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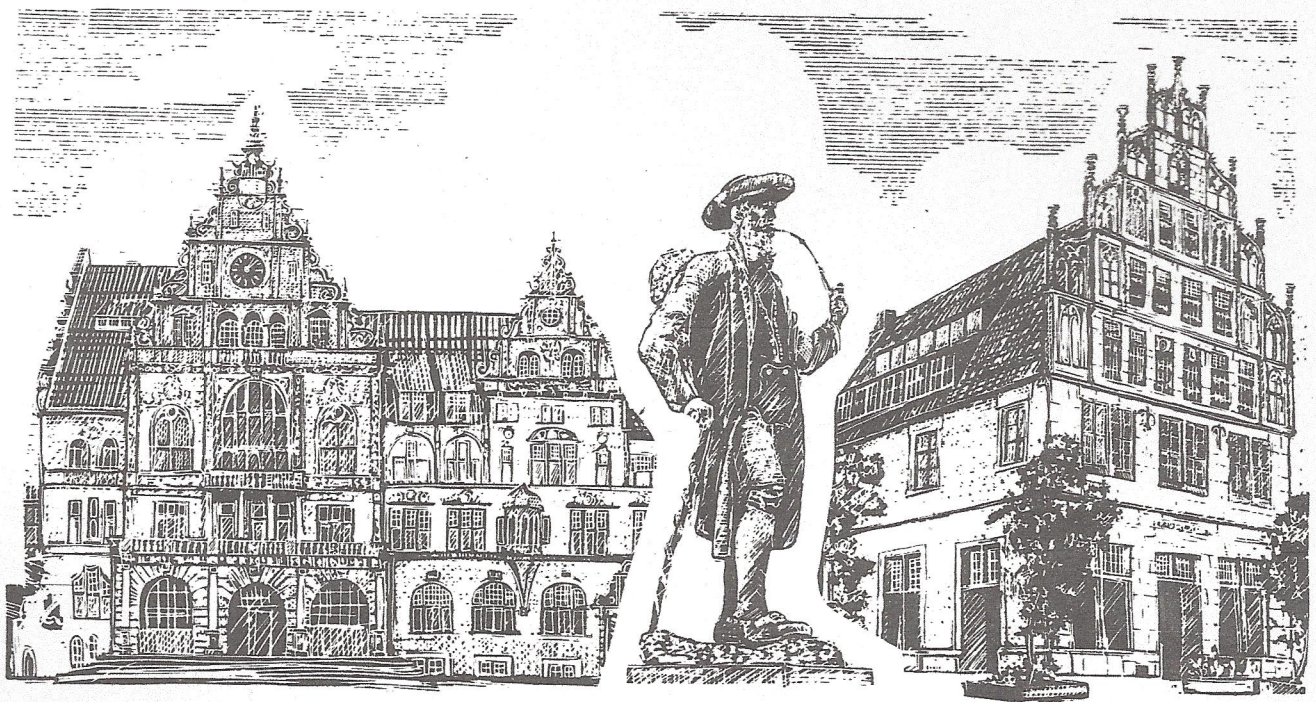## Institute of Mathematical Economics

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A Simple Game Model of Kidnapping

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A simple game model of kidnapping

by Reinhard Selten [*]

Hostage taking situations such as kidnapping a rich person in order to extort ransom money undoubtedly have some game theoretical aspects. In the following a very simple game model will be developed which cannot claim to be more than a first attempt to gain some insight into the strategic problems faced by a kidnapper - he will be called player K - and by the hostage's family, called player F who has to pay the ransom money.

The two-person game between K and F begins with a choice of player K who has to decide whether he wants to go ahead with his plan or not. This choice is modelled by a binary decision variable b:

$$(1) \qquad b = \begin{cases} 0 & \text{Kidnapping does not take place} \\ 1 & \text{Kidnapping takes place} \end{cases}$$

The game ends if K selects b=0. If he selects b=1, he kidnaps the hostage and takes him to a hidden place unknown to player F and to the police. He then announces a ransom money demand D.

At this point it becomes necessary to look at the negotiation process between K and F which results if F is willing to pay but wants to reduce the amount. We are going to model this negotiation process in the simplest possible way: Player F

makes an offer C, the amount he is willing to pay. Then player K either decides to accept C and to release the hostage or he kills the hostage.

This very simple description of the negotiation process should not be taken literally. Actually there may be some bargaining involving the reduction of initial demands and the increase of initial offers but eventually player K will take a firm stand and ultimatively demand D and player F will then have to make a final offer C.

Why should player K ever decide to execute his threat to kill the hostage? He cannot improve his situation by doing so. We can safely assume that he does not like the idea of killing. Nevertheless, his threat has some credibility. One must fear that under the strain of emotional pressure the kidnapper may react violently to an unsatisfactory offer in spite of the fact that this is against his long run interests. Therefore, we must expect that with a positive probability α the kidnapper will perceive an offer C < D as an aggressive act and a strong frustration to which he will react violently by the execution of his threat.[1]

It is reasonable to suppose that the probability α will depend on how high C is in relation to D. The danger will be greatest for C=0 and it will be virtually non-existent for C=D. In order to keep the analysis simple, we assume that α can be described by a linear function of C/D :

(2) $\qquad \alpha = a(1 - \dfrac{C}{D}) \quad$ for $0 \le C \le D$

where a is a constant with

(3) $\qquad 0 < a < 1$

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[1] This assumption conforms to the well known frustration aggression hypothesis [2]. For our purposes it is not important whether an aggressive reaction to frustation is a learned response or not and whether aggression is a necessary consequence of frustration or not.

If non-rational emotional pressures do not result in the
execution of the threat, player K still can make a rational
decision to execute his threat. This possibility is formally
modelled by a binary decision variable e:

$$(4) \qquad e = \begin{cases} 0 \text{ release of hostage for ransom C} \\ 1 \text{ execution of threat} \end{cases}$$

The analysis of the model will confirm our informal argument
that it is never rational for player K to choose $e = 1$.

After the release of the hostage or the execution of the
threat, the police will try to find the kidnapper and to
capture him. It is assumed that this attempt will be success-
ful with probability q, where

$$(5) \qquad 0 < q < 1$$

One might consider the possibility that the probability of
detection q depends on whether the hostage has been killed
or not; this will not be done here.

The players must attach utility values to the possible out-
comes of the game: These payoffs are described by figure 1.
The numbers w, x, y and z are positive constants. Several
simplifying assumptions are implied by the table in figure 1.

First, utilities of K and F are assumed to be linear in money.
Obviously, this is unlikely to be strictly true but in the
framework of this very simple model it seems to be inadequate
to burden the analysis with more complicated functional forms.

Second, several factors which may influence the players' utili-
ties have been neglected, namely player K's cost of preparing
the kidnapping and player F's non-monetary disutilities other
than those incurred by the hostage's life. Thus, player F does
not attach any value to the capture of the kidnapper.

Third, we assume that in the case where the kidnapper is caught after the release of the hostage, the ransom money is recovered and given back to F. Therefore, the utilities for this case do not depend on C.

Outcome	Payoffs	
	K	F
Kidnapping does not take place	O	O
Release of hostage for ransom payment C kidnapper not caught	C	-C
Kidnapper caught after release of hostage	-x	O
Kidnapper not caught after execution of threat	-y	-w
Kidnapper caught after execution of threat	-z	-w

Figure 1: Payoffs

The kidnapper's disutility of being caught can be expected to be increased by the execution of the threat. Therefore we assume:

$$(6) \qquad z \geq x$$

Formally the model is an extensive game with perfect information. At every point in the course of a play both players know the complete previous history. A short description of the game, where the decisions are listed in the sequential time order of their occurrence, is given in the following summary of the rules.

Rules

1. Player K chooses between b = O and b = 1. If he selects b = O, the game ends and both players receive payoffs O.

2. If player K selects b = 1, he has to announce a demand D > O.

3. After player K has announced D player F must make an offer $O \leq C \leq D$.

4. After the offer C has been made, a random choice decides whether a non-rational execution of player K's threat occurs or not. The probability α of a non-rational execution of player K's threat is given by (2).

5. If a non-rational execution of the threat does not occur, player K chooses between e = O and e = 1. If he selects e = O, the ransom C is paid and the hostage is released. If he selects e = 1, he (rationally) executes his threat.

6. After the release of the hostage or the execution of the threat a final random choice decides whether the kidnapper is captured or not. The probability of capture is q. After this random choice the game ends with payoffs according to figure 1.

Solution concept: The game is played non-cooperatively. It is natural to analyse the game with the help of the concept of a perfect equilibrium point in pure strategies. For the purposes of this paper, it is sufficient to define a perfect equilibrium point as a strategy combination with the property that not only in the game as a whole but also in every subgame no player can improve his payoff by a deviation from his equilibrium strategy if he expects the other players to stick to their equilibrium strategies. [2]

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[2] This is the original definition of a perfect equilibrium point, first proposed in [4] and generalized to behavior strategies in [5]. The refined concept of [6] is not considered here.

As we shall see the game of this paper generally has a unique-
ly determined perfect equilibrium point which can be found
by analysing the game from behind in the well known dynamic
programming fashion. The choices prescribed by the perfect
equilibrium point will be called "optimal".

The optimal choice of e: We first look at the subgames
which begin with player K's choice of e. Let V_0 be his ex-
pected payoff if he selects e = 0 and let V_1 be his expected
payoff if he selects e = 1.   These expectations are computed
as follows:

(7) $V_0 = (1-q)C - qx$

(8) $V_1 = -(1-q)y - qz$

In view of $C \geq 0$, $y > 0$ and $z \geq x$ and $0 < q < 1$ we always
have

(9) $V_0 > V_1$

This shows that e = 0 is the optimal choice of e. Player K
will never rationally decide to execute his threat.

The optimal choice of C: In the subgame which begins with
player F's choice of C, player F knows that player K will
choose e = 0. Under this condition the expected value of his
utility is as follows:

(10) $U = -(1-\alpha)(1-q)C - \alpha w$

With the help of (2), this yields:

(11) $U = -a(1-q)\frac{C^2}{D} + (\frac{aw}{D} - (1-a)(1-q))C - aw$

Equation (11) shows that U is a strictly concave quadratic
function of C. In order to determine the optimal value $\bar{C}$ of C
we compute $\partial U / \partial C$.

(12)
$$\frac{\partial U}{\partial C} = -2a(1-q)\frac{C}{D} + \frac{aw}{D} - (1-a)(1-q)$$

Equation (12) shows that U assumes its maximum at

(13)
$$C = \frac{w}{2(1-q)} - \frac{1-a}{2a} D$$

if this value of C is in the interval $0 \le C \le D$. This is the case if D is in the closed interval between the following criti-cal values D_1 and D_2.

(14)
$$D_1 = \frac{a}{1+a} \cdot \frac{w}{1-q}$$

(15)
$$D_2 = \frac{a}{1-a} \cdot \frac{w}{1-q}$$

For $D < D_1$ the derivative $\partial U/\partial C$ is positive in the whole interval $0 \le C \le D$. Similarily $\partial U/\partial C$ is always negative in this interval for $D > D_2$. Therefore the optimal offer $\bar{C}$ is given by (16):

(16)
$$\bar{C} = \begin{cases} D & \text{for } 0 < D \le D_1 \\[2ex] \dfrac{w}{2(1-q)} - \dfrac{1-a}{2a} D & \text{for } D_1 \le D \le D_2 \\[2ex] 0 & \text{for } D \ge D_2 \end{cases}$$

Note that with increasing D, the optimal offer $\bar{C}$ first increases up to $D = D_1$ and then decreases until it becomes 0 at $D = D_2$. In the interval $0 \le D \le D_1$ player F eliminates the danger of the execution of the threat by yielding to player K's demand. In the interval $D_1 \le D \le D_2$, the reduction of α obtained by an additional money unit added to C is the lower, the higher D is.

This explains that there the optimal offer $\bar{C}$ is decreased by an increase of D. For $D \geq D_2$ the influence on α is so small that it appears to be useless to offer anything at all.

The optimal choice of D: We now look at the subgame which begins with player K's choice of D. Player K knows that player F will select his offer optimally and that later he himself will choose $e = 0$. We want to determine player K's payoff expectation V under this condition. Let $\bar{\alpha}$ and $\bar{V}_0$ be the values which α and V_0 assume at $C = \bar{C}$, respectively. We have

$$(17) \qquad V = (1-\bar{\alpha})\bar{V}_0 + \bar{\alpha}V_1$$

In order to find the optimal value of D it is necessary to discuss the behavior of V as a function of D in the regions below D_1, between D_1 and D_2 and above D_2. For $C = D$ we have $\bar{\alpha} = 1$. This yields

$$(18) \qquad V = (1-q)D - qx \qquad \text{for } 0 < D \leq D_1$$

Here V is an increasing function of D. We now look at the interval $D_1 \leq D \leq D_2$. In order to show that there V is a decreasing function of D we first observe that V is a decreasing function of $\bar{\alpha}$ if $\bar{V}_0$ is kept constant. This is a consequence of (9). In the interval $D_1 \leq D \leq D_2$ an increase of D decreases $\bar{C}$ and $\bar{C}/D$ and thereby decreases $\bar{V}_0$ and increases $\bar{\alpha}$. The effect of an increase of D on V can be traced by first adjusting only $\bar{\alpha}$ and keeping $\bar{V}_0$ constant - thereby V is decreased - and then adjusting $\bar{V}_0$, whereby V is further decreased.

For $D \geq D_2$ the variables $\bar{C}, \bar{\alpha}, \bar{V}_0$ and therefore also V become constant.

We have seen that V as a function of D is first increasing up to D_1, then decreasing up to D_2 and then constant. It follows that the optimal value $\bar{D}$ of D is assumed at D_1:

(19) $\bar{D} = \dfrac{a}{1+a} \cdot \dfrac{w}{1-q}$

Player K's optimal demand $\bar{D}$ can be characterized as the highest demand such that player F's optimal offer coincides with the demand. The probability α of a non-rational execution of the threat vanishes if the game is played optimally.

The optimal choice of b: Let $\bar{V}$ be the value of V assumed at the optimal value $\bar{D}$ of D. Equations (18) and (19) yield:

(20) $\bar{V} = \dfrac{a}{1+a} w - qx$

Obviously the optimal choice $\bar{b}$ of b is $\bar{b} = 0$ for $\bar{V} < 0$ and $\bar{b} = 1$ for $\bar{V} > 0$:

(21) $\bar{b} = \begin{cases} 0 & \text{for } \dfrac{a}{1+a} w < qx \\[2mm] 1 & \text{for } \dfrac{a}{1+a} w > qx \end{cases}$

In the border case $\bar{V} = 0$ both $b = 0$ and $b = 1$ are optimal choices. This is the only case where the game fails to have a uniquely determined perfect equilibrium point.

$\bar{V}$ is player K's incentive to engage in the act of kidnapping. Note that the formula for $\bar{V}$ does not contain y and z. This is due to the fact that in the optimal play of the game player K never executes his threat. Nevertheless, it is important for the derivation of the results that y is positive and that (6) holds.

Results: With the exception of the border case $\bar{V} = 0$ the game always has a uniquely determined perfect equilibrium point. The optimal choice of b, D and C is given by (21), (19) and (16), respectively. The optimal choice of e is e = 0. Equation (16) shows how the optimal offer $\bar{C}$ behaves as a function of the demand D. Up to a critical value D_1, the optimal offer is equal to D, then it becomes a decreasing function of D up to another critical value D_2. For $D \geq D_2$ the optimal offer is 0.

The optimal demand $\bar{D}$ is the highest demand, such that the optimal offer is equal to the demand; $\bar{D}$ is the critical value D_1. If all choices are optimal, player K never executes his threat.

Policy conclusions: As long as the crime of kidnapping does occur, it must be the aim of public policy to decrease the incentive to engage in the act of kidnapping. (20) shows that $\bar{V}$ is decreased by a decrease of a or w and by an increase of q or x.

The parameter w which can be interpreted as the value of the hostage's life from the point of view of player F, seems to be outside the range of the influence exerted by public policy.

Player K's disutility x of being caught after the release of the hostage obviously depends on the punishment faced by the kidnapper. Here the policy maker may face the difficulty that a substantial increase of the length of the prison term for kidnapping may not have a noticable influence on x. Whether this is the case or not is an empirical question which cannot be answered here.

The probability of capture q can be increased by the allocation of additional resources to the efforts towards detection. This possibility of decreasing $\bar{V}$ is limited by the availability of resources. It seems to be plausible to assume that a prohibitively high police budget would be needed to secure the capture of the kidnapper with certainty. Interestingly, the policy of increasing q is less effective than one might think, since it also increases player F's chances to get the ransom money back and thereby increases his willingness to pay. An increase of q shifts the critical values D_1 and D_2 to the right and increases the optimal demand $\bar{D}$.

In the extreme case where **aw/ (1+a)** is greater than x, it is impossible to achieve the goal of making $\bar{V}$ negative by an increase of q. In this respect, the model is unrealistic for

high values of q. Later, we shall show how this weakness of the
model can be removed by the introduction of an upper limit M
of player F's ability to pay.

The parameter a is not completely outside the range of influence
exerted by public authority. The way in which the authorities
advise player F to handle a kidnapping case may help to decrease
this psychological parameter. Seemingly unimportant details
may have an important effect on the kidnapper's emotional state
and thereby on the parameter a. Everything must be done in
order to make it easy for the kidnapper to view his situation
in a rational way. For this purpose, it may be important to
communicate with the kidnapper in a non-aggressive way which
does not enhance his fears and reduces his emotional stress.

Introduction of a limit of player F's ability to pay: The
basic model can be modified by the introduction of an upper
limit M of player F's ability to pay. In the modified model,
rule 3 is replaced by the following rule 3a, whereas all the
other rules remain unchanged:

3a.   After player K has announced D, player F must
       make an offer $0 \leq C \leq$ min (D,M)

Obviously e = O is optimal in the modified model, too.
The optimal offer $\bar{\bar{C}}$ is determined as follows:

$$(22) \qquad \bar{\bar{C}} = \min (\bar{C}, M)$$

This follows by the strict concavity of U. In view of (22)
it is clear that the optimal demand $\bar{\bar{D}}$ for the modified model
is as follows:

$$(23) \qquad \bar{\bar{D}} = \min (\bar{D}, M)$$

Finally the incentive $\bar{V}$ to engage in the act of kidnapping is
replaced by a modified incentive $\bar{\bar{V}}$:

$$(24) \qquad \bar{\bar{V}} = \min (\bar{V}, (1-q)M - qx)$$

In the modified model the optimal choice of b is b = 0 for $\bar{\bar{V}} < 0$
and b = 1 for $\bar{\bar{V}} > 0$.

Equation (24) shows that for

(25) $q > \dfrac{M}{M+x}$

the value of $\bar{\bar{V}}$ is always negative, regardless of the values
assumed by a and w.

As long as the optimal demand $\bar{\bar{D}}$ is smaller than M, the effects
of small parameter changes are the same as in the unmodified
model.

Extension of the model: The basic model looks at kidnapping
as a two-person game between the kidnapper and the hostage's
family. Actually, there are many potential kidnappers and many
potential victims. Additional insight can be gained by an extended
model which explicitly includes all these potential participants.

Let k be the number of potential kidnappers, numbered from 1 to k
and let m be the number of potential hostages, numbered from 1 to
m. Each potential kidnapper is characterized by different payoff
parameter x_i, y_i and z_i and a different value w_j is associated
to each of the potential hostages, such that the assumptions of
the basic model are satisfied. The parameters a and q are assumed
to be the same for all possible kidnapping cases.

According to the basic model kidnapper i's incentive to take
hostage j is given by

(26) $\bar{V}_{ij} = \dfrac{a}{1+a} w_j - qx_i$

In order to exclude the border case possibility of non-unique
optimal behavior, we assume that the parameters w_j and x_i are
such that the following is true:

(27) $\quad\quad\quad\quad\quad \bar{v}_{ij} \neq 0$ for $i=1,\ldots,k$ and $j=1,\ldots,m.$

Define

(28) $\quad h_{ij} = \begin{cases} 0 & \text{for } \bar{v}_{ij} < 0 \\ 1 & \text{for } \bar{v}_{ij} > 0 \end{cases}$

If potential kidnapper i contemplates the kidnapping of potential
hostage j, the value of h_{ij} will decide whether he actually will
go ahead with his plan. The kidnapping will occur for $h_{ij} = 1$
and it will not occur for $h_{ij} = 0$

We do not assert that a potentially profitable kidnapping with
$\bar{v}_{ij} > 0$ necessarily will occur. Potential kidnapper i must first
turn his attention to his opportunity to take hostage j before he
even begins to find out whether his incentive $\bar{v}_{ij}$ to do so is
positive or not. Ordinarily many criminal and non-criminal
opportunities with a chance of profitability will compete for
his attention and there will be only a small probability that
he spends his limited planning and decision efforts on any one
of them.

Let p_t be the probability for the event that at a given period
of time t potential kidnapper i will contemplate the kidnapping
of potential hostage j. For the sake of simplicity we assume
that this probability is the same for all possible pairs i,j.
Time is viewed as a succession of discrete time periods $t = 0,1,\ldots$
Let n_t be the number of kidnapping cases in period t. We assume
that a profitable kidnapping opportunity which is contemplated
in period t - 1 will be realized in period t. Define

(29) $\quad\quad\quad\quad\quad H = \sum_{i=1}^{k} \sum_{j=1}^{m} h_{ij}$

The variable H is the number of profitable kidnapping opportuni-
ties. If k and m are large and p_t is small, n_t will be very near
to its expected value which can be approximated as follows:

$$(30) \qquad n_t = p_{t-1} H$$

Here we assume that in every period t every potential kidnapper contemplates at most one of his opportunities and we neglect the unlikely possibility that two potential kidnappers turn their attention to the same potential hostage.

It must be emphasized that the attention focusing process is viewed as a psychological mechanism outside the control of rational thinking. At this point, an important element of bounded rationality enters our theoretical considerations [3]. Only after the attention has been focused rational calculations begin to determine behavior.

It is reasonable to assume that p_t is a function of n_t. If more kidnapping cases are observed and reported by the media, a potential kidnapper will be more aware of his possibilities. He will be more likely to think of a feasible plan and to consider its consequences. Therefore, we make the following assumption:

$$(31) \qquad p_t = f(n_t)$$

where p_t is a monotonically increasing differentiable function. (30) and (31) together yield a first order difference equation for n_t:

$$(32) \qquad n_t = Hf(n_{t-1})$$

Since p_t is a probability the function f is bounded from below and above. This has the consequences that the limit of $f(n)$ for $n \to \infty$ exists. Define

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[3] The concept of bounded rationality has first been introduced by H.A. Simon [7]. Relatively few efforts have been made towards economic theorizing on the basis of this concept, e.g. in [1], [3] and [9]. Existing microeconomic theory is almost exclusively built on the neoclassical view of economic man as an optimizing decision maker.
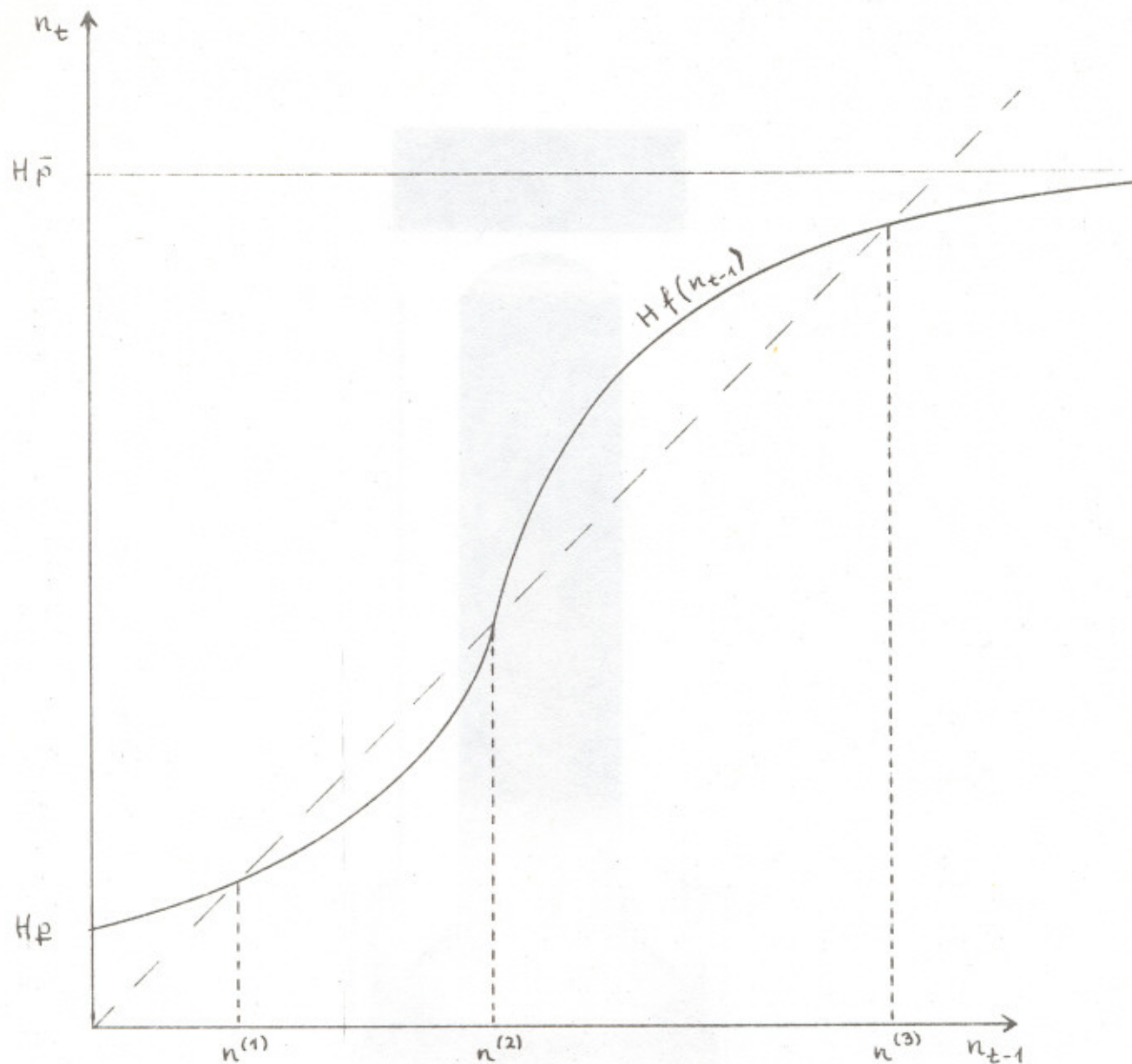
Figue 2: Graphical representation of the difference equation (32).

(33) $$\underline{p} = f(0)$$

(34) $$\bar{p} = \lim_{n \to \infty} f(n)$$

It is reasonable to assume that we have

(35) $$0 < \underline{p} < \bar{p} < 1$$

and that the shape of the function f is similar to that of a logistic curve. The situation is illustrated by figure 2. The intersections of the curve with the 45°-degree line corres- pond to stationary solutions.

(36) $$n_t = n^{(i)}$$

In the example of figure 2 we find three such stationary solutions. Our assumptions secure that at least one stationary solution always exists.

If the process starts with an initial value n_0 such that $n_1 = Hf(n_0)$ is above the 45°-degree line, then the process will converge to the lowest stationary solution above n_0. Similarily, if $n_1 = Hf(n_0)$ is below the 45°-degree, the process will converge to the highest stationary solution below n_0. This shows that only those stationary solutions are locally stable which correspond to intersections from above to below. In the case of figure 2 these are the stationary solutions $n^{(1)}$ and $n^{(3)}$. The stationary so- lution $n^{(2)}$ is unstable and is never reached by a process which does not begin there.

Policy conclusions: An increase of H results in an upward shift of the curve Hf; if the shift is sufficiently small the inter- sections from above to below are moved to the right and the stable stationary solutions will be increased. It is not surprising that an increase of the number H of profitable opportunities has the long run effect of increasing the number of observed cases. The short run effect on the next period's number of observed cases

has the same direction but the long run effect is always stronger than the short run effect.

A special situation arises if an intersection disappears as a consequence of an increase of H. Suppose, for example, that in figure 2 the process has converged to $n^{(1)}$ and that from now on H begins to increase very slowly. In order to have something specific in mind we imagine that an increasing lack of police resources results in a decrease of the probability of detection q and thereby increases the number of profitable opportunities H. As H is increased and Hf is shifted to the above, $n^{(1)}$ and $n^{(2)}$ move towards each other until they meet and finally vanish. Once this happens the process which up to now was attracted to a slowly moving $n^{(1)}$ drastically changes its character since now it is attracted by the much higher stationary solution $n^{(3)}$. This explains why without any apparent reason the number of cases which has grown slowly for some time may suddenly begin to grow at an alarming rate.[4]

Suppose that special police measures are taken in order to reduce H to its previous level. If such measures do not come soon enough they may fail to bring the process back to $n^{(1)}$ in spite of the fact that H returns to the same value as before. Instead of this the process may converge to $n^{(3)}$.

There is only one way to move the number of observed cases from $n^{(3)}$ to the more desirable equilibrium $n^{(1)}$: a temporary reduction of H below the value where $n^{(2)}$ and $n^{(3)}$ vanish. This low level must be upkept long enough to permit the process to come sufficiently near to $n^{(1)}$. Afterwards the police efforts may be relaxed and H may be allowed to return to its previous level.

A parameter change which increases or decreases $\bar{V}$ will move the number of profitable opportunities H in the same direction.

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[4] This phenomenon may be called a catastrophe in the sense of Thom $\lfloor 8 \rfloor$. In view of the simplicity of our case we have avoided the explicit use of catastrophe theory.

In this sense the policy conclusions derived from the basic model
can be transferred to the extended model.

The extended model may be of interest beyond the subject of
kidnapping. The explanation of the number of observed cases
by a dynamic model involving a probability of opportunity re-
cognition and the number of profitable opportunities may be
applicable to other criminal activities.

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