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The Equal Division Kernel: An Equity Approach to Coalition Formation and Payoff Distribution in n-Person Games

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Summary

Most existing models of coalition formation and payoff distribution in groups rest upon normative considerations and are ambiguous in their predictions insofar as they don't determine which of several coalitions will most probably result. The paper scetches the basic features of a model derived from social psychological exchange- and equity-theory which predicts coalitions and payoff distributions for a variety of situations. The evaluation of the model by the results of several experiments indicates that it provides a reasonable starting point for further theoretical developments that are based on empirical studies.

Zusammenfassung

Die meisten Modelle zur Koalitionsbildung und Gewinnaufteilung in Gruppen beruhen auf normativen Überlegungen und sind insofern unbestimmt in ihren Vorhersagen als sie nicht angeben, welche der verschiedenen Koalitionen sich bilden werden. Der vorliegende Artikel skizziert die grundlegenden Charakteristiken eines Modells, das - aufbauend auf der sozialpsycholigischen Austausch- und Equitytheorie - Koalitionen und Gewinnaufteilungen für eine Vielzahl von Situationen vorhersagt. Die Bewertung des Modells anhand der Ergebnisse verschiedener Experimente erweist den Ansatz als eine akzeptable Ausgangsbasis für weitere, auf empirischen Untersuchungen beruhenden,theoretische Entwicklungen.

The Equal Division Kernel: An Equity Approach to Coalition Formation and Payoff Distribution in n-Person Games. *)

by

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1. Introduction

Conflicting interest among the members of a group or among parties are commonplace in social interactions.
Making decisions under the condition of interpersonal conflict belongs to the daily routines not only of business men and politicians but also of teachers, social workers, officials, scientists, ect. In conflict situations where cooperation promises to be fruitful, the effect often depends on the formation of coalitions, i.e. subgroups of participants who decided to coordinate their actions in order to utilize their joint resources.

The increasing interest of social scientists in the analysis of characteristic function games indicates that this type of games is regarded as a useful abstraction of a coalitional situation that provides a basis for experimental analysis of the process and the result of coalition formation (Rapoport 1979). Although in social psychology most previous research has concentrated on weighted majority games (Gamson 1964), this restriction is neither desirable nor necessary.

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As Rapoport (1979) points out:
games in characteristic funtion form are sufficiently general to allow the study of three central issues, namely 1) which coalitions are most likely to form, 2) how the payoffs to a coalition will be allocated by its members, and 3) how will the bargaining process develop from the first contacts and offers to its final result?

We will not review the results of experiments with characteristic function games, however, it appears important to stress two main results that strikingly often occur. Firstly, it can be observed that coalitions with the strongest player are more often formed than coalitions excluding the strongest player. Especially coalitions among the two strongest players appear attractive. Secondly, the strongest player, to the benefit of the other players, usually gets less than he should get according to most accepted solution concepts.

In the following we will sketch the main features of a new solution concept that involves those two findings as a consequence. The concept is developed on the basis of social psychological equity theory and related empirical findings.

Before proceeding to the basic ideas of the concept we shall briefly describe the features of the characteristic function game and discuss some examples of such games and some solution concepts.

2. Prerequisites

2.1 Basic definitions and notations

The games we consider are given by a set $N = \{1,2,...,n\}$ (the players), a set P of subsets of N (the set of permissible coalitions) and a value function v, which assigns a real number $v(S)$ to any permissible coalition S. We assume that P contains at least all one person coalitions that $v(\emptyset) = 0$ and that $v(S) \geq \Sigma_{i \in S} v(i)$ for all $S \in P$. We call v superadditive iff $v(S \cup T) \geq (S) + v(T)$ whenever $S \cap T = \emptyset$.

In any play the players can form one or several disjoint permissible coalitions with $S_i \cap S_j = \emptyset$ for all $i \neq j$ and with $\cup S_i = N$ (isolated players are treated as one-player coaltions). A set $C = (S_1,\ldots,S_n)$ of coalitions that meet these conditions is called a <u>coalition structure</u>. In any coalition the players must agree upon the division of the joint payoff $v(S)$. So the results of a play can be described by a <u>payoff configuration</u>, i.e. a pair

$$(2.1) \quad (x,C) = (x_1,\ldots,x_n;\ S_1,\ldots,S_n),$$

where $C = (S_1,\ldots,S_n)$ is a coalition structure and $x = (x_1,\ldots,x_n)$ is a vector of payoffs of the players with $\Sigma_{i\in S_i} x_i = v(S_i)$ for all $S_i \in C$ and $x_i \geq v(i)$ for all $i \in N$.

For any $S \in P$ we define the equal share-value

$$(2.2) \qquad e(S) = \frac{v(S)}{|S|},$$

where the symbol $|S|$ denotes the number of players in S.

2.2 Normalized games

Sometimes it is convenient to normalize games in such a way that the $v(i)$-values are subtracted from all coalition values, i.e.

$$(2.3) \quad v'(S) = v(S) - \Sigma_{c\in S} v(i), \quad \text{for all } S \in N.$$

We call v' the O-normalized version of v. In O-normalized games all one person coalitions have the value $v'(i) = 0$.

It should be noted that the O-normalization changes the equal share values $e(S)$ of the coalitions $S \subseteq N$ ($e'(S) = v'(S)/|S| = e(S) - \Sigma_{i\in S} v(i)/|S|$).

Since the solution concept (we suggest in the following paragraphs) is based on the equal share values, it is important to know if subjects analyse a non-0-normalized game directly or via its corresponding 0-normalized version, i.e. if they consider the payoff which can be distributed within the coalition S or if they only consider the exceeding amount $v'(S)$.

In the data section of this paper we present the results of two experimental games with $v(i) > 0$. The distribution of payoffs in those games seem to justify the analysis in terms of v' rather than of the v-function.

2.3 Some examples of games

In an inessential n-person game the value of all coalitions correspond to the sum of the individual values (for an example see game a) in table 1. In such cases it is not profitable to form coalitions. A version of an essential game is given by the superadditive game b) in table 1. The type of games in table 1b) to 1d) have most often been investigated in experimental literature.
Since in 1c) and 1d) individual values are set equal to 0 all players are equal, on the basis of their individual values $v(i)$. However, they differentially contribute to the success of a coalition.

The game in table 1b) can be partitioned into an unessential game, 1e) with $v''(S) = \Sigma_{i \in S} v(i)$ and a 0-normalized game 1f) with values $v'(S)$. Game 1f) gives the excess of game 1b) over the unessential game 1e).

Table 1: Simple three-person games

	v(1)	v(2)	v(3)	v(12)	v(13)	v(23)	v(123)
a)	10	3	2	13	12	5	15
b)	10	3	2	17	13	8	19
c)	0	0	0	17	13	8	19
d)	0	0	0	17	13	8	23
e)	10	3	2	13	12	5	15
f)	0	0	0	4	1	3	4

2.4 The quotas of a game

In games of type 1c), 1d) and 1f) the contribution w_i of a player to the coalitional success can be calculated by solving the system of equations which is for 3-person games given by the two person coalitions: $v'(1,2) = w_1 + w_2$, $v'(1,3) = w_1 + w_3$, $v'(2,3) = w_2 + w_3$. The w_i- values are called the quotas of the game. The value of the coalition N might again differ from the sum of quotas, e.g. in example 1d) $v'(1,2,3) > w_1 + w_2 + w_3$.

To extend the notion of a quota to more general cases, we define a generalized quota game as a characteristic function game with a unique vector $w = (w_1, \ldots, w_n)$ such that (1) w is uniquely determined by some equations of the type $\Sigma_{i\in S}\, w_i = v'(S)$ with $S \subseteq N$ and (2) $\Sigma_{i\in S}\, w_i \geq v'(S)$ for all $S \subseteq N$, $S \neq N$. [1)]

In the following we assume that $w_1 \geq w_2 \geq \ldots \geq w_n$ in order to have the players ordered according to their strength as defined by quotas.

The simplest solution suggested for quota games is the idea that players will somehow calculate the quotas and then divide the stake of a coalition accordingly. This solution concept does not predict which coalition will be formed, however, being in a certain coalition S with $\Sigma_{i\in S}\, w_i = v'(S)$ players will get their quotas.

For three-person quota games the concept of quota solution can be given a dynamic interpretation, namely the bargaining set interpretation (cf. Aumann & Maschler, 1964). Against a quota solution in a three-person quota game no player has any argument that cannot be neutralized by a counterargument. Consider that player 2 in game 1c) argues against the quota division (11,6) in the coalition (1,2) by suggesting a more

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1) This definition is a little bit simpler and more general than others, as for instance that of von Neumann and Morgenstern (1943).

equalized distribution, say (10,7), threatening that he will leave the coalition otherwise and form a coalition with player 3 with a payoff configuration [0,7,1; 1,(2,3)]. Then player 1 can easily counter this argument by suggesting [11,0,2; (1,3),2]. He can keep his 11, simultaneously offering player 3 more than he gets with player 2. It should, however, be noted again that the bargaining set interpretation does not predict which coalition will be formed in a quota game.

2.5 The Kernel concept

The Kernel (cf. Davis & Maschler, 1965) can be understood as a kind of arbitration process considering the different bargaining power of the players. It may be introduced as follows:

To analyze the stability of a payoff configuration (x,C) one considers two arbitrary players i, j, who are members of the same coalition $S \in C$. It is presumed that player i will check all permissible coalitions which would include him but not player j in order to evaluate how much more (or less) he might get compared with his present payoff x_i in the best among those alternative coalitions, assuming that the others members of the new coalition would be content with their payoffs in (x,C). In other words, player i threatens to change to a coalition which does not include player j. The additional amount which player i can get more in the best alternative permissible coalition under the given assumptions is called the maximum surplus of i over j. The Kernel concept demands that the relations between the payoffs of any two players in any coalition $S \in C$ shall be such that the maximum surpluses of the players are pairwise equal. To assure that every player gets a least $v(i)$ the maximum surplus of a player i is permitted to be greater than that of another player j if $x_i = v(i)$.

The maximum surplus, thus, represents the maximal amount player i can gain (or the minimal amount he will lose) by withdrawing from S and joining another coalition which does not require the consent of j with the understanding that the other members of that coalition will be satisfied with getting the same amount they had in the different coalitions which they were members of. Of course, the Kernel solution can be applied to non quota games as well.

3. Psychological Analysis

In the following we intend to derive some statements and describe some behavioral consequences which are based on empirical results in gaming experiments. By this approach we hope to shorten the gap between social psychological research on coalition behavior and economical as well as mathematical models of coalition forming behavior (cf. von Neumann-Morgenstern solution or the Aumann-Maschler bargaining set). As Selten (1977) stresses mathematical models usually define equilibrium points or equilibrium sets as theoretical solutions of the game. Predictions based on non equilibrium behavior must be unstable, so to say self-destroying, since at least one player is tempted then to deviate from the agreement, if he believes that the other players obey the prescriptions. On the other hand, one cannot expect human bargaining behavior to be perfectly rational and even not restrictedly rational (e.g. in the sense of the bargaining set). From empirical psychological findings one would tend to conclude that persons learn simple rules and the application of social norms to the situation rather than abstract principle leading to an equilibrium. Especially in two- or more-person settings the essence of a learning process is more likely to be a simple rule of thumb than a elaborated device. In our analysis we, therefore, stress on chracteristics of the game structure which can easily be recognized or calculated by the participants.

Compared with existing mathematical models the social psychological equity theory (cf. Homans 1961, Adams 1963, Walster 1976) as a general theory of sharing behavior offers a different approach to the prediction of payoff allocation. A widely accepted notion of the equity theory proposes that persons who bring comparable inputs into a social relationship should get comparable outputs. In a n-person game situation, where the different roles are not assigned on the basis of merits, this simply means that the stake of the coalition is divided equally among the participants. However, as Homans (1976) discusses in detail, even recent developments of the equity theory do not sufficiently pay attention to the conditions of power within a group structure. - We hope that our proposal will render an alternative to the known mathematical equilibrium concepts on one side and the crude equality-prediction of equity theory for equal costs and investments on the other side. Nevertheless, we will start our considerations with a typical equity statement:

i) Equality: In any actual relationship between persons of similar status an equal division of the stake will result provided that persons' inputs (other than status) also do not differ.

One might conclude that an application of that rule to a three-person game should result in an equal share within the grand coalition. But statement i) does not exclude that two person coalitions will form. Furthermore, though we maintain that participants will strive to obey that requirement - equal share is not the only behaviorally relevant principle. Without sufficiently analysing the consequences for the actual output configuration equity theory implicitly or explicitly asserts (cf. e.g. Walster 1976):

ii) Payoff maximization: Each participant will try to maximize his payoff.

The two principles are partly incompatible. The equal share demand will hamper the individual in getting more than that amount in the actual relationship. However, the partner with better bargaining alternatives will experience a stronger conflict because an equal share is less than he possibly could achieve by exploiting his bargaining advantage. For the weaker partner an equal division is a fairly good result because it gives him more than he can expect from a tough bargaining process. Demanding less than he might possibly get by tough bargaining, the stronger player is able to convince other players to join him.

In the next section we will sketch our ideas of how participants cognitively structure the situation, and on the basis of those considerations we will predict an expected range of payoffs for each coalition.

We will then analyse which coalitions will most probably result. By introducing the concepts of substitutional and nested coalitions we then have to revise our predictions for payoff distribution in the different coalitions.
Referring to experimental results in bargaining- and reward allocation situations (Crott et al 1976, Crott et al 1978) as a third statement which relates the other two, we formulate:

iii) Dominance relation between equality and payoff maximization: Principle i) is behaviroally more stringent than principle ii).

By principle ii) we expect each player trying to find coalition partners since in essential games two- or more person coalitions provide the possibility of a better payoff. The stronger player in an asymmetrical game may try to buy into a coalition by offering a certain amount of his possible gain in order not to endanger the final agreement by bargaining tough. As he also accepts principle i) and as the partner will usually demand an equal share he will take this common reference point as a basis for arbitration. But because of principle ii) he will use his better bargaining position as an argument against his actual partner.

4. The Equal Division Kernel

4.1 The concept

As in the kernel concept we suppose that - given a tentative coalition S within a coalition structure C of a payoff configuration (x,C) - some of the players in S might consider forming a different coalition. Again as in the kernel concept we consider it reasonable that members of the prospective new coalition tend to evaluate the amount that they would supposingly get in that coalition. The kernel concept assumes that a player i evaluates the new coalition by assuming that the other players would be content with what they presently get. Consequently, this means that players being in a one-person coalition and receiving nothing would be content with zero payoff in the new coalition either. This does not seem reasonable from a behavioral point of view. It appears more likely that each player assumes that his partners will aspire to an adequate share in the new coalition. Since it is postulated that equal division is a strong social demand in actual interpersonal relations one should expect that all partners consider an equal division as a possible payoff configuration for the new coalition though strong players may think of an equal share as of the least they should get and weak players may regard it as the upper limit of their share. Anyhow, the equal division may serve as the behaviorally relevant distribution that is accepted by all players at least as a starting point for a reasonable bargaining.

Consequently, the concept of Equal Division Kernel (EK) proposes that in a first step of analysis players regard an equal share in the actual coalition as orientation marks. In a second step of analysis they realize that simply dividing equally seems not necessarily adequate in the face of different bargaining possibilities. With other words, player i compares with player j by calculating how much more (or less) he might get in the best alternative coalition which would include him but not j provided that the alternative coalition divides the stake equally.

As in the original kernel concept we suppose that these possible gains (or losses) are the same for any two players within a coalition S of a payoff configuration (x,C).

The concept of an Equal Division Kernel as presented here is related to Thibaut & Kelleys' (1958) notion of a comparison level for alternatives: Persons who are engaged in an actual relationship tend to check alternative relationships taking the most promising of them as a basis for comparison with what they actually get.

We do not intend to present the EK concept formally in this paper. The properties of the EK are with several regards similar to those of the kernel. A more detailed algebraic treatment and specification of the concept will be given later (Albers & Crott, in prep.). Here we are mainly interesting in sketching some characteristics of the EK.

Summarizing what has been said, we state some definitions starting with the definition of the maximum surplus in the sense of the Equal Division Kernel concept.

Definition 1: The maximum surplus of player i over player j in a coalition S of C is

$$(4.1) \qquad t_{ij} := \max_{\substack{S \subset N \\ i \in S \\ j \notin S}} (e(S) - x_i)$$

The maximum surplus therefore represents the maximum amount player i can gain (or the minimal amount he can lose) by withdrawing from $(x;C)$ and joining a coalition S that does not require the consent of j (since $j \notin S$) with the understanding that he himself and all other members of S will be content with $e(S)$.

Definition 2: A coalition S of C is defined to be balanced with respect to $(x;C)$, if for any two players i and j in S

$$(4.2) \qquad (t_{ji} - t_{ij})x_i \leq 0 \quad \text{and} \quad (t_{ij} - t_{ji})x_j \leq 0.$$

In a balanced coalition any two players who get more than $x_i = 0$ have the same maximum surplus over the other. From Definition 2 we can see that there are three possible cases of balance, namely

$$(4.3)\quad \begin{array}{l} t_{ij} = t_{ji} \\ t_{ij} < t_{ji} \text{ and } x_i = 0 \\ t_{ij} > t_{ji} \text{ and } x_j = 0 \end{array}$$

Definition 3: The Equal Division Kernel EK(v') of a game with characteristic funtion v' is the set of all payoff configurations having only balanced coalitions.

We use the notation v' for the considered value function to point out that our definition is formulated for O-normalized games. For general games we simply assume that players will determine the EK values on the basis of the normalized game and then add their individual values to the EK values.

4.2 Some Properties of the Equal Division Kernel

The values of the Equal Division Kernel (EK-values) for any coalition in any game can be calculated by starting from an arbitrary payoff configuration and computing the surpluses for all pairs of players i and j in the present coalition under consideration with respect to all other possible alternative coalitions. Then the x-values must be changed in a way that the maximum surpluses are pairwise equal. This would,however,require tedious computational work even for games with a small number of player and, therefore, it seems adequate to develop some systematic devices for the calculation of the EK-values.

Throughout this paragraph we consider a payoff configuration (x,C) in the Equal Division Kernel and two players i,j in the same coalition $S\in C$.

Since by definition it is demanded that the values of the EK for any two players i and j should be choosen such that the maximum surpluses are equal (except from $x_i = 0$ or $x_j = 0$) we can state

Remark 1: For any (x,C) in the Equal Division Kernel and any two players i,j with $x_i > 0$ and $x_j > 0$ being members of the same coalition $S \in C$, we have

(4.4) $$x_i - x_j = \Delta_{ij}$$

where

(4.5) $$\Delta_{ij} = t_{ij} - t_{ji} = \max_{\substack{S \subseteq N \\ i \in S \\ j \notin S}} e(S) - \max_{\substack{S \subseteq N \\ j \in S \\ i \notin S}} e(S)$$

So the payoff differences of the players in S only depend on their best alternative coalitions and are independent of the bargaining history.

Since the payoffs of the players in S sum up to $v'(S)$ it follows:

Remark 2: The payoff x_i of a player i in a solution (x,C) in the Equal Division Kernel only depends on the coalition $S \in C$ which contains i.

This is an important feature of the EK since the EK value of any player only depends on his coalition and the characteristic function of the game, regardless what happens to the other player, i.e. regardless of the coalition structure, resulting besides S.

According to remark 1 the differences in payoffs x_i and x_j should correspond to the differences Δ_{ij} between the best-alternatives for those two players. In order to derive a general computational device we let player i compare his payoff with all other players of his present coalition:

$$\Delta_{ij} = x_i - x_j$$

$$\Delta_{ik} = x_i - x_k$$

$$\Delta_{il} = x_i - x_l$$

etc.

Summing over all players j who are members of the present coalition S we get

(4.6) $\Sigma_{j\in S}\, \Delta_{ij} = (|S| - 1)\, x_i - (v'(S) - x_i) = |S|x_i - v'(S).$

From this follows:

<u>Remark 3</u>: If (x,C) is the Equal Division Kernel and $i \in S \in C$ and $x_i > 0$ for all $i \in S$ then

(4.7) $x_i = e(S) + \frac{1}{|S|}\, \Sigma_{j\in S}\, \Delta_{ij}.$

We can specify remark 3 for <u>three-person quota games</u>. Here only pair coalitions must be considered as comparison coalitions since the grand coalition does not exclude anyone.

<u>Remark 4</u>:For three-person quota games (with $v(i) = 0$ for all i)

(4.8) $\Delta_{ij} = (v'(i,k) - v'(j,k))/2 = (w_i - w_j)/2$

and for any (x,C) in the equal division kernel

(4.9) $x_i = e(S) + \frac{1}{2}\, (w_i - \frac{1}{|S|}\, \Sigma_{j\in S}\, w_j)$

In many cases the structure for the game is such that for all players the coalitions for which $\max_{S\subsetneq N, i\in S, j\notin S}\, e(S)$ is obtained have the same number s of players for all $i,j\in N$. Thus for n-person quota games with this property, as a generalization of formula 4.9, we can derive an expression which can be computed more conveniently than formula 4.7:

<u>Remark 5</u>: If the coalitions for which $\max_{i\in S}\, e(S)$ is obtained have the same number s of players for all $i\in N$, we get for any (x,C) in the EK

(4.10) $x_i = e(S) + \frac{1}{2}\, (w_i - \frac{1}{s}\, \Sigma_{j\in S}\, w_j)$

From the definitions of the EK follows immediately

Remark 6: The Equal Division Kernel does not depend on the labelling of the players.

Moreover, with respect to linear equivalence:

Remark 7: If one multiplies the values of a characteristic function game v' with a real number a such that $\hat{v}'(S) = av'(S)$ for all $S \subseteq N$ then one gets Equal Division Kernel solutions $(\hat{x},C)$ corresponding pairwise with the solutions (x,C) of the original game v' such that $\hat{x}_i = ax_i$ for all $i \in N$.

Referring to our demand that computational devices should be easy in order to be behaviorally relevant, we can give the EK-values for three-person quota games an intuitive meaning.

1) Players first consider the $e(S)$ values of the coalitions. Each player i then thinks of the surplus he would have in another coalition which would include him, but exclude his comparison partner j, under the assumption that this coalition will divide the stake equally. He looks for the best alternative coalitions for himself and for j and then subtracts the $e(S)$ values of those two coalitions. In a three-person quota game this operation will result in $(w_i - w_j)/2$ and this difference shall be equal to the difference in the final payoffs of player i and j. - The comparison procedure can also be interpreted as a balance of threat. Player 1 may threaten to leave e.g. (1,2) in order to form (1,3). Player 2 may use (2,3) as a threat alternative. Both players realize that they can easily convince player 3 by offering him an equal share. There is a balance of threat if the loss (gain) involved by a chance is equal for both players.

2) The second interpretation is only slightly different. By this we simply stress the often observed fact that an equal division in the grand coalition has a pre-eminence compared to other solutions. Participants first consider a grand coalition with an equal division of the stake. Then they think of how they might possibly bribe other players

to join them into smaller coalitions. For the two stronger players it is evident that a coalition among themselves provides the highest equal share value. In the third stage players think about their bribing potential compared with other players in the sense of interpretation .

4.3 Relations to other solution concept based on equal share

Selten (1972) considered a solution concept which is also based on the equal share distribution. He defined a solution range in the following way:

Definition: A payoff configuration (x,C) is in the Equal Division Core, if for all $i \in S \in C$

(4.11) either $x_i \geq e(T)$ for all $T \subseteq N \setminus (S \setminus \{i\})$ or $\{i\} \in C$ and $x_i = v(i)$.

This idea can be interpreted in such a way that an isolated player $\{i\} \in C$ gets $x_i = v(i)$ while a player i in a coalition $i \in S \in C$ computes his equal division value for all coalitions T that do not need the support of his partners in S . In a solution player i must get at least as much as in the best of these alternatives.

It follows immediately from the definition that for an arbitrary coalition $S \subseteq N$ there is a payoff configuration (x,C) in the Equal Division Core with $S \in C$ iff $\Sigma_{i \in S} E_i(S) \leq v(S)$ where

(4.12) $E_i(S) := \max \{e(T) \mid T \subseteq N, S \cap T = \{i\}\}$

Moreover, a payoff distribution $x_S = (x_i \mid i \in S)$ can be extended to a payoff configuration (x,C) in the Equal Division Core (with $x_i = x_{Si}$ for all $i \in S$ and with $S \in C$) iff

(4.13) $\Sigma_{i \in S} x_i = v(S)$ and $x_i \geq E_i$ for all $i \in S$

The set of all such vectors x_S is an $((S)-1)$-dimensional simplex, all edges of which have the same length. The barycenter of this simplex (which coincides with the center of gravity) can be extended to a payoff configuration in the equal Division Kernel and it is the only vector x_S in the simplex with this property.

So for a fixed coalition S the Equal Division Kernel selects the center of gravity out of the set of vector x_S which can be extended to a solution (x,C) in the Equal Division Core. If such a vector x_S does not exist the EK concept gives additional solutions.

Recently Komorita (1979) has proposed a framework for an equal excess model which is also based on equal share as the initial expectation of the participants. It predicts that the outcomes change stepwise, round by round, from an equal share in the pre-negotiation phase to an asymptotic payoff distribution which can deviate considerably from an equal split. Except for the first round in pair coalitions, the prediction of Komorita's model in general differs from that of the equal division kernel. Since the underlying philosophy stresses power differences rather than justice considerations, especially for the later stages of the bargaining the equal excess predictions depart more and more from the range of solutions defined by the equal share and the equal division kernel concept.

Both models, Selten's equal division core and Komorita's equal excess model, provide interesting implications for experimental comparisons with the equal division kernel concept.

5. Descriptive Generalization of the Equal Division Kernel

5.1 Equal Division Kernel dominance relations

As described now the EK concept does not explicitly suggest which of several possible coalitions will actually result in a bargaining process. However, based on the idea of the EK a certain dominance relation can be formulated. We say that a payoff configuration (x,C) is EK-dominated by a coalition T if the EK values of T give all members of the new coalition T more than they get in the different coalitions they presently belong to.

It appears plausible also to construct an equal share dominance relation since according to our assumption players in the first step regard the equal share values of a coalition. Which alternative players in actual situations will use depends on the degree in which they anticipate the bargaining process in the sense of the EK, with other words, it appears to be an empirical question rather than a theoretical one. In this paper, in the following, we will apply EK-dominance as a basis of analysis.

5.2. Reference coalitions

We propose that undominated coalitions will serve as reference coalitions in the bargaining process. Since no member has a chance to get more in another coalition according to the EK criterion, players will consider the reference coaltion in the first place. For three-person quota games, e.g., we can derive from formula 4.10 that either the strongest players $(1,2)$ or all players $(1,2,3)$ are the reference coalition. Neither player 1 nor player 2 can get a higher EK value in any pair coalition. Coalition $(1,2,3)$ will be preferred if its value $v(N)$ is at least $w_1+w_2+w_3+1/4\ (w_1+w_2-2w_3)$. So this can only happen if $v(N)$ is more than the sums of the quotas.

5.3 Substitutional and nested coalitions

Our notion of a reference coalition does however not necessarily mean that the reference coalition will finally result. Let us assume that the two strongest players constitute a reference coaltion. It seems reasonable that they define their level of aspiration somewhere in the range between the equal share value and the equal division kernel value for this reference coalition. That depends on the degree to which the players insist on a differentiation according to bargaining power in the sense of the equal division kernel. However, there is no reason to assume that players really form the reference coalition.

In a three-person game, for instance, player 1 might form a coalition (1,3) but demand the payoff he would get in the reference coalition (1,2) . The resulting payoff distribution in (1,3) usually gives player 1 more and player 3 less than his EK value in coalition (1,3). In many cases this is the only way for player 3 to find a coalition partner.

We call coalitions which are formed in place of the reference coalition substitutional coalitions presuming that members of the reference coalition will get the e(S) - or the EK value of the reference coalitions.

A third category of coalitions are so-called nested coalitions using a term introduced by Tack (1979). In a nested coalition another coalition or coalitions seem to be built in. A nested coalition might develop when players cannot agree on the division of the stake within a reference or a substitutional coalition, e.g. because player 1 asks for the EK value in (1,2) whereas player 2 insists on the e(S) value. Then both might decide to extend the coalition by player 3 to secure that both can get their demands or even more.

5.4 Predictions for coalitions structures and payoff distributions

Summarizing the foregoing arguments we can predict which coalitions will most probably be formed and how the stake will be devided among the participants. Since otherwise we would need a more general measure of strength, we restrict our predictions to quota games, reminding to the fact that most games that have raised the interest of experimenters are quota games. For quota games we assume that the strongest player is a member of a reference coalition.

1) The strongest player will never be excluded from a final coalition. Since he has the highest quota, he has to give away the comparably highest amount to his coalition partners by accepting the e(S) or the EK-value in the coalition. Therefore, other players will strive to join him.

2) The strongest player in any final coalition will get an amount somewhere between the equal share calue e(S) and his Equal Division Kernel value in the reference coalition.

3) The coalition of all playwer will only appear in the form of a nested coalition or as a reference coalition.

We admit that those 'predictions' are derived post hoc by looking to the data of several experiments and that thes depend heavily on our notion of reference coalitions.

6. Analysis of Data

6.1 General remarks

Theories of fair reward allocation are conceptionalized, more or less explicitly, for interacting groups, i.e. distributional justice is not supposed to be a strong factor under conditions or highly restricted communication (cf. Homans 1961). Ss playing a game in a computarized situations, without face to face contact and without the opportunity to exchange arguments, interacting by offers and counteroffers only will presumably not tend to realize fair outcome distributions.One reason for that might be the observation that the need for a just reward allocation is felt mainly by players which are 'disadvantaged' by the structural conditions of the situation (cf. Leventhal & Anderson 1970). Ss must be allowed to express their attitude towards certain outcome distributions.

The conditions under which we expect the EK concept to have predictive value may be summarized as follows:

1) Ss should be completely informed about the characteristic function of the game. This is a precondition for Ss to be able to evaluate the situation adaquately, i.e. also to develop adaquate distributional suggestions.

2) Ss should not play different roles in the same game over a period of sessions with the same partners. Otherwise the strategy to differentiate between outcomes within one session but to equalize outcome within the whole sequence might dominate the bargaining process.

3) No communication channel structure should be imposed on the interaction process. Each player should be allowed to contact whom-ever he wants.

4) Communication should be possible at least at the level of written messages (not necessary nonverbal communication and/or verbal communication).

6.2 Communication vs no communication: The Murnighan & Roth study

In order to justify our differentiation between communication conditions we reanalyse data of an experiment that realized different communication conditions (Murnighan % Roth 1977).

Among other factors the degree of communication was varied: Ss either could send written messages via the experimenter to any other S or not. In the first case the experimenter either kept these messages secret or announced them openly. Each group of Ss played the game 12 times, each player keeping his role in the game for all 12 sessions.

The game in this study was presented as a market game consisting of three players, each of whom is the owner of one shoe.

Player 1 owns a right shoe while player 2 and 3 each own one left shoe. Single shoes have no value but a complete pair of shoes can be sold for 100 points. Thus no player alone has the power to earn any income but any coalition of players that can assemble a pair of shoes has the power to earn 100 points. In characteristic function form the game can be modeled:

$$v(1) = v(2) = v(3) = v(2,3) = 0$$

$$v(1,2) = v(1,3) = v(1,2,3) = 100$$

Table 2 gives the results for 36 groups, each group playing the game 12 times. For our analysis we define an ES/EK and a quota range. The ES/EK range is varying between an equal share and the equal division kernel value. To allow a rough comparison with the quota (bargaining set) solution we define an equally wide range for the quota solution, i.e. we count solutions between 50 and 75 for the ES/EK concept and solutions between 75 and 100 for the quota concept (taking exactly 75 as an undecidable case). In order to get independent measures for the 36 groups we label a group as preferring the ES/EK solution over the quota solution when within the 12 sessions the group more often agreed within the range 50-75 than within the range 75-100.

Table 2

Results of a three-person game (Murnighan & Roth 1977)

Game type	Results[1]		
	within quota range	within ES/KS range	undecidable or grand coalition
no messages	7 (79)	10 (113)	1 (24)
secret or announced messages	0 (26)	18 (164)	0 (26)

[1] the frequencies in brackets refer to single plays.

In table 2 the numbers in parentheses represent the dependent cases (12 x 36) in which the solution lies within the ES/EK range, the quota range and undecidable cases (i.e. cases outside of both ranges, 75 : 25 split) as well as grand coalitions.

There is a tendency to settle more quota range solutions under the condition of no communication (7 vs 10) than under the conditions of communication (0 vs 18) (X^2 = 6.87 df-1, p<0,01). Other experiments that realize only the no communication condition also show a tendency towards quota-, Bargaining set solutions (Kahan & Rapoport, 1974, Medlin, 1975).

6.3 Results of experiments with communication possibilities

In table 3 the results of 10 studies satisfying our conditions 1 to 4 are summarized for coalitions (1,2). In games 9 and 10 no actual communication between participants took place, however, Ss simultaneously payed in different roles (one shot bargaining) such that they had to consider the partners' aspirations in their own interest.
Coalitions (1,2) and (1,3) are analytically identical for games 1 to 8. Therefore, (1,3) is presented as (1,2) in those games. Since (2,3) coalitions rarely happen we will not comment on them here. Characteristics for the grand coalition are basicly different from game to game, thus we will not analyzse them either. - We only include studies with at least five observations per game. Additionally, we will only consider studies where the difference between the EK value and the quota is not smaller than 5% of the value of the coalition in order to avoid the inference of Ss' tendency to round off their demands. Table 3 reports the frequencies of solutions 1) nearest to the quota, 2) nearest to an equal share and 3) nearest to the equal division kernel.

Table 3: Comparison of results of several experiments with equal share, equal division kernel and quota solution

Game type [1]	Quota [2] (range)	Equal share	Equal Division K.	Number of pair coalit.	Results nearest to: Quota	Equal share [3]	Equal Div.K.	Other pair coal. than (1,2) [4]
	for 1 in (1,2)							
(1,2), (1,3), (2,3), (1,2,3) 50 50 0 0 Maschler (1978)	50.0	25.0	37.5	11	4	4 (3)	3	0
(1,2), (1,3), (2,3), (1,2,3) 60 60 10 0 Maschler (1978)	55.0	30.0	42.5	24	1	7 (1)	12	4
(1,2), (1,3), (2,3), (1,2,3) [5] 2 2 1 2,0 or 2,5 or 6,0 Kaufmann & Tack (1970)	4.5	4.0	4.25	24	7	10 (6)	5	2
Albers (in prep.) (1,2), (1,3), (2,3), (1,2,3) 1.800 1.800 1.200 1.800	1200	900	1050	5	0	3 (0)	2	0
(1,i) [6], (2,3,4,5) [7] 40 40 Selten & Schuster (1970)	30.0	20.0	25.0	7	1	1 (0)	5	-
(1,2), (1,3), (2,3), (1,2,3) 120 120 60 120 Frassini et al. (1967)	90.0	60.0	75.0	14	0	9 (4)	5	0

Game type 1)	Quota 2)	Equal share for 1 in (1,2)	Equal Division K.	Number of pair coalit.	Results nearest to: Quota	Equal share 3)	Equal Div.K.	Other pair coal. than (1,2) 4)
(1,i) 6), (2,3,4) 7) 1.000 1.000 Albers (1978) Version C	666.0	500.0	583.0	6	2	0 (0)	4	-
(1,i) 6), (2,3,4,5,) 7) 1.000 1.000 Albers (1978) Version B	750.0	500.0	625.0	19	6	1 (0)	12	-
(1,2), (1,3), (2,3), (1,2,3) 60 55 35 75 May & Crott (in prep.)	40.0	30.0	35.0	18	2	11 (10)	4	1
(1,2), (1,3), (2,3), (1,2,3) 65 55 40 80 May & Crott (in prep.)	40.0	32.5	36.25	17	0	11 (10)	6	0
					23	56 (34)	58	7

1) All studies are presented in terms of v'functions
2) Quotas according to the generalized quota-concept (see page 5)
3) Numbers in parantheses give the frequencies of exact equal share solutions
4) No agreement and coalitions other than (1,2),((1,3), respectively)
5) In Kaufmann & Tack study Ss played different strategically equivalent versions of the game. For the present purpose we are only interested in the conditions 'empty core' and 'one element core', since for his 'big core' condition the grand coalition would be the reference coalition in terms of the v'function
6) $i \in \{2,3,4\}$
7) Other coalitions where possible, however, these are the minimal winning coalitions

The games investigated by Kaufmann & Tack (1975) and by Albers (in preparation) are games of type b) in table 1 that means there are individual values greater zero ($v(i) > 0$). By counting the number of cases we assumed that Ss would subtract the individual values in advance and then analyze the rest game (like table 1f)). Though in a few cases (3) Ss apparently disregard the individual values, the pertitioning of the game into an inessential game and a game with $v'(S) = v(S) - \Sigma_{i \in S} v(i)$ seems to be an adaquate description of payoff allocation for the behavior in pair coalitions.

From table 3 one can infer that there are more solutions nearest either to an equal share (56) or to the equal division kernel (58) than to quotas (23).

In table 4 the different kinds of coalitions are reported. We can see that the majority of pair coalitions are coalitions with the strongest player 1.

Table 4: Types of coalitions with 10 games of table 3

	(1,2) or (1,3)		(2,3) or coalition among other small players	Non minimal winning-coalitions - grand coalitions - no agreement
games 1 - 8	103		6	55
	(1,2)	(1,3)	(2,3)	grand coalition or no agreemt.
games 9 and 10	32	2	1	25
	137		7	80

6.4 The role of the reference coalition

Finally, we would like to present the results of an experiment which underlines the necessity of incorporating the idea of dominance relations and reference coalitions into the EK concept. Consider a four person game in which all pair coalitions get 50 , but in which there exists only one three person coalition (2,3,4) that earns 120 (s. table 5). For the coalition (1,i) the EK predicts 17.5 for player 1 and 32.5 for any of the possible partners in coalition (1,i). However, intuitively we do not even expect player 1 to get 17.5. A smaller share is also predicted by the reference coalition idea. Since coalition (2,3,4) for all three participants dominates all other coalitions by criterion of EK, they will realize (2,3,4) with 40 points for each player or they will form a substitutional coalition with player one which gives them at least 40.

Maschler (1978) has investigated the game mentioned above where $v(1,i) = 50$ for $i \in \{2,3,4\}$, $v(2,3,4) = 120$ and a similar one where $v(1,i) = 50$ for $i \in \{2,3,4\}$, $v(2,3,4) = 111$. Table 4 shows the results of those games in all nine cases of pair coalitions;player i gets at least his equal share value for the coalition (2,3,4) within the substitutional coalition (1,i), i.e. 40 or 37, respectively.

Table 5

Results of two four person games (Maschler 1978)

Game type	results for pair coalitions				results for the three person coalition		
v(1,i) = 50;	x_1	x_i	x_j	x_k	x_2	x_3	x_4
	5	45	25	25	40	40	40
v(2,3,4) = 120	5	45	25	25			
	9	41	25	25			
	9	41	25	25			
v(1,i) = 50	10	40	25	25			
	12	38	0	0			
v(2,3,4) = 111	5	45	25	25			
	10	40	25	25			
	10	40	25	25			

6.5 Conclusions

The data presented in table 2 - 5 give a certain support for the ES/EK concept since results more often fall into the ES/EK range than into the quota range. Furthermore, it can be stated that - as predicted - the majority of coalitions include the strong player. However, it is apparent that in order to effectively support equal share type of solution concepts much more should be known about the cognitive structuring of the situation by the participants, their aspirations and expectations as well as the process of offers and counteroffers leading to the final result. With other words, experiments should deliver more informations on the players' perception of the situation and the dynamics of bargaining rather than presenting solutions and frequencies of coalitions only.

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