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Alfred Zauberman

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H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

NOTE ON ASSIMILATION OF OPTIMAL CONTROL THEORY AND RAMIFICATIONS IN ECONOMIC PLANNING THEORY

Alfred Zauberman
London School of Economics,
pro.temp. Institut für Mathematische Wirtschaftsforschung,
University of Bielefeld,
West Germany

I.

Introduction.[1] [2] This Note offers a few informal remarks on a subject, which one feels, deserves a fuller, deeper and rigourized investigation. It is concerned with the assimilation by, the mathematically-oriented, economic planning of the optimal-control-theoretic approaches and techniques - a process which by now has one decade's history. By itself the process can be seen in the perspective to correspond to a longer-run logic of advance in the conceptual and instrumental equipment of the planning theory and in its application.

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1) This Note was produced when I was Visiting Professor at the Institute of Mathematical Economics, University of Bielefeld. I am grateful to the Director of the Institute Professor Reinhard Selten, for the aid extended.

2) In reference in this Note the titles of my recently appeared books are abbreviated as follows:
Aspects of Planometrics (London University Press 1967) - "Aspects"
Mathematical Theory in Soviet Planning (Oxford University Press, 1976) - "Mathematical Theory"
Differential Games and other Game-theoretic Topics (New York University Press, 1975) - "Differential Games"

For one may detect a logical progress in what was initiated by the absorption of the Leontievian apparatus for securing feasibility of the economy's activities - first static then dynamic; next - of the apparatus for the selection of the "best" plan-programming solution - again, first static (Kantorovich, Dantzig), and then placed into the time dimension (with the extensions from the original linear to non-linear situation and processes); more - this associated with the expliciting of the system's guidance by one or many controllers in the pursuance of his (or their) objectives (Pontryagin and associates; Bellman). All this - paralleled by a movement from a strict deterministic to a probabilistic stand. It seems tenable that thus presented without qualifications the process of assimilation we are talking about must appear gradually to improve the planner's potentialities in performing his task.

Although our observations stop at the point marked by the acceptance into the body of planning theory of the two principal constructs of the post-classical calculus of variations - the Pontryaginian and the Bellmanian - we shall <u>passim</u> refer to some of their further elaborations of relevance from our angle. Indeed in the present context of the gradual progress towards better "capturability"of "real world" and of the planner's task, we may exemplify this straight away by pointing to one, very important one, among reformulations of the constructs: that by Gabasov and Kirillova [3] - of the Maximum Principle in terms of variational derivatives

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[3] R. Gabasov, M.Kirillova, <u>Printsip Maksimuma v Teorii Optimalnogo Upravlenya,1974</u> ; see also by the same, <u>Kachestvennaya Teoriya Optimalnyk Protsesov,1971</u>

and variations of the second order, patently something closer to the "variational" handling by the planner of the plan-alternatives confronted.

Having suggested in a general **fashion that,** and in what sense, the assimilation discussed has enriched planning theory we shall concentrate on some of the specific matters diminating today the problematics of the planning theory. It seems that they can be conveniently put under the following headings: (1) time horizon, (2) goals pursued, (3) parametric steering of economy, (4) stability of, and stationarity in the system's advance, (5) path of planned advance, (6) multicontrol,, (7) scale and complexity, (8) indeterminacy, (9) simulative methods, (10) advance in control theory, (11) systemic extension.

II

(1) Time horizon. The dilemma of setting the horizon has posed itself to planning theory from its very inception. To put it loosely, it arises from a conflict between the nature of planning as a free-end operation ("the end of the world" not being assumed), and the familiar technical difficulties in solving non-finite programmes.

The Maximum Principle has offered a heuristic relief in the first place by permitting to build up a theory of a sliding horizon (continual revision). Goldman's models [4] typi-

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4) S.M. Goldman, Journal of Political Economy, 1969

fy it. (Atypical is the approach in Smirnov's [5] Maximum-Principle-based design where the plan's time horizon is a function of the discount factor; and in turn the latter - a function of time).

Moreover, the Principle has given the framework for a rigorous treatment of infinity. To recall, the solution offered by Pontryagin and assoc. themselves holds essentially for autonomous systems; but not otherwise, in particular it does do so where a time discount is adopted - as shown by Shell [6] (who applies the familiar method of "partitioning" the free-right-hand time dimension and expanding the middle part): it breaks down at the transversality condition.

However, as it would seem, the approximate plan-oriented solution - such as offered by Moiseyev[7] - based on the Hukuhara (1934) theorem - may do the job for the purpose. The approach rests on the idea of a preliminary "asymptotic processing" of the initial state; then controls are probed where "the point at infinity" becomes a "regular point". One of the advantages is a possible avoidance of the tricky task of finding the auxiliary problem for the necessary conditions of the Maximum-Principle kind.

The work of Moiseyev - control-theoretic-algorithmically oriented - is likely to bring (at least as far as economic planning is concerned) closer to a solution the search, which in fact was

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5) A.D. Smirnov, Modelirovanye i Prognozirovanye Sotsyalisticheskogo Vosproivodstva, 1970

6) K.Shell, "Application of Pontryagin's Maximum Principle to Economics" in H.W. Kuhn, G.P. Szegö, Eds., Mathematical System Theory and Economics, p.I, 1969

7) I.N. Moiseyev, Zhurnal Vychislitelnoy Matematiki i Fiziki, no.4, 1974

started in economic theory before the optimal-control phase (with such fundamental contributions as those of von Weizsäcker [5] and Koopmans [6].

(2) Goals pursued. The problem of the objective pursued - specifically its substance as distinct from its formalism - can be considered as a datum for the planner. Admittedly from the plan-constructional angle he will not be indifferent to this substance where it has an impact on the form of the plan statement. This would be true, say, for the question of tenability or otherwise of Arrow's impossibility theorem [7]; or the measurability of the objective function: thus e.g. the formulation of the utility objective as one of von-Neumann-Morgenstern type or the matter of interpersonal aggregation (as in Sen [8]) or the issue of discounting the objective (cf (1) above) are immediately pertinent to the plan's "architecture".

That allowed for, it is the technical possibilities that optimal control offers in treating the objective function that are of relevance here. In this context one would mention the freedom of shifting - under Maximum Principle - elements of goal pursued as between the objective function and contraints (and,parenthetically, also the possibilities in constraining

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5) C.C. von Weizsäcker, Review of Economic Studies, 1965

6) T.C. Koopmans, Econometrica 1967

7) K.J.Arrow, Social Choice and Individual Values, 2nd ed. 1963

8) A.K.Sen, Collective Choi e and Social Welfare, 1970; the same Econometrica, 1966 and 1970

the state as well as control variables and handling constraints as either equations or inequalities).

Also the question of multicriteriality belongs here. In the literature of the subject I would draw attention in particular to Salukvadze's [9] generalizations in his study of optimization of control systems under vector-valued performance criteria: technically of considerable interest is the ingenious statement of the variational problem in terms of Mayer which in turn is solved on the Maximum Principle.

(3) Parametric steering of economy. One of the crucial benefits planning theory secured from optimal-control theory is the clarification of the nature and of the placing into time dimension of the Lagrangean-type valuation of constraints. This is the Hamiltonian multiplier in the Maximum Principle, the gradient of the Bellman function in the Optimality Principle formulation: a closed, dynamized analogue of the Lagrange coefficient from dual of a mathematical programme; thus well interpretable by the economic planner as the price in time of factors and of commodities, including that of the "commodity" time itself.

For lack of better context we may note here that by the latter-1960s/1970s various limitations of the Maximum Principle [10] have reveiled themselves to the economist student some severely inhibiting the use, others - depriving the results of economic meaning. They are accountably for by either mathematical or/and "real" facts. On the mathematical side there is first of all

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9) **Salukvadze, Avtomatika i Telemekhanika, No.5, 1972**

10) cf.Zauberman, "A Choice of Control for a Turnpike" in J. Los and M. Los, Eds., The Mathematical Models in Economy, Symposium of the Mathematical Institute, Polish Academy of Sciences, 1974

"notorious" two-point boundary value problem. Among the facts on the economic side there is the absence of admissible controls as a function of phase coordinates in the neighbourhood of the optimal trajectory, convexity of the control domain, concavity of the subintegral, negativity of some variables.

The matter has been treated extensively by Arrow and Kurz [11] (cf also an interesting contribution by Schmidt treating the tricky matter of phase constraints in optimization problems (see Zauberman [12]).

Detecting limitations have induced a search for remedies from the angle of the planning theories' needs. Thus note in the context inquiries such as that by Arkin and Levin [13] into variational problem with integral-valued constraints (integral inequalities) where controls are measurable n-variable functions. Here a postulated continuum of participants leads to abandoning the usual convexity assumptions (for the purpose the familiar A. Lyapunov apparatus is generalized and the Pontryagin principle suitably reformulated). The supposition arrived at is then that assuming a large number of sectors and regions for a plan, the framework a price-instrumented decentralization could be effective no matter what is the formal shape of components, specifically of the functions of utility,

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11) K.J. Arrow, in Mathematics of the Decision Sciences, p.2,1968
K.J. Arrow, M.Kurz, Public Investment, the Rate of Return, and Optimal Fiscal Policy, 1970; M.Kurz, Review of Economic Studies, 1968

12) Zauberman, Mathematical Theory

13) V.I.Arkin, V.Levin, "Varyatsyonnyie zadachi s funktsyami mnogikh peremennykh i model raspredelenya resursov" in B.S.Mityagin, Ed., Matematicheskaya Ekonomika i Funktsyonalnyi Analiz, 1974

local constraints and technologies (the idea is closely related to the present-day theories of continua, core and equilibrium in large-scale economy as represented by Hildenbrand [14]).

(4) <u>Stability of and stationarity in the system's advance</u>. The time-honoured policy-maker's concern about the stability of the economy's behaviour was taken care of initially by the theory of regulation. This was largely due to Phillips. The work was not focused however on the quality of the stability until the contributions by Holt (who adopted the quadratic criterion functional and a regime of the Simon-Theil type).

It is only with the advent of optimal-control theory that the issue has become one of the optimum. Of crucial importance was N.N. Krasovskiy's discovery - in his treatment of the time-optimal control process - of the direct nexus between the functions of Bellman and of Lyapunov: a nexus whose substance is that for a number of problems with certain sign-determined criteria, the Bellman function in a familiar form, itself well-known to be of the Hamilton-Jacobi family, reveals itself as the Lyapunov function. Thus the solution of the optimum control problem in such situations reduces to deriving the "optimal Lyapunov" function. (Parallel to Krasovskiy, Kalman treated the problem in probabilistic terms).

Since then the problems of analytically constructing an optimal - or suboptimal - "regulator" by means of the Lyapunov function (in both deterministic and stochastic versions) has obtained a large literature. One would refer here <u>inter alia</u>

to Kosut,[14] Satyanarayana/Srinath,[15] Lorenz,[16] Popov,[17] V.M.Kuntsevich/M.M.Lychak[18] (in the latter contribution we have a design for a synthesis of optimal and suboptimal discrete systems of control, by means of a Lyapunov function, of a stochastic as well as a deterministic object).

(5) Path of planned advance. Simultaneously the 1960's and 1970's have witnessed the proliferating literature devoted to the tracing out with the help of the Maximum Principle of the optimum trajectory for the economy's stationary advance: the trajectory which reveals itself to be the von Neumann path - as conceived of in Dorfman/Samuelson/Solow[19] design of the "turnpike". The enquiry has crystallized in Shell's study of the problem of consumption-optimal growth for an economy illustrated by a one and two sectors'models with a linear or non-linear objective-functional. The by now celebrated Shell's basic theorem for the planning problem to the effect that the "Pontryagin programme" requires the planner to adopt the Golden Rule of Accumulation ("modified" - allowance made for time discount) for all but a finite amount of time; moreover, that as the length of the planning period increases the fraction of time with a capital ratio differing from that of the Rule, approaches zero. For elaboration of this crucial contribution of optimal-control theory to that of the planner's strategy one is advised to turn to Shell's own writings

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14) R.L.Kosut, in I.E.E.E. Transactions, AC-13, no.5, 1970

15) N.Satyanarayana, M.Srinah, I.E.E.E.Transactions, AC-15, no.5, 1970

16) J.Lorenz, Archiwum Automatyki i Telemechaniki, no.2, 1968

17) V.M.Popov, Giperustoichivost Avtomaticheskikh Sistem.

18) V.M.Kuntsevich, M.Lychak, Avtomatika i Telemekhanika, no.1, 1973.

own writings and to his references to the family of related literature.[20] To be sure this family does not provide the only approach to designing the optimal stationary path to the economy's advance.

Thus, more immediately planning-oriented is the Dyukalov/ Ilyutovich design.[21] Theirs is a design for a problem with continuous time, belonging to the linear dynamic programming or more strictly to Bellman's bottleneck problem. And for the probing into the asymptotic properties of the path it resorts to the method akin to the familiar stability proof for the solution for differential equations - by means of the Lyapunov function (see section 4).

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19) Dorfman, Samuelson, Solow, Linear Programming and Extension; see also J.R.Hicks, M.Morishima, R.Radner, "The Turnpike Theorem" Review of Economic Studies, 1961.

20) K.Shell "Application of Pontryagin's Maximum Principle to Economics" H.W.Kuhn, G.P.Szegö, Eds., Mathematical Systems Theory and Economics, part I, 1969. The planning problems in the paper are conceived as a particular case of those treated by Shell's essay (on optimal programme for capital accumulation) in his contribution to K.Shell, Ed., Essays on the Theory of Optimal Economic Growth, 1967; the latter collection contains contributions by several authors on related matters.

21) A.N.Dyukalov, A.E.Ilyutovich, Avtomatika i Telemekhanika, no.3, 1973.

22) L.V.Kantorovich, V.L.Makarov, "Growth Models and Their Application to Long-term Planning and Forecasting" in T.S.Khachaturov, Ed., Methods of Long-term Planning and Forecasting, 1976.

Again, the basic result is that the vector of phase coordinates on the optimal trajectory spends most of the time close to the turnpike's direction (the turnpike being defined as the trajectory of "balanced" growth - without unemployed capacities) coinciding with the right-hand eigenvector of the matrix of the full capital-intensity of production.

A still stronger planning orientation is to be found in the most recent Kantorovich-Makarov (1976) design[22] for a multi-sector multi-commodities system (although formally it works with a dynamic input-output apparatus plus convex programming, it is in substance an optimal-control design close to the dynamic programming species). And, again, it too leads to tracing a Neumannian kind of a "reasonable" turnpike determinable with given **growth rates** and time discount (where the relationship between the growth rate, the output and capital formation does not prove effectively tractable, a procedure is indicated for the planner to derive an approximation from a sequence of iterations).

(6) Multi-control. One of the aspects of "unitariness" of control characterizing the original optimal-control constructs which has inevitably proved to be unrealistic in planning application is that of a single controller. Hence the motivation for multi-control modelling. To reflect the reality of non-mandatory and mandatory planning economies that would be respectively non-hierarchic and hierarchic designs (reality would require a degree of this property). The substance of the problem is here the coordination of the plans; for the former - preserving the equality of the controllers' status, for the latter - enforcing a hierarchic ordering of the preference scales in the system.

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22) See previous page.

To be sure this is also an area where control-theoreticallv oriented planning theorv has continued the work of the earlier, call it the programming "era" (the massive output of literature in this field is discussed in mv recent books referred to on p. 1. Here I will rather eclecticallv refer for exemplification to the work of Malinvaud[23] in the West, and that of Dudkin and associates[24] and Puqachev and associate[25] in the East: and shall focus rather on the generali ing hierarchic games.

Such being the focus I would single out the work of Emelvanov/ Burkov/ Opoitsev[26] to the extent that it is verv distinctlv planning oriented. That is a model for a svstem where the only meta-plaver defines the rules of the game which otherwise reflects the plaver's interaction. To be more specific, the idea is that inasmuch as the games of the kind discussed are well-known not to have a single solutional concept (Nash equilibrium, Pareto-optimum, maximum guaranteed result, are but a few from the range), it is reasonable to determine uniquely the plaver's behaviour; the choice made is for the Nash point claimed to have secured experimental validation.

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23) see in particular the planning-oriented contributions bv E.Malinvaud, "Decentralized procedures for planning" in E.Malinvaud/M.O.L.Bachrach, Eds., Activitv Analysis in the Theorv of Growth and Planning, 1967: and the same Canadian Journal of Economics, Feb., 1968.

24) I.A.Vakhutinskiv, Kh.M.Dudkin, B.A.Shchennikov, Izvestva Akademii Nauk SSSR - economic series 6, 1973 and references therein.

25) V.Pugachev, A.K.Pitelin, Ekonomika i Matematicheskie Metodv, no.6, 1974 and references therein.

26) S.V.Emelvanov, V.N.Burkov, V.I.Opoitsev in Avtomatika i Telemekhanika, no.1, 1974 and reference to the Seminar on large-scale systems, 1973.

To move one step further on in plan construction, generalization of optimal control has led the architects of plan-modelling to n-persons differential games. There are so far only a few elaborations which should be indicated here.

In the first place reference must be made to the work of mathematicians connected with the Computation Centre of the USSR Academy of Sciences. Central here is the idea that in formalizing the working of a hierarchic organization (posing and solving the problems of its management, that of aggregation among others) the "natural" apparatus is provided by the <u>non</u>-antagonistic game; a construct associated with the name of Germeyer.[27)]

An important contribution in this field is owed to Moiseyev[28)] concerned with the balanced - "acceptable" "degree" of decentralization. A strong emphasis is on the, inevitably less than necessary amount of information, and the equally inevitable degree of behavoural indeterminancy as a characteristic of the largeness of scale plus complexity. (The line of reasoning is broadly this: at some point of largeness the collecting and processing of information becomes unfeasible, or the system reaches an intolerable degree of indeterminancy (see our section 8). Where thus the centralist mechanism breaks down the alternative is one in which the handling of information and the decision-making is carried out in several paralleling processes). The described characteristics and in addition "informal factors" of the organization

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27) Yu. B.Germeyer, <u>Igrovyie Reshenya v Issledovanyi Sistem</u>, 1974 (?)

28) I.Moiseyev, <u>Kibernetika</u>, no.6, 1973.

are dealt with in Moiseyev by a whole range of "unconventional" non-conflicting game designs: up to, and including, a multiperson differential game leading to optimal-control-theoretic solutions. A facet of the Moiseyev hierarchic game is the importance attached to the sequence of moves with precedence accorded to the higher-level players. Note that game-theoretic approach as understood here differs substantially from the "traditionalist" one based on the block-programming methods.

We should point out that "hierarchization" of a controlled system creates some dilemmas of its own. One concerns determining the pattern on which the hierarchic system should be ordered. An interesting study in this direction is that by Pavlovskiy.[29] His approach is based on the theory of groups: group properties of a controlled dynamic system and a phase-organizational structure. To this end groups of transformations of such a system are considered, and the groups are employed to explore aggregation - its"admissibility" and its modelling - and "natural" hierarchic ordering.

Further, in the class of differential game we draw attention to the Starr/Ho study[30] - for the great importance of both its general analysis and its explicit economic exemplification. One of its outstanding merits is the rigorous analysis of solutional potentialities, specifically those based on Nash equilibrium,

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29) Yu.N.Pavlovskiy, Zhurnal Vychislitelnoy Matematiki i Matematicheskoy Fiziki, no.4, 1974: also, the same, ibidem, no.6, 1971-

30) A.W.Starr, Y.C.Ho, Journal of Optimization Theory and Applications, nos.3 and 4, 1969.

minimax and non-inferior set of strategies. And from our angle of considerable interest is the Starr/Ho demonstration of features <u>absent</u> in the optimal differential-game theoretic approaches, specifically difficulties encountered in a transplant from control theory of what is prevalently believed to be the obvious ideas suitable for the purpose: those of the Optimality Principle and on the relation between open-loop and closed controls deserve particular attention.

Of the few algorithmically oriented constructs so far produced in this field I would mention the differential-game-theoretic model for coordination of conflicting controls by Gadziyev.[31)] The model is formed by interacting sub-systems: the controlled, the controlling, and the observation sub-systems, making in substance a two-level hierarchic planned mechanism. The controlled sub-system is described by the probability distribution of the output vector depending partly on controlled and partly on uncontrolled parameters; the values of the latter in general represent realization of components of some stochastic vector. One of the alternatives treated is that of a non-cooperative n-person games. Nash equilibrium strategies are pursued by the controlled units and they are next used by the command in formulating the extremal problem in the space of a "correcting vector". Specific cases considered are those with regressive control for Gaussian payoff functions and with Gaussian property of the controlled system.

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31) M.Yu. Gadzhiyev, <u>Avtomatika i Telemekhanika</u>, no.5, 1972.

We further draw attention to the Pau[32] report on the application of two algorithms for the numerical solution of differential games used in the formulation of a national economic plan; the algorithms are employed by the author for the search of the optimal Nash strategies in the treatment of a macro-plan for the Danish economy.

(7) Scale-complexity as related to "Decentralization".

To supplement what has been said hitherto on the scale-complexity issue, here a few observations on "decentralization " - a heading under which that issue is, as often as not, confusingly discussed in the context of economic planning. Whatever the terms employed they are meant in this particular context to relate to the largeness of "substantive" (i.e. other-than-time) dimensions; and to the intricacy phenomena and situations (both intrinsically interrelated, and, increasingly so under the growing operational and technological complexity of economies controlled. If anywhere, here is a case where the tenet on quantity turning into quality holds. Mathematically the problems of "decentralization" rest to a considerable extent on the theory of decomposition and its ramifications, (cf.Zauberman)[33].

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32) L.F.Pau, "Differential Games among Sectors in an Economy", in IFAC/IFORS, International Conference on Dynamic Modelling, 1973.

33) A.Zauberman, Aspects..., 1967; Mathematical Theory......,1967 (chapter 2); Mathematical Decentralization, (forthcoming).

Now the philosophy of optimal control as originally designed presupposes a unitary explicited steering in pursuance of a unitary system of preferences (only in the latter but not in the former property is it akin to the wellknown Radner-Marschak[34] concept of the "team" and this makes all the difference).

Such being the motivation it could not provide a decisive relief to the planner's notorious handicaps under our heading.

More specifically we may refer ourselves here to one of the key facets of the issue - that of aggregation of the plan (aggregation and disaggregation) in Theil's sense,[35] i.e. of behavioural relations, of preferences and of volumes (physical and numeraire terms). There has been a massive and almost continuous output of work - particularly algorithmic work - on the topic; only part of this is in debt to control theory as such. Some references have been made in the closely related context of multi-control, others will be given in the sequel; from a broader large-scale and control-theoretic angle we may point to the early admirable study by T.Marschak,[36] the writings of Aoki[37] (control

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[34] J.Marschak, R.Radner, Economic Theory of Teams, 1972.

[35] cf. H.Theil "Alternative approaches to the aggregation problem" in E.Nagel, D.Suppes, A.Tarski, Ed., Logic, Methodology and Philosophy of Science, 1962.

[36] Th. Marschak, Econometrica, 1959.

[37] M. Aoki, IEEE, Transactions on Automatic Control, AC-13/3, 1968, Operations Research, 18/3, 1970: and the same, "Elements of the large-scale mathematical programming", Management Science Institute, University of California, Los Angeles, Dec. 1968.

(control of large-scale dynamic systems by aggregation) and of Geoffrion (in a generalization of elements of the large-scale programming).[38] Severe handicaps in "exact" treatment have induced of late some attempts at an approximate one - by a stochastic approach. Also some help to the methodology of aggregation might be, with good reason, expected from progress in the theory of pattern recognition (see on their relationship Blin, Fu, Whinston, Moberg).[39]

What has been said calls, however, for some qualifications to the effect that optimal control theory (a) has helped the progress of the theory of the planner's parametric tools for indirect steering of the economy; (b) has led, in its logical advance, to the emergence of theories of multi-controlled systems; (c) has given - if not a direct conceptual or instrumental help - at least a broad inspiration to a variety of theories providing components to the discipline of large-scale systems (on this we touch passim; and specifically in the section on systemic extensions.

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[38] A.A.Geoffrion, "Elements of large-scale mathematical programming", Management Science Institute, University of California, Los Angeles, Dec.1968: also the same, Operations Research, 18/3, 1970.

[39] see J.M.Blin, K.S.Fu, A.B.Whinston, K.B.Moberg, Journal of Cybernetics, no.4, 1974

Excursus to sections 6 and 7.

By way of a background a few more general points on coping with largeness. The notion is far from crystallized in literature (nor is, consequently, its theory - cf. section 11 on its "discipline"). The disagreements go as far as stressing by at least a few students, that the concept of largeness is by no means antithetic to that of smallness. There is more agreement on the intractability of the large system with the border-line between largeness and complexity very blurred indeed. (We shall not worry about the definitions which are circular).

There are, in literature, various approaches in dealing with the phenomenon of largeness, broadly understood, of a controlled system. Some students rely - for the purpose - on =traditionalist" regulation (in particular Laplace transformations, frequency methods, and so on) and classical and post-classical (Maximum Principle, Optimality Principle) calculus.

The handicap is that generally "conventional" methods of control presuppose the knowledge of a quite often, large number of parameters and/or of outputs as a function of control signals. And prevalently such a priori information is unavailable.

The informational aspect apart, there is the no less troublesome one - of a computational barrier. As often as not treating the large-scale system in toto exceeds the capacity of the modern computer, and presumably will do so over the fore-seeable future.

Against this background there is the empirically supported intuitionistic (rather than abstract-theoretically argued) tenet that partitioning of the system into sub-systems is operationally helpful. It is usually assumed that there would be interaction between these sub-systems; and also - that signals directed to one of the sub-systems do influence the states of other sub-systems, coupled with it as well. One of the possible approaches to the problem confronted is to decouple the systems - as far as feasible.

Further, one of the approaches providing a framework which permits the formalization of the sub-systems' independence, with hierarchic structure of the system as a whole, is that of multilevel dynamic games, in particular as designed by Flerov.[40)]

Finally note that large-scale systems are a domain of certain striking analogies between the "physical" (e.g.biological) and "non-physical" worlds. Hence suggestions for handling, say, certain large-scale economic phenomena by resorting to analogy from what is known on the former rather than, or in supplementing, "conventional" theories. The possibilities offered in this direction to the theory of planning by the theory of automata are of particular interest (see remarks in our section 11).

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40) Yu. A. Flerov, in Journal of Cybernetics, no. 3, 1974.

(8) Indeterminancy. Planning with deterministic models, in a world inherently affected by uncertainties - a phenomenon characteristic of nearly a half-century of its history - is, in our submission - explainable by the poor "state of art". Inasmuch as control theory has tended to raise that state, it has also influenced the drive towards a more realistic approach. Significantly the last few years have seen the development in Soviet planning thought of the, undoubtedly fruitful idea of planning as an operation to be inexorably carried out under conditions of "indeterminancy" ("zone of indeterminancy").[41] The notion is wider than that of uncertainty as the latter is usually understood, for it very broadly embraces what we would describe as deficient informativeness, be this with respect to the retrodiction or prediction, or - also the present states. The broadness of the conception permits allowing for some features immanent in the planning mechanism, including a variety of types of errors more or less inevitably entailed. (To some extent one may discover a kinship between the notion discussed and that of fuzziness in the sense of Zadeh/Bellman,[42] i.e. that of imprecision, although the latter, unlike the former, tends to an emphasis on the

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[41] see on this N.G. Lavrov, Yu.G. Markov, in Ekonomika i Matematicheskiye Metody, no.5, 1975; also A.A. Markov, A.S. Makarova, A.N. Zeyliger, ibidem no.5, 1970.

[42] L.A. Zadeh, and R. Bellman, Management Science, no.4, 1970

contrast between it and randomness, as usually understood, (see Bling on the "Fuzzy Planner").[43] Or one could go further in this direction and establish the family relationship with the still more general theories based on the "logic of the inexact concepts" (as in Goguen).[44] I should think that the theory of indeterminancy in planning stands a chance to gain by borrowing some of the formalism of these theories.

To anticipate some points to be touched upon in the sequel, we may note the acceptance of the intuitive tenet that the control of virtually any object does not require a full and exact knowledge of all magnitudes determining its state at each time-points and of its kinetic equations, generally a full mathematical cognition; nor does it demand complete information on exogenous influences on, and signals directed to the object controlled. Expanding on this point would bring us to the area we have categorized as that of " **systemic** extensions" (section 11) - to the theories branching out of that of optimal control and resting on the idea of the on-line absorption of information, supplementing and/or correcting the minimally needed initial information: the self-learning and adaptive system supported by pattern recognition.[45]

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43) "Fuzzy Planner", B.Bling in <u>Journal of Cybernetics</u>, no.3, 1974.

44) A.Goguen, <u>Synthese</u>, 1968 pp 325 ff.

45) H.Tanaka, T.Okuda, K.Asai, <u>Journal of Cybernetics</u>, no.3, 1974

Here we shall limit ourselves to hints at several elements in the instrumentation of the non-deterministic planning in its relation to optimal control (possibly though not exactly in the order of their assimilability). Starting from some simple resorting to "certainty equivalents" in plan-programmes as introduced in the latter 1950's by Simon[46] and Theil[47] with the use of dynamic programming. (We may pause at this point to recall Bellman's claim that in the usual multistage dynamic-programming formulation of decision processes, the deterministic and probabilistic processes lend themselves to statement in the same conceptual, analytic and computational fashion;[48] this claim I am prepared to accept. Extensions and variants of the approaches indicated here have led in recent years to some noteworthy results in quantified treatment of problems of considerable size and complexity - with quite a few random variables. (For a new method resting on application of the "z-transform" cf. a recent paper by Hay and Holt.[49] It has also an interesting outline of progress and alternatives over the past two decades in the work of Bellman, Theil, Simon, Kalman, Muth, Holt). Next come the stochastic "translations" of the deterministic plan model. Recent work of Dynkin and his associates deserves

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46) H.A.Simon, Econometrica, 1956

47) H. Theil, Econometrica, 1957; the same, Economic Forecasts and Policy, 1970.

48) R.Bellman, in A.Prekopa, Ed., Applications of Mathematical Economics, Budapest, 1965.

49) G.A. Hay, C.C.Holt, Econometrica, 1975.

particular note. In Dynkin[50] we have what in substance (mathematical) is a dynamic-programming related model: it is built up of a sequence of functions possessed of "nice" properties which, as the adopted criteria function reaches its extremum, form the economy's optimal plan. Once the random element is introduced, with postulated known probability distribution, the objective becomes an expectation of the extremand; and the "embedded" dynamic planned price expectations too are arrived at for the economy. One of the purposes of Dynkin's modelling appears to be to demonstrate the degree of analogy from the familiar deterministic plan-programme elements of the corresponding probabilistic constructs. (Note in the context that the Dynkin modelling of optimal control is supplemented by his treatment of the problems of preference relationships,as between behaviour variants - under conditions of indeterminancy. Dynkin et assoc. have of late modified the familiar Arrow theorem[51] by formulating and proving the necessary and sufficient conditions for the existence of the entailed utility function and probability distribution.[52]

One may place next a plan model with explicit stochastic differential equations, with a measurable stochastic process prevalently postulated to be Brownian motion, that is zero mean

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50) E.B.Dynkin, <u>Doklady</u>, USSR Academy of Sciences, 1971 vol.200, no.3

51) K.J.Arrow, <u>Essays in the Theory of Risk Bearing</u>, 1971.

52) E.B.Dynkin, A.I. Ovsevevich, <u>Ekonomika i Matematicheskiye Metody</u>, no.2, 1975.

Gaussian with stationary independent increments. This class of constructs has a number of variants differing in their optimal-control mechanism and stochasticity and planning realism - one finds them well represented in Dreyfus[53] and in Tintner/Sengupta.[54] The alternatives include both open-loop and closed-loop and feedback designs, though prevalently the latter. (A bracket might be suitably opened here for a remark on the planning theorist's "bias" for Brownian motion and Gaussian processes. It is accounted for not only by their obvious relatively greater mathematical tractability but also the empirically observed fact that on the whole they do not violate too drastically the requirements of planning realism. Talking of the planning modeller's special favour we should include here the Markovian-world assumption, most often - that of Markovian chains. (Note here the kinship of this assumption with Optimality Principle)). It is motivated by the consideration that the assumption agrees with the architect's of a plan's actual approach. Incidentally the actual procedure of the plan designer and implementer accounts also for the Bayesian-type approach.

Realistically important in planning-model-design is one that aims at <u>approximation</u> to optimal control. Broadly speaking, in a qualitative sense such approximation deals with the convergence of sequences of random variables; its helpfulness is recognized particularly in problems of feedback control and structural estimation;

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53) S.E.Dreyfus, "Introduction to Stochastic Optimization and Control" in F.Karrenan, Ed., <u>Stochastic Optimization Control</u>, 1967-

54) G.Tintner, J.K.Sengupta, <u>Stochastic Economics</u>, 1972.

and also - filtering (see below).[55] Constructs, based on such approximation, acceptably suboptimal, have been effectively applied in modelling stability of a system's advance.[56]

A potential candidate for the application in planning is of course the fully-fledged stochastic optimal-control model, linear and non-linear. (Some of our references would be to Aoki, Aström, Bryson/Ho, Meditch, Charlotte Striebel, Bellman).[57] By this we mean the apparatus solutionally relying on optimal control - predominantly a version, continuous or discrete, of Bellmanian programming with a feedback - equipped with tractable mathematical instrumentation for filtering and prediction; at the present state of art that would be Kalman's celebrated device - presumably in the Kalman-Bucy[58] formalization(the antecedent Wiener-Kolmogorov-theoretic apparatus could suit the purpose, but - as it is known - there could arise difficulties in the solving of the integral Wiener-Hopf equation).

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[55] cf. P.R.Schultz, "Some Elements of Approximation Theory and its Application to a Control Problem" in C.Leondes, Ed., Modern Control System Theory.

[56] M.Athans, P.Falb, Optimal Control - An Introduction to the Theory and its Applications, 1966: and also E. Tse, M.Athans, on adaptive stochastic controls in Proceedings of the 1970 IEEE Symposium on Adaptive Processes, IEEE, 1970.

[57] see M.Aoki, Optimization of Stochastic Systems, 1967: Karl Aström, Introduction to Stochastic Control, 1970; A.E.Bryson, Y.C.Ho, Applied Optimal Control, 1969; Meditch, Stochastic Optimal Linear Estimation and Control, 1969; Charlotte Striebel, Optimal Control of Discrete Time Stochastic Systems, 1975; in related contexts R.Bellman, Adaptive Control Processes, 1961

[58] R.E.Kalman, R.S.Bucy, Journal of Basic Engineering (Transactions of ASME, Ser.D.), 1961; for the proliferating literature, see R.K.Mehra, "Credibility Theory and Kalman Filtering with Extensions", IIASA, 1975 and references therein.

There is in literature hardly any record of an actual planning-oriented model of the "fully-blown" kind in the sense just indicated. But one can point to a mathematically less exacting version - with a plausible claim to sufficient tractability. Our reference is to Kendrick.[59] His is a design for approximational techniques in stochastic control, using Chow's plan macroeconomic model where some variables are turned into a stochastic-Markovian form. The stochastic-control system with a quadratic performance-criterion has a two-stage solution (stage 1: random variables set to mean values; nonlinear problem solved; stage 2: first-order Taylor expansion of the equations about the optimal (deterministic) path carried out; the quadratic-linear perturbation solved with a matrix Riccati routine; stochastic feedback control secured).

At this point we digress to draw attention to a debate in literature - of considerable interest to the planning theorist - on the issue: how "economical" is the application of stochastic control at this stage of its theoretical development? Cogent reasoning pro and contra of its application are presented (not without qualifications) by, respectively, Chow and Pindyck.[60]

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[59] D.Kendrick, "Stochastic Control in Macroeconomic Models" in IFAC/IFORS, International Conference on Modelling and Control of National Economies, 1973.

[60] G.C.Chow, "How much could be gained by optimal stochastic control policy", Annals of Economic and Social Measurements, 1972, and R.S.Pindyck's counter-argument; see also Chow's paper on the effect of uncertainty on optimal control policies in International Economic Review, 1973; further, Chow's Analysis and Control of Dynamic Economic Systems, 1975, and Pindyck's Optimal Planning for Economic Stabilization, 1975.

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Finally, as a pendant to what was said on the role, actual and potential, of the differential game in our field (section 6) a word or two pro-memoria may be added here on its probabilistic version. The relevance of the advances in this discipline - still in its early gestation stage - is obvious; the reference in particular may be made to the writings of A.Friedman,[61] Basar/Ho,[62] and Varaiya[63] (in the latter's latest contribution, in addition to demonstration of some "curiosities" on sufficiency and necessity of conditions the interested student finds a pertinent indication of desirable lines of further research).

Modelling that is sufficiently matured for the purposes of planning theory can be found in particular in the area of the two-person pursuit-evasion game with a stochastic element. For the planner the evader would "embody" all the indeterminancy in his plan whatever its source; also elements that are non-measurable and/or unplannable with which he has to cope (the obvious references here are N.N.Krasovskiy,[64] Kolmogorov/Mishchenko/Pontryagin[65])

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[61] A.Friedman, Differential Games, 1971

[62] T.Basar, Y.C.Ho, Journal of Economic Theory, no.7, 1974

[63] P.Varaiya, "N-Player Stochastic Differential Games" (?)

[64] N.N. Krasovskiy, Igrovyie Zadachi o Vstreche Dvizheniy, 1970; the same, Teoriya Upravlenya Dvizenyem, 1968.

[65] E.F.Mischenko, Avtomatika i Telemekhanika, no.9, 1972.

as well as the Isaac's classic;[66] some general observations can be read in Zauberman.[67] [68]

(9) Simulative methods. For reasons of mainly a technical nature - which hardly need enlarging upon - the trend in plan modelling shows unmistakably a favour for simulative systems (one particular consideration is to secure a way for bypassing difficulties in more rigourized methods for evading or mitigating indeterminancy including that arising from complexity). As a matter of fact the relationship between optimal control theory and simulative procedures for some not over-sophisticated plan problems (specifically a typical linear-quadratic problem with Leontievian dynamic system and a Bellmanian-type extremand) had been examined by Livesey.[69] His broad suggestion would be that for complex problems rather than presenting a solution to the control problem, optimal control formalism provides the framework within which it can be stated; the numerical solutions would be reached by employing computer algorithms being in substance simulating tests in search of the adopted extremand. (The Livesey inquiry is related to Stone's Cambridge plan modelling).[70]

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66) R.Isaacs, Differential Games, 1965.

67) A. Zauberman, Differential Games......, 1975

68) see also T.Bewley, E.Molberg, "The Theory of Stochastic Games with Zero-stop Probabilities", Harvard University, Technical Report, January 1976; it carries very up-to-date bibliography of the subject.

69) D.A.Livesey, "Control Theory and Input-Output Analysis", International Journal of Systems Sciences, no.3, 1971; cf.also the same, Economic Journal, Sept.1971 (plan design for the U.K.)

70) J.R.N.Stone, A Programme for Growth, nos.5, 6, 9, respectively 1965, 1965, 1970.

Generally speaking the work in this field is by its nature empirically oriented. The purpose being what it is, its quality should be appraised by operational suitability. From our angle we refer ourselves at least to two studies of this class, the mathematical formalism of which is control-theoretically based. One is the outline of model simulation presented in the Swedish contribution to the December 1974 conference of the Economic Commission of Europe, United Nations.[71] Mathematically it rests on Pontryagin's formalism. Its economic substance is a search for a strategy for levelling out, over time, disparity in development rank between two parts of the country - as defined by some criteria. One of its attractions to the students is the ingenuity in expressing economic facts in the "language" of the Maximum Principle. It seems to be a good example of the real-life application of the Principle on a modest scale. As against this the other case of our reference is, to our knowledge, the most ambitious attempt as yet. This is the build-up by Dyukalov, Ivanov, Tokarev of an effective system,[72] closed or semi-closed, for simulating complexes of the "object+management" kind at the all-economy level. Here man's intervention is confined to elements least susceptible to algorithmization (such as changes in the measurement framework, in the planning schemata and in the feedback connections). Other elements are worked out with the help of computers: that comprises

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71) Examples of Model Simulation, Swedish Contribution to the Dec.1974 ECE, U.N.conference, No.ECE/EC AD7.

72) A.M.Dyukalov, Yu.N.Ivanov, V.V.Tokarev, Avtomatika i Telemekhanika no.12, 1973.

also matters of both the operational control and the actual plan construction. The work stands out for the admirably careful elaboration of the routines for plan-implementation in production, consumption and management. This is also true of the mathematical elaboration of the various particular components of the system; first of all - of its overall mathematical statement of the plan construction (by Dubovskiy/Ivanov/Dyukalow/Tokarev/Uzdemir/Fatkin.[73]) That has the format of optimal-control problem, for a semi-infinite time interval (reduced to finiteness), with time adopted as the minimand, with Leontievian dynamic balance equations (resting on a quadratic constant matrix of full capital intensity). A noteworthy feature of the plan-problem, as devised, are mixed constraints on both the controls and the phase coordinates; and among the constraints there is the "guaranteed" level of consumption. In the system the capacity-stock is the coordinate; the vectors of incremental "intensity" of capacities and of their underemployment - are the mixed controls (with a vector of an excess in terminal capacity over the desired magnitude as a control parameter). The use of the Maximum Principle is facilitated by the handling of boundedness of the controls for values of the phase coordinates at each time-point, as well as the assumed linearity for constraints and of the linearity of relationships between the phase coordinates. The test of optimality (sufficient as well as necessary conditions and those of transversality) is carried out in an original fashion over the relationships of the duals - duals that is of both the phase

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[73] cf. S.V.Dubovskiy, A.N.Dyakalow, Yu.N.Ivanov, V.V.Tokarev, A.P.Uzdemir Yu.M. Fatkin, Avtomatika i Telemekhanika, no.8, 1972.

coordinates and those of controls - "impulses".

(As it appears the model was experimented with, numerically, for a five-sectors economy by means of Birger-Charnyi iterative methods (related to that of Krylov/Chernousko)[74]: its computer implementation entails storing current phase coordinates with a file of Boolean variables).

In this school of thought economic planning is (or should be) mathematically concerned with a variational problem. A variational principle, and a noteworthy procedure have been offered by some members of the team: these rest on an analogy with variational methods known from mechanics. (These constructs have formalized in this direction the use of policy instruments - such as e,g, taxes - for steering the economic system as close as possible to some "ideal" path. The mathematical approach is found in the Hamiltonian and the Euler-Lagrange equations).

The scope, the scale, the quality of the design, the originality of some of the mathematical approaches, make the progress in the project discussed one of interest to students of assimilation of optimal control in our field. (We have in mind the progress in mathematization and in experimenting; in the former - probing into the possibility of getting rid of or relaxing some of the severe assumptions would be worth watching).

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74) E.S.Birger, V.I.Charnyi, Avtomatika i Telemekhanika, no. 12, 1973, And I.A. Krylov, F.L. Chernousko, Zhurnal Vychislitelnoy Matematiki i Fiziki, No. 1/1972
S.V. Dubovskiy, A.P. Uzdemir, Avtomatika i Telemekhanika, no. 6, 1974

A class by itself is formed by econometric exercises resorting to stochastic simulations. Here belongs the noteworthy Cooper-Fisher[75] work on the non-linear econometric model (financial policy design) with stochastic simulations.

Finally there is a kinship between the models of simulation and those of a man-machine dialogue. In fact in quite a few simulative constructs we see an incorporated dialogue of this kind. Typically this work may be taken to be represented by such scheme as that of the "Electre" (the abbreviation stands for "elimination et choix traduisant la realitê").[76] The evolutive-simulational constructs such as that by Likhtenshtein,[77] can be in a sense placed in this class. (A good analytical survey of the area of the man-machine in decision-making procedures will be found in Larichev).[78] Characteristically enough, having modelled his search for an answer to the question of degree of "decentralization" in terms of a hierarchic differential game, Moiseyev too discovers that an effective application of his ideas will depend in the last instance on the theorist's ability to design a suitable simulative system resting on a man-machine dialogue.[79] It seems pertinent here to draw attention to attempts at modelling man's behaviour in

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[75] J.P.Cooper, S.Fisher, Econometrica, 1975.

[76] B.Sussman/P.Buffet/J.P.Gremy/M.Marc in Metra, no.2, 1967.

[77] V.E.Likhtenshtein, Ekonomika i Matematicheskiye Metody, no.6, 1971.

[78] O.I.Larichev, Avtomatika i Telemekhanika, no.12, 1971. See also A.Siegel/J.J.Wolf, Man-machine Simulation Models, 1969.

[79] Moiseyev, Kibernetika, no.6, 1973, op.cit.

decision-making in controlling an economic system: in information processing and control proper. Of approaches tried one will single out the imaginative study by David Kleinman (its starting base is the analysis of man-vehicle systems).[80] In studying human decision-making the characteristic elements are borrowed from Kalman's filter-predictor theory: an "internal model" is being built up.

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[80] David L. Kleinman, in NBER 2nd Stochastic Control Conference, Annals of Economic and Social Measurement, 1974.

Supplementary remarks to sections 8 and 9.

The Cooper-Fischer studies are one of the very scarce cases of algorithmically oriented control-theoretic plan modelling where both nonlinearities and randomness of economic reality are handled.[81)]

The substance of the methods resides in estimating the mean and the variances of the dynamic responses of the nonlinear system to adjustments of the policy-instrument variables; in this way a linear representation of the model with random coefficients is obtained; then such linear stochastic version is handled in optimization. The procedure is designed in five steps viz. (1) nominal path for control instruments are selected (possibly through solution of a deterministic control problem, (2) a linear stochastic "underlying" model is treated; there the stochastic disturbances (of the instrument variables) around the nominal path are taken to have the same variance as the historical one, (3) the information is summarized in linear, random-coefficients regressions (formed of autoregressive-moving average models: these represent the target variables as functions of (lagged) instruments and target variables) (4) quadratic loss function is employed to secure for optimal feedback, control rules (for linearized version of the model represented by linear regressions); (5) - a check is carried out for the rules in the original design.

[81)] P.Cooper, S.Fischer, op.cit. and the same in NBER 2nd Stochastic Control Conference, op.cit.

Even this brief outline bears out some of the differences between the Cooper-Fischer construct and the so far "established" line (in the latter we have calculations of optimal path for the nominal deterministic path with parameters and error terms set at their mean; then the model is linearized at the nominal path; then the linear version is used for deriving optimal feedback controls in terms of deviations of variables from that path).

The Cooper-Fischer design has been made use of in the treatment of the monetary and fiscal policies in the St.Louis econometric model. It is claimed that the model has proved equally effective in treating large and small systems.

We may place on record also the Holbrook method[82] for the control of a large nonlinear stochastic system. The emphasis here is on simplicity, low cost and practicability in coping with these three handicaps. No exact solution is here expected but accuracy contended to be obtainable at a cost (reducing convergence criteria and size of perturbations). Among several interesting mathematical-technical facets of the study is the way the Newton-Raphson procedure is applied by the author. The model which is nonlinear is approximated by a linear function and the second derivatives are ignored.

As against Holbrook's emphasis on the immediate use in practice we find as the objective in Athan's modelling of the discrete, linear-quadratic-Gaussian stochastic control problem,

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82) R.S.Holbrook, in NBER 2nd Stochastic Control Conference, op.cit. 1974. M.Athans, in Annals of Economic and Social Measurement, no.4, 1972.

the offer of a unified philosophy of the design: (the focus is, however, on the non-engineering aspects of the theory because, as the authors defensibly argue, there are inherent differences in the approach to control problems by an engineer and by a policy-making economist, in particular with regard to on-line (real time) feedback control of an uncertain, usually nonlinear, physical process).

In pursuance of the philosophy we have here a design with a deterministic optimal control and perturbation control, and - a stochastic state estimation and linearized stochastic control; that leads to an overall closed-loop control system.

The exercise has a solution of the adopted deterministic linear quadratic model; then the deterministic design is analyzed from the angle of uncertainty and sensor constraints; thence it proceeds to the issue of estimating the state variables and the uncertain physical process - on the basis of past measurements via the Kalman-Bucy theory; then the stochastic estimator and deterministic controller are "hooked" together to secure the compensator - the "translator" of actual sensor measurements to commanded control inputs. As interestingly observed the (N) Kalman filters are used to generate firstly the probability that each econometric model is the correct one, and secondly - a weighted probabilistic average for the parameters of the econometric model to be used by the central agency, based upon the individual parameter estimates of each of the econometric models. (The help from the Kalman filtering method in the economist's problem was not fully appreciated until not long ago; one of the causes was that where measurements are corrupted by white noise that method entails estimation of state variables. But of late it has been persuasively shown that the Kalman algorithm can be an effective aid in economic problems which involve refining parameter estimates and their variances).[83]

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[83] M.Athans, NBER Stochastic Control Symposium, 1974.

(10) Advance in control theory. Until this point the remarks (though not all) have been made as it were under a tacit status quo assumption: the assumption, that is, that the theory of optimal control, absorbable into planning theory, has reached the stage of its definitiveness and cannot be expected to change materially. In fact the sense of such definitiveness quite characteristic of the initial stage of its assimilation into the body of the planning theory has been by now refuted. This is largely connected with the growing awareness of the limitations of the post-classical control-theoretic apparatus and the trend towards a generalized theory ("schema") of dynamic optimality (a matter discussed by me at some length in a paper "Beyond the Maximum Principle"[84] and my book Differential Games......). One of the approaches here rests on the conception of embedding the optimal control problem in an abstract problem of extremization of a functional on subsets of the topological space; thence general sets of a relative extremum are sought. Truly seminal in this field have been the impressive parallel - and mutually influencing themselves, contributions in the East and West: - by Halkin and Neustadt,[85] also Polak,[86] and associates and followers - in the West. (Halkin and Neustadt arrived, by the end of the 1960's,

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[84] in Jahrbuch der Wirtschaft Ostenropas, München, 1976.

[85] L.W.Neustadt, Journal of Mathematical Analysis and Applications, 1960; the same, SIAM, Journal Control, 1966 and 1967. H.Halkin, Journal of Analytical Mathematics, 1963; the same, Bulletin of American Mathematical Society, 1967; the same, Journal of Mathematical Analysis and Applications.

[86] A.F.Polak, in G.B.Dantzig, A.F.Veinott, Eds.,Mathematics of Decision Sciences, vol.2; also M.D.Canon, C.D.Callum, E.Polak, Optimal Control and Mathematical Programs, 1970.

at an abstract framework for the theory of optimization: a general theory, supported by applicational investigation); and in the East by Dubovitskiy and Milyutin,[87)] Pshenichnyi,[88)] and followers.

The advances have been observed not without reservations as to some of their symptoms. It has been noted in particular that in certain respects it means a movement backwards: towards the position of the classical calculus (see in particular the illuminating critical surveys by Gabasov and Kirillova)[89)] I am inclined to think that what we are witnessing is rather, as it were a"dialectical" advance. And am prepared to accept appraisal by Milyutin[90)] of the advances in the field, in his recent stock-taking paper, to the effect that the general schemes in fact change with every newly posed problem of optimal control. This is taken to be not accidental: considered to relate to the basic question in the inquiry which is the effective application. The work is indeed far from completion (Milyutin singles out, as waiting for solutions, such areas as that of the minimax, of the multivariable functions, and of the higher-orders conditions).

[87)] A.Ya.Dubovitskiy, A.A.Milyutin, in Doklady, USSR Academy of Sciences, vol.149, no.4, 1963: and the same Avtomatika i Telemekhanika, no.12, 1963; also Doklady, vol.180, no.6, 1969.

[88)] B.N.Pshenichnyi, Neobkhodimyie Usloviya Ekstreumuma, 1969.

[89)] Gabasov and Kirillova, Avtomatika i Telemekhanika, no.2, 1972.

[90)] A.A.Milyutin, Uspyekhi Matematicheskikh Nauk, no.5, 1970.

Contemporary advances in the field rely to a considerable extent on a functional analysis. A major contribution in this has been made by Gabasov and Kirillova.[91]

Gabasov/Kirillova take credit too for an admirable exploration of the area where the Maximum Principle type of test breaks down; and where therefore this test is shifting to the much trickier zone of higher-order conditions. Also they should be credited with the illuminating critical surveying of and contributing to the fields of singular controls;[92] an important contribution has been made also by D.H.Jacobson associates and followers.[93]

Surely since the revelations in the mathematics of cosmic navigation nobody applying optimal control in planning can take the "irregularities" of optimal control to be freakish oddities not meriting intensive study (as was habitual not long ago). All then that this section conveys is the feeling that we may be still only at the threshold in the era of a mathematical optimal-steering theory: that of optimal plan-steering of an economy in particular.

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[91] Gabasov, Kirillova, see references on p.2 and particularly Journal SIAM Control, 1967 and Journal of Optimization Theory and Applications, no.2, 1971.

[92] cf. analytical tour-d'horizon of the area of higher-order conditions for optimality and of singular controls, in R.Gabasov, F.M.Kirillova, V.A.Srochko, N.V.Tarasenko, in Avtomatika i Telemekhanika, nos.3 and 5, 1971.

[93] D.H.Jacobson, J.L.Speyer, Journal of Mathematical Analysis and Applications; inter al. cf. also in the context E.J.Messerli and E.Polak, SIAM, Journal of Control, no.2, 1969

Remark to section (10) (motivated by the 1976 JET Symposium[94] with which I became acquainted after the present Note was completed). The Hamiltonian, borrowed from equations of motion in mechanics, had penetrated economic thinking a decade ago via Pontryagin's Maximum Principle to become a basic control-theoretic concept (and thereby in a "natural" fashion planning-theoretic concept.) The Symposium, and particularly its central paper - by Cass and Shell[95] - reflect a new stand: viewing the "Hamiltonian Dynamic System" as a competitive system in the sense that it derives from the perfect-foresight, zero-profit, asset-market clearing equations which arise in a descriptive growth theory (consistent with the Malinvaud "tradition" in efficiency pricing and the Euler and Pontryagin conditions). Such a novel idea of "Hamiltonian economics", crystallizing around 1975, would seem to suggest a severing of the hitherto intimate tie, in contemporary economics, between the Hamiltonian and the steering of an economic system. [The control term, the key element in Pontryagin, fades out in the Cass-Shell formalism. One could say that the latter generalizes that of the Pontryagin with the control vector, u (t)=0]. This may be so, but even then by-products of progress, in what is now called "Hamiltonian economics", are still likely to benefit control theory; and - to this extent, the theory of planning. Indeed such expectations are corroborated by certain results yielded by the

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94) cf. Journal of Economic Theory, no.1, 1976

95) D.Cass, K.Shell, "Introduction to Hamiltonian Dynamics in Economics", ibidem. A kin to approach is that by F.G. Adams and E. Burmeister, in JEEE Transactions on Systems. Man, and Cybernetics, January 1973.

Symposium itself, of crucial relevance for the planning theory. I would mention here some sufficient as well as necessary optimality conditions established; and first of all, findings on the system's optimal stability. [Thus Cass/Shell and Rockafellar[96] make heavy use of monotonicity properties of the Lyapunov function (potentialities of Poincaré's overall stability theorems are also indicated). And Brock/Scheinkmann are highlighting a general Lyapunov method as especially useful in the stability analysis of optimal path generated by optimal control problems in capital theory.[97] This agrees with the trend in thinking I have indicated (in section 4).]

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96) D.Cass, K.Shell, op.cit. ; R.T.Rockafellar, "A Growth Property in Concave-Convex Hamiltonian Systems", ibidem.

97) W.A.Brock, J.A.Scheinkman, "Global Asymptotic Stability of Optimal Control Systems with Applications to the Theory of Economic Growth, ibidem.

(11) Systemic Extensions. Whereas the preceding section dealt with advances "beyond Maximum Principle" (of actual or potential relevance for planning theory), but still within the "established" formalism of optimal control, this one is dealing with those outside that formalism itself. Very tentatively then in a preliminary fashion - for lack of a better one - we give them a portmanteau heading, "systemic extensions": a common heading, although they make a rather heterogenous group. Heterogenous as they are, they do belong to the area of general systems theory and specifically theory of large-scale systems. We note the doubts expressed by some students[98] as to whether one can expect that the handling of the large-scale problem will ever acquire a single consolidated language and mathematical theory. Now, admittedly some of the work which thematically does belong here is framed in the more or less "established" formalisms. For this good examples are some well-known general-system-theoretic contributions by Mesarovic and associates concentrating on the problems of aggregation;[99] also the less known (but deserving of recognition)- by McFadden[100] on the controllability of "decentralized" macroeconomic systems (with the persuasive reference to the Fisher-Fuller theorems on matrix stabilization and convergence of linear iterative processes).

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98) N.N.Krasovskiy, N.N.Moisievev, Tekhnicheskaya Kibernetika, no.5, 1967.

99) M.D.Mesarovic, D.Macko, Y.Takahara, Theory of Hierarchical Multilevel Systems, 1970; also the same "Mathematical Theory of General Systems and some Economic Problems", in H.W.Kuhn, G.P.Szegö Mathematical Systems Theory and Economics, I, 1969.

100) D.McFadden, "On the Controllability of Decentralized Macroeconomic Systems" in Kuhn and Szegö, op.cit.

It seems that, if anything, it is the specific mathematical language that increasingly integrates this domain of inquiry. Indeed for most of the work under the present title it is the more "modern" algebraic methodology that is jointly characteristic. Central to it is the theory of the automaton (or machine) with semi-groups. Now the automata theory - at the first glance - does appear somewhat alien to the matter of control, optimal or otherwise. However, some students have been convincing in their contention that familiar concepts basic to the latter do lend themselves to an intuitive attractive formulation of the former. Arbib in particular[101) has done a good deal for what he describes as the "rapprochment" of the automata theory with control theory. Indeed by now control-theory has the benefit of a reformulation in the automata-theoretic terms. We have then in these terms what in fact is an insight-deepening generalization of the "traditional" (time optimal) statement for the central optimal-control problem: one of the transfers of the automaton - possibly finite - from the initial-state point to within the "tolerance" of the reachable terminal state, such as to secure the minimum of the cost function (the tolerance automaton is the familiar "trick" to secure continuity).

What we have said about the "translation" of the theory's central problem is true of that of its other key elements. We have in mind here specifically what, since Kalman's celebrated findings at the start of the 1960's, has been recognized as the effective

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[101)] thus see M.A.Arbib in Automatica, no.3, 1966. What follows relies largely on his contribution to R.E.Kalman, P.L.Falb, A.A.Arbib, Topics in Mathematical System Theory, 1969.

touchstone of the system's operational abilities: the theory of controllability and observability(and, for that matter, of constructibility) has a "natural" generalization in automata-theoretic conceptual frame.

Similarly, with the use of the notions of automata and semigroups, a generalization of the system's structure and decomposition theory has been elegantly secured. In the automaton discourse the issue posed is that of decomposing an automaton into a series, and of parallelly composing simpler machines. When a semigroup is associated with each machine the discovery is made along this path that a machine is always decomposable into simpler machines, except when it is a "flip-flop", or has a simple group for its semigreup ("flip-flop" to recall, is broadly a two-state identity-reset machine).

Our last remark will point to the development owed to Tsetlin's celebrated initiative: of automaton-collectives. By now a good deal has been achieved in the study of the collective behavour of autonomous objects with explicited local interests.[102)] The reader will notice that we have limited ourselves here to matters of obvious interest to planning theory.[103)]

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102) cf.section(11); for examples of collective behaviour of automata see a contribution by V.Varshavskiy - on models of such behaviour - in P.Suppes, L.Henkin, A.Joja, G.C.Moisil, Eds., Logic, Methodology and Philosophy of Sciences, IV, 1973.

103) the aspect is discussed at certain length in my paper on planning under indeterminancy, expected to appear before long.

Among other lines of progress in the theorv of automata we single out those which focus on randomness. Some notable advances have been made in this domain over the past decade and a half-since Tsetlin first initiated the study[104] in the behaviour of finite automata in environment randomlv reacting to their actions (with the use of the (by now familiar) concept of "expediency".

We shall just allude to a few contributions of interest from our angle, supplementing what has been said here before on the coping with incomplete information. Potentialities of a large class of automata in random search processes have been investigated by several students. Thus Pozniak[105] examined the case of incomplete information where the criterial function can represent a certain average with distribution unknown. This is but one of the ways for solving the problem of stochastic optimization by means of random search. Our particular angle being what it is, we are interested here in the role of stochastic automata theory for that of adaptive systems (the concept of such a system is far from being generally agreed upon; we may accept a definition which requires a system to be equipped for monitoring the state and performance variables with the view to adjusting the controls). More particularly this applies to learning systems. Algorithmically the latter may be conceived of as combining the work of pattern recognition, identification and filtering as well as that of control.

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104) M.L.Tsetlin, Avtomatika i Telemekhanika, no.10, 1961

105) A.S.Pozniak, Avtomatika i Telemekhanika, no.12, 1972

Attention may be drawn in this context to the work of Fu and his associates on stochastic automata: inquiry into the behaviour of automata as models of adaptive and learning controllers and their application in the synthesis of learning systems. (Also his overview and assessment of prospects in learning control systems).[106) 107) 108)]

The theory of complexity is yet another major beneficiary in the developments discussed in this section. It has by now a considerable bibliography.

For our purpose the choice of reference is the crucial contribution to literature of the subject by Gottinger:[109)] we subscribe to its guiding premiss to the effect that what is waiting for elaboration is the mathematical theory of complexity seen as an intrinsic property of a dynamic system: a theory formalized in such a way that agrees with the intuitive essence of the notion and captures its empirically detectable phenomena; our"bias" would naturally tend towards its "phenomenology" related to optimally controlled, specifically planned economic systems and this "bias" is satisfied in Gottinger's treatment of the matter. (One will

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106) Y.T.Chien, K.S.Fu (on Bayesian learning and stochastic approximation), IEEE Transactions - on Sciences and Cybernetics,no.1,1967. K.S.Fu (on stochastic automata as modes of learning systems) in J.T.Tou, Ed., Computer and Information Sciences, 1967; cf.also K.S.Fu, Sequential Methods in Pattern Recognition and Machine Learning, 1968.

107) see K.S.Fu, in IEEE Transactions, Automatic Control, no.2, 1973.

108) for an analysis of the relationship between optimality and adaptivity of control see A.A.Krasovskiy, Journal of Cybernetics, no.3, 1974: cf. also on learning algorithms Ya.Z.Tsypkin, Journal of Cybernetics, no.2, 1973.

find in Gottinger a distinction between the two facets of complexity - that of design and of control complexity, and an analysis of their relationship of relevance for a social system's dynamics: stability, adaptation, and other properties). More fundamentally we also accept Gottinger's philosophy of the subject, especially so with respect to its status vis-à-vis the theory of probability: while the latter is conceivably a measure of uncertainty in particular situations, complexity would be rather related to the measure of understanding of the elements of a system's behaviour. Such being the adopted position one notes Gottinger's pioneering elaboration of a fully-fledged algebraic theory of complexity in dynamic systems. Parenthetically, our reference presents also a generalized model for simulation of the "action potential" with postulated quantitative accuracy - one of direct pertinence for stability theory: that potential being inherently complex, independent of the finite-state machine by which it is simulated. [Note in our context Gottinger's observation to the effect that whereas it is largely the continuous-time, dynamic system postulating what one could call "nice" properties of the relevant relations that much of control theory is concerned with, it is the "computational" techniques, in the sense of embracing dynamic progrmmming, that have a closer link with a system's algebraization; algebraic techniques - operating in abstract group theory, homological algebra, category

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109) H.W.Gottinger, "Toward an Algebraic Theory of Complexity in Dynamic Systems" (circulated draft, May 1976) and literature cited therein: see also the same "Complexity and Catastrophe - Applications of Dynamic System Theory", Western Management Science Institute, University of California, Los Angeles, working paper 246, 1976.

theory - are essentially employed in discrete-time dynamic systems. Computational potentialities apart, such operations as decomposition and partition are intuitively related to some of the familiar key-issues of the control and planning theories.]

[By way of a footnote we take note of the authoritative appercu - by Krasovskiy, Gavrilov, Lyetov, Pugachev[110] of recent advances and present-day work by leading Soviet research bodies in the area indicated, specifically in the various directions of the theory of learning systems and their application. We understand that a set of effective recursive algorithms has been evolved for learning with pattern recognition; these are said to include algorithms based on methods of stochastic approximation, of potential functions, and also on approximation to optimal statistical algorithms and sequential statistical analysis. Further - still with reference to the source quoted - we note that the fundamentals of a general statistical theory of optimization of learning system has been elaborated; that methods have been designed for optimal learning based on the theory of statistical solution and theory of groups; that start has been made with bringing into the theory of learning systems - a "real" teacher; one that would implement the given process with the "ideal" correctness. (All this apparently passed a phase of computer modelling). However, the authoritative contributors make it clear that in spite of the theoretical successes in the domain of recognition and classification, the application of learning systems in control is only in its initial state; that indeed "so far even the main areas of purposeful application of such systems have not revealed themselves"(op.cit.,p.25)

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110) N.N.Krasovskiy, M.A.Gavrilov, A.M.Lyetov, V.S.O.Pugachev in Vestnik Akademii Nauk, no.8, 1970.

As it appears, modelling cognitive processes apart, one of the tasks adopted for the nearest future is the formulation of "concrete" principles for construction and for application of the findings in problems of controlling the economy. Students of assimilation of new ideas and methods in planning theory will follow the implementation of the task with understandable interest.

III.

Marginal Remarks. One or two remarks will be made at the margin of this Note on the relative pace of assimilation discussed here in planning theory and in planning.

An observer will be inclined to think that the penetration into the planning theory of optimal control approaches, its methods and techniques, has exercised a certain impact on planning. This has been confined, however, largely to the less sophisticated elements. Some of the most difficult areas of planning - such as those dealing with the vast dimensions or indeterminancy - have received relatively rather less relief as yet. On the whole it is the very-long-range strategic planning that is the prime beneficiary. Current planning shows itself rather less immediately "penetrable" by formal "novelties". Very broadly it does seem tenable that the gap between the levels of formalization of planning theory and planning tends to widen. Consequently also the time-lag between the absorption of the new formalism by planning theory and by actual planning work tends observably to grow. To be sure this is true of what had been happening in the pre-optimal control phase of the formalism, but it has been so *a fortiori* since optimal control theory has offered itself to planning as one extending the potentialities of effective optimization: and again it is still more so since the theory of optimization has moved into the phase which we have described as that "beyond the Maximum Principle". One hardly needs to stress that the rapid progress in the several *ramifications* (mentioned here) of control theory - ramifications working on the plane of a growingly exacting abstraction - accentuates the disparities of levels.

As could be only expected, the lag and the gap give rise to two views. One would maintain that as they deepen and widen they may result in an insulation of planning from theoretical achievements in control theory, its ramifications and extensions. The other view argues that the growing lag and gap but testify to the remarkable intellectual vigour of the theoretical thinking in the field; and that ultimately it is bound to be beneficial to the process of assimilation with which we are concerned. There is hardly a way of rigorous reasoning in favour of one view mentioned as against the other. This writer is inclined to accept the second school's stand; and in this he is prepared to invoke analogy from the history of advance in other fields of science - natural or social; once such stand is taken the developments discussed here may be believed to carry a promise as well as a challenge.