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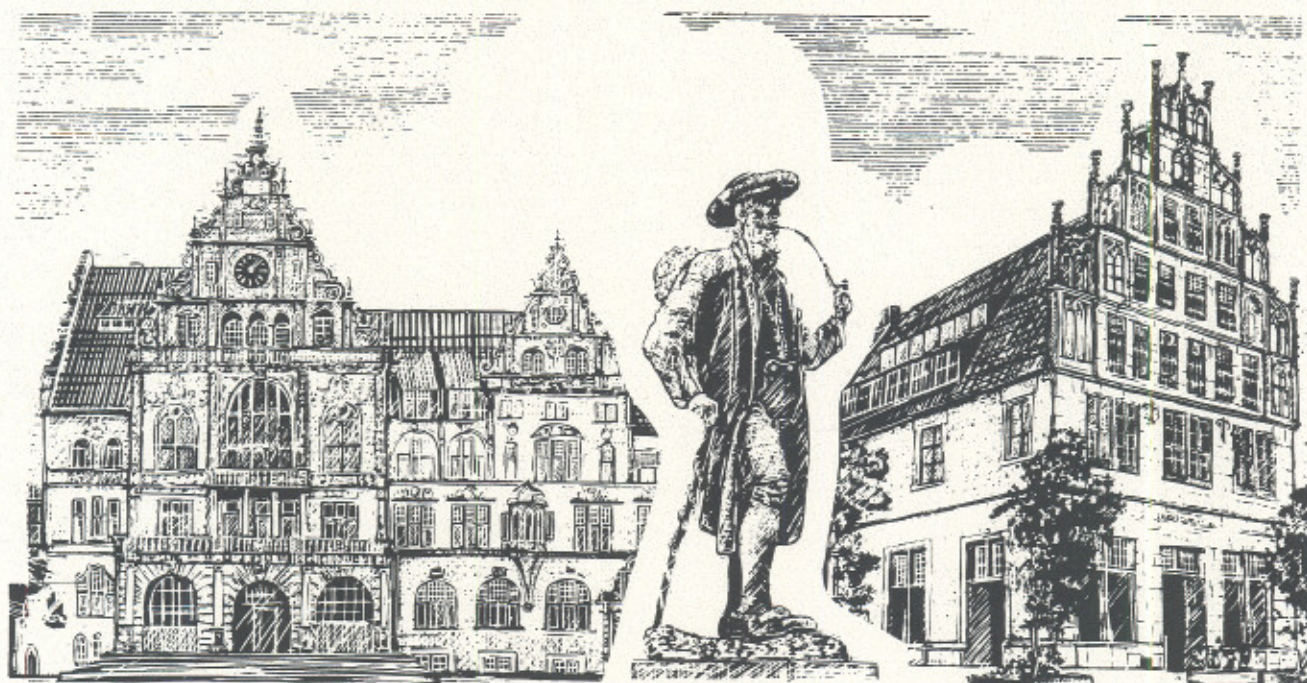
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Values of Non-Sidepayment Games
and their Application
in the Theory of Public Goods

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SECTION 0 : INTRODUCTION

Let $\Omega = \{1, \dots, n\}$ denote the "set of players" and let $\underline{P} = \mathcal{P}(\Omega) = \{S \mid S \subseteq \Omega\}$ be the system of coalitions of players. ($\mathcal{P}(\cdot)$ denotes the power set.) If $v : \underline{P} \rightarrow \mathbb{R}$ satisfies $v(\emptyset) = 0$ then we write $v \in \mathcal{W}$ and call $(\Omega, \underline{P}, v)$ a sidepayment game.

Similarly, a mapping $V : \underline{P} \rightarrow \mathcal{P}(\mathbb{R}^n)$ constitutes a game without sidepayments $(\Omega, \underline{P}, V)$ if V satisfies certain regularity conditions. Essentially, this means that $V(S)$ is a nonempty, closed and comprehensive subset of \mathbb{R}_S^n (the subspace of \mathbb{R}^n spanned by the coordinates $i \in S$); for the details see e. g. [14]. \mathcal{V} denotes the space of mappings obeying the conditions as indicated.

There is a natural imbedding

$$\mathcal{W} \rightarrow \mathcal{V}$$

defined by

$$V^v(S) = \{x \in \mathbb{R}_S^n \mid \sum_{i \in S} x_i \leq v(S)\}$$

The set $\mathcal{A} \subseteq \mathcal{W}$ of additive mappings is identified with \mathbb{R}^n ; thus $x \in \mathbb{R}^n$ and the mapping

$$x : \underline{P} \rightarrow \mathbb{R}, \quad x(S) = \sum_{i \in S} x_i \quad (S \in \underline{P}),$$

are regarded as the same objects.

The Shapley-value (SHAPLEY [15]) is a mapping

$$\phi : W \rightarrow A = \mathbb{R}^n$$

which intuitively associates a "fair" or "expected" payoff to every game. Thus, given $v \in W$, $\phi_i(v)$ is player i 's expected payoff and $(\phi_1(v), \dots, \phi_n(v)) = \phi(v)$ represents a "fair" distribution of money or utility available from the game via cooperation of the "grand coalition" Ω .

There are several systems of axioms characterizing the Shapley value as well as formulas in order to perform a computation. A possible way of definition is as follows (cf. [15]).

Define $v = e^T \in W$ (for $T \in \underline{P}$) by

$$e^T(S) = \begin{cases} 1 & (S \supseteq T) \\ 0 & (S \not\supseteq T) \end{cases}$$

(the "unanimous game" for coalition T) and put

$$\phi(e^T) = \mu^T \quad (T \in \underline{P})$$

where $\mu^T \in A$ denotes uniform distribution over T ($\mu_i^T = \frac{1}{|T|} 1_T(i)$). The e^T ($T \in \underline{P}$) constitute a basis of W , hence $v \in W$ allows for a unique representation

$$(1) \quad v = \sum_{T \in \underline{P}} c_T e^T$$

Now put

$$(2) \quad \phi(v) = \sum_{T \in \underline{P}} c_T \phi(e^T) = \sum_{T \in \underline{P}} c_T \mu^T$$

I. e., the Shapley value is linear extension of uniform distribution.

Observe that the coefficients in (1) are obtained by "Möbius inversion":

$$c_T = \sum_{S \subseteq T} (-1)^{t-s} v(S)$$

where t and s generically denote the size of sets T and S ($t = |T|$, $s = |S|$).

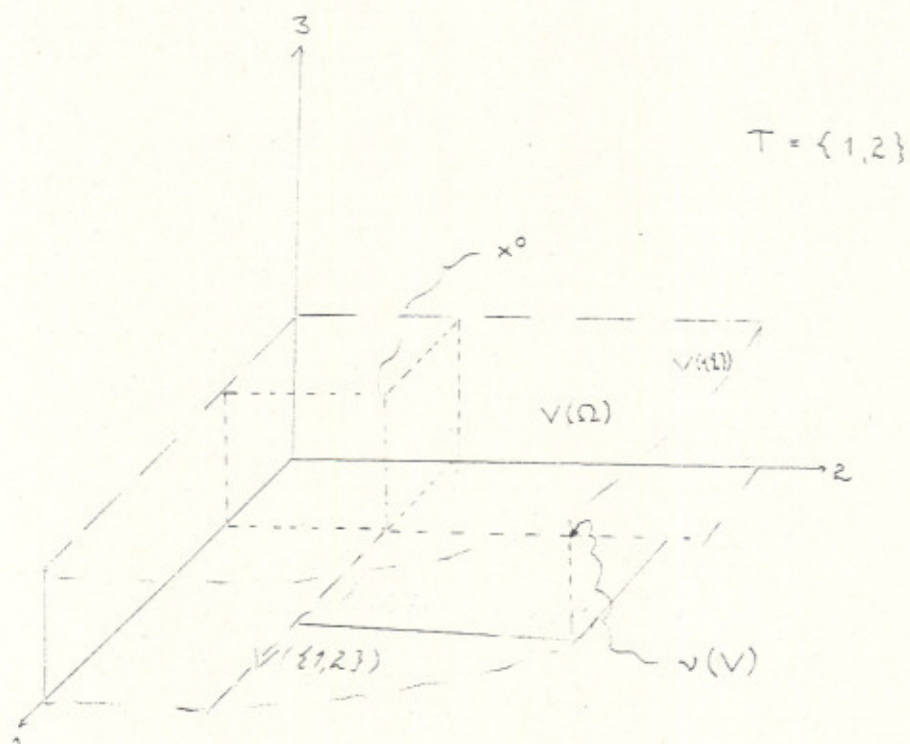
Conceivably, (2) represents the fact that ϕ is a linear mapping. Linearity of the "fair value" (also required by the axiomatic definition in SHAPLEY [15]) is a somewhat dubious property. It may be justified in a more or less convincing manner within the framework of sidepayment games. However, it has to be abandoned whenever attempts are made to generalize ϕ to non-sidepayment games.

For $T \in \underline{P}$, $A \subseteq \mathbb{R}_T^n$ (closed and comprehensive), $x^0 \in \mathbb{R}^n$ with $x_T^0 \in A$, let $V = E_{T,A,x^0} \in \mathcal{W}$ be defined as follows:

$$\begin{aligned} V(\{i\}) &= \{te^i \mid t \leq x_i^0\} & (i \in \Omega) \\ V(S) &= \sum_{i \in S}^{\oplus} V(\{i\}) & (S \not\supseteq T) \\ V(S) &= A \oplus \sum_{i \in S-T}^{\oplus} V(\{i\}) & (S \supseteq T) \end{aligned}$$

E_{T,A,x^0} is the unanimous game (of 1st kind) of T over A with x^0 as threat point.

(For the details see [14].)



For general $V \in \mathcal{V}$, the threat point $\underline{x}(V) \in \mathbb{R}^n$ is defined by

$$\underline{x}_i(V) = \max \{t \mid te^i \in V(\{i\})\}$$

and, of course, $\underline{x}(E_{T,A,x^0}) = x^0$. The class

$$V^0 = \{V \in \mathcal{V} \mid V = E_{T,A,x^0} ; T \in \underline{P} ; \\ A \text{ convex} ; x^0 \in \mathbb{R}^n ; x_T^0 \in A\}$$

admits of the NASH-value (NASH [6])

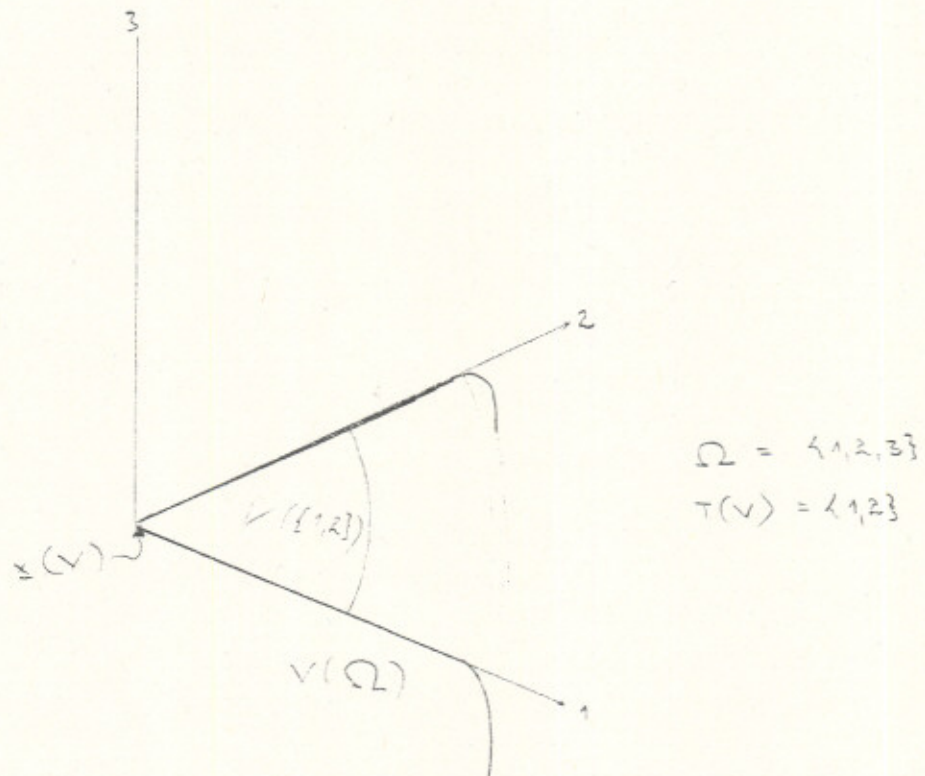
$$v : V^0 \rightarrow \mathbb{R}^n .$$

$v(V)$ is readily obtained as the maximizer of a function g^V over the Pareto optimal and individually rational points of $V(\Omega)$; here

$$g^V(x) = \prod_{i \in T(V)} (x_i - \underline{x}_i(V))$$

and $T(V)$ is the set of those players that may possibly exceed their threatpoint payoff in a suitable coalition.

v is a nice mapping which, in particular, commutes with permutations of the players and affine transformation of utility.



Given the natural embedding of W into V , it is seen at once that ϕ and v coincide on the ("small") system of unanimous "sidepayment" functions of the type E_{T,A,x^0} . However, it is of larger interest to define a value ψ on some system $V^* \subseteq V$ such that ψ coincides with v and ϕ on their domains of definition respectively. Several authors did address themselves to this question, see e. g. HARSANYI [1],[2], MIYASAWA [5], SHAPLEY [16], OWEN [8].

We shall give a joint representation of the first three of these values in sections 1 and 2. Sections 3 and 4 will serve to sketch a possible application.

Remark 0.1

For $T \in \underline{P}$, $\alpha \in \mathbb{R}$, $x^0 \in \mathbb{R}^n = /A$, define

$$e^{T,\alpha,x^0} := (\alpha - x^0(T))e^T + x^0$$

Then ϕ satisfies

$$(4) \quad \phi(e^{T,\alpha,x^0}) = (\alpha - x^0(T))u^T + x^0 \quad (T \in \underline{P})$$

and

$$(5) \quad \phi(v_T) = \phi(e^{T, v(T), \sum_{S \not\subseteq T} (-1)^{t-s} \phi(v_S)}) \quad (T \in \underline{P})$$

Moreover, ϕ is uniquely defined by (4) and (5).

(v_T denotes the restriction of $v \in W$ on $T \in \underline{P}$).

For the proof see e. g. [12].

Note that (4) and (5) define ϕ recursively without using the fact that ϕ is linear on W .

In fact, defining ϕ on (generalized) unanimous games e^{T, α, x^0} by (4) is immediately appealing.

(5) however requires the players to somehow compute

$$(6) \quad t^T = t^T(v, \phi) = \sum_{S \subsetneq T} (-1)^{t-S} \phi(v_S)$$

as a result depending on values of the same game restricted to proper subcoalitions. Thus, values (utilities) are added (for each player separately) but not games (functions v).

Note also that $t^T(v, \phi)(T) \leq v(T)$ is not necessarily required. But, fortunately, ϕ is defined for all $v \in W$, superadditive or not and (4) takes no notion whether $x^0(T) \leq \alpha$ holds true or not. For the recursive definition (5) it suffices to have a primitive version of ϕ defined only on the unanimous games e^{T, α, x^0} available - but this version must necessarily be defined for $\alpha < x^0(T)$ and for $x^0(T) \leq \alpha$.

SECTION 1 : GENERATION OF VALUES

As e^{T, α, x^0} corresponds to E_{T, A, x^0} in the sidepayment framework, it is close at hand to generalize (4) and (5) via replacing $e^{...}$ by $E_{...}$. However, it is then feasible that the recursive definition takes advantage of the fact that $x_T^0 \in A$ is not necessarily true, i. e. the "threat point" is not feasible.

Therefore, in what follows, let $V^{00} \subseteq V$ be such that

$$(1) \quad V^{00} \subseteq \{E_{T, A, x^0} \mid T \in \underline{P}, x^0 \in \mathbb{R}^n, A \subseteq \mathbb{R}_T^n \text{ convex}\}$$

(the definition of E_{T, A, x^0} makes no use of a requirement " $x_T^0 \in A$ "). V^{00} satisfying (1) is called a suitable subclass of V . A mapping $\varphi : V^{00} \rightarrow \mathbb{R}^n$ is called a primitive concept (for V^{00}) and (V^{00}, φ) will sometimes be termed to be a primitive pair.

THEOREM 1.1.

Let (V^{00}, φ) be a primitive pair. Then there is a unique subclass $V^\varphi \subseteq V$, $V^{00} \subseteq V^\varphi$ and a mapping $\psi = \psi^\varphi : V^\varphi \rightarrow \mathbb{R}^n$ such that

$$(2) \quad \psi(V) = \varphi(V) \quad (V \in V^{00})$$

$$(3) \quad \psi(V_T) = \varphi(E_{T,V(T), \sum_{S \subsetneq T} (-1)^{t-s} \psi(V_S)}) \quad (V \in V^\varphi)$$

ψ^φ inherits all invariance properties from φ .

Again, V_T denotes the restriction of V on T ; a suitable definition of this (not violating the regularity conditions) is found in [14].

Let us write

$$(4) \quad t^T(V, \psi) := \sum_{S \subsetneq T} (-1)^{t-s} \psi(V_S) ,$$

such that (3) reads

$$\psi(V_T) = \varphi(E_{T,V(T), t^T(V, \psi)})$$

in accordance with (5) and (6) of SECTION 0 .

The Proof of the Theorem is rather straightforward. Existence and uniqueness follow via the recursive definition implied by (3); the class V^φ is given by all those function guaranteeing that during the recursion the term $\varphi(E_{\dots})$ on the right hand side of (3) is always well defined, i. e., that E_{\dots} is an element of V^{00} . The fact that "invariance properties" (i. e., invariance under affine transformations of utility and under permutations of the players) are inherited takes some computations which are performed during the proof of Theorem 4.11. and Lemma 4.12. of [12].

SECTION 2 : CHARACTERIZATION OF VALUES

The aim of this section is to show that all values mentioned in SEC. 0 are generated by the appropriate primitive pair via Theorem 1.1.

Consider

$$\hat{V} := V^0 \cup \{E_{T,A,x^0} \mid T \in \underline{P}, A \text{ convex}, x^0 \in \mathbb{R}^n, \\ x_T^0 \notin A, \{x \in \mathbb{R}_T^n \mid x \notin A, x \leq x_T^0\} \text{ bounded}\}$$

Then we may rephrase Theorem 4.13. of [12] as follows.

THEOREM 2.1.

There is a mapping

$$v : \hat{V} \rightarrow \mathbb{R}^n$$

such that $v|_{V^0}$ is the Nash-value.

v is invariant under affine transformations of utility and permutations of the players,

thus generating a pair (V^v, Ψ^v) with the same properties. In addition, v obeys the

"Zeuthen-Nash-principle", i. e., for

$V = E_{T,A,x^0} \in \hat{V}$, $\bar{x} := v(V)$ and \bar{h} the normal vector at the Pareto surface of A in \bar{x} , we have

$$(1) \quad \bar{h}_i = \frac{1}{\bar{x}_i - x_i^0} \text{ const} \quad (i \in T(V));$$

signaling that utility at $\bar{x} = v(V)$ is equal for every (essential) player if it

is compared at the local rate of transfer defined by the tangency hyperplane at $\bar{x} = v(V)$.

Thus, ψ^v represents the following philosophy: Coalitions successively compute values and threatpoints (blisspoints) according to (2) and (3) of Section 1, but the "surplus" compared to the threat point (or the "deficit" compared to the blisspoint) in any coalition is distributed according to the "Zeuthen-Nash-principle".

Note that those ideas also appear in MIYASAWA [5]. However, his definition uses an underlying normal form. Also, MIYASAWA's value is a correspondence which frequently ends up to cover the individually rational part of the Pareto surface. In addition, no invariance properties are proved. However, sometimes ψ^v is a selecting function of this correspondence.

For $a \in \mathbb{R}^{n+}$, $a \neq 0$, let $a^{-1} = (\frac{1}{a_1}, \dots, \frac{1}{a_n})$

where $\frac{1}{a_i} := 0$ iff $a_i = 0$. Also, let

$f_a(x) = ax$ ($x \in \mathbb{R}^n$) and denote by

$$(2) \quad M_A f_a = \{\hat{x} \in A \mid f_a(\hat{x}) \geq f_a(x) \quad (x \in A)\}$$

the set of maximizers of the function f_a taken over some subset A of \mathbb{R}^n .

Given

$$V = E_{T,A,x^0} \in \hat{V}$$

define

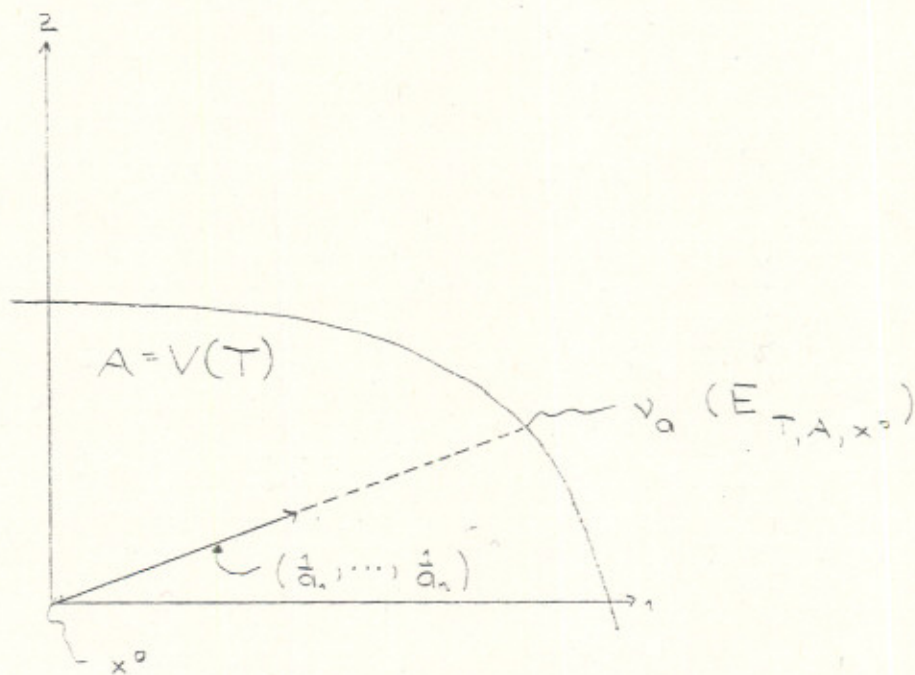
$$\begin{aligned} \tau(V) &= \tau_a(V) = \max \{t \in \mathbb{R} \mid x_T^0 + ta_T^{-1} \in A\} \\ &= \max \{t \in \mathbb{R} \mid \underline{x}_T(V) + ta_T^{-1} \in V(T)\}, \end{aligned}$$

provided $a_T \neq 0$, and

$$\tau(V) = 0 \quad \text{for } a_T = 0.$$

A mapping $\varphi = v_a : \hat{V} \rightarrow \mathbb{R}^n$ is then given by

$$\begin{aligned} (4) \quad v_a(V) &= x^0 + \tau_a(V)a_T^{-1} \\ &= \underline{x}(V) + \tau_a(V)a_T^{-1} \quad (V = E_{T,A,x^0}). \end{aligned}$$



Clearly, (\hat{V}, φ) is a primitive pair.

THEOREM 2.2.

Let $\psi_a := \psi^{v_a}$ be generated by (\hat{V}, v_a) via Theorem 1.1. Assume, for some $\bar{a} \in \mathbb{R}^{n+}$ and $V \in \mathbb{V}^{v_a}$

(5) $\psi_{\bar{a}}(V) \in M_{V(\Omega)}^f_{\bar{a}}$.

Then $\psi_{\bar{a}}(V)$ is an Harsanyi-value (say, in the sense of Section 6, CH. IV, of [14]).

PROOF.

1st Step. We shall assume that $a > 0$ within the first two steps of the proof. If $a \geq 0$, then the statements below hold true with due modifications, however, a great many technicalities will burden the proof. Similarly, in step 3, we shall assume $\bar{a} > 0$ by the same reasoning.

Now, let $a > 0$, $a \in \mathbb{R}^{n+}$ be arbitrary. Define $L = L^a$ to be the linear transformation $\mathbb{R}^n \rightarrow \mathbb{R}^n$ satisfying

$$x = (x_1, \dots, x_n) \rightarrow (a_1 x_1, \dots, a_n x_n)$$

For any $V \in \mathbb{V}$, L induces a mapping $LV \in \mathbb{V}$ (see SEC. 2, CH IV, of [14]). Since the threatpoint commutes with affine transformations, we find in view of the definitions (3) and (4) (with $e = (1, \dots, 1)$):

$$\begin{aligned}
 \tau_e(LV) &= \max \{t \mid \underline{x}_T(LV) + te_T^{-1} \in LV(T)\} \\
 &= \max \{t \mid L_T(\underline{x}_T(V)) + tL_T a_T^{-1} \in L_T(V(T))\} \\
 (6) \quad &= \max \{t \mid L_T(\underline{x}_T(V) + ta_T^{-1}) \in L_T(V(T))\} \\
 &= \max \{t \mid \underline{x}_T(V) + ta_T^{-1} \in V(T)\} \\
 &= \tau_a(V) \quad ,
 \end{aligned}$$

and similarly

$$\begin{aligned}
 v_e(LV) &= \underline{x}(LV) + \tau_e(LV)e_T^{-1} \\
 &= L(\underline{x}(V)) + \tau_a(V)L(a_T^{-1}) \\
 (7) \quad &= L(\underline{x}(V) + \tau_a(V)a_T^{-1}) \\
 &= L(v_a(V)) \quad .
 \end{aligned}$$

Next, since ψ^φ inherits all invariance properties from φ , it is not hard to see that (7) implies for $\psi_a = \psi^{v_a}$:

$$(8) \quad \psi_e(LV) = L(\psi_a(V)) \quad .$$

Note that (8) is not generally true but for $L = L^a$. However, (8) is a consequence of Theorem 1.1. (or rather of Theorem 4.11. of [12]) and a direct proof would have to repeat the computations offered in [12].

2nd Step. Let us recall the content of Definition 6.2., CH IV, of [14]. Accordingly, given a suitable $V \in \mathcal{V}$, $w = w^V \in W$ is defined recursively as follows:

1. $w(\{i\}) := x_i(V) := \rho(\{i\}) \quad (i \in \Omega).$

2. Given $w(S)$ and ρ_S for all $S \in \underline{P}$, $|S| < s$, define, for $T \in \underline{P}$, $|T| = s$:

$$t^T := \sum_{S \subsetneq T} \rho_S \mu^S$$

$$\rho_T := \max \{ \rho \in \mathbb{R} \mid t^T + \rho \mu^T \in V(T) \}$$

$$w(T) := \sum_{S \subseteq T} \rho_S$$

(Again, $\mu^T \in A$ is uniform distribution.)

Obviously, this is a construction we have employed previously. In fact, Remark 6.3. of CH. IV, [14], shows that

$$t^T = t^T(V, \psi) \quad (T \in \underline{P})$$

for $\psi = \psi_e$. Defining

$$x^T = \sum_{S \subseteq T} \rho_S \mu^S \quad (T \in \underline{P})$$

we therefore obtain similarly and by the same Remark 6.3. that

$$(9) \quad \phi(w^V) = x^{\circ}(\Omega) = \psi_e(V)$$

3. Step. As V was arbitrary, we may replace V by LV where $L = L^a$ is taken to be the same linear transformation as in step 1.

Thus, (9) implies

$$(10) \quad \psi_e(LV) = \phi(w^{LV})$$

Combining (8) and (9) we obtain

$$(11) \quad L(\psi_{\bar{a}}(V)) = \phi(w^{LV}) \quad .$$

Now, if $\bar{a} \in \mathbb{R}^{n+}$ is such that ($\bar{a} > 0$ and)

$$\psi_{\bar{a}}(V) \in M_{V(\Omega)}^{\bar{a}}$$

(see (5)), then, in addition, the following holds true. Put $\bar{x} := \psi_{\bar{a}}(V)$ for the moment.

Then, by (5)

$$\begin{aligned} \bar{a}\bar{x} &= \max \{ \bar{a}x \mid x \in V(\Omega) \} \\ &= \max \left\{ \sum_{i \in \Omega} L(x)_i \mid x \in V(\Omega) \right\} \quad (L = L^{\bar{a}}) \\ (12) \quad &= \max \{ y(\Omega) \mid y \in LV(\Omega) \} \\ &= v^{LV}(\Omega) \quad . \end{aligned}$$

Here, v^V generally denotes the element of V obtained by admitting sidepayment in V ("maximal joint utility"). Thus,

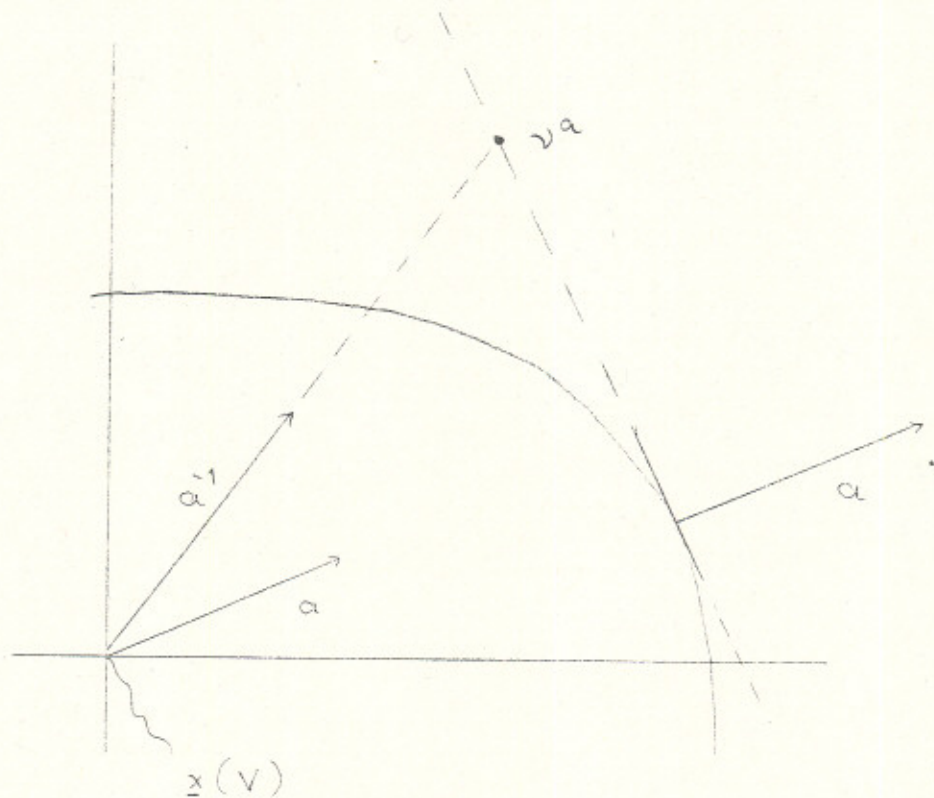
$$\begin{aligned} (13) \quad w^{LV}(\Omega) &= \phi(w^{LV})(\Omega) = L(\bar{x})(\Omega) \\ &= \bar{a}\bar{x} = v^{LV}(\Omega) \quad . \end{aligned}$$

We now call upon Theorem 6.4., Ch IV, of [14]. This Theorem states that (11) and (13) are necessary and sufficient in order to identify $\bar{x} = \psi_{\bar{a}}(V)$ as an Harsanyi value, q. e. d.

The next value concept to be considered requires the existence of $\max \{ax \mid x \in V(T) = A\}$ for all $V = E_{T,A,x^0}$ in the domain of definition of the primitive concept. Let \mathcal{V}^* denote the system of those $V = E_{T,A,x^0}$ for which A is compactly generated. For $V = E_{T,A,x^0}$ define

$$(14) \quad v^a(V) := \underline{x}(V) + (v^{LV}(T) - \underline{x}(LV)(T))a_T^{-1}$$

for $a \geq 0$, $a_T \neq 0$, and $L = L^a$. Of course, $v^a(V) := \underline{x}(V)$ if $a_T = 0$.



THEOREM 2.3.

Let $\psi^a := \psi^{v^a}$ be generated by (\bar{V}, v^a) via Theorem 1.1. Suppose, for some $\bar{a} \in \mathbb{R}^{n+}$, $\bar{a} \neq 0$, and $V \in \mathcal{V}^{v^a}$

$$(15) \quad \psi^{\bar{a}}(V) \in V(\Omega) .$$

Then $\psi^{\bar{a}}(V)$ is a Shapley value. (See SHAPLEY [16], also Section 5, CH IV of [14].)

PROOF.

1st Step. Note that

$$(16) \quad \begin{aligned} v^e(V) &= \underline{x}(V) + (v^V(T) - \underline{x}(V)(T))e_T^{-1} \\ &= x^0 + (v^V(T) - x^0(T))e_T^{-1} \\ &= x^0 + (v^V(T) - x^0(T))\mu^T . \end{aligned}$$

Comparing (16) with (4) of section 0,

we find

$$(17) \quad v^e(V) = \phi(e^{T, v^V(T); x^0})$$

whenever $V = E_{T, A, x^0}$ (and, of course, $v^V(T) = \max \{x(T) \mid x \in A\}$). As is easily established

$$(18) \quad v^V = v^{E_{T, A, x^0}} = e^{T, v^V(T); x^0} ,$$

thus (17) implies

$$(19) \quad v^e(V) = \phi(v^V) \quad (V = E_{T, A, x^0} \in \bar{V}) .$$

Next, we want to verify that

$$(20) \quad \psi^e(V) = \phi(v^V) .$$

This is done by proving

$$(21) \quad \psi^e(V_T) = \phi(v_T^V)$$

via induction on $|T|$. Indeed, (21) follows from (19) for $|T| = 1$ and the induction step is

$$\begin{aligned}
 \psi^e(V_T) &= v^e(E_T, V(T), \sum_{S \subseteq T} (-1)^{t-s} \psi^e(V_S)) \\
 &= v^e(E_T, V(T), \sum_{S \subseteq T} (-1)^{t-s} \phi(v_S^V)) \\
 (17) \quad &= \phi(e_{T, v^V(T)}, \sum_{S \subseteq T} (-1)^{t-s} \phi(v_S^V)) \\
 &= \phi(v_T^V) \quad (\text{by (5) of section 0}).
 \end{aligned}$$

This proves (20). Of course, inferring (20) from (19) is nothing but to apply the simple fact that ψ^e inherits invariance properties from v^e .

2nd Step. Using the defining equation (14) we verify at once

$$\begin{aligned}
 v^e(LV) &= \underline{x}(LV) + (v^{LV}(T) - \underline{x}(LV)(T))e_T^{-1} \\
 (22) \quad &= L(\underline{x}(V)) + (\dots)L(a_T^{-1}) \\
 &= L(v^a(V)),
 \end{aligned}$$

and calling again upon the inheritance properties of ψ^a and v^a , we conclude that (22) implies

$$(23) \quad \psi^e(LV) = L(\psi^a(V)).$$

3rd Step. Suppose now, $\bar{a} \in \mathbb{R}^{n+}$ satisfies the additional requirement of our theorem, i. e. (15).

Using (23) and replacing V by LV in (20), we find

$$(24) \quad L_{\Psi^{\bar{a}}}(V) = \Psi^e(LV) = \Phi(V^{LV}) .$$

Moreover $\Psi^{\bar{a}} \in V(\Omega)$ implies

$$(25) \quad L_{\Psi^{\bar{a}}} \in LV(\Omega) .$$

However, according to SEC. 5, Ch IV, of [14], (24) and (25) characterize a Shapley value, q. e. d.

Remark 2.4.

The existence of $\bar{a} \in \mathbb{R}^{n+}$, $\bar{a} \neq 0$, as specified in theorems 2.2. and 2.3. is, of course, established via a fixed point theorem.

SECTION 3. LOCATION CONFLICTS

In the course of introducing the system \hat{V} we have clearly left superadditive territory. For an unanimous game $V = E_{T,A,x^0}$ with $x_T^0 \notin A$, it is not the threatpoint which is represented by x^0 . Rather, this is a situation where all players $i \in \Omega$ might in fact prefer x^0 to anything feasible but are forced to retreat to some point on the feasible set. Thus x^0 will be called a blisspoint. The primitive concepts discussed in section 3 as well as the values generated by them are applicable to blisspoint situations. In sections 4 and 5 we shall sketch a possible economic background for "values of blisspoint games". The details will be found in [13].

DEFINITION 3.1.

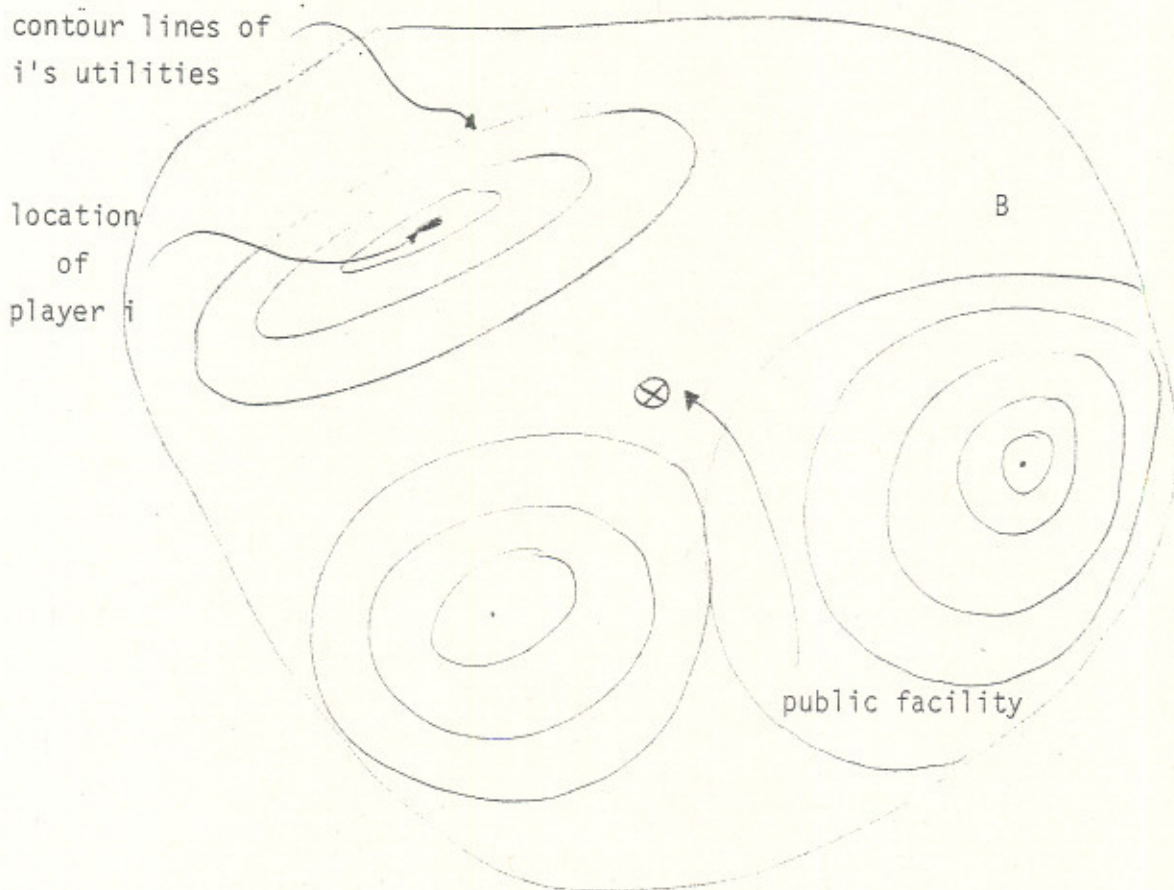
A location conflict is a triple

$$\Sigma = (\Omega, B, U)$$

where $\Omega = \{1, \dots, n\}$ represents the set of players, $B \subseteq \mathbb{R}^1$ is convex and closed (the "planning area") and $U = (u^i)_{i \in \Omega}$ is a family of "utility functions", $u^i : B \rightarrow \mathbb{R}$, which are concave, bounded, and continuous such that every u^i has satiation points, i. e.

(1) $M_B u^i \neq \emptyset$.

Intuitively, players $i \in \Omega$ have been found eligible to receive a public facility (swimming pool, park) located within their vicinity (i. e. B). Each player prefers the object to be closely located to his own location (the satiation points of his utility). Drawing the contour lines of the players' utilities, we obtain the following familiar picture.



The problem for the players (or a planner appointed) is to find a "fair location" for the object.

Such a problem might be treated by methods of Game Theory as discussed in sections 1 and 2. Indeed, there is a natural transformation of the location area into the utility space by means of the utility functions. If Z denotes the set of location conflicts, the mapping $Z \rightarrow V$ may e. g. be defined by $\Sigma \rightarrow V_\Sigma$ where

$$(2) \quad V_\Sigma(S) = \{x \in \mathbb{R}_S^n \mid \exists y \in B, x_i \leq u^i(y) \ (i \in S)\}.$$

$V_\Sigma(S)$ represents the utilities available for coalition S if planning is performed only with regard to the needs of S . (The usual interpretation of Game Theory according to which S "commands" the utility vector in $V(S)$ via cooperation is no more feasible.)

To make (2) more appealing, we might as well replace B by a family $(B_S)_{S \in \underline{P}}$ of planning areas, each one available to each coalition. However, this has no bearings concerning our general discussion.

Given a value ψ , defined on a subset $V^0 \subseteq V$, it is conceivable that $\psi(V_\Sigma)$ denotes the "fair distribution of utility" (provided, however, $V_\Sigma \in V^0$). If $\psi = \psi^\varphi$ then $V_\Sigma \in V^\varphi$ has to be

accounted for.). In the present context we are even more interested in "fair location".

No confusion will arise by writing

$$\psi(\Sigma) := \{y \in B \mid (u^i(y))_{i \in \Omega} = \psi(V_\Sigma)\} .$$

Thus, $\psi(\Sigma)$ denotes the value locations of Σ . In general, this version of ψ is defined on a subset \mathcal{L}^0 of \mathcal{L} only.

On the other hand, the values developed in section 3 are not the only ones available. "Values for location conflicts" may be developed axiomatically or as functions representing certain aspects of public choice theory. We refer to values studied by A. OSTMANN [7] and W. F. RICHTER [9],[10],[11], see also [13].

In any case let us assume that a version of a value ψ defined on some $\mathcal{L}^0 \subseteq \mathcal{L}$ is given in order to consider an even more specialised version of a blisspoint problem connected to general equilibrium theory.

SECTION 4. PUBLIC GOODS

DEFINITION 4.1.

An economy

$$\mathcal{W} = (\Omega, \mathcal{X} \times \mathbb{R}^{1+}, U, A, b, \mathcal{Y})$$

is defined by means of the following data:

$\Omega = \{1, \dots, n\}$ represents the set of players;

$\mathcal{X} = \mathbb{R}^{m+} \times \mathbb{R}$ represents bundles of private goods; the last coordinate being monetary (a booking account);

\mathbb{R}^{1+} represents bundles of public goods;

$U = (u^i)_{i \in \Omega}$, $u^i : \mathcal{X} \times \mathbb{R}^{1+} \rightarrow \mathbb{R}$ represents the "utility functions" of players $i \in \Omega$;

$A = (a^i)_{i \in \Omega}$, $a^i \in \mathcal{X}$ represents the initial allocations of players $i \in \Omega$ (private goods only);

$b \in \mathbb{R}^{1+}$ represents an initial outfit of public goods;

$\mathcal{Y} \subseteq \mathbb{R}^{m+1} \times \mathbb{R}^{1+}$ represents the aggregate production set of the public sector.

As this section again is only providing a sketch of a model, we shall not be too specific with respect to the conditions imposed upon the data; however, u^i ($i \in \Omega$) should at least be continuous, monotone, and concave, while \mathcal{Y} is assumed to be at least closed, convex, and contain 0. There is also a notion of taxes in our model. Taxation is used as a means to decentralize the decision of producing a certain bundle of public goods.

DEFINITION 4.2.

A taxation scheme $C = (c^i)_{i \in \Omega}$ is specified by a set of continuous, monotone, and convex (at least) mappings

$$c^i : \mathbb{R}^{1+} \rightarrow \mathbb{R} \quad (i \in \Omega).$$

If $\mathcal{C} := \{C \mid C \text{ is a taxation scheme}\}$, then $\mathcal{C}^0 \subseteq \mathcal{C}$ is said to be a taxation policy.

A taxation policy is an exogenous quantity: society has certain notions in advance about those tax schemes it considers applicable by customs, tradition, or institutional regulations.

Let us mention two particular versions of a taxation policy. Clearly

$$(1) \quad \mathcal{Q} := \{C = (c^i)_{i \in \Omega} \mid \exists q^i \in \mathbb{R}^{1+}, \alpha_i \in \mathbb{R} : c^i(y) = q^i y + \alpha_i \quad (y \in \mathbb{R}^{1+})\}$$

is the affine taxation policy. A taxation scheme

$C \in \mathcal{Q} \subseteq \mathcal{C}$ may assign different tax rates to the various players. If this is considered to be at variance with the standards of society, then e. g.

$$(2) \quad \mathcal{Q}^0 := \{C = (c^i)_{i \in \Omega} \mid \exists q \in \mathbb{R}^{1+}; \\ \alpha \in \mathbb{R} : c^i(y) = qy + \alpha \ (y \in \mathbb{R}^{1+})\}$$

may be chosen as the taxation policy; here the rates are equal for all players, thus \mathcal{Q}^0 is the equalizing affine taxation policy.

Now, let \mathbb{P}^{m+1} denote the price simplex in \mathbb{R}^{m+1} (prices for private goods). Given $p \in \mathbb{P}^{m+1}$ and a real number w , representing the wealth of player i , as well as a bundle of public goods $y \in \mathbb{R}^{1+}$, player i will maximize his utility with respect to private goods within his budget constraints, i. e., he will assess a utility

$$(3) \quad u^{pi}(y, w) := \max \{u^i(x, y) \mid px \leq w\}$$

to the pair (y, w) (provided the "max" exists at all). If a taxation scheme is at hand, then his wealth may be computed according to his initial holdings $a^i \in \mathcal{X}$, and the benefit of the initial public good b has to be

$$(4) \quad w = pa^i + c^i(b) - c^i(y) ,$$

assuming that he pays $c^i(y)$ towards the public bundle y .

Thus, given prices p and a tax scheme C , player i 's utility attached to a public goods bundle y is actually

$$(5) \quad \begin{aligned} \hat{u}^{pc^i i}(y) &= \overset{o}{u}^{pi}(y, pa^i + c^i(b) - c^i(y)) \\ &= \max \{u^i(x, y) \mid x \in X, \\ &\quad px \leq pa^i + c^i(b) - c^i(y)\} \end{aligned}$$

Of course, the existence of $\hat{u}^{pc^i i}$ has to be established. The essential requirements are positive prices ($p > 0$) and increasing marginal costs (e. g., c^i strictly convex) versus decreasing marginal utility (e. g., u^i strictly concave). More details may be found in [13]. If the necessary requirements are satisfied for all members C of a tax policy $\mathcal{C}^0 \subseteq \mathcal{C}$, then \mathcal{W} and \mathcal{C}^0 are said to be compatible.

Let us quote

LEMMA 4.3.

If \mathcal{W} and \mathcal{C}^0 are compatible, then, for $p > 0$, $p \in \mathbb{P}^{m+1}$ and $C \in \mathcal{C}^0$, $\hat{u}^{pc^i i}$ is continuous and concave. Moreover

$$M_{\mathbb{R}^{1+}} \hat{u}^{pc^i i} \neq \emptyset$$

i. e., $\hat{u}^{pc^i i}$ has satiation points ($i \in \Omega$).

For the details see ZECKHAUSER-WEINSTEIN [17] or [13]. Clearly, the lemma states that player i 's utility ($\hat{u}^{\dots i}$) will have satiation points if, for large

public bundles the marginal cost to him with respect to the public goods eventually exceeds his marginal utility ($u^i(\cdot, \cdot)$) w.r.t. the public goods.

Let us write $\hat{U}^{PC} = (\hat{u}^{PC^i})_{i \in \Omega}$

Then we have

COROLLARY 4.4.

Let \mathcal{W} and \mathcal{C}^0 be compatible. Then, for $p \in \mathbb{P}^{m+1}$, $p > 0$ and $C \in \mathcal{C}^0$

$$\Sigma^{PC} := (\Omega, \mathbb{R}^{1+}, \hat{U}^{PC})$$

is a location conflict.

Clearly, players $i \in \Omega$ will in general have conflicting notions about which public bundle should be chosen. Given prices and taxes, this depends in fact on their utilities \hat{u}^{PC^i} ($i \in \Omega$), the satiation points of which might well be different. In the light of our analysis of location conflicts in section 3, it is suggested that the "fair" public bundle to be produced is just the fair value of the location conflict. The notion of "value", again, is exogenous. Society has to agree upon which idea of fairness should be applied, i. e., a "value" ψ should be specified in advance.

Given ψ and assuming that Σ^{pC} is within the domain of definition of ψ , the "fair" bundle is $\psi(\Sigma^{pC})$ once p and C are fixed. Now the question occurs as to whether $y = \psi(\Sigma^{pC})$ is feasible in the sense that it can be obtained by the production technology. For, by construction of \hat{u}^{pc^i} , each player will maximize w.r.t. the private goods. The maximizing bundles of private goods together with the "fair" public bundle should constitute a feasible state of the economy. As this cannot be expected a priori, we are thus led to the following definition of equilibrium.

Denote by $\hat{X}(p, c^i, y)$ the set of maximizers in (5), i. e.,

$$\hat{X}(p, c^i, y) = \{x \in X \mid u^i(x, y) = \hat{u}^{pc^i}(y)\}$$

(assuming that existence problems are taken care of by "compatibility"-requirements. In fact, in many cases \hat{X} is a u.h.c. correspondence).

DEFINITION 4.4.

Let \mathcal{M} be an economy and let ψ be a value defined on $Z^0 \subseteq Z$. Also, let $C^0 \subseteq C$ be a taxation policy, $\bar{X} = (\bar{x}^i)_{i \in \Omega}$ ($\bar{x}^i \in X$) a collection of bundles of private goods and $\bar{y} \in \mathbb{R}^{1+}$ a bundle of public goods. Furthermore, let $\bar{p} \in \mathbb{P}^{m+1}$ be a price system for private goods. Then

$$(\bar{p}, \bar{c}, \bar{x}, \bar{y}) \in \mathbb{P}^{m+1} \times \mathbb{Q}^0 \times \mathcal{X}^n \times \mathbb{R}^{1+}$$

is said to be a ψ - \mathbb{Q}^0 -equilibrium if the following holds true:

1. $\bar{p} > 0$.
2. \mathcal{W} and \mathbb{Q}^0 are compatible...
3. $\Sigma \bar{p} \bar{c} \in \mathcal{L}^0$.
4. $\bar{y} = \psi(\Sigma \bar{p} \bar{c})$.
5. $\bar{x}^i \in \mathcal{X}^i(\bar{p}, \bar{c}^i, \bar{y})$ ($i \in \Omega$).
6. $(\Sigma_{i \in \Omega} \bar{x}^i - a^i, \bar{y} - b) \in \mathcal{Y}$.

As a first example, let us consider the LINDAHL-equilibrium ([3], see [4] for a survey). Values Ψ , as discussed in the previous sections, enjoy the property that the value coincides with a blisspoint if the blisspoint is feasible (or with the threatpoint if the threatpoint is Pareto optimal). The LINDAHL-equilibrium assigns different tax rates to the various players but thereafter every player maximizes his utility w.r.t. public and private goods given his budget constraints, i. e., he achieves his blisspoint of the corresponding location conflict. Indeed, it is not hard to prove

THEOREM 4.5.

Let \mathcal{W} be an economy and let \mathbb{Q} denote the affine taxation policy (see (1)). Then a LINDAHL-equilibrium is a ψ - \mathbb{Q} -equilibrium

for every ψ satisfying

$\psi(V) = \underline{x}(V)$ whenever $\underline{x}(V)$ is Pareto optimal and feasible.

Imagine however that the (general) affine taxation is not feasible by institutional reasons and that the players have agreed upon the decision that only equal tax rates should be applied. In other words, consider the equalizing affine taxation policy Q^0 (see (2)). Now, every taxation scheme $C = (c^i)_{i \in \Omega} \in Q^0$ is of the form

$$c^i(y) = qy + \alpha$$

for certain $q \in \mathbb{R}^{1+}$, $\alpha \in \mathbb{R}$. Moreover, the budget restriction employed on the right hand side of (5) reads

$$\begin{aligned} px &\leq pa^i + c^i(b) - c^i(y) \\ &= pa^i + qb - qy \end{aligned}$$

or

$$px + qy \leq pa^i + qb .$$

Obviously, α does not enter at all, hence it is sufficient to describe $C \in Q^0$ by the associated q . We may then write Σ^{pq} instead of Σ^{pC} and an obvious meaning is attached to the phrase that

$$(p, q) \rightarrow \psi(\Sigma^{pq})$$

is required to be continuous.

THEOREM 4.6.

Let \mathcal{M} be an economy and let Q^0 denote the equalizing affine taxation policy. If $(p,q) \rightarrow \psi(\Sigma^{pq})$ is continuous, then there is a ψ - Q^0 -equilibrium (given further conditions concerning compatibility of \mathcal{M} and Q^0 and concerning appropriate properties of γ) .

The details of conditions and proofs are stated in [13]. It should be noted that a sufficient condition for ψ is in fact that

$$(p,q) \rightarrow \psi(\Sigma^{pq})$$

is an u.h.c. and convexvalued correspondence.

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