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John-ren Chen

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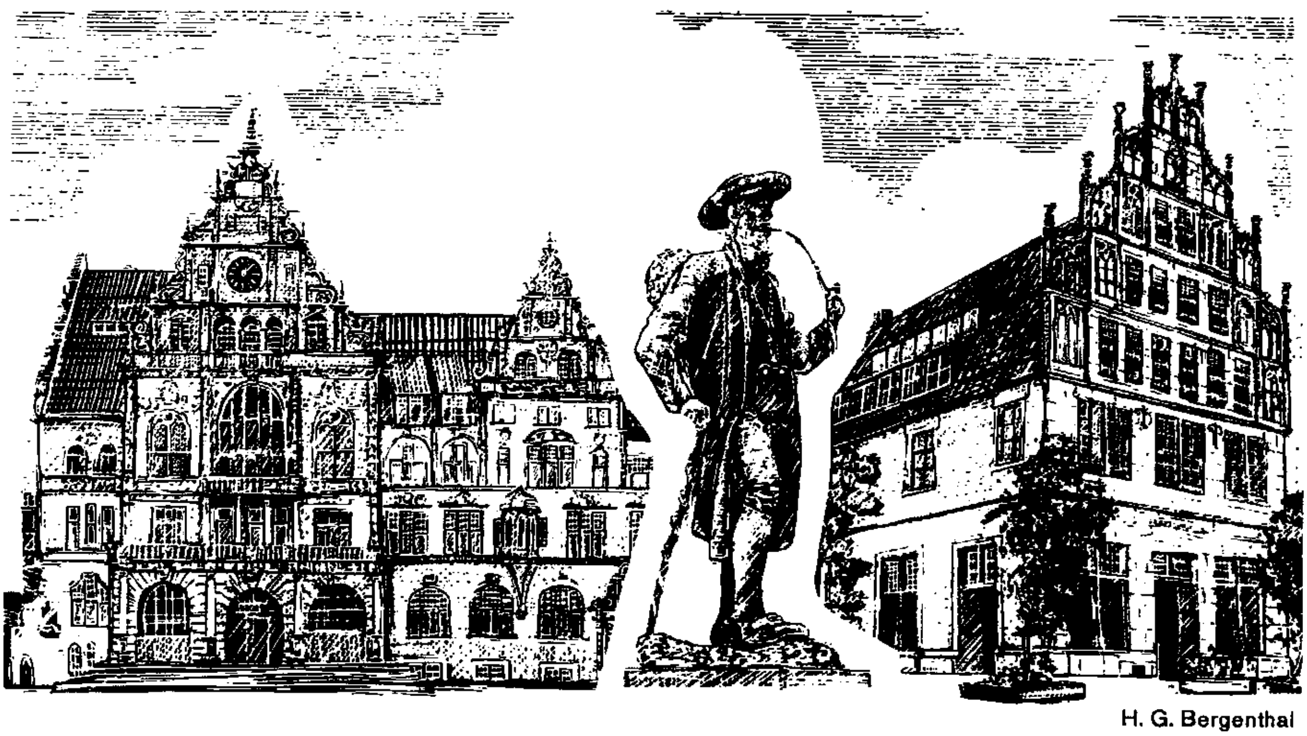

H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

Dr. John-ren Chen

The Effects of the Partially Pegging Exchange Rate Policy in a Small Open Economy

I. Introduction:

Since 1973 the floating exchange rate is practised by the most developed market-economy countries instead of the Bretton Woods system of fixed exchange rates. But many countries, especially the (small) developing countries, tie their exchange rate to a special foreign currency, generally to that of their most important trading country. For example South Korea and Taiwan peg constant exchange rates of their currencies to the US Dollar. [1] Austria ties the foreign value of Shilling to the German Mark.

In the European Common Market some member countries keep constant exchange rates between their currencies (the so-called European snake).

For convenience we shall call the country which ties the foreign value of her currency to one special other currency "the pegging country", the country of this special currency as "the pegged country" and the other country as "the floating" and their currency "the pegging currency, "the pegged currency" and "the floating currency" respectively.

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1) see Black [1976]
Formally Taiwan gave up the pegging policy in 1978. Really the same policy is practised.

While the exchange rate between the pegging and the pegged currency is fixed the exchange rates between the floating and the pegged currency as well as between the floating and the pegging currency are flexible. We shall call the policy of the pegging country to keep constant exchange rate of her currency to the pegged currency as "pegging exchange rate policy" or shortly "pegging policy".

In the most developing countries the forward foreign exchange market does not exist. Hence using the pegging policy in a floating world seems to be needed for maintaining and promoting foreign trade of the developing countries. The first problem of the pegging policy is to choice the pegged currency. In case if there is a special foreign country with dominant position in the foreign economic relations of the pegging country the problem of choicing pegged country seems to be simple. In other cases the choice problem of pegging policy has to be considered carefully.[1)] USA is the most important market of Taiwanese export products. More than 45 percent (average of the last 10 years) of export from Taiwan is delivered to the USA. But, on the other hand, Japan has a share of more than 40 percent (average of the last 10 years) of Taiwanese Import. In a previous paper [2] the author points out the special interlacing character of the Taiwanese foreign trade. The fact that Taiwan pegged its exchange rate to the US Dollar seems to be due mainly to the reserve currency position of the US Dollar after the Second World War.

In this paper we shall study the effects of pegging policy in a small open economy. A country is small in the sense that it cannot sensibly influence the economic activity of the other countries. Hence there are no feed-back effects to the economic activity in the small country through her foreign economic relations.

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1). Black [1976]

In section 2 a model will be constructed to study the pegging policy. While the aggregate demand and the money market are described by a usual macroeconomic model we shall study the classical and the Keynesian hypothesis of aggregate supply.

In section 3 we study the short-run effects of the stabilization policy as well as variation in exchange rate in the classical model with constant capital stock and given international income transfer.

In section 4 the stability of the long-run equilibrium in which the capacity effect of net investment and the international-income-transfer-effect of net international capital movements are considered.

In section 5 the short-run effects of the stabilization policy as well as changing in exchange rate in the Keynesian model are studied.

In a concluding remarks we discuss the problem of choosing pegged currency.

II. The Model

Symbols:

Y : national income (in constant prices of domestic product)
C : consumption (in constant prices ")
T : Tax (in constant prices)
I : Investment (in constant prices)
G : government expenditure (")
r : interest rate
X^{ab}: export to country b in constant prices
X^{ac}: export to country c in constant prices
X : total export in constant prices
M^{ab}: import from country b in constant prices
M^{ac}: import from country c in constant prices
M : total import
q : price of one unit of pegged currency in pegging currency
u : price of one unit of floating currency in pegging currency
e : price of one unit of pegged currency in floating currency
π : price level of the pegging country
P : price of output of the pegging country
Z^{ab}: net income transfer to the pegged country
Z^{ac}: net income transfer to the floating country
Z : total net income transfer
$\bar{L}$: nominal quantity of money
L : cash balance
H : balance of payments
D : balance of trade in constant prices
V^{ab}: net capital movements to the pegged country
V^{ac}: net capital movements to the floating country
V : total net capital movements
W : wage rate

N : employment

Q : output

E : disposable income in constant price level

P^b : price of output of the pegged country

P^c : price of output of the floating

Y^b : national income of the pegged country in constant prices

Y^c : national income of the floating country in constant prices

r^b : interest rate in the pegged country

r^c : interest rate in the floating country

$D^{ab} = X^{ab} - M^{ab}$

$D = D^{ab} + D^{ac}$

$U^{ab} = P\,X^{ab} - P^b\,q\,M^{ab}$

$U^{ac} = P\,X^{ac} - P^c\,u\,M^{ac}$

$U = U^{ab} + U^{ac}$

The Superscript is used if necessary:

for a : the pegging country

b : the pegged country

c : the floating country

Notation: Subscript is used for partial derivatives, e.x.

$$X^{ab}_{P^b} = \frac{\partial X^{ab}}{\partial P^b} \quad \text{etc.} \quad \text{and} \quad \dot{K} = \frac{dK}{dt}$$

The Structural functions of the model are as following:

(1) $Y = C + I + G + D$

(2) $C = C\,(\overset{+}{E}, \overset{-}{r})$

(3) $E = Y - D + U + Z/P - T$ (disposable income)[1]

(4) $I = I\,(\overset{-}{r})$

(5) $X^{ab} = X^{ab}\,(\overset{-}{P}, \overset{+}{P^{b}}, \overset{+}{Y^{b}}, \overset{+}{q}) = M^{ba}$

(6) $X^{ac} = X^{ac}\,(\overset{-}{P}, \overset{+}{P^{c}}, \overset{+}{Y^{c}}, \overset{+}{u}) = M^{ca}$

(7) $M^{ab} = M^{ab}\,(\overset{+}{P}, \overset{-}{P^{b}}, \overset{+}{Y}, \overset{-}{q}) = X^{ba}$

(8) $M^{ac} = M^{ac}\,(\overset{+}{P}, \overset{-}{P^{c}}, \overset{+}{Y}, \overset{-}{u}) = X^{ca}$

(9) $\bar{L} = L\,(\overset{+}{Y}, \overset{-}{r}, \overset{+}{P})$

(10) $V^{ab} = V^{ab}\,(r^{a} \overset{+}{-} r^{b})$

(11) $V^{ac} = V^{ac}\,(r^{a} \overset{+}{-} r^{c})$

(12) $H = P\,X - q\,P^{b}\,M^{ab} - u\,P^{c}\,M^{ac} + V + Z$

(13) $Q = F\,(N, K), \quad F_{N} > 0, \quad F_{NN} < 0, \quad F_{K} > 0, \quad F_{KK} < 0,$

$$F_{KN} = F_{NK} > 0$$

(14) $\frac{W}{P} = F_{N}$

(15) $N = N\,(\overset{+}{\frac{W}{P}})$

(16) $N = N\,(\overset{+}{W})$

(17) $Y = Q + Z$

(18) $\dot{K} = I$

(19) $\dot{Z} = r\,V$ [2]

1) Chen [1978]

2) The total net foreign indebtedness (or claims) at time t is defined as

$$\frac{Z^{i}}{r^{i}} = \int_{0}^{t} V^{i}\,(\tau)\,d\tau$$

(20) $D^{ab} = X^{ab} - M^{ab}$

(21) $D^{ac} = X^{ac} - M^{ac}$

(22) $D = D^{ab} + D^{ac}$

(23) $U^{ab} = P\ X^{ab} - q\ P^{b}\ M^{ab}$

(24) $U^{ac} = P\ X^{ac} - u\ P^{c}\ M^{ac}$

(25) $U = U^{ab} + U^{ac}$

(26) $Z = Z^{ab} + Z^{ac}$

(27) $V = V^{ab} + V^{ac}$

(28) $u = \frac{q}{e}$

(29) $\Pi = \alpha_1\ P + \alpha_2\ q\ P^{b} + \alpha_3\ u\ P^{c}$

where $\sum_i \alpha_i = 1, \qquad \alpha_i \geq 0$

The model described by the above structural functions is an usual macroeconomic model for an open economy with explicit consideration of the bilateral trade relations.

The functions (1) to (8) describe the aggregate demand of an open economy in an usual Keynesian model. While the functions (1) to (4) characterize the domestic absorption of the model, the functions (5) to (8) describe the foreign trade relations, especially the bilateral trade relations of the pegging country. The import or the export between any two trading countries is assumed to depend on the output price of both countries, the bilateral exchange rate between the currencies of the trading countries, and on the real income of the importing country. The assumption of these double price adjustment function in the business of international trade is due to the following two reasons: First, while the change in exchange rate influences the import price and export price at the same time, in the same direction and with the same percentage for both import and export country but generally the change in the output import prices of the trading countries does not occur at the same time or with the same percentage and in general only partial in one of the trading countries; secondly, the effects of the change in output prices and in exchange rate occur at different time. The reaction of the price level and the exchange rate to the change of the economic environment are not the same.

(12) is the balance of payments. Because of the pegged exchange rate policy the balance of payments is not always equal to zero. H is an endogenous variable in our model. The exchange rate between the pegging and the pegged currency q is fixed by the government of the pegging country.

(13) is a linear homogenous production function of the neoclassical type. In the short-run the capital stock is given and the

output will depend only on the labor employment.

(14) is a demand function for labor input according to the principle of marginal productivity.

(15) is the classical labor supply function in which the labor supply is assumed to depend on the real wage rate.

(16) is the Keynesian labor supply function in which the labor supply is assumed to depend on the given monetary wage rate.

(17) defines the real national income. We shall assume that the income transfer to the other countries is denominated in terms of the pegging currency.[1] If the pegging country has to pay interests,etc. to the other countries then it has to produce more than its national income. Otherwise the output will be smaller than the national income.

(18) and (19) describe the "capacity effect" of net investment and the "income transfer effect" of the international capital movements,respectively.

The equations (20) to (27) are several definitions which are used to simplify our presentation.

(23) and (24) are the bilateral trade balances to the pegged and the floating country. (25) defines the multilateral trade balance of the pegging country.

(27) describes the multilateral capital balance of the pegging country where the bilateral capital balance is assumed to depend on the real interest difference of the countries considered (see (10) and (11)).

(9) describes the money market equilibrium in the pegging country. The demand for money (nominal) is assumed as usually to be a function of the real interest, the price level and the real

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1). Chen [1978]

income. The money supply is considered as an exogenous variable in our model.

(28) arises under the condition of foreign exchange arbitrage without transaction cost.

(29) defines the price level in the pegging country, where α_i are the constant weights. The price level is just the weighted average of the output price of the pegging, the pegged and the floating country. A depreciation has in general an inflationary effect even if the output price in all three countries of the world considered remain constant.

In the following analysis we assume:

(i) The exchange rate q is given (pegged policy). But the exchange rates e and u are flexible.

(ii) There exists an equilibrium for the model.

(iii) Initially we have $e = u = q = P = P^b = P^c = 1$

(iv) The pegging country has net "indebtedness" denominated in terms of its currency both to the pegged and the floating country.[1)]

(v) $X_e^i - M_e^i - M^i > 0$, (Marshall-Lerner-Condition)[2)]

(vi) The traded goods are not inferious

(vii) $-\dfrac{\partial x^{ij}}{\partial p^i} = \dfrac{\partial x^{ij}}{\partial p^j} = \dfrac{\partial x^{ij}}{\partial e^{ij}}$

(viii) The growth rate of potential labor is exogenous given

$$\frac{\dot{\bar{N}}^i}{\bar{N}^i} = n^i$$

(ix) The outputs of the pegging, pegged and floating country are assumed to be imperfect substitute to each other.

(x) The production function is linear homogenous with decreasing marginal productivity.

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1) In [2] we show the meaning whether the net foreign indebtedness and the net income transfer are denominated in domestic or foreign currency for the effects of stabilization policy and international transmission of business cycles.

2) We have to point out that the Marshall-Lerner condition in our model is different from that used in the usual form.

III. The Classical Economy: Full Employment Case

A small country has no marked influences on the economic activity of the other countries and on the price of the world market. Hence for the small pegging country the economic activity of the pegged country and the floating country as well as the exchange rate between their currencies are assumed to be given.

A. The Short-Run Effects

At first we shall study the short-run effects. In this case the capacity effects of net investment and the "income-transfer-effect" of international capital movements are neglected. The labor potential is assumed to be given.

In a classical economy of the small pegging country the real output does not depend on the price level. Since in the employment, the real wage and the real output can be determined by the functions (13) to (15) independently from the other functions of the model.[1)]

Since the variables Y^i, Q^i, r^i, P^i (for $i=b,c$) and e are given we can derive one price-interest-rate-schedule from the goods market (we shall call this the GM-schedule) and one from the money market (we shall call this the MM-schedule), respectively.

(30) $-A_{11}\, dr + A_{12}\, dP = A_{10}$ (GM-schedule)

(31) $A_{21}\, dr + A_{22}\, dP = A_{20}$ (MM-schedule)

with : $A_{11} = C_r + I_r < 0$; $A_{12} = D_q + Z > 0$

$A_{21} = L_r - V_r < 0$; $A_{22} = L_p + U_q > 0$

1) Bowers & Baird [3] . P.196. The classical dichotomy as D. Patinkin calls it.

$$D_q = X_q - M_q > 0$$

$$U_q = X_q - M_q - M = X_q^{ab} + X_u^{ac} - X_q^{ba} - X_u^{ca} - X^{ba} - X^{ca} > 0$$

(Marshall-Lerner)

$$A_{10} = - \left[1-C_Y(1-T_Y)+M_Y\right] dY - D_u^{ac}\, de + D_q\, dq + D_q^{ab}\, dP^b +$$

$$D_u^{ac}\, dP^c + X_Y^{ab}\, dY^b + X_Y^{ac}\, dY^c + d\,Z + d\,G$$

$$A_{20} = U_q^{ab}\, dP^b + U_u^{ac}\, dP^c + U_q\, dq - U_u^{ac}\, de - (M_Y+L_Y)dY + d\bar{L} +$$

$$X_Y^{ab}\, dY^b + X_Y^{ac}\, dY^c - V_r(dr^b + dr^c) + dZ$$

Solving this equation system we have:

$$(32) \qquad dr = \frac{A_{12}A_{20} - A_{22}A_{10}}{\Delta}$$

$$(33) \qquad dP = \frac{A_{11}A_{20} + A_{21}A_{10}}{\Delta}$$

where $\Delta = A_{11}A_{22} + A_{12}A_{21} < 0$

The GM-schedule has negative slope while the MM-schedule has positive slope as shown in Fig. 1

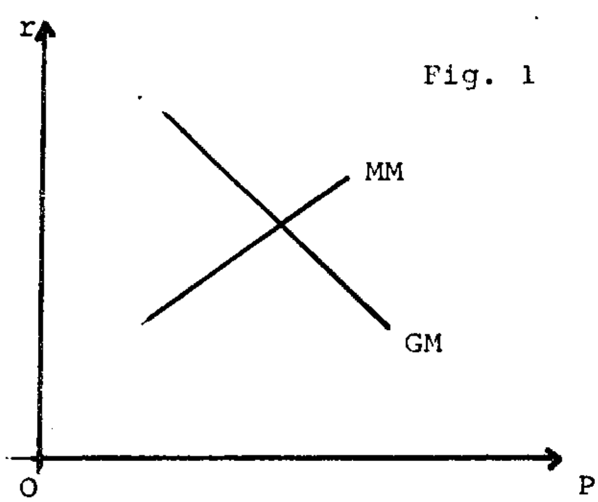

Fig. 1

For comparative static study we derive the solution for the balance of payments of the pegging country as following:

$$(34) \qquad dH = \frac{-1}{\Delta}\left[(U_q A_{21}+V_r A_{22})A_{10}+(U_q A_{11}-V_r A_{12})A_{20}\right] + U_q^{ab}\, dP^b + U_u^{ac}\, dP^c + U_q\, dq - U_u^{ac}\, de - M_Y\, dY + X_Y^{ab}\, dY^b + X_Y^{ac}\, dY^c - V_r(dr^b+dr^c) + d\, Z$$

We shall analyse at first the effects of fiscal and monetary policy as well as of the change in the exchange rates q and e for the case that the net international income transfer (interest payment) and the net foreign indebtedness(claims) of the pegging country are denominated in terms of the pegging currency and then for the case that the net international income transfer and the net foreign indebtedness of the pegging country are denominated terms of the pegged and the floating currency.

In this case the effects of fiscal and monetary policy are similar to those under the regime of fixed exchange rates if the "Marshall-Lerner condition" holds.

A depreciation of the pegging currency to the pegged currency improves the balance of payments of the pegging country, if the Marshall-Lerner condition holds and the international income transfer of the pegging country is not negative. Hence the Marshall-Lerner condition is necessary and sufficient for a positive effect of an exchange rate depreciation on the balance of payments of the pegging country, if this has zero net international income transfer.

A surprising result of this short-run analysis is that an appreciation of the floating currency is neither necessary nor sufficient for an improvement in the balance of payments

Tab. 1: Some Comparative Static Results for the Small Pegging Country (I)

dependent Variables / indep. Variables	dP	dr	dH
dG	> 0	> 0	< 0
$d\bar{L}$	> 0	< 0	> 0
dq	$\frac{1}{\Delta}(A_{11}D_q+A_{21}U_q) > 0$	$\gtreqless 0$, if $L_p \gtreqless \frac{Z\ U_q}{D_q}$	$\frac{(U_qA_{11}-D_qV_r)L_p+L_rU_qZ}{\Delta} > 0$, if $Z \geq 0$
de	$\gtreqless 0$, if $\frac{D_u^{ac}}{M^{ac}} \lesseqgtr \frac{A_{11}}{A_{11}+A_{21}}$	$\gtreqless 0$, if $1-\frac{M^{ac}}{D_u^{ac}} \gtreqless \frac{A_{22}}{A_{12}}$	$\frac{(V_rD_u^{ac}-A_{11}U_u^{ac})L_p-Z\ U_u^{ac}\ L_r}{\Delta}$
dZ	> 0	$\gtreqless 0$, if $A_{22} \gtreqless A_{12}$ x)	$\frac{(A_{11}-V_r)L_p+(M+Z)L_r}{\Delta} > 0$, if $M > -Z$
dP^b	$\gtreqless 0$, if $\frac{D_u^{ab}}{M^{ab}} \gtreqless \frac{A_{21}}{A_{11}+A_{21}}$	$\frac{Z\ U_q^{ab} - L_p\ D_q^{ab}}{\Delta}$	$\frac{(A_{11}U_q^{ab}-V_rD_q^{ab})L_p+Z\ U_q^{ab}\ L_r}{\Delta}$
dP^c	$\gtreqless 0$, if $\frac{D_u^{ac}}{M^{ac}} \gtreqless \frac{A_{11}}{A_{11}+A_{21}}$	$\frac{Z\ U_u^{ac} - L_p\ D_u^{ac}}{\Delta}$	$\frac{(A_{11}U_u^{ac}-V_rD_u^{ac})L_p-Z\ U_u^{ac}\ L_r}{\Delta}$
dr^b, dr^c	< 0	> 0	< 0

x) Due to the stability condition $A_{12} > A_{22}$ Hence $\frac{dr}{dZ} < 0$

of the pegging country even if the Marshall-Lerner condition holds for the multilerteral trade balance of the pegging country. An appreciation of the floating currency will improve the balance of payments of the pegging country, if (a) the Marshall-Lerner condition is fulfilled especially for the bilateral trade balance both to the pegged and the floating country, (b) the pegging country has no initial deficit of bilateral trade balance to the floating country or this initial deficit is sufficiently small and (c) the pegging country does not transfer income to other countries. But these conditions are not generally fulfilled for the pegging country. Even if the conditions (a) and (c) are fulfilled but not the condition (b), i.e. if the pegging country has initially a deficit trade balance to the floating country an appreciation in the currency of the floating country to that of the pegged country will deteriorate the balance of payments of the pegging country (instead of an improvement of balance of payments as generally expected).

Another surprising result of this short-run analysis is the effect of change in exchange rate of the floating currency to the pegged currency on the price of output in the pegging country. Assuming the normal demand functions for the bilateral trade relations [1] between the pegging and the floating country a depreciation of the floating currency to the pegged currency will have inflationary effect on the price level in the pegging country, if the following condition holds: [2]

$$(35) \qquad \frac{D_u^{ac}}{M^{ac}} < \frac{A_{11}}{A_{11}+A_{21}}$$

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1) i.e. $X_u^{ac} > 0$ and $M_u^{ac} < 0$

2) Notes : $\frac{A_{11}}{A_{11}+A_{11}} < 1$ and $\frac{dP}{de} \gtrless 0$ if $A_{11}U_u^{ac} \gtrless - A_{21}\, D_u^{ac}$

This condition seems not to be fulfilled for the case $M^{ac} - X^{ac} > 0$, i.e. the pegging country has a bilateral trade deficit to the floating country initially. Hence an appreciation of the floating currency to the pegged currency would have deflationary effect on the price level in the pegging country if the pegging country has an sufficiently large deficit of bilateral trade balance to the floating country. This is a surprising result since an appreciation of the floating currency to the pegged currency means also a depreciation of the pegging currency to the floating currency.

An inflation in the pegged country will transmit to the pegging country, if [1)]

$$\frac{D_u^{ab}}{M^{ab}} > \frac{A_{21}}{A_{11}+A_{21}} \tag{36}$$

This condition will hold if the pegging country has an sufficiently large bilateral trade surplus to the pegged country (i.e. $X^{ab} > M^{ab}$) initially.

An increase in the output price of the floating country influences the bilateral balance of payments of the pegging country to the floating country similarly as a depreciation of the floating currency to the pegged country and hence to the pegging country. An increase in the output price of the floating country will therefore transmit to the pegging country if the following condition holds: [2)]

$$\frac{D_u^{ac}}{M^{ac}} > \frac{A_{11}}{A_{11}+A_{21}} \tag{37}$$

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1) Assuming normal demand function for both countries

$$\frac{dP}{dP^b} \gtrless 0, \quad \text{if} \quad -A_{11}U_q^{ab} \gtrless A_{21}D_q^{ab}$$

2) $\frac{dP}{dP^c} \gtrless 0, \quad \text{if} \quad -A_{11}U_u^{ac} \gtrless A_{21}D_u^{ac}$

This condition will not be fulfilled if the pegging country has sufficiently large deficit of bilateral trade balance to the floating country. Hence an increase in the output price of the floating country as well as a depreciation of the floating currency to the pegged currency will have a negative effect on the output price of the pegging country, if the deficit of bilateral trade balances of the pegging country to the floating country is sufficiently large.

Assuming that the Marshall-Lerner condition holds for the global foreign trade relations of the pegging country and that the pegging country has a zero global trade balance then an increase in the output price either of pegged country or the floating country can have a negative effect on the output price of the pegging country.

This can be shown as following:

For $U_q = U_q^{ab} + U_u^{ac} > 0$ and $X = M$

if the Marshall-Lerner condition is not fulfilled for the bilateral trade to the pegged country then it must hold for the bilateral trade to the floating country, i.e. if $U_q^{ab} < 0$ then for the Marshall-Lerner condition to be fulfilled for the global trade relations we must require $U_u^{ac} > 0$.

Otherwise if $U_u^{ac} < 0$ then we have to require that $U_q^{ab} > 0$ for the Marshall-Lerner condition for the global foreign trade relations of the pegging country.

If $U_q^{ab} < 0$ so that $\frac{dP}{dP^b} < 0$ holds, then due to

$U_u^{ac} > 0$ we have $\frac{dP}{dP^c} > 0$, since $-A_{11}U_u^{ac} > A_{21}D_u^{ac}$

Otherwise if we have $U_u^{ac} < 0$, so that $\frac{dP}{dP^c} < 0$ holds, then due to $U_q^{ab} > 0$ we have $\frac{dP}{dP^b} > 0$ since

$$-A_{11}U_u^{ab} > A_{21}D_u^{ab}$$

Our proposition has been proved.

In the above analysis we assume that both the foreign assets (i.e. foreign bonds owned by the pegging country) and the international income transfer of the pegging country are denominated in terms of the pegging currency. As we point out in an another paper[Chen 1978] that in this case the change in exchange rate has neither real wealth effect nor real income effect. Otherwise if the foreign assets and the international income transfer of the pegging country a change in exchange rate has a real wealth effect and a real income effect.

IV. Keynesian Economy:

We are now going to study the problem of partial pegging exchange rate policy in a small country with the Keynesian labor supply function, i.e. the supply of labor is a function of money wage rate which is assumed as given. In this case the function (12) is replaced by

(16) $$N = N(\overset{+}{W}) \quad \text{with} \quad N_W > 0$$

From functions (13), (14) and (16) we can derive an aggregate supply function as following:

(38) $$Q = Q(\overset{-}{W}, \overset{+}{P}) \qquad Q_W < 0, \quad Q_P \gtrless 0$$

From the other functions of our model we can derive the aggregade demand function

(39) $$Y = Y(P, P^b, P^c, Y^b, Y^c, G, q, e, r^b, r^c, Z)$$

with

$$Y_P = -\frac{\Delta}{\Lambda}$$

$$\Delta = A_{11} A_{22} + A_{12} A_{21} < 0$$

$$\Lambda = A_{11} b_1 + A_{21} b_2 < 0$$

$$b_1 = M_Y + L_Y$$

$$b_2 = 1 - C_Y (1 - T_Y) + M_Y$$

$$Y_e = -\frac{\Psi_2}{\Lambda} \qquad Y_{P^b} = \frac{\Psi_3}{\Lambda}$$

$$Y_g = \frac{\Psi_1}{\Lambda} \qquad Y_{P^c} = \frac{\Psi_2}{\Lambda}$$

$$Y_Y b = \frac{\Psi_4 X_Y^{ab}}{\Lambda} > 0$$

$$Y_Y c = \frac{\Psi_4 X_Y^{ac}}{\Lambda} > 0$$

$$Y_Z = \frac{\Psi_4}{\Lambda} > 0 \; ; \quad Y_G = \frac{A_{11}}{\Lambda} > 0 \; ; \quad Y_{\bar{L}} = \frac{A_{21}}{\Lambda} > 0$$

$$Y_r b = Y_r c = \frac{-A_{21} V_r}{\Lambda} < 0$$

$$\Psi_1 = A_{11} U_q + A_{21} D_q < 0$$

$$\Psi_2 = A_{11} U_u^{ac} + A_{21} D_u^{ac}$$

$$\Psi_3 = A_{11} U_q^{ab} + A_{21} D_q^{ab}$$

$$\Psi_4 = A_{11} + A_{21} < 0$$

We have now to solve our model simultanously.

$$(40) \quad dY = \frac{Q_P}{\Omega} \Big[\Psi_1 dq - \Psi_2 (de - dP^c) + \Psi_3 dP^b + \Psi_4 (X_Y^{ab} dY^b + X_Y^{ac} dY^c) + (\Delta + \Psi_4) dZ - A_{21} V_r (dr^b + dr^c) + A_{11} dG + A_{21} d\bar{L} + \Delta Q_W dW \Big]$$

where $\Omega = \Delta + Q_P \Lambda < 0$

$$(41) \quad dP = \frac{1}{\Delta} \Big[\Psi_1 dq - \Psi_2 (de - dP^c) + \Psi_3 dP^b + A_{11} d\bar{L} + A_{21} dG + \Psi_4 (X_Y^{ab} dY^b + X_Y^{ac} dY^c + dZ) - V_r (dr^b + dr^c) - \Lambda dY \Big]$$

(42) $$dN = \frac{1}{W}(dY-dZ-F_K dK-NdW)(1+\frac{K\ F_{KN}}{W})$$ 1)

where $F_{KN} > 0$

(43) $$dr = -\frac{1}{\Delta}\Big[(b_1A_{12}-A_{22}b_2)dY-b_3dq-b_4de-b_3dP^b-b_4dP^c- b_5(X_Y^{ab}dY^b+X_Y^{ac}dY^c)+A_{22}dG-A_{12}d\bar{L}-b_5dZ+A_{12}V_r(dr^b+ dr^c)\Big]$$ 2)

where $b_1 = M_Y + L_Y > 0$; $b_2 = 1 - C_Y(1-T_Y) + M_Y > 0$

$b_3 = A_{12}\ U_q - A_{22}\ D_q$; $b_4 = A_{12}U_u^{ac} - A_{22}D_u^{ac}$

$b_5 = A_{12} - A_{22}$

---

1) Because of linear homogenity of the production function $Q=N\ F_N+K\ F_K$ and $N=N(W)$, $F_N = \frac{W}{P}$, $Y=Q+Z$

$Y=N\ F_N+K\ F_K+dZ$

$=N\ \frac{W}{P}+K\ F_K+dZ$, $N= \frac{P}{W}(Y-Z-K\ F_K)$

$(1+\frac{K\ F_{KN}}{W})dN= \frac{1}{W}(dY-dZ-F_KdK- \frac{P}{W}\ N\ F_NdW)$

Since $Q_N= \frac{W}{P}$, $dN= \frac{1}{W}(dY-dZ-F_KdK-N\ dW)(1+ \frac{K\ F_{KN}}{W})$

2) $\frac{\partial r}{\partial Y} > 0$, if $A_{12}(M_Y+L_Y) > A_{22}\left[1-C_Y(1-T_Y)+M_Y\right]$

The model is stable if $A_{12} > A_{22}$, so that $\frac{\partial r}{\partial Y} > 0$ if $L_Y > 1-C_Y(1-T_Y)$ holds.

This condition seems to be always fulfilled.

$$(44)\quad dH = \frac{1}{\Delta}(h_1h_2+M_Y \quad +h_2b_1)dY+\left[U_q-\frac{1}{\Delta}(h_1D_q+h_2U_q)\right] dq$$

$$+(\frac{1}{\Delta}h_3-U_u^{ac})de-\frac{1}{\Delta}\ (h_1D_q^{ab}+h_2U_q^{ab})-U_q^{ab}\ dP^b$$

$$-(\frac{1}{\Delta}h_3-U_u^{ac})dP^c-(\ \frac{1}{\Delta}h_4-1)(X_Y^{ab}dY^b+X_Y^{ac}dY^c+dZ)$$

$$-\ \frac{1}{\Delta}(h_1dG+h_2d\bar{L})+(\frac{1}{\Delta}h_2-1)V_r(dr^b+dr^c)$$

where $h_1= U_qA_{21}+V_rA_{22}$; $h_2= U_qA_{11}-V_rA_{12} < 0$

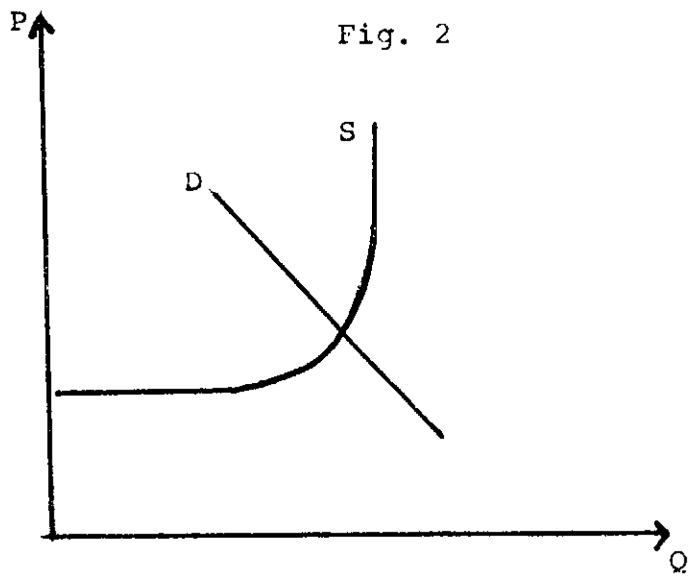
$$h_3= h_1D_u^{ac} + h_2U_u^{ac} \ ; \quad h_4= h_1 + h_2$$

$$(45)\quad dU = -U_qdP+U_q^{ab}dP^b+U_u^{ac}dP^c+U_qdq-U_u^{ac}de+X_Y^{ab}dY^b$$

$$+X_Y^{ac}dY^c-M_YdY$$

The equilibrium of the Keynesian model can be described by the Fig. 2. The schedule of aggregate demand has negative slope while the schedule of aggregate supply is assumed to be non-negative.

Fig. 2



According to Keynes the aggregate supply will become completely elastic if there is "unemployment" in the economy and completely inelastic if there is full employment. [1] In the real world these two extreme cases seem rarely to be observed. We assume in our model that the slope of aggregate supply is non-negative . Between the two extreme cases there is a range in which the aggregate supply is neither completely elastic nor completely inelastic. Obviously,the classical model can be considered as a special case of the Keynesian model with full employment.

The effects of stabilization policy and exchange rate policy for the case of full employment in the Keynesian model are similar to those in the classical model with given real output and employment in the short-run (See Tab. 1 & 2). In the following analysis we shall consider only the cases other than that of full employment.

A. The Effects of Stabilization Policy:

The effects of stabilization Policy (monetary and fiscal policy) on the real income, price level, trade balance and employment depend on the employment situation. [2]

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1) Keynes (1936)
We shall call the case of completely elastic aggregate supply the case of "unemployment", while the case of completely inelastic aggregate supply the case of "full employment". The real world seems to lie between these two extreme cases.

2) The impact effects of stabilization policy and exchange rate policy on the balance of payments are complicated by the fact that their influences on the interest rate and therefore on the international capital movements are not clearly cut.

Tab. 2: Some Results of Comparative Static Analysis: Keynesian Model

	dY	dP	dN x)	dU
dG	> 0	> 0	> 0	< 0
$d\bar{L}$	> 0	> 0	> 0	< 0
dq	$Q_P \frac{\Psi_1}{\Omega} > 0$	$\frac{\Psi_1}{\Omega} > 0$	$\frac{\Psi_1}{\Omega} \frac{Q_P}{W} w$	$U_q (1- \frac{\Psi_1}{\Omega}) - M_Y Q_P \frac{\Psi_1}{\Omega}$
de	$- \frac{\Psi_2}{\Omega} Q_P$	$- \frac{\Psi_2}{\Omega}$	$- \frac{Q_P}{W} \frac{\Psi_1}{\Omega} w$	$\frac{\Psi_2}{\Omega}(U_q + M_Y Q_P) - U_u^{ac}$
dZ	$\frac{\Delta + \Psi_4 Q_P}{\Omega} > 0$	$\frac{\Psi_4 - \Lambda}{\Omega} > 0$	$\frac{\Psi_4 - \Lambda}{\Omega} \frac{Q_P}{W} w > 0$	$-U_q \frac{\partial P}{\partial Z} - M_Y \frac{\partial Y}{\partial Z} + 1$
dP^b	$Q_P \frac{\Psi_3}{\Omega}$	$\frac{\Psi_3}{\Omega}$	$\frac{Q_P}{W} \frac{\Psi_3}{\Omega} w$	$U_q^{ab} - (U_q + Q_P M_Y) \frac{\Psi_3}{\Omega}$
dP^c	$Q_P \frac{\Psi_2}{\Omega}$	$\frac{\Psi_2}{\Omega}$	$\frac{Q_P}{W} \frac{\Psi_1}{\Omega} w$	$- \frac{\Psi_2}{\Omega}(U_q + M_Y Q_P) + U_u^{ac}$
dr^b, dr^c	< 0	$- \frac{V_r}{\Delta}(1+ \frac{A_{21}\Lambda}{\Omega} Q_P)$	< 0	< 0
dW	< 0	> 0	< 0	$(U_q \frac{\Lambda}{\Delta} - M_Y) \Delta Q_W \frac{Q_P}{\Omega}$

x) $w = 1 + \frac{K F_{KN}}{W}$

So long as there is "unemployment" the stabilization policy will have positive effects on real income and employment; and when there is "full employment" the stabilization policy will influence the price level positively. But the stabilization policy will always effect the trade balance negatively, since the stabilization policy will influence positively either on the real income or on the price level or on both and either of these effects stimulates the import demand or decrease the export supply. Hence an expansive monetary or fiscal policy has an impact deteriorating effect on the trade balance.

B. The Effects of Appreciation and Depreciation of the Pegging Currency to the Pegged Currency:

(i) A change in the exchange rate of the pegging currency to the pegged currency has a positive effect on the real income

(46) $$\frac{\partial Y}{\partial q} = \frac{\Psi_1}{\Omega} Q_P > 0$$

For $\lim\limits_{Q_P \to 0} \frac{\partial Y}{\partial q} = 0$ and $\lim\limits_{Q_P \to \infty} \frac{\partial Y}{\partial q} = \frac{\Psi_1}{\Lambda} > 0$ and

$\frac{d}{dQ_P}\left(\frac{\partial Y}{\partial q}\right) = \frac{\Psi_1}{\Omega}\left(1 - \frac{Q_P \Lambda}{\Omega}\right) > 0$, the effect is greater the higher the elasticity of the aggregate supply.

(ii) A depreciation of the pegging currency to the pegged currency has inflationary effect with the exception of completely elastic aggregate supply

From $\frac{\partial P}{\partial q} = \frac{\Psi_1}{\Delta}\left(1 - \frac{\Lambda}{\Omega} Q_P\right)$ we have

$$\frac{\partial P}{\partial q} > 0, \text{ if } 1 - \frac{\Lambda}{\Omega} Q_P > 0$$

we can show that this condition holds always,

$$\text{since} \quad \frac{1}{Q_P} > \frac{\Lambda}{\Omega} \quad \text{i.e.} \quad \frac{1}{Q_P} > \frac{1}{\frac{\Lambda}{\Omega} + Q_P}$$

We can also see that the inflationary effect of a depreciation of the pegging currency to the pegged currency is greater, the lower the elasticity of aggregate supply.

$$\text{since} \quad \frac{d\left(\frac{\partial P}{\partial q}\right)}{d\, Q_P} = - \frac{\Psi_1 \Lambda}{(\Omega)^2} < 0$$

(iii) A depreciation of the pegging currency to the pegged currency will improve the "global" trade balance of the pegging country, if the following condition holds:

$$(47) \qquad U_q > \frac{\Psi_1 \, M_Y \, Q_P}{\Omega - \Psi_1} = \frac{M_Y \, Q_P \, D_q \, A_{21}}{A_{11}(L_P + L_Y) + A_{21}(b_2 + Z)} > 0$$

This can be proved as follows:

$$\frac{\partial U}{\partial q} = - U_q \frac{\partial P}{\partial q} + U_q - M_Y \frac{\partial Y}{\partial q}$$

$$= U_q \left(1 - \frac{\Psi_1}{\Omega} \right) - M_Y \, Q_P \, \frac{\Psi_1}{\Omega}$$

$$\text{Hence} \quad \frac{\partial U}{\partial q} \gtrless 0 \quad \text{if} \quad U_q \gtrless \frac{\Psi_1 \, M_Y \, Q_P}{\Omega - \Psi_1}$$

$$\text{where} \quad 1 > \frac{\Psi_1}{\Omega} \quad \text{or} \quad \Omega - \Psi < 0 \quad \text{and} \quad \Psi_1 < 0$$

$$\text{therefore} \quad \frac{\Psi_1 \, M_Y \, Q_P}{\Omega - \Psi_1} > 0$$

Furthermore, if $U_q > \dfrac{\Psi_1 \, M_Y \, Q_P}{\Omega - \Psi_1}$, then

$$M_Y \, Q_P \, \Psi_1 > U_q \, (\Omega - \Psi_1)$$

Settung now Ω and Ψ_1 into the above inequality, we have:

$$U_q > \frac{M_Y \, Q_P \, D_q \, A_{21}}{A_{11}(L_P+L_Y)+A_{21}(b_2+Z)} > 0 \qquad \text{(48)} \qquad \text{(Q.E.D.)}$$

The Marshall-Lerner condition, i.e. $U_q > 0$, is necessary but not sufficient for improvement in the trade balance of the pegging country by depreciating its currency to the pegged currency.

(iv) <u>It is more difficult for the pegging country to improve its "global" trade balance by depreciating its currency to the pegged currency the higher the marginal propensity to import of the pegging country is.</u>

To prove this proposition we differentiate

$\Gamma = \dfrac{\Psi_1 \, M_Y \, Q_P}{\Omega - \Psi_1}$ partially with respect to M_Y

$$\frac{d\Gamma}{dM_Y} = \frac{\Psi_1 \, Q_P}{(\Omega - \Psi_1)^2}\left[\Omega - \Psi_1 - M_Y \, Q_P (A_{11}+A_{21})\right] \qquad \text{(49)}$$

$$= \frac{\Psi_1 \, Q_P}{(\Omega - \Psi_1)^2}\left[(\Delta - \Psi_1)+Q_P A_{11} L_Y + Q_P A_{21}(b_2 - M_Y)\right] > 0$$

where $\Delta - \Psi_1 < 0$ and

$$b_2 - M_Y = 1 - C_Y \, (1 - T_Y) > 0 \qquad \text{(Q.E.D.)}$$

(v) <u>It is more difficult for the pegging country to improve its "global" trade balance by depreciating its currency to the pegged currency the higher the elasticity of its aggregate supply is</u>.

This can easily be seen by differentiating partially in respect to Q_P :

(50) $$\frac{d\Gamma}{dQ_P} = \frac{(\Delta - \Psi_1)\Psi_1 \; M_Y}{(\Omega - \Psi_1)^2} > 0$$ (Q.E.D.)

This is due to the fact that the higher the elasticity of the aggregate supply is the higher is the positive effect of a depreciation of the pegging currency to the pegged currency on the real income and this has a negative effect on the global balance of trade.

(vi) <u>A depreciation of the pegging currency to the pegged currency will increase the real output and the employment in the pegging country</u>

Since $$\frac{\partial Q}{\partial q} = \frac{\partial Q}{\partial P}\frac{\partial P}{\partial q} > 0$$

and

$$\frac{\partial N}{\partial q} = \frac{Q_P}{W}\frac{\Psi_1}{\Omega} > 0$$

C. The Effects of Change in Exchange Rate of the Floating Currency to the pegged Currency:

(i) An appreciation of the floating currency to the pegged currency and hence to the pegging currency has non negative effect on the real income and the employment, if

(51) $$\frac{D_u^{ac}}{M^{ac}} > \frac{A_{11}}{A_{11} + A_{21}}$$

or if the Marshall-Lerner condition holds for the bilateral trade balance of the pegging country to the floating country.

Since $\frac{\partial Y}{\partial e} = - \frac{Q_p}{\Omega} \Psi_2$ $\qquad \frac{\partial Y}{\partial e} \leq 0$

if and only if $\Psi_2 = A_{11}U_u^{ac} + A_{21}D_u^{ac} < 0$

i.e. $\frac{D_u^{ac}}{M^{ac}} > \frac{A_{11}}{A_{11}+A_{21}}$

If $U_u^{ac} > 0$ (the Marshall-Lerner condition for the bilateral trade balance of the pegging country to the floating country) holds, then $\Psi_2 < 0$ is always fulfilled, since $A_{11}U_u^{ac} < -A_{21}D_u^{ac}$

(ii) An appreciation of the floating currency to the pegged currency and hence to the pegging currency has inflationary effect if the Marshall-Lerner condition holds for the bilateral trade balance of the pegging country to the floating country.

(iii) An appreciation of the floating currency to the pegged currency and hence to the pegging currency will improve the balance of trade of the pegging currency, if the following condition holds:

$$(52)\qquad U_u^{ac} < \frac{A_{21}D_u^{ac}(U_q+Q_P\ M_Y)}{A_{11}(L_P+Q_PL_Y)+A_{21}(A_{12}+b_2Q_P)}$$

This assertion can easily be proved, since

$$(53)\qquad \frac{\partial U}{\partial e} = \frac{\Psi_2}{\Omega}(\ U_q + M_Y\ Q_P\ ) - U_u^{ac}$$

$$= -\ U_u^{ac}\frac{A_{11}(L_P+Q_PL_Y)+A_{21}(A_{12}+b_2Q_P)}{\Omega} + \frac{A_{21}D_u^{ac}(U_q+M_YQ_P)}{\Omega}$$

Therefore

$$\frac{\partial U}{\partial e} \lesseqgtr 0, \quad \text{if} \quad U_u^{ac} \gtreqless \frac{A_{21}D_u^{ac}\ (\ U_q + M_Y\ Q_P\ )}{A_{11}(L_P+Q_PL_Y)+A_{21}(A_{12}+b_2Q_P)}$$

Remark: If the pegging country has sufficiently large deficit of bilateral trade balance to the floating country initially such as that U_u^{ac} is negative, then an appreciation of the floating currency will likely deteriorate the bilateral trade balance of the pegging country to the floating country. Recently several discussions on the effect of the Yen appreciation in US Dollar on the bilateral trade balance of Taiwan to Japan seem to assert that Taiwan would improve her bilateral trade balance to Japan. But according to our proposition the reverse seems to be expected because Taiwan has a high trade deficit to Japan tradionally.

D. The Effects of International Income Transfer :[1]

(i) The international income transfer has positive effect on the real income

Since $$\frac{\partial Y}{\partial Z} = \frac{\Delta + \Psi_4 Q_P}{\Omega} > 0 ;$$

(ii) The international income transfer cannot have positive effect on the price level, if the marginal propensity to import is greater than the marginal propensity to consume

From $$\frac{\partial P}{\partial Z} = -\frac{\Lambda - \Psi_4}{\Omega}$$

$$\frac{\partial P}{\partial Z} \gtreqless 0 \qquad \text{if} \qquad -\Psi_4 \gtreqless -\Lambda$$

$$\text{i.e.} \qquad -A_{11}-A_{21} \gtreqless -b_1A_{11}-b_2A_{21}$$

$$(1-b_2)\frac{A_{21}}{A_{11}} \gtreqless b_1$$

$$\frac{\partial P}{\partial Z} < 0 , \quad \text{if} \quad 1-b_2 = C_Y(1-T_Y)-M_Y < 0$$

$$\text{or if} \quad M_Y > C_Y(1-T_Y) \qquad \text{(Q.E.D.)}$$

where M_Y is marginal propensity to import
$C_Y(1-T_Y)$ is marginal propensity to consume.

In other words, the international income transfer has deflationary effect if the marginal propensity to import is greater than the marginal propensity to consume. Since the marginal propensity to import is generally smaller than the marginal propensity to consume, an international

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1) In this section we consider only the case that the international income transfer is denominated in terms of the pegging currency.

income transfer will normally have inflationary effect. This inflationary effect enables real transfer in goods from an international income transfer.

(iii) <u>The international income transfer cannot have positive effect on the output and therefore on the employment if the marginal propensity to import is greater than the marginal propensity to consume</u>

Since $$\frac{\partial Q}{\partial Z} = - \frac{\Lambda - \Psi_4}{\Omega} Q_P$$

similar to the effect of international income transfer on the price level, we can show

$$\frac{\partial Q}{\partial Z} < 0 \ , \quad \text{if} \quad M_Y > C_Y \ (1-T_Y)$$

From the fact that the marginal propensity to import is generally not greater than the marginal propensity to consume a negative effect of international income transfer on the output seems not to be expected.

(iv) <u>An international income transfer will in general deteriorates the balance of trade.</u> Since an international income transfer has generally positive effect on real income as well as on price level. Both of these will deteriorates the balance of trade.

E. The Effects of a Change in Output Price of the Pegged Country:

(i) An increase in the output price of the pegged country has positive effect on the real income in the pegging country if the Marshall-Lerner condition holds for the bilateral trade balance of the pegging country to the pegged country.

(ii) An increase in the output price of the pegged country has a positive effect on the output price of the pegging country if the Marshall-Lerner condition holds for the bilateral trade balance of the pegging country to the pegged country.

Since $\frac{\partial Y}{\partial p^b} = \frac{Q_p}{\Omega} \psi_3$, $\frac{\partial Y}{\partial p^b} > 0$ if and only if $\psi_3 < 0$

and $\frac{\partial P}{\partial p^b} = \frac{\psi_3}{\Omega}$, $\frac{\partial P}{\partial p^b} > 0$ if and only if $\psi_3 < 0$

but $\psi_3 < 0$, if $U_q^{ab} > 0$ and $D_q^{ab} > 0$

where $D_q^{ab} > 0$ is assumed to hold (Q.E.D.)

(iii) An increase in the output price of the pegged country will improve the bilateral trade balance of the pegging country to the pegged country or deteriorate the bilateral trade balance of the pegged country to the pegging country if the following condition holds

(54) $$U_q^{ab} > \frac{(U_q+Q_PM_Y)A_{21}D_q^{ab}}{A_{11}(L_P+L_YQ_P)+A_{21}(A_{12}+b_2Q_P)} > 0$$

Since

$$\frac{\partial U}{\partial p^b} = -(U_q+Q_P M_Y)\frac{\Psi_3}{\Omega} + U_q^{ab}$$

$$= U_q^{ab}\left(1-\frac{U_q+Q_P M_Y}{\Omega}A_{11}\right)-(U_q+Q_P M_Y)\frac{A_{21}D_q^{ab}}{\Omega}$$

$$= \frac{A_{11}(L_P+Q_P L_Y)+A_{21}(A_{12}+b_2 Q_P)}{\Omega}U_q^{ab} - \frac{(U_q+Q_P M_Y)A_{21}D_q^{ab}}{\Omega}$$

Hence $\frac{\partial U}{\partial p^b} \gtreqless 0$, if

$$U_q^{ab} \gtreqless \frac{U_q+Q_P M_Y)A_{21}D_q^{ab}}{A_{11}(L_P+Q_P L_Y)+A_{21}(A_{12}+b_2 Q_P)} > 0 \qquad \text{(Q.E.D.)}$$

F. The Effects of Change in Output Price of the Floating Country

(i) An Inflation in the floating country influences the real income of the pegged country positivelly, if the Marshall-Lerner condition holds for the bilateral trade balance of the pegged country to the floating country

Since $\frac{\partial Y}{\partial p^c} = \frac{Q_P}{\Omega}\cdot\Psi_2$

$$\frac{\partial Y}{\partial p^c} \gtreqless 0 , \quad \text{if} \quad \Psi_2 \lesseqgtr 0$$

For $U_u^{ac} > 0$, we have $\Psi_2 < 0$ and hence

$$\frac{\partial Y}{\partial p^c} < 0 \qquad \text{(Q.E.D.)}$$

(ii) An increase in the output price of the floating country will transmit to the output price of the pegging country, if the Marshall-Lerner condition holds for the bilateral trade balance of the pegging country to the floating country

Since $\frac{\partial P}{\partial P^c} = \frac{\Psi_2}{\Omega}$

If $U_u^{ac} > 0$, then $\Psi_2 < 0$ and therefore $\frac{\partial P}{\partial P^c} > 0$

(iii) An increase in the output price of the floating country will improve the bilateral trade balance of the pegging country to the floating country, i.e. deteriorate the bilateral trade balance of the floating country to the pegging country, if the following condition holds:

$$(55) \qquad U_u^{ac} > \frac{A_{21} D_u^{ac} \; (U_q + Q_P M_Y)}{A_{11}(L_P + Q_P L_Y) + A_{21}(A_{12} + b_2 Q_P} > 0$$

Since $\frac{\partial U}{\partial P^c} = - \frac{\Psi_2}{\Omega} (U_q + M_Y Q_P) + U_u^{ac}$

$$= U_u^{ac} \frac{A_{11}(L_P + Q_P L_Y) + A_{21}(A_{12} + b_2 Q_P)}{\Omega} - \frac{A_{21} D_u^{ac} (U_q + M_Y Q_P)}{\Omega}$$

hence $\frac{\partial U}{\partial P^c} \gtreqless 0$, if

$$\frac{A_{21} D_u^{ac} \; (U_q + M_Y Q_P)}{A_{11}(L_P + Q_P L_Y) + A_{21}(A_{12} + b_2 Q_P)} \gtreqless 0 \qquad \text{(Q.E.D.)}$$

V. The Stability of the Steady-State Equilibrium:

In the long-run we have to consider the capacity effect of investment as well as the income transfer effect of international capital movements. Assuming that the capital-labor-ratio is constant in the long-run equilibrium the stability of our model can be studied from the following system of differential equations: [1)]

$$(56) \qquad \dot{K} = I\,(r) - n\,K$$

$$(57) \qquad \dot{Z} = r\,V\,(r)$$

Linearizing these differential equations around the equilibrium and using the solution (32) for dr we have the linearized homogenous differential equations after some manipulation:

$$(58) \qquad \dot{K} = \frac{I_r}{\Delta}\left[(b_{11}A_{22}-b_{21}A_{12})-n\right]dK + \frac{I_r}{\Delta}(b_{22}A_{12}-b_{12}A_{22})\,dZ$$

$$(59) \qquad \dot{Z} = \frac{z}{\Delta}(b_{11}A_{22}-b_{21}A_{12})\,dk + \frac{z}{\Delta}(b_{22}A_{12}-b_{12}A_{22})\,dZ$$

where $b_{11} = \left[1-C_Y(1-T_Y)+M_Y\right] Q_K > 0$

$b_{12} = C_Y(1-T_Y)-M_Y$

$b_{21} = (M_Y+L_Y)Q_K > 0$

$b_{22} = 1-M_Y-L_Y$

$z = r\,V_r+V > 0$

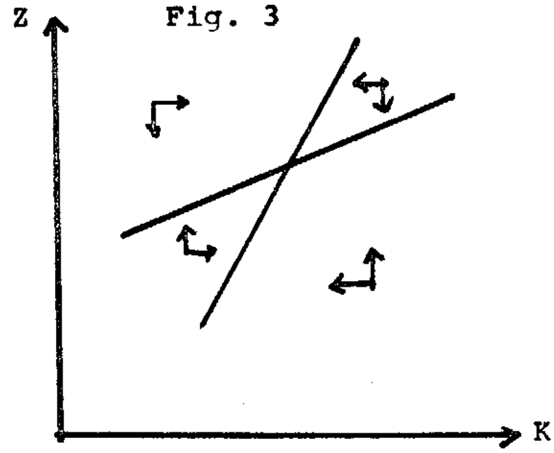
---

1) If the capital-labor-ratio $k=\frac{K}{L}$ is hold constant in the long-run, i.e. $\dot{k}=0$, then the capital stock has to keep the same growth rate as that of labor, i.e. $\frac{\dot{K}}{K}= n$
Hence the equilibrium growth of capital stock is described by $\dot{K}= n\,K$

From the above system of homogenous differential equations the local stability of the long-run equilibrium (steady-state growth path) can be shown by the phase diagram in Fig. 3

Both the schedule of $\dot{K} = 0$ and $\dot{Z} = 0$ have positive slope. But the schedule of $\dot{Z} = 0$ has steeper slope than that of $\dot{K} = 0$, if the growth rate of labor force is positive. This is the case shown in Fig. 2.

Fig. 3



The up- and downward arrows show the direction of time path in the two regions divided by the schedule of $\dot{Z} = 0$ for $\frac{Z}{\Delta}(b_{22}A_{12}-b_{12}A_{22}) < 0$. The right-and leftward arrows show the direction of time path in the two regions divided by the schedule of $\dot{K} = 0$ for $\frac{I_r}{\Delta}(b_{11}A_{22}-b_{21}A_{12}) - n < 0$

We shall now prove:

"The long-run equilibrium is local stable, if the following condition holds

$$\frac{\sigma \alpha I_r}{\Delta}(M + Z - L_p) > n \text{ "}$$

where $\quad \sigma = \frac{Q}{K}$ output-capital-ratio

$\alpha = \frac{K\,Q_K}{Q}$ capital share

The system of first order differential equations is stable, if the following conditions are fulfilled:

(61) $\frac{I_r}{\Delta}(b_{11}A_{22}-b_{21}A_{12})-n+\frac{z}{\Delta}(b_{22}A_{12}-b_{12}A_{22}) < 0$ and

(62) $-\frac{zn}{\Delta}(b_{22}A_{12}-b_{12}A_{22}) > 0$

These conditions hold if the following conditions are fulfilled:

$$b_{22}A_{12}-b_{12}A_{22} > 0$$

$$b_1A_{12}-b_2A_{22} > \frac{n}{I_r}\frac{\Delta}{Q_K}$$

where $b_1 = M_Y + L_Y$

$b_2 = 1 - C_Y (1-T_Y) + M_Y$

$b_{22} = 1 - b_1$

$b_{12} = 1 - b_2$

If the above two conditions are fulfilled then we have

$$Q_K \frac{I_r}{\Delta}(A_{12} - A_{22}) > n \qquad \text{or}$$

$$Q_K \frac{I_r}{\Delta}(M + Z - L_P) > n \qquad \text{Q.E.D.}$$

Remark: Since the annual growth rate of labor is generally lower than 0,03. The stability condition is to be fulfilled if $M + Z > L_P$ holds.

VI. Concluding Remarks:

In this paper we study the partially pegging exchange rate policy of a small open economy whose economic activities have no feed-back effects through the world market, since the economic activities of the other countries cannot be influenced by the small country.

The effects of stabilization policy in the pegging country are not influenced by the partially pegging policy. Generally an increasing in government expenditure (or an expansive policy) as well as in money supply will have positive effects on the real income, output as well as employment. The effect of a depreciation of the pegging currency on the trade-balance or the balance of payments in the pegging country depend mainly on the elasticities of import demand in the pegging, the pegged and the floating country. The usual Marshall-Lerner condition for the global trade is in general not sufficient for an improving trade balance by a depreciation of the pegging currency.

Important results are observed in the bilateral trade balance between the pegging, the pegged and the floating country. In this relation the Marshall-Lerner condition for the bilateral trade balance is important for the effects appreciation or depreciation of the floating currency on the trade balance or the balance of payments in the pegging country. But the Marshall-Lerner condition seems difficult to hold for the bilateral trade balance of the pegging country, even if this is fulfilled for the global trade balance. Therefore it is probable that an appreciation of the floationg currency may have negative effect on the trade balance of the pegging country, especially if the bilateral trade between the pegging and the floating country is not equalized initially. Hence from economic view points the pegging country has to choice that country as pegged country to which the bilateral trade of the pegged country is characterized by "low" elasticities. In this case the bilateral trade balance will rarely be affected by changing in exchange rate.

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