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Structural Characteristics of Economic
Models: A Study in Complexity

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1. Introduction

In the past twenty years we have experienced a tremendous ad-
vancement of tools and techniques in analyzing, diagnosting,
predicting and controlling economic processes. Sophisticated
models have been built that claim to predict future economic
trends of micro and macro-economic variables, largely facili-
tated by the availability of large-scale computational re-
sources. These models also form the basis of policy recom-
mendations such as the Brookings models or similar large-scale
econometric models. There have been some major improvements
in the design, estimation, statistical structure, testability
of economic model-building as have been in decentralized de-
cision-making, distributed computation and hierarchical con-
trol. Yet, there is still one major link missing which we
consider as one of the most fundamental properties of any mo-
del-building in the large that one has not come to grips
with the structural constraint of complexity, e.g. the infor-

mation processing limits arising in the control of dynamic systems. Also in a recent survey on large-scale systems (N.R. Sandell et al [1]) it is stated that an adequate measure of complexity is lacking for control systems and that a major task of constructing a unified theory of decentralized control is to include a formal measure of system complexity. We focus on this structural constraint of complexity by designing models that explicitly cope with this issue.

2. Preliminary Considerations

From a methodological view there are two ways to understand and predict the behavior of (discrete-time) dynamic systems. The _first_ is to evaluate changes in the future by past performance, to project realizations in the past into the future, what may be summed up by 'prediction by trends'. Intuitively, this is based on the idea that any system's behavior 'cannot escape the past to shape the future', that any evolving system develops its own memory which conditions it to past events. The more data are compiled from the past the more the memory activates its own self-organized dynamics. This appears to exclude purely random behavior in the past, exhibited by a random sequence, hence the occurrences of events do not seem to be statistically independent. The _second_ is by constructing a model which is considered to be a representative mapping of the system under investigation. A model attempts to capture the basic technical, structural and behavior characteristics of the underlying system and on this ground estimates its potentialities for future development. This is referred to as 'prediction by models'. Now 'prediction

by trends' assumes regularity of the process as if social and economic processes satisfy laws of statistical regularity. It comes very close to conceiving the world of being in a state of 'disorganized complexity' according to W. Weaver [2], meaning that we can view courses of events essentially as 'random sequences' and that the statistical law of large numbers is the basic concept to predict future behavior. Already F.A. Hayek [3] remarked that economic processes cannot be observed in a statistical fashion. The problem with a statistical assessment of complexity, or disorganized complexity, lies in the attempt to average over a large number of random variables, as represented by the mean or expected value, which, indeed, treats all phenomena of the system alike or as uniform whereas in reality there exist different structural relationships between elements that bear different characteristics and that may have a non-negligible impact on the behavior of the entire system. Hence, it is very doubtful that economic systems can be explained on the basis of pure random phenomena.

I have shown elsewhere (Gottinger [4]) that this implies that economic systems would perform as if they were infinitely large systems. Instead, I have pointed out that economic systems are much more akin to finite complex systems' (FCS), i.e. systems that are much too complex to get explicit solutions for them, as is the case in 'simple' systems, nor is the number of parts large enough or are the parts homogeneous enough to be able to pass to the limit as in infinitely large systems. An FCS has some peculiar properties, not shared by its better worked out counterparts that makes it difficult to analyze, understand and predict its behavior. It appears to be (1) highly sensitive

in responding to changes or disturbances of its environment,
(2) strongly interdependent with regard to actions of its
components, (3) following a threshold discretionary type
of behavior with qualitative jumps, (4) partially or locally
controllable inducing global effects. In fact, these pro-
perties can be observed in modern economic systems that
render them a higher degree of instability as similar
behavior patterns are traced in ecosystems (R.M.May [5]).
Hence behavior of an FCS is not of a purely random character.
The reason for this is that it reveals certain imperfections
like lagged responses, maladjustment, non-stochastic depen-
dence between parts, discontinuity that appears not to be
compatible with randomness.

3. A General Approach to Dynamic Systems and Complexity

The approach starts with the basic identification problem. The
technicalities have been treated elsewhere (Gottinger [6],[7]).
Given some natural complex system, a black box, where we only
observe outputs and inputs over time but we are ignorant about
what is happening 'inside', e.g. about local properties or
parameters of transition. Is it possible to find an 'artificial'
system that simulates the original natural system? Systems
we have in mind in this context are those which respond in real-
time to their environments just to stay alive (bull fighting
a bear). Ecological systems (bird colonies) or biological systems
(metabolism of cells) constitute systems striving for survival,
also all types of competitive economic systems challenged by
adversary environments belong to this category. In general,
extreme notions, such as survival or non-survival (death),
which are characteristic for pure forms of competitive systems

are involved. Here interest is focused on global properties of dynamic systems. The design orientation follows from the identification process, e.g. by taking interconnecting part of the artificial system 'hooking' them together in order to simulate the 'black box'. The approach is algebraic, since it starts from finite-state dynamic systems, e.g. sequential machines, the general characteristics of which are described by:

1. A set of inputs, e.g. those changing parameters of the environment which will affect the system behavior in a predictable way.

2. A set of outputs, i.e. those parameters which act upon the environment leaving observable changes in the relationship between the system and the environment.

3. A set of states, i.e. those internal parameters which determine the relationship between inputs and outputs and which may reveal all necessary information embodied in the past.

4. The state transition function which determines the dynamics of how the state will change when the system is fed by various inputs.

5. The output function which determines what output the system will yield with a given input when in a given state.

There are some obvious advantages, theoretical and practical ones, to use an algebraic approach. First, algebra is a natural tool because it emphasizes the design of a system as a collection of objects. Second, algebra is natural for computational work, and this is an important factor in applications. Modern computers accept digital instructions and those in turn require an algebraization of systems. Third, algebraic system theory emphasizes qualitative aspects of systems to the extent that we are interested in properties such as survival or breakdown. This is achieved by determining complexity bounds of the

system's design (design complexity). By the fact that algebraic
system theory is related to computational structures we are
in the position to construct a complexity theory for dynamic
systems which is also amenable to applications.

Systems of that sort reveal a natural bound of complexity.
Complexity here has two facets, computational complexity and
structural complexity.

Structural complexity refers to the inherent capability of
parts of the system, of what they are able to perform.
Computational complexity refers to the length of computation
given by the extent of interconnection between parts. The
most important distinction is that between design and con-
trol complexity.

Under design complexity it is understood that complexity (num-
ber) associated to the transformation process in which full
use of the system potential is made. Design complexity can
only be achieved if a system is working without faults, if the
components live up to computability requirements, if the parts
function reliably. Under control complexity we understand that
specific complexity number which keeps the entire system or
at least part of it under complete control. Only if design and
control complexity coincide stable configurations in the state
and output space will result, or the system runs smoothly.
The relation between design and control complexity is an in-
dication for stability or harmony of a dynamic system.

4. Decomposable Systems

We wish to explore the nature of the interdependence in economic
systems, to show how these systems might be decomposed into
their component parts for analytic purposes, and to relate

the results to the choice of models in policy analysis and projection.

By the previous arguments and those provided by the algebraic theory of machines we are advised that there must be general principles of decomposition which arise in the design process of algebraic (computational) systems. From these general principles of decomposition we emerge with a natural theory and measure of complexity which is structural (intrinsically related to the parts, the basic building blocks) and computational, e.g. addressed to the computational links between these parts. We will see how this framework is useful for approaching more specific type questions in economic model building.

The core for the investigation in the present section is provided by the Ando-Fisher theorem [8] on the decomposibility and independence in dynamic economic systems.

The Ando-Fisher theorem and related results deal with general ways how to decompose a system into its component subsystems for purposes of analysis and prediction. They distinguish between (completely) decomposable and nearly (completely) decomposable systems. In the latter case, a nearly decomposable system consists of subsystems where each is causally dependent on the rest of the system but the rest of the system is only weakly dependent (or weakly coupled) on (with) each subsystem. In contrast, a decomposable system only allows for within systems dependencies but ignores intersystems dependencies altogether. The Ando-Fisher theorem asserts, for linear dynamic systems, that

(1) provided inter-systems dependencies are sufficiently
weak (up to a negligible degree) relative to intra-
systems dependencies, then, in the short run, the
relative behavior of a nearly decomposable system be-
comes almost indistinguishable to that of a decomposable
system.

(2) provided inter-systems dependencies are sufficiently
weak relative to intra-system dependencies, then also
in the long run relative behavior of the nearly decomposable
system and the decomposable system is approximately the
same even though their behavior in terms of absolute le-
vels and rates of change may be very different. In other
words, if we give a nearly decomposable system enough run-
ning time, and that even when influences having been neg-
lected have had time to make themselves fully felt, the
relative behavior of the variables within any subsystem
will be approximately the same as would have been the
case had those influences never existed.
Now there are obviously two critical factors in the va-
lidity of the last result:

(a) the degree of approximation depends on the number
of computational cycles (e.g. running time) and is cer-
tainly a result in the limit,

(b) the degree of approximation depends on a prepostu-
lated, sufficiently weak linkage among the subsystems,
below a certain threshold value where the system moves
continuously in an orderly manner. This however presupposes
a very large number of subsystems, acting uniformly on
an equal power base, as in the classical model of economic

equilibrium.

Both assumptions (a) and (b) are hard to defend in the
context of analyzing real-life social and economic systems.
For these systems, as real-time systems, we simply haven't
got enough running time to force this approximation upon
the system's behavior. Time is a scarce and valuable re-
source.

Second, there are only a few, relatively large subsystems,
the relative behavior of which have a great potential
impact on the entire system, and where the intra-system
behavior is not limited to the subsystem itself.

In fact, we would very much argue in the opposite direction.
The characteristics of large-scale complex systems are
such that they are very sensitive to discretionary be-
havior of their subsystems and that actions, outcomes and
consequences of these subsystems very often induce snow-
ball-effects that pervade other subsystems and enforce
actions, perturbations, maladjustments, all sorts of reactions
that deeply affect smoothness, regularity, stability, control -
lability which are highly unpredictable because of the non-
linearities involved.

5. Systems, Modelling and Complexity

Let us start by describing a dynamic system very much like
an automaton. The ingredients are given by

an internal state vector $z_t = [z_{1,t}, z_{2,t}, \ldots z_{n,t}]$

an external state vector $x_t = [x_{1,t}, x_{2,t}, \ldots x_{n,t}]$

representing exogeneous factors, driving the system from
'outside', and not incorporated in the decomposition.

An <u>output</u> function δ that maps strings of inputs, say $(a_1, \ldots a_n)$ into single outputs $b_1, b_2, \ldots b_n$ which enter as inputs into other parts (components) of the system, the <u>state</u> <u>transition</u> function λ with

$$z_t = \lambda(z_t, z_{t-1}, \ldots, x_t), \quad \text{given in}$$

difference equation form.   The function λ, without any specification yet, represents the hypothesis about the process, inferred from the observations of real world systems. The external state vector x_t can be conceived as a <u>primary</u> input factor (stimulus) which sets the system into motion, but which itself may be suitably partitioned as to which component is primarily affecting the process.

The behavior of the system is generated by the set of time series of the z_{it} which are produced as the model generates successive state descriptions plus the external states given exogeneously.

Now in the chartist approach to studying the future of systems it is assumed that the connections between different components either do not exist or are sufficiently weak that they can be safely ignored.   The value of any component z_{it} depends only on its previous values and possibly random disturbances u, thus predicting a future course of actions or events means extrapolating past performance.

In a comprehensive modelling approach, as suggested here, at least some of the possible interdependencies among components exist in general.   The behavior of each component may depend on its past behavior as well as on other variables in the system, the corresponding exogeneous factors x_{it},

i = 1,...,n, respectively, and possibly the random disturbances u,
i.e.

$$z_{it} = \lambda(z_{i,t-1}, \; z_{i,t-2}, \ldots, z_{k,t}, \; z_{k,t-1}, \; \ldots, \; x_{it}, u) \quad k \neq i.$$

Three immediate problems in the model procedure arise from the
Decomposition Scheme exhibited in Fig. 1

(1) As the number of components and sufficiently strong con-
nections among components increase the behavior of the
system becomes increasingly obscure by complex interactions
which resemble very much non-linearities in total system's
behavior in correspondence with size.

(2) The structure and size of the components themselves present
a potential source of complexity depending on whether and
to which extent the component system is sensitive to dis-
turbances, errors, threshold phenomena, etc.

(3) As the number of components and interdependencies in the
system enhances, increasingly longer sequences of calcu-
lations are required to deduce the behavior of the system
which results in computational complexity.

The solutions of these three problems would enable us to de-
termine the complexity of the system on-line, as it is running
from some initial time to some target time in the future. But
knowing the complexity would permit us to design control stra-
tegies which are effective in guiding the system toward re-
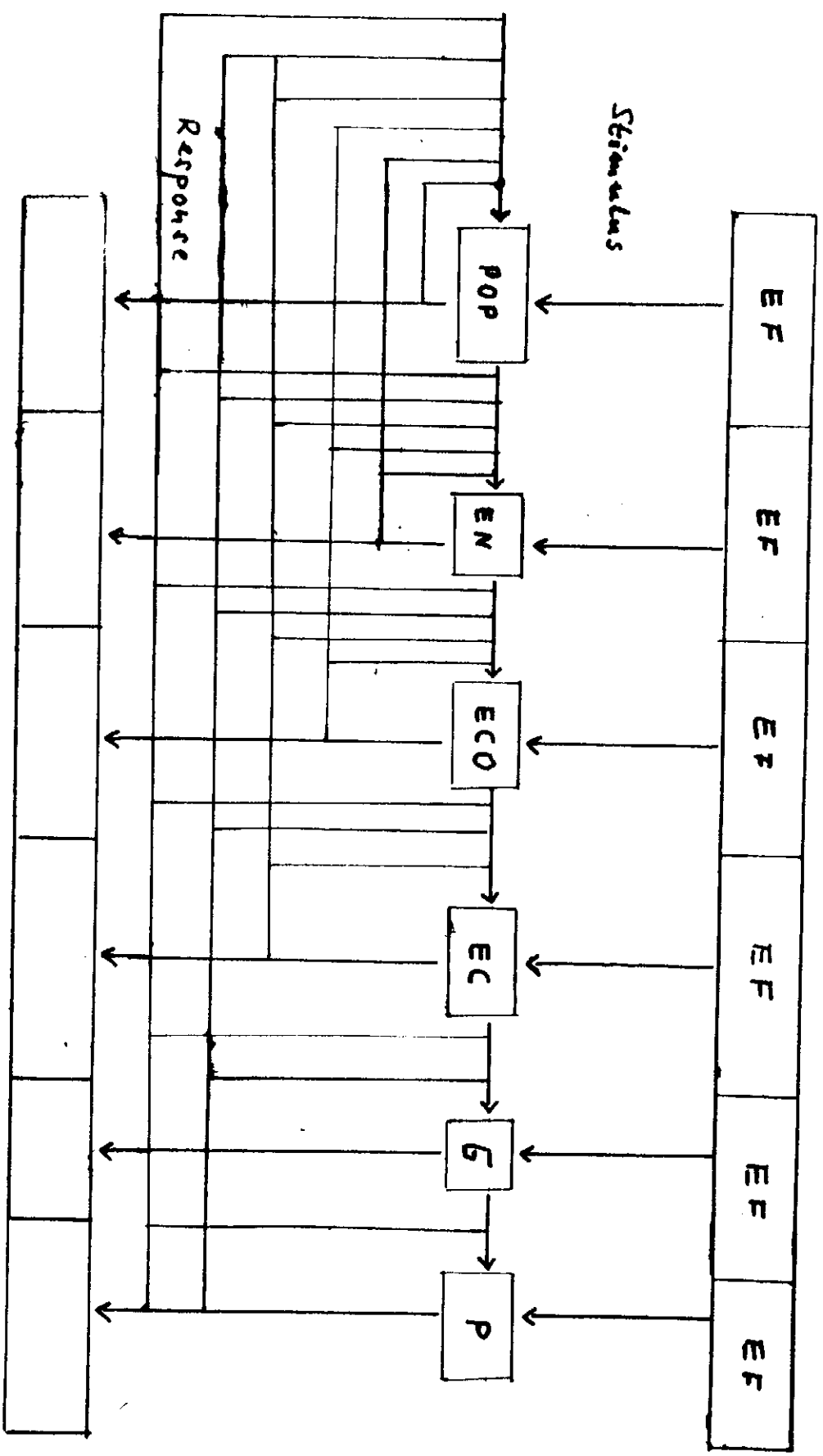lative stability or harmony.

Fig. 1 Cascade Form of Decomposition

Therefore to understand the complexity of such systems we should be able to understand the strongly connected, coupled nature of its subsystems. For this purpose we need a measure of complexity that reflects the structural performance of each of the connected subsystems in terms of state space configurations plus the number of computational links that are established among the various subsystems and that reflect the richness of state representations in the global trajectory space of the entire system.

Illustration

Take the population subsystem [POP] consisting of initial eight states, e.g. $z = (z_1, z_2, \ldots, z_8)$, then after inputs are fed into the [POP] system a new state configuration obtains .
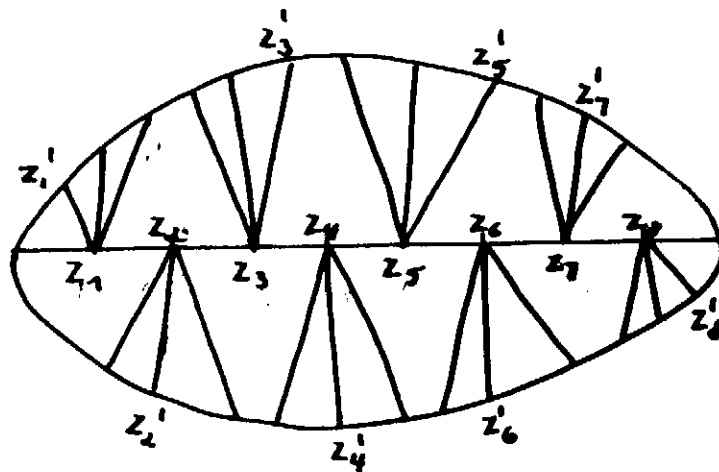

Fig. 2   [POP] - Subsystem

A simple measure of structural complexity, in accordance to Gottinger [6], [7], could be given by the number

$$\#_s(z) = \sum_{i=1}^{8} \overline{\#_s}(z_i) = \sum_{i=1}^{8}$$ (length of time needed to reach a satisfactory state) x (number of feasible states to be attainable).

On the other hand, the computational complexity indicates the number of links between various subsystems times the number of interactions that ensue until the computational cycle (in real-time) is completed, as roughly indicated only for one specific subsystem, say [POP], (for all other subsystems likewise) in the following illustration

In the satellite system, connecting all neighboring subsystems, the computational complexity related to the [POP] subsystem amounts to

$$\#_c([POP]) = (\# \text{ links to } [EN] + \# \text{ links to } [ECO]+...+\# \text{ links to } [P])$$
$$= \sum_{i=1}^{n} \#_i (\text{links to } i).$$

Then the total computational complexity comprising all links among all subsystems within the satellite system is given by
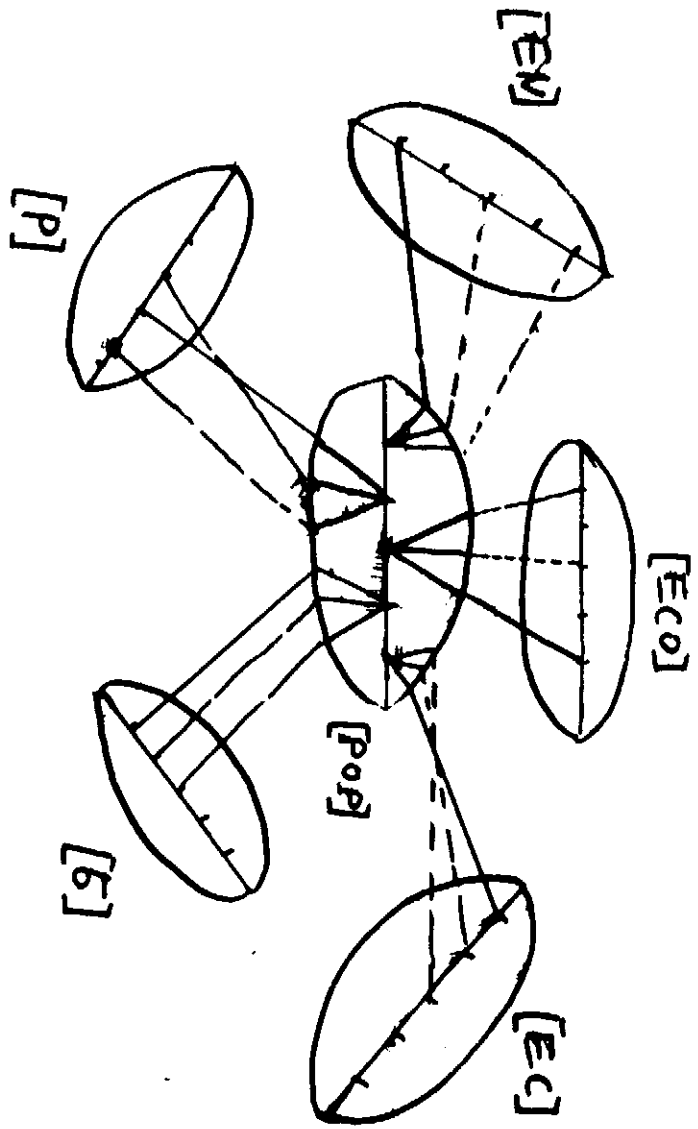
Fig. 3 Satellite System
Links Indicate Computational Complexity

$$\#_c([POP],[EN],[ECO],[EC],[G],[P]) = \#_c(Z_F,F) = \sum_{k=1}^{6} \#_k(Z_{Fk},F_k)$$

Using the notation, let Z_F be the state space over which the entire finite state system is running, let F be all feasible semigroups of transformations acting on this space, then in accordance with decomposition results of algebraic finite state systems we establish a comprehensive complexity measure comprising of structural and computational complexity, e.g.

$$C(Z_F,F) = \Pi_{k=1}^{m} \#_s(Z) \#_c(Z_F,F),$$

and this is the measure we use in analyzing the structure of a particular economic model.

6. Structure of the Model

We wish to explore the nature of interdependences in societal system to show how systems might be decomposed into their component parts for analytical purposes and to relate the results to the choice of models in policy analysis and projection. According to what has been illustrated in Fig. 1 we choose a kind of partition of the overall system into parts that comprise the main activities of complex societal systems - this is very much in spirit of earlier attempts at modelling complex political systems for purposes of simulating their behavior, see R.D. Brunner and G.D. Brewer [9]. The components of this model consist of the set of variables and parameters listed under the detailed description later.

[POP] - the population subsystem is relatively specialized to the growth and distribution (density) of the population N_t, being composed of the urban and rural population.

[EN] - the energy subsystem is singled out as the major resource
sector which could be of mixed private-public activities, and
where the energy basket consists of the output E_t of primary
and secondary energy resources generated.

[ECO] - the environmental system puts environmental restrictions
(water and air pollution) on growth of production in urban
and industrial areas.

[EC] - the economic subsystem is relatively specialized to
the production and distribution of economic goods and services
within the private sector.

[G] - the government subsystem comprises all activities of
the public sector which are directed at providing goods and
services demanded by the market place or generated by poli-
tical considerations.

[P] - the political subsystem is specialized to the production
of changes in the size and distribution of mass support for
the government MV_t to permit majority rule and conservation
of power, furthermore, determines the consumptive and distri-
butive characteristics of government expenditures G_t. The
relationships in the model are grouped according to subsystems,
these relationships are hypotheses about the way in which each
variable changes as a function of the others, with the mag-
nitudes of the changes being determined in part by the para-
meters.

The degree of connectedness or interdependence implied in
the relationships can be explored by noting the presence or
absence of causal links among the variables. Call this the
structural connectedness of the model. In fact, there are direct
causal and indirect causal connections (links) between the
output and input variables, depending on whether they occur
within the components blocks or are cross-connected among
different blocks. The static description of the structural
connectedness underestimates the degree of connectedness
among variables as the model operates through time. For example,
since government revenue R_t is merely a function of gross
output Y_{t-1}, variation in G_t within one real-time cycle is
limited to the variation in this variable. However, if the
model is operated through time, the number of variables
causally connected to R_t increases. Thus while R_t is a direct

function of Y_{t-1} (or some aggregates thereof), it is also

an indirect function through this variable of C_{t-1}, I_{t-1},

G_{t-1}, and through these variables R_t is an indirect function

of several other variables, etc. Obviously, the power of

the connectedness matrix increases in correspondence with

an increse in the number of real-time cycles of the model.

The analysis of structural connectedness has obvious limi-

tations for the chartist approach to prediction. If the

analysis suggests that even in a system that is loosely

connected in its static description every variable in the

system may ultimately depend upon every other variable as

the system runs on-line through time, then a simple extra-

polation of trends which ignores these dependencies may

indeed be highly misleading. In a situation where the num-

ber of outside factors tend to be increasing to infinity,

we would well expect some statistical regularity in offset-

ting influences among the factors, and, indeed, a chartist

approach may be a good thing to follow by neglecting these

dependencies. This would correspond to view the real si-

tuation as a state of 'disorganized complexity'. However,

in the course of previous arguments, the case of an FCS

would not support this point. Then a serious modelling

and prediction effort would have to take into account in-

direct effects that may have a highly nonlinear character. Of

course, there is the possibility that in a particular system

or in a particular application of the model, certain of these

dependencies may be sufficiently weak that they can be safe-

ly ignored. If this were the case, the chartist's approach

which ignores these dependencies and the comprehensive modelling approach taking them more fully into account may lead approximately to the same conclusions.

7. The Model's basic sets of relationships

(a)   [POP]-Subsystem

Rate of population change

p(rho) $(\overset{\cdot}{N})$ = α_1 (average number of children per family) exogeneous
  + β_1 (number of women in child-bearing age)) factors
  + λ_1 (availability of birth control devices) [EF]
    and liberal abortion policies)

  + δ_1 (expected average family income,
    given some current level of family   [EC]
    income)

  + ε_1 (population supporting energy con-   [EN]
    sumption level)

  + η_1 (population density index level)   [ECO]

  + ζ_1 (provision of public goods and ser-
    vices relevant to child rearing acti-   [G]
    vities) .

  + θ_1 (level of wide-spread political satis-)
    faction and trust in political insti-   [P]
    tutions)

(b)   [EN] - Subsystem

Supply of Energy

$S(E_t)$ = α_2 (availability of domestic energy resources))
  + β_2 (energy provision through imports)   )
    [EF]
  + λ_2 (potential of mobilizing new energy   )
    sources via technology)

$+ \delta_2$ (price expectations and market prices)   )   [EC]

$+ \varepsilon_2$ (level of manpower directed toward energy production)   )   [POP]

$+ \eta_2$ (upper limits of energy provision by environmental factors, i.e. strip-mining, health and environmental hazards, pollution)   )   [ECO]

$+ \xi_2$ (degree of government intervention in terms of price or quantity regulation)   )   [G]

$+ \theta_2$ (political interference in the industrial organization of the energy industry)   )   [P]

$D(E_t) = \mu N_t + \lambda Y_t$, energy consumption is proportional to total population and to the level of G.N.P.

$\Delta(D(E_t) - D(E_{t-1})) = \mu(N_t - N_{t-1})$, the rate of energy consumption is proportional to the rate of population change.

(c) [ECO] - Subsystem

Abatement Performance (measured in terms of level of exhaust emission)

$AP = \alpha_3$ (index of industrial distribution)   )

$+ \beta_3$ (environmental immersion factors)   )

$+ \lambda_3$ (level of environmental technology)   )   [EF]

$+ \delta_3$ (G.N.P. as related to regional and sectoral industrial activity)   )   [EC]

$+ \varepsilon_3$ (rate of population change and structural population shifts from rural to urban sectors)   )   [POP]

$+ \xi_3$ (government regulation and industrial incentive structure in terms of emission limits, waste disposal, land use zoning regulations, noise restrictions)   )   [G]

+ η_3 (energy supply and mixture of energy) [EN]
  sources))
)

+ θ_3 (effectiveness of government control,)
  efficiency/inefficieny ratio of super-) [P]
  vision))

(d) [EC] - Subsystem

Links of G.N.P. aggregates by definition:

$Y_t = C_t + I_t + G_t + F_t$ (F_t = foreign trade, given exogeneously).

$C_t = m_t (1-\tau) Y_{t-1} \dot{N}$,

$I_t = I_{t-1} \dot{N} + r(\dot{C} \dot{N})^{+f(G_t)}$, $\dot{C} = dC/dt$

where f represents the extent of government expenditure restricting private investment, to take care of welfare - state limits on private investment activity.

Growth of G.N.P.

$\dot{Y} = \alpha_4$ (resource endowment))
)
  + β_4 (R & D level))
)
  + λ_4 (level of technology used)) [EF]
)
  + δ_4 (minimal level of energy supply needed) [EN]
  to support growth))

  + ϵ_4 (minimal abatement performance AP = ma-)
  ximal pollution, noise, waste disposal) [ECO]
  limit))

  + η_4 (size of government investment activi-) [G]
  ties))

  + ξ_4 (rate of population change related to) [POP]
  productive employment))

  + θ_4 (government guidance and organizatio-) [P]
  nal support))

(e) [G] - Subsystem

Government revenue is proportional to G.N.P. Y_t via the

proportionality factor τ (tax rate).

$$R_t = \tau \, Y_t.$$

Government expenditure

$G_t = \alpha_5$ (international structural changes in terms of balance-of-payments surplus/ deficit)   [EF]

$+ \beta_5$ (rate of population change and age structure of society)   [POP]

$+ \lambda_5$ (energy supply $S(E_t)$ and R & D expenditures on new energy sources)   [EN]

$+ \delta_5$ (abatement performance: safety and environmental regulations)   [ECO]

$+ \varepsilon_5$ (level of economic and industrial activity)

$+ \eta_5$ (employment and capacity level)

$+ \xi_5$ (index of inflation)   [EC]

$+ \theta_5$ (political support by major segments of the population (social groups))   [P]

(f) [P] - Subsystem

Majority of votes from all or some segments of the population to achieve majority rule in Parliament.

$MV_t = \alpha_6$ (psychological conditions of trust, honesty and stability in society)   [EF]

$+ \beta_6$ (rate of population change)

$+ \lambda_6$ (change of migration patterns from rural to urban areas)

$+ \delta_6$ (occupational shifts)   [POP]

$+ \varepsilon_6$ (energy supply and distribution of energy resources)   [EN]

$+ \eta_6$ (level of pollution affecting major social groups and key industrial areas)   [ECO]

+ ξ_6 (composition of rate of G.N.P. in)
    relation to political goals agreeing) [EC]
    with major interests of the popula-)
    tion))

+ θ_6 (provision of government services and)
    collective goods)) [P]

8. Evaluation of Complexity

Now the static description of the structural connectedness
underestimates the degree of interaction among subsystems
as the interaction operates through time, this is true since
the various feedback- and feedforward mechanisms reinforce
the interaction in a nonlinear way. We consider the connected
system as a real-time type on-line dynamic system, see Martin [10],
that starts operating at a specific initial state for which
sufficient information is available, and which operates on an
equal length time interval of one year.   The selection of one
year (in a real-time computational cycle) is arbitrary and used
only for illustration purposes although parameter changes in
this model appear to have only longer term impacts.   Then the
the number of links to be estimated (pertaining to computational
complexity) amounts to the following set-up. The entire system
consists of altogether six 'modules', [POP], [EN], [ECO], [EC],
[G], [P], plus the external environment, exogeneous factors [EF].
Each module consists of at least five linking factors to other
modules, neglecting the exogeneous factors [EF] which are consi-
dered to be predetermined, the information of which is only di-
rectly accessible by the module concerned.   Each of these five
linking factors or parameters depend upon at most eight factors

in some other module, therefore indirectly involving [EF].

In the first case we refer to <u>direct links</u>, in the second to

<u>indirect links</u> of each of the modules involved.

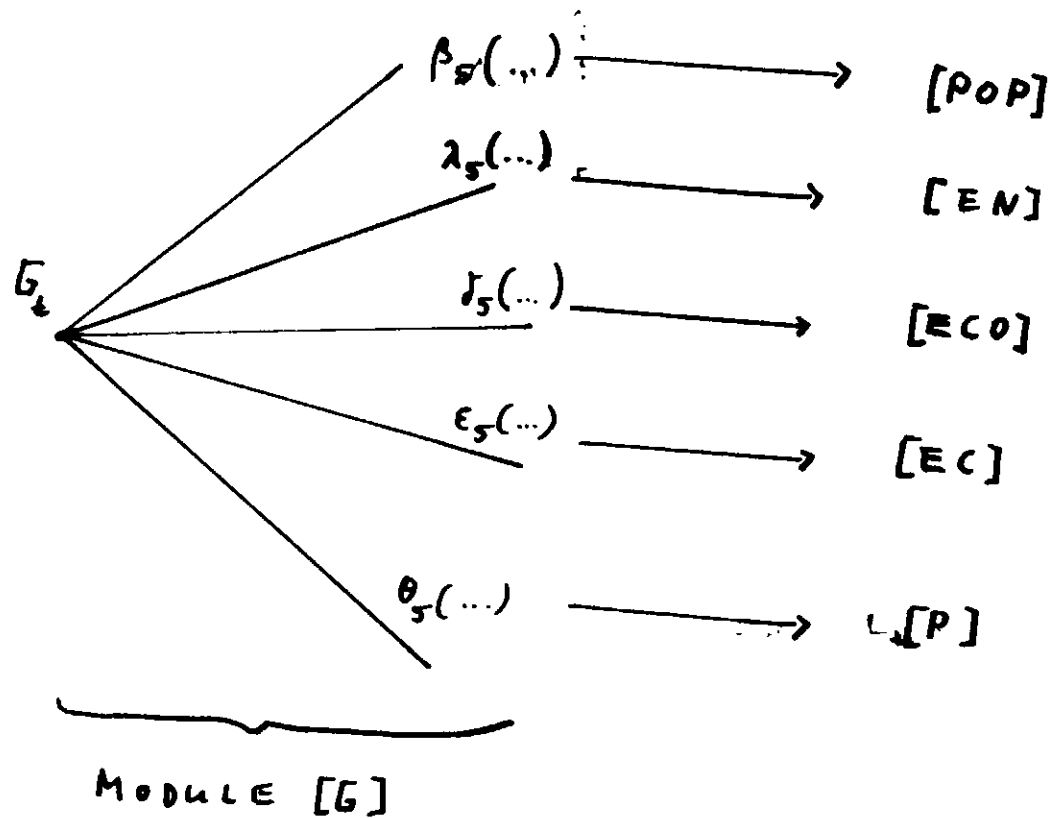For illustration look at the determination of G_t in Module [G]:

Fig. 4   Determination of G_t in Module [G] by direct links to other modules.

With six active decision-making modules operating the total number of structural parameters to be computed amounts to at least 240 structural parameters, e.g. five direct links given by the parameters of each module times eight indirect links for each such parameter in some other module times six, the number of interactive modules.

Now granting that at least 240 structural parameters be estimated, and suppose a response cycle of yearly data for a period of a ten year planning model is used, then in view of the non-linearities of the system interactions as the system operates through time and extends the length of indirect links, we can show, by complete enumeration, that we arrive at a total number of 240^{10} structural links which is a dramatic increase. In other words, the same number of structural parameters would have to be estimated in order to reach a control complexity that fits the design complexity of the entire system - a formidable task. (Still, it would be even more dramatic if we relate the response cycle to quarterly or even monthly data.) Now this holds for a problem-solving mechanism acting as a brute-force search, as built into large-scale computer programs, in which

(i)   the modules do not provide any structural complexity, i.e. no intrinsic problem-solving power as given by a special heuristic ,

(ii)   there is complete ignorance on the controller's side concerning the parameter variations in the dynamic process.

In other words, this corresponds to a centralized controller who perceives the modules simply as 'black boxes' with no self-steering capability and who at each real-time response cycle is ignorant about (or does not learn about) parameter variations that ensue.

In the alternative case the controller may activate the problem-solving power, heuristic search capabilities of the modules where each module, as a decentralized unit, by itself intrinsically computes all indirect links, thereby decreasing the computational burden of the controller. Hence the computational process reduces to direct links only, at least five for each module, so that for each response cycle 5 x 6 = 30 structural parameters have to be estimated as compared to 240. This amounts to 30^{10} structural links in a ten years period. In fact, the reduction of the number 240^{10} to 30^{10} could be assigned to the 'smartness' or 'structural complexity' of the modules, precisely the structural complexity amounts to $240^{10} - 30^{10}$, the residual in the reduction of computational complexity. Therefore, we see how a tradeoff between structural complexity and computational complexity evolves in such an interactive model.
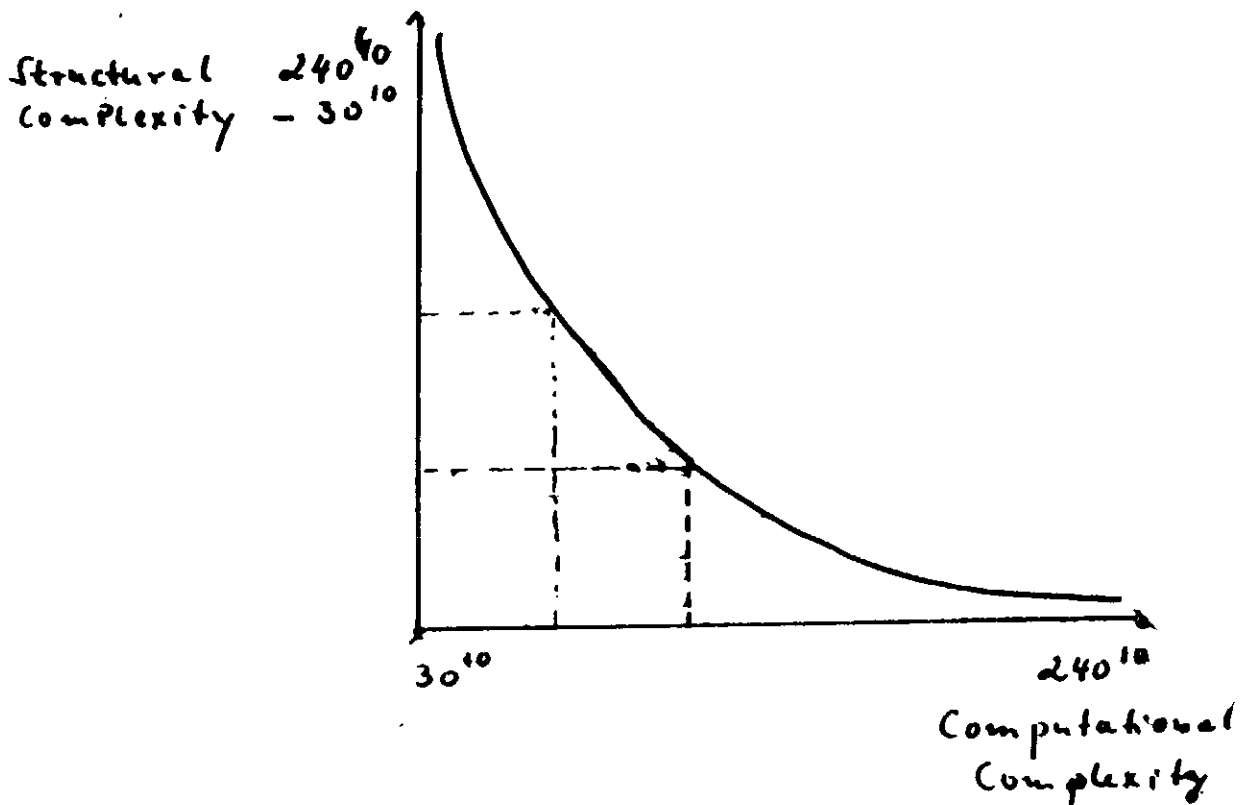

Fig. 5 Complexity Tradeoff

9. Conclusions

The most interesting aspect of this analysis for problem-solving methodologies pertains to the distinction between design and control complexity. If the design complexity of the system amounts to 240^{10} structural links, the only way to cope successfully with the control of such systems is to use a decomposition that makes most out of the intrinsic computational capabilities of the modules, their smartness or sophistication, much in the same way as chess playing programs become smarter to match master chess players by building into the programs some sophisticated heuristics.

This sheds new light on the question of centralization vs decentralization of decision-making in organizations, in particular in view of these results one could hardly hope to achieve rational centralized economic planning against market-type decentralized mechanisms (see F.A. Hayek [11]). At least the need for decentralization is now widely acknowledged in Soviet work on program-planning. As N.N. Moiseev and A.G. Schmidt [12] put it: 'it is not difficult to see that no matter how advanced the level of the techniques for data processing and data transmission may be, a certain level of decentralization in management will always be necessary.'

Furthermore, in general mixed-type economies, one can conclude that government regulation will easily find its limit of workability because of the computational burden that ensues.

Another point worth of consideration is that the analysis of structural connectedness had obvious implications for the chartist approach to projection. Since even in a system that is loosely connected in its static description, every parameter in

the system may ultimately depend upon every other parameter as the system operates through time. A simple extrapolation of trends which ignores these dependencies may indeed be highly misleading. If in a particular system it turns out that most of these dependencies (links) are sufficiently weak so that they can be safely ignored (according to the Ando-Fisher Theorem), the projections of the chartist who neglects these dependencies and the fundamentalist who takes them fully into account may be approximately the same. But even 'simple' systems which are only loosely connected in their static description are highly interconnected in their dynamic behavior.

Let's close with a methodological note.

Throughout this paper we attempted to answer the question how complexity appearing as a structural constraint on large-scale dynamic economic systems can be successfully handled from a controller's point of view. For answering this question it appears necessary to know what units of decomposition are most appropriate for designing and understanding complex systems. The sheer amount of computation, reflected by a measure of computational complexity, prohibits the decision-making or information-processing power of any human or artificial controller, yet to be designed. This indicates that by proper decomposition we should strive for mobilizing the problem-solving potentiality of systems components, that helps to reduce the computational burden of the controller. These systems components are supposed to activate useful heuristics at a local or decentralized level that involves sophistication, creativity, non-routine

problem-solving vs standard operating programs under repetitive situations as required in purely arithmetical tasks. It is reasonable to assume that market-type processes, e.g. market clearing functions, taking place in well-defined subunits, provide such a useful heuristic: they substitute structural for computational complexity. This is an issue that drives at the core of discussion of human problem-solving and the principles of bounded rationality as presented by March and Simon [13], ch. 7.

ADDENDUM

The results of this paper seem to contradict claims such as that price systems require essentially the same calculations as command systems or that command and price systems are informationally equivalent. According to S.A. Marglin ('Information in price and command systems of planning', in: J. Margolis and H. Guitton, eds., Public Economics, New York: MacMillan 1969, chapter 3, pp. 54-77) the information aspect is not essential for the distinction between price and command systems. Neglecting the incentive aspect, our analysis shows , on the other hand, that except for relatively small systems, in general, decentralized systems are more likely to cope with complex tasks than centralized systems. The reason appears to be that the market is a heuristic device, capable of searching for a 'good solution', whereas the centralized unit has to activate an algorithm that completely relies on an enumeration technique. A more differentiated viewpoint has been taken by H. Oniki ('The Cost of Communication in Economic Organization', Quarterly Journal of Economics 86, Nov. 1974, pp. 529-550) who argues that centralized economic systems may perform better, e.g. with lower cost of communication, if only a low degree of accuracy is required (measured by the 'allocation error' as a departure from the efficient allocation), and if the system is relatively small in the number of interacting units, variables, etc.. However, this applies only to tiny but not real economies so that - for all practicable purposes - command and price mechanisms appear not to be informationally equivalent, agreeing with our lines of reasoning. But we suggest even a stronger statement: not merely 'costs of communication' will be higher in a centralized system but beyond a certain threshold of complexity, centralized systems will not be able to control physically the economy without simultaneously achieving the same performance as a comparable decentralized system (of the same size). K.J. Arrow ('Limited Knowledge and Economic Analysis', American Economic Review 64, March 1974, pp. 1-10) in a presidential address to the American Economic Association reemphazises the information-economizing effect of decentralized systems. To quote him: 'But what was left obscure is a more definite measure of information and its costs, in terms of which it would be possible to affect superiority of the price system over a centralized alternative'.

This paper is an attempt to provide such a mesure, a measure of complexity.

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