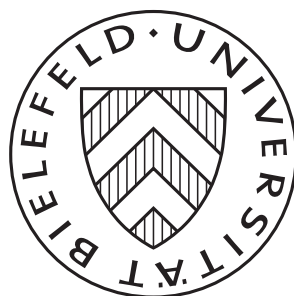


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Information Transmission in Hierarchies

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Abstract

We analyze a game in which players with unique information are arranged in a hierarchy. In the lowest layer each player can decide in each of several rounds either to pass the information to his successor or to hold. While passing generates an immediate payoff according to the value of information, the player can also get an additional reward if he is the last player to pass. Facing this problem while discounting over time determines the player's behavior. Once a successor has collected all information from his workers he starts to play the same game with his successor.

We state conditions for different Subgame Perfect Nash Equilibria and analyse the time it takes each hierarchy to centralize the information. This allows us to compare different structures and state which structure centralizes fastest depending on the information distribution and other parameters. We show that the time the centralization takes is mostly affected by the least informed players.

Keywords : communication network; dynamic network game; hierarchical structure; information transmission.

JEL Classification : D83, C72, C73.

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1 Introduction

In this paper we introduce a model of information transmission where players send their unique information to successors who then do the same. In each hierarchy the players have several periods in which they can decide to pass their information to their successor and receive utility according to the value of the information, or to wait and hold the information privately. The incentive for holding is given by a reward which is granted to the player who passes the last piece of information to the successor. A disadvantage of holding is the discount of utility over time. Once a successor got the information from all his predecessors, he can pass the information to his own successor, facing the same problem.

An application for our model can be found in nearly every company. Once the head of a department requires information, all the heads of the sub-departments will collect the information from their workers. These workers will sooner or later deliver all the information they have. The last player who delivers the information, transmits the critical information the supervisor is missing and for that receives an reward additional to the value of his information.

The idea that all the information are perfect complements might sound strange first, but there are several examples that can motivate it. The one that comes to mind is a mathematical proof. It requires all kind of different ideas and information, and as long as there is just a tiny part missing, it is worthless. Similar the composition of a machine can serve as an example: If not every part and explanation is available, building the machine most likely is going to fail.

Another example is a research proposal for funding. It requires that the administration is doing their part and also the scientific part needs to be done, because all forms have to be completed. Otherwise the proposal will not be considered.

A complete different example comes from the politics. If for example a city wants to change a law it often needs the unanimous agreement of all districts. All those districts generate some different payoff from this law. We could for example think about a certain quota for kindergartens that needs to be satisfied in each part of the city. Passing the information in this case is equivalent to signing the law. Of course the more residential areas with a low number of kindergartens benefit most, while districts in which the quota is already fulfilled or more industrial regions, do not benefit a lot. If we scale this up and think that first the city needs the districts agreement and then the region needs all cities to pass the law, before it is a national-wide law, we have already an example for a hierarchy with several layers. Our model will work in a way that even a single player can delay the information centralization.

A real life example for this were the CETA negotiations between Canada and the European Union in 2016. Just Wallonia, a region of Belgium, decided not to sign the treaty and by that blocked the entire process. Only after some changes were made, the representatives in the Wal-

lonian parliament voted for the agreement. We can see these changes as some additional utility given as an incentive.

In our model the last player who delivers the information gets an additional reward. In addition to the example mentioned before, we can find several other reasons why it is reasonable. The most simple one, in the example of a principal collecting all the information, is that the one who delivers the crucial part is most likely best remembered by the principal and gets benefits for the remainder of his employment.

We can also motivate the reward by bargaining between successor and predecessor: If the successor has collected nearly all the information and the remaining player knows that he has the last part, he has a lot of bargaining power and by that can achieve an additional payoff.

In most team projects and also in sports we find a similar reward system. It is a team effort to score a goal, but most often one player is rewarded for it more than the others. It is like that in football, where the player that scores is celebrated most and maybe gets additional salary for it, while he wouldn't be able to net a goal, without the player doing the last pass or the players before.

In this paper we define the game, where the players can either pass or hold, we derive the equilibria and study the effects of different parameters. Starting with a simple model with just two layers, i.e. one successor and several predecessors, we show that no matter the deadline the successor will either collect all information in the first, second or last period. The first surprising result is that the successor may even prefer long deadlines over shorter ones, because it speeds up the process of centralization. We give a detailed analysis of the impact of all parameters on the equilibrium and on the duration of the game.

We state all the Subgame Perfect Nash Equilibria (SPNE), the duration of the game, compare different hierarchies and illustrate our results. For hierarchies with several layers we also state the SPNE and the optimal behavior of the players. We compare different structures and show that there is no hierarchical structure that always centralizes the information fastest. Even if for one set of parameters one hierarchy centralizes faster than all others, even a slight change in the parameters may change this.

We show that in a setting with a hierarchy consisting of just two layers the value of information of the two least informed players affects the time it takes until the information is centralized, but the value of information of all other players has no impact on the duration. An increase in the information value of the two least informed players yields to a decrease in the duration. The same effect can be achieved by a decrease of the additional reward. Surprisingly also a longer deadline can speed up the game.

The total time it takes the principal to centralize the information in a multi-layer model is af-

ected by the same parameters, but in addition also by the network structure. Not only a change in the parameters can decrease or increase the duration, but also changes in the hierarchical structures can do so. We show that different structures are optimal for different information distributions and for different sets of parameters.

An important foundation for this model is the paper of Hagenbach (2011). She models information transmission in a network where all players together aim to centralize information. At the same time each player wants to be the one who centralizes all the information and by that get some additional utility. While we focus on (directed) tree networks, Hagenbach gives general results depending on the (undirected) network structure. The main difference is the motivation and utility of each player. We model utility such that once a player passed his information he is receiving an utility according to the value. Also the role of successors is quite different. While in Hagenbach the players, who are connected to leaves, have an advantage in collecting all information, we look more at the situations where the agents are passing to their successors.

Hierarchical structures, as we use them, can be found in many different articles with different usages. There are many papers on hierarchies which focus on different solution and which compare these concepts, e.g. Demange (2004), Álvarez-Mozos et al. (2015) and van den Brink, Dietz, et al. (2016). van den Brink and Steffen (2008) analyze the power that comes with positions and the arrangement of positions in hierarchies, see also van den Brink and Steffen (2012). They take into account the role of the decision making mechanism and focus on the dominance relation between different players in a hierarchy. Closer to our model is the work of Garicano (2000). The author deals with knowledge production and transmission in networks. While players in a low level of the hierarchy solve simple problems, more complex knowledge is held by the specialist in upper levels. He shows that this split of tasks is optimal, but the firm have to give additional incentives if the complexity is not observable. An empirical work on production in networks was done by Garicano and Hubbard (2007).

In the networks in our paper, we also have players who act as intermediaries. The literature on the role of intermediaries in networks grew in the last years. Manea (2015) models the reselling of a good, which also might be information, in a network until the good reaches the final buyer. The utility generated by the players comes from bargaining over the price. The author studies differences between intermediaries who bargain with players on the same layer of the network and those who interact with players from different levels. Siedlarek (2015) focuses more on the competition between different routes that a given good can take through the network from the source to the final buyer. Other papers dealing with trade and intermediaries in networks are for example Blume et al. (2009) and Choi et al. (2014). Manea (2016) gives an overview over different models of bilateral trade, while the survey of Galeotti and Condorelli (2016) focuses on the role of the intermediaries in networks.

With Sah and Stiglitz (1986) and Radner (1993) a whole new literature on organizational economics arose. In Radner (1993) the managers of a firm are the information processors. The author studies the efficiency of different structures under specific circumstances. We do similar comparisons of structures in our model.

Jehiel (1999) deals with a similar problem. He models a communication structure in which a decision maker needs to centralize information to make a decision about a project. The utility of all players is given by a share of the surplus generated by that project, which incentivizes them to work as a team. After the decision is made, the decision maker is fired if he made a bad decision. The author gives conditions under which a communication structure is optimal for players who want to communicate their private information. Other models on information transmission, but without the networks aspect, can be found in Lewis and Sappington (1997) and Levitt and Snyder (1997). Lewis and Sappington (1997) derives a way for an agent to acquire information optimally, while Levitt and Snyder (1997) focuses on information transmission in a principal-agent model.

A connection between communication and networks is done in Ambrus et al. (2013). The authors create a model in which communication in a network takes place by cheap-talk between different intermediaries.

Our paper is structured as follows: In Section 2 we start with a model with just two layers of hierarchy. In this model we first give examples and then use the same ideas to derive all equilibria. We focus on the time it takes the principal to collect all information and analyze the impact of all parameters on this duration. In Section 3 we move to a model with more than two layers. Again we start with some examples and a small set of players. Furthermore, we state general results and we give some more results depending on additional conditions. The results from the two-layer model help with the analysis here. We discuss two different extension possibilities in Section 4. Section 5 concludes. All proofs are relegated to the appendix.

2 Two layer model

In this part we focus our attention on networks that just consist of two different layers of hierarchy. There is one successor/principal and a set of predecessors/agents $N = \{1, \dots, n\}$. We assume that there are at least two players with information. Each of the agents $i \in N$ has an information item with value $x_i > 0$. Without loss of generality we say that $x_1 \geq x_2 \geq \dots \geq x_n > 0$ holds.

The game has a finite deadline of $T > 1$ periods. Starting from period $t = 1$ each player can decide to pass his information in that period or hold it. We denote the action player i took at time t by $a_i^t \in \{P, H\}$ and limit our attention to pure strategies.

Passing the items generates a utility according to the value of information x_i . Once a player passed at time τ_i , he is eliminated from the game, i.e. $a_i^t = H \forall t > \tau_i$. All players make their decision simultaneously. The player who passes his information last gets an additional reward $R \in (0, n \cdot (x_{n-1} - x_n) + (n - 2) \cdot x_{n-1})$. The upper bound for the reward depends not only on the number of players, but also on the difference between the value of information of the two least informed players. The larger the difference is, or the larger the number of players is, the larger the reward R can be. It is not the value of information of the least informed players, but his difference to the value of information of the second least informed players that limits the reward. Additionally, in larger organizations with more players, the reward can be higher. In an extension in Section 4 we relax this assumption.

If several players pass the last information in the same period the reward is split equally between them. All players discount over time by the same discount factor $\delta \in (0, 1)$. Holding the information generates no utility for the players, i.e. $u_i = 0$. The present value of player i 's utility is as follows:

$$u_i(x_i) = \delta^{\tau_i - 1} \cdot \begin{cases} x_i & , \text{ if } \exists j \in N, j \neq i \text{ such that } \tau_j > \tau_i \\ x_i + \frac{R}{\ell} & , \text{ if } \forall j \in N : \tau_j \leq \tau_i \text{ with } \ell = \{j \in N \mid \tau_j = \tau_i\} \end{cases}$$

Our aim is to find the Subgame Perfect Nash Equilibrium depending on δ, R, T and $(x_i)_{i \in N}$. We assume that in a situation where several players have an incentive to pass the information, the players with more valuable information pass first.

Remark 1. *All players will pass their information at one point. In particular in $t = T$ all the players who have not passed their information will pass.*

Even in the very last period the players generate a utility strictly larger than 0 from passing, because $\delta > 0$ holds.

Remark 2. *As soon as there is only one player left who has not passed, this player will pass immediately.*

In the situation where there is just one player left, that player does not face any competition, but gets the reward R for himself for sure. Since the value of his information decreases over time, he will pass as soon as possible.

2.1 Two players, two period example

Let us start with a simple example of just two players and two periods, i.e. $T = 2$. We have already noted that in the last period $t = T$ all remaining players will pass, so it remains to analyze the players in $t = 1$.

	P	H
P	$x_1 + \frac{R}{2}$ $x_2 + \frac{R}{2}$	x_1 $\delta \cdot (x_2 + R)$
H	$\delta \cdot (x_1 + R)$ x_2	$\delta \cdot (x_1 + \frac{R}{2})$ $\delta \cdot (x_2 + \frac{R}{2})$

In this game player 1 chooses the row, while player 2 selects the column. We can simplify the conditions, because we have sorted the players such that $x_1 \geq x_2$ holds and we get the following Nash equilibria:

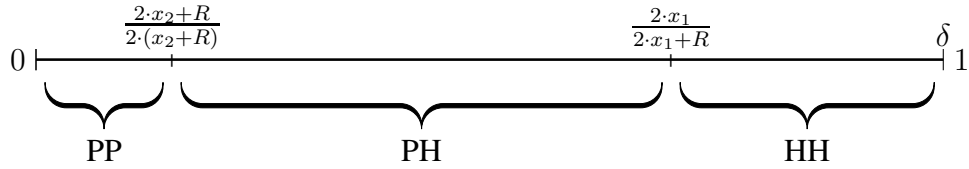


Figure 1: Subgame Perfect Nash Equilibria

We can see that for a low value of δ both players prefer to pass in the first period, for a medium value player 2 prefers to wait to get the reward for himself, while for a high value of δ both players will wait until the second period and share the reward. This matches our intuition that players get more patient when the discount factor is larger. There is no area in which player 1 holds his information, but player 2 passes, because this interval would be included in the second interval and we assume that in that case always the players with the more valuable information passes.

2.2 n players, two period example

Similarly to the previous example we can search for the SPNE in a game with n players and two periods. In a later part we show the relation to games with any finite number of periods. Again we know that in $t = T = 2$ all the remaining players are going to pass their information, which gives all players the choice between the following utilities:

$$\begin{aligned}
 u_i(\text{P} \mid \text{all other players pass}) &= x_i + \frac{R}{n} \\
 u_i(\text{H} \mid \text{all other players pass}) &= \delta \cdot (x_i + R) \\
 u_i(\text{P} \mid j \leq n - 1 \text{ players hold}) &= x_i \\
 u_i(\text{H} \mid j \leq n - 1 \text{ players hold}) &= \delta \cdot (x_i + \frac{R}{j+1})
 \end{aligned}$$

With these utilities we can write down the game in matrix form and get the first proposition to state all the SPNE:

Proposition 1 (SPNE for $T = 2$).

1. All players pass their information in $t = 1$ if and only if $\delta \in \left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$.
2. One player holds his information, while all other players pass in $t = 1$, if and only if $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)$.
3. $i \in \{2, \dots, n-1\}$ players hold their information, while all other players pass in $t = 1$, if and only if $\delta \in \left(\frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}, \frac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R}\right)$.
4. All players hold their information in $t = 1$ if and only if $\delta \in \left(\frac{n \cdot x_1}{n \cdot x_1 + R}, 1\right)$.

The complete SPNE profile, if $i \in \{1, \dots, n\}$ players pass in $t = 1$, is that

- the players 1 to $(n - i)$ player pass in $t = 1$ and by definition hold in $t = 2$ and
- the players $(n + 1 - i)$ to n hold in $t = 1$ and pass in $t = T$.

With this proposition we can sort all SPNE depending on δ . With a higher δ more players will hold their information in the first period, while for a low δ they prefer to pass sooner.

Subgame Perfect Nash Equilibria depending on regions similar to those can be found in many later parts of this paper. Again we can illustrate the $(0, 1)$ interval:

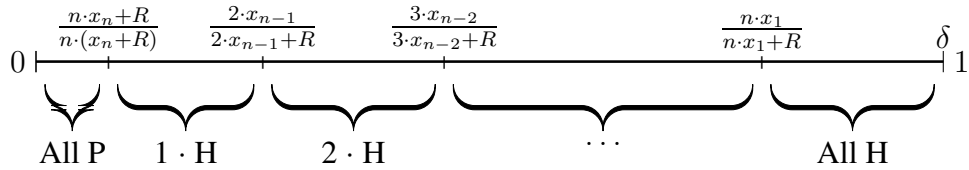


Figure 2: Different equilibria depending on δ

In Figure 2 we see that the number of players who hold their information in $t = 1$ is increasing in δ . By the assumption we made on the reward R we get that the equilibrium is unique, because

$$\frac{n \cdot x_n + R}{n \cdot (x_n + R)} < \frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R} < \frac{3 \cdot x_{n-2}}{3 \cdot x_{n-2} + R} < \dots < \frac{n \cdot x_1}{n \cdot x_1 + R} \text{ holds.}$$

2.3 n players, T periods

In this general setting with T periods we first state some more results about the behavior of the players. These results then help us to state the SPNE.

Remark 3. *In equilibrium no player will hold his information first and then pass it later without getting a share of the reward.*

The reason for this remark is simply given by the discounting. Passing in the first period generates an utility of x_i , while passing at time t' without getting any part of the reward gives only $\delta^{t'-1} \cdot x_i$. The same holds for all other periods. If the player passes in period t , he gets $\delta^{t-1} \cdot x_i$, for any period $t' > t$ he only gets $\delta^{t'-1} \cdot x_i < \delta^{t-1} \cdot x_i$.

Proposition 2.

There is no SPNE in which $j > 1$ players hold their information in $t = 1$ and then pass it in any $t' \neq T$.

This proposition states that there is never an equilibrium in which at least two players hold their items in the first period and then collect the reward together in the following period, but instead they will wait until the last period to pass. With the help of this proposition we can list all the possible SPNE:

Corollary 1.

There can exist only the following SPNE:

1. *All players pass in $t = 1$.*
2. *Players 1 to $n - 1$ pass in $t = 1$, just player n holds in $t = 1$ and passes in $t = 2$.*
3. *Players 1 to $n - i$ ($i \in (1, n - 1)$) pass in $t = 1$, players $n + 1 - i$ to n hold in $t = 1$ and pass in $t = T$.*
4. *All players hold in $t = 1$ and pass in $t = T$.*

By backward induction we can state also conditions for each of the SPNE as in Proposition 1.

Proposition 3 (SPNE).

1. *All players pass in $t = 1$ if and only if $\delta \in \left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$.*
2. *One player holds his information, while all other players pass in $t = 1$, if and only if $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$. This one player then passes in $t = 2$.*
3. *$i \in \{2, \dots, n - 1\}$ players hold their information in $t = 1$, while all other players pass in $t = 1$, if and only if $\delta \in \left(\left(\frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}\right)^{\frac{1}{T-1}}, \left(\frac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R}\right)^{\frac{1}{T-1}}\right)$. The players who have hold in $t = 1$, also hold in all $t \in \{2, \dots, T - 1\}$ and pass in $t = T$.*
4. *All players hold their information in $t = 1$ if and only if $\delta \in \left(\left(\frac{n \cdot x_1}{n \cdot x_1 + R}\right)^{\frac{1}{T-1}}, 1\right)$. All players will also hold in $t \in \{2, \dots, T - 1\}$. They pass in $t = T$.*

The complete SPNE profile is:

- If all players pass in $t = 1$, they hold by definition for all $t > 1$.
- If one player (player n) holds in $t = 1$, the remaining players hold by definition for all $t > 1$, while this player n passes in $t = 2$ and then also holds by definition for $t > 2$.
- if $i \in \{3, \dots, n\}$ players pass in $t = 1$, is that
 - the players 1 to $(n - i)$ player pass in $t = 1$ and by definition hold in $t > 1$ and
 - the players $(n + 1 - i)$ to n hold in $t \in \{1, \dots, T - 1\}$ and pass in $t = 2$.

We can see that these results are similar to Proposition 1. Only the upper bound of the second case and the boundaries of the third and fourth cases got the exponent $\frac{1}{T-1}$. This is caused by the fact that if more than one player holds in $t = 1$, the players wait for the last period to pass and so they have to discount $T - 1$ times. It is obvious that the equilibrium is still unique, because the first boundary is not changed and all others are increased in the same way.

Duration

Our aim is to compare different hierarchies and see in which hierarchy the information is centralized fastest or slowest. We have seen in Proposition 2 that all the games have either a duration of 1, 2 or T periods. We can modify Proposition 3 easily to focus on the duration of the game:

Corollary 2.

1. *The game ends after 1 period if and only if $\delta \in \left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$.*
2. *The game ends after 2 periods if and only if $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$.*
3. *The game ends after T periods if and only if $\delta \in \left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$.*

It is to notice that the duration only depends on the parameters R , T and δ and the value of information of the two least informed players x_n and x_{n-1} . The intuition behind is as follows: In $t = 1$ all players will only pass if no one has an incentive to deviate. Because of the order of players ($x_1 \geq \dots \geq x_n$) and our assumption that the more informed players pass first, the least informed player may deviate first and so his value of information influences the boundary between a duration of one and two periods.

In the case that player n holds, the remaining players have to pass to achieve a duration of two periods. Again, because of the order of players, the boundary is defined by the value of information of the least informed of the remaining players, player $n - 1$. If he decides to hold in $t = 1$ as well, the players who hold in the first period will wait until the last period to pass, as stated in Proposition 2. In that case the duration is T .

2.4 Impact of the parameters

The previous corollary gives us the boundaries for the duration of the game. We can see that the reward R , the deadline T , the number of players n and the information distribution $(x_i)_{i \in N}$ have an impact on the duration. In this part of the paper we show the impact of the different parameters on the duration. This allows us to compare two hierarchies in the later part. We will do the comparison for hierarchies with the same number of players and for hierarchies with a different number of players. Some results are illustrated in the last part of this section.

Effect of the information distribution

In this part we want to give a short analysis on how the information distribution affects the duration. We fix δ , R , T and n and compare different information distributions, to see under which distribution the information is centralized fastest.

Let X be the value of all information in the hierarchy with n players, i.e. $X = \sum_{i=1}^n x_i$. The interval of δ for a duration of one period becomes maximal when $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$ is maximal, so for given n and R an increase in x_n increases the interval. By that we get that the interval is maximal if the information is distributed equally between all players, i.e. $x_i = \frac{X}{n}$ for all players i .

Even for this information distribution the discount factor can be such that the duration is longer than one period. To maximize the interval which yields a duration of two periods we have to maximize $\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}$ for a given n , R and T , while we no longer care about the lower bound $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$. As already described the lower boundary decreases with an decrease of x_n , so to minimize this boundary, x_n should be minimal. Therefore it is $x_n = \epsilon$ with $\epsilon > 0$. To maximize the upper bound of the interval we have to maximize x_{n-1} . As before we see that an equal information distribution yields the best result: $x_i = \frac{X - \epsilon}{n-1}$ for $i \neq n$.

We see that for equally distributed information the game ends even for higher δ after one period. If δ is too high then a duration of one period cannot be achieved by any distribution. In that case the information distribution in which one player gets the smallest possible piece of information (ϵ) and the remaining players have information with the same value is optimal. Under this distribution the interval with a duration of two periods is maximized.

We should notice that also a long deadline T can be beneficial for the principal. With increasing T also $\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}$ increases, so the interval in which the game ends after 2 periods increases, while the interval where the game ends after T periods decreases.

Effect of the deadline

We have already seen that with an increase of the deadline the interval

$\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ becomes larger. It is also interesting to analyze which intervals decrease by an increase of T .

Proposition 4 (Effect of an increase in T).

An increase in the deadline T has the following effects:

1. The interval $\delta \in \left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$ is unchanged.
2. The upper bound of the interval $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ increases and so the size of the interval increases as well.
3. Both boundaries of the interval $\delta \in \left(\left(\frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}\right)^{\frac{1}{T-1}}, \left(\frac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R}\right)^{\frac{1}{T-1}}\right)$ increase, while the interval becomes smaller.
4. The lower bound of the interval $\delta \in \left(\left(\frac{n \cdot x_1}{n \cdot x_1 + R}\right)^{\frac{1}{T-1}}, 1\right)$ increases, so the whole interval decreases in size.

This result states that for an increasing deadline the size of the interval for δ in which the game ends in two periods increases, while all the intervals in which the game ends after T periods become smaller. This means that, if we increase the deadline T to T' , there is an interval in which under the old deadline T the duration is T periods, while under the new deadline T' , the duration is only 2 periods. So an increase in T can decrease the duration from T periods to 2 periods.

Effect of the reward

A decrease of the reward R has a different effect than the change of the previous two parameters. The interval $\left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$ increases, so the interval in which the duration is one period becomes larger. Also the upper bound of $\left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ increases, but the overall effect depends on the remaining parameters. For some combination of the information distribution and T the interval grows, while for other the size decreases.

The interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$ diminishes. So all in all a decrease of R speeds up the information transmission. This fits our intuition, because with a lower reward R the players have less incentive to hold.

Effect of the number of players

To analyze the impact of a change of the number of players on the duration, we also have to take the information distribution into account. We do a comparison of two different hierarchies with a different number of players in the next section. To analyze the pure effect of the number of players, we assume that the value of information of the two least informed players, i.e. x_n

and x_{n-1} do not change. So this effect can be seen as the adding of well informed players: The derivative of $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$ with respect to n is $-\frac{R}{n^2(x_n + R)}$, which is negative. This means that an increase of n decreases the size of the interval in which the duration is one period and increases the interval with a duration of two periods. The number of players also has an effect on the different SPNE, but it has no further effect on the duration.

Summary

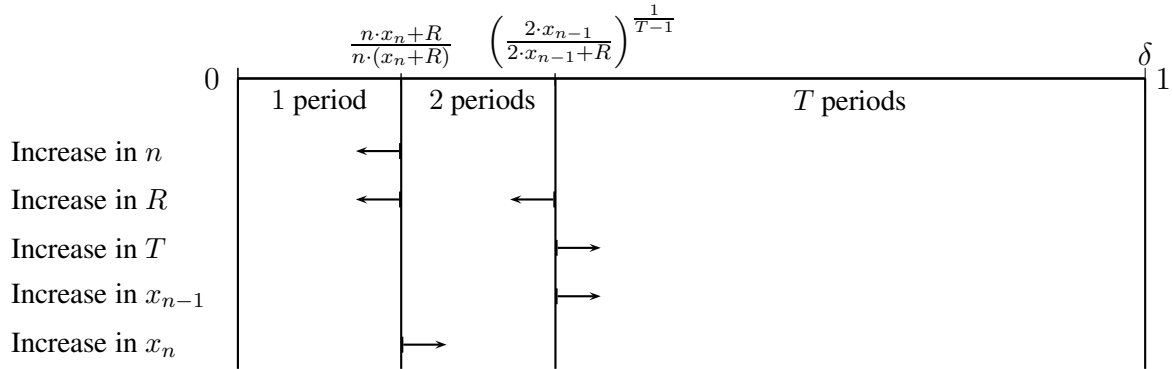


Figure 3: Summary of the impact of all parameters on the duration

Figure 3 sums up all the changes that come with an increase of a single parameter. We can see that only a change in the reward R influences both boundaries, while increases in all other parameters just change a single boundary. We can use this result to compare two different hierarchies.

2.4.1 Comparison of two hierarchies

Even with just two layers of hierarchies we have seen that there are several effects on the duration. To prepare the comparison of larger hierarchies we first have to start with the comparison of two-layer hierarchies with the same deadline T and reward R . In a second step we compare two hierarchies with a different number of players.

Comparison of two hierarchies with the same number of players

We will start with the comparison of two hierarchies with the same number of players n . Let us denote the values of information in the first hierarchy by x_1, \dots, x_n and in the second by y_1, \dots, y_n . Without loss of generality we can assume that $y_n \geq x_n$, i.e. that the least informed player of the second hierarchy is not less informed than the one of the first hierarchy. For different values of δ we get different durations for both structures, some of the following cases are shown in the Figures 4, 5 and 6. In each figure, above the axis we see the duration of the first hierarchy, while below the duration of the second hierarchy is shown.

1. For $\delta \in \left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$ the game ends after 1 period for both structures.
2. For $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \frac{n \cdot y_n + R}{n \cdot (y_n + R)}\right)$ the second structure still just takes one period, while in the other hierarchy the information is centralized slower. We have to separate between two cases:
 - If $\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}} > \frac{n \cdot y_n + R}{n \cdot (y_n + R)}$ holds the first hierarchy takes two periods. This is shown in Figures 4 and 5.
 - Otherwise there exists an interval $\left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the first structure needs two periods and the interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, \frac{n \cdot y_n + R}{n \cdot (y_n + R)}\right)$ in which it takes T periods. This combination is displayed in Figure 6.
3. For $\delta \in \left(\frac{n \cdot y_n + R}{n \cdot (y_n + R)}, \left(\frac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ the second structure needs two periods for the centralization.
 - If $y_{n-1} > x_{n-1}$ holds there exists an interval $\left(\frac{n \cdot y_n + R}{n \cdot (y_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the first structure also takes two periods and one interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, \left(\frac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the duration of the first structure is T (see Figures 4 and 6).
 - Otherwise the first structure also needs just two periods (see Figure 5).
4. For $\delta \in \left(\left(\frac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$ the game with the second hierarchy ends after T periods.
 - If $y_{n-1} > x_{n-1}$ holds the first structure has the same duration (see Figure 4 and Figure 6).
 - Otherwise there is an interval $\left(\left(\frac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the duration of the first structure is just two periods, while on the remaining interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$ both hierarchies need T periods (see Figure 5).

It is clear that all combinations are possible and only the comparison of the two least informed players in both hierarchies determines in which the information is centralized faster. Three combinations are illustrated below.

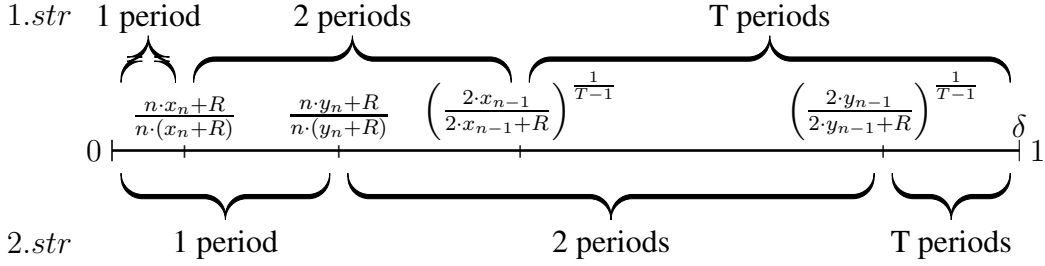


Figure 4: First combination

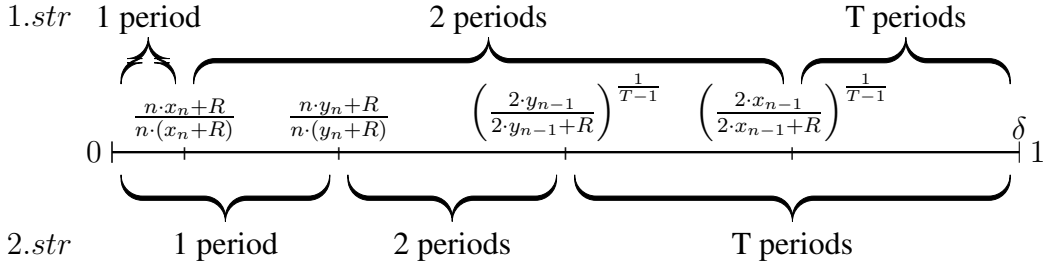


Figure 5: Second combination

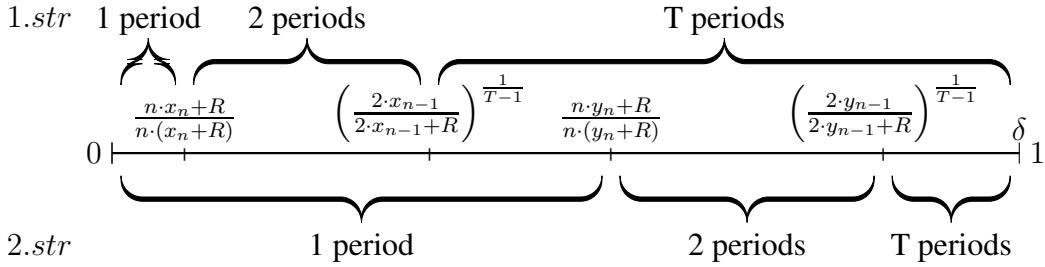


Figure 6: Third combination

Comparison of two hierarchies with a different number of players

If we compare two hierarchies with a different number of players, the analysis gets more complex, but basically we still have to compare the lower two bounds.

Let there be n players in the first hierarchy with information x_1, \dots, x_n and m players in the second structure with information y_1, \dots, y_m . Without loss of generality we can assume $m > n$.

If in addition also $x_n > y_m$ holds we get $\frac{m \cdot y_m + R}{m \cdot (y_m + R)} < \frac{n \cdot x_n + R}{n \cdot (x_n + R)}$. In that case, if δ is in the interval $\left(0, \frac{m \cdot y_m + R}{m \cdot (y_m + R)}\right)$ both structures need just one period, while for the interval $\left(\frac{m \cdot y_m + R}{m \cdot (y_m + R)}, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$ just the first hierarchy needs a single period, while the other structure needs two or T periods, depending on the other boundary. If this additional assumption does not hold, i.e. $x_n < y_m$ is true, we cannot make any general statement. The second lowest boundary is independent of x_n and y_m . For $x_{n-1} > y_{m-1}$ we get that $\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}} > \left(\frac{2 \cdot y_{m-1}}{2 \cdot y_{m-1} + R}\right)^{\frac{1}{T-1}}$ holds. Therefore there

exists an area $\left(\left(\frac{2 \cdot y_{m-1}}{2 \cdot y_{m-1} + R} \right)^{\frac{1}{T-1}}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R} \right)^{\frac{1}{T-1}} \right)$, where the first structure takes two periods, while the duration in the second structure is T periods. Again all combinations are possible.

Different deadlines or rewards

As soon as we compare two hierarchies which have not the same deadlines T and which also may not have the same rewards R , we have to look at the effect of T and R on the boundaries of the SPNE. In Proposition 4 we have already seen that an increase of T increases the boundary at which the duration changes from 2 to T periods. It remains to analyze how an increase of R shifts the intervals for δ :

$\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$ has a negative first derivative with respect to R , so this boundary is strictly decreasing in R . Obviously also $\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R} \right)^{\frac{1}{T-1}}$ is decreasing in R , so both upper boundaries decrease and create a larger interval in which the game takes T periods to end. We see that an increase in R has a different effect than an increase in T .

Graphics

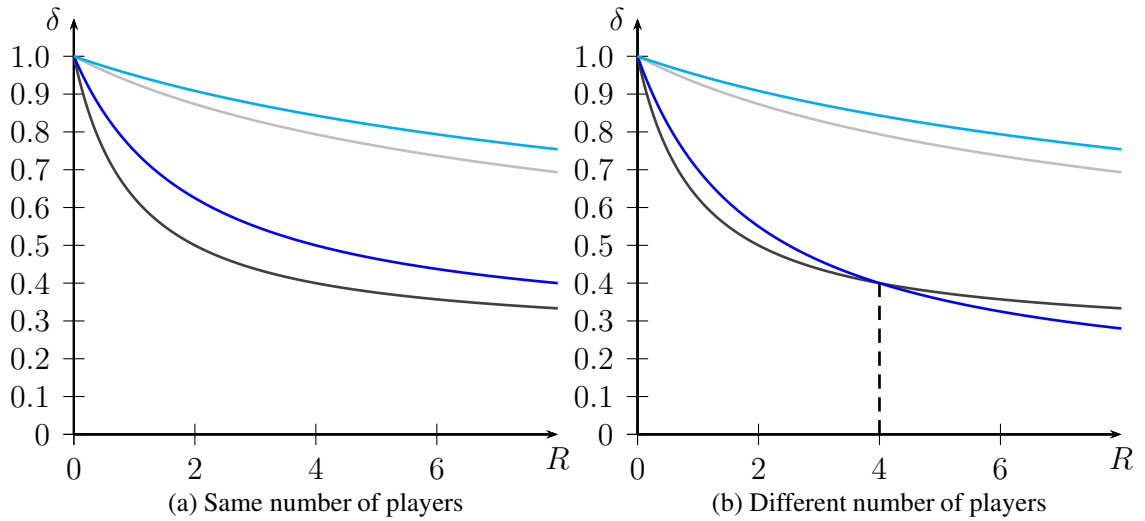


Figure 7: Duration of two different hierarchies depending on R and δ

The figure above shows the differences between the comparison of hierarchies with the same number of players and with different number of players. The gray curves characterize the boundaries for the first hierarchy (with 4 players), while the blue graphs display those of the second hierarchy. The lighter curves are the boundaries between two and T periods, while the darker curves are the first boundary. This means that below the dark gray (blue) curve the first (second) hierarchy needs one period to centralize the information, between the dark and the light curve it takes two periods and above the light one, T periods.

We have fixed the deadline as $T = 4$. In the first hierarchy we have 4 players with $x_4 = 1$ and $x_3 = 2$. On the left hand the second hierarchy also has four players, but they have more valuable information with $y_4 = 2$ and $y_3 = 3$. We see that in the left figure the lines just intersect for $R = 0$. For all values of R the areas stay the same: Below the dark gray line both hierarchies need one period, between the dark gray and dark blue the first hierarchy takes two periods, while the second still centralizes the information in one period. In the area between the dark blue and the light gray curves both take two periods, between the light gray and light blue the first hierarchy needs T periods, while the second just takes two periods and in the area above the light blue graph none of the two hierarchies centralizes the information before T .

So even with a change in R , as long as the information distribution and the deadline stay constant these areas just shift, but none vanishes and no new area is created.

This is different if we change the number of players in the second hierarchy to 10. For values of R below 4 we still have the same areas as before, but then the dark gray and dark blue curves cut and create a new area: In the area that is bound by the dark blue curve from below and by the dark gray function from above the first hierarchy finishes in one period, while the second needs two periods. In this example we see that the comparison of two structures with a different number of players depends more on the parameters, in this case on R .

If we instead would change the number of players in the first hierarchy from 4 to 10, the dark gray curve would shift downwards, but no other changes will occur. The remaining analysis for figure 7 (b) is the same as for (a), because the second boundary does not depend on the number of players.

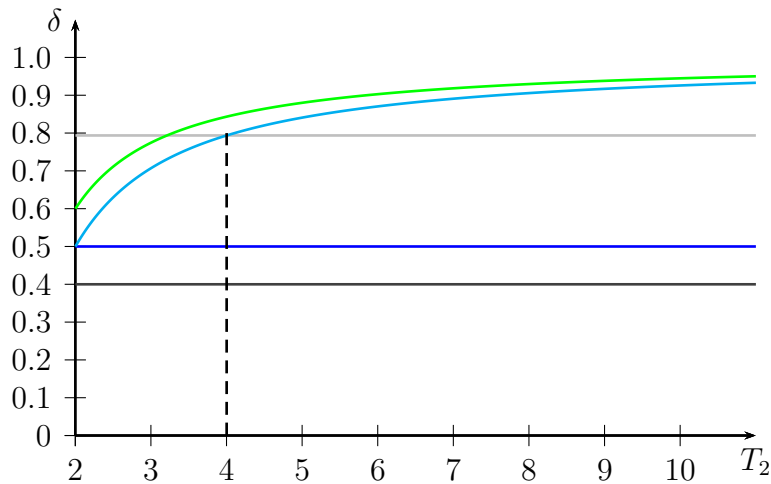


Figure 8: Duration of two different hierarchies depending on T_2 and δ

Figure 8 displays the impact of the deadline on the intervals for the SPNE: Again we compare two hierarchies. We have fixed $R = 4$. The boundaries for the SPNE of the first hierarchy are shown by the gray lines. In this setting the hierarchy contains 4 players with $x_n = 1$ and

$x_{n-1} = 2$ and has a deadline of $T_1 = 4$. Below the dark gray curve all players pass in $t = 1$, so the duration is one period. Between the dark and light gray lines the duration is two periods and above the light gray it is $T_1 = 4$.

The second structure also has four players and also the same reward, but the least informed player has information of value $y_n = 2$. The deadline is T_2 which is displayed on the x-Axis. Similar to the first hierarchy below the dark (blue) line the information centralization takes only a single period, above it takes two periods at least. The light blue curve shows the boundary between two and T_2 periods for $y_{n-1} = 2$, below the duration of the game in the second hierarchy is two periods, above it takes T_2 rounds. We see that for $T_2 = 4 = T_1$ the values of both hierarchies for the upper bound are the same. For lower values of T the boundary for the second hierarchy is lower, for larger values it is larger.

For a small deadline, e.g. $T_2 = 3$ there is an area where the second structure centralizes the information slower than the first one, while for large values the centralization works faster. The green curve is a simple shift from $y_{n-1} = 2$ to $y_{n-1} = 3$. We observe that the curve shifts upwards as we have already noticed in the figures before.

3 Multi layer model

In this Section we extend our model to allow for more than just two layers of hierarchy. Coming from the two layer model, we can use the results to analyze larger hierarchies with several layers. The main difficulty we face in this part is that those players who are the principals for some other players, also have to pass the information. We assume that they can only do it as soon as they have collected all information from their predecessors. Let the set of total players in the hierarchy be N , with $|N| = n \geq 2$. Each player $i \in N$ has some first period when he can pass, we denote this entry time by t_i . For the players who are leafs it is obviously that $t_i = 1$ holds and they can pass the information from the first period on. If player j collects all information in period τ he can pass them in the next period, i.e. $t_j = \tau + 1$. We assume that the entry times are public knowledge. If the players would not know if there is a competitor already in the game, or if one will enter later, there is more uncertainty. This type of uncertainty is analyzed in Bobtcheff et al. (2016).

A player who is not a leaf can either have some information from the beginning or not. In both cases we simply add the value of information he gets from his agents to the value of information he has initially. By that the players on a higher level of the hierarchy have higher value of information than the lower players.

Each hierarchy with more than two layers, consists of different sub-hierarchies. By $L = \{1, \dots, \ell\}$ we denote the set of two-layer sub-hierarchies. We start the numbering with the lowest levels and then move upwards. So sub-hierarchy 1 always consists of some of the play-

ers who are leaves in the complete structure, while sub-hierarchy ℓ is always the one at the top. Each sub-hierarchy $h \in L$ has some deadline T_h and reward R_h . If a branch of the entire structure has the deadline T_h , we assume that the structure containing this branch has a deadline $T' > T_h + 1$. This assumption holds for all branches and ensure that each player has some time left to pass the information after receiving them. The more interesting cases arise if we assume that for each layer of hierarchy we add at least three more periods.

Let N_h be the set of players in one sub-hierarchy, by $N_h^t \subseteq N_h$ we denote the set of players in this branch who have already collected all their information, i.e. $N_h^t = \{i \in N_h \mid t_i \leq t\}$. For the competition only the players who have not passed their information yet are important. Let $M_h^t \subseteq N_h^t$ be the set of players who have centralized all information from their predecessors and have not passed to their successor yet. If $a_i^t \in \{P, H\}$ is the strategy of player i , then we can write M_h^t as $M_h^t = \{i \in N_h^t \mid a_i^\tau = H \forall \tau \in \{t_i, t - 1\}\}$.

In contrast to the two-layer model we do not sort player primary based on their value of information, but on their entry time. Player 1 of a sub-hierarchy is the player who has the lowest entry time, while player n enters last. If two or more players have the same entry time we sort those players by value of information as before. By this we still get the same order for the two-layer model. In the situation where several players have an incentive to pass the information, we still assume that the players with the more valuable information pass the information. If there are more players with the same value, we select those to pass the information, with a smaller entry time.

We can repeat some Remarks from the two layer model:

Remarks.

- *All players will pass their information at one point.*
- *In $t = T_h$ all the players from sub-hierarchy h , who have not passed their information will pass.*
- *As soon as there is only one player left in a sub-hierarchy, who has not passed and no other player will enter afterwards, this player will pass immediately.*
- *In equilibrium no player will hold his information first and then pass it later without getting a share of the reward.*

For the hierarchies where all the players enter in with the beginning of the game, i.e. $t_n = 1$, the boundaries of the SPNE are the same as before. Only for the higher layer hierarchies we have to take into account those players who enter later.

3.1 SPNE in higher layers

The last player who enters into a sub-hierarchy is player n , he enters at time t_n . With just some minor adjustments of Proposition 3, we get the SPNE in period $t = t_n$. One of these adjustments

comes with the different entry times.

Corollary 3 (SPNE in a sub-hierarchy at $t = t_n$).

Let denote the value of information of player $i \in N_h$ by x_i .

The Subgame Perfect Nash Equilibria at $t = t_n$ depend on δ .

1. All players pass in t if and only if $\forall i \in M_h^t : \delta < \frac{|M_h^t| \cdot x_i + R_h}{|M_h^t| \cdot (x_i + R_h)}$ holds.
2. A single player holds in t if and only if $\exists j : \delta > \frac{|M_h^t| \cdot x_j + R_h}{|M_h^t| \cdot (x_j + R_h)}$ and
 $\forall i \neq j : \delta < \left(\frac{2 \cdot x_i}{2 \cdot x_i + R_h} \right)^{\frac{1}{T_h - t}}$.
3. $k \in \{2, |M_h^t| - 1\}$ players hold in t and pass in T_h if and only if $\exists J \subset M_h^t$ with $|J| = k$
and $\forall j \in J : \delta > \left(\frac{k \cdot x_j}{k \cdot x_j + R_h} \right)^{\frac{1}{T_h - t}}$ and $\forall i \in M_h^t \setminus J : \delta < \left(\frac{(k+1) \cdot x_i}{(k+1) \cdot x_i + R_h} \right)^{\frac{1}{T_h - t}}$.
4. All players hold in t and pass in T_h if and only if $\forall i \in M_h^t : \delta > \left(\frac{|M_h^t| \cdot x_i}{|M_h^t| \cdot x_i + R_h} \right)^{\frac{1}{T_h - t}}$
holds.

These changes in the conditions are necessary to adjust to the fact that the players may enter at different times and we do not sort the players by their value of information any longer. In case 2 of the Corollary it is the least informed player that holds the information, i.e. j is such that $x_j \leq x_i \forall i \neq j$ and $\forall i \neq j$ with $x_j = x_i, t_j \leq t_i$ holds. Similar in case 3: $\forall i \in M_h^t \setminus J$ and $\forall j \in J : x_i \leq x_j$ holds and $\forall i \in M_h^t \setminus J$ with $\exists j \in J$ such that $x_i = x_j; t_j \leq t_i$ holds.

This corollary just defines the remaining players' behavior starting from period t_n , but we also want to characterize the players behavior before. The problem that arises for the higher layers is the following: There exist two different behaviors of players: There are players who enter in one period and pass as soon as possible without getting a share of the reward and those who wait to get the reward. The game tree in Figure 9 shows the choices of player 1, where d, e, f and g stand for different duration and i, j, k and l for the different number of players who share the reward.

As we can see in Figure 9, only if player 1 decides to pass in the period he enters, his payoff is not affected by the decision of the other players. If he decides to hold, he will hold until the last period. Depending on the decision of the other players (just player 2 in the figure) and the equilibrium, player 1 will get a different utility. Still the game is solvable by backward induction, where the equilibrium depends on δ and follows the assumptions we made.

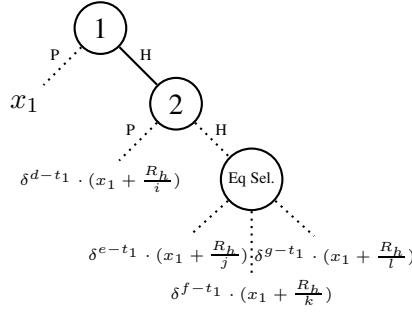


Figure 9: Example of a decision tree

3.2 Comparison Examples

In this part we compare different hierarchical structures. The main question we want to address is "which hierarchy centralizes the information fastest?". We will show that the answer to this questions depends on the parameters. Even the change of a single parameter will change in which hierarchical structure the information is centralized fastest. To show that we make the following simplifications: We assume that for all sub-hierarchies the reward R and deadline T are the same, furthermore the value of information of the least informed player is set to 1, for the second-least informed player to 2 and is increasing in steps of 1 for the remaining players. The smallest example we can start with, is a model with only three players. There are only three different ways how to arrange three informed players, while satisfying the assumptions we made so far. With the introduction of a fourth informed player, there are already 25 possible hierarchical-structures, which we compare in the second part.

3 Players

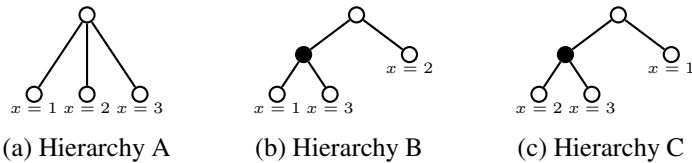


Figure 10: All possible multi-layer hierarchies for 3 players

Figure 10 shows the three different ways how to arrange the three informed players in a hierarchy. While in the flat hierarchy the positions of all players are the same, it is different if we install one uninformed intermediary (filled node). Then the only feasible possibilities are that the players with value of information 1 and 3 or those with information 2 and 3 pass their items to that intermediary. It is not possible to let the players 1 and 2 be in that part of the hierarchy, because this would contradict our assumptions we made on R in Section 2.

We assume that the total deadline for the hierarchy is $2 \cdot T$ and in the hierarchies B and C the deadline for the sub-hierarchy is T .

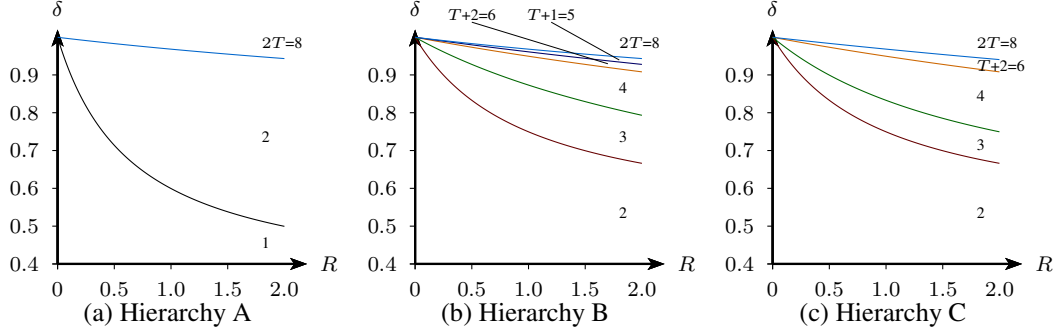


Figure 11: $T = 4$

In the figure above we can see the duration of the different hierarchies, depending on δ and R . For the flat hierarchy the only possible values are one period, two periods and $2T$ periods. As soon as there are three layers the duration can be either two, three, four, $T + 1$, $T + 2$ or $2T$ periods. While the first three values are just possible if the sub-hierarchy works fast, $T + 1$ and $T + 2$ just occur if the sub-hierarchy has a duration of T periods.

In Figure 11(b) we can observe something that has never happened in the two-layer model: An increase of δ or R yields to a decrease of the duration. Above the orange line the lower hierarchy needs T periods to centralize the information. Below the dark-blue curve the player with $x = 2$ will wait and then get the reward for himself in period $T + 2$, but above the dark-blue curve the intermediary, which then has a value of information equal to 4, will also hold. This would lead to a duration of $2T$ periods. If δ and R are not high enough, so in this example below the light-blue curve, the player with $x = 2$ decides not to wait for his opponent to centralize the information, but to pass in the very first period. Then the intermediary faces no competition when he finally has centralized the information and passes immediately, yielding to a duration of only $T + 1$ periods. In other words between the two blue curves the incentive to wait for the player $x = 2$ is not large enough.

Comparing the different structures leads to an interesting insight: There are no values for δ and R for which Hierarchy B is faster than Hierarchy A. This is quite obvious for the parameters, where Hierarchy A needs less than $2T$ periods. For both, Hierarchy A and B, the change to $2T$ periods (the light-blue curve) is defined by $\left(\frac{4.0}{4.0+R}\right)^{\frac{1}{7}}$.

Still Hierarchy A is not always faster than Hierarchy C, because for Hierarchy C the light-blue curve is defined by $\left(\frac{10}{10+R}\right)^{\frac{1}{3}}$, which is larger than $\left(\frac{4.0}{4.0+R}\right)^{\frac{1}{7}}$. This means in the interval $\left(\left(\frac{4.0}{4.0+R}\right)^{\frac{1}{7}}, \left(\frac{10}{10+R}\right)^{\frac{1}{3}}\right)$, the duration of Hierarchy C is less than the duration of the flat hierarchy.

4 Players

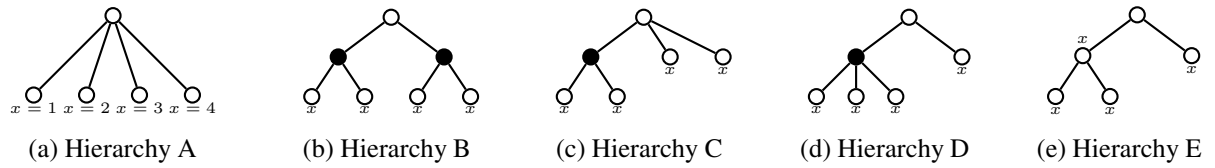


Figure 12: All possible multi-layer hierarchies for 4 players

In Figure 12 we see the different hierarchical structures that are possible for four players. Starting from Hierarchy B it makes a difference which player has which value of information. If we add all the different possibilities we end up with 25 different possibilities. In Hierarchy E all four players have information from the first period, this means that the intermediary is informed. In the Hierarchies B, C and D only the leafs have information, but the intermediaries (filled nodes) are uninformed.

With the value of information of the players as 1, 2, 3 and 4, there are two feasible possibilities to arrange the players in Hierarchy B, six in Hierarchy C, four in Hierarchy D and 12 in Hierarchy E. Note that in Hierarchy B it is not possible that the players with $x = 1$ and $x = 4$ report to the same intermediate, because of the assumption we made about R . For some of these possibilities there is an easy way to exchange two players, while keeping the structure, to speed up the centralization. By that we can already delete 10 cases which are never faster than others. The following 15 cases remain:

- 1) Hierarchy A
- Hierarchy B with:
 - 2) $x = 1$ and $x = 2$ reporting to the same intermediary
 - 3) $x = 1$ and $x = 3$ reporting to the same intermediary
- Hierarchy C with:
 - 4) $x = 1$ and $x = 2$ reporting to the intermediary
 - 5) $x = 1$ and $x = 3$ reporting to the intermediary
 - 6) $x = 1$ and $x = 4$ reporting to the intermediary
 - 7) $x = 2$ and $x = 4$ reporting to the intermediary
 - 8) $x = 3$ and $x = 4$ reporting to the intermediary
- Hierarchy D with:
 - 9) $x = 4$ not reporting to the intermediary
 - 10) $x = 2$ not reporting to the intermediary
 - 11) $x = 1$ not reporting to the intermediary

- Hierarchy E with:

- 12) $x = 2$ and $x = 3$ reporting to intermediary $x = 1$
- 13) $x = 2$ and $x = 4$ reporting to intermediary $x = 1$
- 14) $x = 3$ and $x = 4$ reporting to intermediary $x = 1$
- 15) $x = 3$ and $x = 4$ reporting to intermediary $x = 2$

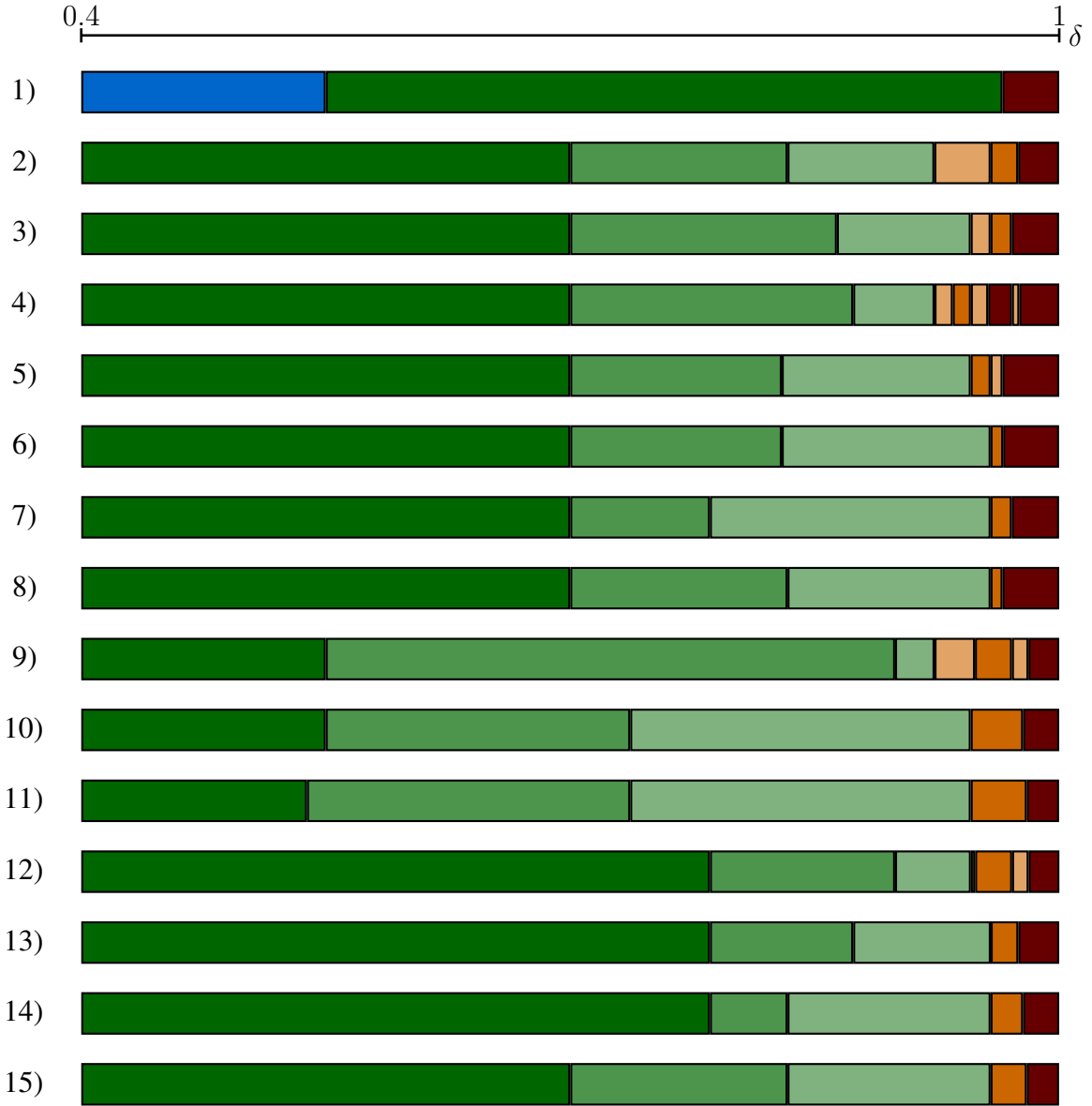


Figure 13: Duration of all 15 cases with $T = 5$ and $R = \frac{3}{2}$.

Colors: 1 period, 2 periods, 3 periods, 4 periods, $T+1$ periods, $T+2$ periods, $2T$ periods

Figure 13 shows the duration from all those 15 cases, depending on δ . The jumps from one duration to another differ between the cases. We show only the interval $(0.4, 1)$, because the duration for lower values of δ is the same as for 0.4 for all hierarchies.

For low values of δ the duration of each hierarchy is low, until it increases at certain boundaries.

For the non-flat structures the lowest duration is 2 periods, then it increases to 3 and then to 4. While for some hierarchies there is a jump directly from 4 to $T + 2$ periods, in others there is a region with a duration of $T + 1$ periods in between. We can also see that in the asymmetric hierarchies the same effect occurs as in the three player example: With an increase in δ the duration decreases to $T + 1$. This effect happens in case 4), 5), 9) and 12) and again is caused by the fact that one of the players without intermediary has no incentive to wait and to get into the competition with the intermediary. In Hierarchy C, where there are two players directly connected to the principal these jumps can even happen twice, as we can see in 4).

If we compare all these 15 cases we see that some cases are weakly dominated: 2), 3), 6), 7) and 8) are never faster than 13). The structures 4), 9), 10) and 11) are weakly dominated by 12). As we may argue that a flat hierarchy should not be feasible, we ignore the dominance from case 1). By that the remaining cases are 1), 5), 12), 13), 14) and 15).

In most applications it seems unreasonable to assume that all players can pass directly to the principal. For example in the political case, there will be no system where each city reports directly to the administration of the whole country. We can observe this for example in Germany, where most cities or rural districts are under the administration of a state. The exceptions are the areas of Hamburg, Bremen and Berlin, which are not only cities, but also states. We can find such asymmetries fitting for our Hierarchies C, D and E.

Also in the work of Radner (1993) and Jehiel (1999), the flat hierarchy is avoided. While Radner models it with a maximal capacity for each player, Jehiel introduces a probability of information loss for this case. Our way is similar to Radner, as we do not consider hierarchies in which many agents report to one player. In the example with only three players, we already mentioned that there exists a region of δ where the flat hierarchy is not the one with the smallest duration. The same holds with four players as we can see in the figure above.

For the remainder of this example, we will focus on the hierarchies with three layers.

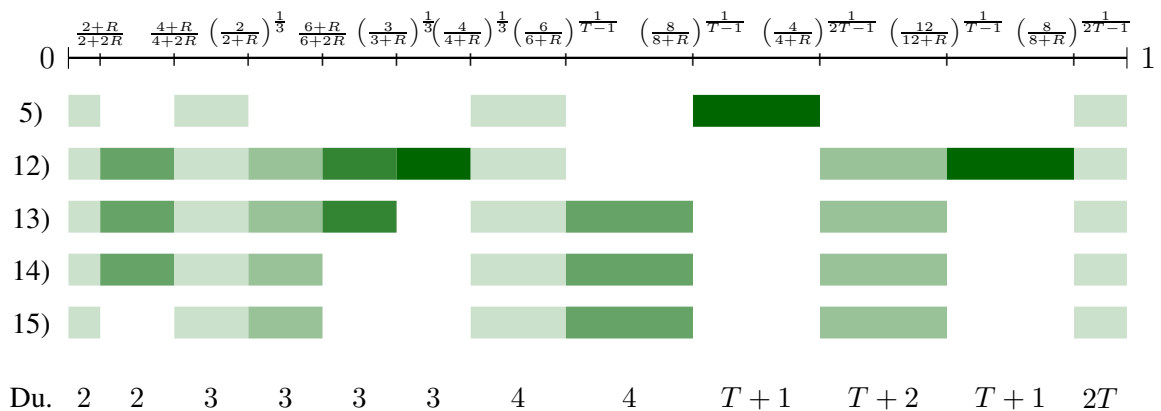


Figure 14: Duration of the remaining five cases with $T = 5$ and $R = \frac{3}{2}$

Figure 14 shows the intervals in which the remaining five hierarchical structures centralize the

information fastest. Below it is written which is the shortest duration possible for that interval. While a dark-green bar means the hierarchy is the unique fastest one, lighter shades show the intervals in which other hierarchies are also the fastest. We want to point out that in Figure 14 we modified the δ -Axis such that all intervals are clearly visible. The real sizes of the intervals do not correspond to the size shown in the figure.

It is important to point out that we do not make any statement about the duration of a hierarchy in an interval where it is not the fastest. For example in the second last interval the duration of the hierarchies 5), 13), 14) and 15) differs between $T + 2$ and $2T$ periods.

From this figure we can clearly see that only the hierarchies 5) and 12) have some intervals in which this hierarchy is the unique fastest to centralize. Furthermore whenever the hierarchies 15) is the fastest, so is 14) and the same holds for 14) compared to 13). This means that for all intervals in which 14) or 15) have the shortest duration, so does the hierarchy from case 13). The reason why we did not delete 14) and 15) before is that they dominate 13) for some values, but for these values 5) or 12) are even faster. Since we just focus on the fastest hierarchies, we can neglect 14) and 15) for the rest of the analysis. The remaining three structures are the following:

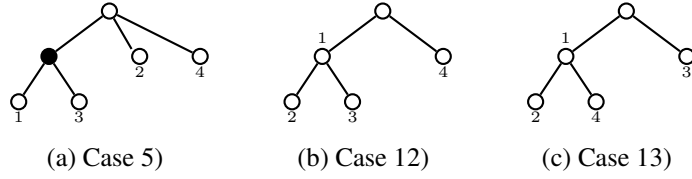


Figure 15: Fastest three-layer hierarchies for 4 players

For most values of δ the hierarchy displayed in case 12) (b) has either the shortest duration or shares the shortest duration with another hierarchy. Just for the interval $\left(\left(\frac{6}{6+R} \right)^{\frac{1}{T-1}}, \left(\frac{8}{8+R} \right)^{\frac{1}{T-1}} \right)$ hierarchy 13) is faster, and in $\left(\left(\frac{8}{8+R} \right)^{\frac{1}{T-1}}, \left(\frac{4}{4+R} \right)^{\frac{1}{2T-1}} \right)$ hierarchy 5) is the fastest.

The reasons for this are as follows: Hierarchy 13) is faster than 12), because in that specific interval the sub-hierarchy of case 13) still needs just two periods, while the sub-hierarchy in case 12) already needs T periods. This is simply caused by the fact that in case 13) the second player in the sub-hierarchy has a value of information $x = 4$, while it is only $x = 3$ in case 12). The dominance of case 5) for the interval defined above is due to the different structure. In this interval the sub-hierarchy of case 5) needs T periods, but the players connected directly to the principal ($x = 2$ and $x = 4$) pass their information already in the first period, so that the intermediary faces no competition once he received the information. So the duration is only $T + 1$ periods. In the other two remaining hierarchies for those values of δ the one player who reports directly to the principal still would wait, which leads to a duration of $T + 2$ periods.

From this example we can learn that there is always an interval in which the flat hierarchy is not optimal and if we ban the flat hierarchy in different asymmetric hierarchies the information is centralized fastest, depending on δ .

3.3 General results

The previous examples have shown that the hierarchical structure plays an important role in the speed of centralization. In this part, we will give some more general results. We will show that some hierarchies can never be optimal and give more detailed analysis for a special case.

Proposition 5 (Uninformed Intermediary).

Any hierarchy that contains a sub-hierarchy with an uninformed intermediary and at least three players directly connected to this intermediary is weakly dominated.

Proposition 6 (Informed Intermediary).

In a sub-hierarchy with one intermediary and $n - 1 \geq 2$ players who pass to this intermediary, the only non-dominated sub-hierarchies are those where either the least informed player or the second-least informed player is the intermediary.

These two propositions already permit to exclude several structures for our further analysis. We can neglect all hierarchies which have a sub-hierarchy with an uninformed intermediary who has at least three agents reporting to him. The simple reason for this is that we can get the same or a smaller duration by installing one of these agents as the (informed) intermediary. We have already seen in the four player example that Hierarchy D, which contains these kind of sub-hierarchy, is always dominated.

On the other hand we know that the centralization of information works fastest when the intermediary has only little information. Let us compare a sub-hierarchy with an informed intermediary, which has more information than two of his agents to those sub-hierarchies where one of the least informed players is the intermediary. Taking the least informed player as an intermediary yields to the result that the boundaries (for δ) for a duration of one and two periods both shift upwards and by that make the process faster, because both x_{n-1} and x_n of the sub-hierarchy increase. If we take the second-least informed player instead, it shifts only the boundary between 2 and T periods upwards, but by that also improves the speed. The arguments for those are the same as in Section 2.4. A more detailed analysis can be found in the proofs of Proposition 5 and 6. It is important to note that the structures described in those propositions still can be faster than other, completely different hierarchies, but they are never the unique fastest.

Impact of the parameters

Figure 14 shows two main differences between the two-layer model and the multi-layer model. Even though it is just the example with four players, we see that the duration is no longer increasing in δ . Furthermore the boundaries of the different intervals are more complex than before. This makes it impossible to state general results on the impact of an increase of δ and R on the duration. We still can make statements about the impact of the number of players and the information distribution.

If we add a well informed player to a sub-hierarchy, the duration of this sub-hierarchy does not increase, but may decrease. The reason for this is the same as before: The addition of a well informed player, i.e. $x_i > x_{n-1} > x_n$ is equivalent to an increase in the number of players n . By that the boundary for the duration between one and two periods gets shifted upwards, which decreases the duration for certain values of δ . Similarly, an increase in x_{n-1} or x_n shifts the boundaries such that the duration decreases for a specific interval of δ , while it stays the same for the remaining values.

Simplification

Our model allows for more results if we simplify the reward R and the deadline T . For this part of our analysis we do the same as in the examples: We focus on the same reward R for all hierarchies, i.e. $R_h = R$ and on the same deadline for all sub-hierarchies. This means that the lowest hierarchies have a deadline of T periods, the ones that include this sub-hierarchy have $2T$ periods and so on. By this the entire hierarchy has a deadline equal to $(\#layers-1) \cdot T$.

Proposition 7.

Assume that the least informed player in the highest-level hierarchy has more valuable information than the second-least informed player of any other hierarchy.

If the highest-level hierarchy needs the entire deadline when all agents of this hierarchy arrive at the same time, then each sub-hierarchy with at least two players needs their entire sub-deadline.

Proposition 8.

The total duration is minimal (equal to $(\#layers-1)$) if the following three conditions are true:

- 1. The lowest level hierarchies have a duration of just a single period*
- 2. The number of players is non-increasing from bottom to top*
- 3. Each asymmetric linked player has at least the value of information as the least informed player in the lowest level of the hierarchy.*

These two results give us a possibility to check for the entire duration of a hierarchy just by checking certain sub-hierarchies. If the highest part needs the entire deadline, we know that each sub-hierarchy needs the entire time available. If, on the other hand, the lowest levels just have a duration of one period, we get that the duration of the entire structure is minimal.

4 Extensions

In the previous sections we have created a basic model for information transmission in hierarchies. While there may be several ways to extend this model, we want to discuss two important extensions in this section. The first extension relaxes the assumption we made on the reward R . In the second extension we introduce shared information.

4.1 Extension 1: Unbound reward

For the entire paper we were assuming that the reward R is not larger than $n \cdot (x_{n-1} - x_n) + (n - 2) \cdot x_{n-1}$. This assumption ensures that even in a setting with just two periods, all different types of SPNE exist, i.e. $\frac{n \cdot x_n + R}{n \cdot (x_n + R)} < \frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}$ holds. If we relax this assumption and select a larger reward, this is no longer true. The equilibrium in each period and the SPNE will no longer be unique, unless we specify an equilibrium selection rule.

The only equilibrium that can coexist with another equilibrium for the same values of the parameters is the equilibrium in which all players pass in the first period. At the same time there is no more interval in which just one player holds his information in the first period. For higher values of δ , but which are below $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$, there will be two equilibria: On the one hand the same as before, where some players hold their information in the first period, and in addition the equilibrium in which all players pass in $t = 1$. If R is very large and if the discount factor is close to 1, there can even be a region in which there is an equilibrium with all players holding in the first period and at the same time an equilibrium with all players passing in the first period. Naturally two different ways of equilibrium selection exist. We can either always select the equilibrium where more players hold or where more players pass. In the first case the analysis stays quite similar to the one we did with the assumption on R . The only difference is that in a two-layer model the duration changes directly from a single period to T periods, but there is no interval for δ with a duration of 2 periods.

This is not the case if we always select the equilibrium in which all players pass. For values of δ larger than $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$, some players will hold their information in the first period. In this new setting Proposition 2 no longer holds. This means that even if two or more players hold their information in $t = 1$, they will not necessarily wait until T to pass. In this case we get additional conditions for the SPNE, which describe the players behavior starting from the second period. Furthermore, the duration not necessarily will be strictly increasing in δ .

We also need similar conditions for other equilibrium selection methods, as long as not always the equilibrium where most players hold gets selected.

4.2 Extension 2: Shared information

While in some examples the uniqueness of the information of the different players comes naturally, one can also argue that in some cases there should be shared information. A researcher might propose the same idea for different projects, or a mathematical proof gives a stronger result than necessary.

For simplification we will focus on a two-layer model again. We include the possibility that two players have some shared information in addition to their unique information. Only the player who passes the information first gets the payoff according to the value of the shared information.

We assume that if both players pass at the same time, they split the payoff equally. In Section 2.4 we have described the impact of an increase in the value of information on the duration. We have seen that only if the value of one of the two least informed players is changed, the duration is affected. In this situation the result will be similar. If at least one of the players $n - 1$ and n has shared information, the duration may decrease. Still the effect is different than an increase in x_n (or x_{n-1}), because the players do not get this additional payoff for sure. We still assume that if several players have an incentive to pass the information, the players with the more valuable unique information pass first.

Let player j and k have the shared information with value $y > 0$. Without loss of generality we can assume that $x_j \geq x_k$ holds. To have a duration of a single period all players need to prefer passing over holding in the first period, i.e.

$$\begin{aligned} \forall i \in N \setminus \{j, k\} : \quad & x_i + \frac{R}{n} > \delta(x_i + R) \\ \Leftrightarrow & \delta < \frac{n \cdot x_i + R}{n \cdot x_i + R} \end{aligned}$$

$$\begin{aligned} \forall i \in \{j, k\} : \quad & x_i + \frac{R}{n} + \frac{y}{2} > \delta(x_i + R) \\ \Leftrightarrow & \delta < \frac{n \cdot x_i + R}{n \cdot x_i + R} + \frac{y}{2 \cdot (x_i + R)} \end{aligned}$$

We can see that this boundary is not changed if it is not the least informed player n , who has the additional shared information. In a similar way we can also get the other boundaries, which result in five cases. In Figure 16 we can see how the shared information changes the duration. If the shared information is split between two players that are both not the least or second least informed player, there is no change at all. This benchmark is shown in the first case.

In the second case the second least informed player has the shared information. This increases the upper boundary of the interval with a duration of two periods. So there exists an interval in which the information is centralized in two periods, while in the benchmark it takes T periods.

The boundary does not only depend on x_{n-1} , but also on x_{n-2} . Even in the benchmark $\delta < \left(\frac{2 \cdot x_{n-2}}{2 \cdot x_{n-2} + R}\right)^{\frac{1}{T-1}}$ has to hold, but as we described in Section 2, this is always satisfied for $\delta < \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}$.

In the third case the third least informed player also has the shared information, so the boundary between a duration of 2 and T periods depends on x_{n-3} . This condition ensures that the players 1 to $n - 3$ prefer to hold and will not deviate. In some cases the boundary between 2 and T periods will be the same for the second and third case, there is also the possibility that the boundary is higher in the third case.

The change in the fourth case is different. In that case the least informed player has shared information, while the second least informed player does not. This shifts up the upper bound for the interval in which the duration is one period. Both boundaries increase if the least and the

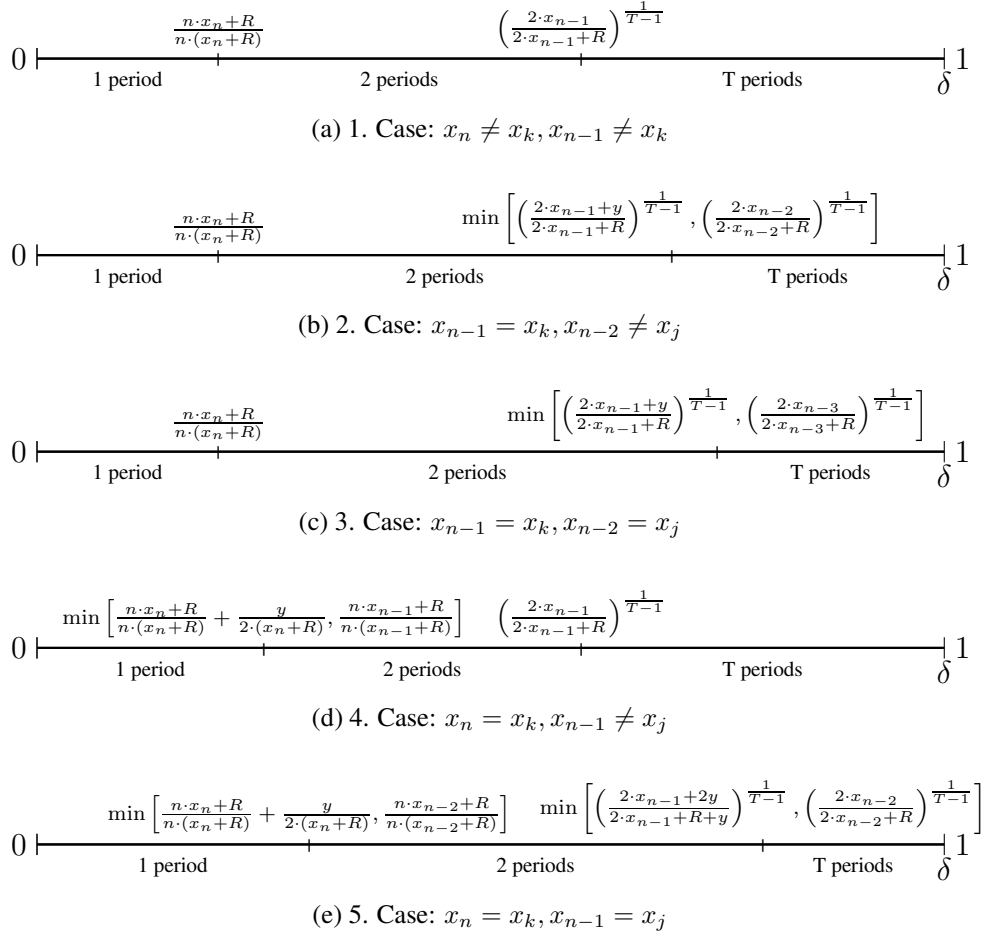


Figure 16: Duration in case of shared information

second least informed player share the information. In that case the increase of the boundary between 2 and T periods is stronger than in the second case. Whether the increase in the third or fifth case is stronger depends on several variables. The first boundary of the fifth case can also be larger than that of the fourth case.

We can see that as soon as at least one of the players n and $n - 1$ has shared information, the boundaries change. We still need the conditions to hold for all other players, so we use the minimum of both conditions.

These changes are different than those we did in Section 2.4. If we just increase x_n or x_{n-1} , we observe a different change. A comparison of these changes shows that for some values of the parameters the duration decreases more if the players have shared information, while for other values the duration decreases stronger if just one of the players has additional information.

5 Conclusion

In this paper we have analyzed a model of hierarchies in which information flows from the leafs to the root. Each player has unique information and faces the problem to transmit this information to his predecessor at the best time. While late passing comes with the disadvantage of the discounted value of information the last player to pass gets an additional payoff.

For the two-layer model we have shown that the information in a hierarchy with two layers is always centralized in either one, two or T periods. The parameters such as the number of players, the additional reward, the deadline and the information distribution all have different effects on the duration. The most surprising result may be that an increase of the deadline T can lower the duration from T to two periods. Furthermore, we have compared different hierarchies in combination with a different information distribution and we have visualized the importance of the number of players and different deadlines for the duration of two hierarchies.

In a hierarchy with several layers we have proposed a model in which the players can only pass their information once they have collected all information from their agents. The players' problem of timing is more complex, because they have to take into account that other players may centralize their information later and then enter the game. For the period when all players have entered, we have given conditions for all SPNE, for the players who have entered the game before, we have stated the problem they face.

We have compared all different hierarchical structures for three and four players and have shown that not only the parameters have an impact on the duration, but also that the structure and the information distribution play an important role. Even in a general multi-layer model, some hierarchies and some arrangements of players can never be optimal, while the dominance of the remaining possibilities depends highly on the discount factor and the reward.

In the entire paper we make one crucial assumption, which is the upper limit on the reward R . In one of the extensions we have seen that weakening this assumption creates intervals for δ in which the equilibrium is not unique. Defining an equilibrium selection rule or a mechanism and then studying the model again, can be a nice first step to a more general model. A similar change would arise if we allow the players to have different discount factors.

We have already shown the impact of shared information between two players on the duration in a two-layer model. This can be extended even more, either by letting different pairs of players share information, or even by shared information between more than two players.

Other extension possibilities can be found easily by slight modifications of the utility function: Can we replicate the results if the reward is not split equally, but according to the value of information? How do the results change if the reward is depending on the value of information? These questions should be answered to analyze the behavior of players who have some kind of fixed wage and a variable wage, depending on their work.

Appendix

Proof of Proposition 1.

Player j 's utility is given by:

$$\begin{aligned}
 u_j(\text{ Pass } \mid \text{ all other players pass }) &= x_j + \frac{R}{n} \\
 u_j(\text{ Hold } \mid \text{ all other players pass }) &= \delta \cdot (x_j + R) \\
 u_j(\text{ Pass } \mid i \text{ players hold }) &= x_j \\
 u_j(\text{ Hold } \mid i \text{ players hold }) &= \delta \cdot (x_j + \frac{R}{i+1}) \\
 u_j(\text{ Pass } \mid \text{ all other players hold }) &= x_j \\
 u_j(\text{ Hold } \mid \text{ all other players hold }) &= \delta \cdot (x_j + \frac{R}{n})
 \end{aligned}$$

1) This implies that all players prefer to pass over hold if and only if $\forall j \in \{1, \dots, n\}$:

$$\begin{aligned}
 x_j + \frac{R}{n} &> \delta \cdot (x_j + R) \\
 \Leftrightarrow \delta &< \frac{n \cdot x_j + R}{n \cdot (x_j + R)}
 \end{aligned}$$

With $x_1 \geq x_2 \geq \dots \geq x_n$ we get $\delta < \frac{n \cdot x_n + R}{n \cdot (x_n + R)}$

2) The players 1 to $n - 1$ prefer to pass and player n prefers to hold if and only if $\forall j \in \{1, \dots, n - 1\}$:

$$\begin{aligned}
 x_j &> \delta \cdot (x_j + \frac{R}{2}) \\
 \Leftrightarrow \delta &< \frac{2 \cdot x_j}{2 \cdot x_j + R}
 \end{aligned}$$

and for player n we get the opposite result for 1). Then with $x_1 \geq x_2 \geq \dots \geq x_n$ we get the interval of δ .

3) The players 1 to $n - i$ prefer to pass, while the players $n + 1 - i$ to n prefer to hold if and only if:

$\forall j \in \{1, \dots, n - i\}$:

$$\begin{aligned}
 x_j &> \delta \cdot (x_j + \frac{R}{i+1}) \\
 \Leftrightarrow \delta &< \frac{(i+1) \cdot x_j}{(i+1) \cdot x_j + R}
 \end{aligned}$$

and $\forall j \in \{n + 1 - i, \dots, n\}$:

$$\begin{aligned}
 \delta \cdot (x_j + \frac{R}{i}) &> x_j \\
 \Leftrightarrow \delta &> \frac{i \cdot x_j}{i \cdot x_j + R}
 \end{aligned}$$

With $x_1 \geq x_2 \geq \dots \geq x_n$ we get $\delta \in \left(\frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}, \frac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R} \right)$

4) All players prefer to hold if and only if $\forall j \in \{1, \dots, n\}$:

$$\begin{aligned} \delta \cdot \left(x_j + \frac{R}{n}\right) &> x_j \\ \Leftrightarrow \delta &> \frac{n \cdot x_j}{n \cdot x_j + R} \end{aligned}$$

With $x_1 \geq x_2 \geq \dots \geq x_n$ we get $\delta > \frac{n \cdot x_1}{n \cdot x_1 + R}$ □

Proof of Proposition 2.

Let us first assume that there are only 3 periods, i.e. $T = 3$.

If in $t = 2$ there are i players left, the discount factor has to be such that all pass. As there is only one period left the SPNE are as in Proposition 1. To not have the equilibrium where all players hold their information we need $\delta < \frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}$.

In $t = 1$ we need that i players hold and $(n-i)$ players pass. In that situation the utility of player $n+1-i$ is $u_{n+1-i}(\text{H}) = \delta \cdot \left(x_{n+1-i} + \frac{R}{i}\right)$. If this player passes he gets $u_{n+1-i}(\text{P}) = x_{n+1-i}$. So the player prefers to hold if and only if $\delta \cdot \left(x_{n+1-i} + \frac{R}{i}\right) > x_{n+1-i}$ which is equivalent to $\delta > \frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}$ and contradicts the behavior in $t = 2$.

So it is impossible that the remaining players all pass before the deadline and by Remark 3 also no subset of players will pass, therefore all remaining players will wait until the last period to pass their information.

This also does not change for any other $T > 3$, because if we require that all the remaining i players pass in $t = 2$, they will do so in $t = 3$ as well. This generates the same utility for all i players if they all hold as in the setting with $T = 3$. □

Proof of Proposition 3.

The proof follows the same steps as the proof of Proposition 1, but we have to discount the utility for holding $T - 1$ times, because of Proposition 2. □

Proof of Proposition 4.

1) Obviously the interval is not affected by T .

2) The lower boundary stays unchanged. The exponent of the higher boundary decreases (between 0 and 1) and since the base is smaller than one, the whole term increases.

3) By the same idea of 2) both values increase. It remains to show that the difference between the upper and lower boundaries decreases: The size of the interval is

$$\frac{[(i+1) \cdot x_{n-i}]^{\frac{1}{T-1}} \cdot [i \cdot x_{n+1-i} + R]^{\frac{1}{T-1}} - [i \cdot x_{n+1-i}]^{\frac{1}{T-1}} \cdot [(i+1) \cdot x_{n-i} + R]^{\frac{1}{T-1}}}{[(i+1) \cdot x_{n-i} + R]^{\frac{1}{T-1}} \cdot [i \cdot x_{n+1-i} + R]^{\frac{1}{T-1}}}$$

We can rewrite that as $\frac{a^{\frac{1}{x}} - b^{\frac{1}{x}}}{c^{\frac{1}{x}}}$ with $c > a > b > 0$. Derivation yields

$$\frac{1}{c^{\frac{1}{x}} * x^2} \left[b^{\frac{1}{x}} \cdot (\ln(b) - \ln(c)) - a^{\frac{1}{x}} \cdot (\ln(a) - \ln(c)) \right]$$

It remains to show that this term is negative. Obviously $\frac{1}{c^{\frac{1}{x}} * x^2}$ is positive, so we need to show that

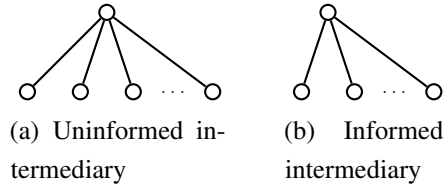
$$b^{\frac{1}{x}} \cdot (\ln(b) - \ln(c)) < a^{\frac{1}{x}} \cdot (\ln(a) - \ln(c))$$

holds. As $b < a$ implies $b^{\frac{1}{x}} < a^{\frac{1}{x}}$ and also $\ln(b) < \ln(a)$ we see that the inequality holds and the size of the interval is decreasing with T .

4) Obviously the lower boundary is increasing, which decreases the interval. □

Proof of Proposition 5.

We compare the following two structures:



In hierarchy (a) with the uninformed intermediary there are n agents connected to the intermediary, while in hierarchy (b) there are only $n - 1$ players, because we have selected one of the players to become the intermediary.

For simplification we sort the players by value of information, i.e. $x_1 \geq x_2 \geq \dots x_n > 0$. The duration for the left hierarchy is:

- 1 period if and only if $\delta < \frac{n \cdot x_n + R}{n \cdot (x_n + R)}$
- 2 periods if and only if $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)^{\frac{1}{T-1}}} \right)$
- T periods if and only if $\delta > \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)^{\frac{1}{T-1}}}$

If we take player $i \neq n, i \neq n - 1$ as the intermediary in hierarchy (b) we get:

- 1 period if and only if $\delta < \frac{(n-1) \cdot x_n + R}{(n-1) \cdot (x_n + R)}$
- 2 periods if and only if $\delta \in \left(\frac{(n-1) \cdot x_n + R}{(n-1) \cdot (x_n + R)}, \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)^{\frac{1}{T-1}}} \right)$
- T periods if and only if $\delta > \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)^{\frac{1}{T-1}}}$

Clearly the boundary between 1 and 2 periods is higher for hierarchy (b).

If we take player n or $n - 1$ as the intermediary also the other boundaries shift:

- 1 period if and only if $\delta < \frac{(n-1) \cdot x'_n + R}{(n-1) \cdot (x'_n + R)}$
- 2 periods if and only if $\delta \in \left(\frac{(n-1) \cdot x'_n + R}{(n-1) \cdot (x'_n + R)}, \frac{2 \cdot x'_{n-1}}{2 \cdot (x'_{n-1} + R)} \frac{1}{T-1} \right)$
- T periods if and only if $\delta > \frac{2 \cdot x'_{n-1}}{2 \cdot (x'_{n-1} + R)} \frac{1}{T-1}$

By x'_n and x'_{n-1} we denote the value of information of the two least informed players after removing the former player n or $n - 1$. If we take player n as the intermediary we have $x'_n = x_{n-1}$ and $x'_{n-1} = x_{n-2}$ and if we take player $n - 1$ we get $x'_n = x_n$ and $x'_{n-1} = x_{n-2}$. Obviously the two least informed players in hierarchy (b) have at least the same value of information as those in hierarchy (a). We have described this effect already in the first part of Section 2.4. With this reasoning we get that the boundaries shift upwards and so hierarchy (b) weakly dominates hierarchy (a). \square

Proof of Proposition 6.

As we have seen several times (e.g. in the previous proof) the duration of a hierarchy depends on the value of information of the two least informed players. If we remove one of these players, because we install him as the intermediary and add the previous intermediary who has more valuable information, it is beneficial for the duration. This step is equivalent to giving one of the two least informed players more information. \square

Proof of Proposition 7.

We assume there are k -levels of the hierarchy. Assume that the players of the highest level of the hierarchy all enter at time $\tau \leq (k - 2) \cdot T$. Then the total duration is $(k - 1) \cdot T$ if and only if $\delta > \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)} \frac{1}{(k-1) \cdot T - \tau}$, where x_{n-1} denotes the value of information of the player who has the second least valuable information.

For all other sub-hierarchies the respective x_{n-1} has to be lower than that of the highest level hierarchy, because there are also other players who have information. By that we get that a value of δ that implies the longest duration in the highest part of the hierarchy, also implies the longest duration in all sub-hierarchies. \square

Proof of Proposition 8.

The lowest hierarchies all have a duration of one period if $\delta < \frac{n \cdot x_n + R}{n \cdot (x_n + R)}$ holds for all these hierarchies. With condition 2 we get that n is increasing if we move to higher-levels, which makes this boundary higher. Since the intermediaries add the value of information of their agents, also x_n increases going from the bottom to the top. Together with the third condition we then get that for all hierarchies δ is such that the duration is only one period. \square

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