatively small and the highest possible return on one's project is comparatively far from the average, which is intuitively very reasonable. Yet we have to impose harsher condition on function ϕ, since its second order derivative is lack of some nice property. Thus, we consider in the following assumptions below in order to make sure the uniqueness of the threshold values.

Assumption 3.2. $\int_{\underline{v}}^{\bar{v}} vh(v)dv + c < \bar{v}$.

Assumption 3.3. $\exists!\ \hat{R}$ *such that* $\phi(\hat{R}) = \hat{R}$.

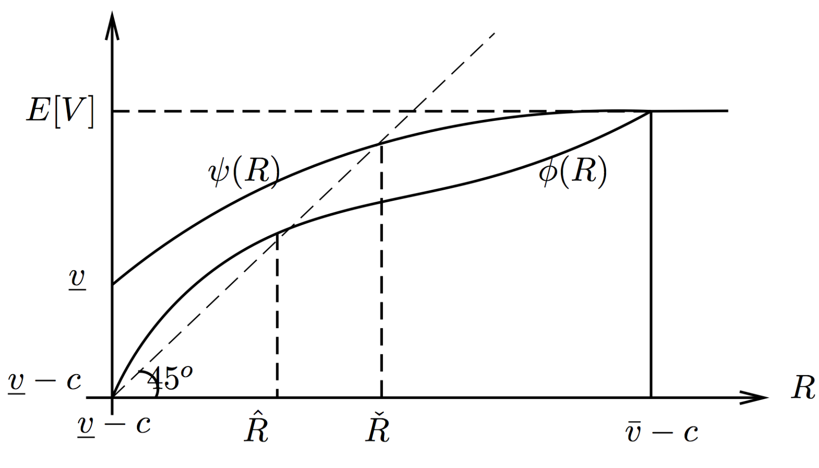

Figure 3.5: Graphs of two key functions ϕ and ψ.

Since $\phi(R) < \psi(R)$ for all $R < \bar{v} - c$, the following inequality holds: $\hat{R} < \check{R}$ (see Figure 3.5 and the proof is shown in the appendix). And by focusing on the cases in which the equilibrium is stable (off the thresholds), we can now characterize the equilibrium with respect to the value of the risk-free interest rate.

Proposition 3.4. *Assume that the preceding assumptions hold. Then,*

1. *if $R_0 < \hat{R}$, there exists a unique opaque equilibrium where all projects are financed, and the demand for loans is $H(\bar{v}) = 1$;*

2. *if $R_0 > \check{R}$, there exists a unique transparent equilibrium where projects such that $v \geq R_0 + c$ reveal information and are financed; the demand for loans is $1 - H(R_0 + c)$;*

3. *if $\hat{R} < R_0 < \check{R}$, there exist two stable equilibria. One equilibrium is opaque, all projects are opaque and financed; the demand for loans is $H(\bar{v}) = 1$. The second one is transparent, borrowers with projects $v \geq R_0 + c$ reveal information and are financed; the demand for loans is $1 - H(R_0 + c)$.*

Due to the monotonicity of the functions ϕ and ψ, it is easy to see that $R_0 < \hat{R}$ is equivalent to $\phi(R_0) > R_0$. By Proposition 3.1, there exists uniquely the opaque equilibrium. Similarly, $R_0 > \check{R}$ implies that $\psi(R_0) < R_0$, which in turn implies that there is only transparent equilibrium according to Proposition 3.2. And the third case corresponds to the conditions stated in Proposition 3.3, where multiple equilibria exist.

More precisely, when the risk-free interest rate R_0 is low enough, i.e., lower than the smaller threshold $\hat{R}$, there exists only opaque equilibrium; when R_0 is very high, there can only be transparent equilibrium. This is in line with our intuition. When the funding cost is low, banks tend to lower their lending standards, take more risk and invest without adequate information on the projects, some of which may be unprofitable. On the other hand, they become more cautious when the funding cost is high, and therefore only invest in those opaque and qualified projects. Besides, when the interest rate is within some moderate range of values, it can be either type of the equilibrium, reflecting different tightness of the credit granting. Hence, if we consider the opacity of the credit market as the proportion of opaque borrowers among all the borrowers that are financed, we can also see that the opacity of the credit market is basically decreasing - from 1 to 0, as the risk-free interest rate r_0 increases.

Here in our model, we have simplified the basic setup, which includes the full knowledge of the return of the borrower and the complete revelation of their private information. Though this yields some discontinuity of the credit demand and the market opacity with respect to the risk-free interest, the main property of the model is kept that the demand of the credit tends to be decreasing in the interest rate, while the transparency of the credit market is increasing as the fundamental interest rate rises.

3.4 Credit supply and market transparency

Consider banks obtain funds from their depositors at the risk-free interest rate r_0, and all the previous analysis have been done under the assumption that each bank faces a perfectly elastic supply of funds at an exogenous risk-free interest rate r_0. Now we consider r_0, thus R_0, as endogenous.

From Proposition 3.4, we know how the demand of loan is related to the risk-free interest rate. Here we further assume that the supply of loan from the banks - corresponding to consumers' savings, $S(R_0)$, is increasing in R_0. In a competitive credit market, R_0 is a result of equating the supply and the demand of loans in the credit market. Therefore, any change in the credit supply affects the value of R_0, and further influence the market equilibrium and the opacity of the market (see Figure 3.6).

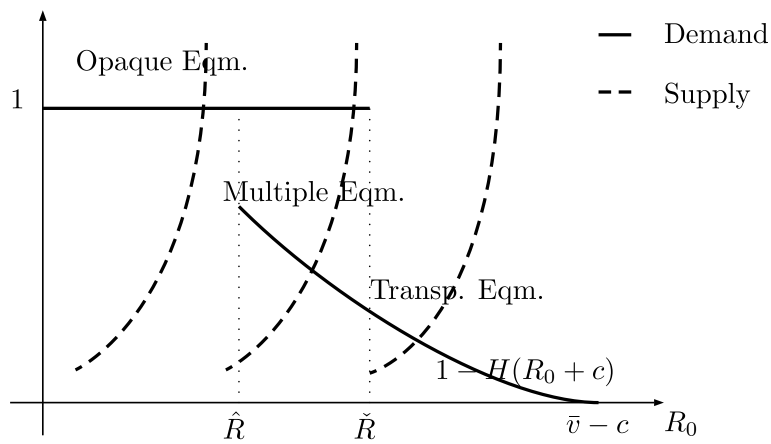

Figure 3.6: Demand and supply of loans

During the period when credit is easy to obtain (due to the broader inflow of funds and larger willingness of banks' lending), as we had experienced before the financial crisis burst, interest rate is relatively low, and the funding cost of a lender is also low. This allows lenders to take more risk of investing in opaque but possibly unprofitable projects. More interestingly, as the credit supply becomes tighter, with the interest rate falls in the range of $(\hat{R}, \check{R})$, banks may still have the incentive to issue loans to opaque projects, since the cost is not very high, though it is also likely that they only provide funding to those transparent and qualified borrowers if they have less confidence in the market and are more concerned about defaults in the future, resulting a transparent equilibrium at the end. That is, it might be either an opaque or a transparent equilibrium. In such a situation, there can be some jump from one equilibrium to the other, and if, for example, a jump from an opaque equilibrium occurs, we can observe a decrease in credit supply together with a decrease in the interest rate.

However, a decrease in the interest rate is more often in line with an increase in credit supply, since the cost of providing loans is smaller. Therefore, what is shown in our model provides us a theoretical explanation for credit crunch, where the reduction of credit happens without an increase of the interest rate. In our model, lenders may still choose to invest in opaque projects when the situation of the credit market is no longer suitable for opaque lending, i.e. when $R > \hat{R}$. Thus, we can experience an opaque equilibrium if all lenders coordinate in this more aggressive way, in the sense that lenders issue loan contracts to opaque but possibly unprofitable projects. Yet due to the existence of multiple equilibria, when lenders become less confident about the market and realize this excessive willingness of "irresponsible" and "inappropriate" lending, they may cut off the credit supply to opaque borrowers and end up in the transparent equilibrium. The jump from the opaque equilibrium to the transparent one happens, credit crunch occurs. Credit is reduced even the interest rate is getting lower, which results in less projects funded and less production in the economy, as we saw during the financial crisis.

3.5 Macroeconomic dynamics

In this subsection we incorporate the static model into an OLG economy. At each date $t = 0, 1, ...$ a continuum of agents of unity mass are born. Each agent lives for three periods: youth, adulthood and old age. When he is young, the agent is endowed with a project that requires one unit of funding to proceed in the next period; and during the old age, the agent is retired. Assume that the agent consumes only in the last two periods.

A generation-t agent lives during period $t-1$, t and $t+1$[9]. In period $t-1$, the agent owns a project of return v, which is privately observed by himself. He applies at a bank for one unit of loan with the repayment of R_t in period t, while R_t is determined in $t-1$. As in the static model, he can, according to his knowledge of v, decide whether or not to disclose the return on his project and to be certified/monitored during the implementation of the project with bearing an extra cost $c > 0$ in period t. We then call an agent a transparent borrower if he decides to certify his project; and he is referred to as an opaque borrower if he chooses not to reveal the information.

[9]Period t refers to a period starting at date t and ending just before date $t+1$.

In period t, the agent carries out the project if it is funded[10] and produces v. The income from the project, denoted by $P_t(v)$, is $max\{v - R_t, 0\}$ if he is opaque, and $max\{v - R_t - c, 0\}$ if he is transparent; R_t is predetermined in period t. Moreover, he can also earn an exogenous labor income w, no matter his project is accomplished or not. Let $I_t(v) := w + P_t(v)$, denoting the total income of the agent. The agent allocates his total income between current consumption c_t and savings s_t, which will, accumulated with a factor R_{t+1}, support his consumption when he is retired in period $t+1$, denoted by d_{t+1}.

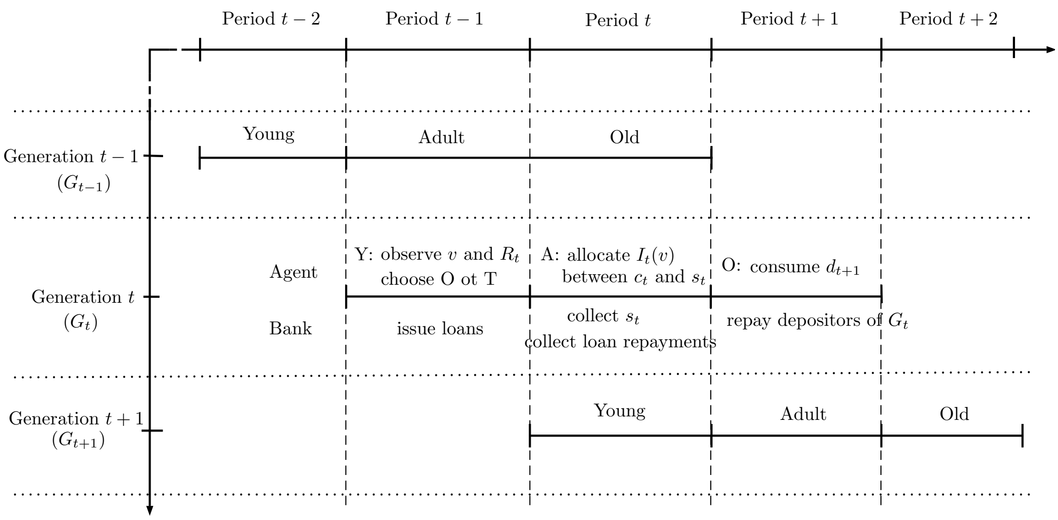

Figure 3.7: Timing of the OLG model.

Financial intermediaries, such as banks, collect savings and finance the projects when it is profitable. Banks have no more information on the projects than the distribution of v, denoted by $H(v)$, if borrowers do not disclose it; the corresponding density is denoted by $h(v)$. Banks have no operating costs and are in perfect competition. The time structure of the OLG model is presented in Figure 3.7.

The agent maximizes his CES utility by choosing the optimal levels of consumptions. That is,

$$\max_{(c_t, d_{t+1})} u(c_t, d_{t+1}) = (c_t)^{\frac{\sigma-1}{\sigma}} + \beta \, (d_{t+1})^{\frac{\sigma-1}{\sigma}} \text{[11]}$$

[10] No private benefit.

[11] Note that c_t, s_t, as well as d_{t+1}, are all determined by the income, which is a function of project return v, yet we omit v for the moment.

subject to the budget constraints:

$$c_t + s_t = I_t(v) \quad \text{and} \quad d_{t+1} = s_t R_{t+1}.$$

where β , $\sigma > 1$.

From the constraints, we have $c_t = I_t(v) - s_t$ and $d_{t+1} = s_t R_{t+1}$, so we can rewrite the optimization problem as

$$\max_{s_t} \ u(I_t(v) - s_t, s_t R_{t+1}) = (I_t(v) - s_t)^{\frac{\sigma-1}{\sigma}} + \beta \ (s_t R_{t+1})^{\frac{\sigma-1}{\sigma}}.$$

First order condition gives

$$-\frac{\sigma - 1}{\sigma} \ (I_t(v) - s_t)^{-\frac{1}{\sigma}} \ + \ \beta \ \frac{\sigma - 1}{\sigma} \ (s_t R_{t+1})^{-\frac{1}{\sigma}} \ R_{t+1} = 0 \ ,$$

which yields the optimal savings as:

$$s_t = \frac{I_t(v)}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \ . \tag{3.19}$$

Thus, the aggregate savings in period t, S_t, is given by :

$$S_t = \int_{\underline{v}}^{\bar{v}} s_t(v) h(v) dv = \frac{Y_t}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \ , \tag{3.20}$$

where $Y_t = \int_{\underline{v}}^{\bar{v}} I_t(v) h(v) dv$ is the aggregate income earned by generation t during adulthood. The aggregate savings S_t constitute the supply of the credit market in period $t + 1$.

The uncertain part of Y_t, thus of the total credit supply S_t, comes from the income of the projects, which depends on whether the agents' loan applications are approved or not, as well as on the type of the agent as a borrower, i.e. a transparent one or an opaque one. According to the analysis of the static model shown in Proposition 3.4, there exists opaque equilibrium when $R_t < \check{R}$ and transparent equilibrium when $R > \hat{R}$, with $\hat{R} < \check{R}$.

Therefore, for an opaque equilibrium, where all projects are opaque and financed, we have

$$Y_t^O = \int_{\underline{v}}^{\bar{v}} \big(w + (v - R_t)\big) \ h(v) \ dv = w + \int_{\underline{v}}^{\bar{v}} v \ h(v) \ dv - R_t. \tag{3.21}$$

with $R_t < \check{R}$; while in an transparent equilibrium, only borrowers with $v \geq R_0 + c$

are financed and produce, the aggregate income from the production is then given by

$$Y_t^T = w + \int_{R_t+c}^{\bar{v}} (v - R_t - c)h(v)dv = w + \int_{R_t+c}^{\bar{v}} vh(v)dv - (R_t + c)\big(1 - H(R_t + c)\big). \tag{3.22}$$

Denote the aggregate savings in an opaque and a transparent equilibrium as S^O and S^T, respectively. Then, the supply of loans if it is in an opaque equilibrium, denoted as L^{sO}, is

$$L_t^{sO} = S_t^O = \frac{w + \int_{\underline{v}}^{\bar{v}} v\ h(v)\ dv - R_t}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}}\ , \tag{3.23}$$

and the supply of loans in a transparent equilibrium, denoted as L^{sT}, is

$$L_t^{sT} = S_t^T = \frac{w + \int_{R_t+c}^{\bar{v}} vh(v)dv - (R_t + c)\big(1 - H(R_t + c)\big)}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}}\ . \tag{3.24}$$

The equilibrium is determined by both the loan supply, L^s, and the loan demand, denoted by L^d, where the supply corresponds to the aggregate income as we have discussed above; the demand for credit depends on which type of equilibrium will occur in the following period. According to Proposition 3.4, the loan demand is unity when $R_{t+1} < \check{R}$ and the equilibrium in $t+1$ is opaque, while when $R_{t+1} > \hat{R}$ and the equilibrium in $t+1$ is transparent, the demand for loans is $1 - H(R_{t+1} + c)$. That is,

$$L_t^{dO} = 1 \tag{3.25}$$

and

$$L_t^{dT} = 1 - H(R_{t+1} + c). \tag{3.26}$$

Consequently, the dynamics of the interest rate is driven by the following four equations, with equating the credit supply and demand for each period in four cases.

1. When there are opaque equilibria in both period t and $t + 1$, we have $L_t^{sO} = L_t^{dO}$, i.e.,

$$\frac{w + \int_{\underline{v}}^{\bar{v}} vh(v)\ dv - R_t}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \ = \ 1,$$

which can be written as,

$$1 + \beta^{-\sigma} R_{t+1}^{1-\sigma} = w + \int_{\underline{v}}^{\bar{v}} v h(v) dv - R_t, \tag{3.27}$$

with $R_t < \check{R}$ and $R_{t+1} < \check{R}$.

2. When the equilibrium is opaque in period t and transparent in period $t+1$, we have $L_t^{sO} = L_t^{dT}$, i.e.,

$$\frac{w + \int_{\underline{v}}^{\bar{v}} v h(v)\ dv - R_t}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \ = \ 1 - H(R_{t+1} + c),$$

which can be written as

$$\big(1 - H(R_{t+1} + c)\big)(1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}) = w + \int_{\underline{v}}^{\bar{v}} v h(v) dv - R_t, \tag{3.28}$$

with $R_t < \check{R}$, and $R_{t+1} > \hat{R}$.

3. When the equilibrium is transparent in period t while opaque in period $t+1$, we have $L_t^{sT} = L_t^{dO}$, i.e.,

$$\frac{w + \int_{R_t+c}^{\bar{v}} v h(v) dv - (R_t + c)\big(1 - H(R_t + c)\big)}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \ = \ 1,$$

which can be written as,

$$1 + \beta^{-\sigma} R_{t+1}^{1-\sigma} = w + \int_{R_t+c}^{\bar{v}} v h(v) dv - (R_t + c)\big(1 - H(R_t + c)\big), \tag{3.29}$$

with $R_t > \hat{R}$, and $R_{t+1} < \check{R}$.

4. When there are transparent equilibria in both period t and $t+1$, we have $L_t^{sT} = L_t^{dT}$, i.e.,

$$\frac{w + \int_{R_t+c}^{\bar{v}} v h(v) dv - (R_t + c)\big(1 - H(R_t + c)\big)}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \ = \ 1 - H(R_{t+1} + c),$$

which can be written as,

$$\big(1-H(R_{t+1}+c)\big)(1+\beta^{-\sigma}R_{t+1}^{1-\sigma}) = w+\int_{R_t+c}^{\bar{v}} vh(v)dv-(R_t+c)\big(1-H(R_t+c)\big), \tag{3.30}$$

with $R_t > \hat{R}$, and $R_{t+1} > \hat{R}$.

R_t is predetermined in period t, then given R_0, the four difference equations (3.27), (3.28), (3.29) and (3.30) can completely capture the evolutions of the interest rate.

To simplify the expression of the dynamics of R_t, we define the following functions:

$$\begin{aligned}
F^O(R) &:= w + \int_{\underline{v}}^{\bar{v}} v\, h(v)dv - R;\\
F^T(R) &:= w + \int_{R+c}^{\bar{v}} v\, h(v)dv - (R+c)\big(1-H(R+c)\big);\\
G^O(R) &:= 1+\beta^{-\sigma}R^{1-\sigma};\\
G^T(R) &:= \big(1-H(R+c)\big)\ (1+\beta^{-\sigma}R^{1-\sigma}).
\end{aligned}$$

It is straightforward to see that all the four functions are decreasing in R, and moreover, we have

$$\begin{aligned}
F^O(R)-F^T(R) &= \int_{\underline{v}}^{R+c} vh(v)dv + (R+c)[1-H(R+c)] - R\\
&= \phi(R) - R \quad \begin{cases} >0, & R<\check{R};\\ <0, & R>\check{R}.\end{cases}
\end{aligned}$$

and

$$G^O(R)-G^T(R) = (1+\beta^{-\sigma}R^{1-\sigma})\ H(R+c) > 0.$$

Let S_t denote the state of the economy in period t, and $S_t \in \{O, T\}$. Then the dynamics of R_t can be summarized by a recurrence relation of the pair of sequences (R_t, S_t). That is,

$$G^{S_{t+1}}(R_{t+1}) = F^{S_t}(R_t), \tag{3.31}$$

where $S_t = O$ when $R_t < \check{R}$, $S_t = T$ when $R_t > \hat{R}$ and $S_t = O$ or T when $\hat{R} < R_t < \check{R}$.

3.5.1 Existence of stationary states

A stationary state corresponds to a fix point of the dynamic system described in (3.31), i.e. a pair value of the interest rate and the state of the economy, (R, S). Since the state can be either of opaque, O, or transparent, T, we have two possible stationary states stated as follows:

$$G^O(R) = F^O(R) \ , \quad \text{or} \quad G^T(R) = F^T(R),$$

where R must accord with the state according to Proposition 3.4.

Hence, the existence of the opaque regime requires that there exists $R < \check{R}$ such that $G^O(R) = F^O(R)$. i.e.,

$$1 + \beta^{-\sigma} R^{1-\sigma} = w + \int_{\underline{v}}^{\bar{v}} v\ h(v)dv - R. \tag{3.32}$$

We can see that both F^O and G^O are decreasing, and F^O is linear with a slope of -1 while G^O is convex to the origin. Thus, to ensure the two curves to intersect with each other we have a necessary condition for the existence of the opaque equilibrium, as stated in the following proposition.

Proposition 3.5. *If there exists a stationary state which is opaque, then we have*

$$\frac{\sigma(\sigma-1)}{\beta}^{\frac{1-\sigma}{\sigma}} \leq w + \int_{\underline{v}}^{\bar{v}} vh(v)dv - 1. \tag{3.33}$$

We can see from condition (3.33) that a higher value of w is more favorable to ensure an opaque stationary state. That is, the higher the wage is, the more likely an opaque stationary state exists. In fact, the higher the wage is, the larger the aggregate savings, and thus the credit supply, are. A larger supply tends to push the interest rate lower and the banks are more aggressive to issue loans. Therefore, opaque and even unqualified projects can also be financed. A state with opaque equilibrium is more likely to happen. Moreover, to ensure a solution to equation (3.32) which is smaller than $\check{R}$, it is sufficient to have $F^O(\check{R}) > G^O(\check{R})$.

And the transparent steady state is implied by the existence of $R > \hat{R}$ such that

$$\big(1-H(R+c)\big)\ (1+\beta^{-\sigma}R^{1-\sigma}) = w + \int_{R+c}^{\bar{v}} v\ h(v)dv - (R+c)\big(1-H(R+c)\big). \tag{3.34}$$

It is easy to check that $R = \bar{v} - c$ is a solution of equation (3.34)when $w = 0$ -

both sides of the equation are equal to zero. By the continuity, there also exists transparent steady state when $w > 0$ in a neighborhood of $R = \bar{v} - c$.

The intuition follows similar argument as above. The low wage reduces the credit supply, which in turn causes more cautiousness of the lending, and only transparent projects can have a chance to be approved.

3.5.2 Simulations with a uniform distribution

Now we assume that v is uniformly distributed in the interval $[\gamma, \gamma + \delta]$, thus we have $h(v) = 1/\delta$ when $v \in [\gamma, \gamma + \delta]$ and zero elsewhere. Then functions F^S and G^S become

$$\begin{aligned}
F^O(R) &= w + \gamma + \frac{\delta}{2} - R; \\
F^T(R) &= w + \frac{(\gamma + \delta - R - c)^2}{2\delta}; \\
G^O(R) &= 1 + \beta^{-\sigma} R^{1-\sigma}; \\
G^T(R) &= \frac{\gamma + \delta - R - c}{\delta} (1 + \beta^{-\sigma} R^{1-\sigma}).
\end{aligned}$$

The above four functions govern the motion of the interest rate and decide the type of the state of the economy in each period. Now we can see the interest rate and the equilibrium evolve over time as well as the changing of market opacity. Take Figure 3.8 for example. Starting with an initial interest rate level which is smaller than $\hat{R}$, the only possibility is an opaque state. Thus, start with (R_0, O), according to (3.31), we have

$$G^{S_1}(R_1) = F^O(R_0), \quad \text{which implies} \quad S_1 = T.$$

Since $G^O(R) \neq F^O(R_0)$ for any R, the transparent state is the only possible state for the next period - period 1. Then, we can in turn find R_2, which can equate G^{S_2} to $F^T(R_1)$ (note that $S_1 = T$), and determine both the interest rate and the state for period 2, and so on so forth.

As we discussed in the previous subsection, there exists a transparent stationary state when $w = 0$. By choosing a parameter setting with $\gamma = 1, \delta = 4,\ w = 0,\ c = 0.3,\ \beta = 0.5,\ \sigma = 1.5$, as shown in Figure 3.8, we see that the economy converges into a transparent equilibrium in the long run. In fact the F functions show also the level of aggregate output, and therefore the convergence shown in Figure 3.8 describe a steady state with null output, although it seems unattractive to us. This is again in line with the analysis above, which shows that the equilibrium

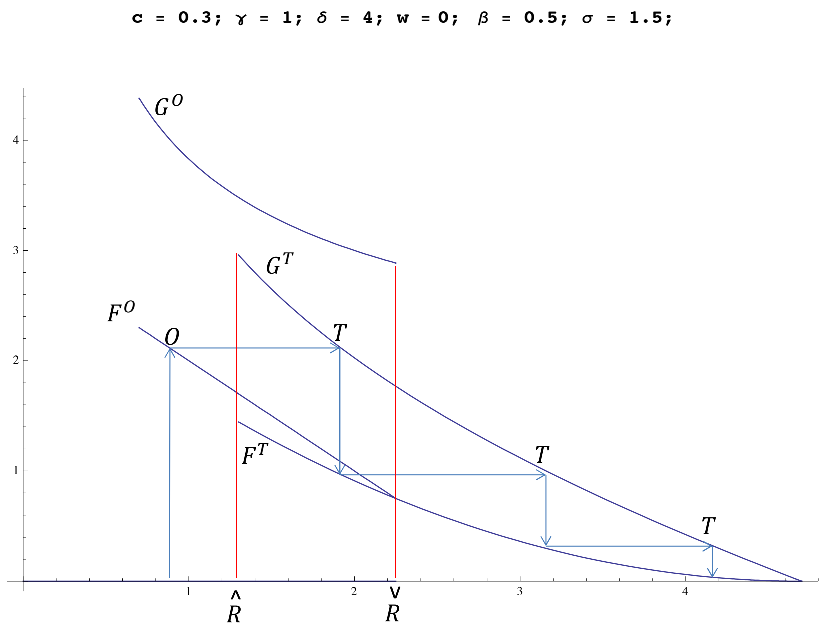

Figure 3.8: Simulation with a uniform distribution.

interest rate is $R = \bar{v} - c$. That is, although transparent projects could be financed, there is only a mass zero of borrowers (with $v \geq R + c = \bar{v}$) that are qualified and therefore reveal information.

Note that the wage goes directly into the F functions and thus can shift the level of F functions by taking different values. Hence, by raising the wage to $w = 0.5$, we have a more meaningful case, in which the market converges in the long run to a transparent equilibrium with considerable output (see Figure 3.9). In such a state, all projects are transparent. But as we have seen in the static model, the interest rate is high and the production is in a relatively low range - the transparency of the credit market is achieved with a compromise of the aggregate output.

We have seen that the wage plays an important role on determining the level of F functions (while the G functions remain the same with different wages). Hence, we can imagine that some significant changes of the wage may lead to different long run states, which is true when we increase the wage to $w = 1.9$. In Figure 3.10, even starting from a high level of interest rate, the market goes from a transparent state to an opaque stationary state in the long run.

What we have seen here is in accordance with the analysis in the previous subsection. When the wage is very low, credit is constrained as a result of fewer

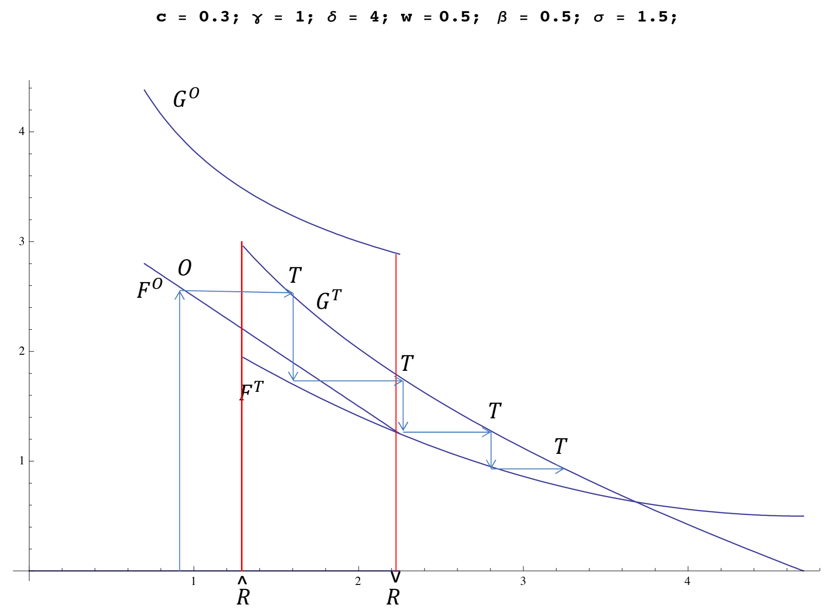

Figure 3.9: Simulation with a uniform distribution with $w = 0.5$.

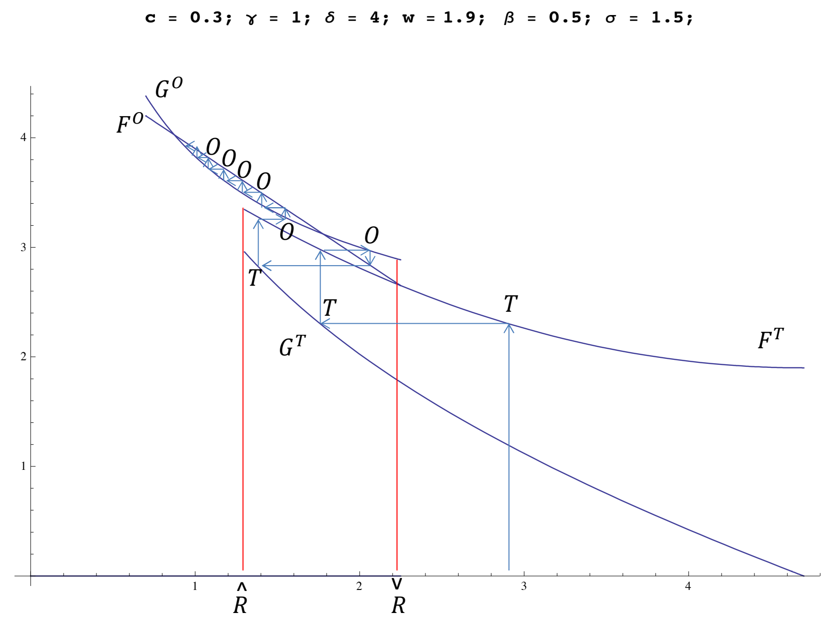

Figure 3.10: Simulation with a uniform distribution with $w = 1.9$.

savings which constitute the credit supply, and there only exists one transparent

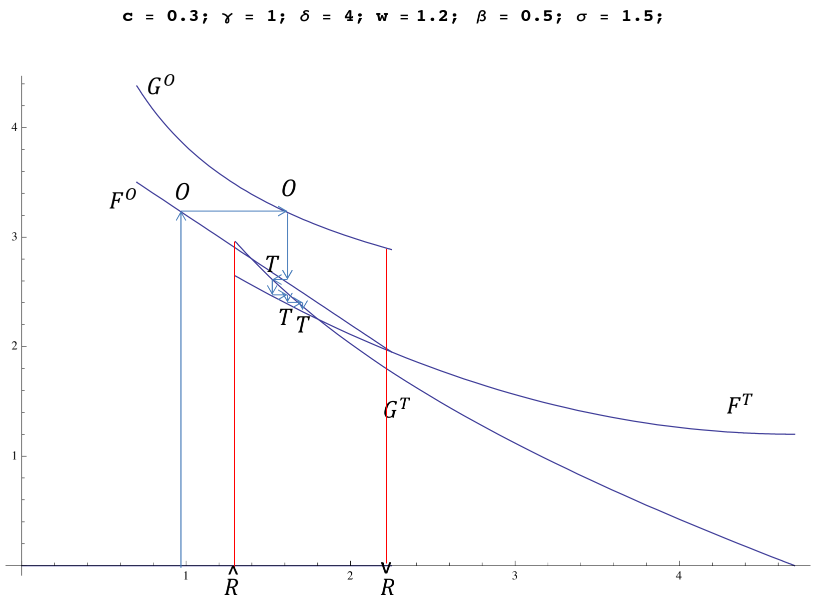

Figure 3.11: Simulation with a uniform distribution with $w = 1.2$.

stationary equilibrium with relatively high interest rate in the credit market; production is hampered since limited projects can be funded and the necessary certification cost brings in further inefficiency in the economy. Yet when the wage is high enough, credit supply is adequate and opaque equilibrium occurs with higher level of production and low interest rate.

With some wage of moderate level the dynamics may evolve without a certain pattern, especially when the interest rate falls into the intermediate range, where there exists uncertain path that the evolution may follow. Besides, the state may change between opaque and transparent equilibrium. This more interesting case is as shown in Figure 3.12, where we can observe cyclical switches between opaque and transparent states when the interest rate is moderate. Although such a path is not stable, it could illustrate some permanent fluctuations of the credit market and, in turn, of the real economy. Moreover, as the output in an opaque equilibrium is always higher than that in a transparent one, a switch from an opaque state in the credit to a transparent one therefore also indicates economic contraction.

By adjusting the value of the wage w, which contributes to the credit supply, we show similar patterns here as in the static framework, though from a dynamic perspective. In addition to the indeterminacy we see in the static model, the dynamic model further demonstrates the instability of financial market and fluctuations of

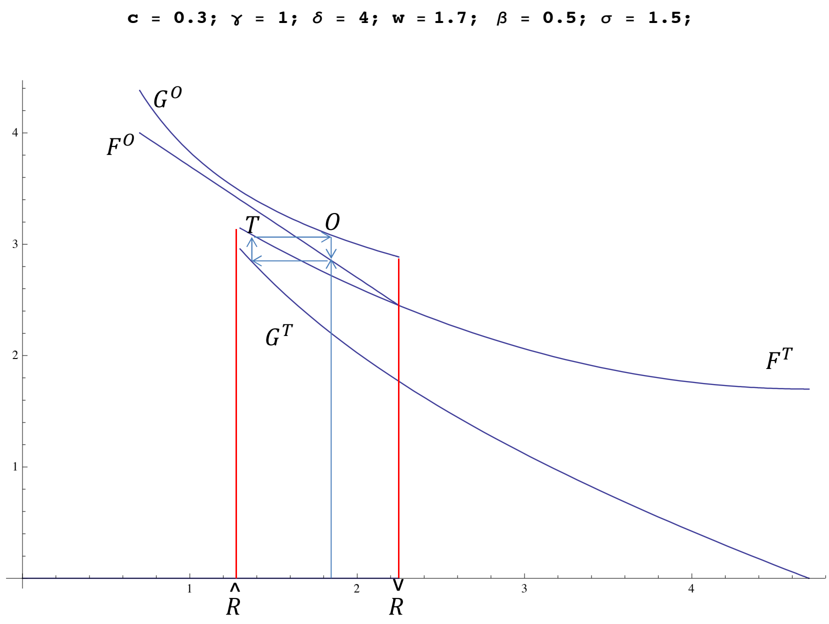

Figure 3.12: Simulation with a uniform distribution with $w = 1.7$.

the real economy caused by the imperfection of the credit market.

3.6 Conclusion

We characterize the equilibrium in a competitive credit market, where a borrower's profitability is private information to oneself and he could choose to disclose it through a costly but dissipative signal. The existence and the type of the equilibrium depends on the interest rate - when the interest rate is very low, the exists only an opaque equilibrium, and when the interest rate it high enough, there only exists a transparent equilibrium; in addition, in the intermediate level, there can be multiple equilibria. Hence, a close relationship between the opacity of the credit market and the fundamental funding cost can also be established. In addition, by endogenizing the interest rate, the interaction between credit supply and demand shows us distinctive states in the equilibrium, where an easing monetary policy corresponds to a larger credit supply, and therefore, more opacity in the market. More interestingly, when the banks tend to issue more loans, and thus to finance opaque projects even when the interest rate is not low enough, the possibility of jumping from one type of the equilibrium to the another may occur. Credit crunch can be observed when there is a jump from an opaque equilibrium to a transparent one.

Moreover we further examine the dynamic interaction with the market degree of opacity, the interest rate is endogenized by extending the model to an OLG context. We show that, by changing the level of wages and thus the credit supply, the market is likely to converge to either an opaque or a transparent equilibrium, and for some configurations of parameters there exist permanent oscillations between two different regimes, which provides us a theoretical support of the (in)stability of the credit market and further indicates a possible way of explaining the credit crunch during the financial crisis.

Appendix 3.A More propositions on multiple equilibria

Proposition 3.6 (Multiple Equilibria with Half-Opaque Equilibrium). *Assume that the following equality holds:*

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + R_0\big(1 - H(R_0 + c)\big) = R_0. \tag{3.35}$$

Then there exist 2 types of equilibria:

1. *an opaque equilibrium, where all borrowers are opaque and funded by loans with repayment* $R_1^* < R_0 + c$*.*

2. *half-opaque equilibrium where* $R_1^* > R_0 + c$*,* $R_2^* = R_0$*. The borrowers with* $v \geq R_0 + c$ *reveal information while the others, with* $v < R_0 + c$*, remain opaque, but all are funded.*

In a half-opaque equilibrium, we can see that the opacity of the credit market is

$$\alpha_{Half-O} = \frac{H(R_0 + c)}{1} = H(R_0 + c).$$

Also note that the expected profit of a lender on the opaque market is independent of the value of R_1 once R_1 exceeds $R_0 + c$. Therefore, we could have equilibria with different repayments of opaque funding, with the same opacity on the credit market and the same productions in the real sectors.

Proposition 3.7 (Multiple Equilibria with Unstable Opaque Equilibrium). *Assume that the following equality holds:*

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)\big(1 - H(R_0+c)\big) = R_0 \tag{3.36}$$

Then there exist 2 possible equilibria:

1. *a transparent equilibrium, where only borrowers with $v \geq R_0 + c$ reveal information and are funded, while the others remain opaque and are not financed.*

2. *an (unstable) opaque equilibrium where $R_1^* = R_0 + c$. All borrowers remain opaque and are financed.*

In fact, good borrowers are indifferent between being opaque or transparent when $R_1^* = R_0 + c$. The existence of opaque equilibrium implies that $h_O = h$ for any v and that the gain of a lender on the opaque market, Z_O, can equal to the funding cost R_0. That is,

$$Z_O(R_1^*) = \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)\big(1 - H(R_0+c)\big) = R_0$$

However, good borrowers may also choose to reveal information due to the indifference. Once it happens, the profit of a lender on the opaque market will fall below R_0, and the credit market cannot clear any longer and the equilibrium collapse. More precisely, it switches to the transparent equilibrium which is stable.

Appendix 3.B Proofs of propositions.

Proof of Proposition 3.1. First we show that if condition (3.15) holds true, there exists an equilibrium which corresponds to Case *3* of Definition 3.1. That is, $Supp(h_T) = \emptyset$ - no borrower reveals information, and all projects can be financed in the equilibrium at a loan repayment $R_1^* < R_0 + c$ such that $\int_{\underline{v}}^{R_1^*} vh(v)dv + R_1^*\big(1 - H(R_1^*)\big) = R_0$. This is true since

$$\int_{\underline{v}}^{R_1^*} vh(v)dv + R_1^*\big(1 - H(R_1^*)\big) < \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)\big(1 - H(R_0+c)\big),$$

while the right hand side is larger than $\int_{\underline{v}}^{R_0+c} vh(v)dv + R_0[1 - H(R_0+c)]$, and therefore larger than R_0 according to condition (3.15).

Next, we show that under condition (3.15), the opaque equilibrium, i.e., $Supp(h_T) = \emptyset$, is the only possible equilibrium. Suppose that there would exist some equilibrium in which $Supp(h_T) \neq \emptyset$, then by definition we would have $R_2^* = R_0$ and $Supp(h_T) \subset [R_0 + c, \bar{v}]$.

Moreover, we would have for one possibility that $Z_O(R_1) < R_0$ for any $R_1 \geq R_0$, i.e.,

$$Z_O(R_1) = \frac{\int_{\underline{v}}^{R_1} v h_O(v) dv + R_1\big(H_O(\bar{v}) - H_O(R_1)\big)}{H(\bar{v})} < R_0. \tag{3.37}$$

Therefore, no lender would issue loans to opaque borrowers and no good borrower would choose to be opaque: $Supp(h_O) \not\subset [R_0 + c, \bar{v}]$. So, we have $h_O = h$ for any $v \leq R_0 + c$ and zero otherwise, and thus $H_O(\bar{v}) = H(R_0 + c)$. Then, the lender's (potential) expected gain from the opaque market by charging R_1, $Z_O(R_1)$, becomes

$$Z_O(R_1) = \frac{\int_{\underline{v}}^{R_1} v h(v) dv + R_1\big(H(R_0 + c) - H(R_1)\big)}{H(R_0 + c)}$$

with $R_1 \leq R_0 + c$, and it reaches its maximum value at $R_1 = R_0 + c$. According to (3.37), we have $\frac{\int_{\underline{v}}^{R_0+c} v h(v) dv}{H(R_0+c)} < R_0$. That is,

$$\int_{\underline{v}}^{R_0+c} v h(v) dv + R_0\big(1 - H(R_0 + c)\big) < R_0,$$

which contradicts condition (3.15).

If we would be in the other case of $Supp(h_T) \neq \emptyset$, i.e. there exists $R_1^* \geq R_0 + c$ such that $Z_O(R_1^*) = R_0$. Note that if $R_1^* > R_0 + c$, $Supp(h_O) \not\subset [R_0 + c, \bar{v}]$, then we have for $R_1^* > R_0 + c$,

$$Z_O(R_1^*) = \frac{\int_{\underline{v}}^{R_1^*} v h_O(v) dv + R_1^*\big(H_O(\bar{v}) - H_O(R_1^*)\big)}{H(R_0 + c)} = \frac{\int_{\underline{v}}^{R_0+c} v h(v) dv}{H(R_0 + c)} = R_0$$

which is a contradiction to condition (3.15).

If $R_1^* = R_0 + c$, $Supp(h_O) \supset [R_0 + c, \bar{v}]$, then we have $h_O = h$ for any $v < R_0 + c$, and $h_O = h - h_T$ for any $v \geq R_0 + c$, with $h_O,\ h_T \geq 0$. Then, we have

$$\begin{aligned} R_0 = Z_O(R_1^*) = Z_O(R_0 + c) &= \frac{\int_{\underline{v}}^{R_0+c} v h(v) dv + (R_0 + c)\big(H_O(\bar{v}) - H(R_0 + c)\big)}{H_O(\bar{v})} \\ &> \frac{\int_{\underline{v}}^{R_0+c} v h(v) dv}{H(R_0 + c)} \end{aligned}$$

which is also a contradiction to condition (3.15).

Therefore, neither of the cases of $Supp(h_T) \neq \emptyset$ is true, and the existence of the opaque equilibrium is unique under condition (3.15).

Now we show that if there only exists an opaque equilibrium where no borrower reveals information but all are financed, with the equilibrium repayment $R_1^* \leq R_0 + c$ such that $\int_{\underline{v}}^{R_1^*} vh(v)dv + R_1^*\big(1 - H(R_1^*)\big) = R_0$, then condition (3.15) must hold true. Suppose not, i.e. $\int_{\underline{v}}^{R_0+c} vh(v)dv + R_0\big(1 - H(R_0 + c)\big) \leq R_0$. Then we have either

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)\big(1 - H(R_0 + c)\big) < R_0$$

or

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)\big(1 - H(R_0 + c)\big) \geq R_0.$$

For the first case, for any $R_1 \leq R_0 + c$,

$$\int_{\underline{v}}^{R_1} vh(v)dv + R_1\big(1 - H(R_1)\big) \leq \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)\big(1 - H(R_0 + c)\big) < R_0$$

which contradicts the existence of the opaque equilibrium. In the other case, the inequality assures the existence of a value of $R_1^* \geq R_0 + c$ such that $Z_O(R_1^*) = R_0$ for some h_O and h_T with $h_O + h_T = h$ for any v, which violates the uniqueness of the existence of opaque equilibrium. ■

Proof of Proposition 3.2. First we show that under condition (3.16) there exists an equilibrium in which all good borrowers reveal information and get financed and all those with $v < R_0 + c$ choose to be opaque and cannot obtain any loan. Consider some density function $h_O = h$ for any $v < R_0 + c$ and zero elsewhere, and some $h_T = h$ for any $v \geq R_0 + c$ and zero elsewhere, we have $h_O + h_T = h$ for any v. In such a case, the expected gain of a lender investing in opaque projects is

$$\begin{aligned} Z_O(R_1) \leq Z_O(R_0 + c) &= \frac{\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)\big(H(R_0 + c) - H(R_0 + c)\big)}{H(R_0 + c)} \\ &< \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)\big(1 - H(R_0 + c)\big) < R_0. \end{aligned}$$

That is, $Z_O(R_1) < R_0$ for any $R_1 > R_0$. By definition, the equilibrium corresponding to Case *2*.a - the transparent equilibrium - exists.

Then, we show that the transparent equilibrium uniquely exists under condition (3.16). Suppose not, then we have either $Supp(h_T) \neq \emptyset$ with some $R_1^* \geq R_0 + c$

such that $Z_O(R_1^*) = R_0$, or $Supp(h_T) = \emptyset$ with some some $R_1^* \leq R_0 + c$ such that $Z_O(R_1^*) = R_0$. Note that in either of the alternatives, opaque projects could also be funded. We will show that it can not be true under condition (3.16).

If $R_1^* > R_0 + c$, then all borrowers with $v > R_0 + c$ reveal information, so $h_O \not\subset [R_0 + c, \bar{v}]$, i.e., $h_O = h$ for any $v < R_0 + c$ and zero elsewhere. Thus, we have for any $R_1^* > R_0 + c$

$$Z_O(R_1^*) = \frac{\int_{\underline{v}}^{R_0+c} vh(v)dv}{H(R_0 + c)} < \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)(1 - H(R_0 + c)) < R_0.$$

Or if $R_1^* = R_0 + c$, it is indifferent for good borrowers to be transparent or opaque, but the highest profit that a lender could get from the opaque market, if it exists, is when all good borrowers are opaque - no loss due to the certification cost. That is, the maximum expected gain of a lender is, with $h_O = h$ for any v,

$$maxZ_O(R_1^*) = \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)(1 - H(R_0 + c)) < R_0.$$

And if $R_1^* < R_0 + c$, all good borrowers would also choose to be opaque, i.e., $h_T = \emptyset$, the expected gain of a lender also has a upper boundary since Z_O is increasing. That is,

$$Z_O(R_1^*) \leq Z_O(R_0 + c) = \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)(1 - H(R_0 + c)) < R_0.$$

Since none of the possible value of R_1^* can yield $Z_O(R_1^*) = R_0$, it is impossible to have other type of equilibria, and the transparent equilibrium is unique.

Now we show that if the transparent equilibrium uniquely exists, we have $\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)(1 - H(R_0 + c)) < R_0$. Suppose it is not the case, i.e.

$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)(1 - H(R_0 + c)) \geq R_0. \tag{3.38}$$

Consider first that the strict inequality holds true. Note that, for any $R_1 < R_0 + c$, no borrower reveals information, so $h_O = h$ for any v, and we have

$$Z_O(R_1) = \int_{\underline{v}}^{R_1} vh(v)dv + R_1\big(1 - H(R_1)\big) < \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0 + c)(1 - H(R_0 + c))$$

Then under the strict inequality of (3.38), there exists some $R_1^* < R_0 + c$, such that $Z_O(R_1^*) = R_0$.

If
$$\int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)\big(1 - H(R_0+c)\big) = R_0,$$
then consider $h_O = h$ for all v (and thus $h_T = 0$), and $R_1^* = R_0 + c$, which guarantee the existence of another equilibrium (corresponding to *3.* of Definition 3.1) other than the transparent one, since we have
$$Z_O(R_1^*) = \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)(1 - H(R_0+c)) = R_0.$$

That is, condition (3.38) violates the uniqueness of the transparent equilibrium in either of the cases. Therefore, condition (3.16) is also the necessary condition of the unique existence of the transparent equilibrium. ■

Proof of Proposition 3.3. As we have shown in the previous propositions, condition (3.17) is sufficient to assure the existence of a transparent equilibrium - the expected gain from the opaque market if it exists is too low for any lender to invest in opaque projects, so good borrowers choose to be transparent and are financed while bad borrowers who do not reveal information cannot be funded. That is, there exists a transparent equilibrium.

Besides, condition (3.18) implies that there exists $R_1^* < R_0 + c$ such that $Z_O(R_1^*) = R_0$ when all borrowers are opaque, and therefore exists an opaque equilibrium.

Then, we show that condition (3.17) and (3.18) imply the third case stated in Proposition (3.3), where $R_1^* = R_0 + c$. By lemma 3.2, borrowers with $v \geq R_0 + c$ are indifferent between reveal information or not. We can see that the more good borrowers choose to be opaque, the higher the profit of a lender is, since more opaque projects imply less cost of dissipative signaling. Thus, the maximum profit a lender may get is when all borrowers are opaque, i.e., $h_O = h$ for all $v \geq R_0 + c$. Thus, we have
$$max\ \pi_L^O = \int_{\underline{v}}^{R_0+c} vh(v)dv + (R_0+c)[1 - H(R_0+c)] > R_0.$$

On the other hand, the minimum of lender's profit on opaque market is when all good borrowers are transparent, i.e., $h_O = 0$ for all $v \geq R_0 + c$. So we have

$$min\ \pi_L^O = \frac{\int_{\underline{v}}^{R_0+c} vh(v)dv}{H(R_0+c)} < R_0.$$

Therefore, there exists a h_O^* (and thus H_O^*) such that

$$\int_{\underline{v}}^{R_0+c} \frac{vh(v)}{H_O^*(\bar{v})}dv + (R_0+c)\frac{(H_O^*(\bar{v}) - H_O^*(R_0+c))}{H_O^*(\bar{v})} = R_0$$

where $h_O^* = h$ for any $v < R_0 + c$ and $0 < h_O^* < h$ for $v \geq R_0 + c$. In this case, all opaque projects can be financed with repayment $R_1^* = R_0 + c$ and all transparent projects are financed with loan repayment R_0. ■

Lemma. *For $\hat{R}$ and $\check{R}$ such that*

$$\begin{aligned}\phi(\hat{R}) &= \int_{\underline{v}}^{\hat{R}+c} vh(v)dv + \hat{R}[1 - H(\hat{R}+c)] = \hat{R}, \\ \psi(\hat{R}) &= \int_{\underline{v}}^{\check{R}+c} vh(v)dv + (\check{R}+c)[1 - H(\check{R}+c)] = \check{R},\end{aligned}$$

we have $\hat{R} < \check{R}$.

Proof. Note that $(\psi(R) - R)' = -H(R+c) < 0$ and $\psi(\check{R}) - \check{R} = 0$, we know $\psi(R) - R \leq 0$ for any $R \geq \check{R}$. Suppose $\hat{R} \geq \check{R}$, we have $\psi(\hat{R}) \leq \hat{R} = \phi(\hat{R})$, which contradicts the fact that $\phi(R) < \psi(R)$ for any R. ■

Proof of Proposition 3.5. We know that an opaque stationary state exists if there is some $R < \check{R}$ such that $G^O(R) = F^O(R)$, i.e.

$$1 + \beta^{-\sigma} R^{1-\sigma} = w + \int_{\underline{v}}^{\bar{v}} v\ h(v)dv - R.$$

Note that $F^O(R)$ has a constant slope -1, there exists a solution to the above equation if $G^O(R^*) \leq F^O(R^*)$, where R^* is such that $(G^O)'(R) = -1$. That is,

$$(G^O)'(R^*) = (1-\sigma)\ \beta^{-\sigma}\ (R^*)^{1-\sigma} = -1.$$

Thus, we have $R^* = (\sigma - 1)^{\frac{1}{\sigma}} \beta^{-1}$, and the condition $G^O(R^*) \leq F^O(R^*)$ becomes

$$1 + \beta^{-\sigma}((\sigma-1)^{\frac{1}{\sigma}}\beta^{-1})^{1-\sigma} \leq w + \int_{\underline{v}}^{\bar{v}} v\ h(v)dv - (\sigma-1)^{\frac{1}{\sigma}}\beta^{-1}.$$

By rearranging we have the condition as in equation (3.33). Note that this is just the condition that ensures the existence of a solution to $G^O(R) = F^O(R)$, the solution is not necessarily smaller than $\check{R}$. Therefore, condition (3.33) is only a necessary but not a sufficient condition of the existence of an opaque stationary state. ■

Chapter 4

Holdup and hiring discrimination with search friction

Coauthored with Mr. Sheng Bi

Abstract

A holdup problem on workers' skill investment arises when employers can adopt discriminatory hiring norm to extract higher than socially optimal profit. When hiring priority is determined both by productivity-dependent (skill level) and -independent characteristics (discrimination), skill investment decision becomes strategic between the discriminated and favored group. We consider frictional markets with posted wage. Depending on market tightness there may be equilibrium or multiple equilibria with different choices of skill investment. With discriminatory hiring, if in equilibrium in which both groups stay high skilled, both are worse off and firms are better off; in any equilibrium in which one group underinvests in skills, the other group remains high skilled and is better off, while firms are worse off. We further discuss on the problem in wage bargaining (fixed sharing rule) context. With bargained wage, similar equilibrium in which the favored group underinvests exists, and firms incur cost for an intermediate range of bargaining power when they discriminate.

4.1 Introduction

A holdup problem arises when some investment is sunk ex ante by one party, and the payoff is shared with that one party's trading partner. Since cost has no other use once sunk, that trading partner will have every incentive to squeeze the profit at the ex post stage. In an important study on such a problem in a labor market with search friction, Acemoglu and Shimer (1999b) shows that with firms' sinking capital and ex post wage bargaining, the equilibrium is always inefficient, since wages paid ex post can be so high such that firms' ex ante incentive of investment is impaired; while if firms are able to post wages to direct workers' search, then the holdup problem to firms' investment no longer appears; the efficiency can be achieved, because wage posting allows workers to observe offers and choose where to apply, and it induces workers to optimize their expected payoff from application by making trade-off between every wage they observe and the probability of obtaining it. Within conventional wage posting framework, we spot another source of inefficiency in a holdup problem where workers sink skill investment cost: when the market is crowded for the firms, by adopting a discriminatory hiring norm, firms are able to expropriate higher profit than socially optimal level, and this has the consequence of discouraging the investment incentives for both the favored and discriminated groups. We analyze the impact of such rent seeking behavior of firms on the structure of market segmentation, and on the workers' skill investment incentives.

When discrimination is absent, the wage posting economy with workers' ex ante skill investment attains efficiency in the equilibria, and we show which equilibrium emerges depends on the rivalry between the log return to skill and the market tightness which measures the degree of market competition. The fundamental reason behind this efficiency result is that skill achievement is a quality which can be legally written into the wage contracts. It is a different story when other (binary) characteristics which are not closely related to productivity, such as gender, race, height, origin etc. enter also into firms' preference. Under equal pay legislation, posted wages can not be conditioned explicitly on these characteristics; however, if firms still select workers according to their preference on these characteristics, a separating equilibrium results where separate firms post different levels of wages, and workers of different groups sort themselves and apply to different wages: the market is then endogenously segregated. On the side of firms, they have incentive to adopt such discriminatory hiring norm, when workers' return to skill investment is sufficiently high; in that case discrimination allows them to grasp higher operating profit than the socially optimal level. On the side of the workers, it proves that both the discriminated and favored group are worse off: for the former, it is because discrimination discretely reduces the labor market opportunity of these workers, who anticipate discrimination, then demand lower wages, which makes them cheaper to hire; for the latter, it is so because when firms are able to hire the discriminated workers cheaply, it is as if firms enjoy larger "market power", which allows them to suppress further the unfavored workers' expected payoff. Naturally, anticipating discrimination, all groups expect lower payoff from search, jeopardizing their skill investment incentives.

A key feature of our study is the multidimensionality of characteristics based on which workers are ranked. On one hand, there is ranking by productivity-dependent type identity: workers are either high skilled (type H) or low skilled (type L); the high skilled have priority to low skilled simply because firms' profit is increasing in productivity. On the other hand, there is ranking by productivity-independent group identity: workers belong either to the favored group (group a) or the discriminated group (group b). The resulting ranking schedule has the following order: $aH \succ bH \succ aL \succ bL$. It reads: given any skill level, group a are preferred to group b; the high skilled is always preferred to the low skilled. Under such an "intertwined" ranking order, the skill investment decision for different groups becomes strategically interdependent. Focusing on Nash pure strategy equilibrium on skill investment, in the wage posting economy, we find that depending on the value of market tightness there can be equilibrium or multiple equilibria on skill investment due to that interdependence. Compared to the case without discrimination, when the market is very crowded (market tightness is small) for the firms, discrimination is profitable for firms and all the workers are worse off; as the tightness further increases, both group can choose low skill and in equilibrium whenever one group underinvest, the other group remains high skilled and is better off, while the firms are worse off with discrimination. In particular, there exists an equilibrium in which the favored group underinvests while the discriminated group remains high skilled; and in this case firms' profits drop since workers' underinvestment in skill leads to lower average productivity compared to the case in which discrimination is absent.

In the economy where wages are bargained (determined according to a fixed sharing rule) after matching hence do not direct search, we find similar equilibrium where the favored group underinvest, hence earn lower expected payoff compared to the case without discrimination within a certain region of bargaining power; in such an equilibrium, surplus is transferred from firms and favored group to discriminated group. Firms' profits are piecewise monotone, because increase of workers' bargaining power can increase workers' incentive of skill investment, hence discretely improves the market skill composition and average productivity. We also find that there is an intermediate range of workers' bargaining power for values of which firms are worse off by discriminating, due to discouraged skill investment from discriminated group. All in all, the key difference between wage posting and wage bargaining is that the actual wage now exogenously pegs on the productivity, and firms can no longer manipulate their market power by translating their discriminatory preference into constantly lower wages.

Job search process is an important channel through which discrimination keeps functioning in the labor market. Several papers have highlighted the impact of discrimination through job search channel to the wages gaps. To name a few, Pendakur and Woodcock (2010) show that the existent glass ceilings for the immigrant and minority workers may be attributed by large measure to the poor access to the jobs in high-wage firms; As well, in an important article from Ritter and Taylor (2011), they show that most of the disparity in unemployment rate could not be explained by cognitive skills that emerge at an early stage, although for wage gap it could be the case. This result concerning the unemployment disparity is confirmed by the finding that this disparity is still significant even for workers of similar skill levels.

Our work is most closely related to the directed search literature[1]. In this literature, search frictions are derived endogenously through agents' sequential strategic interactions. Taking into account strategic interaction allows the search externality to be internalized. The resulting economy remains competitive, albeit with a non-Walrasian market structure, and prices play an allocative role to achieve efficiency. To the best of our knowledge, among the discrimination literature with search friction, only two of them are built upon wage posting context. Lang, Manove, and Dickens (2005, hereafter LMD) show that a discriminatory hiring rule could lead to labor market segmentation and significant wage gap with even a negligible difference in productivity; however, their discriminated group turn out to have lower unemployment rate, which is in sharp contrast with evidence. Merlino (2012) aims at improving the result of LMD (2005). He considers further the pre-matching investment from the firms' side, and obtains technology dispersion and realistic unemployment gap. His results rely on the strong assumption that there is more discrimination in the high technology sector, and he is silent on the workers' skill levels. Our paper differs from theirs, in that our focus is to analyze

[1]This literature is sometimes also termed as wage posting game with coordination friction

how hiring discrimination could distort the structure of market segmentation, and workers' skill investment incentives.

While the setup of wage bargaining (no information of level of wage before matching) is more prevalent, it neglects an important trade-off that the workers make to some extent in the search for jobs: the wage and the probability of obtaining it. This endogenous link between wage and employment probability is especially important, since wages convey information on whether the employers discriminate or not. Having information of wages available before matching, workers are able to adjust accordingly their search strategy to avoid being discriminated. Workers apply to certain wage only when their expected payoff (wage times the employment probability) from this application attains certain level, and a high wage which attracts also the favored group discretely lowers the probability of employment for the discriminated group to such an extent that their expected payoff at these high wage firms does not meet the expected market payoff. This setup is supported by Lang and Lehmann (2012) and Heckman (1998), who mention that workers do not apply randomly and they actually avoid prejudiced employers to some extent, which implies between-group search externality is taken into account by the discriminated workers. Moreover, it is well known that within-group search externality may be prevalent when wages are bargained; while in wage posting context, we are able to abstract from search externality and focus on discrimination.[2] Hall and Krueger (2010) use U.S. data to show that fraction of posted and bargained wages are both around one third. They also document a negative relationship between the education level and precise information concerning the expected pay. Brenzel, Gartner and Shnabel (2013) focus on the employer's side of the study in Germany, and showed that around two thirds of the wages are posted, and the bargained wages are more likely set for those with higher education and qualification. The message is that not only is wage posting a prevalent wage determination process in the labor market, more importantly, it is also dominant in the relatively low skilled sector.[3] Within our context, employers can not post wages contingent on workers' group identity which is irrelevant to productivity, which could be understood as due to the functioning of the equal opportunity legislation.

Literature addressing discrimination problem in random search context is vaster. However, to have tractable such model convenient for linking to evidence, the introduced discrimination is usually taste-based, hence to obtain realistic

[2]By focusing on posted wage, efficiency in wage determination is guaranteed (because strategic interaction is taken into account) and we are able to focus on the effect of discrimination.

[3]It is consistent with our knowledge that the more skilled workers, whose number is comparatively small, usually receive more attention and protections.

outcome may often require making compromise on assuming ex ante differences in parameters governing relevant characteristics. Rosen (1997) is an exception and shows that discrimination can result even if there are no differences across groups. Job opportunities arrive stochastically, minority workers choose reservation productivities above which they accept the job; to avoid majority workers who are always preferred, they choose to accept jobs even with low reservation wages. Although private information is the key element in Rosen (1997)'s model, search externality remains the main channel for the functioning of the discrimination mechanism. Our focus is on how the ranking order of firms contributes to strategic interdependence in workers' skill investment decisions, and search externality is internalized when search is directed.

There is also the important statistical discrimination literature[4] which emphasizes the role of asymmetric information on qualities related to the productivity. One strand of this literature derives group inequalities endogenously even in the absence of ex ante group difference on relevant characteristics. Their mechanism is that decision makers' asymmetric beliefs on relevant characteristics of members for different groups could subsequently dim unfavored agents' incentive on investment on payoff-relevant technology, which in turn justifies the firms initial beliefs. Our context is different from this literature mainly in the point that, instead of relying on the information friction which plays central role in generating the pessimistic outcome, we work through a sequential game where agents could correctly anticipate the pessimistic outcomes, hence choose to react accordingly in a rational way.

The paper is organized as follows. Section 4.2 analyzes the case without discrimination. We then move to the economy with discrimination: section 4.3 explains the basic setups with discrimination; and we then examine how discrimination alters the incentives of workers' skill investment. In section 4.4 we further discuss the problem in a wage bargaining context and provide further remarks on free-entry of firms and heterogeneity of skill investment cost. And finally, section 4.5 concludes this chapter.

4.2 The model without discrimination

We start with a context without hiring discrimination. Consider an economy populated by two types of agents, workers and firms. The number of workers is

[4] We refer readers to the survey from Fang & Moro (2010)

N[5], with the index $i \in \{1, 2, ..., N\}$, and the number of firms is M, with the index $j \in \{1, 2, ..., M\}$. Define the market tightness as $\beta := \frac{N}{M}$.

We introduce a pre-matching skill investment stage into a standard wage posting game. Each job seeker makes a skill investment decision before entering the labor market, and the skill choice is binary - either to be high skilled or low skilled, with a cost E_H and E_L for investing in high skill and low skill, respectively; $E_H > E_L = 0$. A high skilled job seeker who pays E_H is capable of producing y_H, while a low skilled one can only produce y_L. This can be understood in the following way: workers who enter into labor market after a longer period of training (schooling) pay higher opportunity cost and are averagely expected to be more productive, compared to those who spend a shorter period on schooling and enter the labor market at an earlier stage. The workers' skill level is public information, and both the costs $\{E_L, E_H\}$ and productions $\{y_L, y_H\}$ are exogenous.

Firms are ex ante identical. Having observed the skill attainment of job seekers, firms post wages conditional on skills. If firms choose to attract a high skilled worker, they post wage w_H, and the surplus at the ad interim stage is $y_H - w_H$. In case hiring a low skilled worker, firms post wage w_L and the ad interim profit from hiring this worker is thus $y_L - w_L$[6]. Skill level is a characteristic of workers which the wage contracts can be conditioned on. This characteristic is in contrast to other qualities such as gender, race, height, etc., which should not be conditioned on under equal pay legislation; when firms distinguish workers according to these latter qualities, the wage contract becomes "incomplete"[7]; by this, we say that the firm discriminates.

The timing follows a standard wage posting game, augmented with a pre-matching skill investment stage (stage 0). More precisely,

Stage 0: *Workers choose skill level, either L or H, and pay E_L or E_H accordingly.*
Stage 1: *Firms observe the skills of job seekers and announces wage (w_L, w_H).*
Stage 2: *Workers observe wage offers, and choose which wage to apply to.*
Stage 3: *Firms select workers from the received applications, and they select workers with identical skills with equal probability. Then the production is carried on and payoffs are realized.*

[5]As noted by Lang, Manove, and Dickens (2005), the number N could be regarded as the expected number of job seekers from the firms' perspective.

[6]It would be useful to consider the firms adopting a skill-biased technology with productivity y. A high skilled worker succeeds to produce y with probability p_H, and therefore the expected productivity is $y_H = p_H y$, and a low skilled gets the output with probability p_L, and $y_L = p_L y$; $p_H > p_L$. This formulation is adopted by Shi (2002, RES).

[7]Incompleteness of contract is the source of inefficiency for the holdup problem. See Acemoglu and Shimer (1999) for related literature.

We will focus on subgame perfect competitive equilibria (SPCE). Firms choose wages to maximize profits, and workers maximize their expected payoff by choosing firstly the skill level and then which wage to apply to.

4.2.1 Strategies, matching probabilities and payoff functions

In the skill investment stage, workers choose from skill levels L and H. Let α be the fraction of the job seekers who choose to invest in high skill, and the remaining fraction $(1-\alpha)$ is low skilled[8]. Then, having observed the skill levels of workers, firm j posts wage $w^j(\alpha) = (w_H^j, w_L^j)$. Denote $\mathbf{w}(\alpha) = (w^1(\alpha), ..., w^M(\alpha))$. Define a type t job seeker i's strategy as a vector of probabilities $\Theta_t^i(\mathbf{w}) = (\theta_t^{i1}(\mathbf{w}), ..., \theta_t^{iM}(\mathbf{w}))$, where θ_t^{ij} is the probability with which the type t worker i applies to firm j, and $t \in \{L, H\}$. It holds true that $\sum_j \theta_t^{ij} = 1$ for any i and t. Therefore, an equilibrium is characterized by $\Big(\alpha^*, \mathbf{w}^*(\alpha^*), \big(\Theta_t^{i*}(\mathbf{w}^*)\big)_{i=1,2...N}\Big)$.

Since we only consider symmetric equilibria, for a given firm j, θ_t^{ij} has the same value for any type t job seeker, so we denote $\theta_t^{ij} = \theta_t^j$ for any j. As in the literature, it is convenient [9] to proceed with a transformation of variable. We define q, as the *expected number of applications received by the firm*; it is also called *the expected queue length.* Denote q^j as the queue length of firm j, and q_t^j as the queue length of the type t workers in firm j. If a firm attracts both high and low skilled workers, we have $q^j = q_L^j + q_H^j$, where q_L^j and q_H^j are the queue lengths of type L and type H workers applying to firm j, respectively. By definition, q_t^j is equal to the number of workers of type t times their application probability: $q_H^j = N_H \times \theta_H^j$, and $q_L^j = N_L \times \theta_L^j$ for any j, where N_L and N_H are the total number of workers of type L and H respectively.

Job seekers. Job seekers observe all the wages $\mathbf{w}$ announced by the firms, and choose which wage to apply to. To derive the employment probability of a particular job seeker, we first consider the case in which the firm that this worker applies for posts a single wage to attract a single type of workers. Then this type t

[8] Since the skill choice is binary, we can always interpret the profile of all workers' skill levels as a fraction of certain type among all workers. In a large market, α can also be regarded as the probability with which a job seeker chooses to invest in high skill by virtue of the Law of Large Number (for the sake of symmetric equilibrium on the workers' side, we have $\alpha^i = \alpha^{i'}$ for any $i, i' \in \{1, 2, ..., N\}$).

[9] When the number of firms and workers are large, it is no longer convenient to operate with the workers' application strategy θ_i^j, because it will tend to zero in the symmetric mixed strategy equilibrium.

worker's probability of employment in that firm is, for any $t \in \{L, H\}$,

$$Pr_t^{emp} = \frac{1-(1-\theta_t)^{N_t}}{N_t\theta_t} \;\rightarrow \frac{1-e^{-q_t}}{q_t}, \text{ when } N, M \rightarrow \infty,$$ [10]

since $q_t = N_t\theta_t$. Notice that is $\frac{1-e^{-q_t}}{q_t}$ is decreasing in q_t: the higher the expected number of applicants is, the lower the probability that this job seeker can be employed is. When a firm attracts both types of workers, the employment probabilities of high and low skilled workers are,

$$Pr_H^{emp} = \frac{1-e^{-q_H}}{q_H}, \text{ and } Pr_L^{emp} = e^{-q_H}\;\frac{1-e^{-q_L}}{q_L},$$

respectively. Pr_H^{emp} is the same as before due to their priority in the hiring, which represents the within group competition effect, while the employment probability of low skilled workers, Pr_L^{emp}, has an extra term e^{-q_H}, which is the probability with which no high skilled worker applies to this firm, and which governs the between group competition effect.

Note that since q is a function of job seekers' application strategy θ, it depends on, or more precisely, it is induced also by $\mathbf{w}$. We now look more closely into their causal relationship. We first distinguish two terms: (1) each job seeker's *expected income* from applying, u_t, and (2) the *expected "market" income*, U_t. We refer to the *expected income* as the expected payoff that a worker gains when applying to a certain firm, namely, the product of the probability of his being employed by this firm and the wage he gets in the firm. Thus, we have $u_t = Pr_t^{emp}\; w_t$ for $t \in \{L, H\}$. The *expected "market" income* is the level of the expected income that workers have in the equilibrium. Due to the large number of firms and the competition among them, U_t is independent of a single firm's wage posting strategy, and thus any firm's wage posting that provides an expected income lower than the "market" level is unattractive to workers (and therefore this "market" level can also be understood as the reservation wage).

Therefore, a particular type t job seeker is willing to send application to a firm j only if his expected income from applying to the firm, $u_t^j = Pr_t^{emp,j}\; w_t^j$, is greater than or equal to the expected market income U_t. In fact, U_t is also the maximum of all possible expected incomes of type t workers from applying. Since any firm who could offer w_t such that $Pr_t^{emp}\; w_t > U_t$ would attract workers' application away from the original firms, and U_t would not be the expected income level in an

[10]Without a superscript j, we mean to refer to an arbitrarily firm. See appendix for the detailed derivation.

equilibrium. Therefore, we can rule out the case in which $Pr_t^{emp,j}\ w_t^j > U_t$, and we have

$$q_t^j \begin{cases} > 0, & \text{if } Pr_t^{emp,j}\ w_t^j = U_t, \\ = 0, & \text{if } Pr_t^{emp,j}\ w_t^j < U_t. \end{cases} \tag{4.1}$$

Job seekers make trade-off between the wage and the probability of obtaining it. From 4.1, we have the workers' strategy in the job application subgame, $q_t^j(\mathbf{w})$, satisfies

$$q_t^j \begin{cases} > 0, & \text{if } w_t^j > U_t, \\ = 0, & \text{if } w_t^j \leq U_t, \end{cases} \tag{4.2}$$

since the employment probability $Pr_t^{emp,j} < 1$. It is now more straightforward to see that U_t is alike the reservation wage, above which the job seekers are willing to apply to.

Firms. A firm can match with a type t worker if at least one type t worker appears ex post, which happens with probability $Pr_t^{Hir} = 1-(1-\theta_t)^{N_t}$ (the *hiring probability*), since the probability with which no type t job seeker sends application to this firm is $(1-\theta_t)^{N_t}$. Note that $q_t = N_t\theta_t$, we have

$$Pr_t^{Hir} \ \rightarrow \ 1-e^{-q_t}, \ \text{ when } N_t \ \rightarrow \infty.$$

This probability is increasing in q, showing that the more the expected number of applicants, the higher the probability that the firm can fill the vacancy.

Firms post wages to maximize their expected payoff. The expected payoff from attracting a type t worker is the product of the probability of meeting a type t worker and the net profit, i.e., $\pi_t := (1-e^{-q_t}) \times (y_t - w_t)$, for $t \in \{L, H\}$. Given that both types appear in the market, if firms post both w_L and w_H to attract both types, as shown by Shi (2006), they rank the high skilled in priority to the low skilled. Then the total expected payoff *(from attracting both types of workers)* is thus $\pi_{LH} := (1-e^{-q_H})(y_H - w_H) + e^{-q_H}(1-e^{q_L})(y_L - w_L)$, where e^{-q_H} is the probability of no type H job seeker applies to this firm, since the firm considers hiring low skilled workers only when they do not receive any application from the high skilled.

Solution of wage posting subgame. The solution concept we adopt is the subgame perfect competitive equilibrium (SPCE), and one way of solving the equilibrium for the wage posting subgame is as shown in Burdett, Shi and Wright (2001). Firms choose wages to maximize their expected profit, taking into account the best responses of other firms as well as of the job seekers. As we consider a large economy in which a single firm's deviation does not alter the expected market income, firms therefore must offer workers a certain level of expected income - the expected market income, which is taken as given in the stage where a firm maximizes his profit and will later be determined endogenously[11].

Thus, given a skill distribution of skill level α, the firm chooses w_t, $t = L$ or H, to maximize his expected profit π_t, taking the expected market income U_t (the other firms' responses) and the functional relationship between w_t and q_t (job seekers' responses) as given. In the case of $\alpha = 0$ or 1, the firm attracts a single type of workers, and the optimal strategy can be solved by

$$\begin{array}{ll} \max_{w_t} & (1-e^{-q_t})(y_t - w_t) \\ \text{s.to} & \frac{1-e^{-q_t}}{q_t}\, w_t = U_t \end{array} \tag{4.3}$$

for $t \in \{L, H\}$. For a given U_t, solving w_t from the constraint, substituting it into the objective function, and maximizing with respect to q_t, we can obtain an optimal functional relationship between q_t^* and U_t^*. Using this obtained relationship, with the help of the constraint, we then obtain an optimal functional relationship between w_t^* and q_t^*. Besides, since in symmetric equilibrium all firms post the same wages, so that all workers apply to each firm with equal probability, by definition of q we have $q_t^* = \frac{N}{M} = \beta$.

In the case of $\alpha \in (0, 1)$, i.e., there are both high and low job seekers in the market, it is optimal for firms to attract both types at the same time (as shown in Shi (2006)), and the wage posting subgame is solved by

$$\begin{array}{lc} \max_{w_H, w_L} & (1-e^{-q_H})\,(y_H - w_H) + e^{-q_H}\,(1-e^{-q_L})\,(y_L - w_L) \\ \text{s.to} & \frac{1-e^{-q_H}}{q_H}\, w_H = U_H \\ & e^{-q_H}\,\frac{1-e^{-q_L}}{q_L}\, w_L = U_L \end{array} \tag{4.4}$$

Similar as above, solving (4.4), we can have

$$U_L^* = e^{-q_H^* - q_L^*}\, y_L \tag{4.5}$$

and

$$U_H^* = e^{-q_H^*}(y_H - y_L) + e^{-q_H^* - q_L^*}\, y_L \tag{4.6}$$

And in the equilibrium we have $q_H^* = \frac{N_H}{M} = \frac{\alpha N}{M} = \alpha\beta$, $q_L^* = \frac{N_L}{M} = (1-\alpha)\beta$, and

[11] As emphasized also in Lang, Manove and Dickens (2005), it is "a simplification of standard subgame perfection in which aggregate variables are assumed constant with respect to the changes in the strategy of an individual agent".

$q_H^* + q_L^* = \beta$. At last, it is important to remark that q_t depends on w_t continuously, as remarked by Shi (2002). In this way, a marginal change of wage w_t can only lead to a marginal modification on the expected number of applicants q_t.

4.2.2 Decentralized market equilibrium without discrimination

In this subsection, we consider the skill investment decision of job seekers at first stage, and establish the decentralized market equilibrium in the nondiscriminatory regime. Let α^* be the fraction of high skilled job seekers in the equilibrium. There are three cases:

Case (1). $\alpha^* = 1$. *All job seekers are high skilled.*

Case (2). $\alpha^* \in (0, 1)$. *Some job seekers invest in high skill, while the other get low skill.*

Case (3). $\alpha^* = 0$. *All job seekers are low skilled.*

With Case (1) and Case (3), there exists only one type of skill level in the market, and thus there is only one wage posted in equilibrium. However, the market with Case (2) features two skill levels. As in Shi (2006), it is optimal for firms to attract both skill types at the same time, while ranking the high skilled in priority to the low skilled. We now show that the rivalry between the market competition (captured by market tightness β) and the magnitude of the return to skill ratio $\frac{y_H - y_L}{E_H - E_L}$ are crucial in the determination of the equilibrium.

Proposition 4.1 (Return to skills). *Given the return to skill ratio* $\frac{y_H - y_L}{E_H - E_L}$*, define* $\hat{\beta}$ *as* $\frac{y_H - y_L}{E_H - E_L} = e^{\hat{\beta}}$*. Then, we have*

(i) When $0 < \beta \leq \hat{\beta}$*, i.e.* $\frac{y_H - y_L}{E_H - E_L} \geq e^{\beta}$*, there exists a unique equilibrium in which all job seekers choose to obtain high skill, i.e.* $\alpha^* = 1$.

(ii) When $\beta > \hat{\beta}$*, i.e.* $1 < \frac{y_H - y_L}{E_H - E_L} < e^{\beta}$*, there exists a unique* $\alpha^* \in (0, 1)$ *such that* $\frac{y_H - y_L}{E_H - E_L} = e^{\alpha^* \beta}$*, and thus a unique equilibrium with* $\alpha^* \in (0, 1)$.

(iii) When $\frac{y_H - y_L}{E_H - E_L} \leq 1$ *such that* $\frac{y_H - y_L}{E_H E_L} = e^{\hat{\beta}}$ *has no positive solution, there exists a unique equilibrium in which all job seekers are low skilled,* $\alpha^* = 0$.

Proof. See appendix.

When the value of return to skill $\frac{y_H - y_L}{E_H - E_L}$ is sufficiently large compared to e^{β}, which measures the intensity of competition of the market, job seekers find it a dominant strategy to invest in high skill. There is no incentive for them to deviate to low skill investment, and the output is highest among all the equilibria. When the value of $\frac{y_H - y_L}{E_H - E_L}$ is moderate, there exists an equilibrium where job seekers are indifferent from being high skilled or low skilled; all firms find it optimal to attract both skill types and the output is lower compared to the previous equilibrium. At last, when the value of return to skill is sufficiently low, it does not provide them incentive to sink this fixed cost against the risky job search game they are going to play; the equilibrium level of output turns out to be the lowest.

4.2.3 Constrained efficient allocations

The objective of this subsection is to find the efficient allocations in the centralized market, and evaluate whether the decentralized market attains its efficiency. The social planner maximizes the aggregate output, subject to the same matching friction as in the decentralized equilibrium. More precisely, the social planner chooses the fraction of workers to be high skilled, divides firms into different groups to attract distinct compositions of workers, and assigns workers to match with a certain group of firms. With the same matching friction as before, the social planner is restricted to treat workers of the same skill level in the same way, and assures that workers of the same type must match with firms from the same group with the same probability.

Let α^P be the fraction of high skilled workers the social planner chooses, $\alpha^P \in [0, 1]$. Note that if the optimal arrangement $\alpha^{P*} = 1$, all job seekers are high skilled, and only one type of firms exists - those which attract high skilled workers. It is similar for $\alpha^{P*} = 0$. If $\alpha^{P*} \in (0.1)$, there are both high and low skilled job seekers and it is optimal for the planner to assign all firms to post wages for both the high and low skilled (as shown in Shi (2006)). Furthermore, in the last case the planner can also manage the priority of firms' hiring differently skilled workers - whether to prefer high skilled to low skilled or otherwise. Let x^P be the probability of the firms ranking high skilled worker in priority to the low skilled. And q_t^P is the expected number of applicants in a firm, $t \in \{L, H\}$, which governs how the planner assigns workers' applications.

Thus, the social planner's problem is to maximize the aggregate output as follows.

$$\max\ (1 - e^{-q_H^P})\big(e^{-q_L^P} + x^P(1 - e^{-q_L^P})\big)y_H + (1 - e^{-q_L^P})\big(e^{-q_H^P} + (1 - x^P)(1 - e^{-q_H^P})\big)y_L \\ - \beta\ (\alpha^P E_H + (1 - \alpha^P)E_L).^{12} \qquad (4.7)$$

If at least one high skilled visits a certain firm, with probability $(1 - e^{-q_H})$, the firm hires a high skilled worker, either with probability 1 when no low skilled worker shows up, which happens with probability e^{-q_L}, or with probability x^P if there is at least one low skilled worker who shows up at the same firm, which occurs with probability $(1-e^{-q_L})$; it is similar for the case with low productivity. Since firms and workers of the same skill are all identical from the planner's perspective, we have $q_H^{P*}=\alpha^{P*}\beta$ and $q_L^{P*}=(1-\alpha^{P*})\beta$, and the above objective includes all cases with different values of α^P. Solving the problem, we can see that the optimal ranking is that firms always prefer high skilled workers, i.e., $x^{P*}=1$, and we have the following proposition.

Proposition 4.2 (Social Optimality)**.** *The skill investment and the labor allocation are socially optimal.*

Proof. See appendix.

In the proof, we can also verify that the threshold $\hat{\beta}^P$ for skill investment of the social planner coincides with $\hat{\beta}$ in the decentralized economy. That is, when $0<\beta\leq\hat{\beta}=log\frac{y_H-y_L}{E_H-E_L}$, it is socially optimal that workers all invest in high skills; when $\beta>\hat{\beta}$, it is socially optimal to that a fraction of $\alpha^{P*}=\alpha^*$ workers invest in high skill, while the rest invest in low skills; and when $\frac{y_H-y_L}{E_H-E_L}\leq 1$ such that $\hat{\beta}$ has no positive solution, all invest in low skill.

In the rest of the paper, we mainly focus on the first case, so that whenever workers are discouraged to underinvest, it is due to the effect of discrimination.

Assumption 4.1. *Assume that* $\frac{y_H-y_L}{E_H-E_L}\geq e^{\beta}$*. That is,* $0<\beta\leq\hat{\beta}$*.*

Under above assumption, all workers choose to be high skilled due to the high rate of skill return. And the firms' and workers' expected income are summarized in the following corollary.

Corollary 4.1. *Under Assumption 4.1, we have that the expected profit of firms in the equilibrium is* $\pi_H^*=(1-e^{-\beta}-\beta e^{-\beta})\,y_H$*, the wage of workers in the equilibrium is* $w_H^*=\frac{\beta e^{-\beta}}{1-e^{-\beta}}\,y_H$*, and the expected income of workers is* $U_H^*=e^{-\beta}y_H$*.*

Proof. See appendix.

4.3 The model with hiring discrimination

We now introduce discrimination. Consider an economy where workers can be partitioned into two groups, group a and group b, according to certain trait which is irrelevant to productivity. Gender, for example, is such one possible binary partition of labor force. Denote γ as the fraction of group a workers, and the fraction of group b workers is $1-\gamma$. The two groups of workers are ex ante identical in all other aspects.

Discrimination modifies the matching functions of agents. Specifically, in order to formulate discrimination, we introduce a term x called hiring (ranking) rule specified by firms. To be precise, x could be understood as the probability with which the group a workers are selected when both groups are present. Notice that when job seekers consider their probability of being hired, they have to take into account of the impact from the competition with the other group. The probability that a group a worker is employed by this firm (regardless of skill difference for the moment) is

$$F_a(q_a, q_b, x) := \frac{1-e^{-q_a}}{q_a}\left(e^{-q_b} + x(1-e^{-q_b})\right).$$

Analogously, the probability that a group b worker is employed by this firm is

$$F_b(q_a, q_b, x) := \frac{1-e^{-q_b}}{q_b}\left(e^{-q_a} + (1-x)(1-e^{-q_a})\right).$$

Then the part $\frac{1-e^{-q_a}}{q_a}$ and $\frac{1-e^{-q_b}}{q_b}$ capture the within group competition, while the remaining parts with x capture the between group competition.

Notice that when $x = 1$, firms hire group b workers only when none of the group a workers is present. Firms always prefer group a to group b, although they have identical productivity. This is what we consider as discrimination of hiring. The employment probability for group a and group b workers become, respectively,

$$F_a(q_a, q_b, 1) = \frac{1-e^{-q_a}}{q_a} \tag{4.8}$$

and

$$F_b(q_a, q_b, 1) = \frac{1-e^{-q_b}}{q_b} \cdot e^{-q_a}. \tag{4.9}$$

Another interesting example is $x = \frac{1}{2}$. The employment probabilities of the two groups become symmetric, i.e.,

$$F_a(q_a, q_b, \frac{1}{2}) = \frac{1-e^{-q_a}}{q_a}\left(e^{-q_b} + \frac{1}{2}\,(1-e^{-q_b})\right)$$

and

$$F_b(q_a, q_b, \frac{1}{2}) = \frac{1-e^{-q_b}}{q_b}\left(e^{-q_a} + \frac{1}{2}\,(1-e^{-q_a})\right).$$

Note that when $q_a = q_b$, the employment probabilities of both groups are equal, $F_a(q_a, q_b, \frac{1}{2}) = F_b(q_a, q_b, \frac{1}{2})$, which is the case we consider as the one without discrimination, and $x = \frac{1}{2}$ is therefore considered as the hiring rule without discrimination for such a case. Furthermore, we have $F_a(q_a, q_b, 1) \geq F_a(q_a, q_b, \frac{1}{2})$ for any $(q_a, q_b) \in \mathbb{R}^2_+$. That is, the employment probability of workers from the preferred group (group a) is higher under discrimination than that in the case without discrimination. Similarly, we can see that $F_b(q_a, q_b, 1) \leq F_b(q_a, q_b, \frac{1}{2})$ for any $(q_a, q_b) \in \mathbb{R}^2_+$, i.e., the employment probability of the discriminated group (group b) is lower when there is hiring discrimination.

In fact, the employment probability of group a is increasing in x, whereas that of group b decreases with x. Thus, for given $(q_a, q_b) \in \mathbb{R}^2_+ \setminus (0, 0)$, there exists $\hat{x} \in (0, 1)$ such that $F_a(q_a, q_b, \hat{x}) = F_b(q_a, q_b, \hat{x})$. We then say $\hat{x}$ is the hiring rule without discrimination[13].

Indeed, x measures the intensity of the discriminatory preference. Given q_a and q_b, for $x \in [0, \hat{x})$ firms discriminate group a workers, and for $x \in (\hat{x}, 1]$ firms discriminate group b workers. The closer x approaches to the extremes of the interval $[0, 1]$, the more intensive the hiring discrimination is. And the employment probability of the preferred (discriminated) group is always increasing (decreasing) in the intensity of the discrimination.

In the rest of paper, we focus on the case $x = 1$ such that group a achieves absolute priority to group b, which we refer to as *strong discrimination.*

4.3.1 The case of strong discrimination: $x = 1$

Formally, we introduce two assumptions as Merlino (2012), which help to introduce some heterogeneity that is not productivity-relevant among the labor pool.

Assumption 4.2. *Firms are not allowed to post wages which are dependent on the group identity.*

Assumption 4.3. *Firms prefer group a job applicants in the sense that firms only hire workers from group b when group a workers are not present, i.e. $x = 1$.*

Same as the case without discrimination, workers apply to a firm only when they can obtain the *expect market income* from applying to that firm. Note that under Assumption 4.1, all workers choose to be high skilled if there were no discrimination, and we denote in this section the expected market income of high skilled job seekers

[13] Note that $\hat{x}$ is a function of (q_a, q_b), while q_a and q_b in turn depend on the composition of the two groups of workers, i.e., γ.

from group a and group b as U_{aH} and U_{bH}, respectively. Thus, firms offer wage w_H aiming at attracting high skilled workers, by (4.8) and (4.9), under the following constraints

$$U_{aH} = F_{aH}(q_{at}, q_{bH}) \cdot w_H = \frac{1 - e^{-q_{aH}}}{q_{aH}} \, w_H \text{ }^{14}, \tag{4.10}$$

and

$$U_{bH} = F_{bH}(q_{aH}, q_{bH}) \cdot w_H = \frac{1 - e^{-q_{bH}}}{q_{bH}} \, e^{-q_{aH}} \, w_H. \tag{4.11}$$

With the introduction of discrimination, workers may have the incentive to deviate to low skill even under Assumption 4.1. We start with the case where both group a and group b choose to be high skilled, and then proceed to find the equilibrium of the wage posting game with skill investment, in which workers may choose to invest low skills. Yet before this, we first review the results of Lang, Manove, and Dickens (2005), where they study the case with discrimination but no difference in workers' skill levels (or productivity).

4.3.2 Existing results revisited and reinterpreted

In a context where there are two groups of workers with identical productivity (skill level) and firms strongly prefer one group, group a, to the other, group b [15]. First, LMD show that any subgame perfect competitive equilibrium (SPCE) is separating.

Separating equilibrium. LMD show that there is no wage to which both groups of job seekers apply. More precisely, no wage can maximize firms' profit while attracting both groups of workers (with both constraints (4.10) and (4.11) binding) simultaneously. The equilibrium is separating. That is, *there are some firms posting a higher level of wage attracting only the preferred group (group a), whereas the rest of firms offering a lower wage only attracting the discriminated group b* (see Proposition 2 in LMD). Notice that the discriminated group has always the choice of applying to the high wage; however, they choose not to do so, because they anticipate discrimination in these firms. The most essential results of LMD (2005) are summarized as follows:

(i) For the firms attracting group a workers, the expect profit in the equilibrium

[14] We omit the argument $x = 1$ in F_{aH} and F_{bH}.

[15] We refer to this identical skill level as high skill, H, and later add an extra H in the subscripts (for example, use aH and bH instead of a and b), in order to make the transition to the next subsection more visable.

is given by

$$\pi^*_{aH} = (1 - e^{-q^*_{aH}} - q^*_{aH} e^{-q^*_{aH}})\ y_H, \tag{4.12}$$

and we have

$$U^*_{aH} = e^{-q^*_{aH}}\ y_H, \tag{4.13}$$

$$w^*_{aH} = \frac{q^*_{aH}\ e^{-q^*_{aH}}}{1 - e^{-q^*_{aH}}}\ y_H. \tag{4.14}$$

(ii) For the firms attracting group b workers, the expect profit in the equilibrium is

$$\pi^*_{bH} = (1 - e^{-q^*_{bH}})\ (1 - e^{-q^*_{aH}}) y_H, \tag{4.15}$$

and we have

$$w^*_{bH} = U^*_{aH} = e^{-q^*_{aH}}\ y_H, \tag{4.16}$$

$$U^*_{bH} = \frac{1 - e^{-q^*_{bH}}}{q^*_{bH}} w^*_{bH} = \frac{1 - e^{-q^*_{bH}}}{q^*_{bH}}\ e^{-q^*_{aH}}\ y_H. \tag{4.17}$$

(See equations (22)-(27) in LMD, and a sketch of the proofs of the above results is provided in the appendix.)

Note that separating equilibrium requires firms be indifferent between attracting group a and group b. That is,

$$\pi^*_{aH} = \pi^*_{bH}, \tag{4.18}$$

Let δ denote the fraction of firms who attract group a, then a fraction of $1 - \delta$ firms post low wage and attract group b. The above indifference condition helps to determine δ^* and in turn q^*_{aH} and q^*_{bH}, since $q_{aH} = \frac{\gamma N}{\delta M} = \frac{\gamma}{\delta}\beta$ and $q_{bH} = \frac{(1-\gamma)N}{(1-\delta)M} = \frac{(1-\gamma)}{(1-\delta)}\beta$.

(iii) Furthermore, we have $\delta^* < \gamma$,

$$q^*_{aH} > \beta > q^*_{bH}, \quad \text{and} \quad \frac{q^*_{aH}}{d\beta} > 0,\ \frac{q^*_{bH}}{d\beta} > 0. \tag{4.19}$$

Here we make some remarks on the features of the separating equilibrium. Firstly, the resulted equilibrium allocations are incentive compatible. For any particular group b job seeker, by deviating to applying for w^*_{aH}, the best they could get is $e^{-q^*_{aH}}\ w^*_{aH}$ (when none of the group a workers shows up in the firm he deviates to apply to). However, this deviating payoff is strictly lower than sticking to applying to w^*_{bH} since

$$e^{-q^*_{aH}}\ w^*_{aH} \overset{(4.14)}{=} e^{-q^*_{aH}} \cdot \frac{q^*_{aH}\ e^{-q^*_{aH}}}{1-e^{-q^*_{aH}}}\ y_H \overset{(4.13)}{=} \frac{q^*_{aH}\ e^{-q^*_{aH}}}{1-e^{-q^*_{aH}}}\ U^*_{aH}$$
$$\overset{(4.18)}{=} e^{-q^*_{bH}}\ U^*_{aH} \overset{(4.16)}{=} e^{-q^*_{bH}}\ w^*_{bH} < \frac{1-e^{-q^*_{bH}}}{q^*_{bH}} w^*_{bH} \overset{(4.17)}{=} U^*_{bH}. \qquad (4.20)$$

As for any particular group a job seeker, by deviating to apply for w^*_{bH}, the best they can get is $w^*_{bH} = U^*_{aH}$ by (4.16), which is not larger than that he can get if he does not deviate. Secondly, we do not have the reservation wage structure, which requires that workers apply to any wage that is higher than the certain reservation value. In this separating equilibrium, group b job seekers apply merely to the low wage w^*_{bH} but not to w^*_{aH} even it is above their reservation wage. This is because the expected income from applying to the high wage is a strictly dominated strategy for group b: the expected income from applying to w^*_{aH} is too low to match their expected market income U^*_{bH}. Following are several noteworthy properties of such an equilibrium.

Results from LMD (2005): *Compared to the context without discrimination, (1) Both groups have lower expected income; (2) All firms earn higher profits; (3) The expected income of group a and group b are such that* $U^*_{aH} > U^*_{bH}$.

4.3.3 Analysis under our context

In the last subsection, we interpreted the equilibrium of the wage posting subgame given that all workers choose to be high skilled. In this subsection, we study how discrimination leads to different incentives of skill investment for the two groups respectively, and attempt to find the corresponding equilibrium.

An important observation is that the skill decision for group a and group b workers is strategic, and this is a direct consequence of the coexistence of ranking through the productivity-dependent (skill) and productivity-independent (discrimination) traits. Ranking by skills requires that the high skilled worker has the priority; while ranking by productivity-independent traits means that group a has the priority. Although multidimensional characteristics are involved, these two ranking schedules yield a unique market hierarchy:

$$aH \succ bH \succ aL \succ bL.$$

It reads as follows: high skilled group a (aH) is preferred to high skilled group b (bH), who is preferred to low skilled group a (aL), who is then preferred to low skilled group b (bL).[16] The matching probability $\frac{1-e^{-q}}{q}$ captures the intensity of competition within the same type (within type), while the probability e^{-q} captures the intensity of competition from the higher ranked type (between type).

Under Assumption 4.1, all workers choose to be high skilled in the nondiscriminatory regime. Now we see that with hiring discrimination they may have incentive of deviating to low skill. Let α_s be the fraction of group s choosing to be high skilled, for $s = a$ or b. To make the analysis simplified (but without losing the attraction of the model), we assume

Assumption 4.4. *Whenever indifferent, all workers with the same group identity (as a group) choose either high or either low skilled.*

For workers in the same group, whenever indifferent between the two alternatives (L and H), they all randomize towards the same direction: that is to say, we consider the group of workers as a whole, or there is perfect correlation on their skill choices. As a result, α does not represent an individuals probability of choosing high skill, and either $\alpha_s = 1$ or $\alpha_s = 0$. Thus, we have the following four possibilities:

(P1) $\alpha_a = 1$ *and* $\alpha_b = 1$*: group a - high, group b - high;*
(P2) $\alpha_a = 1$ *and* $\alpha_b = 0$*: group a - high, group b - low;*
(P3) $\alpha_a = 0$ *and* $\alpha_b = 1$*: group a - low, group b - high;*
(P4) $\alpha_a = 0$ *and* $\alpha_b = 0$*: group a - low, group b - low.*

To decide the skill investment, workers take into account firms' best response in the wage posting stage to infer the expected income from the application, and compare the payoffs net of the cost of skill investment. In the wage posting subgame, when facing all workers with identical skill level (as in case (P1) and (P4)), firms' optimal strategy is the same as stated in LMD (2005); that is, some firms post a higher wage to attract group a, whereas the remaining firms post a lower wage targeting merely group b. When there are both low and high skilled workers (as in case (P2) and (P3)), firms post wages conditional on skill levels, and it is optimal for firms to attract both skill levels and rank the high skilled in priority to low skilled, as in Shi (2006). We then proceed to find workers' best response

[16] Quantitatively, in terms of employment probability, we have $F_{aH} > F_{bH} > F_{aL} > F_{aL}$ for any $q_{st} > 0$, where $s \in \{a, b\}$ and $t \in \{H, L\}$. That is,

$$\frac{1-e^{-q_{aH}}}{q_{aH}} > e^{-q_{aH}} \frac{1-e^{-q_{bH}}}{q_{bH}} > e^{-q_{aH}-q_{bH}} \frac{1-e^{-q_{aL}}}{q_{aL}} > e^{-q_{aH}-q_{bH}-q_{aL}} \frac{1-e^{-q_{bL}}}{q_{bL}}. \quad (4.21)$$

The inequality comes from the fact that $\frac{1-e^{-q}}{q} > e^{-q}$ and $0 < \frac{1-e^{-q}}{q} < 1$ for any $q > 0$. Note that by defining $\frac{1-e^{-q}}{q} = 1$ for $q = 0$, we can extend (4.21) for all $q_{st} \geq 0$, $s \in \{a, b\}$ and $t \in \{H, L\}$ with weak inequalities.

in the skill investment stage, and in turn the equilibrium in this discriminatory context with skill investment. We will use P1, P2, P3, P4 as the superscript for corresponding equilibrium allocations.

When $\alpha_a = 1$ and $\alpha_b = 1$, workers are composed of type aH and bH. Firms post wages separately, and by (4.13) and (4.17), we have the payoff of group a (i.e. aH) and group b (i.e. bH) are, respectively,

$$V_{aH}^{P1} = e^{-q_{aH}^{P1*}} y_H - E_H, \tag{4.22}$$

and

$$V_{bH}^{P1} = \frac{1 - e^{-q_{bH}^{P1*}}}{q_{bH}^{P1*}} e^{-q_{aH}^{P1*}} y_H - E_H, \tag{4.23}$$

and from (4.19) we have $q_{aH}^{P1*} > \beta$ and $q_{bH}^{P1*} < \beta$.

When $\alpha_a = 1$ and $\alpha_b = 0$, workers are composed of type aH and bL. All firms post two wages to attract both types of workers at the same time. By (4.5) and (4.6), we have the payoff of group a and b are, respectively,

$$V_{aH}^{P2} = e^{-q_{aH}^{P2*}} (y_H - y_L) + e^{-q_{aH}^{P2*} - q_{bL}^{P2*}} y_L - E_H, \tag{4.24}$$

$$V_{bL}^{P2} = e^{-q_{aH}^{P2*} - q_{bL}^{P2*}} y_L - E_L, \tag{4.25}$$

and note that $q_{aH}^{P2*} = \gamma\beta$, $q_{bL}^{P2*} = (1-\gamma)\beta$, and $q_{aH}^{P2*} + q_{bL}^{P2*} = \beta$.

When $\alpha_a = 0$ and $\alpha_b = 1$, workers are composed of different skill levels, aL and bH, and we have similarly

$$V_{aL}^{P3} = e^{-q_{aL}^{P3*} - q_{bH}^{P3*}} y_L - E_L, \tag{4.26}$$

$$V_{bH}^{P3} = e^{-q_{bH}^{P3*}} (y_H - y_L) + e^{-q_{aL}^{P3*} - q_{bH}^{P3*}} y_L - E_H, \tag{4.27}$$

and similarly here we have $q_{aL}^{P3*} = \gamma\beta$, $q_{bH}^{P3*} = (1-\gamma)\beta$, and $q_{aL}^{P3*} + q_{bH}^{P3*} = \beta$.

When $\alpha_a = 0$ and $\alpha_b = 0$, workers are composed of type aL and bL. Both are of the same skill level, firms discriminate and post wages as in LMD (2005), and workers payoffs are

$$V_{aL}^{P4} = e^{-q_{aL}^{P4*}} y_L - E_L, \tag{4.28}$$

and

$$V_{bL}^{P4} = \frac{1 - e^{-q_{bL}^{P4*}}}{q_{bL}^{P4*}} e^{-q_{aL}^{P4*}} y_L - E_L, \tag{4.29}$$

with $q_{aL}^{P4*} > \beta > q_{bL}^{P4*}$.

A pure-strategy Nash equilibrium consists of a profile of skill investment with the property that no single group can achieve a higher payoff by unilateral deviation. The existence of equilibrium depends on the value of β. The payoffs are summarized in table 4.1 in the appendix. By comparing the payoffs under different strategies, we can find the best response of workers in the skill investment. For example, responding to group b choosing to be high skilled, it is optimal for group a to choose high skill if $V_{aH}^{P1} \geq V_{aL}^{P3}$, i.e. $e^{-q_{aH}^*} y_H - E_H \geq e^{-\beta} y_L - E_L$, which is not always true. Note that although both V_{aH}^{P1} and V_{aL}^{P4} are decreasing in β, there could be more than one critical value which equates V_{aH}^{P1} and V_{aL}^{P4}. In order to ensure a single threshold and to avoid unnecessary technical complexity, we further assume that

Assumption 4.5. *For $q_{aH}^*(\beta)$ and $q_{bH}^*(\beta)$ that are solved by (4.18),*

1. *$\exists!\ \hat{\beta}_1$ such that $e^{-q_{aH}^*(\hat{\beta}_1)} y_H - E_H = e^{-\hat{\beta}_1} y_L - E_L$;*
2. *$\exists! \hat{\beta}_2$ such that $\frac{1-e^{-q_{bH}^*(\hat{\beta}_2)}}{q_{bH}^*(\hat{\beta}_2)} e^{-q_{aH}^*(\hat{\beta}_2)} y_H - E_H = e^{-\hat{\beta}_2} y_L - E_L$.*

In fact, if one group chooses to be low skilled, the best response of the other group is always to be high skilled, while the best response to the other's high skill choice depends on the two thresholds (see Figure 4.1). Furthermore, the rise of market tightness β makes workers have stronger incentive to deviate to low skill, and group b is more prone to deviate compared to group a, in the sense that the threshold at which group b begins to contemplate to invest in low skill is lower that for group a.

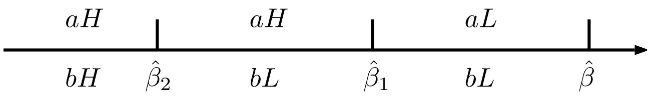

Figure 4.1: Best responses given the other group choosing H.

Focusing only on the pure strategy equilibrium, we formalize the results regarding the existence of equilibrium in the following proposition.

Proposition 4.3. *Under Assumption 4.1 - 4.5, there exist two thresholds $\hat{\beta}_1$ and $\hat{\beta}_2$, with $0 < \hat{\beta}_2 < \hat{\beta}_1 < \hat{\beta}$.*

1. *When $0 < \beta < \hat{\beta}_2$, there exists a unique pure strategy equilibrium in which both group a and group b workers invest in high skill, (aH, bH).*

2. *When $\hat{\beta}_2 < \beta < \hat{\beta}_1$, there exists a unique pure strategy equilibrium in which group a invests in high skill while group b invests in low skill, (aH, bL).*

3. *When $\hat{\beta}_1 < \beta \leq \hat{\beta}$, there exist multiple pure strategy equilibria. Either group a invests in high skill and group b invests in low skill, or group a invests in low skill and group b invests in high skill, (aH, bL) or (aL, bH).*

Interestingly, for values of β close to $\hat{\beta}$, there exists an equilibrium where the preferred group a chooses low skill, while the discriminated group b chooses high skill. We have the following results on the comparison of workers expected payoff and firms profits compared to the case without discrimination.

Note that when the market tightness β meets the thresholds, we can also have the following equilibria. At $\beta = \hat{\beta}_2$, both (aH, bH) and (aH, bL) can be equilibrium; at $\beta = \hat{\beta}_1$, both (aL, bH) and (aH, bL) can be equilibrium.

Corollary 4.2. *Compared the nondiscriminatory regime,*

1. *in (aH, bH) equilibrium, firms always earn higher expected profits, while in (aH, bL) or (aL, bH) equilibrium, firrms earn lower expected profits.*

2. *in (aH, bH) equilibrium, both aH and bH workers earn lower expected payoff, while in (aH, bL) equilibrium group aH (group bL) earns higher (lower) expected payoff and in (aL, bH) equilibrium, group bH (group aL) earns higher (lower) expected payoff.*

Proof. In the Appendix.

4.4 Discussion

4.4.1 Comparision with wage bargaining

Consider an economy with the same discriminatory ranking as previous, but the wage is determined by ex post bargaining after a job seeker meets an employer. In such a context, workers only choose the amount of skills to obtain, buts not where to search. The timing of the economy is now as follows: firstly, workers decide skill levels simultaneously; secondly, workers and firms get matched according to the matching technology; thirdly, the matched worker-firm pair bargain à la Nash to determine how to share the output y.

Denote the bargaining power for all workers as ψ, then workers receive ψy_t and firms receive $(1-\psi)y_t$ (fixed sharing rule). We now focus on the case where ψ is the same for both skill levels. The hiring norm is as previous: $aH \succ bH \succ aL \succ bL$. The employment probabilities for different types of workers are inherited (as in (4.21)), and the expected payoff is the corresponding employment probability times ψy_t net of the skill investment cost. As in the wage posting context, we consider the skill investment under Assumption 4.4, i.e., the whole group choose either high or low skill; and assume that the group a is the majority: $\gamma \geq \frac{1}{2}$[17] .

We focus on Nash equilibrium as the solution concept. In the wage bargaining context, only workers make skill investment decisions, firms do not post wages since ψ is exogenous. Due to the discriminatory rule, the payoffs of skill investment for different groups of workers are interdependent. This renders the skill investment strategic. The equilibrium depends on the bargaining power ψ, as in the following proposition

Proposition 4.4. *Let $\gamma \geq \frac{1}{2}$. There exist four thresholds $\hat{\psi}_{aL,b} \leq \hat{\psi}_{bH,a} < \hat{\psi}_{bL,a} < \hat{\psi}_{aH,b}$, such that*

(1) For $\psi \in [0, \hat{\psi}_{aL,b})$, the unique pure strategy Nash equilibrium is (aL, bL); for $\psi = \hat{\psi}_{aL,b}$, the equilibrium can be (aL, bL) or (aL, bH).

(2) For $\psi \in (\hat{\psi}_{aL,b}, \hat{\psi}_{bH,a})$, the unique pure strategy Nash equilibrium is (aL, bH);

(3) For $\psi \in (\hat{\psi}_{bH,a}, \hat{\psi}_{bL,a})$, there is no pure strategy Nash equilibrium; for $\psi = \hat{\psi}_{bL,a}$, the equilibrium is (aH, bL);

(4) For $\psi \in (\hat{\psi}_{bL,a}, \hat{\psi}_{aH,b})$, the unique pure strategy Nash equilibrium is (aH, bL); at the point $\psi = \hat{\psi}_{aH,b}$, the equilibrium can be (aH, bL) or (aH, bH).

(5) For $\psi \in (\hat{\psi}_{aH,b}, 1)$, the unique pure strategy Nash equilibrium is (aH, bH).

(6) Define the threshold $\hat{\psi}$ of skill investment without discrimination as $\hat{\psi} y_H \frac{1-e^{-\beta}}{\beta} - E_H = \hat{\psi} y_L \frac{1-e^{-\beta}}{\beta} E_L$; then $\hat{\psi}_{bL,a} < \hat{\psi} < \hat{\psi}_{aH,b}$.

According to Proposition 4.4, we can determine the exact values of the queue lengths in the expression. Firms' profit will be piecewise monotone because although ψ increases continuously, the skill composition hence the average productivity of the market improves discretely with respect to this bargaining power. The fact that $\hat{\psi}_{bL,a} < \hat{\psi} < \hat{\psi}_{aH,b}$ suggests that although firms can gather higher profits for $\psi < \hat{\psi}$, they encounter loss for $\psi \geq \hat{\psi}$ compared to the case without discrimination. The reason is that strategic competition between the group a and group b deters the discriminated group's skill investment decision (in the sense that group b may still choose to be low skilled when ψ is sufficiently high), which pulls down the market's average productivity and makes firms' expected profit dim.

[17] $\gamma \geq \frac{1}{2}$ is more empirically relevant when we talk about gender or racial discrimination. The case $\gamma < \frac{1}{2}$ could be also analogously derived.

It is interesting to notice that our simple result that discrimination is costly for firms at high skilled sector (when wages are bargained) questions the plausibility of key assumption of Merlino (2012) that “there is more discrimination in the high technology sector”. Although Merlino (2012) mentioned bunches of empirical evidence in support of this assumption[18], our simple results suggest that firms are simply better off not discriminating when wages are principally bargained, since the loss in profit from discriminating in the high skilled sector may surpass the gain from discriminating in the low skilled sector. All in all, the key difference between wage posting and wage bargaining is that the ex post wage now exogenously pegs on the productivity, and firms can no longer manipulate their market power by translating their discriminatory preference into constantly lower wages.

4.4.2 Free entry

LMD (2005) have shown that their economy under discrimination with workers' identical in productivity can be generalized to take into account firms' free entry. Specifically, we consider a stage where firms sink capital after observing workers' skills. Each firm has different capital cost with $C_1 < C_2 < ... < C_M < y_L$. Then firms which earn expected non-positive profits after the reduction of capital cost would simply not enter into the market. In the paper, we observe that the equilibrium is unique with respect to β, which has a one-one relationship with M - the number of firms in the market, so that the results in the paper could carry through with free entry. All firms in the market expect positive net profits. When there are different skill groups, this result could also carry through, because the equilibrium profit of firms is still an increasing function of β.

4.4.3 Heterogeneity in skill investment cost.

Some preliminary attempts from us suggest that our context could be generalized to a situation where workers are heterogeneous in their skill investment cost (although more complicated): let the low skill investment cost be zero ($E_L = 0$) for all workers, and the high skill investment cost be, for simplicity, of two values $E_{H,1} < E_{H,2}$; there are still two levels of productivity: y_L and y_H. Focus on the corresponding $\hat{\beta}$ and define it as $\hat{\beta} = \log \frac{y_H - y_L}{E_{H,2} - E_L}$. If the contracts can be contingent on $E_{H,1}$ and $E_{H,2}$, the submarkets for type $E_{H,1}$ workers and type $E_{H,2}$ workers are separated, and all the results in the paper carry through for the workers of cost $E_{H,2}$; as for the

[18] See Merlino (2012) page 4 for more relevant reference.

workers of cost $E_{H,1}$, their skill investment cost is lower, hence they have stronger incentive to remain high skilled; then for values of β close to $\hat{\beta} = \log \frac{y_H - y_L}{E_{H,2} - E_L}$, some equilibrium which exists in the $E_{H,2}$ submarket may not exist in the $E_{H,1}$ submarket. If the contracts can not be contingent on $E_{H,1}$ and $E_{H,2}$, both type $E_{H,1}$ and type $E_{H,2}$ are in the same market and will compete; as a result, there may exist a region of β where both high skilled group a and group b, as well as both low skilled group a and group b, exist at the same time. The extent of the skill investment game is in turn larger, because, for example, a particular group "$a, E_{H,1}$"'s skill investment decision should be a best response of other groups: "$a, E_{H,2}$", "$b, E_{H,1}$", and "$b, E_{H,2}$". If heterogeneity in skill investment cost is managed, it is possible to extend the model to multiple skill levels. Shi (2006) shows that in such a model with multiple skill levels free of discrimination, the result that firms always rank the high skilled workers in priority to the workers with lower skills can be generalized. The difficulty under the context with discrimination, as just stated, is on the extent of the game.

4.5 Conclusion

In this paper, we study a holdup problem where firms can use discriminatory hiring norms to extract higher than socially optimal profits. We find that when firms rank workers according to both productivity-dependent and productivity-independent characteristics, skill investment becomes strategic between the discriminated and the favored group. In case where wages are posted, we suggest that depending on the market tightness there may be equilibrium or multiple equilibria on skill investment; in some equilibrium the discriminated group can obtain higher expected payoff compared to the case without discrimination[19]. We also consider fixed sharing rule (bargained wage) and make a comparison. Similar equilibrium, where the favored group underinvests while the discriminated group remains high skilled, exists; however, the discriminated group are in general worse off compared to the case without discrimination in the sense that they may still choose to underinvest when ψ is sufficiently high. Firms' profits are piecewise monotone because the skill composition hence the average productivity of the market improves discretely with respect to the bargaining power, and profit loss may be incurred with discrimination within an intermediate range of bargaining power.

[19] Recall that without discrimination, it is socially optimal.

Appendix 4.A Derivation of matching probabilities.

We now derive a job seeker's matching probability (employment probability) and his expected payoff.

Job seekers. Having observed all the wage $\mathbf{w} = \{w^1, w^2, ..., w^M\}$ announced by the firms, job seekers choose which firm (or wage) to visit (or to apply for). Consider a particular job seeker i's problem, where $i \in \{1, 2, ..., N\}$. The chance of his being employed depends on how many *other* applications arrive in the same firm. This number (of *the other* job seekers who compete with him in the firm) is a random variable which has a realization from the set $\{0, 1, ..., N-1\}$ and has a Binomial distribution.

Let k be the realized number of his competitors. If $k = 0$, which happens with probability $(1-\theta)^{N-1}$,[20] then worker i can be chosen by the firm with probability 1, because this job seeker is the only candidate. If $k = 1$, which happens with probability $(N-1)\ \theta^1(1-\theta)^{(N-1)-1}$, then worker i can be employed with probability $\frac{1}{2}$. Generally, if with k competitors, he can be employed with probability $\frac{1}{k+1}\ C_{N-1}^k\ \theta^k(1-\theta)^{N-1-k}$.

Hence, the employment probability for the workers is

$$Pr^{emp} = \sum_{k=0}^{N-1} \frac{1}{k+1}\ C_{N-1}^k \theta^k (1-\theta)^{N-1-k}.$$

Note that $\frac{1}{k+1}\ C_{N-1}^k = \frac{1}{N}\ C_N^{k+1}$, we have $Pr^{emp} = \frac{1}{N}\sum_{k=0}^{N-1} C_N^{k+1}\ \theta^k(1-\theta)^{N-1-k}$ and

$$\begin{aligned} N\theta\ Pr^{emp} &= \sum_{k=0}^{N-1} C_N^{k+1}\ \theta^{k+1}(1-\theta)^{N-1-k} \\ &= -(1-\theta)^N + \sum_{k=0}^{N} C_N^k\ \theta^k(1-\theta)^{N-k} = 1-(1-\theta)^N. \end{aligned}$$

Thus, the employment probability is

$$Pr^{emp} = \frac{1-(1-\theta)^N}{N\theta}\ ^{21}.$$

[20]Without introducing confusion, we omit the superscript of firm index j and the subscript of worker's type t.

[21]See also Melanie Cao & Shouyong Shi, 2000.

Hence, when the firm only attracts one type of workers, the expected payoff of the job seeker with type-t is $\frac{1-(1-\theta_t)^{N_t}}{N_t\theta_t}\ w_t$, for $t \in \{L, H\}$ (where θ_t actually depends upon $\mathbf{w}$, as we have mentioned above). And when the firm attracts both type of workers, the expected payoff of high-skilled workers is $\frac{1-(1-\theta_H)^{N_H}}{N_H\theta_H}\ w_H$, whereas the expected payoff of low-skilled workers is $(1-\theta_H)^{N_H} \cdot \frac{1-(1-\theta_L)^{N_L}}{N_L\theta_L}\ w_L$.

Appendix 4.B Sketch of the proofs of the results in LMD (2005)

Separating Equilibrium. First we see that it is impossible to have pooling equilibrium when there are two groups of workers with identical skill level under the context of strong discrimination.

First, notice that group a workers only apply for wages higher than U_a, i.e., $q_a > 0$ only if $w > U_a$. Similarly, $q_b > 0$ only if $w > U_b$. Due to the discrimination, we have $U_a > U_b$ since group a workers always have better chances during the hiring process. Therefore, if a firm posting wage w that can attract both groups, the wage must be strictly larger than U_a.

Now we consider the possibility of a pooling equilibrium. Suppose all firms aim to attract both groups of workers, then firms maximize the expected profit with the contraints that ensures the expected market income for both groups of workers. That is, the equilibrium would be the solution of the following program (firms can only pose one wage since there is only one skill level):

$$\begin{array}{ll} \max_w & (1-e^{-q_a})\ (y-w) + e^{-q_a}\ (1-e^{-q_b})\ (y-w) \\ \text{s.to} & \frac{1-e^{-q_a}}{q_a}\ w = U_a \\ & e^{-q_a}\ \frac{1-e^{-q_b}}{q_b}\ w = U_b \end{array}$$

The objective function can be rewritten as

$$(1-e^{-q_a-q_b})\ (y-w) \tag{4.30}$$

From the two contraints, we can verify that $\frac{d(q_a+q_b)}{dw} > 0$ for any $q_a, q_b > 0$. Thus, reducing w can increasing both terms in (4.30). That is, for a firm attracting both groups, decreasing wage (up to as close to U_a as possible) will always yield higher expected profit. Besides, it is obviously not an equilibrium that all firms uniformly attract a single group whenever there are two groups. Thus, pooling equilibrium does not exist.

In addition, we have in such an equilibrium that $w_b^* = U_a^*$. First, note that the discrimination does not affect the prefered group. Thus, as in the nondiscriminatory context, there is a lower limit of wages that group a will apply for, i.e. U_a^* (by (4.2)). Besides, it can be shown that there also exists an upper limit of wages that group b workers will apply for; we denote this threshold as $\hat{w}$. The intuition is: group b can infer that the rise of wage increases the expected number of group a job seekers, and thus leads to more between-group competition and lower employment probability, which is not enough to be compensated by the increase of wages. Then, we can consider firms choice in two intervals : $(U_b^*, U_a^*]$, where only group b applies, and $(U_a^*, \hat{w})$, where both groups apply. We can see that firms' profit is increasing in w in the first case, and decreasing in w in the second case, with continuity at $w = U_a^*$. Therefore, the optimal wage that firms set for group b is $w_b^* = U_a^*$.

Furthermore, note that the expected output in the case without discrimination is $(1-e^{-\beta})y$ while that in the discriminatory case is $\delta^*(1-e^{-q_a^*})y+(1-\delta^*)(1-e^{-q_b^*})y$, with $q_a^* = \frac{\gamma\beta}{\delta^*}$ and $q_b^* = \frac{(1-\gamma)\beta}{1-\delta^*}$, where δ^* is the fraction of the firms which attact group a in the equilibrium. Cancelling out y, we can simply compare $e^{-\beta}$ with $\delta^* e^{-\frac{\gamma\beta}{\delta^*}} + (1-\delta^*)e^{-\frac{(1-\gamma)\beta}{1-\delta^*}}$, which has the minimum value at $\delta^* = \gamma$. That is, $e^{-\beta} \leq \delta^* e^{-\frac{\gamma\beta}{\delta^*}} + (1-\delta^*)e^{-\frac{(1-\gamma)\beta}{1-\delta^*}}$, and therefore, the expected output in the case without discrimination is larger compared to that in the discriminatory case .

Appendix 4.C Proofs of propositions

Proof of Proposition 4.1 (Return to skills). We prove only case *1* here, while the proof of case *2* and *3* are highly similar. Note that

$$\frac{y_H - y_L}{E_H - E_L} \geq e^{\beta} \iff e^{-\beta} y_H - E_H \geq e^{-\beta} y_L - E_L. \tag{4.31}$$

Now we prove that the optimal choice is $\alpha^* = 1$ under condition (4.31).

We prove firstly that the deviation to low skills is not optimal. By this, we prove that a proportion ϵ of workers' deviating to the low-skilled type is suboptimal. And it suffices to show that after deviation, the deviator can not get higher expected payoff. Before deviation, the expected payoff of workers is $e^{-q_H^*} y_H - E_H$, where $q_H^* = \beta$. After deviation, the expected payoff becomes $e^{-q_H^D - q_L^D} y_L - E_L$, where q_t^D is the expected queue length of type-t workers after deviation, $t \in \{L, H\}$; and moreover, we have $q_H^D + q_L^D = \beta$. However, under the condition 4.31, the expected payoff after deviation is weakly lower. Therefore, there is no deviation to low skills and $\alpha^* = 1$ is an equilibrium solution.

Secondly, we show the uniqueness of the equilibrium. Now we show that for the case of $\alpha = 0$ and $\alpha \in (0, 1)$, there will be profitable deviations.

When $\alpha = 0$, the expected payoff of workers is $e^{-\beta} y_L - E_L$. If there is a fraction ϵ deviating to high skilled, then the expected income for the deviator becomes $e^{-\epsilon\beta}(y_H - y_L) + e^{-\beta} y_L - E_H$. Then this expected payoff after deviation is greater than the the expected payoff before deviation because. $e^{-\epsilon\beta}(y_H - y_L) > e^{-\beta}(y_H - y_L) \geq E_H - E_L$. So the deviation is profitable for the deviators. As for the rest of the population $(1-\epsilon)$, their expected payoff is not affected. Hence deviating weakly increases the payoff of all the job seekers.

When $\alpha \in (0, 1)$, the expected income from search is $e^{-\beta} y_L - E_L$ for the low skilled, and $e^{-\hat{\alpha}\beta}(y_H y_L) + e^{-\beta} y_L - E_H$ for the type H job seekers, where $\hat{\alpha}$ should be pinned down by workers' indifference condition $e^{-\hat{\alpha}\beta}(y_H - y_L) = E_H - E_L$. However, this condition is incompatible for all $\alpha < 1$ with our condition $e^{-\beta} y_H - E_H \geq e^{-\beta} y_L - E_L$. So that it is impossible that job seekers are indifferent from being high or low skilled.

All in all, we have proved that when the configuration of parameters is such that $e^{-\beta} y_H - E_H > e^{-\beta} y_L - E_L$, there exists a unique equilibrium in which all job seekers choose to obtain high skill, i.e. $\alpha^* = 1$. ■

Proof of Proposition 4.2 (Social Optimality). The social planner's maximizes the aggregate output as follows.

$$\max\ (1 - e^{-q_H^P})\big(e^{-q_L^P} + x^P(1 - e^{-q_L^P})\big)y_H + (1 - e^{-q_L^P})\big(e^{-q_H^P} + (1 - x^P)(1 - e^{-q_H^P})\big)y_L - \beta\ \big(\alpha^P\ E_H + (1 - \alpha^P)\ E_L\big). \tag{4.7}$$

Note that the equilibrium implies $q_H^P = \alpha^P \beta$ and $q_L^P = (1 - \alpha^P)\beta$, and program (4.7) includes all the cases of different values of α^P. That is, when $\alpha^P = 1$, noting that $q_L^P = 0$, (4.7) becomes

$$\max\ (1 - e^{-q_H^P})y_H - \beta\ E_H. \tag{4.32}$$

When $\alpha^P = 0$, noting that $q_H^P = 0$, (4.7) becomes

$$\max\ (1 - e^{-q_L^P})y_L - \beta\ E_L. \tag{4.33}$$

We solve the optimal ranking rule by taking derivative with respect to x^P in (4.7) (note that x^P is irrelevant in problem (4.32) and (4.33)), which yields

$$(1 - e^{-q_H^P})(1 - e^{-q_L^P})\ (y_H - y_L),$$

which is positive since $y_H > y_L$. Thus, $x^{P*} = 1$, and problem (4.7) reduces to

$$\max \;\; (1 - e^{-q_H^P})y_H + e^{-q_H^P}\,(1 - e^{-q_L^P})y_L - \beta\,\big(\alpha^P E_H + (1-\alpha^P)E_L\big). \qquad (4.34)$$

Substituing $q_H^P = \alpha^P\beta$ and $q_L^P = (1-\alpha^P)\beta$ into (4.34) and we can solve for the social optimal skill investment α^{P*}. Note that $q_H^P + q_L^P = 1$, taking derivative with respect to α^P gives

$$\beta\Big(\, e^{-\alpha^P\beta}\,(y_H - y_L) - (E_H - E_L)\Big). \qquad (4.35)$$

Thus, we know when $\frac{y_H - y_L}{E_H - E_L} \geq e^{\beta}$, (4.35) ≥ 0 for any $\alpha^P \in [0,1]$, and the optimal skill investment is $\alpha^{P*} = 1$. When $\frac{y_H - y_L}{E_H - E_L} \leq 1$, (4.35) ≤ 0 for any $\alpha^P \in [0,1]$, and the optimal skill investment is $\alpha^{P*} = 0$. When $1 < \frac{y_H - y_L}{E_H - E_L} < e^{\beta}$, there exists a unique $\alpha^{P*} \in (0,1)$ such that (4.35) $= 0$, which maximizes the aggregate output. That is, it is optimal for social planner to arrange some workers (of fraction α^{P*}) to have high-skill training while the other (of fraction $1-\alpha^{P*}$) to obtain low-skill training. This is consistent with the decentralized result. ■

Proof of Corollary 4.1 . According to Proposition 4.1, all the workers obtain high skills under Assumption 4.1, it suffices to solve the following program:

$$\begin{array}{ll} \max_{w_H} & \pi_H = (1 - e^{-q_H})\,(y_H - w_H) \\ \text{s.to} & \frac{1-e^{-q_H}}{q_H}\; w_H = U_H \end{array}$$

From the constraint, we know

$$w_H = \frac{q_H\, U_H}{1 - e^{-q_H}}. \qquad (4.36)$$

We substitute out wages w_H from the objective function and maximize with respect to expected number of applicants q_H:

$$\max_{q_H} \;\; \big(1 - e^{-q_H}\big)\, y_H - q_H U_H$$

Taking derivative with respect to q_H gives $U_H^* = e^{-q_H^*} y_H$. By equation (4.36), we have

$$w_H^* = \frac{q_H^*\, e^{-q_H^*}}{1 - e^{-q_H^*}}\; y_H.$$

	bH	bL
aH	(P1) $\left(e^{-q_{aH}^{P1*}} y_H - E_H, \; \frac{1-e^{-q_{bH}^{P1*}}}{q_{bH}^{P1*}} e^{-q_{aH}^{P1*}} y_H - E_H \right)$	(P2) $\left(e^{-\gamma\beta}(y_H - y_L) + e^{-\beta} y_L - E_H, \; e^{-\beta} y_L - E_L \right)$
aL	(P3) $\left(e^{-\beta} y_L - E_L, \; e^{-(1-\gamma)\beta}(y_H - y_L) + e^{-\beta} y_L - E_H \right)$	(P4) $\left(e^{-q_{aL}^{P4*}} y_L - E_L, \; \frac{1-e^{-q_{bL}^{P4*}}}{q_{bL}^{P4*}} e^{-q_{aL}^{P4*}} y_L - E_L \right)$

Table 4.1: Payoffs of the workers with different strategies.

Substituting w_H^* into the objective function, we have

$$\pi_H^* = \left(1 - e^{-q_H^*} - q_H^* e^{-q_H^*}\right) \; y_H.$$

Note that in equilibrium all the workers choose to be high skilled, $q_H^* = \beta$. ■

Proof of Proposition 4.3 . We summarize in Table 4.1 the payoffs of workers with different skill choices, with taking into account the best response of firms in the wage posting stage. Now, we can look into the best response of workers in the very first stage.

Given group b choosing high skill, bH, group a's best response depends on the comparison of $V_{aH}^{P1}(\beta) = e^{-q_{aH}^{P1*}} \; y_H - E_H$ and $V_{aL}^{P3}(\beta) = e^{-\beta} \; y_L - E_L$. Note that q_{aH}^{P1*} is a function of β, which is determined by (4.18) in the wage posting subgame as in LMD; and from (4.19), we know $q_{aH}^{P1*}(\beta) > \beta$. Thus, we have

$$V_{aH}^{P1}(\hat{\beta}) = e^{-q_{aH}^{P1*}(\hat{\beta})} \; y_H - E_H < e^{-\hat{\beta}} \; y_H - E_H = e^{-\hat{\beta}} \; y_L - E_L = V_{aL}^{P3}(\hat{\beta}),$$

while when $\beta = 0$, we know, by Assumption (4.1),

$$V_{aH}^{P1}(0) = y_H - E_H > y_L - E_L = V_{aL}^{P3}(0).$$

Due to the continuity of the V_{aH}^{P1} and V_{aL}^{P3}, there exists a $\hat{\beta}_1 \in (0, \hat{\beta})$ such that $V_{aH}^{P1}(\hat{\beta}_1) = V_{aL}^{P3}(\hat{\beta}_1)$. Moreover, according to Assumption 4.5, $\hat{\beta}_1$ is unique. Then, we have $V_{aH}^{P1}(\beta) > V_{aL}^{P3}(\beta)$ for any $\beta \in (0, \hat{\beta}_1)$, and $V_{aH}^{P1}(\beta) < V_{aL}^{P3}(\beta)$ for any $\beta \in (\hat{\beta}_1, \hat{\beta})$.

Given group b choosing low skill, bL, group a's best response depends on the comparison of $V_{aH}^{P2}(\beta) = e^{-\gamma\beta}(y_H - y_L) + e^{-\beta}y_L - E_H$ and $V_{aL}^{P4}(\beta) = e^{-q_{aL}^{P4*}}\, y_L - E_L$, which is true for any $\beta \in (0, \hat{\beta})$, implying that choosing H is a dominant strategy for group a when group b chooses L. This is because we have, for any $\beta \in (0, \hat{\beta})$,

$$\left(e^{-\gamma\beta}(y_H - y_L) + e^{-\beta}y_L - E_H\right) - (e^{-\beta}y_H - E_H) = (e^{-\gamma\beta} - e^{-\beta})(y_H - y_L) > 0.$$

Thus, for any $\beta \in (0, \hat{\beta})$,

$$V_{aH}^{P2}(\beta) > e^{-\beta}y_H - E_H > e^{-\beta}y_L - E_L > e^{-q_{aL}^{P4*}(\beta)}\, y_L - E_L = V_{aL}^{P4}(\beta),$$

where the second inequality comes from Assumption 4.1, and the last inequality comes from $q_{aL}^{P4*}(\beta) > \beta$.

Similarly, given group a's choice aH, we see that

$$V_{bH}^{P1}(0) = y_H - E_H > y_L - E_L = V_{bL}^{P2}(0)$$

and

$$V_{bH}^{P1}(\hat{\beta}_1) < V_{aH}^{P1}(\hat{\beta}_1) = V_{aL}^{P3}(\hat{\beta}_1) = V_{bL}^{P2}(\hat{\beta}_1)$$

where the inequality in the latter comes from the fact that with discrimination the expected income of group a is larger than that of group b, i.e. $U_a^* > U_b^*$ (in (P1)). Therefore, there is a $\hat{\beta}_2 \in (0, \hat{\beta}_1)$ such that $V_{bH}^{P1}(\hat{\beta}_2) = V_{bL}^{P2}(\hat{\beta}_2)$. And by Assumption 4.5, $\hat{\beta}_2$ is unique. That is, there is a threshold $\hat{\beta}_2 \in (0, \hat{\beta}_1)$ such that $V_{bH}^{P1}(\beta) > V_{bL}^{P2}(\beta)$ for any $\beta \in (0, \hat{\beta}_2)$, and $V_{bH}^{P1}(\beta) < V_{bL}^{P2}(\beta)$ for any $\beta \in (\hat{\beta}_2, \hat{\beta})$.

Also, given group a's choice aL, choosing H is dominant for group a. Since for any $\beta \in (0, \hat{\beta})$,

$$\left(e^{-(1-\gamma)\beta}(y_H - y_L) + e^{-\beta}y_L - E_H\right) - (e^{-\beta}y_H - E_H) = (e^{-(1-\gamma)\beta} - e^{-\beta})(y_H - y_L) > 0.$$

Therefore, for any $\beta \in (0, \hat{\beta})$,

$$V_{bH}^{P3}(\beta) > e^{-\beta}y_H - E_H > e^{-\beta}y_L - E_L > e^{-q_{aL}^{P4*}(\beta)}\, y_L - E_L = V_{aL}^{P4}(\beta) > V_{bL}^{P4}(\beta),$$

where the last inequality comes from the fact that $U_a^* > U_b^*$ in (P4).

To see the equilibrium, we first consider $\beta \in (0, \hat{\beta}_2)$. Neither of the two group choose to invest to high skill as best response, and both aH and bH are the best response to each other - (aH, bH) is the unique equilibrium. When $\beta \in (\hat{\beta}_2, \hat{\beta}_1)$, group b's best response to aH becomes bL - as β increases, group b benefits from deviating to low skill, and aH is also the best response to bL. Therefore, (aH, bL)

is an equilibrium, and it is also the only equilibrium. When $\beta \in (\hat{\beta}_1, \hat{\beta})$, we can see that both (aL, bH) and (aH, bL) form equilibria, and we have multiple equilibria in this circumstance. ■

Proof of Corollary 4.2. (1) Note that firms' profits in the nondiscriminatory case is $(1 - e^{-\beta} - \beta e^{-\beta})\, y_H$, by Corollary 4.1. In (aH, bH) equilibrium, the result is proved in LMD (2005). In (aH, bL) equilibrium, firms' profits are

$$\pi^{P2*}_{aH,bL} = (1 - e^{-q^{P2*}_{aH}} - q^{P2*}_{aH} e^{-q^{P2*}_{aH}})y_H + \Big((q^{P2*}_{aH} + 1)e^{-q^{P2*}_{aH}} - e^{-\beta}(\beta + 1)\Big)y_L.$$

This term is smaller than the profit without discrimination, because

$$\begin{aligned}\pi^{P2*}_{aH,bL} &= (1 - e^{-q^{P2*}_{aH}} - q^{P2*}_{aH} e^{-q^{P2*}_{aH}})y_H + \Big((q^{P2*}_{aH} + 1)e^{-q^{P2*}_{aH}} - e^{-\beta}(\beta + 1)\Big)y_L \\ &< (1 - e^{-q^{P2*}_{aH}} - q^{P2*}_{aH} e^{-q^{P2*}_{aH}})y_H + \Big((q^{P2*}_{aH} + 1)e^{-q^{P2*}_{aH}} - e^{-\beta}(\beta + 1)\Big)y_H \\ &= (1 - e^{-\beta} - e^{-\beta}\beta)y_H,\end{aligned}$$

where the inequality uses the fact that $(x + 1)\, e^{-x}$ is a decreasing function and $q^{P2*}_{aH} < \beta$. The proof for the case of (aL, bH) equilibrium can be analogously reproduced.

(2) Without discrimination, by Corollary 4.1, workers' expected income in equilibrium is $e^{-\beta} y_H$. For the case (aH, bH), it follows from LMD (2005). For the case of (aH, bL). We have

$$V^{P2}_{aH} = e^{-\gamma\beta}(y_H - y_L) + e^{-\beta} y_L - E_H > e^{-\beta} y_H - E_H$$

since we know $e^{-\gamma\beta} > e^{-\beta}$, and

$$V^{P2}_{bL} = e^{-\beta} y_L - E_L < e^{-\beta} y_H - E_H,$$

since $\beta \le \hat{\beta}$. The proof for the case of (aL, bH) equilibrium can be analogously reproduced. ■

Proof of Proposition 4.4. The payoff matrix is as follows in Table 4.2.

Define $\hat{\psi}_{aH,b}$ by $e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\psi y_H - E_H = e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\psi y_L - E_L$.

Define $\hat{\psi}_{aL,b}$ by $\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\psi y_H - E_H = e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\psi y_L - E_L$.

Define $\hat{\psi}_{bH,a}$ by $\frac{1-e^{-\gamma\beta}}{\gamma\beta}\psi y_H - E_H = e^{-(1-\gamma)\beta}\frac{1-e^{-\gamma\beta}}{\gamma\beta}\psi y_L - E_L$.

Define $\hat{\psi}_{bL,a}$ by $\frac{1-e^{-\gamma\beta}}{\gamma\beta}\psi y_H - E_H = \frac{1-e^{-\gamma\beta}}{\gamma\beta}\psi y_L - E_L$.

When $\gamma \ge \frac{1}{2}$, it can be verified that $\hat{\psi}_{aL,b} \le \hat{\psi}_{bH,a} < \hat{\psi}_{bL,a} < \hat{\psi}_{aH,b}$.

	bH	bL
aH	$\left(\frac{1-e^{-\gamma\beta}}{\gamma\beta}\ \psi y_H - E_H,\ e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\ \psi y_H - E_H\right)$	$\left(\frac{1-e^{-\gamma\beta}}{\gamma\beta}\ \psi y_H - E_H,\ e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\ \psi y_L - E_L\right)$
aL	$\left(e^{-(1-\gamma)\beta}\frac{1-e^{-\gamma\beta}}{\gamma\beta}\psi y_L - E_L,\ \frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\ \psi y_H - E_H\right)$	$\left(\frac{1-e^{-\gamma\beta}}{\gamma\beta}\ \psi y_L - E_L,\ e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\ \psi y_L - E_L\right)$

Table 4.2: Workers' Payoffs in wage bargaining context.

(1) We first prove for values of $\psi \in \left[0, \hat{\psi}_{aL,b}\right)$, the unique pure strategy Nash equilibrium is (aL, bL): group a choose low skill, group b choose low skill. Holding group b high skilled, group a choose to be low skilled, because $\psi < \hat{\psi}_{aL,b} < \hat{\psi}_{bH,a}$. Holding group b low skilled, group a choose to be low skilled, because $\psi < \hat{\psi}_{aL,b} < \hat{\psi}_{bL,a}$. Holding group a high skilled, group b choose to low skilled, because $\psi < \hat{\psi}_{aL,b} < \hat{\psi}_{aH,b}$. Holding group a low skilled, group b choose to be low skilled, because $\psi < \hat{\psi}_{aL,b}$. At the point $\psi = \hat{\psi}_{aL,b}$, the equilibrium can be (aL, bH) or (aL, bL).

(2) We prove for values of $\psi \in \left(\hat{\psi}_{aL,b}, \hat{\psi}_{bH,a}\right)$, the unique Nash pure strategy equilibrium is (aL, bH). Holding group b high skilled, group a choose to be low skilled, because $\psi < \hat{\psi}_{bH,a}$. Holding group b low skilled, group a choose to be low skilled, because $\psi < \hat{\psi}_{bH,a} < \hat{\psi}_{bL,a}$. Holding group a high skilled, group b choose to low skilled, because $\psi < \hat{\psi}_{bH,a} < \hat{\psi}_{aH,b}$. Holding group a low skilled, group b choose to be high skilled, because $\psi > \hat{\psi}_{aL,b}$. At the point $\psi = \hat{\psi}_{bH,a}$, the unique equilibrium is (aL, bH).

(3) We prove for values of $\psi \in \left(\hat{\psi}_{bH,a}, \hat{\psi}_{bL,a}\right)$, there is no pure strategy Nash equilibrium is (aL, bH). Holding group b high skilled, group a choose to be high skilled, because $\psi > \hat{\psi}_{bH,a}$. Holding group b low skilled, group a choose to be low skilled, because $\psi < \hat{\psi}_{bL,a}$. Holding group a high skilled, group b choose to low skilled, because $\psi < \hat{\psi}_{bL,a} < \hat{\psi}_{aH,b}$. Holding group a low skilled, group b choose to be high skilled, because $\psi > \hat{\psi}_{aL,b}$. At the point $\psi = \hat{\psi}_{bL,a}$, the unique equilibrium is (aH, bL).

(4) We prove for values of $\psi \in \left(\hat{\psi}_{bL,a}, \hat{\psi}_{aH,b}\right)$, there is a unique pure strategy Nash equilibrium (aH, bL). Holding group b high skilled, group a choose to be

high skilled, because $\psi > \hat{\psi}_{bH,a}$. Holding group b low skilled, group a choose to be high skilled, because $\psi > \hat{\psi}_{bL,a}$. Holding group a high skilled, group b choose to low skilled, because $\psi < \hat{\psi}_{aH,b}$. Holding group a low skilled, group b choose to be high skilled, because $\psi > \hat{\psi}_{aL,b}$. At the point $\psi = \hat{\psi}_{aH,b}$, the equilibrium can be (aH, bL) or (aH, bH).

(5) We prove for values of $\psi \in \left(\hat{\psi}_{aH,b}, 1\right)$, there is a unique pure strategy Nash equilibrium (aH, bH). Holding group b high skilled, group a choose to be high skilled. Holding group b low skilled, group a choose to be high skilled. Holding group a high skilled, group b choose to high skilled, because $\psi < \hat{\psi}_{aH,b}$. Holding group a low skilled, group b choose to be high skilled, because $\psi > \hat{\psi}_{aL,b}$.

(6) At last, notice that $\hat{\psi} = \frac{E_H - E_L}{y_H - y_L} \times \left(\frac{1-e^{-\beta}}{\beta}\right)^{-1}$, $\hat{\psi}_{bL,a} = \frac{E_H - E_L}{y_H - y_L} \times \left(\frac{1-e^{-\gamma\beta}}{\gamma\beta}\right)^{-1}$, and $\hat{\psi}_{aH,b} = \frac{E_H - E_L}{y_H - y_L} \times \left(e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\right)^{-1}$. It is straightforward to verify that $\hat{\psi}_{bL,a} < \hat{\psi} < \hat{\psi}_{aH,b}$ because $\left(\frac{1-e^{-\gamma\beta}}{\gamma\beta}\right)^{-1} < \left(\frac{1-e^{-\beta}}{\beta}\right)^{-1} < \left(e^{-\gamma\beta}\frac{1-e^{-(1-\gamma)\beta}}{(1-\gamma)\beta}\right)^{-1}$. ■

Chapter 5

General Conclusion

This thesis studies the role of information imperfection both from a theoretical perspective and a problem with asymmetric information in the credit market. Besides, market imperfections such as discrimination in labor market are also analyzed here.

First of all, we consider a decision problem with uncertainty in which the decision maker can receive a signal which reveals some of the information about the unknown true state. In such a context, decisions, after signals being observed, are made according to the posterior distributions. The expectation conditional on signals is likely to be crucial, and Ganuza and Penalva's precision criteria seem attractive. For this reason, the connections between precision criterion and traditional criteria are examined. Chapter 2 focuses on the supermodular precision criterion, which is defined on the conditional expectations by the dispersive order. By looking into the intrinsic property of the dispersive order, we show that Blackwell's informativeness criterion does not necessarily imply or be implied by the dispersion of conditional expectations in general discrete cases, while in the monotone decision problems the relationship between dispersion of conditional expectations and some informativeness criterion, which is defined similar as accuracy criterion, could be built up under some conditions. This could provide theoretical basis for applying the dispersive orders directly to decision analysis. Actually, this is just a preliminary study in this topic, and more informativeness and dispersion criteria are under investigation.

In Chapter 3, we consider a signaling problem in a competitive credit market with asymmetric information and focuses on the role of dissipative signals. Borrowers can choose to reveal information on the quality of their projects; signaling is costly, and borne by the borrowers. The existence of equilibrium depends on the interest rate - when the interest rate is very low, the exists only an opaque equilibrium in which all borrowers choose not to reveal information and are financed, and when the interest rate it high enough, there only exists a transparent equilibrium in which only borrowers who reveal information can be funded; in addition, in the intermediate level, there can be multiple equilibria. Therefore, when the interest rate is moderate, it is possible to experience a jump from an opaque equilibrium to a transparent one, where a decrease of interest rate is in line with a decrease of credit supply, which provides us a possible way of understanding the credit crunch. Moreover, the extension of the model into a dynamic OLG context provides us a macroeconomic viewpoint of the instability of credit market: the market is likely to converge to either an opaque or a transparent equilibrium, and for some configurations of parameters there exist permanent oscillations between two different regimes.

Finally, we study a holdup problem where firms can use discriminatory hiring norms to extract higher than socially optimal profits. We find that when firms rank workers according to both productivity-dependent and productivity-independent characteristics, skill investment becomes strategic between the discriminated and the favored group. In case where wages are posted, we suggest that depending on the market tightness there may be equilibrium or multiple equilibria on skill investment; in some equilibrium the discriminated group can obtain higher expected payoff compared to the case without discrimination. We also consider fixed sharing rule (bargained wage) and make a comparison. Similar equilibrium, where the favored group underinvests while the discriminated group remains high skilled, exists; however, the discriminated group are in general worse off compared to the case without discrimination in the sense that they may still choose to underinvest when ψ is sufficiently high. Firms' profits are piecewise monotone because the skill composition hence the average productivity of the market improves discretely with respect to the bargaining power, and profit loss may be incurred with discrimination within an intermediate range of bargaining power.

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Résumé (Summary in French)

Une chose qui est omniprésent dans l'économie est l'imperfection de l'information, par laquelle nous nous référons à une situation dans laquelle les différentes parties d'une transaction ont des informations différentes. Un exemple courant qui se passe dans le marché du travail. Les travailleurs ont une connaissance meilleure sur leurs compétences, leur assiduité et la productivité, tandis que les employeurs peuvent difficilement connaître la véritable qualité des travailleurs potentiels, même si ils sont en mesure d'obtenir des informations sur les candidats tels que le niveau d'éducation. Ces imperfections sont également partagées sur le marché des capitaux: les relations entre les banques et les emprunteurs agissent comme une relation de travail entre les employeurs et les travailleurs. Les emprunteurs sont généralement plus familiers avec leurs projets et possèdent des informations plus précises sur la qualité de leurs projets, alors que les prêteurs comme les banques ont un accès limité à ces informations et ont des difficultés à savoir s'il est probable pour les emprunteurs d'être insolvable. D'autres exemples sont le commerce des marchandises entre un vendeur et un acheteur, la signature d'un contrat d'assurance entre un assureur et son client, et ainsi de suite.

L'impact de l'information peut dépendre de la structure du marché et dans quelle mesure les participants au marché peuvent accéder à l'information. Les problèmes causés par les imperfections de l'information peuvent être atténués quand il y a des signaux qui peuvent révéler l'information cachée, au moins dans une certaine mesure. Par exemple, les employeurs demandent des certificats ou des tests de conception et d'entretiens pour déduire les véritables productivités des travailleurs candidats; les banques procèdent à des analyses d'investissement ou demandent des garanties ou autres exigences des emprunteurs. Les signaux, ou l'information provenant des signaux, jouent un rôle important dans le processus de prise de décision.

Cette thèse se concentre sur le rôle de l'information et se compose de trois chapitres autonomes. Chacun peut être lu indépendamment des autres. Il commence à partir d'un point de vue théorique en examinant les critères d'information, puis considère deux problèmes concernant le marché du crédit et de marché du travail, respectivement.

L'information imparfaite et critères d'information

Dans un problème de décision avec l'asymétrie d'information, il est possible et commun pour la partie des informations inconvénient de recueillir des informations avant que la décision soit prise. Nous pourrions dire que l'information est signalée peu importe si ce recueil d'information est actif ou passif. Les décideurs observent des signaux et actualisent leur croyances et les convictions mises à jour - les croyances postérieures - sont mises à profit pour prendre leurs décisions. Pourtant, les informations contenues dans les signaux ne sont pas toujours vrai et précises, car les signaux et les caractéristiques inconnues ne sont généralement pas parfaitement corrélés. Il est naturel de penser que les signaux plus fiables avec des informations plus précises pourraient contribuer à une performance plus efficace. Pour formuler cette évaluation des signaux, un système d'information ou une structure d'information est ainsi utilisé pour décrire la relation entre les signaux et les états sous-jacents, différents critères d'information sont proposées sur la base du système d'information. En vertu de cela, nous sommes autorisés à examiner de quelle façon collecter des informations moins bruyants et sont donc plus fiable.

Un critère classique a été développé par Blackwell (1951, 1953), qui fait suite à l'intuition qu'un système d'information moins informatif peut être reproduit à partir d'un autre plus informatif en ajoutant des erreurs aléatoirede transmission. Une grande littérature suit le critère de caractère informatif de Blackwell, d'autres critères ont également été introduits (voir Lehmann (1988), Kim (1995), Persico (2000), Jewitt (2007), et Quah et Strulovici (2009)). D'autre part, il existe des critères d'information qui sont définis sur des moyennes conditionnelles relatives à la réalisation des signaux. Vu que ces décisions sont prises après observation des signaux et sont effectués selon les distributions a posteriori, l'espérance conditionnelle sur les réalisations de signal semble être cruciale. Un signal plus informatif est censé avoir un impact plus grand sur les signaux postérieures, et donc les attentes conditionnelles devraient être plus dispersée (voir Ganuza et Penalva (2010)). Ce genre de critères est très intuitif et donc d'intérêt pour nous. La question qui se pose est celle de savoir quel type de critères d'information pourrait être adopté dans un problème de prise de décision avec asymétrie d'information. Par conséquent, la relation entre les critères d'information classiques et la dispersion des espérances conditionnelles est examinées dans le premier chapitre, qui fournit également une base théorique sur la façon de formaliser la révélation de l'information.

Dans le chapitre 2, intitulé l'Informativité et la dispersion des distributions postérieures, je passe en revue certains critères de l'information et prêtes une attention particulière sur les critères de précision qui sont définis sur les espérances conditionnelles par Ganuza et Penalva (2010). Plus précisément, l'étude vise à trouver des liens entre le critère de précision super-modulaire et le critère de Blackwell. Le résultat montre que le critère de Blackwell ne signifie pas nécessairement ou peut être déduit de la dispersion des espérances conditionnelles dans des cas distincts en général, bien que le caractère de Blackwell peut impliquer la dispersion des espérances conditionnelles lorsque le signal est binaire. En outre, le critère de Persico est aussi analysé. Sur la base du critère de Persico, un critère similaire est construit, ce qui peut impliquer la dispersion des espérances conditionnelles sous certaines conditions.

Informations imperfection sur le marché du crédit et la crise du crédit

Dans le marché du crédit, il a compris depuis longtemps que l'asymétrie d'information joue un rôle central dans la détermination des équilibres de marché (depuis Stiglitz et Weiss (1981)). Une grande littérature a étudié l'influence significative de l'asymétrie de l'information sur le marché du crédit, en montrant comment les stratégies et les interactions des prêteurs et des emprunteurs sont déterminées dans les circonstances de l'information asymétrique. Le manque d'information sur les caractéristiques pertinentes des emprunteurs de prêteurs peut entraîner dans le résultat de sous-investissement. Le crédit est dit être rationné dans ce sens. Toutefois, la possibilité pour le revers de la médaille de cette histoire est aussi largement étudiée: le niveau d'investissement peut se révéler au-delà du niveau social efficace (voir De Meza et Webb (1987) et Alberto et Filippo (2013)). Avec l'asymétrie d'information, les prêteurs ont des difficultés à savoir s'il est probable pour les emprunteurs insolvables, ce qui conduit à des aléas moraux et de sélection adverse des problèmes dans le marché financier. Par conséquent, les intermédiaires financiers comme les banques peuvent choisir soit le dépistage ou de surveillance, ou les deux technologies pour atténuer ces problèmes. Nous poursuivons notre étude le long de leurs lignes, mais procédons à partir d'une perspective différente: dans quelle mesure la révélation de l'information sur la qualité des projets impacte le montant financé des projets, et donc, du type d'équilibre?

En faisant référence a la crise financière que nous avons connue, en particulier la crise du crédit en 2007-2008, nous avons vu qu'il a été principalement causé par l'investissement et les activités relatives aux valeurs mobilières opaques tels que les titres adossés à des hypothèques (MBS), Credit Default Swaps (CDS). Les véhicules financiers qui sont structurés de façon compliquée ont été en plein essor. Des crédits ont été proposés même si les investisseurs n'avaient pas suffisamment d'information sur ces projets. Cependant, comme l'éclatement de la crise, les normes de prêt et de crédit serrées sont thésaurisées. Un resserrement du crédit a été répandu dans le marché financier et ce dernier a conduit à des contractions de l'économie réelle (voir Brunnermeier (2009), Acharya et Skeie (2011) et ainsi de suite). Par conséquent, nous considérons un modèle dans un marché du crédit concurrentiel où les emprunteurs ont la possibilité de révéler des informations (en payant un coût) ou le maintien opaque (sans frais), discuter de l'opacité du marché dans l'équilibre, et tenter d'expliquer le resserrement du crédit et l'instabilité du marché financier.

Chapitre 3, un travail conjoint avec Professeur Bertrand Wigniolle, est intitulé Révélation endogène de l'information dans un marché du crédit concurrentiel et resserrement du crédit. Nous considérons une relation prêteur-emprunteur où les emprunteurs ont une meilleure information sur leurs propres projets que les prêteurs et peuvent choisir de divulguer ou non ces informations. La signalisation est coûteuse et est supporté par les emprunteurs. La décision de la révélation d'informations est endogène, et est donc l'opacité du marché. Nous caractérisons l'équilibre par rapport au taux d'intérêt sans risque et nous montrons que l'existence et les caractéristiques de l'équilibre dépendent du niveau du taux d'intérêt sans risque: il existe seulement un équilibre opaque, dans lequel tous les emprunteurs ne révèlent pas l'information et sont financés, lorsque le taux d'intérêt est faible; il existe seulement l'équilibre transparente, dans laquelle seuls les emprunteurs qui possèdent de bons projets révèlent des informations et sont financés, lorsque le taux d'intérêt est assez élevé; et il y a des équilibres multiples où les deux équilibres opaques et transparentes peuvent être possible que lorsque le taux d'intérêt est dans une gamme intermédiaire. L'existence de plusieurs équilibres admet une certaine possibilité de sauter d'un équilibre transparent à un équilibre opaque. Cela fournit un moyen possible d'expliquer le phénomène de resserrement du crédit, nous avons observé dans la crise financière, où une diminution des taux d'intérêt et une réduction de l'offre de crédit se sont déroulés au a la même période.

En outre, nous étendons le modèle à un contexte OLG et examinons la convergence de l'équilibre de long terme. Selon des paramètres différents, le marché est susceptible de faire converger soit à un opaque ou un état stationnaire transparent, et plus intéressant, pour certaines configurations de paramètres il n'y a pas de convergence dans le long terme; nous pouvons avoir des oscillations permanentes entre équilibres opaques et transparentes. Tant le statique et les cadres dynamiques fournir des explications possibles de l'instabilité du marché du crédit et indiquent une voie possible d'expliquer la crise du crédit pendant la crise financière.

Imperfections du marché du travail et la discrimination

Le marché du travail est un autre exemple où l'information imparfaite prévaut. L'asymétrie de l'information affecte de la décision d'embauche, ainsi que celle des travailleurs sur leur attitudes et investissement académique. Un exemple bien connu est la discrimination statistique. Lang et Manove (2011) montrent que, dans l'asymétrie d'information, les Noirs sur-investissent sur des caractéristiques observables (par exemple l'éducation) pour surmonter l'inconvénient de la recherche d'emploi que les employeurs trouvent qu'il est plus difficile d'évaluer les Noirs par rapport au blanc. Les travaux empiriques avec des faits similaires peuvent également être vu dans Rivkin (1995), et Cameron et Heckman (2001).

Pourtant, Lang et Manove (2011) gardent le silence sur la friction du marché du travail capturée par le taux d'emploi, qui devient une autre mesure centrale de la discrimination dans la littérature de nos jours. Le but du quatrième chapitre est donc de proposer une théorie traitable basée sur une branche relativement nouvelle du modèle de recherche pour expliquer pourquoi le groupe fait preuve de discrimination, par rapport au groupe privilégié, peut investir davantage dans les compétences lorsque la discrimination est potentiellement présent dans le marché. Différent du modèle de recherche aléatoire, Moen (1997) ainsi que Shimer (1996) suggèrent que si les entreprises individuelles sont en mesure de poster des salaires afin de maximiser le bénéfice attendu, alors l'allocation sociale optimale des résultats en matière de ressources. La différence essentielle entre ce contexte et recherche aléatoire avec la négociation est de savoir si les informations sur les salaires sont disponible pour les travailleurs. Si oui, alors la stratégie de recherche des travailleurs dépend de ces salaires, ou leur recherche est dirigée par les salaires. En outre, les travailleurs prennent en compte le compromis entre les salaires et la probabilité d'adaptation induite de manière optimale afin de maximiser leur utilité attendue de la recherche; les entreprises perçoivent cette relation qui est induite par leur salaire posté, sujet auquel le profit est maximisé. L'externalité de la recherche peut être internalisée et l'efficacité peut résulter.

Chapitre 4, un travail conjoint avec Sheng Bi, est intitulé Hold-up et la discrimination à l'embauche avec la friction de la recherche d'emploi.Dans le chapitre 4 nous étudions comment la discrimination à l'embauche peut influencée sur la décision d'investissement des compétences des travailleurs dans le modèle de recherche dirigée. Nous voyons qu'un problème de hold-up sur l'investissement de compétences des travailleurs se pose lorsque les employeurs peuvent adopter des normes d'embauche discriminatoires pour réaliser un profit plus élevé que celui socialement optimal. Lorsque la priorité d'embauche est déterminée à la fois par (niveau de compétence) de la productivité dépendante et les caractéristiques -indépendante (discrimination), la décision d'investissement des compétences devient stratégique entre le groupe discriminé et favorisé. Dans les marchés de friction avec salaire posté, l'investissement de compétences en équilibre dépend de la tension du marché. Le groupe discriminé a tendance à sous-investir dans les compétences lorsque le marché se resserre par rapport à celui privilégié, même s'il existe aussi un équilibre où les groupes privilégiés sous-investissent tandis que celui discriminée ne le fait pas. D'ailleurs, nous discutons plus loin sur le problème dans le cadre des négociations salariales (règle de partage fixe). Avec le salaire négocié, l'équilibre similaire existe et les entreprises encourent des coûts pour une gamme intermédiaire du pouvoir de négociation quand elles sont discriminatoires.

Chapitre 2: Informativité et la dispersion des distributions postérieures

L'incertitude existe presque partout dans l'économie, tels que le prix futur d'un stock, le rendement incertain d'un projet sur le marché financier, ou la compétence non-observable d'un travailleur sur le marché du travail. La reconnaissance de l'information imparfaite a eu une influence profonde et a fourni une méthode remarquable pour expliquer les phénomènes économiques et sociaux. Bien que les agents ne peuvent pas être sûr de l'état vrai, ils peuvent, dans la plupart des cas, surmonter l'incertitude par l'obtention d'informations pertinentes qui sont véhiculée dans certains signaux, qui peuvent être dérivée de l'enquête personnelle, suggéré par un expert, acheté de certains institutions, ou même volés. Leur décision sera alors basée sur la connaissance révisé de l'état vrai, c'est-à-dire la croyance postérieure de l'état.

Mais comment peuvent les signaux distinctifs influencer le processus de décision? Comment pourrions-nous juger la qualité de l'information? Pour formuler le caractère informatif, le concept de système d'information ou de la structure de l'information, qui capture la distribution conjointe des signaux et des états sous-jacents, est introduit, et divers critères sont définis sur ceci.

Dans une autre perspective, si nous considérons un cas extrême où les signaux peuvent révéler exactement l'état futur, alors la croyance révisée dépendra entièrement de la valeur réalisée du signal plutôt que la croyance antérieure; si le contenu informationnel du signal est relativement faible, la croyance postérieure sera similaire à la croyance antérieure et différentes réalisations de signaux ne feront pas beaucoup différences dans les croyances postérieures. Cela implique que la distribution a posteriori se disperse dans une moindre mesure sous signaux moins informatifs. Cela pourrait également être étendue à l'utilité espérée de l'agent conditionnelle à des réalisations de signal. Donc plus d'informations devrait conduire à plus de dispersion dans les espérances conditionnelles. En effet, suite à cette

intuition, Ganuza et Penalva (2010) proposent un nouveau type de critères pour évaluer le caractère informatif, qu'ils désignent comme des *critères de précision*.

Ce type de critères de précision sont pratique car les décisions sont souvent prises sur la base des espérances conditionnelles, et ils sont donc utiles pour un large éventail de problèmes dans les décisions économiques, telles que les enchères, les investissements en matière d'éducation, etc., et la formalisation des critères de précision fournit un moyen plus facile à interpréter le caractère informatif de signaux. Néanmoins, il est peu conventionnel d'évaluer le caractère informatif fondé sur les espérances conditionnelles plutôt que directement sur les systèmes d'information sous-jacents. Par conséquent, je tente de combler le fossé entre les critères de précision et les critères de informativité traditionnels, tels que le critère de Blackwell (1951 et 1953) et le critère de Persico (2000), qui sont considérés dans le document présent. Quant aux critères de précision, je ne me concentre ici que sur le critère de précision supermodulaire Ganuza et de Penalva, qui est défini à l'ordre de dispersion.

En outre, il est naturel de supposer qu'un décideur peut déduire un état attendu plus élevé ou un utilité espérée plus élevé de l'observation d'un signal avec une valeur plus élevée, et donc préférer un signal plus élevé. En fait, cette hypothèse intuitive peut être assurée par l'établissement du rapport de vraisemblance monotone (Monotone likelihood ratio property, ou MLRP, en Anglais), qui est applicable à tous les systèmes d'information que nous considérons dans le présent document.

En examinant les propriétés de l'ordre de dispersion, nous voyons qu'en général le critères de Blackwell n'implique pas ou n'est pas impliquée par la dispersion supermodulaire, bien qu'il puisse y avoir un lien entre les deux critères dans un cas binaire dans le sens où il n'y a que des réalisations de signaux binaires. En outre, nous fournissons une condition nécessaire de l'ordre de dispersion des variables aléatoires discrètes ainsi qu'une caractérisation pour le cas des variables aléatoires binaires. De plus, nous considérons le critère de Persico, avec lequel nous construisons un critère similaire. Et le résultat montre que le genre de critères de précision est possible d'être connecté avec la dispersion des espérances conditionnelles (en terme de critère supermodulaire de Ganuza et Penalva).

D'abord, ce article examine le lien entre le critère de précision supermodulaire ($\succ_{sm}$) de Ganuza et Penalva et le critère de linformativité de Blackwell ($\succ_i$). Le

critère de linformativité de Blackwell suit l'intuition qu'un système plus informatif doit avoir une valeur supérieure quelque soit les problèmes de décision en question ou croyances antérieures que les décideurs partagent. D'autre part, un système plus informatif doit être statistiquement suffisant pour pour un système moins informatif, dans le sens que un système moins informatif peut être reproduit à partir d'un système plus informatif en ajoutant une erreur de transmission aléatoire. Cependant, le critère de Ganuza et Penalva, qui est basé sur l'ordre de dispersion, décrit la propriété de dispersion des espérances conditionnelles. L'intuition est qu'un signal plus informatif devrait fournir des révisions plus efficaces pour mettre à jour la croyance. Lorsque les signaux peuvent effectivement réviser la croyance postérieure, l'espérance conditionnelle varie beaucoup avec les réalisations différentes des signaux. Par conséquent, il est naturel de relier la précision des signaux avec la dispersion de l'espérance conditionnelle de signaux.

Ici, je considère la relation entre les deux critères dans un contexte discret. Avant d'entrer dans l'examen de la relation, je fournis d'abord une condition nécessaire de l'ordre de dispersion, ce qui est basé sur la précision supermodulaire. Notons $\geq_{disp}$ que l'ordre de dispersion, on a

Soit $\tilde{x}$ et $\tilde{z}$ deux variables aléatoires discrètes avec les supports $\{x_1, ..., x_n\}$ et $\{z_1, ..., z_n\}$, respectivement, où $x_1 < ... < x_n$ et $z_1 < ... < z_n$. Ensuite, $\tilde{x} \geq_{disp} \tilde{z}$ implique que $Pr(\tilde{x} = x_i) = Pr(\tilde{z} = z_i)$ pour tout $i \in \{1, ..., n\}$.

Je peux vous référer à cette condition nécessaire de l'ordre de dispersion comme la condition de probabilité égale. Et à partir d'un cas binaire, je fournis une caractérisation de l'ordre de dispersion,

Soit $\tilde{x}$ et $\tilde{z}$ deux variables aléatoires discrètes, avec les supports $supp(X) = \{l_x, h_x\}$ et $supp(Z) = \{l_z, h_z\}$, respectivement. Ensuite, $\tilde{x} \geq_{disp} \tilde{z}$ si, et seulement si, (1) $h_x - l_x \geq h_z - l_z$; et (2) $Pr(\tilde{x} = l_x) = Pr(\tilde{z} = l_z)$.

Cette caractérisation comprend la condition de probabilité égale et que la différence entre les deux réalisations possibles est plus grande si la variable aléatoire est plus dispersée. Basé sur la caractérisation ci-dessus je soutiens que le critère de Blackwell peut impliquer le critère de Ganuza et Penalva dans un cas binaire sous la condition de probabilité égale. Plus précisement, considérons systèmes d'information binaires $(\{q_L, q_H\}, \{y_L, y_H\}, F)$ et $(\{q_L, q_H\}, \{y_L, y_H\}, G)$ avec la distribution a priori $(\pi, 1-\pi)$, où

$$F = \begin{pmatrix} 1-p_1 & p_1 \\ p_2 & 1-p_2 \end{pmatrix} \quad et \quad G = \begin{pmatrix} 1-q_1 & q_1 \\ q_2 & 1-q_2 \end{pmatrix}$$

avec $p_1, p_2, q_1, q_2 \in (0,1)$, $p_1 + p_2 < 1$ et $q_1 + q_2 < 1$. Nous avons

$F \succ_i G$ implique $F \succ_{sm} G$ pour les priors $0 < \pi < 1$ tel que $Pr^F(\tilde{y} = y_L) = Pr^G(\tilde{y} = y_L)$.

Ce résultat peut être étendu au cas avec plusieurs états aussi longtemps que le signal est binaire. En outre, tous les résultats peuvent être reproduits à l'utilité espérée des gains, tant que la fonction d'utilité est strictement croissante.

En outre, d'autres critères pour la comparaison les deux différents systèmes d'information ont également été introduites dans la littérature. Par exemple, Lehmann (1988) considère les systèmes d'information avec MLRP, qui est également connu comme la propriété d'affiliation. Pour cette certaine classe de problèmes de décision, Lehmann a proposé ce que l'on appelle le critère de l'efficacité de l'intuition qu'une meilleure information devrait être plus corrélée à l'état vrai. Persico (2000) formalise le critère de Lehmann, où le caractère informatif d'un système d'information est défini de la manière suivante. Inspiré par le critère de Persico, nous considérons maintenant deux systèmes d'information avec des signaux $\tilde{x}^\eta$ et $\tilde{x}^\theta$, respectivement, ce qui satisfait la propriété: $f^\theta(T_v(x)|\tilde{v} \leq v) = f^\eta(x|\tilde{v} \leq v)$, $\forall v \in \mathcal{V}$, où $T_v'(x) < 1 \, \forall x \in (0,1)$. Je montre que les systèmes d'information avec la propriété ci-dessus peuvent impliquent la dispersion de l'espérance conditionnelle, bien que les restrictions soient dures et difficiles à interpréter.

Pour résumer, avec la recherche dans la propriété intrinsèque de l'ordre de dispersion, cet article met l'accent sur le critère de précision supermodulaire, qui est défini sur la dispersion de l'espérance conditionnelle sur les réalisations de signal. Je tente de relier ce critère de précision pour l'autre critère d'information. Bien que le critère de précision soit intuitivement attirant, nous ne pouvons pas voir les liens bien au-delà du cas binaire; tandis que dans les problèmes de décision monotone la rapport entre la dispersion de l'espérance conditionnelle et un critère, qui est défini semblable à le critère de précision de Perciso, peut être construit sous certaines conditions.

Chapitre 3: Révélation endogène de l'information dans un marché du crédit concurrentiel et resserrement du crédit

Nous avons traversé la grande instabilité financière pendant la crise financière mondiale de 2007-2008, dont la cause a été principalement attribuée à des titres adossés à des créances hypothèques (Mortgage-backed securities en anglais). Les fondements posés dans la période pré-crise sont l'environnement de taux d'intérêt faible et une dégradation des normes de crédit. Des véhicules d'investissement structurés étaient en plein essor et l'étendue de la titrisation a amené à une opacité excessive dans le marché financier. Pourtant, comme l'éclatement de la crise, le capital des institutions financières s'est érodé et, dans le même temps, les normes de prêt se sont resserrées. Même en injectant de la liquidité venant des banques centrales, les banques ont commencé à thésauriser des fonds et sont devenues réticentes à prêter, même entre elles. Les écarts des taux interbancaires en Europe sont montés en flèche atteignant un pic d'environ 200 points en Septembre 2008, tandis qu'aux états-Unis, ces écarts peuvent monter jusqu'à 500 points de base, si on se réfère au niveau d'environ 10 points de base avant la crise (Heider et al., 2009). Un resserrement du crédit a été étendu par les agents économiques, et ce resserrement a conduit à des contractions de l'économie réelle - les prix des actifs ont chuté, le chômage a augmenté, et la croissance des productions s'est embourbée (voir Brunnermeier (2009), Acharya et Skeie (2011) et ainsi de suite).

L'asymétrie d'information entre les emprunteurs et les prêteurs pourrait être une des raisons pour expliquer le prêt inapproprié. Ayant l'avantage de l'information, les emprunteurs peuvent recourir à un moyen coûteux pour révéler leurs informations afin d'augmenter les chances d'être financés. Avec cette hypothèse clé de divulgation coûteuse d'information, nous sommes en mesure de représenter l'existence de différents équilibres, et même l'existence d'équilibres multiples. En outre, à partir du modèle nous sommes en mesure d'observer que les emprunteurs non qualifiés peuvent aussi être financés s'ils restent opaques et lorsque le marché du crédit est relativement souple. Ainsi, l'asymétrie d'information et la révélation coûteuse des informations pourraient être un moyen possible d'expliquer la crise du crédit que nous avons connue et la volatilité du marché du crédit.

Dans cet article, nous considérons un problème de signalisation sur le marché du crédit concurrentiel. Dans notre modèle, les emprunteurs, qui cherchent des fonds pour financer leurs projets, ont des informations privées concernant le retour sur leurs propres projets, et ils l'option entre divulguer leur information monnayant un coût ou rester opaque. Les banques, qui collectent des dépôts et servent en tant que prêteurs, facturent des taux d'intérêt en fonction du type des emprunteurs (opaque ou transparent). En endogénéisant le taux d'intérêt et en tenant compte de l'offre de crédit imparfaitement élastique, nous montrons que, selon les différentes valeurs du taux d'intérêt sécuritaire, il peut exister trois types d'équilibre - l'équilibre opaque dans lequel tous les emprunteurs ne révèlent aucune information et sont financés, l'équilibre transparent dans lequel seuls les emprunteurs qui possèdent de bons projets révèlent des informations et sont financés, et les équilibres multiples, où les équilibres à la fois opaque et transparent peuvent être possibles. En outre, nous élargissons le contexte à un cadre dynamique en endogénéisant le taux d'intérêt à travers un modèle à générations imbriquées, où l'épargne de la période t constitue l'offre de crédit sur le marché du crédit à la période $t+1$. A chaque période, l'économie peut être soit dans un équilibre opaque soit dans un équilibre transparent. Nous fournissons la dynamique du marché du crédit, qui est régie par l'évolution de deux variables - le taux d'intérêt et le type de l'état. Les deux états stationnaires opaques et transparents peuvent éventuellement se produire dans le long terme.

Depuis le travail pionnier de Stiglitz et Weiss (1981), une vaste littérature a étudié le rôle joué par l'asymétrie d'information sur la détermination des équilibres du marché de crédit. L'absence d'information des prêteurs dur les caractéristiques des emprunteurs pourrait résulter à un sous-investissement. Le crédit est alors dit rationné. Cependant, la possibilité de l'autre cas de figure est aussi largement étudiée : le niveau d'investissement pourrait excéder le niveau social efficace. Par exemple, De Meza et Webb (1987) montrent que si les rendements escomptés sur le projet peuvent différer, un surinvestissement pourrait survenir sous des hypothèses plausibles sur la fonction de distribution du rendement du projet. Un papier récent de Alberto et Filippo (2013) renforce cette orientation dans un contexte dynamique sur comment la sélection adverse favorise fortement l'investissement, l'accumulation de capital et l'afflux e capitaux. Nous poursuivons notre étude en ce sens, mais nous procédons avec une perspective différente : comment sont affectés les équilibres si on tient compte de l'option des emprunteurs à révéler l'information sur les rendements de leur projet.

Comme dans le cas des problèmes de signal en marché de crédit, des études peuvent remonter à Leland et Pyle (1977), qui considèrent un problème similaire et supposent un signal à travers le choix de la structure financière. De plus, un traitement classique du problème de sélection adverse sur le marché de crédit peut être vu notamment dans Besanko et Thakor (1987), où l'agent principal conçoit des contrats qui l'auto-sélection. D'autres études peuvent être observées dans Milde et Riley (1988), Cremer et Khalil (1992), et Tirole (2006). Toutefois, dans notre modèle, nous simplifions le problème de prise de décision de l'agent principal (la banque) sans perdre de vue les découvertes essentielles de cette littérature.

Concernant l'étude sur les dynamiques macroéconomiques dans le marché de crédit, des résultats similaires se trouvent dans Azariadis et Smith (1998), établis en considérant un secteur de production et en se focalisant sur la manière dont le stock de capital varie, et par conséquent bascule entre différents équilibres. Dans notre papier, nous nous intéressons plus sur le marché de crédit et entamons avec la décision prise par des emprunteurs et prêteurs typiques, qui fournissent plus de fondements microéconomiques.

Le résultat principal dans le modèle statique est que le niveau du taux d'intérêt sans risque détermine le type de l'équilibre sur le marché de crédit, and donc l'opacité du marché. Nous considérons un marché de crédit constitué de deux types d'agent : entrepreneurs et investisseurs. Les entrepreneurs (ou emprunteurs) ont besoin de mobiliser des capitaux pour lancer leurs projets, tandis que les investisseurs (ou prêteurs) leur accordent des prêts. Emprunteurs et prêteurs sont neutres face au risque. Chaque emprunteur a besoin d'une unité de fonds pour procéder avec son projet, qui rapporte un rendement aléatoire V. Les emprunteurs sont néanmoins hétérogènes ; le rendement du projet varie suivant les emprunteurs et constitue une information privée pour le détenteur du projet. La divulgation d'information est onéreux. Ce coût est exclusivement à la charge de l'emprunteur. Les emprunteurs qui divulguent les informations sont appelés transparents, et ceux qui ne le font pas sont dits opaques. Les prêteurs connaissent précisément le rendement réalisé dès que l'information est communiquée ; dans le cas contraire, ils ne possèdent pas plus d'information sur le rendement en dehors de la distribution de V. Les prêteurs (banques) accordent des prêts qui doivent être remboursés avec intérêt à la fin de la période. Les remboursements requis par les prêteurs sont différents pour les emprunteurs transparents et opaques.

Nous montrons qu'il existe un équilibre transparent lorsque le taux d'intérêt sans risque est élevé, tandis qu'un taux d'intérêt faible induit un équilibre opaque. Plus précisément, lorsque le taux d'intérêt sans risque est assez faible, c'est-à-dire en dessous du seuil inférieur, seul un équilibre opaque peut avoir lieu. Ceci est en concordance avec notre intuition. Lorsque le coût de financement est faible, les banques ont tendance à réduire leurs critères de prêt, prennent plus de risque et investissent sans l' information adéquate sur les projets, certains d'entre eux peuvent être non rentables. Par ailleurs, lorsque le taux d'intérêt se trouve sur une gamme de valeurs modérées, chacun des deux types d'équilibre peut survenir, reflétant l'étroitesse de l'octroi de fonds. Par conséquent, si nous considérons l'opacité du marché de crédit comme étant la propension des emprunteurs opaques parmi les emprunteurs financés, nous pouvons aussi constater que l'opacité du marché de crédit diminue considérablement, de zéro à un, alors que le taux d'intérêt sans risque s'accroit.

Dans notre modèle, nous avons simplifié le modèle de base, qui inclut la connaissance parfaite du rendement de l'emprunteur et une révélation complète de son information privée. A travers ceci apparaît une discontinuité de la demande de crédit l'opacité du marché en fonction du taux d'intérêt sans risque, la propriété principale du modèle est maintenue : la demande de crédit tend à diminuer avec le taux d'intérêt, tandis que la transparence du marché de crédit est croissante quand le taux d'intérêt fondamental croît.

Il est intéressant d'observe et de s'adresser sur l'existence d'équilibres multiples, puisque cela pourrait clarifier l'indétermination sur le marché de crédit. Pendant la période où il est facile d'obtenir du crédit (grâce à un plus large afflux de fonds et une plus grande volonté des banques à prêter), comme ce que nous avons connu avant que la crise financière ne s'éclate, le taux d'intérêt est relativement faible et le coût de financement d'un prêteur est aussi faible. Cela permet aux prêteurs de prendre plus de risque en investissant dans des projets opaques mais éventuellement non profitables. Comme l'offre de crédit devient plus serrée quand le taux d'intérêt diminue jusqu'à un certaine gamme intermédiaire, les banques pourraient toujours être incitées à octroyer des prêts à des projets opaques, puisque le coût n'est pas très élevé, même s'il se pourrait aussi qu'elles n'octroient des prêts qu'aux emprunteurs transparents et qualifiés, si leur confiance au marché est moindre et se elles sont plus concernées par les futurs défauts de paiement, qui à la fin résultent à un équilibre transparent. Cela veut dire que c'est un équilibre qui pourrait être soit opaque soit transparent. Dans une telle situation, il pourrait y avoir un saut d'un équilibre à un autre, et si un saut à partir d'un équilibre opaque survient, nous pouvons observer une diminution de l'offre de crédit accompagnée d'une diminution du taux d'intérêt. Par conséquent, le modèle nous apporte une éventuelle explication théorique concernant la pénurie de crédit, qui aboutit à moins de projets financés et moins de production dans l'économie, comme observé pendant le crise financière.

De plus, nous intégrons le modèle statique dans une économie à générations imbriquées. A chaque période, un continuum d'agents de masse unité sont nés. Chaque agent vit sur trois périodes : jeune, adulte et vieux. Quand il est jeune, l'agent est doté d'un projet qui requiert une unité de fonds pour procéder à la période suivante. Comme dans le modèle classique, il peut décider entre de dévoiler ou non le rendement de son projet et d'être certifié/ suivi pendant l 'implémentation du projet en supportant un coût additionnel à la période suivante. Pendant l'age adulte, l'agent peut effectuer le projet s'il est fiancé et la production est réalisée. En supposant que l'agent consomme seulement pendant les deux dernières périodes, il alloue son revenu total, pendant son age adulte, entre la consommation actuelle et l'épargne qui financera sa consommation quand il sera à la retraite.

Les intermédiaires financiers, comme les banques, encaissent les épargnes et financent les projets quand c'est profitable. Les banques ne possèdent aucune autre information sur les projets que la distribution, si les emprunteurs ne le divulguent pas. Les banques n'ont pas de coûts d'exploitation et sont en concurrence parfaite.

A partir d'une telle intégration de modèle static de base dans le cadre des générations imbriquées, nous formons la dynamique du taux d'intérêt sans risque, en montrant qu'il peut y avoir une convergence vers différents types d'équilibre à long terme, et en réglant certains paramètres, il peut y avoir des oscillations cycliques entre équilibres opaques et transparents. Les résultats des simulations avec différents niveaux de salaire sont en accord avec l'analyse dans le modèle statique. Lorsque le salaire est très bas, le crédit est restreint car il y a moins d'épargne qui constitue l'offre de crédit, et il n'existe qu'un équilibre stationnaire transparent avec un taux d'intérêt relativement élevé sur le marché de crédit ; la production est entravée puisqu'un nombre limité de projets sont financés et le coût nécessaire de certification apporte plus d'inefficacité à l'économie. Pourtant, lorsque le salaire est suffisamment élevé, l'offre de crédit est suffisante et l'équilibre opaque se produit avec un haut niveau de production et un taux d'intérêt faible. Avec un certain salaire à niveau modéré, la dynamique peut évoluer sans suivre un certain rythme, surtout quand le taux d'intérêt tombe dans la gamme intermédiaire, où l'évolution peut suivre un chemin incertain. En outre, l'état peut changer entre équilibres opaques et transparents, et l'on peut même observer des commutations cycliques entre les états opaques et transparents. Bien qu'un tel chemin ne soit pas stable, il pourrait illustrer quelques fluctuations permanentes du marché du crédit et, à son tour, de l'économie réelle. De plus, comme la production dans un équilibre opaque est toujours plus élevée que celle dans un équilibre transparent, le passage d'un état opaque dans un crédit à un état transparent indique également une contraction économique.

Pour résumer, une caractéristique de notre modèle est l'incorporation du signal sur la décision de dissipation. Dans le modèle statique avec un taux d'intérêt endogène, en dépit de la configuration simple que nous avons choisie, la décision de révéler des informations est endogénéisée, l'opacité du marché est donc aussi endogénéisée. Contrairement à la littérature avec asymétrie d'information sur le marché du crédit, les emprunteurs dans notre modèle peuvent choisir d'être transparents ou opaques selon le contexte financier qu'ils connaissent - à savoir, le niveau du taux d'intérêt sécuritaire et la volonté des prêts de banques. Ainsi, nous sommes en mesure de caractériser l'équilibre par rapport à un taux d'intérêt de marché exogène sécuritaire, et par conséquent, d'aborder le lien entre le coût fondamental de financement et le type de l'équilibre, ainsi que l'opacité du marché et de la production globale. Une autre caractéristique est que nous abordons les fluctuations à long terme, en plus de l'indétermination à court terme. Par conséquent, nous sommes en mesure de démontrer que les commutations entre les types d'équilibre et le resserrement du crédit peuvent expliquer les fluctuations économiques.

Chapitre 4: Hold-up et la discrimination à l'embauche avec la friction de la recherche d'emploi

Un problème de hold-up se pose lorsque les investissements sont coulés ex ante par une partie, et la récompense des investissements est partagée avec partenaire de cette partie. Comme le coût n'a pas d'autre utilisation une fois coulé, alors le partenaire commercial de la partie qui investit aura tout intérêt à presser le bénéfice au stade ex post. Dans une étude importante sur un tel problème dans un marché du travail avec la recherche d'emploi, Acemoglu et Shimer (1999b) montrent que, si les entreprises font les investissements capitaux ex ante et les salaires sont déterminés ex post par négociation, l'équilibre est toujours inefficace, puisque les salaires versés aux travailleurs peuvent être si élevés de telle sorte que l'incitation ex ante de l'investissement des entreprises soit nuite; tandis que si les entreprises sont en mesure d'annoncer des salaires avant la recherche d'emploi, les salaires peuvent diriger la recherche d'emploi des travailleurs, alors le problème de holdup de l'investissement à des entreprises ne se déroule non plus; l'efficacité peut être atteinte, car si les salaires sont annoncés aux travailleurs, les travailleurs peuvent observer ces offres et choisir où à postuler, donc les travailleurs vont maximiser leur utilité espérée de la recherche d'emploi aux entreprises en faisant un compromis entre chaque salaire observé et la probabilité de l'obtenir pour tous les salaires. Dans un cadre conventionnel de l'annonce de salaire, nous apercevons une autre source d'inefficacité dans un problème de holdup où les travailleurs paient le coût d'investissement de compétences: lorsque le marché est bondé pour les entreprises (le nombre des entreprises est grand), en adoptant une norme à l'embauche discriminatoire, les entreprises sont capable d'exproprier un niveau de profit qui est supérieur au niveau socialement optimal; il s'avère que dans cette situation, les incitations de l'investissement pour le groupe des travailleurs favorisés et le group des travailleurs discriminés sont tous les deux affectées négativement. Nous analysons l'impact d'un tel comportement de la recherche de rente des entreprises sur la structure de la segmentation du marché, et sur les incitations à l'investissement de compétences des travailleurs.

RÉSUMÉ

Lorsqu'il n'y pas de discrimination, l'économie de l'annonce de salaire avec les investissements ex ante de compétences des travailleurs atteint efficacité dans les équilibria, et nous pouvons montrer que quel équilibre émerge dépend de la comparaison des valeurs entre le rendement logarithmique de l'investissement de compétences et la tension du marché (le rapport travailleurs/entreprises), qui mesure l'intensité de concurrence sur le marché du travail. Et nous montrons que les décisions des travailleurs sur l'investissement de compétences sont socialement optimales. La raison fondamentale de ce résultat de l'efficacité est que les salaires peuvent dépendre explicitement du niveau des compétences (le niveau des compétences est une qualité qui peut être légalement écrite dans les contrats salariaux). Il est une autre histoire lorsque d'autres caractéristiques qui ne sont pas fortement liés à la productivité, comme le sexe, la race, taille, origine, etc. entrent également dans la préférence des entreprises. Conformément à la législation sur l'égalité salariale, les salaires annoncés sont interdits d'être conditionné explicitement à ces caractéristiques; toutefois, si les entreprises toujours sélectionnent les travailleurs selon leur préférence sur ces caractéristiques binaires, un équilibre séparateur peut se dérouler, et dans un tel équilibre les différentes entreprises annoncent différents niveaux de salaires, et les travailleurs de différents groupes se trient et postulent à des salaires différents: le marché est alors ségrégé de façon endogène. Du côté des entreprises, elles ont intérêt à adopter une telle norme de l'embauche d'emploi discriminatoire, lorsque le rendement de l'investissement de compétences des travailleurs est suffisamment élevé; dans ce cas, la discrimination leur permet de saisir un niveau de profit qui est plus élevé que le niveau socialement optimal, et les travailleurs vont toujours choisir d'être qualifiés. Du côté des travailleurs, il se révèle que l'utilité espérée du groupe des travailleurs discriminés et l'utilité espérée du group des travailleurs favorisés sont tous les deux inférieures au niveau socialement optimal: concernant le groupe discriminé, il est parce que la discrimination réduit discrètement l'opportunité de trouver un emploi dans le marché du travail pour ces travailleurs, qui, anticipent la discrimination, exigent des salaires plus bas, ce qui les rend moins cher d'embaucher du point de vue des entreprises; concernant le groupe favorisé, il est ainsi parce que lorsque les entreprises sont capable d'embaucher les travailleurs discriminés à moindre coût, il est comme si les entreprises bénéficient un plus grand niveau de pouvoir de marché

elles peuvent menacer les travailleurs du groupe favorisé que s'ils n'acceptent le salaire bas, elles vont embaucher les travailleurs du groupe discriminé. Naturellement, anticipant la discrimination, tous les groupes attendent un niveau d'utilité espérée inférieure de la recherche d'emploi, ce qui affecte de façon négative leurs incitations à l'investissement de compétences.

Une caractéristique importante de notre étude est la multi-dimensionnalité des caractéristiques des travailleurs basées sur laquelle les travailleurs sont classés. D'une part, on considère le classement par l'identité de type qui dépend de la productivité des travailleurs: les travailleurs sont soit qualifiés (type H) soit peu qualifiés (type L); les travailleurs qualifiés ont la priorité par rapport aux travailleurs peu qualifiés simplement parce que cette priorité (ou ce classement) donne aux entreprises des profits plus élevés. D'autre part, on considère également le classement par l'identité du groupe qui est indépendante de la productivité: les travailleurs appartiennent soit au groupe favorisé (groupe a) soit au groupe discriminé (groupe b). Le classement final a l'ordre suivant: $aH \succ bH \succ aL \succ bL$. On peut le comprendre de la façon suivante: étant donné un niveau de compétence, les travailleurs du groupe a sont préférés aux travailleurs du groupe b; au même temps, les travailleurs qualifiés sont toujours préférés aux travailleurs peu qualifiés. Selon un tel classement, la décision de l'investissement de compétences des groupes différents devient stratégiquement interdépendante. On se concentre sur l'équilibre de Nash en stratégies pures sur les investissements de compétences des travailleurs dans l'économie de l'annonce des salaires, et nous constatons que selon la valeur de la tension du marché il peut y avoir d'équilibre ou des équilibria multiples sur l'investissement de compétences en raison de l'interdépendance stratégique. Par rapport au cas où la discrimination est absente, lorsque le marché est très bondé pour les entreprises (ou la tension du marché est faible), la discrimination est rentable pour les entreprises en ce sens que les entreprises peuvent gagner un niveau de profit supérieur au niveau socialement optimal, et tous les travailleurs sont moins bien en termes d'utilité espérée; lorsque le niveau de la tension du marché augmente, tous les deux groupes peuvent choisir de sous-investir dans les compétences et à l'équilibre chaque fois qu'un groupe sous-investissent, l'autre groupe restent qualifiés et vont recevoir un niveau de l'utilité espérée supérieur au niveaux socialement optimale, alors que les profits des entreprises sont inférieurs

par rapport au cas où la discrimination est absente. En particulier, il existe un équilibre où le groupe des travailleurs favorisés sous-investissent, tandis que le groupe des travailleurs discriminés choisissent de rester qualifiés; et dans ce cas les profits des firmes sont inférieurs au niveau socialement optimal, parce que le fait qu'il y a des travailleurs qui sous-investissent dans les compétences conduit à une baisse de la productivité moyenne de l'économie par rapport au cas où la discrimination est absente.

Dans l'économie où les salaires sont négociés (ou déterminés selon une règle de partage de surplus fixe) après une firme et un travailleur se rencontrent et donc ne dirigent pas la recherche d'emploi des travailleurs, nous trouvons des équilibres similaires par rapport à l'économie où les salaires sont annoncés. En particulier, on a un équilibre où les travailleurs favorisés sous-investissent, donc gagnent un niveau de surplus espéré inférieur par rapport au cas où la discrimination est absente dans une certaine région du pouvoir de négociation des travailleurs; dans un tel équilibre, le surplus est transféré des entreprises et des travailleurs favorisés aux travailleurs discriminés. Les profits d'entreprises sont monotones et décroissantes par morceaux, parce que l'augmentation de pouvoir de négociation des travailleurs peut accroître leur incitation de l'investissement des compétences, et donc améliorer discrètement la composition des compétences de marché et la productivité moyenne de l'économie. Nous constatons également qu'il y a une gamme intermédiaire du pouvoir de négociation des travailleurs pour valeurs desquelles les profits des entreprises sont inférieurs par rapport au niveau socialement optimal lorsqu'elles discriminent, en raison de l'investissement de compétences découragé de groupe des travailleurs discriminés. Dans l'ensemble, la différence principale entre le contexte de l'annonce des salaires et le contexte de la négociation salariale est que le salaire réel négocié dépend d'une manière exogène sur la productivité, et les entreprises ne peuvent plus manipuler leur pouvoir de marché en traduisant leur préférence discriminatoire sur les salaires constamment inférieurs.

La recherche d'emploi est un moyen important par lequel la discrimination fonctionne au marché du travail. Plusieurs articles ont mis en évidence l'impact de la discrimination à travers le canal de recherche d'emploi sur les inégalités salariales. Pour en nommer quelques uns, Pendakur et Woodcock (2010) montrent que les plafonds de verre existants pour les travailleurs immigrés et des minorités

peuvent être attribués par une large mesure à leur faible accès aux emplois dans les entreprises à hauts salaires; En outre, dans un article important du Ritter et Taylor (2011), ils montrent que la plupart de la disparité du taux de chômage ne pouvait être expliquée par les compétences cognitives qui émergent à un stade précoce, bien que pour l'écart salarial, il pourrait être le cas. Ce résultat concernant la disparité entre les taux de chômage est confirmé par la constatation que cette disparité est encore importante, même pour les travailleurs de niveaux de compétences similaires.

Notre étude est plus étroitement liée à la littérature de recherche d'emploi dirigée. Dans cette littérature, les frictions liées à la recherche d'emploi sont dérivées de façon endogène par les interactions stratégiques séquentielles des agents. Tenant compte de l'interaction stratégique permet que l'externalité de la recherche d'emploi soit internalisée. L'économie résultant demeure concurrentielle, mais avec une structure de marché non-Walras, et les prix jouent un rôle d'allocation pour atteindre l'efficacité. Au mieux de nos connaissances, dans la littérature de la discrimination avec les frictions à la recherche d'emploi, seulement deux d'entre eux sont construits sous le contexte d'annonce des salaires. Lang, Manove, et Dickens (2005, ci-après LMD) montrent qu'une règle d'embauche discriminatoire pourrait conduire à une segmentation du marché du travail et un écart salarial important avec même une différence négligeable de la productivité; cependant, le groupe des travailleurs discriminés se révèlent avoir un taux de chômage inférieur, ce qui est en contraste frappant avec des évidences empiriques. Merlino (2012) vise à améliorer le résultat de LMD (2005). Il considère en outre l'investissement pré-marché du côté des entreprises, et obtient la dispersion de la technologie et l'écart de chômage réaliste. Ses résultats reposent sur l'hypothèse forte qu'il y a plus de discrimination dans le secteur de la haute technologie, et il est silencieux sur les niveaux de compétences des travailleurs. Notre papier diffère de la leur, en ce que notre objectif est d'analyser comment discrimination à l'embauche pourrait fausser les incitations à l'investissement de compétences des travailleurs et la structure de la segmentation du marché.

Dans cet article, nous commençons avec le cadre sans discrimination, où nous considérons un modèle de recherche dirigée, et on ajoute une phase de l'investissement de compétences des travailleurs. Plus précisément, à l'étape 0

de ce jeu, les travailleurs choisissent leurs niveaux de compétence, soit L ou H, et paient les coûts correspondants; à l'étape 1, les entreprises observent les compétences des travailleurs, et elles annoncent les contrats salariaux dans lesquels les entreprises peuvent spécifier quel niveau de compétences des travailleurs elles veulent embaucher; à l'étape 2, les travailleurs observent les offres salariaux et peuvent choisir où à postuler (ou quelle firme à visiter); à l'étape 3, les entreprises observent combien de postulations elles ont reçues et sélectionnent les travailleurs parmi ces demandes reçues, et elles choisissent des travailleurs qui ont de compétence identique avec une probabilité égale. Ensuite, la production est réalisée et les gains sont réalisés. Nous nous concentrons sur équilibres de Nash parfait par sous-jeux concurrentiels (subgame perfect competitive equilibria, SPCE, en anglais). Les entreprises choisissent les salaires pour maximiser les profits, et les travailleurs maximisent leur gain espéré en choisissant d'abord le niveau de compétence et puis quelle entreprise à postuler.

Dans un tel contexte avec investissement de compétences et sans discrimination, nous montrons que la décision optimale de l'investissement des compétences des travailleurs dépend de la comparaison entre le rendement logarithmique de l'investissement des compétences et la tension du marché. Lorsque la valeur du rendement logarithmique des compétences est suffisamment grande par rapport à la tension du marché, qui mesure l'intensité de la concurrence du marché, les demandeurs d'emploi trouvent que l'investissement dans de hautes compétences est une stratégie dominante. Dans ce cas, le bien-être social est plus élevé parmi tous les équilibres. Lorsque la valeur du rendement logarithmique des compétences est modérée, il existe un équilibre où les demandeurs d'emploi sont indifférents d'être qualifiés ou peu qualifiés; toutes les entreprises trouvent qu'il est optimal d'attirer les deux types de compétences en même temps et le bien-être social est au niveau plus faible par rapport à l'équilibre précédent. Enfin, lorsque la valeur du rendement logarithmique de l'investissement des compétences est suffisamment faible, il ne fournit pas aux travailleur suffisamment d'incitation d'investir pour être qualifiés; le niveau du bien-être social à l'équilibre se révèle être le plus faible.

Ensuite, nous analysons le modèle où la discrimination est présente. Nous considérons une économie où les travailleurs peuvent être divisées en deux groupes, le groupe a et le groupe b, selon certaine caractéristique qui est sans rapport avec la

productivité, par exemple, le sexe. Les deux groupes sont ex ante identiques dans tous les autres aspects. Les entreprises n'embauchent les travailleurs du groupe b que dans le cas où aucun travailleur du groupe a est présent. En se concentrant uniquement sur l'équilibre de Nash en stratégies pures, nous formalisons les résultats concernant l'existence d'équilibre comme suivants

Il existe deux seuils $\hat{\beta}_1$ *et* $\hat{\beta}_2$*, avec* $0 < \hat{\beta}_2 < \hat{\beta}_1 < \hat{\beta}$*.*

Lorsque $0 < \beta < \hat{\beta}_2$*, il existe un équilibre de Nash en stratégies pures unique dans lequel le groupe a et le groupe b investissent tous les deux dans hautes compétences.*

Lorsque $\hat{\beta}_2 < \beta < \hat{\beta}_1$*, il existe un équilibre de Nash en stratégies pures unique dans lequel le groupe a investit dans hautes compétences, cependant le groupe b investit dans faible niveau de compétences.*

Lorsque $\hat{\beta}_1 < \beta \leq \hat{\beta}$*, il existe des équilibres de Nash en stratégies pures multiples. Soit groupe a investit dans hautes compétences et le groupe b investit dans faible niveau de compétences, ou le groupe a investit dans les compétences faibles et le groupe b investit dans hautes compétences.*

Nous voyons que si un groupe choisit d'être peu qualifiés, la meilleure réponse de l'autre groupe est toujours de choisir à être qualifiés, tandis que la meilleure réponse au choix de hautes compétences de l'autre dépend des deux seuils. En outre, lorsque la tension du marché augmente, les travailleurs ont plus forte incitation à dévier à faible niveau de compétences, et le groupe discriminé est plus enclin à dévier par rapport au groupe favorisé, dans le sens que le seuil à partir duquel le groupe b commence à envisager d'investir à faible niveau de compétences est plus faible pour le groupe b que pour le groupe b.

Nous discutons aussi le problème dans une économie où le salaire est déterminé par la négociation après un travailleur et un employeur se rencontrent avec le même classement discriminatoire. Dans un tel contexte, les travailleurs ne choisissent que le niveau de compétences, mais pas où à postuler. Le timing de l'économie est désormais le suivant: tout d'abord, les travailleurs décident simultanément les niveaux de compétence; en suite, les travailleurs et les entreprises se rencontrent selon une technologie d'appariement; troisièmement, le travailleur et l'entreprise qui forment une paire déterminent comment partager la production par négociation à la

Nash. La norme d'embauche est identique comme précédent: $aH \succ bH \succ aL \succ bL$. Les probabilités d'emploi pour les différents types de travailleurs sont héritées, et le gain espéré est le produit de la probabilité d'emploi et son salaire moins le coût d'investissement de compétences. Comme dans le cadre de l'annonce de salaire, nous considérons l'investissement de compétences sous l'hypothèse que tout le groupe choisit soit d'être qualifiés soit d'être peu qualifiés, et nous supposons que le groupe a est la majorité. En raison de la règle discriminatoire, les utilités de l'investissement des compétences pour différents groupes de travailleurs sont interdépendantes. Cela rend encore la décision de l'investissement stratégique. Selon les valeurs différentes du pouvoir de négociation, les équilibres similaires au contexte précédent de l'annonce de salaire existent. Par exemple, il existe un équilibre où le groupe favorisé n'investit pas, tandis que le groupe discriminé reste hautement qualifié. Le groupe discriminé est en général moins bien par rapport au cas où la discrimination est absente, dans le sens qu'ils peuvent toujours choisir à sous-investir lorsque le pouvoir de négociation est suffisamment élevé. Les profits des entreprises sont monotones par morceaux parce que la composition des compétences (donc la productivité moyenne) du marché améliore discrètement à l'égard du pouvoir de négociation, et pour un niveau intermédiaire de pouvoir de négociation, le profit des entreprises est inférieur par rapport au niveau correspondant à l'économie où la discrimination est absente.