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1. Introduction

Since the so called "oil-embargo" in 1973 the energy-problem has received particular attention in economics. The analysis of the influences of energy on economic activities can be undertaken by several methods, such as input-output analysis, etc.[1)] It appears to be difficult to use the neoclassical macroeconomic model to explain the influences of the change of energy price or of the limited energy supply, because the usual neoclassical production function with two production factors (capital and labor forces) does not consider the energy problem explicitly. This limitation of the neoclassical macroeconomic model can be removed by the introduction of a different kind of production function which does not assume full employment of the capital stock. This production function may be called a capacity utilization production function. There, production depends on employment and on the effective capital stock which may be smaller than the available capital stock. It is assumed that the available supplies of energy and productive materials limit the degree of capacity utilization and thereby determine the effective capital stock.

In this paper I want to show the influences of the energy price on aggregate income, price level, employment, wage rate and degree of capacity utilization. I shall concentrate on the supply side of the economy, since the effects of energy on macroeconomic activity seem to occur

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* I am indebted to Professor Reinhard Selten for helpful comments and to Professor Lawrence Nitz who read the manuscript. Any errors are solely my responsibility.

1) Tintner, Gerhard, Deutsch, Edwin and Rieder, Robert (1975)

through aggregate supply.

The energy price is an exogeneous variable in the model, since I do not try to explain the determination of the energy price but the influences of a change of the energy price. In practice in many economies the energy price is set by the government exogeneously.

2. The Model:

Suppose the production of the economy considered can be described by the following equation:

$$Y = F(B,L) \qquad \ldots (1)$$

with $F_B > 0$, $F_L > 0$, $F_{BB} < 0$, $F_{LL} < 0$

$F_{BL} = F_{LB} > 0$; F is homogeneous of degree one

where the symbols have the following meaning:

B: capital input; Y: aggregate product

L: labor input; K: capital stock

We use the following notations:

$$F_B = \frac{\partial Y}{\partial B} \cdot \quad F_{BB} = \frac{\partial^2 Y}{\partial B^2} \text{ etc.}$$

$$B = \lambda \cdot K \qquad \ldots (2)$$

with $1 \geqq \lambda > 0$

Where λ is the "coefficient of capacity utilization". For $\lambda = 1$ the existing capital stock is fully used in production. For the sake of simplicity we assume $0 \leqq \lambda \leqq 1$, although one might wish to consider cases where $\lambda > 1$ is permitted, in order to describe a state of "capacity overutilization".

If the effective demand of the economy considered is always high enough for full capacity employment, the coefficient of capacity utilization is assumed to be:

$$\lambda = \min \{ 1, \frac{J}{K} \} \qquad \ldots (3)$$

Where J is the energy used in the production. The unit of energy is chosen in such a way that one unit of energy is required for the utilization of one unit of capital.

The input of energy is necessary for the use of the capital stock in the production process. In the short run the capital stock is fixed. Whether the existing capital is fully utilized in the production process, is dependent on the energy input.

Suppose the energy price and the monetary wage rate are given; then the demand of energy for production can be described by:

$$J = \min \{ (Y - \frac{W}{P} L) / q, K\} \qquad \ldots (4)$$

Where P = the price level of production
W = the monetary wage rate
q = the real energy price where the price level of production is used as numeraise

Equation (4) says that the energy requirement for production is proportional to the capital stock employed.

The energy requirement for production is equal to the existing capital stock if the capital stock is fully employed in production. The energy requirement is derived under the condition that the marginal productivity of energy input is equal to the real energy price, if the existing capital stock is not fully employed.[1)]

Suppose further that the demand for labor is such that the marginal productivity of labor is equal to the real wage rate

$$F_L\ (B,L) = \frac{W}{P} \qquad \ldots (5)$$

While generally there are no discrepancies between the classical and the Keynesian school with respect to the labor demand functions the literature is filled with debates as to whether the supply of labor depends on the money wage rate or the real wage rate. I do not wish to discuss the labor supply in detail, but I shall use three markedly different cases of labor supply to illustrate the effect of the energy price on macroeconomic activity[2)]:

(i) Case A: the labor supply is an increasing function of the real wage rate:

$$L = N(\frac{W}{P}) \qquad \ldots (6)$$

with $\frac{dL}{d(\frac{W}{P})} = N' > 0$

(ii) Case B: the labor supply is an increasing function of the money wage rate

$$L = M(W) \qquad \ldots (7)$$

with $\frac{dL}{dW} = M' > 0$

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1) If $\frac{J}{K} < 1$, $Y = F(L,J)$; $F_J = q \Rightarrow J = B = (Y - L \cdot \frac{W}{P}) / q$
because of the constant returns to scale we have: $Y = L \cdot F_L + J \cdot F_J$ so that $F_J = (Y - L \cdot F_L)/J = q$. Substituting $F_L = \frac{W}{P}$ gives $J = (Y - \frac{W}{P} \cdot L)/q$.

2) See: Allen, R.G.D. (1968)
Bowers, D.A. & Baird, R.N. (1971)

(iii) Case C: As in case B the labor supply is a function of the money wage rate, but the wage rate is fixed exogeneously at a level where the labor supply is greater than the labor demand.

$$W = W^{O} \qquad \ldots (8)$$

Our model will be completed by the aggregate demand function:[1]

$$Y = H(P,G) \qquad \ldots (9)$$

with $H_p < 0$, $H_G > 0$, $H_Q > 0$

where G is given exogeneously.

And the equilibrium condition for aggregate demand and aggregate supply is given:[2]

$$Y = S(P) = H(P) \qquad \ldots (10)$$

i.e. the aggregate income is realized at the price level where the aggregate demand is equal to the aggregate supply.

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1) The aggregate demand function (9) is derived from the commodity market and the money market.

See: Bowers, David A. and Baird, Robert N. (1971)

2) The aggregate supply function (S) is a function the price level and derived from the production function and the labor market. The function S differs according to the distinct cases of labor supply assumed by (6), (7) or (8).

See: Bowers & Baird (1971)

3. The Short-run influences of energy

In the short-run the available labor supply $\bar{B}$ and the capital stock are given. The investment has only demand effects.

The short-run solution of our model can be classified into two different cases:

(3A) Case one: $\lambda = 1$

This is the usual case considered in the macroeconomic literature.[1)]

The aggregate income and the price level are determined as shown in Fig.1. The labor employment can be calculated easily if we substitute P^* and Y^* in the labor supply and labor demand function.

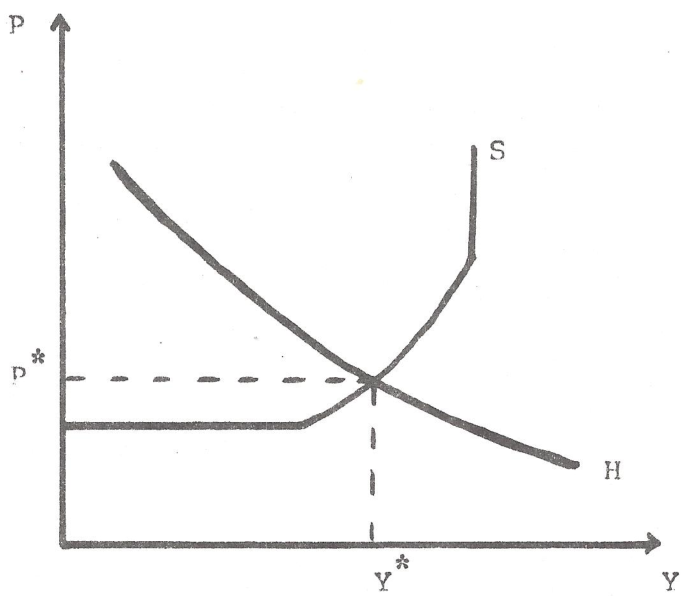

Fig. I

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1) Allen, R.G.D. (1968): Chapter 6 & 7
Bowers, David A. & Baird, Robert N. (1971), Chapter 10.

For given $\frac{W}{P}$ and K we can deduce the energy price for the case: $\lambda = 1$, i.e. the capital stocks are fully employed. From (4) we have

$$(Y-\frac{W}{P}\cdot L)/\sigma \cdot K \geq 1 \qquad \text{or}$$

$$\sigma \leq (Y-\frac{W}{P}\cdot L) \;/\; K = \frac{1}{k}(\frac{Y}{L} - \frac{W}{P})^{1)} \qquad \ldots (11)$$

Because of the linear homogenity of the production function, if labor is rewarded by its marginal productivity, so

$$Y - L\cdot F_L = Y - \frac{W}{P} \cdot L = F_K \cdot K$$

The condition (11) is the same as

$$\sigma \leq F_K \qquad \ldots (11a)$$

Condition (11a) is quite clear:
if the energy price is not higher than the marginal productivity of capital then all capital stock available will be employed in the production process.

In this case K and J are constant; the equilibrium aggregate income, price level and labor employment are determined uniquely by well behaved aggregate demand and supply functions.

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1) Here $k = \frac{K}{L}$ is the capital-labor ratio.
This condition is derived from $Y - \frac{W}{P}L - \sigma K \geq o$, i.e. the existing capital will be used for production if it entails no loss. $(Y-\frac{W}{P}L)/K \geq \sigma$.

Let $A = (Y-\frac{W}{P}L)/K$, we see

$$\frac{\partial A}{\partial K} = \frac{F_K\cdot K+F_L\cdot L-F-F_{KL}LK}{K^2} = -\frac{L\cdot F_{KL}}{K} < 0$$

for $F = F_K\cdot K+F_L\cdot L$ (constant return to scale)

In this case (J=K) an increase of energy price will not reduce the aggregate product and the labour employment and the price level will not be influenced if the condition (11a) is not disturbed.

The reduction of energy quantity at the given energy price will reduce the agregate income, labor employment suppose J=K is assumed before quantity reduction. Due to this reduction the available energy quantity becomes insufficient to allow the full employment of the capital stock. The price level will increase since the marginal productivity of labor is reduced as a consequence of the dimished capital stock employed in production due to the energy shortage. With a given monetary wage rate the labor employment decreases for the same reason.

(3 B) Case 2: $\lambda < 1$

In this case the existing capital capacity cannot be fully used for production because of a "high energy price", i.e. $q > F_K$ or because of "energy shortage", i.e. the required energy quantity at the given energy price for full employment of the existing capital stocks is not available.

I shall now examine the influence of an increase of the energy price on the macroeconomic activity. The effects will be investigated for the three different cases of labor supply chosen for illustration.

(i) Case 2A:

The functions (1) to (5), (6), (9) and (10) will be used for the following discussion.

If the energy price is higher than the marginal productivity of the fully employed capital stock, then the demand for labor and for energy for production are determined simultaneously by the real wage rate and energy price. This can be shown easily. The aggregate income, labor employment, energy, the price level and other endogeneous variables are determined by the following simultaneous equations:[1)]

$$Y - F(L,J) = 0 \qquad \ldots (1a)$$

$$L - N\{F_L\ (B,L)\} = 0 \qquad \ldots (12)$$

$$F_J\ (L,J) = q \qquad \ldots (4a)$$

$$Y - H\ (P,G) = 0 \qquad \ldots (7)$$

To analyse the short-run effects of a rise of energy price we differentiate the above simultaneous equations at the equilibrium values:

$$dP = \frac{D_1}{D} \qquad \ldots (13)$$

$$dL = \frac{D_2}{D} \qquad \ldots (14)$$

$$dJ = \frac{D_3}{D} \qquad \ldots (15)$$

$$DY = H_P \cdot \frac{D_1}{D} + H_G\ d\,G \qquad \ldots (16)$$

$$dW = \frac{W}{P} \cdot dP + \frac{P}{N'}\ dL \qquad \ldots (17)$$

1) Using structural functions of the labor market: $F_L = \frac{W}{P}$ and $L = N(\frac{W}{P}) = N(w)$, where $w = \frac{W}{P}$, we have $L = N(F_L)$.

We substitute (7) in the (1a) and take total differentials of the equation system:

$$H_p\ dP - F_L dL - F_J dJ = -\ H_G dG$$

$$F_{LJ}\ dL + F_{JJ}\ dJ = dq$$

$$(1-N'F_{LL})dL - N'F_{LJ}\ dJ = 0$$

Where $D = H_p \left(- F_{JJ} - N' (F_{LJ}^2 - F_{LL} F_{JJ})\right) < 0$

since $H_p < 0$, $F_{JJ} < 0$, $N' > 0$

and $F_{LJ}^2 - F_{LL} F_{JJ} < 0$

$$D_1 = H_G \left(F_{LJ}^2 \; N' + F_{JJ} (1-N'F_{LL})\right) dG$$

$$- \left[N' F_L F_{LJ} + F_J (1-N' F_{LL})\right] dq$$

$$D_2 = - H_p N' F_{LJ} \, dq$$

$$D_3 = - (1-N' F_{LL}) H_p \, dq$$

From equations (13) to (17) we see:

$$\frac{\partial P}{\partial q} = \frac{-1}{D}\left[N' F_L F_{LJ} + F_J (1-N'F_{LL})\right] > 0 \quad \ldots \quad (18)$$

$$\frac{\partial P}{\partial G} = \frac{H_G}{D}\left(F_{JJ} + N' (F_{LJ}^2 - F_{LL} F_{JJ})\right) > 0 \quad \ldots \quad (19)$$

because $H_G > 0$

$$\frac{\partial L}{\partial q} = \frac{-H_P}{D} \cdot N' F_{LJ} < 0 \quad \ldots \quad (20)$$

$$\frac{\partial L}{\partial G} = 0 \quad \ldots \quad (21)$$

$$\frac{\partial J}{\partial q} = \frac{-H_P}{D} (1-N' F_{LL}) < 0 \quad \ldots \quad (22)$$

$$\frac{\partial J}{\partial G} = 0 \quad \ldots \quad (23)$$

$$\frac{\partial Y}{\partial q} = \frac{-H_P}{D} \left(N' F_L F_{LJ} + F_J (1-N' F_{LL})\right) < 0 \quad \ldots \quad (24)$$

$$\frac{\partial Y}{\partial G} = H_G \cdot \{\frac{H_P}{D}\left(F_{LJ}^2\ N' + F_{JJ}\ (1-N'\ F_{LL})\right) + 1\} \quad \ldots (25)$$

$$\frac{\partial Y}{\partial G} \gtreqless 0, \quad \text{if } \frac{H_P}{D}\left(F_{LJ}^2\ N' + F_{JJ}(1-N'\ F_{LL})\right] \lesseqgtr 1 \quad \ldots (26)$$

$$\frac{\partial W}{\partial q} = \frac{-1}{D} \cdot \{\frac{W}{P} \cdot \left(N'\ F_L\ F_{LJ} + F_J(1-N'\ F_{LL})\right] + H_P\ N' F_{LJ}\} \ldots (27)$$

$$\frac{\partial W}{\partial q} \gtreqless 0, \quad \text{if } \frac{W}{P} \cdot \frac{\partial P}{\partial q} > - \frac{P}{N'}, \frac{\partial L}{\partial q} \quad \ldots (28)$$

$$\frac{\partial W}{\partial G} = \frac{W}{P}\frac{H_G}{D}\left(F_{JJ} + N'(F_{LJ}^2 - F_{LL}\ F_{JJ})\right] > 0 \quad \ldots (29)$$

From the above results we can summerize the short-run effects of a change of the energy price for the case that the capital stock is not fully employed ($\lambda < 1$):

„A rise of the energy price entails:

(a) a rise of the price level of the aggregate income;
(b) a reduction of the labor employment
(c) a reduction of the employment of the capital stock
(d) a reduction of the aggregate income
(e) the monetary wage rate will increase, remain the same or decrease, according to whether the effect of energy price change on the price level of the aggregate income is larger, the same or smaller than the effects of energy price change on the labor employment."

<u>(ii) Case 2B:</u>

Instead of (6) we use now the function (7) for the forthcoming analysis: $L - M(P \cdot F_L) = 0$ ... (12a)

We take total differentials of (1a), (12a), (4a) and (7). This yields [1)]

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1) The equation system derived from the total differentiations is as follows:

$$H_p dP - F_L\ dL - F_J\ dJ = - H_G\ dG$$

$$F_{JL}\ dL + F_{JJ}\ d_J = dq$$

$$- M'\ F_L\ dP + (1-M'\ P\ F_{LL})\ dL - M'P\ F_{LJ}\ dJ = 0$$

$$dP = \frac{D_1'}{D'} \qquad \dots (30)$$

$$dL = \frac{D_2'}{D'} \qquad \dots (31)$$

$$dJ = \frac{D_3'}{D'} \qquad \dots (32)$$

$$dY = H_P \frac{D_1'}{D'} + H_G\, dG \qquad \dots (33)$$

$$dW = P \cdot (F_{LL}\, dL + F_{LJ}\, dJ) + F_L\, dP \qquad \dots (34)$$

where $D' = - H_P \left[F_{JJ} + M' P (F_{LJ}^2 - F_{JJ} F_{LL}) \right]$

$$+ M' F_L (F_L F_{JJ} - F_J F_{JL}) < 0$$

$$D_1' = H_G \cdot \left[F_{JL}^2 M' \cdot P + F_{JJ} (1 - M' P F_{LL}) \right] dG$$

$$- \left[F_L M' P F_{LJ} + F_J (1 - M' \cdot P \cdot F_{LL}) \right] d\sigma$$

$$D_2' = - (F_J M' F_L + H_p M' P F_{LJ})\, d\sigma$$

$$+ H_G F_{JJ} M' F_L\, dG$$

$$D_3' = \left[F_L^2 M' - H_p (1 - P \cdot M' \cdot F_{LL}) \right] dq$$

$$- H_G F_{JL} M' F_L\, d_G$$

From these equations we derive:

$$\frac{\partial P}{\partial q} = \frac{-\left\{F_L \; M' \; P \; F_{LJ} + F_J \; (1 - M' \; P \; F_{LL})\right\}}{D'} > 0 \quad \ldots (35)$$

$$\frac{\partial P}{\partial G} = \frac{H_G \left\{F_{JL}^2 \; M' \; P + F_{JJ} \; (1 - M' \; P \; F_{LL})\right\}}{D'} > 0 \quad \ldots (36)$$

$$\frac{\partial L}{\partial q} = \frac{-\,(F_J \; M' \; F_L + H_P \; M' \; P \; F_{LJ})}{D'} \quad \ldots (37)$$

$$\frac{\partial L}{\partial G} = \frac{H_G \; F_{JJ} \; M' \; F_L}{D'} > 0 \quad \ldots (38)$$

$$\frac{\partial J}{\partial q} = \frac{F_L^2 \; M' - H_P (1 - P \; M' \; F_{LL})}{D'} < 0 \quad \ldots (39)$$

$$\frac{\partial J}{\partial G} = \frac{-H_G \; F_{JL} \; M' \; F_L}{D'} > 0 \quad \ldots (40)$$

$$\frac{\partial Y}{\partial q} = -H_P \; \frac{\left\{F_L \; M' \; P \cdot F_{LJ} + F_J \; (1 - M' \; P \; F_{LL})\right\}}{D'} < 0 \quad \ldots (41)$$

$$\frac{\partial Y}{\partial G} = H_P \; H_G \; \frac{\left\{F_{JL}^2 \; M' \; P + F_{JJ} (1 - M' \; P \; F_{LL})\right\}}{D'} > 0 \quad \ldots (42)$$

$$\frac{\partial W}{\partial q} = -\frac{1}{D'} \; (F_{LJ} \cdot H_P \cdot P + F_L \; F_J) \quad \ldots (43)$$

$$\frac{\partial W}{\partial G} = \frac{F_L \; H_G \; F_{JJ}}{D'} > 0 \quad \ldots (44)$$

From these results we see that the effect of the direction of the energy price change on employment and the money wage rate are not clear cut. These results may be explained in the following terms:

(a) The labor demand is a function of the real wage rate. The change of the real wage rate is determined by the change in the money wage rate and the price level. While the price level increases with a rise in the energy price the money wage rate also increases with a rise in the energy price. Therefore the change in demand for labor due to a rise in the energy price is dependent on the percentage of change of the price level and of the money wage rate.

(b) The price level will change more markedly than the money wage the lower the elasticity of aggregate demand. Labor employment will increase as a result of a raising of the energy price, if [1]

$$\tau > \eta \qquad \ldots (45)$$

where τ is the elasticity of substitution between factors

and $\eta = - \frac{P}{Y} \frac{\partial H}{\partial P}$ (the elasticity of aggregate demand)

Condition (45) means that the price level increases due to a raise of the energy price if the elasticity of substitution between factors is greater than the elasticity of aggregate demand.

Analogously the condition for the reaction of the money wage rate for a change of the energy price is given by

(c) $\frac{\partial W}{\partial q} \gtreqless 0$, if $\tau \gtreqless \eta$ ... (45a)

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1) See Appendix

(iii) Case 2c:

The function

$$P \cdot F_L = W^o \qquad \ldots (12b)$$

is used instead of (12) for the following analysis.[1)]

$$dP = \frac{D_1''}{D''} \qquad \ldots (46)$$

$$dB = \frac{D_2''}{D''} \qquad \ldots (47)$$

$$dJ = \frac{D_3''}{D''} \qquad \ldots (48)$$

$$dY = H_p \frac{D_1''}{D''} + H_G \, dG \qquad \ldots (49)$$

where $D'' = P \cdot H_p \, (F_{JL}^2 - F_{JJ} F_{LL}) + F_L (F_J F_{JL} - F_L F_{JJ}) > 0$

$$D_1'' = P \cdot H_G \cdot (F_{JJ} F_{LL} - F_{JL}^2) \, dG + P(F_J F_{LL} + F_L F_{LJ}) d\alpha$$

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1) The system of equations derived from total differentiation is as follows:

$$H_p \, dP - F_L \, dL - F_J \, d_J = - H_G \, dG$$

$$F_{JL} \, dL + F_{JJ} \, dJ = d\alpha$$

$$F_L \, dP + PF_{LL} \, dL + PF_{LJ} \, dJ = 0$$

$$D_2'' = - H_G F_L F_{JJ} dG - (F_J F_L - H_P \cdot P \cdot F_{LJ}) dq$$

$$D_3'' = - (H_P P \cdot F_{LL} + F_L^2) dq + F_L F_{JL} H_G dG$$

From above equations we derive:

$$\frac{\partial P}{\partial q} = \frac{P(F_J F_{LL} + F_L F_{LJ})}{D''} \quad \ldots (50)$$

$$\frac{\partial P}{\partial G} = \frac{P \cdot H_G (F_{JJ} F_{LL} - F_{LJ}^2)}{D''} > 0 \quad \ldots (51)$$

$$\frac{\partial L}{\partial q} = - \frac{(F_J F_L + H_P P \cdot F_{LJ})}{D''} \quad \ldots (52)$$

$$\frac{\partial L}{\partial G} = - \frac{H_G \cdot F_L \cdot F_{JJ}}{D''} > 0 \quad \ldots (53)$$

$$\frac{\partial J}{\partial q} = - \frac{H_P P F_{LL} + F_L^2}{D''} < 0 \quad \ldots (54)$$

$$\frac{\partial J}{\partial G} = \frac{F_L F_{JL} H_G}{D''} > 0 \quad \ldots (55)$$

$$\frac{\partial Y}{\partial q} = H_P \cdot \frac{P(F_J F_{LL} + F_L F_{LJ})}{D''} \quad \ldots (56)$$

$$\frac{\partial Y}{\partial G} = \frac{H_G F_L (F_J F_{JL} - F_L F_{JJ})}{D''} > 0 \quad \ldots (57)$$

Besides the input of energy the influence of change in the energy price on the other endogeneous variables in the model is not unique. The main reason for this result lies in the assumption of labor unemployment at a constant wage rate: The process of a change in the energy price can be explained as follows:

(a) An increase of the energy price reduces the energy input directly. The aggregate product decreases and the price level rises immediately, due to the decreased energy input.

(b) Labor will be substituted for capital and at the given monetary wage rate

(c) $\frac{\partial P}{\partial q} \gtreqless 0$, according to $0{,}5 \gtreqless \frac{J \cdot f_k}{Y}$ ... (58)

If the term $\frac{J \cdot f_k}{Y}$ in (58) is interpreted as the share of capital returns in the aggregate income, then the condition (58) means that the price level will increase with an increase in the energy price if the capital return share of aggregate income is less than 0.5.

(d) The effect of a change in energy price on aggregate income is just the opposite of the effect on the price level, namely:

$$\frac{\partial Y}{\partial q} \gtreqless 0, \text{ according to } 0.5 \gtreqless \frac{J \cdot f_k}{Y} \quad \dots (59)$$

(e) $\frac{\partial L}{\partial q} \gtreqless 0$, according to $\eta \gtreqless \tau$ ... (60)

This is merely a reversal of condition (45)

4. Summary:

The influences of an increase of the energy price on the aggregate income, the price level, the employment, the wage rate, and the coefficient of capacity utilization are investigated in a static macroeconomic model with a capacity utilization production function. The discussion examines principally aggregate supply, since this is the locus of the effects of energy on macroeconomic activity. The energy price is considered an exogeneous variable.

The input of energy in the production process of a modern economy originates mainly in the engagement of machinery capital. The use of mechanical capital in the production process is only possible if there are sufficient energy and materials available. The input of energy in the production process is proportional to the capital stocks engaged in the production.Energy is a limitational production factor since without energy the mechanical capital cannot be used in the production process, but additional energy input has no productive effect if no more capital can be engaged.

The condition for the input of the energy in the production process is discussed under three different assumptions about the labor supply.

Two specific cases must be considered explicitly:

The case in which the capital stock is used fully in the production process and the case in which some capital stock is not engaged in production. In the first case the effects of a change of the energy price on the macroeconomic activity can be neglected, while in the latter case the influence of a change of the energy price must be explicitly examined.

The influence of an increase in the energy price on the macroeconomic activity (for the case $\lambda < 1$) can be summarized under the alternate assumptions about labor supply in the following table:

endogeneous variables \ labor supply	the labor supply depends on the real wage rate	the labor supply depends on the money wage rate	the wage rate fixed exogeneously
the energy input	-	-	-
the price level	+	+	$\left.\begin{matrix}+\\o\\-\end{matrix}\right\}$ if $0.5 \gtreqless \frac{J \cdot f_k}{Y}$
the aggregate income	-	-	$\left.\begin{matrix}+\\o\\-\end{matrix}\right\}$ if $0.5 \gtreqless \frac{J \cdot f_k}{Y}$
the labor employment	-	$\left.\begin{matrix}+\\o\\-\end{matrix}\right\}$ if $\tau \gtreqless \eta$	$\left.\begin{matrix}+\\o\\-\end{matrix}\right\}$ if $\eta \gtreqless \tau$
the money wage rate	$\left.\begin{matrix}+\\o\\-\end{matrix}\right\}$ if $\frac{W}{P}\frac{\partial P}{\partial q} \gtreqless -\frac{P}{N_W}\frac{\partial L}{\partial q}$	$\left.\begin{matrix}+\\o\\-\end{matrix}\right\}$ if $\tau \gtreqless \eta$	o

Notes:
+ for positive effects
o for null effects
- for negative effects

Appendix:

Y = F(L,J) is a production function of homogenity of degree one with the usual assumptions on marginal productivities of factors

$$Y = L \cdot f(k) \qquad \ldots (A1)$$

where $k = \frac{J}{L}$ capital intensity

$$F_L = \frac{\partial F}{\partial L} = f - k\,f' > 0 \qquad \ldots (A2)$$

$$F_J = \frac{\partial F}{\partial J} = f' > 0 \qquad \ldots (A3)$$

where $f' = \frac{df}{dk}$

$$F_{LJ} = F_{JL} = -\frac{J}{L^2}\,f'' > 0 \qquad \ldots (A4)$$

where $f'' = \frac{d^2f}{dk^2}$

$$F_{LL} = \frac{k^2}{L} \cdot f'' < 0 \qquad \ldots (A5)$$

$$F_{JJ} = \frac{1}{L}\,f'' < 0 \qquad \ldots (A6)$$

The elasticity of substitution between factors (τ) is:[1)]

$$\tau = -\frac{f'\,(f-kf')}{kff''} \gtrless 0 \qquad \ldots (A7)$$

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1) See: Allen, R.G.D. (1968) P.48

(A) Conditions (45) and (45a)

We have

$\frac{\partial L}{\partial \sigma} \gtreqless 0$ if $\qquad (F_J F_L + H_p \cdot P \cdot F_{JL}) \gtreqless 0$

Equations (A1) to (A6) yield $\quad \eta = - H_p \frac{P}{Y} \geqq 0$

$$F_J F_L + H_p \cdot P \cdot F_{JL} = f' (f - kf') + \eta k f f'' \quad \text{... (A8)}$$

Hence

$F_J F_L + H_p \cdot P F_{JL} \gtreqless 0$ if

$$f' \cdot (f-kf') \gtreqless \eta k f f'' \quad \text{... (A9)}$$

or

$$- \frac{f'(f-kf')}{kff''} \gtreqless \eta$$

i.e. $\tau \gtreqless \eta$ ... (A10)

(B) Condition (58)

We have

$$\frac{\partial P}{\partial q} \gtreqless 0 \quad \text{if} \qquad F_J \, F_{LL} + F_L \, F_{LJ} \gtreqless 0$$

Equations (A2), (A3), (A4) and (A5) yield

$$F_J \, F_{LL} + F_L \, F_{LJ}$$

$$= f' \cdot \frac{J^2}{L^3} f'' - (f - kf') \frac{J}{L^2} f''$$

$$= - \frac{k}{L} f'' \cdot (f - 2kf') \qquad \text{... (A11)}$$

$$\text{Hence} \quad \frac{\partial P}{\partial q} \gtreqless 0 \quad \text{if} \qquad f \gtreqless 2 \, kf' \qquad \text{... (A12)}$$

$$\text{or} \quad f \cdot L \gtreqless 2 \, J \, f'$$

$$Y \gtreqless 2 \, J \, f'$$

$$0.5 \gtreqless \frac{J \cdot f'}{Y} \qquad \text{... (A13)}$$

$\frac{J \cdot f'}{Y}$ may be interpreted as the capitalist-share of the aggregate income.

(C) Condition (60)

We have

$$\frac{\partial L}{\partial \sigma} \gtreqless 0 \quad \text{if} \quad - H_D \cdot P \cdot F_{LJ} \gtreqless F_L F_J \qquad \ldots \text{(A14)}$$

Equations (A2), (A3) and (A4) yield

$$- \eta \cdot f \cdot k \cdot f'' \gtreqless f' \cdot (f - k f') \qquad \ldots \text{(A15)}$$

or $$\eta \gtreqless \tau \qquad \ldots \text{(A16)}$$

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