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Fertilizers and Development of a
Dual Economy

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Fertilizers and Development of a Dual Economy.

I. Fertilizers and Sectoral Interdependence of Dual Economy:

In a recent paper [1,2,3] we studied the significance of natural fertilizer (produced and used by an agricultural economy) and chemical fertilizer for the development of a purely agricultural economy. In a neoclassical model we show that the input of fertilizers increases not only the net food production but also stimulates the long-run growth of the purely agricultural economy. In a purely agricultural economy chemical fertilizer should be imported.

In the literature of dual economy the sectoral relationship seems to be characterized by the aspect of consumer demand, or the so-called problem of marketable surplus, i.e. the agricultural products which are not consumed by the agricultural sector are exchanged with industrial products. The sectoral relationship in the aspect of production is generally neglected hitherto. Really, the production aspect of the sectoral relationship in a dual economy is not less important since the problem of economy development is mainly a problem of increasing productivity and production capacity.

Not only in the developed economics but also in the underdeveloped countries the contribution of chemical fertilizer for the increasing agricultural productivity is unquestionable. Hence the industrial sector could stimulate the agricultural productivity by delivery of chemical fertilizer. If this is done, then the net agricultural production for the agricultural sector is higher than the one without chemical fertilizer. Furthermore, the marketable surplus of the agricultural sector will be higher since not only the industrial product bought for consumption by the agricultural sector but also the chemical fertilizer should be paid by the agricultural products. Hence this production aspect of sectoral relationship has important effect on the development of a dual economy. In this study we shall consider this point explicitly.

In order to compare our results with those of Jorgenson and the classical dual model we shall use a neoclassical dual economic model of Jorgenson type modified by explicit treatment of chemical fertilizer as a production factor for the agricultural sector. In a closed dual economy without international economic relations chemical fertilizer has to be produced by the industrial sector. We shall assume for simplicity that the product of the industrial sector is an aggregate which could be used for consumption, investment as well as fertilizer for the agricultural production. The production of the agricultural sector will be higher if more fertilizers are employed. Hence more labor force can be employed in the industrial sector if more fertilizers are used in the agricultural sector.

Throughout this study the constant quantity of land in the dual economy is assumed. Therefore, only two factors, i.e. labor and fertilizer, are treated for the agricultural production.

Furthermore, we assume that the development of the underlining purely agricultural economic system is characterized by a steady growth equilibrium and population is growing at its physiological maximum rate. In a steady growing agricultural economic system agricultural surplus per agricultural labor force (s) comes into being. The agricultural surplus (s) is defined by

$$s = y - y^+$$

where y is the agricultural production per agricultural labor force and y^+ is the level of output of food necessary to bring about the physiological maximum growth rate of population.

In the neoclassical dual economy of Jorgenson labor may be free from the land at a rate which is sufficient to absorb the agricultural surplus [6,7]. This assumption implies:

$$1 > \frac{A}{P} = \frac{y^+}{y} > 0$$

where A is agricultural employment. P is the labor force of the dual economy.

In Section II we present a neoclassical dual model with an modified agricultural production function in which chemical fertilizer is considered explicitly.

In Section III we study the development of a labor surplus economy in sense of Fei-Ranis [5].

In Section IV we study the development of a dual economy with endogenous wage rate.

In both sections II and IV the significance of use of chemical fertilizer are shown by comparing our results with those of Fei-Ranis model and Jorgenson model.

II. The Model :

As usually, in the literature the dual economy is characterized by an agricultural and an industrial sector. The agricultural production is described by a production function of modified Cobb-Douglas type as following: ¹⁾

$$(1) \quad Y = e^{\lambda t} (1+F)^{\alpha} A^{\beta} \quad \text{with } \lambda, \alpha, \beta > 0 \text{ and } \alpha + \beta \leq 1$$

where Y is the agricultural (food) output. F is the input of chemical fertilizer and A is the labor input; λ is the rate of neutral technical progress; α and β are elasticities of output.

In a purely agricultural economy without international trade as well as in a closed dual economy in which chemical fertilizer cannot be produced no input of chemical fertilizer is possible. In these cases F is equal to zero. These cases are considered hitherto in the literature [4,5,6,7,8]. In the dual economy considered the agricultural sector is assumed to be built up by farms which use their own land and labor force in the production. Chemical fertilizer is the only strange factor of pro-

1) The quantity of land is assumed constant and neglected here.

duction used in the agricultural sector. In assuming that the farms try to maximize their income the input of fertilizer can be described by the following equation:

$$(2) \quad F = \alpha q^{-1} y^{-1}$$

where q is the price of chemical fertilizer or industrial product in terms of agricultural product (i.e. terms of trade).

The growth rate of population (\hat{P}) which is assumed to be identified by the labor force is exogenous equal to

$$(3) \quad \hat{P} = \varepsilon > 0 \quad \text{where} \quad \hat{P} = \frac{dP}{dt} \frac{1}{P} = \frac{\dot{P}}{P}$$

As in the Jorgenson model [6] the food demand is simply described by

$$(4) \quad Y = P y^+$$

where y^+ is the given per capita food demand.

The agricultural sector is now completely described.

The industrial production is characterized by a production function of Cobb-Douglas type:

$$(5) \quad X = e^{at} K^b M^{1-b}; \quad a, b > 0$$

where X is the industrial output. K is the input of capital stock and M is the labor input.

Under the assumption of profit maximization the input of industrial labor is given by:

$$(6) \quad M = \frac{(1-b)qX}{w}$$

where w is the wage rate in terms of agricultural product.

Following Jorgenson [6] the income of industrial labor and the income of agricultural sector are used only for consumption. The investment is therefore equal to the capitalist income of the industrial sector:

$$(7) \quad \dot{K} = I = bX$$

where I is the investment in the industrial sector. For simplification the depreciation of capital will be neglected. The labor force is allocated either in the agricultural or in the industrial sector, i.e.

$$(8) \quad P = M + A$$

In the dual economic model with surplus labor (classical model) A is just the labor force which is not employed in the industrial sector. In the neoclassical dual economic model (8) is an equilibrium condition for the labor market.

The national income of the dual economy considered (measured in units of agricultural product) is:

$$(9) \quad Y - qF + qX = (1-\alpha)Y + q(X+I)$$

The saving in terms of agricultural products is equal to:

$$(10) \quad Y - qF + qX - q(1-b)X - (Y-qF) = bqX$$

Our dual model is now completely described.

Made use of (1) and (2) we have:

$$(11) \quad Y = \left[e^{-\lambda t} (\alpha q^{-1})^\alpha A^\beta \right]^{\frac{1}{1-\alpha}}$$

From (4) and (11) we have the following relation between the terms of trade and the labor input in the agricultural sector:

$$(12) \quad A = \left\{ e^{[(1-\alpha)\epsilon - \lambda]t} \alpha^{-\alpha} [y^+ P(0)]^{1-\alpha} q^\alpha \right\}^{\frac{1}{\beta}}$$

This shows that the more labor will be put in the agricultural production the higher is the price of chemical fertilizer since chemical fertilizer can be substituted by labor force.

Now the origin of time is taken as the point of the emergence of the industrial sector. Hence

$$(13) \quad P_0 y^+ = P_0^\beta \quad \text{or} \quad y^+ = P_0^{\beta-1}$$

In consideration of (13) the labor input in the agricultural

sector is given by:

$$(14) \quad A = A_0^{1-\alpha} \left\{ e^{[(1-\alpha)\epsilon-\lambda]t} \alpha^{-\alpha} \right\}^{\frac{1}{\beta}}$$

where the fact $A_0 = P_0$ is taken into account.

III. Development of a Labor Surplus Economy:

In this section we shall treat a dual economy in which the marginal productivity of agricultural labor is less than the institutional wage rate. This is a case of Fei-Ranis-Lewis labor surplus economy. Throughout this section we shall follow the tradition of the classical dual-economic model that the wage rate is given institutionally. (We shall not consider the case of zero marginal productivity of agricultural labor of Fei-Ranis since the main results of this section can be carried over without much correction.)

At the beginning we shall describe the short-run solution of our model. In the short-run the investment has no effect on capital stock and the population is given.

Made use of (6), (8) and (14) we have:

$$(15) \quad K - [(1-b)w^{-1}e^{at}q]^{-\frac{1}{b}} [P - A_0^{1-\alpha} e^{\frac{(1-\alpha)\epsilon-\lambda}{\beta}t} \alpha^{-\frac{\alpha}{\beta}} q^{\frac{\alpha}{\beta}}] = 0$$

Since the last term of (15) represents just the labor employed in the industrial sector which is always positive, except at the time point $t = 0$, therefore the capital stock is positive for $t > 0$, or the critical minimum effort criterion is permanently violated, i.e. if $\bar{M} < \epsilon$ ²⁾. From (15) the terms of trade can be solved.

The short-run solution of our model can be shown in Fig.1. Since the capital stock of the industrial sector and the total labor forces of the dual economy are given, the industrial output and employment can be determined with isoquant of the industrial sector as shown in the quadrant (a) of Fig. 1.

2) If this is the case the industrial sector will disappear in the long-run.

Fig. 1

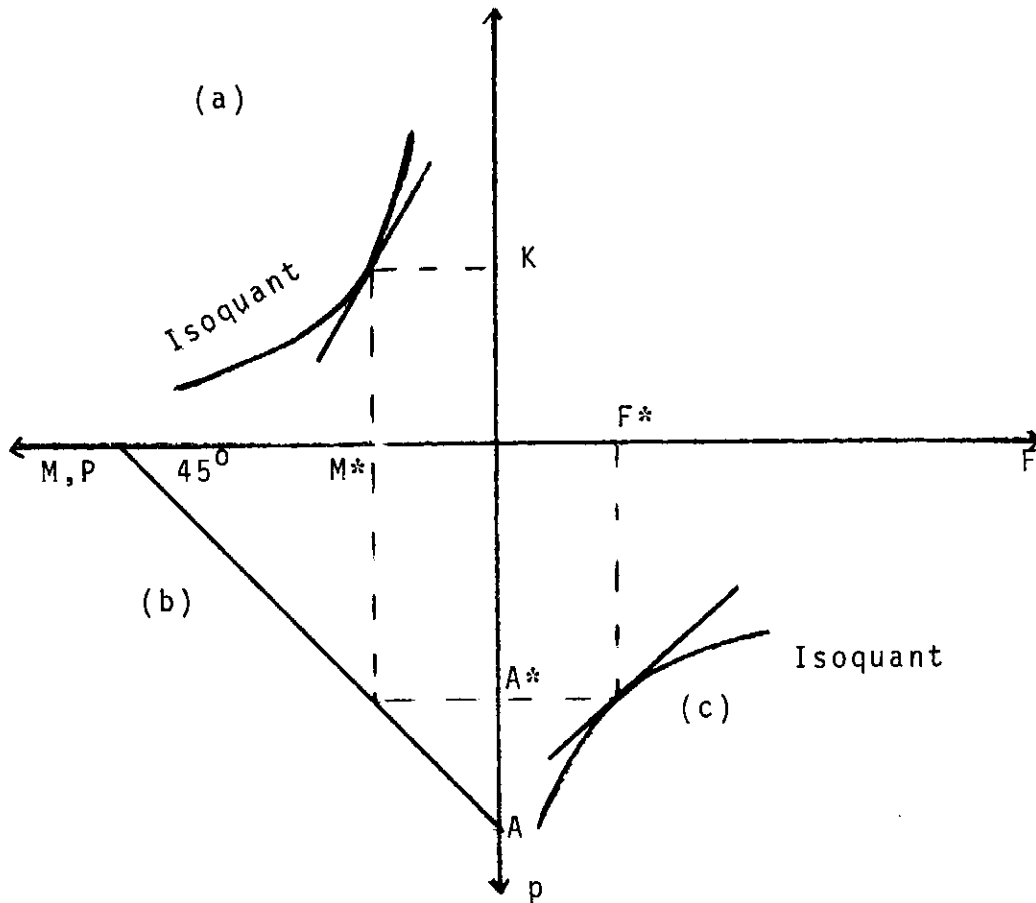


Fig. 1

Using a 45°-line in the quadrant (b) to represent the population we can determine the allocation of labor in the dual economy after the employment of the industrial sector is known. At last, we can determine the demand for chemical fertilizer and the agricultural output with known agricultural employment and the price of agricultural product with the tool of isoquant as shown in quadrant (c).

The investment can be determined also since the industrial output is known. Other variables of the model can be easily solved.

$$(16) \quad X = e^{\frac{a}{b}t} [(1-b)q w^{-1}]^{\frac{1-b}{b}} K$$

$$(17) \quad M = [(1-b) w^{-1} e^{at} q]^{\frac{1}{b}} K$$

$$(18) \quad \dot{K} = I = b e^{\frac{a}{b}t} [(1-b)w^{-1}q]^{\frac{1-b}{b}} K$$

The long-run properties or the pattern of development of the labor surplus economy can be described by the following points:

(i) The development of the average productivity of industrial labor:

$$(19) \quad \hat{X} - \hat{M} = -\hat{q}$$

This is due to the reason that more labor intensive technique of production will be employed if the terms of trade improve for the industrial product since the real wage rate in terms of industrial product reduces.

(ii) The development of capital intensity:

$$(20) \quad \hat{K} - \hat{M} = -\frac{a}{b} - \frac{1}{b} \hat{q}$$

As in the classical model the capital intensity falls during the development process. In our model the capital intensity falls more sharply if the terms of trade turn in favor of the industrial product as explained above.

(iii) The development of capital output ratio:

$$(21) \quad \hat{X} - \hat{K} = \frac{a}{b} + \frac{(1-b)}{b} \hat{q}$$

The capital output ratio decreases during the process of economic development as in the classical model. But in our model the decreasing rate of the capital output ratio is higher than the growth rate of the terms of trade in favor of industrial product.

These properties of development of a labor surplus economy are identical to those of the classical dual economic model if the terms of trade remain constant during the process of economic development.

The development pattern of the classical model is mainly due to the constant real wage rate. If the wage rate is given in terms of industrial product the real wage rate paid to the industrial labor does not change during the development. In this case, the development pattern of the industrial sector in our model will be identical to those of the classical model.

The critical minimum effort criterion in our model is:

$$(22) \quad \hat{M} - \epsilon = \frac{a}{b} + \frac{1}{b} \hat{q} + \hat{K} - \epsilon > 0$$

This condition can be fulfilled if the terms of trade do not deteriorates against the industrial product since the growth rate of capital seems to increase during the development:

$$(23) \quad \frac{d\hat{K}}{d\hat{t}} \frac{1}{\hat{K}} = \frac{a}{b} + \frac{1-b}{b} \hat{q}$$

Hence the critical minimum effort criterion will be fulfilled (even if it is not realized at the beginning period of development) if the terms of trade do not deteriorate against the industrial product.

From the above discussion we find the importance of the terms of trade for the dual economy considered. It is not easy to solve the long-run development of the terms of trade. Therefore, we shall consider some important points of the development of the terms of trade.

At first, we know that chemical fertilizer will be used for agricultural production if it is advantageous for the farms to do so, i.e. if

$$(24) \quad Y - qF > e^{\lambda t} A^{\beta} \quad \text{with } F > 0$$

If this condition holds then the input of chemical fertilizer will increase the agricultural income. This condition is fulfilled if

$$(Y - qF)(e^{\lambda t} A^{\beta})^{-1} = (1 - \alpha) Y \frac{1}{1 - \alpha} + q e^{-\lambda t} A^{-\beta} > 1$$

or if

$$(25) \quad \alpha(1 - \alpha) \frac{1 - \alpha}{\alpha} e^{\lambda t} A^{\beta} > q$$

Thus chemical fertilizer will be used for the agricultural production if the price of chemical fertilizer (price of industrial product) is not too high.

In the following discussion we shall assume that the condition (25) and thus (24) are always fulfilled. Therefore, chemical fertilizer is used in the agricultural sector. Really, this will be the case since the higher the price of chemical fertilizer the more labor will be used in the agricultural production.

Because of the starr demand function for food assumed in our model the agricultural production should always be equal to $P_0 e^{\epsilon t}$. From the isoquant the following condition has to be hold:

$$(26) \quad dA = \frac{1}{\beta} \frac{A}{q} dq \quad \text{or} \quad \frac{dA}{A} \frac{q}{A} = \frac{1}{\beta} > 0 \quad \text{for } 1 > \beta > 0$$

Due to use of chemical fertilizer for the agricultural production some labor forces used in the agricultural sector can be set free for the industrial production because of the starr demand function for food. In order these labor forces can be employed in industrial sector, the real wage (in terms of industrial product) must fall. Since the institutional wage rate is fixed in terms of agricultural product, the terms of trade must improve for the industrial product in order to increase the industrial employment.

This can be easily seen from (17)

$$(27) \quad \frac{\partial M}{\partial q} \frac{q}{M} = \frac{1}{b} > 1 \quad \text{since } 1 > b > 0$$

Thus the demand of the industrial sector for labor employment is elastic with respect to the real wage rate. From (26) we find that the demand of the agricultural sector for labor is also elastic with respect to the price of industrial product. β is the elasticity of agricultural output with respect to the labor input and b is the elasticity of industrial output with respect to capital input. For reasonable values of these two parameters we could accept that $\beta > b$ and that the elasticity of labor demand of the industrial sector with respect to the price of industrial product is higher than that of the agricultural sector. Hence for an increasing price of industrial product the increase in the industrial labor forces will be higher than that of the agricultural employment. Thus we show the following lemma:

" An improvement of the terms of trade for the industrial product will influence the employment structure of the labor surplus economy considered positively, i.e. the

percentage of the industrial employment will increase. "

In the labor surplus economy treated here the use of chemical fertilizer will set a part of agricultural labor free from the agricultural production. In order these labor forces could be employed in the industrial sector the terms of trade has to be improved for the industrial product and analogously the real wage rate in terms of industrial product has to be decreased. This increase in the price of industrial product will have an indirect positive effect on the demand for labor of both sectors of the labor surplus economy considered. But because of the higher elasticity of the demand for the industrial labor force the employment structure will finally be improved. Thus we could conclude

Conclusion 1: " In a labor surplus economy the employment structure will be improved by use of chemical fertilizer in the agricultural production. "

Following the fact that chemical fertilizer is used in the agricultural production the national income of the labor surplus economy considered according to (9) is equal to

$$Y - qF + qX$$

which is higher than the national income of the same economy if chemical fertilizer is not used in the agricultural production. In this last case, the agricultural production is equal to $e^{\lambda t} A^{\beta}$ which would be lower than $Y - qF$ if the same quantity of labor is used. But because of the starr demand function for food assumed in our model the quantity of labor input is lower if chemical fertilizer is used in the agricultural production. These labor forces set free from the agricultural production will be employed in the industrial sector. Since the labor productivity in the industrial sector is higher than that in the agricultural sector. Therefore the total labor income will be higher. Thus we arrive the following conclusion

Conclusion 2: " In a labor surplus economy the national income will be higher if chemical fertilizer is used in the agricultural production. "

Due to the fact that the agricultural production is equal to the rigid food demand. The increase in national income is contributed mainly by the increase in the industrial production and partly due to the improvement of the terms of trade for the industrial product. Therefore, investment in the industrial sector will be higher since the capitalistic income which is used for investment is also increased.

Conclusion 3: "The amount of investment in a labor surplus economy will be higher if chemical fertilizer is used in the agricultural production. "

In the discussion on development policy it is often argued that the low wage rate policy is a favor for the economic development. In our study we show that the improvement of the terms of trade for the industrial product and, therefore, a decrease in the real wage rate in terms of industrial products would improve the employment structure. But unlike the discussion hitherto we find out a limitation for the price of industrial product. ³⁾ Surely, we should be careful to interpretate our findings. We would point out that in our model the industrial product is considered as an aggregate. A distinction between chemical fertilizer and other industrial products is not made in our study here.

In the long-run chemical fertilizer will be used for the agricultural production if

$$(28) \quad \lambda + \epsilon > \bar{q}$$

The growth rate of food demand is identical to that of labor force (ϵ). Since the rate of technological innovation (λ) is assumed to be positive, an increase in the terms of trade for industrial product is possible for a successful development of a labor surplus economy. As in the proposition 1 above shown an improvement of the terms of trade for the industrial product during the development process will accelerate the im-

3) Above this chemical fertilizer will not be used in the agricultural production.

provement of the employment structure of the labor surplus economy and arrive earlier the turning point to the phase of full commercialization. In the classical dual economic model with given institutional wage rate the terms of trade remain constant [7] . In our model the price of industrial product would be higher than that of the classical model in which chemical fertilizer is not used in the agricultural production. It is easy to see that if chemical fertilizer is not used for the agricultural production the growth rate of agricultural labor force in order to satisfy the food demand of the growing population is equal to

$$(29) \quad \hat{A} = \frac{\epsilon - \lambda}{\beta}$$

But if chemical fertilizer is used this growth rate will be equal to

$$(30) \quad \hat{A} = \frac{(1-\alpha)\epsilon - \lambda}{\beta} \quad \text{for } \hat{q} = 0, \text{ i.e.}$$

if the terms of trade remain constant. It is easy to see that the growth rate in the latter case is lower than that in the first case.

If the price of fertilizer increases with a growth rate of the population then the growth rate of agricultural labor force in order to satisfy the food demand of the growing population is identical in both case, i.e.

$$(31) \quad \hat{A} = \frac{\epsilon - \lambda}{\beta} \quad \text{for } \hat{q} = \epsilon$$

It is interesting to see that the growth rate of capital stock and of the industrial employment will increase during the development if the terms of trade remain constant.

$$(32) \quad \frac{d\hat{M}}{dt} \frac{1}{\hat{M}} = \frac{d\hat{K}}{dt} \frac{1}{\hat{K}} = \frac{a}{b} > 0 \quad \text{if } \hat{q} = 0$$

From the above discussion we arrive the following conclusion

Conclusion 4: "The turning point to the full commercialization of a labor surplus economy will be arrived earlier if chemical fertilizer is

used for the agricultural production with constant terms of trade. "

To study the dynamic property of our labor surplus economy we transform now the fundamental differential equation in terms of capital intensity as follows:

$$(33) \quad \dot{k} = z_1 b e^{at} k^b - z_2 k$$

$$\text{where } z_1 = \frac{\beta}{\alpha b + \beta} \text{ with } 1 > z_1 > 0, \quad k = \frac{K}{M}$$

$$z_2 = z_1 \left\{ 1 + \frac{l_1 [a\alpha + \lambda - \epsilon(1-\alpha)]}{\beta} \right\}$$

$l = \frac{P}{M}$ and $l_1 = \frac{A}{M}$ (both are calculated with values at the initial equilibrium)

Clearly $l > l_1 > 0$.

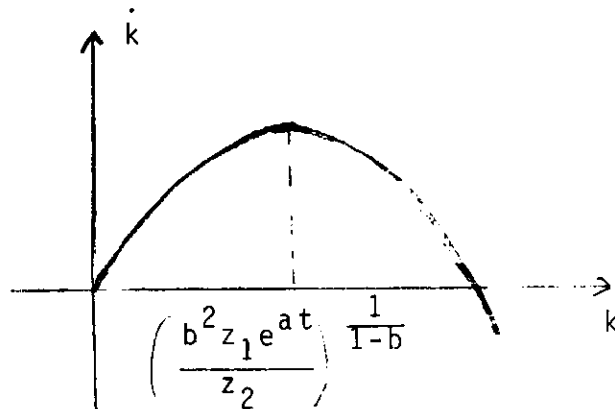
Because of the assumption that the underlying agricultural sub-model has a steady growth equilibrium z_2 is always positive.

The phase diagram of (33) can be shown as Fig. 2. The not trivial stationary solution is:

$$(34) \quad k^* = (z_1 b e^{at} z_2^{-1})^{\frac{1}{1-b}}$$

\dot{k} increases up to $(b^2 z_1 e^{at} z_2^{-1})^{\frac{1}{1-b}}$

arrives maximum at there and decreases after there.



To solve the differential equation (33) we transform it in terms of $K = v \frac{1}{b-1}$ as follows:

$$(35) \quad \dot{v} = -z_2 v + (1-b) z_1 b e^{at}$$

This is a linear differential equation with non-constant coefficients.

We shall not solve the differential equation explicitly. But in order to compare with Fei-Ranis model we give the fundamental differential equation for a labor surplus economy without chemical fertilizer as follows:

$$(36) \quad \dot{k} = b e^{at} k^b - z_3 k \quad \text{with } z_3 = \frac{\beta \epsilon + 1 + \lambda - \epsilon}{\beta} > 0$$

The non-trivial stationary solution of this differential equation is:

$$(37) \quad k^{**} = (b e^{at} z_3^{-1})^{\frac{1}{1-b}}$$

To compare (34) and (37) we find:

$$(38) \quad \frac{z_2}{z_1} - z_3 = \alpha \lambda_1 (\alpha + \epsilon) \beta^{-1} > 0.$$

Hence we could conclude this explanation as follows:

Conclusion 5: "In the stationary solution the capital intensity of the industrial sector in a labor surplus economy is lower if chemical fertilizer is used for the agricultural production. "

Thus we can expect the capital intensity of the industrial sector in a labor surplus economy will be lower if chemical fertilizer is introduced in the economy.

IV. Development of a dual economy with endogenous wage rate:

In this section we shall treat the wage rate as an endogenous variable. In expecting that agricultural workers respond to wage differentials between industry and agriculture only if industrial wages are greater than agricultural income, where agricultural income includes both wages and rent, we assume:

$$(39) \quad \sigma w = \frac{(1-\alpha)Y}{A} = (1-\alpha)y, \quad \sigma > 1$$

where σ denotes the ratio between the agricultural income per man and the industrial wage rate (per man).⁴ The function (1) to (10) and (39) characterize our model of a dual economy with endogenous wage rate. In comparison to Jorgenson [6] the wage rate and the price of industrial product in our model are given in terms of agricultural product. This will not influence the results considerably since both the wage rate and the terms of trade are endogenous both in our and in Jorgenson Model. This and the explicit consideration of chemical fertilizer as a production factor in the agricultural sector characterize the differences between our model and the neoclassical dual model of Jorgenson.

From (6) and (39) we have:

$$(40) \quad w = \frac{(1-b)qx}{M} = (1-b)qx = \sigma(1-\alpha)y \quad \text{and}$$

$$(41) \quad q = \frac{\sigma(1-\alpha)y}{(1-b)x}$$

With the given capital stock and population the model can be solved for the short-run. Other than the surplus labor economy the short-run solution has to be determined simultaneously for the wage rate and the terms of trade. The solution can be described similarly as in Fig. 1.

To solve the model in the short-run we introduce the following variables: (a) capital intensity in the industrial sector $k = \frac{K}{M}$ and (b) capital per man of the dual economy

$$\bar{k} = \frac{K}{P}$$

In the short-run \bar{k} is known. Made use of (8) we have

$$(42) \quad A = P(k-\bar{k})/k$$

since $k-\bar{k} = Ak/P$.

From (11) and (42) we find:

4) Correctly $\sigma w = \frac{(1-\alpha)Y+q}{A}$. To simplify our presentation we shall use $\sigma w = (1-\alpha)y$ where $y = \frac{Y}{A}$.

$$(43) \quad y = \frac{Y}{A} = \{ e^{\lambda t} (\alpha q^{-1})^\alpha [P(k-\bar{k})k^{-1}]^{\alpha+\beta-1} \}^{\frac{1}{1-\alpha}}$$

Substituting $x = e^{at} k^b$ and (43) in (41)

$$(44) \quad \ln q = (1-\alpha) \ln \frac{\sigma(1-\alpha)}{1-b} - \alpha \ln \alpha + [\beta \epsilon + \lambda - (1-\alpha)(a+\epsilon)] t + (\alpha+\beta-1) \ln A_0 \\ + (\alpha+\beta-1) \ln(k-\bar{k}) + [(1-\alpha)(1-b)-\beta] \ln k$$

Made use of (14) and (42) we have

$$(45) \quad \ln q = \ln \alpha A_0^\beta + \frac{(\alpha+\beta-1)\epsilon+\lambda}{\alpha} t + \frac{\beta}{\alpha} [\ln(k-\bar{k}) - \ln k]$$

Since in the short-run \bar{k} is given we can solve the equation system of (44) and (45) for q and k . Thus we can substitute q^* and k^* to calculate other short-run equilibrium values of our model. The equation (44) has negative slope for the agricultural production function has "no increasing returns to scale":

$$\frac{d \log q}{dk} = \frac{1}{q} \frac{1}{k(k-\bar{k})} \{ \bar{k}[(\alpha+\beta-1)+(1-\alpha)b] - k(1-\alpha)b \} < 0 \quad \text{since}$$

$\alpha+\beta \leq 1$ and $k > \bar{k}$.

Otherwise the equation (45) has positive slope:

$$\frac{d \log q}{dk} = \frac{1}{q} \frac{\beta}{k(k-\bar{k})\alpha} \bar{k} > 0$$

Hence the equation system (44) and (45) has unique solution:

$$(46) \quad \ln k = \frac{\alpha+\beta}{\alpha(1-b)-\beta} \ln(k-\bar{k}) - \frac{\alpha}{[\alpha(1-b)-\beta](1-\alpha)} \{ \ln \alpha A_0^\beta - (1-\alpha) \ln \frac{\sigma(1-\alpha)}{1-b} \\ + \alpha \ln \alpha - (\alpha+\beta-1) \ln A_0 + \frac{1}{\alpha} [(a-1)\epsilon + (\beta+\alpha)(1-\alpha)\epsilon - (2\alpha-\beta-1)\lambda + \\ \alpha(1-\alpha)a] t \}$$

The solution of our model with endogenous wage rate can be described similarly as that of the model of the labor surplus economy.

To study the development of a dual economy with endogenous wage rate (neoclassical model) where chemical fertilizer is consider-

ed as input for the agricultural production, we assume that the condition (25) is fulfilled.

If chemical fertilizer is introduced in the neoclassical model of Jorgenson the productivity of agricultural labor increases. Since the demand of the dual economy for food is rigid an increase in productivity of agricultural labor has two immediate impacts on the dual economy: (i) the wage rate of the dual economy will be raised because the wage rate is proportional to the agricultural income per man and (ii) the terms of trade will be improved for the industrial product. The final effects of introducing chemical fertilizer in the agricultural production on the neoclassical dual economy depend on its influence on the real wage rate (in terms of industrial product), i.e. wq^{-1} .

As in the labor surplus model some agricultural labor forces can be set free for industrial production. In order the labor forces which are set free from agricultural production would be employed in the industrial sector the real wage rate (in terms of industrial product) has to decrease since in the short-run the capital stock is given. The decrease in the real wage rate is realized both by an increase in the wage rate in terms of agricultural product because of higher productivity of the agricultural labor and by an improvement in the terms of trade for the industrial product because of the rigid demand function for agricultural product.

In order to realize a decrease in the real wage rate in terms of industrial product the rate of improvement in the terms of trade for the industrial product should be higher than the rate of increase in wage rate (in terms of agricultural product). Really, this will be the case since the demand of the dual economy for agricultural product is perfectly inelastic though the increase in the agricultural product will be compensated by the improvement of terms of trade for the industrial product.

As in the labor surplus economy the demand for agricultural labor is elastic with respect to the price of industrial product. But the elasticity of demand for industrial labor with

respect to real wage rate is higher than that of demand for agricultural labor with respect to the price of industrial product. Hence the employment structure will be improved by introducing chemical fertilizer in the neoclassical dual economy.

Because of the assumption that the productivity of the industrial labor is higher than that of the agricultural labor an improvement in the employment structure the national income will be higher.

From these considerations we see the analogy of the labor surplus economy and the neoclassical dual economy. The propositions 1 to 3 in the last section can be carried over to the neoclassical model in this section.

To study the long-run properties of our model with endogenous wage rate made use the assumption $\hat{Y} = \epsilon$ and the equation (11) we have the growth rate of agricultural labor in order to satisfy the growing food demand due to the growth of population:

$$(47) \quad \hat{A} = \frac{\epsilon(1-\alpha)-\gamma}{\beta} + \frac{\alpha}{\beta} \hat{q}$$

and the growth of the productivity per agricultural labor which is equal to the growth rate of wage rate in terms of agricultural product:

$$(48) \quad \hat{y} = \frac{\lambda - (1-\alpha-\beta)\epsilon}{\beta} - \frac{\alpha}{\beta} \hat{q}$$

Hence with regard to the rigid food demand of the dual economy considered the growth rate of the agricultural productivity per agricultural labor is a decreasing function of the growth rate of the price for industrial product. Recalling the fact that the existence of a steady growth equilibrium of purely agricultural economy is required for the rise of a dual economy the first term on the left-hand side of (48) is positive. Hence the productivity per agricultural labor will increase if the terms of trade remain constant. Thus the wage rate in terms of agricultural product increases during the development of the

dual economy if the terms of trade do not improve for the industrial product.

Made use of (16), (40) and (48) we can show (following long-run properties of our model with endogenous wage rate):

$$(49) \quad \widehat{X-K} = \frac{a\beta - (1-b)[\lambda - (1-\alpha-\beta)\epsilon]}{b\beta} + \frac{(1-b)(\beta+\alpha)}{b\beta} \widehat{q}$$

$$(50) \quad \widehat{K-M} = \frac{\lambda - (1-\alpha-\beta)\epsilon - a\beta}{b\beta} - \frac{1}{b} \frac{\alpha+\beta}{\beta} \widehat{q}$$

$$(51) \quad \widehat{X-M} = \frac{\lambda - (1-\alpha-\beta)\epsilon}{\beta} - \frac{\alpha+\beta}{\beta} \widehat{q}$$

In comparison to (19), (20) and (21) we find out that the development of the output capital ratio ($\widehat{x} - \widehat{k}$) in the model with endogenous wage rate is less clear cut if the terms of trade are constant during the development. In (45) the sign of the first term on the right-hand side is not definite. The development of the capital intensity in the model considered here is also dubious. The industrial production per industrial labor increases if the terms of trade are constant during the development. This is the only clear development of (49) to (51) in our model considered in this section.

In comparison to the labor surplus economy we find that the influences of the development of the terms of trade in the model with endogenous wage rate are significantly higher than in the labor surplus economy case. To compare with the Jorgenson-Model [6] we find that the influences of the development of terms of trade on the long-run properties in our model are also higher than in the Jorgenson-model ⁵⁾.

In the Jorgenson-model we calculate the growth rate of agricultural labor in order to satisfy the growing food demand due to the growth of population as follows:

$$(52) \quad \widehat{A} = \frac{\epsilon - \lambda}{\beta}$$

Made use of (52) and the agricultural production function for

5) In the Jorgenson-model the wage rate and the terms of trade are measured in terms of industrial products. In our model both the wage rate and the terms of trade are measured in terms of agricultural product.

F = 0 we find that

$$(53) \quad \hat{w} = \hat{y} = \frac{\lambda - \epsilon(1-\beta)}{\beta}$$

Due to the assumption of a steady growth equilibrium of the purely agricultural (sub-) model the growth rate of the real wage rate in terms of agricultural product is positive. Made use of (53) and (16) to (18) we have:

$$(54) \quad \hat{X} - \hat{K} = \frac{a\beta - (1-b)[\lambda - \epsilon(1-\beta)]}{b\beta} + \frac{1-b}{b} \hat{q}$$

$$(55) \quad \hat{K} - \hat{M} = \frac{\lambda - (1-\beta)\epsilon - a\beta}{b\beta} - \frac{1}{b} \hat{q}$$

$$(56) \quad \hat{X} - \hat{M} = \frac{\lambda - \epsilon(1-\beta)}{\beta} - \hat{q}$$

Now assuming that the terms of trade remain constant during the development we can show the following conclusion by comparison of (49) to (51) with (54) to (56):

Conclusion 6: " For constant terms of trade during the development in our model

a) the output-capital ratio is smaller

b) the capital-intensity is higher and

c) the average productivity of the industrial labor is higher

if chemical fertilizer is used for the agricultural production

than in Jorgenson model [7]."

The fundamental differential equation of Jorgenson-model can be transformed in a differential equation of capital-intensity of the industrial sector as follows:

$$(57) \quad \dot{k} = be^{at} k^b - z_3 k$$

$$\text{when } z_3 = \frac{\beta\epsilon + 1 - \lambda}{\beta}$$

Note (57) is identical to (36).

The fundamental differential equation of our model with endogenous wage rate can be written in capital-intensity as follows:

$$(58) \quad \dot{k} = z_4 b e^{at} k^b - z_5 k$$

$$\text{where } z_4 = \frac{(\alpha+\beta)}{\alpha+\beta+\alpha l_1 b} > 0$$

$$z_5 = z_4 \frac{(\alpha+\beta)\{\beta\epsilon l_1 + l_1[\lambda - \epsilon(1-\alpha)]\} + \alpha l_1 [a\beta - \lambda(1-\alpha-\beta)\epsilon]}{(\alpha+\beta)\beta}$$

and z_5 is expected to be positive.

The non-trivial stationary solution for (58) is equal to:

$$(59) \quad k^+ = (z_4 b e^{at} z_5^{-1})^{\frac{1}{1-b}}$$

$$z_5 z_4^{-1} - z_3 = \frac{l_1 \alpha \{[(1+\lambda)(\alpha+\beta) - \lambda]\epsilon + a\beta\}}{(\alpha+\beta)\beta} > 0$$

Hence we have shown the following point:

Conclusion 7 : " In a neoclassical dual economy (i.e. with endogenous wage rate) the capital-intensity of the industrial sector in the stationary solution is lower if chemical fertilizer is used for agricultural production. "

At last, we assume the not trivial stationary solutions (34) and (59):

$$(60) \quad z_5 z_4^{-1} - z_2 z_1^{-1} = -\alpha l_1 [\lambda(1-\alpha-\beta)\epsilon + a] / [\beta(\alpha+\beta)] < 0$$

This can be concluded as follows:

Conclusion 8: " If chemical fertilizer is used for the agricultural production then the capital-intensity of the industrial sector in the stationary solution is higher in the dual model with endogenous wage rate than in the labor surplus economy."

V. Summary:

In this paper the chemical fertilizer is considered explicitly as a production factor for the agriculture. In a modified neo-classical dual model we study the significance of the chemical fertilizer for the economic development. Chemical fertilizer is produced in the industrial sector. No distinction between the chemical fertilizer and other industrial products is assumed in this paper.

In order to compare the results of our model with those of Jorgenson and Fei-Ranis in which no fertilizer is considered the Jorgenson model is used in this study with a small correction. In our model the chemical fertilizer is considered explicitly as a production factor in the agricultural sector.

In a later surplus economy we find out:

- (a) the employment structure will be improved,
 - (b) the national income and the per capita income is higher and
 - (c) the amount of investment is higher
- if chemical fertilizer is used for the agricultural production.

In a labor surplus economy the turning point to the full commercialization can be arrived earlier if chemical fertilizer is used.

In a neoclassical dual model with endogenous wage rate the use of chemical fertilizer has similar significance in the short-run as in the model of a labor surplus economy since there is analogy between these two models if chemical fertilizer is considered explicitly in the agricultural production.

In the long-run the influences of the terms of trade on the properties of the neoclassical dual model are stronger if chemical fertilizer is used for the agricultural production.

In the stationary solution the capital intensity of the industrial sector will be lower if chemical fertilizer is used for the agricultural production.

From these findings we could conclude that the use of chemical fertilizer in the agricultural production has significant effects for the economic development.

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