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Carl Christian von Weizsäcker

Modern Capital Theory and the Concept of Exploitation

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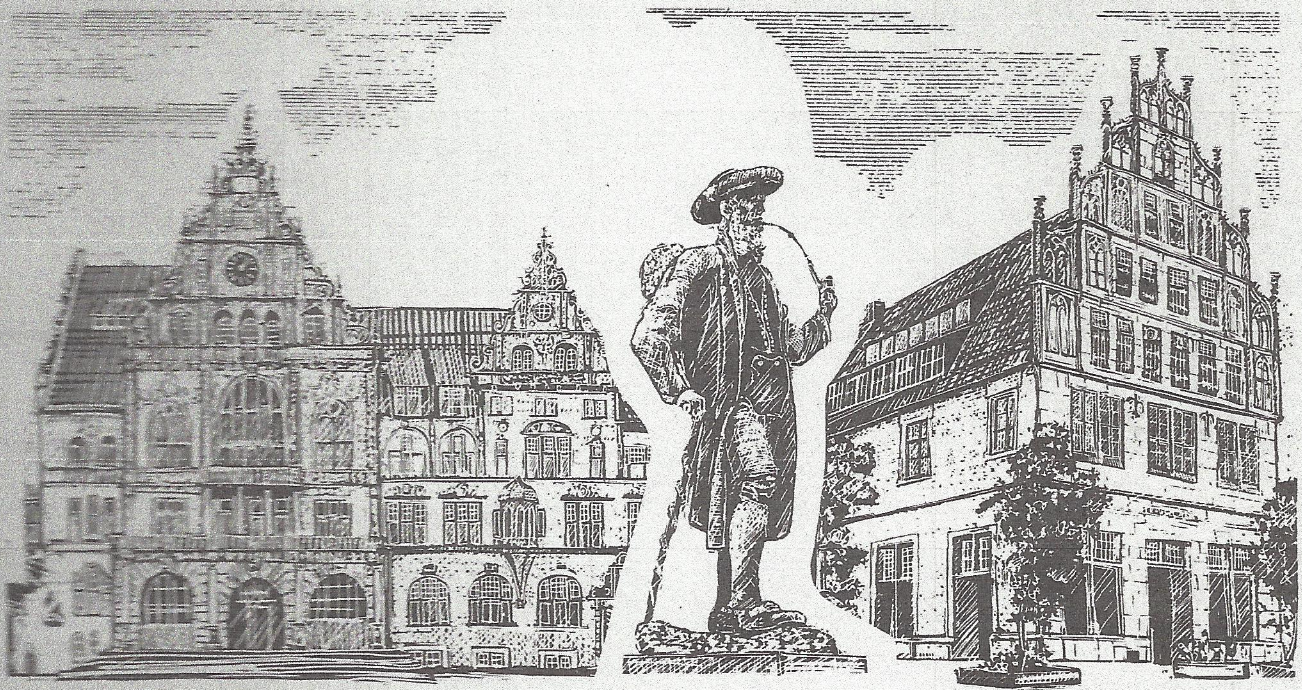


H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

I

The political relevance of the concept of exploitation cannot seriously be denied. Thus for instance a representative of the German Democratic Republic (East Germany) may use this term when he tries to make clear the difference between the situation of the working class in East and West Germany. He may say:"in the capitalist system of the Federal Republic of Germany the workers are exploited, in the German Democratic Republic they are not." But only by serious theoretical analysis are we able to judge when the use of this term in politics is really justified.

In the following I try to discuss the concept of exploitation from the point of view of modern capital theory. I try to confront certain results in capital theory with the Marxian use of this term. The method of discussion will be, as it were, inductive: I will first discuss special cases and then I generalize the applicability of the term in several consecutive steps. This allows me to take up the difficulties involved in a reasonable definition of exploitation one after the other.

In trying to deal with the concept of exploitation we should investigate why and how Marx has introduced it into the realm of serious economic theorising. Exploitation of humans by humans is a phenomenon whose existence common sense would not deny. In the French Revolution at the latest the privileges of the aristocracy were considered as an exploitation of the large majority of the people by a small upper class. These privileges enabled the aristocracy to lead a comfortable life of leasure while others with their labour produced the goods which the aristocratic class consumed.

The abolition of these privileges by the French Revolution, the principles of equality before the law and of freedom of contract, in other words the bourgeois revolution, established a society which in the eyes of many people was free of exploitation. Nobody was any longer forced by law to work for the benefit of others. If he did so, he did it deliberately and he received a compensation representing the value of his work. Thus, many thought, exploitation could no longer prevail.

It was against this ideology that Marx tried to demonstrate the persistence of exploitation in capitalism. The analytical instrument which he used was his theory of value. It can explain how exploitation is possible in a society in which every agent operates on the basis of the freedom of contracts. The value of a commodity is equal or proportional to the amount of labour socially necessary to produce (or reproduce) the commodity. In a capitalist system labour power is also a commodity and its value corresponds to the amount of labour time necessary to reproduce the labour power. But the commodity labour power, in a system which is sufficiently productive, commands more labour time than is necessary to produce the means of subsistence for its own reproduction. Thus the labour time available to the user of the commodity labour power is divided into necessary labour (time) (i.e. necessary to reproduce the labour power) and surplus labour (time) available for the production of goods other than the means of subsistence of the worker. Competition forces every seller to sell his commodity at its exchange value and thus the worker, who is without means of production, must sell his labour power at its value, which corresponds to the necessary labour, whereas it produces more value than is its own. This surplus value is appropriated by the agent who buys the labour power and then is entitled to use it by the terms of the labour contract. He is able to use it effectively if he owns the means of production which the labour power needs to produce value, i.e. if he is a capitalist. Thus, even though there is freedom of contract, there is a class of people who are able to receive goods and services without an effort of their own labour and there is a class of people who receive less than the product of their own labour, just as was the case in the precapitalist epoch. While the form has changed, the substance is still there: capitalism is characterized by the existence of an exploiting and an exploited class.

The change of the form of exploitation, which was mistaken by the bourgeois class as its abolition, was effected by a social, economic and political revolution whereby the ruling class as well as the means of exploitation changed. Exploitation by the aristocratic class was accomplished by formal privileges of this class and by lifelong obligations of the exploited towards the exploiters. Capitalist exploitation is effected by means of the private property of means of production in the hands of the capitalist class. It is interesting to see that large segments of the adherents to Marxist doctrine fell into the same mistake, which Marx uncovered in his time, namely to confuse the prevailing form of exploitation with its substance. It is their belief that exploitation is abolished if private property of the means of production is abolished and the dictatorship of the proletariat is established. On the other hand, Marx' own writing, in as much as it is relevant for a precise analysis of exploitation was almost exclusively concerned with capitalism, so that an identification of exploitation with the existence of a bourgeois class was natural for a loyal Marxist. Thus e.g. Bettelheim, who is convinced that exploitation still prevails in East European countries, defines the ruling class in these countries as the new bourgeoisie which uses mechanisms of exploitation similar to those prevailing under capitalist conditions [1]. In other words: The Marxian treatment of exploitation stresses a specific form of exploitation so much that this is likely to cause confusion between the substance of exploitation and this specific form it takes in a capitalist system.

The substance of exploitation is that there exist groups of people or classes which are able to obtain more goods on a permanent basis than can be produced with the amount of labour provided by the group. It is our purpose to make this definition of exploitation precise by using analytical methods of modern capital theory. But before doing so we must emphasize that exploitation as a phenomenon is consistent with a situation in which every group (or individual) gains from the fact that it can transact business with other groups (or individuals). The existence of gains from trade does not exclude the existence of exploitation. To say otherwise would imply that the whole point of Marx' theory would be lost, it would be a regression into pre-Marxian bourgeois ideology. For, in a system of freedom of contract mutual gains from trade are a precondition for transactions between members of different classes on which a system of exploitation is built: Thus, the worker who sells his labour power to the capitalist gains from this transaction, since he would otherwise starve. Still there exists a class which is able to obtain more commodities than correspond to the labour expenditure of this class.

II

Let us then as a simple example consider an input-output system without substitution and without fixed capital.
If a_{oj} is the amount of labour necessary to produce one unit of good j and if a_{ij} is the amount of good i necessary to produce one unit of good i, if $\bar{w}$ is the wage rate, r is the interest rate and p is the price of good i, then the unit costs of good j are given by the expression

$$(1+r)\left(\sum_{i=1}^{n} a_{ij} P_i + a_{oj}\bar{w}\right)$$

Here we assume that the inputs have to be available one unit period before the output becomes available. For given $\bar{w}$ and r we can look for the prices of the n commodities which cover costs of production. We then have the equations

$$P_j = (1+r) \left(\sum_{i=1}^{n} a_{ij} P_i + a_{o_j} \bar{w}\right) \quad , \quad j = 1, \ldots. n$$

or in matrix notation (A being the input-output matrix, a_o the labour input vector and p the price vector)

$$p = (1+r) \ (p A + a_o \bar{w})$$

from which follows

$$p \ (I - (1+r) \ A) = (1+r) \bar{w} \ a_o$$

or, if (I - (1+r) A) is nonsingular

$$p = (1+r) \ \bar{w} \ a_o \ (I - (1+r) \ A)^{-1}$$

Under suitable conditions, and given that r is sufficiently low, the solution will be nonnegative.

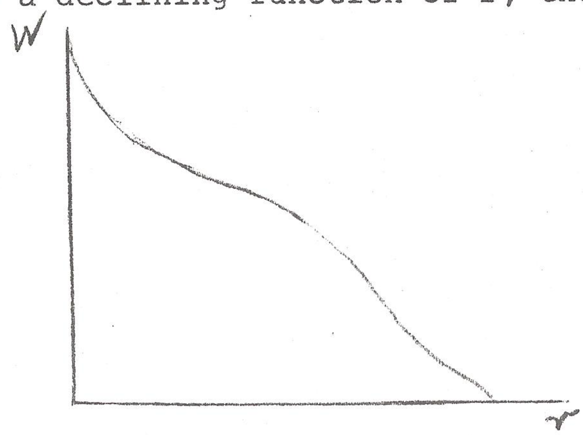
Let us now define a real wage, w, by means of a standard commodity basket containing the quantity s_i of good i and represented by the vector $s = (s_1, s_2, \dots s_n)$.
The price of the standard basket is $\sum_{i=1}^{n} p_i s_i = ps$.
Now, the real wage (with respect to basket s) is defined to be

$$w = \frac{\bar{w}}{ps}$$

i.e. the number of baskets one can buy with the wage rate $\bar{w}$. Computation of w yieds

$$w = \frac{\bar{w}}{(1+r)\ \bar{w}\ a_o\ (I - (1+r)\ A)^{-1} s} = \frac{1}{(1+r)\ a_o\ (I - (1+r)\ A)^{-1} s}$$

It is easy to show that the elements of the matrix $(I-(1+r)A)^{-1}$ are nonnegative and grow with increasing r and then we see that w is a declining function of r, the rate of interest.



Consider now a system with a production technique described by our input output system. We assume that this system grows exponentially at a certain rate g. Consumption from the system, $\bar{c}$ is assumed to be proportional to the standard basket s, so that we have

$$\bar{c} = \lambda s$$

where λ is some scalar.
Let x be the production vector of the system, let y be the commodity input vector, let L be the labour input.
We then have

$$y\ (t-1) = A\ x(t)$$
$$L\ (t-1) = a_o\ x\ (t)$$

or considering the exponential growth of the system

$$y(t) = (1+g)\, y(t-1) = (1+g)\, A\, x(t)$$

$$L(t) = (1+g)\, L(t-1) = (1+g)\, a_o\, x(t)$$

consumption $\bar{c}$ is given by x - y, or

$$\bar{c}(t) = x(t) - (1+g)\, A\, x(t) = (I - (1+g)\, A)\, x(t)$$

For a given consumption vector this is an equation system for x and we get

$$x(t) = (I - (1+g)\, A)^{-1}\, \bar{c}(t)$$

we then have

$$L(t) = (1+g)\, a_o\, (I - (1+g)\, A)^{-1}\, \bar{c}(t)$$

Now, let c be real consumption per hour of work in terms of the standard basket s. Thus

$$c = \frac{\lambda(t)}{L(t)}$$

we then can write

$$\bar{c}(t) = L(t)\, c\, s$$

and so we obtain the equation for c by dividing

$$L(t) = (1+g)\, a_o\, (I - (1+g)\, A)^{-1}\, L(t)\, cs$$

on both sides by L(t). We have

$$c = \frac{1}{(1+g)\, a_o\, (I - (1+g)\, A)^{-1} s}$$

We observe that the function c(g) has the same mathematical form as the function w(r). Thus the consumption-growth pay-off curve is the dual interpretation of the wage interest curve.

This dual relationship can, of course, be generalized to other models. As long as a given technique of production has a wage interest curve which does not depend on the rate of growth of the system, national accounting relations yield from

net national product = consumption + investment
= wages + profit that

$$c + g\, v = w + r\, v$$

where v is the value of capital per man hour per unit period and hence for $r = g$ we have $c(g) = w(g)$ what had to be proved.

III

Let us now consider a model in which different population groups exist. They all grow at the same rate g and then they expand their consumption and their supply of labour also at this rate g. Let the consumption of group k ($k = 1, 2, \ldots m$) $\bar{c}^k(t)$ be proportional to some standard commodity basket $s^k = (s_1^k, s_2^k, \ldots s_n^k)$, i.e.

$$\bar{c}^k(t) = \lambda^k(t)\, s^k$$

We then can express consumption per unit of labour of this group in terms of the standard basket s^k. We write

$$c^k = c^k(s^k) = \frac{\lambda^k(t)}{L_k(t)}$$

Now we define the consumption growth curve of the technique in use in our economy by means of the standard basket s^k and we get a curve $c(s^k, g)$ which obtains a certain value $c^*(s^k)$ at the rate of growth prevailing in the economy.
We can now ask the question: is $c^k(s^k)$ larger or smaller than $c^*(c^k)$? $c^k(s^k)$ larger than $c^*(s^k)$ implies that the group under consideration does not provide enough labour to produce the consumption goods it consumes. It thus can obtain its consumption goods only by exploiting other groups. If $c^k(s^k)$ is smaller than $c^*(s^k)$ then the group provides more labour than is necessary for the production of its own consumption goods. It can be considered to be an exploited group.
Here we introduce an additional assumption: absence of intrinsic joint production and constant returns to scale.
By this we mean the following: let $c^1(t)$, $c^2(t)$, $c^3(t)$ be consumption vectors with $c^1(t) = (1+g)^t \hat{c}^1$, $c^2(t) = (1+g)^t \hat{c}^2$, $c^3(t) = c^1(t) + c^2(t)$. Let $L_i^*(t)$ be the amount of labour necessary in an exponentially growing production system producing the consumption vector $c^i(t) = (1+g)\, \hat{c}^i$. Then we assume $L_3^*(t) = L_1^*(t) + L_2^*(t)$. While this assumption is almost never fulfilled for nonexponential, nonstationary paths,

there are many models in which it is fulfilled for exponential and stationary paths. We call such paths completely synchronized paths.

A good definition of exploitation should imply that whenever there exists an exploiting group there should also exist an exploited one. Is this the case with the definition we just suggested? That this is so can be seen in the following way.
Let L_k^* be the amount of labour necessary to provide the consumption of group k, let L_k be the amount of labour supplied by group k. All these variables grow of course exponentially at the rate g through time. We then by definition have the relations

$$L_k(t) = \frac{\lambda^k(t)}{c^k(s^k)}$$

$$L_k^*(t) = \frac{\lambda^k(t)}{c^*(s^k)}$$

or

$$\frac{L_k(t)}{L_k^*(t)} = \frac{c^*(s^k)}{c^k(s^k)}$$

But in the system as a whole the amount of labour required to produce the consumption goods which the system supplies is equal to the amount of labour the system actually supplies and thus we have: by the additivity of required labour quantities

$$\sum_{k=1}^{m} L_k(t) = \sum_{k=1}^{m} L_k^*(t)$$

Whenever there is a group with $L_k < L_k^*$ there must be another group with $L_k > L_k^*$.
Indeed, we can use the relation between L_k and L_k to define a degree of exploitation R_k. Let

$$R_k = \frac{L_k}{L_k^*} - 1$$

Then a group is exploited, if R_k is positive. It is an exploiting group, if R_k is negative. If for example there is a working class (class 1) and a capitalist class (class 2), which does not supply any labour, then $L_1 = L$ and $L_2 = 0$. The degree of exploitation of the amount of labour necessary to produce the consumption goods of the workers (what Marx calls necessary labour) to the amount of labour going into the production of the capitalists' consumption goods (Marx calls it surplus labour).

If we distinguish between more than two groups we may aggregate them into two classes, class I, the exploited class, and class II the exploiting class. Thus let I and II be two mutually exclusive and exhaustive subsets of the integers from 1 to m (the number of groups) such that

$k \in I$ if and only if $R_k > 0$

$k \in II$ if and only if $R_k \leq 0$

We may then define the aggregated degree of exploitation, R, to be

$$R = \frac{\sum_{k \in I} L_k}{\sum_{k \in I} L_k^x} - 1$$

R would be the degree of exploitation which would come closest to Marx' degree of exploitation. We shall discuss the relation between our definition and Marx' definition below.

IV

We should look at a system with a market economy to see what our definition of exploitation implies. We assume that there exist a uniform wage rate and a uniform profit rate in the economy. Moreover for the moment we assume that the different groups in the economy all consume a multiple of the same standard commodity basket, s. The real wage rate is defined in terms of this commodity basket and so is real consumption per unit of labour expended, c (for the whole economy) and c^k (for the groups). Let v^k be the amount of wealth the group k holds per unit of labour expended (also in terms of the standard basket s). Since wealth in every group grows at the uniform rate g, savings per unit of labour in the group are equal to $g\,v^k$.

We thus obtain the equations

$$c^k + gv^k = w + rv^k$$

and

$$c + gv = w + rv$$

Now, a group belongs to the exploiting class, if $c^k > c$.

We obtain

$$c^k - c = (r-g)\,(v^k - v)$$

Given that $r > g$, a group belongs to the exploiting class if and only if its wealth per unit of labour is greater than the average of the economy. Thus in such a case it is natural to identify the capitalist class with the class of exploiters. They own at least part of the means of production employed together with the labour of those groups whose wealth is below average. If the rate of interest is equal to the rate of growth of the system then $c^k = c$ for every group. No group receives more consumption goods than can be produced by means of its own labour. If the rate of interest is lower than the rate of growth (an unlikely case) then $c^k - c > 0$ implies $v^k < v$. The exploiting groups are now those whose wealth is below average, since their saving out of wages is low compared to the rest of the economy.

As long as the consumption baskets of the different groups are proportional to each other our national accounting equations allow us to compute the degree of exploitation. For then we know of course that the ratio of the value of consumption to required labour is the same in every group and thus required labour can easily be computed from the value of consumption of any single group . Global national accounting data are no longer sufficient if the consumption vectors of different groups are not proportional. Then the value of consumption in current prices need not be an accurate indicator of the amount of labour required to produce the consumption goods. We encounter the same difficulties as are present in Marxist theory and which there are known as the transformation problem. But in principle the computation of the labour requirements of any exponentially increasing flow of consumption goods is no problem. If one takes the standard commodity basket s^k of group k, so that consumption of that group is proportional to s^k then we can express real consumption and the real wage rate in terms of this basket s^k and in principle it is possible to compute the wage-interest curve of the technique of production in use, which at the same time is the consumption-growth curve. Let us then consider the subsystem of the production system which produces the consumption goods for group k and all intermediate products necessary for their production. The capital intensity v_k in this subsystem is given by the equation

$$c\ (s^k) + gv_k\ (s^k) = w\ (r,s^k) + rv_k\ (s^k)$$

where $w\ (r,s^k)$ is the real wage rate at the prevailing interest rate r in terms of the standard commodity basket s^k. This together with the equation

$$c^k\ (s^k) + gv^k\ (s^k) = w\ (r,s^k) + grv^k (s^k)$$

implies

$$c^k\ (s^k) - c\ (s^k) = (r-g)\ (v^k(s^k) - v_k\ (s^k)$$

If $r > g$ then group k exploits the other groups if $v^k(s^k)$, the wealth of group k per unit of labour supplied, is larger than $v_k\ (s^k)$ the amount of capital necessary per unit of labour in the subsystem producing the consumption goods of group k. Thus the results have the same heuristic interpretation as in the case discussed above. We still can identify the exploiting class as being the capitalist class as long as the rate of interest is greater than the rate of growth.

V

Our next step will be a generalization of the concept of exploitation. We will keep the assumption that the economy is evolving at a constant exponential rate of growth g and has a constant technique of production. But we will now consider any group of people (possibly even a single individual) and we will define the degree of exploitation for this group. For our purpose it is sufficient to characterize the group under consideration by the flow of consumption goods which it consumes and the flow of labour it supplies. Let the first be a vector function of time $c^k\ (t) = c_1^k\ (t), c_2^k\ (t), \ldots c_n^k\ (t)$ and the latter a real valued function of time, $L_k\ (t)$.

How can we decide whether the group k is exploited or exploiting? In all likelihood the technique of production will not allow to produce c^k (t) out of the labour flow L_k (t). Can we therefore call the group k exploiters? As we said above it is only useful to call a group an exploiting group if the rest group of the society somehow can be called an exploited group. But it is very likely that the group $\bar{k}$ consisting of all people not in group k is also not able to produce the consumption goods it consumes by means of the labour it supplies. To see this, take for example the input-output technique of production discussed in II. For the vector function c^k (t) we can compute the flow of labour necessary to produce it. It is not difficult to see that I_k^* (t) obeys the formula

$$L_k^* (t) = a_o \left(\sum_{\tau=0}^{\infty} A^\tau c^k (t+\tau+1) \right)$$

In general we have neither $L_k (t) \geq L_k^* (t)$ for all t nor $L_k (t) \leq L_k^* (t)$. Thus in general neither group k nor its complement in the economy are selfsufficient. Both need parts of the labour provided by the other group.

As we have seen, this is different, if the consumption and the labour supply of group k grow exponentially or remain constant through time. This is so, because under a constant technique of production the labour requirements of an exponentially increasing consumption vector grow exponentially at the same rate. This provides the clue for the way in which to define exploitation with respect to group k. I assume that the group has finite extension over time, i.e. L_k (t) = o and c^k (t) = o unless t lies in some finite interval. Now imagine group k to be part of a larger hypothetical group k' with the following properties

$$c^{k'} (t) = \sum_{\tau=-\infty}^{+\infty} (1+g)^{-\tau} c^k (t+\tau)$$

$$L_{k'} (t) = \sum_{\tau=-\infty}^{+\infty} (1+g)^{-\tau} L_k (t+\tau)$$

were g is some growth rate.

In other words group k' (g) consists of overlapping cohorts where each cohort exhibits the same consumption-labour supply pattern-only shifted on the time axis and such that each consecutive cohort is larger by the factor $1+g$ than the preceding one. In this way we construct a hypothetical group k' whose consumption vector and whose labour supply grow exponentially through time at the rate g. Indeed

$$c^{k'}(t+1) = \sum_{\tau=-\infty}^{+\infty} (1+g)^{-\tau} c^k (t+1+\tau) =$$

$$\sum_{\tau'=-\infty}^{+\infty} (1+g)^{-\tau'} (1+g)\, c^k (t+\tau') = (1+g)\, c^{k'}(t)$$

and similarly fo $L_{k'}(t+1)$. We call the group k' the g-exponential extension of k. Now we can compute $L^*_{k'}(t)$ which will grow at the same rate g and thus the degree of exploitation

$$R_{k'} = \frac{L_{k'}(t)}{L^*_{k'}(t)} - 1$$

is independent of t. Now we define the rate of exploitation as a function of g as $R_k(g) = R_{k'}(g)$.
In other words we consider group k to be exploited if and only if the hypothetical group k', consisting only of overlapping groups all of the same kind as k and growing at the rate g, is exploited in the sense defined earlier.

We so far have defined the rate of exploitation R_k as a function of the rate of growth g of the system into which we hypothetically have embedded group k.
There remains some ambiguity , since in general $R_k(g)$ does indeed depend on g. But now the actual economy, of which group k is a part grows at some definite rate, say $\bar{g}$. Is there anything special to be said about $R_k(\bar{g})$? Yes, and indeed what we can say about $R_k(\bar{g})$ seems to me to be sufficiently important to propose that we select $R_k(\bar{g})$ as <u>the</u> rate of exploitation of group k.

We have above required that, whenever there is a group of exploiters in an economy the complement of this group should be considered to be an exploited group and vice versa.
We now introduce the following definition:
Complement group of finite extension of group k is a group $\bar{k}$ such that

$$c^{\bar{k}}(t) = c(t) - c^{k}(t)$$

$$L_{\bar{k}}(t) = L(t) - L_{k}(t)$$

for t in a finite interval containing all points with $c^{k}(t) \neq 0$, or $L_{k}(t) \neq 0$, where $c(t)$ is the total consumption vector and $L(t)$ is total labour supply of the economy; and such that

$$c^{\bar{k}}(t) = o$$

$$L_{\bar{k}}(t) = o$$

for t outside of this interval.

We now show the following result:

1) Given a group k of finite extension in an economy growing at the rate $\bar{g}$. Then for any complement group of finite extention, $\bar{k}$, we have the relation

$$R_{\bar{k}}(\bar{g}) < 0 \quad \text{if and only if} \quad R_{k}(\bar{g}) > 0$$

$$R_{\bar{k}}(\bar{g}) > 0 \quad \text{if and only if} \quad R_{k}(\bar{g}) < 0$$

2) For any $g \neq \bar{g}$ we can construct a group of finite extention, k, such that there exists a complement group of finite extension, $\bar{k}$, in the economy growing at the rage $\bar{g}$ with $R_{k}(g)\, R_{\bar{k}}(g) > 0$

We restrict ourselves to complement groups of finite extension, since exploitation was only defined for groups of finite extension. Our results mean that only by the choice of $R_{k}(\bar{g})$ can we be sure that every exploited group has a complement group which exploits and every exploiting group has a complement group which is exploited.

We first prove 1). We look at the group of finite extension which is the union of group k and its complement of finite extension under consideration. Its consumption is

$C(t)$ for $t_o \leq t \leq t_1$

0 for other t

The labour supply is

$L(t)$ for $t_o \leq t \leq t_1$

0 for other t

when t_o and t_1 are chosen in such a was as to ensure that $C^k(t) = 0$ and $L_{\bar{k}}(t) = 0$ for t not lying between t_o and t_1. Now construct the $\bar{g}$ exponential extension of this union of group and group $\bar{k}$. Its consumption vector $\hat{C}(t)$ and its labour supply $\hat{L}$ can be characterized in two different ways. First we have

$$\hat{C}(t) = \sum_{\tau=-\infty}^{+\infty} (1+\bar{g})^{-\tau} C(t+\tau) =$$

$$= \sum_{\tau=-\infty}^{+\infty} (1+\bar{g})^{-\tau} C^k(t+\tau) + \sum_{\tau=-\infty}^{+\infty} (1+\bar{g})^{-\tau} C^{\bar{k}}(t+\tau)$$

$$= C^{k'}(t) + C^{\bar{k}'}(t)$$

where $C^{k'}(t)$ and $C^{\bar{k}'}(t)$ are the consumption vectors of the exponential extensions of k_1 and $\bar{k}$ respectively. Similarly we have

$$\hat{L}(t) = L_{k'}(t) + L_{\bar{k}'}(t)$$

On the other hand, we obtain

$$\frac{\hat{C}(t)}{\hat{L}(t)} = \frac{\sum_{t=-\infty}^{+\infty} (1+\bar{g})^{-\tau} C(t+\tau)}{\sum_{\tau=-\infty}^{+\infty} (1+\bar{g})^{-\tau} L(t+\tau)} =$$

$$= \frac{C(t) \sum_{\tau=t_o-t}^{t_1-t} (1+\bar{g})^{-\tau} (1+\bar{g})^{\tau}}{L(t) \sum_{\tau=t_o-t}^{t_1-t} (1+\bar{g})^{-\tau} (1+\bar{g})^{\tau}} = \frac{C(t)}{L(t)}$$

Thus consumption and labour supply of the $\bar{g}$-exponential extension of the union of group k and $\bar{k}$ are proportional to consumption and labour supply in the actual economy under consideration. Since for that economy as a whole the labour required to produce the consumption goods is of course equal to the labour supplied, we infer that the same is true for the consumption goods of the $\bar{g}$-exponential extension of the union of group k and group $\bar{k}$. Thus $\hat{L}^{*}(t) = \hat{L}(t)$ and hence the union of k and $\bar{k}$ neither exploits nor is exploited. On the other hand because of the additivity of required labour we have from

$$\hat{C}(t) = C^{k'}(t) + C^{\bar{k}'}(t)$$

that

$$\hat{L}^{*}(t) = L^{*}_{k'}(t) + L^{*}_{\bar{k}'}(t).$$

Now, if

$$R_k(\bar{g}) = \frac{L_{k'}(t)}{L^{*}_{k'}(t)} - 1 > 0$$

then

$$R_{\bar{k}}(\bar{g}) = \frac{L_{\bar{k}'}(t)}{L^{*}_{\bar{k}'}(t)} - 1 =$$

$$\frac{\hat{L}(t) - L_{k'}(t)}{\hat{L}^{*}(t) - L^{*}_{k'}(t)} - 1 = \frac{\hat{L}(t) - L_{k'}(t)}{\hat{L}(t) - L^{*}_{k'}(t)} - 1 < 0$$

and, if $R_k(\bar{g}) < 0$ we similarly arrive at $R_{\bar{k}}(\bar{g}) > 0$. This proves proposition 1).

Proof of proposition 2). Consider the group k characterized by

$$C^k(t) = C(t) \qquad t_o \leq t \leq t_1$$

$$C^k(t) = 0 \qquad \text{for other } t$$

$$L_k(t) = L(t) \qquad t_o \leq t \leq t_1$$

$$L_k(t) = 0 \qquad \text{for other } t$$

That is, group k provides all the labour and consumes all consumption goods in the economy (which grows at rate $\bar{g}$) within a certain interval. Now we compute $R^k(g)$. The g-exponential extension has the following consumption and labour supply

$$C^{k'}(t) = \sum_{\tau=-\infty}^{+\infty} (1+g)^{-\tau} C^k(t+\tau) =$$

$$= \sum_{\tau=t_0-t}^{t_1-t} (1+g)^{-\tau} C(t+\tau) =$$

$$C(t) \sum_{\tau=t_0-t}^{t_1-t} (1+g)^{-\tau} (1+\bar{g})^{\tau}$$

Similarly

$$L_{k'}(t) = L(t) \sum_{\tau=t_0-t}^{t_1-t} (1+g)^{-\tau} (1+\bar{g})^{\tau}$$

and then

$$\frac{C^{k'}(t)}{L_{k'}(t)} = \frac{C(t)}{L(t)}$$

Now we ask about $L^*_{k'}(t)$. There is no reason why it should be equal $L_{k'}(t)$ if $g \neq \bar{g}$. Indeed, we have above already introduced the concept of the growth-consumption payoff. The higher the rate of growth of the system the lower is consumption per unit of labour employed. To express the same thing differently: the higher the rate of growth of the system the higher is the required labour input $L^*_{k'}(t)$ for a given $C^{k'}(t)$.

Thus $L^*_{k'}(t)$ is rising with g. Hence $g < \bar{g}$ implies $L^*_{k'}(t) < L_{k'}(t)$, since the right hand side is equal to $L^*_{k'}(t)$ for $g = \bar{g}$. We then get

$$R_k(g) = \frac{L_{k'}(t)}{L^*_{k'}(t)} - 1 > 0 \quad \text{for } g < \bar{g}$$

$$R_k(g) \qquad < 0 \quad \text{for } g > \bar{g}$$

If we apply a growth rate $g < \bar{g}$ then the group consisting of the total population in the economy between t_o and t_1 is exploited. Now just split this total population into two equal halves. One of these parts, simply having half the consumption vector and half the labour supply is then also exploited. But the same is true of the other half which is its complement of finite extension. This proves proposition 2)

VI

These two propositions are, I believe, sufficient reason to take $R_k(\bar{g})$ as the appropriate definition of exploitation. But it should be made clear that it is not compatible with Marx' definition of exploitation for a capitalist system. It can be shown that the Marxian definition coincides with $R_k(o)$ for a system without technical progress. This I will show now for the input output system already discussed.

The definition of Marx requires that one computes the specific labour contents of the different commodities. The specific labour contents $\pi = (\pi_1, \ldots \pi_n)$ can be computed from the system of equations

$$\pi_j = a_{oj} + \sum_{i=1}^{n} a_{ij} \pi_i \quad , j = 1_i \ldots n$$

or in matrix notation

$$\pi = a_o + \pi A$$

which yields the solution

$$\pi = a_o (I - A)^{-1}$$

The specific labour contents or values are proportional to the prices which would prevail at a zero rate of interest. Consider now a two class society in an exponentially growing economy where one class - the workers - supplies all the labour but does not accumulate wealth, whereas the other class - the capitalists - accumulates and perhaps also consumes. Now let $L_1 (t) = L (t)$ be the supply of labour by workers and let $C^1 (t)$ be the consumption vector of workers. The value of consumption by workers is equal to the value of their labour power and thus the latter is equal to $\pi C^1 (t)$. The Marxian degree of exploitation R^M is given by $R^M = \frac{L (t)}{\pi C^1 (t)} - 1.$

Consider now the workers living in period $\bar{t}$ to be a group for which we want to compute the function $R^1 (g)$ according to our definition. For it consumption is zero except at $\bar{t}$, when it is $C^1 (\bar{t})$. Consumption of the g-exponential extension of this group $C^{1'} (t)$ is given by

$$C^{1'} (t) = \sum_{\tau=-\infty}^{+\infty} (1+g)^{-\tau} C^1 (t+\tau) = (1+g)^{t-\tau} C^1 (\bar{t})$$

Similarly $L_{1'} (t) = (1+g)^{t-\tau} L (\bar{t})$

The labour necessary to produce $C^{1'} (t)$ is then (using the formulas of section II)

$$L^*_{1'} (t) = a_o (I - (1+g) A)^{-1} C^{1'} (t)$$

Putting $g = 0$ we get

$$L^*_{1'} (t) = a_o (I - A)^{-1} C^{1'} (t) = \pi C^{1'} (t) \text{ and}$$

then

$$R^1 (o) = \frac{(1+g)^{t-\bar{t}} L (\bar{t})}{(1+g)^{t-\bar{t}} \pi C^1 (t)} - 1 = \frac{L (t)}{\pi C^1 (t)} - 1 = R^M$$

This discrepancy between the Marxian definition $R^M = R^1 (o)$ and the definition $R^1 (\bar{g})$ should not disturb us. I believe that in the time of Marx the understanding of synchronized growth processes was not yet sufficiently developed to consider a defintion of exploitation appropriate for growing systems. There is, of course, the difficulty that our definition of exploitation yields smaller measures of exploitation of workers in a capitalist system with a growing population; indeed, exploitation of workers cedes to exist at a point when the rate of interest is still positive. Marxists may interpret this as a part of a procapitalist apology. On the other hand, it should be clearly understood that our definition of exploitation tries to extrapolate this concept to noncapitalist systems so that it must be such as to yield reasonable results in these different contexts.

For example consider a socialist economy in which the function of accumulation is performed by the government and not by a capitalist class. Workers receive wages for which they buy consumption goods. Compare this economy with a capitalist economy where capitalists do not consume and also workers do not save. Provided that the techniques of production are the same the level of consumption in both economies are equal and, since consumption is equal to wages, wages are also equal. Surely, nobody would say that the working class is exploited in the socialist regime. On the other hand, using the Marxian definition of exploitation, the working class in the capitalist system is exploited. Since the supply of labour and the level of consumption are the same in both systems and since the technique of production is the same the degree of exploitation according to our definition must be the same and indeed in this example it is zero.
If we want to give both economic systems a fair treatment, I believe the degree of exploitation has to be the same in the two model economies. Extending the Marxian definition from capitalism to socialism would imply that the degree of exploitation is positive in the socialist system as well. Extending the Marxian nonexploitation thesis from socialism would imply that also in the capitalist system there is no exploitation.
But even, if such generalization beyond the capitalist systems were not our aim, we should point out some ambiguity in the Marxian theory of value. As Marx states, the value of the commodity labour power corresponds to the amount of labour necessary to reproduce this labour power. By this is meant the labour time necessary to produce the consumption goods consumed by the workers. But this necessary labour time is only defined after the rate of

growth of the labour force is specified. Marx and the Marxists implicitly put the rate of growth of the labour force equal to zero when they compute the value of the labour power. But this is only consistent with the model of simple reproduction, i.e. with a stationary economy. In a model of extended reproduction the labour time necessary to reproduce the labour power at an extended scale is different from what Marx considers it to be. Indeed it is equal to L_1^* (t) according to our definition, where the index 1 represents the exponentially growing labour force of the Marxian model of extended reproduction. Thus a correct calculation of the "necessary labour time" leads exactly to our definition of exploitation.

VII

The methods to arrive at a precise measure of exploitation which we have used are different from those used by Marx. We have looked at completely synchronised, i.e. stationary or exponentially growing production systems and have compared the amount of labour necessary to "fuel" such systems with the amount of labour supplied by the group harvesting the net output of the synchronised system. Marx' definition does not seem to require that we look at the system as a whole as long as we know the properties of the techniques of production which are used to produce the different commodities. We then seem to be able to compute the Marxian values of the commodities and from there the degree of exploitation. Hence it seems that Marx does not need the assumption of an exponentially growing system in order to give a precise definition of exploitation, whereas our definition does need this assumption.

This impression is largely fallacious, even apart from the point that"necessary labour" is an ambiguous concept unless the rate of growths is specified. In the simple input-output-model discussed above Marxian values can be computed in the Marxian way without regard to the behaviour of the system as a whole. But if such a system deviates from steady state development, the Marxian "socially necessary labour time" to produce a certain commodity becomes economically rather meaningless. For then those labour particles which had to be available when there was abundance of labour are added with equal weight together with labour particles which had to be provided at times of labour scarcity This expression gives little information about the social opportunity costs of the production of this commodity in terms of other commodities. Thus it is not a good basis to measure exploitation.[1]

In addition, this model is of course very special and is useful as an example only, if its results would carry over to more general models. This is not the case here. For example there is the possibility of substitution. Whereas under steady state conditions we can ignore the existence of alternative techniques of production and concentrate on the technique which has been chosen, this is no longer possible outside of steady state paths. For these switches of techniques will probably take place, or at least the mix with which different techniques will be used is not constant through time. But the historical specific labour content of a commodity depends on the techniques in use and thus it changes from period to period. Moreover, at any given time it depends on the techniques in use in the past, and the historical labour cost will not in general coincide with what Marx has called on average socially necessary labour time. This discrepancy is an important point in Marxist doctrine. But what is then the socially necessary labour time in a system with substitution as long as it is not in a steady state equilibrium? Marxist theory has so far not given a clear definition. Clearly it would be unsatisfactory simply to take the mix of techniques which is employed at time t as the basis to compute the socially necessary labour time by way of the equation

$$\pi = a_o \, (I - A)^{-1}$$

where a_o and A are determined by the actual mix of techniques.

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[1] For a discussion of opportunity costs under steady state conditions see C.C. von Weizsäcker, P.A. Samuelson[5] The results given there point towards a modification of the Marxian Theory of value similar to the modification of the concept of exploitation presented here.

For this actual mix of techniques is the result of a dynamic process in which the actual prices (which may be far from any kind of "natural prices", to use the classical expression) dictate the choice of techniques whereas they themselves are largely determined by the supplies of inputs available at the beginning of the period, which are accidental from the point of view of the decisions to be taken in this period and future periods. The actual mix of techniques probably only exists because everybody knows that it will not last over a longer period of time. It would be nonsensical to compute the labour costs to reproduce inputs by means of the ephemeric mix of techniques prevailing today, since they depend on the expected demand for inputs tomorrow when a different mix of techniques will use these inputs. Similar arguments as Marxian doctrine uses against the identification of historical labour costs and socially necessary labour time, can be used against identification of labour costs computed from techniques of production in actual use with socially necessary labour time.

As already stated, a precise definition of socially necessary labour time in Marxian doctrine does not exist. But from the insistence on a difference between this concept and the (disequilibrium) historical labour costs we may derive the hypothesis that the Marxian concept of socially necessary labour time is an equilibrium concept. On average socially necessary labour time is the labour time necessary to produce the commodity under equilibrium (i.e. steady state) conditions, given the techniques of production which are available to the economy under consideration.

If this is so, then the Marxian definition of exploitation is not more general than ours. It can only be defined by reference to a steady state model in which the concept "socially necessary labour time" can be made precise. We are then able to define a concept of socially necessary labour time which corresponds to our definition of exploitation. As was shown above, the Marxian definition of exploitation by means of labour values is identical to our definition $R^1(o)$. In a similar way it can be shown that there exists a definition of labour values which corresponds to the definition $R^k(\bar{g})$.

These labour values are obtained by giving the labour inputs and consumption good outputs the weights $(1+\bar{g})^{-t}$ where t is the time they accrue. To show this we would just have to repeat the proof for the equivalence of Marxian exploitation with $R^1(o)$ by changing the weights of the labour inputs accordingly. The values which result from this computation also coincide with the steady state opportunity costs of the commodities as was shown in [5] and [6].

Hence in our case, just as in the case of Marx, the concept of values can be made precise by reference to some steady state economy. Exploitation can be defined in both cases for non-steady-state economies by using the values defined in a steady state context.

The choice of the reference steady state economy and hence of the weighting system for computing values is ambiguous outside of steady state.

There is no particular reason for giving a system with weights derived from a stationary economy the preference before other steady state economies. A precise definition of exploitation may therefore not be possible. But it may still be possible to give certain upper and lower bounds for the degree of exploitation by considering upper and lower bounds of the trend growth rate of the labour force. Since the labour force exhibits a rather steady growth (which in developed economies no longer deviates very much from a rate of growth equal to zero) this approximation may be quite precise.

VIII

I do not think that it is a mistake if a certain definition can only be made precise under certain assumptions and if we have to work with pragmatic approximations in the context of empirical research. If we would insist that the concepts which we use in empirical research are defined unambiguously for the the economies to which we apply them, nothing of interest could be said about these economies. (To avoid confusion: I do not

mean the practical difficulties of measuring well defined concepts.) All interesting concepts can be made precise only if certain assumptions are fulfilled. I want to mention two examples from every day national accounting work: the concepts of national wealth and of consumption. Under certain conditions of deterministic equilibrium the concept of national wealth is precise: it is for example equal to the historical cost of past net investment and at the same time it is equal to the present value of future rents and quasirents of the capital goods in the economy. But in reality the future is uncertain and we are not in general exactly in an economic equilibrian situation. This implies that historical cost of net investment (if it could be defined unambiguously) is not equal to the owners' expectations about future rents and quasirents. So what is then national wealth? The concept has no longer an unambiguous meaning. Or take consumption. Of the expenditures of households there are some whose function it is to enable the household to earn a higher revenue. They should be separated out of consumption and hence of net income. But there is in general no unambiguous way of doing it.[2] Only under very special circumstances can we solve this problem. But this intrinsic ambiguity is not an argument against the use of these concepts in empirical work.
Of course, the concepts can be split up into many different concepts. There may be the variable "national wealth 1", "national wealth 2", etc. But modern methodology and philosophy of science will make us aware that we can never expect to get concepts which are completely unambiguous under all circumstances. For this would presuppose a model of reality which is no longer distinguishable from reality itself, an idea which belongs into the realm of theology rather than the social sciences.

Ambiguity of concepts is thus not necessarily an argument against their introduction, as long as they can be made

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[2] In practice this involves many income tax problems which in the last resort cannot be solved analytically, but have to be decided politically by the law-makers.

unambiguous in the context of models and theories which have some relevance for reality. These models, which provide unambiguous definitions of a concept or of a set of concepts, usually are equilibrium models (as is seen in the definition of exploitation or of national wealth). It is a fundamental methodological error to believe that equilibrium models can be dispensed with in economics. They are conceptually necessary as a step towards the definition of useful concepts. Of course, at any given stage of development of the science of economics, the existing equilibrium models may no longer be satisfactory. This then points to the necessity to define new concepts by developing new equilibrium models. For a given conceptual framework it is also possible and often necessary to investigate processes which describe disequilibria from the point of view of the models within which the basic concepts have been defined. This will usually end in the definition of new concepts for which new equilibrium models are necessary. The concept of equilibrium is only meaningful with respect to a certain model. It makes no sense to say:"this economy is in equilibrium" or "it is not in equilibrium." Meaningful are only statements like: "in this economy equilibrium exists with respect to the variables which are defined in the context of this or that model." It may be valid to criticize the specific equilibrium models and concepts which orthodox theory prefers, but I believe it is futile to attack equilibrium economics as such. If the present orthodoxy should be replaced by some other orthodoxy, this again will have to work with equilibrium models if it wants to cast theoretical light on phenomena of more than ephemeral importance, unless, of course economic theory is abandoned altogether in favour of the fascinating number games of computer simulation models. While computers nowadays are capable to compute almost as many disequilibrium paths as you want, the human brain seems only to be capable to understand simple, i.e. equilibrium solutions; but fortunately it also has the capacity to form new concepts which help it to interpret disequilibria in an old model as equilibria in the context of a revised model.

To avoid one misunderstanding we should stress that using the instrument of equilibrium models does not imply an implicit decision in favour of stability of the real economic system under consideration. Marx in his time was a master in the use of equilibrium models (cf. e.g. his analysis of the equalisation of profit rates across industries) and yet nobody claims that he was particularly optimistic about the stability of the capitalist system ([2], vol.III).

IX

In this section we want to generalize the concept of exploitation to economies with technical change. The theory of economic growth has predominantly concentrated on process innovations, since problems of measuring economic growth become easier, if final outputs of a later period only contain commodities which were already produced in earlier periods. On the other hand, in reality most advances in technology are in fact connected with the creation of new products, in the sense of the word as it is usually applied. It would therefore be better to obtain a generalization of exploitation which also is useful in the context of product innovations. In order to cover both cases, product innovations as well as process innovations, we make use of the fact that it is not difficult to treat process innovations as a special case of product innovations. We consider an economy in which only product innovations take place. Thus, whenever a technological change has taken place, a new commodity comes into the picture which is the output of the new process of production. Such a model does not preclude the analysis of technological changes which usually are classified as process innovations. If in the process of production of a certain commodity an improvement takes place, we simply interpret the output as being a new commodity which happens to have the same physical characteristics as the old commodity. Thus process innovations are the special case of product innovation in which the new product is a perfect substitute of some old product in every use.

If we now look at some group of people, k, we observe that at the time t they consume a vector $C^k(t)$ which is an element of some infinitly (but countably) dimensional linear vector space. If the group has finite extension (and only for those have we defined exploitation) the consumption of the group can even be described by vectors in a finite dimensional Euclidean vector space, since for every t_1 the number of existing commodities can be considered to be finite, hence also for a t_1 being an upper bound of the extension of group k. If a certain good i is the result of a product innovation which took place in period $t^i - 1$ then the good did not exist in periods $t < t^i$. Thus $C_i^k(t) = o$ for $t < t^i$. Then technical progress in this model can be described by a specification of the period t^i in which the good i becomes available to the economy.

As we have seen above, an unambiguous definition of exploitation requires that we consider a steady state economy. By its very definition a steady state economy extends through time from minus infinity to plus infinity. On the other hand, we only get a fair picture, if we compare the consumption and the labour supply of group k with the production possibilities available at the time the production processes using the labour of group k and providing the consumption goods for group k took place. We therefore have to imagine an artificial economy growing at the rate $\bar{g}$ and using only the relevant techniques of production. This artificial economy thus basically is an economy without technical progress. Our formalisation of technical progress makes it very easy to construct such an economy. We simply have to drop the constraints that good i can only be produced in positive quantities in period t^i or later. We thus look at an economy with a finite number of consumption goods, namely all those actually consumed by group k, and with all those techniques of production which correspond to these goods. Since we only have product innovations and since our "real" economy (of which group k is a part) is in steady state the technique of production for a given good does not change through time and can therefore be identified unambiguously. Thus for our "artifical" steady state economy no technical change exists. Formally, it cannot be distinguished from an economy with a stationary technique of production. As an example, assume that the actual economy can be described by a sequence of input-output coefficient sets .... $(A(t), a_o(t))$, $(A(t+1), a_o(t+1))$, ... such that $A(t+1)$, $a_o(t+1)$ is generated from $A(t)$, $a_o(t)$ by adding new columns to $A(t)$ and new elements to $a_o(t+1)$. This means that only product innovations occur. Then the "artificial" economy has a stationary technique of production characterized by $A(t_1)$, $a_o(t_1)$ for t_1 sufficiently large to be an upper bound on the time extension of group k. Now we construct the $\bar{g}$-exponential extension k' of group k and compute $L_{k'}(t)$, $C^{k'}(t)$ in the usual way. We then obtain $L^*_{k'}(t)$ by using the input-output matrix $A(t_1)$, $a_o(t_1)$ for the computation of the labour requirements. In this way, we arrive at a rate of exploitation R_k even under conditions of technical progress.

Since, by the nature of the problem, we had to transform the "actual" steady state economy into an artificial economy with no technical progress, the implications of our definition for a capitalist steady state economy are formally the same as they are in an economy without technical change. But there is a point of interpretation which should be mentioned. Since in steady state models with no technical progress the wage rate remains constant we also have to look at this model as if the wage rate remained constant. If the actual wage rate rises by x percent per year the relevant rate of profit r is also the nominal rate of interest minus x percent. Since, as was shown, a nonexploitative state exists when r equals $\bar{g}$, we infer that the rates of exploitation in this model are zero whenever the nominal interest rate is equal to $\bar{g}$ plus the rate of growth of nominal wages, x. If now the rate of growth of real wages, as measured in the conventional way, is y percent then we have a rate of inflation of x - y percent and thus a situation of no exploitation prevails, if the real rate of interest is equal to $\bar{g}$ + y which is equal to the real rate of growth of the system. Thus the theorem of nonexploitation of the Golden Rule Path generalizes to a situation with technical progress. It would be wrong to say that the real rate of interest has to be equal to $\bar{g}$ to eliminate exploitation by capitalists.

I think it is worthwhile to discuss the Marxian concept of exploitation under conditions of technical progress. My definition and the Marxian definition are identical for a model with zero population growth with or without technical progress. In other words, an application of the Marxian definition of exploitation to an economy with a stationary labour force, and with a nominal rate of interest equal to the rate of growth of nominal national income has no capitalist exploitation.

It is, of course, irrelevant how we choose our numeraire. The most convenient numeraire is the commodity labour. By definition the wage rate is equal to unity in terms of this numeraire. Since the labour force is constant the national wage bill is also constant through time. In steady state conditions the distribution of national income is constant and hence the ratio of national income to the wage bill is constant. But then national income (always in terms of the numeraire labour) is constant, i.e. its rate of growth is zero. By assumption the rate of profit is equal to the rate of growth of national income, hence it is also zero. Thus nominal net income of capitalists is zero and national net income is equal to the wage bill. Moreover, under steady state conditions the ratio of national wealth to national income is constant and thus national wealth is itself constant. Nominal net investment is zero, thus consumption is equal to national income and to the wage bill.
Now, since the wage rate is a constant and since the rate of profit in this accounting scheme is zero, competitive prices of commodities reflect their specific labour contents. If we identify these specific labour contents with what Marx calls socially necessary labour time then the prices reflect the Marxian values. Thus net accumulation in value terms and the mass of surplus value are zero, since net investment and profits are zero in terms of the prevailing price system. (It is of course always possible to make the nominal

rate of profit equal to zero, by choosing an appropriate price deflator. But this does not mean that we always can discuss away surplus value. This is only legitimate if at a nominal zero profit rate the wage rate is constant).

Marxists may perhaps not like this result, since it implies that the capitalist system with a positive rate of profit in bourgeois accounting terms may exhibit no exploitation. The necessity of exploitation for the viability of capitalism was one of Marx' important points.[3] Marxists may try to avoid this conclusion by denying that even under steady state conditions as outlined above the "historical" labour content of commodities can be identified with what Marx calls socially necessary labour time.[4] But, as far as I can see, this does not help, as long as we stick to the accounting requirement that, if capitalists abstain from private consumption, the net addition to the value of capital must be equal to the mass of surplus value. At a wage rate of unity the wage bill and the consumption bill in terms of market prices are equal to the total labour time expended during any period. They are then equal to the total net value creation during the same period. If there is exploitation the value of the consumption goods consumed by the workers must be less than total net value creation and then less than the historical labour content of the consumption goods. So let Δ be the difference between the historical labour content of consumption goods and the value of the consumption goods. Then Δ labour time units are expended per period which do not create value. But then, of course, total net value creation in the economy is also reduced by Δ and we again have the result that the value of the consumption goods acquired by workers is equal to the total net value creation, and again there is no surplus value.

The "stylized facts" of present day developed economies are that the rate of technical progress is substantial whereas the rate of growth of labour time expended is close to zero. Thus the Marxian definition, as interpreted here, and our definition of exploitation are quantitatively not very different for such countries.

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3) For this cf. the forthcoming book of Morishima on Marx.

4) This criticism has actually been raised against my argument by Prof. W. Vogt, Regensburg and Dr.B.Schefold,Basel, in private communication.

X

This section is devoted to a discussion of exploitation under conditions where the additivity of steady state labour requirements no longer holds. We call it the case of intrinsic joint production. There are well known cases of joint production, such as for instance simple models of fixed capital,where the additivity of steady state labour requirements still holds. I believe it is a reasonable definition to speak of intrinsic joint production whenever the additivity of steady state labour requirements is no longer valid. It is easy to show that additivity of steady state labour requirements is a necessary and sufficient condition for the non-substitution theorem to hold. By dropping this additivity assumption prices become a function of the demand structure in the economy. Also steady state labour requirements for a commodity cannot unambiguously be defined in the case of intrinsic joint production.

But this is no reason to give up thinking about labour requirements. We observe in reality that, while there are intrinsic joint products we do have a lot of independent firms. This fact is related to the informational advantages of decentralisation. I propose that we consider the phenomenon of joint products in this context of decentralisation. The following approach may be useful as a first step in this direction. For every commodity there are available different techniques of isolated production of outputs (possibly by throwing away joint products which accrue in the production process). Due to indivisibilities there are certain minimum output quantities which have to be produced with every technique, if production with this technique takes place at all. But now inputs may be saved, if certain outputs are produced jointly. This tends to make joint production more efficient than isolated production. But joint production also tends to make the size of minimum operation larger than it would otherwise be.
If there is a cost of control of operations which grows more than in proportion to size of operations we may arrive

at a theory of the optimum firm, similar to those which already exist and have been developed decades ago (cf.eg. [4]). Due to indivisibilities of production processes the optimum degree of joint production will also be limited if one wants to avoid too large sizes of firms.
This suggests the following revision of the definition of labour requirements. Instead of looking at labour requirements for different commodities which no longer can be defined properly under conditions of intrinsic joint production we should look at labour requirements of the outputs of production establishments (i.e. firms) producing independently of each other. We should simply look at the flows of values between firms, from firms to different consumer groups and at the flows of labour from different consumer groups to the firms.

Assume that all firms are growing exponentially. Then let l_j be the direct labour inputs of firm j, let Zij be the flow of goods from firm i to firm j, let c_i be the production of consumption goods of firm i, let π_i be the total direct and indirect labour requirements of firm i. If we now allocate the labour requirements to the different value flows out of firm i in proportion to their size we get the following equations:

$$\pi_j = l_j + \sum_{i=1}^{n} \frac{Z_{ij}}{Z_i} \pi_i$$

or writing $q_{ij} = \dfrac{Z_{ij}}{Z_i}$

$$\pi_j = l_j + \sum_{i=1}^{n} q_{ij} \pi_i$$

in matrix notation, putting $(q_{ij}) = Q$

$$\pi = l + Q$$

$$\pi(I - Q) = l \quad \pi = l\,(I - Q)^{-1}$$

This will have a positive solution, if suitable conditions are fulfilled. The labour requirements of the consumption goods produced by firm i are then

$$\frac{c_i}{Z_i} \pi_i$$

Correspondingly, the labour requirements of the consumption of group k (which also grows exponentially) are $\sum_i \frac{c_{ik}}{z_i} \pi_i$ where c_{ik} is the value of consumption goods delivered from firm i to group k.
If neither firms nor groups grow exponentially we again can apply the device of the $\bar{g}$-exponential extension in order to obtain the degree of exploitation. But it should be pointed out that it is no longer quite legitimate to "isolate" the group's consumption in this way completely from the consumption of other groups. The price (in terms of labour) at which any given group is able to obtain its consumption goods may depend on the consumption habits of other groups which buy the joint products of those commodities preferred by the group under consideration. Thus the demand of the poor for low quality meat reduces the supply price of high quality meat mainly purchased by the rich, and the other way round, since the two kinds of meat are joint products. Both groups would have to pay more for their meat, if the other group did not exist.

This, of course, is a well known case of gains from trade. As was pointed out in the introduction, it would be wrong to use such gains from trade as an escape from the acknowledgement that there exists exploitation. Therefore, we have to compute the volume of required labour, given that the gains from trade connected with joint products prevail.

There is one other point of criticism against this extension of the definition of exploitation. In treating the deliveries of a firm as if they were a homogeneous commodity, we weigh the different commodities by their market prices. As is known from the Marxian transformation problem in the case of absence of joint products, prices do not exactly represent labour values (whether we measure them in the Marxian way or in the way proposed above). Thus we are making a certain mistake in the measurement of labour requirements by using market prices as weights. Whereas in the case of absence of intrinsic joint production the nonsubstitution theorem states that prices do not depend on the structure of demand, this is different here. The principle by which we

compute labour values is basically that we compute the steady state opportunity costs of the commodities, or, which comes to the same thing, compute the prices of the commodities prevailing in a situation of no exploitation - keeping the techniques of production the same as they are used in the actual situation under consideration. But given joint products, both methods yield the result that labour values of commodities depend on the structure of demand. And demand depends on the distribution of income. Thus the values themselves depend on the distribution of income.
In principle it is possible to solve the value equations in a steady state general equilibrium model which specifies the demand behaviour of all groups. As can be shown (see [6]) the Golden Rule path (i.e. a path without capitalist exploitation) is always an Arrow-Debreu-equilibrium and thus computation of values is possible. By using these values as weights in the sales of joint product firms to different consumer groups a "correct" computation of degrees of exploitation may be possible. For all practical purposes this procedure is too complicated and we thus propose to use actual prices as weights as was outlined above. As Ricardo already pointed out, the approximation of labour values by prices is quite a good one ([3], chapter 1). This supports our procedure.
But it should be pointed out that a correct computation of values, even if we have intrinsic joint production, is possible if we use our definition of values. This is not the case for Marxian values, since there exists in general no zero interest rate general equilibrium solution of the model.

XI

Marxian doctrine provides not very convincing solutions to the "problem of reduction", as Marxists call it, i.e. the problem coming from the fact that there are different kinds of labour which usually receive different wages. Marx proposes to reduce different kinds of labour to "simple" labour by weighing them with their wage rates. But these wage rates are endogenously determined in the context of a general equilibrium system. Thus, if, due to a rise in the rate of interest, relative wages change (wages of people with a special skill requiring a long education will rise in relation to other wages as the rate of interest rises) while the physical quantities of different labour inputs remain the same, the Marxian "values" of commodities change. Wage differentials cannot completely be explained by differences in the costs of education and training for the different skills. To a certain extent these differentials have rent character, since genetic factors or other social (but not economic) factors limit the extent to which the supply of any given skill can be varied. But this implies again that wage differentials depend on the structure of demand. The Marxian reduction to simple labour and hence Marxian values can only be determined in a general equilibrium framework specifying the demand behaviour of all economic agents.

Marx' emphasis lay on capitalist exploitation. Any discussion of exploitative relations between different strata of "workers" would have blurred his comparatively simple theory of exploitation. The problem of wage differentials was a nuisance for him and he tried to get rid of it in such a way as not to disturb his analysis of capitalist exploitation. He, therefore did not propose to call a man an exploiter, if he received the tenfold wage of a simple worker. He rather preferred the fiction that this man was providing 10 hours of effective work whenever he worked for an hour. To justify this approach Marx tried to explain wage differentials as the result of differences in the cost of training. To the extent that this hypothesis is empirically valid, his approach is alright in principle. But even then the actual wage structure would still have to be transformed into a wage structure purified from profit elements which consist of the implicit interest on the educational capital accumulated in a person. There is no reason why the tendency towards an equalisation of profit rates should not apply to human capital as well. The actual, nonpurified wages thus contain parts which in Marxian terms are surplus value. Thus, if we accept the explanation of wage differentials by Marx it has the important consequence that the dividing line between the classes of society becomes blurred. There are large and increasing sections of employees who at the same time are members of the exploited class of the wage dependent workers and are members of the capitalist class of exploiters.

But I believe that not all wage differentials can be explained by differentials in training costs. I therefore propose that we solve the problem of different kinds of labour in the context of exploitation by giving them equal weights. We thus measure the quantity of labour in terms of time units. This, I believe, should be in the spirit of Marx, who has taught us to see that in spite of all the differences between the specialised human activities they are all representations of the same thing, human labour. Of course, in the production process the different skills are not perfect substitutes of each other. The division of labour makes everyone dependent on the work done by specialists other than himself. But this again is a case of gains from trade (or cooperation) which do not preclude the existence of exploitation. If we want to define exploitation by the existence of a group or groups of people who receive more human labour (embodied in consumption goods) than they supply, it is sensible to apply equal weights to each hour of labour time, independent of its specific characteristics.

This definition of exploitation in the presence of different kinds of labour should be of interest in the discussion of wage differentials and income distribution. Most countries in the world rely on material incentives to stimulate productivity of labour and the efficient allocation of resources. This implies the existence of efficiency related wages and incomes in general. It is at present impossible for industrialized countries to abandon material incentives without the effect of a severe drop in productivity of labour and, therefore, the standard of life. Given the mentality of most economic agents in most countries of the world, exploitation, as it comes with the system of material incentives, seems to be unavoidable. Capitalist exploitation seems to be comparatively easy to overcome: sufficient accumulation of capital by the government can drive the rate of profit down to the rate of growth of the system where capitalist exploitation no longer exists. Exploitation due to material incentives, including wage differentials, is a phenomenon which unites socialist and capitalist countries. Its quantitative importance is already much larger than capitalist exploitation. The main difference between western and eastern countries is not so much the predominance or otherwise of private property of means of production. For the latter is neither a necessary nor a sufficient condition for capitalist exploitation. The main difference between the systems is this: the western system accepts and fosters the individualistic attitudes of economic agents and thus accepts the existence of material incentives and exploitation as final and necessary. The stress on individualism makes individual freedom consistent and even helpful for the economic system. The East European system abolished large parts of individual and political freedom in a grand effort to develop a communist classless society in which material incentives and thus exploitation become unnecessary. On its way towards this final goal the system uses material incentives and hence exploitation to increase productivity and efficiency. Whether this final goal ever will be obtained remains to be seen. The ideological rift between the Eastern European countries and China is largely a quarrel about the role material incentives should play in the development of a socialist society. The Chinese try to replace material incentives by other means to foster growth. Again it remains to be seen how successful they will be. Whatever the drawbacks of the Chinese system may be, it should be clear that the average rate of exploitation in China is not as large as in some of the East-European countries.

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