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FINANCIAL INTERMEDIATION, THE ENVIRONMENT, AND ECONOMIC GROWTH

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α Technology parameter relating R&D and success-probabilities

$\bar{A}$ Pre-industrial level of atmospheric CO_2-concentration

ℓ Subindex identifier of consumer type $\ell \in \{w, i, e\}$, w=workers, e=entrepreneurs, i=innovators

η_{ϑ_j} Probability of innovation success on capital good line ϑ_j in intermediate sector j, $\vartheta_j \in \Theta_j$, $j \in \{m, z\}$

ι Mass of innovators in sector $_j$, $\iota \in [0, 1]$

κ Specific CO_2 emissions per unit of dirty output

λ_j Parameter of innovation "size" in sector j, $j \in \{m, z\}$

μ Technology parameter in intermediate production function

ϕ Share of emissions decaying at a geometric rate

ϕ_0 Share of immediately absorbed CO_2-emissions

ϕ_L Share of permanent CO_2-emissions

θ Parameter of climate damages

Θ_j Set of capital good lines in sector j, $j \in \{m, z\}$, $\Theta_j := [0, 1]$

ε Elasiticity of substitution in final output production function

ϑ_j Capital good line ϑ_j in intermediate sector j, $\vartheta_j \in \Theta_j$, $j \in \{m, z\}$

ζ Technology parameter representing research costs

A Atmospheric CO_2-concentration

A_2 Non-permanent emissions

A_1 Permanent emissions

C Total consumption

C_ℓ Aggregate consumption of consumer type ℓ

D Aggregate deposit demand

D_j Aggregate credit supply in sector j, $j \in \{m, z\}$

E Aggregate CO_2 emissions

H Aggregate R&D investment

h_{ϑ_j} R&D expenditure on capital good line ϑ_j in intermediate sector j, $\vartheta_j \in \Theta_j$, $j \in \{m, z\}$

H_j Aggregate R&D expenditures in sector j, $j \in \{m, z\}$

I_ℓ Income of consumer type ℓ

J Index identifier of intermediate sector-specific production factors: $J \in \{M, Z\}$

J Index representing factor inputs in intermediate production, $J = M$ for $j = m$ and $J = Z$ for $j = z$

j Subindex identifier of intermediate sector-specific variables: $j \in \{m, z\}$

L_j Sectoral employment levels, $j \in \{m, z\}$

M Production factor used in intermediate sector m

m (Green) Intermediate sector

p Price of final output

p_j Price of intermediate good Y_j, $j \in \{m, z\}$

p_{ϑ_j} Price of capital good ϑ_j

Q_j Average sectoral capital goods quality index, $j\{m, z\}$

q_{ϑ_j} Current quality level of capital good ϑ_j

r Deposit interest rate

R_j Interest rate on business credit in sector j, $j \in \{m, z\}$

S Aggregate savings = Total deposit supply

t Subindex of discrete time

V_j Expected discounted average profits from innovation in sector $j \in \{m, z\}$

X Aggregate capital goods investment

X_j Aggregate capital goods investment in sector j

x_{ϑ_j} Quantity of capital good ϑ_j

Y Final output

Y_j Intermediate output in sector $j \in \{m, z\}$

Z Production factor used in intermediate sector z

z (Dirty) Intermediate sector

Introduction

Economic growth is one foundation pillar of modern democratic market economies, because it helps reduce poverty, increases levels of employment, contributes to a rise in living standards through public services such as education, and health care, and reduces governmental budget deficits. Over the last five decades, the annual mean growth rate of the world's real Gross Domestic Product (GDP) has been 3.5%. In order to achieve similar rates of economic growth in the future and maintain current welfare levels, global policy makers have to eliminate several threats. Among the most pressing policy challenges the world faces today are: in the short term, the task to overcome the aftermath of the global financial and governmental debt crisis that started with the collapse of Lehman Brothers in September 2008 and is not yet completely resolved.

In the medium term, a second threat to growth may come from the fact that economic growth is associated with an increasing consumption of energy and that energy sources to a great extent are exhaustible resources. The ambitious task here is to redirect the global economy in a way that shifts the energy supply, currently based on limited fossil fuels, towards the use of non-exhaustible renewable energy sources. Closely connected to this is a third threat that could lead to great economic damages in the long term: climate change. Consumption of fossil fuels, i.e. their combustion, emits greenhouse gases, which are detrimental to the earth's climate. Since economic growth is associated with an increasing consumption of fossil fuels, global carbon emissions have significantly increased since 1900: By 2010 the amount of carbon emissions had increased more than seventeenfold on the amount emitted in 1900, and one and a half time on the figure for 1990. If greenhouse gases continue to increase in the future, the effects of human-induced climate change such as hurricanes, floods and periods of aridity will occur more frequently with corresponding negative effects on economic growth. To alleviate these growth-damping effects, the industrialized nations have to decouple economic growth and further increases of emissions. Consequently policy makers have to support the development of new abatement and emission-extensive technologies.

In general, the process of developing and inventing new products or technologies of higher quality is costly, since some capital may be needed to finance R&D expenditures in order to even develop a new product. Then, some more capital is required to finance the cost of producing this new commodity. On the one hand, innovative individuals lack the necessary capital to finance their business start up by themselves if they have been already successful in generating a new commodity. On the other hand, even established firms at least sometimes need external capital to finance their R&D expenditures. Basically, these different businesses can raise capital to finance their investments either by asking lenders for loans or by issuing bonds/stocks on financial markets.

Empirical observations support the idea that business R&D investment is partially financed externally through loans. What is more, the data show an increasing share of external capital in R&D expenditures: First, total business loans divided by total business investment in the U.S. fluctuated around 85% between 1960-2011 while the ratio of domestic credit to GDP increased from 78% in 1960 to 193% in 2011.[1] Second, in the same time period, the share of research and development expenditures (R&D) on total investment expenditures in the U.S. increased from 8% to 20%. Third, long-term interest rates on credit lending declined from 18% in 1980 to 3% in 2008. Since the relationship between interest rates and investment is negative, the pattern of R&D expenditures and interest rates indicates that aggregate R&D expenditures respond to interest rates. This already suggests a relationship between financial intermediation and technical change: the lower the loan rate on business credit, the greater is the amount of firms' R&D expenditures and therefore the rate at which new innovations arrive. Hence, the rate of technological advance presumably responds to conditions on the credit market.

Moreover, empirical observations suggest technical changes to be biased towards certain factors: in almost all western industrialized nations, the prices of the two production factors capital and labor have shown diverging characteristics over the past 150 years. While the rental rate of capital has been approximately constant, the wage rate has continuously increased. This evolution in factor prices suggests technological changes to be labor-augmenting. Therefore technical change is not factor-neutral and takes different "directions".

In addition, loan interest rates differ across industries: on the one hand, capital markets allocate capital to sectors or countries where the highest return and thus the largest growth potential can be expected. On the other hand, interest rates also reflect the risk inherent in an investment project that investors (i.e. banks) would have to bear. Since different investment projects compete for loans, projects linked with higher risk have to accept higher loan rates to compensate for the higher risk involved. Consequently, profit-maximizing firms who are free to direct their innovation effort to a certain industry ceteris paribus choose the sector linked with the lowest risk of default, because the associated credit costs are the lowest. Taken together with the negative relationship between interest rates on business credit and the size of R&D investment, this could indicate that banks and credit might also affect the direction of technical change in a systematic way.

To analyze to what extent banks influence the direction of firms' innovation effort is worthwhile because, given the link between financial intermediation and directed technical change, policies that aim at forcing technical changes towards a certain direction should take into account the effects of financial markets in general and banks in particular: Suppose, for instance, the aim of an environmental regulation is to support the development of new technologies based on renewable energy sources, in order to reduce the dependence on exhaustible fossil fuel energy and simultaneously to abate emissions from fossil fuel consumption, which is the main source of man-made climate change. In general, profit-maximizing firms engaging in innovation direct their R&D effort to the sector promising the highest return. Especially, a negative external long-term effect from carbon emissions on the climate does not directly affect the firms' short-term return on innovation. Hence, whether a sector is emission-intensive ("dirty") or emission neutral ("green"), plays no role in the decision of firms to which sector they direct their inno-

[1]Domestic credit to private sector refers to financial resources provided to the private sector, such as through loans, purchases of nonequity securities, and trade credits and other accounts receivable, that establish a claim for repayment.

vation effort, and firms do not automatically invest in the development and invention of eco-friendly commodities. To make sure that they nevertheless target the “green” sector, environmental policy has to create profit incentives for them to develop new eco-friendly technologies that would otherwise not be invented.

If, however, firms are capital constrained, they need to raise external capital to finance R&D expenditures and - after successful innovation - some more external capital to finance the production of the newly-invented commodity. Policies that affect the determinants of expected profits should consequently take into account the influence of credit markets on these determinants.

Here are two simple examples: Assume there are two sectors, a “dirty” and a “green” sector. Firms decide to which sector they direct their innovation effort based on expected profits. Environmental regulation seeks to direct innovations to the “green” sector. Now suppose first that ceteris paribus the risk of a default on credit obligations in the “green” sector is greater than in the “dirty” sector. Hence, redit costs would be greater in the “green” sector, since the interest rate on business credit is higher to compensate for the higher risk. Consequently, the expected profits are lower in the “green” sector, due to greater credit costs. An environmental regulation that does not consider the effect of the credit market on firms’ profits may be ineffective in the sense that intervention is too weak to create an incentive for firms to direct their effort towards the development of “green” commodities. This would imply that the consideration of the credit market would make a stronger environmental policy intervention necessary, compared to the situation without considering the impact of credit on the profits of capital-constrained firms. If the results of my thesis turn out this way, then banks inhibit the response to environmental regulation.

Secondly, suppose that ceteris paribus the risk of default on credit obligations in the “green” sector is lower than in the “dirty” sector, but expected profits are nevertheless such that without intervention, firms would innovate on technologies for the “dirty” sector. Then, the credit cost to a firm who plans to invent a new commodity would be greater in the “dirty” sector, since the interest rate on business credit here is higher to compensate for the higher risk. In this case, an environmental regulation that does not consider the effect of the credit market on firms’ profits is too strong. A less strong intervention would still be sufficient to create an incentive for firms to direct their effort towards the development of “green” commodities. If the results in this study turn out that way, then banks promote the response to environmental regulation. Either way, given that firms need credit to finance their ventures, the examples show that credit interest rates affects the direction of technical change.

Therefore, the purpose of my study is to analyze the role of banks in directing endogenous technical change with regard to climate change and economic growth. My thesis also analyzes the question to what extent existing climate policy evaluations alter if credit is considered among the determinants of directed R&D investment. To achieve this purpose theoretical economic models will be constructed that account for environmental constraints and financial intermediation.

The first part explores the relationship between private sector lending and the rate and direction of technical change: Profit-seeking individuals establish new firms and invest in research and development to absorb temporary monopoly profits from patent protected productivity improvements on existing capital goods. These quality improvements are the driving force behind economic growth. The capital goods are used in two different intermediate sectors as inputs to production. Innovators direct R&D to capital goods in one of the two intermediate sectors. Therefore, the direction of technical change is endogenous. Yet they lack the capital resources to finance investments. Banks supply the necessary capital. In simplified terms, the research question of this first part is: *Do banks influence the direction of technical change?* To answer this research question, we construct an *endogenous directed technical change growth model* and explicitly incorporate *a banking sector*.

The second part examines the role of banks in the process of innovation and the invention of ”green technologies” in an economy with environmental constraints. The two intermediate sectors now differ with respect to their levels of pollution: one sector is ”green”, the other one ”dirty”. A by-product of dirty sector production are carbon emissions that lead to negative effects on aggregate output through climate change. The second part answers the exemplary question: *What roles do banks play in directing technical change towards a ”green” economy?* To answer this question, we build on the results of the first part and develop a model of endogenous directed technical change, financial intermediation and additionally account for an environmental externality.

The third part evaluates different climate policy instruments with respect to their effects on economic growth, their cost efficiency and their effectiveness in achieving a postulated environmental target. Any climate policy instrument seeks to direct technical change away from the ”dirty” sector towards the ”green” sector; probably at the cost of (temporary) negative effects on economic growth. This part answers the stylized research question: *What policy rules are best suited to help the economy move towards a path of sustained and green economic growth?* Here, we conduct numerical model simulations based on the more general findings of parts one and two.

The dissertation is organized as follows. The next chapter gives an overview of previous approaches, methodology, relates my study to the relevant literature and states some empirical facts. The first main part, consisting of chapters two to four, explores the general relationship between financial intermediation, credit and the direction of technical change. The second part, consisting of chapters five to seven, introduces environmental constraints into the model developed in part one. In the third part, consisting of chapters eight and nine, the study analyzes different climate policies and conducts quantitative model simulations. The last chapter draws a number of conclusions and outlines possible model extensions.

1. Concepts, Methodology and Literature

Let us start with some fundamentals on technological progress, differences between factor-augmenting and factor-biased technical change, the influence of financial intermediation on economic development according to previous approaches, and the relationship between economic growth and the environment. Throughout the rest of this study the words "environment" and "climate" will be used as synonyms and the challenge of reducing CO_2 should be seen as a special case of the general challenge to reduce environmental degradation and pollution. In this regard "environmental policy" here means in fact "climate policy". Note that these preliminary remarks serve only to highlight the motivation for the present dissertation and provide some concepts that will be used throughout the rest of this thesis. They are by no means complete with respect to the existing literature in the different fields of research.

1.1. Technological Change

Suppose aggregate output of an economy Y can be described by a function of labor L, a production factor Z and a technology index Q that is $Y = \mathcal{F}(L, Z, Q)$.[2] Independent of how Q enters this function, we can presume $\partial Y/\partial Q > 0$: An increase in the technology index Q parallels improving technology levels, i.e. *technological progress*. In this regard, the relevant literature distinguishes the following three special forms of technological progress.

If the function is given by $Y = \mathcal{F}(L, QZ)$, then technological change will be purely Z-augmenting. If we assume Z to represent physical capital, then this form implies purely capital augmenting technical change, which is commonly referred to as *Solow-neutral* technical change. In this form, technical change leaves the factor share of labor $\frac{\partial \mathcal{F}}{\partial L}\frac{L}{Y}$ unchanged provided that the wage rate (the marginal product of labor $\frac{\partial \mathcal{F}}{\partial L}$) is constant.

If $Y = Q\mathcal{F}(L, Z)$ holds, technical change guarantees the ratio of the marginal factor productivities of Z and L ($\frac{\partial Y}{\partial Z}, \frac{\partial Y}{\partial L}$) to remain unchanged, if the ratio of Z and L is kept constant. The literature commonly refers to this form as *Hicks-neutral* technical change.

If finally the aggregate production function takes the form $Y = \mathcal{F}(QL, Z)$, then technical change is purely labor-augmenting, which is commonly referred to as *Harrod-neutral* technical change. If again Z represents physical capital, then technical change leaves the factor share of capital $\frac{\partial \mathcal{F}}{\partial Z}\frac{Z}{Y}$ unchanged provided that the interest rate (here the marginal product of capital $\frac{\partial \mathcal{F}}{\partial Z}$) is constant.[3]

Due to the following two reasons, the latter formulation is probably the most accepted one in macroeconomics. First, empirical observations show that technological progress has been labor-augmenting over the past 150 years, i.e. the two key factors, capital and labor show diverging characteristics: while real wages – the marginal product of labor –

[2]Formally, the economy is described by an aggregate production function $\mathcal{F} : \mathbb{R}^3_+ \longrightarrow \mathbb{R}_+$, $Y = \mathcal{F}(L, Z, Q)$, where $\mathcal{F}$ is assumed to be continuous, non-decreasing in all variables, homogeneous of degree zero and concave.

[3]For a formal proof of the equivalence of labor-augmenting technical change and Harrod-Neutrality, see Uzawa (1961).

continuously increased, the rental rate of capital – the marginal product of capital – remained almost constant, which indicates technical progress to be labor-augmenting (see Acemoglu (1999), Barro and Sala-i Martin (2004)). Secondly, only labor augmenting technical change is consistent with the existence of a long-run steady state growth rate in the one-sector neoclassical growth model developed by Solow (1956) and Swan (1956)).[4].
In Solow's model of economic growth, technological change is – alongside population or labor force growth – the driving force behind sustained long-run economic growth. A shortcoming of this formulation is that especially technological change is exogenous and thus not explained by the model. Moreover the model fails to account for entrepreneurship that might catalyze economic growth, i.e. the model formulation lacks a sound microfoundation.
In order to overcome these shortcomings, new models were developed, known today as *endogenous growth models*.[5] Roughly speaking, the endogenous growth literature implemented the idea of Schumpeter (1912) that profit-maximizing individuals engage in R&D in order to attain – at least temporary – monopoly profits based on patent-protected innovations. This search for profits creates an incentive to innovation activities that induce technical change, which represents the engine of economic development.

1.2. Factor-Biased Technical Change

However, despite their strength and influence on macroeconomic theory, endogenous growth literature does not allow technological change to be *biased* or *directed* to one factor. This contradicts the empirical observation that technological progress benefits some factors and/or sectors of production more than others. So while endogenous growth literature ascertain innovations to respond to policy rules in general, the literature on directed technical change sharpened these findings and showed that technical progress is neither factor neutral nor sector neutral. The idea that technical changes should be directed is not new. In their *New view on technological change*, Atkinson and Stiglitz (1969) stated
"[...] *the Government should be concerned not merely with the level of investment or output, but must make sure that firms are directed towards the "right" technique on long-run considerations."*
However, this idea was not pursued until Acemoglu (1998, 1999, 2002) developed a comprehensive framework that can be used to study empirical phenomena such as the question why technical change over the past 60 years has been skill-biased, to determine the effect of factor-biased technical change on the income gap between rich and poor countries, and to what extent oil prices induced energy-saving innovations and so on.[6]
Using this framework, economists can, for instance, evaluate environmental policies that aim at directing technical changes towards the development of new "green" technologies, i.e. technologies based on renewable energy sources or emission-neutral technologies.
In order to delineate the notion of biased technical change and to distinguish this idea

[4] A complete proof of Uzawa's theorem can be found in Jones and Scrimgeour (2004)

[5] See among many others Lucas (1988), Romer (1990), Rebelo (1991) and Rivera-Batiz and Romer (1991), Aghion and Howitt (1992), Young (1993) and Grossman and Helpman (1993).

[6] See Acemoglu (2002), Hassler et al. (2012) and Acemoglu et al. (2012).

from the concept of factor-augmenting technical change, consider the aggregate production function of the form introduced above:[7]

$$Y = \mathcal{F}(M, Z, Q),$$

where M and Z are the factors of production and again Q represents a quality index. Then in contrast to M-*augmenting* technical change, technical change is M-*biased* if it increases the relative marginal product of factor M compared to Z:

$$\frac{\partial \frac{\partial \mathcal{F}(M,Z,Q)/\partial M}{\partial \mathcal{F}(M,Z,Q)/\partial Z}}{\partial Q} \geq 0,$$

where $\partial \mathcal{F}(M, Z, Q)/\partial M$ is the marginal product of labor and $\partial \mathcal{F}(M, Z, Q)/\partial Z$ is the marginal product of factor Z. Conversely, technological change is Z-biased if

$$\frac{\partial \frac{\partial \mathcal{F}(M,Z,Q)/\partial Z}{\partial \mathcal{F}(M,Z,Q)/\partial M}}{\partial Q} \geq 0.$$

In order to further clarify the concept, consider the aggregate constant elasticity of substitution (CES) production function:

$$Y = \left[\gamma (Q_M M)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(Q_Z Z)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where M, Z are specified as above, $\varepsilon \in]0, \infty[$ is the elasticity of substitution, $\gamma \in]0, 1[$ is a distribution parameter that describes the relative importance of M and Z for the production of Y, Q_M and Q_Z are factor (or sector) specific technology indices. Depending on the value of ε, the CES-production function can take the following three forms:

- if $\varepsilon = 0$, then the production function is Leontieff, in other words the factors of production M and Z cannot be substituted for each other,
- if $\varepsilon = 1$, then the production function is Cobb-Douglas,
- if $\varepsilon = \infty$, then M and Z are perfect substitutes.

In the remainder of this thesis, any two goods (or factors) will be considered *gross substitutes* if their corresponding elasticity of substitution is strictly greater than one ($\varepsilon > 1$). They will be referred to as *gross complements* whenever the elasticity of substitution is strictly less than one ($\varepsilon < 1$), since to distinguish between $\varepsilon > 1$ and $\varepsilon < 1$ is most important for this analysis.

By construction, Q_M is M-augmenting and Q_Z is Z-augmenting. Whether technical change

[7] Since Acemoglu developed the formal framework of factor biased technical change, the following explanations borrow heavily from Acemoglu (2002).

is M-biased or Z-biased depends on the elasticity of substitution ε: The relative marginal product of the factors M and Z is given by

$$\frac{\partial Y/\partial M}{\partial Y/\partial Z} = \frac{1-\gamma}{\gamma}\left(\frac{Q_Z}{Q_M}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{Z}{M}\right)^{-\frac{1}{\varepsilon}}$$

This relative marginal product of Z is decreasing in the relative supply of factors $\frac{Z}{M}$. This is due to the usual substitution effect, leading to a negative relationship between relative supplies and relative prices. The more abundant factor is substituted for the less abundant one. This holds independent of whether the relationship between the two factors is gross substitutability or gross complementarity.
In contrast, the response of the relative marginal product to changes in the productivity of Z, Q_Z, depends on the elasticity of substitution ε: If $\varepsilon > 1$, an increase in the relative productivity Q_Z/Q_M increases the relative marginal product of Z. If $\varepsilon < 1$, an increase in Q_Z relative to Q_M decreases the relative marginal product of Z. Consequently, if the two factors are gross substitutes, Z-augmenting technological change is also Z-biased. In contrast, if the two factors are gross complements, Z-augmenting technological change is M-biased. If $\varepsilon < 1$, an increase in the productivity of Z increases the demand for M by more than the demand for Z and consequently, the marginal product of M increases relative to the marginal product of Z.

1.3. Financial Intermediation and Economic Development

Concerning the role of financial markets in economic development,"pre-crisis" macroeconomics offered surprisingly deviating positions.[8] The first group of economists suggest that financial markets had at the most a minor relevance in the sense that the development in the financial sector follows "real"-sector activity (Robinson (1952)) or that macro-economic research overestimates the role of financial development when exploring the determinants of economic growth (Lucas (1988)).[9]
The second group of economic researchers go one step further and proclaim their view on financial matters with respect to economic growth by simply ignoring it (Chandavarkar (1992), Meier and Seers (1984) and Stern (1989)). This was probably the most popular view in macroeconomics until the great financial and economic crisis engulfed the global economy from 2008. Ever since, macroeconomic theory has come under fundamental criticism: because economists failed to consider financial markets in their macroeconomic models, they were unable to predict the crisis. Anyhow, the response of macroeconomics towards this kind of criticism is ambiguous. Some prominent mainstream economists have lent their voices to this growing chorus of criticism, many others continue to adhere to their earlier views.[10]
The third and last group of researchers build on the works of Schumpeter (1912), who

[8] The literature cited in the following is by no means exhaustive and provides only examples of the different positions regarding finance and economic growth. But in terms of actual opinions, it is nevertheless complete. For a complete review the reader may turn to the references given in Levine (2004).
[9] Lucas (1988) p. 6 states that economists "over-stress" financial matters for economic development.
[10] Dutt (2010)

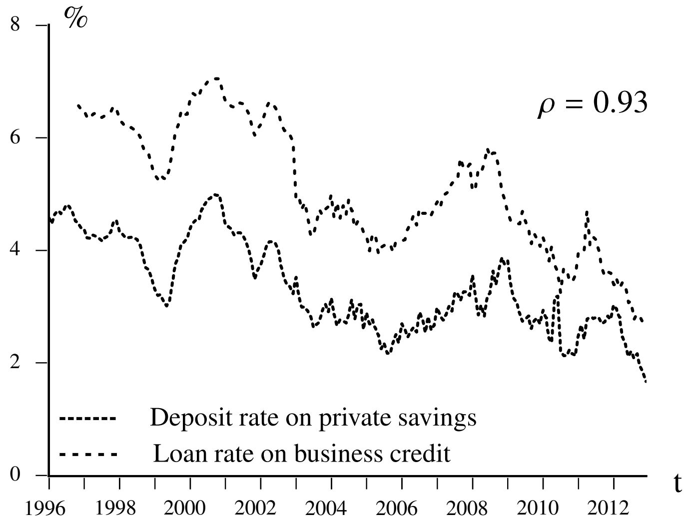


FIGURE 1: Long term interest rates in Germany, Jan.1996-Dec.2012

stated that financial intermediaries have to be considered in the process of economic growth, because they evaluate and finance entrepreneurs and help them invent new products. To name the most prominent ones among many others, Schumpeter (1912), John Hicks (1969), Boyd and Prescott (1986), King and Levine (1993), and Levine (1997) assign financial intermediation an active role in economic development that cannot be ignored when exploring the mechanics of economic growth. However, the channels of this active influence are still a matter of scientific debate.[11]

Independent of these different views on financial intermediation and macroeconomics, empirical observations on (credit and deposit-) interest rates feature different properties that are especially interesting for the present thesis.

Figure 1 shows the interest rates on long-term deposits paid to private households and on firms' medium to long-term credit demand per month in Germany from Jan. 1996 to Dec. 2012.[12] In the upper right-hand corner of the picture, I also stated the correlation coefficient of the two time series, $\rho = 0.93$ which indicates a high positive, almost perfect linear relationship between the two rates of interest. The data suggests the existence of an interest rate spread, meaning a differences between interest rates paid to private household deposits and interest rates charged on business credit. Classical financial intermediaries make money from taking deposits, paying households an interest rate, pool these deposits and supply them as credit to firms with greater interest rates. The interest rate spread is the profit from financial intermediation.

Another interesting empirical observation is that interest rates on business credit differ across sectors. Table 1 states the development of loan rates on long term credit paid by

[11] Summarizing the state of research, Ross Levine (2004 p.3) says that "*We are far from definitive answers to the questions: Does finance cause growth, and if it does, how?*

[12] Source: Deutsche Bundesbank

firms in different industrial sectors in Germany from 1980 to 2010. The numbers in the table show two important features for the purposes of this thesis. First, loan rates charged on credit differ across industries. Second, but less important for the purpose of this study, loan rates show similar patterns over time: between 1980 and 1990 loan rates declined and then continuously increased from 1995 until 2010.
Especially in the period 2000-2004, where the bust of the "dotcom"-stock market bubble hit the global economy, loan rates were higher compared to those of the period 1990-1999. Assuming that interest rates on loans reflect the risk inherent in any investment project, this suggests higher risk premiums were charged on long-term business credit during 2000-2004.

TABLE 1: Loan rates for selected industries in Germany, 1980-2010 in %

Sector \ Time period	80 - 84	85 - 89	90 - 94	95 - 99	00 - 04	05 - 10
Chemicals & Chemical products	10.66	6.62	8.27	7.54	13.44	14.42
Food, Beverage & Tobacco	7.31	4.67	5.53	4.30	5.06	6.10
Non-metallic mineral products	8.38	5.49	5.99	6.23	7.30	8.05
Machinery & Equipment	5.51	3.29	5.30	5.73	9.20	11.61
Basic metals & fabr. metal products	18.17	10.45	9.66	6.00	6.45	7.73
Rubber& Plastic	10.64	6.51	8.34	6.64	8.22	9.29
Wood & Paper, Print	7.68	5.19	5.80	4.31	4.54	6.31
Textiles & Clothes etc.	7.14	4.37	5.09	4.59	4.94	5.67

Source: Statistisches Bundesamt, Deutsche Bundesbank, own calculations

In 2005-2010 the loan rates on business credit rose even higher compared to the period 2000-2004. While in 2008 and 2009 the great financial crisis lead to a global recession, from 2010 a phase of slow economic recovery and uncertainty has prevailed until today. This can be interpreted as empirical evidence that banks charge higher risk premium on business credit whenever there is a higher probability of default.

1.4. Economic Growth, the Environment and Technical Change

Similar to the controversy about the role of financial intermedation in promoting economic growth, the positions regarding the relationship between economic growth and the environment differ widely. In somewhat oversimplifying terms, the opinions can be divided into two different groups: One group of economists focus on the remaining and often serious environmental problems of today. Adherers to this position see the emergence of new pollution problems, the failure to successfully reduce global carbon emissions and the continuing dependence of the global economy on exhaustible resources. The second group of researchers consider the long history of improvement in living standards and refer to the enormous improvements in air quality in cities, urban sanitation, life expectancy et cetera, all made possible by technological advance.
Over the last decades, the fact that the natural resource base of the planet is limited was

viewed as the main source of limits to economic growth.[13] However, in the recent past it has become more and more apparent that limits to economic growth may also arise from natures' limited ability to act as a sink for human pollution, especially with respect to greenhouse gas emissions.[14] This pollution may lead to limits in economic growth because further environmental degradation makes more intensive clean-up or abatement efforts necessary that lower the return on investment. An even more pessimistic view is that economic growth may be limited because further damages to the ecosystem lead to a point of no return where the ecosystem deteriorates beyond repair and the global economy moves to a less productive long-run steady state.

Whether there are serious limits to growth has long been a matter of scientific debate and has not yet been completely resolved: Starting with Meadows et al. (1972) see the subsequent contributions by Solow (1973) followed by Meadows et al. (1991), then Nordhaus (1992). Slightly more recently, Stokey (1998) showed that environmental constraints can create endogenous limits to economic growth, while Aghion and Howitt (1998) showed that this may not be the case if "eco-friendly" innovations are allowed for.

Closely connected to this, another scientific debate sparked around the question of how optimal policy should respond to the task of dealing with climate change and limited natural resources given that economic growth is negatively affected to some extent in a laissez-faire future, i.e. without any environmental regulation. More precisely, differences lie in the answers to the questions of how strong intervention should be in order to avoid a climate catastrophe, should intervention be temporary or permanent, what are the long run implications, and are there any costs involved in delaying climate policies into the future?

To name only representative studies for the different opinions, from among approaches assuming exogenous technology, Stern (2007) calls for decisive and immediate governmental intervention and Stern (2009b) argues that intervention needs to be in place permanently even though the induced economic costs are significant. A slighty more optimistic view with modest control in the short-term and limited stronger intervention in the long-term is suggested in Nordhaus (2008). Approaches assuming endogenous technology are for instance provided by Bovenberg and Smulders (1995, 1996): The authors develop a growth model featuring endogenous progress in abatement technologies, where the environment is modeled as a renewable resource. They find an optimal tax on pollution that rises at the growth rate of pollution-augmenting knowledge. Goulder and Schneider (1999) also study endogenous innovations in abatement technologies. Popp (2002) documents an influence of energy prices on energy saving innovations by using patent data from 1970 to 1994. Buonanno et al. (2003) study the implications of the Kyoto Protocol within an endogenous technical change model. Popp (2004) introduces directed innovation in the energy sector and suggests in a calibration exercise that ignoring directed technical change might lead to an overestimation of the true costs of environmental regulation. Building on these approaches, Acemoglu et al. (2012) construct a systematic model framework of endogenous directed technical change to analyze the impact of different types of

[13] See "Limits to Growth" by Meadows et al. (1972).

[14] See for instance Intergovernmental Panel on Climate Change (2007).

environmental regulation. Within the empirically relevant parameter space, they find that in order to avoid damages in the form of climate change, immediate environmental intervention is indeed necessary. They suggest using carbon taxes only gradually and instead rely more on medium-term-oriented subsidies to R&D in emission-extensive sectors.
However, with regard to the present dissertation, none of the work just quoted considers the impact of credit markets on technical change in general nor on the direction of eco-friendly innovations in particular.

FINANCIAL INTERMEDIATION AND DIRECTED TECHNICAL CHANGE

Introduction

The following first part concentrates on the general role of banks in determining the rate and direction of technical change. The goal is to develop a micro-founded general equilibrium model framework to examine the impact of financial intermediation on the determinants of directed technology innovation. In this regard, the present chapter serves as a foundation for the more applied research work of part two and three, where additional environmental constraints will be introduced.

The developed model combines elements from two different strands of literature and is therefore related to each of them. First, the present study builds on the literature on endogenous directed technological change (Acemoglu (1998, 1999, 2002, 2007)): the approach in its turn builds on the work of Schumpeter (1912), Romer (1990), Aghion and Howitt (1992), Grossman and Helpman (1993) and features some types of profit maximizing individuals who engage in research and development (R&D) in order to attain profits from inventing new commodities of greater quality. By extension, in the endogenous directed technical change approach, innovative firms target different sector- or factor specific technologies. The key feature is that expected profits from innovation determine the choice to which sector or factor the innovative firms direct their R&D effort in the first place. Consequently, the direction of innovations and thus technical change is now endogenous.

Second, the thesis draws on the literature on financial intermediation and economic development (Schumpeter (1912), Boyd and Prescott (1986), King and Levine (1993) and Levine (1997)). This strand of literature is more or less entirely based on the work of Schumpeter (1912), who stated that financial intermediaries have to be considered in the process of economic growth, because they evaluate and finance entrepreneurs and thereby help to invent new products. To understand the relation of this thesis to the second strand of literature, suppose innovative firms have no internal capital resources to finance their investment projects. Given that innovation is costly, firms need external capital. A bank supplies these necessary capital resources. Now suppose for the sake of simplicity that the expected profits from innovation across sectors differ only in the credit costs (for instance due to different interest rates charged on credit liabilities). In this situation, the sector linked with the lowest credit cost ceteris paribus yields the highest expected return on innovation. According to the directed technical change literature, expected profits from innovation determine to which sector firms direct their innovation effort. Thus profit maximizing firms ceteris paribus always choose the sector linked with the lowest cost of credit. This already suggests an influence of banks on the direction of technical change.

The literature on endogenous technical change cited above, however, does not consider this sketched role of financial intermediation in directing technical change. Therefore, the model presented next builds on the findings of these two different strands of research and extends existing approaches by introducing credit constraints and financial intermediaries into the model framework developed by Acemoglu (2002).

Accordingly the rest of this part is organized as follows: the next section presents the model. Here, we state the different decision problems, derive conditions for optimal behavior and market clearing. Afterwards, I define and describe the equilibrium, and present the key result, the determinants of technical change taking account of the bank sector (section 3). Part one ends with a critical conclusion (section 4).

2. The Model

We start with a brief overview of the different model components and their corresponding interactions in the different markets. Then we derive conditions of optimal behavior for banks, producers and consumers. Then we derive aggregate sectoral and macroeconomic variables and market clearing conditions.

2.1. The Economy

The economy evolves infinitely in discrete time t, $t \in \{0, 1, 2, ...\}$ and in each t, a continuum of overlapping generations populate the economy, where each generation's life span being divided into two periods: In their first period of life, people are young and in the second period, they are old. Assume that (i) at the end of each period, old individuals are replaced by new born young individuals, so that each member of the old generation has exactly one descendant in the subsequent period and ii) the population of the initial (period 0) old generation is identical in number to the young population of period 0. This formulation implies that the population is constant over time. So at each point in time, two different generations populate the economy. Also, the two generations of consumers are heterogeneous: Each generation consists of workers, entrepreneurs and innovators who differ in terms of their access to investment projects and consumption profiles.
The other entities in the economy are: a sector of financial intermediation and a "real" side of goods production. The production side exhibits a downstream structure: during the first stage, firms produce a variety of capital goods, the second stage contains two heterogeneous intermediate production sectors and during the last stage, firms produce one unique final good. The final good can be consumed, transferred into future periods and invested in capital goods production or in R&D. The final or consumption good serves as the numeraire: all prices, returns, payments and costs are measured in terms of the consumption good. Overall, the considered economy contains the following types of "agents":

- Banks,
- Firms (final goods, intermediate goods, capital goods), and
- Consumers (workers, entrepreneurs and innovators)

who act in three different market types

- Goods markets (Final output, intermediate goods, capital goods),
- Capital markets (Deposit market, credit market), and
- Factor markets.

Figure 2 shows the different model entities and their mutual market interactions schematically.

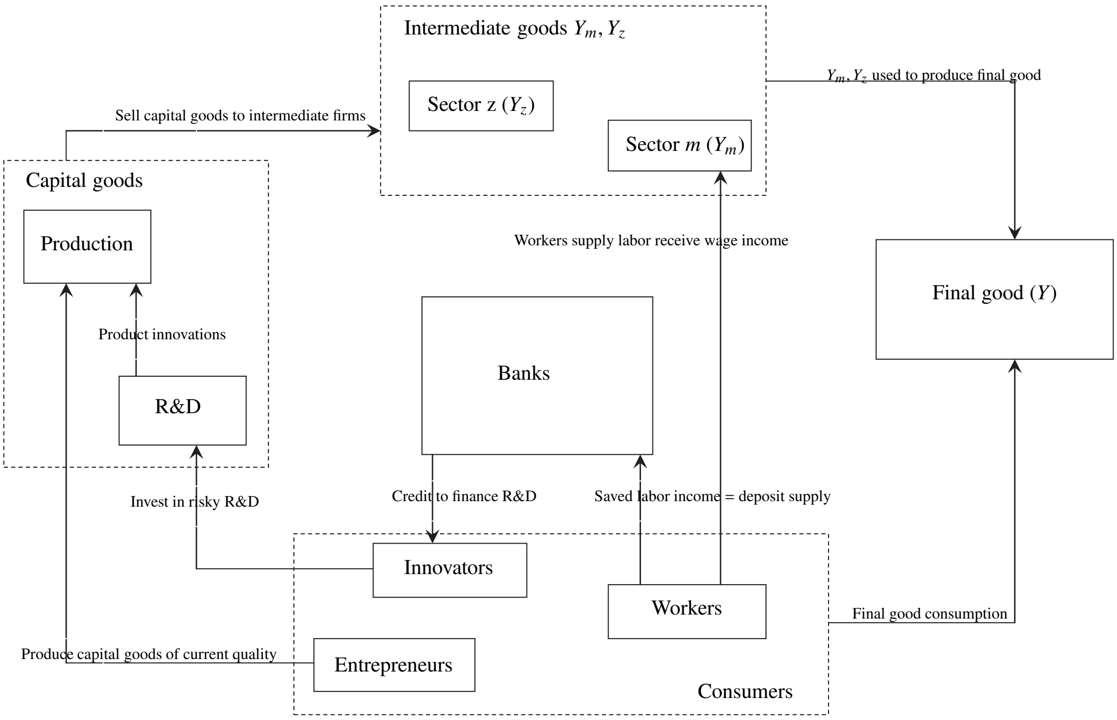


FIGURE 2: The structure of the model

Banks
The banking sector, located in the center of figure 2, interacts with two different agents on two different markets: First, they interact with the workers on the market for deposits: in each period, the banks demand the savings of the workers and repay these deposits plus interest back to the workers in the following period. Second, the banking sector supplies business credit to innovators on the credit market to finance R&D at a certain rate of credit interest. Market conduct in capital markets, i.e. the deposit market and the credit market, are perfectly competitive. Therefore the bank sector takes deposit interest rates and credit interest rates as exogenously given and chooses sectoral credit supply and deposit demand to maximize expected profits from financial intermediation.

Goods Production
Moving right from the center and proceeding counter-clockwise to the left, figure 2 shows the goods production side of the economy. In total, five types of goods are produced in three production stages: one final good, two intermediate goods and two continua of sector specific capital goods. The different production stages are connected in an upstream manner.
In the first stage, a large number of identical firms produce the unique final good, using the two different goods from the upstream intermediate stage. The inputs are instantaneously converted to output, so in this production stage, profit maximizing decisions are static.

The final output producers interact with four different agents in two markets: i) They supply their output to the consumers in the final goods market, ii) They represent the demand side in the intermediate goods market. The final goods market and the intermediate goods market are perfectly competitive. Final output producers choose production levels and input quantities to maximizes profits for given factor input prices and output prices.
The second downstream stage, the intermediate stage, consists of two different sectors, denoted by m and z. In either sector, a large number of identical, price taking firms produce a sector specific good and supply this output on the intermediate goods market. In this work, the subindex j To avoid confusion, note that the firms in the intermediate and the final output stage are completely different firms. Similar to final output production, intermediate firms convert inputs instantaneously into output. Hence no dynamic or intertemporal decisions occur in this intermediate stage. Production here needs a sector-specific production factor and a continuum variety of capital goods (or machines) that complement this factor. The factors used in sector m and z are denoted by the capital letters M and Z respectively. Let the index $J \in \{M, Z\}$ identify the two factors used in intermediate production. By construction, production in sector m is M-intensive and in sector z, Z-intensive.[15] Both factors are constant in supply. Beside the interaction with the final output producers, the firms in each intermediate sector interact with two other model agents in two markets: Firms in sector m use labor and a continuum variety of capital goods complementing labor, so they represent the demand side in i) the labor market and ii) in the "labor-complementing capital goods market". Firms in sector z use factor Z and a continuum variety of capital goods complementing this factor Z. Hence, these firms represent the demand side in i) the market for factor Z and ii) in the "Z-complementing capital goods market". All intermediate firms take the factor input prices as exogenously given. The model then derives intermediate good supply and capital goods demand endogenously from static profit maximization.
In the third production stage, entrepreneurs and innovators produce sector specific capital goods and supply these "machines" to the intermediate firms. Young innovators represent the demand side on the credit market: they are capital-constrained and thus need capital resources to finance R&D expenditures. The capital goods exhibit different levels of quality. Naturally, better-quality goods are more productive in manufacturing intermediate goods. So this rise in the quality represents technical progress in the two intermediate sectors. In contrast to the intermediate and final output production stages, decisions during this stage cover two periods of time and are thus intertemporal: in the first of two decision periods, innovators decide i) how much to spend on R&D and ii) the direction of R&D in order to potentially improve the quality of an existing capital good. The innovation process takes one period of time. Successful innovators sell the newly invented capital good monopolistically on the market for capital goods in the subsequent period. Entrepreneurs sell those capital goods where innovation was unsuccessful or even did not take place. Together they represent the capital goods supply side. Profits from capital goods selling are redistributed to the innovators and entrepreneurs.

[15] Note that factor M represents labor and Z is another factor of production that remains unspecified throughout the first part. However one can think of Z as representing energy for more concreteness.

Consumers
The consumer side in the model is represented by the workers, entrepreneurs, and innovators. Thereby, each consumer lives for two consecutive periods. Workers are endowed with one unit of labor time. Over their lifetime, workers interact with three other model entities on three different markets. In their first period of life – when young – they inelastically supply their unit of labor on the labor market to the intermediate firms of sector *m* and earn an income from wage payments. They deposit their income in a bank account, where deposits yield a certain rate of interest in the next period. Workers only consume in the second period of life and, therefore, wish to transfer their current wealth into the next period. For this purpose, they supply their labor income to the deposit market and receive a deposit rate on their savings.
Entrepreneurs receive patents to produce capital goods of current quality for the two intermediate sectors. When old, they consume the profits from capital goods selling.
Innovators invest in R&D to improve the quality of existing capital goods. If successful, they receive a patent to produce the new capital good of higher quality. R&D investment is financed by credit. Young innovators initially select either one of the two intermediate sectors to potentially invent a new capital good of greater quality for. The expected profits from innovation determine whether technical progress is directed to one or the other or both sectors, because innovators choose those sectors that promise the highest return. This search for sectoral profits is the engine that drives directed innovations. In their second period of life, old innovators then receive the profits from monopolistic capital goods supply. Since individuals have no bequests, they spend the profits entirely on final goods consumption.
The following table 2 summarizes all supply and demand interactions of the different model entities:

\ Market	Goods markets			Capital markets		Factor markets
Agent	Y	Y_J	X_J	H	D	J
Workers	Demand	-	-	-	Supply	Supply
Entrepreneurs	Demand	-	Supply	-	-	-
Innovators	Demand	-	Supply	Demand	-	-
Final output	Supply	Demand	-	-	-	-
Intermediates	-	Supply	Demand	-	-	Demand
Capital goods	Demand	-	Supply	-	-	-
Banks	-	-	-	Supply	Demand	-

Y = Final output market, Y_J = Intermediate output market, X_J = Capital goods market, H = Credit market, D = Deposit market, J = Factor markets.

TABLE 2: Supply-and-demand interactions in the different markets

Next, we state the different decision problems in detail and derive the first order optimality conditions. We start with the description of the banking sector. The study then presents the goods production side and finally states the consumers' decisions.

2.2. Banks

This study deals with "banks", because a bank represents the classic example of a "financial intermediary". Nevertheless, other institutions like credit unions, financial advisers or brokers, insurance companies or pension funds are also financial intermediaries who perform functions similar to banks in the context of the present study: here, a bank or financial intermediary a) brings together borrowers and lenders and b) performs certain functions that will be specified below. In order to highlight the role of banks with respect to the intensity and direction of firms R&D effort, all financial transactions in the economy are entirely intermediated, no "private" borrowing or lending is possible (Diamond (1984), Williamson (1986)).
This section provides answers to the following exemplary questions: What is the relationship between loan rates paid by innovators and deposit rates paid to workers? Do sectoral interest rates on business credit differ?

2.2.1. Preliminaries

Crucial to the general relevance of financial intermediation in macroeconomic models is some type of imperfect information. Otherwise, financial intermediation would be irrelevant to economic activity.[16] Here the relevant model entities have no unrestricted access to the available information when they make their decisions. More precisely, the economy features the following *ex-ante* information imperfection: Neither the banking sector nor individual innovators themselves know in advance, whether they are successful in the research lab. It is only public information that innovators are successful with a certain probability that depends on the intermediate sector, they decided to potentially innovate for, so the investment projects feature idiosyncratic risk. Since innovators have limited capital resources, they need credit to finance their projects. Consequently, this lack in information implies that the banking sector allocates some credit resources to innovators who will fail in the innovation process and thus default from credit liabilities.
According to this set up, borrowers (innovators) and lenders (the banking sector) have the same amount of information in advance. This is in contrast to the literature on borrowing and lending relationships in partial or general equilibrium models.[17] This symmetry in the ex-ante information deficit rules out problems of *adverse selection* that are typically associated with ex-ante informational asymmetries. A lender suffers adverse selection when he is not capable of distinguishing between projects associated with different credit risks when allocating credit.[18].
Furthermore, innovators who have been successful in the research lab cannot hide their

[16]According to the Miller-Modigliani theorem (Modigliani and Miller (1958)), economic decisions do not depend on financial structure in a setting of perfect capital markets. This implies that the addition of "banks" to this setup would have no consequence for real activity (see for instance Bernanke and Gertler (1985)). For a formal proof of banks' redundancy in a simple Arrow-Debreu competitive equilibrium with perfect capital markets the reader is referred to Freixas and Rochet (2008) p. 7-11.

[17]This literature is far too extensive to be listed here. An overview of the functions of financial intermediaries can be found in Bernanke and Gertler (1985), Levine (1997); Levine and Zervos (1998).

[18]An analysis of adverse selection problems in financial intermediation can be found for instance in Boyd and Prescott (1986), Leland and Pyle (1977)

success from the banks, claim unsuccessful R&D and then deviate from the credit liability incurred. Hence, the present study does not consider *moral hazard* problems either, typically associated with ex-post information asymmetries in the credit market. By moral hazard, the literature means the borrower's ability to put the funds to uses different from those agreed upon with the lender, who is hindered by his lack of information and control over the borrower.[19] Accordingly, in this study, there is no need for the banks to design credit contracts that satisfy participation constraints (the innovators must have an incentive to demand credit) and incentive constraints (the innovators must be willing to act in the best interests of the banks). What is more, innovators are not allowed to default on credit, i.e. take the borrowed capital and run. So there are no problems of commitment in this model framework.[20]

On the basis of this information set-up, the banking sector negotiates with innovators in the credit market and with workers in the deposit market. The banking sector supplies credit to the innovators in one period and receives credit plus loan interest rates in the subsequent period. Thus, interest rates compensate for credit default risk inherent in business projects: if innovators fail to develop a new capital good of higher quality, the capital is lost completely and innovators default on credit liabilities. Since the chance of successful innovation depends on the sector innovators decided to potentially innovate for, the likelihood of successful innovations depends on the direction of R&D. So the investment projects contain idiosyncratic and, even more importantly, sector-specific risks. From the banks' perspective this implies risk of default on credit to be sector-specific and thus banks penalize higher risk projects with a higher loan interest rates. Thus the model features sector-specific interest rates on business credit.

Members of the working population save their labor income when young to finance consumption when old. They take this income, enter the deposit market to store their savings in a bank account and then receive a certain deposit interest rate.

In principle, workers could also lend their savings directly to innovators. However, private savings are typically small compared to business investment projects and more importantly, different projects of innovators contain idiosyncratic risks. Thus, the return on any individual investment project and hence the income from savings would contain risks. Workers are not willing to bear these idiosyncratic risks. So in order to diversify these risks, they would have to lend their savings to a very large number of different firms. And before that, workers would have to evaluate each individual investment project by themselves, which is impossible considering their limited amount of time and capital. Therefore, banks who specialize in these activities emerge.

The banking sector collects the savings from workers. Then these savings are pooled and the capital resources are used to finance large scale business investments. Through

[19]For an analysis of asymmetric information and moral hazard see for instance Leland and Pyle (1977), Grossman and Stiglitz (1980).

[20]For an analysis of this topic see Krueger and Uhlig (2006): The authors analyze dynamic equilibrium risk sharing contracts between profit-maximizing intermediaries and a large pool of ex-ante identical agents facing idiosyncratic income uncertainty that makes them heterogeneous ex-post. After having obserevd their income, agents can walk away from the contract, while the intermediary cannot, i.e. there is one-sided commitment.

financing a large number of different projects with stochastically independent returns, banks fully diversify the idiosyncratic risk and the workers receive a deterministic rate of interest on their "portfolio" or deposits. To sum up, in this study the banking sector performs four important functions.[21] These are:

- pool a large number of small amount private savings,
- provide resources for business investment,
- diversify risk and
- evaluate prospective entrepreneurs (innovators) and finance the most promising ones.

2.2.2. Decisions

Suppose, a continuum of identical banks operate under perfect competition. Each bank collects and pools a large number of small-amount savings from young workers in any period t and refunds savings plus interest to workers in the following period $t+1$, where r_{t+1} denotes the interest rate paid on workers' deposits. Banks compete for workers' savings by simultaneously offering deposit contracts that promise a certain rate of interest. Since the banks are identical and competitive, they all offer the same deposit interest rate and so workers' return on deposits is identical across banks.[22] Therefore workers are indifferent between the different bank offers and they distribute their deposits across the individual banks.

The banks use these capital resources to finance business investment projects. Innovators borrow capital to invest in R&D. Due to sector-specific credit risk, each banks charges a sector-specific loan interest rate on credit liabilities, denoted by $R_{j,t}$, $j \in \{m, z\}$. No matter what individual bank is considered, all borrowers representing an identical risk of credit default are offered loan contracts with correspondingly identical loan interest rates to compensate for the inherent risk. Consequently, borrowers are indifferent between offered contracts and they also distribute equally across the individual banks. In the sequel, we describe the decisions of the continuum of individual banks in an aggregate, decision problem of a single, price taking representative bank.

The decision problem

This representative bank faces idiosyncratic uncertainty in credit supply: An innovator (the borrower) trying to improve the quality of a J-complementary capital good in industry ϑ_j, $j = \{m, z\}$, is successful in innovation with a sector-specific probability $\eta_j(h_{\vartheta_j})$, where h_{ϑ_j} represents R&D-effort measured in units of the final good.

[21] See also Boyd and Prescott (1986), King and Levine (1993), and Levine and Zervos (1998).

[22] Any deposit contract containing a deposit rate $\tilde{r}_t$ smaller than r_t cannot exist in equilibrium, because any bank that would offer such a contract would loose customers to competing banks that offer the higher deposit rate r_t. A bank that would offer a contract with a deposit rate $\tilde{r}_t$ higher than r_t would make negative profits. Hence, the banks all offer the same deposit interest rate r_t.

However, the returns of the individual innnovators' investment projects are stochastically independent and consequently, for a large number, in fact a continuum of different projects, these idiosyncratic risks are completely diversified. Therefore, the randomness in the returns vanishes in aggregate terms and the aggregate banks' decision problem becomes completely deterministic.[23]

Since $\eta_m \neq \eta_z$ holds true in general, expected return from financial intermediation depends on the total size of capital resources devoted to either sector m or z. Denote total credit supply in sector j at time t by $D_{j,t}$. Credit demand in sector j is given by R&D expenditures in sector j, denoted as $H_{j,t}$. The aggregate banking sectors' deposit demand is D_t. Aggregate deposit cost to the banking sector equal $r_{t+1}D_t$. Total deposit supply is given by aggregate savings S_t. We assume that total deposit demand in period t determines total credit supply in $t+1$, so

$$D_t = \sum_{j\in\{m,z\}} D_{j,t+1}. \tag{2.1}$$

With credit interest rates of $R_{j,t}$, $j = \{m,z\}$, the return of funding equals $\eta_m R_{m,t+1} D_{m,t} + \eta_z R_{z,t+1} D_{z,t}$. In a competitive environment, the bank sector takes the interest rates r_t, $R_{m,t}$ and $R_{z,t}$ as given. The banking sector solves the following optimization problem:[24]

$$\max_{(D_{m,t},D_{z,t})\in\mathbb{R}_+^2} \left\{ \sum_{j\in\{m,z\}} \eta_j R_{j,t+1} D_{j,t} - r_{t+1} \sum_{j\in\{m,z\}} D_{j,t} \right\} \tag{2.2}$$

A solution to (2.2) satisfies the following first order optimality conditions:

$$R_{m,t+1} = \frac{r_{t+1}}{\eta_m} \quad \text{and} \quad R_{z,t+1} = \frac{r_{t+1}}{\eta_z}. \tag{2.3}$$

These conditions state the relationship between sectoral loan interest rates $R_{j,t}$, $j \in \{m,z\}$ and the deposit interest rate r_{t+1} and shows that loan rates contain the inverse of the success-probabilities η_m^{-1}, η_z^{-1} as risk premiums: the greater the chance of success in innovation, the lower is the corresponding rate of interest on business credit. Moreover, one can derive the following relationship between sectoral credit interest rates:

$$\eta_m R_{m,t+1} = \eta_z R_{z,t+1}. \tag{2.4}$$

This simply gives the condition that in equilibrium, the expected loan interest rates in the two intermediate sectors have to be equal.

[23]In principle, the underlying law of large numbers argument holds only for averages of a countable sequence of random variables, where as the model here features a continnuum of random varibales. However, Uhlig (1996) showed how to obtain a law of large number for a continuum of uncorrelated random variables.

[24]This optimization problem is linear in the choice variables $D_{m,t}$ and $D_{z,t}$. This implies that the bank would supply infinite credit if the expected return is greater than the costs, since then each additional unit of credit would generate a higher profit. If the costs are greater than the expected return, credit supply would be zero. So in equilibrium the banks are indifferent, if the expected return from financial intermediation equals the costs of financial intermediation. Thus the bank makes zero profits in equilibrium.

Conditions (2.3) and (2.4) explain the relationship between loan rates paid by innovators and deposit rates paid to households and answers the question whether sectoral equilibrium loan rates differ from each other or not. The answers constitute the main results of this section and thus the following two propositions sum up these findings.

Proposition 2.1. *Whenever banks finance innovative activities, an interest rate spread between deposit rates paid to workers and loan rates paid by innovators exists:* $r_t < R_{j,t}$ *for* $j = \{m, z\}$*, i.e. interest rates on business credit exceed the interest rate on deposits. This interest rate spread remains in place even in a financial market characterized by perfect competition and is therefore not a result of monopoly distortion.*

Proof: see appendix A.
Moreover, the condition stated in equation (2.4) shows the relationship between sectoral interest rates charged on credit:

Proposition 2.2. *In all periods* $t \geq 0$*, the interest rates on private sector lending* $R_{m,t}$ *and* $R_{z,t}$ *used to finance R&D can vary between intermediate sectors m and z.*

Proof: See appendix A.
On the one hand, equation (2.4) is an equilibrium condition for the bank sector. The aggregate bank is indifferent between funding innovators in sector m or z as long as expected returns on sectoral funding are equal. In the case of $\eta_m > \eta_z$ the probability of successful innovation and therefore the probability of debt repayment in the next period is greater for innovators directing R&D to sector m than for those who direct R&D to sector z. Since the bank bears the risk of "losing money" a lower probability of success implies a larger risk of default on credit liabilities. Sectoral interest rates on loans reflect these different risks and include a risk premium. In the case considered here, the risk premium and thus the loan rate is higher for innovators who direct their innovation effort to sector z. Without different loan rates, banks would always prefer the lowest risk borrowers, in this case innovators' investments in sector m. In other words, a higher interest rate enables higher risk borrowers to compete for capital resources.
On the other hand, innovators are indifferent between capital goods invention in the two intermediate sectors as long as the expected returns on innovation effort are equal across m and z. In this case, innovators accept higher equilibrium interest rates paid on funds in one sector and still direct innovation effort towards that sector.
To compute market clearing deposit- and credit interest rates and optimal sectoral credit, we also need (deposit and credit) market clearing conditions and results from optimal producer and consumer behavior. Therefore, optimal sectoral credit quantities and market clearing credit- and deposit interest rates will be derived in section 2.6.

2.3. Firms

Consider now the economy's production side. Goods production occurs in three different stages that are linked in a "downstream" manner: in the final stage, firms combine two inputs from two intermediate sectors to produce the final output of the economy. During the second production stage, firms produce the sector specific intermediate good by combining two input factors: a sector-specific factor and a continuum variety of capital goods which complement this sector-specific factor. In both of these two stages, firms operate under perfect competition. In contrast to that, the undermost and first stage exhibits imperfect competition. Here, entrepreneurs and innovators produce sector-specific capital goods and engage in Bertrand competition. Innovators devote resources to invent capital goods of higher quality. If successful, they become the entrepreneur on that capital good line. I describe this competition in detail below. The innovation activity is the engine of economic growth. I derive answers to the following exemplary questions: Does private sector lending affect R&D investment and what determines the probability of successful innovation? To save on notation, I suppress time subscripts as long as no confusion arises.

2.3.1. Capital Goods

In the undermost stage, innovators engage in research and development in order to increase the quality of existing capital goods. If successful in the research lab, they invent a "new" capital good used in the intermediate production process. In capital good lines where innovators were unsuccessful or even were not present, entrepreneurs produce capital goods with lower quality using existing technologies and supply these goods to intermediate firms.

Innovations

Industrial R&D-investment and thus the process of innovation requires resource inputs and responds to profit incentives. On the one hand, firms may engage in research to significantly reduce the production costs of already invented commodities. This is defined as *process innovation*. On the other hand, firms may invest in R&D to invent an entirely new commodity, which is defined as *product innovation*. With respect to product innovation, one can further distinguish two different forms: if a newly invented commodity performs similar functions compared to those performed by already existing products, but offers greater *quality*, the newly invented good and the existent good are *vertically* related. If a newly invented good provides new functions and thus expands the *variety* in consumption or the production set, the relation is *horizontal*.

In this thesis, innovators perform R&D to improve the *quality* of capital goods, i.e. innovations are vertically related to existing capital goods. Hence, the developed model equates economic growth with the rise of average capital goods quality. I abstract from any expansion of variety. This can be done, since the general results of this study do not depend on any assumption as to whether the relation between existing and newly invented products is vertical or horizontal.

In order to ensure the possibility of permanent economic growth, every capital goods quality can be improved an unlimited number of times. There is a set of product lines or industries, each member of this set representing one line of capital good of probably infinite different qualities. Since the variety of capital goods does not expand, the set of industries or varieties is constant through time and I impose the following assumption with regard to the set of capital goods:

Assumption 2.1. *Let the index $j = \{m, z\}$ denote the two intermediate sectors and let Θ_j denote the set of industries or capital good lines in sector j. Then each $\vartheta_j \in \Theta_j$ corresponds to a different capital good line (machine type). The set of different industries*

Θ_j is fixed through time and hence, Θ_j can be normalized to 1, i.e. $\Theta_j := [0,1]$ *for* $j = \{m, z\}$.

Accordingly, in each sector j a continuum of capital goods exists, whose mass is normalized to unity. Thereby, each product line ϑ_j complements sector-specific production factor $J \in \{M, Z\}$. So machines of type ϑ_m (type ϑ_z) cannot be used in sector z (sector m) production. In every industry ϑ_j, the number of vertically differentiated varieties or qualities is given by $q(n_{\vartheta_j})$, whereby $n_{\vartheta_j} := \{0, 1, 2, ...\}$ denotes the n^{th} product generation in industry ϑ_j complementing sector $j = \{m, z\}$. *Different product lines complementing the same factor substitute each other imperfectly. Within any product line, goods of different quality substitute each other perfectly.* In order to avoid an overload of notation, I make the following:

Assumption 2.2. *Let $q(n_{\vartheta_j})$ be defined as above. Then the quality of each new generation of J-complementary capital goods is exactly λ_j times the quality of the preceding product generation. Hence,*

$$q(n_{\vartheta_j}) = \lambda_j q(n_{\vartheta_j} - 1) \quad \textit{for all } n_{\vartheta_j} = 0, 1, 2, ... \textit{ and } \vartheta_j \in \Theta_j, \lambda_j > 1.$$

To save on notation, we write q_{ϑ_j} instead of $q(n_{\vartheta_j})$ and simply mean the current quality level of capital good line ϑ_j.[25]
The "size" of the innovation on product lines ϑ_j differs with respect to the target sector $j = \{m, z\}$. Therefore λ_j is sector-specific and $\lambda_m \neq \lambda_z$ might hold. In this study, λ_j is exogenously fixed. Consequently, capital goods producers treat λ_j as a parameter and the amount of additional services provided by the newly invented product generation compared to the previous generation does not depend on the size of resources devoted to R&D.[26]
These different forms of a step-by-step rise in the quality of a commodity are commonly referred to as innovation on a "quality ladder": whenever a firm is successful in the research lab and develops a blue print of a higher quality product, it "jumps" one step up the quality ladder. Figure 3 plots schematic quality ladders for exogenously fixed and endogenous, R&D-dependent increases in product line quality q_{ϑ_j}. In the pictures, the horizontal axis plots different product lines ϑ_j. The vertical axis plots the corresponding

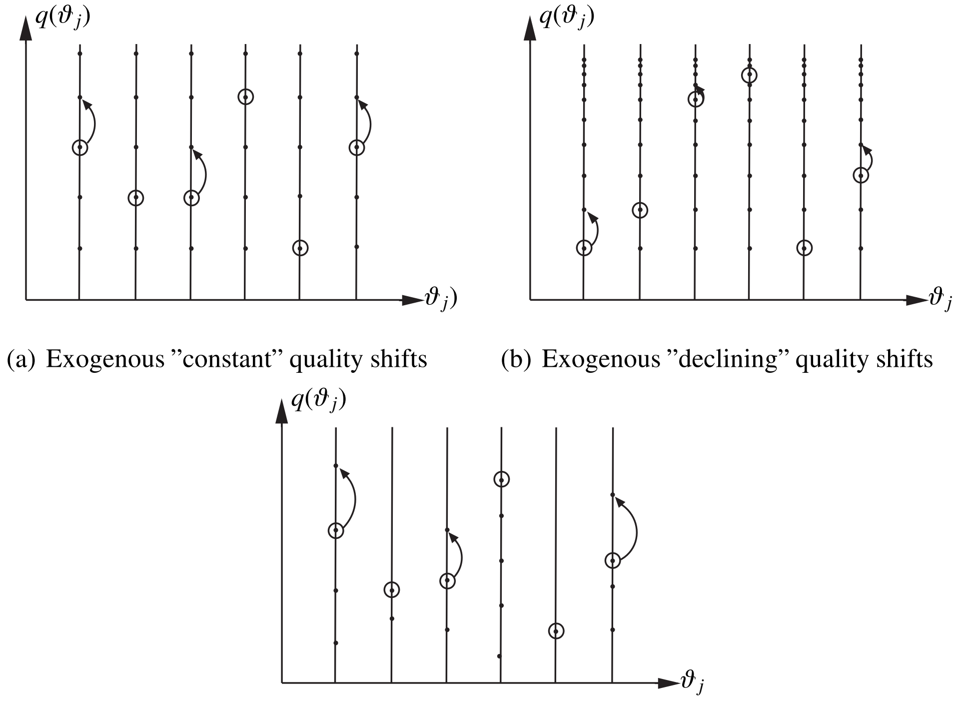
[25] For a constant rise in product quality λ_j, the number of previous innovations can always be calculated from the current level of product quality:

$$q(\vartheta_j) = \lambda_j^{n_{\vartheta_j}}.$$

Taking the logarithm leads to

$$n_{\vartheta_j} = \frac{log(q(\vartheta_j)}{log(\lambda_j)}.$$

[26] Alternatively, λ_j could decrease with every newly invented product generation. This formulation would take into account the increasing difficulty of finding new services with each invention of a new, more service providing commodity.Finally, λ_j could also vary endogenously. Then the size of the rise in quality from $q(n_{\vartheta_j} - 1)$ to $q(n_{\vartheta_j})$ would depend on R&D spending.



(a) Exogenous "constant" quality shifts (b) Exogenous "declining" quality shifts

(c) Endogenous "variable" quality shifts

FIGURE 3: Schematic representation of different quality ladders

quality q_{ϑ_j}. One vertical ray corresponds to one product line or industry and on each ray, the dots denote different product generations. The arrows denote successful product innovations by some research lab in that industry. In this case, the corresponding product quality jumps up one step on the quality ladder. In picture a), the dots are equidistantly distributed along each ray. Hence, a successful innovation increases the quality by the same amount, independent of the product position on the ladder and the amount of resources devoted to R&D. In picture b), the quality jumps are also exogenous. In contrast to picture a) the size of a step on the quality ladder is smaller, the higher the "leading-edge" product quality is. In this case, the rise in the amount of services decreases with the number of previous innovations. In picture c), the quality-jump-size is endogenous. Here, q_{ϑ_j} responds to R&D: the amount of additional services depends on the amount of resources devoted to R&D. Consequently, firms face a trade-off between the higher costs of more ambitious research and the extra profits that could be earned with greater product quality.
In this study, however, the rise in product quality is exogenously fixed and constant. First, this formulation keeps the model complexity as low as possible. Second and more importantly, the direction of innovation efforts is determined by the following two forces, which do not depend on assumptions as to whether capital goods quality changes exogenously or endogenously: i) the relative probability of successfully developing new capital goods, as given by η_z/η_m and ii) the relative size of the jump in capital goods quality in the two intermediate sectors m and z in the case of successful innovation, as given by λ_z/λ_m. In

contrast, it is not at all important whether the size of jumps in product quality declines with the number of previous innovations, or if the size of quality jumps responds to R&D expenditures.
Last but not least, there are different formulations of the necessary inputs in the innovation process. i) The R&D-process could demand labor as the only input, ii) R&D could be such that labor and some additional goods are required or iii) only one physical good is transformed into R&D output. Moreover the process could additionally demand some time units et cetera. Note that each formulation carries specific implications for the growth rate of the economy. In this study, innovators borrow a certain amount of the consumption good from banks to perform R&D. No other inputs are required. The R&D process involves only the final good being used in generating new innovations. According to Rivera-Batiz and Romer (1991), this specification is referred to as the 'lab equipment' specification.

Entrepreneurs and Innovators
Each intermediate sector uses a continuum of different capital goods in the production process. Each capital good is supplied monopolistically either by an entrepreneur or by an innovator. Entrepreneurs hold patents to produce capital goods of existing quality. Innovators try to invent new generations of product with higher quality in some of the capital good lines. If innovation is successful, the innovators receive a patent on that innovation and they supply the newly invented good of higher quality to the intermediate firms. The entrepreneur on that product line makes zero profit. If innovation is not successful, the entrepreneur exclusively sells the capital good with existent quality level to the intermediate sector. Key feature here is, that innovators decide to which sector they direct their R&D effort on the basis of expected profits. This is the engine of endogenous directed technical change.
The competition between entrepreneurs and innovators in detail can be described as follows. In capital good lines where innovators are present, entrepreneurs and innovators operate in a market with Bertrand competition: no matter what sector $j \in \{m, z\}$ or capital good line ϑ_j, marginal and average production costs are identical and entrepreneurs and innovators compete by setting prices simultaneously. Thereby, each entrepreneur initially holds a patent on the blueprint to produce the current highest quality capital good in capital good line ϑ_j. Innovators can observe the product characteristics of this "leading-edge" capital good and engage in R&D to invent new capital goods of higher quality. Although competitors set prices equal to marginal costs and so product prices are identical in principle. However, if an innovator develops a new capital good of higher quality, this supplied good offers greater product quality compared to previous generations of this specific product. Here, I consider an institutional setup, where inventors of new "leading-edge" capital goods receive a one-period patent for the production and sale of that good. Abstracting from any search- or transaction costs, intermediate sectors want to buy capital goods of the highest quality standard, given identical prices for all generations of that product. Consequently, the firm offering the highest quality or equivalently the firm demanding the lowest quality-adjusted price within any one industry ϑ_j gains the complete market demand and the "leading-edge" capital good in any industry is again monopolistically

supplied. Therefore, positive monopolistic profits are realized by successful innovators and by those entrepreneurs who operate on a capital good line, where innovation was not successful or did not take place at all. Together this implies that competition between innovators and entrepreneurs takes place on an individual level.
With respect to the size of the quality jump after innovation λ_j, I impose the following assumption:

Assumption 2.3. *Independent of the sectors $j = \{m, z\}$ innovators direct their innovation effort to, the size of the innovation is* drastic *if λ_j is sufficiently high to guarantee the innovator the unrestricted monopoly profit. This holds true if the price set after innovation is less than or equal to the marginal cost of production:*

$$p_{\vartheta_j} \leq \psi \quad \textit{for all } n_{\vartheta_j} = 1, 2, ..., \quad \vartheta_j \in \Theta_j, \quad j \in \{m, z\}.$$

The innovations are drastic in the sense that entrepreneurs with the "old" production technology can not compete with an innovator that has the "new" technology when the innovator chooses a monopoly price. In this regard, the new capital good of higher quality makes the old capital good obsolete. One could also assume innovations to be non-drastic in the sense that $p_{n_{\vartheta_j}} > \psi$. Then the innovator would practice *limit pricing* in equilibrium by setting the price equal to the rivals marginal costs.[27] The results here do not depend on whether innovations are drastic or non-drastic. More importantly a successful innovation leads to (temporary) monopoly power, which ensures positive profits from innovation and thus encourages research activities in the first place. One can show that this contrasts to firms' research activities if they operated in a perfectly competitive market. In fact, the next proposition shows that

Proposition 2.3. *In a capital goods market characterized by perfect competition, producers have no incentive to devote costly innovation effort to any target sector $J = \{M, Z\}$.*

Proof see appendix A.
So new born innovators choose an intermediate sector m or z to direct R&D to and -if the innovation is successful- to sell capital goods to. Note that they are indifferent between these target sectors as long as expected profits in both sectors are equal. After a target sector has been chosen, young innovators need credit to finance R&D investment, since this venture is costly and they lack the necessary capital resources. Thus financial intermediaries lend the capital to them at a certain rate of interest.[28]

[27] A detailed analysis of this topic can be found in Grossman and Helpman (1993), Mas-Colell et al. (1995).

[28] This lack of resources is in contrast to standard models of endogenous growth (Rivera-Batiz and Romer (1991), Romer (1990)) and growth models of factor biased technical change (Acemoglu (2002)), where each potential entrepreneur takes ψ from the jar of final goods. The assumption of capital-constrained innovators has been used in different strands of research. For an analysis regarding endogenous growth and finance see for instance King and Levine (1993). Moreover, Williamson (1986) assume external project finance to analyze optimal debt contracts under asymmetric information. As in Williamson (1986, 1987), Bernanke and Gertler (1989), the formulation presented here implies a fixed project size. In other words the decision, which sector the individual chooses to produce machines for, does not depend on the project size and the size of a project remains constant over time. A variable loan size is considered for instance in Gale and Hellwig (1985).

Then, innovators in sector j distribute across the different capital good lines ϑ_j. Each innovator observes the characteristics of her current state of the art product with quality $q_{\vartheta_j} := q_{n_{\vartheta_j}}$ and spends h units of the final good for R&D to improve the quality of that commodity to $q'_{\vartheta_j} = \lambda_j q_{\vartheta_j}$. If successful, the development of a leading edge commodity allows an innovator to sell the commodity exclusively in the following period. At the end of that period, the corresponding patent protection ends. Independent of the intermediate sector $j = \{m, z\}$ a capital good is produced for, the unit costs are given ψ. Without loss of generality, I normalize $\psi \equiv 1$.
Each individual innovator needs credit equal to finance R&D expenditures so she borrows h units of the final good in period t and returns this loan plus the interest charged on that credit back to the bank in period $t+1$. To indicate the dependence of R&D expenditures on credit interest rates and also on current quality levels of the corresponding good that is planned to be improved, we denote period t R&D expenditures as $h_{\vartheta_{j,t}}(R_{j,t+1})$. Accordingly, credit contracts between innovators and banks are signed on an individual level.
Innovators and entrepreneurs face a two-stage decision process within two consecutive periods of time. Innovators' first decision period is the research and planning period. In this stage, they decide on R&D expenditures. Young innovators engage in costly R&D if the net present value of expected future profits is at least as large as the net present value of total expenditures. If this is the case, then they ask banks to finance this venture. Young entrepreneurs do not undertake any actions in the first decision period. Old innovators then set the profit maximizing price of their capital good, given that they were successful in the first period. Old entrepreneurs set the price in the second period given that innovation on their product line was not successful or even did not take place at all. In the following, we refer to innovators and entrepreneurs who supply a capital good as "capital good producers".
So in the second stage, capital goods producers determine the optimal price at which they are going to sell the capital goods to the intermediate sectors. Given intermediate demand for capital goods, this price determines the profit flow in the second period and thus determines the net present value of profit in the first stage.
Consider an arbitrary period t. This period t is referred to as the "first" decision period and the subsequent period $t+1$ is the "second" decision period. The decision problem is solved recursively. First we establish the optimal price for a capital goods producer to set in the second decision period, $t+1$, given that either the person succeeded in the research lab in the previous period t if is an innovator or, if he is an entrepreneur, given that no innovator tried to improve the quality of his capital good or tried but failed. Afterwards, we derive individual and sectoral R&D expenditures and compute the probability of successful innovation.

Profit maximization
Ignoring time subscripts for the moment, the demand for the capital good ϑ_j in the intermediate sector j is given by

$$x_j(p_{\vartheta_j}) = \left(\frac{\mu p_j}{p_{\vartheta_j}}\right)^{\frac{1}{1-\mu}} q_{\vartheta_j} J, \qquad j \in \{m, z\}, \tag{2.5}$$

where p_{ϑ_j} denotes the price of the capital good, which is the choice variable of the capital goods producer and will be determined next. q_{ϑ_j} is the quality level of capital good ϑ_j, p_j denotes the price of the intermediate good in sector j and $J = \{M, Z\}$ represents the sector-specific factor in sector $j \in \{m, z\}$, all are given to the monopolistic producer. Equation (2.5) above is determined at the intermediate stage and corresponds to equation (2.21b) in the next section. For the moment it is enough to note that the demand $x(p_{\vartheta_j})$ is decreasing in its price p_{ϑ_j}.
Capital goods producers take the demand above as given and maximize profits. The second period decision problem of an innovator in period $t+1$ then reads:

$$\max_{(p_{\vartheta_j,t+1})\in\mathbb{R}_+} \left\{ \left(p_{\vartheta_j,t+1} - 1\right) x_{j,t+1}(p_{\vartheta_j}) - R_{j,t+1} h_{\vartheta_{j,t}}(R_{j,t+1}) \;\middle|\; x_{j,t+1}(p_{\vartheta_{j,t+1}}) = (2.5) \right\} \tag{2.6}$$

and the decision of an entrepreneur is given by:

$$\max_{(p_{\vartheta_j,t+1})\in\mathbb{R}_+} \left\{ \left(p_{\vartheta_j,t+1} - 1\right) x_{j,t+1}(p_{\vartheta_j}) \;\middle|\; x_{j,t+1}(p_{\vartheta_{j,t+1}}) = (2.5) \right\} \tag{2.7}$$

These optimization problems are identical in the second decision stage and differ only by the term $R_{j,t+1} h_{\vartheta_{j,t}}$ representing the cost of R&D. As the time index indicates, this term is determined in the first decision stage and thus plays no role in the determination of the profit maximizing price. The first order optimality condition with respect to $p_{\vartheta_{j,t+1}}$ gives the profit maximizing monopoly price of a capital good ϑ_j:

$$p_{\vartheta_j,t+1} = \frac{1}{\mu}, \tag{2.8}$$

which is a constant markup over marginal cost and equal across industries ϑ_j.[29][30]
The resulting flow of monopoly profit for an innovator then can be computed as

$$\pi^{(i)}_{\vartheta_j,t+1} = \bar{\pi}_{j,t+1} q_{\vartheta_{j,t+1}} - R_{j,t+1} h_{\vartheta_{j,t}}(R_{j,t+1}) \tag{2.9}$$

and the monopoly profit for an entrepreneur e reads

$$\pi^{(e)}_{\vartheta_j,t+1} = \bar{\pi}_{j,t+1} q_{\vartheta_j,t+1}, \tag{2.10}$$

where $\bar{\pi}_{j,t+1} := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{j,t+1}^{\frac{1}{1-\mu}} J_{t+1}$. Since the supply of factor M (no population growth) and Z is constant, the influence of factors $J = \{M, Z\}$ on profits is constant.

[29] Nevertheless, I keep ϑ_j as "arguments" in the expression in order to distinguish the price of a capital good from the price of the intermediate good $p_{j,t+1}$ and also to indicate that the price corresponds to a product line.

[30] The monopoly price would depend on the loan rate R_j if capital goods producers would also lack the resources to finance production inputs. In this case, marginal production costs would equal to the loan rate. In this regard, capital goods producers would face a dynamic decision problem: banks provide the capital resources to finance production inputs in the "first" period. Firms can pay back credit only after selling the capital good in the "second" period and thus have to pay the interest rate on credit. Hence, the monopoly price for a capital good would contain the interest rate R_j, since each additional unit of production must be financed by credit.

R&D and the probability of successful innovation
In the first decision stage, innovators determine the amount of their R&D expenditure and so the amount of capital they demand as credit from the banks. Innovators who are successful in the research lab, receive a patent to produce the newly invented good. However, an innovator's patent protects the invented good for one period only. Therefore, the net present value of future profits from innovation in industry ϑ_j on a J-complementary capital good is

$$V_{\vartheta_j,t} = \frac{\mathbb{E}_t[\pi^{(i)}_{\vartheta_j,t+1}]}{r_t} \tag{2.11}$$

where $\mathbb{E}_t$ denotes the expectation of future outcomes in period t, $\pi^{(i)}_{\vartheta_j,t+1}$ is given in equation (2.9) and r_t is the interest rate which is potentially time varying.
In contrast to the second decision stage described previously, individual innovators and individual entrepreneurs face idiosyncratic risks during the first decision stage: From the innovators' perspective the return is random, since R&D effort is successful only with a certain probability. From the entrepreneurs view, there are two possible sources of uncertainty in the return: first, since the unit mass of entrepreneurs/capital goods in each sector face a unit mass of innovators in total, there exist capital good lines which remain without an innovator. On those capital good lines, entrepreneurs produces the capital good with current quality using the existing technology. Second, if an innovator is present in a capital good line, the innovator probably fails to innovate. Then entrepreneurs also produce the existent capital good. This setup is such that competition between entrepreneurs and innovators takes place on an individual level and thus each person faces an idiosyncratic risk of return.
However, to get deterministic income profiles for all individuals nevertheless, I assume that individuals form "alliances" to protect themselves against these idiosyncratic risks: Innovators within each intermediate sector j form "R&D syndicates", entrepreneurs in both sectors together form an "entrepreneur association".
Since realizations of individual innovation attempts are independent and thus uncorrelated, a fraction of the projects will be successful while the rest of the projects will be unsuccessful. Then one could in principle conclude, applying a law of large numbers type of reasoning that the fraction of successful projects in sector $j \in \{m, z\}$ is equal to the ex-ante probability of successful innovation η_j, leading to aggregate average innovator profits of $\eta_j\Pi_j$, while the fraction of unsuccessful projects is equal to the ex-ante probability of innovation failure $1 - \eta_j$.
Moreover, using the same type of reasoning, the ex-ante probability of an innovator trying to improve the quality of capital good in sector j is equal to the frequency or mass of innovators in sector m, denoted by $\iota_m := \iota$ and sector z, denoted by $\iota_z := 1 - \iota$. So entrepreneurs expected return is equal to $(1 - \iota\eta_m)\Pi^e_m + (1 - (1 - \iota)\eta_z)\Pi^e_z$. One has to be careful here, however, because the typical laws of large numbers apply to the average of a countable sequence of random variables, whereas the model here features a continuum of capital goods/entrepreneurs and innovators (each normalized to unity) and so the η_j's describe the "average" of a continuum of random variables. Based on the findings of Uhlig (1996), who shows how to obtain a law of large numbers for a continuum of uncorrelated

random variables we can conclude that the η_j's and the ι_j's are well defined in the present context (see also Acemoglu (2009)). In this way, an alliance allows individuals to trade the high return with idiosyncratic risk for a deterministic but lower return, i.e. an average return (per sector).
Now, remember that quality steps are fixed and equidistantly distributed along the quality ladder for any capital goods line ϑ_j. Consequently, increasing R&D expenditures do not increase the size of the jump in capital goods quality. Instead, the size of this expenditure determines the probability of successfully developing a capital good of larger quality. Naturally, larger R&D outlays make successful innovation more likely. More precisely, increasing R&D expenditures could increase the probability of success, but with a diminishing marginal effect. One could also assume the marginal effect on this probability to increase with larger R&D outlays. This relationship also could be proportional. Diminishing returns to R&D seem plausible and most of semi-endogenous growth as well as, to some extent, Schumpeterian models are based on this assumption as well. In contrast to this frequent assumption, Madsen (2007), for instance, analyzed the returns to R&D in OECD countries and concluded that the null hypothesis of constant returns to R&D cannot be rejected. In this study, the effect of increasing R&D expenditures to the probability of success diminishes by assumption.[31]
Additionally, the number of previous inventions n could influence the probability of successful innovation for a given amount of R&D spending. On the one hand, a higher n could make R&D more difficult. For a given level of research expenditures, this implies a declining probability of success with an increasing number of previous innovations n. On the other hand, fixed steps of quality improvement could become easier due to learning effects making R&D effort less expensive.
The next assumption specifies the relationship between R&D expenditures, product generation n_{ϑ_j} and the probability of successful innovation:

Assumption 2.4. *The individual probability of successful innovation η_{ϑ_j}, $j = \{m, z\}$ is a strictly concave function of R&D expenditures and is defined as*

$$\eta_{\vartheta_j} := h^{\alpha}_{\vartheta_j}\phi_{\vartheta_j}, \qquad j = \{m, z\}, \tag{2.12}$$

where $\alpha \in]0, 1[$ and

$$\phi_{\vartheta_j} := \frac{1}{\zeta} q^{-\alpha}_{\vartheta_j} \tag{2.13}$$

captures the effects of the capital good's current quality ladder position.[32]

[31]The equilibrium analysis becomes much more complicated in case of a diminishing marginal effect of rising R&D and success probabilities, because the relationships between the model variables become nonlinear. Although the general relationship between banks and the direction of technical change does not depend on the specific modeling assumptions of how the innovation process relates to R&D expenditures, I chose the simplest model framework consistent with economic intuition.

[32]Since the mapping from R&D to success-probability is unbounded in general, the defined η's are not probabilities in a strict sense, i.e. they do not necessarily take values between zero and one. We account for this and set the parameters appropriately, so that the values of success-probabilities lie between zero and one.

With this formulation, innovators treat η_{ϑ_j} as a choice variable.[33] In the first decision period, an innovator takes the interest rates r_t, $R_{j,t+1}$, the price of the intermediate good $p_{j,t}$, the profit maximizing price determined in the second decision stage $p_{\vartheta_{j,t+1}}$, quality level $q_{\vartheta_j,t+1}$ factors J_{t+1} and the function z_{ϑ_j} as given. An innovator in capital goods line ϑ_j chooses R&D expenditures $h_{\vartheta_j,t}$ to solve the following optimization problem

$$\max_{h_{\vartheta_j,t}\in\mathbb{R}_+}\left\{\frac{\eta_{\vartheta_j}}{r_t}\Big(\bar{\pi}_{j,t+1}q_{\vartheta_j,t+1}-R_{j,t+1}h_{\vartheta_j,t}\Big)\ \mid\ \eta_j=h_j^{\alpha}z_{\vartheta_j}\right\} \tag{2.14}$$

A solution to (2.14) satisfies the following first order conditions

$$\alpha\left(\bar{\pi}_{j,t+1}q_{\vartheta_j,t+1}-R_{j,t+1}h_{\vartheta_j,t}\right)=R_{j,t+1}h_{\vartheta_j}. \tag{2.15}$$

Solve for $h_{\vartheta_j,t}$ gives

$$h_{\vartheta_j,t}=\frac{\alpha}{1+\alpha}\frac{\bar{\pi}_{j,t+1}}{R_{j,t+1}}q_{\vartheta_j,t+1}. \tag{2.16}$$

This expression implies that innovators direct a constant fraction of (expected) profits to R&D such that the marginal benefit from one additional unit of R&D expenditure is equal to the marginal additional individual expected profit from capital goods selling. Thereby individual research depends positively on the quality level q_{ϑ_j}, on the return from capital goods selling $\bar{\pi}_{j,t+1}$ and on the coefficient α. An increase in α leads to a higher marginal return of research, since the higher α, the larger is the effect of one additional unit of R&D on the probability of innovation success. An increase in any of the other two variables q_{ϑ_j} and $\bar{\pi}_j$ results in higher (expected) profits from capital goods production. Hence, the flow of resources into innovation effort increases. For the purpose of this thesis, the most important result implied by equation (2.16) is a negative influence of the loan interest rate $R_{j,t}$ on R&D outlays. This suggests an influence of banks on R&D investment.
Using optimal R&D expenditures given in (2.16), expected profits for an innovator on capital good line ϑ_j then read

$$\pi^{(i)}_{j,t+1}=\frac{1}{1+\alpha}\eta_j\bar{\pi}_{j,t+1}q_{\vartheta_j,t+1}. \tag{2.17}$$

To derive the sectoral probabilities of innovation success use definition 2.4, insert individual R&D-expenditures and the expression for ϕ_{ϑ_j} gives

$$\eta_{\vartheta_j}=\eta_j=\zeta^{-1}\left(\frac{\alpha}{1+\alpha}\frac{\bar{\pi}_{j,t+1}}{R_{j,t+1}}\right)^{\alpha} \tag{2.18}$$

Thereby, the function $\phi_{\vartheta_j}:=\frac{1}{\zeta}q^{-\alpha}_{\vartheta_{j,t+1}}$ captures the effects of the current position on the quality ladder. The present study assumes that successful innovation becomes more difficult the more product generations have been previously invented. The function ϕ now states that this difficulty increases in proportion to the additional output that would be produced

[33] For a different formulation, where aggregate R&D expenditures determine the probability of success and individuals take the probability as given, see for instance Barro and Sala-i Martin (2004).

in case of successfully increasing the product quality from $q_{\vartheta_{j,t}}$ to $\lambda_j q_{\vartheta_{j,t}}$ between t and $t+1$. The parameter ζ represents a cost of research measured in units of the final consumption good.
Equation (2.18) shows the determinants of the probability of successful innovation and constitutes the answer to the second question formulated at the beginning of section 4.3: First, note that the probability of success is specific with respect to target sector j. Moreover, in this formulation, it is independent of the quality level and hence equal for all innovators across industries ϑ_j within the same intermediate sector j. On the one hand, this probability increases with greater returns $\bar{\pi}_j$. Higher expected returns imply larger aggregate R&D outlays. Since R&D expenditure relates proportionally to η_j by definition, the probability of successful innovation increases.
On the other hand, the influence of research costs ζ on the probability of success is negative: Ceteris paribus, the larger the cost of doing research, the lower is the probability of success and thus the expected profits from innovation per unit of R&D spending.
The probability of research success also decreases with the loan rate R_j. However, the expression for η_j in eq. (2.18) so far is incomplete, because R_j itself depends on η_j, and the term $\bar{\pi}_{j,t+1}$ contains the endogenous price p_j for the intermediate good Y_j. I postpone this derivation of the explicit formula for η_j until section five and instead highlight the result implied by (2.18):

Remark 2.1. *Banks influence the sectoral probabilities of successful innovation through the loan rate R_j.*

2.3.2. Intermediates
The intermediate stage consists of the two different sectors, denoted by $j \in \{m, z\}$ and the sector-specific factors are identified by the index $J = \{M, Z\}$. In both sectors, a large number of identical firms produce the sector specific intermediate good under perfect competition. The firms in this intermediate stage make no inter-temporal decisions. Denote the intermediate output of sector j by Y_j. Production in sector j combines two input factors: sector specific factors $J = \{M, Z\}$, constant in supply and a continuum variety of different capital goods (machines) x_{ϑ_j} of different quality q_{ϑ_j}, where $q_{\vartheta_j} := q(n_{\vartheta_j})$ denotes the quality of the n^{th} generation of machine ϑ_j in sector j (at time t) and x_{ϑ_j} denotes the quantity input of capital good ϑ_j in sector j (at time t). Capital goods depreciate fully after use.[34]
The good Y_m is M-intensive, where M represents labor and the good Y_z is Z-intensive.[35] In the following, I will be unspecific about what Z represents, but for more concreteness one can think of Z to represent some form of energy input. Since within either intermediate sector, firms are identical, aggregate sectoral production equals the sum of the functions of individual firms and individual profit-maximizing decisions can be represented by two single representative aggregate decision problems (one per sector). The production function is defined as:

[34]According to Acemoglu (2002), slow depreciation of capital goods has no effects on the balanced growth path equilibrium. It affects only the speed of the transitional dynamics.

[35]I refer to a good Y_j to be J-intensive, whenever its production uses factor J and capital goods complement this factor J where here $J = \{M, Z\}$.

Assumption 2.5. *In any period t, intermediate sector $j = \{m, z\}$ produces an intermediate good Y_j using the technology*

$$Y_j = \mathcal{G}(x_{\vartheta_j}, J) = \Big(\int_0^1 q_{\vartheta_j}^{1-\mu} x_{\vartheta_j}^{\mu} d\vartheta_j\Big) J^{1-\mu}. \tag{2.19}$$

Production function $\mathcal{G}$ is linear homogeneous, quasi-concave and twice continuously differentiable. In each intermediate sector, all capital goods are imperfect substitutes. That is, each line of capital good ϑ_j is a necessary input in the intermediate production process. The production function implicitly assumes that at any point in time t, only one quality of any capital good is used. This assumption does not cause any loss of generality, because within any one industry ϑ_j, goods of different qualities are perfect substitutes for each other. Naturally, higher quality capital goods are more productive in manufacturing intermediate goods. However, perfect competition among firms in intermediate sectors implies that the price equals minimum marginal production costs. Therefore, only those capital goods ϑ_j which go for the lowest quality-adjusted price are used in equilibrium intermediate production. This implies the exclusive use of the "leading-edge" (highest quality) capital good of each industry ϑ_j.
Decisions of intermediate stage firms are static, so we suppress time subscripts in the following derivations. The firms in the two sectors m and z maximize profits taking the price of their product, p_m, p_z, the rental prices of the machines, denoted by $p_j(m_{\vartheta_j})$, $j = \{m, z\}$, as well as the quality of machines, q_{ϑ_j} and the input prices w_M, w_Z of the factors M and Z as given. The decision problem of a representative firm in sector $j = \{m, z\}$ reads

$$\max_{(x_{\vartheta_j}, J) \in \mathbb{R}_+^2} \quad \Big\{ p_j Y_j - w_J J - \int_0^1 p_{\vartheta_j} x_{\vartheta_j} d\vartheta_j \ \mid\ \mathcal{G}(x_{\vartheta_j}, J) = (2.19) \Big\} \tag{2.20}$$

A solution to (2.20) satisfies the following first order optimality conditions which equate prices and marginal products of each production factor for all $t \geq 0$

$$(1-\mu) p_{j,t} J_t^{-\mu} \Big(\int_0^1 q_{\vartheta_j,t}^{1-\mu} x_{\vartheta_j,t}^{\mu} d\vartheta_j\Big) \ = \ w_{J,t}, \tag{2.21a}$$

$$\mu p_{j,t} J^{1-\mu} \Big(q_{\vartheta_j,t}^{1-\mu} x_{\vartheta_j,t}^{\mu-1}\Big) \ = \ p_{\vartheta_j,t}. \tag{2.21b}$$

Demand for capital goods of type ϑ_j of quality q_{ϑ_j} in sector $j = \{m, z\}$ can be derived from (2.21b) as

$$x_{\vartheta_m,t} = \Big(\frac{\mu p_{m,t}}{p_{\vartheta_m,t}}\Big)^{\frac{1}{1-\mu}} q_{\vartheta_m,t} M_t. \tag{2.22a}$$

$$x_{\vartheta_z,t} = \Big(\frac{\mu p_{z,t}}{p_{\vartheta_z,t}}\Big)^{\frac{1}{1-\mu}} q_{\vartheta_z,t} Z_t. \tag{2.22b}$$

These equations imply that the quantity of used capital goods is increasing in the corresponding price of the intermediate good, p_m, p_z and in the firms' employment of the sector-specific factor $J = \{M, Z\}$ The capital goods use in sector j is decreasing in the price of the capital good p_{ϑ_j}. Intuitively, a greater price for the product increases the value of the marginal product of all factors, including that of capital goods. This encourages intermediate firms to purchase more capital goods. A greater level of factor employment J implies more use of the capital goods, hence capital goods demand raises. Moreover, because the demand curve for the capital goods is downward sloping, a higher cost implies lower demand. Finally, note that the conditional demand functions for the machines are linear in the two sector-specific factors M and Z and in the level of capital goods quality quality. This feature simplifies the equilibrium analysis in section five.

2.3.3. Final Output

In the final output stage, a large number of profit-maximizing firms compete to produce a consumption good. Similar to the intermediate sector, all the firms in the final output sector are identical. Then, the aggregate or total production function is just the sum of the functions of the individual firms and the individual profit maximizing decisions can be converted into one aggregate decision problem of a single representative firm. Production technology is given in the next assumption:

Assumption 2.6. *The final good is produced using the production technology* $\mathcal{F} : \mathbb{R}_+^2 \to \mathbb{R}_+$

$$Y = \mathcal{F}(Y_m, Y_z) = \left(\gamma Y_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_z^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.23}$$

where $\varepsilon \in [0, \infty[$ *represents the constant elasticity of substitution between the inputs used in the production process,* $\gamma \in]0, 1[$ *is a distribution parameter and* Y_m, Y_z *represent intermediate input quantities.*

Final output is produced using a standard CES-function and $\mathcal{F}$ maps nonnegative inputs to nonnegative levels of output. The property of linear homogeneity states that $\mathcal{F}(\lambda Y_m, \lambda Y_z) = \lambda \mathcal{F}(Y_m, Y_z)$ and thus $\mathcal{F}$ features constant returns to scale in Y_m, Y_z. Furthermore, linear homogeneity in conjunction with quasi-concavity implies $\mathcal{F}$ to be concave.[36] If $\varepsilon = \infty$, Y_m and Y_z are perfect substitutes. If $\varepsilon = 1$, the production function will be Cobb-Douglas and if $\varepsilon = 0$, aggregate production will be Leontieff. If $\varepsilon > 1$, then the two factors are referred to as *gross substitutes*. If $\varepsilon < 1$, I refer to Y_m and Y_z as *gross complements*. The single representative firm in final output production takes the distribution parameter $\gamma > 0$, prices of the final good p, and of intermediate inputs p_m, p_z, and the production technology stated in (2.23) as given and solves the following optimization problem:

$$\max_{(Y_m, Y_z) \in \mathbb{R}_+^2} \left\{ p\mathcal{F}(Y_m, Y_z) - p_m Y_m - p_z Y_z \quad | \quad \mathcal{F}(Y_M, Y_Z) = (2.23) \right\} \tag{2.24}$$

[36]In general, aggregate production functions in models with technological progress contain a term that measures technological advances, i.e. $Y \approx \mathcal{F}(Y_m, Y_z, A)$, with an argument A representing technology levels. In order to keep the model as simple as possible, I abstract from any form of technical progress in this stage of production. Hence, the technological relationship between necessary inputs and output does not change through time. However, this is not equivalent to an overall constant relationship between input factor shares. If the inputs are substitutes the shares of these factors might change in reaction to a change in relative prices, but this is solely due to substitutions, not technological advances.

A solution to (2.24) satisfies the following first order conditions which equate prices and marginal products of each production factor for all $t \geq 0$:

$$p_{m,t} = \gamma\Big(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\Big)^{\frac{1}{\varepsilon-1}} Y_{m,t}^{-\frac{1}{\varepsilon}} \qquad (2.25a)$$

$$p_{z,t} = (1-\gamma)\Big(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\Big)^{\frac{1}{\varepsilon-1}} Y_{z,t}^{-\frac{1}{\varepsilon}}. \qquad (2.25b)$$

The consumption good serves as the numeraire. Hence, the price of this consumption good is normalized to $p \equiv 1$ in all periods $t \geq 0$. The normalization of the final output price implies a normalization of intermediate goods prices. The following proposition describes this equivalence in price normalization of final output and intermediate goods prices for the case of N different intermediate goods. The case of two intermediate goods, as used in the present study, is then a special case of this proposition of $N = 2$.

Proposition 2.4. *Suppose, the production of final output uses $N \in \mathbb{N}$ different intermediate inputs Y_i, $i = 1, 2, ..., N$. Then the normalization of final output prices p is equivalent to setting the weighted sum of intermediate goods prices $\sum_{i=1}^{N}(\gamma_i^{\varepsilon} p_i^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}$ equal to 1 in all periods. This weighted sum is referred to as the "ideal" price index or Fisher price index.*

Proof: See appendix A.
In this study, the final goods sector combines $N = 2$ different intermediate goods. The ideal price index in this case is given by

$$p = \Big(\gamma_m^{\varepsilon} p_m^{1-\varepsilon} + \gamma_z^{\varepsilon} p_z^{1-\varepsilon}\Big)^{\frac{1}{1-\varepsilon}} = \Big(\gamma^{\varepsilon} p_m^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_z^{1-\varepsilon}\Big)^{\frac{1}{1-\varepsilon}} \equiv 1. \qquad (2.26)$$

2.4. Heterogeneous consumers

In each period a continuum of young consumers is born. Each consumer lives for two periods. In the first period of life, people are young and in the second period, they are old. Assume that (i) at the end of each period, old consumers are replaced by new born young consumers, so that each member of the old generation has exactly one descendant in the subsequent period and ii) the population of the initial (period 0) old generation is numerically identical to the young population of period 0. This formulation implies that the population is constant over time.[37]
The young generation is indexed by the superscript "y", members of the old generation are indexed with an "o". In this setting, the attribute of being a worker, an entrepreneur or an innovator is preassigned. So at each point in time, two different generations populate the economy and each generation consists of workers (w), entrepreneurs (e), and

[37]The two-period setting corresponds to a period length of 30 – 40 years and implies a lifetime of 60 to 80 years. Alternatively one could assume a larger number of life periods. This would imply a larger number of generations at each point in time and the complexity of the consumer sector would increase: within a multi-period overlapping generations setting of $n \in \mathbb{N}$ consecutive periods of life one would have to control $2n$ different decision problems faced by the individuals. This would add a large amount of realism but would also make the analysis much more difficult, contributing very little to dealing with the task in hand, i.e. to analyze the influence of bank's credit lending on the rate and direction of technical change.

innovators (i), who differ in terms of their access to investment projects and consumption profiles. The subindex $\ell \in \{w, e, i\}$ identifies the different consumer types. Preferences over consumption are identical for all consumers and equal to:

Assumption 2.7. *Consumption preferences can be represented by an utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$, which is defined as*

$$U(c^y_{\ell,t}, c^o_{\ell,t+1}) := \beta u(c^o_{\ell,t+1}), \tag{2.27}$$

β is the individual discount factor and the "instantaneous" utility function $u : \mathbb{R}_+ \to \mathbb{R}$, is logarithmic: $u(c) := log(c)$,

So utility is defined as a function of only second period consumption.

Workers
At each point in time a continuum of mass M of young workers is born. Each one is endowed with one unit of labor time. Workers supply their labor inelastically to the labor market when young and receive a "real" wage rate of $w_{M,t}$. At the beginning of their second period of life, when old, they retire. Workers only consume in the second period of life and, therefore, wish to transfer their current wealth into the next period. For this purpose, they supply their labor income to the deposit market and receive a deposit rate of r_{t+1} on these savings. Given labor income of $w_{M,t}$ and since the entire income is saved, we have $s_t = w_{M,t}$, and old workers have income of $r_{t+1}s_t$. So the budget constraint of a worker equals

$$c^{(o)}_{w,t+1} = r_{t+1}s_t. \tag{2.28}$$

Denote aggregate consumption of old workers as $C^{(o)}_{w,t}$ and aggregate savings as S_t, then the aggregate budget constraint of the old working population is given by

$$C^{(o)}_{w,t+1} = r_{t+1}S_t = r_{t+1}w_{M,t}M_t =: I^{(o)}_{w,t+1}. \tag{2.29}$$

Old workers do not care about the wealth of their descendants and leave no bequests. Since utility is strictly increasing in consumption quantities, old workers spend their income entirely for consumption of the final good.

Entrepreneurs
A continuum of mass 2 of young entrepreneurs enters the economy at the beginning of each period t. Each entrepreneur receives a patent to produce capital goods of current quality for one of the two intermediate sectors. Then the mass of entrepreneurs holding patents for capital goods in each intermediate sectors $j \in \{m, z\}$ equals unity. Recall that the continuum of capital goods used in each intermediate sector has unit mass, so this ensures identical masses of capital good lines and entrepreneurs. Hence each entrepreneur holds a patent for one capital good line and consequently on each capital good line one entrepreneur is present.
Entrepreneurs form an "entrepreneur association" to protect themselves from idiosyncratic risks in profits. Since realizations of individual innovation attempts are independent

and thus uncorrelated, a fraction of the projects will be successful while the rest of the projects will be unsuccessful. Then using a law of large numbers type of reasoning for a continuum of random variables (see Uhlig (1996)), the fraction of successful projects in sector $j \in \{m, z\}$ is equal to the ex-ante probability of successful innovation η_j and the fraction of unsuccessful projects is equal to the ex-ante probability of innovation failure $1 - \eta_j$. Moreover, using the same type of reasoning, the ex-ante probability that an innovator tries to improve the quality of a certain capital good in sector j is equal to the frequency or mass of innovators in sector m, denoted by $\iota_m := \iota$ and sector z, denoted by $\iota_z := 1 - \iota$. Entrepreneurs make profits in capital good lines without an innovator or where innovators' R&D is unsuccessful. Together this implies an income for the entrepreneur association of

$$\bar{\Pi}^e_t = (1 - \eta_m \iota)\Pi^e_{m,t} + (1 - (1 - \iota)\eta_z)\Pi^e_{z,t} \tag{2.30}$$

The aggregate budget constraint of the entrepreneurs is then given by

$$C^{(o)}_{e,t} = \bar{\Pi}^e_t =: I^{(o)}_{e,t} \tag{2.31}$$

Old entrepreneurs also consume their income completely, since utility strictly increasing in consumption.

Innovators

Each period, a continuum of innovators is born whose mass is normalized to one. Young innovators engage in R&D to improve the quality of existing capital goods. Young innovators need credit to finance R&D expenditures and banks supply the necessary capital resources. If successful, they receive a one period patent on that innovation and produce that capital good with greater quality instead of the entrepreneur on that capital good line. Innovators direct their investment to capital goods innovation for either intermediate sector on the basis of expected profits. Denote the share of innovators directing R&D investment to sector m by ι, then the share of innovators investing in R&D in sector z equals $1 - \iota$, where $\iota \in [0, 1]$ can vary in equilibrium.

Similar to the entrepreneurs, innovators group in an "alliance" to protect themselves from idiosyncratic risks in profits. Within one intermediate sector $j = \{m, z\}$ innovators form an "R&D syndicate". Again, since investment projects contain idiosyncratic risk, we can apply a law of large number argument for a contiuum of random variables, and conclude that the fraction of successful projects is equal to the ex-ante probability of successful innovation η_j. Aggregate average profits for the R&D syndicate in sector j are given by $\eta_j \Pi^{(i)}_j$, so the income for innovators in sector $j \in \{m, z\}$ reads

$$\bar{\Pi}_{j,t} = \iota_j \eta_j \Pi_{j,t} \tag{2.32}$$

Old innovators also spend their income from monopolistic capital goods selling completely for consumption and leave no bequests for their descendants. Note that the aggregate income of the innovators is sector-specific since innovators within one intermediate sector form a "R&D-Syndicate". Therefore, the aggregate income of the syndicate depends also on the number of innovators ι_j in sector j, since the more innovators try to improve the quality of capital goods in one sector, the greater is the income of the R&D-syndicate. This implies a budget constraint for innovators in sector j equal to

$$C^{(o)}_{i,j,t} = \bar{\Pi}^{i}_{j,t} =: I^{(o)}_{i,j,t}. \tag{2.33}$$

Total consumption of all innovators is then given by

$$C^{(o)}_{i,t} = C^{(o)}_{i,m,t} + C^{(o)}_{i,z,t}. \tag{2.34}$$

2.5. Aggregation

So far, the study presented the microeconomic decisions of all individuals in the economy. Next , we compute the aggregate sectoral and macroeconomic variables of the economy. At the sectoral level, we need a quality index, describing the average development of capital goods quality. Then, one can derive R&D spending, capital goods investment and the sector specific outputs as functions of the average sectoral quality index, denoted by $Q_{m,t}$ in sector m and $Q_{z,t}$ in sector z respectively. The aggregate sectoral quality index for $t \geq 0$ is defined as:[38]

$$Q_{j,t} := \int_0^1 q_{\vartheta_j,t} d\vartheta_j \quad \text{for } j = \{m, z\}. \tag{2.35}$$

Index Q_j is a combination of the various q_{ϑ_j}'s and increases in the q_{ϑ_j}'s affect aggregate sectoral output to the extent that they raise $Q_{j,t}$. The index of aggregate quality in this economy is the average of qualities, since each intermediate sector uses a continuum of capital goods whose mass is normalized to unity (other aggregation types that reflect for instance a CES-aggregator type can be found in Barro and Sala-i Martin (2004) P.324 eq. (7.15) or Acemoglu (2009)). The aggregator is linear homogeneous, which captures the plausible feature that if we increase all single productivities by a number $a > 0$ then the average sectoral productivity will increase by a.

Although the q_{ϑ_j}'s of each product line ϑ_j are stochastic, we will argue now that the randomness in microeconomic quality improvements vanishes in the sectoral and macroeconomic variables and the average sectoral quality $Q_{j,t}$ is completely deterministic: This holds, because realizations of the quality increases of different capital goods are independent and thus uncorrelated. Then one could in principle conclude, applying a law of large numbers type of reasoning that $Q_{j,t}$'s are deterministic. One has to be careful here, however, because the typical laws of large numbers apply to the average of a countable sequence of random variables, whereas the model here features a continuum of capital goods (normalized to unity) and so $Q_{j,t}$ describes the "average" of a continuum of random variables. However, based on the findings of Uhlig (1996), who shows how to obtain a law of large numbers for a continuum of uncorrelated random variables, we can conclude that the $Q_{j,t}$'s are well defined in the present context (see also Acemoglu (2009)).

The dynamics of the average sector-specific quality improvements can now be derived as follows: for any industry ϑ_j in sector $j = \{m, z\}$, the probability of success is the same

[38] See for instance Acemoglu et al. (2012).

and is given by (2.18). One can show that the expected change in the average quality of capital goods in sector j between time t and $t+1$ is given by

$$\mathbb{E}_t[Q_{j,t+1} - Q_{j,t}] = \iota_{j,t}\eta_{j,t}\Big(\lambda_j - 1\Big)Q_{j,t}.^{39} \tag{2.36}$$

Thereby, $\iota_m := \iota$ and $\iota_z := 1-\iota$, $\iota \in [0,1]$ enter this expression, since quality probably increases only in those capital good lines where innovators are present. If for instance $\iota = 1$, all innovators direct their R&D effort to sector m and the quality of all capital goods used in sector z remain constant.

Using the law of large numbers argument given above we can conclude that the average sectoral capital goods quality develops deterministically over time - even though individual R&D projects exhibit stochastic returns, so that the expectations operator in the equation above can be dropped and the dynamic development of average sectoral capital goods quality $Q_{m,t}$ and $Q_{z,t}$ is given by

$$\frac{Q_{m,t+1} - Q_{m,t}}{Q_{m,t}} = \iota_t\eta_{m,t}\left(\lambda_m - 1\right) \qquad \text{for all } t \geq 0, \tag{2.37a}$$

$$\frac{Q_{z,t+1} - Q_{z,t}}{Q_{z,t}} = (1-\iota_t)\eta_{z,t}\left(\lambda_z - 1\right) \qquad \text{for all } t \geq 0, \tag{2.37b}$$

where by assumption (2.2), $\lambda_j > 1$ so $\iota_{j,t}\eta_{j,t}\left(\lambda_j - 1\right) > 0$ and $Q_{j,0}$ is some given initial quality level. According to equations (2.37a) and (2.37b), the dynamic development of the average quality of the capital goods in each intermediate sector can be represented by a deterministic, linear first order difference equation. Denote the economy-wide average capital goods quality as Q_t. Then, Q_t is defined as the weighted average of the two sectoral quality indices $Q_{m,t}$ and $Q_{z,t}$, where the weights are given by the distribution parameter of the intermediate inputs $Y_{m,t}$, $Y_{z,t}$ in final goods production, γ and $1-\gamma$:

$$Q_t = \gamma Q_{m,t} + (1-\gamma)Q_{z,t}. \tag{2.38}$$

It is important to bear in mind that the average quality of sector-specific capital goods used in the production of the two intermediate goods Y_m and Y_z are different, allowing technical change to be biased or directed to one intermediate sector. The average quality of capital goods Q_m and Q_z, determine aggregate productivity, while the quotient Q_z/Q_m

[39] This relation can be derived as follows:

$$\begin{aligned}
\mathbb{E}_t[Q_{j,t+1} - Q_{j,t}] &= \iota_{j,t}\eta_{j,t}\Big[\int_0^1 q'_{\vartheta_j,t+1}d\vartheta_j - \int_0^1 q_{\vartheta_j,t}d\vartheta_j\Big] \\
&= \iota_{j,t}\eta_{j,t}\Big[\int_0^1 \lambda_j q_{\vartheta_j,t}d\vartheta_j - \int_0^1 q_{\vartheta_j,t}d\vartheta_j\Big] \\
&= \iota_{j,t}\eta_{j,t}\Big(\int_0^1 q_{\vartheta_j,t}d\vartheta_j\left(\lambda_j - 1\right)\Big) \\
&= \iota_{j,t}\eta_{j,t}\left(\lambda_j - 1\right)Q_{j,t}.
\end{aligned}$$

determines the relative productivity of the factor Z.
Aggregate sectoral R&D expenditures can be computed from individual R&D expenditures of an innovator in industry ϑ_j, given in (2.16), as

$$H_{j,t} = \frac{\alpha}{1+\alpha}\frac{\bar{\pi}_{j,t+1}}{R_{j,t+1}}\iota_{j,t}Q_{j,t+1}, \qquad j \in \{m, z\}, \tag{2.39}$$

where still $\iota_{m,t} := \iota_t$ and $\iota_{z,t} := 1 - \iota_t$. The expression shows that aggregate sectoral R&D is increasing in the average sectoral capital goods quality $Q_{j,t+1}$, decreasing in the credit interest rate $R_{j,t+1}$ and increasing in the mass of innovators per sector $\iota_j \in [0, 1]$. In this regard ι_t controls the direction (and also rate) of technical change: for instance, if $\iota_t = 1$, all innovation effort would be directed to sector m and consequently, R&D expenditures in sector z would be zero. From (2.37a) and (2.37b) then follows that only the quality of capital goods in sector m changes over time, while the quality of capital goods in sector z remains constant, i.e. technical change would occur only in sector m.
Total R&D in period t is simply given by the sum of aggregate sectoral R&D:

$$H_t = H_{m,t} + H_{z,t}. \tag{2.40}$$

Spending on capital goods in sector j can be derived from (2.22a) and (2.22b):

$$X_{j,t} = \int_0^1 x_{\vartheta_j,t}d\vartheta_j = \mu\bar{\pi}_{j,t}Q_{j,t} \tag{2.41}$$

and aggregate capital goods spending is the sum over sectoral capital goods spending equal to

$$X_t = X_{m,t} + X_{z,t}. \tag{2.42}$$

Finally, aggregate savings are given by total labor income (income of factor M_t) of the workers

$$S_t = w_{M,t}M_t \tag{2.43}$$

and aggregate consumption in period t is given by consumption of old workers, old entrepreneurs and old innovators:

$$C_t = \sum_{\ell} C^{(o)}_{\ell,t}, \qquad \ell \in \{w, e, i\}. \tag{2.44}$$

2.6. Market clearing

The previous sections derived conditions of optimal behavior of banks, firms and consumers. The remaining requirement to describe an equilibrium of the economy is that all markets clear.

Capital markets

Total deposit supply is given by aggregate savings of young workers S_t. Market clearing on the deposit market requires that workers' aggregate savings are equal to the deposit demand of the representative aggregate bank:

$$S_t \stackrel{!}{=} D_t \qquad \forall t \geq 0. \tag{2.45}$$

Credit demand in each sector $j = \{m, z\}$ is given by aggregate sectoral R&D expenditures (eq. (2.39)). Market clearing requires that in each sector, credit supply is equal to credit demand:

$$D_{j,t} \stackrel{!}{=} H_{j,t} \qquad j = \{m, z\}, \forall t \geq 0. \tag{2.46}$$

Moreover, the bank cannot supply more capital to the innovators than received from the workers, so we impose the condition that total deposit demand today determines total credit supply tomorrow:

$$D_t = \sum j = \{m, z\} D_{j,t+1}. \tag{2.47}$$

Factor markets
The supply of the factors M and Z is constant. The firms in the two intermediate sectors represent aggregate factor demand. Market clearing on the market for factor $J = \{M, Z\}$ then requires

$$J_t^S \stackrel{!}{=} J_t^D \qquad J \in \{M, Z\}, \forall t \geq 0. \tag{2.48}$$

Goods markets
The entrepreneurs and innovators represent the supply side of capital goods, the demand side is given by the intermediate firms in the two sectors. For each capital good ϑ_j, market clearing requires:

$$x^S_{\vartheta_j,t} \stackrel{!}{=} x^D_{\vartheta_j,t} \qquad \vartheta_j \in [0, 1], j \in \{m, z\}, \forall t \geq 0. \tag{2.49}$$

Market clearing on the markets for the intermediate goods requires

$$Y^S_{j,t} \stackrel{!}{=} Y^D_{j,t} \qquad j \in \{m, z\}. \tag{2.50}$$

Market clearing on the final goods market requires that final goods demand (consumption, investment in R&D and machine production) equals supply and gives the economy's resource constraint:

$$Y_t = X_t + H_t + C_t \qquad \forall t \geq 0, \tag{2.51}$$

where C_t is given in (2.44), X_t is given in (2.42) and H_t is given in (2.40).
Finally, we need a "market clearing" condition for the mass of innovators. For all $t \geq 0$:

$$\iota_{m,t} + \iota_{z,t} = 1. \tag{2.52}$$

3. Equilibrium

The individual demand and supply decisions of firms, banks and consumers together with market clearing constitute the basis of the following equilibrium analysis. In this section, I emphasize the influence of banks on the direction of technology development in order to answer the central research question formulated at the beginning of the first part: *Do banks influence the direction of technical change?*
The remainder of this chapter is structured as follows: section 3.1 defines and characterizes the equilibrium. Next the study analyzes the determinants of directed technical change (3.2) and derives equilibrium properties (3.3). This section ends with a short comparative static analysis of parameter changes on equilibrium variables (3.4).

3.1. Definition and characterization

For a given quantity of available resources in the economy, its distribution for a) different purposes such as consumption, production and investment purposes and its distribution b) across time is called *allocation*. Then, roughly speaking, an allocation constitutes an *equilibrium*, if -for a given set of market prices- each individual consumer, firm and bank responds optimally to the (optimal) decisions of all other entities in the economy. Accordingly, an *equilibrium* with financial intermediation is defined as:

Definition 3.1. *An equilibrium of the economy considered here is an allocation*

$$\mathbf{A} = (Y_t, C_t, M_t, Z_t, (Y_{j,t}, X_{j,t}, H_{j,t}, \iota_{j,t}, (q_{\vartheta_j,t}, x_{\vartheta_j,t})_{\vartheta_j \in [0,1]})_{j \in \{m,z\}}, D_{t+1})_{t \geq 0}$$

and a price system

$$\mathbf{P} = (r_t, w_{j,t}, R_{j,t}, p_{j,t}, p_{\vartheta_j,t})_{t \geq 0}$$

such that:

(i) The allocation is consistent with the production technologies (2.19), (2.23), and the market clearing conditions/resource constraints (2.45), (2.46), (2.48), (2.49), (2.50), (2.51), and (2.52).

(ii) Banks behave optimally, i.e. equations (2.3) and (2.4) hold for all $t \geq 0$.

(iii) Producers behave optimally, i.e. equations (2.8), (2.16) , (2.21) hold for all $t \geq 0$.

(iv) Consumers behave optimally with (profit) incomes determined by (2.29), (2.30) and (2.32) for all $t \geq 0$.

(v) Average capital goods qualities in sector $j \in \{m, z\}$ *evolve according to (2.37).*

Recall that the process of R&D effort for the individual innovator is stochastic. Therefore, the quality of capital good ϑ_j in sector $j = m, z$, is random and so are the corresponding prices, quantities and values. However, since the outcomes from random individual "experiments" are stochastically independent, the uncertainty vanishes in aggregate terms

and the corresponding sectoral and macroeconomic variables are non-stochastic. This implies deterministic time paths of the aggregate sectoral and macroeconomic variables and the equilibrium analysis becomes completely deterministic.
It is useful to derive properties of some equilibrium variables that will be used in the following analysis. So in a first step, we compute aggregate production of the two intermediate goods $Y_{m,t}$ and $Y_{z,t}$ as functions of the factors M_t, Z_t, output prices $p_{m,t}, p_{z,t}$ and aggregate sectoral quality indices $Q_{m,t}, Q_{z,t}$, insert the intermediate firms equilibrium demand schedules for capital goods $x_{m,t}, x_{z,t}$ stated in equations (2.22a) and (2.22b) into the production functions of good Y_m and good Y_z given in assumption 2.5. This yields for sector m

$$Y_{m,t} = \frac{\mu_0}{\mu} p_{m,t}^{\frac{\mu}{1-\mu}} M_t Q_{m,t} \tag{3.1}$$

and for sector z

$$Y_{z,t} = \frac{\mu_0}{\mu} p_{z,t}^{\frac{\mu}{1-\mu}} Z_t Q_{z,t}, \tag{3.2}$$

where $\mu_0 := (1-\mu)\mu^{\frac{2\mu}{1-\mu}}$. Note that the integral parts in the production functions vanish due to the definition of the aggregate or average sectoral quality index.[40] Taking the ratio of equations (3.1) and (3.2) gives

$$\frac{Y_{z,t}}{Y_{m,t}} = \left(\frac{p_{z,t}}{p_{m,t}}\right)^{\frac{\mu}{1-\mu}} \frac{Q_{z,t}}{Q_{m,t}} \frac{Z_t}{M_t}. \tag{3.3}$$

In a second step, we combine the optimality conditions of the final good sector stated in (2.25). This yields the relative factor price of the two inputs as functions of the relative factor demand. Let $\tilde{p}$ denote the relative price of the two input factors, the equilibrium relative pricing scheme reads

$$\tilde{p}_t = \frac{p_{z,t}}{p_{m,t}} = \frac{1-\gamma}{\gamma}\left(\frac{Y_{z,t}}{Y_{m,t}}\right)^{-\frac{1}{\varepsilon}} = \tilde{\gamma}\tilde{Y}_{j,t}^{-\frac{1}{\varepsilon}}, \tag{3.4}$$

where $\tilde{\gamma} := \frac{1-\gamma}{\gamma}$, $\tilde{Y}_{j,t} := \frac{Y_{z,t}}{Y_{m,t}}$. The lower the price of each factor, the higher is the corresponding demand. The greater the supply of factor Y_z relative to Y_m (so the greater $\tilde{Y}_{j,t}$), the lower is the relative price $\tilde{p}$. Moreover, changes in the relative price level in response to changes in the relative factor supply depend on the elasticity of substitution ε.
Now we use this equilibrium relative price to eliminate the term $\tilde{Y}_{j,t} = Y_{m,t}/Y_{z,t}$ from equation (3.3). Solve the resulting expression for $\tilde{p}_t$ gives the expression for the relative price of the two goods $Y_{m,t}$ and $Y_{z,t}$ as a function of the relative factor supply and the relative average quality or physical productivity of the sector-specific capital goods:

$$\tilde{p}_t = \left(\tilde{\gamma}^{-\varepsilon}\tilde{Z}_t\tilde{Q}_t\right)^{-\frac{(1-\mu)}{\sigma}} \tag{3.5}$$

[40]The average sectoral quality of capital goods is defined in equation (2.35) as

$$Q_{j,t} := \int_{\vartheta_j \in \Theta_j} q_{\vartheta_j} d\vartheta_j \quad \text{for } j = \{m, z\}.$$

where $\tilde{Z}_t := Z_t/M_t$, $\tilde{Q}_t := Q_{z,t}/Q_{m,t}$ and $\sigma := 1 + (1-\mu)(\varepsilon - 1)$. The relative price given is decreasing in the relative supply and relative productivity. This implies a constant relative price if relative factors and relative productivities are constant. The latter holds for instance if the growth rates of average sectoral capital goods are identical. The expression also coincides with the result in equation (16) p. 790 in Acemoglu (2002), except that in this thesis, a) the relative technology is given by the productivity or quality of a constant quantity of sector specific capital goods and not by an increasing variety of capital goods. As in the standard directed technical change model of Acemoglu (2002), σ is the elasticity of substitution between the two factors M and Z, since one can show that

$$\sigma = -\Big(\frac{dlog(\frac{w_Z}{w_M})}{dlog(\frac{Z}{M})}\Big)^{-1}. \tag{3.6}$$

It will turn out in brief that the elasticity of substitution has a crucial role on the direction of technical change. If the two intermediate goods Y_m and Y_z are gross substitutes ($\varepsilon > 1 \Leftrightarrow \sigma > 1$), then the two factors M and Z are gross substitutes. If Y_m and Y_z are gross complements ($\varepsilon < 1 \Leftrightarrow \sigma > 1$) then M and Z are gross complements.
In a third step we derive the equilibrium credit and deposit interest rates and sectoral credit quantities. Note that the stated equations all contain various endogenous variables on their right hand side that will be determined next, so the treatment is incomplete at this stage. To get the equilibrium deposit interest rate r_t use (2.45) together with (2.47), (2.46), (2.39), and (2.3), we get

$$r_{t+1} = \frac{\alpha}{S_t}\Big(\iota_t \mathbb{E}_t[\Pi_{m,t+1}] + (1-\iota_t)\mathbb{E}_t[\Pi_{z,t+1}]\Big) \tag{3.7}$$

where $\mathbb{E}_t[\Pi_{j,t}]$ denotes sectoral average expected profits for innovators containing endogenous intermediate output prices, capital good qualities and success-probabilities. I determine the expressions in detail below.
The equilibrium interest rate on deposits is decreasing in aggregate savings which is equivalent to deposit supply, and increasing in sectoral expected profits from innovation. Insert (3.7) into (2.3) to get the credit interest rates for sector j :

$$R_{j,t+1} = \frac{\alpha}{\eta_j S_t}\Big(\mathbb{E}_t[\Pi_{m,t+1}] + \mathbb{E}_t[\Pi_{z,t+1}]\Big), \qquad j \in \{m, z\}. \tag{3.8}$$

To see the influence of prices and factors on equilibrium credit interest rates, insert $\eta_{j,t}$ given in (3.11) into (2.3):

$$R_{j,t} = \Big(\frac{1+\alpha}{\alpha}\Big)^{\frac{\alpha}{1-\alpha}} \frac{r_t^{\frac{1}{1-\alpha}}\zeta}{\big(p_{j,t}J_t\big)^{\frac{1}{1-\alpha}}}, \quad \alpha \in]0,1[, \quad j \in \{m, z\}, \quad J \in \{M, Z\}. \tag{3.9}$$

This shows that credit interest rates are increasing in the deposit interest rate r_t, decreasing in the corresponding intermediate output price $p_{j,t}$, and also decreasing in the sector-specific factor $J_t = \{M, Z\}$.
Take the ratio of (3.8) to compute the relative credit interest rate $\tilde{R}_t$:

$$\tilde{R}_t = \frac{R_{z,t}}{R_{m,t}} = \frac{\eta_{m,t}}{\eta_{z,t}} = \tilde{\eta}_t^{-1}, \tag{3.10}$$

so the relative credit interest rate is inversely related to the ratio of success-probabilities in innovation effort.
Now use (2.18) and (2.3), the success-probability in sector j reads

$$\eta_{j,t} = \zeta^{-1}\Big(\frac{\alpha}{1+\alpha}\frac{\bar{\pi}_{j,t+1}}{r_{t+1}}\Big)^{\frac{\alpha}{1-\alpha}}, \qquad j \in \{m, z\}, \tag{3.11}$$

where $\bar{\pi}_{j,t} := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{j,t}^{\frac{1}{1-\mu}} J_t$.
To get equilibrium credit in sector j, use the market clearing conditions in the capital markets (2.45) and (2.46) together with the condition that total deposit demand in t determines total credit supply in $t+1$ (2.47). Then, credit in sector m reads

$$D_{m,t-1} = \frac{S_{t-2}}{1+\tilde{R}_t\tilde{\pi}_t\tilde{Q}_t\tilde{\iota}_t}, \tag{3.12}$$

which can be solved for

$$D_{m,t-1} = \frac{\iota_{t-1}R_{z,t}\bar{\pi}_{m,t}Q_{m,t}S_{t-2}}{\iota_{t-1}R_{z,t}\bar{\pi}_{m,t}Q_{m,t} + (1-\iota_t)R_{m,t}\bar{\pi}_{z,t}Q_{z,t}}. \tag{3.13}$$

Similar, credit demand in sector z is given by

$$D_{z,t-1} = \frac{S_{t-2}\tilde{R}_t\tilde{\pi}_t\tilde{Q}_t\tilde{\iota}_t}{1+\tilde{R}_t\tilde{\pi}_t\tilde{Q}_t\tilde{\iota}_t}, \tag{3.14}$$

which is equivalent to

$$D_{z,t-1} = \frac{(1-\iota_{t-1})R_{m,t}\bar{\pi}_{z,t}Q_{z,t}}{\iota_{t-1}R_{z,t}\bar{\pi}_{m,t}Q_{m,t} + (1-\iota_{t-1})R_{m,t}\bar{\pi}_{z,t}Q_{z,t}}S_{t-2}. \tag{3.15}$$

The epxressions in (3.13) and (3.15) reveal that credit in sector $j \in \{m, z\}$ is increasing in aggregate savings and shares of innovators ι_j. Credit in sector m is lower for greater $R_{m,t}$, greater (lower) for greater $R_{z,t}$, increasing in $Q_{m,t}$ and decreasing in $Q_{z,t}$.
The ratio of sectoral creditor relative credit in sector z is then equal to:

$$\tilde{D}_t = \tilde{\iota}_t\tilde{p}_{t+1}^{\frac{1}{1-\mu}}\tilde{Z}_{t+1}\tilde{Q}_{t+1}\tilde{R}_{t+1}. \tag{3.16}$$

It will turn out next that the right hand side equals relative expected profits from innovation. This is intuitively clear since sectoral credit is equal to sectoral R&D expenditures and the latter are constant share of expected profits.

3.2. Determinants of Directed Technical Change

Let us now analyze the determinants of directed technology development on a balanced growth path equilibrium (BGP). More precisely, I derive the determinants of directed innovations and then analyze how these determinants response to changes in relative factor supply. After presenting the economics of the direction of technical change, I show the existence and uniqueness of the BGP-equilibrium. Here, a balanced growth equilibrium is characterized as an equilibrium, where all variables grow at constant rates.

The demand for innovations
In the following we analyze how the production side of the economy determines the return to different types of innovation – the demand for innovation. The next section then discusses the other side of this equation, the cost of different innovations or the supply side of innovations.
Average profits from capital goods selling for old innovators in sector m read

$$\Pi_{m,t} = (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{m,t}^{\frac{1}{1-\mu}} M_t Q_{m,t} \tag{3.17}$$

and in sector z

$$\Pi_{m,t} = (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{z,t}^{\frac{1}{1-\mu}} Z_t Q_{z,t}. \tag{3.18}$$

The net present discounted value of expected profits for young innovators in sector j, denoted by $V_{j,t}$, $j \in \{m, z\}$, are then given by

$$V_{j,t} = \frac{\mathbb{E}_t[\Pi_{j,t+1}]}{r_t}. \tag{3.19}$$

one can show that the V's are equal to

$$V_{m,t} = \mu_0 p_{m,t+1}^{\frac{1}{1-\mu}} M_{t+1} Q_{m,t+1} R_{m,t+1}^{-1} \qquad V_{z,t} = \mu_0 p_{z,t+1}^{\frac{1}{1-\mu}} Z_{t+1} Q_{z,t+1} R_{z,t+1}^{-1}. \tag{3.20}$$

where $\mu_0 := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}}$. So the *larger $V_{z,t}$ in relation to $V_{m,t}$, the greater is the reward to develop Z-augmenting capital goods* (see Acemoglu (2002) p.789). The two profit equations given in (3.20) suggest that capital goods producers' profits increase with greater output prices, with greater factor use and with greater productivity in the intermediate sectors and decrease with larger interest rates on business credit. The latter effect simply follows from the relationship between credit interest rates and success-probabilities given in (2.3).
To show and discuss these determinants in more detail, take the ratio of $V_{z,t}$ and $V_{m,t}$. This gives

$$\frac{V_{z,t}}{V_{m,t}} = \underbrace{\left(\frac{p_{z,t+1}}{p_{m,t+1}}\right)^{\frac{1}{1-\mu}}}_{\substack{\text{price}\\ \text{effect}}} \times \underbrace{\frac{Z_{t+1}}{M_{t+1}}}_{\substack{\text{market size}\\ \text{effect}}} \times \underbrace{\left(\frac{R_{z,t+1}}{R_{m,t+1}}\right)^{-1}}_{\substack{\text{risk}\\ \text{effect}}} \times \underbrace{\left(\frac{Q_{z,t+1}}{Q_{m,t+1}}\right)}_{\substack{\text{productivity}\\ \text{effect}}}. \tag{3.21}$$

The conclusions drawn from equation (3.21) represent the central results of part one. The next two theorems comprehend these main findings.

Theorem 3.1. *The direction of technical change –whether technical change will favour relatively scarce or abundant factors– is determined by four different market forces: the* price effect*, the* market size effect*, the* productivity effect *and the* risk effect*.*

- The *price effect*: Since $\mu \in]0,1[$, the relative profitability of inventing new Z-augmenting capital goods is increasing in the relative price $p_{z,t+1}/p_{m,t+1}$. Therefore, the higher this relative price is, the greater is the return on developing new

Z-complementary technologies (Acemoglu (2002) p.789).
Naturally, relatively scarce factors are relatively more expensive. Thus, the price effect directs technical changes to technologies or sectors that complement scarce input factors and thus command higher commodity prices.

- The *market size effect*: The relative profitability of inventing Z-complementary technologies inreases in the relative supply of the factors Z_{t+1}/M_{t+1}. Therefore, the larger this relative factor supply, the greater is the return on the development of new Z-complementary technologies (Acemoglu (2002) p.789).
In this study, the market for a technology is determined by the factors that use this technology. An increase in the supply of a factor leads to a larger market for capital goods that complement this factor. The market size effect encourages innovations in sectors that use the more abundant factor. Hence, this effect works in the opposite direction compared to the price effect.

- The *risk effect*: The risk effect results from the fact that $V_{z,t}/V_{m,t}$ decreases in $R_{z,t+1}/R_{m,t+1}$. The lower the relative probability of successful innovation in sector z, η_z/η_m, the higher is the corresponding relative loan rate R_z/R_m and the risk of default on external credit reflected in the interest rate on loans.[41] Thus, the risk effect directs innovations to sectors with a higher probability of research success. This illustrates the significance of the assumption, how the current η_J's relate to the number of previous innovations n_{ϑ_j} in any industry ϑ_j and sector m, z with respect to the direction of technical change.

- The *productivity effect*: The ratio $V_{z,t}/V_{m,t}$ increases in $Q_{z,t+1}/Q_{m,t+1}$ and thus the productivity effect encourages innovations in sectors with a higher productivity.

The result established in theorem 3.1 resembles the effects stated amongst others in Acemoglu (2002) plus the risk effect. Altogether the net effect of these partly counteracting forces determines the direction of technical change. Thereby, the risk effect stated in equation (3.21) results from the explicit consideration of financial intermediation and represents the innovation of this thesis. The following theorem establishes the second fundamental result:

Theorem 3.2. *If innovators are capital constrained and R&D is financed by credit, then banks influence the direction of technical change through the risk effect.*

First, credit interest rates contain a risk premium given by the inverse of the probability of successful innovation. The risk premia and the loan rates relate inversely to another: the lower the probability of successful innovation in any one sector, the higher is the

[41] Recall that the relationship between the ratio of loan rates and success-probabilities equals

$$\tilde{R} := \frac{R_z}{R_m} = \frac{r}{\eta_z} / \frac{r}{\eta_m} = \frac{\eta_m}{\eta_z} = \left(\frac{\eta_z}{\eta_m}\right)^{-1} =: \tilde{\eta}^{-1}.$$

risk of default on credit obligations by innovators who try to invent new capital goods in that sector. Consequently, banks compensate for the higher risk and demand increasing risk premia and thus charge innovators higher interest rates on credit. A relatively higher interest rate – in, say, sector z – ceteris paribus implies relatively increasing credit costs for innovators whose plan is to invent new Z-complementary capital goods. To keep profits constant, innovators would have to cut R&D investment. This would result in a lower probability of success in sector z. The expected profitability of developing new Z-complementary capital goods would decline and innovators would direct R&D effort to the relatively less expensive sector: The presence of banks adds an additional component to the determinants of directed technical change through the relative interest rate charged on loans $R_{z,t}/R_{m,t}$, which is equivalent to the inverse of the probabilities of successful innovation $(\eta_{z,t}/\eta_{m,t})^{-1}$.
Second, R&D expenditures are proportional to (expected) profits. Credit costs determine equilibrium profits and thus the amount of resources devoted to R&D in equilibrium. This in turn influences indirectly the probability of successful innovation and thus the *rate* of technical change in both sectors.
Theorem 3.2 provides an answer to the first stylized research question: *Do banks influence the direction of technical change and if so how?*. The second part of this question can also be answered with the explanations carried out previously: Banks influence the direction of technical change through the evaluation of innovators and by funding the most promising ones.
As a side note, the literature on induced innovation (e.g. Hicks (1932), Habakkuk (1962), Kennedy (1964), Dranakis and Phelps (1965), Samuelson (1965)) states that relative factor prices influence the type of technological progress. In particular, the literature argues that innovations are directed at "more expensive" factors. In the discussion here, I confine myself to the role of (output)-prices, market size effects et cetera. However, we can show the similarity between the present approach and a formulation that considers factor costs as determinants to develop new technologies rather than output prices: Combine the capital goods demand stated in equations (2.22a) and (2.22b) with the first order conditions of the intermediate firms with respect to the factors M and Z given in (2.21a). Using these expressions we can rewrite (3.17), (3.18) as

$$\Pi_m = \mu w_M M \quad \text{and} \quad \Pi_z = \mu w_Z Z. \tag{3.22}$$

Then one can express the relative profitability of developing Z-complementary capital goods in terms of factor costs w_M, w_Z and market sizes M and Z and the relative profitability of developing new Z-complementary capital goods then reads

$$\frac{\Pi_z}{\Pi_m} = \frac{w_Z}{w_M}\frac{Z}{M}. \tag{3.23}$$

Equation (3.23) indicates a higher incentive to innovate for factors that are more expensive. This result shows the equivalence of the approach presented here (and for instance by Acemoglu (2002)) that considers output prices, and the approach in the induced innovation literature cited above, which concentrates on factor input prices.

For purposes of a compact notation, I employ the following notation to all relative equilibrium variables from now on: Let a_m and a_z denote arbitrary sector-specific variables for sector m and sector Z respectively. Then we define the *relative variable with respect to sector* Z as $\tilde{a} := \frac{a_z}{a_m}$.
Under this convention, relative expected profits of innovation in sector z, given in equation (3.21), equal

$$\tilde{V}_t = \tilde{p}_{t+1}^{\frac{1}{1-\mu}} \tilde{Z}_{t+1} \tilde{Q}_{t+1} \tilde{R}_{t+1}^{-1}. \tag{3.24}$$

To get explicit formulations of relative profits and relative productivity, we have to eliminate endogenous prices $p_{m,t}$, $p_{z,t}$ and also loan rates $R_{m,t}$ and $R_{z,t}$.
Recall that relative credit interest rates for sector z are given by

$$\tilde{R}_{t+1} = \frac{R_{z,t+1}}{R_{m,t+1}} = \frac{\eta_{m,t}}{\eta_{z,t}} = \tilde{\eta}_t^{-1} \tag{3.25}$$

which is the inverse of the relative probability of successful innovation in sector z. Using (3.11), insert $\bar{\pi}_j := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_j^{\frac{1}{1-\mu}} J$ for $j \in \{m, z\}$, and take the ratio gives

$$\tilde{\eta}_t = \left(\frac{\bar{\pi}_{z,t+1}}{\bar{\pi}_{m,t+1}} \right)^{\frac{\alpha}{1-\alpha}} = \left(\tilde{p}_{t+1}^{\frac{1}{1-\mu}} \tilde{Z}_{t+1} \right)^{\frac{\alpha}{1-\alpha}}. \tag{3.26}$$

Note that the factors M_t and Z_t are constant in supply, so as long as the relative price $\tilde{p}_t$ is constant, $\tilde{\eta}_t$ is also constant and we can ignore time subscripts. Moreover, $\tilde{\eta}_t$ is independent of the levels of Q_m and Q_z and equals the ratio of sectoral profits. The latter results simply from the fact that R&D expenditures are a constant fraction of profits and R&D determines the probability of success.
To highlight the role of the risk effect in determining the direction of technical change, we eliminate the relative price $\tilde{p}_t$, as given in (3.5), from (3.26) in a first step. The risk effect then reads

$$\tilde{R}_{t+1} = \tilde{\eta}_t^{-1} = \left(\tilde{\gamma}^{\varepsilon} \tilde{Z}_{t+1}^{\sigma-1} \tilde{Q}_{t+1}^{-1} \right)^{-\frac{\alpha}{(1-\alpha)\sigma}}, \tag{3.27}$$

where $\sigma := 1 + (1-\mu)(\varepsilon - 1)$ is the elasticity of substitution between the two factors $M$and Z.[42] Inspection of (3.27) reveals that the response of the risk effect to an increase in the relative supply of factors $\tilde{Z} := Z/M$ depends on the size of σ (since $1 - \alpha > 0$). In fact (3.27) implies the following proposition:

Proposition 3.1. *If factors M and Z are gross complements,* $\varepsilon < 1 \Leftrightarrow \sigma < 1$*, the risk effect increases with an increase in the relative supply of factor* $\tilde{Z}$*.*
If factors M and Z are gross substitutes, $\varepsilon > 1 \Leftrightarrow \sigma > 1$*, the risk effect decreases with an increase in the relative supply of factor* $\tilde{Z}$*.*

[42] For the purpose of this study it is important to distinguish the cases where $\varepsilon \lesseqgtr 1$ and thus $\sigma \lesseqgtr 1$. For the relevant parameter values of ε and σ, the exponent is always positive, i.e. $1/(1-\alpha)\sigma > 0$, since $\alpha \in]0, 1[$ and $\sigma > 0$ by assumption.

After elimination of $\tilde{p}_t$, as given in (3.5), relative expected profits from innovation in sector z, given in equation (3.21), read

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{\sigma-1}\tilde{Q}^{\sigma-1}\tilde{R}^{-\sigma}\right)^{\frac{1}{\sigma}}. \tag{3.28}$$

Ignore the risk effect in equation (3.28) for the moment (and suppose that $\tilde{R} = 1$). The parameter σ has a crucial impact on the direction of technical changes.If $\sigma > 1$, M and Z are gross substitutes and an *increase* in the relative factor supply $\tilde{Z} := Z/M$ (either because the supply of Z increases or equivalently, the supply of M decreases) will *increase* the relative profitability of inventing Z-complementary capital goods $\tilde{V}_t$. On the one hand, if $\tilde{Z}$ increases, factor Z becomes relatively more abundant. This translates into a larger market for the capital goods that complement Z, which *increases* $\tilde{V}$ (this is referred to as the "market-size effect"). On the other hand, if $\tilde{Z}$ increases, factor M becomes relatively scarce and thus relatively more expensive. This translates into higher prices of goods that use M-complementary capital goods in the production process and thereby increase the profit from inventing those M-complementary capital goods. Thus $\tilde{V}$ *decreases* (this represents the "price-effect").

Consequently, the price effect and the market-size effect work in opposite directions. If $\sigma > 1$, $\tilde{V}$, as given in (3.28), *increases with an increase* in $\tilde{Z}$ and we can conclude that the market size effect dominates the price effect. If $\sigma < 1$, $\tilde{V}$ *decreases with an increase* in $\tilde{Z}$ and the price effect dominates the market-size effect. Together this implies that the parameter σ regulates whether the price effect dominates the market size effect.

Next consider the response of relative expected profits to a change in the relative factor supply $\tilde{Z}$, given that the risk effect is an additional determinant in the direction of technical changes (and compare this case to the hypothetical situation above, where we treated the risk effect $\tilde{R}$ as equal to 1). For that, insert $\tilde{R}$ from equation (3.27) into (3.28):

$$\tilde{V}_t = \left(\gamma^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\tilde{Q}_{t+1}^{-\varphi}\right)^{\frac{1}{\sigma(1-\alpha)}}, \qquad \varphi := 1 - \sigma(1-\alpha). \tag{3.29}$$

Inspection of (3.29) shows first that relative expected profits basically respond to an increase in the relative factor supply $\tilde{Z}$ in the manner described above: If $\sigma > 1$, $\tilde{V}_t$, as given in (3.29), *increases with an increase* in $\tilde{Z}$ and the market size effect dominates the price effect. If $\sigma < 1$, $\tilde{V}_t$ *decreases with an increase* in $\tilde{Z}$ and the price effect dominates the market-size effect. So the result derived by Acemoglu (2002) that the elasticity of substitution plays a crucial role in determining the direction of technical change does not change if we consider capital constrained firms and financial intermediation.

Additionally, comparing (3.28) (and still assume $\tilde{R} = 1$ there) with (3.29) reveals that the response of relative profits to an increase in $\tilde{Z}$ changes, if we additionally account for the risk effect in the determinants of technical change:

Proposition 3.2. *Relative expected profits $\tilde{V}_t$ including the risk effect respond stronger to an increase in relative factor supply $\tilde{Z}$. This result holds independent of the size of the elasticity of substitution.*

Proof: See Appendix 1.
Taken together, propositions 3.1 and 3.2 imply that the elasticity of substitution between factors regulates the way in which innovators' incentive to invent Z-complementary capital goods responds if the relative factor supply changes in the first place and additionally, this response changes if the risk effect of private sector lending enters the determinants of directed technical change. Especially the latter result has some implications for the assessment of policy rules that aim at directing technological advances towards a certain factor or sector.[43]
To see another important aspect of the elasticity of substitution, consider the relative price $\tilde{w}$ of the (relative) factors Z and M, $\tilde{Z}$. The relative factor reward can be computed as[44]

$$\tilde{w} = \tilde{\gamma}^{\frac{\varepsilon}{\sigma}} \tilde{Z}^{-\frac{1}{\sigma}} \tilde{Q}^{\frac{\sigma-1}{\sigma}}. \tag{3.30}$$

This relative factor price is decreasing in the relative supply $\tilde{Z}$, since $\sigma > 0$. This consitutes the usual substitution effect: the more abundant factor is substituted for the less abundant one, and has a lower marginal product.[45]

The supply of innovations
So far, the equilibrium analysis has studied how the production side of the economy determines the return on different types of innovations. In the next step, we consider the other side of this equation and analyze the costs of different innovations. As stated in the model section, only the final good is being used to generate innovations. This formulation corresponds to the "lab equipment" formulation in Rivera-Batiz and Romer (1991).[46] This approach implies that future relative costs of innovations are unaffected by the composition of current R&D and therefore remain constant.
Recall that innovators choose a sector to innovate for on the grounds of expected profits. To have innovators who are willing to invest in R&D in both sectors, expected profits from improving the productivity of the capital goods in both sectors have to be equal:

$$V_{m,t} = V_{z,t}. \tag{3.31}$$

[43] For instance, suppose that Y_m uses -instead of labor- a "clean" production factor in the sense that production in sector m has no negative external effect on the environment. Suppose also that the production of Y_z uses a "dirty" input Z, so the use of Z is associated with some form of pollution. Then the aim of environmental policy would be to direct technical changes away form improving the capital goods complementary to Z and instead direct innovations towards improving the productivity of capital goods complementary to M. For instance, the German Renewable Energy Sources Act ("Erneuerbare Energien Gesetz" (EEG 2009)) is an environmental policy that aims at directing technological changes towards "green" technologies, i.e. technologies that complement renewable energy resources such as wind, solar energy, geothermal energy etc. In light of the result derived above that the risk effect has a catalyzing effect on profit incentives, questions such as how long renewable energy technologies should be subsidized before releasing them into competition, should perhaps be answered differently. Parts two and three of this thesis present a detailed analysis of this topic.

[44] For a derivation of the expression given in (3.30), see appendix A1.

[45] See Acemoglu (2002) p. 790.

[46] The Lab Equipment assumes that investment in equipment or in laboratories is all that is required for research. So new capital goods and ideas are created using the final good.

The condition states that in terms of expectation, it is equally profitable to invest in generating innovations on M-and Z- complementary capital goods. Hence the average productivity of the sectoral capital goods $Q_{m,t}$ and $Q_{z,t}$ can both grow along the balanced growth path. Now resolve the condition above as $\tilde{V}_t$, insert equations (3.27), (3.28) and resolve for $\tilde{Q}$ gives

$$\tilde{Q} = \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{\varphi}}, \tag{3.32}$$

where $\varphi := 1-\sigma(1-\alpha)$. The notable feature is that with the direction of technical change endogenized, the relative productivity of technology $\tilde{Q}$ is determined by the relative factor supply $\tilde{Z}$ and the elasticity of substitution σ. Moreover, the expression for $\tilde{Q}$ implies the following proposition:

Proposition 3.3. *If the two factors are gross substitutes and $\frac{1}{1-\alpha} > \sigma > 1$, an increase in $\tilde{Z}$ will raise $\tilde{Q}$. Hence, equation (3.32) implies that if these two conditions hold, the physical productivity of the relatively more abundant factor tends to be higher.*
If the two factors are gross complements, $\sigma < 1$, an increase in $\tilde{Z}$ will lower $\tilde{Q}$. Thus, equation (3.32) implies that the physical productivity of the relatively more scarce factor tends to be higher if $\sigma < 1$.

Recall from section 2.2 that technological change is biased towards a factor if it increases the marginal product of that factor in comparison to the marginal product of the other factor. Since $\sigma > 1 \Leftrightarrow \varepsilon > 1$, a higher level of the relative physical productivity $\tilde{Q}$ corresponds to Z-biased technical change, because if $\varepsilon > 1$, the relative marginal product of Z, $\tilde{w}$ as given in equation (3.30), is increasing in $\tilde{Q}$. Thus Z-augmenting technical change is also Z-biased and technological advances will be endogenously biased in favor of the more abundant factor: with gross substitutability an increase in $\tilde{Q}$ is relatively biased toward Z.
In case of gross complementarity ($\sigma < 1$), the condition $\sigma < 1 < \frac{1}{1-\alpha}$ is automatically fulfilled and the result stated in the proposition above holds generally. However, since $\sigma < 1 \Leftrightarrow \varepsilon < 1$, the decrease in the relative physical productivity translates into higher value of the relative marginal product $\tilde{w}$, as given in (3.30). Therefore, even if $\sigma < 1$, technological advances will again be endogenously biased in favor of the more abundant factor: with gross complementarity, a decrease in $\tilde{Q}$ is relatively biased toward Z. Taken together this gives us the

Proposition 3.4. ***Relative equilibrium bias**: Consider the directed technical thange model with capital constraints and financial intermediation. Then, if $\sigma < \frac{1}{1-\alpha}$ an increase in the relative abundance of the factor Z, denoted by $\tilde{Z} := Z/M$, always induces technological change relatively biased toward factor Z.*

According to this proposition, the induced technological change increases the relative marginal product of the factor becoming more abundant. This holds, as long as $\sigma \neq 1$. If instead $\sigma = 1$, the elasticity of substitution between factors is equal to one, and technological change is not biased towards any one of the factors. Since marginal factor productivities are equal to factor prices, the relative equilibrium bias therefore describes

the impact of technology on relative factor prices at given factor proportions. Note that this equilibrium bias is not a result of the banks' financial intermediation. So in other words, the implementation of capital constrained firms and a bank sector does not alter the findings of the standard directed technical change model introduced by Acemoglu (2002), p. 792, but adds a component to the determinants of technology bias (see also Acemoglu (2007)).
Finally, the result is confined to the case considered here, where technologies are factor augmenting. So the equilibrium bias crucially depends on the assumptions about the set of possible technologies. If we include non-factor-augmenting technologies into the set of possibilities, examples can be constructed, where an increase in the abundance of a factor induces technology to be biased against this factor and the stated equilibrium bias no longer remains valid.[47]

3.3. Properties of the balanced growth path equilibrium

Next, we derive final expressions for the sectoral probabilities of successful innovation, interest rates etc., derive the rate of output growth and show the properties of the balanced growth path equilibrium.
First, we take the relative price of the intermediate goods given in (3.5) and insert (3.32):

$$\tilde{p} = \frac{p_z}{p_m} = \left(\tilde{\gamma}^{\varepsilon(1-\alpha)}\tilde{Z}^{\alpha}\right)^{-\frac{1-\mu}{\varphi}}, \tag{3.33}$$

where $\varphi := 1 - \sigma(1-\alpha)$. Solve this equation for p_z (p_m) and plug the result into the normalized price term of the final good (equation (2.26)),

$$P = [\gamma^{\varepsilon} p_m^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_z^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \equiv 1. \tag{3.34}$$

This gives the final expressions for the prices of the good Y_m:

$$p_{m,t} = \bar{W}_t^{\frac{1}{\varepsilon-1}}\left(\gamma^{\varepsilon(1-\alpha)}M_t^{\alpha}\right)^{-\frac{1-\mu}{\varphi}} \tag{3.35}$$

and of the good Y_z:

$$p_{z,t} = \bar{W}_t^{\frac{1}{\varepsilon-1}}\left((1-\gamma)^{\varepsilon(1-\alpha)}Z_t^{\alpha}\right)^{-\frac{1-\mu}{\varphi}}. \tag{3.36}$$

where $\bar{W}_t := \left(\gamma^{\varepsilon}M_t^{\sigma-1}\right)^{\frac{\alpha}{\varphi}} + \left((1-\gamma)^{\varepsilon}Z_t^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}$ and $\varphi := 1-\sigma(1-\alpha)$. The equations indicate that intermediate output prices are constant as long as the factors M and Z are constant.
After elimination of these price terms the probability of successful innovation, as given in (3.11), read

$$\eta_{m,t} = \frac{\alpha_0^{\frac{\alpha}{1-\alpha}}}{\zeta}\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{-\frac{\alpha}{\varphi}}\left(\frac{\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{r_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3.37}$$

[47] For a detailed analysis of the necessary and sufficient conditions of (relative and absolute) equilibrium bias of technology development, see Acemoglu (2007).

in sector m and

$$\eta_{z,t} = \frac{\alpha_0^{\frac{\alpha}{1-\alpha}}}{\zeta}\left((1-\gamma)^{\varepsilon} Z_{t+1}^{\sigma-1}\right)^{-\frac{\alpha}{\varphi}}\left(\frac{\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{r_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3.38}$$

in sector z, where $\alpha_0 := \alpha/(1+\alpha)$. Still, these expressions contain the economy's interest rate which will be determined below.

To solve for the balanced growth path equilibrium, we need equations a) relating the growth rate of sectoral output to sectoral R&D expenditures and b) relating the rate of aggregate growth to sectoral output growth rates. The growth rates of sectoral or average capital goods quality were given in equations (2.37a) and (2.37b):

$$\frac{Q_{m,t+1} - Q_{m,t}}{Q_{m,t}} = \iota_t \eta_{m,t} (\lambda_m - 1) \qquad \text{for all } t \geq 0 \tag{3.39}$$

for sector m and

$$\frac{Q_{z,t+1} - Q_{z,t}}{Q_{z,t}} = (1-\iota_t)\eta_{z,t} (\lambda_z - 1) \qquad \text{for all } t \geq 0 \tag{3.40}$$

for sector z. The sectoral output as a function of average sectoral capital goods quality were derived in (3.1) and (3.2):

$$Y_{m,t} = \mu^{\frac{2\mu}{1-\mu}} p_{m,t}^{\frac{\mu}{1-\mu}} M_t Q_{m,t}, \tag{3.41}$$

$$Y_{z,t} = \mu^{\frac{2\mu}{1-\mu}} p_{z,t}^{\frac{\mu}{1-\mu}} Z_t Q_{z,t}. \tag{3.42}$$

Since equations (3.35) and (3.36) reveal that intermediate output prices are constant over time, as long as the factors M and Z are constant over time, equations (3.41) and (3.42) state the relationship between sectoral output and R&D expenditures over time:

$$\frac{Y_{m,t+1} - Y_{m,t}}{Y_{m,t}} = \frac{Q_{m,t+1} - Q_{m,t}}{Q_{m,t}} \tag{3.43}$$

and

$$\frac{Y_{z,t+1} - Y_{z,t}}{Y_{z,t}} = \frac{Q_{z,t+1} - Q_{z,t}}{Q_{z,t}}. \tag{3.44}$$

This implies that the growth rates of sectoral output equal the rate of change of average sectoral capital goods quality.

Before we get final expressions for the equilibrium deposit interest rate and also rates of aggregate output growth, we establish a result in the following Lemma that helps to simplify the rest of the analysis.

Lemma 3.1. *Innovators are indifferent between intermediate sectors m and z, i.e.*

$$V_{m,t} = V_{z,t} \qquad t \geq 0, \tag{3.45}$$

if the growth rates of average sectoral capital goods qualities are identical:

$$g_{m,t} := \frac{Q_{m,t+1}}{Q_{m,t}} - 1 = \frac{Q_{z,t+1}}{Q_{z,t}} - 1 =: g_{z,t} \qquad t \geq 0. \tag{3.46}$$

Proof: See Appendix A.
The condition (3.45) states that in expected terms, it is equally profitable to invest in generating innovations on M-and Z-complementary capital goods. In this case, innovators are indifferent and R&D is directed to both sectors. Hence, the average productivity of the sectoral capital goods $Q_{m,t}$ and $Q_{z,t}$ can both grow. In fact, the lemma proves that in this case, the growth rates of $Q_{m,t}$, denoted by $g_{m,t}$, and $Q_{z,t}$, denoted by $g_{z,t}$ are identical and we can define

$$g_t := g_{m,t} = g_{z,t}. \tag{3.47}$$

Now take the deposit interest rate, as given in equation (3.7):

$$r_{t+1} = \frac{\alpha}{S_{t-1}}\Big(\iota_t \mathbb{E}_t[\Pi_{m,t+1}] + (1-\iota_t)\mathbb{E}_t[\Pi_{z,t+1}]\Big), \tag{3.48}$$

where $\mathbb{E}_t[\Pi_{j,t+1}] = \frac{\alpha}{1+\alpha}(1-\mu)\mu^{\frac{1+\mu}{1-\mu}}\eta_t \tilde{p}_{t+1}^{\frac{1}{1-\mu}}\tilde{Z}_{t+1}\tilde{Q}_{t+1}$. After elimination of endogenous prices etc., the equilibrium deposit interest rate reads:

$$r_{t+1} = \alpha_0 \bar{W}_{t+1}^{\frac{\alpha}{\sigma-1}}\left(\frac{\mu(1+g_t)^2}{\zeta\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}}\right)^{1-\alpha}, \tag{3.49}$$

where still $\alpha_0 := \alpha/(1+\alpha)$, $\bar{W}_t := \left(\gamma^{\varepsilon}M_t^{\sigma-1}\right)^{\frac{\alpha}{\varphi}} + \left((1-\gamma)^{\varepsilon}Z_t^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}$, $\varphi := 1-\sigma(1-\alpha)$. We use this deposit interest rate to get final expressions for the success-probabilities and equilibrium credit interest rates. First, insert (3.7) into (3.37) and (3.38). The probability of successful innovation in sector m then equals

$$\eta_{m,t} = \left(\frac{\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu(1+g_t)^2\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{1-\alpha}{\varphi}}}\right)^{\alpha} \tag{3.50}$$

and in sector z

$$\eta_{z,t} = \left(\frac{\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu(1+g_t)^2\left((1-\gamma)^{\varepsilon}Z_{t+1}^{\sigma-1}\right)^{\frac{1}{\varphi}}}\right)^{\alpha}. \tag{3.51}$$

Now we are in a position to derive equilibrium credit interest rates. Recall that credit interest in sector m was given by

$$R_{m,t+1} = \alpha\frac{\bar{\pi}_{m,t+1}Q_{m,t+1}}{S_{t-1}}\left(\frac{1}{1+\alpha} + \mathbb{E}_t[\tilde{\Pi}_{t+1}]\right) \tag{3.52}$$

and in sector z credit interest equals

$$R_{z,t+1} = \alpha\frac{\bar{\pi}_{z,t+1}Q_{z,t+1}}{S_{t-1}}\left(\frac{1}{1+\alpha} + \frac{1}{\mathbb{E}_t[\tilde{\Pi}_{t+1}]}\right) \tag{3.53}$$

where $\mathbb{E}_t[\tilde{\Pi}_{t+1}] = \tilde{\eta}_t\tilde{p}_{t+1}^{\frac{1}{1-\mu}}\tilde{Z}_{t+1}\tilde{Q}_{t+1}$. Insert prices, success-probabilities and aggregate savings from (2.43), this leads directly to the following credit interest rates. In sector m, the single representative bank demands a credit interest rate equal to

$$R_{m,t+1} = \frac{\alpha(\mu(1+g_t)^2}{(1+\alpha)} \tag{3.54}$$

and in sector z, innovators pay an interest rate on credit equal to

$$R_{z,t+1} = \frac{\alpha\mu(1+g_t)^2}{(1+\alpha)}\left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}. \tag{3.55}$$

To see that the growth rate of sectoral output $g_t := g_{m,t} = g_{z,t}$ is also the rate of final output, consider the production function for final output given in (2.23) for period $t+1$:

$$Y_{t+1} = \left(\gamma Y_{m,t+1}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_{z,t+1}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3.56}$$

Using the relationship between the dynamic development of average capital goods quality and intermediate output in sector $j = \{m, z\}$ as given in (3.43) and (3.44) final output in period $t+1$ reads

$$Y_{t+1} = \left(\gamma\Big((1+g_{m,t})Y_{m,t}\Big)^{\frac{\varepsilon-1}{\varepsilon}} + \Big((1-\gamma)(1+g_{z,t})Y_{z,t}\Big)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3.57}$$

Since $g_{m,t} = g_{z,t} = g_t$ we get

$$Y_{t+1} = (1+g_t)\left(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = (1+g_t)Y_t. \tag{3.58}$$

This shows that final output also grows with a rate equal to g_t. Furthermore, on the balanced growth path, output and consumption growth must be equal.[48] This argument can be proved by showing that consumption growth of old workers, entrepreneurs and innovators is identical and equal to the rate of output growth.[49]

[48]In equilibrium, the condition $Y_t = C_t + X_t + Ht$ holds for all $t \geq 0$. Then, in a BGP, Y, C, X, H must all grow at the same rate: Suppose, consumption grows at a maximum rate (compared to X, H) on the right-hand side of this equation. Then, this must also be the rate at which output grows, because otherwise, one side of the equation would become negligible asymptotically and the equation cannot hold. Now, substract consumption C_t from both sides. We then get $Y_t - C_t = X_t + H_t$, where the left-hand side is strictly positive and grows at a rate greater than all terms on the right-hand side. This is clearly impossible. It follows that all variables on the right-hand side have to grow at the rate of consumption growth. Therefore, all variables grow at the same rate.

[49]Just take the consumers' income profiles for two consecutive periods, use the dynamic development of capital goods quality together with the fact that consumers spend their income entirely, it follows that:

$$\frac{C_{W,t+1}^{(o)}}{C_{W,t}^{(o)}} = \frac{C_{e,t+1}^{(o)}}{C_{e,t}^{(o)}} = \frac{C_{i,t+1}^{(o)}}{C_{i,t}^{(o)}} = 1 + g_t.$$

The equilibrium mass of innovators in sector $j = \{m, z\}$ is given by:[50]

$$\iota_{m,t} = \frac{\eta_{z,t}\bar{\lambda}_z}{\eta_{m,t}\bar{\lambda}_m + \eta_{z,t}\bar{\lambda}_z}, \tag{3.59}$$

$$\iota_{z,t} = \frac{\eta_{m,t}\bar{\lambda}_m}{\eta_{m,t}\bar{\lambda}_m + \eta_{z,t}\bar{\lambda}_z}, \tag{3.60}$$

where $\bar{\lambda}_j := \lambda_j - 1$ and λ_j represents the exogenous and constant sector-specific rise of quality in case of innovation. After elimination of the $\eta_{j,t}$'s the mass of innovators in sector m reads:

$$\iota_{m,t} := \iota_t = \frac{\bar{\lambda}_z}{\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m}, \tag{3.61}$$

and in sector z the mass of innovators equals

$$\iota_{z,t} := 1 - \iota_t = \frac{\bar{\lambda}_m}{\bar{\lambda}_m + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{-\alpha}{\varphi}}\bar{\lambda}_z}. \tag{3.62}$$

Finally, the equilibrium growth rate of the economy is given by an implicit expression:

$$g_t(1+g_t)^{2\alpha} = \frac{\bar{\lambda}_z\bar{\lambda}_m}{\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m}\left(\frac{\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu}\right)^{\alpha}. \tag{3.63}$$

Note that the term on the right hand side is constant over time, as long as the factors M and Z are constant. Define this term as

$$\bar{\Lambda} := \frac{\bar{\lambda}_z\bar{\lambda}_m}{\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m}\left(\frac{\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu}\right)^{\alpha}. \tag{3.64}$$

So in equilibrium, the growth rate has to be such that the equality condition in (3.63) holds. This is equivalent to the problem of finding the root(s) of the function

$$F(g_t) := g_t(1+g_t)^{2\alpha} - \bar{\Lambda}. \tag{3.65}$$

The economy's growth rate is unique if the function $F(\cdot)$ has only one root. The following Lemma proves the conditions under which F has one zero point. We use the results to characterize the balanced growth path equilibrium afterwards.

[50] This expressions can be computed as follows. We use the result of Lemma 3.1 that given innovation in both sectors, the growth rates have to be equal: $g_{m,t} = g_{z,t}$. Insert the corresponding terms for the endogenous sectoral growth rates given in (3.39) and (3.40)

$$\iota_t\eta_{m,t}(\lambda_m - 1) = (1-\iota_t)\eta_{z,t}(\lambda_z - 1)$$

and simply solve this expression for ι_t to get the expression in the text.

Lemma 3.2. *Let the function F given in (3.65) be defined on the interval $[g_u, g^o]$, with $0 < g_u < g^o$. Then for appropriate values of g_u, g^o there exist one g_t^* such that $F(g_t^*) = 0$.*

Proof: See appendix A.
Together, these derivations prove the following

Proposition 3.5. *Consider the directed technological change model described above. Then, there exists a unique balanced growth path equilibrium in which the relative technologies are given by*

$$\tilde{Q}^* = \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{\varphi}}, \quad \text{with } \varphi := 1 - \sigma(1-\alpha). \tag{3.66}$$

The interest rate of the economy is given by (3.49), and consumption, sectoral and aggregate output grow at the rate stated in (3.63).[51]

3.4. Comparative statics

It is also useful, to analyze the comparative static influence of (some of) the exogenous parameters on the rate of aggregate economic growth g^*.
The implicit growth rate g_t^* depends positively on the parameter $\bar{\Lambda}$: the greater $\bar{\Lambda}$ the higher has g_t^* to be in order to fulfill equation (3.63). So we have to analyze the response of $\bar{\Lambda}$ to parameter changes to get the response of g_t.
First, the parameter λ_j measures the size of a quality increase in sector $j \in \{m, z\}$ in case of successful innovation. Since $\bar{\lambda}_j := \lambda_j - 1$, the following influence of $\bar{\lambda}_j$ on g_t^* holds:

Proposition 3.6. *The parameter $\bar{\Lambda}$ increases with the "size" of innovation in sector $j \in \{m, z\}$, given by $\bar{\lambda}_j$. Hence, the greater $\bar{\lambda}_j$, the higher is the equilibrium growth rate g_t^*.*

Proof: See appendix A.
An increase in λ_j leads to a larger quality gaps between newly invented and existing capital goods. Since the growth rate is given by the average rise in capital goods quality, this directly implies a greater rate of economic growth.
Second, consider the influence of the cost of research on $\bar{\Lambda}$ respectively g_t^*, given by ζ:

Proposition 3.7. *$\bar{\Lambda}$ is decreasing in the cost of research ζ. Therefore, the greater ζ, the lower is the equilibrium growth rate g_t^*.*

Proof: See appendix A.
A greater cost of research certeris paribus leads to lower probabilities of innovation success. For given masses of innovators per sector, the mass of those who are successful in the research lab declines. So on average, the change in sectoral capital goods quality declines. Since in this study, the engine of economic growth is the rise capital goods quality, this already implies the stated influence of ζ on g_t^*.
Third, μ affects $\bar{\Lambda}$ respectively g_t^*, where μ is a technology parameter in the intermediate production function and can be interpreted as the factor cost share of capital goods in total intermediate production costs and we have:

[51] See also Acemoglu (2009) p. 509

Proposition 3.8. $\bar{\Lambda}$ *and so the equilibrium growth rate* g_t^* *is increasing in* μ*.*

Proof: See appendix A.
A greater μ ceteris paribus leads to larger capital goods demand. This implies increasing profits from capital goods supply and so returns on R&D increase. Since R&D effort is a constant fraction of profits, this leads to greater R&D spending. The probability of innovation-success increase and so on average, changes in sectoral capital goods quality rises. This already implies a greater rate of economic growth g_t.

4. Concluding Remarks I

The formal framework presented in this part combines the results of existing models of a) endogenous directed technical change and b) endogenous growth models featuring financial intermediation to analyze the impact of credit and banks on the rate and more importantly on the direction of technical change.
In general, endogenous directed technical change models allow innovations to be directed towards different factors or sectors: firms can invest resources to develop new technologies complementing a particular factor or sector. Thereby, the relative profitability of generating innovations for these specific technologies constitute the incentive to invent higher quality commodities, i.e. relative profits determine the direction of technical change.
I confirm and extend the findings of current research and show that four market forces determine the relative profitability of the different types of technologies:

- the price effect, which encourages innovations on technologies that use the more expensive (scarce) factors;
- the market size effect, which directs innovations towards technologies that use the more abundant factor;
- the productivity effect, which increases the incentive to develop technologies for sectors with a higher productivity;
- and the risk effect, which creates incentives to develop technologies complementing factors where the risk of failure during the innovation process is lower.

The price effect and the market size effect are not a result of capital constraints and financial intermediation. These two forces work in opposite directions. The former directs innovations towards technologies using relatively more scarce (and therefore more expensive) factors and the latter creates a bias towards technologies complementing relatively more abundant (and therefore less expensive) factors.
This risk effect however results from capital constraints and financial intermediation and represents the first major result of this thesis: if innovators are capital constrained and therefore need credit to finance their business expenditures, cost minimizing behavior then directs innovation effort to those sectors or factors, where the cost of credit is lower. Since banks demand higher loan interest rates on projects with greater risk of default, this implies ceteris paribus that the risk effect directs innovations to sectors with lower risk of failure during the innovation process.
Furthermore, the elasticity of substitution between the factors has a crucial role in determining the relative power of the price and the market size effect. If the two intermediate goods and, therefore, the two factors are gross substitutes, the market size effect is relatively more powerful than the price effect. Therefore technical changes are directed towards the factor with the larger market. In contrast, if the two intermediate goods and therefore also the two factors are gross complements, the price effect is relatively more powerful and technical changes are directed towards the relatively more scarce factor.
This role of the elasticity of substitution remains unaltered if the risk effect is an additional

component in the determinants of directed technical change. The presence of capital constraints and banks, however, influences the response of relative profits of technology invention to an increase in relative factor supply: If the factors are gross substitutes, relative profits including the risk effect respond more strongly to an increase in the relative factor supply compared to relative profits without the risk effect and if the factors are gross complements, relative profits including the risk effect responds less strongly. This response is quantitatively even stronger if we consider the risk effect in the determinants of directed technical change. So the presence of capital constraints and banks lead to a stronger response of relative profits to factor supply changes.

Finally, the analysis also yields the “relative equilibrium bias hypothesis”: irrespective of the elasticity of substitution, an increase in the relative supply of a factor always induces technological change that is biased in favor of that factor. This implies that the market size effect is always relatively more powerful than the price effect.

A. Appendix: Mathematical proofs part I

Proposition 2.1

Proof. This follows simply from the two equations in (2.3) for sector m and Z: In both expressions, the denominator on the right hand side is a product of variables with values in $[0,1]$ for η_J and values in $]0,1[$ for Λ respectively and therefore the denominator itself is a number between 0 and 1. Thus, the stated wedge between interest rates $R_{J,t+1} > r_{t+1}$ holds for all t, with $J = \{M, Z\}$. □

Proposition 2.2

Proof. The proof is straightforward: In general $\eta_M \neq \eta_Z$ is possible. Without loss of generality, suppose $\eta_M > \eta_Z$. In order to ensure the relationship stated in equation (2.3), $R_{M,t} < R_{Z,t}$ has to hold. □

Proposition 2.3

Proof. To simplify notation I suppress the index J in the following proof. Let $X = x(p(m,\vartheta))$ denote the demand of the capital good line by some intermediate sector, where $p(m,\vartheta)$ denotes the price of the commodity and X the demand at this price. In section 4.3.2 I show that $x(\cdot)$ is strictly decreasing, differentiable and satisfies $x(1) > 0$, where the marginal cost of production $\psi \equiv 1$. Finally, let the elasticity of demand be defined as

$$\sigma(p(m,\vartheta)) \equiv -\frac{p(m,\vartheta)\frac{\partial x(p(m,\vartheta))}{\partial p(m,\vartheta)}}{x(p(m,\vartheta))}$$

with $\sigma \in]1,\infty[$. Suppose a large number of firms $N \in \mathbb{N}$ have access to the same production technology and they produce ϑ with the same quality $q(m)$. Suppose now, an arbitrary firm, say firm 1, gets access to a research technology such that this firm can improve the quality of ϑ from $q(m)$ to $q(m+1)$ without any uncertainty, by expanding $\varphi > 0$ units of effort. Finally let the innovation be nonrival and nonexcludable due to a lack of a working patent law.

Then, the profits for the different firms are as follows: Before firm 1 invents the new commodity, all firms use the same production technology. Therefore, the equilibrium price equals marginal costs of production $p(m,\vartheta) = 1$. The profit of firm $i = 1,...,N$ is then $\Pi_A^{(i)} = (p(m,\vartheta) - 1)x(p(m,\vartheta))^{(i)} = 0$, where A simply denotes the equilibrium before innovation. If firm 1 pays φ and innovates, then due to non-excludability all other firms will also produce the new commodity with quality $m+1$. Therefore, the equilibrium quality adjusted price equals marginal costs $p(m+1,\vartheta) = 1$. Hence, the profit of firm $i = 2,...,N$ is $\Pi_B^{(i)} = (p(m+1,\vartheta) - 1)x(p(m+1,\vartheta))^{(i)} = 0$. The net profit of firm 1 is $\Pi_B^{(1)} = (p(m+1,\vartheta)-1)x(p(m+1,\vartheta))^{(1)} - \varphi = -\varphi < 0$. Thus $\Pi_A^{(1)} > \Pi_B^{(1)}$. So if firm 1 would innovate, it would have to bear the fixed cost of research but could not charge a price to cover these costs and would therefore lose money. Consequently there is no incentive to innovate in a competitive market for capital goods. □

Proposition 2.4

Proof. i) Take the relative demand for the intermediate goods stated in equation (2.25) for any two goods i, j, solve for Y_i and insert the result into the total production costs:

$$\sum_{i=1}^{N} p_i Y_i = \sum_{i=1}^{N} p_i \Big(\frac{\gamma_j}{\gamma_i}\Big)^{-\varepsilon} \Big(\frac{p_i}{p_j}\Big)^{-\varepsilon} Y_j = \gamma_j^{-\varepsilon} p_j^{\varepsilon} Y_j \sum_{i=1}^{N} \gamma_i^{\varepsilon} p_i^{1-\varepsilon} = \Big(\frac{p_j}{\gamma_j}\Big)^{\varepsilon} Y_j \sum_{i=1}^{N} \gamma_i^{\varepsilon} p_i^{1-\varepsilon}. \tag{A.1a}$$

ii) Since the production of the final output uses N intermediate inputs, equation (2.23) changes to

$$Y = \Big(\sum_{i=1}^{N} \gamma_i Y_i^{\frac{\varepsilon-1}{\varepsilon}}\Big)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Use this expression and the relative factor demand from (2.25) for any two goods i, j to manipulate the firms return PY:

$$\begin{aligned} PY &= P\Big(\sum_{i=1}^{N} \gamma_i Y_i^{\frac{\varepsilon-1}{\varepsilon}}\Big)^{\frac{\varepsilon}{\varepsilon-1}} = P\Big(\sum_{i=1}^{N} \gamma_i \Big(\Big(\frac{\gamma_j}{\gamma_i}\frac{p_i}{p_j}\Big)^{-\varepsilon} Y_j\Big)^{\frac{\varepsilon-1}{\varepsilon}}\Big)^{\frac{\varepsilon}{\varepsilon-1}} \\ &= P\Big(\sum_{i=1}^{N} \gamma_i \Big(\frac{\gamma_j}{\gamma_i}\frac{p_i}{p_j}\Big)^{1-\varepsilon} Y_j^{\frac{\varepsilon-1}{\varepsilon}}\Big)^{\frac{\varepsilon}{\varepsilon-1}} = P\Big(\Big(\frac{p_j}{\gamma_j}\Big)^{\varepsilon-1} Y_j^{\frac{\varepsilon-1}{\varepsilon}} \sum_{i=1}^{N} \gamma_i^{\varepsilon} p_i^{1-\varepsilon}\Big)^{\frac{\varepsilon}{\varepsilon-1}} \\ &= P\Big(\frac{p_j}{\gamma_j}^{\varepsilon} Y_j \Big(\sum_{i=1}^{N} \gamma_i^{\varepsilon} p_i^{1-\varepsilon}\Big)^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned} \tag{A.1b}$$

Now combine the right hand sides of equations (A.1a) and (A.1b) to get

$$P\Big(\sum_{i=1}^{N} \gamma_i^{\varepsilon} p_i^{1-\varepsilon}\Big)^{\frac{\varepsilon}{\varepsilon-1}} = \sum_{i=1}^{N} p_i^{1-\varepsilon} \gamma_i^{\varepsilon}.$$

Solve for P gives the final result and proves the corollary stated in the text:

$$P = \Big(\sum_{i=1}^{N} p_i^{1-\varepsilon} \gamma_i^{\varepsilon}\Big)^{\frac{1}{1-\varepsilon}} \equiv 1. \tag{A.1c}$$

□

Proposition 3.2

Proof. The proof simply follows from comparison of the exponents in the two expressions: The statement is true, if $\frac{1}{\sigma} < \frac{1}{\sigma(1-\alpha)}$ which holds if $\alpha > 0$. Since by assumption $\alpha \in]0, 1[$, the condition is always fulfilled. □

Lemma 3.1

Proof. Suppose condition (3.45) holds and

$$V_{m,t} = V_{z,t} \qquad t \geq 0. \tag{A.2}$$

This is equivalent to

$$\tilde{V}_t = 1 \qquad t \geq 0. \tag{A.3}$$

To prove the statement in the Lemma, we have to show that this implies

$$g_{m,t} := \frac{Q_{m,t+1}}{Q_{m,t}} - 1 = \frac{Q_{z,t+1}}{Q_{z,t}} - 1 =: g_{z,t} \qquad t \geq 0. \tag{A.4}$$

The relative productivity of sectoral average capital goods quality was given in (3.32):

$$\tilde{Q}_t = \left(\tilde{\gamma}^{-\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{1-\sigma(1-\alpha)}} \quad \forall t \geq 0. \tag{A.5}$$

Solve for $Q_{z,t}$:

$$Q_{z,t} = \left(\tilde{\gamma}^{-\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{1-\sigma(1-\alpha)}} Q_{m,t} \quad \forall t \geq 0. \tag{A.6}$$

The growth rates $g_{m,t}$ and $g_{z,t}$ are equal to

$$\frac{Q_{m,t+1}}{Q_{m,t}} = 1 + \iota\eta_m\left(\lambda_m - 1\right) \qquad \text{for all } t \geq 0 \tag{A.7}$$

and

$$\frac{Q_{z,t+1}}{Q_{z,t}} = 1 + (1-\iota)\eta_z\left(\lambda_z - 1\right) \qquad \text{for all } t \geq 0. \tag{A.8}$$

Since (A.5) holds for all $t \geq 0$, we have

$$Q_{z,t+1} = \left(\tilde{\gamma}^{-\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{1-\sigma(1-\alpha)}} Q_{m,t+1}. \tag{A.9}$$

Now insert (A.9) into the enumerator and (A.6) into the denominator on the left-hand side of (A.8) yields:

$$1 + g_{z,t} = \frac{Q_{z,t+1}}{Q_{z,t}} = \frac{\left(\tilde{\gamma}^{-\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{1-\sigma(1-\alpha)}} Q_{m,t+1}}{\left(\tilde{\gamma}^{-\varepsilon}\tilde{Z}^{\sigma-1}\right)^{\frac{1}{1-\sigma(1-\alpha)}} Q_{m,t}} = \frac{Q_{m,t+1}}{Q_{m,t}} = 1 + g_{m,t}. \tag{A.10}$$

Hence $g_{m,t} = g_{z,t}$ and we can conclude that if the productivity of capital goods in both sectors grows, the growth rates are identical. This proves the statement in the lemma. □

Lemma 3.2
Proof. The function F is defined on the interval $[g_u, g^o]$ with $0 < g_u < g^o$ and according to (3.65) given by

$$F(g_t) := (g_t(1 + g_t)^{2\alpha} - \bar{\Lambda}. \tag{A.11}$$

First, note that $F(\cdot)$ is continuous on its domain and strictly increasing in its argument g_t. Then, for a given parameter value $\bar{\Lambda} > 0$, we can choose a g_u such that $g_u < \bar{\Lambda}$. Then $F(g_u) < 0$. Since $\bar{\Lambda}$ is constant, we can find a g^o with $g^o > \bar{\Lambda}$. Then $F(g^o) > 0$. Together this implies $F(g_o)F(g^u) < 0$ and we can conclude using the intermediate value theorem that g_t^* in $]g_u, g^o[$ exist with $F(g_t^*) = 0$. This already shows that g_t^* is a root of F. Uniqueness of g_t^* follows, since F is strictly increasing in g_t. Therefore, the growth rate of the balanced growth equilibrium is unique. □

Derivation of $\tilde{\omega}$ (equation (3.30))

Proof. The first order optimality conditions for the factors Z and Z and in intermediate goods production are given by

$$(1-\mu)p_m M^{-\mu}\Big(\int_0^1 q_{\vartheta_m}^{1-\mu} x_{\vartheta_m}^{\mu} d\vartheta_m\Big) = w_M, \tag{A.12a}$$

$$(1-\mu)p_z Z^{-\mu}\Big(\int_0^1 q_{\vartheta_z}^{1-\mu} x_{\vartheta_z}^{\mu} d\vartheta_z\Big) = w_Z. \tag{A.12b}$$

The capital goods demand functions of type ϑ_j of quality q in sector $j = \{m, z\}$ are equal to

$$x_{\vartheta_m} = \left(\mu^2 p_m\right)^{\frac{1}{1-\mu}} q_{\vartheta_m} M. \tag{A.13a}$$

$$x_{\vartheta_z} = \left(\mu^2 p_z\right)^{\frac{1}{1-\mu}} q_{\vartheta_z} Z. \tag{A.13b}$$

Inserting these capital goods demand functions into the corresponding equations (A.12a), (A.12b) respectively and take the ratio yields:

$$\tilde{w} := \frac{w_z}{w_m} = \left(\frac{p_z}{p_m}\right)^{\frac{1}{1-\mu}}\left(\frac{Q_z}{Q_m}\right) = \tilde{p}^{\frac{1}{1-\mu}}\tilde{Q}. \tag{A.14}$$

Insert $\tilde{p}$ from (3.5) gives (3.30) in the text:

$$\tilde{w} = \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}^{-1}\tilde{Q}^{\sigma-1}\right)^{\frac{1}{\sigma}}. \tag{A.15}$$

□

Proposition 3.6

Proof. To prove the proposition, simply calculate the partial derivative of $\bar{\Lambda}$ with respect to $\bar{\lambda}_m$ and $\bar{\lambda}_z$:

$$\frac{\partial\bar{\Lambda}}{\partial\bar{\lambda}_m} = \frac{\bar{\lambda}_z^2}{\left(\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m\right)^2}\left(\frac{\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}(1-\mu)\mu}\right)^{\alpha} = \frac{\bar{\lambda}_z\bar{\lambda}_m^{-1}\bar{\Lambda}}{\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m} > 0. \tag{A.16}$$

and

$$\frac{\partial\bar{\Lambda}}{\partial\bar{\lambda}_z} = \frac{\bar{\lambda}_z\bar{\lambda}_m^{-1}\bar{\Lambda}\left((1-\gamma)^{\varepsilon}Z_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}}{\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m} > 0. \tag{A.17}$$

Thus the greater the size of innovation λ_j, the greater is $\bar{\Lambda}$ and so the equilibrium growth rate g_t^*. □

Proposition 3.7

Proof. To prove the proposition, we compute the partial derivative of $\bar{\Lambda}$ with respect to ζ:

$$\frac{\partial \bar{\Lambda}}{\partial \zeta} = \frac{-(1-\alpha)\zeta^{-(2-\alpha}\bar{\lambda}_z}{\bar{\lambda}_z + \left(\tilde{\gamma}^{\varepsilon}\tilde{Z}_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{\lambda}_m}\left(\frac{\left(\gamma^{\varepsilon}M_{t+1}^{\sigma-1}\right)^{\frac{\alpha}{\varphi}}\bar{W}_{t+1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}(1-\mu)\mu}\right)^{\alpha} = -\frac{(1-\alpha)\bar{\Lambda}}{\zeta} < 0. \tag{A.18}$$

□

Proposition 3.8

Proof. The partial derivative of $\bar{\Lambda}$ with respect to μ gives:

$$\frac{\partial \bar{\Lambda}}{\partial \mu} = -\frac{\alpha\bar{\Lambda}}{\mu} < 0, \tag{A.19}$$

so larger values of μ imply lower values of $\bar{\Lambda}$ and thus the larger μ the lower is the equilibrium growth rate g_t^*. □

Part II

THE ENVIRONMENT, BANKS, AND DIRECTED TECHNICAL CHANGE

Introduction

Economic growth is associated with increasing consumption of fossil fuels and their combustion emits greenhouse gases, which are detrimental to the earth's climate. Global carbon emissions from fossil fuel combustion and cement production have significantly increased since 1900. Emissions increased by over 17 times between 1900 and 2010 and by about 1.5 times between 1990 and 2010.[52] If greenhouse gases (GHG's) continue to increase in the future, the effects of human induced climate change such as hurricanes, floods and periods of aridity will occur more frequently with corresponding negative effects on economic growth.

To alleviate these growth-damping effects, the industrialized nations are faced with the ambitious task to decouple economic growth and further increases of emissions. This decoupling can be achieved for instance through (a combination of): (i) the invention of emission-extensive or even emission-neutral production technologies, (ii) a shift in sectoral industrial production from GHG-intensive sectors towards GHG-extensive sectors, (iii) within production sectors, the substitution of emission intensive production inputs by emission neutral inputs (for instance the substitution of electricity from fossil fuels by electricity from wind, water, nuclear and solar power etc.). The present study concentrates on (ii), the shifts in intermediate production from carbon intensive to carbon extensive production, i.e. *inter-sectoral structural change*, especially analyzing the role of financial intermediation.

This is well worth analyzing, because the previous parts suggest a systematic link between the rate and direction of technical change and financial intermediation. Therefore, environmental or climate policies that aim at shifting technical changes from "dirty" sectors or factors towards "green" sectors/factors, have to take the effects of financial intermediation into account as well.[53] Thus we build on the findings of part one and adjust the developed model to additionally account for environmental constraints and examine the following stylized research question of part two: *What role do financial intermediaries play in directing technical change towards "green" technologies?*.

Thereby, the second part combines elements from different strands of literature and is therefore related to each of them. The basis of the first part and thus also of the current second part is the literature on endogenous directed technological change (Acemoglu (1998, 2002, 2007)): innovators have a choice to increase the quality of capital goods in either a "green" or a "dirty" intermediate sector. Second, the explicit consideration of financial intermediation builds on the findings of Schumpeter (1912), Boyd and Prescott (1986), King and Levine (1993), Levine (1997) that banks do matter for economic development. Here, innovators lack the capital resources to finance their ventures and banks provide necessary funds. A consequence of these results is that banks might also matter for the direction of "green" technical change. Third, the thesis is connected to the growing

[52] Data source: U.S. Department of Energy, Carbon Dioxide Information Analysis Center.

[53] In the sequel, the study uses the terms "green" and "dirty" to describe an economy that is characterized by low levels of atmospheric carbon concentration and small damages to aggregate output respectively an economy with high levels of carbon concentration in the atmosphere with corresponding large damages to aggregate output.

literature on quantitative general equilibrium models of climate change (Nordhaus (1994, 2008), Hassler et al. (2012), Krusell and Smith (2009), Acemoglu et al. (2012), Golosov et al. (2014)). I follow these studies and implement a simple model of the atmospheric carbon cycle within the general equilibrium model developed in the first part.
The rest of the second part is structured as follows: The next section presents the model and derives the microeconomic decision problems (section 5). Afterwards, the study defines and describes the corresponding equilibrium and derives the answer to the stylized research question formulated above (section 6). This second part ends with some concluding remarks (section 7).

5. The Model

We start again with a brief overview of the model and highlight the changes in the formal framework in comparison to the model develpoed in part one. It will turn out that most of the adjustments apply to the economy's production side.

5.1. The Economy

Still, the economy evolves infinitely in discrete time t, $t \in \{0, 1, 2, ...\}$ and in each t, a continuum of overlapping generations populate the economy, whose life span being divided into two periods. Population is constant over time. So at each $t \geq 0$, two different generations populate the economy. Each generation consists of workers, entrepreneurs and innovators who differ in terms of their access to investment projects and consumption profiles. Hence the model features heterogeneous consumers.
The other entities in the economy are: a banking sector and a "real" side of goods production. The production side exhibits a downstream structure: during the first stage, firms produce a variety of capital goods, the second stage contains two heterogeneous intermediate production sectors and during the last stage, firms produce one unique final good. The final good can be consumed, transferred into future periods and invested in capital goods production or in R&D. The final or consumption good serves as the numeraire: all prices, returns, payments and costs are measured in terms of the consumption good. The type of "agents" and market types remain unchanged and three different types of "agents":

- Banks,
- Firms (final goods, intermediate goods, capital goods), and
- Consumers (workers, entrepreneurs and innovators)

act on three different market types:

- Goods markets (Final output, intermediate goods, capital goods),
- Capital markets (Deposit market, credit market), and a
- Labor market.

Figure 4 shows the different model entities and their mutual market interactions schematically. Red lines and red tagged areas indicate model changes compared to the model presented in part one.

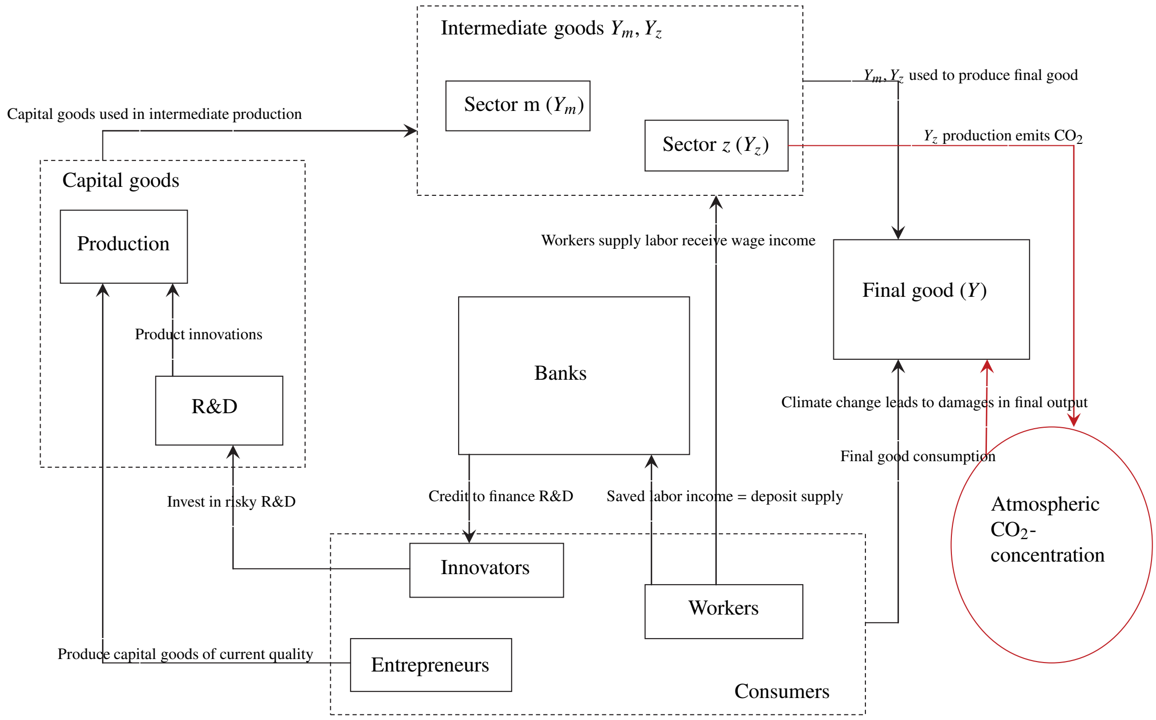


FIGURE 4: The structure of the model

Banks

Financial intermediation is represented by a single aggregate competitive bank. The formulation of financial intermediation remains unchanged compared to the model developed in part one: The banking sector demands the savings of young workers and repay these deposits plus interest back to then old workers in the following period. The savings are pooled and as credit supplied to the innovators. The bank takes deposit interest rates and credit interest rates as exogenously given and chooses sectoral credit supply and deposit demand to maximize expected profits from financial intermediation.

Goods Production

In the first stage, firms combine the two different goods from the upstream intermediate stage to produce the unique final good. Still, inputs are instantaneously converted to output, so decisions are static in this stage. Final output producers choose production levels and input quantities to maximizes profits and take factor input prices and output prices as given. Note that in contrast to the first part, final output is negatively affected by CO_2-emissions. This modeling of a negative external effect allows us to use the model framework and general results of the first part to analyze the role of banks, if technical change needs to be directed away from carbon-intensive industrial production towards carbon-extensive, or even carbon-neutral goods production.

In the two sectors of the second downstream intermediate stage, price taking firms pro-

duce a sector specific good and supply this output to the intermediate goods market. To avoid confusion, note that the firms in the intermediate and the final output stage are completely different firms. Intermediate firms convert inputs instantaneously into output. Hence no intertemporal decisions occur in this intermediate stage. Firms in this stage take wage and machine prices as given and set production quantities to maximize profits.
In comparison to the model of part one, two major differences apply in the intermediate stage: First, intermediate production now emits carbon. More precisely, while production of the good Y_m is "green" in the sense that it creates no carbon emissions, production of Y_z is now "dirty", because it emits a constant quantity of carbon per unit of output. Emissions enter the atmosphere and increase global carbon concentration. This leads to a rise in global mean temperature with negative effects on the economy. These negative effects are measured in percentage damage to final output. Second, and of less importance, sectoral production in both sectors no longer uses the two different factors M_t (labor) and Z_t (another unspecified production factor). Instead, both firms use labor, denoted by $L_{j,t}$, $j \in \{m, z\}$ and still a continuum variety of sector-specific capital goods (machines) x_{ϑ_j} of different quality q_{ϑ_j}. Note that labor here is not of different quality in the sense that the wages in the two sectors differ. So in the sequel, intermediate firms in both sectors pay the wage w_t to the workers. This adjustment helps to simplify the analysis.
In the third production stage, no model adjustments are necessary. Still, entrepreneurs and innovators produce sector specific capital goods and supply these "machines" to the intermediate firms. Young innovators need credit to finance R&D expenditures. Capital goods exhibit different levels of quality. Naturally, better-quality goods are more productive in manufacturing the intermediate goods. The rise in quality represents technical progress.

Consumers

Consumers in the model are the workers, entrepreneurs, and innovators. We adjust the model and introduce lump-sum taxes levied on old workers' income to finance subsidies on R&D and lump-sum transfers to old workers from tax revenue on dirty production. The rest of the consumer side remains unchanged: each consumer lives for two consecutive periods. Workers are endowed with one unit of labor time and supply their unit of labor on the labor market to the intermediate firms of sector m and z, earn an income from wage payments and save their entire income to finance second period of life consumption. Hence, workers transfer their current wealth into the next period.
Entrepreneurs receive patents to produce capital goods of current quality for the two intermediate sectors. When old, they consume the profits from capital goods selling.
Innovators invest in R&D to improve the quality of existing capital goods. Investments are financed by credit. Young innovators initially select either one of the two intermediate sectors to potentially invent a new capital good of greater quality for. The expected profits from innovation determine whether technical progress is directed to one or the other or both sectors, because innovatiors choose those sectors that promise the highest return. This search for sectoral profits is the engine that drives directed innovations. In their second period of life, old innovators then receive the profits from monopolistic capital goods supply. Since individuals have no bequests, they spend the profits entirely on final goods consumption.

Before we consider the adjustments and decision problems in detail, we describe how aggregate carbon emissions behave over time and illustrate how aggregate emissions affect final output and economic growth.

5.1.1. The Carbon Cycle

In this study, I assume that the effect of emissions is to raise the atmospheric CO_2-concentration, that a larger concentration of CO_2 increases the global mean surface temperature, and the rise in temperature then translates into economic damages.

Naturally, parts of the carbon emitted into the atmosphere by burning fossil fuels immediately exit into the biosphere and the surface oceans, while other parts decay slowly in time or even remain permanently in the atmosphere. This reduction, absorption and emission of carbon is commonly referred to as the (atmospheric) carbon cycle.[54] There is a large number of studies featuring various degrees of integration between carbon emissions, climate changes and economic development. A review of these studies is beyond the scope of this thesis. The interested reader may start with Nordhaus (2011) and the references therein. With respect to the degree of integration, my approach to modelling economic development and climate change can be seen as a rather lean *integrated assessment model* of climate change, since for instance the depreciation structure of carbon will be linear by assumption and the response of global mean temperature to the carbon concentration in the atmosphere here will be immediate.

The next assumption specifying this approximation of the carbon cycle in the atmosphere, is taken from Golosov et al. (2014) and based on the work of Nordhaus (1994, 2008) and Mendelsohn et al. (1994).

Assumption 5.1. *Let T denote the first date when emission started and denote total carbon emissions in period t by E_t. Then, the atmospheric carbon concentration in period t, A_t, is determined as follows:*

$$A_t - \underline{A} = \sum_{\tau=0}^{t+T} (1 - d_\tau) E_{t-\tau}, \tag{5.1}$$

where $\underline{A}$ is the pre-industrial level of atmospheric carbon concentration, $1 - d_\tau \in]0, 1[$ is the amount of carbon emitted τ periods into the future and still left in the atmosphere. The latter is determined by

$$1 - d_\tau = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^\tau, \tag{5.2}$$

where $\phi_L \in]0, 1[$ is the share of carbon emissions that remains permanently in the atmosphere, $(1-\phi_L)\phi_0 \in]0, 1[$ is the fraction of the remainder that quickly exits the atmosphere and $\phi \in]0, 1[$ is the (geometric) rate of decay of carbon concentration over time.[55]

[54] See for instance Falkowski et al. (2000)

[55] This formulation of the atmospheric carbon concentration is a generalized version of the formulation frequently used in the literature on economic growth and climate change. In many studies, the atmospheric carbon concentration is given by $A_{t+1} = \mu E_t + (1-\delta)A_t$ in discrete time and $\dot{A} = \mu E - \delta A$ in continuous time,

As in Golosov et al. (2014), the process of carbon concentration can be represented by a recursive vector representation where $A_{1,t}$ denotes permanent carbon in the atmosphere, and $A_{2,t}$ denotes carbon that depreciates at rate ϕ. Given the sequence of emissions $\{E\}_{t\geq 0}$ which are determined below, assumption 5.1 implies that the climate state evolves as

$$A_{1,t} = A_{1,t-1} + \phi_L E_t, \tag{5.3a}$$

$$A_{2,t} = (1-\phi)A_{2,t-1} + (1-\phi_L)\phi_0 E_t. \tag{5.3b}$$

The atmospheric CO_2-concentration is the sum of permanent and non-permanent emissions in the atmosphere and given by

$$A_t = A_{1,t} + A_{2,t}. \tag{5.4}$$

In general, there are several ways in which climate change harms economic development in practice. To name only a few, through a rise in sea-levels, floods, droughts, wildfires and extreme storms, climate change could destroy essential infrastructure such as homes, roads, bridges, railroad tracks, airport runways, power lines, dams, levees and seawalls. Moreover, disruptions in daily life related to climate change can mean lost work and school days and harm trade, transportation, agriculture, fisheries, energy production, and tourism. Severe rainfall events and snowstorms can cause power outages, snarl traffic, delay air travel and so on.
In this study I assume that CO_2-emissions do not affect production directly, but only indirectly by affecting the climate of the earth, which in turn damages economic development through the side effects mentioned. So to connect economic development with carbon concentration, we need a map from the stock of carbon dioxide in the atmosphere A_t to aggregate output Y_t, where this connection could in general be either positive or negative and in the sequel, I use the word "damage" in this sense.[56] One can conceive of this map as depicting the relation between carbon emissions and economic damages in of two steps. The first one relates the global carbon concentration in the atmosphere to climate, commonly represented by mean global temperature. The second one then relates climate to economic damages, the latter being measured as a percentage of final output. Accordingly, an increase in the atmospheric carbon concentration first would lead to a higher surface temperature and this increase in the temperature would affect economic growth

where the parameter μ represents the absorbtion rate and δ represents the inverse of the lifetime of CO_2. The difference between the formulation used here and the one frequently used in the literature comes from the assumption that some parts of the carbon emissions remain permanently in the atmosphere. For a fraction of permanent emissions equal to zero ($\phi_L = 0$) our formulation of the atmospheric carbon concentration reproduces the mentioned dynamic (difference or differential equation) form of the atmospheric carbon concentration. Therefore, the version used here contains the formulation commonly used in the literature as a special case.

[56]Roughly speaking, to analyze the effects of climate change on economic development, economist assume that CO_2-pollution (directly or indirectly) either affects (i) the utility of the consumers, or (ii) goods production function(s). Here, climate change affects the productivity of final output. In contrast to this, for instance Acemoglu et al. (2012) assume that consumers' utility depends positively on the quality of the environment, where goods production is associated with a decrease in the quality of the environment. So in their work, environmental degradation goes along with negative effects on consumers utility.

for instance through more extreme weather situations.[57] However, I skip this mapping from carbon concentration to global mean temperature by formulating economic "damages" directly as a function of atmospheric carbon concentration:

Assumption 5.2. *The damage function* $D : [\underline{A}, \infty[\longrightarrow [0, 1[$ *is differentiable, strictly increasing in the atmospheric carbon concentration* A_t *and defined as*

$$D(A_t) := 1 - e^{-\theta(A_t - \underline{A})}, \tag{5.5}$$

where $\theta \geq 0$ *is a scale parameter and* $\underline{A}$ *is the pre-industrial level of the atmospheric carbon concentration.*[58]

This function shows that the damage is higher, the higher the deviation of the actual global atmospheric carbon concentration A_t is from the pre-industrial carbon concentration level $\underline{A}$. Moreover, this assumption implies the global mean temperature and thereby the damage function to respond immediately to atmospheric carbon. One could also assume some delay in the reaction of temperature to carbon concentration; for instance Nordhaus (1994) assumes a slower temperature dynamic with respect to carbon concentration.
Climate scientists typically refer to a *climate catastrophe*, if the rise of global mean surface temperature reaches or lies above some critical level (e.g. a rise in global mean temperature of 3 degrees Celsius by the year 2100). Since this study does not consider the global mean temperature explicitly and the effects of climate change are measured in percentage deterioration of aggregate output, let us define a climate catastrophe slightly differently by putting forward the economic effects of climate change rather than the change of climate or temperature itself:

Definition 5.1. *A* climate catastrophe *occurs if* $D(A_t) > \bar{b}$ *for some* $t < \infty$ *and* $\bar{b} > 0$.

$D(A_t)$ represents the damages to final output in period t measured in percent and $\bar{b}$ represents a critical upper bound of those damages. So in the present study, the global economy faces a climate catastrophe, if the percentage damages to global GDP caused by high levels of atmospheric carbon concentration exceed a certain level (for instance if $\bar{b} = 10\%$, then a climate catastrophe occurs if climate change shrinks global economic output by more than 10%).
It remains to show how the different potential threats of climate change, which are encapsulated in the single damage function, translate into economic performance. By assumption, it is the economy's production side that emits CO_2 and it is also goods production that bears the damages associated with increasing atmospheric carbon concentration.

5.2. Firms

In the first production stage, now global warming affects final output negatively through the damage function formulated above. In the second stage of intermediate output, emissions of carbon originate in one of the two sectors. In the third stage, entrepreneurs and innovators produce capital goods that are used in intermediate production.

[57] See for instance Greiner et al. (2009).
[58] See Nordhaus (1994, 2008)

5.2.1. Final output

The final output sector contains a large number of identical and profit maximizing firms producing the unique consumption good. In addition, final output production is now negatively affected by the amount of atmospheric carbon concentration, as given in the assumptions 5.2 and 5.1 above. This leads to the following production function for final output:

Assumption 5.3. *The final good is produced using the production technology $\mathcal{F} : \mathbb{R}^2_+ \to \mathbb{R}_+$*

$$Y_t = (1 - D(A_t))\mathcal{F}(Y_{m,t}, Y_{z,t}) = (1 - D(A_t))\left(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5.6}$$

where $D(A_t) := 1 - e^{-\theta(A_t - \underline{A})}$ is the damage function, $\varepsilon \in [0, \infty[$ represents the constant elasticity of substitution between the inputs used in the production process, $\gamma \in]0, 1[$ is a distribution parameter and Y_m, Y_z represent intermediate input quantities.

In this stage, firms make no inter-temporal decisions. If $\varepsilon = \infty$, Y_m and Y_z are perfect substitutes. If $\varepsilon = 1$, the production function will be Cobb-Douglas and if $\varepsilon = 0$, aggregate production will be Leontieff. Throughout the rest of this thesis, we impose the following assumption on the elasticity of substitution:

Assumption 5.4. *The two intermediate goods Y_m and Y_z are gross substitutes ($\varepsilon > 1$).*

First, an elasticity of substitution $\varepsilon > 1$ between clean and dirty (energy-) production is frequently used in the literature. For instance Löschel et al. (2009) and also Acemoglu et al. (2012) assume gross substitutability between dirty and clean (energy-) inputs. In addition, an elasticity of substitution smaller than one would imply that especially the dirty input is an essential production input.

Given damage-adjusted productivity the firm in the final output sector takes the distribution parameter $\gamma > 0$, the price of the final good p, the prices of the two intermediate inputs p_m, p_z and the production technology stated in (5.6) as given. The profit maximization of the final goods sector (in period $t \geq 0$) reads:

$$\max_{(Y_m, Y_z) \in \mathbb{R}^2_+} \left\{ pY - (p_m Y_m + p_z Y_z) \quad | \quad Y = (5.6) \right\} \tag{5.7}$$

A solution to (5.7) satisfies the following first order conditions which equate prices and marginal products of each production factor for all $t \geq 0$:

$$p_{m,t} = P(1 - D(A_t))\gamma\left(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} Y_{m,t}^{-\frac{1}{\varepsilon}} \tag{5.8a}$$

$$p_{z,t} = P(1 - D(A_t))(1-\gamma)\left(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} Y_{z,t}^{-\frac{1}{\varepsilon}}. \tag{5.8b}$$

The consumption good serves as the numeraire. Hence, the price of the final output or consumption good p is normalized to $p_t \equiv 1$ in all periods $t \geq 0$. The normalization of the final output price implies a normalization of the prices of the intermediate goods.

Corollary (2.4) in part one describes this equivalence in price normalization of final output and intermediate goods prices for the case of N different intermediate goods. The case of two intermediate goods, as used in the present study, is then a special case of this corollary for $N = 2$ and we get:

$$p = \left(\gamma_m^{\varepsilon} p_m^{1-\varepsilon} + \gamma_z^{\varepsilon} p_z^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} = \left(\gamma^{\varepsilon} p_m^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_z^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \equiv 1. \tag{5.9}$$

5.2.2. Intermediate Goods Production

Recall that this study assumes that product innovations are vertically related to existing products. This means that newly invented products perform similar functions compared to those performed by already existing products, but offer greater *quality*. So the developed model equates economic growth with the rise of average capital goods quality. Thus, every capital goods' quality can be improved an unlimited number of times to ensure the possibility of permanent economic growth.

Capital goods are used in the economy's second downstream stage of goods supply, the intermediate stage. The intermediate stage consists of the two different sectors m and m. In both sectors, firms produce the intermediate goods denoted by Y_j, $j \in \{m, z\}$. Production inputs in both sectors are labor, denoted by L_j, $j \in \{m, z\}$ and a continuum variety of sector-specific capital goods (machines) x_{ϑ_j} of different quality q_{ϑ_j}, where $q_{n_{\vartheta_j}}$ denotes the quality of machine ϑ_j of generation $n = 0, 1, 2, ...$ in sector j (at time t) and x_{ϑ_j} denotes the input of capital good ϑ_j in sector j (at time t). Note that labor here is not of different quality in the sense that the wages in the two sectors differ. So in the sequel, intermediate firms in both sectors pay the wage w_t to the workers. The depreciation rate of the capital goods is set to 100%, i.e. they depreciate fully after use.

The set of product lines or industries, where each member of this set representing one line of capital good of probably infinite different qualities is constant through time. We impose the following assumption with regard to the set of capital goods:

Assumption 5.5. *Let the index $j = \{m, z\}$ denote the two intermediate sectors and let Θ_j denote the set of industries or capital goods in sector j. Then each $\vartheta_j \in \Theta_j$ corresponds to a different capital goods line (machine type). The set of different industries Θ_j is fixed through time and hence, Θ_j can be normalized to 1, i.e. $\Theta_j := [0, 1]$ for $j = \{m, z\}$.*

Consequently the continuum of capital goods in each intermediate sector has unit mass. Capital goods of type ϑ_m (ϑ_z) cannot be used in production of the good Y_m (Y_z). With regard to the capital goods quality we make the following:

Assumption 5.6. *Let $q(n_{\vartheta_j})$ be defined as above. Then the quality of each new generation of J-complementary capital goods is exactly λ_j times the quality of the preceding product generation. Hence,*

$$q(n_{\vartheta_j}) = \lambda_j q(n_{\vartheta_j} - 1) \quad \textit{for all } n_{\vartheta_j} = 0, 1, 2, ... \textit{ and } \vartheta_j \in \Theta_j, \lambda_j > 1.$$

A larger quality of the capital goods increases the *productivity* of manufacturing the intermediate goods. In other words, a newly invented capital good or machine with higher

quality produces a larger quantity of the intermediate good per unit of factor input compared to the previous generation of product.
If intermediate production would require the combustion of some type of fossil fuel, a rise in the quality of the capital goods could additionally imply an increase in the productivity of the machines with respect to fossil fuel consumption or "energy". That means, each newly invented machine would decrease the amount of energy used per unit of output. As a consequence, carbon emissions per unit of output, based on fossil fuel combustion, would decrease with each newly invented machine.
However, in this study I assume that intermediate production emits carbon, but fossil fuels are no additional input to production. This helps to keep the complexity of the following calculations tractable. More precisely, while production of Y_m is "green" in the sense that it creates no carbon emissions, production of Y_z is now "dirty", because it emits $\kappa > 0$ units of carbon per unit of output. Denote the total amount of carbon emitted into the atmosphere in period t measured in gigatonnes of carbon by E_t, this implies

$$E_t = \kappa Y_{z,t}. \tag{5.10}$$

Here, the parameter κ simply describes how polluting the production of the intermediate good Y_z is. For high values of κ a given level of production goes along with high emissions and vice versa.
Although emissions enter the developed model, fossil fuels are not an explicit factor of intermediate production. Hence, technical change increases the productivity of machines, while according to equation (5.10), the emission intensity of intermediate production remains constant over time. Consequently, technical changes do not lead to the development of new technologies causing lower emissions per unit of output in the present study.[59]
The aggregate sectoral production function is defined as:

Assumption 5.7. *In any period t, the intermediate sector $j = \{m, z\}$ produces an intermediate good Y_j using the technology*

$$Y_j = \mathcal{G}(x_{\vartheta_j}, L_{j,t}) = \Big(\int_0^1 q_{\vartheta_j}^{1-\mu} x_{\vartheta_j}^{\mu} d\vartheta_j\Big) L_j^{1-\mu}, \qquad j \in \{m, z\}. \tag{5.11}$$

Since intermediate goods markets are competitive, firms in the two sectors m and z take the price of their product, p_j, the rental prices of the machines, denoted by p_{ϑ_j}, the quality of machines, q_{ϑ_j}, $j = \{m, z\}$ and the wage $w_{j,t}$ as given. The decision problem of a

[59]This approach of a constant "emission-to-output-ratio" can be found for instance in Acemoglu et al. (2011). The authors assume constant emissions per unit of output, when analyzing the direction of technical change from dirty to clean technologies in a general equilibrium quality ladder model featuring climate change. Other studies assume that technical change alters the emissions-to-output ratio. In their analysis of taxes on fossil fuels in general equilibrium, Golosov et al. (2014) model energy as an input to production explicitly and endogenous technological change here affects the energy productivity of the firms and therefore the emissions from combustion of fossil fuels as well. Acemoglu and Robinson (2012) analyze the direction of technical change and the environment and assume that a rise in the quality of the capital goods lowers the level of environmental degradation that is associated with production. Consequently technical change is "emissions-saving".

representative firm in sector $j = \{m, z\}$ then reads

$$\max_{(x_{\vartheta_j}, L_j)\in\mathbb{R}_+^2} \left\{ p_j Y_j - w_j L_j - \int_0^1 p_{\vartheta_j} x_{\vartheta_j} d\vartheta_j \mid Y_j = (5.11) \right\} \tag{5.12}$$

A solution to the optimization problem (5.12) must satisfy the following first order conditions:

$$(1-\mu) p_j L_j^{-\mu} \left(\int_0^1 q_{\vartheta_j}^{1-\mu} x_{\vartheta_j}^{\mu} d\vartheta \right) = w_j, \tag{5.13a}$$

$$\mu p_j L_j^{1-\mu} \left(q_{\vartheta_j}^{1-\mu} x_{\vartheta_j}^{\mu-1} \right) = p_{\vartheta_j}. \tag{5.13b}$$

Equations (5.13a) and (5.13b) state that the production factor prices equal their marginal factor productivities. The demand for capital goods in sector $j = \{m, z\}$ can be derived from (5.13b) as[60]

$$x_m(p_{\vartheta_m}) = \left(\frac{\mu p_m}{p_{\vartheta_m}} \right)^{\frac{1}{1-\mu}} q_{\vartheta_m} L_m. \tag{5.14a}$$

$$x_z(p_{\vartheta_z}) = \left(\frac{\mu p_z}{p_{\vartheta_z}} \right)^{\frac{1}{1-\mu}} q_{\vartheta_z} L_z. \tag{5.14b}$$

Intermediate capital goods demand is increasing in the output price p_j, in quality levels q, and employment L_j and decreasing in capital good prices p_{ϑ_j}. These results are quite similar to those of the first part, except for the fact that instead of the general factors M and Z, intermediate firms in both sectors now use labor L_m and L_z and therefore, the demand for capital goods depends on sectoral employment.

5.2.3. Capital Goods Production

The capital goods production stage is similar to the one of the first part. Entrepreneurs and innovators supply the capital goods monopolistically to the intermediate firms. Within any product line ϑ_j either an entrepreneur or an innovator is the exclusive capital good supplier. Still we assume that entrepreneurs hold patents to produce capital goods of existing quality and innovators try to invent new generations of product with higher quality in some of the capital good lines. If innovation is successful, the innovators receive a patent on that innovation and they supply the newly invented good of higher quality to the intermediate firms. The entrepreneur on that product line makes zero profit. If innovation is not successful, the entrepreneur exclusively sells the capital good with existent quality level to the intermediate sector. Key feature here is, that innovators decide to which sector they direct their R&D effort on the basis of expected profits. This is the engine of endogenous directed technical change.

Entrepreneurs and innovators operate in a market with Bertrand competition: no matter what sector $j \in \{m, z\}$ or capital good line ϑ_j, marginal and average production costs

[60]Note that these capital goods demand functions contain the endogenous variables L_m and L_z and are therefore not classical demand functions.

are identical and entrepreneurs and innovators compete by setting prices simultaneously. Thereby, each entrepreneur initially holds a patent on the blueprint to produce the current highest quality capital good in capital good line ϑ_j. Innovators can observe the product characteristics of this "leading-edge" capital good and engage in R&D to invent new capital goods of higher quality. Although competitors set prices equal to marginal costs and so product prices are identical in principle. However, if an innovator develops a new capital good of higher quality, this supplied good offers greater product quality compared to previous generations of this specific product. Here, I consider an institutional setup, where inventors of new "leading-edge" capital goods receive a one-period patent for the production and sale of that good. Without any search- or transaction costs, intermediate sectors want to buy capital goods of the highest quality standard, given that prices for all generations of that product are identical. Consequently, the good offering the highest quality or equivalently the good demanding the lowest quality-adjusted price within any one industry ϑ_j gains the complete market demand and the "leading-edge" capital good in any industry is again monopolistically supplied. Therefore, positive monopolistic profits are realized by successful innovators and by those entrepreneurs who operate on a capital good line, where innovation was not successful or did not take place at all. Note that competition between innovators and entrepreneurs takes place on an individual level.
With respect to the size of the rise in quality after innovation, we again assume that:

Assumption 5.8. *Independent of the sectors $j = \{m, z\}$ innovators direct their innovation effort to, the size of the innovation is* drastic *if λ_j is sufficiently high to guarantee the innovator the unrestricted monopoly profit. This holds true if the price set after innovation is less than or equal to the marginal cost of production:*

$$p_{\vartheta_j} \leq \psi \text{ for all } \vartheta_j \in \Theta_j, \, j = \{m, z\}.$$

New born innovators choose either the "green" or the "dirty" intermediate sector direct R&D to and -if the innovation is successful- to sell capital goods to. Innovators in sector then j distribute across the different capital good lines ϑ_j. Then each innovator observes the characteristics of her current state of the art product with quality $q_{\vartheta_j} := q_{n_{\vartheta_j}}$ and spends h units of the final good for R&D to improve the quality of that commodity to $q'_{\vartheta_j} = \lambda_j q_{\vartheta_j}$. If successful, innovators hold the patent on the blueprint to produce this capital good and allows them to sell the commodity exclusively in the following period. At the end of that period, the corresponding patent protection ends. Production costs of capital goods equal ψ units of the final good. Without loss of generality, I normalize $\psi \equiv 1$, so entrepreneurs and innovators can convert final goods into capital goods on a one to one basis.
Each individual innovator needs credit equal to finance R&D expenditures so she borrows h units of the final good for given credit interest rate. To indicate the dependence of R&D expenditures on credit interest rates and also on current quality levels of the corresponding good that is planned to be improved, we denote period t R&D expenditures as $h_{\vartheta_{j,t}}(R_{j,t+1})$. Accordingly, credit contracts between innovators and banks are signed on an individual level.
Similar to the framework of part one, innovators and entrepreneurs face a two-stage decision process within two consecutive periods of time. Innovators' first decision period is

the research and planning period. In this stage, they decide how much to spend on R&D to maximize expected profits. Entrepreneurs' first decision period is a planning period. They decide how much to produce in the second period given that innovation on their product line was not successful or even did not take place at all. In the following, we refer to all innovators and entrepreneurs who supply a capital good as "capital good producers".
In the second decision stage, capital goods producers determine the optimal price at which they are going to sell the capital goods to the intermediate sectors. Given intermediate demand for capital goods, this price determines the profit flow in the second period and thus determines the net present value of profit in the first stage.
Consider an arbitrary period t. This period t is referred to as the "first" decision period and the subsequent period $t + 1$ is the "second" decision period. The decision problem is solved recursively. First we establish the optimal price for a capital goods producer to set in the second decision period, $t+1$, given that either he succeeded in the research lab in the previous period t if he is an innovator or if he is an entrepreneur, given that no innovator tried to improve the quality of his capital good or tried but failed. Afterwards, we derive individual and sectoral R&D expenditures and compute the endogenous probability of successful innovation.

Profit Maximization
Ignoring time subscripts for the moment, the demand for the capital good ϑ_j in the intermediate sector j is given in (5.14a) and (5.14b) and reads

$$x_j(p_{\vartheta_j}) = \left(\frac{\mu p_j}{p_{\vartheta_j}}\right)^{\frac{1}{1-\mu}} q_{\vartheta_j} L_j, \qquad j \in \{m, z\}, \tag{5.15}$$

where p_{ϑ_j} denotes the price of the capital good, which is the choice variable of the capital goods producer and will be determined next.
The rest of the derivations are identical to those of part one: Capital goods producers take the demand above as given and maximize profits. The second period decision problem of an innovator in period $t + 1$ then reads:

$$\max_{(p_{\vartheta_j,t+1})\in\mathbb{R}_+} \left\{\left(p_{\vartheta_j,t+1} - 1\right) x_{j,t+1}(p_{\vartheta_j}) - R_{j,t+1} h_{\vartheta_{j,t}}(R_{j,t+1}) \;\middle|\; x_{j,t+1}(p_{\vartheta_{j,t+1}}) = (5.15)\right\} \tag{5.16}$$

and the decision of an entrepreneur is given by:

$$\max_{(p_{\vartheta_j,t+1})\in\mathbb{R}_+} \left\{\left(p_{\vartheta_j,t+1} - 1\right) x_{j,t+1}(p_{\vartheta_j}) \;\middle|\; x_{j,t+1}(p_{\vartheta_{j,t+1}}) = (5.15)\right\} \tag{5.17}$$

These optimization problems are identical in the second decision stage and differ only by the term $R_{j,t+1} h_{\vartheta_{j,t}}$ representing the cost of R&D. As the time index indicates, this term is determined in the first decision stage and thus plays no role in the determination of the profit maximizing price. The first order optimality condition with respect to $p_{\vartheta_{j,t+1}}$ gives the profit maximizing monopoly price of a capital good ϑ_j:

$$p_{\vartheta_j,t+1} = \frac{1}{\mu}, \tag{5.18}$$

which is a constant markup over marginal cost and equal across industries ϑ_j.
The resulting flow of monopoly profit for an innovator then can be computed as

$$\pi^{(i)}_{\vartheta_j,t+1} = \bar{\pi}_{j,t+1} q_{\vartheta_{j,t+1}} - R_{j,t+1} h_{\vartheta_j,t}(R_{j,t+1}) \tag{5.19}$$

and the monopoly profit for an entrepreneur e reads

$$\pi^{(e)}_{\vartheta_j,t+1} = \bar{\pi}_{j,t+1} q_{\vartheta_j,t+1}, \tag{5.20}$$

where $\bar{\pi}_{j,t+1} := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{j,t+1}^{\frac{1}{1-\mu}} L_{j,t+1}$.

Optimal R&D and success-probabilities
In the first decision stage, innovators determine the amount of R&D expenditures they need as credit from the banks. The formulation here is identical to part one. The net present value of future profits from innovation in industry ϑ_j is equal to

$$V_{\vartheta_j,t} = \frac{\mathbb{E}_t[\pi^{(i)}_{\vartheta_j,t+1}]}{r_t} \tag{5.21}$$

where $\mathbb{E}_t$ denotes the expectation of future outcomes in period t, $\pi^{(i)}_{\vartheta_j,t+1}$ is given in equation (5.19) and r_t is the interest rate which is potentially time varying.
During the first stage, innovators and entrepreneurs face idiosyncratic risks. For an individual innovator, the return is random, since R&D effort is successful only with a certain probability. The return for an entrepreneur contains two sources of uncertainty: first, since the unit mass of entrepreneurs/capital goods in each sector face a unit mass of innovators in total, there exist capital good lines which remain without an innovator. On those capital good lines, entrepreneurs produces the capital good with current quality using the existing technology. Second, if an innovator is present in a capital good line, the innovator probably fails to innovate. Then entrepreneurs also produce the existent capital good. This setup is such that competition between entrepreneurs and innovators takes place on an individual level and thus each person faces an idiosyncratic risk of return.
I again assume that individuals form “alliances” to protect themselves against these idiosyncratic risks to get deterministic income and consumption profiles for all individuals nevertheless: Innovators in the green sector m and innovators in the dirty sector z each form an ”R&D syndicate”, so there exist two different R&D-syndicates. All entrepreneurs in both sectors together form one “entrepreneur association”.
Since realizations of individual innovation attempts are independent and thus uncorrelated, a fraction of the projects will be successful while the rest of the projects will be unsuccessful. Then one could in principle conclude, applying a law of large numbers type of reasoning that the fraction of successful projects in sector $j \in \{m, z\}$ is equal to the ex-ante probability of successful innovation η_j, leading to aggregate average innovator profits of $\eta_j \Pi_j$, while the fraction of unsuccessful projects is equal to the ex-ante probability of innovation failure $1 - \eta_j$.
Moreover, using the same type of reasoning, the ex-ante probability of an innovator trying to improve the quality of capital good in sector j is equal to the frequency or mass

of innovators in sector m, denoted by $\iota_m := \iota$ and sector z, denoted by $\iota_z := 1 - \iota$. So entrepreneurs expected return is equal to $(1 - \iota\eta_m)\Pi_m^e + (1 - (1 - \iota)\eta_z)\Pi_z^e$. The η_j's and the ι_j's are well defined in the present context, although the model here features a continuum of capital goods/entrepreneurs and innovators (each normalized to unity) and so the η_j's describe the "average" of a continuum of random variables (See Uhlig (1996) and also Acemoglu (2009)). In this way, an alliance allows individuals to trade the high return with idiosyncratic risk for a deterministic but lower return, i.e. an average return (for innovators the average return is sector-specific).
The next assumption specifies the relationship between R&D expenditures, generation of product n_{ϑ_j} and the probability of successful innovation:

Assumption 5.9. *The individual probability of successful innovation η_{ϑ_j}, $j = \{m, z\}$ is a strictly concave function of R&D expenditures and is defined as*

$$\eta_{\vartheta_j} := h_{\vartheta_j}^{\alpha}\phi_{\vartheta_j}, \qquad j = \{m, z\}, \tag{5.22}$$

where $\alpha \in]0, 1[$ and

$$\phi_{\vartheta_j} := \frac{1}{\zeta} q_{\vartheta_j}^{-\alpha} \tag{5.23}$$

captures the effects of the capital good's current quality ladder position.[61]

With this formulation, innovators treat η_{ϑ_j} as a choice variable.[62] In the first period of decision, an innovator takes the interest rates $r_t, R_{j,t+1}$, the price of the intermediate good $p_{j,t}$, the profit maximizing price determined in the second decision stage $p_{\vartheta_{j,t+1}}$, quality level $q_{\vartheta_j,t+1}$ factors J_{t+1} and the function z_{ϑ_j} as given. An innovator in capital goods line ϑ_j chooses R&D expenditures $h_{\vartheta_j,t}$ to solve the following optimization problem

$$\max_{h_{\vartheta_j,t} \in \mathbb{R}_+} \left\{ \frac{\eta_{\vartheta_j}}{r_t} \left(\bar{\pi}_{j,t+1} q_{\vartheta_j,t+1} - R_{j,t+1} h_{\vartheta_j,t} \right) \;\middle|\; \eta_j = h_j^{\alpha} z_{\vartheta_j} \right\} \tag{5.24}$$

A solution to (5.24) satisfies the following first order conditions

$$\alpha \left(\bar{\pi}_{j,t+1} q_{\vartheta_j,t+1} - R_{j,t+1} h_{\vartheta_j,t} \right) = R_{j,t+1} h_{\vartheta_j}. \tag{5.25}$$

Solve for $h_{\vartheta_j,t}$ gives

$$h_{\vartheta_j,t} = \frac{\alpha}{1 + \alpha} \frac{\bar{\pi}_{j,t+1}}{R_{j,t+1}} q_{\vartheta_j,t+1}. \tag{5.26}$$

This expression implies that innovators direct a constant fraction of (expected) profits to R&D such that the marginal benefit from one additional unit of R&D expenditure is equal to the marginal additional individual expected profit from capital goods selling. For the

[61] Since the mapping from R&D to the probability of success is unbounded in general, the defined η's are not probabilities in a strict sense, i.e. they do not necessarily take values between zero and one. We account for this and set the parameter appropriately, so that the success-probability values lie between zero and one.

[62] For a different formulation, where aggregate R&D expenditures determine the probability of success and individuals take the probability as given, see for instance Barro and Sala-i Martin (2004).

purpose of this thesis it is more important to note a negative influence of the loan interest rate $R_{j,t}$ on R&D outlays. This suggests an influence of banks on R&D investment.
Using optimal R&D expenditures given in (2.16), expected profits for an innovator on capital good line ϑ_j then read

$$\pi^{(i)}_{j,t+1} = \frac{1}{1+\alpha}\eta_j\bar{\pi}_{j,t+1}q_{\vartheta_j,t+1}. \tag{5.27}$$

To derive the sectoral probabilities of innovation success use definition 5.9, insert individual R&D-expenditures and the expression for ϕ_{ϑ_j} gives

$$\eta_{\vartheta_j} = \eta_j = \zeta^{-1}\left(\frac{\alpha}{1+\alpha}\frac{\bar{\pi}_{j,t+1}}{R_{j,t+1}}\right)^{\alpha} \tag{5.28}$$

Thereby, the function $\phi_{\vartheta_j} := \frac{1}{\zeta}q^{-\alpha}_{\vartheta_{j,t+1}}$ captures the effects of the current position on the quality ladder. The present study assumes that successful innovation becomes more difficult the more product generations have been invented previously. The function ϕ now states that this difficulty increases in proportion to the additional output that would be produced in case of successfully increasing the product quality from $q_{\vartheta_{j,t}}$ to $\lambda_j q_{\vartheta_{j,t}}$ between t and $t+1$. The parameter ζ represents a cost of research measured in units of the final consumption good. The probability of research success decreases with the loan rate R_j. However, the expression for η_j in eq. (5.28) so far is incomplete, because R_j depends itself on η_j, and the term $\bar{\pi}_{j,t+1}$ contains the endogenous price p_j for the intermediate good Y_j. I postpone this derivation of the explicit formula for η_j until section five and instead highlight the result implied by (5.28):

Remark 5.1. *Banks influence the sectoral probabilities of successful innovation.*

5.3. Banks

Consider next the decisions of the banking sector. No fundamentel changes compared to part one occur in this stage. Similar to the first part, financial intermediation is represented by a single price taking aggregate bank. The set up in the credit market is such that borrowers (innnovators) and lenders (the banking sector) have the same amount of information in advance about the risk inherent in the projects. This symmetry in the ex-ante information deficit rules out problems of *adverse selection* that are typically associated with ex-ante informational asymmetries. Furthermore, innovators who have been successful in the research lab cannot hide their success from the banks, claim unsuccessful innovation and then deviate from the credit liability incurred. Hence, the present study does also not consider *moral hazard* problems, typically associated with ex-post information asymmetries in the credit market. What is more, innovators are not allowed to default on credit, i.e. take the borrowed capital and run away. So there are no problems of commitment in this model framework. On the basis of this information set-up, the banking sector negotiates with the innovators on an individual level in the credit market and with the workers in the deposit market.
Members of the working population save their labor income when young to finance consumption when old. They take this income share and enter the deposit market to store

their savings in a bank account and receive a certain deposit interest rate. In principle, workers could also lend their savings directly to entrepreneurs. However, private savings are typically small compared to business investment projects and more importantly, the different projects of entrepreneurs contain idiosyncratic risks. Thus, the return on any individual investment project and hence the income from savings would contain risks. Risk-averse workers are not willing to bear these idiosyncratic risks. So in order to diversify these risks, they would have to lend their savings to a very large number of different firms. And before that, workers would have to evaluate each individual investment project by themselves, which is impossible with a limited amount of time and capital. Therefore, banks who specialize in these activities emerge.
The banking sector collects the savings from a large number of workers. Then these savings are pooled and the capital resources are used to finance large scale business investments. Through financing a large number of different projects with stochastically independent returns, the banks fully diversify the idiosyncratic risk and the workers receive a deterministic rate of interest on their "portfolio".

The decision problem
The banks use these capital resources to finance business investment projects. Innovators borrow capital to invest in R&D. Due to sector-specific credit risk, each banks charges a sector-specific loan interest rate on credit liabilities.
The representative bank faces idiosyncratic uncertainty in credit supply: An innovator (the borrower) trying to improve the quality of a sector-specific capital good in industry ϑ_j, in sector $j = \{m, z\}$, is successful in innovation with a sector-specific probability $\eta_j(h_{\vartheta_j})$, where still h_{ϑ_j} represents R&D-effort measured in units of the final good.
However, the returns of the individual innnovators' investment projects are stochastically independent and consequently, for a large number, in fact a continuum of different projects, these idiosyncratic risks are completely diversified. Therefore, the randomness in the returns vanishes in aggregate terms and the aggregate banks' decision problem becomes completely deterministic.[63]
Since $\eta_m \neq \eta_z$ holds true in general, the expected return from credit lending depends on the total size of capital resources devoted to the green sector m or the dirty z. Credit supply in sector j at time t is denoted by $D_{j,t}$. Credit demand in sector j is given by R&D expenditures in sector j, denoted as $H_{j,t}$. The aggregate banking sectors' deposit demand is D_t. This implies aggregate deposit cost for the banking sector equal to $r_{t+1}D_t$. Total deposit supply is given by aggregate savings S_t. We assume that total deposit demand in period t determines total credit supply in $t + 1$, so

$$D_t = \sum_{j\in\{m,z\}} D_{j,t+1}. \tag{5.29}$$

With credit interest rates of $R_{j,t}$, $j = \{m, z\}$, the return of funding equals $\eta_m R_{m,t+1} D_{m,t} + \eta_z R_{z,t+1} D_{z,t}$. The bank takes the interest rates r_t, $R_{m,t}$ and $R_{z,t}$ as given and chooses credit

[63]In principle, the underlying law of large numbers argument holds only for averages of a countable sequence of random variables, where as the model here features a continnuum of random varibales. Uhlig (1996) showed how to obtain a law of large number for a continuum of uncorrelated random variables.

quantities to solve the following optimization problem:

$$\max_{(D_{m,t},D_{z,t})\in\mathbb{R}_+^2} \left\{ \sum_{j\in\{m,z\}} \eta_j R_{j,t+1} D_{j,t} - r_{t+1} \sum_{j\in\{m,z\}} D_{j,t} \right\} \tag{5.30}$$

A solution to (5.30) satisfies the following first order optimality conditions:

$$R_{m,t+1} = \frac{r_{t+1}}{\eta_m} \quad \text{and} \quad R_{z,t+1} = \frac{r_{t+1}}{\eta_z}. \tag{5.31}$$

These conditions state the relationship between the sectoral loan interest rates $R_{j,t}$, $j \in \{m,z\}$ and the deposit interest rate r_{t+1} and shows that the loan rates contain the inverse of the success-probabilities η_m^{-1}, η_z^{-1} as risk premiums: the greater the chance of success in innovation, the lower is the corresponding rate of interest on business credit. Moreover, one can derive the following relationship between sectoral credit interest rates:

$$\eta_m R_{m,t+1} = \eta_z R_{z,t+1}. \tag{5.32}$$

This simply gives the condition that in equilibrium, the expected loan interest rates in the two intermediate sectors have to be equal.
Conditions (5.31) and (5.32) explain the relationship between the loan rates paid by innovators and the deposit rates paid to workers and we get:

Proposition 5.1. *Whenever banks finance innovative activities, an interest rate spread between deposit rates paid to workers and loan rates paid by innovators exists: $r_t < R_{j,t}$ for $j = \{m,z\}$, i.e. the interst rate on business credit exceeds the interest rate on deposits. This interest rate spread remains in place even in a financial market characterized by perfect competition and is therefore not a result of monopoly distortion.*

Proof: See Appendix B.
Moreover, the condition stated in equation (5.32) shows the relationship between sectoral interest rates charged on credit:

Proposition 5.2. *In all periods $t \geq 0$, the interest rates on private sector lending $R_{m,t}$ and $R_{z,t}$ used to finance R&D can vary between intermediate sectors m and z.*

Proof: See appendix B.
On the one hand, equation (5.32) is an equilibrium condition for the bank sector. Banks are indifferent between funding capital goods producers in the dirty and the green sector, as long as the expected returns on sectoral funding are equal. In the case of $\eta_m > \eta_z$ the probability of successful innovation and therefore the probability of debt repayment in the next period is greater for innovators directing R&D to the green sector m than for those who direct R&D to the dirty sector z. Since the bank bears the risk of "losing money" a lower probability of success implies a larger risk of default on credit liabilities. The interest rates on loans reflect these different risks and include a risk premium. In the case considered here, the risk premium and thus the loan rate is higher for innovators who direct their innovation effort to sector z. Without different loan rates, banks would

always prefer the lowest risk borrowers, in this case innovators' investments in sector *m*. In other words, a higher interest rate enables higher risk borrowers to compete for capital resources.
On the other hand, innovators are indifferent between capital goods invention in the two intermediate sectors as long as the expected returns on innovation effort are equal across *m* and *z*. In this case, innovators accept higher equilibrium interest rates paid on funds in one sector and still direct innovation effort to that sector.
Finally, the conditions given in (5.31)-(5.32) replicate the results of part one perfectly. This gives the followign remark:

Remark 5.2. *Suppose banks fund potential entrepreneurs indiscriminately. Then pure profit maximization considerations in financial intermediation ignore the presence of environmental constraints and sectoral credit lending strategies are independent of the negative externality associated with dirty sector production.*

In other words, the implementation of carbon emissions into the model has no effect on the microeconomic decisions of the banks. This should be intuitively clear, because the banks supply capital resources to the innovators in the two different sectors on the basis of profit maximization considerations. Consequently, banks fund capital good producers in those sectors, with the highest expected return on funding. Whether capital goods producers demand credit to invent new technologies for emission intensive or extensive intermediate sectors is completely irrelevant to the bank in this framework.

5.4. Consumers

Consumers in the model are the workers, entrepreneurs, and innovators. In each period a continuum of young consumers is born. Each consumer lives for two periods. Population is constant over time. The young generation is indexed by the superscript "y", members of the old generation are indexed with an "o". In this setting, the attribute of being a worker, an entrepreneur or an innovator is preassigned. So at each point in time, two different generations populate the economy and each generation consists of workers (w), entrepreneurs (e), and innovators (i), who differ in terms of their access to investment projects and consumption profiles. The subindex $\ell \in \{w, e, i\}$ identifies the different consumer types. Preferences over consumption are identical for all consumers and equal to:

Assumption 5.10. *Consumption preferences can be represented by an utility function* $U : \mathbb{R}^2_+ \to \mathbb{R}$*, which is defined as*

$$U(c^y_{\ell,t}, c^o_{\ell,t+1}) := \beta u(c^o_{\ell,t+1}), \tag{5.33}$$

β *is the individual discount factor and the "instantaneous" utility function* $u : \mathbb{R}_+ \to \mathbb{R}$*, is logarithmic:* $u(c) := log(c)$*,*

So utility is defined as a function of only second period consumption.

Workers
At each point in time a continuum of young workers is born whose mass is now normalized to unity. Each worker is endowed with one unit of labor time. Workers supply their labor inelastically to the labor market when young and receive a "real" wage rate of w_t. At the beginning of their second period of life, when old, they retire. Workers only consume in the second period of life and, therefore, wish to transfer their current wealth into the next period. For this purpose, they supply their labor income to the deposit market and receive a deposit rate of r_{t+1} on these savings. Given labor income of w_t and since the entire income is saved, we have $s_t = w_t$, and old workers have income of $r_{t+1} s_t$.
This leads to the following aggregate budget constraint of old workers:

$$C^{(o)}_{w,t+1} = r_{t+1} S_t = r_{t+1} w_t =: I^{(o)}_{w,t+1}. \tag{5.34}$$

Old workers do not care about the wealth of their descendants and leave no bequests. Since utility is strictly increasing in consumption quantities, old workers spend their income entirely for consumption of the final good.

Entrepreneurs
The formulation of the entrepreneurs remains unchanged. A continuum of mass 2 of young entrepreneurs enters the economy at the beginning of each period t. Each entrepreneur receives a patent to produce capital goods of current quality for one of the two intermediate sectors. Then the mass of entrepreneurs holding patents for capital goods in each intermediate sectors $j \in \{m, z\}$ equals unity. Recall that the continuum of capital goods used in each intermediate sector has unit mass, so this ensures identical masses of capital good lines and entrepreneurs. Hence each entrepreneur holds a patent for one capital good line and consequently on each capital good line one entrepreneur is present. Entrepreneurs form an "entrepreneur association" to protect themselves from idiosyncratic risks in profits. Since realizations of individual innovation attempts are independent and thus uncorrelated, a fraction of the projects will be successful while the rest of the projects will be unsuccessful. Then using a law of large numbers type of reasoning for a continuum of random variables (see Uhlig (1996)), the fraction of successful projects in sector $j \in \{m, z\}$ is equal to the ex-ante probability of successful innovation η_j and the fraction of unsuccessful projects is equal to the ex-ante probability of innovation failure $1 - \eta_j$. Moreover, using the same type of reasoning, the ex-ante probability that an innovator tries to improve the quality of a certain capital good in sector j is equal to the frequency or mass of innovators in sector m, denoted by $\iota_m := \iota$ and sector z, denoted by $\iota_z := 1 - \iota$. Entrepreneurs make profits in capital good lines without an innovator or where innovators' R&D is unsuccessful. Together this implies an income for the entrepreneur association of

$$\bar{\Pi}^e_t = (1 - \eta_m \iota)\Pi^e_{m,t} + (1 - (1 - \iota)\eta_z)\Pi^e_{z,t} \tag{5.35}$$

The aggregate budget constraint of the entrepreneurs is then given by

$$C^{(o)}_{e,t} = \bar{\Pi}^e_t =: I^{(o)}_{e,t} \tag{5.36}$$

Old entrepreneurs also consume their income completely, since utility strictly increasing in consumption.

Innovators

The formulation of the innovators remains unchanged also. Each period, a continuum of innovators is born whose mass is normalized to one. Young innovators engage in R&D to improve the quality of existing capital goods. Young innovators need credit to finance R&D expenditures and banks supply the necessary capital resources. If successful, they receive a one period patent on that innovation and produce that capital good with greater quality instead of the entrepreneur on that capital good line. Innovators direct their investment to capital goods innovation for either intermediate sector on the basis of expected profits. Denote the share of innovators directing R&D investment to sector m by ι, then the share of innovators investing in R&D in sector z equals $1-\iota$, where $\iota \in [0,1]$ can vary in equilibrium.

Similar to the entrepreneurs, innovators group in an "alliance" to protect themselves from idiosyncratic risks in profits. Within one intermediate sector $j = \{m, z\}$ innovators form an "R&D syndicate". Again, since investment projects contain idiosyncratic risk, we can apply a law of large number argument for a contiuum of random variables, and conclude that the fraction of successful projects is equal to the ex-ante probability of successful innovation η_j. Aggregate average profits for the R&D syndicate in sector j are given by $\eta_j \Pi_j^{(i)}$, so the income for innovators in sector $j \in \{m, z\}$ reads

$$\bar{\Pi}_{j,t} = \iota_j \eta_j \Pi_{j,t} \tag{5.37}$$

Old innovators also spend their income from monopolistic capital goods selling completely for consumption and leave no bequests for their descendants. Note that the aggregate income of the innovators is sector-specific since innovators within one intermediate sector form a "R&D-Syndicate". Therefore, the aggregate income of the syndicate depends also on the number of innovators ι_j in sector j, since the more innovators try to improve the quality of capital goods in one sector, the greater is the income of the R&D-syndicate. This implies a budget constraint for innovators in sector j equal to

$$C_{i,j,t}^{(o)} = \bar{\Pi}_{j,t}^{i} =: I_{i,j,t}^{(o)}. \tag{5.38}$$

Total consumption of all innovators is then given by

$$C_{i,t}^{(o)} = C_{i,m,t}^{(o)} + C_{i,z,t}^{(o)}. \tag{5.39}$$

5.5. Aggregation

The aggregate sectoral quality index for $t \geq 0$ is defined as in part one:

$$Q_{j,t} := \int_0^1 q_{\vartheta_j,t} d\vartheta_j \quad \text{for } j = \{m, z\}. \tag{5.40}$$

The aggregator is linear homogeneous, which captures the plausible feature that if we increase all single productivities by a number $a > 0$ then the average sectoral productivity will increase by a.

Although the q_{ϑ_j}'s of each product line ϑ_j are stochastic, we will argue now that the randomness in microeconomic quality improvements vanishes in the sectoral and macroeconomic variables and the average sectoral quality $Q_{j,t}$ is completely deterministic: This

holds, because the realizations of the quality increases of different capital goods are independent and thus uncorrelated. Then, we can conclude that the $Q_{j,t}$'s are deterministic based on Uhlig (1996) who showed how to obtain a law of large numbers for a continuum of uncorrelated random variables.
The dynamics of the average sector-specific quality improvements can be derived as

$$\mathbb{E}_t[Q_{j,t+1} - Q_{j,t}] = \iota_{j,t}\eta_{j,t}\Big(\lambda_j - 1\Big)Q_{j,t}. \tag{5.41}$$

Thereby, $\iota_m := \iota$ and $\iota_z := 1 - \iota$, $\iota \in [0, 1]$ enter this expression, since quality probably increases only in those capital good lines where innovators are present. If for instance $\iota = 1$, all innovators direct their R&D effort to the green sector m and the quality of all capital goods used in dirty sector production would remain constant.
Using the law of large numbers argument given above to conclude that the average sectoral capital goods quality develops deterministically over time - even though individual R&D projects exhibit stochastic returns - the expectations operator in the equation above can be dropped and the dynamic development of average sectoral capital goods quality $Q_{m,t}$ and $Q_{z,t}$ is given by

$$\frac{Q_{m,t+1} - Q_{m,t}}{Q_{m,t}} = \iota_t\eta_{m,t}(\lambda_m - 1) \qquad \text{for all } t \geq 0, \tag{5.42a}$$

$$\frac{Q_{z,t+1} - Q_{z,t}}{Q_{z,t}} = (1 - \iota_t)\eta_{z,t}(\lambda_z - 1) \qquad \text{for all } t \geq 0, \tag{5.42b}$$

where by assumption (5.6), $\lambda_j > 1$ so $\iota_{j,t}\eta_{j,t}\left(\lambda_j - 1\right) > 0$ and $Q_{j,0}$ is some given initial quality level. According to equations (5.42a) and (5.42b), the dynamic development of the average quality of the capital goods in each intermediate sector can be represented by a deterministic, linear first order difference equation. Denote the economy-wide average capital goods quality as Q_t. Then, Q_t is defined as the weighted average of the two sectoral quality indices $Q_{m,t}$ and $Q_{z,t}$, where the weights are given by the distribution parameter of the intermediate inputs $Y_{m,t}$, $Y_{z,t}$ in final goods production, γ and $1 - \gamma$:

$$Q_t = \gamma Q_{m,t} + (1 - \gamma)Q_{z,t}. \tag{5.43}$$

It is important to bear in mind that the average quality of sector-specific capital goods used in the production of the two intermediate goods Y_m and Y_z are different, allowing technical change to be biased or directed to one intermediate sector. The average quality of capital goods Q_m and Q_z, determine aggregate productivity, while the quotient Q_z/Q_m determines the relative productivity of the dirty sector z.
Aggregate sectoral R&D expenditures can be computed from individual R&D expenditures of an innovator in industry ϑ_j, given in (5.26), as

$$H_{j,t} = \frac{\alpha}{1 + \alpha}\frac{\bar{\pi}_{j,t+1}}{R_{j,t+1}}\iota_{j,t}Q_{j,t+1}, \qquad j \in \{m, z\}, \tag{5.44}$$

where still $\iota_{m,t} := \iota_t$ and $\iota_{z,t} := 1 - \iota_t$. The expression shows that aggregate sectoral R&D is increasing in the average sectoral capital goods quality $Q_{j,t+1}$, decreasing in the credit

interest rate $R_{j,t+1}$ and increasing in the mass of innovators per sector $\iota_j \in [0,1]$. In this regard ι_t controls the direction (and also rate) of technical change: for instance, if $\iota_t = 1$, all innovation effort would be directed to sector m and consequently, R&D expenditures in sector z would be zero. From (5.42a) and (5.42b) then follows that only the quality of capital goods in sector m changes over time, while the quality of capital goods in sector z remains constant, i.e. technical change would occur only in sector m.
Total R&D in period t is simply given by the sum of aggregate sectoral R&D:

$$H_t = H_{m,t} + H_{z,t}. \tag{5.45}$$

Spending on capital goods in sector j can be derived from (5.14a) and (5.14b):

$$X_{j,t} = \int_0^1 x_{\vartheta_j,t} d\vartheta_j = \mu \bar{\pi}_{j,t} Q_{j,t}, \tag{5.46}$$

where $\bar{\pi}_{j,t}$:=. Aggregate capital goods spending is the sum over sectoral capital goods spending equal to

$$X_t = X_{m,t} + X_{z,t}. \tag{5.47}$$

Finally, aggregate savings are given by total labor income of the workers (whose mass is normalized to unity)

$$S_t = w_t \tag{5.48}$$

and aggregate consumption in period t is given by consumption of old workers, old entrepreneurs and old innovators:

$$C_t = \sum_{\ell} C^{(o)}_{\ell,t}, \qquad \ell \in \{w, e, i\}. \tag{5.49}$$

5.6. Market clearing

Finally, the remaining requirement to describe an equilibrium of the economy is that all markets clear.

Capital markets

Total deposit supply is given by aggregate savings of young workers S_t. Market clearing on the deposit market requires that workers' aggregate savings are equal to the deposit demand of the representative aggregate bank:

$$S_t \overset{!}{=} D_t \qquad \forall t \geq 0. \tag{5.50}$$

Credit demand in each sector $j = \{m, z\}$ is given by aggregate sectoral R&D expenditures (eq. (5.44)). Market clearing requires that in each sector, credit supply is equal to credit demand:

$$D_{j,t} \overset{!}{=} H_{j,t} \qquad j = \{m, z\}, \forall t \geq 0. \tag{5.51}$$

Moreover, the bank cannot supply more capital to the innovators than received from the workers, so we impose the condition that total deposit demand today determines total credit supply tomorrow:

$$D_t = \sum j = \{m, z\} D_{j,t+1} \quad \forall t \geq 0. \tag{5.52}$$

Labor market
Instead of the two more general factors M and Z, intermediate firms in part one used as production inputs, now both intermediate sectors use labor, denoted by Lm and L_z. The mass of workers is normalized to unity, so market clearing on the labor market then requires

$$L_{m,t} + L_{z,t} = 1 \quad \forall t \geq 0. \tag{5.53}$$

Goods markets
The entrepreneurs and innovators represent the supply side of capital goods, the demand side is given by the intermediate firms in the two sectors. For each capital good ϑ_j, market clearing requires:

$$x^S_{\vartheta_j,t} \stackrel{!}{=} x^D_{\vartheta_j,t} \qquad \vartheta_j \in [0,1], j \in \{m,z\}, \forall t \geq 0. \tag{5.54}$$

Market clearing on the markets for the intermediate goods requires

$$Y^S_{j,t} \stackrel{!}{=} Y^D_{j,t} \qquad j \in \{m,z\}. \tag{5.55}$$

Market clearing on the final goods market requires that final goods demand (consumption, investment in R&D and machine production) equals supply and gives the economy's resource constraint:

$$Y_t = X_t + H_t + C_t \qquad \forall t \geq 0, \tag{5.56}$$

where C_t is given in (5.49), X_t is given in (5.47) and H_t is given in (5.45).
Finally, we also need a "market clearing" condition for the mass of innovators. For all $t \geq 0$:

$$\iota_{m,t} + \iota_{z,t} = 1. \tag{5.57}$$

6. Equilibrium

The following general equilibrium analysis emphasizes the role of banks in directing technical change, when industrial production emits greenhouse gases that cause climate change. In the sequel, the study refers to an economy that is characterized by relatively low levels of atmospheric carbon concentration and corresponding low damages to aggregate output, i.e. a "green" economy. In contrast to that, an economy that is characterized by relatively high levels of carbon concentration in the atmosphere and corresponding large damages to aggregate output is referred to as a "dirty"economy. With this understanding, "green" intermediate sector m represents a sector that is carbon neutral and "dirty" intermediate sector z represents a sector where production emits carbon.
This chapter derives the answer to the stylized research question formulated at the beginning of part two: *What role do financial intermediaries play in directing technical change towards a "green" economy?*
Note that the results of this part will be used in the third part to analyze different environmental policy instruments and also for a quantitative assessment of financial intermediation and environmental regulation. In this regard, the following results can be characterized as *laissez-faire equilibrium* outcomes, i.e. we analyze a decentralized equilibrium without any policy intervention. Therefore, part two and three are more closely related to each other than each of them is to part one.
The rest of this chapter is structured as follows: The following section derives the equilibrium of the economy where firms, banks and consumers behave optimally and markets clear, given that production emits CO_2-emissions that causes climate change.
Then we state the determinants of "green" technical change and analyze the influence of financial intermediation on the development of "green" technologies.

6.1. Properties of equilibrium

The following definition of equilibrium is similiar to the one defined in the first part, except that now climate variables such as emissions, atmospheric CO_2-concentration, and damages from climate change are considered. The induced equilibrium allocation constitutes an important benchmark in the subsequent discussion. It is clear that this solution will, in general not constitute a Pareto optimal outcome due to the climate externality in production and monopolistic competition in innovation effort.

Definition 6.1. *An equilibrium of the economy considered here is an allocation*

$$\mathbf{A} = (Y_t, C_t, (Y_{j,t}, X_{j,t}, H_{j,t}, L_{j,t}, \iota_{j,t}, (q_{\vartheta_j,t}, x_{\vartheta_j,t})_{\vartheta_j \in [0,1]})_{j \in \{m,z\}}, D_{t+1})_{t \geq 0}$$

and a price system

$$\mathbf{P} = (r_t, w_{j,t}, R_{j,t}, p_{j,t}, p_{\vartheta_j,t})_{t \geq 0}$$

such that
(i) The allocation is consistent with the production technologies (5.11), (5.6), and the market clearing conditions/resource constraints (5.50), (5.51), (5.53), (5.54), (5.55), (5.56), and (5.57).
(ii) Banks behave optimally, i.e. equations (5.31) and (5.32) hold for all $t \geq 0$.

(iii) Producers behave optimally, i.e. equations (5.18), (5.26) , (5.13) hold for all $t \geq 0$.
(iv) Consumers behave optimally with (profit) incomes determined by (5.34), (5.35) and (5.37) for all $t \geq 0$.
(v) Average capital goods qualities in sector $j \in \{m, z\}$ evolve according to (5.42).
(vi) Climate variables evolve according to (5.3) with emissions given by (5.10) and climate damages in (5.6) determined by (5.4) and (5.5).

Recall that the process of R&D effort for the individual innovator is stochastic. Therefore, the quality of capital good ϑ_j in sector $j \in \{m, z\}$ is random and thus the corresponding prices, quantities and values. However, since the outcomes from random individual "experiments" are stochastically independent, the uncertainty vanishes in aggregate terms and the corresponding sectoral and macroeconomic variables are non-stochastic. This implies deterministic time paths of the aggregate sectoral and macroeconomic variables and the equilibrium analysis becomes completely deterministic.
Similar to the first part, we analyze the direction of technology development on a balanced growth path. Recall that we view a balanced growth path to be a trajectory such that all variables grow at constant rates.
Again, we start with properties of some equilibrium variables that will be used throughout the rest of the analysis. First, we compute aggregate production of the two intermediate goods $Y_{m,t}$ and $Y_{z,t}$ as functions of output prices $p_{m,t}$, $p_{z,t}$, labor inputs $L_{m,t}$, $L_{z,t}$ and aggregate sectoral quality indices $Q_{m,t}$, $Q_{z,t}$. Insert the equilibrium demand schedules for capital goods $x_{m,t}$, $x_{z,t}$ stated in equations (5.14a) and (5.14b) into the production function of the intermediate goods given in (5.11):

$$Y_{m,t} = \mu^{\frac{2\mu}{1-\mu}} p_{m,t}^{\frac{\mu}{1-\mu}} L_{m,t} Q_{m,t}, \tag{6.1}$$

$$Y_{z,t} = \mu^{\frac{2\mu}{1-\mu}} p_{z,t}^{\frac{\mu}{1-\mu}} L_{z,t} Q_{z,t}. \tag{6.2}$$

The ratio of (6.1) and (6.2) is then equal to

$$\frac{Y_{z,t}}{Y_{m,t}} = \left(\frac{p_{z,t}}{p_{m,t}}\right)^{\frac{\mu}{1-\mu}} \frac{L_{z,t}}{L_{m,t}} \frac{Q_{z,t}}{Q_{m,t}}. \tag{6.3}$$

For purpose of compact notation, we use the following convention throughout the rest of this equilibrium analysis:
For any pair of sectoral variables a_M for sector m and a_Z for sector Z define the *relative variable with respect to sector* Z by $\frac{a_Z}{a_M} =: \tilde{a}$.
With this convention, (6.3) reads

$$\tilde{Y} = \tilde{p}^{\frac{\mu}{1-\mu}} \tilde{L} \tilde{Q}. \tag{6.4}$$

Now we use the first order conditions of the final good sector, as stated in (5.8), to eliminate the term $\tilde{Y}$ from (6.4). This gives us the expression for the relative price of the two intermediate goods Y_m and Y_z as a function of relative factor supply and relative productivity:

$$\tilde{p}_t := \frac{p_{z,t}}{p_{m,t}} = \left(\tilde{\gamma}^{-\varepsilon} \tilde{L}_t \tilde{Q}_t\right)^{-\frac{(1-\mu)}{\sigma}} \tag{6.5}$$

where $\tilde{\gamma} := \frac{1-\gamma}{\gamma}$, and $\sigma := 1 + (1-\mu)(\varepsilon - 1)$. By construction, $\sigma > 1$ whenever $\varepsilon > 1$. Note that this relative price is decreasing in relative employment $\tilde{L}$ and relative productivity of capital goods $\tilde{Q}$.
Furthermore, we can express the relative price in terms of the relative productivity $\tilde{Q}$. Combine the demand functions for the capital goods (equation (5.13b)) with the first order conditions for labor (equation (5.13a)) yields:

$$\tilde{p}_t = \tilde{Q}_t^{-(1-\mu)}. \tag{6.6}$$

This equation simply shows that the input produced by means of more productive capital goods will be relatively cheaper.
Next we derive equilibrium credit and deposit interest rates, and sectoral credit quantities. Note that the stated equations all contain various endogenous variables on their right hand side that will be determined next, so the treatment here is incomplete. To get the equilibrium deposit interest rate r_t use (5.50) together with (5.52), (5.51), (5.44), and (5.31), we get

$$r_{t+1} = \frac{\alpha}{S_t}\Big(\iota_t \mathbb{E}_t[\Pi_{m,t+1}] + (1-\iota_t)\mathbb{E}_t[\Pi_{z,t+1}]\Big) \tag{6.7}$$

where $\mathbb{E}_t[\Pi_{j,t}]$ denotes sectoral average expected profits for innovators containing endogenous intermediate output prices, capital good qualities and success-probabilities. I determine the expressions in detail below.
The equilibrium interest rate on deposits is decreasing in aggregate savings which is equivalent to deposit supply, and increasing in sectoral expected profits from innovation. Insert (6.7) into (5.31) to get the credit interest rates for sector j :

$$R_{j,t+1} = \frac{\alpha}{\eta_j S_t}\Big(\mathbb{E}_t[\Pi_{m,t+1}] + \mathbb{E}_t[\Pi_{z,t+1}]\Big), \qquad j \in \{m, z\}. \tag{6.8}$$

Take the ratio of (6.8) to compute the relative credit interest rate $\tilde{R}_t$:

$$\tilde{R}_t = \frac{R_{z,t}}{R_{m,t}} = \frac{\eta_{m,t}}{\eta_{z,t}} = \tilde{\eta}_t^{-1}, \tag{6.9}$$

so the relative credit interest rate is inversely related to the ratio of success-probabilities in innovation effort.
Now use (5.28) and (5.31), the success-probability in sector j reads

$$\eta_{j,t} = \zeta^{-1}\left(\frac{\alpha}{1+\alpha}\frac{\bar{\pi}_{j,t+1}}{r_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}, \qquad j \in \{m, z\}, \tag{6.10}$$

where $\bar{\pi}_{j,t} := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{j,t}^{\frac{1}{1-\mu}} L_{j,t}$.

6.2. Determinants of green technical change

Average profits from capital goods selling for old innovators in sector m read

$$\Pi_{m,t} = (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{m,t}^{\frac{1}{1-\mu}} L_{m,t} Q_{m,t} \tag{6.11}$$

and in sector z

$$\Pi_{m,t} = (1-\mu)\mu^{\frac{1+\mu}{1-\mu}} p_{z,t}^{\frac{1}{1-\mu}} L_{z,t} Q_{z,t}. \tag{6.12}$$

The net present discounted value of expected profits for young innovators in sector j, denoted by $V_{j,t}$, $j \in \{m, z\}$, are then given by

$$V_{j,t} = \frac{\mathbb{E}_t[\Pi_{j,t+1}]}{r_t}. \tag{6.13}$$

one can show that the V's are equal to

$$V_{m,t} = \mu_0 p_{m,t+1}^{\frac{1}{1-\mu}} L_{m,t+1} Q_{m,t+1} R_{m,t+1}^{-1} \qquad V_{z,t} = \mu_0 p_{z,t+1}^{\frac{1}{1-\mu}} L_{z,t+1} Q_{z,t+1} R_{z,t+1}^{-1}. \tag{6.14}$$

where $\mu_0 := (1-\mu)\mu^{\frac{1+\mu}{1-\mu}}$. So the *greater $V_{z,t}$ relative to $V_{m,t}$, the greater is the reward for innovators to develop dirty capital goods* (see Acemoglu (2002) p.789). The two profit equations given in (3.20) suggest that capital goods producers' profits increase with greater output prices, with greater factor use and with greater productivity in the intermediate sectors and decrease with larger interest rates on business credit. The latter effect simply follows from the relationship between credit interest rates and success-probabilities given in (2.3).
To show and discuss these determinants in more detail, take the ratio of $V_{z,t}$ and $V_{m,t}$. This gives

$$\frac{V_{z,t}}{V_{m,t}} = \underbrace{\left(\frac{p_{z,t+1}}{p_{m,t+1}}\right)^{\frac{1}{1-\mu}}}_{\substack{\text{Price}\\ \text{effect}}} \times \underbrace{\frac{L_{z,t+1}}{L_{m,t+1}}}_{\substack{\text{Market size}\\ \text{effect}}} \times \underbrace{\left(\frac{R_{z,t+1}}{R_{m,t+1}}\right)^{-1}}_{\substack{\text{Risk}\\ \text{effect}}} \times \underbrace{\left(\frac{Q_{z,t+1}}{Q_{m,t+1}}\right)}_{\substack{\text{Productivity}\\ \text{effect}}}. \tag{6.15}$$

The higher this ratio, the greater is the incentive for innovators to develop new technologies for the dirty sector z, so it is more profitable to direct innovation effort towards dirty technologies. The four different market forces stated in equation (6.15) shape the incentive to innovate on clean versus dirty technologies: *The price effect* (captured by the term $(p_z/p_m^{\frac{1}{1-\mu}}$) directs technology innovations towards the sector with higher prices; *the market size effect* (captured by the term L_z/L_m encourages innovations in the sector with the greater employment and therefore the larger market for capital goods; *the productivity effect* (captured by the term Q_z/Q_m) pushes innovation effort towards the sector with the greater productivity; *the risk effect* (captured by the term $(R_z/R_m)^{-1}$) encourages innovations in the sector with a lower credit interest rate and comes from the additional consideration of capital constrained innovators and capital supplying banks in the economy.
This suggests that if innovative firms are capital constrained and financial intermediaries finance R&D expenditures, then the resulting additional term in the determinants of green technical change should be considered in policy evaluation. For instance, if (environmental) policies seek to direct technical change towards the development of new carbon low technologies - via subsidies on green R&D or taxes on dirty inputes etc.- the instruments should consider the risk effect in firms relative profitability of developing these green commodities.

6.3. Laissez-faire equilibrium

In this subsection, we briefly characterize equilibrium labor decisions and intermediate output prices and derive the growth rate of the economy in a *laissez-faire* equilibrium outcome, i.e. a decentralized equilibrium without any policy intervention as defined in 6.1. Note that we relax the condition of innovation and thus growth in both sectors and additionally allow "corner solutions", where innovations are exclusively directed to either the green or the dirty intermediate sector. Here, we focus on the implications of these cases of laissez-faire equilibrium for climate change in view of definition 5.1. The analysis of "corner solutions" is important, since climate policy intervention in part three seeks to direct technical change -possibly completely- towards "green" technologies.

With the notational simplification introduced previously, the relative profitability of directing R&D to the dirty sector z becomes

$$\tilde{V}_t = \tilde{p}_{t+1}^{\frac{1}{1-\mu}} \tilde{L}_{t+1} \tilde{Q}_{t+1} \tilde{R}_{t+1}^{-1}. \tag{6.16}$$

First, eliminating the endogenous relative price given in (6.5), the relative expected profitability of directing R&D towards the dirty sector becomes

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon} \tilde{L}_t^{\sigma-1} \tilde{Q}_t^{\sigma-1} \tilde{R}^{-\sigma}\right)^{\frac{1}{\sigma}}. \tag{6.17}$$

Relative credit interest rates – the risk effect – still equals

$$\tilde{R}_{t+1} = \frac{R_{z,t+1}}{R_{m,t+1}} = \frac{\eta_{m,t}}{\eta_{z,t}} = \tilde{\eta}_t^{-1} = \left(\tilde{p}_{t+1}^{\frac{1}{1-\mu}} \tilde{L}_{t+1}\right)^{\frac{\alpha}{1-\alpha}}. \tag{6.18}$$

so the relative credit interest rate is inversely related to the ratio of success-probabilities in innovation effort, given in (6.10) and endogenously determined by sectoral R&D expenditures. Inserting (6.18) into (6.17) relative expected profits from innovation in the dirty sector read:

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon} \tilde{L}_t^{\sigma-1} \tilde{Q}_t^{-\varphi}\right)^{\frac{1}{1-\varphi}}, \qquad \varphi := 1 - \sigma(1-\alpha). \tag{6.19}$$

To get relative sectoral labor as a function of relative productivity, take the relative price from (6.5), solve for $\tilde{L}$ and insert $\tilde{p}$ from (6.6):

$$\tilde{L}_t = \tilde{\gamma}^{\varepsilon} \tilde{Q}_t^{\sigma-1}. \tag{6.20}$$

This equation simply states that if the two intermediate inputs are gross substitutes $\varepsilon > 1$ (and therefore $\sigma > 1$), the sector employing a greater share of people is also the sector featuring greater aggregate productivity.

Using (6.20), the expected relative profitability of innovation in the dirty sector finally can be written as

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon} \tilde{Q}_{t+1}^{\sigma-1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{6.21}$$

The next proposition then can be derived from (6.21):

Proposition 6.1. *In a "laissez-faire equilibrium", innovation at time $t \geq 0$ occurs in the "green" sector m only if $Q_{m,t} > \tilde{\gamma}^{\frac{\varepsilon}{\sigma-1-\alpha}}(1+\eta_{m,t}\bar{\lambda}_m)^{-1}Q_{z,t}$, in the "dirty" sector z only if $Q_{z,t} > \tilde{\gamma}^{-\frac{\varepsilon}{\sigma-1-\alpha}}(1+\eta_{z,t}\bar{\lambda}_z)Q_{m,t}$ and in both sectors if $Q_{z,t}/Q_{m,t} = \tilde{\gamma}^{\frac{-\varepsilon}{\sigma-1-\alpha}}$.*

Proof: See Appendix B.
Accordingly, since innovators direct their investment to the sector that promises the higher return from innovation, three situations implied in proposition 6.1 can occur:

$$(i)\ \tilde{V}_t > 1, \quad (ii)\ \tilde{V}_t = 1, \quad (iii)\ \tilde{V}_t < 1. \tag{6.22}$$

If $\tilde{V}_t > 1$, innovation in the dirty sector is more profitable than innovation in the green sector. Hence, all innovators direct their R&D to sector z and so $\iota_t = 0$ and innovation occurs only in the dirty sector.
If $\tilde{V}_t = 1$, innovation is equally profitable in the two sectors and consequently innovators are indifferent. A mass of ι_t innovators direct their R&D to sector m and a mass of $1 - \iota_t$ direct innovation effort to sector z and innovation occurs simultaneously in both sectors. This is similar to the situation in part one.
If $\tilde{V}_t > 1$, innovation in the green sector is more profitable than innovation in the dirty sector. Hence, all innovators direct their R&D to sector m and so $\iota_t = 1$ and innovation occurs only in sector m. In part three, we will concentrate on the first two situations, however, since in the latter scenario, policy intervention is obsolete, because the conditions are such that in this "corner solution" only green technologies grow.
The normalized price term of the final good was given in equation (5.9):

$$P = [\gamma^{\varepsilon} p_m^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_z^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \equiv 1. \tag{6.23}$$

Taking the relative price of the intermediate goods given in (6.6), solving for p_z and inserting the result into the previous expression gives prices of good Y_m:

$$p_{m,t} = \left(\frac{Q_{z,t}^{1-\sigma}}{\hat{W}_t}\right)^{\frac{1}{1-\varepsilon}} \tag{6.24}$$

and of good Y_z:

$$p_{z,t} = \left(\frac{Q_{m,t}^{1-\sigma}}{\hat{W}_t}\right)^{\frac{1}{1-\varepsilon}}, \tag{6.25}$$

where $\hat{W}_t = \gamma^{\varepsilon} Q_{z,t}^{1-\sigma} + (1-\gamma)^{\varepsilon} Q_{m,t}^{1-\sigma}$. To get levels of sectoral employment, use (6.20), solve for L_z and insert this result into the labor market clearing condition (5.53):

$$L_{m,t} = \frac{\gamma^{\varepsilon} Q_{z,t}^{1-\sigma}}{\hat{W}_t} \tag{6.26}$$

and

$$L_{z,t} = \frac{(1-\gamma)^{\varepsilon} Q_{z,t}^{1-\sigma}}{\hat{W}_t}. \tag{6.27}$$

These expressions are used below to derive equilibrium variables for the case of i) innovation in both sectors and ii) the "corner solution" of dirty innovation only.

Green and dirty innovations
First, consider the situation of innovation on green and dirty capital goods. To have innovators who are willing to invest in R&D in both sectors, expected profits from improving the productivity of capital goods in both sectors have to be equal and thus for $t \geq 0$, the condition

$$\tilde{V}_t = 1 \tag{6.28}$$

must hold, where $\tilde{V}_t$ is given in equation (6.21). Recall that lemma (3.1) showed if $\tilde{V}_t = 1$, then intermediate sector growth rates are identical, so $g_{m,t} = g_{z,t} =: g_t$.
Insert endogenous prices and employment levels into (6.7) to compute the equilibrium deposit interest rate:

$$r_t = \frac{\alpha\gamma^{\varepsilon}}{1+\alpha}\left(\frac{\mu(1+g_t)^2}{\zeta}\right)^{1-\alpha}\left(\frac{Q_{z,t}}{\hat{W}_t^{\frac{1}{1-\sigma}}}\right)^{1+\alpha-\sigma}, \tag{6.29}$$

where $\hat{W}_t = \gamma^{\varepsilon} Q_{z,t}^{1-\sigma} + (1-\gamma)^{\varepsilon} Q_{m,t}^{1-\sigma}$. Note that r_t is constant, since the term $Q_{z,t}/\hat{W}_t^{\frac{1}{1-\sigma}}$ is constant as long as $Q_{m,t}$ and $Q_{z,t}$ grow at identical rates.
Inserting endogenous prices, employment levels and the deposit interest rate into (6.10), sectoral success-probabilities are given by

$$\eta_{m,t} = \left(\frac{Q_{z,t}\hat{W}_t^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu(1+g_t)^2}\right)^{\alpha} \tag{6.30}$$

and

$$\eta_{z,t} = \left(\frac{\tilde{\gamma}^{\frac{\varepsilon}{1-\alpha}}\hat{W}_t^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu(1+g_t)^2}\right)^{\alpha}\left(\frac{Q_{m,t}^{2-\sigma}}{Q_{z,t}^{1+\alpha-\sigma}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{6.31}$$

Credit interest rates can be computed from (5.32), (6.29), (6.30), and (6.31):

$$R_{m,t} = \frac{\alpha\gamma^{\varepsilon}\mu(1+g_t)^2}{1+\alpha}\frac{Q_{z,t}}{\hat{W}_t^{\frac{1}{1-\sigma}}} \tag{6.32}$$

and

$$R_{z,t} = \frac{\alpha\gamma^{\varepsilon}\mu(1+g_t)^2}{(1+\alpha)\hat{W}_t}\left(\frac{Q_{z,t}^{1+\alpha-\sigma}}{Q_{m,t}^{(2-\sigma)\alpha}}\right)^{\frac{1}{1-\alpha}}. \tag{6.33}$$

The mass of innovators in sector m is given by ι_t and in sector z by $1-\iota_t$. In equilibrium, the term $\iota_t \in]0,1[$ can be computed as

$$\iota_t = \frac{\bar{\lambda}_z}{\bar{\lambda}_z + \tilde{\gamma}^{-\frac{\varepsilon\alpha}{(1-\alpha)(1+\alpha-\sigma)}}\bar{\lambda}_m}. \tag{6.34}$$

Intermediate production equals

$$Y_{m,t} = \mu^{\frac{2\mu}{1-\mu}}\gamma^{\varepsilon}\left(Q_{z,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma} Q_{m,t} \tag{6.35}$$

$$Y_{z,t} = \mu^{\frac{2\mu}{1-\mu}}(1-\gamma)^{\varepsilon}\left(Q_{m,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma} Q_{z,t} \tag{6.36}$$

These two expressions imply

$$\frac{Y_{m,t+1}}{Y_{m,t}} = \frac{Q_{m,t+1}}{Q_{m,t}} =: g_{m,t} \qquad \frac{Y_{z,t+1}}{Y_{z,t}} = \frac{Q_{z,t+1}}{Q_{z,t}} =: g_{z,t}, \tag{6.37}$$

so sectoral output grows with the rate of average sector capital goods quality (because the terms in brackets are constant as long as $Q_{m,t}$ and $Q_{z,t}$ grow at identical rates). Since $g_{m,t} = g_{z,t} =: g_t$ if $\tilde{V}_t = 1$ (lemma 3.1), intermediate output growth rates are identical. Equation (5.10) implies that final output grows also at the rate g_t.
This equilibrium growth rate g_t is given by an implicit expression:

$$g_t(1+g_t)^{2\alpha} = \frac{\bar{\lambda}_z\bar{\lambda}_m}{\bar{\lambda}_z + \tilde{\gamma}^{-\frac{\varepsilon\alpha}{(1-\alpha)(1+\alpha-\sigma)}}\bar{\lambda}_m}\left(\frac{Q_{z,t}\hat{W}_t^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu}\right)^{\alpha} \tag{6.38}$$

Note that the term on the right hand side is constant over time, as long as growth of $Q_{m,t}$ and $Q_{z,t}$ is identical (which follows from Lemma (3.1). Define this term as

$$\hat{\Lambda} := \frac{\bar{\lambda}_z\bar{\lambda}_m}{\bar{\lambda}_z + \tilde{\gamma}^{-\frac{\varepsilon\alpha}{(1-\alpha)(1+\alpha-\sigma)}}\bar{\lambda}_m}\left(\frac{Q_{z,t}\hat{W}_t^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu}\right)^{\alpha} \tag{6.39}$$

Similar to part one, in equilibrium with innovation in both sectors, the growth rate has to be such that the equality condition in (6.38) holds. This is again equivalent to the problem of finding the root(s) of the function

$$\hat{F}(g_t) := g_t(1+g_t)^{2\alpha} - \hat{\Lambda}. \tag{6.40}$$

The economy's growth rate is unique if the function $\hat{F}(\cdot)$ has only one root. The following lemma – similar to lemma 3.2 in part one – states the conditions under which $\hat{F}$ has one zero point. We use these results to characterize the implications on climate change afterwards.

Lemma 6.1. *Let F given in (3.65) be defined on the interval* $[g_u, g^o]$, *with* $0 < g_u < g^o$. *Then for appropriate values of* g_u, g^o *there exist one* g_t^* *such that* $F(g_t^*) = 0$.

Proof: See Appendix B.
Finally, to get the implications for CO_2 emissions and climate change if innovation occurs in both sectors, take the relationship between emissions and dirty sector production given in (5.10)

$$E_t = \kappa Y_{z,t}. \tag{6.41}$$

This shows that carbon emissions grow also at the rate g_t and we can conclude:

Proposition 6.2. *Suppose,* $\tilde{V}_t = 1$ *for* $t \geq 0$ *and* $\sigma > 1$. *In this case, a a laissez-faire equilibrium exists where innovation occurs in both sectors and intermediate outputs, final output, and* CO_2*-emissions grow at identical rates.*□

Equations (5.3a), (5.3b), and (5.4) state the relationship between CO_2 emissions E_t and atmospheric CO_2-concentration A_t:

$$A_{1,t} = A_{1,t-1} + \phi_L E_t, \tag{6.42a}$$
$$A_{2,t} = (1-\phi)A_{2,t-1} + (1-\phi_L)\phi_0 E_t, \tag{6.42b}$$
$$A_t = A_{1,t} + A_{2,t}. \tag{6.42c}$$

So rising emissions increase the carbon concentration in the atmosphere and we get the following result:

Proposition 6.3. *If $\tilde{V}_t = 1$ and $\sigma > 1$. Then, although innovation occurs in the dirty sector z and in the green sector m, the laissez-faire equilibrium runs into a climate catastrophe, according to definition 6.1, where losses in aggregate output are above a critical bound $\bar{b} > 0$.*□

This result directly follows from the fact that rising emissions over time increase atmospheric carbon concentration A_t (equations (5.3a)-(5.4)) and that damages to aggregate output $D(A_t)$ increase with increasing A_t (equation (5.5)).
So even if both sectors co-exist and grow over time at identical growth rates, the economy runs into a bad climate state, featuring a high atmospheric carbon concentration and large damages (in fact, damages above $\bar{b}$) to final output due to climate change. In this regard, the "size" of the dirty sector is too large.

Dirty sector innovations
Consider first the "corner solution" where innovation is completely directed to sector z. With regard to the implications for climate change, the result will be similar to the scenario of growth in both sectors, i.e. the economy will run into a climate catastrophe. This is unsurprising since in the previously considered case of simultaneous growth, production levels in sector z were already too large and now, dirty intermediates gain even more weight in final output production.
To have innovators who are willing to invest in dirty technologies only, expected profits from improving the productivity of capital goods in sector z have to be greater than those of sector m and therefore, for $t \geq 0$, the condition

$$\tilde{V}_t > 1 \tag{6.43}$$

must hold, where $\tilde{V}_t$ is again given in equation (6.21). The equilibrium deposit interest rate is given by

$$r_t = \frac{\alpha(1-\gamma)^{\varepsilon}}{(1+\alpha)Q_{m,t}^{\sigma-1-\alpha}}\left(\frac{\mu(1+g_{z,t})^2}{\zeta}\right)^{1-\alpha}\left(\frac{\hat{W}_{t+1}^{\sigma-2}}{\hat{W}_{t-1}^{1-\alpha}}\right)^{\frac{1}{\sigma-1}}, \tag{6.44}$$

where $\hat{W}_t = \gamma^{\varepsilon}Q_{z,t}^{1-\sigma} + (1-\gamma)^{\varepsilon}Q_{m,t}^{1-\sigma}$.
Innovators in sector z are successful in research with a probability equal to

$$\eta_{z,t} = \left(\frac{Q_{m,t}\hat{W}_{t-1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu(1+g_{z,t})^2}\right)^{\alpha}. \tag{6.45}$$

Banks demand credit interest in sector z equal to

$$R_{z,t} = \frac{\alpha(1-\gamma)^{\varepsilon}\mu(1+g_{z,t})^2}{(1+\alpha)Q_{m,t}^{\sigma-1}}\left(\frac{\hat{W}_{t+1}^{2-\sigma}}{\hat{W}_{t-1}}\right)^{\frac{1}{\sigma-1}}. \tag{6.46}$$

Intermediate production still equals

$$Y_{m,t} = \mu^{\frac{2\mu}{1-\mu}}\gamma^{\varepsilon}\left(Q_{z,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma} Q_{m,t} \tag{6.47}$$

and

$$Y_{z,t} = \mu^{\frac{2\mu}{1-\mu}}(1-\gamma)^{\varepsilon}\left(Q_{m,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma} Q_{z,t}. \tag{6.48}$$

One can show that dirty sector production grows at a rate equal to $g_{z,t}$, while green intermediate production grows at a rate equal to $(1 + g_{z,t})^{2-\sigma}$, even though the productivity of green capital goods is constant. If $\sigma > 1$, however, the fraction of green intermediate input use in total intermediate production decreases, since dirty sector production grows faster compared to green sector production.[64]
This implies for CO_2 emissions and climate change that carbon emissions also grow at the rate $g_{z,t}$ and we can conclude:

Proposition 6.4. *Suppose, $\tilde{V}_t > 1$ for $t \geq 0$ and $\sigma > 1$. In this case a laissez-faire equilibrium exists where innovation occurs in sector z only. Dirty intermediate output and CO_2-emissions grow at a rate equal to $\eta_{z,t}\bar{\lambda}_z$.*□

Since rising emissions increase the carbon concentration in the atmosphere, this immediately implies:

Proposition 6.5. *If $\tilde{V}_t > 1$ and $\sigma > 1$. Then, a laissez-faire equilibrium with dirty innovation runs into a climate catastrophe, according to definition 6.1, where losses in aggregate output are above a critical bound $\bar{b} > 0$.*□

This result directly follows from the fact that rising emissions over time increase atmospheric carbon concentration A_t (equations (5.3a)-(5.4)) and that damages to aggregate output $D(A_t)$ rise with increasing A_t (equation (5.5)).

Green sector innovations
Finally, consider the "corner solution" where the conditions are such that innovations are completely directed to sector m. To have innovators who are willing to invest in green technologies only, expected profits from improving the productivity of capital goods in sector m have to be greater than those of sector z and therefore, for $t \geq 0$, the condition

$$\tilde{V}_t < 1 \tag{6.49}$$

[64]If Y_z grows at a rate equal to $1+g_{z,t}$, then Y_m grows at a rate $(1+g_{z,t})^{2-\sigma}$. Then $1+g_{z,t} > (1+g_{z,t})^{2-\sigma} \Leftrightarrow 1 > 1/(1+g_{z,t})^{\sigma-1}$. This condition is fulfilled if $\sigma > 1$.

must hold, where $\tilde{V}_t$ is given in equation (6.21). The equilibrium deposit interest rate is then given by

$$r_t = \frac{\alpha\gamma^{\varepsilon}}{(1+\alpha)Q_{m,t}^{\sigma-1-\alpha}}\left(\frac{\mu(1+g_{m,t})^2}{\zeta}\right)^{1-\alpha}\left(\frac{\hat{W}_{t+1}^{\sigma-2}}{\hat{W}_{t-1}^{1-\alpha}}\right)^{\frac{1}{\sigma-1}}, \tag{6.50}$$

where $\hat{W}_t = \gamma^{\varepsilon}Q_{z,t}^{1-\sigma} + (1-\gamma)^{\varepsilon}Q_{m,t}^{1-\sigma}$.
Innovators in sector z are successful in research with a probability equal to

$$\eta_{m,t} = \left(\frac{(1-\gamma)^{\varepsilon}Q_{z,t}\hat{W}_{t-1}^{\frac{1}{\sigma-1}}}{\zeta^{\frac{1-\alpha}{\alpha}}\mu(1+g_{m,t})^2}\right)^{\alpha}. \tag{6.51}$$

Banks demand credit interest in sector m equal to

$$R_{m,t} = \frac{\alpha\gamma^{\varepsilon}\mu(1+g_{mt})^2}{(1+\alpha)Q_{z,t}^{\sigma-1}}\left(\frac{\hat{W}_{t+1}^{2-\sigma}}{\hat{W}_{t-1}}\right)^{\frac{1}{\sigma-1}}. \tag{6.52}$$

Similar to the"corner solution“ of dirty innovation only, intermediate production in case of exclusively green innovations equals

$$Y_{m,t} = \mu^{\frac{2\mu}{1-\mu}}\gamma^{\varepsilon}\left(Q_{z,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma}Q_{m,t} \tag{6.53}$$

and

$$Y_{z,t} = \mu^{\frac{2\mu}{1-\mu}}(1-\gamma)^{\varepsilon}\left(Q_{m,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma}Q_{z,t}. \tag{6.54}$$

One can show that in this scenario, green sector production grows at a rate $g_{m,t}$, while dirty intermediate production grows at a rate equal to $(1+g_{m,t})^{2-\sigma}$, even though the productivity of dirty capital goods is constant. If $\sigma > 1$, however, the fraction of dirty intermediate inputs used in final output production decreases, since the production of y_m grows faster than the production of Y_z.[65]
This implies for CO_2 emissions and climate change that carbon emissions also grow at the rate $(1+g_{z,t})^{2-\sigma}$ and we can conclude:

Proposition 6.6. *Suppose,* $\tilde{V}_t > 1$ *for* $t \geq 0$ *and* $\sigma > 1$. *In this case a laissez-faire equilibrium exists where innovation occurs in sector m only. Dirty intermediate output and* CO_2*-emissions "grow" at a rate equal to* $(1+\eta_{m,t}\bar{\lambda}_m)^{2-\sigma}$.□

Note that the growth rate $(1+\eta_{m,t}\bar{\lambda}_m)^{2-\sigma}$ is not necessarily greater than one. For $\sigma < 2$ this holds and in this case emissions rise. This immediately implies:

Proposition 6.7. *If* $\tilde{V}_t > 1$ *and* $\sigma < 2$. *Then, a laissez-faire equilibrium with green innovation runs into a climate catastrophe, according to definition 6.1, where losses in aggregate output are above a critical bound* $\bar{b} > 0$.□

[65]If Y_m grows at a rate equal to $1+g_{m,t}$, then Y_z grows at a rate $(1+g_{m,t})^{2-\sigma}$. Then $1+g_{m,t} > (1+g_{m,t})^{2-\sigma} \Leftrightarrow 1 > 1/(1+g_{m,t})^{\sigma-1}$. This condition is fulfilled if $\sigma > 1$.

The argument is similar to the previous case of dirty innovation: rising emissions over time increase atmospheric carbon concentration A_t (equations (5.3a)-(5.4)) and damages to aggregate output $D(A_t)$ rise with increasing A_t (equation (5.5)).
One interpretation for the case where $\sigma < 2$ and dirty production increases although only green capital goods improve over time is that improvements in ”green“ technologies also correspond to improvements in the technology of final good production which uses them as inputs; the final good, in turn, is an input for the dirty sector because machines employed in this sector are produced using the final good; hence, technical change in sector m creates a force towards the expansion of sector z.
These three cases combined with the fact that the risk effect enters the determinants of directed technical change represent a major argument for the analysis of part three: to protect the economy from running into a climate catastrophe, some form of policy intervention is indeed necessary, and in this the role of credit and financial intermediation has to be taken into account.

7. Concluding Remarks II

The macroeconomic model presented in the second part of this dissertation is a first step towards a comprehensive framework that can be used for theoretical and quantitative analysis of the interactions between climate change, endogenous technology and financial intermediation. This part formulates a dynamic general equilibrium endogenous technical change model of the world, treated as a uniform region and inhabited by two generations of consumers, where there is a global externality from emitting carbon. This externality is a by-product of using fossil fuel or generally a ”dirty” good as an input into production. The formal framework complements to the existing literature on quantitative general equilibrium models of climate change by introducing capital constrained firms and credit supplying financial intermediaries together with a simple model of the atmospheric carbon cycle into the model.

The analysis lead to two major outcomes. One result of the second part was that banks influence the direction of “green” technical change. This influence manifests itself through the presence of the *risk effect* in the determinants of directed private innovation effort. The risk effect encourages innovations in those sectors, where the risk of failure during the innovation process is lower.

The long run properties of the laissez-faire equilibrium found in the analysis are related to the rate of substitutability between CO_2 intensive or ”dirty“ inputs and CO_2 neutral or ”green“ inputs. When the two intermediate inputs are sufficiently substitutable and the CO_2 neutral sector is initially relatively less productive, then the economy always runs into a climate catastrophe here defined as damages to GDP above a critical threshold. Hence, additional market forces associated with profit maximizing financial intermediaries do not change the general outcome that under laissez-faire, the economy always runs into a state of high atmospheric CO_2 concentration and significant damages to GDP. This is intuitive, since financial intermediaries finance the ventures of firms in those sectors with the highest expected return from credit lending and especially do not consider the negative emission externality associated with dirty sector production in their lending strategy. And this already implies the second outcome: policy intervention is indeed necessary to avoid a climate catastrophe.

B. Appendix: Mathematical proofs part II

Proposition 6.1

Proof. [66]

In the following proof, I characterize the equilibrium allocations of innovators in the two sectors m and z for the case of gross substitutability between intermediate goods Y_m and Y_z (i.e. $\varepsilon > 1$), and provide a proof of proposition 6.1.

The equilibrium relative profitability is given in (6.21) and reads:

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon}\tilde{Q}_{t+1}^{\sigma-1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{B.1}$$

Use development of average sectoral capital goods quality (B.1) can be written as:

$$\tilde{V}_t = \tilde{\gamma}^{\frac{\varepsilon}{1-\alpha}}\left(\frac{1+(1-\iota_t)\eta_{z,t}\bar{\lambda}_z}{1+\iota_t\eta_{m,t}\bar{\lambda}_m}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}}. \tag{B.2}$$

The mass of innovators is given by $\iota_t \in [0,1]$. For $\iota_t \in [0,1]$, we can define a function $f(\iota_t)$ as

$$f(\iota_t) := \tilde{\gamma}^{\frac{\varepsilon}{1-\alpha}}\left(\frac{1+(1-\iota_t)\eta_{z,t}\bar{\lambda}_z}{1+\iota_t\eta_{m,t}\bar{\lambda}_m}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}} \tag{B.3}$$

and rewrite $f(\iota_t) = \tilde{V}_t$. Clearly, if $f(0) > 1$, then $\iota_t = 0$ is an equilibrium; if $f(1) < 1$, then $\iota_t = 1$ is an equilibrium; and finally if $f(\iota_t^*) = 1$ for some $\iota_t^* \in]0,1[$, then ι_t^* is an equilibrium. Given these observations, we have to distinguish three possible situations:

1. If $\frac{\sigma-1-\alpha}{1-\alpha} > 0$, then $f(\iota_t)$ is strictly decreasing in ι_t. Then it immediately follows that: (i) if $f(0) > 1$, then $\iota_t = 0$ is the unique equilibrium and all innovators direct R&D towards the dirty sector; (ii) if $f(1) < 1$, then $\iota_t = 1$ is the unique equilibrium and all innovators direct R&D towarss the green sector; (iii) if $f(1) > 1 > f(0)$, then by continuity there exists a unique $\iota_t^* \in]0,1[$ such that $f(\iota_t^*) = 1$, which is the unique (interior) equilibrium and innovation occurs in both sectors.

2. If $\frac{\sigma-1-\alpha}{1-\alpha} < 0$, then $f(\iota_t)$ is strictly increasing in ι_t and we have: (i) if $1 < f(1) < f(0)$; then $\iota_t = 0$ is the unique equilibrium; (ii) if $f(1) < f(0) < 1$; then $\iota_t = 1$ is the unique equilibrium; (iii) if $f(1) < 1 < f(0) < 1$; then there we have an interior equilibrium with a unique $\iota_t^* \in]0,1[$ such that $f(\iota_t^*) = 1$.

3.If $\frac{\sigma-1-\alpha}{1-\alpha} = 0$, then $f(\iota_t) \equiv f$ is a constant. If $f > 1$, then $\iota_t = 0$ is the unique equilibrium; if $f < 1$, then $\iota_t = 0$ is the unique equilibrium.

Together this characterizes the allocation of innovators across sectors and implies the results in proposition 6.1. □

Lemma 6.1

Proof. The function F is defined on the interval $[g_u, g^o]$ with $0 < g_u < g^o$ and according to (3.65) given by

$$F(g_t) := (g_t(1+g_t)^{2\alpha} - \bar{\Lambda}. \tag{B.4}$$

[66]The proof follows the arguments given in Acemoglu et al. (2012) appendix A.

First, note that $F(\cdot)$ is continuous on its domain and strictly increasing in its argument g_t. Then, for a given parameter value $\bar{\Lambda} > 0$, we can choose a g_u such that $g_u < \bar{\Lambda}$. Then $F(g_u) < 0$. Since $\bar{\Lambda}$ is constant, we can find a g^o with $g^o > \bar{\Lambda}$. Then $F(g^o) > 0$. Together this implies $F(g_o)F(g^u) < 0$ and we can conclude using the intermediate value theorem that g_t^* in $]g_u, g^o[$ exist with $F(g_t^*) = 0$. This already shows that g_t^* is a root of F. Uniqueness of g_t^* follows, since F is strictly increasing in g_t. Therefore, the growth rate of the balanced growth equilibrium is unique. □

Part III

CLIMATE POLICY ANALYSIS

Introduction

The previous section revealed that under certain conditions, the decentralized equilibrium leads into a climate catastrophe, meaning that at some point in time, losses per period in aggregate output due to climate change will lie above a critical level.
In the context of this thesis, the aim of any environmental policy instrument is to reduce the damages to aggregate output resulting from climate change. So the third and last part of this thesis analyzes which environmental policy instruments can help avoid a climate catastrophe, especially given that banks have an influence on the direction of R&D effort. This work highlights the importance of the *risk effect* for the direction of green technical changes. Recall that this risk effect encourages innovations in those sectors where the interest rate on business credit is lower, which in the present framework is equivalent to saying that the risk effect encourages innovations in those sectors where the chances of successful innovation are greater. I also examine to what extent the consideration of financial intermediaries affect the results of the existing literature on directed technical progress and climate change. Given the findings of parts one and two, the third part answers the stylized research question: *What policy rule helps the economy move towards a path of sustained and green economic growth?*
The remainder of this part is organized as follows. Section 8 briefly shows how simple policy interventions can prevent a climate catastrophe and clarifies the role of banks in these results. Different measures to evaluate climate policy intervention are considered. This section characterizes the equilibrium of the economy for a given climate policy and analyzes optimal climate policy in this setup. Section 9 contains a numerical simulation example. This dissertation ends with a conclusion and an outlook on possible future research directions (10).

8. Climate Policy

In order to avoid a climate catastrophe as defined in 5.1, some type of policy intervention is necessary, since under laissez-faire, the economy runs into such a state. In this connection, the need for policy intervention does not hinge on whether financial intermediation is explicitly considered, but the results with respect to the speed of adjustment, the temporary cost of policy intervention, the strength of intervention et cetera may alter when the *risk effect* associated with private sector lending is taken into account. This could imply changing results with respect to environmental policy evaluation when we consider capital constrained innovators and credit markets.

8.1. Directed Technical Change and a Climate Catastrophe

For an initial and preliminary analysis of such a policy intervention, suppose a government subsidizes R&D in the green sector by a subsidy proportional to green sector profits (financed through a lump-sum tax on consumers income). The results with respect to the role of banks would be identical if instead of a subsidy on green R&D, the government would impose a tax on the profits from research in the dirty sector. So from a formal perspective, it is irrelevant for the results of this thesis whether the government uses a tax or a subsidy in order to direct technical changes towards the green sector. However, it is important from an economic perspective, because the resources for a subsidy need to be raised first. For instance, the German Renewable Energy Act (EEG) aims to promote the development of environmentally friendly technologies through a feed-in tariff, i.e. a legally guaranteed payment for electricity produced by green energies such as solar, wind, biomass or small hydro power plants that is being fed into the national electricity grid and is financed through a renewable energy surcharge paid by firms and private households. To retain some analogy to the empirically observed policy, I assume that the government subsidizes research in the green sector. Assume for the moment that initially, innovators' expected profits in sector z are greater than those in sector m:

$$V_{z,0} > V_{m.0}. \tag{8.1}$$

Then, with regard to the relative expected profits of different periods, the following lemma holds:

Lemma 8.1. *Let $V_{z,t^\circ} \geq V_{t^\circ}$ hold for some $t^\circ \geq 0$. Then we have $V_{z,t} \geq V_{m,t}$ for all $t \geq t^\circ$.*

Proof: See appendix C.

Remark 8.1. *From lemma 8.1 directly follows that if $V_{z,t^\circ} < V_{m,t^\circ}$ holds for some $t \geq 0$, then $V_{t,z} < V_{m,t}$ holds for all $t \geq t^\circ$.*

So if expected profits are such that in some t, innovators direct their investment to either one sector or are indifferent between sectors respectively, then expected profits in all subsequent periods will be such that innovators direct their investment to either one sector or are indifferent between sectors respectively.
Given (8.1), Lemma 8.1 implies

$$V_{z,t} > V_{m,t} \text{ for all } t \geq 0. \tag{8.2}$$

If initially, young innovators direct R&D to sector z, then all future generations of young innovators in all subsequent periods will direct R&D to dirty capital goods improvement. This implies on the one hand, that without intervention, innovation in the green sector is absent. On the other hand, this also implies that a government could in principle introduce a subsidy proportional to profits in the green sector that is sufficiently high to redirect innovators' investment decisions towards the green sector. In period $t \geq 0$, the government could introduce a subsidy rate denoted by d_t, so that

$$V_{z,t} < (1 + d_t)V_{mt}, \tag{8.3}$$

which is equivalent to

$$d_t > \tilde{V}_t - 1. \tag{8.4}$$

Inserting the expected relative profits from (6.21) into equation (8.4), using the dynamics of capital goods quality (5.42) and solving for d_t, gives a lower bound for the subsidy rate d_t:

$$d_t \geq \hat{d}_t \equiv \left[\left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} \left(\frac{1 + (1 - \iota_t)\eta_{z,t}\bar{\lambda}_z}{1 + \iota_t \eta_{m,t}\bar{\lambda}_m} \right)^{\sigma-1-\alpha} \left(\frac{Q_{z,t}}{Q_{m,t}} \right)^{\sigma-1-\alpha} \right]^{\frac{1}{1-\alpha}} - 1. \tag{8.5}$$

Note that this lower bound decreases with the mass of innovators in sector m, denoted by ι_t. Thus the greatest lower bound for the subsidy d_t is equal to

$$d_t \geq \hat{d}_t \equiv \left[\left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} \left(1 + \eta_{z,t} \bar{\lambda}_z \right)^{\sigma-1-\alpha} \left(\frac{Q_{z,t}}{Q_{m,t}} \right)^{\sigma-1-\alpha} \right]^{\frac{1}{1-\alpha}} - 1. \tag{8.6}$$

This implies:

Proposition 8.1. *The implementation of a subsidy on R&D in the green sector m equal to* $d_t = \hat{d}_t + \nu$*, where* $\hat{d}_t$ *is given by equation (8.6) and* $\nu > 0$ *is an arbitrary small positive number, is sufficient to achieve the desired objective of redirecting innovation effort and thus technical change away from the dirty sector towards the green sector.*

This proposition shows the relevance of endogenous directed technical change: governmental intervention can influence profit incentives and thus redirect technical change towards green sectors. Given that the intermediate goods are sufficiently substitutable, once the green sector is sufficiently advanced, market forces of profit maximizing product innovation will guarantee that only the productivity of the green, emission free capital goods will be further developed. This result does not depend on whether the risk effect enters the determinants of directed technical change. It will turn out in the next section, however, that the size of d_t, sufficient to redirect innovators' investment decisions, does depend on the risk effect.
Whether intervention can be temporary or needs to be permanent in order to avoid a climate catastrophe depends on the value of ε as the next proposition shows:

Proposition 8.2. *If the two intermediate goods are gross substitutes and* $\varepsilon \in [\frac{2-\mu}{1-\mu}, \infty[$*, a temporary intervention- the implementation of a subsidy on R&D in sector m for a finite number of periods t- will limit climate change and prevent a climate catastrophe as defined in 5.1.*

Proof: See Appendix C.
We can conclude that intervention can redirect all research to the green sector. Equation (6.36) however implies that even after this happens, $Y_{z,t}$ and thus E_t will grow at a rate of $(1+\eta_{m,t}\bar{\lambda}_m)^{2-\sigma}$. Nevertheless, if the two intermediate goods Y_m and Y_z are sufficiently substitutable, i.e. $\varepsilon > (2-\mu)/(1-\mu)$ (wich is equivalent to $\sigma > 2$), then it follows from (6.36) and (5.10) that dirty production and CO_2 emissions will not grow. In this case, temporary intervention is sufficient to redirect technical change towards green technologies and thus to limit the effects of climate change.

8.2. Measures of policy intervention

Nevertheless, financial intermediaries might affect environmental policy, because after intervention, the economy needs some time to close the technology gap between the dirty and the green sector. The lenth of time required for adjustment could depend on the risk effect and thus on the consideration of the banks. Moreover, the optimum strength of policy intervention, measured by the size of the temporary subsidy could change; the length of intervention, given by the number of periods with an active intervention, or the

cost of delaying policy intervention into the future, might also alter with the consideration of credit and banks.
Throughout the rest of this section, we assume for the initial level of relative productivity in the two sectors:

Assumption 8.1.

$$\frac{Q_{z,0}}{Q_{m,0}} > \min\left\{\left(\frac{1-\gamma}{\gamma}\right)^{1+\alpha-\sigma}\left(1+\eta_{m,0}\bar{\lambda}_m\right), \left(\frac{1-\gamma}{\gamma}\right)^{1+\alpha-\sigma}\left(1+\eta_{z,0}\bar{\lambda}_z\right)^{-1}\right\}. \tag{8.7}$$

This assumption imposes the condition that initially, the green sector is sufficiently backward or less productive relative to the dirty sector so that in a decentralized laissez-faire equilibrium, the economy starts innovating in the dirty sector.[67] Lemma 8.1 then implies that innovators direct R&D exclusively to the dirty sector in all subsequent periods and thus only the productivity of capital goods used in the dirty sector improves over time. Since in this case, dirty sector output grows at a rate equal to the average quality of dirty sector capital goods, the economy runs into a climate catastrophe under laissez-faire.
If we take a brief look at empirical observations, assumption 8.1 seems to be justified: First, according to the statistics of the International Energy Agency (IEA), the share of fossil fuel based energy in world primary energy consumption equals almost 82%, so the world economy uses mostly CO_2-emitting or dirty energy type. In view of the present model, this implies a relatively large dirty sector z. Second, the share of clean (CO_2-neutral) technologies in world electricity generation account for less than 1/3 (16.3% Hydro, 10.6% Nuclear and 5.7% Renewables). Third, the energy conversion efficiency of clean power plants is lower than in conventional thermal power plants based on fossil fuel.[68]

The strength of intervention
The first measure is given by subsidy rates sufficiently high to redirect R&D towards the green sector. To show the role of credit and banks, this paragraph considers the direction of innovation effort *with the risk effect* (I) and *without the risk effect* (II) and discusses the influence of banks on the size of the subsidy rate, i.e. the *strength of intervention*.[69]
When the risk effect enters the determinants of endogenous directed technical change, the

[67] This assumption is based on intial expected relative profitability of technology development $V_{z,0}/V_{m,0}$]. If initially (in period $t = 0$) the expected (for period $t = 1$) profits from innovation in the dirty sector are greater than the expected profits from innovation in the green sector, i.e. $V_{z,t}/V_{m,t} > 1$, innovators would direct R&D to the dirty sector in all subsequent periods and the economy moves towards a climate catastrophe. See also Acemoglu et al. (2012) page 139.

[68] The German Ministry for Economic Affairs and Energy (2013) reports efficiency conversion rates of about 15% for photovoltaics, 33% for nuclear power and 50% for wind, and 45% for power plants based on fossil fuel.

[69] The hypothetical situation without the risk effect in the determinants of directed technical change can be interpreted as a situation where the risk effect is equal to one. This would occur for instance in the special case where innovation takes place in both intermediate sectors and the "size" of sectoral innovations are identical, i.e. $\bar{\lambda}_m = \bar{\lambda}_z$.

relative expected profitability from innovation effort in the dirty sector is equal to

$$\tilde{V}_t^{(I)} = \left(\frac{V_{z,t}}{V_{m,t}}\right)^{(I)} = \left[\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1+(1-\iota_t)\eta_{z,t}\bar{\lambda}_z}{1+\iota_t\eta_{m,t}\bar{\lambda}_m}\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\sigma-1-\alpha}\right]^{\frac{1}{1-\alpha}}. \tag{8.8}$$

Recall from the previous analysis that subsidy rate d_t in this case (I) must satisfy:

$$d_t^{(I)} \geq \hat{d}_t^{(I)} \equiv \left[\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(1+\eta_{z,t}\bar{\lambda}_z\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\sigma-1-\alpha}\right]^{\frac{1}{1-\alpha}} - 1. \tag{8.9}$$

One can show that relative expected profits of dirty sector R&D without the risk effect are given by

$$\tilde{V}_t^{(II)} = \left(\frac{V_{z,t}}{V_{m,t}}\right)^{(II)} = \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1+(1-\iota_t)\eta_{z,t}\bar{\lambda}_z}{1+\iota_t\eta_{m,t}\bar{\lambda}_m}\right)^{\sigma-1}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\sigma-1}. \tag{8.10}$$

and the subsidy rate in this case (II) must satisfy

$$d_t^{(II)} \geq \hat{d}_t^{(II)} \equiv \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(1+\eta_{z,t}\bar{\lambda}_z\right)^{\sigma-1}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\sigma-1} - 1. \tag{8.11}$$

Equations (8.9) and (8.11) state subsidy rates on profits from innovation for the emission free green sector that are sufficiently high to create an incentive for entrepreneurs to direct their R&D effort solely to that sector. For the purpose of this study it is most important to note that $d_t^{(I)}$ and $d_t^{(II)}$ have different values, indicating an influence of credit and banks on the strength of intervention.
Moreover, for the case of $\sigma > 1 + \alpha$ the subsidy and, thus, the *strength of intervention* has the following properties:

Proposition 8.3. *The strength of intervention d_t is non-decreasing in the productivity gap $\tilde{Q}_t$ and in the elasticity of substitution ε. In addition, increases in d_t are stronger the greater the elasticity of substitution ε is. These properties hold with and without consideration of the risk effect.*

Proof: Appendix C.
The subsidy increases with the technology gap $Q_{z,t}/Q_{m,t}$. An initially greater gap leads to greater initial relative expected profitability of developing new capital goods for the emission intensive sector z. Therefore, the subsidy on green R&D has to be larger in order to redirect innovations to the green sector. In addition, a larger elasticity of substitution ($\varepsilon > 1 \Leftrightarrow \sigma > 1$) leads to a greater initial gap in relative expected profits $\tilde{V}_t$.
For a given level of initial relative expected profits, i.e. an initial profit gap $\tilde{V}_t$, it is straightforward to compute the following relationship between relative profits with and without the risk effect:

$$\tilde{V}_t^{(I)} = \tilde{R}_{t+1}^{-1}\tilde{V}_t^{(II)}. \tag{8.12}$$

This equation implies that the initial profit gap including the risk effect $\tilde{V}_t^{(I)}$ is greater than the initial profit gap without the risk effect $\tilde{V}_t^{(I)}$ if and only if

$$\tilde{V}_t^{(II)} > \tilde{V}_t^{(I)} \quad \Leftrightarrow \quad \tilde{V}_t^{(II)} > \tilde{R}_{t+1}^{-1}\tilde{V}_t^{(II)} \quad \Leftrightarrow \quad 1 < \tilde{R}_{t+1} \quad \Leftrightarrow \quad R_{z,t+1} > R_{m,t+1}. \tag{8.13}$$

This implies that if the credit interest rate is smaller in sector m, the subsidy rate sufficiently high to create an incentive for innovators to direct innovation effort towards sector m can be lower if the risk effect associated with banks and credit enters the determinants of directed technical change. With smaller credit interest rates in sector m, the credit costs in the dirty sector are relatively greater (given an idential credit volume, which is implicitly assumed here). Hence, the expected relative profits from innovation in sector z inclusive credit costs are ceteris paribus smaller compared to the case without the credit costs (and so without the risk effect). Therefore, a subsidy rate that contains these credit costs can be smaller and still be sufficiently high to create the incentive for innovators to direct R&D towards the green sector.
Clearly, this result holds vice versa, whenever credit interest rates in sector z are greater than those in sector m. In this case, the presence of the risk effect implies that policy intervention has to be stronger to redirect innovators R&D effort to the green sector.
The question which situation is empirically more relevant is debatable. One could argue that a sector with a low number of previous product generations offers a higher probability of successful innovation, because it gets more difficult to "find" a product of higher quality, the further up one hase come on the quality ladder. Then credit interest rates would be lower in "less advanced" sectors. One could also mention that the probability of successful innovation is greater in more advanced sectors, because training curves make it easier to invent a new capital good of higher quality the higher the number of previous inventions. Then credit interest rates would be lower in "more advanced" sectors.
For the purpose of this study it is more important to note that credit and banks generally influence the strength of climate policy intervention as long as $\eta_{z,t} \neq \eta_{m,t}$ (which holds in equilibrium) and therefore should be taken into account in climate policy evaluation.
Moreover, according to (8.8) and (8.10) relative profits in situation (I) change with a rate equal to $(1 + \eta_{m,t^\circ}\bar{\lambda}_m)^{\frac{1+\alpha-\sigma}{1-\alpha}}$ and in situation (II) the rate of change equals $(1 + \eta_{m,t^\circ}\bar{\lambda}_m)^{1-\sigma}$. By assumption $\varepsilon > 1 \Leftrightarrow \sigma := 1+(1-\mu)(\varepsilon-1) > 1$. Relative expected profits $(V_{z,t}/V_{m,t})^{(II)}$ thus decline over time. If $\sigma > 1+\alpha$, then relative expected profits including the risk effect $(V_{z,t}/V_{m,t})^{(I)}$ also decline over time.
If $1 < \sigma \leq 1 + \alpha \Leftrightarrow 1 < \varepsilon \leq (1 + \alpha - \mu)/(1 - \mu)$, however, relative expected profits do not decline during time of (temporary) intervention, although capital goods quality increase. This implies that paying regard to credit and banks in the determinants of directed technical change imposes a stronger requirement on the degree of substitutability between green an dirty capital goods; and an omission of this requirement consequently could lead to misleading results in the evaluation of effectiveness of climate policy intervention.

The length of intervention
Closely connected to the previous analysis of *how strongly* climate policy has to intervene is the examination of *how long* governmental intervention should be maintained in order to redirect technical change towards an emission free path of economic growth even without the subsidy.
Let us proceed in a manner similar to the previous paragraph and compare the length of policy intervention with (case (I)) and without (case II) the risk effect. Suppose a government starts regulation in some period $t^\circ \geq 0$ (for instance as stated in proposition 7.5 in

part two, where without such an intervention, technical change is completely directed to dirty capital goods leading to climate change). Then the length of intervention depends on the "profit gap" in t°, $\tilde{V}_{t^\circ}$, i.e. the differences in innovation returns in sectors m and z. Since average productivity of capital goods determine profits, this immediately implies that the "productivity gap" $Q_{z,t^\circ}/Q_{m,t^\circ}$ determines the length of intervention. Clearly, the larger the profit gap, ceteris paribus the longer it takes the economy to adjust towards a state where green innovations occur even without intervention. So the length of intervention depends on the rate at which the profit gap declines. We will show now that this rate depends on whether we consider the risk effect.

Denote the number of periods until the green sector overtakes the dirty sector in terms of innovation profitability by j_t. After j_t periods, innovation returns are such that technical change is directed towards the green sector even without the subsidy. Formally this implies

$$\tilde{V}_{t+j-1}(1 + d_{t+j-1}) \leq 1 \qquad \tilde{V}_{t+j} < 1. \tag{8.14}$$

Propositions 9.1 and showed that temporary climate policy intervention can redirect technical change towards green capital goods improvement, proposition 9.2 stated conditions for intervention to be temporary. Recall that average sectoral quality of capital goods in sector z is freezed during intervention (and also afterwards) so so $Q_{z,t+1} = Q_{z,t}$ for all $t \geq t^\circ$ and average productivity of capital goods in sector m increases and changes over time according to $Q_{m,t+1} = (1 + \eta_{m,t}\bar{\lambda}_m)Q_{m,t}$ for $t \geq t^\circ$, because the mass of innovators in sector m is equal to $\iota_t = 1$ for all $t \geq t^\circ$. In period $t^\circ + j$ the average productivity of capital goods in sector m can be computed recursively as

$$Q_{m,t^\circ+j} = \prod_{n=0}^{j}(1 + \eta_{m,t^\circ+n}\lambda_m)Q_{m,t^\circ}. \tag{8.15}$$

Use this expression to compute the relative expected profits from innovation with the risk effect in period $t^\circ + j$ recursively as

$$\tilde{V}^{(I)}_{t^\circ+j} = \left(\frac{V_{z,t^\circ+j}}{V_{m,t^\circ+j}}\right)^{(I)} = \left[\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\prod_{n=0}^{j}\left(1 + \eta_{m,t^\circ+n}\bar{\lambda}_m\right)^{1+\alpha-\sigma}\left(\frac{Q_{z,t^\circ}}{Q_{m,t^\circ}}\right)^{\sigma-1-\alpha}\right]^{\frac{1}{1-\alpha}}. \tag{8.16}$$

and in situation (II)

$$\tilde{V}^{(II)}_{t^\circ+j} = \left(\frac{V_{z,t^\circ+j}}{V_{m,t^\circ+j}}\right)^{(II)} = \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\prod_{n=0}^{j}\left(1 + \eta_{m,t^\circ+n}\bar{\lambda}_m\right)^{1-\sigma}\left(\frac{Q_{z,t^\circ}}{Q_{m,t^\circ}}\right)^{\sigma-1}. \tag{8.17}$$

Accordingly relative profits in situation (I) change with a rate equal to $(1 + \eta_{m,t^\circ}\bar{\lambda}_m)^{\frac{1+\alpha-\sigma}{1-\alpha}}$ and in situation (II) the rate of change equals $(1 + \eta_{m,t^\circ}\bar{\lambda}_m)^{1-\sigma}$. By assumption $\varepsilon > 1 \Leftrightarrow \sigma := 1 + (1-\mu)(\varepsilon - 1) > 1$. Relative expected profits $(V_{z,t}/V_{m,t})^{(II)}$ thus decline over time. If $\sigma > 1 + \alpha$, then relative expected profits including the risk effect $(V_{z,t}/V_{m,t})^{(I)}$ also decline over time. Clearly, these rates differ, indicating an influence of credit and banks on the length of intervention (the number j_{t° is different in (I) and (II)).

Since again, as in the previous analysis of strength of intervention, if $1 < \sigma \leq 1 + \alpha \Leftrightarrow$

$1 < \varepsilon \leq (1+\alpha-\mu)/(1-\mu)$ relative expected profits do not decline during the time of (temporary) intervention, even though productivity of green capital goods increase over time. This means that paying regard to credit and banks in the determinants of directed technical change imposes a stronger requirement on the degree of substitutability between green an dirty capital goods. Climate policy evaluations without considering the risk effect in the determinants of technical change could probably misjudge the effectiveness of climate policy intervention.
Equations (8.16) and (8.17) implicitly contain the number of necessary time periods until the expected profits from innovation in the green sector are greater than those of the dirty sector even without the subsidy. For an elasticity of substitution $\varepsilon > (1+\alpha-\mu)/(1-\mu)$ and intervention starting in any period $t \geq 1$, the following properties for the length of intervention hold:

Remark 8.2. *The length of intervention is non-decreasing in the "technology gap" $Q_{z,t}/Q_{m,t}$ and the elasticity of substitution ε. These properties hold with and without consideration of the risk effect and are thus independent of credit and financial intermediation.*

First, the length of intervention rises with increasing $Q_{z,t}/Q_{m,t}$. The larger this gap is when governmental intervention starts, the greater are initial relative expected returns on innovation in sector z. Since the "profit gap" $(V_{z,t}/V_{m,t})^{(I)}$ closes at a rate $(1+\eta_{m,t}\bar{\lambda}_m)^{\frac{1+\alpha-\sigma}{1-\alpha}}$, a greater initial technology gap implies a longer time period until the economy reaches a point, where profits from innovation in the green sector are strictly greater than the profits from innovation in the dirty sector even without a subsidy on green R&D.
Second, larger values of ε lead to greater initial gaps in relative expected profits $V_{z,t}/V_{m,t}$. The argument now is similar to the previous one: during the time of intervention, in situation (I), $(V_{z,t}/V_{m,t})^{(I)}$ closes at a rate $(1+\eta_{m,t}\bar{\lambda}_m)^{\frac{1+\alpha-\sigma}{1-\alpha}}$ and in situation (II), $(V_{z,t}/V_{m,t})^{(II)}$ closes with rate $(1+\eta_{m,t}\bar{\lambda}_m)^{1-\sigma}$. Intuitively, if the two inputs are close substitutes, final output production relies mostly on the more productive input, and therefore, productivity improvements in the green sector (taking place during the intervention phase) will have less impact on overall productivity until green capital goods surpass the dirty ones. This holds for both situations (I) and (II).

Cost of adjustment
The third measure of climate policy is the *cost of adjustment* defined as the number of periods necessary for the economy under the policy intervention to reach the same level of output growth as it would have done within one period in the absence of the intervention. The cost of adjustment occurs, because after intervention, the less advanced green sector productivity catches up on the more advanced productivity in the dirty sector and final output increases more slowly than the case where innovation continues to be directed towards the dirty sector.
Suppose governmental intervention starts in an arbitrary period $t^{\circ} \geq 0$. Then we have to analyze the number of periods the economy needs to achieve the same output growth as it would have achieved in just one period in the laissez-faire equilibrium. Denote aggregate output under laissez-faire as $Y_{t^{\circ}}^{(LF)}$ and under period t° intervention as $Y_{t^{\circ}}^{(IN)}$. Formally, this means we are looking for the number of periods denoted by $j \in \mathbb{N}$ the economy needs

until

$$\frac{Y_{t^\circ+1}^{(LF)}}{Y_{t^\circ}^{(LF)}} = \frac{Y_{t^\circ+j}^{(IN)}}{Y_{t^\circ+j-1}^{(IN)}} \tag{8.18}$$

The cost of delay

The last measure is given by the *cost of delay*, i.e. the cost of delaying policy intervention (one period) into the future. Then, the following result holds:

Proposition 8.4. *Suppose the government postpones intervention from period $t^\circ \geq 0$ to period $t^\circ + 1$. Then the subsidy rate of period $t^\circ + 1$ intervention is greater than the subsidy rate that would have been sufficient to redirect innovation effort to the green sector in period t: $d_{t+1} > d_t$.*

Proof: See appendix C.

The proposition shows that delaying intervention is costly, not only because of the continued rise in atmospheric carbon concentration and thus greater economic damages due to climate change in the future, but also because it will make stronger intervention necessary. During the delay of intervention (from t to $t + 1$ or even more periods), the technology gap $Q_{z,t+1}/Q_{m,t+1}$ increases further and, thus, so does the profit gap $V_{z,t}/V_{m,t}$. Hence, the subsidy rate, sufficient to close this gap in profits and thereby redirect entrepreneurs innovation effort to sector m, has to be higher compared to the subsidy rate that would have been sufficient if intervention had not been postponed. Moreover, since the technology gap widens with a delay in intervention, postponed intervention will make longer intervention necessary as well.

Summarizing, this section has established that a simple policy intervention that "redirects" technical change toward emission free sectors or technologies can help to prevent a climate catastrophe. The discussion showed that postponing intervention into the future leads to greater economic costs, not only because it further contributes to climate change, but also because it widens the gap between dirty and green technologies, thereby inducing a longer period of catch-up with slower growth.[70]

8.3. Equilibrium with climate policy

In the following we study optimal (climate) policies which implement a social optimal allocation as an equilibrium allocation. To reduce carbon emissions, a social planner can use a tax on dirty intermediate production $Y_{z,t}$ –a carbon tax– and a subsidy on innovation return in the green sector. Formally, we define these two climate policy instruments as follows:

Definition 8.1. *A carbon tax and a subsidy on "green" R&D are non-negative sequences $\tau = (\tau_t)_{t\geq 0}$ and $d = (d_t)_{t\geq 0}$ where τ_t is the tax rate to be paid per unit of CO_2 in period $t \geq 0$ and d_t is the subsidy rate proportional to the return on "green" capital good innovations in period $t \geq 0$.*

[70] See also Acemoglu et al. (2012).

The emissions tax levied on dirty intermediate input use is paid by firms in the final output sector. The taxes are collected by governmental authorities and passed on as a lump-sum transfer T_t^Y to consumers in equal shares. Subsidies are paid on returns from R&D in sector m and financed by a lump-sum tax T_t^R on consumers income also in equal shares. This formulation of governmental intervention implies slight adjustments in the decision problems of final output and income profiles of consumers.
Final output production uses the technology given in (5.6):

$$Y_t = (1 - D(A_t))\mathcal{F}(Y_{m,t}, Y_{z,t}) = (1 - D(A_t))\left(\gamma Y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_{z,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{8.19}$$

where still $D(A_t) := 1 - e^{-\theta(A_t - \underline{A})}$ is the damage function and A_t represents atmospheric CO_2-concentration and is defined in (5.4). Given damage-adjusted productivity the final output sector also takes prices, taxes and technology parameters as given. The profit maximization inclusive a tax on dirty input use –a carbon tax– reads:

$$\max_{(Y_m, Y_z) \in \mathbb{R}_+^2} \left\{ Y_t - (p_{m,t}Y_{m,t} + (1+\tau_t)p_z Y_{z,t}) \quad | \quad Y_t = (5.6) \right\} \tag{8.20}$$

A solution to (8.20) satisfies the following first order conditions which equate prices and marginal products of each production factor for all $t \geq 0$:

$$p_{m,t} = (1 - D(A_t))\gamma\left(\frac{Y_{m,t}}{Y_t}\right)^{-\frac{1}{\varepsilon}} \tag{8.21a}$$

$$(1+\tau_t)p_{z,t} = (1 - D(A_t))(1-\gamma)\left(\frac{Y_{z,t}}{Y_t}\right)^{-\frac{1}{\varepsilon}}. \tag{8.21b}$$

Note that the solution to (8.20) is identical to the laissez-faire equilibrium of part two if $\tau_t = 0$.
Consumers' preferences over consumption are given in assumption 5.10 and read

$$U(c_{\ell,t}^y, c_{\ell,t+1}^o) := \beta u(c_{\ell,t+1}^o) = \beta \log(c_{\ell,t}^{(o)}), \tag{8.22}$$

where the subindex $\ell \in \{i, w, e\}$ identifies consumption of workers (w), entrepreneurs (e) and innovators (i).
Given labor income of w_t and since the entire income is saved, we have $s_t = w_t$, and old workers have income of $r_{t+1}s_t$. In addition, old workers' income may alter due to transfers and taxes given that a government introduces a tax and transfer scheme. More precisely, carbon taxes levied on the use of dirty intermediate inputs are collected by governmental authorities and passed on as a lump-sum transfer $T_{w,t}^Y$ to old workers. If the goverment subsidizes R&D expenditures, a lump-sum tax $T_{w,t}^G$ is levied on old workers income to finance the subsidy. This leads to the following aggregate budget constraint of old workers:

$$C_{w,t+1}^{(o)} = r_{t+1}S_t + T_{w,t}^Y - T_{w,t}^R = r_{t+1}w_t + T_{w,t}^Y - T_{w,t}^R =: I_{w,t+1}^{(o)}. \tag{8.23}$$

In a similar way, the income of old entrepreneurs and innovators alters. The aggregate budget constraint of the entrepreneurs including taxes proceedings and subsidy costs is then given by

$$C^{(o)}_{e,t} = \bar{\Pi}^{e}_{t} + T^{Y}_{e,t} - T^{R}_{e,t} =: I^{(o)}_{e,t} \tag{8.24}$$

and the budget constraint for innovators in sector j is equal to

$$C^{(o)}_{i,j,t} = \bar{\Pi}^{i}_{j,t} + T^{Y}_{i,t} - T^{R}_{i,t} =: I^{(o)}_{i,j,t}. \tag{8.25}$$

The rest of the decision problems, market clearing conditions and resource constraints remain unchanged and we can define an equilibrium for a given climate tax τ and subsidy d as:

Definition 8.2. *An equilibrium of the economy considered here is an allocation* $\mathbf{A} = (Y_t, C_t, (Y_{j,t}, X_{j,t}, H_{j,t}, L_{j,t}, \iota_{j,t}, (q_{\vartheta_j,t}, x_{\vartheta_j,t})_{\vartheta_j \in [0,1]})_{j\in\{m,z\}}, A_t, D_{t+1})_{t\geq 0}$ *and a price system* $\mathbf{P} = (r_t, w_t, R_{j,t}, p_{j,t}, p_{\vartheta_j,t})_{t\geq 0}$ *such that*
(i) The allocation is consistent with the production technologies (5.11), (5.6), and the market clearing conditions/resource constraints (5.50), (5.51), (5.53), (5.54), (5.55), and (5.56).
(ii) Banks behave optimally, i.e. equations (5.31) and (5.32) hold for all $t \geq 0$.
(iii) Producers behave optimally, i.e. equations (5.18), (5.26) , (5.13) hold for all $t \geq 0$.
(iv) Consumers behave optimally with (profit) incomes determined by (8.23), (8.24) and (8.25) for all $t \geq 0$.
(v) Average capital goods qualities in sector $j \in \{m, z\}$ *evolve according to (5.42).*
(vi) Climate variables evolve according to (5.3) with emissions given by (5.10) and climate damages in (5.6) determined by (5.4) and (5.5).

Thereby $C_t = \sum_{\ell\in\{w,e,i\}} C_{\ell,t}$ is the sum over period t consumption of old workers (w), old entrepreneurs (e) and old innovators (i). Note that the laissez-faire equilibrium defined in part two can be interpreted as a special case of the equilibrium defined here with no taxation ($\tau = 0$) and no R&D subsidy ($d = 0$).
Carbon taxes and green subsidies also affect different equilibrium variables. First the relative price of the two intermediate goods including an emissions tax can be computed from (8.21) as

$$\tilde{p}_t := \frac{p_{z,t}}{p_{m,t}} = \left((1+\tau_t)^{\varepsilon}\tilde{\gamma}^{-\varepsilon}\tilde{L}\tilde{Q}\right)^{-\frac{1-\mu}{\sigma}}, \tag{8.26}$$

where still $\mu \in]0,1[$ and $\sigma := 1 + (1-\mu)(\varepsilon - 1)$. So the relative price is decreasing in τ_t.
Expected profits from innovation, as given in (6.14), equal to

$$\tilde{V}_t = \tilde{p}_{t+1}^{\frac{1}{1-\mu}} \tilde{L}_{t+1} \tilde{Q}_{t+1} \tilde{R}^{-1} \tag{8.27}$$

and the risk effect (eq. (6.9)) equals

$$\tilde{R}_t = \left(\tilde{p}_t^{\frac{1}{1-\mu}} \tilde{L}_t\right)^{-\frac{\alpha}{1-\alpha}}. \tag{8.28}$$

Inserting (8.26) these two expressions become

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon}(1+\tau_t)^{-\varepsilon}\tilde{L}_{t+1}^{\sigma-1}\tilde{Q}_{t+1}^{\sigma-1}\tilde{R}^{-\sigma}\right)^{\frac{1}{\sigma}}. \tag{8.29}$$

and

$$\tilde{R}_t = \left(\tilde{\gamma}^{\varepsilon}(1+\tau_t)^{\frac{\varepsilon}{\sigma}}\tilde{L}_t^{\sigma-1}\tilde{Q}^{-1}\right)^{-\frac{\alpha}{\sigma(1-\alpha)}}. \tag{8.30}$$

So while expected relative profits are decreasing in τ_t, the risk effect is increasing in τ_t. After elimination of $\tilde{R}_t$, $\tilde{V}_t$ reads

$$\tilde{V}_t = \left(\tilde{\gamma}^{\varepsilon}(1+\tau_t)^{-\varepsilon}\tilde{L}_{t+1}^{\sigma-1}\tilde{Q}_{t+1}^{-\varphi}\right)^{\frac{1}{\sigma(1-\alpha)}}. \tag{8.31}$$

where $\varphi := 1-\sigma(1-\alpha)$. This influence of τ_t and since the risk effect is an additional determinant of directed technical change, these expressions play an important role in the following analysis.

8.4. Optimal allocation

In this section we determine an optimal allocation as the solution to a social planning problem which maximizes a weighted utility index of different generations of consumers subject to the constraints imposed by technology, resources, and climate change.
Consider a social planner who chooses a feasible allocation subject to restrictions imposed by technology, factor mobility, and resource constraints. Formally, the planner takes initial capital goods quality $Q_{j,0} > 0$, and the initial climate state A_{-1} as given. Compared to the laissez-faire equilibrium the planning problem incorporates the link between productivity in final production, damage, and climate change. Thus, the decision involves the choice of a feasible allocation defined next.

Definition 8.3. *(i) A feasible allocation is a sequence*

$$\mathbb{A} = (Y_t, C_t, (Y_{j,t}, X_{j,t}, H_{j,t}, L_{j,t}, \iota_{j,t}, (q_{\vartheta_j,t}, x_{\vartheta_j,t})_{\vartheta_j\in[0,1]})_{j\in\{m,z\}}, A_t, D_{t+1})_{t\geq 0} \tag{8.32}$$

which satisfies technologies (5.6), (5.11), and (5.22), resource constraints (5.53), (5.51) $(L), (H), (X), (\iota), (D)$ *(market clearing/ resource constraints)* $(A), (Q)$ *(climate evolution and capital goods quality development) for all t.*

(ii) The set of feasible allocations of the economy considered here is denoted $\mathcal{A}$.

Thereby $C_t = \sum_{\ell\in\{w,e,i\}} C_{\ell,t}$ is the sum over period t consumption of old workers (w), old entrepreneurs (e) and old innovators (i). Since by assumption young individuals do not consume, only old individuals matter for the social planner and we can skip the superindex ”o“ indicating old consumers in the following derivations. The social planner's objective is to maximize welfare which is here given by a weighted sum of utilities of all generations of old consumers (workers entrepreneurs and innovators):

$$W := \sum_{t=0}^{\infty}\omega^t\Big(\sum_{\ell\in\{w,e,i\}}\chi_\ell\beta u(C_{\ell,t})\Big). \tag{8.33}$$

Parameter $\omega \in]0,1[$ represents the constant *social discount factor* and states how the social planner weights the welfare of future generations. The parameter $\beta \in]0,1[$ is the *private discount factor* of the consumers. Moreover, the social planner distributes consumption across different consumers within one generation. The parameters $\chi_\ell \in]0,1[$ for $\ell \in \{w,e,i\}$ with $\chi_w + \chi_e + \chi_i = 1$ represent utility weights for the different consumer types. In principle, for $\omega = 1$ all generations would receive the same weight and there would be no discounting of utility of future generations. However, in this thesis, I leave open the question which values for ω and χ_ℓ would characterize an appropriate choice.[71]
The social planner's optimization problem is given by

$$\max_{\mathbb{A}} \Big\{ \sum_{t=0}^{\infty} \omega^t \Big(\sum_{\ell \in \{w,e,i\}} \chi_\ell \beta u(C_{\ell,t}) \Big) \mid s.t. \;\; \mathbb{A} = (8.32) \in \mathcal{A} \Big\} \tag{8.34}$$

A solution to (8.34) is

$$\mathbb{A}^* = (Y_t^*, C_t^*, (Y_{j,t}^*, X_{j,t}^*, H_{j,t}^*, L_{j,t}^*, \iota_{j,t}^*, (q_{\vartheta_j,t}^*, x_{\vartheta_j,t}^*)_{\vartheta_j \in [0,1]})_{j \in \{m,z\}}, D_{t+1}^*)_{t \geq 0} \tag{8.35}$$

and referred to as the *optimal allocation*. Adopting an infinite-dimensional Lagrangian approach, we can now derive conditions which completely characterize this solution. I provide detailed computations in appendix C. Standard arguments imply that

$$\mathbb{A} = (Y_t, C_t, (Y_{j,t}, X_{j,t}, H_{j,t}, L_{j,t}, \iota_{j,t}, (q_{\vartheta_j,t}, x_{\vartheta_j,t})_{\vartheta_j \in [0,1]})_{j \in \{m,z\}}, A_t, D_{t+1})_{t \geq 0}$$

is a solution to (8.34) if there exist non-negative Lagrange multipliers $\lambda = (\lambda_t)_{t \geq 0}$,

$$\lambda_t := \Big(\lambda_{0,t}, \lambda_{1,t}^A, \lambda_{2,t}^A, \lambda_t^D, \lambda_t^L, \lambda_t^\iota, (\lambda_{j,t}^Y)_{j \in \{m,z\}}, (\lambda_{\vartheta_j,t}^x, \lambda_{\vartheta_j,t}^q, \lambda_{\vartheta_j,t}^h)_{\vartheta_j \in [0,1], j \in \{m,z\}} \Big)$$

such that $(\mathbb{A}, \lambda)$ solve the resulting first order and complementary slackness conditions. For $t \geq 0$, define

$$\Omega_t := \kappa \left(\phi_L \frac{\lambda_{1,t}^A}{\lambda_{0,t}} + (1 - \phi_L)\phi_0 \frac{\lambda_{2,t}^A}{\lambda_{0,t}} \right). \tag{8.36}$$

After eliminating as many Lagrange variables as possible and using the functional forms for technology given in (5.6) and (5.11), and preferences over consumption represented by logarithmic utility, the first order conditions hold for all $t \geq 0$ and can be interpreted as follows.
The first order condition of the social planning problem with respect to $C_{\ell,t}$ gives:

$$\lambda_{0,t} = \chi_\ell u'(C_{\ell,t}) \qquad \ell \in \{w,e,i\}. \tag{8.37}$$

This implies an intra-temporal optimality condition for consumption across different consumers:

$$\chi_w u'(C_{w,t}) = \chi_i u'(C_{i,t}) = \chi_e u'(C_{e,t}). \tag{8.38}$$

[71] See for instance Hillebrand (2012).

The shadow price of intermediate goods use equal

$$\hat{\lambda}_{m,t} = \partial_{Y_m} F(Y_{m,t}, Y_{z,t}) = \gamma Y_t^{\frac{1}{\varepsilon}} Y_{m,t}^{-\frac{1}{\varepsilon}}. \tag{8.39a}$$

$$\begin{aligned}\hat{\lambda}_{z,t} &= \partial_{Y_z} F(Y_{m,t}, Y_{z,t}) - \frac{\lambda^A_{1,t}}{\lambda_{0,t}} \phi_L \kappa - (1-\phi_L)\phi_0 \kappa \frac{\lambda^A_{2,t}}{\lambda_{0,t}} \\ &= (1-\gamma) Y_t^{\frac{1}{\varepsilon}} Y_{z,t}^{-\frac{1}{\varepsilon}} - \kappa \Big(\frac{\lambda^A_{1,t}}{\lambda_{0,t}} \phi_L + (1-\phi_L)\phi_0 \frac{\lambda^A_{2,t}}{\lambda_{0,t}} \Big). \end{aligned} \tag{8.39b}$$

These two equations define the true shadow price of intermediate goods. Key difference between the optimal allocation and the equilibrium equations, as stated in (5.13), is a wedge between the marginal product and the shadow price of $Y_{z,t}$. This implies that the laissez-faire equilibrium equations fail to take the cost of emissions into account and the social planner introduces the wedge equal to $\kappa(\phi_L \lambda^A_{1,t}/\lambda_{0,t} + (1-\phi_L)\phi_0 \lambda^A_{2,t}/\lambda_{0,t})$. This wedge is equal to the emission cost of an additional unit of the dirty input (evaluated in terms of units of the final good at time t).
The first order condition with respect to sectoral capital goods investment $X_{j,t}$ implies:

$$\frac{\lambda^x_{\vartheta_j,t}}{\lambda_{0,t}} = 1 \qquad \forall \vartheta_j \in [0,1], j \in \{m, z\}. \tag{8.40}$$

So in the optimal allocation, shadow prices for all capital good lines in the two intermediate sectors are identical and equal to one. The latter result contrasts to the laissez-faire equilibrium, where capital good producers set prices equal to a mark-up over marginal prodution costs: $1/\mu$. This can also be interpreted as the socially-planned allocation involving a subsidy of μ in the use of capital goods.
To compute optimal capital goods quantities, insert (8.40) into the first order conditions with respect to $x_{\vartheta_j,t}$:

$$x^*_{\vartheta_j,t} = \Big(\mu \hat{\lambda}_{j,t}\Big)^{\frac{1}{1-\mu}} L_{j,t} q_{\vartheta_j,t}. \tag{8.41}$$

Next, eliminate λ^L_t from the first order conditions with respect to $L_{m,t}, L_{z,t}$, this gives

$$\lambda^Y_{m,t} \partial_{L_m} G(L_m, x_{\vartheta_m}) = \lambda^Y_{z,t} \partial_{L_z} G(L_z, x_{\vartheta_z}), \tag{8.42}$$

which ensures *intra-temporal* efficiency of sectoral employment in the optimal allocation.

Optimal emissions tax
To compute the optimal tax on dirty input use, i.e. the optimal CO_2 tax, use the first order conditions with respect to emission components $A_{1,t}$ and $A_{2,t}$:

$$\lambda^A_{1,t} = \lambda_{0,t} \frac{\partial D(A_{1,t} + A_{2,t})}{\partial A} \frac{Y_t}{1 - D(A_{1,t} + A_{2,t})} + \omega \frac{\lambda^A_{1,t+1}}{\lambda_{0,t}}. \tag{8.43a}$$

$$\lambda^A_{2,t} = \lambda_{0,t} \frac{\partial D(A_{1,t} + A_{2,t})}{\partial A} \frac{Y_t}{1 - D(A_{1,t} + A_{2,t})} + \omega(1-\phi) \frac{\lambda^A_{2,t+1}}{\lambda_{0,t}}. \tag{8.43b}$$

These two conditions can be solved forward to obtain

$$\frac{\lambda^A_{1,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty} \omega^n \frac{\lambda_{0,t+n}}{\lambda_{0,t}} \frac{\partial D(A_{t+n})}{\partial A} \frac{Y_{t+n}}{1 - D(A_{t+n})} \tag{8.44a}$$

$$\frac{\lambda^A_{2,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty} \omega^n (1-\phi)^n \frac{\lambda_{0,t+n}}{\lambda_{0,t}} \frac{\partial D(A_{t+n})}{\partial A} \frac{Y_{t+n}}{1 - D(A_{t+n})}. \tag{8.44b}$$

Inserting (8.44a) and (8.44b) into (8.36) and use (8.37) gives

$$\Omega_t = \kappa \sum_{n=0}^{\infty} \omega^n \frac{u'(C_{\ell,t+n})}{u'(C_{\ell,t})} \frac{\partial D(A_{t+n})}{\partial A} \frac{Y_{t+n}}{1 - D(A_{t+n})} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n\Big). \tag{8.45}$$

So the total costs of emitting one additional unit of CO_2 in period t, i.e. the total cost associated with one additional unit of dirty production $Y_{z,t}$ (measured in units of time t consumption) equals the discounted sum of all future climate damages caused by this emission. Note that Ω_t depends on the structural parameters of the model and endogenous model variables in a complicated way. The term (8.45) is the key quantity to incorporate the climate externality into the (shadow) price of dirty input production and thus forms the basis for the optimal tax on CO_2 emissions.
Consider briefly the comparative static influence of the parameters in Ω_t. Emissions per unit of dirty good production, given by parameter κ, as well as greater expected damages given in the function $D(A_t)$, increase Ω_t. The pattern of atmospheric carbon cycle also influences total cost of emissions: the longer emissions remain in the atmosphere (permanently ϕ_L and temporary over some periods $1 - \phi$), the larger is Ω_t. In contrast to this, higher values for the discount factor ω lower total costs of emissions.
Then the social planner can implement an emissions tax, i.e. a tax on the use of dirty input by the final good producer equal to:

$$\tau_t = \frac{\kappa}{\hat{\lambda}_{z,t}} \sum_{n=0}^{\infty} \omega^n \frac{u'(C_{\ell,t+n})}{u'(C_{\ell,t})} \frac{\partial D(A_{t+n})}{\partial A} \frac{Y_{t+n}}{1 - D(A_{t+n})} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n\Big). \tag{8.46}$$

Beside similar influences that parameters have on Ω_t, this tax rate will be higher when the shadow value of emissions quality is greater, when the marginal utility of consumption today is lower, and when the price of dirty input is lower.

Knowledge externality
The optimal allocation also corrects for a knowledge externality, because in the decentralized equilibrium, innovators do not internalize the effects of their research on productivity in the future. Using the functional form for intermediate production G given in (5.11) the partial derivative of G with respect to q_{ϑ_j} can be computed as

$$\frac{\partial G}{\partial q_{\vartheta_j}} = (1-\mu)\mu^{\frac{\mu}{1-\mu}} \hat{\lambda}_{j,t}^{\frac{\mu}{1-\mu}} L_{j,t}. \tag{8.47}$$

The first order conditions of the Lagrange function with respect to q_{ϑ_j} give:

$$\lambda^{(q)}_{\vartheta_j,t}/\lambda_{0,t} = (1-\mu)\mu^{\frac{\mu}{1-\mu}} \hat{\lambda}_{j,t}^{\frac{1}{1-\mu}} L_{j,t} + \omega(1 + \iota_{j,t}\eta_{j,t}\bar{\lambda}_j)\lambda^{(q)}_{\vartheta_j,t+1}/\lambda_{0,t}. \tag{8.48}$$

A marginal increase in quality q_{ϑ_j} in t affects the productivity of G in t and the productivity of that capital good in $t+1$. Since these effects are symmetric accross ϑ_j, this expression can be written as (define $\hat{\lambda}_{q_j,t} := \lambda^{(q)}_{\vartheta_j,t}/\lambda_{0,t}$):

$$\hat{\lambda}_{q_j,t} = (1-\mu)\mu^{\frac{\mu}{1-\mu}}\hat{\lambda}_{j,t}^{\frac{1}{1-\mu}}L_{j,t} + \omega(1+\iota_{j,t}\eta_{j,t}\bar{\lambda}_j)\hat{\lambda}_{q_j,t+1}. \tag{8.49}$$

Intuitively, the shadow value of a unit increase in average productivity in sector $j \in \{m,z\}$ is equal to its marginal contribution to time-t productivity plus its shadow value at time $t+1$ times $1+\iota_{j,t}\eta_{j,t}\bar{\lambda}_j$ (the further productivity increase it enables at time $t+1$). This last term captures the intertemporal "knowledge externality", because in the decentralized equilibrium, innovators do not internalize the effects of their research on productivity in the future.
In the optimal allocation of resources, the social planner allocates innovators towards the sector yielding the higher social gain from innovation. The social value of innovation is measured by the term $\hat{\lambda}_{q_j,t}\eta_{j,t}\bar{\lambda}_j Q_{j,t-1}$, $j \in \{m,z\}$. Using the dynamics of average capital goods quality, we can write:

$$\hat{\lambda}_{q_j,t}\eta_{j,t}\bar{\lambda}_j Q_{j,t} = \eta_{j,t}\bar{\lambda}_j(1+\iota_{j,t}\eta_{j,t}\bar{\lambda}_j)^{-1}Q_{j,t+1}\hat{\lambda}_{q_j,t}. \tag{8.50}$$

Now inserting the right hand side of (8.49) into (8.50) and iterating forwards gives

$$\hat{\lambda}_{q_j,t}\eta_{j,t}\bar{\lambda}_j Q_{j,t} = \eta_{j,t}\bar{\lambda}_j(1+\iota_{j,t}\eta_{j,t}\bar{\lambda}_j)^{-1}(1-\mu)\mu^{\frac{1}{1-\mu}}\sum_{n\geq t}\hat{\lambda}_{j,n}^{\frac{1}{1-\mu}}L_{j,n}Q_{j,n}. \tag{8.51}$$

This implies that the social planner will allocate scientists to the clean sector whenever the ratio

$$\frac{R_{m,t}\bar{\lambda}_z(1+\iota_{z,t}\eta_{z,t}\bar{\lambda}_z)^{-1}\sum_{n\geq t}\hat{\lambda}_{z,n}^{\frac{1}{1-\mu}}L_{z,n}Q_{z,n}}{R_{z,t}\bar{\lambda}_m(1+\iota_{m,t}\eta_{m,t}\bar{\lambda}_m)^{-1}\sum_{n\geq t}\hat{\lambda}_{m,n}^{\frac{1}{1-\mu}}L_{m,n}Q_{m,n}}. \tag{8.52}$$

is smaller than 1. Clearly this expression depends on various endogenous variables, most importantly for our analysis, this ratio depends on the (endogenous) credit interest rates $R_{m,t}$ and $R_{z,t}$, indicating an influence of credit and banks on the size of the subsidy necessary to redirect technical change. We determine this subsidy next.

Subsidy to green R&D
We compute the subsidy on the basis of technology constraints and conditions from the social planning problem. First, we combine the partial derivatives of intermediate production technology, as given in (5.11), with respect to $x_{\vartheta_j,t}, L_{j,t}$ with the first order conditions of the Lagrange function with respect to L_m and L_z:

$$\lambda^{(Y)}_{m,t}\partial_{G_{L_m}}G(L_m, x_{\vartheta_m}) = \lambda^{(Y)}_{z,t}\partial_{G_{L_z}}G(L_z, x_{\vartheta_z}) \tag{8.53}$$

to get

$$\hat{\lambda}_{m,t}^{\frac{1}{1-\mu}}Q_{m,t} = \hat{\lambda}_{z,t}^{\frac{1}{1-\mu}}Q_{z,t}, \tag{8.54}$$

where $\hat{\lambda}_{j,t} := \lambda_{j,t}/\lambda_{0,t}$ for $j \in \{m, z\}$. Next use the first order condition of $\mathscr{L}$ with respect to $Y_{m,t}$, $Y_{z,t}$ for given intermediate production technology. After elimination of all Lagrange multipliers, we get

$$\frac{L_{z,t}}{L_{m,t}} = (1+\tau_t)^{-\varepsilon}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\sigma-1}. \tag{8.55}$$

Insert this result into relative expected profits from innovation in the dirty sector (using the optimality conditions with respect to capital goods x_{ϑ_j}), the equivalent of (6.21) under laissez-faire, then can be written as

$$\frac{V_{z,t}}{V_{m,t}} = (1+d_t)^{-1}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{1-\alpha}}(1+\tau_t)^{\frac{-\varepsilon}{1-\alpha}}\left(\frac{Q_{z,t+1}}{Q_{m,t+1}}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}}. \tag{8.56}$$

Use sectoral capital goods quality dynamics to rewrite this as

$$\frac{V_{z,t}}{V_{m,t}} = (1+d_t)^{-1}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{1-\alpha}}(1+\tau_t)^{\frac{-\varepsilon}{1-\alpha}}\left(\frac{1+(1-\iota_t)\eta_{z,t}\bar{\lambda}_z}{1+\iota_t\eta_{m,t}\bar{\lambda}_m}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\frac{\sigma-1-\alpha}{1-\alpha}}. \tag{8.57}$$

So if the optimal allocation involves $\iota_{m,t} := \iota_t = 1$, the social planner can can choose a subsidy d_t to make this expression smaller than one. Or more explicitly, the planner can set

$$d_t \geq \hat{d}_t \equiv \left(\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}(1+\tau_t)^{-\varepsilon}\left(1+\eta_{m,t}\bar{\lambda}_m\right)^{1+\alpha-\sigma}\left(\frac{Q_{z,t}}{Q_{m,t}}\right)^{\sigma-1-\alpha}\right)^{\frac{1}{1-\alpha}} - 1 \tag{8.58}$$

If the optimal allocation involves $\iota_t \in]0,1[$, then setting d_t to ensure that $V_{z,t}/V_{m,t} = 1$ achieves the desired objective. The next proposition summarizes the previous computations of optimal climate policy and represents a major result of part three of this thesis:

Proposition 8.5. *The socially optimal allocation can be implemented using a tax on emissions, i.e. a tax on dirty input, a subsidy to green R&D and a subsidy for the use of capital goods, whereby all proceeds from taxes and subsidies are redistributed/financed lump-sum.*□

8.5. Properties of climate policy

Let me finish the analysis of optimal climate policy by elaborating the results and relating them to the findings of some earlier studies.

First it is important to note that the main results derived above are a consequence of endogenous and directed technical change. The framework developed here would be without endogenous directed technical change, if – instead of basing the decision to innovate for a certain sector on expected profits – innovators were randomly allocated across the different capital goods in the two sectors. The analysis based on endogenous directed technical change suggests that a temporary subsidy can redirect innovation towards the green sector, whereas without directed technical change such redirecting is not possible, and thus temporary interventions cannot prevent a climate catastrophe.

So far, the above results taken together suggest that governmental intervention should use both a tax on carbon emissions and a subsidy on R&D in the green sector in order to implement a socially optimal allocation. This is intuitive, since the tax deals more directly

with climate externalities by reducing the production of the dirty intermediate good; but also indirectly, because lower production discourages innovators R&D effort in the dirty sector. The use of taxes and subsidies in climate policy is also suggested for instance in Acemoglu et al. (2012).
The size of the subsidy that is necessary to create an incentive for innovators to direct their innovation effort towards the green sector depends on the presence of the *risk effect*. An important implication of this result is that the role of the financial sector should be taken into account when evaluating the effects and costs of climate policy instruments.
Let me relate the results derived above to the findings of the literature on climate change and economic growth. The main difference lies in the answers to the questions of how strong intervention should be in order to avoid a climate catastrophe, should intervention be temporary or permanent and what are the long-term implications and what are the costs of delaying these policies into the future?
The approaches citet below can be subdivided into two different approaches: in the first group of studies technology is exogenous by assumption, in the second group technology is endogenous. Somewhat oversimplifyingly, one could roughly subdivide the first strand of work into three different opinions regarding the derived answers to the question formulated above:

- The first answer, labelled the *Greenpeace* answer by Acemoglu et al. (2012), represents the most pessimistic view that essentially all growth needs to come to an end in order to save the planet.

- The second answer, represented by Stern (2009b) is slighty less pessimistic, but calls for extensive and immediate interventions. Moreover, optimal environmental regulations need to be implemented permanently even though they may induce significant economic costs.

- The third answer, represented by Nordhaus (2008) is even more optimistic and suggests that only limited and gradual intervention is necessary to limit the effects of climate change and thereby affect long-run economic growth only by a modest amount.

The second group, where technology changes endogenously, provide additional answers that can be related to the three positions above

- The fourth answer, represented by the work of Acemoglu et al. (2012), recommends immediate and decisive intervention, because delaying intervention into the future is costly. But due to endogenous directed technical change, a single policy can redirect technical change and thus intervention needs only to be temporary, because once the green, emission extensive technologies have been sufficiently advanced, R&D would be directed towards these technologies without further intervention. This view is even more optimistic than the Nordhaus answer, but holds only under certain conditions: in fact only if substitutability between emission-intensive and clean sectors is sufficiently high.

> However, their work also combines the *Greenpeace* and *Stern* answer, when substitution between emission intensive and clean sectors is not sufficiently high.
> Therefore when evaluating the costs and effectivity of environmental policy, one has to carefully consider the relationships between the different industrial sectors in an economy in order to get precise estimates of the effects of environmental regulation.

With regard to these different positions, I take a position that is most closely connected to the findings of Acemoglu et al. (2012). This is not surprising, since the present work builds on a model framework of endogenous and directed technical change taking environmental constraints into account and implementing credit constraints and a financial sector.
Nevertheless the explicit consideration of banks in the process of technical change suggests an additional answer and hence contributes to the strands of literature stated above. On the one hand, the consideration of financial intermediation leads to an answer that is even more optimistic than Acemoglu et al. (2012): if the substitutability between the two sectors is sufficiently large and the risk effect is strictly smaller than one, financial intermediation helps to direct technical changes towards the green sector and thereby strengthens the impact of environmental policy intervention. This implies that intervention only has to be temporary (this replicates the findings by Acemoglu et al. (2012)) but in addition, the length and the strength of intervention that are sufficient to redirect technical change towards the green sector are even smaller compared to those measures that would have been suggested by an analysis that does not consider the role of financial intermediaries. On the other hand, even if the substitutability between the two sectors is sufficiently large, the answer containing the effects of financial intermediation can nevertheless be less optimistic compared to the most optimistic case found in Acemoglu et al. (2012): If the risk effect is strictly greater than one, the banks slow down the effects of a governmental intervention that aims at redirecting technical change towards green sectors. Although even in this case, intervention only has to be temporary, however the length and the strength of intervention that are sufficient to redirect technical change towards the green sector are greater compared to the measures computed without the role of financial intermediaries. Together, the analysis of environmental policy with explicit consideration of banks in the process of technical change provides an answer that is close to the opinions of Nordhaus (2008) and Acemoglu et al. (2012). However, we suggest that when computing the costs and effects of environmental policies, one should consider the impact of banks on the determinants of directed technical change.

9. Numerical Simulations

The next section states the results of a simple numerical simulation. The objective here is not to provide a comprehensive quantitative evaluation but to highlight the role of financial intermediation in the process of green directed technical change.
In the calibration of the developed model the first step is to relate a model period to real time. Since the consumer side in the current model framework features an overlapping generations structure with two periods of living, one might be inclined to interpret one model period as a rather long time period that measures the living time of one generation,

e.g. a time period of 20 to 40 years. This would be problematic due to the following two points:

- First a comparison of the numerical results with those of other studies is at least difficult -if not impossible. Current studies on climate change or the environment and economic growth typically describe the consumer side by an infinitely lived representative agent (among many others see Nordhaus (2008), Acemoglu et al. (2012), Golosov et al. (2014)). This way of modeling the consumer side enables researchers to relate a model period to any real counterpart, e.g. one model period could represent a quarter, a year, five years etc. So a model period in these studies has a much smaller counterpart in real time, making it difficult to compare these results with those of long term studies.[72]

- Second, the short to medium-term effects of climate policy are simply not appropriately measured in such a setup.

However, Aiyagari (1985) showed in a paper the "observational equivalence" of two-period overlapping generations models and representative agent economies, meaning that under certain conditions, the models lead to identical time paths for aggregate capital, output, consumption, investment, real wage, and the real interest rate. The critical condition for such an equivalence is that the steady state real interest rate is strictly greater than the growth rate of the labor force (Aiyagari (1985) p. 202.). Since in the current framework the population is constant and naturally the interest is positive, the condition for observational equivalent model results is always fulfilled. Hence, it is legitimate to interpret the period in the current overlapping generations model as shorter than initially presumed and to use this framework for analyzing short to medium-term effects of climate policy taking into account the effects of banks on directed technical change. Thereby, we compare three different scenarios:

- a) Laissez-faire without any climate policy,

- b) Immediate (year 2015) intervention using optimal emissions taxes, and

- c) Immediate (year 2015) intervention using optimal emissions taxes and subsidies to green R&D.

9.1. Parameter Calibration

The following numerical analysis takes each model period to be five years in real time and the time horizon is 42 model periods. The model is calibrated to the year 2010, so the horizon is given by the time period 2010 to 2220.[73] In order to focus on the influence of the risk effect, governmental intervention here is given by a subsidy to green R&D to

[72] In Acemoglu et al. (2012) a model period corresponds to five years, in Golosov et al. (2014) a model period represents 10 years.

[73] Comparable studies have a time horizon of 2010 to 2220 Golosov et al. (2014), 300 Years Acemoglu et al. (2012) and about 200 years in Nordhaus (1994).

redirect technical change towards the green sector and the elimination of the emission externality through a “carbond tax” levied on the use of the dirty input in final goods production.

Recall that a *climate catastrophe* here is defined as a situation where climate damages to aggregate output exceed an upper bound. The concrete value of the upper bound is of less importance for the results and thus can be set almost arbitrarily. What is more important, is that damages increase further under laissez-faire. So given a time horizon of 210 years, this study sets the upper bound of climate damages to 5 % of world GDP.

Altogether, we need to calibrate three sets of parameters: the general parameters of preferences and technology, those involving the damage function, and the parameters that describe the carbon cycle in the atmosphere. The next table summarizes all the parameter choices of the numerical example; the rest of this section discusses how the choices were made.

TABLE 3: Parameter decisions for numerical simulation

$\mu = 0.33$	$\gamma = 0.45$	$\varepsilon = 1.6$	$\alpha = 0.06$	$\zeta = 1.6$	$\lambda_m = 1.065$	$\lambda_z = 1.040$
$\bar{A} = 581$	$\beta = 0.961$	$\omega = 0.961$	$\theta = 5.3 \cdot 10^{-5}$	$\phi_L = 0.2$	$\phi_0 = 0.397$	$\phi = 0.0115$

Preferences and technology

The study uses the assumptions made in the model description in part two above: logarithmic preferences, CES final-goods production, CES intermediate-goods production and 100% depreciation of capital goods. Such a depreciation rate is too high for a 5-year model period, presumably even for a period of 10 years. With regard to the effects of depreciation below 100% and directed technical change, Acemoglu (2002) points out that slower depreciation affects the results of directed change models along the transition, but long run growth rates remain unchanged.My concern in this thesis is to analyse the role of banks in the process of technical change. The general results of parts one and two did not consider depreciation rate levels. In this view, the assumed 100% depreciation of capital goods is tolerable for the numerical analysis.

We set the capital goods share $\mu = 0.33$ so the share of national income spend on capital goods (or machines) approximately equals the share of capital, which is commonly set to a value in the interval $[0.3, 0.4]$ in textbooks.[74] The distribution parameter γ is set to 0.45, so the weight of two intermediate goods in final output production is comparatively equal, nevertheless we put a slightly higher weight on dirty inputs since current global GDP mainly uses energy goods and services based fossil fuel (electricity generation, transportation etc).

Estimating the economy wide elasticity of substitution between the two intermediate goods is beyond the scope of this thesis. However, since fossil and non-fossil fuels should be substitutes and not complements, we choose a value of $\varepsilon = 1.6$. This value is low compared to the values for the elasticity of substitution between dirty and clean inputs of

[74]In fact, in 2001 Bernanke and Gürkaynak (2002) estimated a value of 0.36 for the share of capital in aggregate output.

$\varepsilon = 3$ and $\varepsilon = 10$ in Acemoglu et al. (2012), but rather high compared to the values chosen in Golosov et al. (2014) and estimated in Stern (2009a) for different energy inputs.[75]
The analysis sets the parameter representing cost of research $\zeta = 1.6$ and the fraction of profits devoted to R&D,$\alpha = 0.06$. These values ensure that sectoral probabilities of successful innovation are smaller than one and the resulting values for these probabilities imply annual expected default rates of about 1.0% in sector m and 4.4% in sector z. The success-probabilities measure the risk of default from credit obligations. In this regard, the empirically observed default rates justify these parameters: In a study, Giesecke et al. (2011) estimated the default risk in corporate bonds over the last 150 years. They derived an average annual default rate for corporate bonds of 0.304% in the period 1946-2008. And according to the economic data set of the Board of Governors of the Federal Reserve System of the United States, the annual, seasonally adjusted delinquency rate on business loans of all commercial banks was on average 4.4% between 1991 and 2012. So the parameter values in this thesis lie within the range of possible values according to empirical observations.
The size of the jump in the product quality after a new capital good is invented are given by $\lambda_m = 1.065$ and $\lambda_z = 1.040$. This implies a jump in productivity of 6.5% (4%) in sector m (z) whenever a new capital good of higher quality is invented. The study assumes this value for the step size in innovation, because taken together with the success-probabilities, these parameter values imply future annual growth rates of sectoral average capital goods quality of about 2% and therefore, the annual GDP growth rate is approximately equal to 2%. This matches the assumptions of Nordhaus (2008), Golosov et al. (2014) and Acemoglu et al. (2012) of output growth quite well.
The private and social discount factors are set equal to $\omega = \beta = 0.961$. The optimal tax on emissions is a function of the social discount factor ω, where with a higher discount factor there is less weight put on the welfare of future generations. For instance an annual social discount factor of $\omega = 0.999$ implies a social discount rate of 0.1% per annum, which is used by Stern (2007) and a value of $\omega = 0.985$ implies a social discount rate of 1.5% per annum, which is close to the value used in Nordhaus (2008). Golosov et al. (2014) plotted their optimal tax rate for different values of the discount rate and found out that the higher the discount rate, the lower is the optimal tax. However, this thesis does not aim to make a stand with regard to the "best" value for ω and therefore the reported carbon tax should be viewed more in light of the effects of the financial intermediaries rather than an extact quantitative assessment that can be compared with the results of other studies.

The carbon cycle

Burning fossil fuel in order to use the released energy in goods production emits carbon. The emissions then enter the global carbon circulation system. This circulation system

[75]This decision is supported by the findings of a more recent study: Papageorgiou et al. (2013) build on the theoretical work of Acemoglu et al. (2012) and derive estimates for the elasticity of substitution between clean an dirty inputs. Their results support the assumption that the elasticity of substitution between clean and dirty energy inputs exceeds the value of one significantly, both in the electricity generating sector and in non-energy industries. Given this results, they conclude that the economy-wide elasticity can also be expected to exceed the value of one, possibly even two. However, they do not find evidence of an extremely high elasticity (around ten).

contains different reservoirs such as the atmosphere, the terrestrial biosphere, and different layers of the ocean. Between these reservoirs exchanges of carbon emssions occur. Among other greenhouse gases, CO_2 or, more specifically the atmospheric CO_2 concentration, is the key driving force behind climate change. In order to analyze the effects of climate change in the present framework, we need to specify how emissions influence the CO_2 concentration in the atmosphere over time. A good starting point would be to formulate the different carbon reservoirs as a system of linear difference equations as in Nordhaus (2008). However, Golosov et al. (2014) point out that the model specified in Nordhaus (2008) abstracts from important mechanisms that affect the atmospheric carbon concentration in the long run: most importantly this linear formulation abstracts from the so-called Revelle buffer factor (Revelle et al. (1957)) that describes the capacity of the ocean to act as a carbon reservoir in dependence of the accumulated CO_2 in the water. [76] However, this feedback effect can be ignored in economic models, since it is very slow. Thus Golosov et al. (2014) also develop a linear specification of CO_2 emission development, but one with a different interpretation that implies dynamics that are qualitatively different from those in Nordhaus (2008). In addition, the approach taken by Golosov et al. (2014) leads also to quantitatively different results. In fact, their formulation leads to significantly larger effects of man-made emissions on the climate.

In the sequel, we follow the approach developed in Golosov et al. (2014) and assume that (i) a share ϕ_L of carbon emissions remains permanently in the atmosphere, (ii) a share $1 - \phi_0$ of the remainder exits the atmosphere into the biosphere and the surface oceans within a decade; and (iii) a remainder of $(1 - \phi_L)\phi_0$ decays at a geometric rate.

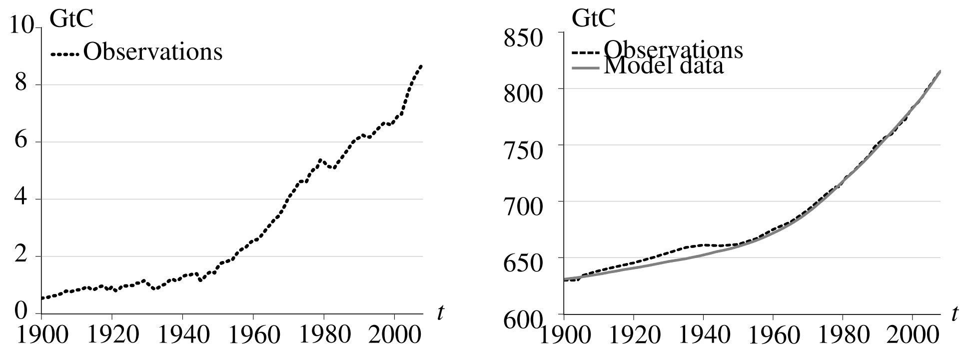
The approach taken here leads to a three parameter formula for a unit of carbon emitted at time 0 that is left in the atmosphere τ periods later equal to: $1-d_\tau = \phi_L+(1-\phi_L)\phi_0(1-\phi)^\tau$. The analysis calibrates the parameters ϕ_L, ϕ_0, ϕ as follows. According to the estimate in the 2007 IPCC report about 20% of any emission pulse will stay in the atmosphere "forever". Hence, we set $\phi_L = 0.2$. Archer (2005) estimated the lifetime of the excess carbon that slowly exists the atmosphere to be 300 years. Since in this study, one model period corresponds to five years, we impose $(1 - \phi)^{60} = 0.5$. This gives $\phi = 0.0115$. The 2007 IPCC report states that about half of the emission pulse to the atmosphere exits within a time scale of 30 years. This implies $d_4 = 1/2$ in the formula above and $1 - 1/2 = 0.2 + 0.8\phi_0(1 - 0.0115)^4$, so $\phi_0 = 0.397$. Finally, the initial condition for the pre-industrial atmospheric carbon concentration is $\underline{A} = 581$ GtC (gigatonnes of carbon).

Part two introduced the following formula to determine the atmospheric CO_2 concentration:

$$A_t = \sum_{\tau=0}^{t+T}(1 - d_\tau)E_{t-\tau} + \underline{A}, \tag{9.1}$$

where T denotes the first date when emissions started. The observed global carbon emissions in GtC and the atmospheric CO_2 concentration is given in figure 5 above, where figure 5 a) shows the evolution of global carbon emissions from 1900 to 2008 in gigatonnes

[76]The accumlation of CO_2 in the oceans acidifies the water, which in turn limits the capacity of the oceans to absorb more CO_2. This can reduce the effective "size" of the oceans as carbon reservoirs (See Golosov et al. (2014)).



(a) World carbon emissions

(b) Atmospheric CO_2 concentration

FIGURE 5: Observations on global carbon emissions and atmospheric CO_2 concentration

and figure 5 b) shows the observed atmospheric CO_2 concentration in the same time period and the data generated by the model equation given above in (9.1). The dynamics implied by the linear depreciation rate for CO_2 used here together with the parameter values given above match the observed evolution of the atmospheric CO_2 concentration over the past century quite well as shown by the close correspondence between the dashed black line representing the oberserved values and the solid grey line representing the CO_2 concentration in the atmosphere implied by the model.

The Damage Function

The damage function used in this thesis follows the exponential damage formulation developed in Golosov et al. (2014). This formulation has the atmospheric CO_2 concentration A as its argument. This is in contrast to other models that typically express damages as a convex function of global temperature (see for instance Nordhaus (2008)). The mapping from CO_2 concentration to damages hence should be interpreted as a composition of two mappings, where the first maps CO_2 concentration into temperature and the second maps temperature into damages. Nordhaus's mapping from global mean surface temperature to damages is specified as

$$1 - D_N(T_t) = \frac{1}{1 + \theta_2 T_t^2},$$

where T is the mean global increase in temperature above the pre-industrial level, with $\theta_2 = 0.0028388$. The damage function D_N is convex for a range of values with some upper bound, for temperature values greater than this upper bound the function is concave.

The standard form for the second mapping from atmospheric carbon concentration A to temperature T is a logarithmic function and given by

$$T_t = T(A_t) = \lambda \log\left(\frac{A_t}{\underline{A}}\right) / \log 2,$$

where $\underline{A} = 581$ GtC is the pre-industrial level of atmospheric CO_2-concentration. The parameter λ represents the sensitivity of the temperature and a standard value is $\lambda = 3.0$ degree Celsius, which means that a doubling of the stock of carbon in the atmosphere

leads to an increase in the global mean temperature by 3 degree Celsius.
The exponential "γ" parameter in the damage formulation of Golosov et al. (2014) is equal to $5.3x10^{-5}$ GtC. We choose our parameter θ -which is the parameter γ in Golosov et al. (2014)- to be the same as theirs. Note that this value is probably too low, since in this study, θ is constant, where in Golosov et al. (2014), the parameter is stochastic and adjusts to new information about damages.
The parameter κ represents specific CO_2 emissions per unit of output and is chosen to link current emissions to the output level of the model. This constant emission parameter might overestimate the amount of emitted CO_2, because this formulation abstracts from changes in the emissions productivity. Hence, we calibrate this parameter in a way that takes this possible overestimation into account.[77]

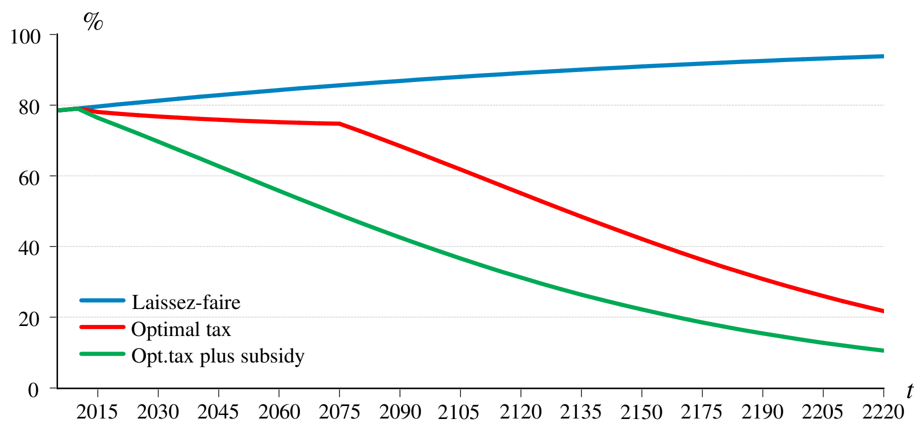


FIGURE 6: Share of dirty input use: laissez-faire vs intervention

Finally, the two intermediate inputs Y_{mt}, $Y_{z,t}$ here represent fossil fuel based energy production (dirty production $Y_{z,t}$) and non-fossil fuel based energy production (green production $Y_{m,t}$). The initial values for dirty and green production are calibrated to the average of 2005-2015 fossil and non-fossil energy production in world primary energy supply according to US Energy Information Administration.

9.2. *Implications for the Future: Climate, Damages, Output and the Role of Financial Intermediation*

Given the assumptions and results of the previous sections, we can now generate quantity paths for output, climate damages, dirty and green intermediate input use and the different

[77]Note that "fossil fuel" is a composite mainly consisting of coal, oil and natural gas. Each of these fuels have a different carbon content (for instance coal has a carbon content of 716KgC/ton coal and crude oil has 846 KgC/ton oil). The carbon content of the composite good "fossil fuel" should therefore be a weighted average of the carbon content of coal, oil and gas, where the weights for this average are given by the relative production of one energy source to total fossil fuel production. This implies that the parameter κ could change in the future if for instance the relative importance of natural gas in total fossil fuel production increases. Thereby, changes in κ due to technological changes are not consered yet.

policy measures etc. for the laissez-faire equilibrium and the equilibrium with governmental intervention in order to analyze the role of financial intermediation in the process of green technical change. Basically, we subdivide the results into two parts: First we show the general effects of climate policies on the paths of the different economic variables, taking financial intermediation into account; and here compare the results of the laissez-faire case to the case of immediate intervention. Second, we show the differences in the results by comparing the policy measures, namely the strength and length of intervention, the cost of adjustment and the cost of delay, for the cases with and without financial intermediaries.
Figure 6 depicts the use of dirty energy under laissez-faire and in the two cases of immediate intervention. Comparing the paths of dirty energy use in the three situations, we find that immediate intervention leads to a much lower use compared to laissez-faire. This is plausible, since the technology gap between the dirty and the green input is initially wide enough to direct productivity-increasing innovations solely to the dirty sector. Hence, fossil fuel use increases over time under laissez-faire. Comparing the two intervention-scenarios, we find that dirty input consumption steadily decreases over time

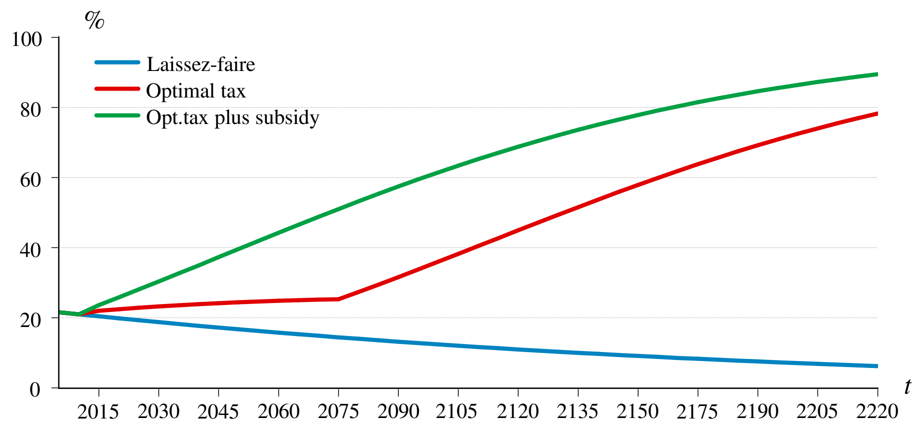


FIGURE 7: Share of green input in total intermediate input use: laissez-faire vs intervention

in both scenarios. Interestingly, however, the use of a carbon tax reduces dirty energy use over the next 50 years only gradually, while a climate policy using a carbon tax and a subsidy to green R&D leads to an immediate decline of dirty energy use. Although the optimal tax discourages profits from dirty capital good innovations, closing the profit gap between dirty and green capital goods takes time. So over the next 50 years, innovations still would be directed to dirty technologies, even though the carbon tax is introduced. A subsidy on green innovations can help to further shrink dirty energy input use. This is because immediate intervention using additionally green subsidies freezes the existing productivity level of the dirty sector and redirects innovations towards the green sector. So the subsidy affects future emissions while at the same time, the tax controls current emissions from dirty energy use.
Inversely related to these results are the shares of non-fossil fuel use in the two scenarios

illustrated in figure 7. While in the laissez faire case, the share of inputs supplied by the green sector declines from 20% in 2015 to around 15% in 2090 to be less than 10% after 100 years from now. Immediate intervention leads to increasing shares of green input consumption over time. However, even though the switch to green innovations is immediate, if intervention uses taxes and subsidies to green R&D, it takes much longer (over 100 years) for 90% of inputs to be supplied by the green sector.

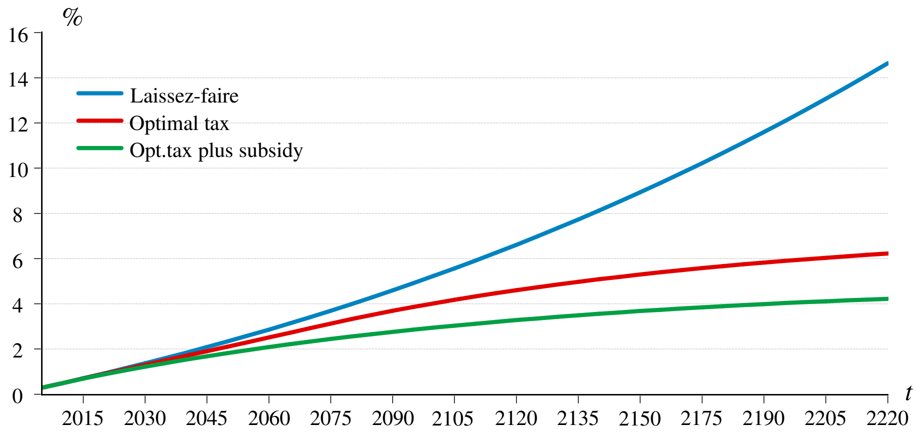


FIGURE 8: Damages as percentage of GDP: laissez-faire vs intervention

The paths for total damages in percent of final output are plotted in figure 8. The gains from immediate intervention through a subsidy on green R&D and an optimal tax on the use of fossil fuel are significant. In the short and medium term (up to 2050) the gains are small. However, they permanently grow over time. Around 2100, damages from climate change are almost 6% of GDP in the laissez-faire scenario rather than 4% in the optimal tax and 2.8% in the optimal tax and subsidy intervention scenario. By 2200, damages will have increased to almost 14% under laissez-faire while immediate intervention keeps damages lower at 6% in the optimal tax and 4% in the optimal tax and subsidy scenario. This is approximately similar to the results of Golosov et al. (2014), who stated damages in 2200 of above 10% in the laissez-faire regime and 1.5% in the optimal allocation.
Using the relation between the atmospheric CO_2 concentration A and the temperature T as described above, where T depends logarithmically on A, we can also compute the paths of global mean temperature in the two scenarios. Figure 9 illustrates these findings. Without immediate climate policy intervention, the global mean surface temperature will have increased by 3 degrees Celsius within the next 100 years. Immediate intervention with a carbon tax on the optimal use of CO_2 intensive dirty goods leads to a global warming of only 2.5 degrees Celsius. Climate policy using an optimal CO_2 tax and subsidies to green R&D can limit temperature increases to 2 degrees Celsius. In the year 2200 the corresponding temperature increases are 7 degrees under laissez-faire and 3.8 (2.8) degrees under optimal tax (optimal tax plus subsidy) intervention.

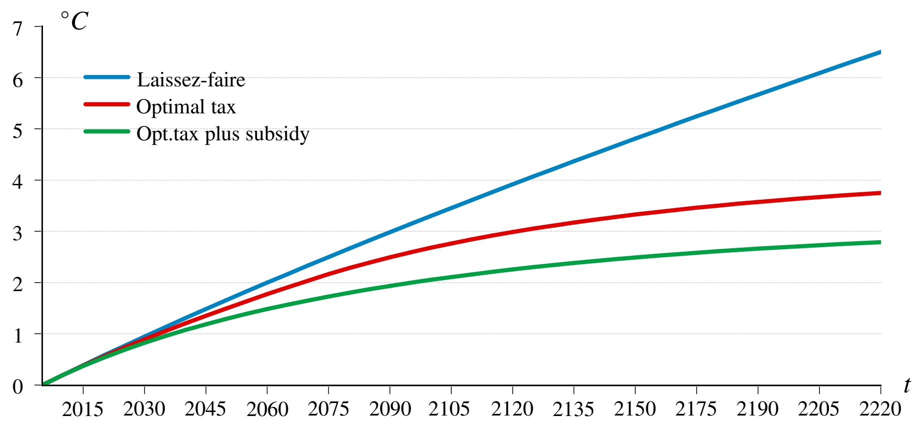


FIGURE 9: Global mean surface temperature: laissez-faire vs intervention

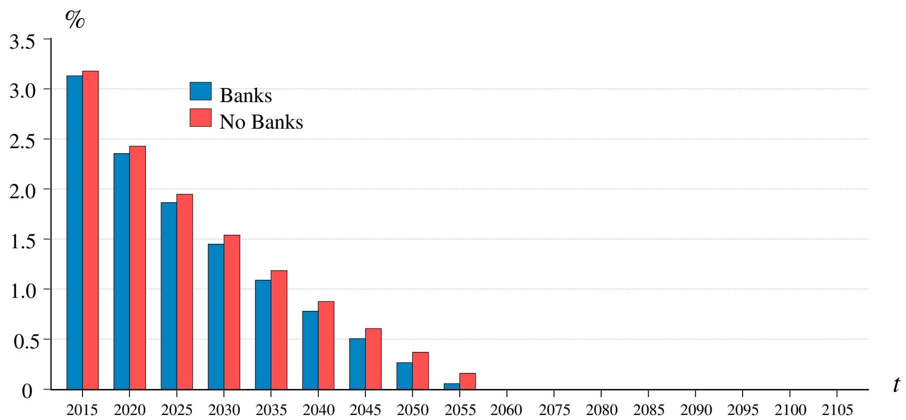


FIGURE 10: Subsidy on green R&D in % of net-output: banks vs no banks

The role of credit and banks

We use different policy measures introduced previously such as the strength of intervention, the length of intervention, the cost of adjustment, and the cost of delay to analyze the effects of credit and banks on climate policy. Thereby we take scenario c), where a government introduces the optimal CO_2 tax and subsidizes returns on R&D directed to green capital goods, and compare the results with those that would have occured if the effects of credit and banks on the direction of technical change would be ignored.

First consider the strength and length of intervention, both determined by the subsidy on green R&D return. Recall that the subsidy is introduced to redirect innovations from dirty to green technologies and remains in place as long as relative expected profits from innovation in the dirty sector are greater or equal to profits from green innovation without this subsidy. The number of periods necessary to create profit incentives to green R&D even without a subsidy is the length of intervention. The size of a subsidy sufficiently

high to create the incentive of green innovations in the first place represents the strength of intervention.
Figure 10 shows the development of the subsidy over time. Comparing results, we find that the consideration of credit and banks in the determinants of technical change has no effect on the length of intervention: in both scenarios, a declining fraction of final output should be used to finance green R&D over the next 50 years. Then green sector technologies overtake dirty technologies in terms of productivity so from then on, innovations are directed to green capital goods even without a subsidy. The decline in the subsidy is due to a decrease in the technology gap, i.e. the relative productivity between the dirty and the green sector declines, since after intervention, the productivity in the dirty sector is freezed, while the productivity in the green sector increases. In contrast to this, the strength of intervention is lower if climate policy accounts for the risk effect in the determinants of directed technical change: Accumulated over the time period of intervention, the cost of financing green R&D are 6.5% higher if climate policy ignores the effect of credit and banks on profit incentives in directed innovations.
Next, the cost of adjustment is given by the number of periods the economy needs to achieve the same output growth as it would have achieved in just one period in the laissez-faire equilibrium. Figure 11 shows the percentage deviation of growth rates from one period of laissez-faire growth in the "bank" and "no bank" case. First the results suggest lower growth rates of aggregate output in case of intervention indicated by the negative percentage deviation from laissez faire growth. Second the cost of adjustment in the bank scenario is much smaller compared to the no bank scenario, since i) the negative percentage values are greater in absolute terms and ii) the number of years with negative percentage output deviations is significantly greater if climate policy ignores the effects from credit and bank. In this case, the model predicts that it takes the economy almost fourty years longer to reach the same level of output growth as it would have achieved in the laissez-faire equilibrium.

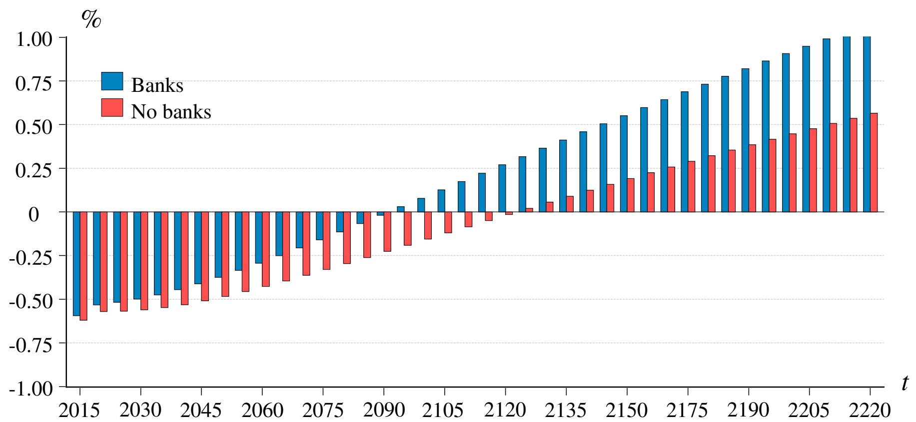


FIGURE 11: Percentage change in net-output growth compared to laissez-faire: banks vs no banks

Figure figure 12 depicts the development of net-of-damage final output production for scenario c), where climate policy uses carbon taxes and subsidies to green R&D, and compares this with net final output that would have occured if climate policy would have ignored the effects from credit and banks on the determinants of directed technical change. We find that 100 years from now, aggregate output is 2.1% greater if climate policy accounts for credit and banks and continues to increase: At the end of the simulation period, this difference in output levels is almost 11%.

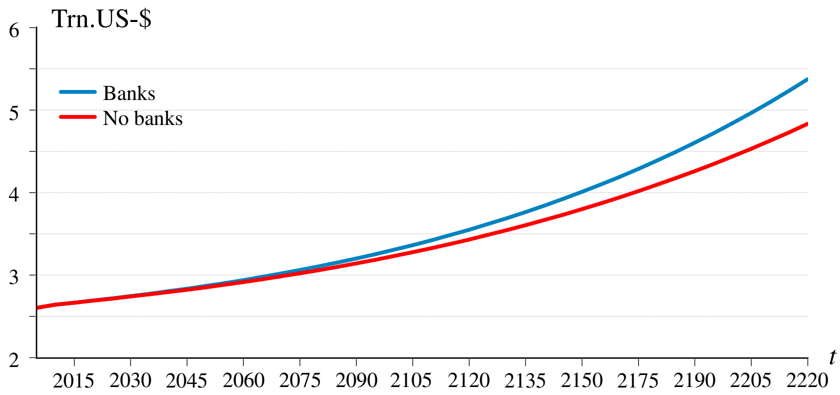


FIGURE 12: Net-output for carbon tax and green subsidy intervention: banks vs no banks

Consider now the cost of delaying climate policy intervention into the future. We relate intervention delay by one, two respectively three decades to the additional economic costs given by losses in aggregate output at the end of the simulation period. A delay here means the number of additional periods where the government keeps the emissions tax and the subsidy on green R&D at zero. We find that a delay of intervention can be substantial no matter whether climate poliy considers the effects of credit and banks. For example, a 10-year delay leads to output losses of approximately 3%. This loss in aggregate output doubles to 6% in case of 20-year delay and triples to output losses of 9% compared to intermediate intervention. Let us close the numerical example with the analysis of the optimal tax on carbon emissions in scenarios b) and c) The optimal tax on the use of dirty input is characterized in equation (8.46). The epxression shows that the tax rate especially depends on the social discount factor ω: the greater ω,
the lower is the tax rate. This relationship implies that it is not exactly clear what the "best" social discount rate is with respect to the emissions tax.[78] Figure 13 plots the CO_2 tax for an annual discount rate of 0.87 for the scenarios b) and c). This discount rate is close to 0.8 which equals the average of the two discount rates used in Stern (2007) and Nordhaus (2008). From figure 13 we see that the optimal tax increases over time. A climate policy using only a carbon tax leads to higher tax rates in the future. Moreover, if climate policy takes into account the risk effect associated with financial intermediaries and capital constrained innovators, this has almost no effect on the optimal tax rate. A

[78]For an analysis of the tax rate as a function of the discount factor, see Golosov et al. (2014) p. 70 f.

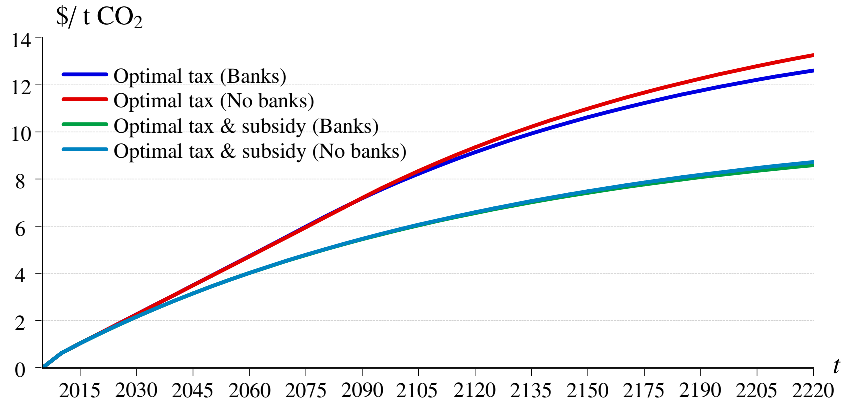


FIGURE 13: Tax on dirty input production: banks vs no banks

marginal shift is still existent (the taxes rates are identical at the beginning of intervention and at the end of the simulation horizon, the tax is 1.6% higher under the no bank regime). This is intuitive, because the tax is equal to the marginal externality damage from emissions and the emission externality is a function of intermediate production, whereas in this framework, credit constraints and thus financial intermediation matters on the downstream capital goods production stage.

In contrast to this, the subsidy to green R&D deals with future emission externalities by re-directing innovators R&D effort away from the dirty and towards the green sector.

Moreover, figure 13 suggests that relying intervention exclusively on a carbon tax to reduce dirty production (current emission externalities) and also stimulate green sector R&D (future knowledge based externalities) would demand a much higher tax on carbon emissions. This implies that an optimal policy relies not only on carbon taxes to avoid a climate catastrophe; one should also use additional instruments that direct innovation towards green technologies, thereby increasing future production by using more productive green technologies.

To relate the numerical tax result found here to available estimates, consider the work of Nordhaus (2008) and Stern (2007) and more recently Acemoglu et al. (2012) and Golosov et al. (2014). The former two amount to a tax of 30$ and 250$ per ton coal, respectively. As already mentioned, one explanation for these deviating results is the use of very different subjective discount rates. Using the same subjective discount rates, Golosov et al. (2014) find an optimal carbon tax of $56.9/ton and $ 496/ ton of coal. A key difference for this higher optimal tax is the depreciation structure of emitted carbon in the atmosphere. The fact that a fraction of carbon remains permanently in the atmosphere implies that more carbon stays and that it stays longer in the atmosphere.

This holds also true for the present findings, since our carbon concentration uses the approach of Golosov et al. (2014). However, our tax rate is nevertheless lower compared to Golosov et al. (2014). This difference can be explained, since the environmental policy in Golosov et al. (2014) relies solely on a carbon tax instead of combining it with a subsidy on green R&D together with the catalyzing effect of financial intermediaries on environmental policies. Acemoglu et al. (2012) showed that without a subsidy (and without financial intermediaries presumably even more) the carbon tax needs to be significantly higher: The carbon tax deals more directly with the current emission externality by reducing the production of the dirty input. In the process the carbon tax also discourages innovation in the emission intensive sector. The subsidy deals with future environmental externalities by directing technical changes towards the green sector, hence the subsidy reduces future damages from carbon emissions.[79]

[79] Another point is that our dirty sector produces "fossil fuel", which represents a composite good of different energy sources with different carbon contents that are not explicitly modeled. So in this study, climate policy taxes the use of the dirty input and hence implicitly the use of emissions from a fossil fuel mix with a corresponding carbon content. This is probably in contrast to Golosov et al. (2014), where a carbon tax is levied on the carbon content of coal, which should be higher compared to the carbon content of a fossil fuel mix including for instance natural gas.

10. Conclusions and Outlook

The objective of this dissertation was to analyze the effect of banks on the direction of endogenous technical change with regard to climate change and economic growth. The thesis answered the question to what extent existing climate policy evaluations alter, if a credit market and capital constraints enter the determinants of directed R&D investment. The first part developed a macroeconomic model to analyze the general relationship between banks and the rate and direction of technical change. Existing approaches of endogenous directed technical change models allow innovations on technologies to be directed towards different factors or sectors: firms can invest resources to develop new sector or factor specific technologies. Accordingy, the relative profitability of generating innovations for these specific technologies shape the direction of technical change. The work of this dissertation complements these existing approaches and shows that when innovating firms lack the necessary capital resources to finance their investments, four market forces determine the relative profitability of the different types of technology: (i) the *price effect*, which encourages innovations on technologies that use more expensive factors; (ii) the *market size effect*, which directs innovations towards technologies that use the more abundant factor; (iii) the *productivity effect*, which increases the incentive to develop technologies for sectors with a higher productivity ; and last but not least (iv) the *risk effect*. This *risk effect* creates incentives to develop technologies complementing sectors where the probability of successful innovation is higher and originates from banks' external funding of innovators' investment projects. Thus the first part showed in a general framework that banks influence the direction of technical change.
Utilizing the findings of the first part, the second part of this thesis formulated a dynamic general equilibrium endogenous technical change model with financial intermediation and an environmental constraint. This constraint is an externality from emitting carbon and modeled as a by-product of using a "dirty" intermediate good as an input to final output production. Here, we emphasized the impact of credit supplying financial intermediaries on the long run properties of a laissez-faire equilibrium with environmental constraints by again assuming that innovating firms in two intermediate sectors were capital constrained. We found that banks influence the direction of "green" technical change through the risk effect. Moreover, the analysis showed for sufficiently substitutable dirty and green intermediate inputs combined with an initially relatively less productive carbon neutral sector that the laissez-faire economy always runs into a climate catastrophe. This suggests in turn that additional market forces associated with financial intermediation do not change the general outcome that without policy intervention, the long run properties of the equilibrium are a high atmospheric carbon concentration associated with significant damages to GDP. This is intuitive, since naturally, the emission externality creates economic costs that are not borne by the polluter. In addition, the lending strategies of profit maximizing banks do not contain this emission externality, i.e. they do not automatically fund investment projects in sectors characterized by lower carbon emissions, but instead they finance the ventures of firms in those sectors that promise the highest expected return from credit lending. Consequently, climate policy intervention is indeed necessary to avoid a climate catastrophe, even when taking into account the role of banks in the process of technical

change.
In the third part, we analyzed optimal climate policies and conducted a simple quantitative evaluation of environmental policy regulation. We found first that in the case of a sufficiently high elasticity of substitution between fossil and non-fossil fuel inputs, a temporary policy intervention by a subsidy on research effort in the carbon neutral sector can redirect technical changes towards the emission neutral sector, thus pushing the economy towards a path of sustainable growth. The consideration of financial intermediaries has no effect on this result. Second, optimal climate policy involves a carbon tax as well as research subsidies, so that taxes need not be excessively used. Third and for the purpose of this study more important, the analysis showed that the consideration of financial intermediaries affects different policy measures quantitatively and should therefore be taken into account in climate policy evaluation. Our estimates of the cost of adjustment in case of immediate intervention, e.g. the lower aggregate output transition phase that results from the catch up of the less productive green sector, is roughly 75 years when we explicitly consider the risk effect and significantly longer (125 years) when we ignore the impact of credit and banks on the direction of technical change. In contrast to this, the length of intervention, i.e. the time period with an implemented subsidy on green R&D that is needed to create an incentive for innovators to direct their R&D investment to the green sector even without the subsidy, is almost identical when we consider the banks. We get a similar outcome for the size of the subsidy, i.e. the strength of intervention that is necessary to create profit incentives for innovation in the green sector. If the risk effect enters the relative profitability of green product innovation this leads to a reduction in the size of the subsidy.
Finally, a delay in policy intervention is costly, meaning that all policy measures increase with postponed intervention, the effect of the banks here is minor. Together this implies that the response of technology to policy, taking credit and banks into account, leads to more optimistic scenarios compared to what emerges from models with endogenous technology without financial intermediaries and even more optimistic results compared to outcomes that emerge from models with exogenous technology. Nevertheless the analysis overall suggests an immediate and strong environmental regulation.
The model framework developed in this thesis eliminates the weakness in existing approaches by accounting for financial intermediation in an endogenous growth model with directed technical change. Still, there are some weaknesses in the present analysis. First, the emissions per unit of output were constant by assumption. This formulation ignores the "technology effect", meaning that technological change leads to falling emissions per unit of output over time. This simplification is in contrast to empirical observations. Hence, the current model might tend to overestimate the quantity of aggregate emissions in the future.
Altogether this dissertation is a first step towards a comprehensive framework that can be used for theoretical and quantitative analysis of the interaction between banks and endogenous (directed) technology in general and economic responses to environmental regulation and climate change considering financial intermediaries in particular. Several extensions to the present setting appear fruitful. First it would be a straightforward idea to extend the financial side in this work from just banks to a general financial market

formulation. Then, one could use this framework to analyze the relationship between financial market development and the direction of technical change. Second, it would be interesting to develop a multi-country version of the model formulated in the second part of this thesis in order to analyze issues of global policy coordination and the problem of "carbon leakage" that describes the problem of globally increasing carbon emissions in response to environmental regulation in some countries, since this policy may induce greater fossil fuel use and hence greater carbon emission in countries that do not implement environmental regulation. One could furthermore extend the energy supply side by explicitly modelling energy sources as in Golosov et al. (2014), implement exhaustible resources and address the issue of whether optimal policy should use price or quantity instruments. Finally one could incorporate aggregate uncertainty (productivity shocks or uncertain climate damages) into the model.

C. Appendix: Mathematical proofs part III

Proof of Lemma 8.1

Proof. Suppose the condition in Lemma 8.1 holds and $V_{z,t^\circ} \geq V_{m,t^\circ}$ for some $t^\circ \geq 0$. This is equivalent to (just insert terms from eq. (6.21)):

$$(1-\gamma)^{\frac{\varepsilon}{\sigma-1-\alpha}} Q_{z,t^\circ+1} \geq \gamma^{\frac{\varepsilon}{\sigma-1-\alpha}} Q_{m,t^\circ+1}. \tag{C.1}$$

Suppose that $V_{z,t} < V_{m,t}$ for some $t > t^\circ \geq 0$.
i) If $V_{z,t^\circ} = V_{m,t^\circ}$, then from Lemma 3.1 $g_{m,t} = g_{z,t} =: g_t$, i.e. sectoral growth rates are identical and

$$Q_{z,t^\circ+1} = (1+g_t^\circ)Q_{z,t^\circ} \quad \text{and} \quad Q_{m,t^\circ+1} = (1+g_t^\circ)Q_{m,t^\circ}. \tag{C.2}$$

Iterate forward to $t > t^\circ$:

$$Q_{z,t+1} = (1+g_t^\circ)^{t+1-t^\circ} Q_{z,t^\circ} \quad \text{and} \quad Q_{m,t+1} = (1+g_t^\circ)^{t+1-t^\circ} Q_{m,t^\circ}. \tag{C.3}$$

One can show that this implies

$$V_{z,t} = (1+g_{t^\circ})^{t-t^\circ} V_{z,t^\circ} \tag{C.4}$$

and

$$V_{m,t} = (1+g_{t^\circ})^{t-t^\circ} V_{m,t^\circ}. \tag{C.5}$$

So from the condition in the Lemma we have $V_{z,t^\circ} \geq V_{m,t^\circ}$ and assumed above that $V_{z,t} < V_{m,t}$. Using (C.4) and (C.5) the latter is equivalent to

$$(1+g_{t^\circ})^{t-t^\circ} V_{z,t^\circ} < (1+g_{t^\circ})^{t-t^\circ} V_{m,t^\circ}, \tag{C.6}$$

which is a contradiction, so $V_{z,t} = V_{m,t}$ if $V_{z,t^\circ} = V_{m,t^\circ}$.
ii) If $V_{z,t^\circ} > V_{m,t^\circ}$, then

$$Q_{z,t^\circ+1} = (1+g_{z,t^\circ})Q_{z,t^\circ} \quad \text{and} \quad Q_{m,t^\circ+1} = Q_{m,t^\circ}. \tag{C.7}$$

Iterate forward to $t > t^\circ$:

$$Q_{z,t+1} = (1+g_{z,t^\circ})^{t+1-t^\circ} Q_{z,t^\circ} \quad \text{and} \quad Q_{m,t+1} = Q_{m,t^\circ}. \tag{C.8}$$

This implies

$$V_{z,t} = (1+g_{z,t^\circ})^{t-t^\circ} V_{z,t^\circ} \tag{C.9}$$

and

$$V_{m,t} = V_{m,t^\circ}. \tag{C.10}$$

So from the condition in the Lemma we have $V_{z,t^\circ} \geq V_{m,t^\circ}$ and assumed above that $V_{z,t} < V_{m,t}$. Using (C.9) and (C.10) the latter is equivalent to

$$(1+g_{t^\circ})^{t-t^\circ} V_{z,t^\circ} < V_{m,t^\circ}, \tag{C.11}$$

which is also a contradiction, since $g_{z,t} > 0$ and $t > t^\circ$ so $V_{z,t} > V_{m,t}$ if $V_{z,t^\circ} > V_{m,t^\circ}$.
i) and ii) together prove the Lemma in the text. □

Proposition 8.2

Proof. We prove the proposition in two steps. First, we show that the introduction of a subsidy can be sufficient to redirect technical changes. In a second step, we show that the size of ε determines whether this intervention can be temporary or permanent.

(i) Suppose a government introduces a subsidy rate given in (8.6) in an arbitrary period $t^\circ \geq 0$. Then, the subsidy has to be implemented until expected profits from innovation in sector m are greater than those of innovating in sector z even without the subsidy. Suppose, this needs $j > 1$ periods. Then we have to show that in period $t^\circ + j$ the condition $V_{m,t^\circ+j} > V_{z,t^\circ+j}$ holds. Inserting terms, expected profits in sector m can be written as

$$V_{m,t} = \bar{\mu}\Big(\gamma^{\varepsilon} Q_{m,t+1}^{\sigma-1-\alpha}\Big)^{\frac{1}{1-\alpha}} = \bar{\mu}\Big(\gamma^{\varepsilon}(1+\iota_t\eta_{m,t}\bar{\lambda}_m)^{\sigma-1-\alpha} Q_{m,t}^{\sigma-1-\alpha}\Big)^{\frac{1}{1-\alpha}} \tag{C.12}$$

and in sector z

$$V_{z,t} = \bar{\mu}\Big((1-\gamma)^{\varepsilon} Q_{z,t+1}^{\sigma-1-\alpha}\Big)^{\frac{1}{1-\alpha}} = \bar{\mu}\Big((1-\gamma)^{\varepsilon}(1+(1-\iota_t)\eta_{z,t}\bar{\lambda}_z)^{\sigma-1-\alpha} Q_{z,t}^{\sigma-1-\alpha}\Big)^{\frac{1}{1-\alpha}}. \tag{C.13}$$

During intervention, i.e. in periods $t^\circ < t < t^\circ + j$,the expected profits from innovation in sector m including the subsidy, are greater than the expected profits from innovation in sector z. Thus $\iota_t = 1$ and

$$Q_{m,t+1} = (1+\eta_{m,t}\bar{\lambda}_m)Q_{m,t}, \quad \text{and} \quad Q_{z,t+1} = Q_{z,t}. \tag{C.14}$$

This implies a decrease in the "productivity gap" $Q_{z,t}/Q_{m,t}$:

$$\frac{Q_{z,t+1}}{Q_{m,t+1}} = \frac{Q_{z,t}}{(1+\eta_{m,t}\bar{\lambda}_m)Q_{m,t}} < \frac{Q_{z,t}}{Q_{m,t}}. \tag{C.15}$$

Proceeding forward, this implies for period $t^\circ + j,\ j > 1$:

$$\frac{Q_{z,t^\circ+j}}{Q_{m,t^\circ+j}} = \frac{Q_{z,t}}{(1+\eta_{m,t}\bar{\lambda}_m)^j Q_{m,t}} < \frac{Q_{z,t}}{Q_{m,t}}. \tag{C.16}$$

Expected profits can be written as

$$V_{m,t^\circ+j} = (1+\eta_{m,t}\bar{\lambda}_m)^{\sigma-1-\alpha} V_{m,t^\circ} \tag{C.17}$$

and

$$V_{z,t^\circ+j} = V_{z,t^\circ}. \tag{C.18}$$

Between t° and $t^\circ + j$ expected profits in sector m increase with a rate $1+\eta_m\bar{\lambda}_m$ per period, while expected profits in sector z remained constant.

So it remains to show that the government can set a subsidy d_{t+j} in period $t + j$ such that

$$V_{m,t^\circ+j}(1+d_{t^\circ+j}) > V_{z,t^\circ+j} \tag{C.19}$$

and

$$V_{t^\circ+j+1} > V_{z,t^\circ+j+1} \tag{C.20}$$

hold.
From (C.19) we get

$$\tilde{V}_{t^\circ} < (1 + d_{t^\circ+j})(1 + \eta_{m,t}\bar{\lambda}_m)^{\frac{j(\sigma-1-\alpha)}{1-\alpha}} \tag{C.21}$$

From (C.20) we get

$$\tilde{V}_{t^\circ} < (1 + \eta_{m,t}\bar{\lambda}_m)^{\frac{(1+j)(\sigma-1-\alpha)}{1-\alpha}} \tag{C.22}$$

These two conditions are equivalent if $d_{t^\circ+j} = (1 + \eta_{m,t}\bar{\lambda}_m)^{\frac{\sigma-1-\alpha}{1-\alpha}} - 1$. So if intervention started in period t° the subsidy rate in period $t^\circ + j$ equals

$$d_{t^\circ+j} = (1 + \eta_{m,t}\bar{\lambda}_m)^{\frac{\sigma-1-\alpha}{1-\alpha}} \tag{C.23}$$

and one can show that the subsidy rate in period t° is given by

$$d_{t^\circ} = (1 + \eta_{m,t}\bar{\lambda}_m)^{j\frac{\sigma-1-\alpha}{1-\alpha}}. \tag{C.24}$$

Clearly, the number of periods j and so the size of the subsidy depends on the "productivity-gap" $Q_{z,t}/Q_{m,t}$: The larger this gap, the greater is j. (ii) Dirty production given in (6.35) equals

$$Y_{z,t} = \mu^{\frac{2\mu}{1-\mu}}(1-\gamma)^{\varepsilon}\left(Q_{m,t}\hat{W}_t^{\frac{1}{\sigma-1}}\right)^{2-\sigma} Q_{z,t}. \tag{C.25}$$

and emissions are given in (5.10):

$$E_t = \kappa Y_{z,t}. \tag{C.26}$$

These two expressions together already imply that emissions E_t do not grow for $\varepsilon > (2-\mu)/(1-\mu) \Leftrightarrow \sigma > 2$, since during intervention and after redirecting R&D to sector m, $Q_{z,t}$ is constant, while at the same time $Q_{m,t}$ grows with rate $(1 + \eta_{m,t}\bar{\lambda}_m)$ and therefore $Y_{z,t}$ and emissions grows with a rate $(1 + \eta_{m,t}\bar{\lambda}_m)^{2-\sigma}$. □

Proof proposition 8.4

Proof. To prove the proposition in the text, first take the strength of intervention for period t° intervention as given in equation (8.9):

$$d_{t^\circ} \geq \hat{d}_{t^\circ} \equiv \left[\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(1 + \eta_{z,t^\circ}\bar{\lambda}_z\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t^\circ}}{Q_{m,t^\circ}}\right)^{\sigma-1-\alpha}\right]^{\frac{1}{1-\alpha}} - 1. \tag{C.27}$$

This conditions for the subsidy holds for all $t \geq 0$, so a delay in intervention of one period to $t^\circ + 1$ implies for the subsidy:

$$d_{t^\circ+1} \geq \hat{d}_{t^\circ+1} \equiv \left[\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(1 + \eta_{z,t^\circ+1}\bar{\lambda}_z\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t^\circ+1}}{Q_{m,t^\circ+1}}\right)^{\sigma-1-\alpha}\right]^{\frac{1}{1-\alpha}} - 1. \tag{C.28}$$

We have to show that $\hat{d}_{t^\circ+1} > \hat{d}_{t^\circ}$ which is straightforward. Insert (C.27), (C.28) and simplify gives:

$$\begin{aligned} \hat{d}_{t^\circ+1} &> \hat{d}_{t^\circ} \Leftrightarrow \\ \left(1 + \eta_{z,t^\circ+1}\bar{\lambda}_z\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t^\circ+1}}{Q_{m,t^\circ+1}}\right)^{\sigma-1-\alpha} &> \left(1 + \eta_{z,t^\circ}\bar{\lambda}_z\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t^\circ}}{Q_{m,t^\circ}}\right)^{\sigma-1-\alpha}. \end{aligned} \tag{C.29}$$

Now recall that with no intervention in period t°, innovators continue to direct R&D to sector z. Thus the average quality of sectoral capital goods in sector z changes by $Q_{z,t^\circ+1} = (1+\eta_{z,t^\circ}\bar{\lambda}_z)Q_{z,t^\circ}$,while the average quality of capital goods in the green sector remains constant: $Q_{m,t^\circ+1} = Q_{m,t^\circ}$. Use this to rewrite the left hand side of the inequality condition above gives

$$\begin{aligned}\left(\left(1+\eta_{z,t^\circ+1}\bar{\lambda}_z\right)\left(1+\eta_{z,t^\circ}\bar{\lambda}_z\right)\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t^\circ}}{Q_{m,t^\circ}}\right)^{\sigma-1-\alpha} &> \left(1+\eta_{z,t^\circ+1}\bar{\lambda}_z\right)^{\sigma-1-\alpha}\left(\frac{Q_{z,t^\circ}}{Q_{m,t^\circ}}\right)^{\sigma-1-\alpha} \Leftrightarrow \\ \eta_{z,t^\circ+1}\bar{\lambda}_z &> 0.\end{aligned}$$

which is always fulfilled, since $\eta_{z,t^\circ+1} > 0, \bar{\lambda}_z > 0$. □

Computing the socially optimal allocation
I adopt a standard Langrangian-type approach to characterize the solution to (8.34). Define the Lagrangian function

$$\begin{aligned}&\mathscr{L}\left(\left(C_t, \left(Y_{j,t}, X_{j,t}, L_{j,t}, \iota_{j,t}, (x_{\vartheta_j,t}, h_{\vartheta_j,t}, q_{\vartheta_j,t})_{\vartheta_j\in[0,1]}\right)_{j\in\{m,z\}}, D_{t+1}, A_{1,t}, A_{2,t}\right)_{t\geq 0}\right) := \\ &\sum_{t=0}^{\infty}\omega^t\Bigg[\beta\sum_{\ell\in\{[w,e,i\}}\chi_\ell u(C_{\ell,t}) + \lambda_{0,t}\Big((1-D(A_{1,t}+A_{2,t}))F(Y_{m,t},Y_{z,t}) - C_t - X_{m,t} - X_{z,t} - D_{t+1}\Big) \\ &\quad + \sum_{j\in\{m,z\}}\lambda^y_{j,t}\Big(G(x_{\vartheta_j,t}, L_{j,t}) - Y_{j,t}\Big) + \sum_{j\in\{m,z\}}\int_0^1\lambda^q_{\vartheta_{j,t}}\Big((1+\iota_{j,t}\eta_{j,t}\bar{\lambda}_j)Q_{j,t} - Q_{j,t+1}\Big) \\ &\quad + \sum_{j\in\{m,z\}}\int_0^1\lambda^h_{\vartheta_{j,t}}\Big(\eta_{j,t} - h^\alpha_{\vartheta_j,t}z_{\vartheta_j}\Big) + \lambda^\iota_t\Big(\iota_{m,t}+\iota_{z,t}-1\Big) + \lambda^L_t\Big(L_{m,t}+L_{z,t}-1\Big) \\ &\quad + \lambda^A_{1,t}\Big(A_{1,t} - A_{1,t-1} - \phi_L\kappa Y_{z,t}\Big) + \lambda^A_{2,t}\Big(A_{2,t} - A_{2,t-1} - (1-\phi_L)\phi_0\kappa Y_{z,t}\Big) \\ &\quad + \lambda^D_t\Big(D_t - H_{m,t+1} - H_{z,t+1}\Big) + \sum_{j\in\{m,z\}}\int_0^1\lambda^x_{\vartheta_j,t}d\vartheta_j\Big(X_{j,t} - \int_0^1 x_{\vartheta_{j,t}}d\vartheta_j)\Big)\Bigg].\end{aligned} \tag{C.30}$$

Then the first order conditions of the social planning problem are for all $t \geq 0$:

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial Y_{m,t}} &: \lambda_{0,t}\partial_{Y_m}F(Y_{m,t}, Y_{z,t}) - \lambda^Y_{m,t} = 0.\\
\frac{\partial \mathscr{L}}{\partial Y_{z,t}} &: \lambda_{0,t}\partial_{Y_z}F(Y_{m,t}, Y_{z,t}) - \lambda^Y_{z,t} - \lambda^A_{1,t}\phi_L\kappa - (1-\phi_L)\phi_0\kappa\lambda^A_{2,t} = 0.\\
\frac{\partial \mathscr{L}}{\partial L_{j,t}} &: \lambda^Y_{j,t}\partial_{L_j}G(L_j, x_{\vartheta_j}) - \lambda^L_t = 0, \qquad j \in \{m, z\}.\\
\frac{\partial \mathscr{L}}{\partial C_{\ell,t}} &: \beta\chi_\ell u'(C_{\ell,t}) - \lambda_{0,t} = 0, \qquad \ell \in \{e, w, i\}.\\
\frac{\partial \mathscr{L}}{\partial X_{j,t}} &: -\lambda_{0,t} + \lambda^x_{\vartheta_j,t} = 0, \qquad \vartheta_j \in [0,1], j \in \{m, z\}.\\
\frac{\partial \mathscr{L}}{\partial x_{\vartheta_j,t}} &: \lambda^Y_{j,t}\partial_{x_{\vartheta_j}}G(L_j, x_{\vartheta_j}) - \lambda^x_{\vartheta_j,t} = 0, \qquad \vartheta_j \in [0,1], j \in \{m, z\}.\\
\frac{\partial \mathscr{L}}{\partial \iota_{j,t}} &: -\lambda^\iota_t + \lambda^q_{\vartheta_j,t}\eta_{j,t}\bar{\lambda}_j Q_{j,t} = 0, \qquad \vartheta_j \in [0,1], j \in \{m, z\}.\\
\frac{\partial \mathscr{L}}{\partial D_{t+1}} &: -\lambda_{0,t} + \omega\lambda^D_{t+1} = 0.\\
\frac{\partial \mathscr{L}}{\partial q_{\vartheta_j,t}} &: \lambda^Y_{j,t}\partial_{q_{\vartheta_j}}G - \lambda^q_{\vartheta_j,t} + \omega(1 + \iota_{j,t+1}\eta_{j,t+1}\bar{\lambda}_j)\lambda^q_{\vartheta_j,t+1} = 0, \qquad \vartheta_j \in [0,1], j \in \{m, z\}.\\
\frac{\partial \mathscr{L}}{\partial h_{\vartheta_j,t}} &: \lambda^q_{\vartheta_j,t}\alpha h^{\alpha-1}_{\vartheta_j,t}\phi_{\vartheta_j,t}\iota_{j,t}\bar{\lambda}_j q_{\vartheta_j,t} + \lambda^h_{\vartheta_j,t}\alpha h^{\alpha-1}_{\vartheta_j,t}\phi_{\vartheta_j,t} - \omega\lambda^D_{t+1} = 0.\\
\frac{\partial \mathscr{L}}{\partial A_{1,t}} &: -\lambda^A_{1,t} + \lambda_{0,t}\frac{\partial D(A_{1,t} + A_{2,t})}{\partial A_1}\frac{Y_t}{1 - D(A_{1,t} + A_{2,t})} + \omega\lambda^A_{1,t+1} = 0.\\
\frac{\partial \mathscr{L}}{\partial A_{2,t}} &: -\lambda^A_{2,t} + \lambda_{0,t}\frac{\partial D(A_{1,t} + A_{2,t})}{\partial A_2}\frac{Y_t}{1 - D(A_{1,t} + A_{2,t})} + \omega(1-\phi)\lambda^A_{2,t+1} = 0.
\end{aligned}$$

Note that $\lambda_{0,t}$ can be interpreted as a shadow price of time t consumption. Thus, the time t shadow price of intermediate good produced in sector $j \in \{m, z\}$ measured in time t consumption goods can be defined as

$$\hat{\lambda}_{j,t} := \lambda^{(y)}_{j,t}/\lambda_{0,t} \qquad j \in \{m, z\}. \tag{C.32}$$

We use this shadow price in the derivations in the main text.

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