

Universität Bielefeld/IMW

**Working Papers
Institute of Mathematical Economics**

**Arbeiten aus dem
Institut für Mathematische Wirtschaftsforschung**

Nr. 49

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A Note on the Hicks Theory of
Strike Bargaining

September 1976



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an der

Universität Bielefeld

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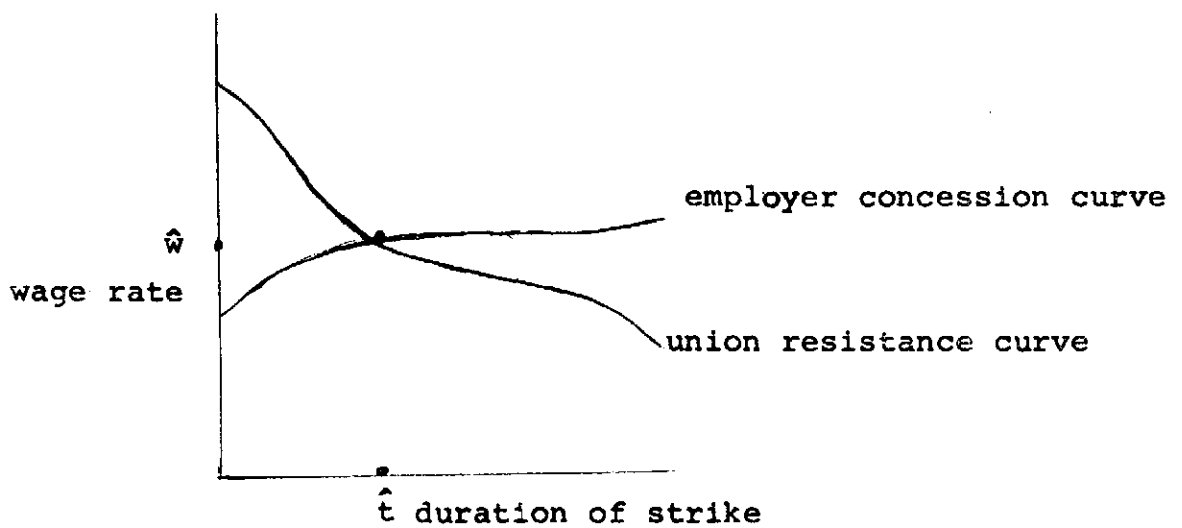
by Peter Rice

Introduction

As part of his general theory of wages, J.R. Hicks develops a theory of strike bargaining. Given the strike costs for labor and management and their initial wage demands, the theory predicts a wage settlement. This wage is then incorporated into a larger theory, being not only the wage settlement of a strike but also the expected wage outcome of bargaining that does not lead to a strike. The strike bargaining theory is not realistic because it ignores the strategic, game theoretic aspect of bargaining. This paper develops a strike bargaining theory as a game and, with an application of limited rationality, predicts a wage settlement (different from Hicks'). This theory fits neatly into Hicks' general theory of wages, replacing the strike bargaining theory.

The Hicks Theory of Strike Bargaining

In the Hicks bargaining theory each of the bargaining agents has two costs which determine his strategy. The employer calculate the cost, C_1 , of a strike of duration t and the cost, W_1 , of agreeing to a wage of w . C_1 is the sum of lost profits and other costs associated with a strike, and W_1 is the cost over the duration of the wage contract of the wage w when compared to an ideal wage rate w_0 . The curve $W_1 = C_1$ is the employer's concession curve and represents the maximum wage w that an employer is willing to pay to avoid a strike of duration t . The union has the cost, C_2 , of lost wages associated with a strike of duration t and a benefit, W_2 , for agreeing to a wage w for the contract period. The union's resistance curve, $W_2 = C_2$, gives the minimum wage rate w that the union would agree to in order to avoid a strike of duration t .



Hicks argues that these two curves intersect at a point (\hat{t}, \hat{w}) . \hat{t} is the expected duration of the strike and \hat{w} the

wage both would agree to in order to avoid such a strike. In order to assert that \hat{w} is a reasonable pre-strike bargain, he argues that, were a strike to occur, each bargainer would keep his wage offers on this curve. For example, the union bargainer would consider wages lost up to time t_0 as irretrievable. Looking at the situation at time t_0 , he is willing to agree to a wage rate w for which $C_2(t-t_0) = W_2(w)$. That is, if the duration of the strike beyond t_0 is $t-t_0$, w is the wage rate that balances the expected cost of continuing the strike and the benefit of the wage w . If the strike were to end at t_0 , w would be w_0 , where $C_2(t_0) = W_2(w_0)$, a point on the union's resistance curve. To agree to a wage $w < w_0$ would be to "sell out" the union membership, and w_0 is clearly acceptable, so the union offers a wage rate of w_0 at time t_0 . The argument that the employer makes wage offers on his concession curve is similar.

The expected result of the strike bargaining process is, therefore, \hat{w} and this justifies \hat{w} as a reasonable pre-strike bargaining result, the idea being that experienced negotiators will expect a wage rate which, according to their experience, would be the result of strike bargaining.

This model of the strike bargaining process ignores the fact that a game is being played and the offers of each bargainer will depend on what has been offered by both parties in the past. The most desirable method of choosing a reasonable pre-strike wage settlement is to solve the strike bargaining game and apply the principal that experienced bargainers will accept the solution as a reasonable pre-strike

settlement.

A Strike-Bargaining Game

As a result of pre-strike bargaining management begins the strike period with a wage offer of V_0 , and the union with a wage demand of U_0 , $U_0 > V_0$. The union's payoff function,

$$f(t,w) = -C_2 + W_2$$

is a C function such that $f_t < 0$, $f_w > 0$. The management payoff function

$$g(t,w) = -C_1 - W_1$$

is a C^1 function such that $g_t < 0$, $g_w < 0$. In the play of the game, the union makes a wage demand of $y(t)$ at each time $t \geq 0$ and management makes a wage offer of $z(t)$. y and z are required to be upper semi-continuous, y is non-increasing and z is non-decreasing. Since y and z are upper semi-continuous, there is a first time \bar{t} such that $y(\bar{t}) \leq z(\bar{t})$ or $\bar{t} = \infty$. If $\bar{t} < \infty$, the game ends at \bar{t} with wage rate $\bar{w} = \frac{1}{2} y(\bar{t}) + \frac{1}{2} z(\bar{t})$.

Because of the discontinuities in y and z , a formulation of this game as a classical differential game presents certain difficulties but a classical formulation of a strategy in a differential game can be applied. Let $\delta > 0$ and define

$$I_n = [(n-1)\delta, n\delta], n = 1, 2, 3, \dots$$

Let $Y_j(Z_j)$ be the set of all upper semi-continuous non-increasing (non-decreasing) real valued functions defined on I_j . Let

$$E_{\delta,1} \in Y_1, \Delta_{\delta,1} \in Z_1,$$

$$\Gamma_{\delta,j} : Y_1 \times Z_1 \times \dots \times Y_{j-1} \times Z_{j-1} \rightarrow Y_j \quad j > 1$$

$$\Gamma_j^\delta : Y_1 \times Z_1 \times \dots \times Y_{j-1} \times Z_{j-1} \times Z_j \rightarrow Y_j$$

$$\Delta_{\delta,j} : Y_1 \times Z_1 \times \dots \times Y_{j-1} \times Z_{j-1} \rightarrow Z_j \quad j > 1$$

$$\Delta_j^\delta : Y_1 \times Z_1 \times \dots \times Y_{j-1} \times Z_{j-1} \times Y_j \rightarrow Z_j.$$

Define $\Gamma_\delta = (\Gamma_{\delta,1}, \Gamma_{\delta,2}, \dots)$

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$$\Delta_\delta = (\Delta_{\delta,1}, \Delta_{\delta,2}, \dots)$$

$$\Delta^\delta = (\Delta_1^\delta, \Delta_2^\delta, \dots).$$

These are the upper and lower δ -strategies for the two players.

Given a pair of δ -strategies $(\Gamma^\delta, \Delta_\delta)$, there are a pair of controls

$y(\Gamma^\delta, \Delta_\delta), z(\Gamma^\delta, \Delta_\delta)$ defined by

$$z_1(\Gamma^\delta, \Delta_\delta) = \Delta_{\delta,1}$$

$$y_j(\Gamma^\delta, \Delta_\delta) = \Gamma_j^\delta(y_1, z_1, \dots, y_{j-1}, z_{j-1}, z_j)$$

$$z_j(\Gamma^\delta, \Delta_\delta) = \Delta_{\delta,j}(y_1, z_1, \dots, y_{j-1}, z_{j-1})$$

$$y(\Gamma^\delta, \Delta_\delta) = y_j \text{ on } I_j$$

$$z(\Gamma^\delta, \Delta_\delta) = z_j \text{ on } I_j$$

The controls $y(\Gamma_\delta, \Delta_\delta^\delta), z(\Gamma_\delta, \Delta_\delta^\delta)$ are defined similarly.

A strategy is a set $\bar{\Gamma} = \{\Gamma^\delta | \delta > 0\}$ or $\underline{\Delta} = \{\Delta_\delta | \delta > 0\}$, etc.

An outcome for a pair of strategies $(\bar{\Gamma}, \underline{\Delta})$ is a pair of functions (y, z) such that there exists a sequence $\delta_k \rightarrow 0$ such that

$$y = \lim_{k \rightarrow \infty} y(\Gamma^{\delta_k}, \Delta_{\delta_k})$$

$$z = \lim_{k \rightarrow \infty} z(\Gamma^{\delta_k}, \Delta_{\delta_k})$$

and y and z are upper semi-continuous. The termination point (\bar{t}, \bar{w}) for an outcome (y, z) is defined as

$$\bar{t} = \inf \{t \mid y(t) \leq z(t)\} \quad (\bar{t} = \infty \text{ if the set is empty})$$

$$\bar{w} = \frac{1}{2}y(\bar{t}) + \frac{1}{2}z(\bar{t}) \quad \text{if } \bar{t} < \infty$$

$$\bar{w} = \frac{1}{2} \lim_{t \rightarrow \infty} y(t) + \frac{1}{2} \lim_{t \rightarrow \infty} z(t) \quad \text{if } \bar{t} = \infty$$

The payoff $P(y, z)$ is a pair of real numbers:

$$\begin{aligned} P(y, z) &= (P_1, P_2) = (f(\bar{t}, \bar{w}), g(\bar{t}, \bar{w})) \text{ if } \bar{t} < \infty \\ &= (-\infty, -\infty) \text{ if } \bar{t} = \infty \end{aligned}$$

(The payoffs for $t = \infty$ guarantee that $t = \infty$ will never be the termination point of a pair of equilibrium strategies.)

A stable equilibrium outcome is a pair (y^*, z^*) which is the only outcome for some pair $(\bar{\Gamma}^*, \bar{\Delta}^*)$ and the only outcome for some pair $(\underline{\Gamma}^*, \bar{\Delta}^*)$ and satisfies

$$P_1(y^*, z^*) \geq P_1(y, z)$$

for any outcome (y, z) of any pair $(\bar{\Gamma}, \bar{\Delta}^*)$ or $(\underline{\Gamma}, \bar{\Delta}^*)$, and

$$P_2(y^*, z^*) \geq P_2(y, z)$$

for any outcome (y, z) of any pair $(\underline{\Gamma}^*, \bar{\Delta})$ or $(\bar{\Gamma}^*, \underline{\Delta})$.

These definitions are a slight alteration of those given by Friedman.

The constant strategy $\Gamma(y)$ associated to a function y is $\Gamma = (\Gamma_1, \Gamma_2, \dots)$, $\Gamma_j = y$ on I_j . A pair (y^*, z^*) is a constant equilibrium outcome when

$$P_1(\Gamma(y^*), \Delta(z^*)) \geq P_1(\Gamma(y), \Delta(z^*)) \text{ for all } y, \text{ and}$$

$$P_2(\Gamma(y^*), \Delta(z^*)) \geq P_2(\Gamma(y^*), \Delta(z)) \text{ for all } z.$$

It is easy to prove that (y^*, z^*) is a stable equilibrium outcome if it is a constant equilibrium outcome.

Classification of Stable Termination Points

If (y^*, z^*) is a stable equilibrium, its termination point will be called a stable termination point. Let \hat{y} and \hat{z} be defined by

$$g(t, \hat{y}(t)) = g(0, U_0)$$

$$f(t, \hat{z}(t)) = f(0, V_0)$$

The conditions on the partials of f and g guarantee that \hat{y} and \hat{z} exist and are class C^1 . From

$$\hat{y} = \int_0^t -g_t/g_w dt + U_0, \quad \hat{z} = \int_0^t -f_t/f_w dt + V_0$$

it is clear that \hat{y} is decreasing and \hat{z} is increasing. Let the termination point for (\hat{y}, \hat{z}) be (\hat{t}, \hat{w}) . If \hat{t} is finite, it is given by

$$\int_0^{\hat{t}} g_t/g_w - f_t/f_w dt = U_0 - V_0$$

The condition that $g_t/g_w - f_t/f_w$ be bounded away from zero would guarantee a finite \hat{t} for all U_0, V_0 . In this case,

$$\hat{w} = \int_0^{\hat{t}} -g_t/g_w dt + U_0 = \int_0^{\hat{t}} -f_t/f_w dt + V_0$$

Define the sets R, R', R'' of termination point by

$$R = \bigcup_{0 \leq t \leq \hat{t}, t < \infty} \{(w, t) \mid \hat{z}(t) \leq w \leq \hat{y}(t)\}$$

$$R' = \bigcup_{0 \leq t < \infty} \{(w, t) \mid \hat{z}(t) > w \geq V_0\}$$

$$R'' = \bigcup_{0 \leq t < \infty} \{(w, t) \mid U_0 \geq w > \hat{y}(t)\}$$

Then the strip $V_0 \leq w \leq U_0$, $0 \leq t$ is the union $R \cup R' \cup R''$.

Theorem If $(t, w) \in R$, it is a stable termination point. If $(t, w) \in R' \cup R''$, it is not a stable termination point.

Proof: Let $(\bar{t}, \bar{w}) \in R$ and define

$$\begin{aligned} \bar{y}(t) &= \hat{y}(t) & 0 \leq t < \bar{t} \\ &= \bar{w} & \bar{t} \leq t \\ \bar{z}(t) &= \hat{z}(t) & 0 \leq t < \bar{t} \\ &= \bar{w} & \bar{t} \leq t \end{aligned}$$

Then (\bar{t}, \bar{w}) is the termination point of the controls (\bar{y}, \bar{z}) .

Assume that management maintains the constant strategy associated with \bar{z} . The payoffs available to the union are the numbers $f(t, \bar{z}(t))$, $0 \leq t < \infty$ or $-\infty$ (if $\bar{z}(t) > V_0$ for all t).

$$\begin{aligned} f(t, \bar{z}(t)) &= f(0, V_0) & \text{for } 0 \leq t < \bar{t} \\ &= f(\bar{t}, \bar{w}) & \text{for } t = \bar{t} \\ &= f(t, \bar{w}) & \text{for } t > \bar{t} \end{aligned}$$

Since $\bar{w} \geq \bar{z}(\bar{t})$, $f_w > 0$, $f_t < 0$,

$$f(\bar{t}, \bar{w}) \geq f(t, \bar{z}(t)), \quad 0 \leq t.$$

The payoff to the union under the constant strategy pair $(\bar{\Gamma}(\bar{y}), \bar{\Delta}(\bar{z}))$ is $f(\bar{t}, \bar{w})$, so the union has no incentive to choose another constant strategy. A similar argument works for management, to prove that (\bar{y}, \bar{z}) is a stable equilibrium outcome.

For the second part of the theorem, let $(\bar{t}, \bar{w}) \in R'$. Let $(\bar{\Gamma}, \bar{\Delta})$ be any strategy pair which has an outcome (y, z) for which (\bar{t}, \bar{w}) is the termination point. Let $\bar{\Gamma}$ be the constant

strategy associated with the function $\hat{y}(t) = V_0, 0 \leq t < \infty$. The termination point for $(\bar{\Gamma}, \bar{\Delta})$ is $(0, \hat{w})$, $V_0 \leq \hat{w} \leq \frac{1}{2}(U_0 - V_0)$ and

$$f(0, \hat{w}) \geq f(0, V_0) \geq f(\bar{t}, V_0) > f(\bar{t}, \bar{w}).$$

Therefore $(\bar{\Gamma}, \bar{\Delta})$ is not an equilibrium under any definition, and (\bar{t}, \bar{w}) is not a stable termination point. A similar argument works for R'' . Finally, note that the payoff $P = (-\infty, -\infty)$ for any termination point with $t = \infty$, and \hat{y} is always worse for the union, so points with $t = \infty$ are never stable termination points.

If $t > 0$, $(0, w)$ is preferred by both players to (t, w) , so $\{(0, w) | V_0 \leq w \leq U_0\}$ is the Pareto Optimal set of stable termination points.

A Solution Theory

As with most bargaining games, this game has too many equilibrium outcomes. At this point a solution theory would usually be invoked to choose one of them as the outcome. However, in games modeling human behavior, it is better to examine expected behavior for a clue to a reasonable solution. These clues lead to behavioral postulates which select one equilibrium outcome.

It is difficult to formulate behavioral postulates because each structure in the definition of a strategy in this game corresponds to a behavioral attitude, and it is difficult to assign a behavioral attitude to a player. For example, if the union has high regard for its own abilities to predict management's next move, it will use the model of the game

where the equilibrium strategies are $(\bar{\Gamma}^*, \bar{\Delta}^*)$. If it feels that the management bargainers are better at guessing subsequent actions, it will use $(\underline{\Gamma}^*, \bar{\Delta}^*)$. If the union tends to react slowly to past moves by the management, it will think in terms of Γ^δ or Γ_δ for large δ . It is possible to avoid these difficulties.

Theorem Let (x^*, y^*) be a stable equilibrium outcome with termination point (\bar{t}, \bar{w}) . Define

$$\begin{aligned} \bar{x} &= x^* & 0 \leq t \leq \bar{t} \\ &= \bar{w} & \bar{t} \leq t \\ \bar{y} &= y^* & 0 \leq t \leq \bar{t} \\ &= \bar{w} & \bar{t} \leq t \end{aligned}$$

Then (\bar{x}, \bar{y}) is a constant equilibrium outcome.

Proof: If not, then one of the players would, for reasons of higher payoff, change to a new constant strategy. Assume this player is the union. It follows that, for some $t_0 < \bar{t}$,

$$f(t_0, \bar{z}(t_0)) = f(t_0, z^*(t_0)) > f(\bar{t}, \bar{w}).$$

Let $(\bar{\Gamma}^*, \bar{\Delta}^*)$ be a strategy pair for which (y^*, z^*) is the only outcome, $\bar{\Gamma}^* = \{\Gamma^\delta\}$, and define

$$\begin{aligned} \Gamma_j^{\prime \delta} &= \Gamma_j^\delta & \text{for } j\delta < t_0 \\ \Gamma_j^{\prime \delta} &= \Gamma_j^\delta & \text{for } (j-1)\delta \leq t_0 < j\delta, t < t_0 \\ \Gamma_j^{\prime \delta} &= \text{constant strategy} & y(t) = z^*(t_0), t > t_0 \\ \bar{\Gamma}' &= \{\Gamma^{\prime \delta}\} \end{aligned}$$

It is clear that the termination point of the outcome of $(\bar{\Gamma}', \bar{\Delta}^*)$ is $(t_0, z^*(t_0))$. Therefore (y^*, z^*) does not satisfy

the requirements for a stable equilibrium outcome.

This theorem and the construction given earlier, which proves that every termination point of a stable equilibrium outcome is attainable with constant strategies, shows that there is a close connection between strategy pairs and constant strategies in this game. This connection justifies the procedure of applying behavioral postulates to constant strategies, and avoids the problem of deciding where in the structure of a strategy the postulate should be applied.

The expected behavior of bargainers in this game has two facets which guarantee a unique outcome. The first is that most bargainers expect, through a sense of fairness, that the opponent will make concessions at least as substantial as they do. When the opponent ceases to make reasonable concessions, the natural (but non-equilibrium) response is to cease to make concessions as well. In the strike bargaining game, concession by the management is measured by the union with $f(t, z(t))$. It is therefore natural to assume that the union will respond to a decreasing $f(t, z(t))$ with $y(t)$ which makes $g(t, y(t))$ decreasing, causing a loss to both players.

Behavioral Postulate I: If (y, z) is a constant equilibrium outcome with termination point (\bar{t}, \bar{w}) , (y, z) will be used only if $y(t) \leq \hat{y}(t)$, $z(t) \geq \hat{z}(t)$ for $0 \leq t \leq \bar{t}$.

Another behavioral characteristic of bargainers is the desire to give away as little as possible while moving toward a reasonable outcome. The amount the union "gives away", for example, is measured by $f(t, y(t))$.

Behavioral Postulate II: For every $t, 0 \leq t \leq \bar{t}$, $f(t, y(t))$ will be a maximum for all $y(t)$ and $g(t, z(t))$ will be a maximum for all $z(t)$.

Theorem: (\hat{y}, \hat{z}) is the only stable equilibrium outcome which satisfies the two behavioral postulates.

Proof: $y \leq \hat{y}$ is required by postulate I, and $f_w > 0$ implies y must be as large as possible from postulate II, so $y = \hat{y}$ for all $t, 0 \leq t \leq \hat{t}$. A similar argument works for z .

The reader will notice that rational behavior in the sense of using an equilibrium strategy is assumed in the theorem, while two non-rational postulates pick out a unique outcome. This is just one example of the application of limited rationality to the analysis of games.

The Game of Incomplete Information

In the foregoing analysis it is assumed that U_0, V_0, f and g are known by both players. In order to determine the conditions under which a strike will actually occur, it is necessary to investigate what happens when some of these factors are unknown to the bargainers. It is assumed that the union knows U_0 and f and the management knows V_0 and g .

It is reasonable to assume that, however bargaining has proceed in the pre-strike period, U_0 and V_0 have been revealed before the strike begins. For **example**, Tietz has discovered that most pre-strike bargaining ends with one or the other bargainer obtaining his "first aspiration level" wage.

Experienced bargainers, knowing this, would end the bargaining with this aspiration level, here called U_0 for the union and V_0 for the management. These are, in some sense, ideal wages which each would set in absence of opposition by the other party. Hicks makes a similar assumption. Therefore, it does not seem reasonable to assume that the union has imperfect information of V_0 or the management of U_0 .

At the end of the pre-strike bargaining and before the beginning of the strike, one or both of the bargainers can be expected to offer a compromise based on an analysis of the game. The union, for example, will guess at g and, knowing f, U_0 and V_0 , will offer the wage outcome \hat{w} as a compromise. If the guess of g is wrong, it may happen that \hat{w} is too high for management to accept because its own guess of the outcome is lower. A strike occurs in this situation.

In the strike itself, the first behavioral postulate ensures that the union will not make demands greater than $\hat{y}(t)$ and the second postulate ensures that the demands will not be less than $\hat{y}(t)$. In fact, the union will begin with a demand which is greater than $\hat{y}(t)$ and the management will refuse to bargain until it is lowered to the acceptable level, $\hat{y}(t)$. In this way, the union avoids too low a demand. As a consequence, the management must inform the union on the nature of its level curve

$$g(t, \hat{y}(t)) = g(0, U_0)$$

Similarly, the union informs management of its level curve, and the strike bargaining proceeds without incomplete in-

formation. Although neither can predict the outcome because information on the level curves is released only as time passes, the play and outcome of the game is identical to the complete information situation.

Conclusion

The game theoretic model of the strike bargaining process presented here, when combined with the argumentation given by Hicks, is a replacement for his theory of industrial disputes. While more realistic in the sense that it recognizes the gaming aspects of the bargaining process, this theory also has some interesting consequences.

Foremost among these is the conclusion that, although instantaneous response to moves by the other bargainer are an integral part of the theory, the game need not be played this way. Since the predicted outcome is a constant strategy pair, the whole plan of the bargaining can be made in advance without allowance for the other player's possible actions. The only situation requiring immediate response occurs when imperfect information on one side must be corrected by the other.

Another consequence is that the union's payoff $f(o, V_o)$ is unaffected by its initial demand U_o , and the management's payoff $g(o, U_o)$ is unaffected by its own initial demand V_o . However, the length of the strike \hat{t} and the negotiated wage \hat{w} depend on management and union payoff functions and initial offers, and are sensitive to such things as union strength, union wealth and worker dissatisfaction. Thus, in the factors

for which this model would be used to analyse strike outcomes,
it offers no surprises.

A consequence worth mentioning is the effect of incomplete information. In most games, incomplete information introduces a complication which alters the player's strategies. In this game, there is no change in the strategies or the outcome. Incomplete information only explains the occurrence of strikes.

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