Universität Bielefeld/IMW

Working Papers
Institute of Mathematical Economics

Arbeiten aus dem
Institut für Mathematische Wirtschaftsforschung

Nr. 101

John-ren Chen

The Partially Pegging Exchange Rate
Problem: A Three-Country Model

September 1980

H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

J O H N - R E N C H E N

The Partially Pegging Exchange Rate Problem : a Three-Country Model

1. Introduction:

The policy of keeping fixed exchange rate to a special foreign currency which is denoted as partially pegging exchange rate policy or shortly pegging policy [Chen, 1980] is used by many countries under the current floating exchange-rate regime, especially by the developing countries to maintain their foreign economic relations. Since in the most developing countries an effective forward foreign exchange market does not exist, a hedging against exchange rate fluctuation on the forward market is imposable in those countries. 1)

The problem of the pegging policy in a small open economy is studied in a recent paper [Chen, 1980] . In this case, the feed-back-effects through the world market on the country considered can be neglected.

The countries of the so-called European snake keep exchange rates fixed between their currencies under the current floating exchange rate regime. These countries can hardly be considered to be "small countries" in the sense mentioned above. Therefore, the meaning as well as the effects of the pegging policy should be studied with explicit consideration of the feed-back effects. To describe the partially pegging exchange rate problem

---

1) It is also not possible to hedge the exchange rate fluctuations by storage of foreign exchange because of foreign exchange control.

sufficiently we need a three-country-model which regards not only the global foreign economic relations of the countries considered but also the bilateral economic relations which characterize the partially pegging policy.

In this paper we shall call the country which pegs the exchange rate of its currency to a special foreign currency as the pegging country, the country of the special foreign currency as the pegged country and the other country as the floating country. The currencies of the three countries will be denoted as the pegging, the pegged and the floating currency, respectively. While the exchange rate between the pegging and the pegged currency is kept fixed, those of the pegging and the floating currency as well as of the pegged currency and the floating currency are flexible.

The partially pegging exchange rate policy will not only effect the economy of the pegging country but also those of the pegged and the floating country. In this paper we shall study the effects of the partially pegging exchange rate policy. In section 2 a macroeconomic three-country model will be set up. To simplify our analysis we shall abstract from the international capital movements and the international income transfer and concentrate the main research on the trade balance. To describe the partially pegging exchange rate problem the bilateral trade relations of the three-country-model will be considered completely.

2. The Model

Symbols:

Y^i : national income of country i in constant price

C^i : consumption of country i in constant price

I^i : net investment of country i in constant price

G^i : government expenditure country i in constant price

r^i : interest rate in country i

X^{ij} : $i \neq j$, export of country i to country j defined in constant price of country i

$M^{ji}=X^{ij}$: import of country j from country i

$e^{ij}=\frac{1}{e^{ji}}$ ($i \neq j$), price of country i's currency in terms of country j's currency ($e^{ij} = 1$ initially)

P^i : output price in country i ($P^i=1$ initially)

$\bar{L}^i$: nominal supply of money in country i

L^i : nominal cash balance in country i

H^i : balance of payments of country i

W^i : money wage rate in country i

N^i : employment in country i

Q^i : real output of country i

π^i : price level in country i ($\pi^i = 1$ initially)

$D^{ij}=X^{ij}-M^{ij}$: trade balance between country i and j ($i \neq j$) in constant prices and exchange rate

$D^i= \sum_j D^{ij}$, $i \neq j$: trade balance of country i in constant price and exchange rates

$$U^{ij}=P^iX^{ij}-P^je^{ij}M^{ij} \quad \text{for} \quad i\neq j, \quad i\neq b$$
$$=P^iX^{ij}-P^j\frac{1}{e^{ij}}M^{ij} \quad \text{for} \quad i\neq j, \quad i=b$$

trade balance between country i and j

$U^i=\sum_j U^{ij}$: trade balance of country i

The upper index is used to denote the country. where

a: the pegging country

b: the pegged country

c: the floating country

Notation: Sudindex is used for partial derivative, e. g.

$$X^{ij}_{P^i} = \frac{\partial X^{ij}}{\partial P^i} \qquad \text{etc.} \qquad \text{and} \qquad \dot{K}^i = \frac{dK^i}{dt}$$

Indices will be neglected where mistakes are not to be expected.

The three-country-model is characterized by the following structural functions:

(1) $Y^i = C^i + I^i + G^i + D^i$

(2) $C^i = C^i\,(Y^i)\,, \qquad 1 > C^i_{Y^i} > 0$

The tax is neglected for shortness. A tax function, $T=T(Y)$ with $1 > T_Y > 0$ can be introduced easily.

(3) $I^i = I^i\,(r^i) \qquad I^i_r < 0$

(4) $X^{ij} = X^{ij}(P^i,P^j,Y^j,e^{ij}), \quad -X^{ij}_{P^i} = X^{ij}_{P^j} = X^{ij}_{e^{ij}} > 0, \quad X^{ij}_{Y^j} > 0$

(5) $X^{ij} = M^{ji} \qquad i \neq j$

(6) $H^i+L^i= L^i(Y^i,r^i,P^i), \quad L^i_{Y^i} > 0, \quad L^i_{r^i} < 0, \quad L^i_{P^i} > 0$

(7) $u = \frac{q}{e}$ [1] (exchange arbitrage without arbitrage costs)

(8) $H^a = P^a X^a - e^{ab} P^b M^{ab} - e^{ac} P^c M^{ac}$

(9) $Q^i = F^i(N^i, K^i)$, $Q^i_{N^i} > 0$, $Q^i_{K^i} > 0$

(10) $w^i = Q^i_{N^i} = \frac{\partial Q^i}{\partial N^i}$ $\quad w^i = \frac{W^i}{P^i}$

(11) $N^i = N^i(w^i)$ $\quad N^i_w > 0$

(12) $N^i = N^i(W^i)$ $\quad N^i_W > 0$

(13) $Y^i = Q^i$

(14) $P^c X^c - \frac{P^a}{e^{ac}} M^{ca} - e^{cb} P^b M^{cb} = 0$

(15) $\pi^i = \sum_j \alpha^i_j e^{ji} P^j$, i=a,b,c where α^i_j the constant weights [2]

Following difinitions will be used

(16) $D^i = \sum_j D^{ij}$

(17) $U^i = \sum_j U^{ij}$

(18) $U^{ij} = \begin{cases} P^i X^{ij} - P^j e^{ij} M^{ij} & \text{for } i \neq j, \quad i \neq b \\ P^i X^{ij} - P^j e^{ji} M^{ij} & i \neq j, \quad i = b \end{cases}$ where

(19) $e^{ij} = \frac{1}{e^{ji}}$ $\quad i \neq j$

1) $q=e^{ab}$, $u=e^{ac}$ and $e=e^{cb}$

e^{ij}=Price of j-th currency (in i-th currency)

2) Note: $e^{ji}=1$ for i=j

The functions (1) to (6) which are used in a macroeconomic model of Keynesian type describe the aggregate demand of goods-markets in our model.

The bilateral trade relations are assumed to depend on the price of the home output and the foreign output as well as on the exchange rate. As we point out in a previous paper [Chen, 1978] both a change in the output price in home and foreign country and a change in exchange rate will influence the comparative price of the trading goods and therefore the trade balance but they have different meaning. Therefore,we consider both of them as determinant factors of the bilateral trade relations. The influence of income on import seems to be obvious.

The equation (7) expresses the relation between the three exchange rates in the three-country-world under the case of costless arbitrage. If the arbitrage is costless then any deviation from equation (7) will induce arbitrage transactions which maintains the relation (7).

Equation (8) is the trade balance of the pegging country. Others than the floating country the trade balance of the pegging country will not be always equalized. The surplus or the dificit of the trade balance will be assumed not to be sterilized and therefore will be fully effective as influence on the nominal money supply.

The aggregate supply of the classical economy is characterized by (9), (10) and (11) where the supply of labor is determined by the real wage rate in comparison to the Keynesian economy which is described by (9), (10) and (12). In the Keynesian economy the labor supply depends on the nominal wage rate which is assumed to be given exogeneously.

Equation (14) describes the balance of trade of the floating country. It is always equalized if the international capital movements and international income transfer are neglected. Due to Walras law the trade balances of our three-country-world are completely described by (6) and (14). Hence the trade balance of the pegged country can be neglected.

(18) defines the general price level which is a weighted average of the price of the home output and the output of the foreign countries.

Definition:

Def. 1 : A model described by the structural functions (1) to (11) and (13) to (18) is called a classical model.

Def. 2 : A model described by the structural functions (1) to (10) and (12) to (18) is called as a Keynesian model.

Def. 3 : To avoid confusion we use the following conventions:

$$D_q^{ab} = \frac{\partial D^{ab}}{\partial q} > 0 \; ; \qquad D_q^{ba} = \frac{\partial D^{ba}}{\partial (\frac{1}{q})} > 0$$

$$D_u^{ac} = \frac{\partial D^{ac}}{\partial u} > 0 \; ; \qquad D_u^{ca} = \frac{\partial D^{ca}}{\partial (\frac{1}{u})} > 0$$

$$D_e^{cb} = \frac{\partial D^{cb}}{\partial e} > 0 \; ; \qquad D_e^{bc} = \frac{\partial D^{bc}}{\partial (\frac{1}{e})} > 0$$

$$U_q^{ab} = \frac{\partial U^{ab}}{\partial q} \; ; \qquad U_q^{ba} = \frac{\partial U^{ba}}{\partial (\frac{1}{q})}$$

$$U_u^{ac} = \frac{\partial U^{ac}}{\partial u} \; ; \qquad U_u^{ca} = \frac{\partial U^{ca}}{\partial (\frac{1}{u})}$$

$$U_e^{cb} = \frac{\partial U^{cb}}{\partial e} \; ; \qquad U_e^{bc} = \frac{\partial U^{bc}}{\partial (\frac{1}{e})}$$

$$D_e^{a} = D_q^{ab} + D_u^{ac} \; ; \quad D_e^{b} = D_q^{ba} + D_e^{bc}$$

$$D_e^{c} = D_u^{ca} + D_e^{cb}$$

$$U_e^{a} = U_q^{ab} + U_u^{ac} \; ; \quad U_e^{b} = U_q^{ba} + U_e^{bc} \; ; \quad U_e^{c} = U_u^{ca} + U_e^{cb}$$

Assumptions:

Assumption 1 : The Marshall-Lerner condition, i.e.

$$U_e^i = X_e^i - M_e^i - M^i > 0 \qquad i=a,b,c$$

holds for the global trade balance.[1)]

Assumption 2 : The exchange rate between the pegging and the pegged currency is given exogenous.

Assumption 3 : There exists an equilibrium with

$$X^i = M^i \quad ^{2)} \qquad \text{for } i = a, b, c$$

$$q = u = e = P^a = P^b = P^c = 1 \quad ^{3)}$$

---

1) The Marshall-Lerner condition need not to hold for the bilateral trade balances, even if it is fulfilled for the global trade balance.

2) The bilateral trade balance need not be equalized.

3) There is no contradiction in this assumption. From (14) we have $e=(P^cX^c)/(P^aM^{ca}/q+P^bM^{cb})=1$, if $X^c=M^c$, and $P^a = P^b = P^c = q = 1$ and if $H^a = P^aX^a-qP^bM^{ab}-uP^cM^{ac}=0$ and $P^a=P^b=P^c=q=u=1$ then $X^a=M^a$

If $H^a = 0$ then $-qH^b = H^a$ thus $H^b = 0$ (See Lemma 2)

Theoretically, we can prove the existence of equilibrium with equalized trade balance U^i for all i and set therefore P^i (for all i) and the exchange rates e, u, and q the value 1.

Assumption 4 :

$$X^{ab} = M^{ba} > 0 \qquad X_q^{ab} = M_q^{ba} > 0$$

$$X^{ac} = M^{ca} > 0 \qquad X_u^{ac} = M_u^{ca} > 0$$

$$X^{ba} = M^{ab} > 0 \qquad X_q^{ba} = M_q^{ab} < 0$$

$$X^{bc} = M^{cb} > 0 \qquad X_e^{bc} = M_e^{cb} < 0$$

$$X^{ca} = M^{ac} > 0 \qquad X_u^{ca} = M_u^{ac} < 0$$

$$X^{cb} = M^{bc} > 0 \qquad X_e^{cb} = M_e^{bc} > 0$$

$$X_{p^a}^{ab} = M_{p^a}^{ba} = -X_q^{ab} < 0 \;; \quad X_{p^b}^{ab} = M_{p^b}^{ba} = X_q^{ab} > 0$$

$$X_{p^a}^{ac} = M_{p^a}^{ca} = -X_u^{ac} < 0 \;; \quad X_{p^c}^{ac} = M_{p^c}^{ca} = X_u^{ac} > 0$$

$$X_{p^b}^{ba} = M_{p^b}^{ab} = X_q^{ba} < 0 \qquad X_{p^a}^{ba} = M_{p^a}^{ba} = -X_q^{ba} > 0$$

$$X_{p^b}^{bc} = M_{p^b}^{cb} = X_e^{bc} < 0 \qquad X_{p^c}^{bc} = M_{p^c}^{cb} = -X_e^{bc} > 0$$

$$X_{p^c}^{ca} = M_{p^c}^{ac} = X_u^{ca} < 0 \qquad X_{p^a}^{ca} = M_{p^a}^{ac} = -X_u^{ca} > 0$$

$$X_{p^c}^{cb} = M_{p^c}^{bc} = -X_e^{cb} < 0 \qquad X_{p^b}^{cb} = M_{p^b}^{bc} = X_e^{cb} > 0$$

$$D_e^a = X_q^{ab} + X_u^{ac} - X_q^{ba} - X_u^{ca} = D_q^{ab} + D_u^{ac} > 0$$

$$D_e^b = -(X_q^{ba} + X_e^{ca} - X_q^{ab} - X_e^{ac}) = D_q^{ba} + D_e^{bc} > 0$$

$$D_e^c = -X_u^{ca} + X_e^{cb} + X_u^{ac} - X_e^{bc} = D_u^{ca} + D_e^{cb} > 0$$

$$U_e^a = X_q^{ab} + X_q^{ba} - X^{ba} + X_u^{ac} - X_u^{ca} - X^{ca} = U_q^{ab} + U_u^{ac} > 0$$

$$U_e^b = -(X_q^{ba} + X_e^{bc} - X^{ab} - X^{ca} - X_q^{ab} - X_e^{cb}) = U_q^{ba} + U_e^{bc} > 0$$

$$U_e^c = X_e^{cb} - X_u^{ca} - X_e^{bc} + X_u^{ac} - X^{ac} - X^{bc} = U_u^{ca} + U_e^{cb} > 0$$

3. Some Lemmas :

Lemma 1 : The bilateral trade balance of the floating country to the pegging country is equal to the negative bilateral trade balance of the floating country to the pegged country.

Proof : This lemma can directly be derivated by function (14). Since the global trade balance of the floating country is always equal to zero (equalized). Therefore

$$P^c X^{ca} - e \frac{P^a}{q} M^{ca} = -(P^c X^{cb} - e P^b M^{cb}) \qquad \text{(Q.E.D.)}$$

Remark : If the floating country has a bilateral trade surplus to the pegged country then a balanced trade balance of the pegging country can be realized only by a bilateral trade deficit to the pegged country.
Initially, we have

$$X^{ab} - M^{ab} = -X^{ac} + M^{ac}$$

$$X^{ba} - M^{ba} = -X^{bc} + M^{bc}$$

$$X^{ab} - M^{ab} = -X^{ac} + M^{ac}$$

Thus initially we find

$$X^{ca} - M^{ca} = M^{ba} - X^{ba}$$

$$= X^{ab} - M^{ab}$$

<u>Lemma 2</u> : The balance of trade of the pegged country is equalized, if and only if the pegging country has equalized balance of trade. The deficit (or surplus) in the balance of trade of the pegged country is always equal to the surplus (or deficit) of the pegging country.

<u>Proof</u> : Since the balance of trade in the floating country is always equalized, according (14)

(i) $P^cX^{ca}-\frac{e}{q}P^aM^{ca}=eP^bM^{cb}-P^cX^{cb}$

The balance of trade in the pegging country given by (8) can be written as:

(ii) $\frac{q}{e}P^cM^{ac}-P^aX^{ac}=P^aX^{ab}-qP^bM^{ab}-H^a$

and that in the pegged country

(iii) $P^bX^{bc}-\frac{1}{e}P^cM^{bc}=H^b-P^bX^{ba}+\frac{1}{q}P^aM^{ba}$

Since the export of the pegging country to the pegged country, etc. is equal to the import of the pegged country from the pegging country, etc. i.e.

$X^{ab}=M^{ba}$ $\quad X^{ac}=M^{ca}$

$M^{ab}=X^{ba}$ $\quad M^{ac}=X^{ca}$

$X^{bc}=M^{cb}$ $\quad M^{bc}=X^{cb}$

From (ii)

$$P^cM^{ac}-\frac{e}{q}P^aX^{ac}=P^cX^{ca}-\frac{e}{q}P^aM^{ca}$$

$$=\frac{e}{q}(P^aX^{ab}-qP^bM^{ab}-H^a)$$

From (iii)

$$e\,(P^bX^{bc} - \tfrac{1}{e}P^cM^{bc}) = e\,P^bM^{cb} - P^cX^{cb}$$

$$= e\,(H^b - P^bX^{ba} - \tfrac{1}{q}P^aM^{ba}) \quad \text{and}$$

$$P^aX^{ab} - qP^bM^{ab} - H^a = qH^b - qP^bX^{ba} + P^aM^{ba} = qH^b + P^aX^{ab} - qP^bM^{ab}$$

Hence $\quad -H^a = q\,H^b \quad$ where $\quad q > 0 \qquad$ (Q.E.D.)

Remark : Since the bilateral trade balance is rarely equalized, therefore in the three-country-model the bilateral trade balance between the pegging country to the pegged country will be in general not equalized. Therefore, the global trade balances of both the pegging and the pegged country seem not to be equalized generally even if the global trade balance of the floating country is always equalized, if the international capital movements and the international income transfers are neglected.

Lemma 3 : Laursen - Metzler effect [4] 1)

A depreciation of the floating currency in order to equalize the trade balance of the floating country has an expansive (stimulating) effect on the economy of the floating country, other things being equal.

Proof : The effect of a depreciation of the floating currency on the balance of trade in the floating country is given as: $U^c_e > 0$ while the effects of a depreciation on the domestic economic activities in the floating country are given as: $D^c_e > 0$ Since $D^c_e = X^c_e - M^c_e$, $U^c_e = X^c_e - M^c_e - M^c > 0$ and $\frac{\partial Y^c}{\partial e} = D^c_e > U^c_e$. Thus a depreciation of the floating currency in order to restore equalization in its trade balance has an expansive effect on the economy of the floating country.

1) Laursen & Metzler - the "price effects" of changes in the exchange rate: with given money incomes and given domestic prices, an increase in import prices will probably increase total expenditure out of a given income (P.294)
[Sohmen E., 1973, P.137-138]

<u>Lemma 4</u> : The following relations hold :

(a) $U^{ij} = - e^{ji}\, U^{ji} = - \dfrac{U^{ji}}{e^{ij}}$ $\quad i,j=a,b,c\ ,\quad i \neq j$

(b) $D^{ij} = - D^{ji}$ $\quad i,j=a,b,c\ ,\quad i=j$

(c) $D_e^{ij} = - D_e^{ji}$ for $i,j=a,b,c\ ,\quad i \neq j$

(d) $U_e^{ij} = - U_e^{ji} + D^{ji}$ $\quad i,j=a,b,c\ ,\quad i \neq j$

(e) $U_e^{ij} = - U_e^{ji}$, if and only if the initial bilateral trade balance between the country i and j is equalized i.e. $D^{ij} = 0$ for $i,j=a,b,c,\quad i \neq j$

<u>Proof</u> : (a) and (b) follow directly from the definitions :

$U^{ij} = P^i X^{ij} - P^j e^{ij} M^{ij}$ and

$U^{ji} = P^j X^{ji} - P^i e^{ji} M^{ji} = P^j X^{ji} - P^i M^{ji} / e^{ij}$

$U^{ij} = - U^{ji} / e^{ij}$ since $e^{ij} = 1 / e^{ji}$

$D^{ij} = X^{ij} - M^{ij} = M^{ji} - X^{ji} = - D^{ji}$

(c) $D^{ij}_{e^{ij}} = X^{ij}_{e^{ij}} - M^{ij}_{e^{ij}} = - (X^{ji}_{e^{ji}} - M^{ji}_{e^{ji}}) = - D^{ji}_{e^{ji}}$ or

$D_e^{ij} = - D_e^{ji}$ (according to definition 3)

(d) $U^{ij}_{e^{ij}} = X^{ij}_{e^{ij}} - M^{ij}_{e^{ij}} - M^{ij} = D_e^{ij} - M^{ij}$

$U^{ji}_{e^{ij}} = - D_e^{ji} - M^{ji} = D_e^{ij} - X^{ij}$

$U^{ij}_{e^{ij}} - U^{ji}_{e^{ij}} = U_e^{ij} + U_e^{ji} = D^{ij}$ or $U_e^{ij} = - U_e^{ji} + D^{ij}$

(according to definition 3)

(e) follows from (c) if $D^{ij} = 0$

<u>Remark</u> : If the Marshall-Lerner condition holds for global trade balance of all three countries, then the Marshall-Lerner condition must hold at least for one of the bilateral trade balance in any country.
Since $U^i = U^{ij} + U^{ik}$ and $U^i_e = U^{ij}_e + U^{ik}_e$
for $i,j,k=a,b,c$ $i=j=k$
If $U^i_e > 0$ or U^{ik}_e or both U^{ij}_e and U^{ik}_e
must be positive.

<u>Lemma 5</u> : The aggregate demand of the model in differentials about the initial equilibrium is given by the following equation system:

$$\begin{bmatrix} A_{11} & -A_{12} & 0 & A_{14} & -A_{15} & -A_{16} \\ -A_{21} & A_{22} & 0 & -A_{24} & A_{25} & -A_{26} \\ -A_{31} & -A_{32} & A_{33} & -A_{34} & -A_{35} & A_{36} \end{bmatrix} \begin{bmatrix} dP^a \\ dP^b \\ dP^c \\ dY^a \\ dY^b \\ dY^c \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \\ A_{30} \end{bmatrix}$$

where $A_{11} = D^{ab}_q + D^{ac}_u U^{cb}_e / U^c_e + (I^a_r / L^a_r)(L^a_P + U^{ac}_u + U^{ab}_q U^{cb}_e / U^c_e)$

$A_{12} = D^{ab}_q + D^{ac}_u U^{cb}_e / U^c_e + (I^a_r / L^a_r)(U^{ab}_q + U^{ac}_u U^{cb}_e / U^c_e)$

$A_{14} = 1 - C^a_Y + M^a_Y + (I^a_r / L^a_r)\ L^a_Y - (1 + U^{ac}_u / U^c_e) M^c_Y\ - D^{ac}_u M^{ac}_Y / U^c_e$

$A_{15} = M^{ba}_Y + (M^{ba}_Y + U^{ac}_e M^{bc}_Y / U^c_e)(I^a_r / L^a_r) + D^{ac}_u M^{bc}_Y / U^c_e$

$A_{16} = M^{ca}_Y + M^c_Y D^{ca}_u / U^c_e + (I^a_r / L^a_r)(M^{ca}_Y U^c_e - M^c_Y) / U^c_e$

$A_{10} = (D^{ab}_q + U^{ac}_e + U^{ab}_e U^{cb}_e / U^c_e) dq + dG^a + (I^a_r / L^a_r) d\bar{L}^a$

$A_{21} = D^{ba}_q + D^{bc}_e U^{ca}_u / U^c_e + (U^{ac}_u + U^{ab}_q U^{cb}_e / U^c_e)(I^b_r / L^b_r)$

$$A_{22} = D_q^{ba}+D_e^{bc}U_u^{ca}/U_e^c+(I_r^b/L_r^b)(L_P^b+U_q^{ab}+U_u^{ac}U_e^{cb}/U_e^c)$$

$$A_{24} = M_Y^{ab}+(M_Y^a-U_e^{ac}M_Y^{bc}/U_e^c)(I_r^b/L_r^b)+D_e^{bc}M_Y^{ac}/U_e^c$$

$$A_{25} = 1-C_Y^b+M_Y^b-D_e^{bc}M_Y^{bc}/U_e^c+(I_r^b/L_r^b)(L_Y^b+M_Y^{ba}+U_e^{ac}M_Y^{bc}/U_e^c)$$

$$A_{26} = M_Y^{cb}+(I_r^b/L_r^b)(M_Y^c-M_Y^{ca}U_e^c)/U_e^c-D_e^{bc}M_Y^c/U_e^c$$

$$A_{20} = (I_r^b/L_r^b)d\bar{L}^b+dG- D_q^{ba}-(I_r^b/L_r^b)(U_u^{ac}U_u^{ca}/U_e^c-U_e^a)+D_e^{bc}U_u^{ca}/U_e^c \; dq$$

$$A_{31} = D_u^{ca} - D_e^c \, U_u^{ca} \, / \, U_e^c$$

$$A_{32} = D_e^{cb} - D_e^c \, U_e^{cb} \, / \, U_e^c$$

$$A_{33} = \frac{I_r^c}{L_r^c} \, L_P^c > 0$$

$$A_{34} = M_Y^{ac} \, M^c \, / \, U_e^c > 0$$

$$A_{35} = M_Y^{bc} \, M^c \, / \, U_e^c > 0$$

$$A_{36} = 1 - C_Y^c - M_Y^c \, M^c \, / \, U_e^c + (I_r^c/L_r^c) \, L_Y^c$$

$$A_{30} = (I_r^c/L_r^c) \, d\bar{L}^c + dG^c - (D_u^{ca} - D_e^c \, U_e^{ca} \, / \, U_e^c) \, dq$$

Proof : The system of aggregate demand is derived from the functions (1) to (8) and (14) to (18). Lemma 1 to 4 are used.

Remark : The paramters A^{ij} for i,j=1,2 and A_{3i} for i=1,2,3 as well as A_{15} and A_{24} are all positive, if the Marshall-Lerner condition holds for all bilateral trade balances.

A_{14}, A_{15} and A_{36} are the inverses of the usual multipliars for the pegging, the pegged and the floating country, respectively. They are in general expected to be positive:

A_{34} and A_{35} are positive $\qquad A_{31} + A_{32} = 0$

$$A_{16} \gtreqless 0 \text{ , for } \frac{L_r^a}{I_r^a}(U_e^c M_Y^{ca} + M_Y^c D_u^{ca}) \gtreqless (M_Y^c - U_e^c M_Y^{ca}) \quad 1)$$

$$A_{26} \gtreqless 0 \text{ , for } \frac{U_e^c}{D_e^{bc}} \cdot \frac{M_Y^{cb}}{M_Y^c}(1 + \frac{I_r^b}{I_r^b}) \gtreqless 1 \quad 1)$$

where $\frac{M_Y^c}{U_e^c} = \frac{\partial e}{\partial Y^c}$ is the partial effect of the national income in the floating country on the exchange rate e.

$$A_{11} - A_{12} = \frac{I_r^a}{L_r^a}\left(L_P^a + U_u^{ac} - U_q^{ab} + \frac{U_q^{ab}U_e^{cb} - U_u^{ac}U_e^{cb}}{U_e^c}\right)$$

$$= \frac{I_r^a}{L_r^a}\left[L_P^a + \frac{(U_u^{ac} - U_q^{ab})(U_u^{ca} + U^{cb}) + (U_q^{ab} - U_u^{ac})U_e^{cb}}{U_e^c}\right]$$

$$= \frac{I_r^a}{L_r^a}\left[L_P^a + \frac{U_u^{ac} - U_q^{ab})U_u^{ca}}{U_e^c}\right]$$

$$A_{22} - A_{21} = \frac{I_r^b}{L_r^b}\left(L_P^b + U_q^{ab} - U_u^{ac} + \frac{U_u^{ac}U_e^{cb} - U_q^{ab}U_e^{cb}}{U_e^c}\right)$$

$$= \frac{I_r^b}{L_r^b}\left[L_P^b + \frac{(U_q^{ab} - U_u^{ac})(U_u^{ca} + U_e^{cb}) + (U_u^{ac} - U_q^{ab})U_e^{cb}}{U_e^c}\right]$$

$$= \frac{I_r^b}{L_r^b}\left[L_P^b + \frac{(U_q^{ab} - U_u^{ac})U_u^{ca}}{U_e^c}\right]$$

$$A_{33} - A_{31} - A_{32} = \frac{I_r^c}{L_r^c} L_P^c - D_u^{ca} - D_e^{cb} + \frac{D_e^c(U_u^{ca} + U_e^{cb})}{U_e^c}$$

$$= \frac{I_r^c}{L_r^c} L_P^c > 0$$

1) It seems realistic to expect that both A_{16} and A_{26} are positive. M_Y^c is marginal propensity to import of the floating country.

Lemma 6

Let $A = (a_{ij})$ be an nxn matrix with dominant diagonal such that $a_{ij} \leqq 0$ for $i \neq j$, where a_{ij} are element of the i-th row, j-th column of the A matrix. Then there exists a unique $X \geq 0$ such that $AX = C$ or equivalently: All the sucessive principal minors of A are positive, that is,

$$|a_{11}| > 0, \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0 \cdots \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & & a_{nn} \end{vmatrix} > 0$$

This is the so-called Hawkins-Simon Theorem. [1)]

Proof: See Takayama, A. [1974, P.383]

Lemma 7: Metzler's Theorem (Morishima, P.18)

If A is a non-negative, indecomposable matrix none of whose row (or column) sums is greater than one, and at least one of whose row (or column) sum is less than one, then the diagonal elements of $[I - A]^{-1}$ are not less than the off-diagonal elements of the corresponding columens (or rows)

---

1) Notations: $A > 0$ if $a_{ij} > 0$ for all i and j
$X \geq 0$ if $X_i \geqq 0$ for all i and $X_i > 0$ for some i

4. The Classical Economy

The classical economy in the short-run is characterized by the structural functions (1) to (11) and (13) to (18). In this case the aggregate output or the aggregate supply is independent on the price level, [1] and the economy is perpetually at full employment. Hence the national income in prices at the initial equilibrium (real income) is determined outside the aggregate demand (dichotomy). [2] Therefore, in the classical case our model can be solved from the following system of equations: [3]

$$\begin{bmatrix} A_{11} & -A_{12} & 0 \\ -A_{21} & A_{22} & 0 \\ -A_{31} & -A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} dP^a \\ dP^b \\ dP^c \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \\ A_{30} \end{bmatrix}$$

Thus the parameter-matrix can be decomposed as:

$$A = \left[\begin{array}{cc|c} A_{11} & -A_{12} & 0 \\ -A_{21} & A_{22} & 0 \\ \hline -A_{31} & -A_{32} & A_{33} \end{array}\right] = \begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix}$$

Since $A_{31} + A_{32} = 0$, the price level in the floating country is not influenced from outside, if and only if $dP^a = dP^b$

1) See e.x. Bowers and Baird [1971, P.196]
2) Patinkin [1965, P.171]
3) compare to Lemma 5

Theorem 1 : If the Marshall-Lerner condition holds for all bilateral trade balances, then the equation system in the classical case has a unique solution.

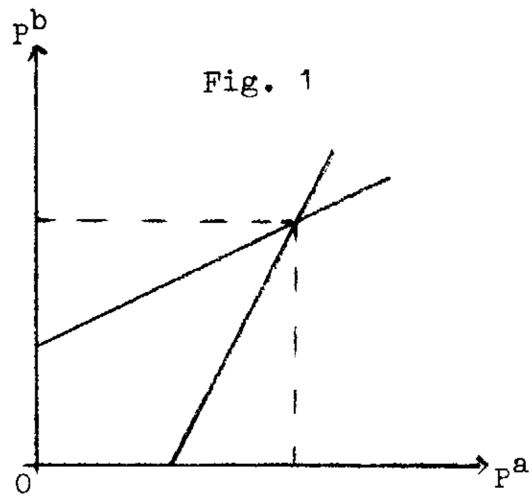
Proof: If the Marshall-Lerner condition holds for all bilateral trade balances, then we have

$$A_{ij} \geqq 0 \quad \text{for all} \quad i \text{ and } j$$

Thus the off-diagonal elements of the matrix A are all non-positive (See Lemma 5)
From Lemma 6 the equation system (of the classical model) has a uniqne solution, if A has dominant diagonal.
From Lemma 5 this is the case, if $(U_q^{ab} - U_u^{ac})$ is sufficiently small. [1)]

Thus the unique solution will be different from zero.

The solution for the subsystem for the pegging and the pegged country can be shown in Fig. 1



1) $A_{11} \gtrless A_{12}$, if $U_e^c L_P^a + (U_u^{ac} - U_q^{ab}) U_u^{ca} \gtreqless 0$

$A_{22} \gtrless A_{21}$ if $U_e^c L_P^b + (U_q^{ab} - U_u^{ac}) U_u^{ca} \gtreqless 0$

<u>Theorem 2</u> : Assuming the Marshall-Lerner condition holds for all bilateral trade balances:

(a) Given an expansive policy in either the pegging or the pegged country, [1] while the other countries do not change their policy, the price level in all three countries must increase, and the price level in the country with the expansive policy will do so by no lower degree i.e.

$$\frac{\partial P^i}{\partial G^i} \geq \frac{\partial P^j}{\partial G^i} \geq 0 \quad \text{and} \quad \frac{\partial P^i}{\partial \bar{L}^i} \geq \frac{\partial P^j}{\partial \bar{L}^i} \geq 0$$

$i,j=a,b,c$

(b) The stabilization policy of the floating country does not transmit to the pegging and the pegged country. But the stabilization policy of the pegging and the pegged country will transmit to the floating as long as there are different inflation rates in the pegging and the pegged country.

<u>Proof:</u> (a) The first part follows from Theorem 1 with $dG^i > 0$ or/and $d\bar{L}^i > 0$ and $dq = 0$ for $i=a,b$.
Let the determinant of the matrix A be det A.
Thus the effects of changing in dG^a , $d\bar{L}^a$ or dG^b , $d\bar{L}^b$ on dP^a , dP^b and dP^c with given policy of the other countries are

$$\frac{\partial P^a}{\partial G^a} = \frac{A_{22}}{\det A} \quad ; \quad \frac{\partial P^b}{\partial G^a} = \frac{A_{21}}{\det A}$$

$$\frac{\partial P^a}{\partial G^b} = \frac{A_{12}}{\det A} \quad ; \quad \frac{\partial P^b}{\partial G^b} = \frac{A_{11}}{\det A}$$

1) i.e. if $dA_{10} > 0$ or $dA_{20} > 0$ with $dq = 0$

$$\frac{\partial P^c}{\partial G^a} = \frac{A_{31}}{A_{33}} \frac{\partial P^a}{\partial G^a} + \frac{A_{32}}{A_{33}} \frac{\partial P^b}{\partial G^b} = \frac{1}{\det A} \frac{A_{22}A_{31} + A_{21}A_{32}}{A_{33}}$$

$$\frac{\partial P^c}{\partial G^b} = \frac{1}{\det A} \frac{A_{31}A_{12} + A_{32}A_{11}}{A_{33}}$$

$$\frac{\partial P^a}{\partial \bar{L}^a} = \frac{I_r^a}{L_r^a} \frac{A_{22}}{\det A} \quad ; \quad \frac{\partial P^b}{\partial \bar{L}^a} = \frac{I_r^a}{L_r^a} \frac{A_{21}}{\det A}$$

$$\frac{\partial P^a}{\partial \bar{L}^b} = \frac{I_r^b}{L_r^b} \frac{A_{12}}{\det A} \quad ; \quad \frac{\partial P^b}{\partial \bar{L}^b} = \frac{I_r^b}{L_r^b} \frac{A_{11}}{\det A}$$

$$\frac{\partial P^c}{\partial \bar{L}^a} = \frac{I_r^a}{L_r^a} \frac{\partial P^c}{\partial G^a} \quad ; \quad \frac{\partial P^c}{\partial \bar{L}^b} = \frac{I_r^b}{L_r^b} \frac{\partial P^c}{\partial G^b}$$

Thus

$$\frac{\partial P^a}{\partial G^a} > \frac{\partial P^b}{\partial G^a} \quad ; \quad \frac{\partial P^b}{\partial G^b} > \frac{\partial P^a}{\partial G^b} \quad ; \quad \frac{\partial P^a}{\partial \bar{L}^a} > \frac{\partial P^b}{\partial \bar{L}^a} \quad ; \quad \frac{\partial P^b}{\partial \bar{L}^b} > \frac{\partial P^a}{\partial \bar{L}^b}$$

To show $\frac{\partial P^a}{\partial G^a} > \frac{\partial P^c}{\partial G^a}$, we have to show that

$$\frac{A_{22}A_{31} + A_{21}A_{32}}{A_{33}} < A_{22}$$

or

$$A_{33}A_{22} - A_{22}A_{31} - A_{21}A_{32} > 0$$

$$A_{22}(A_{33} - A_{31} - A_{32}\frac{A_{21}}{A_{22}}) > 0$$

This condition holds to be true, since $A_{33}-A_{31}-A_{32} > 0$ and $\frac{A_{21}}{A_{22}} < 1$ Analogous $\frac{\partial P^b}{\partial G^b} > \frac{\partial P^c}{\partial G^b}$, since

$$A_{11}(A_{33} - A_{31} - A_{32}\frac{A_{12}}{A_{11}}) > 0$$

(b) This can be seen easily from Lemma 5. Since the matrix A is decomposable, the solutions for dP^a and dP^b do not depend on A_{30}. But otherwise the solution for dP^c depends on A_{10} and A_{20}. (Q.E.D.)

<u>Theorem 3 :</u>

Assuming the Marshall-Lerner condition holds for all bilateral trade balances, then,given a solitary expansive policy of any one of the three countries, the price levels of all three countries will increase with elasticities less than unity.

<u>Proof :</u> The elasticities of dP^a and dP^b to A_{10} or A_{20} are

$$\eta_{P^a/A_{10}} = \frac{\partial P^a}{\partial A_{10}} \frac{A_{10}}{dP^a} = \frac{A_{22}A_{10}}{A_{22}A_{10}+A_{12}A_{20}} < 1$$

$$\eta_{P^b/A_{10}} = \frac{\partial P^b}{\partial A_{10}} \frac{A_{10}}{dP^b} = \frac{A_{12}A_{10}}{A_{11}A_{20}+A_{21}A_{10}} < 1$$

$$\eta_{P^c/A_{10}} = \frac{\partial P^c}{\partial A_{10}} \frac{A_{10}}{dP^c}$$

$$= \frac{(A_{31}A_{22}+A_{32}A_{21})A_{10}}{A_{30}\det A+(A_{31}A_{22}+A_{32}A_{21})A_{10}+(A_{31}A_{12}+A_{32}A_{11})A_{20}} < 1$$

$$\eta_{P^a/A_{20}} = \frac{A_{12}A_{20}}{A_{22}A_{10}+A_{12}A_{20}} < 1$$

$$\eta_{P^b/A_{20}} = \frac{A_{11}A_{20}}{A_{11}A_{20}+A_{21}A_{10}} < 1$$

$$\eta_{P^c/A_{20}} = \frac{(A_{31}A_{12}+A_{32}A_{11})A_{20}}{(A_{31}A_{22}+A_{32}A_{21})A_{10}+(A_{31}A_{12}+A_{32}A_{11})A_{20}} < 1$$

$$\eta_{P^a/A_{30}} = \eta_{P^b/A_{30}} = 0$$

$$\eta_{P^c/A_{30}} = \frac{A_{30}\det A}{A_{30}\det A+(A_{31}A_{22}+A_{32}A_{21})A_{10}+(A_{31}A_{12}+A_{32}A_{11})A_{20}} < 1$$

Theorem 4 : the Lechatelier-Samuelson principle

Assuming the Marshall-Lerner condition holds for all bilateral trade balances, an increase in price level in either of the pegging or the pegged country is less if the price level in the other country is kept constant than if both price levels are permitted to vary.

Proof :

$$\frac{\partial P^a}{\partial A_{10}} \Big| \; dP^b = \text{const} = \frac{1}{A_{11}}$$

$$\frac{\partial P^a}{\partial A_{10}} = \frac{A_{22}}{A_{11}A_{22}-A_{12}A_{21}} = \frac{1}{A_{11} - \frac{A_{12}A_{21}}{A_{22}}}$$

Hence $\frac{\partial P^a}{\partial A_{10}} \Big| \; dP^b = \text{const} < \frac{\partial P^a}{\partial A_{10}}$

$$\frac{\partial P^a}{\partial A_{20}} \Big| \; dP^b = \text{const} = 0 \; ; \; \frac{\partial P^a}{\partial A_{20}} = \frac{A_{21}}{\det A} > 0$$

and

$$\frac{\partial P^b}{\partial A_{20}} \Big| \; dP^a = \text{const} = \frac{1}{A_{22}}$$

$$\frac{\partial P^b}{\partial A_{20}} = \frac{A_{11}}{A_{11}A_{22}-A_{12}A_{21}} = \frac{1}{A_{22} - \frac{A_{12}A_{21}}{A_{11}}}$$

Therefore $\frac{\partial P^b}{\partial A_{20}} \Big| \; dP^a = \text{const} < \frac{\partial P^b}{\partial A_{20}}$

$$\frac{\partial P^b}{\partial A_{10}} \Big| \; dP^a = \text{const} = 0 \; ; \; \frac{\partial P^b}{\partial A_{10}} = \frac{A_{12}}{\det A} > 0$$

(Q.E.D.)

<u>Theorem 5</u> : If the Marshall-Lerner condition holds for all bilateral trade balances, then

(a) a solidary expansive policy of either the pegging or the pegged country has an appreciation effect on the floating currency.

(b) a solidary expansive policy of the floating country has a depreciation effect on the floating currency.

<u>Proof :</u> Since $$de = dP^c - \frac{U_u^{ca}}{U_e^c} dP^a - \frac{U_e^{cb}}{U_e^c} dP^b$$

$$= \left(\frac{A_{31}}{A_{33}} - \frac{U_u^{ca}}{U_e^c} \right) dP^a + \left(\frac{A_{32}}{A_{33}} - \frac{U_e^{cb}}{U_e^c} \right) dP^b + \frac{A_{30}}{A_{33}}$$

for $dY^i = dq = 0$

(a) $$\frac{\partial e}{\partial A_{10}} = \left(\frac{A_{31}}{A_{32}} - \frac{U_u^{ca}}{U_e^c} \right) \frac{\partial P^a}{\partial A_{10}} + \left(\frac{A_{32}}{A_{33}} - \frac{U_e^{cb}}{U_e^c} \right) \frac{\partial P^b}{\partial A_{10}}$$

$$< \left(\frac{A_{31}}{A_{32}} - \frac{U_u^{ca}}{U_e^c} + \frac{A_{32}}{A_{33}} - \frac{U_e^{cb}}{U_e^c} \right) \frac{\partial P^b}{\partial A_{10}} = 0$$

Since $\frac{\partial P^a}{\partial A_{10}} > \frac{\partial P^b}{\partial A_{10}}$ (See Theorem 2)

$$\frac{\partial e}{\partial A_{20}} < \left(\frac{A_{31}}{A_{32}} - \frac{U_u^{ca}}{U_e^c} + \frac{A_{32}}{A_{33}} - \frac{U_e^{cb}}{U_e^c} \right) \frac{\partial P^a}{\partial A_{20}} = 0$$

Since $\frac{\partial P^a}{\partial A_{20}} < \frac{\partial P^b}{\partial A_{20}}$

(b) $\frac{\partial e}{\partial A_{30}} > 0$ (Q.E.D.)

5. The Keynesian Economy: the Unemployment Case

The Keynesian economy in the short-run can be characterized by the structural equations (1) to (10) and (12) to (18). In this section the Keynesian economy with mass unemployment will be studied. In this case the price levels are assumed to be constant given in all three countries. Our model can now be solved from the following system of equations: [1)]

$$\begin{bmatrix} A_{14} & -A_{15} & -A_{16} \\ -A_{24} & A_{25} & -A_{26} \\ -A_{34} & -A_{35} & A_{36} \end{bmatrix} \begin{bmatrix} dY^a \\ dY^b \\ dY^c \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \\ A_{30} \end{bmatrix}$$

This system of equations can be written in the following from:

$$\begin{bmatrix} dY^a \\ dY^b \\ dY^c \end{bmatrix} = \begin{bmatrix} 1-A_{14} & A_{15} & A_{16} \\ A_{24} & 1-A_{25} & A_{26} \\ A_{34} & A_{35} & 1-A_{36} \end{bmatrix} \begin{bmatrix} dY^a \\ dY^b \\ dY^c \end{bmatrix} + \begin{bmatrix} A_{10} \\ A_{20} \\ A_{30} \end{bmatrix}$$

The matrix of the above equation system can be written shortly as (I-A), where the elements A_{ij} for $i \neq j$ and $1-A_{ii}$ for all i and j of this matrix can be interpreted as the share of expenditure of country j for products of country i.

The matrix (I-A) is in general indecompable and with dominant diagonal. It looks like the (I-A)-matrix of the input-output model.

1) See Lemma 6

As in the previous section, we call a stabilization policy as an expansive policy, if $dA_{io} > 0$ for $i = a, b, c$.

Using the Frobenius' theorems or the theorems on the dominant diagonal matrices we can prove the following theorems. As in the previous section, we assume in the course of this section that the Marshall-Lerner condition holds for all bilateral trade balances.

Theorem 6 : The system of Keynesian economy in the mass unemployment case has a unique solution

Proof : Since the matrix (I-A) is indecomposable and with dominant diagonal. According to Lemma 6 there exists a unique solution.

Theorem 7 : Given an expansive policy in the country i with given policy in the other countries, then (a) the real national income in all three countries must increase and (b) that of country i will do so by the largest percentage.

Proof : (a) follows directly from Theorem 6

(b) Since the matrix (I-A) is indecomposable and with dominant diagonal, and since the sum of every row in this matrix is less than one. Hence according to Lemma 7

$$\frac{\partial Y^i}{\partial A_{io}} \geq \frac{\partial Y^j}{\partial A_{io}} \qquad \text{for } i \neq j, \quad i,j=a,b,c$$

(Q.E.D.)

Remark : The Laursen-Metzler paradox does not appear for our three-country model with pegging exchange rate policy. This seems to be an interesting consequence of the pegging exchange rate policy for the international transmission of business cycles.

<u>Theorem 8</u> : Given a solidary expansive policy in the country i (i=a,b,c), the real national income in all countries will increase with elasticities not exceeding unity.

<u>Proof</u> : Let an expansive policy in country i be $A_{io} \rightarrow A'_{io}$ and the new solutions from the system of equation (P.29) be $dY^i \rightarrow (dY^i)'$ and $A_{io} = V_i\ A'_{io}$ and $dY^i = \lambda_i (dY^i)'$ with $1 > V_i$ and $\lambda_i > 0$

$$\eta_{Y^i/A_{jo}} = \frac{(dY^i)' - dY^i}{(dY^i)'} \Big/ \frac{A'_{jo} - A_{jo}}{A'_{jo}} = \frac{(1-\lambda_i)}{(1-V_j)} \quad \frac{V_j}{\lambda_i}$$

and from the equation system (P.29) from the j-th equation

$$dY^j = (I-A)_j\ dY + A_{jo} \quad ^{1)} \qquad \text{or}$$

$$\text{(i)} \quad \lambda_j (dY^j)' = (I-A)_j\ \lambda_i \cdot dY' + V_j\ A_{jo}$$

Suppose $V_j > \lambda_j$: Since $\lambda_j \geqq \lambda_i$ for all i

(Theorem 7), and $X'_j = (I-A)_j\ dY' + A'_{jo}$

we obtain $\lambda_j (dY^j)' < (I-A)_j\ dY' + V_j\ A_{jo}$

which contradicts (i). Hence $V_j \leqq \lambda_j$ and since $1 \geqq \lambda_i \geqq \lambda_j$, i.e. $\eta_{Y^i/A_{jo}} \leqq 1$ (Q.E.D.)

---

1) $(I-A)_j$ = the j-th row of the matrix (I-A)
$dY = (dY^a, dY^b, dY^c)$

<u>Theorem 9</u> : The effect of an expansive policy on the exchange rate of the floating currency:

$$\frac{de}{dA_{10}} \gtreqless 0 \iff M_Y^c \frac{dY^c}{dA_{10}} \gtreqless M_Y^{ac} \frac{dY^a}{dA_{10}} + M_Y^{bc} \frac{dY^b}{dA_{10}}$$

$$\frac{de}{dA_{20}} \gtreqless 0 \iff M_Y^c \frac{dY^c}{dA_{20}} \gtreqless M_Y^{ac} \frac{dY^a}{dA_{20}} + M_Y^{bc} \frac{dY^b}{dA_{20}}$$

$$\frac{de}{dA_{30}} \gtreqless 0 \iff M_Y^c \frac{dY^c}{dA_{30}} \gtreqless M_Y^{ac} \frac{dY^a}{dA_{30}} + M_Y^{bc} \frac{dY^b}{dA_{30}}$$

<u>Proof</u> : These follow directly from

$$de = M_Y^c \, dY^c - M_Y^{ac} \, dY^a - M_Y^{bc} \, dY^b$$

for $dP^i = dq = 0$ for all i

6. The Keynesian Economy

In this section we shall consider the Keynesian economy between the two extreme cases of full employment in the sense that $dY^i = 0$ for all i and mass unemployment in the sense that $dP^i = 0$ for all i. In the Keynesian economy the monetary wage rate is given exogenous. In our model the production is carried out without imported inputs. The system of aggregate supply is given as follows (in deferentials) :

$$dY^i = F^i_P \, dP^i + F^i_W \, dW^i \qquad \text{for} \quad i=a,b,c$$

$$\text{with} \quad F^i_P > 0 \quad \text{and} \quad F^i_W < 0$$

or in other form:

$$dP^i = b_{i1} \, dY^i + b_{i2} \, dW^i$$

$$\text{with} \quad b_{i1} = 1/F^i_P > 0$$

$$b_{i2} = - F^i_W/F^i_P > 0$$

Thus the aggregate supply of each country is determined independently from the other countries. Set these equations in the system of equations in Lemma 5:

$$\begin{bmatrix} B_{11} & -B_{12} & -B_{13} \\ -B_{21} & B_{22} & -B_{23} \\ -B_{31} & -B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} dY^a \\ dY^b \\ dY^c \end{bmatrix} = \begin{bmatrix} B_{10} \\ B_{20} \\ B_{30} \end{bmatrix}$$

where

$$B_{11} = b_{a1} A_{11} + A_{14} > A_{14}$$

$$B_{12} = b_{b1} A_{12} + A_{15} > A_{15}$$

$$B_{13} = A_{16}$$

$$B_{10} = b_{a2} A_{11} dW^{a} + b_{b2} A_{12} dW^{b} + A_{10}$$

$$B_{21} = b_{a1} A_{21} + A_{24} > A_{24}$$

$$B_{22} = b_{b1} A_{22} + A_{25} > A_{25}$$

$$B_{23} = A_{26}$$

$$B_{20} = b_{a2} A_{21} dW^{a} + b_{b2} A_{22} dW^{b} + A_{20}$$

$$B_{31} = b_{a1} A_{31} + A_{34} > A_{34}$$

$$B_{32} = b_{b1} A_{32} + A_{35} > A_{35}$$

$$B_{33} = b_{c1} A_{33} + A_{36} > A_{36}$$

$$B_{30} = b_{a2} A_{31} dW^{a} + b_{b2} A_{32} dW^{b} + b_{c2} A_{33} dW^{c} + A_{30}$$

The above system of equations can also be formulated as follows:

$$\begin{bmatrix} dY^{a} \\ dY^{b} \\ dY^{c} \end{bmatrix} = \begin{bmatrix} 1-B_{11} & -B_{12} & -B_{13} \\ -B_{21} & 1-B_{22} & -B_{23} \\ -B_{31} & -B_{32} & 1-B_{33} \end{bmatrix} \begin{bmatrix} dY^{a} \\ dY^{b} \\ dY^{c} \end{bmatrix} + \begin{bmatrix} B_{10} \\ B_{20} \\ B_{30} \end{bmatrix}$$

and this matrix is called as (I-B)

Assuming that the Marshall-Lerner condition holds for all bilateral trade balances, then the matrix (I-B) is indecomposable and with dominant diagonal. The elements of the matrix (I-B), i.e. $1-B_{ii}$ and B_{ij} for $i \neq j$ can be interpreted as the share of the expenditure of country j for products of country i.
In this section we shall assume that the Marshall-Lerner condition holds for all bilateral trade balances.

Theorem 10: The system of Keynesian economy in the usual case (i.e. the case between the full employment and mass unemployment) has a unique solution.

Proof : Since the matrix (I-B) is indecomposable and with dominant diagonal. According to Lemma 6 there exists a unique solution.

Theorem 11: Given an expansive policy in the country i with given policy in the other countries, then (a) the real national income in all three countries must increase and that of country i will do so by the largest percentage and (b) the price levels in all countries must increase.

Proof : The first part of (a) follows directly from Theorem 10, since

$$dY^i = (B)_i^{-1} A_{io} > 0 \quad \text{for} \quad A_{io} \geq 0 \; , \quad i=a,b,c$$

The second part of (a) holds true since the matrix (I-B) is indecomposable and with dominant diagonal (see Lemma 7).

The effects of expansive policy on the price levels can be seen by

$$\frac{dP^i}{dA_{io}} = b_{i1} \frac{dY^i}{dA_{io}} > 0 \; , \quad \text{for} \quad i=a,b,c$$

$$\text{since} \quad b_{i1} > 0 \quad \text{and} \quad \frac{dY^i}{dY_{io}} > 0$$

<u>Theorem 12</u>: An increasing in the monetary wage rate in the country i with given monetary wage rate in the other countries, then (a) the real national income in all three countries must decrease, and that of country i will do so by the largest percentage; and (b) the price levels in all three countries must increase.

<u>Proof :</u> (a) Analogous to the part (a) of the Theorem 11 with the exception that $b_{i2} < 0$ for all i and theorefore

$$\frac{dY^i}{dW_j} < 0 \quad \text{for all } i \text{ and } j$$

(b) Since $$\frac{dP^i}{dW^j} = b_{i2} \frac{dY^i}{dW^j} > 0$$

for $b_{i2} < 0$ and $\frac{dY^i}{dW^j} < 0$ for all i and j

(Q.E.D.)

7. Concluding Remarks:

In Theorem 1 to 5 the Marshall-Lerner condition is assumed to hold for all bilateral trade balances of the trading countries. In this case, A_{ij} for all i and j are positive. But the Marshall-Lerner condition for bilateral trade balances is necessary for A_{ij} 0. In any case, the A_{ij} for all i and j can be positive if the Marshall-Lerner condition is not fulfilled for some bilateral trade balances. Therefore, the above theorems can also be true even the Marshall-Lerner condition does not hold for some bilateral trade balances.

In general, the Marshall-Lerner condition seems not to hold for all bilateral trade balances, expecially when the deficit in the bilateral trade balance is sufficiently large, for example Taiwan and Austria have large deficit in their bilateral trade balance to Japan and W. Germany, respectively. It seems to be expected that the Marshall-Lerner condition does not hold for these two cases of bilateral trade balance.

References:

1. Black, S. W. [1976]
Exchange Policies for Less Developed Countries in a World of Floating Rates
Seminar Paper No. 53. Institute for International Economic Studies, University of Stockholm

2. Bowers, D. A. and Baird, R. M. [1971]
Elementary Mathematical Macroeconomics
Parentice-Hall, 1971

3. Chen, John-ren [1980]
The Effects of the Partially Pegging Exchange Rate Policy in a Small Open Economy

4. Laursen, L. and Metzler, L. A. [1950]
Flexible Exchange Rates and the Theory of Employment
in: Review of Economics and Statistics

5. Patinkin, D. [1965]
Money, Interest and Prices, An Integration of Money and Value Theory, New York

6. Sohmen, E. [1973]
Wechselkurs und Währungsordnung, Tübingen

7. Takayama, A. [1969]
The Effect of Fiscal and Monetary Policies under Flexible and Fixed Exchange Rates
in: Canadian Journal of Economics