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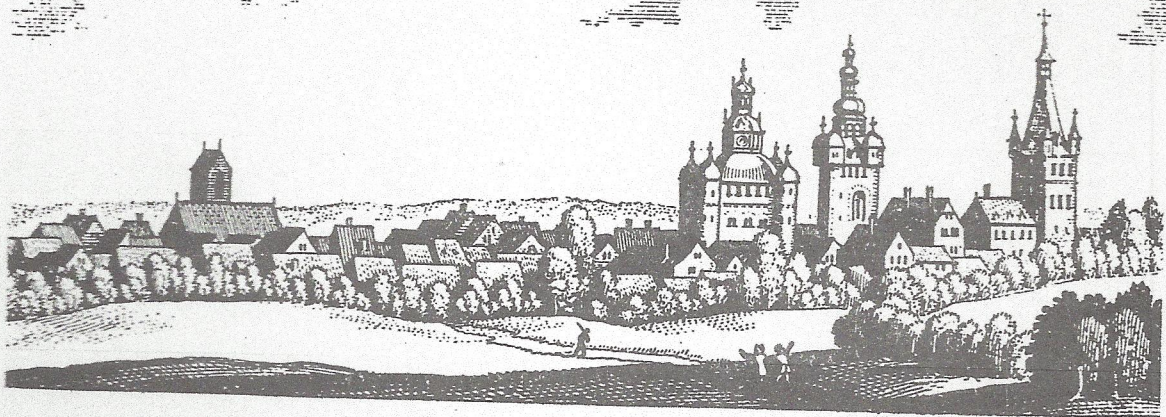
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Morishima on Marx
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The Marxian paradigm is en vogue again. Its fascinating double nature of a scientific system and a political and personal ethic attracts the young intellectuals who seek guidance in an academic world which was left in an anomic vacuum after "truth" was replaced by "hypotheses" and comprehensive systems of thought were driven out by the axiomatic method and particularistic "models." And the alchemy of Hegelian-Marxian dialectic, promising the transformation of the Iron Age of capitalism into the Silver Age of socialism and the Golden Age of the classless society, holds sway over many people who do not find any promise of the Holy Land in the sober teachings of orthodox social and economic science.

It is about time that orthodox economists make themselves acquainted with the Marxian paradigm and to do so in a more than superficial way. Morishima's book on Marx is an excellent guide for mathematically oriented economists who want to engage in such a task. Morishima has written this book for them, trying to convince them of his opinion that Marx should "be ranked as high as Walras in the history of mathematical economics," as he writes in the introduction. Being thus written with a rather sympathetic view of Marx it avoids some of the mistakes orthodox economists tend to make in interpreting Marx, sometimes without bothering to read his books. It may also serve the function of convincing Marxist economists of the usefulness of what some of them surely must have considered the irrelevant mental gymnastics of mathematical capital theory.

The book has five parts dealing with the labour theory of value, the theory of exploitation, the transformation problem, the scheme of reproduction and "Capital and Value." While the first four parts emphasize Marx's positive contribution to economic theory, the last part gives the reasons of Morishima for rejecting the labour theory of value after all. In the following remarks on Morishima's book I cannot possibly give a detailed report of its content. What I shall do instead is to ask a few critical questions from the point of view of orthodox economics, thereby hoping to contribute to Morishima's goal of a fruitful dialogue between orthodox economists and Marxists.

1. Is the existence of a proletariat a necessary condition for a positive rate of profit?

In the fourth chapter of his book Morishima develops a little general equilibrium model of simple reproduction. He thus considers a stationary economy. But, since the model describes a non-capitalist society, it is assumed that workers are provided with means of production at no cost, except that the means of production have to be returned in the same shape in which they have been delivered to the workers. Morishima introduces preferences of the consumers and shows that this stationary economy attains a Walrasian equilibrium, at which the prices of all commodities are proportional to the amount of labour necessary to produce them. He then argues that this proves that profit cannot arise, if the workers do not have to sell their labour power to the owners of means of production. I think that this general conclusion from the results of the model is not valid.

Morishima's computations are, of course, correct. But, I believe, that the assumptions of the model are too restrictive to allow his conclusion. Morishima does not allow the households to make intertemporal decisions. Households, i.e. workers, in this model only maximise utility of consumption within any given period with respect to the budget constraint that the wage rate times the labour supply of the household must be equal to the market value of the household consumption basket.

But why should intertemporal decisions be excluded? Let us modify the model in such a way that intertemporal consumption decisions become possible, but that a proletariat, i.e. a class without means of production, still does not arise. This can be done as follows. Assume we live in a society of peasants. Every peasant owns certain means of production and he has a son who will take over the production process and the means of production after the peasant reaches a certain age. There is free exchange in commodities, but law and custom forbid to buy labour or to

lend money or goods against interest. The law also requires that each peasant bequeathes to his son the means of production which he has inherited from his father. For simplicity assume that land is abundant and hence a free good. The size of population is constant, known techniques of production do not change through time.

There are two consumption goods which can be produced: wheat and wine. A unit of wheat output requires .1 units of wheat input and .9 units of labour input one period earlier. One unit of mature wine requires one unit of labour five periods earlier. In the production of wine there are intermediate products: wine, one year old; wine, two years old; wine, three years old; wine, four years old. These intermediate products as well as finished wine and wheat are freely exchangeable on the market. Now assume that all six commodities are traded at their labour values. In our example all six commodities would then have a price equal to one. If every peasant works for the usual length of time he will earn one unit of money, say one pound. This is independent of whether he produces wine or wheat. If he decides to consume the equivalent of one pound in every year, he has no reason to switch from the production of wine to the production of wheat or vice versa. He then will continue to produce what his father has produced. He inherits five units of wine, each unit of different vintage, if his father was a wine grower, and he will have to bequeath the same to his son. And he will produce wine all through his life.

But now a new era comes, in which wine growers are no longer satisfied with a constant flow of consumption goods through their lifetime. They rather prefer to have more consumption goods now and correspondingly fewer consumption goods later. In that case they will start selling some of the wine which is not yet mature

and thus for a while run down partly their means of production. This enables them to consume more than one pound per year. Later in their life they will buy wine from the market in order to restore the stock they are obliged to bequeath. They are able to do this by not consuming as much as corresponds to their net earnings.

Consider now the economy as a whole. Assume that the families which are obliged to bequeath wine and the families which are obliged to bequeath wheat are such in number that in the earlier virtuous Golden Age, when everybody was satisfied with a constant flow of consumption goods through time, equilibrium prevailed at prices equal to the labour contents of the goods. Assume such equilibrium implied that consumers spent fifty percent of their budget on wheat and fifty percent on wine. But now we are in the Silver Age, and time preference prevails, so that wine growers first run down their stock of means of production and later in life build it up again, at least as long as relative prices do not change. But in a population of wine growers with a balanced age structure this means that the total stock of wine of different vintage would be smaller than it is in the Golden Age. If disproportionalities are to be avoided, the annual production of consumable wine also must be smaller, hence wine will be in excess demand at Golden Age prices. The equilibrium price system of the Silver Age is different. The price of finished wine is higher than the price of wheat. Those, who inherit the means of production to produce wine thus receive a higher net income than those who inherit some wheat to grow more wheat. Demand for finished wine is lower than in the Golden Age, demand for wheat is higher, if demand reacts on price changes. Supply of finished wine is lower, but still equals demand. Wine growers run down their stocks when they are young and build them up again later. Part of the labour devoted to wine production in the Golden Age is now devoted to the production of wheat. The price of wine grows in geometrical

progression with its age towards the price of finished wine. Otherwise it would be profitable to sell unfinished wine whenever the next year's relative rate of appreciation is smaller than the maximum of these rates of appreciation and buy wine of a vintage with the maximum relative rate of appreciation. This would not be an equilibrium situation on the wine market.

It is not difficult to see that the price system of the Silver Age contains an implicit rate of interest on means of production. This is so, even without labour being traded as a commodity and without a class of people who do not own the means of production, which they need to put their labour power to socially useful work. Capitalist income is possible without capitalism.

The whole story has not been told to suggest any semblance to real historical developments. My point is a purely logical one, to refute the belief of Morishima that the existence of a proletariat is a necessary condition for the existence of a positive rate of return on capital and for the discrepancy between market prices and Marxian values. If the story has any other significance it may be this: people interested in worker managed systems should notice that nothing makes it likely that the rationing of capital by means of interest charges becomes superfluous in socialism.

And if consumers are to be charged with prices representing social costs of production there is every reason to pay interest (corresponding to the social rate of return on investment) on the savings of those who partly refrain from spending their current income. Implicit capitalism is likely to continue in a system which believes to have overcome capitalist exploitation.

2. Are Marxian values employment multipliers?

Beginning with Böhm-Bawerk, "bourgeois" critics of Marx had difficulties to understand the function of the difference between values and prices in the Marxian system. Considering the fact that Marx called his values "exchange values" this difficulty is

understandable. One position taken by orthodox economists was that Marxian values were not operationally meaningful, i.e. did not have any measurable counterparts which were of analytical interest. Morishima considers the Marxian labour values to be of direct operational importance in at least three contexts. They are employment multipliers (chapter 1), they measure the rate of exploitation in the economy (chapter 5), and they provide stable weights for the aggregation problem (chapter 8). Here I only want to discuss the values as employment multipliers.

Consider a stationary input output system (to shorten my argument I shall deviate from Morishima's notation), where A is the Leontief matrix of material input requirement coefficients, a_0 is the vector of labour input requirement coefficients. Let q be the vector of Marxian labour values. Then as Morishima argues, there are two ways to compute q . We may solve the equation system

$$q = a_0 + q A$$

implying

$$q = a_0 (I + A + A^2 + \dots) = a_0 (I-A)^{-1}$$

The q 's are then cost prices of a system without interest charges. Hence we may call this method of computing the Marxian values the method of prices, emphasizing their exchange value character. The other method emphasizes that values correspond to the quantity of labour time socially necessary to produce the commodities. We may call it the method of quantities. Consider any net product vector c , the stationary system is required to produce. We ask, what is the amount of labour socially necessary to produce this vector c . If the vector of gross outputs is x , then input requirements are

$$y = Ax$$

$$\text{and hence } c = x - y = (I-A) x$$

from which follows

$$x = (I + A + A^2 + \dots) c = (I-A)^{-1} c$$

The required quantity of labour, L , is then

$$L = a_0 x = a_0 (I-A)^{-1} c$$

The vector $a_0 (I-A)^{-1}$ thus can be considered a vector of employment multipliers which allows to compute L from any given net product vector c . As one can see, these employment multipliers are identical to the exchange values computed with the method of prices. If in a capitalist system the rate of profit is positive, actual exchange values no longer correspond to Marxian exchange values, which therefore could be considered as of merely hypothetical character. Morishima argues that the method of quantities still retains the direct operational meaning of Marxian values as employment multipliers, which are quite analogous to the Keynesian employment multipliers.

Morishima's argument rests on the assumption that the quantity system under consideration is stationary. This assumption is the quantity analogue to the assumption that there exists a zero rate of return on capital. To see this we only have to recall (for a derivation of these formulae see von Weizsäcker and Samuelson [4] or von Weizsäcker [5]) that the price vector is determined by

$$p(r) = (1+r) a_0 [I - (1+r) A]^{-1}$$

i.e. prices depend on the rate of profit, r , for a given nominal wage rate of unity. On the other hand, if the quantity system grows exponentially at a geometric rate of growth g the labour requirements at time t , $L(t)$, corresponding to a consumption vector $c(t)$ at time t are

$$\begin{aligned} L(t) &= (1+g) a_0 [I - (1+g) A]^{-1} c(t) \\ &= p(g) c(t) \end{aligned}$$

The vector of employment multipliers of an exponentially growing system corresponds to the prices prevailing at the rate of profit which is equal to the rate of growth of the system. Thus the Marxian values interpreted as employment multipliers are just as hypothetical as if interpreted as prices. On the other hand, prices at any given rate of profit can be interpreted as employment multipliers or quantities of socially necessary labour in a system whose rate of growth is equal to that rate of profit.

3. Is a positive rate of exploitation necessary for a positive rate of profit?

Chapter 5 and 6 discuss, what Morishima calls the Fundamental Marxian Theorem. The theorem says that a positive rate of profit is not possible without a positive rate of exploitation. He calls it the Fundamental Marxian Theorem because it shows that the viability of capitalism which rests on the existence of a positive rate of profit, depends on exploitation. Morishima invests substantial effort into the proof of the theorem. It is not trivial because the two concepts are defined in two different accounting systems. The degree of exploitation is a concept of the value system, the rate of profit is a concept of the price system. Again our criticism of Morishima's argument does not claim that it is false. It is only not sufficiently general to be of the importance Morishima attaches to it. The crucial assumption which Morishima makes is that the production technology remains constant through time. If we accept technological progress, which a good Marxist must, then things are different. Consider then an economy in which labour productivity grows steadily¹ at a geometric rate γ . The labour force and what "bourgeois" economists call the capital output ratio remain constant. Capitalists do not consume, workers do not save.

¹ The assumption of a steady geometric growth of technology is made for expository simplification. It is not crucial for the argument

Let us first look at the value system. Since the labour force does not grow and - under certain not too restrictive assumptions - the "organic composition of capital" will not change, the stock of capital (constant and variable) in terms of Marxian values does not change. On the other hand the accounting identity (in value terms)

$$\begin{aligned} & \text{surplus value} = \text{net accumulation (in value terms)} \\ & + \text{value of capitalist private consumption} \end{aligned}$$

must hold. (See volume II of "Das Kapital"). By assumption both terms on the right hand side are zero, hence the left hand side is zero and thus according to the Marxian definition the degree of exploitation is zero. On the other hand "bourgeois" theory of growth tells us that under the conditions, which we have specified, the rate of profit is equal to the rate of growth of the system, i.e. equal to γ . Hence the Marxian Fundamental Theorem does not hold.

4. Are Marxian values good aggregators?

Morishima claims that Marxian values are more stable weights for purposes of aggregation of industries than prices would be. Considering that prices for a stable rate of profit can be considered as labour requirements or employment multipliers for a system with a rate of growth equal to this rate of profit, I have doubts whether Morishima's proposition is really valid. Consider the general problem of aggregating several industries into one "department" of the economy. There exist certain structural equations determining values, prices and quantities of industries. Under which conditions exist similar structural equations for the departments, i.e. the more aggregated entities, which of course involve a much smaller number of parameters and hence less information than the structural equations of the disaggregated industries? Morishima shows that, given the relative weights of industries in the output mix of departments there exist

structural equations determining the Marxian values of the composite commodities produced by the department. In general there exist no such aggregated price equations which would be valid for each rate of profit. Structural equations for prices exist, according to Morishima, on the aggregated level, if any department only contains industries with the same value composition of capital.

The first proposition is certainly correct. But it is of limited interest. The value equations are of course equal to the price equations, if the rate of profit is zero. And it is not difficult to see that Morishima's result concerning the aggregated value equation also hold for the price equations for any given fixed rate of profit. But we cannot expect to find parameters for the price equations of the aggregated model which are independent of the profit rate, since the relative prices of industry outputs change with the rate of profit.

There is no particular reason why one should prefer the Marxian values as aggregators to the price system ruling at any fixed rate of interest. Furthermore, if the industry output composition of departments is not fixed, then Morishima's results on aggregation of the value equations no longer hold. What we are mainly interested in is a method of aggregation which is independent of the relative weight of the industries to be aggregated.

Morishima investigates at length the condition of equal value composition of capital in the industries to be aggregated. It is not clear to me how Morishima can claim that this is a sufficient condition for the possibility to obtain aggregated price equations for the departments. An equal value composition of two conventionally defined industries is not a sufficient condition to have a constant ratio of their output prices for varying profit rates. It is therefore not in general possible to find price equations for the department consisting of these two industries which relate the "price" of the output to the prices of the inputs and the rate of

profit and whose parameters are independent of the composition of this output.

I hence do not see Morishima's proposition substantiated that Marxian values are of particular use in the context of the aggregation problem. As we know already from the theory of aggregation, the conditions for exact aggregation are much too severe to be of great interest in most cases. And a far reaching theory of approximate aggregation does not exist. I have doubts whether the Marxian theory of value will be of great use for such a theory.

6. Is Capitalist Accumulation unstable?

It is well known that Marx and the Marxists believe that the process of capitalist accumulation is highly unstable. In the chapter on the "Dynamic Transformation Problem" Morishima discusses a model of capitalist accumulation in terms of a system of difference equations. These difference equations allow the system to evolve along a balanced growth path which Morishima identifies with the famous Marxian model of extended reproduction in volume II of Das Kapital. Outside of the balanced growth path the system of difference equations lead further and further away from the balanced growth path. Morishima considers his model as a formalized analysis of the instability properties of capitalism. The economics of his model is simple, indeed too simple. He assumes that capitalists save a constant fraction of their income. Morishima then considers the system of difference equations following from the input requirement functions. Thus inputs at time t are determined by the outputs of time $t+1$. Among the input requirements are also the means of subsistence of the workers. The level of consumption of capitalists is determined simultaneously with the level of production (and thus the amount of capital required) for an exogenously given rate of profit and an exogenous propensity to consume of capitalists. Thus Morishima arrives at a system of difference equations

$$y(t) = Ay(t+1) + By(t)$$

where $y(t)$ is a vector (in Morishima's model a two dimensional vector) indicating the level of production. This equation allows one to compute levels of production for all $t < T$, given $y(T)$. For any component i we denote by $y_i^*(t)$ the value of y_i on a specified balanced growth path. Now the system has the property that there exists a positive constant c such that for any given $y(T)$ the value $\frac{y_i(t)}{cy_i^*(t)}$ approaches unity as t tends to minus infinity.

This simply means that the relative effect of the future production goal $y(T)$ on $y(t)$ becomes smaller and smaller as $T-t$ becomes larger and larger. This kind of turnpike property is shared by all reasonable production technologies. What Morishima now does is to invert the recursive computation and go from smaller to larger t . He then basically asks the question: which future production vector $y(T)$ is consistent with a given nonequilibrium $y(t)$ for $t < T$? Now, the very same turnpike property of the model implies that any initial imbalance in $y(t)$ will be magnified for increasing t , and indeed will usually quite soon lead towards infeasible, i.e. no longer nonnegative values of $y(t)$. A system which is "stable" for declining t is almost by definition instable for increasing t . Does this say anything about the stability or instability of capitalist accumulation? I do not think so. The question, which future $y(T)$ is consistent with the given system of difference equations and the arbitrarily given initial $y(t)$ is without further economic meaning. Everybody knows that an unbalanced supply of inputs will imply a revision of prices such that the production of commodities in excess supply becomes unprofitable. Their supply therefore will not grow and hence there is a tendency for the excess supply to disappear. Commodities in excess demand will rise in price so that their production is stimulated. All these effects are not included in Morishima's model. In fact, the model does not contain any equation describing how the investment and production decisions of capitalists depend on prices and

expectations. It is thus not sufficiently rich to describe the motion of a capitalist system. This refutation of Morishima's model of instability of capitalist accumulation is of course not equivalent to the statement that capitalist accumulation is a stable process. We need much more complex models, if we want to treat this problem satisfactorily.

Is Morishima right in saying that Marx is as important as Walras in the history of mathematical economics? After having read Morishima I tend to agree with him on this point. May be that our Walrasian education makes us less able to grasp the Marxian approach towards an understanding of complex systems. We should for instance appreciate that the duality of values and prices in Marx is not a mistake but an attempt to analyse a system of interdependent variables without using the now usual instrument of simultaneous equations. For modern economists the Marxian approach may look clumsy, but it has the advantage of enabling one to talk about things like exploitation, which is more difficult in a Walrasian framework. I think we should follow Morishima's advice and read again the great 19th century authors in our field among whom Marx definitely has his place. Morishima's method seems to be to give Marx the benefit of the doubt in comparing him with modern mathematical economics. This, I believe, is the better attitude than the one prevailing in large sections of our profession where Marx is criticised before a serious attempt has been made to understand him. But in so doing Morishima sometimes seems to neglect some of the good points in the bourgeois criticism of Marx, as my review tried to show.

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