

Productivity Growth And International Capital Flows
In An Integrated World

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Chapter 1

General Introduction

The financial globalization for the past decades witnesses the global imbalance phenomenon in which the deficit current accounts by some large advanced economies are continuously financed by some developing economies with the high output growth rates and the scarce capital stocks. On the theoretical ground, the Neo-Classical growth model implies that one economy with scarcity of capital would have a high marginal product of capital and a high autarky interest rate. Therefore, at the integration with the free mobile capital, that country would experience the net total capital inflows so that the domestic interest rate equals to the rest of world's rate (Lucas (1990)). Furthermore, one economy growing faster than the rest of world would also have a higher investment demand and should experience the inflows of net total capitals (Gourinchas and Jeanne (2013)).

The global imbalances are the result of the heterogeneity in the patterns of savings and investments across countries. Indeed, one country experiences an inflow of capital if its saving is less than its investment: that country borrows from the rest of world to finance the excess investment demand. Similarly, one country would lend to the rest of world if its saving is higher than its investment. The Thesis would employ the productivity growth to shed the refresh lights on this heterogeneity across countries.

By a model with the perfect financial market, chapter 2 studies the role of productivity level on the allocation of savings into a portfolio including both the risky and risk-free assets. Both the theoretical model and empirical evidences stress the role of the safe assets accumulation as the main driver of international capital flows. Chapter 3 relaxes the perfect market assumption by introducing the financial frictions on both the saving and investment. These frictions are the key elements of the threshold over which an increase of productivity growth raises the

inflows of net total capitals but below threshold, the reverse holds. Therefore, this chapter can capture the non-linear dependence pattern of net total capital inflows on the productivity growth. Chapter 4 endogenizes the financial friction on the savings to model the simultaneous impacts of the productivity growth on both the saving and investment. If the productivity growth raises the saving more than the investment, one fast-growing economy can experience the outflows of capitals. The Thesis provides the new insights into the pattern of international capital flows over the past decades. Chapter 2 proves that even the advanced economies also demand for the safe assets while most of literature focus on the developing economies as the main drivers of safe assets accumulation. Moreover, Chapter 3 solves the controversy both in theory and empirical evidence on whether there exist an allocation puzzle on the international finance. In particular, the allocation puzzle exists but only for the country which has a too low productivity growth rate (i.e, under one threshold). Finally, chapter 4 proves that, in both developing and advance countries, an increase of productivity growth can lead to outflows of capitals by raising the savings more than investment. Therefore, by combining the high growth rate with the low financial development, the chapter can explain simultaneously the outflows of capitals and the low long-run capital accumulation in one developing economies. Each of these main contributions and the mechanisms underlining them will be presented in details on the next sections.

1.1 Safe Assets Accumulation and International Capital Flows

The international macro-finance literature employs the portfolio choice approach to the current account. Pavlova and Rigobon (2010a) analyze the two-country endowment economies with log utility preference. They realize the valuation gains as the result of the fluctuation on the exchange rate and the differential rates of return between the gross foreign assets and gross foreign liabilities. Since the valuation gains are negatively correlated with the current account, they can offset apart or fully the changes in the current account. Farhi, Caballero and Gourinchas (2008) focus on the safe asset scarcity as the main driver of the pattern of international capital flows. The developing economies exhibit the shortage of financial assets since they have the capitalization rate of the domestic output (i.e, the ratio of financial assets over GDP). That assets' shortage raises the assets' price and depresses the autarky interest rate. At integration, the capital flows

out of developing economies to the advanced economies in seeking the storage of savings. On Chapter 2, we pursue this portfolio approach but focus on the case of Euro zone.

The global imbalance phenomenon in Euro zone are interesting for three reasons. First, all countries use the same currency. Therefore, the exchange of financial assets among countries is based on the difference on the real rates of return rather than the fluctuation of exchange rate and asset prices. Second, both the exporters and importers of capitals are the advanced economies. For instance, Germany, Netherlands usually run the surplus current accounts while France, Italy run the deficit current accounts. Third, the role of risk-free Bonds dominates the Equities and FDI on shaping the pattern of international capital flows. In sum, any model for Euro zone needs to account for the pattern of Bonds but to rule out both the exchange rate fluctuation and the asymmetric financial development levels.

We build up a continuous time two-country economy, based on Turnovsky (1997) where each country issues one risky equity which is the claim on the domestic mean-variance stochastic output process. The mean-variance framework captures the trade-off between the macroeconomic performance and the associated risks. We add one risk-free bond which brings the holder one constant interest rate and is free mobile across countries. Therefore, the portfolio investment is enriched with two risky assets and one risk-free bond. By construction, the model falls into the class of classical portfolio choice model (Merton (1969), Samuelson (1969)) where the agent with the constant-relative-risk-aversion coefficient allocates the constant share of wealth for each asset in one portfolio including both the risky and risk-free assets.

The model has two main implications for the international capital flows. First, a higher productivity level can lead to a higher accumulation of safe assets. The reason is that an increase of productivity level raises both the mean and variance of output growth rate. At a too high productivity level, the increase of variance outweighs the increase of mean. Therefore, the agent would raise the demand for the safe assets to insure against a higher variance of output. Second, there exists the capital gains on the net international investment position, which are driven by the differential productivity levels and the variance of the output. Since the capital gains are negatively correlated with the current account, they can offset the deficit current account. This mechanism is stressed in the literature on the international financial adjustment (Gourinchas and Rey (2007a)).

With these theoretical implications, we carry out the empirical analysis for the case of Euro zone. The data sample is collected by two sources: the main one is

the quarterly data from Eurostat and another one is the annual data from Alfaro, Kalemli-Ozcan and Volosovych (2014) for the robustness check.

The empirical evidences show that a higher productivity level leads to a higher accumulation of safe asset (e.g, Bonds). In particular, 1% increase of the domestic productivity level raises the claim of foreign bonds by 5.6 % over the domestic output. The domestic investment-output ratio, however, is reduced by 0.14%. Therefore, a higher productivity level motivates the agents to reduce the position on the domestic risky investment to have more wealth for the accumulation the foreign risk-free bonds. This result is consistent with the implication of the theoretical model on the safe assets accumulation.

The empirical analysis, however, reveal that the role of capital gains is unimportant on shaping the dependence pattern of capital flows on the productivity levels in Euro zone. As Pavlova and Rigobon (2010a), we measure the real capital gains as the difference between the negative value of current account and the negative change of net international investment position. The evidences show that the productivity level has an insignificant effect on the real capital gains but a significantly negative effect on the negative value of current account as the standard measure of net total capital inflows. Therefore, the real capital gain is not strong enough to shape the dependence pattern of capital flows on the productivity level on Euro zone.

We also decompose the net international investment position into sub-flows by type of assets. The analysis shows that the impact of productivity on the capital flows is mainly driven by the Bonds' flows while its impacts on the Equity and FDI investment flows are quite modest. Therefore, the evidences stress the role of safe assets on shaping the dependence pattern of capital flows on the productivity on Euro zone.

Finally, the theoretical model is flexible to capture the role of safe assets accumulation on the global imbalances between the advanced and developing economies. The advanced economies can borrow the cheap funds by issuing the risk-free bonds to the developing economies to invest on the risky assets with the higher returns. Therefore, the formers can enjoy the exorbitant privilege on their international investment positions. This role of safe assets is presented in the appendix of Chapter 2.

1.2 Non-Linear Pattern of International Capital Flows

There exist the controversy on the pattern of international capital flows. On a sample of Non-OECD countries, Gourinchas and Jeanne (2013) show that the country growing faster tends to export the net total capital to the rest of world. On a larger sample of both Non-OECD and OECD countries, Alfaro, Kalemli-Ozcan and Volosovych (2014), however, find the contradicted result that the fast-growing economies still experience the net total capital inflows. On Chapter 3, we rationalize both these two evidences by proving that there exists one non-linear dependence pattern of international capital flows on the productivity growth rate.

On the empirical analysis, we employ one cross-section dataset of international capital flows for 160 economies, in which each variable is averaged over the time period 1980-2013. The regression shows that the net total capital inflows has a non-linear dependence on the productivity growth rate: at low and high value of productivity growth rate, an increase of productivity raises the net total capital inflows as predicted by the Neo-Classical growth model. At the middle range of value, however, an increase of productivity growth reduces the net total capital inflows (the Allocation puzzle). On the next step, we construct the saving and investment wedge to examine which one can account for the non-linear pattern of capital flows. The result shows that the pattern of capital flows on the saving wedge is non linear while its pattern on the investment wedge is linearly decreasing. Therefore, only the investment wedge can not account for the pattern of international capital flows and we need to take into account both the saving and investment wedges.

On theoretical ground, we build up one multi-country OLG economy to characterize the pattern of cross-border capital flows. With the endogenous world integration interest rate, the capital would flows into the economy which has a higher interest rate at autarky than at integration. Since the interest rate is a function of the domestic productivity growth, labor force growth and financial frictions, the condition on the capital inflows turns out to be the condition on the productivity growth rate. Indeed, if the domestic productivity growth rate passes over one threshold which depends on the world average productivity growth and the financial frictions, the net total capital inflows are increasing on the productivity growth. Below that threshold, the net total capital inflows are decreasing on the productivity growth. In sum, the dependence pattern of the net total capital

inflows on the productivity growth is non linear.

The empirical evidences rely on the construction of productivity level and the financial frictions (saving and investment wedges). First, the real productivity level employed from the updated version of Penn World Table (Feenstra, Inklaar and Timmer (2015)) is on constant 2005 national prices. On this updated version, the labor share is allowed to vary across countries and time periods. Moreover, the capital stock is aggregated over the different types of capitals such as machine, buildings. Each type of capitals has the different price and discount rates. Therefore, the productivity level is more accurate than previous PWT versions and overcomes the controversy on computing the labor share and the capital stocks in the growth accounting literature. Second, the savings wedge is computed based on the data from World Development Indicators on the lending and deposit interest rates for each country. The investment wedge is computed based on the method of Caselli and Feyrer (2007) by which the computation of the marginal product of capital takes into account the price of investment (not price of consumption) and the share of productive capital (which is different to the natural capital such as land). With two key variables, our strategy is to regress the net total capital inflows on the productivity growth and then on the financial frictions to observe which type of friction can account for the pattern of international capital flows.

The theoretical mechanism relies on the construction of the capital taxation. With the lump-sum capital taxation, the mechanism does not depend on the inter-temporal elasticity of substitution coefficient. The reason is that the marginal saving rate is always decreasing on the saving wedge while the investment rate is always decreasing on the investment wedge. On an alternative model in which the taxation is used to finance the public expenditure, the mechanism is still validated excepting for the case that the inter-temporal elasticity of substitution coefficient is low and the saving curve is steeper than the investment curve. On the latter case, the savings is increasing on the financial friction so that one country with high friction level would have high savings supply and then, it runs a surplus current account at integration. This alternative set-up is presented in the Appendix of chapter 3.

1.3 Savings, Capital Accumulation and International Capital Flows

Many models focus on the high savings supply in the developing economies which export the capitals to the rest of world. The idea is to use the financial frictions to depress the autarky interest rate in the economy with the high productivity growth rate. Since the capital flows from the economy with low autarky interest rate to the economy with high autarky interest rate so that the capital accumulation is equalized across countries at integration, the fast-growing economies can still experience the outflows of net total capital. In Angeletos and Panousi (2011), the idiosyncratic risk motivates the precautionary savings and this mechanism is stronger in the developing economies with weak financial systems. Therefore, these economies can have the high savings and low autarky interest rates even with the high productivity growth rates. At integration, the developing economies experience the outflows of net total capital to the advanced economies which are better in insuring against the idiosyncratic risks. In Coeurdacier, Guibaud and Jin (2015), for the developing economies have the more severe borrowing constraints than the advanced economies, the young agents borrow less, the middle-aged and old agents save more than their counterparts in the advanced economies. Therefore, the developing economies would have higher savings and lower interest rate at autarky. At integration, the capital flows from the developing economies to the advanced economies so that the excess savings by the middle-age agents in the former economies serve the excess borrowing by the young agents in the latter economies.

While these aforementioned model can explain the outflows of net total capital from the developing economies which have the high aggregate savings, they also implies the convergence of the long-run capital accumulation across countries on the financial globalization with free mobile capital. However, the data set on the developing economies from 1980 to 2013 on the capital-effective-labor ratio (a measure of long-run capital accumulation) shows that the convergence of capital accumulation level only happens for some developing economies (for instance, Republic of Korea). Most of developing economies, such as China and India, can not converge to the same capital-effective-labor ratio as the advanced economies. This observation suggests that we need to account for the non-convergence of long-run capital accumulation on analyzing the international capital flows.

In Chapter 4, we provide one model of endogenous saving wedge to explain the flow of net total capital from the developing economies which have two crucial

features: (1) a high productivity growth rate; (2) a lower long-run capital accumulation level than the advanced economies.

The model is characterized by two key features. First, the financial friction is endogenously increasing on the domestic productivity growth rate. The government collects the capital taxation to finance the public expenditure which is complementary to the private output. Therefore, a higher productivity growth rate raises the demand for public expenditure and a higher taxation which, in turn, raises the wedge between the domestic and the world interest rate. This feature is different to the model in Buera and Shin (2009) when the productivity growth depends on the exogenous financial friction. On their model, a large-scale reform reduces the financial friction which raises the productivity growth rate by a more efficient allocation of resources. Second, there exists the credit constraint which prevents the domestic firms to borrow such that the domestic marginal product of capital equalizes the interest rate. The interaction between the endogenous financial friction and credit constraint plays an important role on explaining the outflows of capitals from the developing economies.

The pattern of international capital flows relies on the endogenous financial friction. An increase of productivity growth raises simultaneously the investment rate by raising the marginal product of capital and the saving rate by depressing the domestic interest rate. If the saving rate goes up faster than the investment rate, that increase of productivity growth can lead to the outflows of net total capital. The stress on the marginal saving rate differs our paper to Song, Storesletten and Zilibotti (2011) in which the author focus on the aggregate savings. On their paper, an economic transition, which moves the labor force from the financial firms with the low productivity levels to the entrepreneur firms with the high productivity levels, raises the wage rate, then the aggregate savings. At the same time, the reduction on the investment demand by the financial firms pushes down the aggregate investment. Therefore, one transition economy like China can experience simultaneously the high productivity growth rate and the outflows of net total capitals.

The non-convergence of long-run capital accumulation relies on the difference on the tightness of credit constraint between one country and the rest of world. We assume that the rest of world includes the advanced economies with unbinding credit constraint. Therefore, its capital-effective-labor ratio satisfies the condition that the marginal product of capital equals the interest rate. The small open economy in case of binding constraint, however, would converge to one capital-effective-labor ratio which is increasing on the tightness of credit constraint. Therefore, we

find on threshold on the credit constraint such that one small open economy has lower capital-effective-labor ratio than the rest of world at the steady state.

The low long-run capital accumulation in our model is an ex-post measure of capital stock not an ex-ante measure of capital stock such as the low initial capital, which is investigated intensively by other models. In Matsuyama (2004), one poor economy with the low initial capital stock can converge to a lower steady state than one rich economy. Since the economy with lower steady state also has a lower investment demand, the capital flows from the poor to rich economy. The difference on the investment demand also is the key mechanism in Boyd and Smith (1997) where the rich economy has more income to afford the high agency cost associated with the capital investment. In stead of the divergence of initial capital stocks between the rich and poor economies (ex-ante difference), our model focus on the divergence of steady-state capital stocks among the poor economies (ex-post difference). Indeed, with the similar initial capital stocks in 1980s, only some developing economies such as Korea can converge to the same capital-effective-labor ratio of the advanced economies but others such as China and India could not. In sum, the feature of ex-post capital accumulation differs our model to rest of literature on explaining the paradox raised by Lucas (1990) that why capital does not flows to the poor countries.

In order to generate the simultaneous increases of both the saving and investment rates, the model relies on the low value of the inter-temporal substitution coefficient such as the income effect dominates the substitution effects for the impact of the change of interest rate on the marginal saving rate. For other models in which the substitution effect is strong enough, the reduction of interest rate due to the surge of productivity growth rate would reduce the saving rate and amplify the inflows of net total capitals. The empirical literature, however, confirms a low value of the inter-temporal substitution coefficient (Guiso, 2006), which lays out the ground for our analysis.

On the appendix, we provide two extensions for the main model. On the first extension, we allow the government to issue the public debt to finance the public expenditure. The capital taxation on savings, now, is a function of the public expenditure-output and public debt-output ratios. Then, an increase of public debt can raises the taxation and depresses the domestic interest rate. Therefore, the savings in this extended model can raise more than in the main model. And the outflows of capitals can be amplified. On the second extension, we endogenize the productivity level as a concave fuction of the expenditure on Research and Development. But we also rule out the endogenous saving wedge to focus on the

role of R&D expenditure on shaping the pattern of capital flows. The result shows that the capital would flow from the country with low R&D expenditure to the country with high R&D expenditure. By the concave function of productivity level on the R&D expenditure, the country exporting capitals also has a higher productivity growth rate. Therefore, this extended model can explain simultaneously both the high productivity growth rate and outflows of capitals from the developing economies. In sum, the first extension tries to incorporate the public debt on the pattern of capital flows while the second one employs the theory of endogenous growth.

1.4 Outline of Thesis

The rest of Thesis consists three self-contained chapters which can be read independently. Each chapter provides both the theoretical model and the empirical analysis, starting with the role of safe asset accumulation on Chapter 2 to the non-linear pattern of international capital flows on Chapter 3 and the long-run capital accumulation on Chapter 4. The Thesis closes with the general conclusion for future research avenues.

Chapter 2

Global Imbalances with Safe Assets in a Monetary Union

Abstract

In a two-country economy with the stochastic mean-variance output process, the safe assets help the consumers to attain the full risk-sharing across the domestic and foreign risky investments. A higher domestic productivity level raises both the mean and variance of the domestic output, and can lead to a greater accumulation of the safe assets. The empirical analysis on the 19 countries of Eurozone confirms this theoretical implication. Moreover, it also reveals that the accumulation of the safe assets is the main driver of the global imbalances in Eurozone.

2.1 Introduction

The global imbalances in Eurozone are featured by three stylized facts.

Fact 1: The net total capital inflows for each country have been increasing from 1990s (when the European Economic Area is established to allow the free movement of capital across countries) and have boosted up substantially from 2000s when the introduction of one common currency.

Fact 2: Both exporters and importers of capital are the advanced economies. The panel A in Figure 2.1.1 shows that Germany, Netherlands and Austria are the main exporters of capital while France, Italy, Spain are the main importers of capitals. This flow of capital from developed to developed economies is different to the up-hill capital flows from developing to developed economies (Lucas paradox).

Fact 3: The Debt flows dominate the FDI and Portfolio flows on shaping the pattern of capital flows across countries. The panel B demonstrates that the negative of net total capital inflows is driven mainly by the negative of net debt inflows for Germany. This structure of capital flows is the same for Spain (Panel C).

Despite a large body of literature on global imbalances for last decades, there are a few formal structures to analyze the case of Eurozone. On many models, capital can flow out from developing countries with the severe financial friction. Both creditors and debtors in Eurozone, however, do not differ by level of financial friction (Panel A). Some models argue that the advanced economies can have the valuation gains on the international investment position due to the exchange rate fluctuation and differential rate of returns on foreign assets and liabilities. The same currency in Eurozone, however, rules out the effect of the exchange rate on the international investment position. Furthermore, the dominant role of bonds on shaping the pattern of capital flows has not been explored yet (Panel B and C).

The main purpose of this paper is to provide a framework of endogenous portfolio choice to analyze the international capital flows when both foreign assets and liabilities are denominated into one common currency. We stresses the role of the safe assets on shaping the pattern of capital flows. We use this model to show that the dominant features in Figure 2.1.1 can arise from the interaction between the productivity level and the available supply of the safe asset.

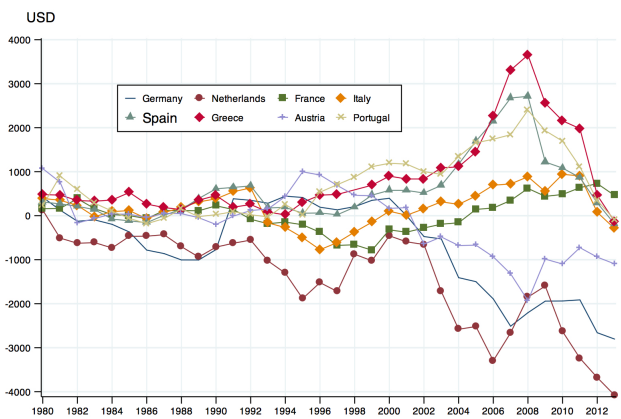
We decompose the net total capital inflows into the flows of the risky investments (FDI and Portfolio) and the risk-free Bonds. To generate the heterogeneity rate of return on the risky assets across countries, the productivity level is allowed to be

different across countries. To analyze the demand for risk-free bonds, we employ a mean-variance output growth process with the risk-averse agents. With an AK production function, a higher productivity level raises both the mean and variance of the rate of return on the risky investment, and motivates the agents to hold the safe asset to insure against the high variance in the risky investments. With these key features, the model sheds light on the motivation behind the flows of both the risky and the risk-free investments across countries with the similar economic fundamentals.

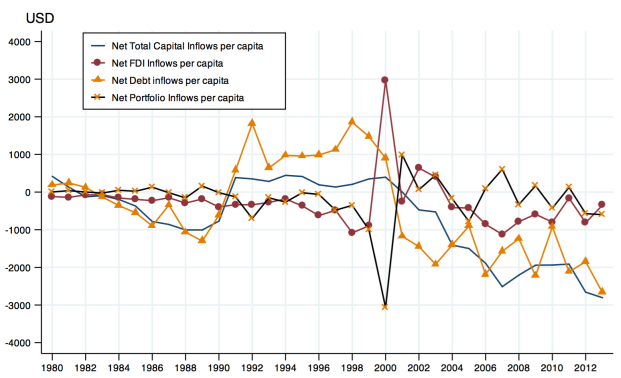
Next, we carry out the empirical analysis on one panel data from Eurostat on 19 countries in Eurozone. The empirical result strongly supports that a higher productivity level results in a greater accumulation of foreign bonds. The empirical test using the data from Alfaro, Kalemli-Ozcan and Volosovych (2014) provides the same result.

The paper offers a theory of the international capital flows across countries in one monetary union. Past papers on the current account adjustment in Eurozone rely on the low saving and high investment rates due to convergence in output per capita (Blanchard and Giavazzi (2002)), on the convergence and growth expectations (Lane and Pels (2012)), on the allocation of imported capital between tradable and non-tradable sectors (Giavazzi and Spaventa (2011)). Some distinguishing elements mark our theory from the aforementioned papers. First, we separate the risky investments' flows (FDI and portfolio) to the risk-free investments' flows (bonds) by allowing the agents to hold a rich portfolio choice including both risky and safe assets. Second, we emphasize the change of net foreign assets rather than the conventional current account to account for the role of the real unexpected capital gains on shaping the cross-border capital flows. Third, we approach the global imbalances with a balance between the theory and the empirical analysis.

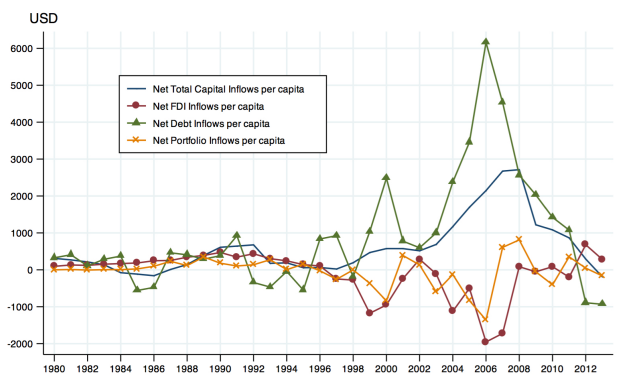
Our paper is related to the literature on the role of the safe asset on the global economy. Farhi and Maggiori (2016) focus on the competition on supplying the safe asset as the key element on the architecture of the international monetary system. Farhi, Caballero and Gourinchas (2008) argue the high supply of the financial assets in advanced economies helps them to attract net total capital flows from developing economies. He, Krishnamurthy and Milbradt (2016) characterize the safe assets based on the float of the sovereign bonds and the fundamentals available to rollover the public debt. Taking the supply side as given, we focus on the demand side of safe assets to shed light on the pattern of international capital flows across countries. Within a mean-variance production process, an increase of



A. Net Total Capital Inflows per capita (-CApc) by country



B. Decomposition of Net Total Capital Inflows per capita: Germany



C. Decomposition of Net Total Capital Inflows per capita: Spain

Figure 2.1.1: GLOBAL IMBALANCES IN EUROZONE

Sources: Alfaro, KalemliOzcan, and Volosovych (2014)

productivity can lead to a greater accumulation of the safe asset if the agent is risk averse enough.

Our work is also related to the growing theoretical macro-finance literature that incorporates endogenous portfolio choice into models of open economy macroeconomy, such as with the asymmetric information (Tille and Wincoop (2010), home bias on equity (Coeurdacier and Rey (2013))). These papers provide a various approximation technique to analyze current account around the steady state. Our paper produces an exact closed-form characterization of the equilibrium. This feature is familiar with Pavlova and Rigobon (2010a)'s pure-exchange economy. Our model, however, shuts down the price adjustment process and elaborates one open production economy with more general utility function.

The paper proceeds as follows. Section 2.2 describes the economic environment and characterizes the equilibrium. Section 2.3 focuses on the role of safe assets. Section 2.4 analyzes the international capital flows. Section 2.5 presents the empirical evidence to support the theory. Section 2.6 concludes and the appendix presents two extended models.

2.2 The model

2.2.1 The economic setting

We work with a continuous-time production economy populated by two countries: Home and Foreign. Home is the core economy with a higher productivity level than Foreign as the periphery one: $a > a^*$.

Technology

Both countries produce one common free mobile good which can be consumed or accumulated as capital and traded in a perfectly integrated world capital market. The flows of outputs at Home (dy) and at Foreign (dy^*) are produced by means of the stochastic linear production functions, using domestic domiciled capitals

$$dy = akdt + ak\sigma dz \tag{2.1}$$

$$dy^* = a^*k^*dt + a^*k^*\sigma^*dz^* \tag{2.2}$$

where $(a, k); (a^*, k^*)$ are productivity levels and capital stocks in Home and Foreign respectively. The parameters $(\sigma; \sigma^*)$ are non-negative constants, representing the variance. The terms $(dz; dz^*)$ represent the proportional productivity shocks

in Home and Foreign. The set-up features the mean-variance analysis (Turnovsky (1997)) to address important trade-offs between the level of macroeconomic performance and the associated risks.

z and z^* are Wiener processes with the increments that are normally distributed with zero mean ($E[dz] = E[dz^*] = 0$) and variance ($E[dz^2] = E[dz^{*2}] = dt$). The productivity shock is assumed to be country-specific: $Cov(dz, dz^*) = 0$.

Beside the two risky assets, there are also the risk-free Bonds which we call the safe assets. The supply of safe assets is exogenous so that the safe interest rate is endogenous determined. The exogeneity of safe assets supply can be interpreted as the monetary union, as a whole, can borrow from the rest of world. The main reason is that, technically, since both Home and Foreign agents have the same risk averse coefficient, there is no motivation for one economy to issue the safe assets (risk-free Bonds) to the other. And the model of endogenous supply of safe assets need to rely on the heterogeneity of risk averse coefficient (Farhi and Maggiori (2016)). The exogenous supply of the safe assets, therefore, is necessary to assure the market clearing conditions. This feature is also the key on Farhi, Caballero and Gourinchas (2008) in which the supply of safe assets is a constant fraction of domestic output. In the Appendix, we present two alternative models which incorporate the heterogeneity on the risk aversion and on the supply of safe assets across countries. These two alternative models, however, are more appropriate to capture the global imbalance between two blocks of economies: advanced and developing economies while our main model focus on the imbalances between the advanced and advanced economies.

Preferences and Portfolio choice

The Home representative consumer holds three assets: domestic risky capital (k^d), foreign risky capital ($k^{d,*}$) and risk-free bonds (b), subject to the wealth (w) constraint.

$$k^d + k^{d,*} + b = w \quad (2.3)$$

Consumers are assumed to purchase output over the instant dt at the nonstochastic rate $c dt$ out of income generated by their holding of assets. Their objective is to select their portfolio of assets and the rate of consumption to maximize the expected value of lifetime utility

$$E \int_0^{\infty} \frac{1}{\gamma} c^\gamma e^{-\beta t} dt \quad (2.4)$$

whereby ($-\infty < \gamma < 1$) and the discount factor satisfies: $0 < \beta < 1$. Note that the relative risk averse coefficient (which is defined as $\frac{-cu''(c)}{u'(c)}$) is constant at $(-\gamma)$ and satisfies $0 < (1 - \gamma) < \infty$.

The stochastic wealth accumulation equation is:

$$dw = w[n^d dR^k + n^{d,*} dR^{k,*} + n^b dR^b] - cdt \quad (2.5)$$

whereby:

$$n^d \equiv \frac{k^d}{w} = \text{portfolio share of the domestic risky capital,}$$

$$n^{d,*} \equiv \frac{k^{d,*}}{w} = \text{portfolio share of the foreign risky capital,}$$

$$n^b \equiv \frac{b}{w} = \text{portfolio share of the risk-free bonds,}$$

$$dR^i = \text{real rate of return on assets } i = (k, k^*, b).$$

The rates of return on the Home capital, Foreign capital and risk-free bonds are:

$$dR^k \equiv \frac{dy}{k} = adt + a\sigma dz \quad (2.6)$$

$$dR^{k,*} \equiv \frac{dy^*}{k^*} = a^* dt + a^* \sigma^* dz^* \quad (2.7)$$

$$dR^b = rdt \quad (2.8)$$

Plugging (2.6), (2.7), (2.8) into (2.5), the stochastic optimization problem can be expressed as being to choose the consumption-output ratio c/w , and the portfolio shares $n^d, n^{d,*}, n^b$ to maximize

$$E \int_0^\infty \frac{1}{\gamma} c^\gamma e^{-\beta t} dt \quad (2.9)$$

subject to the dynamic budget constraint and the wealth constraint:

$$\frac{dw}{w} = \left[an^d + a^* n^{d,*} + rn^b - \frac{c}{w} \right] dt + [an^d \sigma dz + a^* n^{d,*} \sigma^* dz^*] \quad (2.10)$$

$$\equiv \left(\rho - \frac{c}{w} \right) dt + d\bar{w} \equiv \psi dt + d\bar{w} \quad (2.11)$$

$$1 = n^d + n^{d,*} + n^b \quad (2.12)$$

Whereby, we define

$$\rho \equiv an^d + a^* n^{d,*} + rn^b, \text{ as the disposable income,}$$

$\psi \equiv \rho - \frac{c}{w}$, as the deterministic growth rate of wealth accumulation,

$d\bar{w} \equiv n^d a \sigma dz + n^{d,*} a^* \sigma^* dz^*$, as the stochastic part of wealth accumulation.

Similarly, the Foreign agent chooses the consumption rate and the portfolio shares to maximize the lifetime utility:

$$E \int_0^\infty \frac{1}{\gamma} c^{*\gamma} e^{-\beta t} dt \quad (2.13)$$

subject to the dynamic budget constraint and the wealth constraint:

$$\begin{aligned} \frac{dw^*}{w^*} &= \psi^* dt + d\bar{w}^* \\ \psi^* &\equiv an^f + a^* n^{f,*} + rn^{b,*} - \frac{c^*}{w^*} \equiv \rho^* - \frac{c^*}{w^*} \\ 1 &= n^f + n^{f,*} + n^{b,*} \equiv \frac{k^f}{w^*} + \frac{k^{f,*}}{w^*} + \frac{b^*}{w^*} \\ d\bar{w}^* &\equiv n^f a \sigma dz + n^{f,*} a^* \sigma^* dz^* \end{aligned}$$

Each country is characterized by the initial wealth and constant productivity level: (w_0, a) for Home, and (w_0^*, a^*) for Foreign.

2.2.2 Characterization of equilibrium

Definition 2.2.1. *The equilibrium is the list of allocation in consumption and investment $Z := (c, k^d, k^{d,*}, b)$ for Home agent and $Z^* := (c^*, k^f, k^{f,*}, b^*)$ for Foreign agent such that:*

1 Z and Z^* maximize the expected utility (2.4, 2.13) subject to the dynamic budget constraints (2.5, 2.11) respectively.

2 The market clearing conditions on:

2.1 Consumption and investment good: $c + c^* + k + k^* = y + y^*$

2.2 Home capital stock: $k = k^d + k^f$

2.3 Foreign capital stock: $k^* = k^{d,*} + k^{f,*}$

2.4 Bonds: $b + b^* = \bar{b}$

The constrained utility maximization falls into the classical Samuelson-Merton portfolio choice problem where the agent with the constant-relative-risk-aversion utility function allocates the constant portfolio shares among the risky and risk-free assets, which we summarize in the following proposition

Proposition 2.2.1. *Suppose that the productivity shocks are country-specific, then the equilibrium is characterized by the constant portfolio shares, the safe interest rate and the transversality condition.*

$$\begin{aligned}
n^d &= n^f = \frac{1}{(1-\gamma)} \frac{(a-r)}{a^2\sigma^2} \\
n^{d,*} &= n^{f,*} = \frac{1}{(1-\gamma)} \frac{(a^*-r)}{a^{*2}\sigma^{*2}} \\
n^b &= n^{b,*} = 1 - n^d - n^{d,*} \\
r &= \frac{[1/(a\sigma^2) + 1/(a^*\sigma^{*2})] + (1-\gamma)[\bar{b}/(w+w^*) - 1]}{1/(a^2\sigma^2) + 1/(a^{*2}\sigma^{*2})} \\
\rho &= \rho^* = an^d + a^*n^{d,*} + rn^b \\
\frac{c}{w} &= \frac{c^*}{w^*} = \frac{1}{(1-\gamma)} [\beta - \gamma\rho + \frac{1}{2}\gamma(1-\gamma)\sigma_{\bar{w}}^2] \\
\psi &= \psi^* = \frac{1}{(1-\gamma)} [\rho - \beta - \frac{1}{2}\gamma(1-\gamma)\sigma_{\bar{w}}^2] \\
\sigma_{\bar{w}}^2 &= \sigma_{\bar{w}^*}^2 = (n^d)^2 a^2 \sigma^2 + (n^{d,*})^2 a^{*2} \sigma^{*2} \\
\lim_{t \rightarrow \infty} E[w(t)^\gamma e^{-\beta t}] &= 0; \lim_{t \rightarrow \infty} E[w(t)^{* \gamma} e^{-\beta t}] = 0
\end{aligned}$$

Proof. Appendix ■

Merton (1969) and Turnovsky (1997) show that the transversality condition implies a strictly positive consumption over wealth ratio. Since the risk aversion coefficient and the portfolio shares are the same across countries, the trading of assets is only driven by the difference in initial wealth.

Parameters restrictions.

We also needs the restrictions on the parameters for (1) the positive safe interest rate and risk premiums: $(0 < r < a^* < a)$ and (2) the feasible portfolio shares: $(n^d, n^{d,*}, n^b) \in (0, 1)$. Note that $a^* < a$ is by assumption, then the first condition turns out to be $0 < r < a^*$. Then, the portfolio shares are positive. Therefore, the second contion reduces to be (2) $(n^d, n^{d,*}) < 1; 0 < n^b < 1$.

In particular, the condition for the positive safe interest rate ($r > 0$) is as following:

$$(1-\gamma)\left(1 - \frac{\bar{b}}{w+w^*}\right) < \frac{1}{a\sigma^2} + \frac{1}{a^*\sigma^{*2}}$$

This inequality implies that the exogenous supply of safe assets needs to be high enough to meet the demand of safe assets.

$$\frac{\bar{b}}{w+w^*} > 1 - \frac{1/(a\sigma^2) + 1/(a^*\sigma^{*2})}{1-\gamma} \tag{2.14}$$

Another interpretation is that the agents should have a low enough risk averse coefficient:

$$(1 - \gamma) < \frac{1/(a\sigma^2) + 1/(a^*\sigma^{*2})}{1 - \bar{b}/(w + w^*)} \quad (2.15)$$

The condition for the positive risk premiums ($r < a^*$) is as following:

$$(1 - \gamma)\left(1 - \frac{\bar{b}}{w + w^*}\right) > \frac{1}{a\sigma^2}\left(1 - \frac{a^*}{a}\right)$$

The condition can be interpreted as the exogenous supply of safe assets needs to be low enough or the agents should have a high enough risk averse coefficient.

$$\frac{\bar{b}}{w + w^*} < 1 - \frac{1}{a\sigma^2} \frac{1 - a^*/a}{1 - \gamma} \Leftrightarrow (1 - \gamma) > \frac{(1/(a\sigma^2))(1 - a^*/a)}{1 - \bar{b}/(w + w^*)} \quad (2.16)$$

Combining (2.14), (2.15) with (2.16), we end up with two equivalent conditions:

$$\begin{aligned} 1 - \frac{1/(a\sigma^2) + 1/(a^*\sigma^{*2})}{1 - \gamma} &< \frac{\bar{b}}{w + w^*} < 1 - \frac{1}{a\sigma^2} \frac{1 - a^*/a}{1 - \gamma} \\ \frac{(1/(a\sigma^2))(1 - a^*/a)}{1 - \bar{b}/(w + w^*)} &< (1 - \gamma) < \frac{1/(a\sigma^2) + 1/(a^*\sigma^{*2})}{1 - \bar{b}/(w + w^*)} \end{aligned}$$

Next, we find the condition for the feasible portfolio shares. In details,

$$\begin{aligned} n^d < 1 &\Leftrightarrow (1 - \gamma)\sigma^2 a^2 - a + r > 0 \\ n^{d,*} < 1 &\Leftrightarrow (1 - \gamma)\sigma^{*2} a^{*2} - a^* + r > 0 \\ n^b > 0 &\Leftrightarrow \frac{\bar{b}}{w + w^*} > 0 \end{aligned}$$

The first and second inequalities are satisfied because they are the quadratic function with the negative discriminant¹. The last inequality is satisfied by assumption of the positive exogenous supply of safe assets.

Finally, we also assume that $\sigma \geq \sigma^*$. Combining with the assumption that $a > a^*$, this implies that the risk premium on Home risky capital is higher or at least equal to the risk premium on Foreign one²: $(1 - \gamma)a^2\sigma^2 > (1 - \gamma)a^{*2}\sigma^{*2}$. Our result, however, does not depend on this assumption for two reasons. First, the mean-variance framework implies that the Home's risky capital provides a higher mean but also a higher variance than Foreign's. Therefore, the Home agent still

¹ $\Delta = 1 - 4(1 - \gamma)\sigma^2 < 0$; $\Delta^* = 1 - 4(1 - \gamma)\sigma^{*2} < 0$. And $(1 - \gamma)\sigma^2 > 0$; $(1 - \gamma)\sigma^{*2} > 0$. Then, the quadratic functions are always positive.

²For σ is lower enough than σ^* , we can have $a^2\sigma^2 = a^{*2}\sigma^{*2}$, then, $n^d > n^{d,*}$. But for $\sigma \geq \sigma^*$ and $a > a^*$, we can not compare between n^d and $n^{d,*}$.

has the motivation to buy the Foreign asset and bonds to diversify the variance. Second, the existence of the safe assets makes the decision on the portfolio shares by both Home and Foreign to be dependent on the relative return to the safe interest rate, not by the difference on the relative returns between Home and Foreign risky asset. And the risk premium between one risky asset and the risk-free bond would determine the share of wealth on that risky asset. In sum, an agent would buy both the risky and risk-free assets.

Endogenous risk premium.

The risk premium on the Home risky asset is the difference between the Home risky rate of return (a) and the risk-free interest rate (r).

$$a - r = \frac{(a - a^*)/(a^{*2}\sigma^{*2}) + (1 - \gamma)(1 - \bar{b}/(w + w^*))}{1/(a^2\sigma^2) + 1/(a^{*2}\sigma^{*2})} \quad (2.17)$$

As a result, the more scarcity the supply of safe assets (i.e, the ratio $\bar{b}/(w + w^*)$ is lower) raises the risky premium. The reason is the scarcity of safe assets reduces the safe interest rate. Indeed, by using the solution for the safe interest rate, we have:

$$\frac{\partial r}{\partial \bar{b}} > 0 \quad (2.18)$$

Our model provides an alternative explanation for raising risk premium after the 2008 financial crisis as documented by Caballero and Farhi (2014). The crisis has reduced substantially the world supply of safe assets: many assets fall out of AAA ranking group, some sovereign debts become risky because of a higher probability of default. This reduction in the supply of safe assets raises the risk premium by reducing the safe interest rate. Recently, Caballero, Farhi and Gourinchas (2015) show that the scarcity of safe assets at the zero lower bound can push the economy into the safe trap in which the only way to restore the equilibrium is a reduction of aggregate demand.

2.3 Safe Assets and Risk-sharing

The demand for the safe assets arises from the risk-sharing motivation. The existence of the safe assets helps the agents to mitigate the output shocks by providing the risk-free interest rate on any case. Therefore, with the safe assets, the model solution attains the perfect risk sharing between Home and Foreign economy (cf. Obstfeld (1994)).

With a set-up without the mean-variance stochastic production function, the increase of productivity would unambiguously attract more investment. Within a

mean-variance framework, however, a higher productivity level raises both mean and variance of output. Therefore, the household faces the trade-off between higher mean and lower variance of output. Around the equilibrium, we have:

$$\frac{\partial n^d}{\partial a} = \frac{\frac{(a^2\sigma^2 + a^{*2}\sigma^{*2} - (a - a^*)2a\sigma^2}{(1 - \gamma)} - (1 - \frac{\bar{b}}{w + w^*})2a\sigma^2 a^{*2}\sigma^{*2}}{[(a^2\sigma^2) + (a^{*2}\sigma^{*2})]^2}} \quad (2.19)$$

$$\frac{\partial n^b}{\partial a} = -\frac{\partial n^d}{\partial a} \quad (2.20)$$

Therefore, by setting $\frac{\partial n^d}{\partial a} > 0$, we find the condition on the relative risk averse coefficient such that an increase of productivity level raises the safe asset accumulation.

Proposition 2.3.1. *If the agent has a high enough coefficient of relative risk averse, i.e., $(1 - \gamma) > (1 - \bar{\gamma})$, then a higher productivity level raises the safe assets accumulation: $\frac{\partial n^d}{\partial a} < 0$; $\frac{\partial n^b}{\partial a} > 0$. Whereby, $(1 - \bar{\gamma}) \equiv \frac{a^{*2}\sigma^{*2} + 2aa^*\sigma^2 - a^2\sigma^2}{2a\sigma a^{*2}\sigma^{*2}[1 - \bar{b}/(w + w^*)]}$*

If the agent has a low relative risk averse coefficient, an increase of productivity raises the demand for domestic risky investment because she evaluates the gain from the increase of mean to be more than the lost from the increase of variance. And the agent de-accumulates the safe asset to have more funds for the risky asset. If she has a high relative risk averse coefficient, however, an increase of productivity reduces the demand for domestic risky investment because she evaluates the gain from the increase of mean to be less than the lost from the increase of variance. And she demands more the safe assets.

The reason for the existence of the threshold relies on the mean-variance output process. Since the productivity level enters both the mean and variance, its increase raises both the mean and variance of the risky assets (equation 2.6) and of the wealth (2.5). This trade-off motivates an agent to accumulate the safe assets to insure against a higher variance on the growth rate of wealth accumulation.

An interesting result is that the threshold $(1 - \bar{\gamma})$ is increasing on the supply of safe assets (\bar{b}). When supply of safe assets become more scare (i.e., \bar{b} declines), $(1 - \bar{\gamma})$ goes down. The condition that $(1 - \gamma) > (1 - \bar{\gamma})$ tends to be held easier. An increase in the productivity level, therefore, is more likely to raise the demand for the safe asset. This might contribute on explaining the increase of the demand for safe asset for the last decades (for instance, panel B and C in Figure 2.1.1) when the supply of safe assets reduces.

Another interpretation of the previous proposition is that the negative impact of

domestic productivity level on the domestic investment is a marginal effect at a high level of productivity and a high domestic investment-output ratio. The data sample from Eurostat shows that this ratio for 19 economies in Euro area is between 20% and 40%, which is quite high. This observation, in turn, might suggest a great motivation to accumulate the safe assets in Euro area.

Note that an increase of the variance of the output's shock unambiguously reduces the demand for domestic risky investment and raises the demand for the safe assets: $\frac{\partial n^d}{\partial \sigma^2} < 0$ and $\frac{\partial n^b}{\partial \sigma^2} > 0$. There is no threshold since the shock only raises the variance, which in turn only lead to a higher demand for safe assets.

2.4 International Capital Flows

We measure the net total capital outflows by two alternative measures (the current account and the change in the net foreign assets) and show that once the real unexpected capital gains are taken into account, the difference between the two measures can be significant.

2.4.1 Current Account

The conventional measure of the Home's current account, applied in international finance textbooks, is:

$$\text{Current Account} = \text{Trade Balance} + \text{Net Dividend Payments} + \text{Net Interest Payments}$$

The trade balance is the rest of domestic output after being subtracted by the domestic consumption and investment:

$$TB \equiv dy - dk - cdt \tag{2.21}$$

The Home's agent receives the dividend on foreign assets, pays the dividend on foreign liabilities ³.

$$(a^*k^{d,*} + rb - ak^f)dt$$

Therefore, the current account is:

$$CA \equiv dy - dk - cdt + (a^*k^{d,*} + rb - ak^f)dt \tag{2.22}$$

³As in Pavlova and Rigobon (2010a), only the deterministic part of the dividend is accounted for the current account.

2.4.2 Net Foreign Assets

The Net Foreign Asset (NFA) is the difference between total foreign assets ($k^{d,*} + b$) and total foreign liabilities (k^f). Therefore, the change in the net foreign asset (NFA) position of the Home country follows:

$$dNFA = d(k^{d,*} - k^f + b) \quad (2.23)$$

where the first two terms are the Home's investment in the Foreign capital stock minus Foreign's investment in the Home capital stock, and the last term is Home's balance on the bond account. The consistency condition implies that $k = k^d + k^f$. By definition, the Home's wealth equals its portfolio value, $w = k^d + k^{d,*} + b$. Hence, we can rewrite as:

$$dNFA = d[(k^d + k^{d,*} + b) - (k^{f,*} + k^d)] = dw - dk$$

Using the definition of dw on (2.5) and the trade balance accounting on (2.21):

$$\begin{aligned} dw &= k^d dR^k + k^{d,*} dR^{k,*} + b dR^b - c dt \\ &= (ak^d + a^* k^{d,*} + rb) dt + ak^d \sigma dz + a^* k^{d,*} \sigma^* dz^* + TB - dy + dk \end{aligned}$$

But, by equation (4.4.1), $dy = ak(dt + \sigma dz) = a(k^d + k^f)(dt + \sigma dz)$, then:

$$dNFA = TB + (a^* k^{d,*} - ak^f + rb) dt + (a^* k^{d,*} \sigma^* dz^* - ak^f \sigma dz) \equiv CA + KG$$

The change of Net Foreign Assets differs the Current Account by the stochastic component: $KG \equiv (a^* k^{d,*} \sigma^* dz^* - ak^f \sigma dz)$. We label this term as the "real unexpected capital gain"⁴. It is real since we do not have the prices and exchange rate changes. It is unexpected since it would turn to be zero under expectation: $E(a^* k^{d,*} \sigma^* dz^* - ak^f \sigma dz) = 0$, since both dz, dz^* are normally distributed with zero means. This real unexpected capital gain depends on the differential rates of return, on the portfolio shares and also on the variance of productivity shocks across two countries. The capital gain term is positive if the total returns on foreign asset holdings exceeds the return the Foreign country makes on its holdings of Home's assets.

Moreover, the capital gain term can offset the change in the current account. Given the same portfolio shares held by Home and Foreign agent, the extend of the offsetting would be depend on the difference between Foreign and Home productivity levels and on the initial wealth level.

⁴Conceptually, this term is only a part of the "valuation effect" which incorporate both the expected and unexpected fluctuations of the asset prices and exchange rate. Devereux and Sutherland (2010) show that the unexpected capital gain is more important than the expected term on affecting the capital flows.

Proposition 2.4.1. *Under the country-specific output shocks, the correlation between the current account and the capital gain is negative: $\text{corr}(CA, KG) < 0$.*

Proof. Appendix ■

The negative correlation between the current account and the capital gains in the international investment position is emphasized by the "valuation effect" approach to the international financial adjustment by Gourinchas and Rey (2007a), Devereux and Sutherland (2010), Pavlova and Rigobon (2010b). In comparison with their models, ours does not have the price adjustment, therefore there is no exchange rate adjustment. Our model, however, still preserves the unexpected capital gains induced by the differential rates of returns and productivity shocks. The model with exchange rate adjustment is suitable for one country which can improve the deficit current account by the domestic currency devaluation. The model without the exchange rate, however, is more suitable for the countries in one monetary union which cannot improve the deficit current account by adjusting the monetary policy.

2.4.3 Cross-border capital flows and Productivity

Rewriting the $dNFA$ as:

$$\begin{aligned} dNFA &= ((n^{d,*} + n^b)\psi w - n^f \psi^* w^*)dt + ((n^{d,*} + n^b)w d\bar{w} - n^f w^* d\bar{w}^*) \\ &\equiv \mu^{dNFA} dt + dw^{dNFA} \end{aligned}$$

Where we denote the drift part as μ^{dNFA} and the diffusion part as dw^{dNFA} . At the beginning of each instantaneous time period dt , the Home and Foreign agents take the wealth as given, (w, w^*) , and the change in Net Foreign Asset depends on the portfolio shares, on the relative comparison of Home wealth over Foreigner's, and on the realization of production shocks. Around the equilibrium,

$$\frac{\partial \mu^{dNFA}}{\partial a} = ((n^{d,*} + n^b)w - n^f w^*) \frac{\partial \psi}{\partial a} + \left(\frac{\partial n^b}{\partial a} \psi w - \frac{\partial n^f}{\partial a} \psi w^* \right) \quad (2.24)$$

Since $(\partial \psi / \partial a > 0)$, the deterministic growth rate of wealth accumulation unambiguously goes up, which increases the Net Foreign Assets. We label this impact as the *income effect*. The impact of productivity improvement on portfolio shares, however, is ambiguous. With a low coefficient of relative risk averse (i.e, $(1 - \gamma) > (1 - \bar{\gamma})$), both Home and Foreign agents reduce their net claim on bond $(\partial n^b / \partial a < 0)$ and raise the investment on Home's capital stock $(\partial n^f / \partial a > 0)$.

With a high coefficient of relative risk averse, they would increase the net claim on bonds. We label this impact as the *portfolio effect*. We summarize the analysis by the following proposition.

Proposition 2.4.2. *The impact of Home productivity on Net total capital flows depends on the relative magnitude of the income effect and the portfolio effect as characterized by the equation (2.24) .*

The impact of an improvement of the Home's productivity on the capital gains is much clearer. We denote the elasticity coefficient of the Foreign's investment on the Home capital with respect to the Home's productivity as $\varepsilon_a^{n^f} \equiv -\frac{\partial n^f}{\partial a} \frac{a}{n^f}$.

$$\frac{\partial KG}{\partial a} = -n^f w \sigma dz (1 - \varepsilon_a^{n^f}) \quad (2.25)$$

Proposition 2.4.3. *If $(1 - \gamma) < (1 - \bar{\gamma})$ or $((1 - \gamma) > (1 - \bar{\gamma}); 0 < \varepsilon_a^{n^f} < 1)$, then, $\frac{\partial KG}{\partial a} < 0$. Otherwise, $\frac{\partial KG}{\partial a} > 0$.*

Proof. On case that $(1 - \gamma) < (1 - \bar{\gamma}) \Rightarrow \frac{\partial n^f}{\partial a} > 0 \Rightarrow \varepsilon_a^{n^f} \equiv -\frac{\partial n^f}{\partial a} \frac{a}{n^f} < 0$. Then, $(1 - \varepsilon_a^{n^f}) > 0$ and $\frac{\partial KG}{\partial a} < 0$.

On case that $(1 - \gamma) > (1 - \bar{\gamma}) \Rightarrow \frac{\partial n^f}{\partial a} < 0 \Rightarrow \varepsilon_a^{n^f} > 0$. If $(0 < \varepsilon_a^{n^f} < 1)$, $\frac{\partial KG}{\partial a} < 0$.

On other cases, $\frac{\partial KG}{\partial a} > 0$. ■

If the agents have a low coefficient of relative risk averse, an increase of the Home's productivity level raises the Foreign's investment on the risky asset by reducing her net position on the safe assets. Since the Home investment on the Foreign risky assets does not change, the increase of Foreign's investment reduces the real capital gains for the Home agents. This is because the Home economy would have to pay more on her foreign liabilities. This mechanism is same for a high coefficient of relative risk averse but a low elasticity coefficient of the Foreign's investment on Home capitals.

2.5 Empirical Analysis

We carry out the test over two alternative samples of the 19 countries in Eurozone: one quarterly panel data from Eurostat as the main sample and one yearly panel from Alfaro, Kalemli-Ozcan and Volosovych (2014) as the robust sample.

Specification of the empirical model

The empirical analysis aims to find the evidences for the theoretical implications on the role of productivity level on the equilibrium portfolio shares and on the pattern of net total capitals inflows.

For the choice of explanation variables, we rely on the equilibrium portfolio share in the proposition 2.2.1. Indeed, since the risky rates of returns and the safe interest rate affects the equilibrium portfolio shares, we use them as the explanation variables. Moreover, other factors that can affect the portfolio shares are assumed to be uncorrelate to the explanation variables and be included into the error term. For the functional form, we assume that both the portfolio shares and the net total capital flows are the linear functions of the productivity level and the risk-free interest rate in the population sample. In fact, the linear functions are the direct implication by the proposition 2.2.1: since the shares are linear on the explanation variables, the net total capital inflows which are determined by the portfolio shares will also be linear on them. Indeed, the linear solutions for the portfolio shares rely on the AK structure of production function in the theoretical model. Other types of production function would complicate model and may require the numerical solution. So, it is much more difficult to specific one appropriate function form for the empirical model.

Note that since the theoretical model is on the continuous-time, then its implications can be test over the quarterly or yearly data sample.

2.5.1 Descriptive Statistics

Portfolio shares. Conceptually, the portfolio shares on the risky assets are measured by the gross capitals. Indeed, by focusing on one economy, the share of domestic wealth on the foreign risky assets ($(n^{d,*})$) are measured by the gross foreign assets in the risky assets such as FDI and Portfolio investment, scaled by domestic output. And the share of domestic wealth on the domestic risky asset (n^d) is measured by the ratio of domestic gross capital formation over GDP. However, the portfolio share on the safe assets is measured by the net concept. Since one economy can buy foreign bonds and the rest of world can buy the bonds issued by that economy. So, the net position on Bonds = (gross foreign assets - gross foreign liabilities)/GDP. This measure is consistent with our theoretical model when the net position of bonds enters the wealth accumulation.

For each country (called the Home country), we calculate the Home's portfolio shares by scaling total assets by Home's output, and scaling total liabilities by the

sum of output of all other countries (called the Rest of World). This calculation is consistent with the definition of portfolio shares in our theoretical model: the Home's investment on the Foreign's risky capital and on bonds are the ratios over the Home's wealth while the Foreign's investments on the Home's risky capital are the ratio over the Foreign's wealth.

Capital flows. The database from the statistical office of the European Union (Eurostat) is the main source for the international Investment position of which the main categories include the Direct investment, Portfolio investment, Financial derivatives, Other investment and Official reserve assets. The foreign direct investment (*FDI*) data includes equity capital, reinvested earnings and other capital. Portfolio investment data includes equity securities (*PEqt*), bonds and notes (*B*), money market instruments (*M*). The financial derivatives data (*Fin*) includes the financial instruments that are linked to, and whose value is contingent to, a specific financial instrument, indicator or commodity, and through which specific financial risks can be traded in financial markets in their own right. The other investment data (*O*) includes four types of instruments are identified: trade credits, loans, currency and deposits, other assets and other liabilities. Reserves asset data (*Re*) includes the Eurosystem's reserve assets, i.e. the ECB's reserve assets and the reserve assets held by the national central banks of the participating Member States. Each type of assets has total assets and total liabilities on million euros at current price, which are available on a quarterly base starting from 1990Q1 to 2014Q1.

Since Eurostat does not have data from 1980 to 1990, we employ also the updated and extended version of dataset of net private and public capital flows constructed by Alfaro, Kalemli-Ozcan and Volosovych (2014) for the robustness check. We employ the part of data which is explored from International Financial Statistic (IFS). The main categories of capital flows include FDI, portfolio equity investment, and debt inflows. FDI includes greenfield investments, investments into the equity capital of existing companies, reinvesting of earnings, and other types of intercompany debt between affiliated enterprises. Portfolio equity investment includes investments into shares, stock participation, and similar instruments that denote ownership of equity. Debt includes short-term external debt, long-term external debt, and the use of the IMF credit. The FDI and Portfolio equity investment can be considered as the risky assets while the Debt is the safe assets in our theoretical model. The role of debt as the close substitute for the safe assets is also emphasized on Gorton (2010) and on Stein (2011). The net flows for each type of assets from 1980 to 2013 is normalized by the annual nominal GDP

at current price on U.S dollar from World Development Indicators. The data on bonds from the database of Eurostat is included into the debt category in the database of Alfaro et al (2014). Therefore, we use the data on bond flows from Eurostat and on Debt flows from Alfaro et al as proxy for the risk-free asset in our model.

We scale net flows by population, instead of scaling the net total capital flows by GDP like Alfaro et al (2014). First, the scaling over population rules out the country size effect. Second, the scaling by population can help us to focus on the impact of productivity level on net total capital flows. The scaling over output, however, cannot differ the impact of productivity on net capital flows to its impact on output. For instance, a higher productivity level can increase the output which, in turn, decreases the net capital flows per output ratio, even when the net capital flows is unchanged. In details, we use the GDP deflator to convert the net capital inflows into the real value, before dividing over the population. For the sample from Eurostat, the capital flows data is converted into the market price at chain linked volume 2010, in million 2010 euro. For the sample from Alfaro et al, the capital flows data is converted into the constant 2011 national price, in million US dollar.

Table 2.1: DESCRIPTIVE STATISTICS

	Obs	Mean	Std. Dev.	Min	Max
Eurostat quaterly data 1990Q1 – 2014Q1: 19 countries in Eurozone					
Negative change of Net Foreign Asset per capita (<i>negDNFApc</i>) in euro	732	-23.81369	4848.272	-55567.69	37601.01
Negative Current Account per capita (<i>negCApc</i>) in euro	1389	-28.90942	519.8331	-3520.348	2473.101
Net Foreign Direct Investment inflows per capita (<i>FDINetpc</i>) in euro	864	5149.91	67237.34	-146175.6	718425.6
Net Equities investment inflows per capita (<i>PEqtNetpc</i>) in euro	874	104240.6	440754.7	-16669.55	2322798
Net Bonds and Notes inflows per capita (<i>BNetpc</i>) in euro	821	-66670.27	279323.5	-1576895	17347.07
Net Money Market Instrument inflows per capita (<i>MNetpc</i>) in euro	818	-17682.84	74415.34	-539376.8	7726.973
Net Financial Derivatives inflows per capita (<i>FinNetpc</i>) in euro	872	-624.9154	4875.975	-47577.5	20896.96
Net Other investment inflows per capita (<i>ONetpc</i>) in euro	890	-17641.93	98356.04	-653180.3	52996.7
Real labour productivity per hour worked, 2010=100, (<i>A2h</i>)	1478	90.29276	13.58468	41.5	123.3
EMU convergence criterion bond yields (<i>RREU</i>) on %	1769	6.675534	3.758459	1.34	25.4
Alfaro, Kalemli-Ozcan and Volosovych (2014) annual data 1980 – 2013: 19 countries in Eurozone					
Negative change of Net Foreign Asset per capita (<i>negDNFApc</i>) in US dollar	600	299.4898	3997.726	-35049.55	27694.3
Negative Current Account per capita (<i>negCApc</i>) in US dollar	823	-61.27403	1296.041	-9022.555	3652.21
Net Foreign Direct Investment inflows per capita (<i>FDINetpc</i>) in US dollar	768	9.967989	4229.331	-84649.16	36375.12
Net Equities inflows per capita (<i>PEqtNetpc</i>) in US dollar	746	1994.757	17146.98	-44985.32	254336.4
Net total Debt inflows per capita (<i>TDebtNetpc</i>) in US dollar	769	-1674.114	17814.9	-202347.1	90628.8
Productivity level, based on AK production function (<i>A</i>) in US dollar	848	.2622039	.1469731	.0984078	1.283636
Productivity level at constant 2005 national price (US dollar) (<i>TFP</i>) in US dollar	848	.9413703	.139847	.4723747	1.493684
EMU convergence criterion bond yields (<i>RREU</i>) on %	592	6.660203	3.527734	1.4	24.13

Productivity level. For database from Eurostat, we use the real productivity per hour worked, an index data with 2010 = 100. For database from Alfaro et al, we use the data on real productivity level at constant 2011 national price (in US

dollars) from World Penn Table 9.0 (June 2016), an index data with 2011 = 100. We also use the real output and capital stock at constant 2011 national price (in US dollar) to calculate the productivity level as implied by the AK production function, for robustness check.

Risk-free interest rate. We use the Maastricht convergence criterion bond yields for the European Monetary Union as the risk-free interest rate. The data is the interest rates for long-term government bonds denominated in national currencies, gross of tax, with a residual maturity of around 10 years. The bond of the basket is replaced regularly to avoid any maturity drift. Table 2.1 shows descriptive statistics on 19 countries using Euro from Eurostat and Alfaro et al (2014). The negative change of Net foreign assets has a mean of (-23.81) with a standard deviation of (4848.27) in the sample from Eurostat; (-299.5) with (3997.7) in the sample from Alfaro et al. The negative current account has a lower mean in the first sample but a lower mean in the second sample but its standard deviation are more than the negative change of Net foreign assets in both of two samples. Moreover, the data on net capital inflows for the sub categories shows a high variation. The real productivity per hour worked and bond yields in Eurostat shows the variations of (13.58) and (3.75) . On the data sample from Alfaro et al (2014), the productivity level computed by AK production function has lower mean (0.262) and variance (0.146) than the productivity level from PWT 8.1: (0.94) and (0.13) respectively. Therefore, the data set offers rich variation for exploring the relationship between the productivity level and net total capital inflows.

2.5.2 Evidences

We perform the panel data analysis to capture the pattern of capital inflows over time on average across countries. Furthermore, the fixed-effect regression can control for the unobserved heterogeneity which is constant over time in each country.

Safe assets and Risk-sharing

Table 2.2 shows the regression result of the portfolio shares by the rest of world on the Home's productivity level. The main result is that the safe assets (bonds) plays an important role on risk-sharing across countries: a higher productivity level raises the demand for foreign bond and reduces the domestic investment. Column 1 of table 2 demonstrates that 1% of increase on the productivity per hour worked (index with 2011 as base year) raises the ratio of foreign bond over

Table 2.2: Fixed-effect regressions of portfolio shares on Home's productivity: 1990Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	B2y	Invest2y	FDI2ROWlia	PEqt2ROWlia	B2ROWlia	M2ROWlia	Fin2ROWlia	O2ROWlia
Productivity level ($A2h$)	2.811** (1.373)	-0.142*** (0.0118)	0.0672*** (0.0236)	0.0788*** (0.0146)	0.0640*** (0.0223)	0.00580* (0.00310)	0.0487** (0.0208)	0.123*** (0.0141)
Long-term interest rate ($RREU$)	10.43* (5.387)	-0.552*** (0.0485)	-0.234** (0.0941)	-0.00199 (0.0587)	-0.180** (0.0898)	-0.0481*** (0.0125)	-0.267*** (0.0822)	0.334*** (0.0563)
Constant	87.41 (142.3)	39.03*** (1.252)	3.749 (2.452)	0.204 (1.504)	5.564** (2.294)	0.789** (0.319)	-1.123 (2.151)	1.287 (1.466)
Observations	753	1,300	796	878	881	881	806	825
R-squared	0.008	0.124	0.026	0.035	0.019	0.027	0.028	0.098
Countries	17	18	18	18	18	18	18	18

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone. Data on quarterly from 1990Q1 to 2014Q4 from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Assets (*as*) are scaled by Home's GDP (on percent). Liabilities (*lia*) are scaled by sum of GDP over all foreign countries on the Rest of World (on percent). *Invest2y* is ratio of Gross capital formation over GDP (on percent).

domestic output by 2.8%. Column 2 shows that 1% of increase on the productivity level reduces the investment-output ratio by 0.14%. These results are consistent with the theoretical implication that the safe asset is used to insure against the higher variance of domestic output, induced by a higher productivity level.

Column 3 to 9 of table 2 show that a higher productivity level attracts more investment from the Rest of World. Among various type of investment, the portfolio equities experiences the highest increase (0.15%) while financial derivatives the lowest (0.04%). These evidence supports the theoretical result that the domestic productivity level is a driven signal for the foreign investment.

Moreover, the regression results of the long-term interest rate on the portfolio shares are also consistent with the theoretical model. An increase of safe interest rate raises the net purchasing of the safe assets by 10.43% while reduces the investment on the domestic capital by 0.56%. These evidences confirm the theory that an increase of safe interest rate reduces the portfolio shares on the risky assets and raises the share on the safe assets. Column 3 to 9 show that the portfolio investments from the rest of workd are also decreasing on the safe interest rate. In sum, the empirical evidences support the positive effect of safe interest rate on the safe assets accumulation and its negative effect on the risky investments.

Table 2.3 investigates the role of productivity of the Rest of World on the domestic investment on abroad. One common feature is that a higher productivity level on Rest of World motivates domestic agents to raise the investment on abroad, for all type of assets. Among them, the direct investment increases most (17.28%)while

Table 2.3: Fixed-effect regressions of portfolio shares on ROW's productivity: 1990Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
	B2yas	FDI2yas	PEqt2yas	M2yas	Fin2yas	O2yas
ROW's productivity level ($A2hROW$)	8.830*** (1.018)	17.28*** (4.277)	7.329*** (1.650)	2.364*** (0.735)	4.306*** (0.505)	10.64*** (1.574)
Long-term interest rate ($RREU$)	-2.793 (3.418)	-18.14 (15.50)	0.988 (5.780)	3.270 (2.471)	-6.084*** (1.812)	15.42*** (5.733)
Constant	-16.22 (102.4)	-656.3 (430.8)	-160.8 (166.1)	-92.95 (74.00)	-319.3*** (50.86)	-148.5 (158.7)
Observations	753	796	806	752	807	822
R-squared	0.101	0.026	0.025	0.014	0.112	0.056
Countries	17	18	18	17	18	18

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone. Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Assets (*as*) are scaled by Home's GDP (on percent).

money market instrument least (2.36%). Another interesting result is that the magnitude of increase on the demand for the foreign bonds induced by the foreign productivity (8.83% in column 1, table 2.3) is higher than the demand for the foreign bonds induced by the domestic productivity (5.6% in column 1, table 2.2). The regression result proves that Foreign's productivity level determines the decision of Home's households to invest on abroad.

In summary, the productivity level raises the accumulation of the safe assets (Bonds). Moreover, the productivity and the risk-free interest rate jointly determine the portfolio choice in Eurozone.

International capital flows

Table 2.4 reports the fixed-effect regression of quarterly capital flows on real productivity per hour worked, controlling for the interest rate on long-term government's bond. The table reveals that the pattern of capital flows on the safe assets (bonds) is reversal to that of risky assets (FDI and Portfolio equities) and the former dominates the latter on shaping the pattern of net total capital inflows. A higher index of productivity (with 2011 as base year) has an insignificant impact on the inflows of the bond and FDI but a positive impact on the inflows of the portfolio equities (890 euro per capita, in column 5) and other type of investment (1140 euro per capita, in column 7). A higher level of productivity, however, significantly raises the bond (629 euro in column 2), portfolio equities (849 euro in column 6) and other types of investment (1036 euro per capita in column 8). While the portfolio equities and other type of investment experience the positive

Table 2.4: Fixed-effect regressions of capital flows per capita: 1990Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	BInpc	BOutpc	FDIInpc	FDIOutpc	EqInpc	EqOutpc	OInpc	OOutpc	negDNFApc	negCApc	KGpc
Productivity level (<i>A2h</i>)	131.3 (243.1)	629.6*** (153.3)	-943.1 (1,029)	-669.7 (762.7)	890.0* (454.6)	849.6** (331.9)	1,140*** (288.8)	1,036*** (323.4)	-23.17 (24.97)	-6.439*** (1.174)	13.99 (24.96)
Long-term interest rate (<i>RREU</i>)	-2,039** (980.8)	-101.1 (601.7)	-8,647** (4,103)	-6,080** (3,040)	1,111 (1,831)	1,017 (1,302)	3,890*** (1,152)	2,991** (1,288)	36.27 (99.36)	-17.46*** (4.948)	-57.22 (99.30)
Constant	43,120* (25,062)	65,249*** (15,898)	288,105*** (106,921)	243,546*** (79,233)	98,594** (46,889)	6,349 (34,473)	-11,420 (29,994)	22,293 (33,602)	2,036 (2,594)	641.0*** (122.5)	-1,143 (2,592)
Observations	881	753	796	796	878	806	825	822	676	1,208	676
R-squared	0.006	0.025	0.006	0.005	0.004	0.008	0.025	0.015	0.002	0.027	0.001
Number of ncode	18	17	18	18	18	18	18	18	17	18	17

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone, from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (E), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Capital flows are expressed at chain linked volumes 2010, euro per capita. *In* denotes for the inflows, *Out* denotes for the outflows. Net flows = Inflows - Outflows. Regression with Money market instrument and Financial derivatives are insignificant, so they are dropped to save space. $KGpc = DNFApc - CApc$.

net capital inflows, the bond experiences the positive net capital outflows. On net, productivity level raises net total capital outflows, measured by the negative current account (about 6.4 euro per capita in column 10). To conclude, the net total capital outflows are mainly driven by the accumulation of the safe assets. Another finding is that a higher long-term interest rate reduces the net FDI investment inflows but raises net Other type of investment inflows. Therefore, the interest rate has a negative effect on the net total capital inflows. The regression result of the negative change of Net Foreign Assets and capital gain term on the productivity level and the interest rate are insignificant.

Table 2.5 repeats the same regression like Table 2.4 but with the data on capital flows from Alfaro, Kalemli-Ozcan and Volosovych (2014) and the data on productivity level from Penn World Table 9.0 (2016). The table reveals the same result as the previous regression with a quarterly data sample from Eurostat: a higher productivity level raises the net total debt outflows, increases the net FDI and Portfolio equities inflows and pushes up the net total capital outflows. Column 1 and 2 shows that one percent of increase on productivity level index (2011 as base year) results in an increase of 37,314 USD per capita of Debt outflows, which is higher than an increase of 24,386 USD per capita of debt inflows. Column 3 to 6 demonstrates the reversal pattern for FDI and Portfolio net inflows when the total capital inflows is higher than the total capital outflows. Column 8 reports the net total capital outflows is 2559 USD per capita. In sum, the negative relationship between the net total capital inflows and productivity level is mostly driven by the negative relationship between the net total debt inflows and productivity level.

Table 2.5: Fixed-effect regressions of capital flows per capita: 1980-2013

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	TDebtInpc	TDebtOutpc	FDIInpc	FDIOutpc	PEqtInpc	PEqtOutpc	negDNFApc	negCApc	KGpc
Real productivity level (<i>TFP</i>)	24,386*** (6,398)	37,314*** (6,154)	5,590*** (1,383)	5,334*** (1,136)	19,624*** (2,671)	8,123*** (1,401)	-4,472 (3,225)	-2,559*** (627.4)	321.1 (3,224)
Long-term interest rate (<i>RREU</i>)	-57.29 (184.4)	182.0 (177.4)	45.00 (39.87)	-16.56 (32.74)	160.5** (76.93)	7.558 (40.34)	-8.852 (89.61)	-10.41 (18.09)	-9.119 (91.47)
Chinn-Ito openness (<i>kaopen</i>)	800.3 (657.6)	1,266** (632.5)	275.0* (142.1)	201.4* (116.7)	756.5*** (274.5)	256.3* (143.9)	319.3 (328.4)	160.6** (64.49)	-305.3 (343.6)
Constant	-21,585*** (7,051)	-36,500*** (6,782)	-5,263*** (1,524)	-4,473*** (1,252)	-20,496*** (2,943)	-7,751*** (1,543)	4,356 (3,633)	2,356*** (691.5)	-60.18 (3,682)
Observations	383	383	383	383	380	380	355	383	334
R-squared	0.070	0.121	0.062	0.111	0.158	0.125	0.010	0.067	0.004
Number of ncode	17	17	17	17	17	17	17	17	17

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone, based on data of International Financial Statistics (IFS) from Alfaro, Kalemli-Ozcan and Volosovych (2014). Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Total Debt (TDebt). Capital flows are expressed in constant 2005 national price in USD, per capita. *In* denotes for the inflows, *Out* denotes for the outflows. Net flows = Inflows - Outflows.

In the Appendix, we divide the sample into the sub-samples by the year of establishing the Euro area (2000) and the year of global financial crisis (2008). By using the year 2000, we get the same results as the full sample. In Table 2.6 and Table 2.7, a higher productivity level still results in the outflows of net total capitals by raising the safe assets accumulation and reducing the domestic investment-output ratio before 2000. And, in Table 2.8 and Table 2.9, the magnitude of these impact are even greater after 2000. By using the year 2008, the role of safe assets accumulation becomes insignificant after 2008 as in Table 2.12. Moreover, a higher productivity level raises the investment-output ratio before 2008 but reduce it after 2008 as shown in Table 2.13. The occurrence of financial crisis in 2008, followed by the Euro debt crisis in 2009, has reduces the supply of safe assets and changes the impact of productivity level on the safe assets accumulation. Moreover, on the crisis, the decrease of the investment-output ratio can be due to other reasons rather than only the productivity level. In short, the introduction of euro amplifies the role of productivity level on shaping the portfolio investment shares and the pattern of capital flows, but the financial crisis change this role to be insignificant. In summary, the empirical analysis supports the same mechanism that the accumulation of the foreign bonds is the main driver of net total capital outflows from the country with an improvement of the productivity level, excepting after the financial crisis 2008.

2.6 Conclusion

We construct a two-country economy to analyze the role of productivity level on shaping the pattern of international capital flows. The theory implies that a higher productivity level raises the variance of output which motivates households to accumulate more foreign bonds. The empirical analysis over one panel sample of 19 economies on Eurozone supports the theory. Moreover, the pattern of international capital flows on the productivity level is two-way in the sense that a higher productivity level raises net total FDI and Portfolio equities inflows but it induces more net total Bond's outflows.

Our results suggest that along with the policy of improving the domestic productivity level, one country also needs the policy which can reduce the variance of output. Moreover, providing more the safe assets for domestic households can prevent the outflows of domestic savings, which is important for the long-run capital accumulation and economic growth of one country with an increasing path of the productivity level.

Although the theoretical model explains successfully the pattern of the safe assets' flows, it can not account fully the two-way pattern of capital flows. In our model, to accumulate more bonds, both Home and Foreign's households need to reduce the position on risky investments. Our next step would be to handle this problem. Another interesting extension would be to incorporate the exchange rate adjustment by allowing different prices across two countries. In that case, the capital gain term can capture both the differential rates of return and price adjustment across countries.

Appendix

2.A Empirical analysis

We divide the data sample into two sub-samples. The first way is to use 2008 when the global financial crisis occurs. It also cover the time of Euro debt crisis from 2009. The second way is to use 2000 when the Euro becomes the common currency.

The regressions give us more insights into the role of productivity level on the portfolio investment. For the role of safe assets, column (1) in Table 2.6 and Table 2.8 shows that the safe interest rate has a positive impact on the safe assets accumulation. However, the coefficients of the productivity level turn out to be insignificant both before and after 2000. These patterns are the same when we divide the sample using the year of 2008. Column (1) in Table 2.10 and Table 2.12 confirm the positive coefficients of the safe interest rate but the insignificant of the productivity level. These results might suggest that we can find an alternative measure of rate of return on the domestic risky investment, although using the productivity level is consistent for our AK production function. This extension can be done in the future research avenue.

For the domestic risky investment, column 2 in Table 2.6 shows that the coefficient of productivity level on the domestic investment-output ratio is negative before 2000. Table 2.8 illustrates that the magnitudes of this coefficient becomes more stronger after 2000. Next, column 2 in Table 2.10 and 2.12 show that the negative effect of the domestic productivity level on the domestic investment is driven mainly by the sample after 2008: the coefficient is 0.04 before 2008, -0.099 after 2008 and negative for all data sample.

For the pattern of international capital flows, Table 2.7 and 2.9 show that an increase of productivity level raises the outflows of net total capitals by a higher safe assets accumulation in both the sub-sample before and after 2000. Table ?? shows that a higher productivity level raises more the outflows of capitals by more

Table 2.6: Fixed-effect regressions of portfolio shares on Home's productivity: 1990Q1-1999Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	B2y	Invest2y	FDI2ROWlia	PEqt2ROWlia	B2ROWlia	M2ROWlia	Fin2ROWlia	O2ROWlia
A2h	2.608 (1.925)	-0.113*** (0.0324)	0.0208 (0.0207)	0.146** (0.0609)	-0.00692 (0.0308)	-0.00176 (0.00432)	0.00843 (0.00642)	-0.00416 (0.0398)
RREU	8.358* (4.930)	-0.0549 (0.0808)	0.0166 (0.0543)	-0.0912 (0.157)	0.0975 (0.0792)	0.0313*** (0.0111)	0.00643 (0.0172)	0.350*** (0.104)
Constant	-351.5* (181.6)	32.35*** (3.083)	0.189 (1.964)	-8.123 (5.799)	2.436 (2.930)	0.0989 (0.411)	-0.678 (0.609)	2.626 (3.780)
Observations	56	284	72	63	63	63	61	72
R-squared	0.057	0.068	0.029	0.339	0.110	0.379	0.055	0.371
Countries	6	11	8	7	7	7	6	8

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone. Data on quarterly from 1990Q1 to 2014Q4 from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Assets (*as*) are scaled by Home's GDP (on percent). Liabilities (*lia*) are scaled by sum of GDP over all foreign countries on the Rest of World (on percent). *Invest2y* is ratio of Gross capital formation over GDP (on percent).

Table 2.7: Fixed-effect regressions of capital flows per capita: 1990Q1-1999Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Blncp	BOutpc	FDIInpc	FDIOutpc	PEqtInpc	PEqtOutpc	Olncp	OOutpc	negDNFApc	negCApc	KGpc
A2h	-110.8 (104.8)	174.9** (68.31)	130.5*** (40.54)	115.5*** (32.74)	692.8*** (237.6)	225.8** (92.76)	177.8** (88.83)	178.3** (76.00)	123.9 (150.0)	-16.80*** (2.161)	-143.9 (150.5)
RREU	-535.7* (268.5)	106.0 (174.9)	-217.1** (106.5)	-34.59 (86.05)	-411.7 (611.5)	352.7 (238.2)	313.5 (233.4)	91.32 (199.7)	-81.87 (409.3)	-2.452 (4.698)	108.2 (410.4)
Constant	17,679* (9,891)	-11,250* (6,445)	-4,982 (3,853)	-5,715* (3,112)	-44,432* (22,616)	-17,941** (8,832)	-5,826 (8,444)	-5,366 (7,223)	-7,854 (14,102)	1,387*** (200.6)	9,148 (14,142)
Observations	56	56	72	72	63	60	72	72	47	220	47
R-squared	0.104	0.251	0.558	0.427	0.427	0.122	0.065	0.164	0.068	0.300	0.093
Countries	6	6	8	8	7	7	8	8	2	10	2

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone, from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (E), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Capital flows are expressed at chain linked volumes 2010, euro per capita. *In* denotes for the inflows, *Out* denotes for the outflows. Net flows = Inflows - Outflows. Regression with Money market instrument and Financial derivatives are insignificant, so they are dropped to save space. $KGpc = DNFApc - CApc$.

safe assets accumulation and Other types of investment in the sample before 2008. After 2008, Table 2.13 proves that this pattern is the same but we can not detect the source of outflows. In sum, the role of productivity level on the net total capital outflows is the same by two ways of dividing the sample, but we can only trace down the source of outflows by using 2000.

2.B Proofs

Proposition 2.2.1. The method to solve the stochastic optimization problem can be reached in Samuelson (1969), Merton (1969) or Turnovsky (1997). In our model, we are looking for only the interior solution.

Proof. The representative consumer's stochastic optimization problem is to choose

Table 2.8: Fixed-effect regressions of portfolio shares on Home's productivity: 2000Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	B2y	Invest2y	FDI2ROWlia	PEqt2ROWlia	B2ROWlia	M2ROWlia	Fin2ROWlia	O2ROWlia
A2h	1.665 (1.658)	-0.127*** (0.0151)	0.0539* (0.0286)	0.0547*** (0.0165)	0.0598** (0.0246)	0.00332 (0.00354)	0.0583** (0.0249)	0.0933*** (0.0157)
RREU	14.21** (6.218)	-0.785*** (0.0648)	-0.277** (0.108)	0.0325 (0.0645)	-0.256*** (0.0966)	-0.0543*** (0.0139)	-0.317*** (0.0932)	0.313*** (0.0593)
Constant	220.2 (166.8)	38.70*** (1.533)	5.789** (2.885)	2.672 (1.656)	6.837*** (2.470)	1.113*** (0.355)	-1.764 (2.506)	5.089*** (1.585)
Observations	697	1,016	724	815	818	818	745	753
R-squared	0.009	0.159	0.015	0.014	0.017	0.020	0.025	0.074
Countries	17	18	18	18	18	18	18	18

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone. Data on quarterly from 1990Q1 to 2014Q4 from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Assets (*as*) are scaled by Home's GDP (on percent). Liabilities (*lia*) are scaled by sum of GDP over all foreign countries on the Rest of World (on percent). *Invest2y* is ratio of Gross capital formation over GDP (on percent).

Table 2.9: Fixed-effect regressions of capital flows per capita: 2000Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	BInpc	BOutpc	FDIInpc	FDIOutpc	PEqtInpc	PEqtOutpc	OInpc	OOutpc	negDNFApc	negCApc	KGpc
A2h	-490.0 (322.1)	555.9*** (184.2)	-930.6 (1.274)	-670.8 (944.3)	817.3 (535.0)	898.0** (402.2)	971.3*** (355.8)	898.4** (399.0)	-10.60 (29.37)	-4.547*** (1.467)	4.196 (29.33)
RREU	-2.597** (1.208)	-293.0 (690.8)	-9.734** (4.795)	-6.849* (3.553)	1.150 (2.092)	791.5 (1.508)	4.309*** (1.342)	3.306** (1.500)	26.71 (111.0)	-27.12*** (6.271)	-58.17 (110.8)
Constant	-19.277 (32.402)	82.147*** (18.533)	306.584** (128.348)	261.504*** (95.112)	118.617** (53.703)	8.904 (40.507)	12.377 (35.824)	45.427 (40.217)	810.5 (2.956)	505.0*** (148.4)	-134.6 (2.953)
Observations	697	697	724	724	815	746	753	750	629	988	629
R-squared	0.010	0.014	0.006	0.006	0.003	0.007	0.022	0.012	0.000	0.025	0.001
Number of countries	17	17	18	18	18	18	18	18	17	18	17

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone, from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (E), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Capital flows are expressed at chain linked volumes 2010, euro per capita. *In* denotes for the inflows, *Out* denotes for the outflows. Net flows = Inflows - Outflows. Regression with Money market instrument and Financial derivatives are insignificant, so they are dropped to save space. $KGpc = DNFApc - CApc$.

Table 2.10: Fixed-effect regressions of portfolio shares on Home's productivity: 1990Q1-2007Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	B2y	Invest2y	FDI2ROWlia	PEqt2ROWlia	B2ROWlia	M2ROWlia	Fin2ROWlia	O2ROWlia
A2h	0.790 (1.129)	0.0439*** (0.0164)	0.0873*** (0.0136)	0.0934*** (0.0215)	0.0547** (0.0234)	0.00912*** (0.00205)	0.0178*** (0.00409)	0.120*** (0.0213)
RREU	-8.921 (5.946)	0.0402 (0.0661)	0.0769 (0.0722)	-0.0595 (0.121)	0.0228 (0.132)	0.00649 (0.0116)	-0.0156 (0.0230)	0.469*** (0.113)
Constant	374.6*** (124.4)	19.83*** (1.739)	-2.120 (1.498)	-1.647 (2.383)	1.967 (2.592)	-0.218 (0.228)	-1.024** (0.455)	-1.605 (2.346)
Observations	344	832	382	426	429	429	372	391
R-squared	0.024	0.013	0.144	0.084	0.020	0.066	0.105	0.080
Countries	17	18	18	18	18	18	18	18

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone. Data on quarterly from 1990Q1 to 2014Q4 from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Assets (*as*) are scaled by Home's GDP (on percent). Liabilities (*lia*) are scaled by sum of GDP over all foreign countries on the Rest of World (on percent). *Invest2y* is ratio of Gross capital formation over GDP (on percent).

Table 2.11: Fixed-effect regressions of capital flows per capita: 1990Q1-2007Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	BInpc	BOutpc	FDIInpc	FDIOutpc	PEqtInpc	PEqtOutpc	OInpc	OOutpc	negDNFApc	negCApc	KGpc
A2h	-433.5* (233.6)	939.0*** (343.0)	1,134*** (366.5)	1,165*** (383.5)	2,619** (1,016)	2,464*** (846.0)	1,890*** (527.0)	2,262*** (683.7)	-11.93 (34.73)	-3.491* (1.864)	11.80 (34.93)
RREU	-1,540 (1,230)	2,219 (1,806)	4,206** (1,948)	4,768** (2,039)	12,311** (5,743)	11,604*** (4,439)	8,374*** (2,803)	10,489*** (3,635)	70.40 (189.3)	-5.055 (8.002)	-6.303 (190.3)
Constant	-28,574 (25,732)	14,761 (37,778)	-30,469 (40,458)	-31,424 (42,339)	-144,342 (112,693)	-196,650** (93,470)	-111,099* (58,178)	-141,863* (75,521)	915.3 (3,826)	287.2 (196.8)	-1,340 (3,848)
Observations	344	344	382	382	426	372	391	388	291	760	291
R-squared	0.010	0.025	0.026	0.025	0.017	0.026	0.035	0.031	0.003	0.005	0.001
Countries	17	17	18	18	18	18	18	18	14	18	14

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone, from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (E), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Capital flows are expressed at chain linked volumes 2010, euro per capita. *In* denotes for the inflows, *Out* denotes for the outflows. Net flows = Inflows - Outflows. Regression with Money market instrument and Financial derivatives are insignificant, so they are dropped to save space. $KGpc = DNFApc - CApc$.

Table 2.12: Fixed-effect regressions of portfolio shares on Home's productivity: 2008Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	B2y	Invest2y	FDI2ROWlia	PEqt2ROWlia	B2ROWlia	M2ROWlia	Fin2ROWlia	O2ROWlia
A2h	0.245 (2.745)	-0.0987*** (0.0334)	0.0226 (0.0446)	0.0711*** (0.0233)	-0.0261 (0.0242)	-0.00950** (0.00468)	0.0801 (0.0493)	-0.0113 (0.0221)
RREU	15.83** (6.484)	-0.507*** (0.0804)	-0.264** (0.105)	-0.0985* (0.0564)	-0.342*** (0.0586)	-0.0324*** (0.0113)	-0.240** (0.117)	0.217*** (0.0525)
Constant	312.1 (281.9)	33.35*** (3.448)	10.72** (4.574)	2.199 (2.405)	18.69*** (2.499)	2.669*** (0.482)	-3.083 (5.057)	17.56*** (2.270)
Observations	409	468	414	452	452	452	434	434
R-squared	0.015	0.088	0.017	0.032	0.073	0.024	0.019	0.042
Countries	17	18	18	18	18	18	18	18

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone. Data on quarterly from 1990Q1 to 2014Q4 from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (PEqt), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Assets (*as*) are scaled by Home's GDP (on percent). Liabilities (*lia*) are scaled by sum of GDP over all foreign countries on the Rest of World (on percent). *Invest2y* is ratio of Gross capital formation over GDP (on percent).

Table 2.13: Fixed-effect regressions of capital flows per capita: 2008Q1-2014Q4

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	BInpc	BOutpc	FDIInpc	FDIOutpc	PEqtInpc	PEqtOutpc	OInpc	OOutpc	negDNFApc	negCApc	KGpc
A2h	-393.7 (546.3)	229.6 (266.5)	-279.3 (2,078)	-255.1 (1,438)	1,174* (604.7)	787.1* (457.8)	186.3 (645.2)	339.5 (631.1)	-64.27 (58.76)	-11.94*** (2.992)	51.20 (58.51)
RREU	-2,631** (1,291)	-1,129* (629.5)	-11,405** (4,919)	-7,995** (3,404)	-960.6 (1,460)	-885.3 (1,086)	3,208** (1,531)	2,363 (1,497)	-2,392 (143.5)	-30.16*** (7.281)	-27.36 (142.9)
Constant	-18,386 (56,113)	119,628*** (27,367)	296,191 (213,297)	261,868* (147,579)	109,375* (62,323)	24,959 (46,994)	96,310 (66,222)	109,446* (64,777)	6,302 (6,040)	1,293*** (308.4)	-4,905 (6,014)
Observations	409	409	414	414	452	434	434	434	385	448	385
R-squared	0.011	0.011	0.014	0.014	0.011	0.010	0.010	0.006	0.003	0.062	0.002
Countries	17	17	18	18	18	18	18	18	17	18	17

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Samples: 19 countries in Eurozone, from Eurostat. Categories of assets includes Direct investment (FDI), Portfolio equities (E), Bonds and Notes (B), Money market instrument (M), Financial derivatives (Fin) and Other investment (O). Capital flows are expressed at chain linked volumes 2010, euro per capita. *In* denotes for the inflows, *Out* denotes for the outflows. Net flows = Inflows - Outflows. Regression with Money market instrument and Financial derivatives are insignificant, so they are dropped to save space. $KGpc = DNFApc - CApc$.

consumption and portfolio shares to maximize

$$E \int_0^{\infty} \frac{1}{\gamma} c^\gamma e^{-\beta t} dt, \quad -\infty < \gamma < 1$$

subject to the stochastic wealth accumulation equation.

$$\begin{aligned} \frac{dw}{w} &= \left(\rho - \frac{c}{w}\right) dt + d\bar{w} \\ 1 &= n^d + n^{d,*} + n^b \\ \rho &= an^d + a^* n^{d,*} + rn^b \\ dw &= n^d a \sigma dz + n^{d,*} a^* \sigma^* dz^* \\ \sigma_{\bar{w}}^2 &= n_d^2 a^2 \sigma^2 + n_d^{*2} a^{*2} \sigma^{*2} \end{aligned}$$

We define the differential generator of the value function $V(w, t)$ by:

$$L_w[V(w, t)] := \frac{\partial V}{\partial t} + \left(\rho - \frac{c}{w}\right) w \frac{\partial V}{\partial w} + \frac{1}{2} \sigma_{\bar{w}}^2 w^2 \frac{\partial^2 V}{\partial w^2} \quad (2.B.1)$$

Given the exponential time discounting, V can be assumed to be of the time-separable form:

$$V(w, t) = e^{-\beta t} X(w)$$

The formal optimization problem is now to choose c, n_d, n_d^*, n_B to maximize the Lagrangean expression:

$$e^{-\beta t} \frac{1}{\gamma} c^\gamma + L_w[e^{\beta t} X(w)] + e^{-\beta t} \eta [1 - n_d - n_d^* - n_B]$$

Taking partial derivatives of this expression, and canceling $e^{-\beta t}$ yields:

$$\begin{aligned} (c) &: c^{\gamma-1} = X_w \\ (n^d) &: (aX_w w - \eta) dt + cov(d\bar{w}, a\sigma dz) X_{ww} w^2 = 0 \\ (n^{d,*}) &: (a^* X_w w - \eta) dt + cov(d\bar{w}, a^* \sigma^* dz^*) X_{ww} w^2 = 0 \\ (n^b) &: rX_w w - \eta = 0 \\ 1 &= n^d + n^{d,*} + n^b \end{aligned}$$

These equations determine the optimal values for $c/w, n^d, n^{d,*}, n^b, \eta$ as functions of X_w, X_{ww} . In addition, the value function must satisfy the Bellman equation:

$$\max_{c, n^d, n^{d,*}, n^b} \left\{ \frac{1}{\gamma} c^\gamma e^{-\beta t} + L_w[e^{-\beta t} X(w)] \right\} = 0 \quad (2.B.2)$$

We postulate a solution of the form:

$$X(w) = \delta w^\gamma$$

Where δ is the coefficient to be determined. Then, $X_w = \delta\gamma w^{\gamma-1}$; $X_{ww} = \delta\gamma(\gamma - 1)w^{\gamma-2}$. The F.O.Cs become:

$$\begin{aligned} (c) & : \frac{c}{w} = (\delta\gamma)^{1/(\gamma-1)} \\ (n^d) & : (\alpha\delta\gamma w^\gamma - \eta)dt + cov(d\bar{w}, \alpha\sigma dz)\delta\gamma(\gamma - 1)w^\gamma = 0 \\ (n^{d,*}) & : (\alpha^*\delta\gamma w^\gamma - \eta)dt + cov(d\bar{w}, \alpha^*\sigma^* dz^*)\delta\gamma(\gamma - 1)w^\gamma = 0 \\ (n^b) & : r\delta\gamma w^\gamma - \eta = 0 \end{aligned}$$

Substituting for $\left(\frac{c}{w}\right)$ into the Bellman equation (2.B.2) and the value function (2.B.1) leads to:

$$\begin{aligned} \frac{1}{\gamma}[(\delta\gamma)^{1/(\gamma-1)}w]^\gamma - \beta\delta w^\gamma + [\hat{\rho} - (\delta\gamma)^{1/(\gamma-1)}]w\delta\gamma w^{\gamma-1} + \frac{1}{2}\hat{\sigma}_{\bar{w}}^2 w^2 \delta\gamma(\gamma - 1)w^{(\gamma-2)} &= 0 \\ \Rightarrow \left(\frac{\hat{c}}{w}\right) = (\delta\gamma)^{1/(\gamma-1)} = \frac{\beta - \hat{\rho}\gamma - \frac{1}{2}\gamma(\gamma - 1)\hat{\sigma}_{\bar{w}}^2}{1 - \gamma} \end{aligned}$$

The F.O.Cs dividing by $(\delta\gamma w^\gamma)$:

$$\begin{aligned} (n^b) & : r = \frac{\eta}{\delta\gamma w^\gamma} \\ (n^d) & : \left(a - \frac{\eta}{\delta\gamma w^\gamma}\right) dt = (1 - \gamma)cov(d\bar{w}, a\sigma dz) \\ (n^{d,*}) & : \left(a^* - \frac{\eta}{\delta\gamma w^\gamma}\right) dt = (1 - \gamma)cov(d\bar{w}, a^*\sigma^* dz^*) \end{aligned}$$

By substitute the equilibrium value of $\left(\frac{\hat{c}}{w}\right)$ into the Bellman equation, we find:

$$\delta = \frac{1}{\gamma} \left(\frac{\hat{c}}{w}\right)^{\gamma-1}$$

From this system of F.O.Cs, and with the country-specific shocks $(cov(dz, dz^*) =$

0), we get the macroeconomic equilibrium in Home country:

$$\begin{aligned}
n^d &= \frac{1}{(1-\gamma)} \frac{(a-r)}{a^2\sigma^2} \\
n^{d,*} &= \frac{1}{(1-\gamma)} \frac{(a^*-r)}{a^{*2}\sigma^{*2}} \\
n^b &= 1 - n^d - n^{d,*} \\
\frac{c}{w} &= \frac{1}{1-\gamma} \left(\beta - \hat{\rho}\gamma + \frac{1}{2}\gamma(1-\gamma)\hat{\sigma}_w^2 \right) \\
\psi &= \frac{1}{(1-\gamma)} \left(\hat{\rho} - \beta - \frac{1}{2}\gamma(1-\gamma)\hat{\sigma}_w^2 \right) \\
\hat{\rho} &= an^d + a^*n^{d,*} + rn^b \\
\sigma_w^2 &= n^{d2}a^2\sigma^2 + n^{d,*2}a^{*2}\sigma^{*2}
\end{aligned}$$

Similar steps for the Foreign country.

Finally, the safe assets market clearing condition ($b + b^* = n^b w + n^{b,*} w^* = \bar{b}$) implies the safe interest rate as:

$$r = \frac{[1/(a\sigma^2) + 1/(a^*\sigma^{*2})] + (1-\gamma)[\bar{b}/(w+w^*) - 1]}{1/(a^2\sigma^2) + 1/(a^{*2}\sigma^{*2})} \quad (2.B.3)$$

■

Proposition 2.4.1

Proof.

$$\begin{aligned}
CA &= (an_d w + a^*n_d^* w + rn_B w - \frac{c}{w}w - n_d w \psi - n_f w^* \psi^*)dt \\
&\quad + [(1-n_d)n_d w + (1-n_f)n_f w^*]a\sigma dz - [n_d n_d^* w + n_f n_f^* w^*]a^* \sigma^* dz^* \\
KG &= k_d^* a \sigma^* dz^* - k_f \alpha \sigma dz = n_d^* w a^* \sigma^* dz^* - n_f w^* a \sigma dz
\end{aligned}$$

With the assumption that $Cov(dz, dz^*) = 0$,

$$\begin{aligned}
Cov(CA, KG) &= [-n_d^* w (n_d n_d^* w + n_f n_f^* w^*) a^* \sigma^{*2} - n_f w^* ((1-n_d)n_d w + (1-n_f)n_f w^*) a^2 \sigma^2] dt < 0 \\
Var(CA) &= [((1-n_d)n_d w + (1-n_f)n_f w^*)^2 a^2 \sigma^2 + (n_d n_d^* w + n_f n_f^* w^*)^2 a^{*2} \sigma^{*2}] dt \\
Var(KG) &= [n_d^{*2} w^2 a^{*2} \sigma^{*2} + n_f^2 w^2 a^2 \sigma^2] dt \\
corr(CA, KG) &= \frac{cov(CA, KG)}{\sqrt{Var(CA)}\sqrt{Var(KG)}} < 0
\end{aligned}$$

■

2.C Extended model: Safe assets scarcity

We assume that the exogenous supply of safe assets is different across countries. In each country, the share of safe assets over the domestic wealth is different. This share can be interpreted as the capitalization rate of financial assets over domestic income as in Farhi, Caballero and Gourinchas (2008). Indeed, an advanced economy have a higher capitalization rate than the developing economies. Since the capitalization determines the autarky safe interest rate, it shapes the pattern of capital flows. In comparison with an exogenous total supply of safe assets in the main text, this extended model can capture the pattern of capital flows between the advanced economy (Home) and the developing economy (Foreign).

The structure of model is almost the same as in the main text. The only difference is that the exogenous supply of safe assets is δw at Home while $\delta^* w^*$ at Foreign. The key feature is that Home has a higher supply of safe assets:

$$\delta > \delta^* \tag{2.C.1}$$

Autarky equilibrium

At autarky, each agent only allocate her wealth between the domestic risky asset and the safe assets. We focus on the Home economy, but the analysis for Foreign is similar. The equilibrium portfolio shares are:

$$\begin{aligned} n^d &= \frac{1}{1 - \gamma} \frac{a - r}{a^2 \sigma^2} \\ n^b &= 1 - n^d \end{aligned}$$

The market clearing condition on the safe assets is as following:

$$n^b w = \delta w \Leftrightarrow 1 - \frac{1}{1 - \gamma} \frac{a - r^H}{a^2 \sigma^2} = \delta$$

Therefore, the Home autarky safe interest rate is:

$$r^H = a - (1 - \gamma)(1 - \delta)a^2 \sigma^2 \tag{2.C.2}$$

It is clear that the autarky safe interest rate is increasing on the capitalization rate over output (δ): given the demand, a higher supply of safe assets reduces the safe asset's price and therefore, depresses the interest rate.

Similarly, the autarky safe interest rate in the Foreign economy is:

$$r^F = a^* - (1 - \gamma)(1 - \delta^*)a^{*2} \sigma^{*2} \tag{2.C.3}$$

Integration equilibrium

At integration, each agent can invest on the foreign risky asset. The equilibrium portfolio shares are the similar to the model in the main text, but only the equilibrium safe interest rate is modified. In details, one world interest rate (r^w) prevails to clear the world market for safe assets.

$$\begin{aligned} n^d = n^f &= \frac{1}{1-\gamma} \frac{\alpha - r^w}{a^2 \sigma^2} \\ n^{d,*} = n^{f,*} &= \frac{1}{1-\gamma} \frac{\alpha^* - r^w}{a^{*2} \sigma^{*2}} \\ n^b = n^{b,*} &= 1 - n^d - n^{d,*} \end{aligned}$$

The world market clearing condition equalizes the aggregate demand to the aggregate supply of safe assets.

$$n^b w + n^{b,*} w^* = \delta w + \delta^* w^*$$

Therefore, the world safe interest rate is as following:

$$r^w = \frac{[1/(a\sigma^2) + 1/(a^*\sigma^{*2})] - (1-\gamma)(1-\bar{\delta})}{1/(a^2\sigma^2) + 1/(a^{*2}\sigma^{*2})} \quad (2.C.4)$$

whereby, the world supply of safe assets is an average value of safe assets supplies across two countries:

$$\bar{\delta} \equiv \frac{w}{w+w^*} \delta + \frac{w^*}{w+w^*} \delta^* \Rightarrow \delta > \bar{\delta} > \delta^* \quad (2.C.5)$$

We can show that the world interest rate is the linear combination of the autarky safe interest rate. Therefore, the world interest rate is between the highest and lowest autarky interest rate. For $j = H, F$,

$$r^w = \nu + \sum_j \lambda^j \sum_j \mu^j r^j \Rightarrow r^H > r^w > r^F \quad (2.C.6)$$

whereby,

$$\begin{aligned} \nu &\equiv \left(\frac{w^*}{w+w^*} a a^{*2} \sigma^{*2} + \frac{w}{w+w^*} a^* a^2 \sigma^2 \right) / (a^2 \sigma^2 + a^{*2} \sigma^{*2}) \\ \lambda^H &\equiv \frac{a^{*2} \sigma^{*2}}{a^2 \sigma^2 + a^{*2} \sigma^{*2}} \\ \lambda^F &\equiv \frac{a^2 \sigma^2}{a^2 \sigma^2 + a^{*2} \sigma^{*2}} \\ \mu^H &\equiv \frac{w}{w+w^*} \\ \mu^F &\equiv \frac{w^*}{w+w^*} \end{aligned}$$

Exporter of Safe assets

At the world integration, the difference between the supply and demand of safe assets in each country is evaluated at the world safe interest rate:

$$\delta w - n^b w = \delta - \bar{\delta} > 0 \quad (2.C.7)$$

$$\delta^* w^* - n^{b,*} w^* = \delta^* - \bar{\delta} < 0 \quad (2.C.8)$$

Since Home has a higher capitalization of safe assets ($\delta > \bar{\delta} > \delta^*$), its supply of asset is greater than its domestic demand at the integration. Home economy exports the safe assets to the Foreign. Therefore, the Home agent enjoys the positive return on the safe assets which is paid by the Foreign one⁵.

$$r^w(\delta - n^b)w > 0 \quad (2.C.9)$$

On this model, the Home enjoys a positive capital gains by a comparative advantage on providing the safe assets, a mechanism is emphasized by Farhi, Caballero and Gourinchas (2008).

2.D Extended model: Exorbitant privilege

We allow the supply of safe assets is endogenously determined at equilibrium. The idea is employed from Farhi and Maggiori (2016) where one economy with risk-neutral utility issues the bonds to the rest of world whose a risk-averse utility. We extend the model in the main text by allowing the Home agent is a less risk averse than Foreign. We will show that this difference on the risk-taking behavior results in the positive supply of safe assets at equilibrium.

The structure is the same as the model in the main text. Now, the Home agent has a less risk averse coefficient than the Foreign one.

$$(1 - \gamma) < (1 - \gamma^*) \quad (2.D.1)$$

Equilibrium

The equilibrium portfolio shares for Home are as following: The world market clearing condition for the safe assets captures the zero net supply at equilibrium, i.e, total supply is exactly equal to total demand for safe assets.

$$n^b w + n^{b,*} w^* = 0 \quad (2.D.2)$$

⁵By world market clearing condition (2.C.5), $r^w(\delta - n^b)w = r^w(n^{b,*} - \delta^*)$

$$\begin{array}{ll}
\text{Home} & \text{Foreign} \\
n^d = \frac{1}{1-\gamma} \frac{a-r}{a^2\sigma^2}; & n^f = \frac{1}{1-\gamma^*} \frac{a-r}{a^2\sigma^2} \\
n^{d,*} = \frac{1}{1-\gamma} \frac{a^*-r}{a^{*2}\sigma^{*2}}; & n^{f,*} = \frac{1}{1-\gamma^*} \frac{a^*-r}{a^{*2}\sigma^{*2}} \\
n^b = 1 - n^d - n^{d,*}; & n^{b,*} = 1 - n^f - n^{f,*}
\end{array}$$

Plugging the portfolio share $(n^b, n^{b,*})$ into the market clearing condition, we end up with the safe interest rate:

$$r = \frac{\left(\frac{w}{1-\gamma} + \frac{w^*}{1-\gamma^*}\right)\left(\frac{1}{a\sigma^2 + a^*\sigma^{*2}}\right) - (w + w^*)}{\left(\frac{w}{1-\gamma} + \frac{w^*}{1-\gamma^*}\right)\left(\frac{1}{a^2\sigma^2} + \frac{1}{a^{*2}\sigma^{*2}}\right)} \quad (2.D.3)$$

For the positive safe interest rate, the Home agent needs to be risk neutral enough.

$$r > 0 \Leftrightarrow (1-\gamma) < \xi(\gamma^*) \equiv \frac{w}{(w+w^*)\left(\frac{a\sigma^2 + a^*\sigma^{*2}}{a\sigma^2 + a^*\sigma^{*2}}\right) - \frac{w^*}{1-\gamma^*}} \quad (2.D.4)$$

Whereby, the threshold is decreasing on $(1-\gamma^*)$: $\frac{\partial \xi(\gamma^*)}{\partial (1-\gamma^*)} < 0$.

The threshold is intuitive since more risk averse the Foreign agent is (higher $(1-\gamma^*)$), more risk neutral the Home agent should be: $(1-\gamma)$ needs to be lower. This condition would ensure that Home economy would supply the safe assets to Foreign economy. This result is similar to the model by Caballero and Farhi (2014), in which one Neutral agent with the risk-neutral utility issues the safe assets to the Knightian agent with infinite risk averse coefficient, so that the condition (2.D.4) is always satisfied.

Exporter of safe assets

At the integration equilibrium, the Home agent less risk averse ($\gamma > \gamma^*$) would issues the safe asset to the Foreign agent. This intuition follows by the result that:

$$\gamma > \gamma^* \Leftrightarrow (B = n^b w < 0; B^* = -B = n^{b,*} w^* > 0) \quad (2.D.5)$$

It means that the Home agent invests the risky assets more than her wealth while the Foreign investment on the risky assets is less than her wealth.

$$K^d + K^{d,*} = W - B > W \quad (2.4.6)$$

$$K^f + K^{f,*} = W^* - B^* < W^* \quad (2.4.7)$$

The Home economy borrows a fraction of B^* from the Foreign one to invest on the risky assets. With the constant portfolio shares, the differential rate of returns that Home agent gains is:

$$B^*(an^d + a^*n^{d,*} - r) \quad (2.4.8)$$

Plugging the equilibrium portfolio share and the safe interest rate (2.D.3), after some manipulation, we end up with:

$$B^* \left[\frac{1}{(1-\gamma)} \frac{(a-a^*)^2}{(a^2\sigma^2 + a^{*2}\sigma^{*2})} + \frac{\frac{1}{a\sigma^2} + \frac{1}{a^*\sigma^{*2}}}{\frac{1}{a^2\sigma^2} + \frac{1}{a^{*2}\sigma^{*2}}} \frac{w^*(\gamma-\gamma^*)}{(1-\gamma^*)w + (1-\gamma)w^*} + \frac{(w+w^*)(\frac{1}{a^2\sigma^2} + \frac{1}{a^{*2}\sigma^{*2}})^{-1}}{(\frac{w}{1-\gamma} + \frac{w^*}{1-\gamma^*})} \right] > 0 \quad (2.4.9)$$

This term is coined as the exorbitant privilege (Gourinchas, Rey and Govillot (2010)) which is the rent that one country with the reserve currency (like United States) can gain by borrowing with low cost from the Rest of World in order to invest on the risky assets with higher rate of returns. The exorbitant rent is increasing on the value of safe assets (B^*). The rent is also increasing on the Foreign's coefficient of relative risk averse since, when the Foreign agent become more risk averse, she demands more safe assets. And the Home agents can enjoy a higher rent on issuing the safe assets. Finally, the rent is also increasing on the risky rate of returns while decreasing on the safe interest rate. Therefore, a higher differential rates of return between the risky and safe assets raises the exorbitant privilege.

The exorbitant rent for the Home agent arises in our model because of two reasons. The first reason is the heterogeneity in the risk aversion across countries. The Home agent is less risk averse than the Foreign one. Therefore, she is provider of safe assets at equilibrium. This intuition is captured by the positive term $(\gamma - \gamma^*)$. The Foreign agent is more risk averse, then she demands the safe assets at equilibrium. More risk averse the Foreign agent is, more demand for safe assets and higher safe premium that she has to pay for the Home agent. This intuition is captured by the term $(1 - \gamma^*)$. The second reason is the enriched portfolio with two risky assets and one safe assets. As long as there is difference on the rate of returns between the Home and Foreign risky assets, the risk diversification would help the Home agent to allocate her wealth by the most efficient way. This reason is captured by the positive term $(a - a^*)^2$. In sum, the Home agent earns the rent on her investment as the award to her higher risk-taking attitude in comparison with the Foreign counterpart.

Chapter 3

Non-Linear Pattern of International Capital Flows

Abstract

The empirical analysis on one panel sample of 165 countries from 1980 to 2013 reveals one non-linear pattern of dependence of the net total capital inflows on the productivity growth. We rationalize this new finding by building up one multi-country OLG economy to show that there exists one threshold over which an increase of the productivity growth raises the net total capital inflows as implied by the Neo-Classical growth model. Below that threshold, however, the reversal pattern holds (allocation puzzle). The financial frictions and the world average productivity growth rate are the key elements of the threshold.

3.1 Introduction

The past decades witnesses a diversification of the pattern of the international capital flows across the developing and advanced economies (Figure 3.1.1). On one hand, the exporters of capital can be the advanced economies with the low growth rates (Japan (JPN), Germany (DEU), Netherlands (NLD)) or emerging economies with the high growth rates (China (CHN), Korea (KOR), Singapore (SGP)). On other hand, the importers of capital can be the slow-growing advanced economies (United States (USA), United Kingdoms (GBR), Australia (AUS)) or the fast-growing emerging economies (Cyprus (CYP), Thailand (THA), Pakistan (PAK)). The mixed patterns of the net total capital inflows raise the concern over the linear dependence of the net total capital inflows on the productivity growth rate.

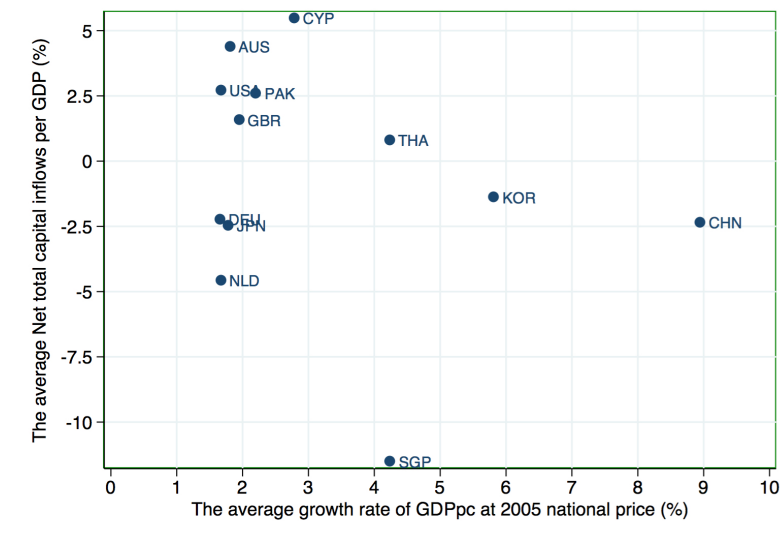


Figure 3.1.1: Pattern of International capital flows by countries

Notes: All variables are averaged over 1980-2013. The net total capital inflows is the negative value of the current account as percentage of GDP and GDPpc is real output-side GDP per capita at constant 2005 national price.

The Neo-Classical growth model predicts that one country growing faster would invest more and experience a net total capital inflow. The recent empirical evidence¹ documents that the fast-growing economies tend to experience a net total capital outflow (Allocation puzzle). Our paper rationalizes both of these two arguments

¹Prasad, Rajan, and Subramanian (2007), Gourinchas and Jeanne (2013)

by showing that the pattern of the net total capital inflows on the productivity growth rate is non-linear: the allocation puzzle exists but there is one threshold over which the prediction by the Neo-Classical growth model works.

We prove the non-linear pattern of international capital flows by both the empirical evidences and the theoretical model. On empirical ground, the analysis of a sample of 165 countries from 1980 to 2013 shows that the allocation puzzle only applies for the middle range of the productivity growth rate. For the low and high range of the productivity growth rate, however, an increase of the growth rate raises the net total capital inflows. In sum, the empirical evidence demonstrates that there exists one threshold over which the net total capital inflow is increasing on the productivity growth rate.

On theoretical ground, we construct one multi-country economy where each country differs by the level of productivity growth and the financial frictions. At autarky, the interest rate depends on the domestic productivity growth rate and financial frictions. At integration, the interest rate is equalized across countries and depends on the world average growth rate and the financial frictions. With the free mobile capital, the capital flows into the country which has a higher interest rate at autarky than at integration. This condition turns out to be one threshold only over which there exists the positive relationship between the net total capital inflows and the productivity growth rate, as implied by the Neo-Classical growth model. In sum, the interaction between the productivity growth rate and the financial frictions can generate one non-linear dependence of the net total capital inflows on the productivity growth rate.

Our paper belongs to the literature on the relationship between the international capital flows and the productivity growth rate (see Gourinchas and Rey (2015) for a recent survey). Lucas (1990) questions why the capital does not flow to the poor countries in which, because of the capital scarcity, the marginal product of capital is higher than in the advanced economies. Gourinchas and Jeanne (2013) raises a deeper question why the capital does not flow to the emerging economies which grow faster and invest more than the advanced economies. Their models, however, are the small open economies. Therefore, there is no implication for the pattern of capital flows across the developing and advanced economies. Moreover, the direction of capital flows is linear in the sense that the net total capital flows can either flow into or out of a small open economy. Our paper proposes an extension of these models to the multi-country economy which takes into account the interaction of the productivity growth rate and the financial friction levels. Therefore, we can build up one threshold on the productivity growth rate to ac-

count for the non-linear pattern of the international capital flows.

There are also many multi-country models to account for the dependence of international capital flows on the productivity growth. In Gertler and Rogoff (1990), the capital market imperfections are endogenously determined by the asymmetric information problem which is less severe in the rich countries. Therefore, the domestic loan rate is lower in the rich than in the poor economies, which implies that the investment demand is higher in the rich than in the poor. In Boyd and Smith (1997), the rich economy also has a higher investment demand because its high income can afford for the agency cost induced by the asymmetric information on lending. In Matsuyama (2004), the rich country having the higher initial capital stock would converge to a higher steady state capital accumulation and also has a higher investment demand than the poor economies. This role of the initial capital stock is quite similar as in Dechert and Nishimura (2012): with an unconvex technology, an optimal path converges to a steady state only if the initial capital stock is above a critical value. The main mechanism of these models is that the capital flows from the poor to rich economies to finance a higher investment demand in the rich economies. Our paper takes into account the differential investment demands but focuses more on the differential saving supplies as the driver of the international capital flows.

Farhi, Caballero and Gourinchas (2008) focus on the savings of the developing economies which have a higher output growth rate than the advanced economies. The supply of financial assets, however, is lower in the developing than in the advanced economies. The low supply of financial assets, in turn, results in a lower interest rate in the developing economies. Therefore, the savings flows from the developing to the advanced economies on seeking the financial assets. While their paper focuses on the supply side, ours focuses on the demand side for the financial asset. Indeed, on country with a low saving wedge (i.e, a low capital taxation), the savings is high and then, the demand for the assets is high. The high demand raises the asset's price and depresses the autarky interest rate. At integration, the capital flows out of that country for a higher foreign interest rate. Therefore, in our model, the heterogeneity on the saving wedge shapes the pattern of international capital flows.

Coeurdacier, Guibaud and Jin (2015) analyze the differential age-saving profiles between the developing (China) and advanced economy (United States) by focusing the borrowing constraint which is tighter on the former. The aging population in China raises the savings by the middle-aged people more than the borrowing by the young people. The case, however, is reversal in the United States. There-

fore, there is excess saving in China and capital flows to US for the interest rate is equalized across countries. The excess saving in the developing economies at autarky is also the key mechanism in Angeletos and Panousi (2011), in which the high level of the idiosyncratic risk motivates the precautionary saving in the developing economies. Our model also generates the high saving rate in the developing economies, but the autarky capital accumulation is not necessary high in the developing economies like the aforementioned papers. In particular, one economy with a low saving wedge and high investment wedge can have both the high savings and low capital accumulation.

The paper is structured as follows. After the introduction, section (3.2) provides the empirical evidence on the non-linear pattern of international capital flows. Then, section (3.3) lays out the model and characterize the equilibrium. Section (3.4) presents the core result on the pattern of international capital flows across countries. And section (3.5) concludes and is followed by the Appendix.

3.2 Empirical analysis

Table 3.1: Fixed-effect regression of the long-run net total capital inflows: 65 Non-OECD countries (1980-2013)

VARIABLES	(1) anegCA2y	(2) anegCA2y	(3) anegCA2y
Output per capita growth rate (<i>aGDPpcgrowth</i>)	-0.825*** (0.295)	-	-
Saving wedge (<i>atau</i>)	-	8.155** (3.824)	-
Investment wedge (<i>agammaPL</i>)	-	-	-0.709*** (0.256)
Openness (<i>akaopen</i>)	-1.399*** (0.518)	-1.836*** (0.546)	-1.118* (0.621)
Constant	3.902*** (0.789)	-1.213 (1.804)	1.723*** (0.593)
Observations	65	63	48
R-squared	0.232	0.188	0.229

Notes: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. (*anegCA2y*) is the average of current account-output ratios (%) over 1980-2013; (*aGDPpcgrowth*) is the average of the output-per-capita growth rates; (*atau*) is the average of saving wedges; (*agammaPL*) is the average of investment wedges; (*akaopen*) is the Chinn-Ito index of capital account openness. Data on the net total capital inflows (*anegCA2y*) is from Alfaro, Kalemli-Ozcan and Volosovych (2014); the output-per-capita growth rate is the per capita value of the real output at constant 2005 national prices from Penn World Table 8.1. The saving wedge = (lending rate - deposit rate)/lending rate. The investment wedge = (PMPKL-lending rate)/PMPKL, where PMPKL is the estimated marginal product of capital taking into account the difference in price of output and capital, and the share of the reproducible capital (Caselli and Feyrer (2007)). Variables are the average value over 1980-2013, on percentage. See data appendix for more details.

Table 3.1 reports the estimation result of the net total capital inflows on the productivity growth rate and on the interest rate wedges for 65 Non-OECD countries, as the sample analyzed by Gourinchas and Jeanne (2013). Column 1 shows that one country with a higher productivity growth rate tends to export the capital, a phenomenon labeled as the allocation puzzle: one country which grows faster and invests more tends to export the capital to the rest of world. Column 2 confirms their result that the net total capital inflow is increasing on the saving wedge while column 3 shows that it is decreasing on the investment wedge. In short, the regression on the sample of Non-OECD countries confirms one linear dependence of the net total capital inflows on the productivity growth rate.

Table 3.2 presents the regression result of the net total capital inflows on the productivity growth rate for a sample including both OECD and Non-OECD countries. Column 1 and 2 show that the coefficient of the productivity growth rate is insignificant, up to its quadratic term. Column 3 demonstrates that the impact of the productivity growth rate on the net total capital inflows is significant only when we consider the impact as one cubic function. In sum, the regression on the full sample proves one non-linear dependence of the net total capital inflows on the productivity growth rate.

Table 3.3 examines the role of the saving wedge on the accounting for the pattern of the net total capital inflows. Column 1 shows that the relationship between the net total capital inflows and the saving wedge follows one cubic function. Column 2 and 3 trace down the pattern of the savings, as the driver of the net total capital inflows (by definition, Net total capital inflows = Investments - Savings). The savings is one cubic function of the wedge, and the sign is reversal to the sign of the net total capital inflows. The table provides the strong evidences that the saving wedge can account for the pattern of the net total capital inflows.

Figure 3.2.1 plots the regression results of the net total capital inflows on the productivity growth rate and the interest rate wedges. On the panel A, the allocation puzzle only applies for the middle range of value of the productivity growth rate. At low or high value, however, an increase of the productivity growth rate raises the net total capital inflows. On the panel B, the net total capital inflows is increasing on the saving wedge only for the middle range of wedge. For low and high saving wedges, it is decreasing on the wedge. Since one country with a high productivity growth rate tends to have a low saving wedge (indeed, for the full sample, $\text{corr}(aGDPpcgrowth, \text{atau}) = -0.3724$), the saving wedge is a strong candidate to explain the relationship between the net total capital inflows and the productivity growth rate. On the panel C, the net total capital inflows has one

Table 3.2: Fixed-effect regression of the long-run net total capital inflows: 169 countries (1980-2013)

VARIABLES	(1) anegCA2y	(2) anegCA2y	(3) anegCA2y
Output-per-capita growth rate ($aGDPpcgrowth$)	0.424 (0.373)	0.0852 (0.669)	1.936** (0.783)
$(aGDPpcgrowth)^2$	-	0.0585 (0.0958)	-1.127*** (0.304)
$(aGDPpcgrowth)^3$	-	-	0.111*** (0.0272)
Constant	2.962*** (0.991)	3.173*** (1.052)	4.327*** (1.043)
Observations	162	162	162
R-squared	0.008	0.010	0.105

Notes: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. ($anegCA2y$) is average of the current account-output ratios over 1980-2013. ($aGDPpcgrowth$) is the average of output-per-capita growth rates over 1980-2013. Data on the net total capital inflows is from Alfaro, Kalemli-Ozcan and Volosovych (2014); the output-per-capita growth rate is the per capita value of the real output at constant 2005 national prices from Penn World Table 8.1. Variables are the average value over 1980-2013, on percentage. See data appendix for more details.

Table 3.3: Fixed-effect regression of the long-run net total capital inflows and savings: 169 countries (1980-2013)

VARIABLES	(1) anegCA2y	(2) aS2GNI	(3) aS2y
Saving wedge ($atau$)	-252.0*** (53.71)	89.40** (43.30)	246.6*** (58.92)
$(atau)^2$	868.6*** (125.3)	-317.3*** (101.1)	-865.6*** (137.6)
$(atau)^3$	-780.8*** (91.03)	271.2*** (73.46)	777.0*** (99.97)
Constant	15.38** (7.776)	19.18*** (6.256)	10.97 (8.515)
Observations	169	166	166
R-squared	0.423	0.104	0.373

Notes: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. ($anegCA2y$) is the average of current account-output ratios over 1980-2013; ($aS2GNI$, $aS2y$) are the average of savings over output ratios over 1980-2013; ($atau$) is the average of saving wedges over 1980-2013. Data on the net total capital inflows is from Alfaro, Kalemli-Ozcan and Volosovych (2014). Two measures of total savings: Gross savings as percentage of Gross National Income (GNI) ($S2GNI$) and Gross savings as percentage of GDP ($S2y$). The saving wedge = (lending rate - deposit rate)/lending rate. All these latter data are from World Development Indicators. Variables are the average value over 1980-2013. See data appendix for more details.

linear relationship with the investment wedge. This proves that only the investment wedge can not fully account for the pattern of the net total capital inflows. We need to take into account both the saving and investment wedges to explain the pattern of the net total capital inflows and the productivity growth rate. The empirical analysis shows that the net total capital inflows has one non-linear relationship with the productivity growth rate. Indeed, there exists one threshold of the productivity growth rate, over which an increase of productivity growth raises the net total capital inflows.

In the next section, we develop a theory of international capital flows taking into account the interaction between the productivity growth rate and the interest rate wedges to be consistent with these empirical evidences.

3.3 Theory

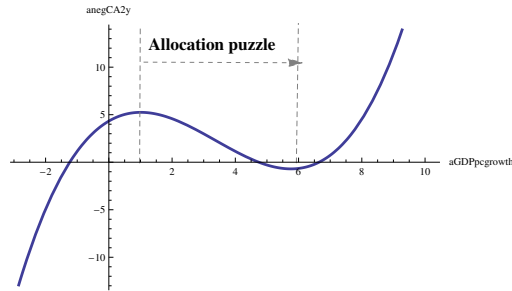
The world economy includes many large open countries, populated with the overlapping generations of households who live for two periods². Each household supplies one unit of labor when young and retires when old. All countries use the same technology to produce one homogeneous good, which is used for consumption and investment, and is traded freely and costlessly. Preferences and production technologies have the same structure and parameter values across countries. The capital is free mobile but labor is immobile across countries. Finally, firms are subject to changes in the country-specific productivity levels and labor forces.

Specification of the theoretical model

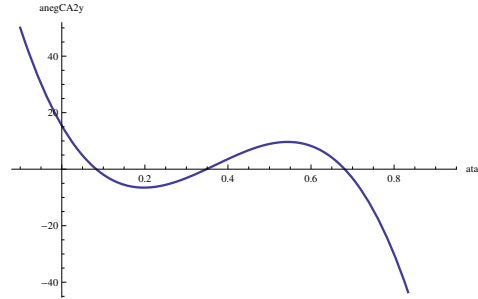
The model is constructed to be consistent with the empirical evidences. The empirical analysis uses the cross-section data sample for about 160 economies, both OECD (advanced economy) and Non-OECD (developing economy). By taking the average over the time period, we want to focus on the long-run measure of variables. By collecting data on 160 economies, we want to capture the pattern of capital flows for both the advanced and developing economies. These two empirical features will be captured by the theoretical model.

On the theoretical model, we employ an OLG framework where each agent lives for two periods: young and old. Since we focus on the steady state of the economy, all the theoretical implications apply for the long run variables, which are proxied

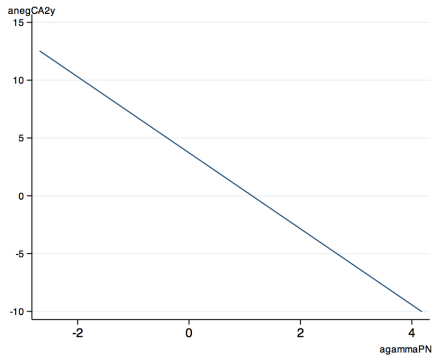
²The model is an extension of small open economy on Gourinchas and Jeanne (2013) to multi-country economy, but with OLG framework like Coeurdacier, Guibaud and Jin (2015).



Panel A: Net total capital inflows and Productivity growth rate



Panel B: Net total capital inflows and saving wedge



Panel C: Net total capital inflows and investment wedge

Figure 3.2.1: The regression result of the Net total capital inflows ($anegCA2y$) on the productivity growth rate ($aGDPpcgrowth$), the saving wedge ($atau$) and the investment wedge ($agammaPL$)

Notes: $anegCA2y = 4.327 + 1.936 * aGDPpcgrowth - 1.127 * (aGDPpcgrowth)^2 + 0.111 * (aGDPpcgrowth)^3$; $anegCA2y = 15.38 - 252 * atau + 868.6 * (atau)^2 - 780.8 * (atau)^3$; $anegCA2y = 3.72 - 3.29 * agammaPN$.

by the average over the time period 1980-2013 in the data sample. On other word, even we use the Ramsey multi-period living agents, we would also focus on the steady state and use the cross-section data sample.

Furthermore, we employ a multi-country model each one of which has an influence on the world equilibrium. This structure allows us to derive the implication for the large-open developed economies and developing economies. This structure is implemented by Coeurdacier, Guibaud and Jin (2015) to analyze the global imbalances between United States and China. Although not all 160 economies in our sample can be large enough to affect the other economies, one large economy can be thought as a combination of small open economies. For example, VietNam and Singapore are the small economies, but the Association of Southeast Asian Nations (ASEAN) can be considered to be a large economy.

In sum, the multi-country OLG model is appropriate to account for the cross-section data analysis over a large sample of countries in the previous empirical section.

3.3.1 Production

For any country j , let K_t denote the capital stock at the beginning of period t , and N_t the total labor input employed in period t . The output is:

$$Y_t = (K_t)^\alpha (A_t N_t)^{(1-\alpha)}$$

where $0 < \alpha < 1$, and A_t is country-specific productivity. The capital stock is fully depreciated and is augmented by investment I_{t-1} , with the law of motion:

$$K_t = I_{t-1}$$

The factor markets are perfect competitive so that each factor, capital and labor, earns its marginal product. Given the rental rate of capital (R_t), the capital stock and the wage rate per unit of labor are as following:

$$\begin{aligned} K_t &= \left(\frac{\alpha}{R_t}\right)^{1/(1-\alpha)} A_t N_t \\ w_t &= (1-\alpha) K_t^\alpha A_t^{(1-\alpha)} N_t^{-\alpha} = (1-\alpha) \left(\frac{\alpha}{R_t}\right)^{\alpha/(1-\alpha)} A_t \end{aligned}$$

Given the expected rental rate of capital at the next period (R_{t+1}^e), the investment demand by firm at time t is:

$$I_t = K_{t+1} = \left(\frac{\alpha}{R_{t+1}^e}\right)^{1/(1-\alpha)} A_{t+1} N_{t+1}$$

Let g_t^A and g_t^N denote the growth rate of productivity and of the size of consecutive cohorts, respectively, so that: $A_t = (1 + g_t^A) A_{t-1}$ and $N_t = (1 + g_t^N) N_{t-1}$.

3.3.2 Consumption

For any country j , one household born in period t earns the competitive wage rate w_t . Let c_t^y and c_{t+1}^o denote the consumption of an agent when young and old respectively. The lifetime utility of a household born in period t is:

$$U_t = u(c_t^y) + \beta u(c_{t+1}^o)$$

with standard isoelastic preferences $u(c) = (c^{1-1/\sigma} - 1)/(1 - 1/\sigma)$ where σ is the intertemporal elasticity of substitution coefficient. The discount factor β satisfies $0 < \beta < 1$.

Let s_t denote the saving at the end of period t of an agent born at t . The sequence of budget constraints is as following:

$$c_t^y + s_t = w_t \quad (3.3.1)$$

$$c_{t+1}^o = R_{t+1}^d s_t + z_{t+1} \quad (3.3.2)$$

When young, each individual works to receive the wage rate. She allocates the income between consumption and saving as deposit into the bank. When old, she receives the deposit interest rate (R_{t+1}^d) on her savings and the income transfer (z_{t+1}) from the government.

Let denote R_{t+1}^l is the lending interest rate which prevails to clear capital market, which equalizes the total saving supplied by households and total investment demanded by firm. We assume that the government levies the tax rate (τ) on the domestic saving. Therefore, each individual only receives the after-tax rate of return paid by the bank.

$$R_{t+1}^d = (1 - \tau)R_{t+1}^l \quad (3.3.3)$$

In general, τ can illustrate the financial frictions (taxation, corruption...) which generate a gap between the lending interest rate (R_{t+1}^l) and the deposit interest rate R_{t+1}^d which accrues to the saving by households.

Plug (3.3.3) into (3.3.2), the consumption at old age of one agent born at t is:

$$c_{t+1}^o = (1 - \tau)R_{t+1}^l s_t + z_{t+1}$$

By solving the utility maximization problem, the wedge enters the Euler equation, then, affects the saving rate by the young agent.

$$\frac{u'(c_t^y)}{u'(c_{t+1}^o)} = \beta(1 - \tau)R_{t+1}^l \Rightarrow s_t = \frac{w_t - (\beta(1 - \tau)R_{t+1}^l)^{-\sigma} z_{t+1}}{1 + \beta^{-\sigma}((1 - \tau)R_{t+1}^l)^{(1-\sigma)}}$$

Therefore, on aggregation, the total supply of savings at time t follows:

$$S_t = N_t s_t = \frac{w_t N_t - (\beta(1 - \tau)R_{t+1}^l)^{-\sigma} N_t z_{t+1}}{1 + \beta^{-\sigma}((1 - \tau)R_{t+1}^l)^{(1-\sigma)}} \quad (3.3.4)$$

3.3.3 Bank

For any country j , one representative bank collects saving to lend to firms. It covers the capital taxation (γ) which is a fraction of total revenue gained by lending to the firms.

$$\Pi_t = R_t L_{t-1}^B - R_t^l D_{t-1}^B - \gamma R_t L_{t-1}^B$$

The deposit market clearing condition requires the total demand of the deposit D_{t-1}^B to be equal to the total supply of the saving by households S_{t-1} . The loans market clearing condition requires the total demand of the investment I_{t-1} to be equal to the total supply of the fund by bank L_{t-1}^B .

Autarky economy

At autarky, the total supply of the fund is equal to the total demand of the deposit. In other words, the total saving supply is equal to the total investment demand. This condition gives the equilibrium lending interest rate (R_t^l).

$$D_{t-1}^B = L_{t-1}^B \Leftrightarrow S_{t-1} = I_{t-1}$$

The zero-profit condition and the autarky interbank market clearing condition imply that the capital taxation (γ) generates an investment wedge between the lending interest rate and the rental rate of capital.

$$R_t = \frac{R_t^l}{(1 - \gamma)}$$

Integration economy

At financial integration, one international bank receives all deposits from households and lends to firms across countries. Then, one world interest rate ($R_t^{l,w}$) prevails such that the world total demand of investments equals to the world total supply of savings.

$$\sum_j S_{t-1}^j = \sum_j I_{t-1}^j$$

The zero profit condition for banks still implies that the domestic rental rate of capital is related to the world lending interest rate by the investment wedge³

$$R_t^j = \frac{R_t^{l,w}}{(1 - \gamma^j)}$$

³One bank collects an amount of deposit M_{t-1}^j from the world market with the interest rate $R_t^{l,w}$ and lends all of this deposit to domestic firm to receive the rental rate of capital R_t^j . With the transaction cost γ^j , the zero-profit condition implies that $R_t^j(1 - \gamma^j) = R_t^{l,w}$.

Therefore, one country with a high investment wedge would suffer a high cost of capital, even the lending interest rate is equalized to the world rate.

3.3.4 Government

The government collects the taxation and transfers all tax revenue to the households⁴. We assume that only the old agents receive transfer. This assumption helps us to focus on the distortion that the taxation generates on the consumption and saving. In the Appendix, we relax this assumption by introducing the public expenditure. In details, the income transfer per each old agent in the country j at autarky follows:

$$z_t = \frac{Z_t}{N_{t-1}} \equiv \frac{\tau R_t^l N_{t-1} s_{t-1} + \gamma R_t I_{t-1}}{N_{t-1}} \quad (3.3.5)$$

whereby, the first term is the tax revenue by taxing the saving and the second term is the tax revenue by taxing the investment.

At integration with one world common lending interest rate, the income transfer is:

$$z_t^j = \frac{Z_t^j}{N_{t-1}^j} \equiv \frac{\tau^j R_t^{l,w} N_{t-1}^j s_{t-1}^j + \gamma^j R_t^j I_{t-1}^j}{N_{t-1}^j}$$

3.3.5 Equilibrium

Autarky equilibrium

Definition 3.3.1. *The autarky temporary equilibrium.*

Given the value determined by the last period (S_{t-1}) and the expected interest rate ($R_{t+1}^{l,e}$), the temporary equilibrium is a list of prices (R_t^l, R_t^d, R_t, w_t) and allocations (c_t^y, c_t^o, s_t, I_t), aggregate variables (K_t, S_t, L_t, Y_t) such that⁵ :

1. Profit maximization and Utility maximization subjected to budget constraint.
2. Market clearing condition:

⁴This approach is also applied by Gourinchas and Jeanne (2013).

⁵Indeed, we have:

- Price: $R_t = \frac{R_t^l}{(1-\gamma)}$; $R_t^d = (1-\tau)R_t^l$ and $w_t = (1-\alpha)A_t \left[\frac{\alpha(1-\gamma)}{R_t^l} \right]^{\alpha/(1-\alpha)} \equiv w(R_t^l)$
- Allocation: $L_t^D = N_t$; $K_t = \left[\frac{\alpha(1-\gamma)}{R_t^l} \right]^{1/(1-\alpha)} A_t N_t \equiv K(R_t^l)$; $Y_t = \left[\frac{\alpha(1-\gamma)}{R_t^l} \right]^{\alpha/(1-\alpha)} A_t N_t$;
 $s_t = \frac{w(R_t^l) - (\beta(1-\tau)R_{t+1}^{l,e})^{-\sigma} z_{t+1}}{1 + \beta^{-\sigma} (R_{t+1}^{l,e} (1-\tau))^{(1-\sigma)}} \equiv s(R_t^l, R_{t+1}^{l,e})$; $c_t^y = w(R_t^l) - s(R_t^l, R_{t+1}^{l,e})$; $c_t^o = R_t^l s_{t-1}$

- a Labor market: $L_t = N_t$
- b Capital market: $K_t = S_{t-1} = N_{t-1}s_{t-1}$
- c Good market: $N_t c_t^y + N_t s_t + N_{t-1} c_t^o = Y_t$

Definition 3.3.2. The autarky inter-temporal equilibrium.

Given an initial lending interest rate ($R_{t=0}^l$), an inter-temporal equilibrium with perfect foresight is a sequence of temporary equilibria that satisfies for all $t > 0$ the condition that:

$$\begin{aligned} R_{t+1}^{l,e} &= R_{t+1}^l \\ K(R_{t+1}^l, \gamma) &= S(R_t^l, R_{t+1}^l, \tau) \end{aligned}$$

Plug the income transfer (3.3.5) into the total saving supply (3.3.4), taking into account the shares over output by labor income ($w_t N_t = (1 - \alpha)Y_t$) and capital income ($R_{t+1} I_{t+1} = \alpha Y_{t+1}$), we end up with the total savings at equilibrium.

$$S_t = \frac{(1 - \alpha)Y_t - [\beta(1 - \tau)R_{t+1}^l]^{-\sigma} \gamma \alpha Y_{t+1}}{1 + [\beta(1 - \tau)]^{-\sigma} (R_{t+1}^l)^{(1-\sigma)}} \quad (3.3.6)$$

With the income transfer by the government, the saving wedge enters the marginal saving rate with the power $(-\sigma)$. Therefore, the saving rate is always decreasing on level of saving wedge, no matter the inter-temporal elasticity of substitution coefficient. As a result, one country with a low saving wedge would have a higher saving supply.

Since the saving-output ratio is always decreasing on the saving wedge for all values of the intertemporal elasticity of substitution coefficient while the investment-output ratio is independent to the intertemporal elasticity of substitution coefficient, we can focus on the case of log utility ($\sigma = 1$) without losing generality.

The autarky interest rate is the solution for the capital market clearing condition⁶, which equalizes the demand for capital (4) and supply of saving (13).

$$R_t^l = \frac{\alpha}{(1 - \alpha)} \frac{Y_t}{Y_{t-1}} \frac{\mu(\gamma, \tau) + (1 - \gamma)}{\vartheta(\tau)} \quad (3.3.7)$$

where $\vartheta(\tau) \equiv \frac{\beta(1 - \tau)}{1 + \beta(1 - \tau)}$ and $\mu(\gamma, \tau) \equiv \frac{\gamma}{1 + \beta(1 - \tau)}$.

⁶For $\sigma \neq 1$, the interest rate solves the equation:

$$\left[1 - \gamma + \gamma \frac{Y_t}{Y_{t-1}}\right] [\beta(1 - \tau)]^{-\sigma} \frac{\alpha}{(1 - \alpha)} (R_t^l)^{(1-\sigma)} - R_t^l + \frac{\alpha(1 - \gamma)}{(1 - \alpha)} \frac{Y_t}{Y_{t-1}} = 0$$

Theorem 3.1. *There exists an unique, stable steady state for an autarky economy. At steady state, $\frac{\partial R^l}{\partial g} > 0$; $\frac{\partial R^l}{\partial \tau} > 0$; $\frac{\partial R^l}{\partial \gamma} < 0$; whereby $g \equiv (1 + g^A)(1 + g^L) - 1$.*

Proof. Appendix ■

Both the productivity growth rate and the interest rate wedges can affect the equilibrium lending interest rate (R^l). Indeed, a higher growth rate raises the demand for investment, which in turn raises the interest rate, given the saving supply. A higher saving wedge reduces the saving supply, which in turn raises the interest rate, given the investment demand. A higher investment wedge, however, reduces the investment demand and depresses the interest rate, given the saving supply. In sum, the interaction of the productivity growth rate and the interest rate wedges can generate the various patterns of the autarky interest rate: one country with a high productivity growth rate does not necessarily have a high interest rate if it also has a low saving wedge and/or a high investment wedge.

The negative relationship between interest rate and saving wedge, however, is not always be held. In the Appendix, we show that in one alternative model where taxation is not transferred to households but to finance public expenditure, the relationship between marginal saving rate and tax depends on the range of value of the inter-temporal elasticity of substitution coefficient. In particular, for $0 < \sigma < 1$, the interest rate is decreasing on saving wedge because a higher saving wedge raises the saving supply, which reduces the autarky interest rate, given the investment demand.

Integration Equilibrium

At integration, capital is free mobile, then interest rates are equalized across countries: $R_t^{l,j} = R_t^{l,w}, \forall j$. Since the interest rate wedges are country-specific, the deposit interest rate and the rental rate of capital are different across countries. Therefore, the supply of saving, the demand for capital and then capital accumulation are different across countries.

Definition 3.3.3. *The integration temporary equilibrium.*

Given the value determined by the last period (S_{t-1}^j) and the expected interest rate ($R_{t+1}^{l,w,e}$), the temporary equilibrium is a list of price ($R_t^{l,w}, R_t^{d,j}, R_t^j, w_t^j$) and allocations ($c_t^{y,j}, c_t^{o,j}, s_t^j, I_t^j$), aggregate variables ($K_t^j, S_t^j, L_t^j, Y_t^j$) in each country j such that⁷ :

⁷Indeed, we have:

1. Profit maximization and Utility maximization subjected to budget constraint in each country.

2. Market clearing condition:

- a Labor market : $L_t^{D,j} = N_t^j$
- b World capital market: $\Sigma_j K_t^j = \Sigma_j S_{t-1}^j \equiv \Sigma_j N_{t-1}^j s_{t-1}^j$
- c World good market: $\Sigma_j (N_t^j c_t^{y,j} + N_t^j s_t^j + N_{t-1}^j c_t^{o,j}) = \Sigma_j Y_t^j$

Definition 3.3.4. The integration inter-temporal equilibrium.

Given an initial world lending interest rate ($R_{t=0}^{l,w}$), an inter-temporal equilibrium with perfect foresight is a sequence of temporary equilibria that satisfies for all $t > 0$ the condition:

$$\begin{aligned} R_{t+1}^{l,w,e} &= R_{t+1}^{l,w} \\ \Sigma_j K^{D,j}(R_{t+1}^{l,w}, \gamma^j) &= \Sigma_j S^j(R_t^{l,w}, R_{t+1}^{l,w}, \tau^j) \end{aligned}$$

The saving-output ratio depends both the saving and investment wedges since the latters determine the income transfer by the government. Moreover, the investment-output ratio only depends on the investment wedge since the latter affects the cost of capitals for firms.

$$\frac{S_{t-1}^j}{Y_{t-1}^j} = (1 - \alpha)\vartheta(\tau^j) - \frac{\alpha}{R_t^{l,w}} \frac{Y_t^j}{Y_{t-1}^j} \mu(\gamma^j, \tau^j) \quad (3.3.8)$$

$$\frac{I_{t-1}^j}{Y_{t-1}^j} = \frac{\alpha(1 - \gamma^j)}{R_t^{l,w}} \frac{Y_t^j}{Y_{t-1}^j} \quad (3.3.9)$$

The world capital market clearing condition, $\Sigma_j I_{t-1}^j = \Sigma_j S_{t-1}^j$, gives the world interest rate.

$$R_t^{l,w} = \frac{\alpha}{(1 - \alpha)} \frac{Y_t^w}{Y_{t-1}^w} \frac{\bar{\mu} + 1 - \bar{\gamma}}{\bar{\vartheta}} \quad (3.3.10)$$

whereby, $\bar{\mu} \equiv \Sigma_j \frac{Y_t^j}{Y_t^w} \mu(\gamma^j, \tau^j)$; $\bar{\gamma} \equiv \Sigma_j \frac{Y_t^j}{Y_t^w} \gamma^j$; $\bar{\vartheta} \equiv \Sigma_j \frac{Y_{t-1}^j}{Y_{t-1}^w} \vartheta(\tau^j)$.

-
- Price: $R_t^j = \frac{R_t^{l,w}}{(1 - \gamma)}$; $R_t^{a,j} = (1 - \tau^j) R_t^{l,w}$ and $w_t^j = (1 - \alpha) A_t^j \left[\frac{\alpha(1 - \gamma^j)}{R_t^{l,w}} \right]^{\alpha/(1-\alpha)}$
 - Allocation: $L_t^j = N_t^j$; $K_t^j = \left[\frac{\alpha(1 - \gamma^j)}{R_t^{l,w}} \right]^{1/(1-\alpha)} A_t^j N_t^j \equiv K(R_t^{l,w}, \gamma^j)$; $Y_t^j = \left[\frac{\alpha(1 - \gamma^j)}{R_t^{l,w}} \right]^{\alpha/(1-\alpha)} A_t^j N_t^j$;
 $s_t^j = \frac{w^j(R_t^{l,w}) - [\beta(1 - \tau^j) R_{t+1}^{l,w,e}]^{-\sigma} z_{t+1}^j}{1 + \beta^{-\sigma} (R_{t+1}^{l,w,e} (1 - \tau^j))^{(1-\sigma)}} \equiv s(R_t^{l,w}, R_{t+1}^{l,w,e})$; $c_t^{y,j} = w(R_t^{l,w}) - s(R_t^{l,w}, R_{t+1}^{l,w,e})$; $c_t^{o,j} = R_t^{l,w} s_{t-1}^j$

Theorem 3.2. *There exists an unique, stable steady state for an integration world economy. And the world interest rate, given by equation (3.3.10), satisfies: $\min_j R_t^{l,j} < R_t^{l,w} < \max_j R_t^{l,j}$.*

Proof. Appendix ■

Since the world interest rate lies between the lowest and the highest autarky rates, the capital flows out of country with a low autarky rate into country with a high autarky rate until the interest rate is equalized across countries. We apply this principle to analyze the pattern of the international capital flows in the next sections.

3.4 International Capital Flows

Let denote B_{t+1}^j as the stock of net foreign assets at the end of period t in the country j . The savings by the young agents is allocated between the net foreign assets and the domestic capital stock: $S_t^j = B_{t+1}^j + K_{t+1}^j$. Therefore,

$$\frac{B_{t+1}^j}{Y_{t+1}^j} = \frac{S_t^j}{Y_t^j} \frac{Y_t^j}{Y_{t+1}^j} - \frac{K_{t+1}^j}{Y_{t+1}^j}$$

Let denote CA_t^j as the country j 's current account at period t , which is the change of net foreign assets between $(t+1)$ and t : $CA_t^j = B_{t+1}^j - B_t^j$. Therefore,

$$\frac{CA_t^j}{Y_t^j} = \frac{B_{t+1}^j}{Y_{t+1}^j} \frac{Y_{t+1}^j}{Y_t^j} - \frac{B_t^j}{Y_t^j}$$

At steady state, the ratios of net foreign assets and current account over output are constant.

$$\frac{\overline{B^j}}{\overline{Y^j}} = \frac{\overline{S^j}}{\overline{Y^j}} \frac{1}{(1+g^{A,j})(1+g^{N,j})} - \frac{\overline{K^j}}{\overline{Y^j}} \quad (3.4.1)$$

$$\frac{\overline{CA^j}}{\overline{Y^j}} = (g^{A,j} + g^{N,j} + g^{A,j}g^{N,j}) \frac{\overline{B^j}}{\overline{Y^j}} \quad (3.4.2)$$

By (3.3.9), the capital-output ratio is as following:

$$\frac{K_t^j}{Y_t^j} = \frac{I_{t-1}^j}{Y_{t-1}^j} \frac{Y_{t-1}^j}{Y_t^j} = \frac{\alpha(1-\gamma^j)}{R_t^{l,w}} \quad (3.4.3)$$

Evaluating (3.3.8) and (3.4.3) at steady state, and plugging the result into (3.4.1):

$$\frac{\overline{B^j}}{\overline{Y^j}} = \frac{\alpha}{R^{l,w}} (\mu(\gamma^j, \tau^j) + 1 - \gamma^j) - \frac{(1-\alpha)\vartheta(\tau^j)}{(1+g^{A,j})(1+g^{N,j})} \quad (3.4.4)$$

The autarky interest rate (3.3.7) at steady state is:

$$R^{l,j} = \frac{\alpha}{(1-\alpha)}(1+g^{A,j})(1+g^{N,j})\frac{\mu(\gamma^j, \tau^j) + (1-\gamma^j)}{\vartheta(\tau^j)} \quad (3.4.5)$$

Using the last equation to rewrite the net foreign assets as:

$$\frac{\overline{B^j}}{Y^j} = \alpha(\mu(\gamma^j, \tau^j) + 1 - \gamma^j)\frac{(R^{l,w} - R^{l,j})}{R^{l,w}R^{l,j}} \quad (3.4.6)$$

Therefore, the net total capital inflows which is measured by the negative current account would be as following:

$$-\frac{\overline{CA^j}}{Y^j} = (g^{A,j} + g^{N,j} + g^{A,j}g^{N,j})\alpha(\mu(\gamma^j, \tau^j) + 1 - \gamma^j)\frac{(R^{l,j} - R^{l,w})}{R^{l,w}R^{l,j}}$$

Therefore, one country experiences a positive net total capital inflow only if its autarky steady state interest rate is higher than the world rate. Moreover, both the interest rate wedges and the growth rates determines the pattern of international capital flows.

$$-\frac{\overline{CA^j}}{Y^j} > 0 \Leftrightarrow R^{l,j} > R^{l,w} \quad (3.4.7)$$

3.4.1 Direction of International Capital Flows

In order to focus on the role of the heterogeneity of the interest rate wedges on the direction of the international capital flows, we assume the symmetric growth rates of total effective units of labor across countries ($g^{A,j} = g^A, g^{N,j} = g^N \forall j$). By (3.3.10), the world interest rate at steady state is:

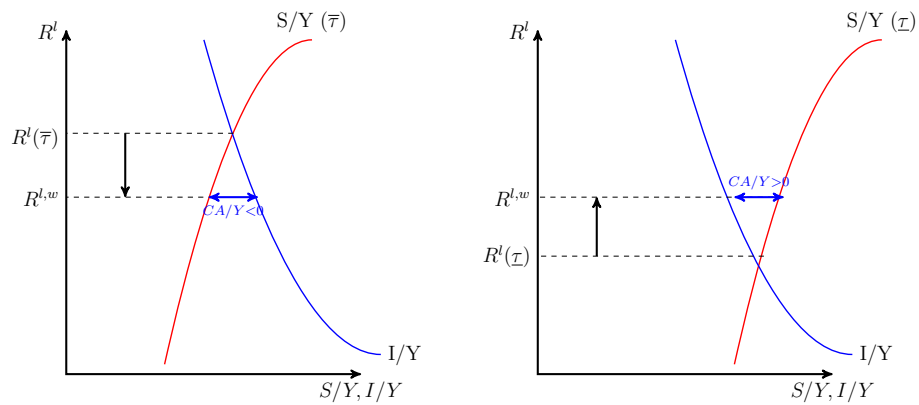
$$R^{l,w} = \frac{\alpha}{(1-\alpha)}(1+g^A)(1+g^N)\frac{\bar{\mu} + 1 - \bar{\gamma}}{\bar{\vartheta}} \quad (3.4.8)$$

Replacing (3.4.5) and (3.4.8) into the inequality (3.4.7), we end up with one condition on the interest rate wedges.

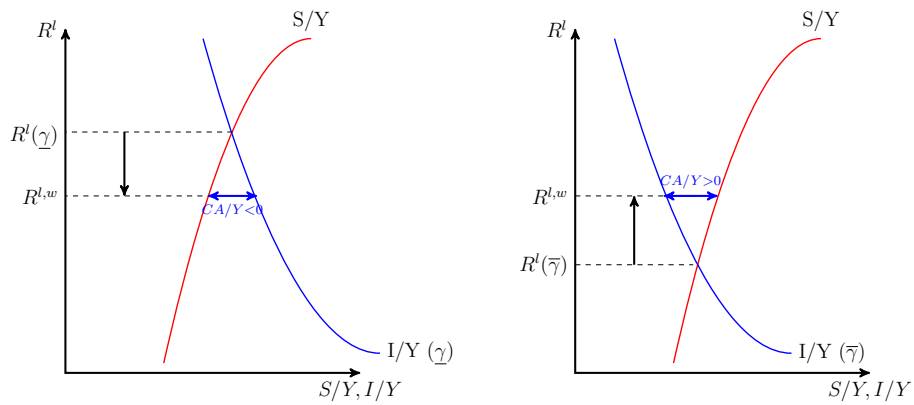
$$-\frac{\overline{CA^j}}{Y^j} > 0 \Leftrightarrow \frac{\vartheta(\tau^j)}{\mu(\gamma^j, \tau^j) + 1 - \gamma^j} < \frac{\bar{\vartheta}}{\bar{\mu} + 1 - \bar{\gamma}}$$

Figure 3.4.1 illustrates the role of the interest rate wedges on determining the direction of the international capital flows by comparing two countries with different levels of the interest rate wedges.

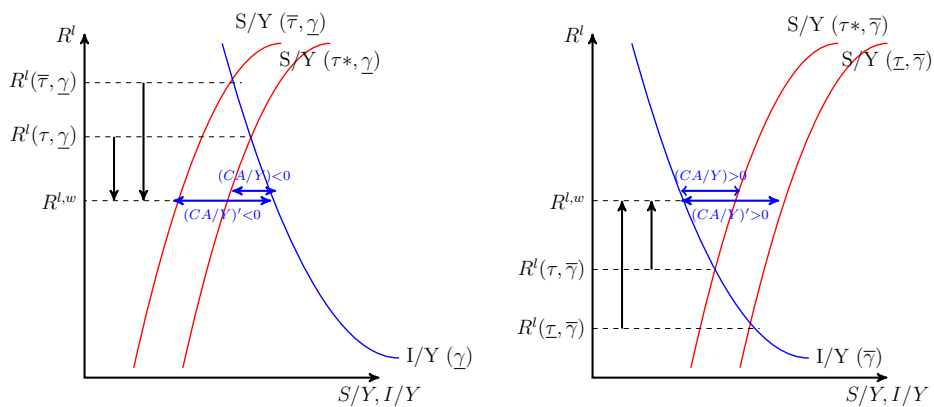
Panel A focuses on the saving wedge: $\tau^j \in [\underline{\tau}, \bar{\tau}]$, and $\gamma^j = 0, \forall j$. The country with a lower saving wedge (graph on the right) would have a higher saving supply at



A: Saving wedge and Current account



B: Investment wedge and Current account



C: Interest rate wedges and Current account

Figure 3.4.1: Direction of International Capital Flows

each value of the interest rate: $(S/Y)(\underline{\tau}) > (S/Y)(\bar{\tau})$. Given the same investment demand curve across two countries, that country has a lower autarky interest rate than the country with a higher saving wedge (graph on the left): $R^l(\underline{\tau}) < R^l(\bar{\tau})$. Therefore, at world integration, the country with a low saving wedge experiences a net total capital outflows since the saving supply is higher than the investment demand at equilibrium world interest rate.

Panel B focuses on the investment wedge: $\gamma^j \in [\underline{\gamma}, \bar{\gamma}]$, and $\tau^j = 0, \forall j$. The country with a higher investment wedge (graph on the right) would have a lower investment demand at each value of the interest rate: $(I/Y)(\bar{\gamma}) < (I/Y)(\underline{\gamma})$. And it would have a lower autarky interest rate than the country with a lower investment wedge: $R^l(\bar{\gamma}) < R^l(\underline{\gamma})$. Therefore, at world integration, the country with a high investment wedge experiences a net total capital outflows.

Panel C takes into account both the saving and investment wedges. For two country with the same saving wedge (τ^*), the country with higher investment wedge ($\bar{\gamma} > \underline{\gamma}$) would has a lower investment demand: the curve $(I/Y)(\bar{\gamma})$ is on the left of the curve $(I/Y)(\underline{\gamma})$. Thefeore, the country with the high investment wedge would export the capitals to the country with the low investment wedge: $(CA/Y)(\tau^*, \bar{\gamma}) > 0 > (CA/Y)(\tau^*, \underline{\gamma})$. Next, if the country with higher investment wedge also has a lower saving wedge ($\tau = \underline{\tau}$), which is confirmed by data that $corr(\tau, \gamma) = -0.1559 < 0$, its savings is higher: the saving curve moves further to the right: $(S/Y)(\underline{\tau}, \bar{\gamma}) > (S/Y)(\tau^*, \bar{\gamma})$. This case is depicted on the right. Similarly, in the country with both lower investment wedge and higher saving wedge, the saving curve moves to the left: $(S/Y)(\bar{\tau}, \underline{\gamma}) < (S/Y)(\tau^*, \bar{\gamma})$. This latter case is depicted on the left. In either case in which the saving wedge become larger or smaller, the flow of net total capital is amplified.

$$\begin{aligned}\tau^* < \bar{\tau} &\Rightarrow |(CA/Y)(\tau^*, \underline{\gamma})| < |(CA/Y)(\bar{\tau}, \underline{\gamma})| \\ \tau^* > \underline{\tau} &\Rightarrow |(CA/Y)(\tau^*, \bar{\gamma})| < |(CA/Y)(\underline{\tau}, \bar{\gamma})|\end{aligned}$$

In summary, the interaction of the saving and investment wedges can generate the various directions of the net total capital inflows across countries.

3.4.2 Non-Linear Pattern of International Capital Fows

With multi-country context, we can generalize the case when countries have the heterogeneity on the productivity growth rate and the investment wedges. By (3.3.10), the world interest rate at steady state with asymmetric growth rates across countries depends on the average growth rates across countries and one

term that captures the financial frictions of all countries.

$$R^{l,w} = \frac{\alpha}{(1-\alpha)}(1+\bar{g})\frac{\bar{\mu}+1-\bar{\gamma}}{\bar{\vartheta}} \quad (3.4.9)$$

whereby, for any variable x^j , $\bar{x} \equiv \sum_j \nu_{t-1}^j x^j$, for $\nu_{t-1}^j \equiv \frac{\hat{y}^j A_{t-1}^j N_{t-1}^j}{\sum_j \hat{y}^j A_{t-1}^j N_{t-1}^j}$.

The interaction between the productivity growth rate and the interest rate wedges enriches the pattern of the international capital inflows.

Proposition 3.4.1. *At integration steady state with asymmetric growth rates, one country imports capital only if its growth rate is high enough (i.e., passing over one threshold):*

$$-\frac{\overline{CA^j}}{Y^j} > 0 \Leftrightarrow (1+g^j) > H(\bar{g}, \gamma^j, \tau^j) \equiv (1+\bar{g})\frac{[\bar{\mu}+1-\bar{\gamma}]/\bar{\vartheta}}{[\beta(1-\tau^j)]^{-1}+1-\gamma^j}$$

Proof. Appendix ■

The proposition justifies the non-linear dependence of the net total capital inflows on the productivity growth rate as documented by the empirical analysis. Let's consider one country growing faster than rest of world ($g^j > \bar{g}$), but has a low saving wedge (for instance, the domestic government subsidizes the saving) and/or a high investment wedge (for instance, the low financial development and the high transaction cost on lending). That country would suffer a very high threshold, since the threshold is decreasing on the saving wedge but increasing on the investment wedge⁸ : ($\frac{\partial H}{\partial \tau^j} < 0$; $\frac{\partial H}{\partial \gamma^j} > 0$). Therefore, only if that country passes this threshold, it experiences positive net total capital inflows, like the prediction of Neo-Classical growth model: one country growing faster would invest more and imports capital from the rest of world. In contrast, one country growing slower than ROW, but with a high saving wedge (for instance, taxing the saving) and/or a low investment wedge (for instance, an advanced financial system) would have a very low threshold. Therefore, if the growth rate is just enough to pass the threshold, that slow-growing economy can still experience a positive net total capital inflow from the rest of world. In sum, the existence of the threshold breaks the linear dependence of the net total capital inflows on the productivity growth rate.

Our result can explain the mixed evidence on the pattern of capital flows across

⁸For the case with only investment wedge, the threshold is: $H(\gamma^j) = \frac{(1+\beta)/\beta - \bar{\gamma}}{(1+\beta)/\beta - \gamma^j}(1+\bar{g})$

countries. The literature on up-hill capital flows postulates that capital flows from developing economies with high output growth rate to advanced economies with low growth rate, while recent empirical evidence (Alfaro, Kalemli-Ozcan and Volosovych (2014), Gourinchas and Jeanne (2013)) finds that the private capital still flows down hill to fast-growing economies and only the public capital flows up-hill. In our model, only the growth rate is not just enough to account for the pattern of capital flows, it is indispensable to take into account its interaction with the financial frictions, captured by the saving and investment wedges.

3.5 Conclusion

Our paper proves the non-linear dependence of the net total capital inflows on the productivity growth rate. On empirical ground, we analyze the cross-section data sample of 165 countries to show that the allocation puzzle about the negative relationship between the net total capital inflows and the productivity growth rate only applies for the middle range of the productivity growth rates. Then, we construct one multi-country OLG open economy to show that there exists one threshold over which an increase of the productivity growth raises the net total capital inflows, as implied by the Neo-Classical growth model.

The threshold is country-specific and increasing on the investment wedge. Therefore, our model implies that the policy directed to increase the productivity growth rate needs to be equipped with the policy which reduces the financial friction on lending to domestic firm (the investment wedge).

Our model can capture the upper threshold but not the lower threshold on the productivity growth rate. On the next step, it is necessary to take into account both these two thresholds. Moreover, decomposing the net total capital inflows into the subflows like the public and private flows, or the debt and the portfolio flows would be an important extension in the future research revenue. For instance, we can analyze the interaction of public and private flows by allowing the government to issue the debt to finance the public expenditure. However, we also need to introduce the uncertainty into the model, such as the risky capital investment, so that the public saving differs the private saving. Without the uncertainty, the households only care the aggregate savings at equilibrium since the arbitrage condition implies the interest rate on public savings equals on the private savings.

Appendix

3.A Data appendix

Descriptive statistics

All variables on the empirical analysis are the average value over time from 1980 to 2013. The data on the net total capital inflows are from the updated and extended version of the dataset of net private and public capital flows constructed by Alfaro, Kalemli-Ozcan and Volosovych (2014). Following these authors, we measure the long-run net total capital inflows as the average over time period (1980-2013) of the negative current account per output ratio on percentage (*anegCA2y*). Moreover, we also use the negative change in the Net Foreign Assets (*anegdNFA*) and the current account per capita (*anegCApc*) as the alternative measures for the robustness check. The results are quite similar.

The data on the productivity growth rates are from the Penn World Table 8.1 (2015). We provide three alternative measures. First, the output per capita growth rate (*GDPpcgrowth*) is computed as the growth rate of the real output at constant 2005 national price (in USD) divided by the total population (explored from World Development Indicators). Second, the real productivity growth rate (*TFPgrowth*) is the growth rate of the real productivity at constant 2005 national price (2005=100). Third, the PPP-adjusted productivity growth rate (*cTFPgrowth*) is the growth rate of the productivity at current PPP price (USA=1). Then, all these growth rates are averaged over 1980 - 2013.

The data on the deposit (R_t^d) and lending interest rate (R_t^l) are from the World Development Indicators from 1980 to 2013. For each country, the saving wedge follows:

$$\tau_{ut} = \frac{R_t^l - R_t^d}{R_t^l}$$

To compute the investment wedge, we need to estimate the marginal product of capital. We follow Caselli and Feyrer (2007) to make four estimations of the

marginal product of capital. On the first estimation, the marginal product of capital is the share of total capital income on the total income (α_w), multiplied by the ratio of the output-side real GDP ($cgdpo$) over the capital stock (ck) (all at current PPPs in million 2005USD). The second estimation takes the share of the productive capital (α_k) on the total income, instead of the share of total capital income. On the third estimation, we employ the first estimation but accounting for the different prices of output (pl_{gdpo}) and capital stocks (pl_k). On the fourth estimation, we employ the second estimation accounting for the different prices of output and capital. In details, the estimation of marginal product of capital follows:

$$\begin{aligned}
 MPKN &= \alpha_w * \frac{cgdpo}{ck} * 100 \\
 MPKL &= \alpha_k * \frac{cgdpo}{ck} * 100 \\
 PMPKN &= \alpha_w * \frac{pl_{gdpo} * cgdpo}{pl_k * ck} * 100 \\
 PMPKL &= \alpha_k * \frac{pl_{gdpo} * cgdpo}{pl_k * ck} * 100
 \end{aligned}$$

Then, the investment wedge is computed as, for instance,

$$gammaN_t = \frac{MPKN_t - R_t^l}{MPKN_t}$$

whereby (R_t^l) is the real lending interest rate. We have four different measures of the investment wedge with four different measures of the marginal product of capital.

Table 3.A.1: DESCRIPTIVE STATISTICS

Variable	Obs	Mean	Std. Dev.	Min	Max
Net total capital inflows (<i>anegCA2y</i>) (%)	184	3.003955	17.98838	-206.175	38.93217
Output-per-capita growth rate (<i>aGDPpcgrowth</i>) (%)	164	1.839335	1.911317	-2.852451	10.20622
Saving wedge (<i>atau</i>)	172	.4847783	.1556737	-.0458484	.9384465
Investment wedge (<i>agammaN</i>), computed by <i>MPKN</i>	121	.5427404	.5810634	-2.213042	3.395791
Investment wedge (<i>agammaL</i>), computed by <i>MPKL</i>	87	-.1884486	2.0008	-11.10505	3.371339
Investment wedge (<i>agammaPN</i>), computed by <i>PMPKN</i>	121	.437458	.7272942	-2.676132	4.175607
Investment wedge (<i>agammaPL</i>), computed by <i>PMPKL</i>	87	-.5635968	2.472551	-12.27182	4.111995
Chinn-Ito openness index (<i>kaopen</i>)	182	.0902266	1.31536	-1.888895	2.389668

The data on the output and capital stock are from Penn World Table 8.1 and the data on the shares of capital income over total income are from Caselli and Feyrer (2007). On the paper, we focus on the investment wedge which is computed by

the fourth method, but also provide the alternative measures for the robustness check.

The openness index of capital account is the Chinn-Ito index from 1980 to 2013. We follow Gourinchas and Jeanne (2013) to use this variable to capture the financial integration of each country.

Table 3.A.1 shows descriptive statistics on the cross-section data sample of 165 countries. All variables are measured by averaging over period of 1980-2013. The net total capital inflows has the mean of (3%) with the standard deviation of (17.98) while the output per capita growth rate has (1.84%) and (1.91%) respectively. In comparison with the output per capita growth rate, the saving wedge has a lower mean (0.48) and a lower standard deviation (0.15). The mean of the investment wedge by taking into account the share of unproducibile capitals (e.g, lands) has a much lower mean than the one by the first measure (-0.56 compared with 0.54). The investment wedge with the unproducibile capitals, however, has a much higher standard deviation (2.47) than the one by the first measure (0.58). In summary, the data set offers rich variation for exploring the relationship between the productivity growth rate and the net total capital inflows.

Empirical analysis

Graph of Net total capitals inflows for 165 economies.

See Figure 3.A.1

Cubic function of capital flows

The regression in the table 3.2 is sensitive to the case that there is a flat regression on the middle range of values of the output-per-capita growth rate and of the saving wedge. We would check whether there is flat in the center or not by running the regression on the middle range of values.

The regression results of the net total capital inflows show that the coefficient of *aGDPpcgrowth* is negative, but insignificant, while the coefficient of *atau* is positive and significant. These results confirm the cubic function of capital inflows on the wedge while they do not rule out the case of flat regression of capital inflows on the productivity growth. However, the graph (3.A.3) of the capital flows over the middle range of productivity shows that there can be a decreasing line. Therefore, the graph can suggest the cubic fuction.

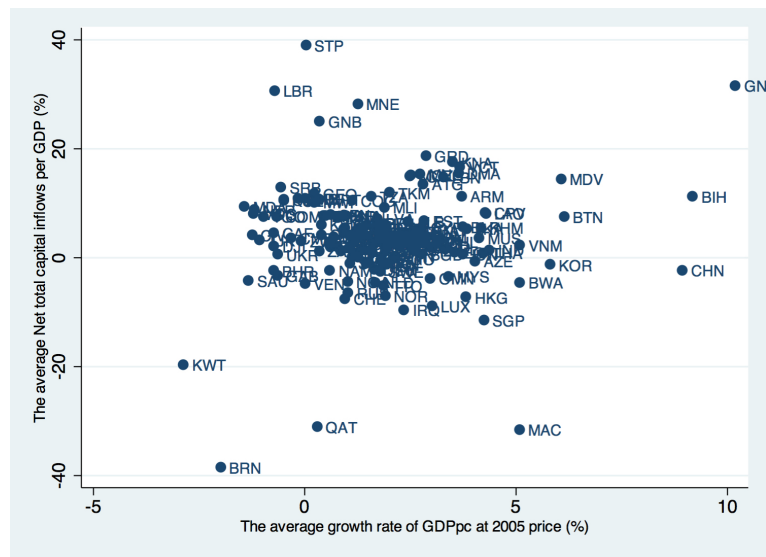


Figure 3.A.1: Pattern of International capital flows by countries: 165 economies
 Notes: All variables are averaged over 1980-2013. The net total capital inflows is the negative value of the current account as percentage of GDP and GDPpc is real output-side GDP per capita at constant 2005 national price.

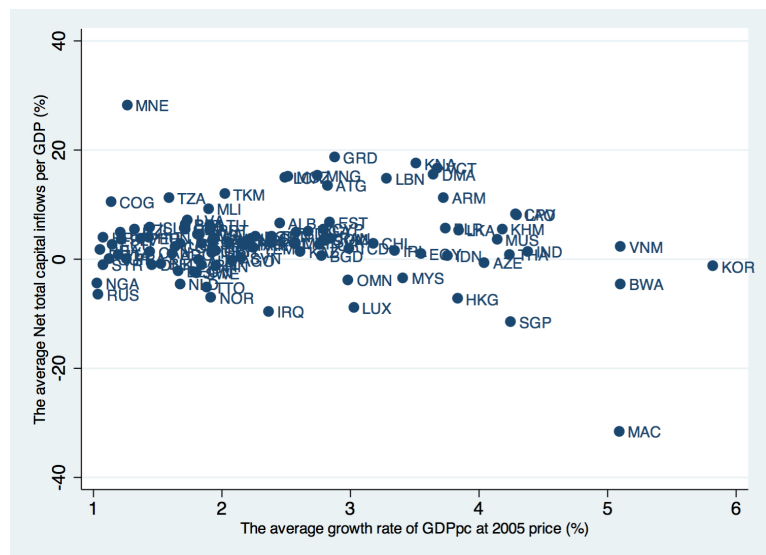


Figure 3.A.3: International capital flows by countries for $aGDPpcgrowth \in [1, 6]$
 Notes: All variables are averaged over 1980-2013. The net total capital inflows is the negative value of the current account as percentage of GDP and GDPpc is real output-side GDP per capita at constant 2005 national price.

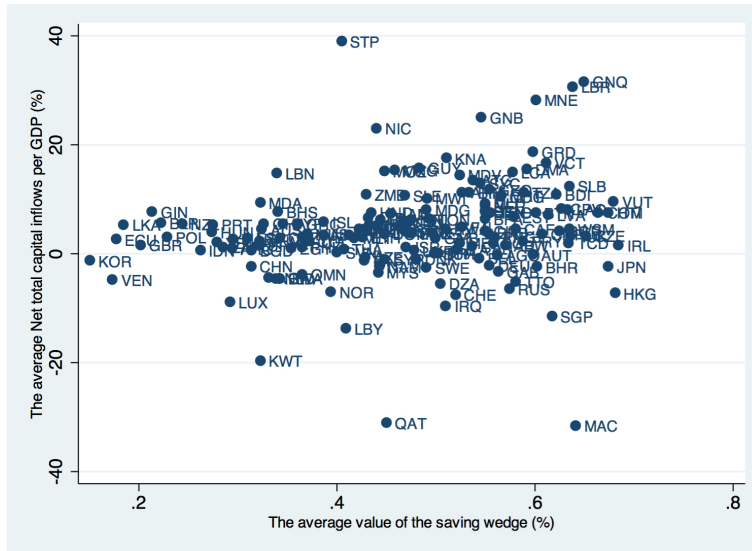


Figure 3.A.5: International capital flows by countries for $\text{atau} \in [0.04, 0.7]$

Notes: All variables are averaged over 1980-2013. The net total capital inflows is the negative value of the current account as percentage of GDP and GDPpc is real output-side GDP per capita at constant 2005 national price.

Table 3.A.2: Regression results of Capital flows on the middle range of value of productivity growth and saving wedge.

$$\begin{array}{l}
 \text{anegCA2}y_t^j = 4.683372 - 0.653251 * \text{aGDPpcgrowth} + u_t^j \\
 p - \text{value} \quad (0.008) \quad (0.312) \\
 \text{Std.Err} \quad (0.6426472) \quad (1.717728)
 \end{array}$$

$$\begin{array}{l}
 \text{anegCA2}y_t^j = -1.652627 + 12.50851 * \text{atau} + u_t^j \\
 p - \text{value} \quad (0.544) \quad (0.027) \\
 \text{Std.Err} \quad (2.718045) \quad (5.602625)
 \end{array}$$

3.B Proofs

Theorem 3.1

Proof. We focus on the case of $\sigma = 1$,

$$\begin{aligned}\Delta(R_{t+1}^l, R_t^l) &\equiv K^D(R_{t+1}^l) - S(R_t^l, R_{t+1}^l) \\ &= \left[(\alpha(1-\gamma))^{1/(1-\alpha)} + \frac{\gamma\alpha[\alpha(1-\gamma)]^{\alpha/(1-\alpha)}(\beta(1-\tau))^2}{1+\beta(1-\tau)} \right] \frac{A_{t+1}N_{t+1}}{(R_{t+1}^l)^{1/(1-\alpha)}} \\ &\quad - \frac{(1-\alpha)(\alpha(1-\gamma))^{\alpha/(1-\alpha)}\beta(1-\tau)}{1+\beta(1-\tau)} \frac{A_t N_t}{(R_t^l)^{\alpha/(1-\alpha)}}\end{aligned}$$

1 Existence of steady state.

$$\lim_{R_{t+1}^l \rightarrow 0} \Delta(R_{t+1}^l, R_t^l) > 0 \text{ and } \lim_{R_{t+1}^l \rightarrow \infty} \Delta(R_{t+1}^l, R_t^l) < 0$$

2 Uniqueness and Global stability.

Since $\frac{\partial \Delta(R_{t+1}^l, R_t^l)}{\partial R_{t+1}^l} \neq 0$, then we can express R_{t+1}^l as a function of R_t^l . Let denote by: $R_{t+1}^l = h(R_t^l)$. By the implicit function theorem,

$$\begin{aligned}\frac{\partial \Delta}{\partial R_{t+1}^l} &= - \left[(\alpha(1-\gamma))^{1/(1-\alpha)} + \frac{\gamma\alpha[\alpha(1-\gamma)]^{\alpha/(1-\alpha)}(\beta(1-\tau))^2}{1+\beta(1-\tau)} \right] \frac{A_{t+1}N_{t+1}}{(1-\alpha)(R_{t+1}^l)^{(2-\alpha)/(1-\alpha)}} < 0 \\ \frac{\partial \Delta}{\partial R_t^l} &= \frac{(1-\alpha)(\alpha(1-\gamma))^{\alpha/(1-\alpha)}\beta(1-\tau)}{1+\beta(1-\tau)} \frac{A_t N_t}{(R_t^l)^{1/(1-\alpha)}} > 0\end{aligned}$$

Then, $\frac{\partial R_{t+1}^l}{\partial R_t^l} > 0$. Therefore, $h(R_t^l)$ is a monotonic increasing function, and it would converge to one positive value if any.

Moreover, by l'Hôpital rule,

$$\lim_{R_t^l \rightarrow \infty} \frac{R_{t+1}^l}{R_t^l} = 0 < 1$$

In sum, $R_{t+1}^l = h(R_t^l)$ is both monotonic increasing and concave function since its graph would be below the 45 degree line for a large enough value of R_t^l . Then, it would converge to one finite positive value $0 < R^l < \infty$.

The comparative statics is straitforward since we have a closed form solution for the interest rate for $\sigma = 1$. ■

Theorem 3.2

Proof.

$$\begin{aligned}
& \Delta(R_{t+1}^{l,w}, R_t^{l,w}) \equiv \Sigma_j K^{D,j}(R_{t+1}^{l,w}, \tau^j, \gamma^j) - \Sigma_j S^j(R_t^{l,w}, R_{t+1}^{l,w}, \tau^j, \gamma^j) \\
&= \left(\frac{1}{R_{t+1}^{l,w}} \right)^{1/(1-\alpha)} \Sigma_j \left[(\alpha(1-\gamma^j))^{1/(1-\alpha)} + \frac{\gamma^j \alpha [\alpha(1-\gamma^j)]^{\alpha/(1-\alpha)} (\beta(1-\tau^j))^2}{1+\beta(1-\tau^j)} \right] A_{t+1}^j N_{t+1}^j \\
&- \left(\frac{1}{R_t^{l,w}} \right)^{\alpha/(1-\alpha)} \Sigma_j \frac{(1-\alpha)(\alpha(1-\gamma))^{1/(1-\alpha)} \beta(1-\tau^j)}{(1+\beta(1-\tau^j))} A_t^j N_t^j
\end{aligned}$$

1 Existence of steady state.

$$\lim_{R_{t+1}^{l,w} \rightarrow 0} \Delta(R_{t+1}^{l,w}, R_t^{l,w}) > 0 \text{ and } \lim_{R_{t+1}^{l,w} \rightarrow \infty} \Delta(R_{t+1}^{l,w}, R_t^{l,w}) < 0$$

2 Uniqueness and Global stability.

Since $\frac{\partial \Delta(R_{t+1}^{l,w}, R_t^{l,w})}{\partial R_{t+1}^{l,w}} \neq 0$, then $R_{t+1}^{l,w}$ is a function of $R_t^{l,w}$: $R_{t+1}^{l,w} \equiv h(R_t^{l,w})$.

By the implicit function theorem, $\frac{\partial \Delta}{\partial R_{t+1}^{l,w}} < 0$ and $\frac{\partial \Delta}{\partial R_t^{l,w}} > 0$. Then, $\frac{\partial R_{t+1}^{l,w}}{\partial R_t^{l,w}} > 0$.

Therefore, $h(R_t^{l,w})$ is a monotonic increasing function, and it would converge to one positive value if any.

Moreover, by l'Hôpital rule,

$$\lim_{R_t^{l,w} \rightarrow \infty} \frac{R_{t+1}^{l,w}}{R_t^{l,w}} = 0 < 1$$

In sum, $R_{t+1}^{l,w} = h(R_t^{l,w})$ is both monotonic increasing and concave function since its graph would be below the 45 degree line for a large enough value of $R_t^{l,w}$. Then, it would converge to one finite positive value $0 < R^{l,w} < \infty$.

For the case of symmetric growth rate across countries,

$$\begin{aligned}
R_t^{l,w} &= \frac{\alpha}{(1-\alpha)} \frac{Y_t^w}{Y_{t-1}^w} \left(\frac{\Sigma_j \lambda^j [\mu(\gamma^j, \tau^j) + 1 - \gamma^j]}{\vartheta} \right) \\
&= \frac{\alpha}{(1-\alpha)} \frac{Y_t^w}{Y_{t-1}^w} \Sigma_j \lambda^j \frac{\vartheta^j}{\vartheta} \left(\frac{[\mu(\gamma^j, \tau^j) + 1 - \gamma^j]}{\vartheta^j} \right) \\
&= \frac{\alpha}{(1-\alpha)} \frac{Y_t^w}{Y_{t-1}^w} \Sigma_j \theta^j \left(\frac{[\mu(\gamma^j, \tau^j) + 1 - \gamma^j]}{\vartheta^j} \right) \\
&= \Sigma_j \theta^j R_t^{l,j}
\end{aligned}$$

Therefore, $\min_j R_t^{l,j} < R_t^{l,w} < \max_j R_t^{l,j}$ ■

Proposition 3.4.1

Proof. At integration steady state, for $\nu_{t-1}^j \equiv \frac{\hat{y}^j A_{t-1}^j N_{t-1}^j}{\Sigma_j \hat{y}^j A_{t-1}^j N_{t-1}^j}$,

$$\begin{aligned} \left(\frac{Y_t^j}{Y_{t-1}^j} \right) |_{ss} &= (1 + g^j) \\ \left(\frac{Y_t^w}{Y_{t-1}^w} \right) |_{ss} &= \frac{\Sigma_j (Y_t^j / A_t^j N_t^j) A_t^j N_t^j}{\Sigma_j (Y_{t-1}^j / A_{t-1}^j N_{t-1}^j) A_{t-1}^j N_{t-1}^j} = \Sigma_j \nu_{t-1}^j (1 + g^j) = (1 + \bar{g}) \end{aligned}$$

$$\begin{aligned} R^{l,j} > R^{l,w} &\Leftrightarrow \frac{\alpha}{(1 - \alpha)} (1 + g^j) \frac{\mu(\gamma^j, \tau^j) + 1 - \gamma^j}{\vartheta(\tau^j)} > \frac{\alpha}{(1 - \alpha)} (1 + \bar{g}) \frac{\bar{\mu} + 1 - \bar{\gamma}}{\bar{\vartheta}} \\ &\Leftrightarrow (1 + g^j) > H(\gamma^j, \tau^j) \equiv (1 + \bar{g}) \frac{[\bar{\mu} + 1 - \bar{\gamma}] / \bar{\vartheta}}{[\mu(\gamma^j, \tau^j) + 1 - \gamma^j] / \vartheta(\tau^j)} \end{aligned}$$

For the case with only investment wedge,

$$\begin{aligned} R^{l,j} &= \frac{\alpha}{(1 - \alpha)} (1 + g^j) [(1 + \beta) / \beta - \gamma^j] \\ R^{l,w} &= \frac{\alpha}{(1 - \alpha)} \left(\frac{Y_t^w}{Y_{t-1}^w} \right) |_{ss} [(1 + \beta) / \beta - \bar{\gamma}] = \frac{\alpha}{(1 - \alpha)} (1 + \bar{g}) [(1 + \beta) / \beta - \bar{\gamma}] \end{aligned}$$

Therefore, $R^{l,j} > R^{l,w} \Leftrightarrow (1 + g^j) > H(\gamma^j) \equiv (1 + \bar{g}) \frac{(1 + \beta) / \beta - \bar{\gamma}}{(1 + \beta) / \beta - \gamma^j}$. ■

3.C Extended model: Public expenditure

We present one model where the government uses the taxation to finance the public expenditure without making the transfer to the households. While the saving rate is always decreasing on the saving wedge with the income transfer, the relationship between marginal saving rate and saving wedge would depend on the intertemporal elasticity of substitution coefficient in the model with the public expenditure.

Since we want to focus on the role of saving wedge on pattern of capital flows, we assume that $\gamma^j = 0, \forall j$. This implies that $R_t^l = R_t$. In comparison with the model in the main text, only the budget constraint of households and good market clearing condition need to be modified to take into account the role of public expenditure.

Consumption

$$\begin{aligned} \max_{c_t^y, c_{t+1}^o, s_{t+1}} U_t &= u(c_t^y) + \beta u(c_{t+1}^o) \\ c_t^y + s_t &= w_t \\ c_{t+1}^o &= R_{t+1}^d s_t = (1 - \tau) R_{t+1}^l s_t \end{aligned}$$

The Euler equation gives the saving rate by the household and the aggregation saving supply are as following:

$$s_t = \frac{w_t}{1 + \beta^{-\sigma} [(1 - \tau) R_{t+1}^l]^{(1-\sigma)}} \Rightarrow S_t = N_t s_t = \frac{w_t N_t}{1 + \beta^{-\sigma} [(1 - \tau) R_{t+1}^l]^{(1-\sigma)}}$$

Therefore, the aggregate saving supply is increasing on the saving wedge if the coefficient of substitution is low. For $\sigma > 1$, the savings is decreasing on the saving wedge, and we end up with the same mechanism on the main text.

$$\frac{\partial(S_t/Y_t)}{\partial\tau} > 0 \Leftrightarrow 0 < \sigma < 1$$

Government

The government uses the capital taxation to finance public expenditure. We assume that government always keeps the balanced budget constraint.

$$G_t = T_t = \tau R_t^l S_{t-1}$$

In comparison with the model with the income transfer, only the market clearing condition on good needs to be modified both at autarky and integration equilibrium.

$$\begin{aligned} N_t c_t^y + N_t s_t + N_{t-1} c_t^o + G_t &= Y_t \\ \Sigma_j (N_t^j c_t^{y,j} + N_t^j s_t^j + N_{t-1}^j c_t^{o,j} + G_t^j) &= \Sigma_j Y_t^j \end{aligned}$$

Autarky equilibrium

The lending interest rate prevails to clear the autarky capital market:

$$\frac{(1 - \alpha)}{1 + \beta^{-\sigma} (1 - \tau)^{1-\sigma} (R_t^l)^{1-\sigma}} = \frac{\alpha}{R_t^l} \frac{Y_t}{Y_{t-1}} \quad (3.C.1)$$

Theorem. *There exists an unique, stable steady state for an autarky economy with public expenditure. All else equals, the interest rate is decreasing on the saving wedge only the substitution coefficient is low: $\frac{\partial R_t^l}{\partial\tau} < 0 \Leftrightarrow 0 < \sigma < 1$.*

Proof.

$$\begin{aligned}
\Delta(R_{t+1}^l, R_t^l) &\equiv K_{t+1} - S_t \\
&= \left(\frac{\alpha}{R_{t+1}^l}\right)^{1/(1-\alpha)} A_{t+1} N_{t+1} - \frac{(1-\alpha)(\alpha)^{\alpha/(1-\alpha)} A_t N_t}{R_t^{\alpha/(1-\alpha)}} \frac{1}{1 + \beta^{-\sigma} (R_{t+1}^l (1-\tau))^{(1-\sigma)}} \\
&\equiv \left(\frac{\alpha}{R_{t+1}^l}\right)^{1/(1-\alpha)} A_{t+1} N_{t+1} - \frac{(1-\alpha)(\alpha)^{\alpha/(1-\alpha)} A_t N_t}{R_t^{\alpha/(1-\alpha)}} \varphi(R_{t+1}^l, \tau)
\end{aligned}$$

1 Existence of steady state.

For $\sigma \geq 1$, $\lim_{R_{t+1}^l \rightarrow 0} \Delta(R_{t+1}^l, R_t^l) > 0$ and $\lim_{R_{t+1}^l \rightarrow \infty} \Delta(R_{t+1}^l, R_t^l) < 0$.

For $0 < \sigma < 1$, $\lim_{R_{t+1}^l \rightarrow 0} \Delta(R_{t+1}^l, R_t^l) > 0$ and $\lim_{R_{t+1}^l \rightarrow \infty} \Delta(R_{t+1}^l, R_t^l) < 0$ if the saving is less elastic to interest rate than investment.

$\lim_{R_{t+1}^l \rightarrow 0} \Delta(R_{t+1}^l, R_t^l) < 0$ and $\lim_{R_{t+1}^l \rightarrow \infty} \Delta(R_{t+1}^l, R_t^l) > 0$ if the saving is more elastic to interest rate than investment.

2 Uniqueness and Global stability

Since $\frac{\partial \Delta(R_{t+1}^l, R_t^l)}{\partial R_{t+1}^l} \neq 0$, then we can express R_{t+1}^l as a function of $R_t^{l,w}$. Let denote by: $R_{t+1}^l = h(R_t^{l,w})$. By the implicit function theorem, if saving is less elastic to interest rate than investment, $\frac{\partial R_{t+1}^{l,w}}{\partial R_t^{l,w}} > 0$. Therefore, $h(R_t^{l,w})$ is a monotonic increasing function, then, it would converge to one positive value if any.

Moreover, by l'Hôpital rule,

$$\lim_{R_t^l \rightarrow \infty} \frac{R_{t+1}^l}{R_t^l} = 0 < 1$$

In sum, $R_{t+1}^l = h(R_t^l)$ is both monotonic increasing and concave function since its graph would be below the 45 degree line for a large enough value of R_t^l . Then, it would converge to one finite positive value $0 < R^l < \infty$.

We can also verify, by the implicit function theorem, that $\frac{\partial R_t^l}{\partial \tau} < 0$ for $0 < \sigma < 1$ and when saving is less elastic to interest rate than investment. In details, at steady state, for $0 < \sigma < 1$, then, the lending interest rate satisfies:

$$G \equiv (1-\alpha)\varphi(R^l, \tau) - \frac{\alpha(1+g)}{R^l} = 0 \Rightarrow \frac{\partial G}{\partial R^l} > 0; \frac{\partial G}{\partial \tau} > 0 \Rightarrow \frac{\partial R_t^l}{\partial \tau} < 0$$

■

A higher saving wedge would depress the deposit interest rate, which in turn raises the marginal saving rate by domestic households on case that $0 < \sigma < 1$. Given the investment demand, a high saving supply requires a lower lending interest rate to satisfy the capital market clearing condition. This feature marks this model to be different to the model with the lump-sum taxation in the main text, in which the country with high saving wedge would have lower saving supply, and a higher autarky interest rate.

Integration equilibrium and International capital flows

The world interest rate is the solution for the world capital market clearing condition.

$$\frac{\alpha}{R_t^{l,w}} \frac{Y_t^w}{Y_{t-1}^w} = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}^w} \frac{(1-\alpha)}{1 + \beta^{-\sigma}(1-\tau)^{(1-\sigma)}(R_t^{l,w})^{(1-\sigma)}} \equiv \sum_j \lambda^j (1-\alpha) \varphi^s(R_t^{l,w}, \tau^j)$$

Where the marginal saving rate is increasing on the saving wedge for $0 < \sigma < 1$: $\frac{\partial \varphi^s(R_t^{l,w}, \tau^j)}{\partial \tau^j} > 0$.

Lemma Suppose that $\bar{\tau} = \max_j \tau^j$ and $\underline{\tau} = \min_j \tau^j$. Then, $R^l(\bar{\tau}) < R^{l,w} < R^l(\underline{\tau})$

Proof.

$$\begin{aligned} \frac{\alpha(1+g)}{R^{l,w}} &= \sum_j \lambda^j \frac{\varphi^s(R^{l,w}, \tau^j)}{\varphi^s(R^{l,j}, \tau^j)} \varphi^s(R^{l,j}, \tau^j) = \sum_j \theta^j \varphi^s(R^{l,j}, \tau^j) = \sum_j \theta^j \frac{\alpha(1+g)}{R^{l,j}} \\ \Rightarrow \min_j R^{l,j} &< R^{l,w} < \max_j R^{l,j} \Rightarrow R^l(\bar{\tau}) < R^{l,w} < R^l(\underline{\tau}) \end{aligned}$$

■

Following the same steps in the main text, we compute the net total capital inflows. The aggregate savings by the young agents at the end of (t) has two components: the net foreign assets (B_{t+1}) and the capital stock (K_{t+1}). Therefore,

$$\begin{aligned} \frac{B_{t+1}^j}{Y_{t+1}^j} &= \frac{N_t^j s_t^{y,j}}{Y_t^j} \frac{Y_t^j}{Y_{t+1}^j} - \frac{K_{t+1}^j}{Y_{t+1}^j} = (1-\alpha) \varphi(R_{t+1}^{l,w}, \tau^j) \frac{Y_t^j}{Y_{t+1}^j} - \frac{\alpha}{R_{t+1}^{l,w}} \\ \frac{CA_t^j}{Y_t^j} &= \frac{B_{t+1}^j - B_t^j}{Y_t^j} \end{aligned}$$

At steady state,

$$-\frac{\overline{B^j}}{Y^j} = \frac{\alpha}{R^{l,w}} - \frac{(1-\alpha) \varphi(R^{l,w}, \tau^j)}{(1+g^{A,j})(1+g^{N,j})} \quad (3.C.2)$$

$$-\frac{\overline{CA^j}}{Y^j} = (g^{A,j} + g^{N,j} + g^{A,j} g^{N,j}) \left(-\frac{\overline{B^j}}{Y^j} \right) \quad (3.C.3)$$

Using the autarky interest rate (3.C.1), the net total capital inflows into one country follows.

$$-\frac{\overline{CA}}{Y} = (g^A + g^N + g^A g^N) \alpha \frac{\varphi^s(R^{l,j}, \tau^j) R^{l,j} - \varphi^s(R^{l,w}, \tau^j) R^{l,w}}{R^{l,w} R^{l,j} \varphi^s(R^{l,j}, \tau^j)}$$

Therefore, one country which has a lower interest rate at autarky than at integration would experience a positive outflows of net total capitals. Indeed, for $0 < \sigma < 1$, a higher saving wedge leads to a higher savings supply, then, a lower autarky interest rate. Therefore, at integration when the interest rate is equalized across countries, the interest rate goes up to the world integration level. At the integration rate, the investment demand is equalized across countries but the saving supply is higher at the country with higher saving wedge. Therefore, the country with a high saving wedge would experience the net total capital outflows since the savings is greater than investment. This mechanism is different to the model in the main text only with case that $0 < \sigma < 1$.

Note that, for $\sigma > 1$, the marginal saving rate is decreasing on the saving wedge, and the country with a higher saving wedge would have a lower saving supply and a higher autaky interest rate. At integration, given the same investment demand across countries, that country would experience net total capital inflows to finance the domestic investment since the domestic savings are lower than the investment. This mechanism is the same as the model in the main text.

The mechanism for $0 < \sigma < 1$, as illustrated by Figure 3.C.1 for $\tau \in [\bar{\tau}, \bar{\tau}]$. At autarky, the country (depicted on the left) with a higher saving wedge ($\bar{\tau}$) also has a higher saving-output ratio and lower autarky interest rate. At integration, that country runs the surplus current account or a net total capital outflows.

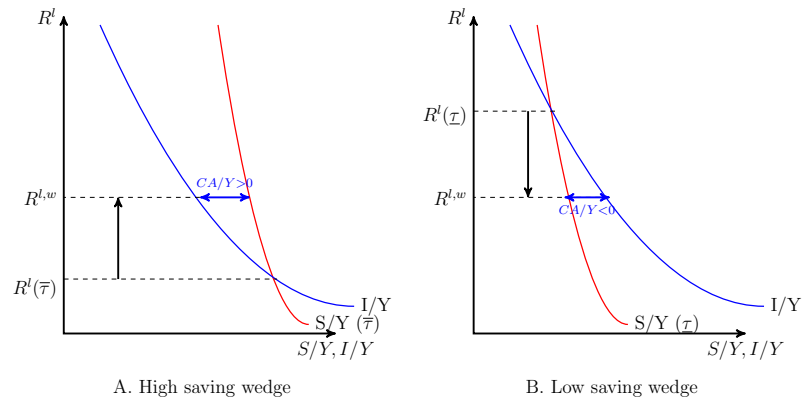


Figure 3.C.1: Saving wedge and Current account with public expenditure

Chapter 4

Saving Wedge, Productivity Growth and International Capital Flows

Abstract

In one open OLG economy, the interaction of endogenous saving wedge and credit constraint can explain the pattern of international capital flows for the emerging economies. A higher productivity growth rate leads to a higher saving by raising the capital taxation and depressing the domestic interest rate. A lower financial development level (tighter credit constraint) reduces the capital accumulation in long run. Therefore, the emerging economies with the high productivity growth and low financial development level can experience (1) the net total capital outflows; (2) the high saving-output ratio and (3) the low long-run capital accumulation.

4.1 Introduction

The international financial integration of the emerging economies whose the high output growth rates into the world economy for the last decades is featured by three stylized facts.

Fact 1: The emerging economies experience the net total capital outflows (e.g, China and Korea in Figure 4.1.1A). Furthermore, the value of net total capital inflows fluctuates over time. The net total capital inflows for China has decreased gradually to the bottom of -9% in 2007, then, it has increased again.

Fact 2: The emerging economies have the higher saving rates than the advanced economies: on average, 40 % in China compared with 16 % in United States (Figure 4.1.1B). Moreover, the saving-output ratios increase substantially over time in the emerging economies.

Fact 3: Some emerging economies have the persistently low capital-effective-labor ratio than the advanced economies even after many years of integration: about 55000 USD in China compared with about 280000 USD in United States in 2013 (Figure 4.1.1C). Some economies, however, can converge to the same ratio with the advance economies (in Korea, from 40000 USD in 1980 to 210000 USD in 2013).

The combination of the high saving rate and low capital accumulation in the capital exporters whose the high output growth rates (the fast-growing or the emerging economies) raises the challenge for the existing theoretical models. Many models can generate the high savings (fact 2) and low interest rate in the fast-growing economies so that these economies experience the net capital outflows at integration (fact 1). These models, however, cannot account for the fact 3 since the high saving also results in the high capital accumulation. Other models can depress the investment demand on the fast-growing economies so that they converge to a low long-run capital accumulation (fact 3) and export capital to the rest of world (fact 1). These models, however, ignore the high saving rates on the emerging economies (fact 2). In sum, there is still a research gap on the literature for the explanation of these three facts simultaneously.

Our paper presents the theory of the endogenous financial friction to rationalize these facts together. In the benchmark open OLG economy, the wedge between the domestic interest rate on saving and the world rental rate of capital is increasing on the domestic output growth rate. A higher growth rate not only increases the investment but also the saving by raising the wedge and depressing the domestic interest rate. Then, one economy which grows faster can experience the

outflows of net total capital if the saving increases faster than the investment. Therefore, the benchmark model can account for the highly correlated between the productivity growth and capital flows not only in the developing economies but also in the advanced economies. In the extended economy with the credit constraint (a measure of financial development level) for the domestic firms, the capital accumulation is increasing on the tightness of the constraint. Therefore, the heterogeneity on the tightness of the constraint generates the difference on the long-run capital accumulation levels across countries. By combining the high output growth rate and low financial development level, our model can explain the high saving rate (fact 2) and low capital accumulation (fact 3) in the emerging economies who export the capital to the rest of world (fact 1).

The paper belongs to the literature on the international capital flows (see ? for a recent survey). Some papers focus on the transition economies by employing one small open economy which takes the interest rate as exogenous given. Buera and Shin (2009) analyze the large-scale reforms which reduce the financial friction on investment and improve the contract's enforcement in one small open economy. The reform raises the productivity growth by a more efficient allocation of economic resources and at the same time, it also results in the outflows of net total capital since the saving rate surges but the investment rate increases with lag. The financial friction is endogenous determined by the productivity growth rate in our model while the productivity growth is endogenously determined by the financial frictions in their model. Therefore, the capital outflows is driven by the productivity growth in our model, instead of the reduction of the financial friction as theirs.

Song, Storesletten and Zilibotti (2011) also explain the high productivity growth rate and capital outflows in an transition economy like China with two types of firms: the entrepreneurial firms with the high productivity level but credit constraint and the financial firms with the low productivity level with the abundant finance. The reform in term of moving of labor force from the financial to entrepreneurial firms raises the wage rates and the aggregate savings through the wealth effect. This reform also results in the net total capital outflows since the aggregate savings increase while aggregate investments decrease because of the reduction of investment demand by the financial firms. Instead of aggregate savings, our model focuses on the change in the marginal saving rate as the driving force of the increasing savings-output ratio and the net capital outflows.

None of two aforementioned papers accounts for the non-convergence of the emerging economies toward the long-run capital accumulation levels of advanced economies.

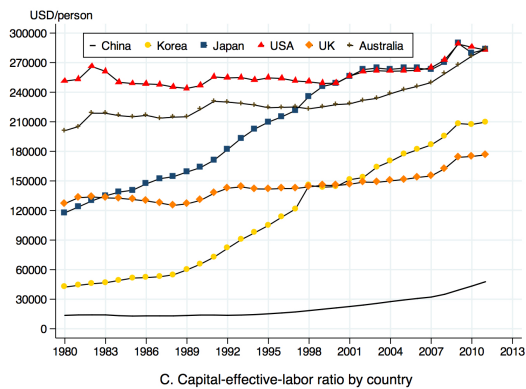
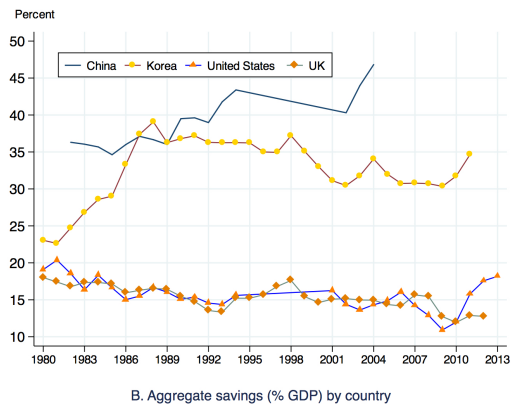
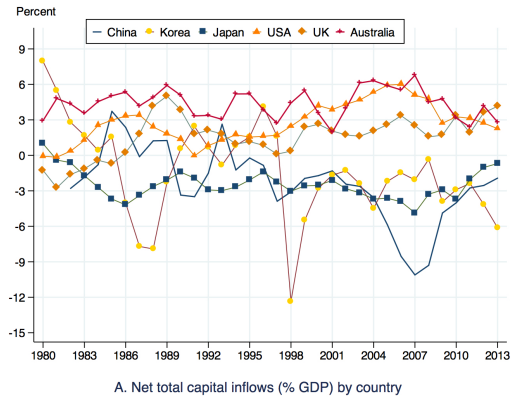


Figure 4.1.1: Three Stylized Facts

Notes: (a) Net total inflows (negative Current account) per Output; (b) Aggregate savings is the sum of private and public savings; both (a) and (b) are from Alfaro, Kalemli-Ozcan and Volosovych (2014); (c) Capital-effective-labor ratio equals to capital stock at constant 2005 national prices (in millions USD) divided by total effective units of labor, which is the productivity level at constant 2005 national prices multiplied by number of people engaged (in millions). Data from PWT 8.1

In comparison with their model, ours captures the increasing saving-output ratio (fact 2) while still explains the outflows of net total capital from the emerging economies (fact 1).

Some papers endogenize the interest rate by building up the multi-country economy. On Coeurdacier, Guibaud and Jin (2015), the borrowing constraint, which is more severe in the developing economies than in the advanced economies, generates the excess savings and low interest rate in the developing economies at autarky. Then, at integration, the capital flows out of these economies for a higher interest rate in the advanced economies. The high saving and low interest rate at autarky economy are also the key implications on Angeletos and Panousi (2011) where the idiosyncratic investment risk motivates the precautionary savings by households. The common mechanism on these model is that the developing economies have a higher capital accumulation and a lower interest rate at autarky than the advanced economies. At integration, the capital flows from the developing to advanced economies to equalize the capital accumulation and the interest rate across countries. This implication is inconsistent to the fact 3 that the some developing economies could not converge to the same long-run capital accumulation with the advanced economies even at integration.

Other multi-country models focus on the investment demand as the driver of international capital flows. On Boyd and Smith (1997), the rich economy has a higher investment demand since it has much more income than the poor one to afford the agency cost on the financial market. Therefore, the capital flows out from the developing economies to the advanced economies for a higher investment demand. The difference in the investment demand across countries is also the key feature on Matsuyama (2004) in which two economies face the same credit constraint converge to asymmetric steady states because of the difference on the initial capital stock. Indeed, the capital flows from the poor to the rich economy since the latter has a higher steady state capital accumulation. These two papers, however, do not account for the fact 2 that the exporters of capital also have the high saving-output ratios.

There are two key features that distinguishes our paper to these multi-country models. First, our model can explain simultaneously the high saving-output ratio and low capital accumulation on the emerging economies who export the capital to the rest of world. Second, while the financial friction is exogenous in the aforementioned papers, it is endogenously determined by the productivity growth rate in our model. Therefore, the net total capital inflows can fluctuate over time as presented in Figure 4.1.1.A.

The paper is related to the literature on the puzzling high correlation between saving and investment on the open economy (Feldstein and Horioka (1979)). On Carroll, Overland and Weil (2000), the utility depends partly on how consumption compares to a habit stock determined by past consumption. Therefore, an increase on the output growth rate can cause an increase on the saving rate by limiting the motive of consumption smoothing. In stead of using the habit formation, Kraay and Ventura (2000) employ the classical Merton-Samuelson portfolio choice approach to the current account. On their model, the agent with CRRA utility function would allocate the constant fraction of wealth between the riskfree and risky assets. Therefore, a positive income shock can raises both the wealth accumulation growth rate (saving) and the portfolio investment. Our model, however, employs one endogenous saving wedge to explain the high correlation between saving and investment. This key opens the channel through which the output growth affects both the saving and investment rates. Indeed, a high output growth rate raises the investment to keep the domestic marginal product of capital to be equal to the world rental rate of capital. Moreover, a higher growth rate raises the saving by depressing the domestic rate of return on the saving. Therefore, both the saving and investment go up for a higher growth rate.

The paper also makes contribution on the literature on interest rate spread, such as Campbell and Shiller (1991), Bernanke and Blinder (1992), Ho and Saunders (1981). Our different position is to emphasize the capital taxation as the key to create the gap between lending interest rate for the firms and the deposit interest rate for the households. Since the government collects tax to finance a public expenditure which is complementary to private output, an increase of future output growth rate raises the tax rate and widens the interest rate spread. Therefore, the future output growth rate is an important predictor for the interest rate spread, not vice versa like the rest of literature.

The paper is organized as follows. After the introduction, section 4.2 provides the definition of interest rate wedge and the empirical analysis on the relationship between productivity growth and international capital flows. Section 4.3 and 4.4 lay out the theoretical models to explain the empirical evidence. Section 4.5 concludes and is followed by Appendix.

4.2 Saving wedge: Definition

Gourinchas and Jeanne (2013) define the saving wedge as the difference between the domestic interest rate (R^j) and world interest rate (R^w) which is exogenous

for one small open economy j .

$$R^j = (1 - wedge^j)R^w$$

They calibrate the wedge such that the net total capital flows predicted by model fit the cross-section data sample of Non-OECD countries. The result is that the wedge is higher in African economies and lower for emerging Asian economies (for instance, China and Korea): countries with low productivity growth rate tax saving while countries with high productivity growth rate subsidy saving. Moreover, the wedge is decreasing on the level of financial development and on the income per capita.

The saving wedge can be one type of financial friction that creates the gap between the rental rate of capital for domestic firms (which equals to world interest rate for one small open economy without credit constraint) and the rate of return on savings for households. Since there does not exist one world lending interest rate across countries, we can not compute the exact measure of saving wedge by data. In fact, we can only observe the deposit interest rate ($R_t^{d,j}$) which is the rate of return on saving and the lending interest rate ($R_t^{l,j}$) which is the cost of capital in one country j at time t . Then, we define the wedge as the relative gap between the lending and deposit interest rates.

$$wedge_t^j \equiv \frac{R_t^{l,j} - R_t^{d,j}}{R_t^{l,j}}$$

On the theoretical ground, the wedge, as one type of financial friction, should be decreasing on output growth rate and on the income levels. Indeed, one country with the high economic growth or income level can be affordable to finance a better but costly financial system (Lucas (1988), Greenwood and Jovanovic (1990)) or better institutional quality (Levine, Loayza and Beck (2000), Rajan and Zingales (1998)). A higher level of financial development can, in turn, reduce the financial friction level, such as the cost induced by the asymmetric information like state-verifying cost, agency cost (Hubbard (1998), Gertler and Rogoff (1990), Bernanke and Gertler (1989)). Moreover, a higher level financial development can push up the economic growth an efficient allocation of the capital to the most productive project (Beck and Levine (2004), King and Levine (1993)). To sum up, the theory suggests that, on controlling for the level of financial development, the economic growth and income level should have negative impacts on the saving wedge.

4.3 Empirical evidences

We carry out the empirical tests for a sample of 169 countries from 1980 to 2013. The objective is to use the saving wedge and the financial development level to shed the new lights on the relationship between the productivity growth rate and the net total capital inflows. We perform the fixed-effect panel data analysis to capture the pattern of capital inflows over time and also to control for the unobserved heterogeneity which is constant over time in each country.

Productivity growth and International capital flows: the role of saving wedge

Table 4.3.1: Fixed-effect estimation results of saving wedge: 1980-2013.

VARIABLES	(1) wedge	(2) wedge	(3) wedge
Time trend (<i>t</i>)	0.0137*** (0.000498)	0.0118*** (0.000368)	0.0118*** (0.000383)
Financial development level (<i>FinDev</i>)	-0.00143*** (0.000204)	-0.00155*** (0.000183)	-0.00156*** (0.000178)
Openness (<i>kaopen</i>)	0.0148*** (0.00372)	0.0234*** (0.00303)	0.0162*** (0.00312)
Productivity growth rate (<i>TFPgrowth</i>)	0.00239*** (0.000805)		
Output-per-capita growth rate (<i>GDPpcgrowth</i>)		0.000847* (0.000486)	
Output (<i>GDP</i>)			3.03e-08*** (6.84e-09)
Constant	-26.85*** (0.990)	-23.12*** (0.732)	-23.08*** (0.762)
Observations	2,191	3,722	3,232
R-squared	0.356	0.307	0.316
Number of countries	99	161	145

Note: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. (*wedge*) is the saving wedge; (*FinDev*) is the financial development level measured by domestic credit to private sector; (*kaopen*) is the Chinn-Ito index of openness; (*TFPgrowth*) is the productivity growth rate; (*GDPpcgrowth*) is the output-per-capita growth rate and (*GDP*) is the output measured as Gross Domestic Product. Productivity is at constant national prices (2005=1). Output is the real GDP at constant 2005 national prices (in mil. 2005USD). Output-per-capita is the real GDP at constant 2005 national prices (in mil. 2005USD) per number of persons engaged (in millions).

Table 4.3.1 investigates the determinants of the interest rate wedge. In all these columns, the wedge is decreasing on the level of financial development, a confirmation that the wedge can illustrate for the level of financial friction. Column 1 shows that the wedge is increasing on the productivity growth. Column 2 and column 3 demonstrate that the wedge is still increasing on the output-per-capita

Table 4.3.2: Fixed-effect estimation results of the net total capital inflows: 1980 - 2013.

VARIABLES	(1) annual_negCA2y_ifs	(2) annual_negCA2y_ifs	(3) annual_negCApc_ifs	(4) annual_negCApc_ifs
Time trend (<i>t</i>)	-0.0942*** (0.0167)	-0.0664*** (0.0201)	-7.816*** (1.760)	-11.28*** (2.484)
Saving wedge (<i>wedge</i>)		-4.461*** (0.831)		-278.2** (109.5)
Productivity growth rate (<i>TFPgrowth</i>)	-0.250*** (3.452)	-1.025 (3.252)		
Output growth rate (<i>GDPgrowth</i>)			-14.48*** (320.4)	-97.77 (372.4)
Constant	190.1*** (33.32)	136.0*** (40.02)	15,588*** (3,512)	22,550*** (4,934)
Observations	2,934	2,219	4,237	3,172
R-squared	0.031	0.040	0.011	0.018
Number of countries	109	103	162	151

Note: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Two alternative measures of the annual net total capital inflows: one as the ratio over GDP (*negCA2y_ifs*) and other as the ratio over Population (*negCApc_ifs*); (*TFPgrowth*) is the productivity growth rate; (*GDPgrowth*) is the output growth rate. Productivity is at constant national prices (2005=1). Output is the real GDP at constant 2005 national prices (in mil. 2005USD).

growth rate and the output level. These results prove that the direct impact of economic growth rate on the saving wedge is positive, on controlling for the level of financial development.

The positive dependence of the wedge on the productivity growth rate is contradicted to the prediction by theory on the financial friction. This puzzling fact, however, does not reject that the wedge is one type of financial friction but only reveals that the wedge can contain other indicators which are positively dependent on the productivity growth.

Since a higher productivity growth rate raises the saving wedge, we can expect the wedge to be one channel for the growth rate to affect the savings and then the net total capital inflows. Following Alfaro, KalemliOzcan, and Volosovych (2008), the strategy is to add the saving wedge into the regression of the net total capital inflows on the productivity growth rate. If the coefficient of the productivity growth rate turns to be insignificant, we conclude that the saving wedge can account for the impact of the productivity growth on the net total capital inflows. We employ this strategy in the next regression.

Table 4.3.2 shows the role of the saving wedge on solving the puzzling positive relationship between the net total capital inflows and the productivity growth rate (Allocation puzzle). Column 1 shows that the 1 % of increase on the productivity growth rate reduces 0.25 % of the net total capital inflows: one country

Table 4.3.3: Fixed-effect estimation results of output-effective-labor ratio ($K2AL$) and capital-effective-labor ratio ($Y2AL$).

VARIABLES	(1) K2AL	(2) K2AL	(3) Y2AL	(4) Y2AL
Time trend (t)	955.6*** (51.72)	545.5*** (95.27)	338.4*** (12.34)	227.3*** (21.23)
Financial development level ($FinDev$)	206.3*** (20.66)	218.9*** (19.32)	43.27*** (4.929)	53.55*** (4.306)
Productivity growth rate ($TFPgrowth$)	-614.2*** (8,806)	-490.1*** (8,206)	-92.95*** (2,101)	-63.84*** (1,829)
$Human_capital$		24,001*** (4,175)		5,190*** (930.4)
$Government_size$		639.8*** (124.3)		-149.3*** (27.71)
Constant	-1.821e+06*** (102,764)	-1.073e+06*** (181,256)	-644,000*** (24,518)	-433,236*** (40,391)
Observations	3,008	2,973	3,008	2,973
R-squared	0.229	0.282	0.328	0.407
Number of countries	109	109	109	109

Note: robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. ($Y2AL$) is the output-effective-labor ratio and ($K2AL$) is the capital-effective-labor ratio; ($FinDev$) is the financial development measured by domestic credit to private sector; ($TFPgrowth$) is the productivity growth rate; $Human_capital$ is the index of human capital per person; $Government_size$ is the public expenditure per output ratio. Capital-effective-labor ratio equals to capital stock at constant 2005 national prices (in million USD) divided by total effective units of labor, which is the productivity level at constant 2005 national prices multiplied by number of people engaged. Output-effective-labor ratio equals to GDP at constant 2005 national prices (in million USD) divided by total effective units of labor.

which grows faster tends to export the capital to the rest of world. Column 2 demonstrates that the addition of the saving wedge makes the coefficient of the productivity growth to be insignificant. And, a higher saving wedge reduces the net total capital inflows. Therefore, the regression results prove that the saving wedge is crucial to explain the allocation puzzle. Column 3 and 4 repeat the same exercise for the other measures of net total capital inflows and economic growth. The increase of 1 % on the output growth rate raises 14.48 % of net total capital outflows per capita. Adding the saving wedge, however, makes the coefficient of output growth rate to be insignificant. And the wedge has a significantly negative impact on the net total capital inflows. Therefore, the regression also supports the important role of the saving wedge on solving the Allocation puzzle.

Capital accumulation: the role of financial development

Table 4.3.3 examines the role of financial development on determining the capital accumulation. Column 1 shows that a higher level of financial development increases the long-run capital accumulation, as measured by the capital-effective-

labor ratio. Column 2 demonstrates that the result is maintained when we control for the productivity growth rate, the government' size and the human capital ¹. Column 3 and 4 prove that a higher financial development level also raises the output-effective-labor ratio, even controlling for the government's size and the human capital. On short, the financial development level has a positive impact on the long-run capital accumulation and output.

We summarize the empirical evidence as three stylized facts:

1. The saving wedge is increasing on the productivity growth rate. This puzzling fact reveals that the wedge contains the variable which is increasing on the productivity growth.
2. The saving wedge is an important channel for productivity growth rate to affect the cross-border capital flows.
3. The long-run capital accumulation is increasing on the financial development level.

In the next section, we develop one benchmark model of the international capital flows based on the endogenous saving wedge to be consistent with the first two empirical stylized facts. Indeed, we will show that in one country, an increase of productivity growth can raise the net total capital outflows. On the extended economy, we add the credit constraint for firms as the measure of financial development level to explain the long-run capital accumulation. Then, we will show that the capital can flows out of the country with the high productivity growth rate and low capital-effective-labor ratio.

4.4 Benchmark Model: Productivity growth and International capital flows

The model is one small open economy which take the world interest rate as exogenous given. There is one homogeneous good, which is used for consumption and investment, and is traded freely and costlessly. Capital is free mobile between the country and the rest of world but labor is immobile, and firms are subject to changes in the productivity and labor force.

¹Levine, Loayza and Beck (2000a, b) find that the human capital and government size have positive impact on capital accumulation and output

Specification of the theoretical model

The theoretical model is constructed to be consistent with the empirical evidences, which are based on a panel-data yearly sample for 160 economies. The time dimension on the panel sample allows us to study the dynamics of the dependence pattern of capital flows on the productivity growth over time. And the cross-section dimension on the panel sample helps us to capture the common pattern of capital flows among the different countries. These two features are both taken into account on the construction of the theoretical model.

The theoretical model is one small open OLG economy which takes the interest rate as exogenous given by the world capital market. With the exogenous interest rate, the model is ready to capture the dynamics of the interested variables over time, such as the saving and investment rates. This approach is also employed by Gourinchas and Jeanne (2013) to analyze the pattern of capital flows on the productivity growth rate. Moreover, the heterogenous agents (young and old) of OLG helps us to focus on the distortion induced by the capital taxation on the marginal saving rate. Indeed, since an agent only lives for two period, the future income become unplegible. Therefore, she can not use the future incomes by the incomming young agents to smooth her consumption, one key mechanism in the Ramsey representative agent framework. On other word, the Ricardian equivalence on the irrelevant timming of taxation does not hold on our model. In sum, the small open OLG model is appropriate to analyze role of the capital taxation on shaping the pattern of capital flows on the productivity growth.

There is also a problem with the OGL framework. In details, the model is not a perfect replication of the yearly time series. Indeed, since each agent lives for two periods (young and old), each time period is appropriated of 30 years in the real life time. This structure is at odd with the yearly frequency on our data sample. Althouth a multi-period OLG economy can capture this time frequency, the cost is that we need to rely on the numerical solution rather than an analytical solution in the two-period OLG economy. Therefore, by comparing between the benefits and cost of an small open OLG model, we conclude that the model is still a convenient framework to capture the dynamic pattern of capital flows on the productivity growth.

Furthermore, the small open economy can be an easy way to approach the large open economy. Indeed, the analysis of one large open economy is in the middle between one closed economy (which fully affect the world equilibrium) and one small open economy (which can not affect the world equilibrium). Therefore, using one small open economy to analyze the case of large economy will miss the partial

effect of that economy on the world equilibrium. The advantage, however, is one tractable framework which helps us to focus on the other endogenous variables rather than the equilibrium interest rate. This approach is employed by Song, Storesletten and Zilibotti (2011) for one small open economy version of China. In sum, the small open economy in our theory can be an appropriate set-up to capture the pattern of capital flows for a sample of many economies although we admittedly can not capture fully this pattern.

4.4.1 Production

The domestic output is produced by one representative firm which employs the capital (K_t) and labor (N_t) with the constant-return-scale technology.

$$Y_t = K_t^\alpha (A_t N_t)^{(1-\alpha)} \hat{g}_t^\eta \quad (4.4.1)$$

The production function is à la Barro and Sala-I-Martin (2002) in which the public expenditure (\hat{g}_t) is one type of production factor. In details, $\hat{g}_t = \frac{G_t}{A_t N_t}$, whereby G_t is the public expenditure.

Let's define $\kappa_t = \frac{K_t}{N_t}$ as the capital-labor ratio and $\bar{\kappa}_t = \frac{K_t}{A_t N_t}$ as the capital-effective-labor ratio. Then, the output-effective-labor ratio is:

$$\bar{y}_t \equiv \frac{Y_t}{A_t N_t} = \bar{\kappa}_t^\alpha \hat{g}_t^\eta \quad (4.4.2)$$

The profit maximization problem for the firm facing the world exogenous interest rate (R^w) and the wage rate (w_t) is as following:

$$\Pi_t = K_t^\alpha (A_t N_t)^{(1-\alpha)} \hat{g}_t^\eta - R^w K_t - w_t N_t \quad (4.4.3)$$

With depreciation rate ($0 < \delta < 1$), the law of motion for capital follows:

$$K_t = I_t + (1 - \delta)K_{t-1} \quad (4.4.4)$$

With the perfect market for factor of production, the interest rate equals to the marginal product of capital.

$$R^w = \alpha \bar{\kappa}_t^{(\alpha-1)} \hat{g}_t^\eta \quad (4.4.5)$$

Therefore, the economy's total demand for capital takes the form:

$$K(R^w, A_t, N_t) = A_t N_t \bar{\kappa}_t = A_t N_t \left(\frac{\alpha}{R^w} \right)^{1/(1-\alpha)} \hat{g}_t^{\eta/(1-\alpha)} \quad (4.4.6)$$

and the corresponding real wage is equal to the marginal product of labor:

$$w_t = (1 - \alpha)A_t \left(\frac{\alpha}{R^w} \right)^{\alpha/(1-\alpha)} \hat{g}_t^{\eta/(1-\alpha)} \quad (4.4.7)$$

Plugging (4.4.6) into (4.4.1), the output is:

$$Y_t = A_t N_t \left(\frac{\alpha}{R^w} \right)^{\alpha/(1-\alpha)} \hat{g}_t^{\eta/(1-\alpha)} \quad (4.4.8)$$

4.4.2 Consumption

There are N_t newborn agents at t , each supplies one unit of labor at young and retires at old. Let denote (c_t^y, s_t^y) as the consumption and saving by the young agent at t and (c_{t+1}^o) as the consumption by that agent at old.

The utility maximization problem is:

$$\begin{aligned} \max_{(c_t^y, c_{t+1}^o, s_t^y)} & u(c_t^y) + \beta u(c_{t+1}^o) \\ c_t^y + s_t^y &= w_t \\ c_{t+1}^o &= R^w s_t^y - t_{t+1} \end{aligned}$$

Whereby the utility function $u(c) = \frac{c^{1-1/\sigma}}{1-1/\sigma}$ with the inter-temporal elasticity of substitution coefficient σ and the discount factor satisfies $0 < \beta < 1$. We will focus on the case that $(0 < \sigma < 1)$ which implies that the income effect dominates the substitution effect. This range of value is consistent with literature on asset pricing (Guisarri (2006) for one discussion) and many empirical papers (Hall (1988), Ogaki and Reinhart (1998)) which estimate the coefficient of substitution below 1.

The t_{t+1} is the capital taxation levied by the government. We assume that the government follows the residence principle in taxation: tax based on where the agents live, no matter the source of income is from the domestic or foreign markets. We also impose that the taxation is proportional to the capital income:

$$t_{t+1} = \tau_{t+1} R^w s_t^y \quad (4.4.9)$$

The Euler equation is:

$$u'(c_t^y) = \beta R^w (1 - \tau_{t+1}) u'(c_{t+1}^o)$$

Therefore, the capital taxation affects the marginal saving rate:

$$s_t^y = \varphi^s(\tau_{t+1}) w_t \quad (4.4.10)$$

where we define the marginal saving rate

$$\varphi^s(\tau_{t+1}) \equiv \frac{1}{1 + \beta^{-\sigma} [R^w(1 - \tau_{t+1})]^{(1-\sigma)}}$$

For $0 < \sigma < 1$, the marginal saving rate is increasing on the capital taxation (i.e, $\partial\varphi(\tau_{t+1})/\partial\tau_{t+1} > 0$) since a higher tax reduces the rate of return on the savings, then raises the marginal saving rate.

4.4.3 Government

As on Barro and Sala-I-Martin (2002), the government demands for the public good such that its marginal product of expenditure ($\partial Y_t/\partial G_t$) equals to its price, which is 1.

$$G_t = \eta Y_t \quad (4.4.11)$$

In order to finance the public expenditure, the government levies the capital taxation:

$$T_t = \tau_t R^w N_{t-1} s_{t-1}^y$$

We assume that the government always keeps the balanced budget².

$$\tau_t R^w N_{t-1} s_{t-1}^y = \eta Y_t \quad (4.4.12)$$

Replacing (4.4.7) into (4.4.10), and the result into (4.4.12), we get an equation determining the endogenous capital taxation:

$$\tau_t R^w \varphi^s(\tau_t) = \frac{\eta}{(1 - \alpha)} \frac{Y_t}{Y_{t-1}} \quad (4.4.13)$$

Therefore, by the implicit function theorem, the capital taxation is increasing on the output growth rate.

$$\frac{\partial \tau_t}{\partial (Y_t/Y_{t-1})} > 0$$

Since the public expenditure is complementary to the private output, a higher output growth rate raises the demand for the public expenditure. With a balanced budget constraint, the government needs to raises the capital taxation.

²The balanced government's budget constraint differs our theory to the literature on international taxation where government involves in debt and has to issue the bond and increase tax to finance public debt

4.4.4 Equilibrium

Plugging the public expenditure (4.4.11) into (4.4.8), the output is:

$$Y_t = \left(\frac{\alpha}{R^w} \right)^{\alpha/(1-(\alpha+\eta))} \eta^{\eta/(1-(\alpha+\eta))} A_t N_t \quad (4.4.14)$$

Taking into account that (4.4.11) implies $\hat{g}_t = \eta \bar{y}_t$ and using (4.4.5), we have the capital-effective-labor ratio:

$$\bar{\kappa} = \left(\frac{\alpha}{R^w} \right)^{(1-\eta)/(1-(\alpha+\eta))} \eta^{\eta/(1-(\alpha+\eta))} \quad (4.4.15)$$

Therefore, the demand for capital is as following:

$$K_t = \bar{\kappa}_t A_t N_t = \left(\frac{\alpha}{R^w} \right)^{(1-\eta)/(1-(\alpha+\eta))} \eta^{\eta/(1-(\alpha+\eta))} A_t N_t \quad (4.4.16)$$

Finally, replacing (4.4.11) into (4.4.7), the wage rate is:

$$w_t = (1 - \alpha) \left(\frac{\alpha}{R^w} \right)^{\alpha/(1-(\alpha+\eta))} \eta^{\eta/(1-(\alpha+\eta))} A_t \quad (4.4.17)$$

4.4.5 International capital flows

With the productivity and labor force growth rates, the steady state is constructed by scaling all key variables by output.

Dividing (4.4.16) by (4.4.14), the capital-output ratio is:

$$\frac{K_t}{Y_t} = \frac{\alpha}{R^w} \quad (4.4.18)$$

Plug (4.4.18) into (4.4.4) and scaling by the output, we end up with the share of investment in current output.

$$\frac{I_t}{Y_t} = \frac{K_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} - (1 - \delta) \frac{K_t}{Y_t}$$

By (4.4.14), the output growth rate is as following.

$$\frac{Y_{t+1}}{Y_t} = (1 + g_{t+1}^A)(1 + g_{t+1}^N) \quad (4.4.19)$$

Therefore, we end up with:

$$\frac{I_t}{Y_t} = [(1 + g_{t+1}^A)(1 + g_{t+1}^N) - 1 + \delta] \frac{\alpha}{R^w} \quad (4.4.20)$$

The investment-output ratio is increasing on the productivity and labor force growth rates since they raise the demand for capital to keep the marginal product of capital to be equal to the world exogenous interest rate. It is decreasing on the interest rate since the latter is the cost of capital for domestic firms.

Using the solution for the wage in (4.4.17) and the assumption of zero earning in old age, a typical date t young agent saves the amount.

$$s_t^y = (1 - \alpha) \left(\frac{\alpha}{R^w} \right)^{\alpha/(1-(\alpha+\eta))} \eta^{\eta/(1-(\alpha+\eta))} A_t \varphi^s(\tau_{t+1}) \quad (4.4.21)$$

Let denote (S_t^y, S_t^o) as the stocks of the aggregate savings by the young and old agents at the end of t . Since $s_t^o = -s_{t-1}^y$, we have:

$$\begin{aligned} S_t^y &\equiv N_t s_t^y = (1 - \alpha) \varphi^s(\tau_{t+1}) Y_t \\ S_t^o &\equiv N_{t-1} s_t^o = (1 - \alpha) \varphi^s(\tau_t) Y_{t-1} \end{aligned}$$

The stock of the aggregate savings (S_t) at the end of t is the sum of the savings by the young and old agents.

$$\frac{S_t}{Y_t} = \frac{N_t s_t^y - N_{t-1} s_{t-1}^y}{Y_t} = (1 - \alpha) \varphi^s(\tau_{t+1}) - \frac{(1 - \alpha) \varphi^s(\tau_t)}{(1 + g_t^A)(1 + g_t^N)} \quad (4.4.22)$$

The saving-output ratio depends on the expected productivity and labor force growth rates through the capital taxation. Indeed, a higher growth rate raises the capital tax rate and depresses the domestic rate of return on saving. Therefore, for $0 < \sigma < 1$, a higher future productivity growth rate raises the marginal saving rate by the young agents.

The saving of the young has two components, net foreign assets (B_{t+1}) and the capital stock that will be used in the production in the next period.

$$S_t^y = B_{t+1} + K_{t+1} \quad (4.4.23)$$

Therefore, by using (4.4.21), (4.4.19) and (4.4.18), the net stock of foreign assets, measured as a fraction of output, is:

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{S_t^y}{Y_t} \frac{Y_t}{Y_{t+1}} - \frac{K_{t+1}}{Y_{t+1}} = \frac{(1 - \alpha) \varphi^s(\tau_{t+1})}{(1 + g_{t+1}^A)(1 + g_{t+1}^N)} - \frac{\alpha}{R^w} \quad (4.4.24)$$

The current account is the change in the net stock of foreign assets .

$$\frac{CA_t}{Y_t} = \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} - \frac{B_t}{Y_t} \quad (4.4.25)$$

At steady state of constant productivity and labor force growth rates (g^A, g^N), the output growth rate is constant, then by (4.4.13), the capital tax rate is constant ($\tau_t = \tau_{t+1} = \tau$). Moreover, all key variables scaled by the total effective units of labor are constant. Therefore, the steady-state net foreign assets at the beginning of period is:

$$\frac{\overline{B}}{Y} = \frac{(1 - \alpha)\varphi^s(\tau)}{(1 + g^A)(1 + g^N)} - \frac{\alpha}{R^w} \quad (4.4.26)$$

Replacing (4.4.26) into (4.4.25), the steady-state current account to finance the net foreign assets is as following.

$$\frac{\overline{CA}}{Y} = (g^A + g^N + g^A g^N) \frac{\overline{B}}{Y} \quad (4.4.27)$$

To analyze the impact of an increase on the productivity growth rate, we assume that the economy has an initial zero net foreign assets position at the productivity growth rate ($g^{A,*}$), that is $\frac{\overline{CA}}{Y}(g^{A,*}) = \frac{\overline{B}}{Y}(g^{A,*}) = 0$. Then, we carry out the Taylor expansion to get the net total capital inflows (measured by the negative current account) for an increase of the productivity growth rate from $g^{A,*}$ to g^A .

$$-\frac{\overline{CA}}{Y}(g^A) \approx \frac{\partial(-\overline{CA}/Y)}{\partial g^A} \Big|_{g^{A,*}} (g^A - g^{A,*}) \quad (4.4.28)$$

By taking into account the dependence of the marginal saving rate on the productivity growth rate through the capital taxation, the change of the net total capital inflows is:

$$\frac{\partial(-\overline{CA}/Y)}{\partial g^A} \Big|_{g^{A,*}} = (g^{A,*} + g^N + g^{A,*} g^N) \frac{\partial(-\overline{B}/Y)}{\partial g^A} \Big|_{g^{A,*}} \quad (4.4.29)$$

Therefore, all the change in the net total capital inflows are from the change in the net foreign asset stock.

$$\frac{\partial(-\overline{B}/Y)}{\partial g^A} \Big|_{g^{A,*}} = \frac{(1 - \alpha)}{(1 + g^{A,*})(1 + g^N)} \frac{\varphi^s}{g^{A,*}} \left[\frac{g^{A,*}}{(1 + g^{A,*})} - \epsilon_{g^{A,*}}^\varphi \right] \quad (4.4.30)$$

Where we define the elasticity coefficient of the marginal saving rate by the young agent w.r.t the productivity growth rate as: $\epsilon_{g^{A,*}}^\varphi \equiv \frac{\partial \varphi^s}{\partial g^{A,*}} \frac{g^{A,*}}{\varphi^s}$. For $0 < \sigma < 1$,

$\frac{\partial \varphi^s}{\partial g^{A,*}} > 0$ and therefore, $\epsilon_{g^{A,*}}^\varphi > 0$.

Placing (4.4.30) into (4.4.29), then the result into 4.4.28, we end up with:

$$-\frac{\overline{CA}}{Y}(g^A) = \frac{(g^{A,*} + g^N + g^{A,*} g^N)}{(1 + g^{A,*})(1 + g^N)} \frac{(1 - \alpha)\varphi^s}{g^{A,*}} \left[\frac{g^{A,*}}{(1 + g^{A,*})} - \epsilon_{g^{A,*}}^\varphi \right] (g^A - g^{A,*}) \quad (4.4.31)$$

A higher productivity growth rate has two effects that tends to offset each other. The traditional channel is that a higher growth rate reduces the share of the saving by *current* young agents over the *following* period's output. Indeed, a higher productivity growth rate leads to a higher investment demand to keep the marginal product of capital to be equal to the world exogenous rental rate of capital. Given the share of savings by current young agents over current output, a higher productivity growth rate makes the saving by current young agents to be not enough for a higher investment demand. Therefore, the country needs to borrow from the rest of world to finance the increase of investment demand. We label this effect as the *Neo-Classical growth effect*, which implies that one country growing faster would experience net capital inflows to finance domestic investment, given the domestic saving is unaffected by the growth rate.

One new channel emerges within our model with the endogenous saving wedge. A higher productivity growth rate raises the share of the savings by *current* young agents over the *current* output. Since the public expenditure is complementary to the the private output, a higher growth rate requires the government to raise the capital taxation in order to finance a higher public expenditure. A higher tax rate, in turn, depresses the domestic interest rate and raises the marginal saving rate for the low value of the inter-temporal substitution coefficient. Therefore, the savings by the young agents goes up. Given the investment demand, the country lends the capital, which is dued to the increase of savings, to the rest of world. We label this effect as the *saving-wedge effect*.

The impact of an increase of the productivity growth rate on the net total capital inflows depends on the relative magnitude of the *Neo-Classical growth effect* and the *saving-wedge effect*, which is summarized on the following proposition.

Proposition 4.4.1. *Suppose that the net foreign assets is zero at the initial productivity growth rate $g^{A,*}$. At a higher productivity growth $g^A > g^{A,*}$, the net total capital inflows is positive if $\epsilon_{g^{A,*}}^\varphi < \frac{g^{A,*}}{(1 + g^{A,*})}$ and negative if $\epsilon_{g^{A,*}}^\varphi > \frac{g^{A,*}}{(1 + g^{A,*})}$.*

Proof. The result is implied directly by the equation (4.4.31). ■

Note that since a higher productivity growth raises both the saving and investment rates, the threshold on the elasticity coefficient of marginal saving rate is increasing on the initial productivity growth rate: $\frac{g^{A,*}}{1 + g^{A,*}}$. Indeed, the condition that

$\epsilon_{g^{A,*}}^\varphi > \frac{g^{A,*}}{1 + g^{A,*}}$ implicitly implies that the saving raises more than the investment. Moreover, without the endogenous saving wedge ($\epsilon_{g^{A,*}}^\varphi = 0$), a higher productivity

growth only raises the investment rate, and then an increase on the inflows of capital.

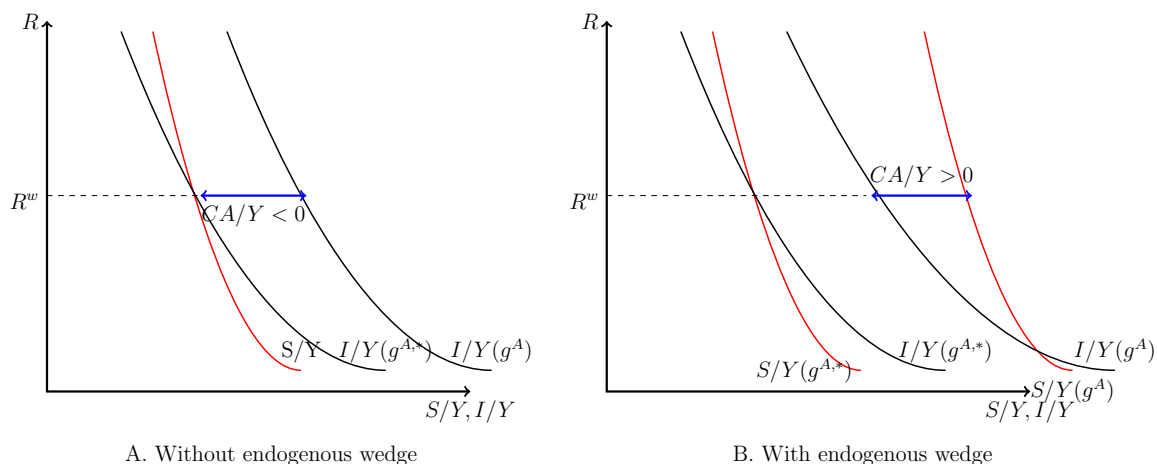


Figure 4.4.1: Saving wedge and Current account

Figure 4.4.1 demonstrates the role of the endogenous saving wedge on shaping the pattern of the international capital flows, given the initial zero capital flows. On panel A, the saving-output ratio is independent on the productivity growth rate. A higher growth rate raises the investment demand by shifting the investment curve to the right. The current account turns to be deficit. On panel B, the saving-output ratio depends on the productivity growth rate. A higher growth rate not only shifts to the right the investment curve (Neo-Classical growth effect) but also the saving curve (Saving-wedge effect). If $\epsilon_{g^A}^{\varphi} > g^{A,*}/(1 + g^{A,*})$, the saving curve moves further than the investment curve, the current account turns to be surplus. Therefore, one fast-growing economy still experiences the outflows of net total capital if the saving-wedge effect overshadows the Neo-Classical growth effect.

Our theory justifies the empirical evidence that a higher growth rate can lead to an increase of the net total capital outflows. Indeed, a higher growth rate raises the investment demand like the prediction by the Neo-Classical growth model. A higher productivity growth rate can also raises the saving by raising the capital tax rate and depresses the rate of return on saving. If the saving supply increases more than the investment demand, the fast-growing economy experiences an increase of net total capital outflows.

The endogenous saving wedge is the key channel through which the productivity growth rate can affect both the savings and the net total capital inflows. The

saving rate is increasing on the productivity growth since the intertemporal elasticity of substitution coefficient is lower than 1. Coeurdacier, Guibaud and Jin (2015) also employ this range of value to prove the outflows of capital from the fast-growing economies with a tight credit constraint for the households.

Since the productivity growth raises both the saving and investment rates, our model can account for the high correlation between the productivity growth rate and the net total capital inflows over time. Figure 4.4.2 illustrates the case of China and United States in which the fluctuation of the productivity growth rate can be the predictor of the net total capital inflows. In China, the positive correlation between the productivity growth and current account is clear over period 1995-2013. In United States, this positive co-movement pattern also appears for some periods of time, such as 1983-1989, or 2007-2013.

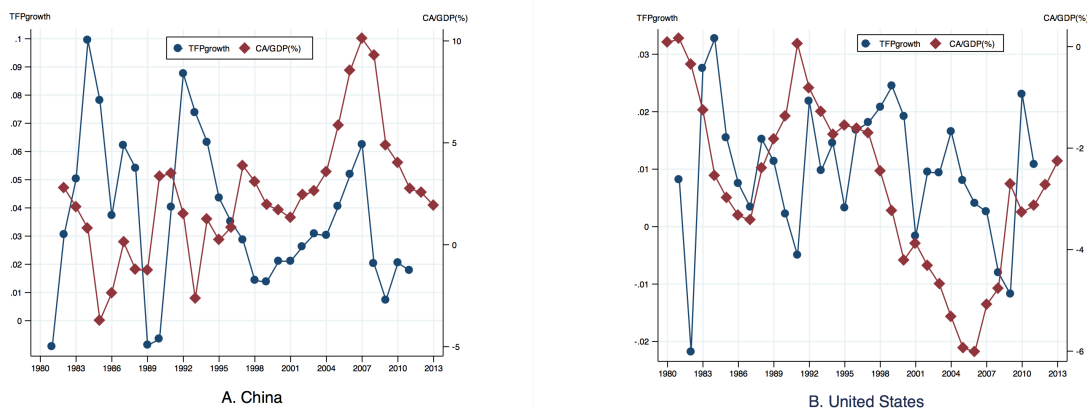


Figure 4.4.2: Net total capital outflows and Productivity growth.

Notes: (a) *TFPgrowth* is gross growth rate of productivity level at constant national prices (2005=1). Data from PWT 8.1; (b) Current account per GDP ratio from Alfaro, KalemliOzcan, and Volosovych (2015).

The benchmark model can explain the dependence pattern of international capital flows on the productivity growth for both the advanced and developing economies. In the next section, we employ the endogenous saving wedge to focus on the pattern of net total capital outflows from the fast-growing country with the low long-run capital accumulation.

4.5 Long-run capital accumulation

We add the credit constraint for the firms in the benchmark model to analyze the role of financial development on the long-run capital accumulation. Only the

profit maximization problem for the firms is different to the benchmark model.

Production

The representative firm faces the credit constraint so that the total value of capital stock can not exceed a constant fraction of output.

$$R^w K_t \leq \theta Y_t \quad (4.5.1)$$

The tightness of the parameter θ measures the financial development level as in Song, Storesletten and Zilibotti (2011). Indeed, a higher θ illustrates for a better financial system in which there are more financial resources for the domestic firms to satisfy their investment demands.

We rewrite the profit maximization for the firms as:

$$\Pi_t = Y_t - R^w K_t - w_t N_t \quad (4.5.2)$$

$$st : R^w K_t \leq \theta Y_t \quad (4.5.3)$$

Let denotes λ_t as the Lagrange multiplier for the credit constraint. With the binding credit constraint, the domestic firms can not satisfy their investment demand. Therefore, the marginal product of capital is greater or at least equal to the world interest rate as the cost of capital:

$$\lambda_t \geq 0; \frac{\partial Y_t}{\partial K_t} \geq R^w \quad (4.5.4)$$

As the benchmark model, the government chooses the public expenditure to maximize the domestic output. Then, the public expenditure contributes a constant fraction of output: $G_t = \eta Y_t$. Therefore, the output is³ :

$$Y_t = (K_t)^\alpha (A_t N_t)^{(1-\alpha)} \hat{g}_t^\eta = \eta^{\eta/(1-\eta)} (\bar{\kappa}_t)^{\alpha/(1-\eta)} A_t N_t \quad (4.5.5)$$

Low capital-effective-labor ratio in long run

We will focus on the case of binding credit constraint. In fact, the constraint would bind for a low value of financial development level (θ) or a high demand of investment (K_t^D) or a combination of both of these two features. By assuming the binding credit constraint, we want to characterize the developing economies with the low financial development levels and the high capital demands because of the domestic scarcity of capitals.

³ $\bar{y}_t = (\bar{\kappa}_t)^\alpha (\hat{g}_t)^\eta = (\bar{\kappa}_t)^\alpha (\eta \bar{y}_t)^\eta \Rightarrow \bar{y} = (\bar{\kappa}_t)^{\alpha/(1-\eta)} \eta^{\eta/(1-\eta)}$. Then, $Y_t = \bar{y}_t A_t N_t$

The binding credit constraint implies that: $K_t = \frac{\theta}{R^w} Y_t$. Dividing both sides by $A_t N_t$, we have:

$$\bar{\kappa}_t = \frac{\theta}{R^w} \bar{y}_t \quad (4.5.6)$$

Plugging (4.5.5) into the last equation, we get the capital-effective-labor ratio at binding credit constraint is as following:

$$\bar{\kappa}^b = \left(\frac{\theta}{R^w} \right) \frac{(1-\eta)}{1-(\alpha+\eta)} \frac{\eta}{\eta 1-(\alpha+\eta)} \quad (4.5.7)$$

The long-run capital accumulation with the binding constraint is decreasing on the world lending interest rate since the latter is the cost of capital for the domestic firms. It is increasing on the financial development level since the latter allows the domestic firms to borrow more to build up the domestic capital stock.

The rest of world is assumed to include the advanced economies with a very high level of financial development such that the credit constraint never binds. Then, its capital-effective-labor ratio in long run is determined such that the marginal product of capital equates the world lending interest rate. Moreover, we allow that the rest of world's production function can have a different share of capital: $\alpha^w \geq \alpha$. By using 4.4.15, we have:

$$\bar{\kappa}^w = \left(\frac{\alpha^w}{R^w} \right) \frac{(1-\eta)}{1-(\alpha^w+\eta)} \frac{\eta}{\eta 1-(\alpha^w+\eta)} \quad (4.5.8)$$

On the next step, we can find one condition on the parameter θ so that one small open economy would have a lower capital-effective-labor ratio in long run than the rest of world, even when the capital is free mobile.

$$\bar{\kappa}^b \leq \bar{\kappa}^w \Leftrightarrow \theta \leq \bar{\theta} \equiv (R^w) \frac{\alpha - \alpha^w}{1 - (\alpha^w + \eta)} (\alpha^w) \frac{1 - (\alpha + \eta)}{1 - (\alpha^w + \eta)} \frac{\eta}{\eta 1 - \eta} \frac{\alpha^w - \alpha}{1 - (\alpha^w + \eta)} \quad (4.5.9)$$

A too low level of financial development generates the persistently low long-run capital accumulation in the long run for one small open economy. Since firms can only borrow a quite low amount of loans from the international financial market, the small open economy could not accumulate enough capital to depress its domestic rate of return on capital to be equal to the world interest rate.

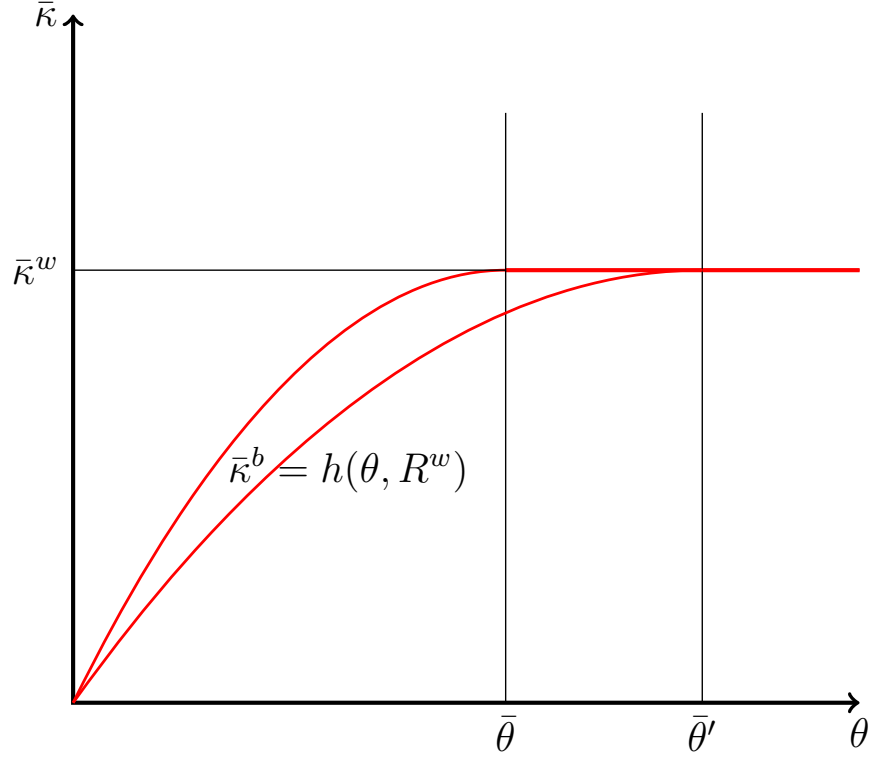


Figure 4.5.1: Financial development and long-run capital accumulation

Figure 4.5.1 illustrates the role of financial development on the long-run capital accumulation⁴ With a low financial development level ($\theta < \bar{\theta}$), one country converges to a lower capital-effective-labor ratio than the world's ratio: $\bar{\kappa}^b < \bar{\kappa}^w$. With a high financial development level ($\theta \geq \bar{\theta}$), one country converges to the world long-run capital-effective-labor ratio. In Figure 4.1.1, the Korea can converge to the same capital-effective-labor ratio as the advanced economies while the China cannot. Our result suggests that the reason for the convergence of Korea can rely on its financial market has been improved to pass the threshold $\bar{\theta}$. If the rest of world has the same share of capital on output ($\alpha^w = \alpha$), the threshold turns out to be exactly the capital share:

$$\bar{\theta} = \alpha \quad (4.5.10)$$

A more interesting result arises if the rest of world has a greater share of capital on output ($\alpha^w > \alpha$). The threshold is decreasing on the world interest rate.

$$\frac{\partial \bar{\theta}}{\partial R^w} < 0 \quad (4.5.11)$$

⁴Below the threshold, $\bar{\kappa}^b$ is a concave function of θ . At the threshold, it is constant at $\bar{\kappa}^w$.

The decrease of world lending interest rate raises the threshold on financial development for the small open economy. With a higher threshold, one country tends to fall into the area of low long run capital-effective-labor ratio. When the world lending interest rate declines over last decades (Figure 4.5.2), there might be more countries fall into the club of low persistent capital accumulation in long run. In Figure (4.5.1), the decline of world interest rate raises the threshold $\bar{\theta}$. Therefore, the countries with $\bar{\theta} \leq \theta < \bar{\theta}'$ would experience a reduction on the capital-effective-labor ratio.

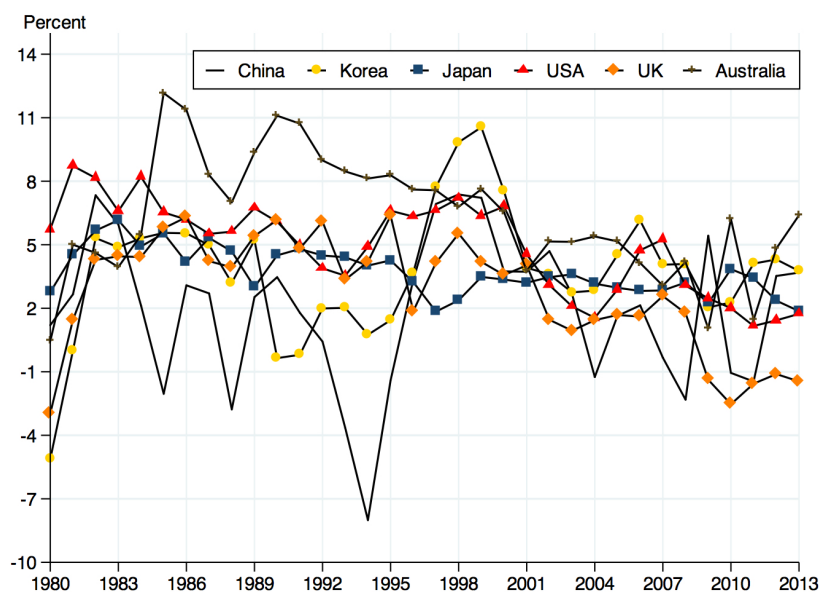


Figure 4.5.2: Real interest rate by country

Note: Real interest rate is the lending interest rate adjusted by the inflation, from World Development Indicators.

International capital flows

The ratio of the capital stock over output now depends on the tightness of credit constraint.

$$\frac{K_t}{Y_t} = \frac{\theta}{R^w}$$

Plugging the above equation into the net foreign assets position (4.4.26), we have:

$$\frac{\bar{B}}{\bar{Y}} = \frac{(1 - \alpha)\varphi^s(\tau)}{(1 + g^A)(1 + g^N)} - \frac{\theta}{R^w} \quad (4.5.12)$$

Therefore, the impact of the productivity growth on the net foreign assets position, then, on the capital flows is the same as the benchmark model. If a higher productivity growth raises the saving more than the investment, it would result in an outflows of net total capitals. In sum, within the framework of binding credit constraint, one fast growing economy can experience both (1) the outflows of net total capitals since the savings raises more than the investment and (2) the low long-run capital accumulation since its low financial development level.

4.6 Conclusion

With the exogenous lending interest rate, the flow of capitals is the difference between the savings-output and the investment-output ratios. The savings-output ratio goes up for the decrease of the deposit interest rate endogenously induced by a higher productivity growth rate. The investment-output ratio also raises for a surge in the rate of return on the capital. Therefore, the capital can flow out of a fast-growing economy if the savings-output ratio increases more than the investment-output ratio (Figure 4.1.1 *A*).

If its level of financial development is too low (below a threshold), one developing economy converges to a lower long-run capital-effective-labor ratio than the advanced economies' ratio (Figure 4.1.1 *C*).

In the appendix, the addition empirical test are carried out to test the robustness of the empirical evidences. Moreover, we present two extentions for the benchmark model: one with the public debt and one with endogenous productivity growth.

For the future research avenue, we can extend the model to analyze the capital flows through the banking system. Since the banking capital flows are driven by both the risky and risk-free assets, introducing the risky investment could generate the motivation for cross-border portfolio flows.

Appendix

4.A Data Appendix

Descriptive statistics

Data on international capital flows is extracted from the updated and extended version of dataset of net private and public capital flows constructed by Alfaro, Kalemli-Ozcan and Volosovych (2014). The net capital inflows is measured by the minus of the annual current account divided by output, denoted by (*negCA2y_ifs*) or divided by total population, denoted by (*negCApc_ifs*).

The productivity level at constant 2005 USD is explored by the Penn World Table 8.1 (updated version 2015). On this updated version, the productivity level is computed with three novelty. First, the share of factor income is different across countries and across time. Second, the physical capital stocks is decomposed by type of assets, each of which has one specific depreciation rate. These are combined with the human capital index which takes into account average year of schooling to result in total factor productivity. One key feature is that the production employed in PWT 8.1 is Harrod-neutral production function, which is also exactly the one we use in the model. Therefore, we prefer this measure on our analysis. In comparison with the approach by Gourinchas and Jeanne (2013), Caselli and Feyrer (2007), the computation of productivity level does not take into account the natural land and capital. However, on case that these natural capital does not change much over time, the productivity level from PWT 8.1 still validates the theoretical implication in our model because we use the growth rate of productivity, not its level. Moreover, Alfaro, Kalemli-Ozcan and Volosovych (2014) use the growth rate of output per capita, in stead of productivity growth rate, on explaining the pattern of capital flows across countries. That measure does not differ the source of growth rate induced by the productivity to the one induced by the capital accumulation. In our paper, the focal point would be on the role of productivity growth rate on pattern of capital flows across countries. But we

also use an alternative measure as the output per capita growth rate from Alfaro, Kalemli-Ozcan and Volosovych (2014) database if necessary.

The output and capital stock at constant 2005 national prices (in millions USD) and number of people engaged are from Penn World Table 8.1(2015). Then, the capital-effective-labor ratio, a measure of capital accumulation, equals to the real capital stock divided by the total effective units of labor, which is the productivity level multiplied by the number of people engaged. And the output-effective-labor ratio equals to the real output divided by total effective units of labor.

The level of financial development is measured by there alternatives ratios from World Development Indicators: (1) Domestic credit to private sector as the percentage of GDP; (2) Private credit by deposit money banks and other financial institutions to GDP and (3) Private credit by deposit money banks to GDP. We would focus on the first measure on the analysis, but, the regressions with two other alternative measures of financial development give the similar results.

The interest rate wedge is computed from the data on lending and deposit interest rate, which are from World Development Indicators 2016. The terms and conditions attached to these rates differ by country⁵. Therefore, even we can construct one common world interest rate based on the weight that equalizes to the share of output of each country over the world economy, the result would be no accurate. Then, our strategy is to use directly the domestic lending interest rate to compute the interest rate wedge. This measure can still validate our set-up where the wedge illustrates the financial friction between the cost of capital that domestic firms have to pay and the rate of return on saving that domestic households receives.

Other independent variables includes (1) the government size, measured by the ratio of public expenditure to output ratio from WDI; (2) index of human capital per person based on years of schooling (Barro/Lee, 2012) and returns to education (Psacharopoulos 1994) from PWT 8.1. These two variables are proved to have a positive impact on economic growth (Levine, Loayza and Beck (2000)). And the openness of one country is measured by the Chinn-Ito index of capital account openness, which measures the extensity of capital controls based on the information from the IMF's Annual Report on Exchange Arrangement and Exchange Restrictions. It combines variables which indicates the presence of multiple exchange rate, the restrictions on current account transactions, restrictions

⁵In details, the data appendix in WDI states that: "Countries use a variety of reporting formats, sample designs, interest compounding formulas, averaging methods, and data presentations for indices and other data series on interest rates".

Table 4.A.1: Descriptive Statistics

Variables	Obs	Mean	Std. Dev.	Min	Max
Net total capital inflows per output (<i>annual_negCA2y ifs</i>)	4973	3.751759	13.55661	-304.0221	240.4958
Net total capital inflows per capital (<i>annual_negCApc ifs</i>)	5100	-91.29572	1661.487	-28141.84	16484.09
Saving wedge (<i>wedge</i>)	4198	.4768908	.292787	-11.49206	.998
Financial Development (<i>FinDev</i>)	5339	55.48431	50.53333	-114.6937	367.6798
Output-effective-labor ratio (<i>Y2AL</i>)	3369	33582.95	30395.53	984.8127	276551.6
Capital-effective-labor ratio (<i>K2AL</i>)	3369	95312.51	86905.77	971.2172	685227.4
Productivity growth rate (<i>TFPgrowth</i>)	3259	.1053784	.534612	-66.07364	52.59184
Output growth rate (<i>GDPgrowth</i>)	4875	3.57206	.61251	-66.11992	106.2798
Public expenditure-output ratio (<i>Government_size</i>)	5489	16.72256	8.274957	0	156.5315
Chinn-Ito index of Capital account Openness (<i>kaopen</i>)	5462	.0736314	1.559328	-1.888895	2.389668
Human capital	4064	2.315615	.5850802	1.086181	3.618748

on capital account transactions and requirement of the surrender of export proceeds.

Table 4.A.1 provides summary statistics on the panel sample of 160 countries from 1980 to 2013. There is considerable variation for each variable. For examples, the net total capital inflows per output has a mean of -3.75% with a standard deviation of about 13.5% . The net total capital inflows per capita exhibits a mean of (-91 USD) with standard deviation of (1661 USD). The saving wedge has a mean of 0.47 , with about 0.29 of standard deviation. Similarly, the productivity growth rate and output growth rate also exhibit quite large deviation, with means of 0.1% and 3.5% respectively. Thus, the data set offers rich variation for exploring the relationship between productivity growth and net total capital inflows.

Empirical Analysis

We conduct the regressions for two subsamples: (1) the sample when the outliers of wedge is dropped out; and (2) the sample of only Non-OECD.

Table 4.A.2 shows the regression result when ruling out the outlier observation on the saving wedge. The wedge is still decreasing on the financial development level. Moreover, it is increasing on the productivity growth, although the coefficient on *GDPpcgrowth* is insignificant. On Table 4.A.3, the wedge still has a negative effect on the net total capital inflows (column 2). Other columns, however, turn to be insignificant. This can be because the observations which have the dropped value of wedges below 0 and above 0.9 also have the productivity growth rate which are highly correlated to the net total capital inflows. Therefore, dropping the outliers also reduces the significance of the regression results.

Table 4.A.4 and 4.A.5 show the regression results on the sample of Non-OECD

countries. The wedge now is only increasing on the financial development level and on the output. On the regressions of net total capitals inflows, the wedge, however, still has the negative impacts on the inflows of capitals.

In summary, the regressions shows that the results in the main text are still quite robust to the existence of outliers and to a smaller sample without OECD economies. This, in turn, provides the ground for the theoretical model in the main text.

Table 4.A.2: Fixed-effect estimation results of saving wedges: 1980 - 2013, for ($0 < wedge < 0.9$).

VARIABLES	(1) wedge	(2) wedge	(3) wedge
t (time trend)	0.0121*** (0.000453)	0.0108*** (0.000326)	0.0107*** (0.000349)
FinDev (Financial Development)	-0.00117*** (0.000185)	-0.00138*** (0.000163)	-0.00135*** (0.000163)
kaopen(Chinn-Ito index of openness)	0.0113*** (0.00327)	0.0169*** (0.00263)	0.0120*** (0.00278)
TFPgrowth (Productivity growth)	0.00129* (0.000720)		
GDPpcgrowth (Output per capita growth)		0.000295 (0.000422)	
GDP (Output)			3.09e-08*** (6.06e-09)
Constant	-23.66*** (0.899)	-21.09*** (0.649)	-20.89*** (0.694)
Observations	2,098	3,568	3,126
R-squared	0.348	0.317	0.319
Number of countries	99	161	145

Note: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. (*wedge*) is the saving wedge; (*FinDev*) is the financial development level measured by domestic credit to private sector; (*kaopen*) Chinn-Ito index of openness; (*TFPgrowth*) is the productivity growth rate; (*GDPpcgrowth*) is the output-per-capita growth rate and (*GDP*) is the output. Productivity is at constant national prices (2005=1). Output is the real GDP at constant 2005 national prices (in mil. 2005USD). Output-per-capita is the real GDP at constant 2005 national prices (in mil. 2005USD) per number of persons engaged (in millions).

Table 4.A.3: Fixed-effect estimation results of the net total capital inflows: 1980-2013 for ($0 < wedge < 0.9$).

VARIABLES	(1) annual_negCA2y_ifs	(2) annual_negCA2y_ifs	(3) annual_negCApc_ifs	(4) annual_negCApc_ifs
t (time trend)	-0.116*** (0.0177)	-0.0474** (0.0213)	-11.00*** (1.953)	-9.468*** (2.329)
wedge (saving wedge)		-5.954*** (1.052)		-144.3 (119.3)
TFPgrowth (Productivity growth)	-0.00439 (0.0341)	0.0106 (0.0340)		
GDPgrowth (Output growth)			2.378 (3.425)	2.573 (3.429)
Constant	233.1*** (35.31)	98.92** (42.31)	21,905*** (3,899)	18,905*** (4,621)
Observations	2,117	2,117	3,059	3,059
R-squared	0.021	0.036	0.011	0.011
Number of countries	103	103	151	151

Note: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Two alternative measures of the annual net total capital inflows: one as the ratio over GDP (*negCA2y_ifs*) and other as the ratio over Population (*negCApc_ifs*); (*TFPgrowth*) is the productivity growth rate; (*GDPgrowth*) is the output growth rate. Productivity is at constant national prices (2005=1). Output is the real GDP at constant 2005 national prices (in mil. 2005USD).

Table 4.A.4: Fixed-effect estimation results of saving wedges: Non-OECD economies, 1980 - 2013.

VARIABLES	(1) wedge	(2) wedge	(3) wedge
t (time trend)	0.0122*** (0.000461)	0.0110*** (0.000332)	0.0108*** (0.000351)
FinDev (Financial development)	-0.00168*** (0.000214)	-0.00192*** (0.000185)	-0.00187*** (0.000184)
kaopen (Chinn-Ito openness)	0.0107*** (0.00344)	0.0165*** (0.00274)	0.0104*** (0.00290)
TFPgrowth (Productivity growth)	0.000927 (0.000709)		
GDPpcgrowth (Output-per-capita growth)		1.39e-05 (0.000420)	
GDP (Output)			3.21e-08*** (5.97e-09)
Constant	-23.94*** (0.918)	-21.44*** (0.660)	-21.07*** (0.698)
Observations	1,677	3,130	2,692
R-squared	0.370	0.326	0.330
Number of countries	79	141	125

Note: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. (*wedge*) is the saving wedge; (*FinDev*) is the financial development level measured by domestic credit to private sector; (*kaopen*) Chinn-Ito index of openness; (*TFPgrowth*) is the productivity growth rate; (*GDPpcgrowth*) is the output-per-capita growth rate and (*GDP*) is the output. Productivity is at constant national prices (2005=1). Output is the real GDP at constant 2005 national prices (in mil. 2005USD). Output-per-capita is the real GDP at constant 2005 national prices (in mil. 2005USD) per number of persons engaged (in millions).

Table 4.A.5: Fixed-effect estimation results of the net total capital inflows: Non-OECD economies, 1980-2013

VARIABLES	(1) annual_negCA2y_ifs	(2) annual_negCA2y_ifs	(3) annual_negCApc_ifs	(4) annual_negCApc_ifs
t (Time trend)	-0.102*** (0.0217)	-0.0142 (0.0261)	-2.083 (1.839)	0.864 (2.190)
wedge (Saving wedge)		-7.829*** (1.338)		-285.9** (115.6)
TFPgrowth (Productivity growth)	0.000640 (0.0380)	0.0189 (0.0378)		
GDPgrowth (Output growth)			1.585 (3.047)	1.921 (3.047)
Constant	205.5*** (43.26)	34.01 (51.89)	4,136 (3,672)	-1,619 (4,344)
Observations	1,681	1,681	2,623	2,623
R-squared	0.014	0.034	0.001	0.003
Number of countries	82	82	130	130

Note: standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Two alternative measures of the annual net total capital inflows: one as the ratio over GDP (*negCA2y_ifs*) and other as the ratio over Population (*negCApc_ifs*); (*TFPgrowth*) is the productivity growth rate; (*GDPgrowth*) is the output growth rate. Productivity is at constant national prices (2005=1). Output is the real GDP at constant 2005 national prices (in mil. 2005USD).

4.B Extended model: Public debt

Government

We allow the government to issue the debt to finance the public expenditure. Denote B_{t+1}^G to be the value of public bonds at the end of t . We assume that the government can borrow at the same interest rate as the domestic firms⁶. Then, the public budget constraint is:

$$B_{t+1}^G - B_t^G = T_t + r^w B_t^G - G_t \quad (4.B.1)$$

Whereby the net interest rate satisfies: $R^w = 1 + r^w$.

We focus on the case in which the government follows the policy of constant debt-output ratio:

$$\frac{B_t^G}{Y_t} = -\bar{d} \quad (4.B.2)$$

Dividing (4.B.1) by Y_{t+1} , we have:

$$\bar{d} \left(R^w - \frac{Y_{t+1}}{Y_t} \right) = \frac{T_t}{Y_t} - \frac{G_t}{Y_t}$$

Indeed, from the main text,

$$\begin{aligned} T_t &= \tau_t R^w N_{t-1} s_{t-1}^y \\ G_t &= \eta Y_t \\ \frac{Y_{t+1}}{Y_t} &= (1 + g_{t+1}^A)(1 + g_{t+1}^N) \end{aligned}$$

Therefore, we end up with:

$$\frac{\eta R^w}{(Y_t/Y_{t-1})} \frac{\tau_t}{1 + \beta^{-\sigma} [R^w(1 - \tau_t)]^{(1-\sigma)}} - \left(R^w - \frac{Y_{t+1}}{Y_t} \right) \bar{d} - \eta = 0 \quad (4.B.3)$$

The implicit function theorem implies that: $\frac{\partial \tau_t}{\partial (Y_t/Y_{t-1})} > 0$ and $\frac{\partial \tau_t}{\partial \bar{d}} > 0$.

⁶Without the uncertainty, the non-arbitrage condition would imply the net rates of return on two different assets need to be equal. This assumption also is consistent with the residence principle for the capital taxation. Indeed, the households would have the same after-tax rate on return when they buy the public bonds or invest on the domestic firms.

International capital flows

Let denote B_{t+1} as the net stock of foreign assets at the end of t . The young agents allocates their saving into the domestic capital stock, the public bonds and the foreign assets.

$$S_t^y = K_{t+1} + B_{t+1}^G + B_{t+1} \quad (4.B.4)$$

Therefore, the net foreign asset is:

$$B_{t+1} = S_t^y - K_{t+1} - B_{t+1}^G \quad (4.B.5)$$

Scaling the last equation by the output and taking into account (4.B.2), we end up with:

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{\eta}{1 + \beta^{-\sigma}[R^w(1 - \tau_{t+1})]^{(1-\sigma)}} \frac{1}{(1 + g_{t+1}^A)(1 + g_{t+1}^N)} - \frac{\alpha}{R^w} - \bar{d} \quad (4.B.6)$$

Evaluating the (4.B.6) at steady state, we end up with:

$$\frac{\bar{B}}{\bar{Y}} = \frac{\eta}{1 + \beta^{-\sigma}[R^w(1 - \tau)]^{(1-\sigma)}} \frac{1}{(1 + g^A)(1 + g^N)} - \frac{\alpha}{R^w} - \bar{d}$$

Therefore, the net total capital inflows is

$$-\frac{\bar{CA}}{\bar{Y}} = (g^A + g^N + g^A g^N) \left(-\frac{\bar{B}}{\bar{Y}} \right) \quad (4.B.7)$$

Around the initial steady state of zero net foreign assets, the impact of an increase of the domestic public debt on the net total capital inflows is as following.

$$\frac{\partial(\bar{-B}/\bar{Y})}{\partial \bar{d}} = \frac{(1 - \alpha)}{(1 + g^A)(1 + g^N)} \frac{\varphi^s}{\bar{d}} \left[\frac{(1 + g^A)(1 + g^N)\bar{d}}{(1 - \alpha)\varphi^s} - \epsilon_d^\varphi \right] \quad (4.B.8)$$

$$\frac{\partial(\bar{-CA}/\bar{Y})}{\partial \bar{d}} = (g^A + g^N + g^A g^N) \frac{\partial(\bar{-B}/\bar{Y})}{\partial \bar{d}} \quad (4.B.9)$$

whereby, the elasticity coefficient of the marginal saving rate by young agents w.r.t the public debt is:

$$\epsilon_d^\varphi \equiv \frac{\partial \varphi^s}{\partial \bar{d}} \frac{\bar{d}}{\varphi^s} > 0$$

Therefore, if the elasticity coefficient of the marginal saving rate by young agents w.r.t the public debt is great enough, an increase of public debt raises the net total capital outflows because of the increase of private savings.

$$\epsilon_d^\varphi > \frac{(1 + g^A)(1 + g^N)\bar{d}}{(1 - \alpha)\varphi^s} \Rightarrow \frac{\partial(\bar{-B}/\bar{Y})}{\partial \bar{d}} < 0 \quad (4.B.10)$$

In summary, we can observe that one country with a higher public debt runs the surplus current account and exports the net total capital flows if the elasticity coefficient of the marginal saving rate w.r.t the public debt is high enough. Like the benchmark model, the mechanism relies on the endogenous saving wedge: a higher public debt raises the capital tax rate, depresses the domestic interest rate and raises the saving by young agents.

On the alternative set-up where the taxation is lump-sum and levied into the young's labor income, a higher public debt only raises the net total capital inflows, no matter the inter-temporal substitution coefficient. In details, we only need to modify the utility maximization problem:

$$\max_{(c_t^y, c_{t+1}^o, s_t^y)} u(c_t^y) + \beta u(c_{t+1}^o) \quad (4.B.11)$$

$$c_t^y + s_t^y = w_t - \tau_t \quad (4.B.12)$$

$$c_{t+1}^o = R^w s_t^y \quad (4.B.13)$$

Euler equation gives the saving function:

$$s_t^y = \frac{w_t - \tau_t}{1 + \beta^{-\sigma} R^{w(1-\sigma)}} \Rightarrow S_t^y = \frac{w_t N_t - N_t \tau_t}{1 + \beta^{-\sigma} R^{w(1-\sigma)}} = \frac{w_t N_t - T_t}{1 + \beta^{-\sigma} R^{w(1-\sigma)}}$$

On aggregation, the total savings by young agents scaled by the output is:

$$\frac{S_t^y}{Y_t} = \frac{\eta}{1 + \beta^{-\sigma} R^{w(1-\sigma)}} - \frac{T_t}{Y_t} \quad (4.B.14)$$

The public budget constraint is:

$$B_{t+1}^G = R^w B_t^G + T_t - G_t$$

We also assume that the public debt over output ratio is constant: $\frac{B_t^G}{Y_t} = -\bar{d}$.

Then, scaling the public budget constraint by the output at $(t + 1)$, we have:

$$\frac{T_t}{G_t} = \bar{d} \left(R^w - \frac{Y_{t+1}}{Y_t} \right) + \eta \quad (4.B.15)$$

Placing (4.B.15) into (4.B.14), the aggregate saving by young agents is:

$$\frac{S_t^y}{Y_t} = \frac{\eta}{1 + \beta^{-\sigma} R^{w(1-\sigma)}} - \bar{d} \left(R^w - \frac{Y_{t+1}}{Y_t} \right) - \eta \quad (4.B.16)$$

Therefore, an increase of the public debt from (\bar{d}^*) to (\bar{d}) leads to the change in the saving-output ratio as:

$$\frac{\bar{S}}{\bar{Y}}(\bar{d}) \approx \frac{\bar{S}}{\bar{Y}}(\bar{d}^*) + \frac{\partial \bar{S}/\bar{Y}}{\partial \bar{d}}(\bar{d} - \bar{d}^*) \quad (4.B.17)$$

whereby, given the conventional view that the gross interest rate is greater than the gross output growth rate: $R^w > (1 + g^A)(1 + g^N)$,

$$\frac{\partial(\overline{S/Y})}{\partial \bar{d}} = -[R^w - (1 + g^A)(1 + g^N)] < 0 \quad (4.B.18)$$

Therefore, the saving-output ratio goes down:

$$\frac{\overline{S}}{\overline{Y}}(\bar{d}) < \frac{\overline{S}}{\overline{Y}}(\bar{d}^*) \quad (4.B.19)$$

Therefore, an increase of public debt reduces the disposable income and the savings, raises the inflows of net total capital. Note that the value of σ does not affect the result.

4.C Extended model: capital flows in the club of convergence

We develop one two-country economy to employ the endogenous growth theory to explain the pattern of international capital flows. The main idea is from the theory on the club of convergence by Aghion, Howitt, Brant-Collett and Garca-Pealosa (1998). Only the countries which invest on the research and development (R&D) can converge to the world technology frontier and belong to the club of convergence. In this club, the countries which are at the frontier have a high expenditure on R&D to invent the new technology and push forward the world frontier. The countries which are far from the frontier only need a lower expenditure on R&D since it only needs to imitate the technology at the frontier. Moreover, further from the world frontier one country is, higher growth rate that country experiences since the imitation of technology is more easier than the invention of new technology.

To capture the idea of the club of convergence, we assume that the public expenditure on R&D is exogenous and is different across countries. Moreover, the productivity level is a concave function of the public expenditure on R&D. And the government levies the income taxation to finance the public expenditure. The assumption that only the government invests on research and development simplifies the model and is ready to be extended for the role of the private involvement in R&D.

There are two periods: 0 and 1. Two countries: Home (H) and Foreign (F). There is one good which is used for consumption and investment and its price is normalized to be 1. The population is normalized to 1 in each country. The capital is free

mobile across two countries but the labor is immobile. The country is denoted by $j = H, F$.

Production

The production takes place in the period 1. It follows a CRS function:

$$Y^j = A^j(K^j)^\alpha(N^j)^{(1-\alpha)}$$

The firm in the country j solves the profit maximization problem:

$$\Pi^j = Y^j - R^j K^j - w^j N^j$$

The factor market is perfect competitive, each factor earns its marginal product. With full depreciation rate and the labor supply being normalized to 1, the capital demand and the wage rate are as following

$$K^j = \left(\frac{\alpha A^j}{R^j} \right)^{1/(1-\alpha)} \quad (4.C.1)$$

$$w^j = (1-\alpha)A^j \left(\frac{K^j}{N^j} \right)^\alpha = (1-\alpha)(A^j)^{1/(1-\alpha)} \left(\frac{\alpha}{R^j} \right)^{\alpha/(1-\alpha)} \quad (4.C.2)$$

Consumption

One representative agent who supplies one unit of labor in the country j . She chooses the consumption and saving rates to maximize the life-time utility subject to the budget constraint.

$$\begin{aligned} U_0^j &= u(c_0^j) + \beta u(c_1^j) \\ st : c_0^j + s^j &= w_0^j - T^j \\ c_1^j &= R^j s^j + w^j \end{aligned}$$

At period 0, the agent receives an exogenous endowment w_0^j . She has to cover the taxation imposed by the domestic government. Then, she allocates the disposable income between consumption and saving. At period 1, she receives the wage rate and the interest rate on the savings. Then, she consumes all available income. Since the taxation is only levied in the period 0, it only affect the disposable income and does not affect the marginal saving and consumption rates. Therefore, we would employ the log utility function without losing the generality. The Euler equation gives the saving rate as:

$$s^j = \frac{\beta}{1+\beta}(w_0^j - T^j) - \frac{w^j}{R^j(1+\beta)} \quad (4.C.3)$$

Government

The government collects the tax to finance the public expenditure on R&D. The public expenditure is exogenous at Z^j . It keeps the balanced budget:

$$Z^j = T^j \quad (4.C.4)$$

The Home economy is assumed to be an advanced economy at the world technology frontier while the Foreign one is a developing economy which is far from the frontier. Therefore, Home's public expenditure is higher than Foreign's.

$$Z^H > Z^F \quad (4.C.5)$$

Endogenous productivity growth

The productivity level is assumed to be a concave function of the expenditure on the research and development. This function captures the idea of the decreasing return to scale on the R&D sector. Moreover, it also assures the convergence of productivity growth rate.

$$A^j = (Z^j)^\Phi; 0 < \Phi < 1 \quad (4.C.6)$$

Autarky equilibrium

At autarky, the saving is equal to the investment demand.

$$s^j = K^j \quad (4.C.7)$$

Plugging the wage rate (4.C.2) and the public balanced budget (4.C.4) into the saving function (4.C.3), we have the supply of savings as:

$$s^j = \frac{\beta}{1 + \beta} (w_0^j - Z^j) - \frac{(1 - \alpha)}{(1 + \beta)} \alpha^{\alpha/(1-\alpha)} Z^{j\Phi/(1-\alpha)} R^{j1/(\alpha-1)} \quad (4.C.8)$$

Then, we replacing the public balanced budget into the investment demand (4.C.1),

$$K^j = \alpha^{1/(1-\alpha)} Z^{j\Phi/(1-\alpha)} R^{j1/(\alpha-1)} \quad (4.C.9)$$

The autarky equilibrium implies the interest rate as:

$$R^j = \frac{\alpha \left(\frac{\beta}{1 + \beta} \right)^{(\alpha-1)} Z^{j\Phi} (w_0^j - Z^j)^{(\alpha-1)}}{\left(1 + \frac{1 - \alpha}{\alpha} \frac{1}{1 + \beta} \right)^{(\alpha-1)}} \quad (4.C.10)$$

Therefore, the interest rate is increasing on the R&D expenditure:

$$\frac{\partial R^j}{\partial Z^j} = R^j \left(\frac{\Phi}{Z^j} + \frac{1 - \alpha}{w^j - Z^j} \right) \quad (4.C.11)$$

Since the taxation reduces the disposable income for the consumer, a higher tax results in a lower supply of savings. Therefore, given the investment demand, the interest rate needs to be higher to clear the capital market. Note that the dependence of interest rate on the R&D expenditure is independent on the structure of the function of productivity level. Whenever $\Phi > 0$, $\frac{\partial R^j}{\partial Z^j} > 0$.

Integration equilibrium

At integration, the capital is free mobile across countries. The world interest rate (R^w) prevails to clear the world capital market:

$$s^H + s^F = K^H + K^F \quad (4.C.12)$$

Therefore, the world interest rate is:

$$R^w = \frac{\alpha \left(\frac{\beta}{1 + \beta} \right)^{(\alpha-1)}}{\left(1 + \frac{1 - \alpha}{\alpha} \frac{1}{1 + \beta} \right)^{(\alpha-1)}} \left[\frac{(w_0^H - Z^H) + (w_0^F - Z^F)}{(Z^H)^{\Phi/(1-\alpha)} + (Z^F)^{\Phi/(1-\alpha)}} \right]^{(\alpha-1)} \quad (4.C.13)$$

Then, we can show that the world interest rate is related to the autarky interest rates by an decreasing function $F(x) = x^{1/(\alpha-1)}$.

$$(R^w)^{1/(\alpha-1)} = \lambda^H (R^H)^{1/(\alpha-1)} + \lambda^F (R^F)^{1/(\alpha-1)} \quad (4.C.14)$$

Whereby, $\lambda^j \equiv \frac{(Z^j)^{\Phi/(1-\alpha)}}{\sum_j (Z^j)^{\Phi/(1-\alpha)}}$. And $0 < \lambda^j < 1$; $\sum_j \lambda^j = 1$. The world interest rate is between the lowest autarky rate and highest autarky rate:

$$\min_j (R^j) < R^w < \max_j (R^j) \quad (4.C.15)$$

International capital flows

For each country j , the savings at the end of period 0 is divided between the net foreign assets (B_1^j) and the capital stock (K_1^j). Given the zero initial net foreign asset $B_0^j = 0$, the current account is:

$$CA^j \equiv B_1^j = s^j - K^j$$

Evaluating the saving function (4.C.3) and the capital investment (4.C.1) at the world interest rate, taking into account the autarky interest rate (4.C.10), the net total capital inflows depends on the difference between the autarky and world interest rates.

$$-CA^j = \alpha^{1/(1-\alpha)}(Z^j)^{\Phi/(1-\alpha)}\left(1 + \frac{1}{1+\beta} \frac{1-\alpha}{\alpha}\right) \left[\frac{1}{(R^w)^{1/(1-\alpha)}} - \frac{1}{(R^j)^{1/(1-\alpha)}} \right]$$

Since the Home economy has a higher R&D expenditure than the Foreign and the interest rate is increasing on the R&D expenditure, the Home economy would experience an inflows of net total capitals.

$$-CA^H > 0 \Leftrightarrow R^H > R^w > R^F \Leftrightarrow Z^H > Z^F \quad (4.C.16)$$

By the concave function of the productivity level, at higher R&D level, the productivity growth is lower.

$$g^j \equiv \frac{dA^j}{A^j} = \frac{\Phi}{Z^j} \quad (4.C.17)$$

Therefore, since Home's expenditure on R&D is higher, its productivity growth rate is lower than Foreign's.

In sum, the Home country with a high R&D expenditure can have both an inflows of net total capital and the low productivity growth. Indeed, since the Home economy is at the world frontier and needs a higher R&D expenditure to inovate the new technology. Therefore, it also needs to impose a higher income taxation than the Foreign. The high taxation, in turn, reduces the supply of savings and raises the autarky interest rate. At world integration, the Home's economy experiences the inflows of net total capitals to compensate the shortage of savings. On other words, the flow of capitals from the developing to advanced economies is used to finance the higher expenditure on R&D, which is necessary for the advanced economies to push forward the world technolofy frontier.

The developing economies, in our model, are willing to export capitals since they only need a lower R&D expenditure to immitate the new technology at the frontier. We also conjecture that when one developing economy approaches the world technology frontier, its R&D expenditure increases. Therefore, both the outflows of net total capitals and the productivity growth rate reduce when one developing economy approach the world technology frontier.

Chapter 5

General Conclusion

The Thesis focuses on the dependence pattern of the international capital flows on the productivity growth. Chapter 2 builds up one two-country stochastic economy with two risky assets and one risk-free bond, then carries out the empirical test for the panel sample of Euro zone. We show that a higher productivity level can raise the accumulation of safe assets (i.e, risk-free Bonds) and the safe assets's flows is the main driver of international capital flows in Euro zone. Chapter 3 proves one non-linear dependence pattern of net total capital inflows on the productivity growth by both the multi-country OLG model and the cross-section analysis of 160 economies. Indeed, at the low and high range of value of productivity growth, a higher growth rate leads to a higher inflow of net total capital but at the middle range of value, a higher growth rate reduces the inflows of net total capital. Chapter 4 focuses on the net total capital outflows from the emerging economies which have both the high productivity growth rates and the low long-run capital accumulation. In the benchmark small open economy with the endogenous saving wedge, a higher productivity growth can raise both the saving and investment. Therefore, if the saving goes up more than the investment, one fast-growing economy experiences an outflow of capitals. By adding the credit constraint for firms, the long-run capital accumulation is increasing on the financial development level. Therefore, there exist one threshold of credit constraint below which one economy converges to the lower capital accumulation level than the rest of world's.

The model on Chapter 2 can be extended to account for the role of safe asset scarcity on shaping the cross-border capital flows. For the last decades, United States turns from being the world banker who provides the loans to the rest of world to being the world venture capitalism who borrows the cheap funds from the rest of world in order to makes the foreign investments with the higher rates of re-

turn such as Equities and FDI (Gourinchas and Rey (2007b)). This phenomenon raises the question why the rest of world (mostly the developing economies) is willing to lend the cheap funds to US even with low interest rate? Farhi and Maggiori (2016) construct one two-country stochastic economy when one Home country supplies the safe assets to meet the demand by Foreign. The Home country enjoys the excess return on the risky investment while Foreign one smooths consumption over time. Since both of two economies can access the same risky investment, their model does not capture the difference on the rates of return on the risky investment between US and the rest of world. Therefore, by adding the mechanism underlying the supply of safe assets into the model with the differential rates of return across countries in Chapter 2, we hope to characterize the role of US as the world venture capitalism.

The analysis of Chapter 3 can be extended to separate the public and private capital flows. We can allow the government in each country collects taxation and issues debt to finance the public expenditure. With the policy of constant debt-output ratio, the model can characterize the capital taxation needed to finance the debt. The taxation, in turn, can affect the savings and then, the inflows of net total capital. To analyze both public and private flows, we also need to allow both the international capital market for the public debt and for the private investment. Moreover, we also need to add another feature (such as the uncertainty on the domestic investment) so that the public debt differs the domestic investment. To do this, we might use the approach of Bruno and Shin (2014) when they employ the banking sector to analyze the gross banking capital flows.

The model of endogenous saving wedge in Chapter 4 can be extended to study the capital controls which have played one central role on the debate on the financial globalization (Frenkel, Razin and Yuen (1996)). The recent strong monetary policy implemented by the advanced economies central banks limits the room for the multilateral coordination and raises the demand for the capital control as the natural instrument for the macro and financial stability (Blanchard (2016)). On the empirical ground, Cowan and Gregorio (2007) find that the capital control contributes on promoting the economic growth in Chile while Prasad and Wei (2007) argue the capital control has been implemented successfully to affect the composition of the capital flows in China. We can decompose the capital taxation on Chapter 4 into the the taxation on making the foreign investment and the taxation on receiving the foreign investment from the rest of world. The separation between the taxation on the inflows and outflows of capitals can account for role of the capital controls on the gross capital flows in one open economy.

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