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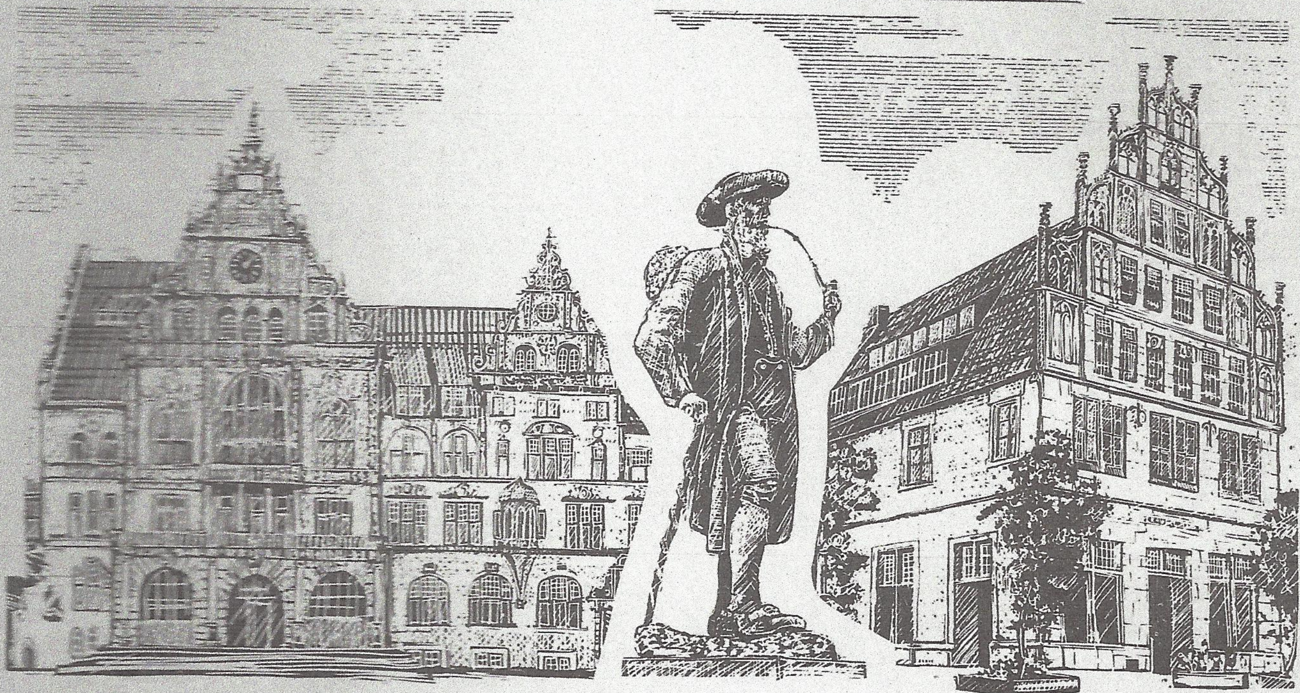

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A Solution Theory for the Finite Negotiation Problem

by

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Introduction

The bargaining problem, in which two bargainers choose an outcome from a convex set of alternatives, has been widely discussed. We develop in this paper[1] a solution theory for the problem when the set of alternatives is discrete and comes from a non-cooperative game.

The problem and solution theory are discussed in the first section and developed in sections 2 and 4. Section 3 is the description of an equivalent problem, section 5 compares the theory with a theory derived from the Nash bargaining theory, and two applications are made in sections 7 and 8.

1. The Finite Negotiation Problem

In general, negotiation or bargaining between two parties involves a discussion between the parties of possible outcomes in a specific situation. The negotiation or bargaining is successful if they agree on an outcome. A bargaining problem is usually understood to allow the possibility of mixed outcomes, that is outcomes which are probability mixtures of pure outcomes. (See,for example, Nash 1950 and 1953). For this reason, we will use the term negotiation problem when such probability mixtures are not allowed. There are many such situations, in fact a mixed outcome is usually accepted only when there is a common, infinitely divisible

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[1] Prepared for presentation at the Reisensburg International Symposium on 'Decision Theory and Social Ethics', June 1976. The author gratefully acknowledges helpful discussions with R. Selten.

unit of exchange which continuously represents utility. Most négotiators would reject as nonsense an outcome on a yes-no vote unless the outcome was yes or no.

In discussions between nations on questions such as tarifs or exchange rates, a bargaining situation results, while discussions on questions such as whether or not to go to war, change policies or present ambassadors, mixed outcomes make no sense and a (finite) negotiation situation develops.

A negotiation problem is a finite set of logically distinct actions for each player and a set of utilities for each player, one utility for each possible assignment of actions to players. We will work with two players, denoting their sets of actions as

$$\text{player 1} : \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$$
$$\text{player 2} : \{\beta_1, \beta_2, \ldots, \beta_m\}$$

With each pair of actions (α_i, β_j) is associated two utilities, one for each player

$$\text{player 1} : (\alpha_i, \beta_j) \to a_{ij}$$

$$\text{player 2} : (\alpha_i, \beta_j) \to b_{ij}$$

The utilities are real numbers and the natural order on the real numbers denotes preference. We will also assume that no two outcomes have the same utility:

$$a_{ij} = a_{rs} \Rightarrow i = r \text{ and } j = s,$$

$$b_{ij} = b_{rs} \Rightarrow i = r \text{ and } j = s.$$

This is not as restrictive as it seems as people usually find some reason, no matter how small, to differentiate between discrete outcomes. Moreover, the set of problems in which two or more outcomes are equal is nowhere dense in the set of all problems with a natural topology.

A solution theory for negotiation problems is usually understood to be a function which assigns one outcome to each problem and satisfies several axioms. Two axioms which are commonly expected are symmetry and Pareto optimal axioms:

Symmetry If (α_i, β_j) is the solution to the problem $(\{\alpha_1,\dots,\alpha_n\}, \{\beta_1,\dots,\beta_n\}, \{(a_{ij}, b_{ij})\})$ then the solution to the problem $(\{\beta_1,\dots,\beta_n\}, \{\alpha_1,\dots,\alpha_n\}, \{b_{ji}, a_{ji})\})$ is (β_j, α_i).

Pareto optimality If (α_i, β_j) is the solution and (α_r, β_s) is another outcome such that one player prefers (α_r, β_s), then the other player prefers (α_i, β_j).

It is unhappy that there can be no solution to the finite negotiation problem which satisfies these two axioms. In figure 1 a situation is given in which (0,0) cannot be a solution because of the Pareto optimal condition, and neither (1,2) nor (2,1) can be the solution because of the symmetry condition ((1,2) in the square (α_2, β_1) means $a_{21} = 1$ $b_{21} = 2$.)

	β_1	β_2
α_1	(0,0)	(2,1)
α_2	(1,2)	(0,0)

Figure 1

Because of this example, we abandon the search for a solution theory given by axioms and turn to a dynamic model in which an outcome is chosen through a formal negotiation process.

It is assumed that, because of mutual trust or some outside mechanism, statements made by participants in the formal negotiation process which bear on their activity will be believed by the other parties.Thus, a negotiator may state that it may rain tomorrow or that it would be in the other persons' best interest to take action β_3, but these statements, having nothing to do with what he will or will not do, will not advance the negotiation process. However, a statement by player 1 to the effect that he will take action α_1 if player 2 takes action β_1 will be believed by player 2 and will advance the negotiations . In essence, by uttering $\beta_1 \rightarrow \alpha_1$, it becomes a true statement, and it is a requirement of the theory that such commitment power exists. This power may exist because of a gentleman's agreement or by the intervention of an enforcement agency, but most often it exists because the failure to honor a commitment will be detected by the other player soon enough for him to retaliate in fashion. The process begins with one player making a statement, then the other player, then the first and so on in rotation until only one outcome is logically possible. The two players then take the actions associated to these outcomes and the session is over. The only restrictions on the statements are that they advance the negotiating process and do not involve logical contractictions.

The process is therefore a game in which moves are statements. To calculate the equilibrium strategies in this game it is necessary to give a formal interpretation of the foregoing paragraph, and this leads immediately to the inherent complexity and logical problems of everyday language. What we do is to describe a formal game which (we hope) captures the idea of such negotiation without unduly restricting the complexity of everyday discourse.

2. The Negotiation Game

Player 1 has a set of atomic statements

$$\{\beta_j \rightarrow \sim\alpha_i\} \quad i = 1,\ldots,n, j = 1,\ldots,m$$

and player 2 has a set of atomic statements

$$\{\alpha_i \rightarrow \sim\beta_j\} \quad i = 1,\ldots,n, j = 1,\ldots,m$$

A move for a player is a statement constructed from that player's atomic statements using $\wedge$, $\vee$ and parentheses. "$\wedge$" means "and", "$\vee$" means "or", "$\rightarrow$" means "implies" and "$\sim$" means "not". The statement α_i means that player 1 will take action α_1. Let P be the conjunction of all moves made by player 1 previous to a move, and Q the conjunction of all moves made by player 2 previous to the same move. Play terminates when there is exactly one pair (α_i, β_j) such that P, Q and $\alpha_i \wedge \beta_j$ are consistent.

A move P' for player 1 is illegal if:

i) $P \wedge P' \leftrightarrow P$.

ii) There exists a j such that $P \wedge P' \rightarrow \bigwedge_i (\beta_j \rightarrow \sim\alpha_i)$, unless $Q \rightarrow \bigwedge_i (\alpha_i \rightarrow \sim\beta_j)$.

iii) Three does not exist a sequence of legal moves $Q', P'', Q'', \ldots$ which terminates the game.

There are a similar set of rules for player 2. Rule i) guarantees that the move advances the negotiation process, so that play may terminate in a finite number of moves. Rule ii) makes it illegal to rule out the action β_j of the other player unless he has already done so himself. It also keeps the player from committing the illogical $\beta_j \rightarrow (\alpha_1 \wedge \alpha_2)$ and from refusing to negotiate $(\sim\alpha_1 \wedge \sim\alpha_2 \wedge \ldots \wedge \sim\alpha_n)$. (If refusal to negotiate is possible, it is an action and is included in the list of actions.)

Rule iii) is the statement that a move is illegal unless it is one of a sequence of legal moves which lead to a successful termination.
These rules and the finite nature of the game ensure a termination.

Rule ii) and iii) were chosen with an eye to satisfying the common feeling that a negotiator may not force the other player into not doing something he would otherwise be free to do or into a logical paradox from which there is no escape.

It remains to demonstrate that there does exist a sequence of legal moves which leads to a termination. In fact, it is easy to show that for any i and j, there is a sequence of legal moves terminating in exactly (α_i, β_j). Let $P_i = \bigwedge_s \bigwedge_{r \neq i} (\beta_s \rightarrow \sim\alpha_r)$ and $Q_j = \bigwedge_{s \neq j} (\alpha_i \rightarrow \sim\beta_s)$, then (α_i, β_j) is the only pair for which P_i, Q_j and $\alpha_i \wedge \beta_j$ are consistent, and the moves are easily seen to be legal.

In translating everyday language into moves, the formally described moves may appear to be excessively detailed, but there are some formulas which make the job relatively easy:

$$(\beta_j \to \alpha_i) \equiv \bigwedge_{k \neq i} (\beta_j \to \sim\alpha_i)$$

$$\sim\alpha_i \equiv \bigwedge_{r} (\beta_r \to \sim\alpha_i)$$

$$\alpha_i \equiv \bigwedge_{j \neq i} \sim\alpha_j$$

$$\beta_1 \to (\alpha_1 \vee \alpha_2) \equiv (\beta_1 \to \alpha_1) \vee (\beta_1 \to \alpha_2)$$

Thus, $P_i = \alpha_i$, $Q_i = (\alpha_i \to \beta_j)$, which are the common "I will do α_i " and "If you do α_i, I will do β_j." In translating common language, it is usual to discount impossible or contradictory statements or parts of statements and verify the logical content before proceeding. "If you threaten to do β_1 when I do α_1, then I will do α_2" is $(\alpha_1 \to \beta_1) \to \alpha_2$, which is equivalent to $(\alpha_1 \wedge \alpha_2) \vee (\beta_2 \to \alpha_1)$ and we would discount the impossible $\alpha_1 \wedge \alpha_2$ and verify the logical content of $\beta_2 \to \alpha_1$. Actually, the statement given as $(\alpha_1 \to \beta_1) \to \alpha_2$ is often misinterpreted from "If you do β_1 when I do α_1, then I will do α_2", which really means that player 1 has two actions, α_1 and α_2, which are not logically distinct (he may do both), and the original actions should have been $\alpha_1' = \alpha_1 \wedge \sim\alpha_2$, $\alpha_2' = \alpha_1 \wedge \alpha_2$, $\alpha_3' = \sim\alpha_1 \wedge \sim\alpha_2$, $\alpha_4' = \sim\alpha_1 \wedge \alpha_2$. This short discussion gives an indication of the problems and pitfalls of the translation problem.

3. An Equivalent Game

To make the structure of this game more intuitive to the reader, we construct an quivalent game. Let A be an **nXm** bi-matrix where each entry is $A_{ij} = (1,1)$. A move consists of "choosing" some subsets of the set of all entries in A. Thus a move by the first player, who moves first, might be $\{A_{11}\}$ and $\{A_{34}, A_{42}, A_{41}\}$. The choice of $\{A_{11}\}$ corresponds to $\beta_1 \to \sim\alpha_1$ and the other set to $(\beta_4 \to \sim\alpha_3) \vee (\beta_2 \to \sim\alpha_4) \vee (\beta_1 \to \sim\alpha_4)$, so the whole move is

$$(\beta_1 \to \sim\alpha_1) \wedge ((\beta_4 \to \sim\alpha_3) \vee (\beta_2 \to \sim\alpha_4) \vee (\beta_1 \to \sim\alpha_4)).$$

Since every statement made from atomic statements using $\wedge$, $\vee$ and parentheses may be rewritten as groups of atomic statement connected by $\vee$ and the groups connected by $\wedge$ (a proof is easily constructed from the identity $(p \wedge q) \vee r \leftrightarrow (p \vee r) \wedge (q \vee r)$), each move in the original game corresponds to a unique move in the new game, and vice versa. When a singleton set is chosen by a player, the entry he corresponds to is changed by changing the 1 to a 0. Thus, if player 2 chooses $\{A_{ij}\}$, $A_{ij} = (,1)$ is changed to $(,0)$. The entries which are unchanged remain as possible outcomes.

The three rules translate as

i) A move must contain at least one set which does not contain a previously chosen set.

ii) Player 1 may not unilaterally alter all entries in a column, or player 2 all entries in a row.

iii) Each move must be a member of a sequence of moves which lead to termination.

The outcome of a sequence of moves is the last unchanged entry, $A_{ij} = (1,1)$ and the payoffs are, as before, a_{ij} and b_{ij}.

From this game it is clear that moves which consist only of non-singleton subsets do not move the game forward except in the sense that they use up subsets and eventually force the choice of singleton sets. For this reason, some negotiators will not allow such moves, claiming that the statement

$$(\beta_1 \rightarrow \sim\alpha_1) \vee (\beta_2 \rightarrow \sim\alpha_2)$$

is a tautology in the context of the game. As will be seen later, outlawing such moves does not change the equilibrium outcome.

4. A Solution Theory

Given a negotiation problem $(\{\alpha_1,\ldots,\alpha_n\},\{\beta_1,\ldots,\beta_m\},\{(a_{ij},b_{ij})\})$ and a choice of a player to make the first move, the game we have described yields a perfect equilibrium outcome. We will show that this outcome is unique and that there is an equilibrium strategy pair which is conceptually simple. For these reasons, we will call this outcome a solution to the negotiation problem.

Let player 1 be designated as the player to make the first move. The pair (α_r,β_s) is a G-outcome if there exist integers $i_1,i_2,\ldots,i_m$ such that $1 \leq i_k \leq n, k = 1,\ldots,m, i_s = r$ and $b_{rs} > b_{i_j j}$ for $j \neq s$. The maximal G-outcome is the G-outcome for which a_{rs} is maximal. The numbers a_{rs} are distinct, so the maximal G-outcome is unique. Let

$$P_1 = (\beta_1 \rightarrow \alpha_{i_1}) \wedge (\beta_2 \rightarrow \alpha_{i_2}) \wedge \ldots \wedge (\beta_m \rightarrow \alpha_{i_m})$$

be player 1's first move. Player 2 can move

$$Q_1 = \beta_s$$

and (α_r, β_s) will be the outcome, so P_1 and Q_1 are legal. After the move of P_1, the only possible outcomes are (α_{i_1}, β_1), $(\alpha_{i_2}, \beta_2), \ldots, (\alpha_{i_m}, \beta_m)$, so it is rational in this subgame for player 2 to choose the outcome in this list which maximizes his payoff, which is (α_r, β_s). Therefore, against a rational player, player 1 can guarantee any G-outcome, including the maximal G-outcome. Suppose that (α_u, β_v) is the outcome of strategies S for player 1 and T for player 2, and that this outcome is not a G-outcome. Then there does not exist a sequence $i_1, \ldots, i_m$ such that $i_v = u$ and $b_{uv} > b_{i_j j}$ for $j \neq v$. For some j it follows that $b_{uv} < b_{ij}$ for all i. Let T' be the strategy for player 2 of playing β_j at the first move. This move is legal against all legal first moves by player 1 and leads to one of the outcomes (α_i, β_j), $i=1, \ldots, n$. Since $b_{uv} < b_{ij}$ for all i, it is rational for player 2 to change his strategy from T to T', and (α_u, β_v) is not a perfect equilibrium outcome.

The game, being finite with perfect recall, has a perfect equilibrium in pure strategies (Selten 1973), and only G-outcomes can be perfect equilibria. Since there exists a unique maximal G-outcome which player 1 can guarantee as an outcome, it must be the unique equilibrium outcome. It is not difficult to see that the strategies P_1 and Q_1

are equilibrium strategies. (We have not given a complete strategy for each player, which would normally include responses to non-equilibrium moves. The conceptual simplicity of the strategies P_1 and Q_1 argue against the possibility that serious negotiators would not play them.) As mentioned previously, we will call this unique perfect equilibrium outcome the <u>solution of the negotiation problem for player 1</u>. There is, as well, a solution for player 2 which is generated by the negotiation problem and the specification of player 2 as first player to move.

Although we have no axiomatic solution theory, it is interesting that the solution has several properties which are characteristic of bargaining theories.

<u>Pareto Optimality</u>. Let (α_i,β_j) be the solution of a negotiation problem for player 1 and let (α_u,β_v) be an outcome such that $a_{uv} > a_{ij}$ and $b_{uv} > b_{ij}$. Since (α_i,β_j) is a G-outcome, there is a sequence $i_1,\ldots,i_m$ such that $i_j = i$ and $b_{ij} > b_{ik}$ for $k\neq j$. Let $i'_v = u$. The sequence $i_1,\ldots,i'_v,\ldots,i_m$ leads to the G-outcome (α_u,β_v) since $b_{uv} > b_{ij} > b_{ik}$, $k \neq v,j$. However, (α_i,β_j) is a maximal G-outcome, yet $a_{uv} > a_{ij}$, a contradiction. It follows that (α_i,β_j) is Pareto optimal.

<u>Rationality</u>. If (α_i,β_j) is an equilibrium in the non-cooperative game which is the negotiation problem, then it is a G-outcome in the negotiation game through the sequence $i_k = i$, $k = 1,\ldots,m$. Since the solution is a maximal G-outcome, it is rational for a player who moves first to negotiate rather than to play the non-cooperative game.

Symmetry. If (α_r,β_s) is the solution for player 1 in the negotiation problem $(\{\alpha_i\},\{\beta_j\},\{(a_{ij},b_{ij})\})$, then (β_s,α_r) is the solution for player 2 in the problem $(\{\beta_j\},\{(\alpha_i\},\{(b_{ji},a_{ji})\})$.

Invariance under Order-preserving Transformation of Utilities.

This is clear from the fact that only the relative sizes of the utilities determine the solution. Also, note that all problems of interpersonal comparison of utilities are avoided.

Negative Response to Restriction of Alternatives.

If one or more of the actions of player 1 are removed from the problem, the set of G-outcome remains the same or becomes smaller, so the utility value of player 1's solution to player 1 does not increase. Unfortunately, anything may happen to the utility value of player 1's solution to player 2.

Independance of Irrelevant Alternatives.

The solution does not satisfy the axiom of independance of irrelevant alternatives given by Nash. His axiom is: if a problem and subproblem are given and the solution to the problem is in the subproblem,then the solutions to problem and subproblem are the same. In figure 2 is a negotiation problem with four actions for each player. The solutions are (α_2,β_1) for player 1 and (α_1,β_2) for player 2. The subproblem given by the actions $\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\beta_3$ contains both solutions, yet the solution to the subproblem is (α_1,β_1) for both players. In fact, the strategies α_4 and β_4 are not irrelevant, as it is these strategies that

	β_1	β_2	β_3	β_4
α_1	(14,14)	(3,15)	(4,9)	(13,11)
α_2	(15,3)	(10,10)	(5,8)	(2,12)
α_3	(9,4)	(8,5)	(7,7)	(1,6)
α_4	(11,13)	(12,2)	(6,1)	(0,0)

Figure 2

allow the players to threaten certain outcomes and force the solution to be more to their advantage.

5. Comparison with the Nash Bargaining Theory

The Nash bargaining theory can be applied to the negotiation problem by taking as the bargaining set S the convex hull of the points (a_{ij},b_{ij}), all i and j and the threat point a as an agreed-upon equilibrium in the non-cooperative game of the negotiation problem. The Nash solution to the discrete problem would be the point (a_{ij},b_{ij}) closest to the Nash solution of the bargaining problem. The rational for this solution is that the negotiators would only bargain if there is an advantage over the non-cooperative outcome and that, although the outcome must correspond to pure strategies, they can think in terms of von Neuman-Morgenstern anticipated utility in the bargaining process.

A prisoner's dilemma situation is a negotiation problem with two actions for each player in which $a_{21} < a_{11} < a_{22} < a_{12}$ and and $b_{12} < b_{11} < b_{22} < b_{12}$. There is a single, pure strategy equilibrium in the non-cooperative game with outcome (a_{11},b_{11}). The Nash solution to the discrete bargaining problem and the solution of the negotiation problem for players 1 and 2 are all (a_{22},b_{22}), so in this important case the two solution theories do not differ.

However, in figure 3 is a problem in which the Nash solution is (α_1,β_1), the solution for player 1 is (α_2,β_1) and the solution for player 2 is (α_1,β_2).

	β_1	β_2	β_3
α_1	(8,8)	(4,9)	(1,7)
α_2	(9,4)	(5,5)	(2,6)
α_3	(1,1)	(6,2)	(3,3)

Figure 3

In comparing the two theories, one notes that the solution theory given here does not use a threat point. In fact, except for situations in which negotiation is forced by one party on another, the threat always exists that one of the players will refuse to negotiate and instead, play an equilibrium strategy in the non-cooperative game. We assume that both parties have analyzed the situation and determined that negotiation would be profitable, and henceforth do not threathen to withdraw from negotiations. (The problem of figure 2 would not be negotiated, while the one in figure 3 would be). The Nash theory assumes that withdrawal from negotiations is a constant threat. Therefore, in this point, the two theories differ only in form.
A more substantial difference between them is the way they incorporate the strategic possibilities of the original non-

cooperative game into the theory. The Nash theory totally ignores the strategic setup (except in the selection of the threat point) while the theory given here retains some of it. We feel that a realistic theory must not ignore strategic situations as they are clearly influential in negotiations between people.

6. Example from International Negotiation

For an example of the application of the solution theory to international politics, we take the confrontation between the United States and the Soviet Union that occurred in October 1962, called the Cuban missile crisis. Aerial intelligence established the existence of nuclear missles in Cuba, and the president and his advisors met secretly to plan a strategy for removing them. Strategically, the situation was that the U.S. could muster conventional and nuclear forces at short notice, while Russia could only send nuclear forces to the Carribean on short notice. The president had essentially three actions which he could take, one of which was a bombing of Cuba to destroy the missiles, followed by an invasion to ensure destruction. Given the Soviet's strategic position, a nuclear war would be a likely result of such an action. Another possible action was a semi-military move designed to halt deployment of missiles and serve notice of intent, and a naval blockade was a clear choice. The third possibility was a series of diplomatic moves, essentially a non-military response, designed to force Moscow to remove the missiles.

The possible Soviet responses to these actions were essentially to maintain or withdraw the missiles. There was, at the time no other area of contention between the powers where the Soviets could retaliate in kind to U.S. action in Cuba.

The situation as described is essentially a non-cooperative game. To specify the game, we must make some assumptions as to utility levels associated with various outcomes. First, in the case of diplomatic moves, Moscow has both options open, but the use of this move by Washington precludes the use of either military or semi-military moves at a later date because, given time to move in conventional military reinforcements, Moscow could block the effectivity of such moves. Invasion may be assumed to lead to war. With this understanding, the Soviet Union has only one possible response, namely war, and the missiles would be gone at the end, as well as a substantial part of the rest of civilization. In the event of a blockade, Moscow could choose to maintain or withdraw, but Washington would insist that a withdrawal be accomplished in short order, or invasion would follow, the point being to not allow them time to move in ships. Regardless of Moscow's response, the U.S. has the option to withdrawing the blockade, maintaining it or invasion.

In the evaluation of outcomes, we assume that war has the lowest value for both sides. For non-war outcomes, the Soviet Union is happiest if the missiles are maintained, and generally is pleased when Washington takes its least aggressive response. The president, on the other hand, is happiest when the Soviets withdraw against the smallest threat. These rules of thumb allow us to write down in extensive form the non-cooperative game which describes the situation.

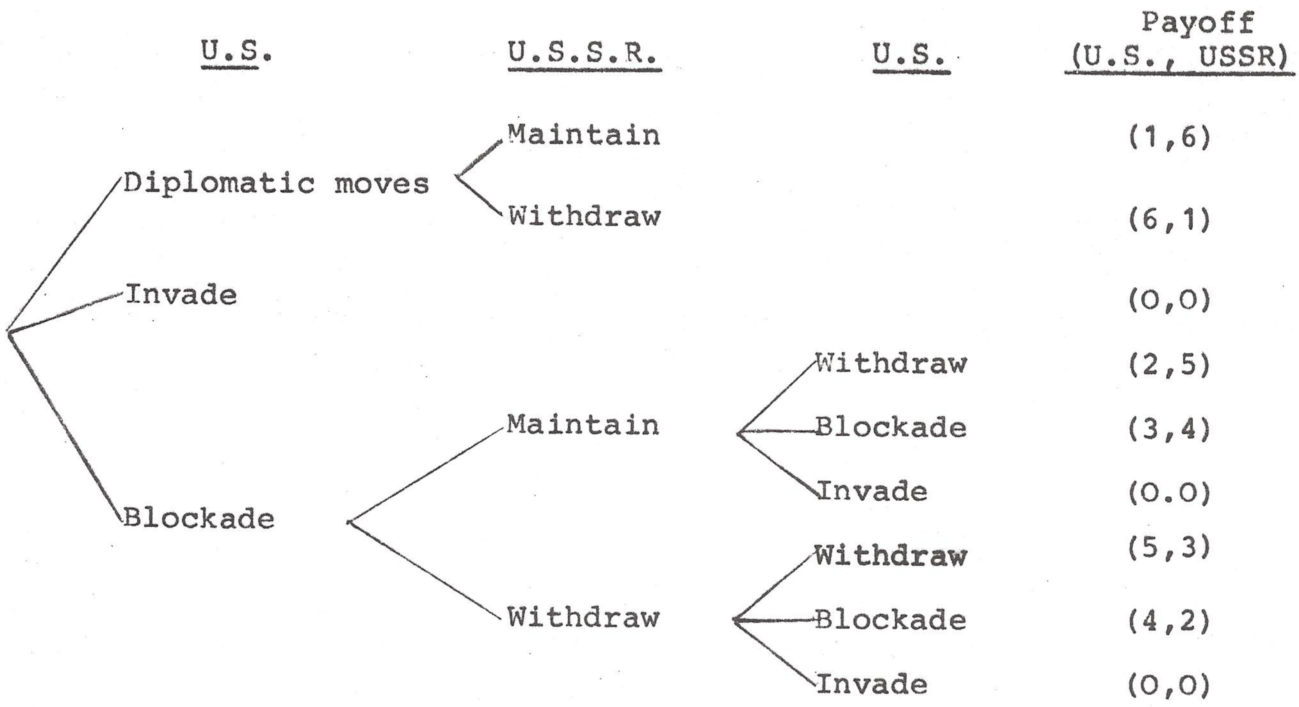

Figure 4

Figure 4 is the game, and it is easy to see that the non-cooperative equilibrium is (3,4), associated with the strategies of blockade and maintain the blockade by Washington and maintain the missiles by Moscow. This did not occur. To explain what did take place, (the U.S. blockaded, Moscow withdrew and the U.S. withdrew the blockade) we investigate the negotiation problem.

The hot-line communication system between Washington and Moscow made negotiation possible. We distinguish between hot-line negotiations which are secret and instant, and diplomatic negotiations which are public and require time to set up and execute. As we will see, the U.S. can gain an advantage over the equilibrium outcome if they negotiate and the U.S. moves first, but Russia can do better by not negotiating.In this case, the U.S. can announce a threat,i.e. make the first move in the negotiation game, and Moscow is forced to listen, so negotiation can be forced by the U.S. and they will have the first move. The existence of the hot-line and the fact that Moscow does not know that Washington knows about the missiles contribute to this situation.

It remains to be seen whether statements made in the negotiation can be believed. First, the value of the hot-line is that it is respected by both sides as a means to effective communication. This makes it unlikely that either side would knowingly lie while using it, since such action might well poison it. However, a statement by the president to the effect that he would invade unless the missiles are withdrawn would probably not be believed. In figure 5, we have the negotiation problem, the normal form of the game in figure 4. Reference to figure 4 should clarify the notation. Note that not all utilities are different. While it should be possible to assign different utilities to the outcome of war based on the steps that lead to it, it is unnecessary to do so in this case, since war is used only as a threat.

	M	W
D	(1,6)	(6,1)
I	(0,0)	(0,0)
BW	(2,5)	(5,3)
BB	(3,4)	(4,2)
BI	(0,0)	(0,0)

Figure 5

If the threat M → I could be believed, the U.S. could achieve its best outcome of (D,W). If the Soviets could move first, they could obtain their solution, (BB,M), which is also the equilibirium. But neither of these is the case. What was suspected by Washington, and proved to be true in the playing out of the game, is that if the blockade move is first made in the non-cooperative game, and the subgame beginning with this position is put up to negotiation, the threat of invasion would be believed and result in a better outcome for Washington.

	M	W
W	(2,5)	(5,3)
B	(3,4)	(4,2)
I	(0,0)	(0,0)

Figure 6

In figure 6, which is the negotiation problem of the subgame of figure 4 beginning with the U.S. move of blockade, the solution for the U.S. is (W,W) obtained with the threat M → I. The analysis we have made leads to the recommendation that the U.S. initiate the blockade and communicate the threat (M → I) $\wedge$ (W → W), whereupon the Soviets would withdraw which is exactly what did happen.

The reader may wish to compare this analysis of the Cuban missile crisis with that given by Brams. Brams uses metagame theory to resolve the conflict, but we feel that the negotiation theory better describes the mental processes of the protagonists in this case.

7. Implications for Social Choice Theory

Whether it is desirable or not, there are many cases of control of a public good by more than one office or agency which exercise their control independantly of each other. For a specific example, the macro economic position of the United States economy is controled by the Federal Reserve Board and the President and Congress. The Federal Reserve Board, working through a traditionally strong chairman, has control of the buying and selling of instruments of federal debt. (It also controls the discount rate and the reserve requirement for federal banks, but these controls are not normally used to control the economy.) When they buy federal debt from banks that hold it, the interest rate is driven down and more money is made available for loans to the public, while selling federal debt drives up interest rates and reduces the amount of money available for loans. The first is called an easy money policy, while the second is called a tight money policy. In general, tight money reduces inflation. Partly because this tool has a negative power on inflation but, in the absence of other forces, cannot be used to decrease unemployment, and partly because of the traditional republican conservatism of bankers, the Federal Reserve tends to match high utility with low inflation rate, and vice versa, minimizing unemployment as an important utility consideration.

The president and congress, on the other hand, view unemployment as more important. They control the national debt and taxation policy which can be used effectively to decrease unemployment but in the absence of other factors is not too effective at controling inflation. If we assume a situation where the president and congress are in general agreement on policy and inflation and unemployment are both high, a situation of conflict between the two agencies occurs.

It may not seem accurate to limit policy choices of either side to two discrete choices but public discussions of the conflict often come down to just that, and we will be concerned with the public choice aspect of the outcome. So we assume that the Federal Reserve Board can choose between tight and easy money, and the president between stimulation and restriction of the economy. Easy money and stimulation means increased inflation and lowered unemployment, while tight money and restriction has the opposite effect. The outcomes for the other two mixtures of policies depend on specifics of the economy. It is certainly not impossible to have the situation pictured in figure 7. This is the prisoner's dilemma, and it is unarguable that both parties are better off under negotiation.

	Stimulation	Restriction
Tight money	(2,2)	(4,1)
Easy money	(1,4)	(3,3)

Figure 7

If it is assumed that the public utility function combines these two utility scales in some way, it is likely that the negotiated outcome is better from a public choice standpoint than the non-cooperative equilibrium.

We have chosen the values in figure 7 to give an extreme example. What happens in general, and is a negotiated solution always better than a non-cooperative solution? To have a standard against which to measure the performance of the two possibilities, let us assume that public utility is measured by the sum of the individual utilities. Upon scanning the Rapoport-Guyer list of 78 different 2 x 2 games, we find 19 cases in which at least one of the negotiated solutions do not maximize the public utility. The worst of these is given in figure 8, where both solutions are (3,2) and the public maximal outcome is (2,4). The difference, (2+4)-(3+2) = 1, is smaller than the difference between non-cooperative equilibrium and public maximum in prisoner's dilemma, (3+3) - (2+2) = 2, and this is not the worst that can happen.

(3,2)	(4,1)
(1,3)	(2,4)

Figure 8

The conclusion is, to minimize the worst possible divergence between outcome and public choice maximum, enforce negotiation between independant regulatory powers.

This does not solve the public choice problem completely, as there are cases (see figure 3) where the negotiated solution differs greatly from the public choice maximum. There is, however, a prescription from the theory that can help here. When a player moves first and has a threat strategy, he can obtain a high utility for himself, and this usually goes along with a lower utility for the other player. When the threat strategy is removed, the principal of negative response to restriction of alternatives says that his maximum obtainable utility goes down. If the strategy that is removed does not figure in this maximum outcome, the Pareto optimal principal says that the utility for the other player must go up. This means that an effective control of the utility difference between negotiating players will be reduced when threat strategies which offer low utility to both are removed.

Another control factor is the decision of who has the right to make the first move. This is usually ruled by circumstances. For example, the Federal Reserve Board has the power to move first because it exercises almost instant control and can change policies quickly, while the president is tied to a yearly budget and his controls are less effective for immediate action. To reduce the power of the Federal Reserve to always move first (i.e. announce its threat), it would be sufficient to make retaliation by the president more easy, say through budget control of the Board. In general, controls over the decision of who moves first are in the hands of the lawmakers.

8. Conclusions

We have developed a solution theory for finite negotiation problems which prescribes a simple and intuitively reasonable strategy for both parties, and therefore probably describes the thought processes of negotiators. As seen from examples, it describes actual outcomes between experienced negotiators, but perhaps the most useful aspect of this and similar theories is that it points the way to the most effective controls of negotiation situations. It remains to develop such a theory for problems which mix the continuous and discrete problems of bargaining and negotiation.

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