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R&D Investments under Endogenous Cluster Formation*

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Abstract

We study investments in R&D and the formation of R&D clusters of firms which are competitors in the market. In a three stage game, firms first decide on the budget allocated to their R&D department, then form research clusters and finally compete in quantities. The second stage cluster formation is modeled by the unanimity game introduced in Bloch(1995). We show that for any distribution of R&D investments, an equilibrium of the second stage cluster formation exists and is generically unique up to a permutation of firms which chose the same investment. Restricting to two investment levels in the first stage, we provide a complete characterization of the equilibria of the three stage game. We show that for some range of investment costs, equilibria with no-investment co-exist with equilibria where a large fraction or even all firms invest in R&D. Furthermore, in the high-investment equilibrium firms over–invest compared to a scenario where research clusters are ex-ante fixed and also compared to the welfare optimum.

JEL Classifications: C71, C72, L13, O30
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1 Introduction

R&D Cooperations among firms play a crucial role in many industries (see e.g. Hagedoorn, 2002; Powell et al., 2005; Roijakkers and Hagedoorn, 2006). Examples include the formation of research joint ventures, the exchange of information, and the share of laboratories or facilities. Although empirical studies show that the majority of R&D cooperations between firms are vertical (i.e. with suppliers or customers), also a large number of horizontal cooperations between competing firms is observed, where this type

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of cooperation is most frequent in high-technology sectors (see Miotti and Sachwald, 2003). Recent examples for horizontal R&D cooperations include the Global Hybrid Cooperation between GM, Daimler, Chrysler, and BMW for the development of hybrid cars, the cooperation between Sony and Samsung for the development of TFT-LCD screens, or the cooperation between Lenovo and NEC to develop tablet computers.

Knowledge and technology transfer from the partners is a main motivation for firms to enter horizontal R&D cooperations and therefore the choice of the partners is of crucial importance (see Arranz and de Arroyabe, 2008; Miotti and Sachwald, 2003). As pointed out e.g. in Belderbos et al. (2004), the firms which mostly benefit from incoming spillovers for being far from the technological frontier might not have access to these spillovers since they are not attractive as cooperation partners. Empirical studies based on data from numerous countries and sectors have consistently found a positive relationship between the R&D intensity and the degree of R&D cooperation of firms (see e.g. Veugelers, 1997; Kaiser, 2002; Becker and Dietz, 2004; Franco and Gussoni, 2014), where some of these studies explicitly refer to permanent R&D investments or the existence of fully staffed R&D labs (e.g. Franco and Gussoni (2014)). Such positive correlation seems at odds with standard models of innovation incentives in the presence of knowledge spillover to competitors (see e.g. D'Aspremont and Jacquemin, 1989; Kamien et al., 1992), which predict that an increase in the intensity of the knowledge exchange (typically captured by a spillover parameter) reduces the R&D investments of the firms[1]. However, as has been shown in Kamien and Zang (2000), in an extended model which incorporates that the absorptive capacity of firms is positively influenced by own R&D, an increase in the spillover parameter leads to stronger R&D incentives as long as the elasticity of the absorptive capacity with respect to own R&D is sufficiently large. In light of these results and, more generally, in the extensive literature on absorptive capacity started by Cohen and Levinthal (1989), the empirical evidence about the positive relationship between R&D investments and R&D cooperation has been mainly interpreted as evidence that firms need own R&D activities to profit from R&D cooperations.

However, the formation of an R&D cooperation requires the agreement of all partners, which means that the R&D expertise of a firm, determined by (permanent) R&D expenditures, does not only influence the incentives of the firm to enter R&D cooperations, but also determines whether potential partners are willing to enter such an agreement with the firm. This aspect of the formation of R&D cooperations has so far been neglected in the theoretical literature and this paper makes a first step to fill this gap. In particular, we consider a Cournot oligopoly where firms choose their (long-term) level of R&D investment before they form R&D clusters. Firms within the same cluster receive spillovers from all cluster members and the sum of own R&D and incoming spillovers determines the marginal production costs of a firm. Although in

[1]Consistent with the literature on R&D networks, to be reviewed below, in this paper we interpret R&D cooperations as an agreement to share (parts of) the R&D results with the partners. The literature on R&D joint ventures initiated by D'Aspremont and Jacquemin (1989); Kamien et al. (1992) typically also considers the effect of cooperating by jointly determining the level of R&D investments of all partners with the goal of maximizing joint profits of the partners. In the empirical literature these different types of cooperations usually cannot be distinguished. Many studies are based on European Community Innovation Survey (CIS) data and in CIS questionaires cooperations are defined in a broad sense including an informal exchange of information.

the main body of the paper we restrict attention to cost reducing process R&D, which is in accordance with the majority of the theoretical literature on R&D cooperation, we show in Appendix A that all our findings also apply to a model in which firms engage in quality improving product innovation such that the type of innovation (cost reduction or quality improvement) is not important for our results.

The main innovative aspect of our analysis is that we explicitly consider interplay between the firms' R&D decisions and the process by which the R&D clusters among potentially heterogeneous firms with repect to R&D investments are formed. To model the formation of clusters we employ a non-cooperative game, which is a version of the unanimity game first introduced in Bloch (1995). Our approach captures that firms choosing a high level of R&D investments do not only thereby reduce their production costs, but become more attractive for potential partners since members of a cluster with high investing firms will receive a larger amount of spillovers. In order to focus on this aspect of the choice of own R&D investment, we abstract from any dependence of the absorptive capacity of a firm on own R&D spending.

Formally, we consider a non-cooperative three stage game, where in the first stage firms choose between two levels (high/low) of cost-reducing R&D investment, in the second stage they engage in the unanimity game in order to determine the profile of R&D clusters, and in the third stage, after the spillovers in all consortia have been realized, firms compete with respect to quantities. We fully characterize the investment patterns and the structure of the R&D clusters arising in the subgame perfect equilibria for different values of the R&D investment costs. With respect to the emerging structure of the R&D clusters we show that under weak conditions all firms will be arranged in exactly two clusters such that one of these clusters may even be heterogenous, i.e. consisting of both low and high investors.Investing high increases the probability to participate in the more attractive cluster consisting of a larger number of high investors and thereby to profit from the corresponding spillovers. For a large range of the number of high investing firms in the population this effect is stronger the more other firms in the industry choose a high R&D level, and, based on this effect, strategic complementarities between the R&D investment decisions of the firms arise. Whereas for sufficiently small and sufficiently large investment costs a unique equilibrium pattern with all respectively none of the firms investing high arises, we show that for a large intermediate range of investment costs a no–investment equilibrium co-exists with an equilibrium where a large fraction or even all firms choose high level of R&D.

Furthermore, we show that the endogenous formation of clusters has a substantial positive effect on R&D incentives. In a scenario where firms are exogenously grouped into clusters the range of investment costs values where no investment is the unique equilibrium is substantially larger. In particular, there is a range of investment cost values such that in the unique equilibrium of the game with exogenous consortia no firm invests although the only equilibrium profile under endogenous formation of consortia implies full investment.

Comparing equilibrium outcomes with the welfare optimum, it turns out that the emerging clusters are too small from a welfare perspective. Due to the strategic complementarity between firms' R&D decisions, distortions of investment incentives relative to the social optimum in both directions can occur. On the one hand, for a considerable range of investment costs over–investment arises in a sense that there is an equilibrium with high investment of all or at least a large fraction of the firms, whereas no investment would be optimal from a welfare perspective. On the other hand, for smaller values of investment costs, profiles without any investment can emerge in equilibrium although welfare is maximized if all firms choose a high R&D level.

Due to these distortions in both directions, policy relevant mechanisms to increase welfare may be discussed. In the case of under–investment, sketched above, a small change in investment costs, e.g. due to R&D subsidies, can induce an abrupt increase in the level of R&D investment and vice versa. Moreover, measures to foster R&D exchange in clusters may lead to an increase in the investment incentives and can also overcome under–investment scenarios. Our analysis, moreover, generates several empirically testable implications about the relationship between certain industry characteristics and the size and heterogeneity of horizontal R&D consortia.

The present paper substantially extends the theoretical literature on R&D cooperations since it is the first contribution to provide a general analytical characterization of emerging R&D cooperation structures in an oligopoly setting with endogenous choice of R&D effort and an arbitrary number of firms. There is a body of literature which studies the formation of cooperation structures between competitors. Most closely related to our model are Goyal and Moraga-Gonzalez (2001) and Greenlee (2005) who also consider settings where both the choice of R&D effort and the formation of cooperation structures are endogenous. Goyal and Moraga-Gonzalez (2001) restrict attention to binary cooperations and characterize stable R&D networks in this setting under the assumption that all firms have an identical number of cooperation partners. A general analysis, not relying on the assumption of a regular R&D network, is provided only for the special case of three firms. Greenlee (2005), instead, provides a partial analytical characterization together with a numerical analysis of the shape of R&D consortia generated through the unanimity game in a setting where firms endogenously choose their R&D effort. Both Goyal and Moraga-Gonzalez (2001) and Greenlee (2005) differ from our setup by assuming that the firm's choice of R&D investment occurs after the cooperation structure has been settled. In this sense these papers deal with short term R&D decisions, whereas we are concerned about the decision about the long term (permanent) R&D level, like the size and endowment of the firm's R&D department.

Our contribution also extends the paper by Bloch (1995), where the outcome of the unanimity game is characterized in a Cournot oligopoly setting where marginal costs of a firm are entirely determined by the pure size of its consortium. In particular, investments in R&D are not modeled in Bloch (1995). In our setting, the analysis in Bloch (1995) corresponds to a scenario where all firms have identical levels of R&D investment. We show in the more general case of firms with potentially heterogeneous investements that different structures emerge, but reproduce the findings of Bloch (1995) as a special case of our analysis. Incorporating endogenous and potentially heterogeneous investment levels, our results can also be used to understand the robustness of the qualitative insights from Bloch (1995) with respect heterogeneity of firms' investments.

Moreover, there are several studies on the formation of bilateral R&D collaborations between homogeneous firms which abstract from endogenous determination of R&D investments. It is shown in Goyal and Joshi (2003), König et al. (2012) and Dawid and Hellmann (2014) that group structures (where all firms within a group are connected) emerge which resembles the structure that emerges from the cluster formation cases. In an analogous framework, Westbrock (2010) studies efficient networks and concludes that the welfare maximizing structures may have similar structures where, however, the sizes of groups differ from the stable structures.

The paper is organized as follows. Our model is introduced in Section 2, in which we also characterize the equilibrium outcome of the Cournot competition stage. Section 3 provides an analysis of the equilibria in the cluster formation stage and the resulting equilibrium investment patterns are examined in Section 4. In Section 5 we provide a welfare analysis of our findings and we conclude in Section 6. In Appendix A we briefly outline a variant of our model where firms invest in product rather than process innovation, to which our results also apply. All proofs are given in Appendix B.

2 The Model

An oligopoly of a set $N = \{1, \ldots, n\}$ of ex ante identical[2] firms engage in a three stage game. Firms first choose permanent R&D efforts, then form R&D clusters and finally compete in the market by choosing quantities of a homogeneous product.[3]

When investing in R&D, firms make long-term and irreversible investment decisions, like building facilities, investing in a lab, or committing a budget to a permanent R&D fund. For simplicity, we assume that the investment decision is binary, such that firms can either invest high or low. We denote by $x(i) \in \{\underline{x}, \bar{x}\}$ the R&D effort of firm i. Choosing to invest high, $x(i) = \bar{x} > \underline{x} \geq 0$, implies costs of $\xi > 0$, whereas the costs of low effort $\underline{x}$ are normalized to zero. In what follows we denote by $\mathbf{x} = (x(1), \ldots, x(n))$ the profile of R&D effort.

Firms may cooperate with other firms to lower their production costs. To do so, firms form clusters where research is shared. Each firm can only participate in one such cluster, or can stay singleton. Hence, the cluster structure or profile of R&D clusters[4], denoted as $\mathbf{A} = (A_1, \ldots, A_K)$, is a partition of the set of firms, i.e. $A_k \subseteq N \ \forall k = 1, \ldots, K$, $\bigcup_{k=1}^{K} A_k = N$, $A_k \cap A_j = \emptyset \ k, j = 1, \ldots, K, j \neq k$. The cluster to which firm i belongs will be referred to as $A(i)$.

We assume that marginal production cost is constant and that R&D has a cost reducing effect and is shared within the respective clusters. That is, incoming spillovers in their cluster contribute to the cost reduction of firms. Thus marginal cost of firm i is given by

$$c(i, \mathbf{x}, \mathbf{A}) := \bar{c} - \gamma\Big(x(i) + \beta \sum_{\substack{j \in A(i) \\ j \neq i}} x(j)\Big), \ i = 1, .., n, \tag{1}$$

where $\bar{c}$ is the base cost (pre-innovation) cost level, the parameter $\gamma > 0$ measures the marginal effect of R&D effort on marginal costs and $0 < \beta < 1$ captures the intensity of knowledge exchange within a cluster. We assume that marginal costs in the absence of any R&D effort are below the reservation price on the market, i.e. $\alpha > \bar{c}$. Whenever the context is clear, we will also denote $c(i) = c(i, \mathbf{x}, \mathbf{A})$ to save notation.

[2] At the end of Section 4 we briefly discuss the effect of heterogeneous investment cost ξ.

[3] When we interpret R&D as product innovation rather than process innovation, products are differentiated while marginal costs are homogeneous, see Appendix A. Both model formulations lead to the same results.

[4] In order to avoid confusion with the variables denoting firms' marginal cost we denote the clusters by A_k rather than C_k. This notation is motivated by Bloch (1995), where what we call clusters is denoted as associations.

Whereas the model described here interprets R&D as process innovation effort reducing marginal costs, in Appendix A we show that a model where R&D effort influences product quality, rather than marginal costs, yields qualitatively analogous results. Hence, the assumption that firms engage in process rather than product innovation is not crucial for our analysis. We focus on the process innovation case following the established literature on R&D network respectively cluster formation which has consistently treated the process innovation case.

Producing quantities of the homogeneous product $q(i)$, $i \in N$, firms face linear inverse demand given by

$$P(Q) = \alpha - Q,\ \alpha > 0,$$

where P denotes the price and $Q = \sum_{i=1}^{n} q(i)$ total quantity.

Since we focus on long-term or permanent R&D investments, cluster formation can adapt much faster. Hence, we model the timing of the choices by the following three stages.

Stage 1: Effort Choice
All firms simultaneously choose their R&D effort $x(i) \in \{\bar{x}, \underline{x}\}$. The effort profile $\mathbf{x}$ becomes public knowledge at the end of the stage.

Stage 2: Cluster Formation
Firms non-cooperatively form R&D clusters. To model the cluster formation process we employ the unanimity game introduced in Bloch (1995). The unanimity game models the cluster formation process as a sequential game where firms propose clusters according to a given rule of order. We assume that the rule of order, i.e. a permutation of firms $\rho : N \to N$, is chosen from the set $\Pi = \{\rho : N \to N | \rho(i) < \rho(j) \text{ if } x(i) > x(j)\}$ with equal probability. The lowest firm in order ρ then proposes a set of firms as the first cluster. All firms included in the proposal are then asked according to the order ρ whether they agree to join the cluster. If all firms in the proposal agree to join, the cluster forms, the firms leave the game, and the lowest remaining firm in the order ρ proposes the next cluster. If one of the firms in the proposal disagrees to join, then all firms remain in the game and the next proposal is made by the firm who first disagreed to join. This procedure is repeated until all firms have joined a cluster. The resulting cluster profile $\mathbf{A}$ becomes public knowledge. Assuming that firms with high R&D effort propose clusters before low investors substantially simplifies the analysis. Furthermore, for sake of simplicity we abstract from discounting between stages of the unanimity game.

Stage 3: Quantity Choice
Firms simultaneously choose quantities given the profile of marginal costs determined by the R&D effort choices and the formed clusters, see (1). Standard calculations yield that under the assumption of a sufficiently large α the Cournot equilibrium in the 3rd stage is given by

$$q^*(i, \mathbf{x}, \mathbf{A}) = \frac{\alpha - (n+1)c(i, \mathbf{x}, \mathbf{A}) + \sum_{j \in N} c(j, \mathbf{x}, \mathbf{A})}{n+1} \tag{2}$$

and the profits read $\pi^*(i, \mathbf{x}, \mathbf{A}) = \left(q^*(i, \mathbf{x}, \mathbf{A})\right)^2 - \xi \mathbb{1}_{x(i)=\bar{x}}$. To abbreviate notation we will also denote firm i's quantities and profits by $q^*(i)$, and $\pi^*(i)$, respectively.

In order to analyze the game described above we focus on the subgame perfect equilibria of the game and therefore apply backward induction. With respect to the unanimity game in general, Bloch (1996) shows that there exists a subgame perfect equilibrium with the property that all firms always accept a proposal as long as rejecting would not result in a strictly higher payoff.[5] In what follows we restrict attention to this type of subgame perfect equilibrium in the unanimity game.

3 Cluster Formation

When forming the R&D clusters according to the unanimity game, interesting effects arise. Firms face the tradeoff between achieving a cost advantage through the incoming spillovers and allowing other firms a cost advantage by reducing the cost of other cluster members while sharing the research within the cluster. This tradeoff is also present in Bloch (1995). In our model, because firms are heterogeneous with respect to their R&D effort chosen in the first stage, the net effect under this tradeoff depends on the profile of the cluster and the investment level of the considered firm.

To understand above effects, let us, thus, inspect the payoff implied by the Cournot quantities in the third stage (2), resulting from a given pattern of investment $\mathbf{x}$ and given cluster structure $\mathbf{A}$. In what follows we denote by h respectively l the number of high (low) investors in the firm population. Whenever we refer to these numbers excluding firm i we indicate this as h^{-i}, respectively l^{-i}, while a subscript A restricts the respective numbers to cluster $A \in \mathbf{A}$. Plugging (1) into (2) and simplifying, we get,

$$\begin{aligned}\pi(i) = &\tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(nx(i) - h^{-i}\bar{x} - l^{-i}\underline{x}) \\ &+ \gamma\beta\Big((n - h^{-i}_{A(i)} - l^{-i}_{A(i)})(h^{-i}_{A(i)}\bar{x} + l^{-i}_{A(i)}\underline{x}) + h^{-i}_{A(i)}(\bar{x} - x(i)) - l^{-i}_{A(i)}(x(i) - \underline{x}) \\ &- \sum_{A_k \neq A(i)} (h_{A_k}((h_{A_k} - 1)\bar{x} + l_{A_k}\underline{x}) + l_{A_k}(h_{A_k}\bar{x} + (l_{A_k} - 1)\underline{x}))\Big)\Big]^2 - \xi \mathbb{1}_{x(i)=\bar{x}}. \qquad (3)\end{aligned}$$

Since Cournot quantities are anticipated in the third stage, firms try to optimize (3) in the cluster formation process. A closer inspection of (3), hence, turns out to be very useful for understanding the logic of the cluster formation process. First, note that the expression on the right hand side of the first line only captures the effects of the direct cost reductions generated by the R&D investments of all firms and as such is independent from the cluster profile. The effects of spillovers on the profit of firm i is given in the second and third line. The second line corresponds to the spillovers arising in the cluster of firm i, and consists of a positive term stemming from spillovers received by firm i and two negative terms describing the spillovers obtained by the other firms in the cluster. Finally, the third line depicts the effects of the spillovers in all other clusters on firm i's profit, having a cost reducing effect for other firms and, via the price channel, a negative effect for firm i's profit. Moreover, the third line also includes the costs of investment and therefore contains only negative terms.

[5]This observation follows from Proposition 2.4 in Bloch (1996) where it is shown that every subgame perfect equilibrium of the unanimity game with discounting is also a subgame perfect equilibrium in the game without discounting if the discount factor is sufficiently close to 1.

When a firm $i \in N$ is selected to propose a cluster and contemplates which firms to include in the proposal, the marginal effect of adding an additional firm which otherwise might end up in a different cluster plays a crucial role. Hence, consider the impact of moving one firm j from a cluster $A(j) \neq A(i)$ to cluster $A(i)$. Since such a move does not affect investment costs of firm i and profit net of investment costs is the square of firm i's quantity we can restrict attention to the induced change in equilibrium quantity $q^*(i)$. This change in quantity in response to a move of firm j from $A(i)$ to $A(j)$ can be calculated to be

$$\begin{aligned}\Delta q^*(i) \;=\; & \tfrac{\gamma\beta}{n+1}\Big(nx(j) - (h_{A(i)} + l_{A(i)} - 1)x(j) - (h_{A(i)}\bar{x} + l_{A(i)}\underline{x}) + (h^{-j}_{A(j)} + l^{-j}_{A(j)})x(j) \\ & +(h^{-j}_{A(j)}\bar{x} + l^{-j}_{A(j)}\underline{x})\Big)\,. \qquad (4)\end{aligned}$$

Adding firm j from a cluster $A(j)$ to $A(i)$ has an effect on both i's and j's spillovers, as well as on all firms' spillovers within the respective clusters. First, firm i experiences additional spillovers by adding j where the size depends on the R&D effort of j captured by the first term in the brackets of (4). However, all other firms within i's cluster are also enjoying these spillovers which are given by the second term and firm j receives the spillovers from the whole cluster (third term). These two terms are negative since a cost reduction of other firms lead to higher quantities of these firms, thus, lower the price and decrease the equilibrium quantities (and hence profit) of i. Note that both of these terms increase in absolute value with the size of $A(i)$ since increasing the number of firms in i's cluster means that more firms receive the additional spillovers and j receives more spillovers from those firms. The last two terms of (4) describe the effects of the reduction in spillovers for the remaining members of cluster $A(j)$ and of firm j loosing spillovers from its former cluster. These two effects are positive for the profit of firm i and their size increases with the size of cluster $A(j)$.

Three important observations can be made. First, it is easy to see that $\Delta q^*(i)$ is independent of $x(i)$, implying that whenever it is optimal for a firm to invite an additional firm to its cluster, the same also holds true for all other firms in the same cluster, regardless of their choices of R&D effort. Second, $\Delta q^*(i)$ is an increasing function of $x(j)$, which means that all firms in $A(i)$ prefer to invite a firm j with high R&D effort compared to a member of $A(j)$ with low R&D effort. Third, the incentive to invite a firm j to the own cluster decreases with the size of the own cluster but increases with the size of the current cluster of firm j.

The three observations discussed above provide a clear intuition for the potential structure of the cluster profile in equilibrium.[6] Due to the fact that firms always prefer high R&D firms to join their cluster compared to low R&D firms, it is intuitive that low R&D firms are only included in a cluster proposal if no more high R&D firms are available. Hence, there can be at most one cluster containing heterogeneous firms, i.e. containing both high and low investors. Since $\Delta q^*(i) > 0$ implies $\Delta \pi^*(i) > 0$, the strategy determining which clusters arise becomes clear. We solve the game via backward induction by supposing that $n - m$ firms have already formed clusters and then determine conditions under which all remaining m firms join one cluster. That is, we determine under which conditions a singleton low effort firm j with $|A(j)| = 1$,

[6] Although the intuition is very straightforward, the derivation of the subgame equilibria of the unanimity game is quite involved, see proof of Proposition 1.

is accepted in a cluster of size $|A(i)| = m-1$. Note that this corresponds to the case where the incentive to add j is minimal since by (4) the incentive is decreasing in the size of $A(i)$ and decreasing in the investment $x(j)$ while increasing in the size of $A(j)$. Hence, we get from (4) for m remaining firms with h_m remaining high investors and $l_m \neq 0$ low investors (thus, $m = h_m + l_m$) that $\Delta q^*(i) > 0$ if and only if

$$m < \tfrac{n+3}{2} - \tfrac{1}{2}h_m\left(\tfrac{\bar{x}}{\underline{x}} - 1\right) \tag{5}$$

while for m homogeneous firms, i.e. $l_m = 0$ or $h_m = 0$, $m < \frac{n+3}{2}$. Hence, by restricting to integers, we may state that m firms with $l_m \neq 0$ join, if and only if $m \leq \left\lceil \frac{n+1}{2} - \frac{1}{2}h_m\left(\frac{\bar{x}}{\underline{x}} - 1\right)\right\rceil$ where $\lceil y \rceil$ is the largest integer smaller than $y \in \mathbb{R}$. To see the latter note that as long as $\Delta q^*(i) > 0$ cluster $A(i)$ has an incentive[7] to add firm j. Thus, to have $\Delta q^*(i) \leq 0$ we need $m \geq \left\lceil \frac{n+3}{2} - \frac{1}{2}h_m\left(\frac{\bar{x}}{\underline{x}} - 1\right)\right\rceil$.

From (5), we can immediately conclude that the grand coalition, i.e. a cluster comprising of all firms, can never be a subgame perfect equilibrium (from now on SPE) for $n \geq 3$. Further, we also cannot have more than three clusters forming in equilibrium. The intuition for the latter is simple. First, by above reasoning, for any pattern of effort choice in the first stage $\mathbf{x}$, we get at most one mixed cluster. Second, any homogeneous cluster not limited by the number of available firms (i.e. a cluster where the proposal would not change even if an additional firm of that type would become available) will consist of at least $\left\lceil \frac{n+1}{2} \right\rceil$ members which immediately implies that there cannot be more than one such homogeneous cluster. Hence, there can exist at most three different clusters in equilibrium.

One additional fact to note from (5) is that the maximal size of a mixed cluster (here when all firms join, but more generally for all mixed clusters) is decreasing in the ratio $\bar{x}/\underline{x}$. Thus, the more homogeneous low and high investors become, the more can we expect to see the formation of two clusters in equilibrium. Proposition 1 confirms this intuition and shows that only two clusters emerge in equilibrium if the ratio between $\bar{x}$ and $\underline{x}$ is not too large. In particular, to simplify the analysis, in what follows, we make the following assumption:

Assumption 1. *The ratio of R&D effort between high and low investors ($\bar{x}/\underline{x}$) is bounded above by* 2*.*

It should be noted that we only consider firms who are active in R&D and (apart from their R&D choice) are symmetric. Hence, restricting the analysis to scenarios where the variance in R&D levels is not too large does not seem to be overly restrictive. Furthermore, although the technical complexity would substantially increase, the qualitative mechanisms driving our results would hardly be affected if we relax Assumption 1.

Proposition 1. *For any profile of investment* $\mathbf{x}$*, there exists a stationary SPE of the cluster formation game. All SPE result in the formation of two clusters* $\mathbf{A} = (A_1, A_2)$*. The number of high and low investors in each cluster are generically unique and are a*

[7] Note that throughout our analysis we assume that a firm is not added to a cluster proposal if the other firms in the proposal are indifferent between including and excluding that firm in the cluster.

function of the total number of high investors h such that

$$h_{A_1}(h) = \begin{cases} h & \text{if } h \leq \tilde{\tilde{h}} \\ \left\lceil \frac{(2n+h-1)\bar{x}+(n-h)\underline{x}}{4\bar{x}} \right\rceil & \text{else} \end{cases}$$

$$l_{A_1}(h) = \begin{cases} \left\lceil \frac{(3(n-h)-1)\underline{x}-h\bar{x}}{4\underline{x}} \right\rceil & \text{if } h \leq \tilde{h} \\ 0 & \text{else} \end{cases}$$

where $\tilde{h} = \frac{(3n-1)\underline{x}}{3\underline{x}+\bar{x}}$, $\tilde{\tilde{h}} = \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}$. *Furthermore,* $h_{A_2}(h) = h - h_{A_1}(h)$ *and* $l_{A_2}(h) = n - h - l_{A_1}(h)$.

The proof of this proposition together with all other proofs are presented in Appendix B. The main difficulty is to exclude other subgame perfect equilibria in the unanimity game by Bloch (1995). We first assume that all proposals are accepted and some clusters have already formed such that the game is at a stage where only two clusters form. Then we use backward induction to show that indeed all proposals are accepted which is mainly due to symmetric incentives of high and low effort firms, see also (4). Finally, we show that under Assumption 1, the stage at which the game gives rise to only two clusters, is actually the stage, when the game starts. The composition of firms in these two clusters then follows.

Proposition 1 implies that essentially three different types of cluster constellations can emerge. If the number of high investors is small, then all these high investors together with a subset of the low investors form the first cluster and all remaining low investors join for the second cluster. If, on the contrary the number of high investors is sufficiently large, then the first cluster contains only high investors and the second cluster is mixed between high and low investors. For an intermediate range of the number of high investors the two types of investors sort into two homogeneous clusters. It is quite intuitive that the thresholds separating the first scenario from the case where all high investors join the same cluster decreases with the size of the ratio $\bar{x}/\underline{x}$ since the incentives for high investors to include a low investor in their cluster decrease. Similarly, the threshold separating the case with two homogeneous cluster from the scenario where the second cluster is mixed, also decreases with $\bar{x}/\underline{x}$. The intuition for this observation is that the incentives of the members of the first cluster to include an additional high investor, thereby preventing this high investor from receiving spillovers from the low investors in the second cluster, decreases as $\bar{x}/\underline{x}$ becomes larger.

In order to gain some additional intuition for the implications of a change in the number of high investors for the size and structure of the emerging clusters let us distinguish between the cases where the homogeneous cluster consists only of low respectively high investors. First, if the homogeneous cluster has only low investors and the other cluster is mixed, an increase of the number of high investors reduces the number of low investors in the mixed cluster, where this reduction is so strong that the overall size of that cluster is weakly[8] reduced. The fact that the inclusion of one additional high investor in the cluster might trigger a reduction of the number of low investors by more than one can be explained as follows. The outgoing spillovers of the low investors

[8]Due to the fact that all cluster sizes are integers they change in discrete steps. Throughout the paper we refer to stepwise decreasing (increasing) functions as weakly decreasing (increasing).

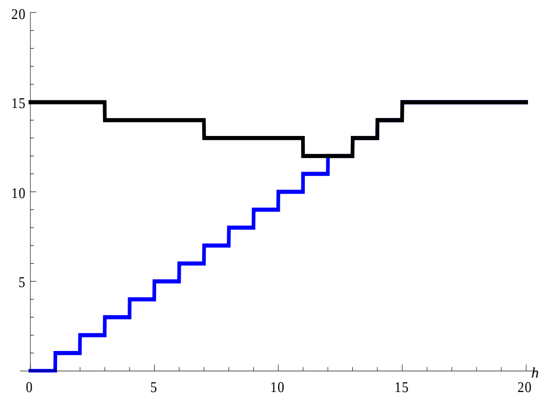


Figure 1: The size of the first cluster (black line) and the number of high investors in that cluster (blue line).

in the cluster remain the same, whereas the spillovers they receive increase due to the exchange of a low with a high investor. Hence, the incentive to have the low investors in the cluster decreases. Secondly, considering the cases where a mixed cluster coexists with a homogeneous cluster of high investors, an increase of the number of high investors induces a (weak) increase in the size of the homogeneous cluster and a (weak) decrease of the number of high investors in the mixed cluster. The underlying rationale is similar to above, namely that due to the exchange of a low investor with a high investors in the mixed cluster, the incentives for members of the homogeneous high investment cluster to transfer one additional high investor to their cluster also increase. The size and structure of the first cluster as a function of the number of high investors is illustrated in Figure 1.[9]

Finally, we note that for the case where all investments are homogeneous (i.e. either $\bar{x} = \underline{x}$ or $h = 0$ or $h = n$) the size of the first cluster is given by $\left\lceil \frac{3n-1}{4} \right\rceil$, which corresponds to the findings in Bloch (1995), where coalition formation in homogeneous populations is analyzed.

4 Effort Choice

In the investment stage, all firms simultaneously choose their R&D effort. In general, the profit of a firm induced by a certain investment profile $\mathbf{x}$ is stochastic due to our assumption that all sequences of proposal orders in the cluster formation game, which satisfy the assumption that high investors propose prior to low investors, have equal probability. Denoting by $\mathbb{E}(\pi(i, x(i), h^{-i})$ the expected profit of firm i with investment level $x(i) \in \{\underline{x}, \bar{x}\}$ if h^{-i} of its competitors choose high R&D investment, it is optimal for firm i to invest high if and only if $\Delta\pi(h^{-i}) := \mathbb{E}(\pi(i, \bar{x}, h^{-i})) - \mathbb{E}(\pi(i, \underline{x}, h^{-i})) > \xi$.

Two main effects determine the investment incentives of a firm: first, the implications of own investment for the expected attractiveness of the firm's cluster, and second, the expected profit increase for a given cluster allocation. Proposition 1 highlights that

[9] In all figures in this paper we use the default parameter setting: $n = 20, \alpha = 35, \bar{c} = 4, \beta = 0.2, \gamma = 0.2, \underline{x} = 1, \bar{x} = 2$.

under our Assumption 1 two clusters emerge, where typically the larger of the two clusters also contains the larger number of high investors. Taking this into account, the expected payoff difference between high and low investment can be written as

$$\Delta\pi(h^{-i}) = \left(p_{A_1}(\bar{x}, h^{-i}+1) - p_{A_1}(\underline{x}, h^{-i})\right)\left(\pi_{A_1}(\bar{x}, h^{-i}+1) - \pi_{A_2}(\bar{x}, h^{-i}+1)\right) \quad (6)$$
$$+ \mathbb{E}_{p_{A_1}(\underline{x},h^{-i})}\left(\pi(\bar{x}, h^{-i}+1)\right) - E_{p_{A_1}(\underline{x},h^{-i})}\left(\pi(\underline{x}, h^{-i})\right) + 2\pi_{A_2}(\underline{x}, h^{-i}).$$

where $p_{A_1}(x,h)$ denotes the probability of a firm with investment x to end up in the cluster A_1 and $\pi_A(x,h)$ gives the profit in cluster A of a firm with investment x, if a total number of h firms have chosen high investment. The term $\mathbb{E}_{p_{A_1}}(\pi(x,h)) = p_{A_1}\pi_{A_1}(x,h) + (1-p_{A_1})\pi_{A_2}(x,h)$ denotes the expected payoff of investing x for a given (fixed) probability p_{A_1} to end up in cluster A_1.

The first of the two main effects is captured in the first line of (6). Ceteris paribus, firms prefer to become a member of the larger cluster with more high investors (i.e. $\pi_{A_1} - \pi_{A_2} > 0$), since this generates stronger incoming spillovers for a firm compared to the smaller cluster with fewer high investors. Clearly, the probability p_{A_1} for a firm to end up in this preferred cluster A_1, depends both on the level of investment of the firm, as well as, the investment pattern of all its competitors. The probability for a firm to end up in the more attractive cluster A_1 can be directly derived from Proposition 1.

$$\begin{array}{lll} p_{A_1}(\bar{x}, h^{-i}+1) = 1, & p_{A_1}(\underline{x}, h^{-i}) = \frac{l_{A_1}(h^{-i})}{n-h^{-i}} & \text{if } h^{-i} \leq \tilde{h} \\ p_{A_1}(\bar{x}, h^{-i}+1) = 1, & p_{A_1}(\underline{x}, h^{-i}) = 0 & \text{if } \tilde{h} < h^{-i} \leq \tilde{\tilde{h}} - 1 \\ p_{A_1}(\bar{x}, h^{-i}+1) = \frac{h_{A_1}(h^{-i}+1)}{h^{-i}+1}, & p_{A_1}(\underline{x}, h^{-i}) = 0 & \text{if } h^{-i} \geq \tilde{\tilde{h}}, \end{array} \quad (7)$$

where $h_{A_1}, l_{A_1}, \tilde{h}$, and $\tilde{\tilde{h}}$ are given in Proposition 1. It is easy to see that both $l_{A_1}(h^{-i})/(n-h^{-i})$ and $h_{A_1}(h^{-i}+1)/(h^{-i}+1)$ are (weakly) decreasing functions of h^{-i}. This establishes that $p_{A_1}(\bar{x}, h^{-i}) - p_{A_1}(\underline{x}, h^{-i})$ is a weakly increasing function of h^{-i} for $h^{-i} \leq \tilde{h}$, but (weakly) decreasing for $h^{-i} \geq \tilde{\tilde{h}}$. Hence, the increase in the probability of ending up in the more attractive cluster, which is induced by high investment, becomes larger the more competitors choose high investment as long as this number does not become so large that high investors might end up in the second cluster. For this range of competitors with high investment the consideration of the probability to become a member of the stronger cluster introduces strategic complementarities into the R&D investment choice of the firms.

However, investment incentives are not entirely driven by the effect of R&D investment on the probability to join the stronger cluster. The expected change of firms' market profit for a given probability to end up in A_1 respectively A_2 influences investment incentives as well. Formally, this is expressed by $\mathbb{E}_{p_{A_1}(\underline{x},h^{-i})}(\pi(\bar{x}, h^{-i}+1)) - \mathbb{E}(\pi(\underline{x}, h^{-i})) > 0$, see (6). The strength of this second effect essentially depends on the expected change in firms' output due to high investment and also the expected level of output, because investment reduces the firm's unit costs of production.

The following Proposition shows that the strategic complementarity sketched above is indeed the dominant force in a sense that for a large range of investment costs extreme patterns (no investment or full investment) prevail in equilibrium and that such extreme equilibria might also co-exist.

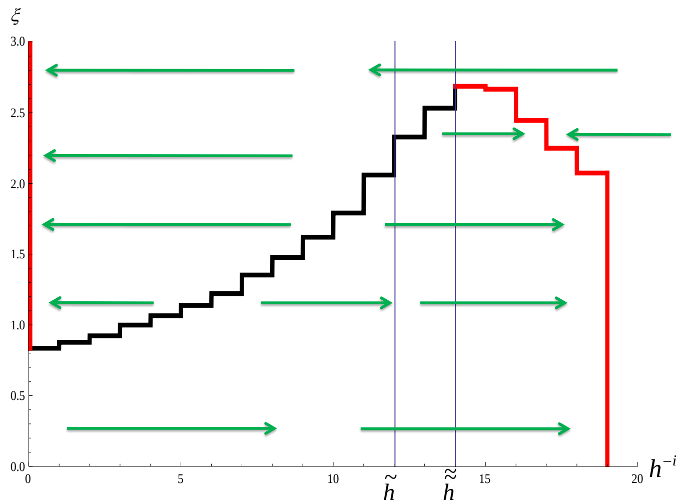


Figure 2: Best response and equilibria on the investment stage.

Proposition 2. *If* $\underline{\beta} := \frac{4\underline{x}}{(n+6)\underline{x}+\bar{x}} < \beta < 1/2$, *then there exist thresholds* $\bar{\xi}, \bar{\bar{\xi}}, \bar{\bar{\bar{\xi}}}$ *with* $\max[\bar{\xi}, \bar{\bar{\xi}}] < \bar{\bar{\bar{\xi}}}$ *such that*

- *For* $\xi < \bar{\xi}$ *there is a unique equilibrium (up to permutation of firms) where the number of firms investing* $\bar{x}$ *is given by* $\bar{h}(\xi) > 0$. *The function* $\bar{h}$ *is constant in* ξ *with* $h(\xi) = n$ *for* $\xi \leq \bar{\bar{\xi}}$ *and weakly decreasing (step-function) in* ξ *for* $\xi > \bar{\bar{\xi}}$.

- *For* $\bar{\xi} \leq \xi \leq \bar{\bar{\bar{\xi}}}$ *an equilibrium where* $\bar{h}(\xi)$ *firms invest* $\bar{x}$ *co-exists with an equilibrium where all firms invest* $\underline{x}$.

- *For* $\xi > \bar{\bar{\bar{\xi}}}$ *there is a unique equilibrium where all firms invest* $\underline{x}$.

The proposition is illustrated in Figure 2, which depicts the best response for a firm on the investment stage depending on the number of high investors among the competitors for different values of investment costs ξ. A green arrow to the left indicates that low investment is the best response, whereas an arrow to the right stands for a best response of high investment. The red lines correspond to equilibria on the investment stage, i.e. situations where the investment of all opponents is optimal given that the considered firm invests according to its best response. The step-function with the inverted U-shape corresponds to the expected change in firm i's market profits if i changes investment from $\underline{x}$ to $\bar{x}$. The figure shows that the qualitative properties of this profit difference is indeed closely related to the difference in the probability to end up in the more attractive cluster. In particular, it can be seen that the incentive to invest increases with h^{-i} for $h \leq \tilde{h}$ and decreases for $h^{-i} \geq \tilde{\tilde{h}}$ where $\tilde{h}$ and $\tilde{\tilde{h}}$ are the bounderies from Proposition 1.

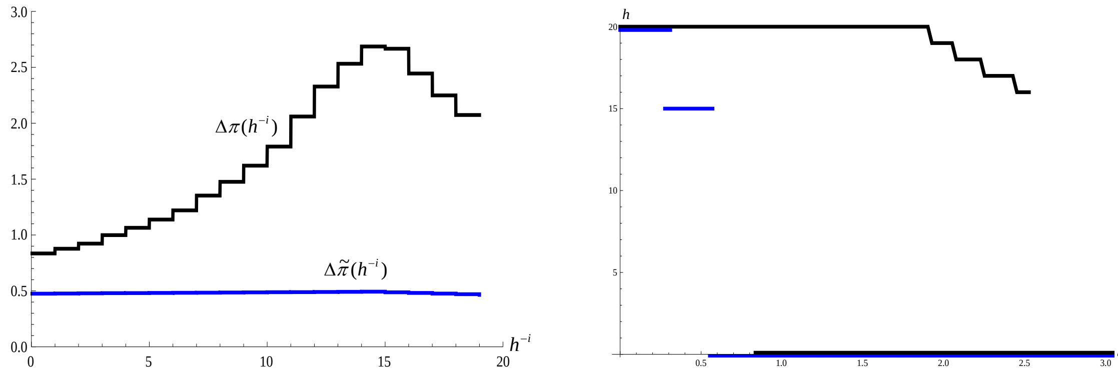
Proposition 2 assumes that the spillover parameter β to be in an intermediate range ($\underline{\beta} < \beta < 1/2$). To understand the implications of a very low spillover parameter $\beta << \underline{\beta}$ on the investment incentives, one can consider the extreme case of $\beta = 0$. In such a scenario, R&D investment decreases only the firm's own marginal production costs but generates no spillovers to other firms. It is well known (see e.g. Qiu, 1997) that under Cournot competition process innovation, investments are strategic substitutes. Hence, for sufficiently small β the firms' investment incentives are decreasing in h^{-i} and, hence, generically a unique equilibrium emerges. On the other hand, if the spillovers become very large ($\beta >> 1/2$), then the incentives stemming from the spillovers in the first (larger) cluster become dominant as the difference in spillovers between the two clusters increase. In such a scenario the main effect of an increase in h^{-i} is that the number of high investors in the first cluster grows. Hence, an increase in h^{-i} increases the spillovers in the larger cluster, where the size of that effect is increasing in β. Thus, investing high becomes more profitable the larger h^{-i} since it increases the probablity of being included in the large cluster. For large β this effect is so strong that strategic complements are satisfied over the whole range of h^{-i}. In this case, only equilibria with no investment and with full investment exist (and they might also co-exist). The most interesting case of the spillover parameter β, which allows also for equilibria with partial investment, is covered in Proposition 2.

The discussion above suggests that the desire to end up in the more attractive larger cluster is the main driving force for the investment behavior of firms. To further illustrate this point we compare the investment incentives in our model in which cluster formation is endogenous with such incentives in a setting in which the allocation of firms to the two clusters is ex-ante fixed. We assume that at most two clusters form, and focus on the maximal possible investment incentives across all possible cluster structures. Formally, we define by $\tilde{\pi}_{\tilde{A}_k}(x, \tilde{\mathbf{A}}, \mathbf{x}(-i))$ the market profit of a firm with investment level x in cluster $\tilde{A}_k, k = 1, 2$ if the profile of clusters is $\tilde{\mathbf{A}} = (\tilde{A}_1, \tilde{A}_2)$ and the investment profile of firm i's competitors $\mathbf{x}(-i)$). The maximal possible investment incentives of a firm given a number h^{-i} of other high investors can be written as

$$\Delta\tilde{\pi}(h^{-i}) := \max_{\tilde{\mathbf{A}}: h^{-i}_{\tilde{A}_1} + h^{-i}_{\tilde{A}_2} = h^{-i}} \left[\tilde{\pi}_{\tilde{A}_1}(\bar{x}, \tilde{\mathbf{A}}, \mathbf{x}(-i)) - \tilde{\pi}_{\tilde{A}_1}(\underline{x}, \tilde{\mathbf{A}}, \mathbf{x}(-i)) \right].$$

Although an analytical characterization of these maximal investment incentives under exogenous cluster allocation of firms is very involved, in Figure 3(a) they are compared numerically to the incentives under endogenous cluster formation. It can be clearly seen that the incentives are substantially larger under endogenous cluster formation. The gap is so large that for a certain range of investment costs ξ the best response of the considered firm under exogeneous cluster allocation is to choose $\underline{x}$ regardless of the investment pattern of the competitors, whereas under endogenous cluster formation it is $\bar{x}$ for all values of h^{-i}.

In order to allow for a more thorough comparison between scenarios with endogenous and exogenous cluster formation, in what follows we will sometimes refer to a scenario with ex-ante given clusters, where the cluster sizes are identical to the ones emerging as equilibrium size under endogenous cluster formation. Given the strategic complementarity between R&D investments of firms in the same cluster (for sufficiently large β) three potential equilibrium constellations might arise under such an exogenous cluster scenario. In addition to equilibria with no investment respectively full investment we can also have equilibria where all firms in the larger cluster A_1 invest, whereas all firms in the smaller cluster A_2 choose $x = \underline{x}$. The number of the high investors in the different types of equilibria under endogenous and exogenous cluster formation is illustrated in Figure 3(b). The figure shows that also under exogenous cluster allocation different equilibria might co-exist. Furthermore, the figure highlights that there is



(a) (b)

Figure 3: Investment incentives (a) and equilibrium number of high investors (b) under endogenous (black) and exogenous (blue) cluster formation.

a range of investment cost values for which the unique equilibrium under endogeneous cluster formation is high investment for all firms, but if clusters of identical size were fixed before the investment stage, then the unique equilibrium would be that all firms choose low investment.

Finally, let us briefly consider a scenario where, contrary to our baseline setting, firms are heterogeneous with respect to the R&D investment cost level ξ. Such heterogeneity might, for example, be based on differences with respect to the level of past R&D activities. For simplicity, let us consider the case where $\bar{n} < n$ firms have investment costs ξ_1 whereas the investment costs of the remaining $n - \bar{n}$ firms is given by $\xi_2 > \xi_1$. In what follows we argue that such heterogeneity may lead to an additional type of equilibrium compared to those described in Proposition 2. Such an equilibrium occurs when all firms with $\xi = \xi_1$ have incentives to invest high if they assume that $\bar{n} - 1$ competitors choose $\bar{x}$ whereas all firms with $\xi = \xi_2$ have incentives to invest low if they assume that $\bar{n}$ competitors choose high R&D. In this equilibrium $\bar{n}$ firms with low investment costs choose $\bar{x}$ and no other firm invests high. If $\bar{n}$ is not too large this implies that in equilibrium the large cluster A_1 consists of high and low investors, whereas the small cluster A_2 contains only firms with low R&D level. Such a scenario cannot occur as equilibrium outcome for homogeneous investment costs. Considering Figure 2 the scenario sketched here corresponds to a value of ξ_1 below the inverse U-shaped step-function for $h^{-i} = \bar{n} - 1$ and ξ_2 above the value of that step-function for $h^{-i} = \bar{n}$.

5 Welfare Analysis

In light of the different investment patterns and cluster profiles emerging under endogenous and exogenous cluster formation the question arises how welfare, consumer surplus and firm profits are affected and how these patterns compare to the social optimum. Given our linear demand function consumer surplus is given by

$$CS = \left(\sum_{i=1}^{n} q(i)\right) - \frac{1}{2}\left(\sum_{i=1}^{n} q(i)\right)^2 - P\left(\sum_{i=1}^{n} q(i)\right)$$

and we obtain for the social welfare function

$$W = \sum_{i=1}^{n} \pi(i) + CS = \sum_{i=1}^{n} (q(i))^2 - h\xi + \left(\sum_{i=1}^{n} q(i) \right)^2 /2. \tag{8}$$

Maximizing this function with respect to the investment pattern and the profile of clusters yields the following Proposition.

Proposition 3. *The following characterizes consumer surplus and welfare maximizing outcomes:*

(i) Consumer surplus is maximal if and only if all firms invest $\bar{x}$ and all join the same cluster.

(ii) If $\alpha - \bar{c}$ sufficiently large, then for all ξ the unique welfare maximizing cluster contains all firms.

(iii) If ξ is sufficiently low, then social welfare is maximized if and only if all firms invest $\bar{x}$ and all join the same cluster.

(iv) If ξ is sufficiently large, then social welfare is maximized if and only if all firms invest $\underline{x}$ and all join the same cluster.

Consumer surplus is maximized if the market price is minimized, which under Cournot competition corresponds to the minimization of average marginal costs. Hence, for consumer surplus to be maximal, R&D effort and spillovers must be maximized. Therefore, a single cluster in which all firms invest high is optimal from a consumer surplus perspective (point (i) of Proposition 3).

Considering welfare, the tradeoff between the costs of R&D investments and their return in terms of cost reduction have to be considered. If all firms have identical R&D effort, then from a social perspective the total cost reduction is clearly maximal if all firms join the same cluster, which maximizes spillovers. This explains parts (iii) and (iv) of Proposition 3. If firms are heterogeneous with respect to their R&D effort including low investors in a cluster of high investors has not only the spillover induced positive effect discussed above, but also induces a larger output for the low investor compared to a scenario where it would stay in isolation.[10] Hence, it is no longer obvious that a single cluster is welfare maximizing. However, part (ii) of Proposition 3 shows that the direct spillover effect always dominates if the market size is sufficiently large and therefore under such a condition the generation of a single cluster always maximizes welfare.

Combining Proposition 3 with Proposition 1 shows that the profile of clusters emerging in equilibrium is generically inefficient. This insight is also illustrated in Figure 4, in which the welfare maximum is compared to social welfare of the different types of equilibria under exogeneous and endogeneous cluster formation. Equilibrium welfare is always strictly below the maximum and it is obvious that this inefficiency stems from the profile of clusters since at least for very low and very high investment costs the welfare maximizing investment pattern coincides with that arising in equilibrium.

[10]This effect is closely related to the well-known fact that reduction of marginal costs of firms with low market shares in Cournot competition can be welfare reducing, see Lahiri and Ono (1988).

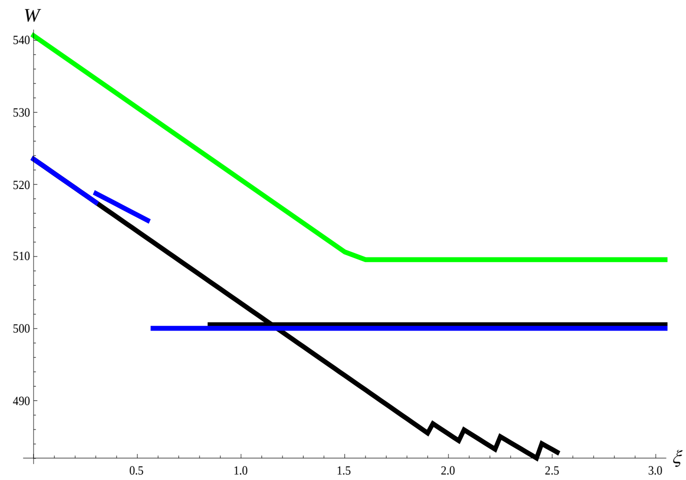
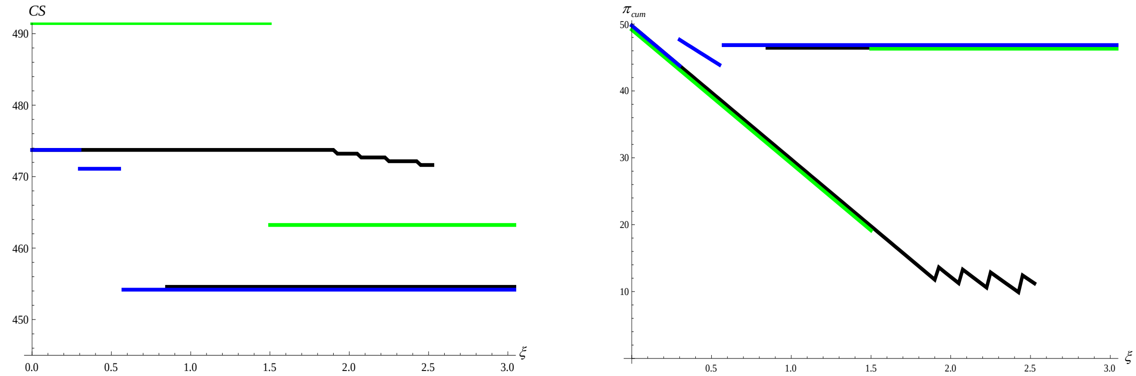


Figure 4: Maximal welfare (green) and welfare under equilibria with endogenous (black) and exogenous (blue) cluster formation.

Comparing the welfare generated in equilibria with endogenous and exogenous cluster formation, Figure 4 shows that the effect of endogenous cluster formation on welfare is ambiguous. On the one hand, as discussed above, there is a range of investment cost values where under endogenous cluster formation there exists a unique equilibrium with high investment whereas under exogenous cluster formation only low investment is done. In such a scenario welfare is substantially larger under endogenous cluster formation. On the other hand, there is also a range of investment cost levels where under exogeneous cluster allocation of firms only the firms in the large cluster invest high whereas all other invest low. Such an investment profile generates higher welfare compared to the full investment profile emerging under endogenous cluster formation because a large share of output is produced by the low cost firms in the larger cluster and for the relatively low output produced in the small cluster the saved investment costs outweigh the aggregate reduction in production costs that would result from full investment of the small cluster firms.

Furthermore, Figure 4 shows that in the upper range of investment cost levels, for which an equilibrium with high investment exists under endogenous cluster formation, such an equilibrium generates welfare which is not only substantially below the welfare maximum but also below that of the unique equilibrium under exogenous cluster formation, which corresponds to the zero investment equilibrium. Welfare maximization requires zero investment in this parameter range, which means that endogenous cluster formation can yield massive overinvestment in equilibrium. Intuitively this inefficiency is triggered by the tournament like structure. All firms have strong incentives to end up in the larger cluster due to the endogeneity of the difference in payoffs between the clusters driven by the strategic complementarity.[11]

[11]Lazear and Rosen (1981) show in the framework of labor contracts that tournament schemes, in which the firm chooses the price structure and prices are independent from workers' investment, can induce efficient investment. In a related setting with endogeneous determination of the price structure and asymmetric information about investment Zabojnik and Bernhard (2001) show that underinvestment in equilibrium results. The main difference between our setting and these contributions is that



(a) (b)

Figure 5: Consumer surplus (a) and total firm profits (b) under welfare maximizing choice of investments and profile of clusters (green) as well as under equilibria with endogenous (black) and exogenous (blue) cluster formation.

Figure 5(b) shows that total industry profit is always larger if firms are ex-ante allocated to clusters. As should be expected, the high investment incentives under endogenous cluster formation are however desirable from a consumer perspective and consumer surplus is for all values of investmenst costs (weakly) larger in the case of exogenous cluster formation (see Figure 5(a)).

6 Conclusions

The main contribution of this paper is to improve our understanding of the strategic relationship between firms' R&D investment decisions and their participation in R&D clusters. From a theoretical perspective, we go beyond the current state of the literature by developing and analyzing a framework which allows to characterize the equilibrium profiles of both R&D investment and R&D cluster formation in a setting with an arbitrary number of competitors and no symmetry assumptions with respect to the number of cooperation partners of firms. Our analysis shows that in equilibrium generically unique cluster profiles emerge which are characterized by a strong heterogeneity between clusters with respect to size and R&D investment while within clusters, the heterogeneity of R&D levels are small. In particular, it is shown that in case of heterogeneous firm investments the majority of high investors is always included in the largest cluster. Overall, our model predicts a positive relationship between the level of firms' R&D activity and the number of cooperation partners, and therefore is able to provide a theory-based explanation for a large set of empirical findings pointing towards such a positive relationship (e.g Veugelers, 1997; Becker and Dietz, 2004). The model also makes the empirically testable prediction that R&D cooperations are stratified in a sense that the variance of R&D levels within clusters is lower than that in the entire population.

the payoffs obtained in the two clusters are positively affected by own investment and marginal returns from investment are larger in the cluster generating higher payoffs.

Furthermore, we show in this paper that the endogenous cluster formation process implies stronger investment incentives, compared to a scenario where allocation of firms to clusters is ex-ante fixed, and generates strong strategic complementarities with respect to the firms' investment decisions. Hence, for a large range of investment cost values a no-investment equilibrium co-exists with an equilibrium in which (almost) all firms choose a high R&D level. Welfare maximization would require a full investment profile for a substantial part of the investment cost range where the no-investment equilibrium exists. These insights have clear policy implications. First, the observation that firms which anticipate that their R&D level influences their cluster membership invest more, thereby moving the investment profile closer to the social optimum, provides justification for policy measures, like technology and cooperation platforms, which foster the exchange of information between firms and the continuous adjustment of cooperation structures. Second, our analysis suggests that in scenarios where no-investment and full investment equilibria coexist, the introduction of a (potentially small) public R&D subsidy, which moves the level of R&D investments required from the firms below the threshold $\bar{\xi}$ can have a strong positive effect by inducing a transition to the equilibrium where all firms invest high.

Our analysis is based on a number of simplifying assumptions whose implications should be critically examined. If the assumption of a binary investment decision would be relaxed by allowing firms to invest any level between $\underline{x}$ and $\bar{x}$ we would not expect any qualitative changes in our results because the investment complementarities should induce firm investments at the boundaries of the considered interval although the complexity in the characterization of the SPE in the second stage increases considerably. If we would allow firms to enter individual cooperation agreements with selected competitors rather than joining a cluster, the resulting analysis would require the characterization of equilibrium network structures among general profiles of heterogeneous firms. This technically and conceptionally demanding task is left for future research. Finally, in this paper we have abstracted from the effects of R&D investment on a firm's absorptive capacity. Considering such effects might substantially affect the qualitative findings obtained here. Again, future work should be able to address this issue.

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Appendix

A An Oligopoly Model with Product Innovation

Here, we briefly outline an oligopoly model where products are vertically differentiated and R&D activities of firms lead to changes in product quality due to product innovation. We show that this simple model formulation yields equilibrium profit functions of firms which have a completely analogous functional form as the ones resulting from the process innovation model used in the main body of the paper. Hence, all results concerning firm investment and formation of clusters derived in the paper are also valid in this product innovation setting.

Like in the main body of the paper, we consider an oligopoly of a set $N = \{1, ..., n\}$ of ex ante identical firms which engage in a three stage game. Firms first choose permanent R&D efforts, then form R&D clusters and finally compete in the market by choosing quantities of their product. The R&D effort $x(i) \in \{\underline{x}, \bar{x}\}$ of firm i is invested in product innovation and influences the quality of the product. Choosing to invest high, $x(i) = \bar{x} > \underline{x} \geq 0$, implies costs of $\xi > 0$, whereas the costs of low effort $\underline{x}$ are normalized to zero. Firms form clusters in the same way as described in Section 2 and the quality of the product of firm i is then given by

$$u(i) = \bar{u} + \gamma\Big(x(i) + \beta \sum_{\substack{j \in A(i) \\ j \neq i}} x(j)\Big). \; i = 1, .., n, \tag{9}$$

To simplify notation we normalize $\bar{u}$ to zero. Marginal production costs of the firms, which are assumed to be constant and identical across firms, are denoted by $\bar{c} > 0$.

Demand on the market is generated by a representative consumer with the utility function (expressed in monetary units)

$$U(q(1), \dots, q(n)) = \sum_{j \in N} (\alpha + u(j))q(j) - \frac{1}{2}\left(\sum_{j \in N} q(j)\right)^2 - \sum_{i \in N} p(j)q(j),$$

where quality and prices are given parameters from the consumer's perspective.

In the third stage of the game all product qualities are common knowledge and firms simultaneously choose their quantities. Prices are then adjusted such that the market clears, which means that the vector of chosen quantities $(q(1), \dots, q(n))$ maximizes the consumer's utility function. The corresponding first order conditions yield

$$p(i) = \alpha + u(i) - \sum_{j \in N} q(j).$$

Taking this into account, the market profit of firm i can be written as

$$\tilde{\pi}(i) = \left(\alpha + u(i) - \sum_{j \in N} q(j) - \bar{c}\right) q(i)$$

and standard calculations yield the equilibrium quantities

$$q^*(i) = \frac{\alpha - \bar{c} + (n+1)u(i) - \sum_{j \in N} u(j)}{n+1}$$

and market profits

$$\tilde{\pi}^*(i) = \frac{\left(\alpha - \bar{c} + (n+1)u(i) - \sum_{j\in N} u(j)\right)^2}{(n+1)^2}.$$

Inserting (9) into this expression yields that the overall profit of the firm is given by (3) and hence coincides with the one derived in the process innovation model considered in the main body of the paper. Therefore, all derived results also hold for the product innovation model sketched here.

B Proofs

Proof of Proposition 1. We show the result in three steps. First, in Lemma 1 we use backward induction to calculate the number $\tilde{m}$ such that all remaining firms join one cluster and deduce from that the maximial number m such that a proposal is made which (upon acceptance) results in a number of remaining firms smaller or equal $\tilde{m}$ implying that two coalitions form. That proposal is made under the assumption that all other firms will join one coalition. Second, we show in Lemma 2 that any rational proposal is accepted by all firms included in the proposal. Finally, we show in Lemmas 3 and 4 that it is indeed optimal to propose a cluster such that all remaining firms join one coalition if the difference between high and low effort firms is bounded, i.e. $\bar{x} \leq 2\underline{x}$. In particular it is shown, that it is not profitable for the proposer to suggest a smaller cluster in order to induce the remaining firms to split up into more than one cluster after formation of the proposed cluster. This implies that under Assumption 1 always two clusters form. The size of these clusters and the number of high and low investors in each of them then follow directly from setting $m = n$ and $h_m = h$ in Lemma 1.

Lemma 1. *Assume that all cluster proposals are accepted. Then, for m remaining firms such that among these h_m invest high and l_m invest low and numbers $l^*(h_m, l_m) := \left\lceil \frac{(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}}{4\underline{x}} \right\rceil$ and $h^*(h_m, l_m) := \left\lceil \frac{(n-1+2h_m+l_m)\bar{x}+l_m\underline{x}}{4\bar{x}} \right\rceil$, the following cluster is proposed under the assumption that all players outside the proposal join one cluster.*

- *If $0 < h_m \leq \frac{(n-1-2l_m)\underline{x}}{\bar{x}+\underline{x}}$, then a coalition of all remaining players is proposed.*
- *If $\frac{(n-1-2l_m)\underline{x}}{\bar{x}+\underline{x}} \leq h_m \leq \frac{(n-1+2l_m)\underline{x}}{\bar{x}+\underline{x}}$, then a coalition of h_m high investors and l^* low investors is proposed.*
- *If $\frac{(n-1+2l_m)\underline{x}}{\bar{x}+\underline{x}} \leq h_m \leq \frac{(n-1+l_m)\bar{x}+l_m\underline{x}}{2\bar{x}}$, then a coalition of h_m high investors and no low investors is proposed.*
- *If $\frac{(n-1+l_m)\bar{x}+l_m\underline{x}}{2\bar{x}} \leq h_m$, then a coalition of h^* high investors and no low investors is proposed.*
- *If $h_m = 0$ and $l_m \leq \left\lceil \frac{n+1}{2} \right\rceil$ a coalition of all remaining players is proposed.*
- *If $h_m = 0$ and $l_m > \left\lceil \frac{n+1}{2} \right\rceil$ a coalition l^* of low investors is proposed.*

Proof of Lemma 1. Suppose $n-m$ firms have already formed clusters and let it be firm i's turn to propose the next cluster. Denote the cluster structure that has been formed before i proposes by $\mathbf{A}(-i)$. Since by assumption all proposals have been accepted and because it is assumed that the rule of proposal order ρ is such that high effort firms have a lower rank than low effort firms, i is either a high investor or there are only low investors left in the game.

First, suppose that $h_m > 0$, i.e. i is a high investor. Since by assumption the firms outside the proposal form one cluster, i faces the optimization problem to propose an optimal cluster $A(i)$ consisting of $\tilde{h}$ high investors and $\tilde{l}$ low investors such that the other $m-\tilde{h}-\tilde{l} \geq 0$ firms form one cluster, denoted by $\bar{A}$ such that the cluster structure is given by $\mathbf{A} = (\mathbf{A}(-i), A(i), \bar{A})$.

Since maximizing profit is equivalent to maximizing quantities and, in the maximization problem, quantities of i are only influenced by the spillovers from the last two clusters $A(i)$ and $\bar{A}$, we get

$$\begin{aligned}
\underset{A(i)\subset N\setminus A(-i)}{\arg\max}\ \pi(i,\mathbf{x},\mathbf{A}) &= \underset{A(i)\subset N\setminus A(-i)}{\arg\max}\ q(i,\mathbf{x},\mathbf{A}) \\
&= \underset{\tilde{h}\leq h_m,\tilde{l}\leq l_m}{\arg\max} \Big[n((\tilde{h}-1)\bar{x}+\tilde{l}\underline{x}) - (\tilde{h}-1)((\tilde{h}-1)\bar{x}+\tilde{l}\underline{x})) - \tilde{l}(\tilde{h}\bar{x}+(\tilde{l}-1)\underline{x}) \\
&\quad -(h_m-\tilde{h})\big((h_m-\tilde{h}-1)\bar{x}+(l_m-\tilde{l})\underline{x}\big) - (l_m-\tilde{l})\big((h_m-\tilde{h})\bar{x}+(l_m-\tilde{l}-1)\underline{x}\big)\Big].
\end{aligned}$$

The proposer chooses first from the high effort firms and then from the low effort firms since a high effort firm is always preferred to a low effort firm and it is assumed that the remainding firms form one cluster.

Suppose first that the proposal includes only high effort firms, i.e. $A(i) = (h_{A(i)}, 0)$ which implies $\bar{A} = (h_m - h_{A(i)}, l_m)$. Since marginal profit of adding other firms to the own cluster $A(i)$ is decreasing in the size of the cluster $|A(i)|$, we get the optimal number of high effort firms in $A(i)$ is the largest integer h^* such that marginal profit of adding the last firm is positive. Thus, we get h^* as the largest integer satsifying,

$$\begin{aligned}
& \pi(i,\mathbf{x},((h^*,0),(h_m-h^*,l_m))) - \pi(i,\mathbf{x},((h^*-1,0),(h_m-h^*+1,l_m))) > 0 \\
\Leftrightarrow\quad & n\bar{x} - (h^*-2)\bar{x} - (h^*-1)\bar{x} + (2(h_m-h^*)+l_m)\bar{x} + l_m\underline{x} > 0 \\
\Leftrightarrow\quad & \frac{1}{4\bar{x}}\bar{x}(n+3+2h_m+l_m) + l_m\underline{x} > h^*.
\end{aligned}$$

Hence, the optimal coalition size is given by

$$h^* := \left\lceil \tfrac{(n-1+2h_m+l_m)\bar{x}+l_m\underline{x}}{4\bar{x}} \right\rceil$$

To be consistent with the assumption that no low investors are selected, we need $h^* \leq h_m$ which is equivalent to $\frac{(n-1+2h_m+l_m)\bar{x}+l_m\underline{x}}{4\bar{x}} \leq h_m$ since h_m is an integer. Hence,

$$h^* \leq h_m \ \Leftrightarrow\ h_m \geq \frac{(n-1+l_m)\bar{x}+l_m\underline{x}}{2\bar{x}} =: h_m^3$$

Thus, for $h_m \geq h_m^3$, i proposes the cluster $A(i) = (h^*, 0)$.

Now consider the case that $h^* < h_m$. Therefore, choosing h^* high investors for the cluster $A(i)$ is no longer feasible. Since high effort firms are more attractive as partners, i will, hence, select all high effort firms. Additionally low investors may also be included. Again, since marginal profit of adding other firms to the own cluster $A(i)$ is decreasing in the size of the cluster $|A(i)|$, we get the optimal number of low effort firms l^* by solving,

$$\begin{aligned} & \pi(i,\mathbf{x},((h_m,l^*+1),(0,l_m-l^*-1))) - \pi(i,\mathbf{x},((h_m,l^*),(0,l_m-l^*))) = 0 \\ \Leftrightarrow \quad & n\underline{x} - (h_m+l^*-1)\underline{x} - h_m\bar{x} - l^*\underline{x} + 2(l_m-l^*-1)\underline{x} = 0 \\ \Leftrightarrow \quad & \tfrac{1}{4\underline{x}}(n-1-h_m+2l_m)\underline{x} - h_m\bar{x} = l^* \end{aligned}$$

Thus, the optimal coalition size is given by

$$l^* := \left\lceil \tfrac{(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}}{4\underline{x}} \right\rceil$$

To ensure that the number selected is feasible, we need $0 \leq l^* \leq l_m$ which is equivalent to $0 \leq \frac{(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}}{4\underline{x}} \leq l_m$ since 0 and l_m are integers. Hence,

$$\begin{aligned} 0 \leq l^* \Leftrightarrow h_m &\leq \frac{(n-1+2l_m)\underline{x}}{\bar{x}+\underline{x}} & =: h_m^2 \\ l_m \geq l^* \Leftrightarrow l_m &\geq \frac{(n-1)\underline{x} - h_m(\bar{x}+\underline{x})}{2\underline{x}} \\ \Leftrightarrow \frac{h_m(\bar{x}+\underline{x})}{2\underline{x}} &\geq \frac{(n-1-2l_m)\underline{x}}{2\underline{x}} \\ \Leftrightarrow h_m &\geq \frac{(n-1-2l_m)\underline{x}}{\bar{x}+\underline{x}} & =: h_m^1 \end{aligned}$$

Thus, for $h_m^1 \leq h_m \leq h_m^2$, i proposes the cluster $A(i) = (h_m, l^*)$. It follows that for $h_m \geq h_m^2$ no low effort firms are included in $A(i)$, which implies that for $h_m^2 \leq h_m \leq h_m^3$ i proposes the cluster $A(i) = (h_m, 0)$. On the other hand, for $h < h_m^1$, i's proposal includes all remaining firms, i.e. $A(i) = N \setminus \mathbf{A}(-i)$, see also derivation in (5).

Finally, the case where there are no high effort firms, $h_m = 0$ corresponds to the case when there are no low effort firms since in both cases all firms are symmetric, completing the proof of Lemma 1.

□

Lemma 2. *Suppose that $\bar{N} \subset N$ have already formed clusters and it is i's turn to propose the next cluster. If firm i proposes a payoff maximizing cluster $A(i) \subset N$, in the sense that i's payoff is maximized among all continuation payoffs following any accepted proposal $\tilde{A}(i) \subset N$, then proposal $A(i)$ is accepted by all firms $j \in A(i)$.*

Proof. Suppose that at some point of the game a firm i is to propose a cluster. In other words, either i is the first to propose or it proposes after the clusters $A_1, \ldots, A_l$ have already been formed. Now, firm i, which is the first of the remaining firms according to the rule of order, proposes a cluster $A(i)$, which upon acceptance results in a continuation subgame with a set $\tilde{N} \subseteq N$ of firms. Furthermore, we assume that $A(i)$ is chosen in a way that it maximizes the profit for firm i induced under the assumption that its current proposal is accepted and the subgame perfect equilibrium is followed in the continuation subgame. For further reference we observe that this optimality property implies that $q(i;\mathbf{x},\mathbf{A}) \geq q(i;\mathbf{x},\tilde{\mathbf{A}})$, where $\mathbf{A}$ denotes the cluster profile induced by $A(i)$ and $\tilde{\mathbf{A}}$ a cluster profile induced by some alternative proposal $\tilde{A}(i)$ at the current stage.

Consider now a firm $j \in A(i),\ j \neq i$ with $x(i) = x(j)$. Clearly, the payoff of j under this proposal is identical to that of i. Assume that j rejects the proposal. This would only be rational if j could obtain a strictly higher payoff by offering an alternative proposal $\tilde{A}(j)$.[12] If $i \in \tilde{A}(j)$ then the payoff of i in this alternative proposal would be identical to that of j, which implies that the original proposal $A(i)$ would not be optimal for i, which is a contradiction to our assumption. If $i \notin \tilde{A}(j)$ then consider instead the proposal $\tilde{A}(i) = \tilde{A}(j)\backslash\{j\}\cup\{i\}$ by firm i. Comparing the subgames after $\tilde{A}(j)$ and $\tilde{A}(i)$ have been accepted, it turns out that both are identical up to permutation of players, since the number of high and low effort firms remaining are identical, and the rule of order is preserved since we assumed that high effort firms have a lower rank than low effort firms. Hence, we can conclude that the payoff of i in $\tilde{A}(i)$ is identical to the payoff of j in $\tilde{A}(j)$. This again yields a contradiction to the assumption that $A(i)$ is the optimal proposal for firm i.

Consider now a firm $j \in A(i),\ j \neq i$ with $x(i) \neq x(j)$. Given our assumption that high investors propose before low investors in the rule of order we must have $x(i) = \bar{x}$ and $x(j) = \underline{x}$. Assume that j rejects the proposal. This would only be rational if j could obtain a higher payoff by offering an alternative proposal $\tilde{A}(j)$. Similar to above we distinguish between the cases where $i \in \tilde{A}(j)$ and $i \notin \tilde{A}(j)$.

In case $i \in \tilde{A}(j)$ let us denote by $\mathbf{A}$ and $\tilde{\mathbf{A}}$ the unique[13] cluster profiles induced by the acceptance of proposals $A(i)$ and $\tilde{A}(j)$. Further, denote by $\Delta q(i) := q(i,\mathbf{x},\tilde{\mathbf{A}}) - q(i,\mathbf{x},\mathbf{A})$, respectively $\Delta q(j)$ the differences in quantities for the two firms between the cases where $\tilde{A}(j)$ is accepted and $A(i)$ is accepted. Since j rejects $A(i)$, it must strictly prefer the outcome induced by $\tilde{A}(j)$ and since profits (net of investment costs) are given by the square of the quantities, we must have $\Delta q(j) > 0$.

Thus,

$$
\begin{aligned}
&(n+1)\Delta q(i) \\
&\quad= -n\Big(c(i;\mathbf{x},\tilde{\mathbf{A}}) - c(i;\mathbf{x},\mathbf{A})\Big) + c(j;\mathbf{x},\tilde{\mathbf{A}}) - c(j;\mathbf{x},\mathbf{A}) + \sum_{m\neq i,j} c(m;\mathbf{x},\tilde{\mathbf{A}}) - c(m;\mathbf{x},\mathbf{A}) \\
&\quad= \gamma\beta\Big[-n\Big((h_{\tilde{A}(j)}-1)\bar{x} + l_{\tilde{A}(j)}\underline{x} - (h_{A(i)}-1)\bar{x} - l_{A(i)}\underline{x}\Big) + \Big(h_{\tilde{A}(j)}\bar{x} + (l_{\tilde{A}(j)}-1)\underline{x} \\
&\qquad -h_{A(i)}\bar{x} - (l_{A(i)}-1)\underline{x}\Big)\Big] + \sum_{m\neq i,j} c(m;\mathbf{x},\tilde{\mathbf{A}}) - c(m;\mathbf{x},\mathbf{A}) \\
&\quad= \gamma\beta\Big[-(n-1)\Big((h_{\tilde{A}(j)} - h_{A(i)})\bar{x} + (l_{\tilde{A}(j)} - l_{A(i)})\underline{x}\Big)\Big] + \sum_{m\neq i,j} c(m;\mathbf{x},\tilde{\mathbf{A}}) - c(m;\mathbf{x},\mathbf{A}) \\
&\quad= (n+1)\Delta q(j) > 0.
\end{aligned}
$$

Therefore, we obtain a contradiction to the assumption that proposing $A(i)$ is optimal for firm i.

[12]Note that is shown in Bloch (1996) that there exists a subgame perfect equilibrium with the property that all firms always accept a proposal as long as rejecting would not result in a strictly higher payoff (see Proposition 2.4 in Bloch (1996)).

[13]By unique, we mean up to a permutation of firms which invest identically. Thus quantities of i and j are uniquely determined. We get the uniqueness by backward induction and acceptance of proposals in case of indifference.

As a next step we consider the case where $i \notin \tilde{A}(j)$, but there exists a firm $k \in \tilde{A}(j)$ with $x(k) = \bar{x}$. In case $k \in A(i)$ we immediately obtain $\Delta q(k) < 0$ since k is of the same type as i and therefore $A(i)$ must have been optimal for k. This implies that also $\Delta q(j) < 0$, which contradicts the optimality of $\tilde{A}(j)$ for j. Consider now the case where $k \notin A(i)$. For proposal $\tilde{A}(j)$ to be strictly preferred by firm j to $A(i)$ we must have $\Delta q(j) > 0$. Analogous to above, this implies $\tilde{\Delta} q(k) > 0$, where $\tilde{\Delta} q(k)$ denotes the difference in quantity for firm k between proposal $\tilde{A}(j)$ and proposal $A(k) = (A(i) \setminus \{i\}) \cup \{k\}$. Hence $A(k)$ would not be optimal for firm k, but since firm k is of the same type as firm i this would contradict that $A(i)$ is optimal for firm i.

Finally, consider the case where $i \notin \tilde{A}(j)$ and there does not exist $k \in \tilde{A}(j)$ with $x(k) = \bar{x}$. In other words, the counter proposal consists of only low effort firms. As above denote by $\tilde{N}$ the remaining firms (before i's proposal) and $\tilde{h}$ and $\tilde{l}$ the number of high respectively low remaining investors. Assume that $\tilde{h}$ is low enough such that i's optimal proposal $A(i)$ (conditional on acceptance) also contains low investors (otherwise a homogeneous coalition is proposed which is always accepted, see above).

We show that no counterproposal $\tilde{A}(j)$ with $x(k) = \underline{x}$ for all $k \in \tilde{A}(j)$ increases j's payoff by induction over the remaining low investors $\tilde{l}$ for given $\tilde{h}$. Clearly for $\tilde{l} = 1$ such an $\tilde{A}(j)$ yields $|\tilde{A}(j)| = 1$ and thus lower profits. In what follows, we show that under the assumption that for l low investors with $l < \tilde{l}$ no such profitable counterproposal $\tilde{A}(j)$ exists, no profitable counterproposal $\tilde{A}(j)$ also exists for $\tilde{l}$ low investors. To the contrary, suppose that for $l = \tilde{l}$ there is a profit increasing counterproposal $\tilde{A}(j)$ which is hence accepted. After formation of $\tilde{A}(j)$ all proposals are accepted by assumption above and, hence, cluster sizes are given by Lemma 1. It is easy to see that with $|\tilde{N} \setminus \tilde{A}(j)| \geq n/2$ the profit of the members of $\tilde{A}(j)$ would increase if they add a high investor to their cluster. Following identical arguments to above this would imply that the profit of j in $A(i)$ would be higher than in $\tilde{A}(j)$.

Based on this we restrict attention to the case where $|\tilde{N} \setminus \tilde{A}(j)| < n/2$. Again, by induction hypotheses, after formation of $\tilde{A}(j)$, all proposals are accepted and we are in the case of Lemma 1. If all firms in $\tilde{N} \setminus \tilde{A}(j)$ join one cluster $\tilde{A}(i) = \tilde{N} \setminus \tilde{A}(j)$, then $\tilde{A}(j)$ clearly cannot be optimal since $(\tilde{A}(j) \setminus \{k\} \cup \{i\})$ yields higher profits for all firms in $\tilde{A}(j) \setminus \{k\}$. This follows from the fact that $x(k) = \underline{x}$ for all $k \in A(j)$ and $x(i) = \bar{x}$ and the observation that for any firm exchanging a low investor in the own cluster with a high investor from another cluster increases the firm's quantity. Since we know that $A(i)$ generates the highest profits for i (and, hence, for j) among all mixed clusters, this implies that $A(i)$ yields higher profits compared to $\tilde{A}(j)$ for firm j. Hence, consider the formation of two clusters among the firms in $\tilde{N} \setminus \tilde{A}(j)$. Lemma 1 then implies that these are $\tilde{A}_2 := (\tilde{h}, l^*(\hat{l}))$ where $l^*(\hat{l}) := \frac{(n-1-\tilde{h}+2(\tilde{l}-\hat{l}))\underline{x}-\tilde{h}\bar{x}}{4\underline{x}}$ and $\tilde{A}_3 := (0, \tilde{l} - \hat{l} - l^*(\hat{l}))$. Let $\tilde{\mathbf{A}} := (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$ denote the resulting cluster structure in the game of remaining firms $\tilde{N}$ induced by the proposal $\tilde{A}_1 := \tilde{A}(j) = (0, \hat{l})$. Consider the alternative counterproposal $A'(j) = (1, \hat{l} - 1)$ which results in the cluster structure $\mathbf{A}' := (A'_1, A'_2, A'_3)$ with $A'_1 := A'(j)$. Hence by Lemma 1 , $A'_2 := (\tilde{h} - 1, l^*(\hat{l} - 1)$ and $\tilde{A}_3 := (0, \tilde{l} - \hat{l} - l^*(\hat{l} - 1))$. Note that from Lemma 1 we get that $l^*(\hat{l} - 1) = l^*(\hat{l}) + \frac{3}{4} + \frac{1}{4}\frac{\bar{x}}{\underline{x}}$. Calculating $\Delta q(j) := q(j, \mathbf{x}, \tilde{\mathbf{A}}) - q(j, \mathbf{x}, \mathbf{A}')$, we get:

$$\Delta q(j) = \tfrac{\gamma}{(n+1)} \Big(ns(j, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_1, k \neq j} s(k, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_2} s(k, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_3} s(k, \mathbf{x}, \tilde{\mathbf{A}})$$

$$- ns(j,\mathbf{x},\mathbf{A}') + \sum_{k\in A_1',k\neq j} s(k,\mathbf{x},\mathbf{A}') + \sum_{k\in A_2'} s(k,\mathbf{x},\mathbf{A}') + \sum_{k\in A_3'} s(k,\mathbf{x},\mathbf{A}')\Big)$$

where $s(i,\mathbf{x},A) = \beta\sum_{k\in A(i)\setminus\{i\}} x(k)$ denotes the spillovers experienced by $i\in N$ in an cluster structure $\mathbf{A}$. We get for the difference in total spillovers in each cluster:

$$\begin{aligned}
\Delta S(A_1) :=& ns(j,\mathbf{x},\tilde{\mathbf{A}}) - \sum_{k\in\tilde{A}_1,k\neq j} s(k,\mathbf{x},\tilde{\mathbf{A}}) - ns(j,\mathbf{x},\mathbf{A}') + \sum_{k\in A_1',k\neq j} s(k,\mathbf{x},\mathbf{A}') \\
=&\beta\Big(n(\hat{l}-1)\underline{x} - (\hat{l}-1)^2\underline{x} - n\big((\hat{l}-2)\underline{x}+\bar{x}\big) \\
&+ (\hat{l}-2)\big((\hat{l}-2)\underline{x}+\bar{x}\big) + (\hat{l}-1)\underline{x}\Big) \\
=& -\beta(n-\hat{l}+2)(\bar{x}-\underline{x}) \\
\Delta S(A_2) :=& -\sum_{k\in\tilde{A}_2} s(k,\mathbf{x},\tilde{\mathbf{A}}) + \sum_{k\in A_2'} s(k,\mathbf{x},\mathbf{A}') \\
=& -\beta\Big(\tilde{h}\big((\tilde{h}-1)\bar{x} + l^*(\hat{l})\underline{x}\big) + (l^*(\hat{l})-1)\big(\tilde{h}\bar{x} + (l^*(\hat{l})-1)\underline{x}\big) \\
&+ (\tilde{h}-1)\big((\tilde{h}-2)\bar{x} + l^*(\hat{l}-1)\underline{x}\big) + (l^*(\hat{l}-1)-1)\big((\tilde{h}-1)\bar{x} \\
&+ (l^*(\hat{l}-1)-1)\underline{x}\big)\Big) \\
=&\beta\tfrac{1}{4}(\bar{x}-\underline{x})\Big(-\tfrac{n}{2} + \tilde{h}(\tfrac{3}{2}\tfrac{\bar{x}}{\underline{x}} - \tfrac{5}{2}) - \tilde{l} + \hat{l} + \tfrac{5}{2} - \tfrac{3}{4}\tfrac{\bar{x}-\underline{x}}{\underline{x}}\Big) \\
\Delta S(A_3) :=& -\sum_{k\in\tilde{A}_3} s(k,\mathbf{x},\tilde{\mathbf{A}}) + \sum_{k\in A_3'} s(k,\mathbf{x},\mathbf{A}') \\
=&\beta\Big(\big(l-\hat{l}-l^*(\hat{l})\big)\big(l-\hat{l}-l^*(\hat{l})-1\big)\underline{x} \\
&- \big(l-\hat{l}-l^*(\hat{l}-1)+1\big)\big(l-\hat{l}-l^*(\hat{l}-1)\big)\Big)\underline{x} \\
=&\beta\Big(\big(l^*(\hat{l}-1)-l^*(\hat{l})-1\big)\big(l^*(\hat{l}-1)+l(\hat{l})-2(\tilde{l}-\hat{l})\big)\Big) \\
=& -\tfrac{\beta}{4}(\bar{x}-\underline{x})\Big(-\tfrac{n}{2} + \tilde{h}(\tfrac{\bar{x}}{2\underline{x}} + \tfrac{1}{2}) + \tilde{l} - \hat{l} - \tfrac{1}{4} - \tfrac{\bar{x}}{4\underline{x}}\Big)
\end{aligned}$$

Thus we get:

$$\begin{aligned}
\Delta q(j) =& \tfrac{\gamma}{(n+1)}(\Delta S(A_1) + \Delta S(A_2) + \Delta S(A_3)) \\
=& \tfrac{\gamma\beta}{(n+1)}\tfrac{\bar{x}-\underline{x}}{4}\Big(-4n + \tilde{h}(\tfrac{\bar{x}}{\underline{x}} - 3) + 6\hat{l} - 2\tilde{l} - \tfrac{7}{2} - \tfrac{1}{2}\tfrac{\bar{x}}{\underline{x}}\Big)
\end{aligned}$$

Thus, for $\frac{\bar{x}}{\underline{x}} - 3 < 4$ the above bracket is clearly negative since $n > \tilde{h} + \hat{l}$ and $\tilde{l} > \hat{l}$. Hence, if $\frac{\bar{x}}{\underline{x}}$ is not too large, in particular $\bar{x} < 7\underline{x}$, then $\Delta q(i) < 0$ and thus the counterproposal cannot have been optimal, implying that for small enough $\bar{x}$ every proposal will be accepted.

□

Lemma 3. *If $h < \lceil\frac{n-1}{2}\rceil$, then every equilibrium of the unanimity game results in the formation of two clusters.*

Proof. First note that by assumption on the rule of order, the first proposer is a high investor. Further by assumption of the Lemma, $h < \lceil \frac{n-1}{2} \rceil$, implies that all high investors will be included in the first proposal, since otherwise marginal utility of adding a high effort firm is always positive which cannot be optimal since all proposals are accepted, see derivation in (5). This implies, that at most three coalitions form, since after all high effort firms joined the first cluster A_1, there are only low effort firms left which form at most two coalitions, see also Bloch (1995). Since there may be also low effort firms included in the first proposal, we get the following cluster structure: $\mathbf{A} = (A_1, A_2, A_3)$ with $A_1 = (h, l_1)$, $A_2 = (0, l_2)$ and $A_3 = (0, l_3)$. We show here that the last cluster is empty, i.e. $l_3 = 0$ for $\bar{x} \leq 2\underline{x}$.

Given $l - l_1$ remaining low effort firms after the first coalition forms, we can calculate the size of A_2 to be the largest integer such that

$$\pi_{A_2}\Big(i, \mathbf{x}, \mathbf{A}\Big) - \pi_{A_2}\Big(i, \mathbf{x}, \big(A_1, A_2 - (0,1), A_3 + (0,1)\big)\Big) \quad > 0$$

and therefore the optimal value of l_2 is given by $l_2^*(l_1) := \left\lceil \frac{n+2(l-l_1)-1}{4} \right\rceil$, see Lemma 1. Note that $l_3^* = l - l_1 - l_2^*(l_1)$. It is easy to see that, given $l - l_1$ remaining firms after the first proposal, if $l_i^*(l_1+1) < l_i^*(l_1)$, $i \in \{2,3\}$, then $l_j^*(l_1+1) = l_j^*(l_1)$ and $l_j^*(l_1+2) < l_j^*(l_1+1)$ as well as $l_i^*(l_1+2) = l_i^*(l_1+1)$ $j \in \{2,3\}$, $j \neq i$. Thus, when firms are added to the first cluster and A_2 and A_3 are non-empty, then these firms are added in alternating order from A_2 and A_3.

Hence the first firm i in order ρ (implying $x(i) = \bar{x}$) chooses l_1 as the largest integer such that

$$\begin{aligned} \pi_{A_1}\Big(i, \mathbf{x}, \big((h, l_1), A_2, A_3\big)\Big) - \pi_{A_1}\big(i, \mathbf{x}, \big((h, l_1 - 2), A_2 + (0,1), A_3 + (0,1)\big)\big) > 0 \\ \Leftrightarrow \quad 2\left(n\underline{x} - h(\bar{x} + \underline{x}) - 2l_1\underline{x} + l_2\underline{x} + l_3\underline{x} + 3\underline{x}\right) > 0 \end{aligned}$$

This implies

$$l_1^*(h) := \max\left\{0, \left\lceil \frac{n\underline{x} - h(\bar{x}+\underline{x}) + l\underline{x}}{3\underline{x}} \right\rceil\right\}, \qquad (10)$$

which is solved by substituting $l_3 = l - l_1 - l_2$. Note that it is necessary for three coalitions to form that $l_3^*(h) = n - h - l_1^*(h) - l_2^*(l_1^*(h)) > 0$ which implies that $n - h - l_1^*(h) = n - |A_1| > \lceil \frac{n+1}{2} \rceil$ (compare also to Lemma 1). Note that (10) implies that the size of A_1, given by $h + l_1^*(h) = \left\lceil \frac{2n\underline{x} - h(\bar{x} - \underline{x})}{3\underline{x}} \right\rceil$, is decreasing in h (for an illustration, see also Figure 1). Hence choosing h maximal under the assumption $h < \lceil \frac{n-1}{2} \rceil$ yields the minimal size of A_1 which implies $h = \frac{n-2}{2}$ and n is even for $|A_1|$ to be minimal under our assumption. We then get this size of A_1 by calculating $l_1^*(h)$:

$$\begin{aligned} l_1^*(\tfrac{n-2}{2}) &= \max\left\{0, \left\lceil \frac{1}{3}\Big(n - \frac{n-2}{2}\left(\frac{\bar{x}}{\underline{x}} + 1\right) + n - \frac{n-2}{2}\Big)\right\rceil\right\} \\ &= \max\left\{0 \left\lceil \frac{1}{6}\Big(2(n+2) - \left(\frac{\bar{x}}{\underline{x}}\right)(n-2)\Big)\right\rceil\right\} \end{aligned}$$

which ist obviously positive due to $\bar{x} \leq 2\underline{x}$, see Assumption 1. Hence $|A_1| = h + l^*(h) > \lceil \frac{n-1}{2} \rceil$ and hence no three cluster outcome can be supported as an equilibrium for $h < \lceil \frac{n-1}{2} \rceil$.

□

Lemma 4. *If $h \geq \lceil \frac{n-1}{2} \rceil$, then every equilibrium of the unanimity game results in the formation of two clusters.*

Proof. First, note that for $\lceil \frac{n-1}{2} \rceil \leq h \leq \lceil \frac{n+1}{2} \rceil$ the first cluster A_1 will include all high effort firms (see Lemma 1 by setting $l_m = n - h$ and using $\bar{x} \geq \underline{x}$), implying that no high effort and only $l_m < n - \lceil \frac{n-1}{2} \rceil = \lfloor \frac{n+1}{2} \rfloor$ low effort firms remain after the first proposal. These form one coalition by Lemma 1, see also derivation in (5). Hence, three coalitions are only possible if $h > \lceil \frac{n+1}{2} \rceil$ such that the first proposal does not include all high effort firms.

Thus, consider the formation of three clusters A_1, A_2, and A_3 such that A_1 consists of only high effort firms $A_1 = (h_1, 0)$ with $h_1 \geq \lceil \frac{n+1}{2} \rceil$ and, hence, $h - h_1 \leq \frac{n-1}{2} \leq \frac{(n-1+l)\bar{x}+l\underline{x}}{2\bar{x}}$. Therefore, A_3 cannot include any high effort firms by Lemma 1.

To summarize, the only way that three coalitions can be supported in equilibrium is to have $A_1 = (h_1, 0)$, $A_2 = (h - h_1, l_2)$ and $A_3 = (0, l_3)$ if $h \geq \lceil \frac{n-1}{2} \rceil$. To the contrary, suppose that these three coalitions indeed form. Denoting by $h_2(h_1)$, $l_2(h_1)$, and $l_3(h_1)$ the number of high respectively low effort firms in coalition 2 and 3 for a given h_1, we get (trivially) $h_2(h_1) = h - h_1$ and, in equilibrium, by Lemma 1, $l_2(h_1) = \left\lceil \frac{(n-1-(h-h_1)+2l)\underline{x}-(h-h_1)\bar{x}}{4\underline{x}} \right\rceil$ and, trivially, $l_3(h_1) = n - h - l_2(h_1)$. Again for $\mathbf{A} = (A_1, A_2, A_3)$ to be an equilibrium outcome, the first proposal must be such that it maximizes payoff under the expectation that these three coalitions form. Note that the quantity of the proposing firm i (lowest ranked firm in order ρ) choosing h_1 is given by

$$\begin{aligned} q_{A_1}\big(i, \mathbf{x}, \mathbf{A}\big) =& \frac{\gamma\beta}{n+1}\Big[n(h_1-1)\bar{x} - (h_1-1)^2\bar{x} - h_2(h_1)\big[(h_2(h_1)-1)\bar{x} + l_2(h_1)\underline{x}\big] \\ & - l_2(h_1)[h_2(h_1)\bar{x} + (l_2(h_1)-1)\underline{x}] - l_3(h_1)(l_3(h_1)-1)\underline{x}\Big] + C, \end{aligned}$$

where C is a constant which is independent from the cluster profile $\mathbf{A}$. Since profit is strictly increasing in the quantity, the optimal choice of h_1 is determined by the first order condition

$$\begin{aligned} \tfrac{\partial q_{A_1}(i,\mathbf{x},\mathbf{A})}{\partial h_1} =& 0 \\ \Leftrightarrow \qquad 0 =& n\bar{x} - 2(h_1-1)\bar{x} + (h-h_1)\big[\bar{x} - \tfrac{\bar{x}+\underline{x}}{4}\big] + \big[(h-h_1-1)\bar{x} + l_2(h_1)\underline{x}\big] \\ & - \tfrac{\bar{x}+\underline{x}}{4\underline{x}}\big[(h-h_1)\bar{x} + l_2(h_1)\underline{x} - \underline{x}\big] + l_2(h_1)\big[\bar{x} - \tfrac{\bar{x}+\underline{x}}{4}\big] + 2(l - l_2(h_1))\tfrac{\bar{x}+\underline{x}}{4} - \tfrac{\bar{x}+\underline{x}}{4} \\ \Leftrightarrow \qquad 0 =& (n+1-2h)\bar{x} + (h-h_1)\big[4\bar{x} - \tfrac{\bar{x}+\underline{x}}{4}\tfrac{\bar{x}+\underline{x}}{\underline{x}}\big] + l_2(h_1)\big[\underline{x} - 4\tfrac{\bar{x}+\underline{x}}{4} + \bar{x}\big] + 2l\tfrac{\bar{x}+\underline{x}}{4} \\ \Leftrightarrow \qquad 0 =& (n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x} + (h-h_1)[4\bar{x} - \tfrac{(\bar{x}+\underline{x})^2}{4\underline{x}}] \\ \Leftrightarrow \qquad h_1^*(h) =& \tfrac{4\underline{x}}{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}\big[(n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\big] + h \end{aligned}$$

And hence,

$$\Leftrightarrow \qquad h_2^*(h) = - \tfrac{4\underline{x}}{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}\big[(n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\big]$$

As pointed out above, we need $h_2^*(h) > 0$ if $h \geq \frac{n-1}{2}$, in order to have $\mathbf{A}$ as an equilibrium outcome. Hence,

$$0 > (n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}$$

$$\Leftrightarrow \qquad h > \frac{(3\bar{x}+\underline{x})n+2\bar{x}}{5\bar{x}+\underline{x}} =: \bar{h} \tag{11}$$

Moreover, we must have $h_2^* > h_m^1 = \frac{(n-1-2l)\underline{x}}{\bar{x}+\underline{x}}$ in order to have A_3 non-empty, see Lemma 1. Using $l = n-h$, we get the condition

$$h_2^*(h) - h_m^1 = -\frac{4\underline{x}}{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}\left[(n-h)(\bar{x}+\frac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\right] - \frac{((n-1)-2(n-h))\underline{x}}{\bar{x}+\underline{x}} > 0 \tag{12}$$

The left-hand side of (12) is non-increasing in h if

$$0 \geq \frac{5\bar{x}+\underline{x}}{2} - 2\frac{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}{4\underline{x}}\frac{\underline{x}}{\bar{x}+\underline{x}} \qquad \Leftrightarrow \qquad 3\underline{x} \geq \bar{x}$$

Hence, for $3\underline{x} \geq \bar{x}$, (11) and (12) cannot be simultaneously satisfied. To see this, note we must have $h \geq \bar{h}$ by (11) and we get $h_2^*(\bar{h}) = 0$ implying that the left-hand side of (12) is negative for $h = \bar{h}$. Since it is, moreover, decreasing in h for $3\underline{x} \geq \bar{x}$, (12) can then not be satisfied. Thus, for $3\underline{x} \geq \bar{x}$ and $h \geq \frac{n-3}{2}$, there cannot exist three coalitions which are supported by a subgame perfect equilibrium. □

To wrap up, we have first characterized optimal cluster profiles for m remaining firms if firms expect all other firms to join one cluster. That proposals are indeed accepted by all players is shown by backward induction in Lemma 2 under the condition that $7\underline{x} > \bar{x}$. Finally, in Lemmas 3 and 4, it is shown that for $2\underline{x} > \bar{x}$, three (and trivially also more) clusters cannot be supported by a subgame perfect equilibrium. Hence, any subgame perfect equilibrium consist of two clusters, and, hence, the sizes and composition of these two clusters are given in Lemma 1. Setting $h_m = h, l_m = n-h$ in Lemma 1 yields the expressions for the cluster sizes as well as $\tilde{h}, \tilde{\tilde{h}}$ in the Proposition. This completes the proof.

□

Proof of Proposition 2. We show the proposition in three steps by considering the investment incentives of firms, i.e. the marginal return on investment. First, if the number of other high effort firms h^{-i} is low then incentives are increasing in h^{-i} for large enough values of β which is shown in Lemma 5. For large values of h^{-i}, the investment incentives are decreasing (Lemma 6) if β is not too large, while for intermediate values, there is a unique maximum (Lemma 7). Together these Lemmas imply the Proposition.

Lemma 5. *If $\beta > \underline{\beta} := \frac{4\underline{x}}{(n+6)\underline{x}+\bar{x}}$, then for $h^{-i} \leq \frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}} - 1$ expected return on investment is increasing in h^{-i}.*

Proof. It follows from (7) that for $h^{-i} \leq \frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}} - 1$ all high investors participate in the first cluster, i.e. $p_{A_1}(\bar{x}, h^{-i}+1) = 1$. Since $l_1 = l_{A_1}(h^{-i}+1)$, the profit of a high investor is given by:

$$\pi(\bar{x}, h^{-i}) = \frac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(n-h^{-i})(\bar{x}-\underline{x}) + \gamma\underline{x} + \gamma\beta\Big((n-h^{-i})(h^{-i}\bar{x}+l_1\underline{x})$$
$$- l_1((h^{-i}+1)\bar{x} + (l_1-1)\underline{x}) - (n-h^{-i}-l_1-1)(n-h^{-i}-l_1-2)\underline{x}\Big)\Big]^2 - \xi$$

The derivative with respect to h^{-i} yields:

$$\frac{\partial\pi(\bar{x},h^{-i})}{\partial h^{-i}} = \frac{2\gamma q(\bar{x})}{(n+1)^2}(\bar{x}-\underline{x})\Big[-1 + \frac{\beta}{4}\Big(n + h^{-i}(\frac{\bar{x}}{\underline{x}} - 1) + 6 + \frac{\bar{x}}{\underline{x}}\Big)\Big]$$

where $q(\bar{x})$ denotes the optimal quantity of a high investment firm. If instead firm i chooses low investment, she will join A_1 with probability $p_{A_1}(\underline{x}, h^{-i}) = \frac{l_1}{n-h^{-i}}$ and A_2 with probability $1 - p_{A_1}(\underline{x}, h^{-i}) = \frac{n-h^{-i}-l_1}{n-h^{-i}}$. The resulting payoff from low investment in A_1 weighted with the probability of being in A_1 is hence

$$\begin{aligned}\hat{\pi}(\underline{x}, h^{-i}, A_1) =& \tfrac{1}{(n+1)^2} p_{A_1}(\underline{x}, h^{-i}) \Big[\alpha - \bar{c} - \gamma h^{-i}(\bar{x} - \underline{x}) + \gamma \underline{x} + \gamma\beta\Big((n - (l_1 - 1))(h^{-i}\bar{x} + (l_1 - 1)\underline{x})\\ &- h^{-i}((h^{-i} - 1)\bar{x} + l_1\underline{x}) - (n - h^{-i} - l_1 - 1)(n - h^{-i} - l_1 - 2)\underline{x}\Big)\Big]^2,\end{aligned}$$

where $l_1 = l_{A_1}(h^{-i})$. The derivative of $\hat{\pi}(\underline{x}, h^{-i}, A_1)$ with respect to h^{-i} yields:

$$\begin{aligned}\tfrac{\partial\hat{\pi}(\underline{x},h^{-i},A_1)}{\partial h^{-i}} =& 2\tfrac{1}{(n+1)^2}\gamma(\bar{x} - \underline{x}) p_{A_1}(\underline{x}, h^{-i}) q(\underline{x}, A_1)\Big[-1 + \tfrac{\beta}{4}\big(n + h^{-i}(\tfrac{\bar{x}}{\underline{x}} - 1) + 7\big)\Big]\\ &+ \tfrac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}} q(\underline{x}, A_1)^2\end{aligned}$$

where $q(\underline{x}, A_1)$ denotes the quantity produced by a low effort firm in A_1. Considering now the payoff from low investment in A_2, weighted with the probability of being in A_2, we obtain

$$\begin{aligned}\hat{\pi}(\underline{x}, h^{-i}, A_2) =& \tfrac{1}{(n+1)^2}\big(1 - p_{A_1}(\underline{x}, h^{-i})\big)\Big[\alpha - \bar{c} - \gamma h^{-i}(\bar{x} - \underline{x}) + \gamma\underline{x}\\ &+ \gamma\beta\Big(n - (n - h^{-i} - l_1 - 1))((n - h^{-i} - l_1 - 1)\underline{x})\\ &- h^{-i}((h^{-i} - 1)\bar{x} + l_1\underline{x}) - l_1(h^{-i}x + (l_1 - 1)\underline{x})\Big)\Big]^2,\end{aligned}$$

where, again, $l_1 = l_{A_1}(h^{-i})$. The derivative of $\hat{\pi}(\underline{x}, h^{-i}, A_2)$ with respect to h^{-i} yields,

$$\begin{aligned}\tfrac{\partial\hat{\pi}(\underline{x},h^{-i},A_2)}{\partial h^{-i}} =& 2\tfrac{1}{(n+1)^2}\gamma(\bar{x} - \underline{x})(1 - p_{A_1}(\underline{x}, h^{-i}))q(\underline{x}, A_2)\Big[-1 + \tfrac{\beta}{4}\big(-n + h^{-i}(\tfrac{\bar{x}}{\underline{x}} - 1) + 5\big)\Big]\\ &- \tfrac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}} q(\underline{x}, A_2)^2\end{aligned}$$

Note that expected payoff from choosing low investment is given by $\mathbb{E}\big(\pi(\underline{x}, h^{-i})\big) = \hat{\pi}(\underline{x}, h^{-i}, A_1) + \hat{\pi}(\underline{x}, h^{-i}, A_2)$. Thus the expected return on investment $\Delta\pi := \pi(\bar{x}, h^{-i}) - \mathbb{E}\big(\pi(\underline{x}, h^{-i})\big)$ changes with h^{-i} according to

$$\begin{aligned}\tfrac{\partial\Delta\pi}{\partial h^{-i}} =& \tfrac{\partial\pi(\bar{x},h^{-i})}{\partial h^{-i}} - \tfrac{\partial\mathbb{E}\left(\pi(\underline{x},h^{-i})\right)}{\partial h^{-i}} = \tfrac{\partial\pi(\bar{x},h^{-i})}{\partial h^{-i}} - \Big(\tfrac{\partial\hat{\pi}(\underline{x},h^{-i},A_1)}{\partial h^{-i}} + \tfrac{\partial\hat{\pi}(\underline{x},h^{-i},A_2)}{\partial h^{-i}}\Big)\\ =& \tfrac{2}{(n+1)^2}\gamma(\bar{x} - \underline{x})\Big[\big(q(\bar{x}) - \mathbb{E}(q(\underline{x}))\big)\Big(-1 + \tfrac{\beta}{4}\Big(n + h^{-i}\big(\tfrac{\bar{x}}{\underline{x}} - 1\big) + 6 + \tfrac{\bar{x}}{\underline{x}}\Big)\\ &+ p_{A_1}(\underline{x}, h^{-i})q(\underline{x}, A_1)\tfrac{\beta}{4}(\tfrac{\bar{x}}{\underline{x}} - 1) + \big(1 - p_{A_1}(\underline{x}, h^{-i})\big)q(\underline{x}, A_2)\tfrac{\beta}{4}(2n + 1)\Big]\\ &- \tfrac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}}\big(q(\underline{x}, A_1)^2 - q(\underline{x}, A_2)^2\big)\end{aligned}$$

where $\mathbb{E}(q(\underline{x})) = p_{A_1}(\underline{x}, h^{-i})q(\underline{x}, A_1) + \big(1 - p_{A_1}(\underline{x}, h^{-i})\big)q(\underline{x}, A_2)$ is the expected quantity of a low effort firm. We clearly have that the quantity produced by a high effort firm in A_1 exceeds the expected quantity of a low effort firm, i.e. $q(\bar{x}) > \mathbb{E}(q(\underline{x}))$. Hence, $\frac{\partial\Delta\pi}{\partial h^{-i}}$ is positive if $\beta > \frac{4\underline{x}}{(n+7)\underline{x} + (h^{-i}+1)(\bar{x}-\underline{x})}$ since $\frac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}} = -\frac{\left(3+\frac{\bar{x}}{\underline{x}}\right)(n-h^{-i})-4l_1}{4(n-h^{-i})^2} < 0$. Expected return on investment is hence increasing in h^{-i} for $h^{-i} \leq \frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}} - 1$ and $2\underline{x} \geq \bar{x}$ under the condition of $\beta > \frac{4\underline{x}}{(n+7)\underline{x} + (h^{-i}+1)(\bar{x}-\underline{x})}$. This expression is maximized for $h^{-i} = 0$ yielding $\underline{\beta} := \frac{4\underline{x}}{(n+6)\underline{x}+\bar{x}}$. Note that the latter is only a sufficient condition. □

Lemma 6. *If $\beta < 1/2$, then for $h^{-i} \geq \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}$ expected return on investment is decreasing in h^{-i}.*

Proof. If $h^{-i} \geq \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}$ then by Proposition 1, we have that two clusters $A_1 = (h_1, 0)$ and $A_2 = (h - h_1, l)$ form with $h_1 := h_{A_1}(h) = \left\lceil \frac{(2n+h-1)\bar{x}+(n-h)\underline{x}}{4\bar{x}} \right\rceil$. When i chooses $x(i) = \bar{x}$, she will be included in A_1 with probability $p_{A_1}(\bar{x}, h^{-i}+1) = \frac{h_{A_1}(h^{-i}+1)}{h^{-i}+1}$ (see (7)). In this case the payoff of a high investor in A_1 weighted with the probability of being in A_1 will be

$$\begin{aligned}\hat{\pi}(\bar{x}, h^{-i}, A_1) =& p_{A_1}(\bar{x}, h^{-i}+1)\tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(n-h^{-i})(\bar{x}-\underline{x}) + \gamma\underline{x} \\ &+ \gamma\beta\Big((n-(h_1-1))(h_1-1)\bar{x} - ((h^{-i}+1)-h_1)\big((h^{-i}-h_1)\bar{x} + (n-h^{-i}-1)\underline{x}\big) \\ &- (n-h^{-i}-1)\big((h^{-i}+1-h_1)\bar{x} + (n-h^{-i}-2)\underline{x}\big)\Big)\Big]^2 - p_{A_1}(\bar{x}, h^{-i}+1)\xi\end{aligned}$$

Taking the derivative with respect to h^{-i} yields,

$$\begin{aligned}\tfrac{\partial\hat{\pi}(\bar{x},h^{-i},A_1)}{\partial h^{-i}} =& p_{A_1}(\bar{x}, h^{-i}+1)\tfrac{2q(\bar{x},A_1)}{n+1}\gamma(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n(\tfrac{\underline{x}}{\bar{x}}-2) + h^{-i}(1-\tfrac{\underline{x}}{\bar{x}}) + 6 - \tfrac{\underline{x}}{\bar{x}}\Big)\Big] \\ &+ \tfrac{\partial p_{A_1}(\bar{x},h^{-i}+1)}{\partial h^{-i}}\big(q(\bar{x},A_1)^2 - \xi\big)\end{aligned}$$

When i chooses $x(i) = \bar{x}$, she could also end up in A_2 which happens with probability $1 - p_{A_1}(\bar{x}, h^{-i}+1)$. In this case the expected payoff will be

$$\begin{aligned}\hat{\pi}(\bar{x}, h^{-i}, A_2) =& (1 - p_{A_1}(\bar{x}, h^{-i}+1))\tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(n-h^{-i})(\bar{x}-\underline{x}) + \gamma\underline{x} \\ &+ \gamma\beta\Big((n-h^{-i}+h_1)\big((h^{-i}-h_1)\bar{x} + (n-h^{-i}-1)\underline{x}\big) \\ &- (n-h^{-i}-1)\big((h^{-i}-h_1+1)\bar{x} + (n-h^{-i}-2)\underline{x}\big) - h_1(h_1-1)\bar{x}\Big)\Big]^2 \\ &- (1 - p_{A_1}(\bar{x}, h^{-i}+1))\xi\end{aligned}$$

Taking the derivative with respect to h^{-i} yields,

$$\begin{aligned}\tfrac{\partial\hat{\pi}(\bar{x},h^{-i},A_2)}{\partial h^{-i}} =& (1 - p_{A_1}(\bar{x}, h^{-i}+1))\tfrac{2q(\bar{x},A_2)}{n+1}\gamma(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n\tfrac{\underline{x}}{\bar{x}} + h^{-i}(1-\tfrac{\underline{x}}{\bar{x}}) + 8 - \tfrac{\underline{x}}{\bar{x}}\Big)\Big] \\ &- \tfrac{\partial p_{A_1}(\bar{x},h^{-i}+1)}{\partial h^{-i}}\Big(q(\bar{x},A_2)^2 - \xi\Big)\end{aligned}$$

Finally, if i invests low, she will be in A_2 for sure, i.e. $p_{A_1}(\underline{x}, h^{-i}) = 0$. Payoff is then given by

$$\begin{aligned}\pi(\underline{x}, h^{-i}) =& \tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} - \gamma h^{-i}(\bar{x}-\underline{x}) + \gamma\underline{x} + \gamma\beta\Big((h^{-i}+1)\big((h^{-i}-h_1)\bar{x} \\ &+ (n-h^{-i}-1)\underline{x}\big) - (h^{-i}-h_1)\big((h^{-i}-h_1-1)\bar{x} + (n-h^{-i})\underline{x}\big) - h_1(h_1-1)\bar{x}\Big)\Big]^2\end{aligned}$$

Taking the derivative with respect to h^{-i} yields,

$$\tfrac{\partial\pi(\underline{x},h^{-i})}{\partial h^{-i}} = \tfrac{2q(\underline{x},A_2)}{n+1}\gamma(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n(\tfrac{\underline{x}}{\bar{x}}) + h^{-i}(1-\tfrac{\underline{x}}{\bar{x}}) + 7\Big)\Big].$$

Note that expected payoff from choosing high investment is given by $\mathbb{E}(\pi(\bar{x}, h^{-i})) = \hat{\pi}(\bar{x}, h^{-i}, A_1) + \hat{\pi}(\bar{x}, h^{-i}, A_2)$. Thus the expected return on investment $\Delta\pi := \mathbb{E}(\pi(\bar{x}, h^{-i})) - \pi(\underline{x}, h^{-i})$ changes with h^{-i} according to

$$\begin{aligned}
\frac{\partial \Delta\pi}{\partial h^{-i}} =& \frac{\partial \mathbb{E}(\pi(\bar{x}, h^{-i}))}{\partial h^{-i}} - \frac{\partial \pi(\underline{x}, h^{-i})}{\partial h^{-i}} = \frac{\partial \hat{\pi}(\bar{x}, h^{-i}, A_1)}{\partial h^{-i}} + \frac{\partial \hat{\pi}(\bar{x}, h^{-i}, A_2)}{\partial h^{-i}} - \frac{\partial \pi(\underline{x}, h^{-i})}{\partial h^{-i}} \\
=& \big(\mathbb{E}(q(\bar{x})) - q(\underline{x})\big) \tfrac{2}{(n+1)^2} \gamma(\bar{x} - \underline{x}) \Big[-1 + \tfrac{\beta}{4}\Big(-(n - h^{-i})(1 - \tfrac{\underline{x}}{\bar{x}}) + 8 - \tfrac{\underline{x}}{\bar{x}} \Big)\Big] \\
& - \tfrac{2}{(n+1)^2} \gamma(\bar{x} - \underline{x}) \tfrac{\beta}{4} \Big[p_{A_1}(\bar{x}, h^{-i}+1) q(\bar{x}, A_1)(n+2) - (1 - p_{A_1}(\bar{x}, h^{-i}+1)) q(\bar{x}, A_2) n \\
& + q(\underline{x}, A_2)(n + 1 - \tfrac{\underline{x}}{\bar{x}}) \Big] + \tfrac{\partial p_{A_1}(\bar{x}, h^{-i}+1)}{\partial h^{-i}} \big(q(\bar{x}, A_1)^2 - q(\bar{x}, A_2)^2 \big) \\
\overset{(*)}{>}& \big(\mathbb{E}(q(\bar{x})) - q(\underline{x})\big) \tfrac{2}{(n+1)^2} \gamma(\bar{x} - \underline{x}) \Big[-1 + \tfrac{\beta}{4}\Big(-(n - h^{-i})(1 - \tfrac{\underline{x}}{\bar{x}}) + 8 - \tfrac{\underline{x}}{\bar{x}} \Big)\Big] \\
& + \tfrac{\partial p_{A_1}(\bar{x}, h^{-i}+1)}{\partial h^{-i}} \big(q(\bar{x}, A_1)^2 - q(\bar{x}, A_2)^2 \big)
\end{aligned}$$

where, again $\mathbb{E}(q(\bar{x}))$ is the expected quantity produced by a high effort firm. The last inequality $(*)$ holds since $q(\bar{x}, A_1) > q(\bar{x}, A_2)$ and, furthermore, the fact that $p_{A_1}(\bar{x}, h^{-i}+1)$ is decreasing in h^{-i} and, hence, for all $h^{-i} < n-1$ it holds that $p_{A_1}(\bar{x}, h^{-i}+1) \geq p_{A_1}(\bar{x}, n) = \lceil \frac{3n-1}{4n} \rceil > 1 - \lceil \frac{3n-1}{4n} \rceil \geq 1 - p_{A_1}(\bar{x}, h^{-i}+1)$ (see Proposition 1). Thus, if $\beta < 1/2$ then all terms above are non-positive which implies the statement of Lemma 6. □

Lemma 7. *For $h^{-i} \in \left[\frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}}, \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}} - 1\right]$ the expected return on investment is increasing in h^{-i} on the entire interval, decreasing in h^{-i} on the entire interval or has a unique local maximum in the interior of the interval.*

Proof. In order to show the claim of the proposition we prove that the change of the return on investment is concave in h^{-i} on the considered interval. It follows from Lemma 1 that for $h^{-i} \in \left[\frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}}, \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}} - 1\right]$ there are two clusters where all high investors are in the first and all low investors are in the second cluster. Taking this into account the return on investment is given by

$$\begin{aligned}
\Delta\pi =& \pi(\bar{x}, h^{-i}+1, A_1) - \pi(\underline{x}, h^{-i}, A_2) \\
=& \tfrac{1}{(n+1)^2} \Big[\alpha - \bar{c} + \gamma(n - h^{-i})(\bar{x} - \underline{x}) + \gamma\underline{x} + \gamma\beta((h^{-i}(n - h^{-i})\bar{x} - (n - h^{-i} - 1)(n - h^{-i} - 2)\underline{x}) \Big]^2 \\
& - \tfrac{1}{(n+1)^2} \Big[\alpha - \bar{c} - \gamma h^{-i}(\bar{x} - \underline{x}) + \gamma\underline{x} + \gamma\beta(n(n - h^{-i})\bar{x} - h^{-i}(h^{-i} - 1)\bar{x} - (n - h^{-i} - 1)^2\underline{x} \Big]^2.
\end{aligned}$$

Considering the derivative with respect to h^{-i} and collecting terms yields

$$\begin{aligned}
\frac{\partial \Delta\pi}{\partial h^{-i}} = \tfrac{2\gamma}{(n+1)} \Big[& -(\bar{x} - \underline{x})(q(\bar{x}, A_1) - q(\underline{x}, A_2)) + \beta\Big(n\bar{x}(q(\bar{x}, A_1) + q(\underline{x}, A2)) \\
& + ((2n-3)\underline{x} - 2h^{-i}(\bar{x} + \underline{x}))(q(\bar{x}, A_1) - q(\underline{x}, A2)) + (\bar{x} - \underline{x}) q(\underline{x}, A_2) \Big) \Big]
\end{aligned}$$

Furthermore, we have

$$\tfrac{\partial q(\underline{x}, A_2)}{\partial h^{-i}} = -\gamma(\bar{x} - \underline{x}) - \gamma\beta(n\underline{x} + (2h^{-i} - 1)\bar{x} - 2(n - h^{-i} - 1)\underline{x}) < 0,$$

because $h^{-i} > \frac{3(n-1)}{3\bar{x}+\underline{x}}$ implies $(n\underline{x} + (2h^{-i} - 1)\bar{x} - 2(n - h^{-i} - 1)\underline{x}) > 0$. Moreover,

$$\tfrac{\partial (q(\bar{x}, A_1) + q(\underline{x}, A_2))}{\partial h^{-i}} = -2\gamma(\bar{x} - \underline{x}) - \gamma\beta((n - 4h^{-i} + 1)\bar{x} + (3n - 4h^{-i} - 5)\underline{x}) < 0,$$

where $((n-4h^{-i}+1)\bar{x}+(3n-4h^{-i}-5)\underline{x})<0$ again follows from $h^{-i}>\frac{3(n-1)}{3\bar{x}+\underline{x}}$ in combination with $\bar{x}\leq 2\underline{x}$. Finally,

$$\tfrac{\partial(q(\bar{x},A_1)-q(\underline{x},A_2))}{\partial h^{-i}}=\gamma\beta((n-1)(\bar{x}+\underline{x})>0.$$

Taking these observations into account we obtain

$$\begin{aligned}\tfrac{\partial^2\Delta\pi}{\partial(h^{-i})^2}=&\tfrac{2\gamma}{(n+1)}\Bigg[-\underbrace{(\bar{x}-\underline{x})}_{>0}\underbrace{\tfrac{\partial(q(\bar{x},A_1)-q(\underline{x},A_2))}{\partial h^{-i}}}_{>0}+\beta\Bigg(n\bar{x}\underbrace{\tfrac{\partial(q(\bar{x},A_1)+q(\underline{x},A2))}{\partial h^{-i}}}_{<0}\\&+((2n-3)\underline{x}-2h^{-i}(\bar{x}+\underline{x}))\underbrace{\tfrac{\partial(q(\bar{x},A_1)-q(\underline{x},A_2))}{\partial h^{-i}}}_{>0}+\underbrace{(\bar{x}-\underline{x})}_{>0}\underbrace{\tfrac{\partial q(\underline{x},A_2)}{\partial h^{-i}}}_{<0}\Bigg)\Bigg]\\<&0,\end{aligned}$$

where we have used that $h^{-i}>\frac{3(n-1)}{3\bar{x}+\underline{x}}$ induces $((2n-3)\underline{x}-2h^{-i}(\bar{x}+\underline{x}))<0$. □

Lemmas 5–7 then imply that the investment incentives have the shape that is depicted in Figure 2 such that the investment incentives have a unique local maximum. For the sake of the argument, we denote the number of firms which invest high as h^* where this local maximum of $\pi(\bar{x},h^{-i}+1)-\pi(\underline{x},h^{-i})$ is attained. Hence, if costs ξ are low, i.e. $\xi<\bar{\xi}:=\pi(\bar{x},1)-\pi(\underline{x},0)$ then because return of investment dominates its cost, even if no other firm invests, there is a unique equilibrium. The equilibrium is such that $h(\xi)$ firms invest high and $n-h(\xi)$ invest low, where $h(\xi)=n$ if $\xi<\bar{\bar{\xi}}:=\pi(\bar{x},n)-\pi(\underline{x},n-1)$ or $h(\xi)$ solves $\min\{h\in\{h^*,\dots,n\}:\pi(\bar{x},h+1)-\pi(\underline{x},h)<\xi\}$ else. For $\xi>\bar{\xi}$ there is also an equilibrium where no firm invests, since $\pi(\bar{x},1)-\pi(\underline{x},0)<\xi$. Finally, if $\xi>\bar{\bar{\bar{\xi}}}:=\pi(\bar{x},h^*+1)-\pi(\underline{x},h^*)$, then there is no equilibrium where $h(\xi)$ firms invest high, since investment cost exceed the maximal gains of investment for all values of h^{-i}, which concludes the proof.

□

Proof of Proposition 3. (i) Suppose that there are K clusters A_k, $k\in\{1,...,K\}$ and denote $X:=\sum_{i\in N}x(i)$, $X_k=\sum_{i\in A_k}x(i)$, and $a_k:=|A_k|$.Thus, $\sum_{k=1}^{K}a_k=n$. Note that maximizing consumer surplus $CS=Q^2/2$ is equivalent to minimizing the sum of all marginal costs $C(\mathbf{x},\mathbf{A})=\sum_{i=1}^{n}c(i,\mathbf{x},\mathbf{A})$, since $(n+1)Q(\mathbf{x},\mathbf{A})=n\alpha-C(\mathbf{x},\mathbf{A})$. Then we get for total cost:

$$C(\mathbf{x},\mathbf{A})=\sum_{i=1}^{n}\Big(\bar{c}-\gamma x(i)-\gamma\beta\sum_{j\in A(i),j\neq i}x(j)\Big)=n\bar{c}-\gamma X-\gamma\beta\sum_{k=1}^{K}\Big((a_k)^2-1\Big)X_k.$$

Clearly C is minimized if $x(i)=\bar{x}$ for all $i\in N$ and further $a_1=n$. Thus, a single cluster where all firms choose high investments is maximizing consumer surplus.

(ii) Fix some profile of investment $x=(x(1),...,x(n))$ and denote by $\tilde{s}(i,\mathbf{x},\mathbf{A})=\gamma(x(i)+\beta\sum_{j\in A(i)\setminus\{i\}}x(j))$ the cost reduction of firm i due to own R&D investment and incoming spillovers for a profile of clusters $\mathbf{A}\in\mathcal{A}$. Thus, $c(i,\mathbf{x},\mathbf{A})=\bar{c}-\tilde{s}(i,\mathbf{x},\mathbf{A}))$. Denote by $\tilde{S}(\mathbf{x},\mathbf{A}):=\sum_{j\in N}\tilde{s}(j,\mathbf{x},\mathbf{A})$ and, as above, $C(\mathbf{x},\mathbf{A}):=\sum_{j\in N}c(j,\mathbf{x},\mathbf{A})$. This implies that given an investment profile $\mathbf{x}$, we can write welfare, consisting of the sum of firm profits and consumer surplus, for a cluster structure $\mathbf{A} \in \mathcal{A}$ as

$$
\begin{aligned}
W =& \sum_{i=1}^{n} (q(i,\mathbf{x},\mathbf{A}))^2 - h\xi + Q^2(\mathbf{x},\mathbf{A})/2 \\
=& \tfrac{1}{(n+1)^2} \left[\sum_{i=1}^{n} (\alpha - (n+1)c(i,\mathbf{x},\mathbf{A}) + C(\mathbf{x},\mathbf{A}))^2 + (n\alpha - C(\mathbf{x},\mathbf{A}))^2/2 \right] - h\xi \\
=& \tfrac{1}{(n+1)^2} \left[\sum_{i=1}^{n} \left(\alpha - \bar{c} + \gamma \left((n+1)(x(i) + \tilde{s}(i,\mathbf{x},\mathbf{A})) - \tilde{S}(\mathbf{x},\mathbf{A}) \right) \right)^2 + (n(\alpha-\bar{c}) + \tilde{S}(\mathbf{x},\mathbf{A}))^2/2 \right] \\
& - h\xi \\
=& \tfrac{1}{(n+1)^2} \Bigg[(n+2)(\alpha-\bar{c})\tilde{S}(\mathbf{x},\mathbf{A}) + n(\alpha-\bar{c})^2(1+n/2) - \tilde{S}(\mathbf{x},\mathbf{A})^2(n+3/2) \\
& \qquad + (n+1)^2 \sum_{i=1}^{n} \tilde{s}(i,\mathbf{x},\mathbf{A})^2 \Bigg] - h\xi
\end{aligned}
$$

Considering the last expression and taking into account that $\tilde{s}(i,x,A) \leq \gamma\bar{x}(1+\beta(n-1))$ it is obvious that for sufficiently large $(\alpha-\bar{c})$ maximizing W is equivalent to maximizing $\tilde{S}(\mathbf{x},\mathbf{A})$. Since every member of a cluster generates spillovers to all cluster members, we have

$$
\tilde{S}(\mathbf{x},\mathbf{A}) = \gamma \left(\sum_{j=1}^{n} x(j) + \beta \sum_{k=1}^{K} \left((a_k - 1) \sum_{i\in A_k} x(i) \right) \right) = \gamma \left(\sum_{j=1}^{n} x(j) + \beta \sum_{i=1}^{n} (a(i)-1)x(i) \right).
$$

and therefore maximizing $\tilde{S}(\mathbf{x},\mathbf{A})$ is equivalent to maximizing $\sum_{i=1}^{n}(a(i)-1)x(i)$ over all profiles of clusters $\mathbf{A} \in \mathcal{A}$. It is straightforward to see that

$$
\sum_{i=1}^{n} a(i)x(i) \leq n \sum_{i=1}^{n} x(i)
$$

and, since the right hand side corresponds to the case of a single cluster containing all firms, we have shown that welfare is maximized for such a cluster.

(iii) We show that a single cluster with full investment strictly maximizes welfare for $\xi = 0$, which by continuity implies the claim of this part of the Proposition. For $\xi = 0$ welfare can be written as

$$
W(\mathbf{x},\mathbf{A}) = U(Q(\mathbf{x},\mathbf{A})) - \sum_{i=1}^{n} c(i,\mathbf{x},\mathbf{A})q(i,\mathbf{x},\mathbf{A}),
$$

where $U(.)$ is the utility function of the representative consumer. Denote by $\bar{\mathbf{x}} = (\bar{x},\ldots,\bar{x})$ the investment profile where all firms invest high and by $\bar{\mathbf{A}}$ the cluster structure in which all firms are in the same cluster. Since under $(\bar{\mathbf{x}},\bar{\mathbf{A}})$ each firm has maximal own R&D investment as well as maximal incoming spillovers, it is easy to see that

$$
\sum_{j\in N} c(j,\bar{\mathbf{x}},\bar{\mathbf{A}}) = nc_{\min} < \sum_{j\in N} c(j,\mathbf{x},\mathbf{A})
$$

for all $(\mathbf{x}, \mathbf{A}) \neq (\bar{\mathbf{x}}, \bar{\mathbf{A}})$, where $c_{min} = c(j, \bar{\mathbf{x}}, \bar{\mathbf{A}})$ ist the minimal marginal cost value that can be reached by any firm. Note that $c(j, \bar{\mathbf{x}}, \bar{\mathbf{A}})$ is identical across all firms j. Due to $Q(\mathbf{x}, \mathbf{A}) = \frac{n\alpha}{n+1} - \frac{1}{n+1} \sum_{j \in N} c(j, \mathbf{x}, \mathbf{A})$ we conclude that

$$Q(\bar{\mathbf{x}}, \bar{\mathbf{A}}) > Q(\mathbf{x}, \mathbf{A}).$$

Furthermore, $U'(\tilde{Q}) = p(\tilde{Q}) > c_{min}$, for all $\tilde{Q} \in [Q(\mathbf{x}, \mathbf{A}), Q(\bar{\mathbf{x}}, \bar{\mathbf{A}})]$ and therefore

$$\begin{aligned} W(\bar{\mathbf{x}}, \bar{\mathbf{A}}) &= U(Q(\bar{\mathbf{x}}, \bar{\mathbf{A}})) - c_{min} Q(\bar{\mathbf{x}}, \bar{\mathbf{A}}) \\ &> U(Q(\mathbf{x}, \mathbf{A})) - c_{min} Q(\mathbf{x}, \mathbf{A}) \\ &\geq U(Q(\mathbf{x}, \mathbf{A})) - \sum_{j \in N} c(j, \mathbf{x}, \mathbf{A}) q(j, \mathbf{x}, \mathbf{A}) \end{aligned}$$

for all $(\mathbf{x}, \mathbf{A}) \neq (\bar{\mathbf{x}}, \bar{\mathbf{A}})$, where the last inequality follows from $c(j, \mathbf{x}, \mathbf{A}) \geq c_{min}$ for all $j \in N$.

(iv) For ξ large enough, any benefit of investment is dominated by the costs, hence all firms must invest low in the welfare maximum. Given this investment pattern, it is easy to see that no profile of clusters can generate a lower value of marginal production costs than what is obtained by all firms if a single cluster is formed. Taking this into account, an analogous argument to that used in the proof of part (iii) establishes that the generation of a single cluster containing all firms maximizes welfare.

□