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A NONCOOPERATIVE MODEL OF CHARACTERISTIC-FUNCTION BARGAINING

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Cooperative game theory has produced quite a number of solution concepts for games in characteristic-function form. Among the most important theories are the von Neumann-Morgenstern solution (von Neumann-Morgenstern, 1944), the Shapley-value (Shapley, 1953), the bargaining set (Aumann-Maschler, 1964), and the nucleolus (Schmeidler, 1969). The aim of these and many other solution concepts has been normative, rather than descriptive, even if some primarily descriptive theories like equal-share analysis have been proposed (Selten, 1972).

One of the reasons for the multiplicity of cooperative-solution concepts can be seen in the fact that a game in characteristic-function form is an insufficient description of real game situations. The way in which proposals can be made and agreements can be reached is left unspecified. If one wants to run an experiment in characteristic-function games, one has to fix specific rules which regulate the process of negotiation. It is not surprising that different ways of doing this can lead to different results.

The necessity to specify detailed rules arises especially in any attempt to run characteristic-function experiments with highly formalized anonymous interaction and communication.

In the experiments of Kahan and Rapoport, the players interact only by a limited set of teletyped messages, like proposals and decisions to accept or reject the proposals of other players (Kahan-Rapoport, 1974). If one looks closely at their experimental setup, it becomes obvious that the players are really involved in a noncooperative extensive game. This suggests that one should try to analyse situations of this type with the tools of noncooperative game theory.

No attempt will be made here to attack the difficult task to explore the extensive game used by Kahan and Rapoport. Instead of this, a much simpler set of rules will be investigated.

John C. Harsanyi's noncooperative model for the von Neumann Morgenstern solution has shown how much can be gained for the understanding of cooperative game theory by the reinterpretation of cooperative solution concepts in terms of equilibrium points of extensive negotiation games (Harsanyi,1974). The analysis of the noncooperative model introduced here also yields results which are connected to a cooperative solution concept, namely, the theory of stable demand vectors (Albers,1975). The theory of Albers is similar to that of John Cross (Cross,1967), but the stability conditions are different. Both solution concepts start from the same basic idea, to look at demand vectors rather than imputations. Demand vectors have components which do not necessarily sum up to the value of the grand coalition. Another piece of work which should be mentioned in this connection is the unpublished dissertation of Turbay, which presents a theory very near to that of John Cross, but in a mathematically much more satisfactory form (Turbay, 1977).

The bargaining rules underlying the noncooperative model to be introduced in this paper are extremely simple. Formally, the model is an infinite-recursive game with perfect information (Everett, 1954).

It is the aim of the analysis to investigate the stationary equilibrium points of this game. It will be shown that every stationary equilibrium point is connected to a demand vector satisfying two of the three conditions imposed by the theory of Albers. Equilibrium points connected to fully stable demand vectors in the sense of Albers can be characterized by additional intuitively reasonable properties.

1. Characteristic-function games.

A characteristic function is a function v, which assigns a real number $v(C)$ to every subset C of $N = \{1,\dots,n\}$; the set N is the set of players and the subsets of N are called coalitions; the empty coalition $\emptyset$ always has value zero:

(1) $v(\emptyset) = 0$

A characteristic function is zero-normalized if we have

(2) $v(i) = 0$ for $i = 1,\dots,n$

It is called superadditive if the following is true for any two coalitions C and D with $C \cap D = \emptyset$:

(3) $v(C \cup D) \geq v(C) + v(D)$

A zero-normalized superadditive characteristic function is called essential if we have

(4) $v(N) > 0$

Let v be a zero-normalized, superadditive, essential-characteristic function. We say that v has the one-stage property if

(5) $v(C) > 0$ implies $v(N \setminus C) = 0$

The value $v(C)$ of a coalition is interpreted as a sum of money which the players in C can distribute among themselves if they reach an agreement on a payoff division. Generally, one thinks of a characteristic function game as being played in such a way that each player can enter only one coalition. (Other interpretations are possible (Harsanyi, 1963), but will not be discussed here.) The one-stage property has the consequence that only one coalition C with $v(C) > 0$ can be formed. If (5) does not hold, the formation of a coalition with positive value may be only the first stage of the game. The players in $N \setminus C$ can still go on to form further such coalitions.

Neither the theory of Albers nor the bargaining model to be proposed here permits meaningful application to characteristic functions without the one-stage property. A generalization to such games seems to be possible in both cases, but no attempt in this direction will be made here. The investigation will be restricted to characteristic functions with the one-stage property.

The bargaining model will be defined for a zero-normalized superadditive, essential-characteristic function v with the one-stage property. The symbol v will stand for an arbitrary, fixed-characteristic function of this kind. All definitions will be relative to v.

2. Stable demand vectors.

A demand vector $d = (d_1,\ldots,d_n)$ is a vector with n real components. We use the following notational conventions:

(6) $d_C = (d_i)_C$

This means that d_C is the collection of the components d_i of d with $i \in C$.

(7) $d(C) = \sum_{i \in C} d_i$

(8) $F_i(d) = \{C | C \subseteq N,\ i \in C,\ d(C) = v(C)\}$
for $i = 1,\ldots,n$.

Coalitions C with $d(C) = v(C)$ will be called d-compatible. $F_i(d)$ can be described as the set of d-compatible coalitions containing player i .

Stable-demand vectors in the sense of Albers are characterized by three conditions:

(a) Maximality: $d(C) \geq v(C)$ for every $C \leq N$

(b) Feasibility: $F_i(d) \neq \emptyset$ for $i = 1,\ldots,n$

(c) Balancedness: For any two players $i,j \in N$, we have
either $F_i(d) = F_j(d)$
or $F_i(d) \setminus F_j(d) \neq \emptyset$ and $F_j(d) \setminus F_i(d) \neq \emptyset$.

A demand vector d is called <u>semistable</u> if it satisfied (a) and (b); and it is called <u>stable</u> if it satisfies (a), (b), and (c).

Interpretation: A demand vector d can be interpreted as a vector of aspiration levels for payoffs in a coalition. The intuitive idea behind the conditions can be expressed by saying that nobody should be able to raise his aspiration level and noboby should be forced to lower it.

Condition (a) requires that demands are maximal in the sense that nobody can raise his demand without excluding himself from all coalitions which can satisfy the demands of all their members. Condition (b) requires that player i's demand is feasible in the sense that he can propose at least one coalition C with $i \in C$ which can satisfy the demands of its members. Note that coalitions capable of more than that are already excluded by (a). Condition (b) is based on the idea that a player with $F_i(d) \neq \emptyset$ would have to lower his demand in order to make it feasible.

Condition (c) removes the possibility that $F_i(d)$ is a proper subset of $F_j(d)$. If this is the case, player j can propose a d-compatible coalition which excludes i, but player i cannot propose a d-compatible coalition which excludes j. This creates a unilateral dependence of i on j. In the case $F_i(d) = F_j(d)$, both players depend on each other in the sense that neither of them can propose a d-compatible coalition including himself and excluding the other. In the second case permitted by (c), neither of the players depends on the other in this sense. We may say that the dependencies are required to be "balanced". This concept of balancedness is of course different from that which has been introduced in connection with conditions for the nonemptyness of the core (Shapley, 1967).

Albers has proved a theorem which shows that stable-demand vectors always exist (Albers, personal communication).

3. Recursive games.

We shall be interested only in recursive games with perfect information without chance moves. Moreover, we shall impose restrictions on the payoff which further narrow the class of games considered. For the purpose of this paper, it will be convenient to define a recursive game as a game with the properties mentioned above, even if the concept as it has been introduced by Everett is a much more general one (Everett, 1954).

A recursive game $G = (X, x_o, Z, P, A, h)$ has the following constituents:

X	the set of positions, an arbitrary set.
x_o	the initial position, an element of X.
Z	the set of endpoints, a subset of X.
$P = (P_1, \ldots, P_n)$	the player partition, a partition of $X \setminus Z$ into player sets P_i.
A	the choice function, a function which assigns a nonempty choice set $A(x) \subseteq X$ to every $x \in X \setminus Z$.
h	the payoff function, which assigns a payoff vector $h(z) = (h_1(z), \ldots, h_n(z))$ to every $z \in Z$. The function h satisfies a restriction of the form $0 \leq h_i(z) \leq \bar{h}$ for $i = 1, \ldots, n$ and every $z \in Z$ where $\bar{h}$ is a constant. (Infinite plays have zero payoffs.)

Interpretation: At a position $x \in X \setminus Z$, the player i with $x \in P_i$ must choose one of the positions in $A(x)$. The game begins with x_o and ends as soon as an endpoint $z \in Z$ has been reached. $h_i(z)$ is player i's payoff if z is reached. The payoff for an infinite play is zero for all players. The payoff restriction $h_i(z) \geq 0$ is an expression of the idea that we want to model situations where infinite plays are not desirable in the sense that no player can gain anything by behaving in a way which leads to an infinite play.

A recursive game is like a game in extensive form, but it may happen that not all elements of X can be reached from x_o.

Strategies: It will not be necessary to define strategies for individual players. We shall only define global behavior strategies which specify the behavior of all players in every conceivable situation.

Let $G = (X,x_o,Z,P,A,h)$ be a recursive game.
A local strategy b_x at a point $x \in X \setminus Z$ is a probability distribution over $A(x)$ with finite carrier. The probability assigned by b_x to $y \in A(x)$ is denoted by $b_x(y)$. The expression "with finite carrier" means that positive probabilities $b_x(y)$ are assigned to a finite number of choices $y \in A(x)$ only. For every $x \in X \setminus Z$, let B_x be the set of all local strategies.

A global strategy b is a function which assigns a local strategy b_x to every $x \in X \setminus Z$. Let B be the set of all global strategies for G.

Remark: The global strategies defined above are stationary in the sense that behavior depends only on x and not on the whole past history. Stationary equilibrium points, as we shall define them, do have this property; and we are not interested in any other equilibrium points. It can be shown that stationary equilibrium points are stable with respect to nonstationary deviations, too, but this shall not be done here.

Expected payoff: For every $x \in X$ and every global strategy $b \in B$, we define player i's expected payoff $E_i(b|x)$ for b at x. In order to do this, we need some auxiliary definitions. A sequence $x_1,\dots,x_m$ with $x_k \in X$ for $k = 1,\dots,m$ is called a position chain from x_1 to x_m if we have $x_{k+1} \in A(x_k)$ for $k = 1,\dots,m-1$. The probability of a position chain $x_1,\dots,x_m$ if b is played is the product of all $b_{x_k}(x_{k+1})$ with $k = 1,\dots,m-1$. Let $p(x,z,b,m)$ be the sum of all probabilities of position chains from x to z with, at most, m positions if b is played. Player i's expected payoff is defined as follows:

$$E_i(b|x) = \lim_{m\to\infty} \sum_{z\in Z} p(x,z,b,m)h_i(z) \tag{9}$$

for $i = 1,\dots,n$.

Since the b_x have finite carriers, finitely many $z \in Z$ are reached with positive probability by chains of at most m positions. In view of $\bar{h}_i(z) \geq 0$, the sum whose limit is formed in (9) is a nondecreasing function of m . Moreover, this sum is bounded from above by $\bar{h}$. Therefore, the limit in (9) always exists.

Stationary equilibrium point: For $i = 1,\ldots m$, a choice $y \in A(x)$ at a position $x \in P_i$ is called optimal at x , if we have:

$$(10) \quad E_i(b|y) = \max_{w \in A(x)} E_i(b|w)$$

A global strategy b is a stationary equilibrium point of G if for every $x \in X \setminus Z$, the following is true for the local strategy b_x assigned to x by b . Every $y \in A(x)$ with $b_x(y) > 0$ is optimal at x .

Local and global optimality: The definition of a stationary-equilibrium point given above is based on a local optimality condition. It can be shown that local optimality in the sense of (10) implies global optimality. For this purpose, we introduce the following definition: A deviation c of player i frim b is a gloval strategy which differs from b only at positions $x \notin P_i$, i.e., we have $b_x = c_x$ for all $x \in X \setminus Z$ with $x \notin P_i$. We cannot have

$$(11) \quad E_i(c|x) > E_i(b|x)$$

unless for sufficiently great m we have

(12) $\sum_{z \in Z} p(x,z,c,m)h_i(z) > E_i(b|x)$

Let $\bar{m}$ be the first m for which (11) holds. We construct a new game, G_x , which begins at x and has the same rules as G with the exception that every play with more than $\bar{m}$ position is cut off at its $\bar{m}$-th position, $x_{\bar{m}}$, which becomes an endpoint with payoffs $E_j(b|x_{\bar{m}})$ for all players $j = 1,\dots,n$.

If (10) holds, the restriction of b to G_x is an equilibrium point of G_x (It is well known that in games of finite length with perfect information, local optimality in the sense of (10) implies global optimality.). Since player i's equilibrium payoff for the restriction of b to G_x is $E_i(b|x)$, it follows by (10) that (12) cannot hold. We can conclude that the following theorem holds:

<u>Theorem 1</u>: If b is a stationary-equilibrium point of G , then for $i = 1,\dots,n$ we have

(13) $E_i(c|x) \leq E_i(b|x)$

for every $x \in P_i$ and for every deviation c of player i from b .

Subgame perfectness: An equilibrium point is subgame perfect if it induces an equilibrium point on every subgame (Selten, 1965, 1975). We do not want to go into the details of the definition of subgame perfectness as this is not necessary here. Nevertheless, it is worth pointing out that stationarity of an equilibrium point in the sense of the definition given above implies subgame perfectness as the optimality condition (10) refers to the local payoff at x .

4. The noncooperative model of characteristic-function bargaining.

The basic ideas of the bargaining model can best be explained by an informal account of the rules. A precise definition of the recursive game will be given in section 5. The verbal description of the rules is illustrated by the flow chart of Figure 1.

RULES

1. Initiator: A player may find himself in the position of an initiator, who can make a proposal if he wants to. At the beginning of the game, an arbitrarily selected player becomes the initiator (rectangle 1). We think of him as randomly chosen even if the random choice will not be a formal part of the model. The initiator must decide whether he wants to make a proposal (rhomboid 2). If he does not want to do this, he must shift the initiative to another player; this other player then becomes the new initiator (rectangle 3).

2. Proposals: An initiator who does not want to shift initiative must propose a coalition C with $v(C) > 0$, where he is a member, and a payoff division of $v(C)$ among the members of C. The other members of C are called receivers of the proposal. The initiator also must select one of the receivers; this receiver becomes the responder (rectangle 4).

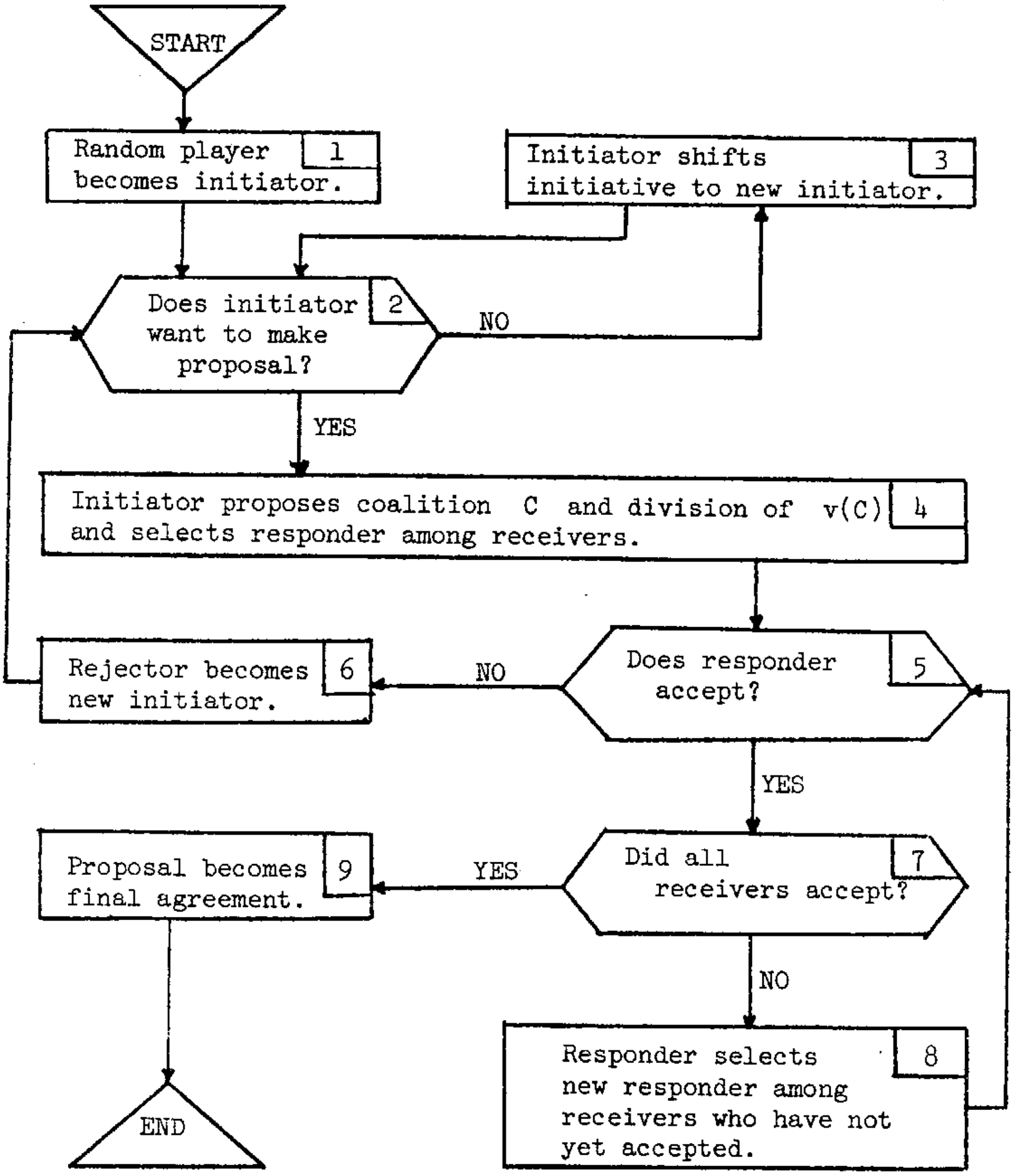

Figure 1: Flow chart of bargaining process.

3. <u>Responder</u>: The responder can either accept or reject the proposal (rhomboid 5). If he rejects, he becomes the new initiator (rectangle 6). The old proposal is erased, and he can make a new one (rhomboid 2). If the responder accepts, it is necessary to ask whether now all receivers have accepted the proposals (rhomboid 7). If this is the case, the game ends. Otherwise, the responder must select a new responder among the receivers who have not yet accepted.

4. <u>End</u>: The game ends after a proposal has been accepted by all receivers. The proposal then becomes the final agreement, and the members of the proposed coalition receive the proposed payoffs. All the other players get zero payoffs. In the case of an infinite play, all players get zero payoffs.

<u>Comment</u>: The rules describe a game with perfect information. This means that at every stage of the game each player knows everything which has happened up to now. Our analysis would not change under less stringent information conditions. An initiator does not have to know anything, and a responder does not have to know more than the proposal and the set of receivers who have not yet accepted. Of course, every player knows the rules of the game.

If one wants to model a process of characteristic-function bargaining, where the players act in sequence rather than simultaneously, one can hardly imagine a significantly simpler scheme.

In a laboratory procedure, the initiator could write the proposal on a card which would then be given to the chosen responder, etc. Obviously, experiments of this type can easily be arranged.

5. The bargaining game.

The rules explained in section 4 lead to a recursive game $G = (X,x_o,Z,P,A,h)$, which will be called the bargaining game. The bargaining game can be described by a list of types of positions, which indicates the player who has to make a decision, the choice set and, in the case of endpoints, the payoffs attached to them. This list is given below:

Position	player, choice set, payoffs
y_i	initiator position in P_i . The choice set $A(y_i)$ contains two kinds of positions: 1) $(j,\{i\},d_C)$ with $i,j \in C$ and $d(C) = v(C) > 0$, $d_C \geq 0$ 2) y_j with $j \in N \setminus \{i\}$.
(j,S,d_c)	responder position in P_j with $j \in C$ and $S \subseteq C \setminus \{j\}$ (The set S is the set of those members of C which have proposed or accepted d_C .) The choice set $A((j,\{i\},d_C))$ contains two kinds of positions:

1) y_j

2) $(k, S \cup \{j\}, d_C)$ for $S \cup \{j\} \subseteq C$

or (C, d_C) for $S \cup \{j\} = C$.

(C, d_C) <u>endpoint</u> with payoffs

$$h_i((C,d_C)) = \begin{cases} d_i & \text{for } i \in C \\ 0 & \text{for } i \notin C \end{cases}$$

The initial position x_o is one of the y_i .

<u>Example</u>: The interpretation of the bargaining game G is clear from the verbal description of the rules. Nevertheless, it may be helpful to illustrate the formalism with the help of a numerical example of a play, shown in Figure 2.

It is assumed that x_0 is player 1's initiator position y_1 . He makes the proposal $(70,10)_{1,3}$, which means that he proposes $C = \{1,3\}$ with payoffs 70 for himself and 10 for player 3. Player 3 rejects the proposal by choosing y_3 and then proposes 40 for player 1 and 30 for player 2 and 30 for himself. He selects player 1 as responder. Player 1 accepts and selects player 2 as responder. Now every member has proposed or accepted the proposal. An endpoints results and the proposal becomes the final agreement.

Characteristic function

$N = \{1,2,3\}$

$v(1) = v(2) = v(3) = 0$

$v(1,2) = 90$, $v(1,3) = 80$, $v(2,3) = 70$

$v(1,2,3) = 100$

Play

1. y_1
2. $(3,\{1\},(70,10)_{1,3})$
3. y_3
4. $(1,\{3\},(40,30,30)_{1,2,3})$
5. $(2,\{1,3\},(40,30,30)_{1,2,3})$
6. $(\{1,2,3\},(40,30,30)_{1,2,3})$

Figure 2: Example of a play of the bargaining game for a numerically given characteristic function v .

6. Stationary equilibrium points of the bargaining game.

We first introduce some further notation which is needed in order to express results and proofs in a convenient way. G will always be the bargaining game.

For any global strategy b and every $x \in X$, let $Z(b|x)$ be the set of all endpoints which are reached with positive probability after x, if b is played.

Further define

(14) $q_i = E_i(b|y_i)$ with $i = 1,\dots,n$

We call q_i player i's quota for b, and we shall refer to the vector $q = (q_1,\dots,q_n)$ as the demand vector for b. Our notational conventions (6), (7), and (8) will also be applied to q.

The number of members of a coalition C is denoted by $|C|$.

Lemma 1: Let b be a stationary equilibrium point of G and let q be the demand vector of b. Then

(15) $q(C) \geq v(C)$

holds for every $C \subseteq N$.

Proof: Suppose $q(C) < v(C)$ and assume $j \in C$.

Define

$$(16) \quad \varepsilon = \frac{1}{|C|}[v(C) - q(C)]$$

Player j can make the following proposal:

$$(17) \quad d_c = (q_i + \varepsilon)_c$$

As we shall see, it is optimal for all receivers of the proposal to accept it. It is clear that a responder must accept it if all others have already accepted it. Rejection would only yield his quota. It follows by induction on the number of receivers who have not yet accepted that all receivers must accept regardless of the order in which they become responders. This shows that player j can get more than q_j at y_j, which contradicts the assumption that b is a stationary equilibrium point.

Lemma 2: Let b be a stationary equilibrium point of G and let q be the demand vector of b. Then

$$(18) \quad E_i(b|y_j) \leq q_i$$

for $i,j = 1,\ldots,n$ and $i \neq j$.

Proof: At y_i, player i can shift initiative to player j and thereby receive $E_i(b|y_j)$.

Lemma 3: Let b be a stationary-equilibrium point of G and let q be the demand vector of b . For every $z \in Z(b|y_j)$, we have $z = (C,q_C)$ for some C and $h_j(z) = q_j$; moreover, for $q_j > 0$, we have $j \in C$.

Proof: Let s be a position sequence from y_j to $(C,d_C) \in Z(b|y_j)$, which is realized with positive probability by b once y_j has been reached. Consider a player $i \in C$. There must be a position $x \in P_i$ on s such that after x there is no initiator position y_m on s ; either we have $x = y_i$ or player i is one of the receivers who has to accept d_C before (C,d_C) is reached.

The expected payoff $E_i(b|x)$ at this position x must be at least q_i , since player i can attain y_i by rejection if $x \neq y_i$. Let α be the probability with which none of the receivers who have not yet accepted d_C will reject d_C if b is played and x has been reached. With probability $1 - \alpha$, one of these receivers will reject d_C . In view of lemma 2, player i gets at most q_i if this happens. We can conclude that the following inequality holds:

(19) $q_i \leq E_i(b|x) \leq \alpha d_i + (1 - \alpha)q_i$

where d_i is player i's component in d_C . Because α is positive, this yields $q_i \leq d_i$. Because the argument can be applied to every $i \in C$ and to every $(C,d_C) \in Z(b|y_j)$, it follows by lemma 1 that we have $d_C = q_C$ for every endpoint $(C,d_C) \in Z(b|y_j)$.

Suppose $q_j = 0$. Then, player j's payoff is zero at every endpoint in $Z(b|y_j)$. Suppose $q_j > 0$. Then for every $(C,q_C) \in Z(b|y_j)$, player j must be a member of C because, otherwise $E_i(b|y_j)$ would have to be smaller than q_j .

Pure stationary-equilibrium points: A local strategy b_x is called pure if it assigns probability 1 to one choice $y \in A(x)$ and zero to all others. A global strategy b is called pure if it assigns a pure local strategy b_x to every $x \in X \setminus Z$. A pure global strategy can be described by the function f , which assigns to every $x \in X \setminus Z$ that $y = f(x) \in A(x)$ which is chosen with probability 1 by the local strategy b_x of b . In agreement with common game-theoretical conventions, we shall identify a pure global stragety b with the function f which corresponds to it in this way. A stationary equilibrium point is called pure if it is a pure global strategy.

Theorem 2: The demand vector q of a stationary equilibrium point b of G is always semistable. If q is a semistable demand vector, then a pure stationary-equilibrium point f of G can be found such that q is the demand vector of f.

Proof: Let b be a stationary-equilibrium point. It follows by lemma 1 that the demand vector q of b satisfies (a). For $q_i = 0$, the set $F_i(q)$ contains $\{i\}$. For $q_i > 0$, condition (b) follows by lemma 3. Consequently, q is semistable.

Now suppose that q is an arbitrary semistable demand vector. We construct a pure stationary-equilibrium point with q as its demand vector: (For an illustration, see section 9.)

$$\text{(20)} \quad f(y_i) = (j,\{i\},q_{C_i}) \quad \text{with} \quad C_i \in F_i(q)$$

$$\text{(21)} \quad f((i,S,d_C)) = \begin{cases} (C,d_C) & \text{for } C = S \cup \{i\} \\ & \text{and } d_i \geq q_i \\ (j,S \cup \{i\},d_C) & \text{for } S \cup \{i\} \subsetneq C \\ & \text{and } d_k \geq q_k \text{ for } k \in C \setminus S \\ y_i & \text{else} \end{cases}$$

An initiator makes a proposal q_{C_i} where he gets his quota.

A receiver accepts if all receivers who have not yet accepted, including himself, get at least their quotas. Therefore, q_{C_i} is accepted by all receivers and q_i is player i's payoff expectation at y_i .

Player i at y_i cannot improve his payoff by a deviation. A proposal d_C with $d_i > q_i$ would have to give $d_j < q_j$ to at least one player $j \in C$. This player would reject the proposal. Obviously, it is optimal to accept a proposal which yields at least q_i . It is also optimal to reject a proposal with $d_i < q_i$ or $d_k < q_k$ for some other player $k \in C \setminus S$, who would later reject the propsal anyhow. Consequently, f is a pure stationary-equilibrium point of G.

7. Additional Properties.

Semistability is a very weak condition on demand vectors. We can conclude from theorem 2 that the bargaining game permits a wide range of possible stationary-equilibrium points. This raises the question whether all of these equilibria are equally plausible as rational prescriptions for playing the game. As we shall see, one receives a much narrower class of stationary-equilibrium points if three reasonable properties are imposed as additional requirements. This class contains stationary-equilibrium points whose demand vectors are stable. Some further definitions and notational conventions are needed before we describe the three properties in detail. In these definitions b will always stand for a fixed stationary-equilibrium point of the bargaining game G. Moreover, $q = (q_1,\ldots,q_n)$ will always be the demand vector of b and b_x will denote the local strategy assigned to x by b.

Effectiveness: A responder position (j,S,d_C) is called effective for b, if b prescribes acceptance for (j,S,d_C) and for all responder positions (i,R,d_C) which might follow (j,S,d_C) if no receiver rejects. This means that for $x = (i,R,d_C)$ with either

(22) $R = S$ and $i = j$

or

(23) $R \supseteq S \cup \{j\}$ and $i \in C \setminus R$

we have

(24) $b_x((k, R \cup \{i\}, d_C)) = 1$

for some $k \in C \setminus (R \cup \{i\})$ or

(25) $b_x((C, d_C)) = 1$ for $R \cup \{i\} = C$

if (j, S, d_C) is effective, the endpoint (C, d_C) will be reached with certainty if b is played. Noreover, (C, d_C) will be reached in the shortest possible way.

The initiator positions y_i are not counted as effective, but endpoints are defined as effective.

Unpunished provocations: A position $x = (j, \{i\}, d_C)$ is called a provocation of i against j if we have:

(26) $d_i > q_i$ and $d_j < q_j$

We say that a provocation is unpunished if in addition to this e have

(27) $E_i(b|x) = q_i$

and

(28) $\quad i \in C \quad$ for all $z = (C, q_C) \in Z(b|x)$

In view of lemma 3, equation (27) implies (28) for $q_i > 0$. For $q_i = 0$, however, (28) is an additional condition.

The definition is motivated by the following ideas: q_i is the maximum player i can get at y_i . A choice of x at y_i will be punished if (27) does not hold as then $E_i(b|x)$ will be smaller than q_i . In the case $q_i = 0$, however, player i cannot be punished in this way. Condition (28) amounts to the assumption that a player prefers to be inside the final coalition rather than outside if payoffs are the same in both cases. This kind of lexicographical preference justifies the idea to look at exclusion from the final coalition as a form of punishment.

Additional properties: We shall look at the consequences of the following three additional properties, which can be imposed on a stationary-equilibrium point b of the bargaining game G .

(S) Shortness property: If for $x \in X \setminus Z$ the choice set $A(x)$ contains at least one position y which is optimal at x

and effective, then every w with $b_x(w) > 0$ is effective.

(M) Mixedness property: For $x = 1,\ldots,n$, the following is true: if $x \in A(y_i)$ is optimal and effective, then $b_{y_i}(x) > 0$.

(P) Provocation property: For any two players i and j, the following is true: If player i has an unpunished provocation against player j, then player j has an unpunished provocation against i.

Interpretation: The shortness property requires that a responder does not reject a proposal which gives him his optimum and will be accepted by all other receivers who have not yet accepted it. This kind of behavior may be interpreted as a secondary preference for shortness of the remainder of the play, where length is not measured by the number of positions, but rather in terms of the number of proposals yet to be made before the final agreement is reached. The preference for shortness is a secondary criterion which does not enter the picture wherever payoffs are different. It only helps to decide between choices with equal payoffs.

The shortness property also applies to an initiator; he is required to make an effective choice if he has one.

The mixedness property says that an initiator must choose every one of his effective choices with positive probability. This may be interpreted by saying that he is indifferent between different effective choices and, therefore, can be expected to randomize among them.

The provocation property can be looked upon as a noncooperative version of the stability condition (c) imposed by Albers. Suppose that (P) does not hold: player i has an unpunished provocation against player j, but player j has no unpunished provocation against player i. Then, player i faces no risk if he tries to get more by using his unpunished provocation. If player j sticks to b , he will reject it; but as we can see by (28), eventually a proposal will be made which includes player i in the coalition and gives him his quota. This means that player i has the power to delay the end of the game as long as he wants, whereas player j does not have this power.

8. Consequences of the additional properties.

In order to prove a theorem which connects the three properties with the stability of the demand vectors, we need several lemmata. In these lemmata, b will always be a stationary equilibrium point of the bargaining game G with the demand vector q.

Lemma 4: If b has property (S), then every x of the form (j,S,q_C) is effective. Moreover, every x of the form $(j,\{i\},q_C)$ is optimal at y_i and effective.

Proof: The second part of the lemma is an immediate consequence of the first one. The first one can be proved by induction on $|C \setminus S| = k$. For $k = 1$, property (S) requires the choice of (C,q_C) since this endpoint is the only effective and optimal choice. If the assertion holds up to $k-1$, it follows that it holds for k, too, since acceptance is a responders only effective and optimal choice.

Lemma 5: Let b have properties (S) and (M). For any two players i and j, the following is true: player i has an unpunished provocation against j if and only if $F_j(q) \subseteq F_i(q)$.

Proof: We first look at the case $q_j = 0$. In this case, player i has no unpunished provocation against player j since he has no such provocation at all. On the other hand, $F_j(q) \subseteq F_i(q)$ does not hold, since $\{j\}$ is in $F_j(q)$ but not in $F_i(q)$. Consequently, the assertion holds for $q_j = 0$. In the following, we shall assume $q_j > 0$.

We first show the necessity of $F_j(q) \subseteq F_i(q)$. Suppose $F_j(g) \setminus F_i(g) \neq 0$. We shall show that every provocation of player i against player j will be punished. Player j will reject the provocation. There are effective and optimal choices at y_j where coalitions are proposed which do not contain player i. In view of (M) , these choices are selected with positive probability. It follows by lemma 3 that player i's payoff will be lower than q_i .

Suppose $F_j(q) \subseteq F_i(q)$. In view of $q_j > 0$, player i can find a provocation against player j. Player j will reject this provocation, but in view of lemma 4, he must make an effective and optimal choice at y_j . This choice will involve the proposal of a coalition which contains player i and of a payoff division of the form q_C . In view of lemma 3, only such choices can be effective. This shows that player i's provocation is unpunished.

<u>Lemma 6</u>: Under the conditions (S), (M) and (P) , the demand vector q of b is stable.

<u>Proof</u>: It follows by lemma 5 and by (P) that the stability condition (c) must be satisfied for q , if b has the properties (S), (M) and (P) .

Theorem 3: The demand vector q of a stationary equilibrium point b of G with the properties (S), (M) and (P) is stable. For every stable demand vector q, there is a stationary equilibrium point b with (S), (M) and (P) such that q is the demand vector of b.

Proof: The first part of the theorem is nothing else than the assertion of lemma 6. Let q be a stable demand vector. Consider a global strategy with the following properties (i) and (ii) :

(i) For $i = 1,\ldots,n$, the local strategies b_{yi} of b assign positive probabilities to all $x \in A(y_i)$ of the form $x = (j,\{i\},q_C)$ and zero probabilities to all other choices.

(ii) At a responder position $x = (i,S,d_C)$, strategy b_x of b always assigns probability 1 to the choice $f((i,S,d_C))$ defined in (21).

We shall show that a global stragey b with (i) and (ii) is a stationary equilibrium point with (S), (M) and (P) .

The equilibrium properties of b can be seen immediately; a choice according to b always leads to the payoff q_i if b is played. Since all endpoints are of the form (C,q_C), an improvement beyond q_i is impossible.

It follows by lemma 4 that (S) and (M) are satisfied by construction. (P) follows by lemma 5.

9. A simple example.

Consider the following zero-normalized 3-person characteristic function game:

(29) $v(1,2) = v(1,3) = v(1,2,3) = 100$

(30) $v(2,3) = 40$

The demand vector $(50,50,50)$ is semistable with respect to v. It is interesting to look at a pure stationary equilibrium point f with $(50,50,50)$ as its demand vector. An equilibrium point of this kind can be constructed with the help of (20) and (21). In our case, there are two such equilibrium points; one with $C_1 = \{1,2\}$ and one with $C_1 = \{1,3\}$. We pick the first one:

(31) $C_1 = \{1,2\}$

(32) $C_2 = \{1,2\}$

(33) $C_3 = \{1,3\}$

We assume that y_1 is the initial position x_o. Why is it impossible for player 1 to get more than 50? Instead of offering 50 to player 2, he could try to get 55 by offering 45 to player 3. What happens according to f, if he does this? Player 3 will reject the proposal in order to make a new one which gives 50 to player 1 and 50 to himself. According to f, player 3 firmly believes that player 1 will accept this proposal.

Whenever player 1 makes a proposal in which he asks for more than 50, the same will happen to him; the proposal will be rejected and an even division of 100 will be proposed to him. This will go on no matter how long he diviates from f in this way.

The position

$$(34) \quad x = (2,\{1\},d_{\{1,2\}})$$

with $d_1 = 55$ and $d_2 = 45$ is an unpunished provocation of player 1 against player 2. Player 2 does not have an unpunished provocation against player 1. Obviously, this is connected with the strange nature of the equilibrium point f .

Suppose we change f in such a way that player 1 offers even divisions of 100 to both players 2 and 3 with equal probabilities. If this is done, the equilibrium properties are lost. Player 3 cannot afford any more to reject an offer of 45 since after a rejection he will leave only a chance of 1/2 to get 50. This shows why property (M) is important.

There is only one stable demand vector for v , namely (80,20,20) . A stationary equilibrium point with the properties (i) and (ii) in the proof of theorem 3 requires that each player i makes offers to both other players with positive probability at his initiator position y_i . None of the players can risk to ask for more than his quota.

10. The first move advantage.

All stationary equilibrium points b of the bargaining game G share an important feature which is worth pointing out. Let player j be that player whose initiator position y_j is the initial position x_0 of G. We say that this player j has the first move. Obviously, player j's expected payoff in the game is $q_j = E_j(b|y_j)$. Each of the other players i gets at most his quota q_i. Whereas, the player with the first move is sure to be in the final coalition, this may not necessarily be true for another player.

In the case where b satisfies (S), (M) and (P), a little more can be said on the advantage of having the first move. Suppose that q_i and q_j are positive and that $F_j(q)$ contains at least one coalition which does not belong to $F_i(q)$. This means that player j does not depend on player i in the sense that j can make a proposal which excludes i. If (M) is satisfied, such proposals will be made with positive probability. This has the consequence that player i's expected payoff $E_i(b|y_j)$ will be smaller than q_i. In this sense, it is advantageous for player j to have the first move.

If we have $F_i(q) = F_j(q)$, it does not matter for the players i and j which of them has the first move as long as in both cases a stationary equilibrium point is played whose demand vector is q. If (S), (M) and (P) hold and if the exceptional case $F_i(q) = F_j(q)$ does not arise, there is a definite advantage in having the first move, if we compare different situations where stationary equilibrium points with the same demand vector q are played.

One may object against the lack of symmetry inherent in the first move advantage. It is very probable that asymmetrics of this kind cannot be avoided in perfect information game models of negotiation. Therefore, one might be tempted to reject such models in favor of conceptualizations of the bargaining process which involve simultaneous choices. It is certainly important to look at such models, too, but it would be premature to discard the perfect information approach altogether.

It is quite plausible that at least in some real situations something like a first-move advantage may be a fact of life. Suppose that there is some profitable opportunity for coalition formation which for some time escapes the attention of the players involved. Assume that one of the players is the first one to recognize the fact that a game can be played. Obviously, he will be the first to approach other players in order to form a coalition, and it is not unreasonable to suppose that this will be advantageous for him.

It would be going too far to follow Schelling's attack on symmetric game models of social phenomena (Schelling, 1960) even if there is some truth in his arguments. Both symmetric and asymmetric models have their place in the description of conflict situations of substantive interest.

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