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Uniqueness of Individual Demand
at almost every Budget via Sard's Theorem

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by

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It is known that preference maximization of economic agents results in significant structural restrictions for the induced individual demand correspondences. More precisely, demand correspondences coincide with demand functions everywhere except possibly on a null set of budgets. This result was first established by Mas-Colell (1976). Further proofs, based on different methods, are due to Mas-Colell and Neuefeind (1977) and Trockel (1983).

Our present aim is it to show that this result in the case of smooth preferences is an immediate consequence of Sard's Theorem.

Let us start by recalling two facts:

1. A given preference relation on a convex subset of a finite dimensional commodity space induces non-unique demand at the same budgets as its convexification.

2. Any non-singleton demand set at some budget for any convex preference relation has an element x in which the Gaussian curvature of the indifference class through x is zero.

Next let us state for reference Sard's Theorem for C^r manifolds $M, N, r > 0$, of the same finite dimension.

<u>Sard's Theorem</u> (cf. p. 42 in Mas-Colell (1985)): The set $C[\phi]$ of critical values of a C^r function $\phi: M \longrightarrow N$, $r > 0$, has measure zero in N.

Following Debreu (1972) and (1976) we denote by P the interior of $\mathbb{R}_+^l$ and consider the C^1 inverse demand function $\phi: P \longrightarrow P$ defined by $\phi(x) = (g_1(x), \ldots, g_{l-1}(x), x \cdot g(x))$.

Here $g: P \longrightarrow S = \{p \in P \mid \|p\| = 1\}$ is a C^1 function defining a monotone preference relation.

According to Debreu (1976) the Jacobian determinant of ϕ at x equals $c(x)\, g_1(x)$, where $c(x)$ denotes the Gaussian curvature at x of the indifference set through x. Now, the set $C[\phi]$ is the set of those budgets $(p,w) \in S \times \mathbb{R}_{++}$ which are images of points with vanishing Jacobian determinant of ϕ, hence with vanishing Gaussian curvature. So, by Sard's Theorem, they build a null set of $S \times \mathbb{R}_{++}$. Regarding fact 2, above, therefore all budgets with associated non-unique demand are contained in a null set of $S \times \mathbb{R}_{++}$.

This shows that demand correspondences derived from smooth monotone preference relations are singleton-valued at almost every budget.

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REFERENCES

Debreu, G. (1972) Smooth Preferences, _Econometrica,_ _40,_ 603 - 615

Debreu, G. (1976) Smooth Preferences: Corrigendum, _Econometrica,_ _44,_ 831 - 832

Mas-Colell, A. (1976) A Remark on a Smoothness Property of Convex Complete Preorders, _J. Math. Econ.,_ _3,_ 103 - 105

Mas-Colell, A. (1985) The Theory of General Economic Equilibrium, Cambridge, Cambridge University Press

Mas-Colell, A. and Neuefeind, W. (1977) Some Generic Properties of Aggregate Excess Demand and an Application, _Econometrica,_ _45,_ 591 - 599

Trockel, W. (1984) On the Uniqueness of Individual Demand at almost every Price System, _J. Econ. Theory,_ _33,_ 397 - 399