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Incomplete Contracts and
Nash–Implementation
in the Case of Two Agents

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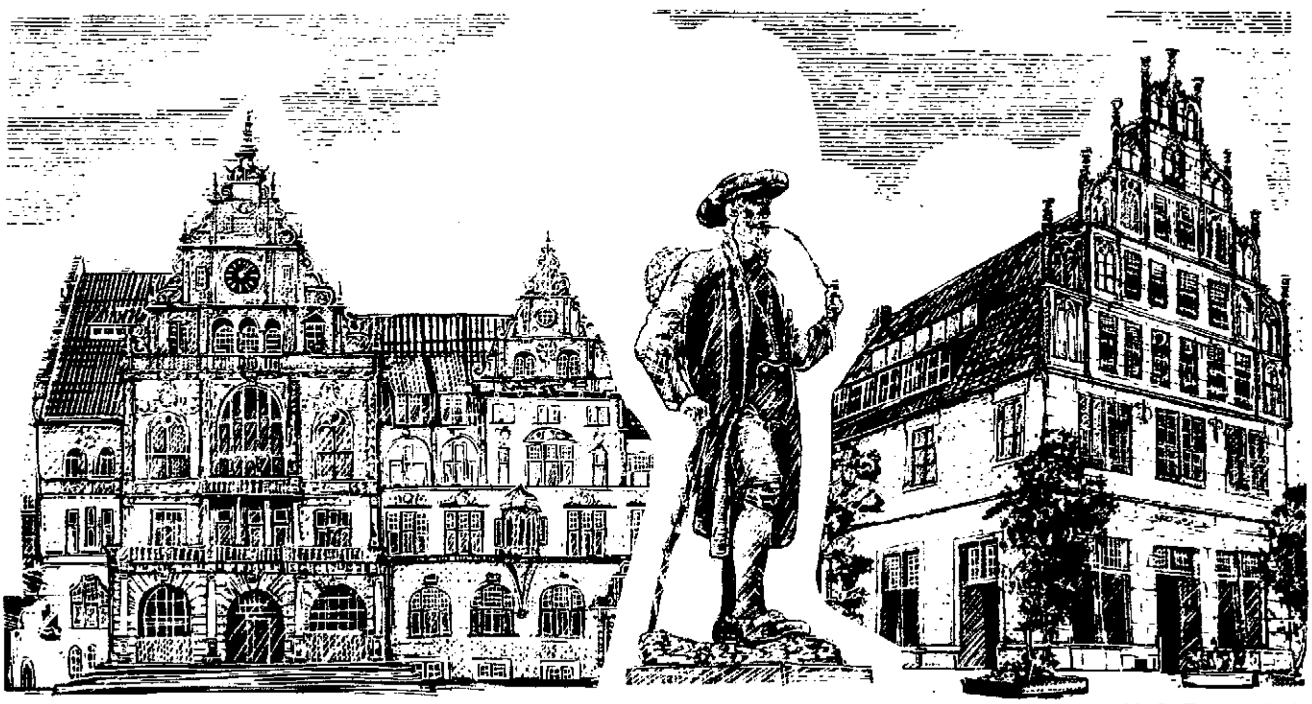

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ABSTRACT

The present paper illustrates the theory of Nash–implementation in the case of two agents by applying the theory of incomplete contracts. An incomplete contract does not determine as exactly as the agents want the outcome in every possible state which will prevail at a later date. A model is designed in which the implementing mechanism is interpreted as an incomplete contract whereas the implemented social choice rule is interpreted as a complete contract. The agents cannot make a complete contract because in the model such a contract is not enforceable by the courts. This is the case because the courts cannot observe the prevailing state on which the result of a complete contract has to depend.

A possibility to introduce the renegotiation of the incomplete contract into the model is analysed because the concept of incomplete contracts is closely connected to the idea of renegotiation. Renegotiation by the agents is advantageous to them because thus they can improve from a Pareto–inefficient outcome of the incomplete contract by agreeing on a Pareto–efficient outcome. A concept of Nash–implementation with renegotiation is investigated. Thereby a kind of revelation principle results. Besides the construction of a mechanism which implements a social choice rule w.r.t. the considered concept of implementation and thus the determination of sufficient conditions for such an implementation are subjects of the paper.

Incomplete Contracts and Nash–Implementation in the Case of Two Agents

The present paper illustrates the theory of Nash–implementation in the case of two agents by applying the theory of incomplete contracts. If the implementing mechanism is interpreted as incomplete contract then it is possible to design a model which is reasonable and consistent for Nash–Implementation of a social choice rule. An essential assumption which enables the use of Nash equilibria is the assumption of complete information of the agents at the date of the use of the mechanism. This assumption can be justified best if there are only few agents (in this case: two agents) in the model. The concept of incomplete contracts is closely connected to the idea of renegotiation of such a contract. Therefore a possibility to introduce the renegotiation of the incomplete contract into the model is analysed in the paper. A concept of Nash–implementation with renegotiation is investigated and an implementing mechanism is constructed.

Complete contracts exactly determine the execution of the contract in advance for every possible state of the world. The two parties who have agreed on the contract do not want to renegotiate such a contract when the state is known because all possible states were considered at the date of contracting. In an incomplete contract not all possible states are distinguished even if the parties want different results for such not distinguished states. Because of the high cost of receiving informations about the existing state it is often not economical to write a contract with conditional statements for every state. In other cases incomplete contracts are caused by high costs for developing provisions for all possible future states even if each occurring state can be easily observed. An incomplete contract can be renegotiated by the parties for correcting the insufficient provisions of the contract.

A contract instead of market exchange is necessary if the parties of an economic relationship make investments that are essentially more valuable within the relationship than outside of it. After the investments the parties are locked into the relationship. Therefore a contract can avoid that the division of the surplus from the relationship after knowing the state does not reflect the ex–ante investment decisions and that these decisions are distorted (cf. Hart (1987), p.3; Holmstrom/Tirole (1987), p.7–19.).

In the following model the state is only observable by the parties of the contract. Other persons cannot observe the state exactly. Therefore the courts that enforce a contract do not know the state of the world precisely. But in the contract actions that are contingent on the states can be specified effectively only if they are enforceable by the courts. Thus the cost of receiving information does not prevent the parties but prevents the courts from knowing the state. In this model a complete contract serves as measure for the incomplete contract. Therefore the costs for developing conditional provisions for all states are not important. The result of the renegotiation depends on the state and therefore it is not contractible in advance.

An example for this model is the cooperation of two firms that are the only suppliers in a market. They invest in a joint information structure for collecting and evaluating informations about the market. It is too expensive for one firm to undertake the total investments. These investments are publically observable. The investments can include information channels, computers and so on. The information structure also consists of knowledge how to handle such informations and the physical things of the information structure (cf. Arrow (1974), p. 39–41). The purchase of this knowledge in courses for the employees is publically observable. This information structure enables the firms to become acquainted with the state of the world especially with data about the production costs of the firms, about the quality of the goods on the market, about possibilities of production and so on. The knowledge of the state cannot be derived from the knowledge about the existence of the information structure. It can be derived only from the well–understood using of this structure. Depending on the existing state the firms have preferences on the possible outcomes that are attainable through decisions of both firms concerning production.

The following model of an economic relationship between two agents is in some points similar to the models of Hart/Moore (1988) and of Green/Laffont (1988).

Let $N = \{1,2\}$ be the set of agents and let X denote the set of possible outcomes. Corresponding to each agent $i \in N$ there is defined a subset $\mathcal{R}_i$ of the set $\mathcal{R}$ of complete, reflexive, transitive binary relations on X. It is assumed that there exists an outcome $x_0 \in X$ with xR_ix_0 for all $x \in X$, all $R_i \in \mathcal{R}_i$ and all $i \in N$. For each element R_i of $\mathcal{R}_i$ relations P_i and I_i on X are defined by $xP_iy \Leftrightarrow \neg yR_ix$, $xI_iy \Leftrightarrow xR_iy$ and yR_ix for all $x,y \in X$ and for all $i \in N$. A finite set Θ of states of the world and a partition $\{\Theta_j \mid j=1,..,k\}$ of Θ are also given. It is assumed that $|\Theta_j| > 1$ for all $j \in \{1,..,k\}$.

Both agents begin a relationship at the date 0. They agree on an incomplete contract about the attainable outcomes and on a complete contract about the relationship–specific investments. The incomplete contract consists in the choice of sets of actions A_1^j, A_2^j and of functions $g^j : A_1^j \times A_2^j \longrightarrow X$ for $j = 1,..,k$. It is explained later on which requirements this contract has to satisfy.

The complete contract records the decision of the agents to effect certain investments. Within the scope of the relationship of the agents these investments are necessary i.d. the investments are prescribed precisely by the prevailing conditions at the date 0 and by the aim of the relationship. The agents can only decide to invest as prescribed or not to invest. In the case of no investments the agents do not begin a relationship and the outcome x_0 is attained at the date 2. In the other case both agents have to effect the prescribed investments by the date 1. These investments are irreversible, publically observable and outside of the relationship without any fundamental value. Therefore the agents are locked into their relationship after these investments. The investments shall be publically observable because otherwise each agent has no incentive to enter the relationship since he cannot be sure at the date 0 that the other agent will effect the necessary investments. Therefore the contract about the investments is enforceable by the courts. The agents agree on the two contracts at the same time because in the case of investments without an incomplete contract there is a risk that they lose their investments. This outcome is in every case for both agents much worser than the worst outcome in X. They agree to invest (i.d. they invest) because thus they can be sure that the attained outcome is determined by the incomplete contract. The relationship and the investments of the agents are worthwhile because of the assumption that for every preference relation of each agent investing and achieving any possible result of the incomplete contract $\{g^j | j = 1 .. k\}$ is not worse than the outcome x_0 which occurs in the case of non–investing. Therefore the strategy of an agent to begin a relationship dominates the strategy not to begin a relationship.
At any time after the agreements at the date 0 the agents can jointly decide to terminate their relationship because nobody else can complain on non–observance of the agreements. Therefore both agents cannot commit themselves at the date 0 to not admit a later agreement on termination.

At the date 1 the state of the world $\vartheta \in \Theta$ occurs and both agents observe this state that is not observable by any other person. But the element Θ_j (and therefore j) of the partition in which the state is included is publically observable. The index j which is observable in the state $\vartheta \in \Theta$ is denoted as $j(\vartheta)$. For each $\vartheta \in \Theta$ it exists a $j(\vartheta) \in \{1, \ldots k\}$ with $\vartheta \in \Theta_{j(\vartheta)}$. This informational assumption can be interpreted as if the agents can become acquainted with the state only because they get specific informations on account of the investments. But the knowledge of the state cannot be derived from the knowledge of the kind of investments. The state ϑ determines a preference relation for each agent i.d. the agent i ($i \in N$) has a relation $R_i(\vartheta)$ that is an element of $\mathcal{R}_i$.

At the date 2 the agent i ($i \in N$) chooses an action $a_i \in A_i^j$. Thus the outcome $g^j(a_1,a_2)$ is attained and it is valued by each agent according to his preference relation. This outcome is enforceable by the courts because the chosen actions of the agents are publically observable. Often the action of an agent in an incomplete contract can be interpreted as a message about the prevailing state.

At the date 3 the agents renegotiate the incomplete contract. They proceed from the attained outcome $g^j(a_1,a_2)$ for jointly approving this outcome. The renegotiation succeeds if the agents agree on an outcome. It is assumed that the successful renegotiation results in an outcome which is Pareto–efficient with respect to the prevailing state and individually rational with respect to $g^j(a_1,a_2)$ and the state. Furthermore it is presumed that the agents do not change an outcome $g^j(a_1,a_2)$ which is already Pareto–efficient with respect to the prevailing state because they cannot improve a Pareto–efficient outcome by renegotiation. Therefore the result of the successful renegotiation depends on the prevailing state. For each state the result of the successful renegotiation only depends on the outcome $g^j(a_1,a_2)$. Therefore the result of all possible successful renegotiations can be described by a function $h : X \times \Theta \rightarrow X$ with the following properties:

1) $h(x,\vartheta)\ R_i(\vartheta)\ x$ for all $x \in X$, for all $\vartheta \in \Theta$ and for all $i \in N$. (Individually rational),

2) $h(x,\vartheta) \in SP(\vartheta)$ for all $\vartheta \in \Theta$ and all $x \in X$ with $SP(\vartheta) = \{z \in X \mid \text{for all } y \in X : z\ I_i(\vartheta)\ y$ for all $i \in N$ or it exists an $i \in N$ with $z\ P_i\ (\vartheta)\ y\}$ for all $\vartheta \in \Theta$ (Pareto–efficient),

3) $h(x,\vartheta) = x$ for all $x \in SP(\vartheta)$ and for all $\vartheta \in \Theta$.

It shall be mentioned that the agents do not determine such a function h. In a renegotiation they only fix one value of this function.
If the agents cannot agree on a result of the renegotiation i.d. if the renegotiation fails, then the result of the incomplete contract can be enforced by the courts. Therefore $g^j(a_1,a_2)$ is the threat point of the renegotiation. In every case successful renegotiation is for each agent at least as good as failing renegotiation.

If no agreements were made at the date 0 then the agents cannot jointly improve the attained outcome x_o.

In principle the agents want to enforce a complete contract $f : \Theta \longrightarrow X$ that gives a Pareto–efficient outcome for every possible state (i.d. $f(\vartheta) \in SP(\vartheta)$ for all $\vartheta \in \Theta$). The function f does not depend on actions of the agents because otherwise the execution of the contract would not be uniquely determined for every state. It is not analysed in this paper how the agents choose this complete contract f. Instead of this complete contract an incomplete contract has to be signed because the state is not observable from third parties. The incomplete contract $\{g^j \mid j=1,..,k\}$ does not depend on ϑ explicitly. It specifies the outcome only in dependence on actions of the agents and on the partition of Θ. The actions are publically observable and the incomplete contract is commonly known. When renegotiation is neglected then the agents are in a normal–form–game with complete information after the agreement on the contract and after the observation of the state ϑ. This game is denoted by (g^j,ϑ). It is assumed that the rational players play a Nash–equilibrium of such a game.
The set of Nash–equilibria (in pure strategies) of the game (g,ϑ) with $g : A_1 x A_2 \longrightarrow X$ and $\vartheta \in \Theta$ is denoted by

$$NE(g,\vartheta) = \{(\bar{a}_1,\bar{a}_2) \in A_1 x A_2 \mid g(\bar{a}_1,\bar{a}_2)\ R_1(\vartheta)\ g(a_1,\bar{a}_2) \text{ for all } a_1 \in A_1 \text{ and } g(\bar{a}_1,\bar{a}_2)\ R_2(\vartheta)\ g(\bar{a}_1,a_2) \text{ for all } a_2 \in A_2\}.$$

$NS_1(g,\vartheta) = \{a_1 \in A_1 \mid \text{it exists } a_2 \in A_2 \text{ with } (a_1,a_2) \in NE(g,\vartheta)\}$ is the set of Nash–equilibrium–strategies of agent 1 in the game (g,ϑ). $NS_2(g,\vartheta)$ is the correspondingly defined set of Nash–equilibrium–strategies of agent 2.

The set of Nash–outcomes of the game (g,ϑ) is denoted by
$NO(g,\vartheta) = \{x \in X \mid \text{it exists } (a_1,a_2) \in NE(g,\vartheta) \text{ with } x = g(a_1,a_2)\}$.

The set of Nash–equilibria depends on the preferences of the agents and therefore on the state so that the result of the contract is influenced indirectly by the state ϑ. In general a game has several Nash–equilibria and the players are not all indifferent between these equilibria. But it can be the case that one player plays a Nash–equilibrium–strategy that does not fit to the equilibrium–strategy played by the other player and that thus an outcome results that is not achievable through a Nash–equilibrium because in general $NS_1(g,\vartheta) \times NS_2(g,\vartheta) \not\subset NE(g,\vartheta)$ for a game (g,ϑ).

In the following it is studied whether this deficiency can be remedied if renegotiation of the incomplete contract is taken into consideration.

At the date 0 the agents know that they will renegotiate the outcome of the incomplete contract at the date 3 because for each agent renegotiation is in every case at least as good as non–renegotiation. But they do not know the result of the renegotiation in advance. At the date 0 they are aware that the final outcome in state $\vartheta \in \Theta$ is the image of the chosen tupel of actions under the composition of the known incomplete contract and an unkown function $h(\cdot,\vartheta) : X \to X$ which is induced by an unkown function $h : X \times \Theta \to X$ with the properties 1),2),3).

If the function $h : X \times \Theta \to X$ were known to the agents then after the observance of the state $\vartheta \in \Theta$ they would face a game in which a tupel $(a_1,a_2) \in A_1^{j(\vartheta)} \times A_2^{j(\vartheta)}$ results in a Pareto–efficient outcome $h(g^{j(\vartheta)}(a_1,a_2),\vartheta)$. This game is denoted as $(h(g^{j(\vartheta)}(\cdot),\vartheta),\vartheta)$.

A game with only Pareto–efficient outcomes is named as a strictly competitive game (cf. Friedman (1986), p. 30–32). In a 2–person–game that is strictly competitive there exist only Nash–equilibria between which both players are indifferent and where any tupel of Nash–equilibrium–strategies of both players yields a Nash–equilibrium. I.d. in a 2–person game (g,ϑ) that is strictly competitive it holds:

4) $NE(g,\vartheta) = NS_1(g,\vartheta) \times NS_2(g,\vartheta)$ and for all $x,y \in NO(g,\vartheta) : x\, I_i(\vartheta) y$ for all $i \in N$.

If the function $h : X \times \Theta \to X$ were known to the agents at the date 0 they would want to construct an incomplete contract $\{g^j \mid j = 1,..,k\}$ with $g^j = A_1^j \times A_2^j \to X$ such that

5) for all $\vartheta \in \Theta$: $NO(h(g^{j(\vartheta)}(\cdot),\vartheta),\vartheta) \neq \phi$ and $NO(h(g^{j(\vartheta)}(\cdot),\vartheta),\vartheta) \subset F(\vartheta)$

   where the correspondence $F : \Theta \twoheadrightarrow X$ is induced by the desired complete contract $f : \Theta \rightarrow X$ such that

6) $F(\vartheta) = \{x \in X \mid x\ I_i(\vartheta)\ f(\vartheta)$ for all $i \in N\}$ for all $\vartheta \in \Theta$.

The consideration of the correspondence makes sense because the agents do not value the element $f(\vartheta)$ and any element of $F(\vartheta)$ differently and because other properties of these elements are not important.

If a set $\{g^j \mid j = 1,..,k\}$ of function satisfies 5) with a given function $h : X \times \Theta \rightarrow X$ then it is said that g implements F in Nash–equilibria with the renegotiation function h.

The complete contract f is also denoted as social choice function (SCF), the induced correspondence as social choice correspondence (SCC).

Thus the mentioned deficiency could be remedied if the renegotiation function were known to the agents at the date 0. But such an informational premise is not very reasonable. If the renegotiation function were known at the date 0 then the agents could deviate from this function and they could renegotiate in another way. It is not possible to enforce a certain renegotiation function at the date 3 because an enforcing automatism or arbiter has to know the prevailing state which is only observable by the agents. Therefore an agent can force to renegotiate in another way for attaining a more preferable outcome.

Because the renegotiation function is not known to the agents at the date 0 the concept of implementation in Nash–equilibria with a given renegotiation function cannot be applied. Instead it is discussed whether a SCC F which is induced by a SCF f can be implemented in Nash–equilibria for all renegotiation functions $h : X \times \Theta \rightarrow X$ with the properties 1), 2), 3) i.d. whether a SCC F can be implemented in Nash–equilibria with renegotiation when it is only known at the date 0 that the renegotiation function satisfies 1), 2), 3). In the following this concept of implementation is simply called "Nash–implementation with renegotiation".

Such an implementation of a SCC F which is induced by a SCF f is possible if it can be guaranteed that the outcome $f(\vartheta)$ is a Nash–outcome of the strictly competitive game $(h(g^{j(\vartheta)}(\cdot),\vartheta),\vartheta)$ for a chosen incomplete contract $\{g^j | j = 1 .. k\}$, for every possible renegotiation function h with 1), 2), 3) and for all $\vartheta \in \Theta$. It can be seen later on that the properties 1), 2), 3) determine a function $h : X \times \Theta \to X$ on a large area because there are only two agents in the model (cf. p.12).

The mentioned result 4) about strictly competitive games implies that for an arbitrary correspondence $G : \Theta \twoheadrightarrow X$ (with $G(\vartheta) \subset SP(\vartheta)$ for all $\vartheta \in \Theta$) which is implementable by a mechanism g in Nash–equilibria with the known renegotiation function h there exists a correspondence $\hat{G} : \Theta \twoheadrightarrow X$ which satisfies: $\hat{G}(\vartheta) \subset G(\vartheta)$ for all $\vartheta \in \Theta$ and $xI_i(\vartheta)y$ for all $i \in N$, all $\vartheta \in \Theta$, all $x,y \in \hat{G}(\vartheta)$ and $\hat{G}$ is implementable by g in Nash–equilibria with the same renegotiation function h. Therefore it is reasonable to consider only SCCs which are induced in the described way by a SCF.

If g implements the correspondence F or the function f in Nash–equilibria then the players can still choose Nash–equilibrium–strategies that do not fit together and that lead to outcomes that are not Nash–outcomes. Therefore renegotiation makes sense in this case.

The same model of an incomplete contract can be used for illustrating the implementation of an arbitrary SCC F in Nash–equilibria without renegotiation (in this case it is not necessary that $F(\vartheta) \subset SP(\vartheta)$ for all $\vartheta \in \Theta$) if the agents do not have the ability to renegotiate. Such a model can also illustrate the implementation of a SCC F (with $F(\vartheta) \subset SP(\vartheta)$ for all $\vartheta \in \Theta$) in undominated Nash–equilibria with renegotiation when it is assumed that the agents choose undominated Nash–equilibria (i.d. Nash–equilibria in undominated strategies) (cf. Palfrey/Srivastava (1986)) instead of Nash–equilibria.

But in the case of implementation in Nash–equilibria without renegotiation an arbitrary correspondence F cannot be interpreted as complete contract because in general F does not determine the execution of the contract uniquely for every state. F can only be identified as an incomplete contract. Because of the high cost for developing provisions for all possible states the agents do not agree on any provisions for a given state $\vartheta \in \Theta$ (i.d. $F(\vartheta) = X$) or they agree not on a unique provision but on a set of possible provisions for this state $\vartheta \in \Theta$ (i.d. $F(\vartheta) \subsetneq X$). After occurrence of such a state ϑ in which

$F(\vartheta)$ is not a singleton a court has to decide which element of the set $F(\vartheta)$ shall be the resulting outcome. Because the decision of the court is not predictable for the agents they cannot consider a complete contract instead of the correspondence F. The incomplete contract F is not of the same type as g because all possible states are assumed to be publically observable in the case of F.

In the following it is assumed that $f(\vartheta) \in SP(\vartheta)$ for all $\vartheta \in \Theta$ and that $k = 1$ i.d. $\Theta_1 = \Theta$. The general case $k \leq |\Theta|$ can be simply reduced to the case $k = 1$.

In the following a kind of revelation principle is discovered for the case of implementation in Nash–equilibria with an a priori known renegotiation function. This kind of principle is also very useful for the case of implementation in Nash–equilibria with renegotiation when the renegotiation function is not known at the date 0.
Now a mechanism $g: A_1 \times A_2 \to X$ and a renegotiation function $h : X \times \Theta \to X$ with the properties 1), 2), 3) are considered.

It is reasonable to require that for all $\vartheta \in \Theta$ at least one Nash–equilibrium of the game $(h(g(\cdot),\vartheta),\vartheta)$ already yields the same outcome in the game (g,ϑ) i.d. without renegotiation. This means that h and g satisfy:

7) for all $\vartheta \in \Theta$ it exists a $s \in NE(h(g(\cdot),\vartheta),\vartheta)$ with $h(g(s),\vartheta) = g(s)$.

It is obvious that the following proposition results:

<u>Proposition 1:</u>
A mechanism $g : A_1 \times A_2 \to X$ and a renegotiation function $h : X \times \Theta \to \Theta$ with the properties 1), 2), 3) are given.
For all $\vartheta \in \Theta$ it yields: if there exists a $s \in NE(h(g(\cdot),\vartheta),\vartheta)$ with $h(g(s),\vartheta) = g(s)$ ($\Leftrightarrow g(s)$ Pareto–efficient in ϑ) then for this s it holds: $s \in NE(g,\vartheta)$.

Therefore that Nash–equilibrium of $(h(g(\cdot),\vartheta),\vartheta)$ yields not only an outcome in the game (g,ϑ). It is also a Nash–equilibrium of the game (g,ϑ) i.d. rational agents can at once achieve this Nash–outcome without renegotiation.

If a SCF f is implemented in Nash–equilibria with renegotiation function h by the mechanism g and if for all $\vartheta \in \Theta$ there exists $(s_1(\vartheta),s_2(\vartheta)) \in NE(h(g(\cdot),\vartheta),\vartheta)$ with $g(s(\vartheta)) = f(\vartheta)$ ($\Rightarrow h(g(s(\vartheta)),\vartheta) = g(s(\vartheta))$) then because of the proposition renegotiation does not induce new Nash–outcomes compared to the Nash–outcomes of (g,ϑ) but it reduces the set of all Nash–outcomes of (g,ϑ) to a singleton. In this case the SCF f is also truthfully implemented in Nash–equilibria by the revelation mechanism $\tilde{g} : \Theta \times \Theta \rightarrow X$ with $\tilde{g}(\vartheta,\phi) = g(s_1(\vartheta),s_2(\phi))$ for all $\vartheta,\phi \in \Theta$, i.d. the strategy–tupel (ϑ,ϑ) is a Nash–equilibrium of the game $(\tilde{g},\vartheta)$ and $\tilde{g}(\vartheta,\vartheta) = f(\vartheta)$ for all $\vartheta \in \Theta$. This is a kind of revelation principle.

If the SCC F is implemented in Nash–equilibria with renegotiation function h by the mechanism g and if for all $\vartheta \in \Theta$ there exists $(s_1(\vartheta),s_2(\vartheta)) \in NE(h(g(\cdot),\vartheta),\vartheta)$ with $g(s(\vartheta)) \in F(\vartheta)$ ($\Rightarrow h(g(s(\vartheta)),\vartheta) = g(s(\vartheta))$) then because of the proposition F is truthfully implementable by the above defined mechanism $\tilde{g}$.

This kind of principle is very helpful for constructing a mechanism which implements a SCC F induced by a SCF f in Nash–equilibria with renegotiation. For it shows that it is reasonable to start with a revelation mechanism g which implements f truthfully in Nash–equilibria (i.d. $g(\vartheta.\vartheta) = f(\vartheta)$ for all $\vartheta \in \Theta$, $f(\vartheta)\ R_1(\vartheta)\ g(\phi,\vartheta)$ for all $\phi,\vartheta \in \Theta$, $f(\vartheta)\ R_2(\vartheta)\ g(\vartheta,\phi)$ for all $\vartheta,\phi \in \Theta$) and to take care that the desired Nash–equilibrium (ϑ,ϑ) in state $\vartheta \in \Theta$ remains a Nash–equilibrium in state ϑ after renegotiation. Because of the informational assumption the properties 1),2),3) must be sufficient for guaranteeing that this Nash–equilibrium remains a Nash–equilibrium after renegotiation. On account of 4) renegotiation eliminates all Nash–equilibria of g in state ϑ with outcomes which are not in $F(\vartheta)$. If it is possible to construct such a g then g implements F in Nash–equilibria with renegotiation. A mechanism g constructed in such a way satisfies 7) if h is any renegotiation function with 1),2),3).
As first step in the sketched project of construction it is analysed whether renegotiation according to an a priori unknown function $h : X \times \Theta \rightarrow X$ with the properties 1),2),3) preserves the orders $f(\vartheta)\ R_1(\vartheta)\ g(\phi,\vartheta)$ and $f(\vartheta)\ R_2(\vartheta)\ g(\vartheta,\phi)$ for arbitrarily chosen $\vartheta,\phi \in \Theta$ with a revelation mechanism g (i.d. $g : \Theta \times \Theta \rightarrow X$). Four cases can be distinguished with respect to the assumption $f(\vartheta)\ R_i(\vartheta) y$ for ($i = 1$ and $y = g(\phi,\vartheta)$) or ($i = 2$ and $y = g(\vartheta,\phi)$).

8)a) It holds: $f(\vartheta)\ P_i(\vartheta)y$ and for $j \in N,\ j \neq i : y\ P_j(\vartheta)\ f(\vartheta)$.
This implies: $h(y,\vartheta)\ P_j(\vartheta)\ f(\vartheta)$ and $f(\vartheta)\ P_i(\vartheta)\ h(y,\vartheta)$ because of $h(y,\vartheta) \in SP(\vartheta)$ and $|N| = 2$.

8)b) It holds: $f(\vartheta)\ P_i(\vartheta)y$ and for $j \in N,\ j \neq i : y\ I_j(\vartheta)\ f(\vartheta)$.
This implies: $f(\vartheta)\ R_i(\vartheta)\ h(y,\vartheta)$ because otherwise $f(\vartheta) \notin SP(\vartheta)$.

8)c) It holds: $f(\vartheta)\ I_i(\vartheta)y$ and for $j \in N,\ j \neq i : f(\vartheta)\ I_j(\vartheta)y$.
This implies: $h(y,\vartheta) = y$ and $y \in F(\vartheta)$.

8)d) It holds: $f(\vartheta)\ R_i(\vartheta)y$ and for $j \in N,\ j \neq i : f(\vartheta)P_j(\vartheta)y$.
In this case the properties 1),2),3) are generally not sufficient to determine the order of agent i between $f(\vartheta)$ and $h(y,\vartheta)$.

The case $f(\vartheta)\ I_i(\vartheta)y$ and for $j \in N,\ j \neq i : y\ P_j(\vartheta)\ f(\vartheta)$ is not possible because this case is in contradiction with $f(\vartheta) \in SP(\vartheta)$.
Therefore in the cases 8)a), 8)b), 8)c) the properties 1), 2), 3) are sufficient for the preservation of the considered orders by renegotiation (i.d. $f(\vartheta)\ R_i(\vartheta)y \Rightarrow f(\vartheta)\ R_i(\vartheta)\ h(y,\vartheta)$ for ($i = 1$ and $y = g(\phi,\vartheta)$) or ($i = 2$ and $y = g(\vartheta,\phi)$)). But in the case 8)d) the function h must be known more precisely to guarantee the preservation of the orders. In the following it is tried to construct the desired mechanism g only by using the cases 8)a), 8)b), 8)c).

Such a construction is possible if there exist $\bar{g}(\phi,\vartheta) \in X$ and $\bar{g}(\vartheta,\phi) \in X$ for all $\vartheta,\phi \in \Theta$, $\vartheta \neq \phi$ with the properties:

9)i) $(f(\vartheta)\ P_1(\vartheta)\ \bar{g}(\phi,\vartheta)$ and $\bar{g}(\phi,\vartheta)\ R_2(\vartheta)\ f(\vartheta))$ or $(f(\vartheta)\ I_i(\vartheta)\ \bar{g}(\phi,\vartheta)$ for all $i \in N)$

9)ii) $(f(\phi)\ P_2(\phi)\ \bar{g}(\phi,\vartheta)$ and $\bar{g}(\phi,\vartheta)\ R_1(\phi)\ f(\phi))$ or $(f(\phi)\ I_i(\phi)\ \bar{g}(\phi,\vartheta)$ for all $i \in N)$

9)iii) $(f(\vartheta)\ P_2(\vartheta)\ \bar{g}(\vartheta,\phi)$ and $\bar{g}(\vartheta,\phi)\ R_1(\vartheta)\ f(\vartheta))$ or $(f(\vartheta)\ I_i(\vartheta)\ \bar{g}(\vartheta,\phi)$ for all $i \in N)$

9)iv) $(f(\phi)\ P_1(\phi)\ \bar{g}(\vartheta,\phi)$ and $\bar{g}(\vartheta,\phi)\ R_2(\phi)\ f(\phi))$ or $(f(\phi)\ I_i(\phi)\ \bar{g}(\vartheta,\phi)$ for all $i \in N)$

Renegotiation preserves the considered orders for any renegotiation function $h : X \times \Theta \rightarrow X$ with 1), 2), 3) because the conditions 9) just combine the cases 8)a), 8)b), 8)c)
(thus it is implied:

9)i) $\Rightarrow f(\vartheta)\ R_1(\vartheta)\ h(\bar{g}(\phi,\vartheta),\vartheta)$

9)ii) $\Rightarrow f(\phi)\ R_2(\phi)\ h(\bar{g}(\phi,\vartheta),\phi)$

9)iii) $\Rightarrow f(\vartheta)\ R_2(\vartheta)\ h(\bar{g}(\vartheta,\phi),\vartheta)$

9)iv) $\Rightarrow f(\phi)\ R_1(\phi)\ h(\bar{g}(\vartheta,\phi),\phi)$).

Because of the conditions 9) the mechanism $\bar{g} : \Theta \times \Theta \rightarrow X$ with $\bar{g}(\vartheta,\vartheta) = f(\vartheta)$ for all $\vartheta \in \Theta$ and formed by the above considered $\bar{g}(\vartheta,\phi)$ with $\vartheta,\phi \in \Theta, \vartheta \neq \phi$ implements the SCF f truthfully in Nash–equilibria and it also implements the induced SCC F in Nash–equilibria with renegotiation.

It is obvious that the conditions 9) are very restrictive. But in the following it is shown that the elements $\bar{g}(\phi,\vartheta)$, $\bar{g}(\vartheta,\phi)$ need not exist for all $\vartheta,\phi \in \Theta$ with $\vartheta \neq \phi$ for implementing F in Nash–equilibria with renegotiation.

For this purpose a classification is considered:
There are six possible orders between $f(\vartheta)$ and $f(\phi)$ for arbitrarily chosen elements $\vartheta,\phi \in \Theta, \vartheta \neq \phi$ according to the preference relations of the agents in the state ϑ. It is remarked how 8) can be applied in each of the six cases to enable the construction of a mechanism g which implements a SCF f truthfully in Nash–equilibria and which also implements the induced SCC F in Nash–equilibria with renegotiation.

a_ϑ) $f(\vartheta)\ P_1(\vartheta)\ f(\phi)$ and $f(\vartheta)\ P_2(\vartheta)\ f(\phi)$.

This case corresponds to 8)d),

b_ϑ) $f(\vartheta)\ I_1(\vartheta)\ f(\phi)$ and $f(\vartheta)\ P_2(\vartheta)\ f(\phi)$.
In this case the definition $g(\vartheta,\phi) = f(\phi)$ would yield:
$f(\vartheta)\ R_2(\vartheta)\ g(\vartheta,\phi)$ and $f(\vartheta)\ R_2(\vartheta)\ h(g(\vartheta,\phi),\vartheta)$ for any $h : X \times \Theta \rightarrow X$ with 1), 2), 3) (because of 8)b)).

c_ϑ) $f(\vartheta)\ P_1(\vartheta)\ f(\phi)$ and $f(\vartheta)\ I_2(\vartheta)\ f(\phi)$.
In this case the definition $g(\phi,\vartheta) = f(\phi)$ would yield:
$f(\vartheta)\ R_1(\vartheta)\ g(\phi,\vartheta)$ and $f(\vartheta)\ R_1(\vartheta)\ h(g(\phi,\vartheta),\vartheta)$ (because of 8)b)).

d_ϑ) $f(\vartheta)\ P_1(\vartheta)\ f(\phi)$ and $f(\phi)\ P_2(\vartheta)\ f(\vartheta)$.
In this case it also holds: $f(\vartheta)\ P_1(\vartheta)\ h(f(\phi),\vartheta)$ and $h(f(\phi),\vartheta)\ P_2(\vartheta)\ f(\vartheta)$ (because of 8)a)). Therefore it would be possible to set: $g(\phi,\vartheta) = f(\phi)$.

e_ϑ) $f(\phi)\ P_1(\vartheta)\ f(\vartheta)$ and $f(\vartheta)\ P_2(\vartheta)\ f(\phi)$.
In this case it also holds: $h(f(\phi),\vartheta)\ P_1(\vartheta)\ f(\vartheta)$ and $f(\vartheta)\ P_2(\vartheta)\ h(f(\phi),\vartheta)$ (because of 8)a)). Therefore it would be possible to set: $g(\vartheta,\phi) = f(\phi)$.

f_ϑ) $f(\phi)\ I_1(\vartheta)\ f(\vartheta)$ and $f(\vartheta)\ I_2(\vartheta)\ f(\phi)$.
In this case it also holds: $f(\vartheta)\ I_1(\vartheta)\ h(f(\phi),\vartheta)$ and $f(\vartheta)\ I_2(\vartheta)\ h(f(\phi),\vartheta)$ (because of 8)c)). Therefore it would be possible to set $g(\vartheta,\phi) = f(\phi)$ or $g(\phi,\vartheta) = f(\phi)$.

The six possible orders (a_ϕ),..,f_ϕ)) according to the preference relations of the agents in the state ϕ are defined analogously and the remarks to each order are formed by exchange of ϑ and ϕ. The combination of the six possibities for ϑ and the six possibilities for ϕ results in 36 different possible orders between the elements $f(\vartheta)$ and $f(\phi)$ according to the relations of both states ϑ,ϕ. (These 36 cases are a_ϑ and a_ϕ, a_ϑ and b_ϕ,...,b_ϑ and a_ϕ,... and so on.)

Depending on these 36 possible cases a mechanism g which implements f truthfully in Nash–equilibria and which also implements the induced SCC F in Nash–equilibria with renegotiation is constructed. Therefore $g(\vartheta,\phi)$, $g(\phi,\vartheta)$ are specified in such a way that it holds for all functions $h : X \times \Theta \rightarrow X$ with 1), 2), 3):

10.1)
$$f(\vartheta)\ R_1(\vartheta)\ g(\phi,\vartheta),\ f(\vartheta)\ R_2(\vartheta)\ g(\vartheta,\phi),$$
$$f(\vartheta)\ R_1(\vartheta)\ h(g(\phi,\vartheta),\vartheta),\ f(\vartheta)\ R_2(\vartheta)\ h(g(\vartheta,\phi),\vartheta)$$

10.2)
$$f(\phi)\ R_1(\phi)\ g(\vartheta,\phi),\ f(\phi)\ R_2(\phi)\ g(\phi,\vartheta)$$
$$f(\phi)\ R_1(\phi)\ h(g(\vartheta,\phi),\phi),\ f(\phi)\ R_2(\phi)\ h(g(\phi,\vartheta),\phi)$$

In the following 19 cases which are summarized in five groups 11.1) – 11.5) the elements $g(\vartheta,\phi)$ and $g(\phi,\vartheta)$ of the mechanism can be specified perfectly by the elements of the set $\{f(\vartheta), f(\phi)\}$.

11.1) It holds f_ϑ and f_ϕ (i.d. $f(\vartheta)\ I_i(\vartheta)\ f(\phi)$ and $f(\vartheta)\ I_i(\phi)\ f(\phi)$ for all $i \in N$).
Then $g(\vartheta,\phi)$ and $g(\phi,\vartheta)$ can be arbitrarily chosen out of the set $\{f(\vartheta), f(\phi)\}$ to satisfiy the conditions 10).

11.2) $\begin{pmatrix} g(\vartheta,\vartheta),\ g(\vartheta,\phi) \\ g(\phi,\vartheta),\ g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta),\ f(\phi) \\ f(\vartheta),\ f(\phi) \end{pmatrix}$ can be chosen if one of the following cases holds: b_ϑ and b_ϕ, b_ϑ and e_ϕ, e_ϑ and b_ϕ, e_ϑ and e_ϕ,
b_ϑ and f_ϕ, e_ϑ and f_ϕ, f_ϑ and b_ϕ, f_ϑ amd e_ϕ.
This results because of the remarks to b_ϑ, b_ϕ, e_ϑ, e_ϕ, f_ϑ, f_ϕ.

11.3) $\begin{pmatrix} g(\vartheta,\vartheta),\ g(\vartheta,\phi) \\ g(\phi,\vartheta),\ g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta),\ f(\vartheta) \\ f(\phi),\ f(\phi) \end{pmatrix}$ can be chosen if one of the following cases holds: c_ϑ and c_ϕ, c_ϑ and d_ϕ, d_ϑ and c_ϕ, d_ϑ and d_ϕ,
c_ϑ and f_ϕ, d_ϑ and f_ϕ, f_ϑ and c_ϕ, f_ϑ and d_ϕ.
This results because of the remarks to c_ϑ, d_ϑ, c_ϕ, d_ϕ, f_ϑ, f_ϕ.

11.4) $\begin{pmatrix} g(\vartheta,\vartheta), g(\vartheta,\phi) \\ g(\phi,\vartheta), g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), f(\phi) \\ f(\phi), f(\phi) \end{pmatrix}$ can be chosen if it holds: f_ϑ and a_ϕ.

10.2) is obviously satisfied. 10.1) is implied by the remark to f_ϑ.

11.5) $\begin{pmatrix} g(\vartheta,\vartheta), g(\vartheta,\phi) \\ g(\phi,\vartheta), g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), f(\vartheta) \\ f(\vartheta), f(\phi) \end{pmatrix}$ can be chosen if it holds: a_ϑ and f_ϕ.

10.1) is obviously satisfied. 10.2) is implied by the remark to f_ϕ.

In the remaining 17 cases it is not possible to use only elements of the set $\{f(\vartheta), f(\phi)\}$ for the specification of $g(\vartheta,\phi)$ and $g(\phi,\vartheta)$. In the following it is assumed that $\bar{g}(\vartheta,\phi)$ satisfies 9)iii) and 9)iv) and that $\bar{g}(\phi,\vartheta)$ satisfies 9)i) and 9)ii). The remaining 17 cases are summarized in seven groups 11.6) – 11.12).

11.6) $\begin{pmatrix} g(\vartheta,\vartheta), g(\vartheta,\phi) \\ g(\phi,\vartheta), g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), \bar{g}(\vartheta,\phi) \\ f(\vartheta), f(\phi) \end{pmatrix}$ can be chosen if one of the following cases holds: a_ϑ and b_ϕ, a_ϑ and e_ϕ, d_ϑ and b_ϕ.

This results because of the remarks to b_ϕ, e_ϕ and because of the conditions 9)iii), 9)iv).

11.7) $\begin{pmatrix} g(\vartheta,\vartheta), g(\vartheta,\phi) \\ g(\phi,\vartheta), g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), f(\vartheta) \\ \bar{g}(\phi,\vartheta), f(\phi) \end{pmatrix}$ can be chosen if one of the following cases holds: a_ϑ and c_ϕ, a_ϑ and d_ϕ, e_ϑ and c_ϕ.

This results because of the remarks to c_ϕ, d_ϕ and because of the conditions 9)i), 9)ii).

11.8) $\begin{pmatrix} g(\vartheta,\vartheta), g(\vartheta,\phi) \\ g(\phi,\vartheta), g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), \bar{g}(\vartheta,\phi) \\ f(\phi), f(\phi) \end{pmatrix}$ can be chosen if one of the following cases holds: c_ϑ and a_ϕ, d_ϑ and a_ϕ, c_ϑ and e_ϕ.

This results because of the remarks to c_ϑ, d_ϑ and because of the conditions 9)iii), 9)iv).

11.9) $\begin{pmatrix} g(\vartheta,\vartheta), & g(\vartheta,\phi) \\ g(\phi,\vartheta), & g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), & f(\phi) \\ \bar{g}(\phi,\vartheta), & f(\phi) \end{pmatrix}$ can be chosen if one of the following

cases holds: b_ϑ and a_ϕ, e_ϑ and a_ϕ, b_ϑ and d_ϕ.

This results because of the remarks to b_ϑ, e_ϑ and because of 9)i, 9)ii).

11.10) $\begin{pmatrix} g(\vartheta,\vartheta), & g(\vartheta,\phi) \\ g(\phi,\vartheta), & g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), & z \\ \bar{g}(\phi,\vartheta), & f(\phi) \end{pmatrix}$ with $z \in \{f(\vartheta), f(\phi)\}$ can be chosen in the

case: e_ϑ and d_ϕ.

This results because of the remarks to d_ϕ, e_ϑ and because of 9)i), 9)ii).

11.11) $\begin{pmatrix} g(\vartheta,\vartheta), & g(\vartheta,\phi) \\ g(\phi,\vartheta), & g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), & \bar{g}(\vartheta,\phi) \\ z, & f(\phi) \end{pmatrix}$ with $z \in \{f(\vartheta), f(\phi)\}$ can be chosen in the

case: d_ϑ and e_ϕ.

This results because of the remarks to d_ϑ, e_ϕ and because of 9)iii), 9)iv).

11.12) $\begin{pmatrix} g(\vartheta,\vartheta), & g(\vartheta,\phi) \\ g(\phi,\vartheta), & g(\phi,\phi) \end{pmatrix} = \begin{pmatrix} f(\vartheta), & \bar{g}(\vartheta,\phi) \\ \bar{g}(\phi,\vartheta), & f(\phi) \end{pmatrix}$ can be chosen if one of the following

cases holds: a_ϑ and a_ϕ, b_ϑ and c_ϕ, c_ϑ and b_ϕ.

This results because of 9)i) – 9)iv).

Therefore the following proposition can be stated:

Proposition 2:

A SCC F which is induced by a given SCF f can be implemented in Nash–equilibria with renegotiation if the following holds:

I) for states $\vartheta,\phi \in \Theta$, $\vartheta \neq \phi$ which fall under the classes 11.7), 11.9), 11.10) there exists a $\bar{g}(\phi,\vartheta)$ which satisfies 9)i) and 9)ii),

II) for states $\vartheta,\phi \in \Theta$, $\vartheta \neq \phi$ which fall under the classes 11.6), 11.8), 11.11) there exists a $\bar{g}(\vartheta,\phi)$ which satisfies 9)iii) and 9)iv),

III) for states $\vartheta,\phi \in \Theta$, $\vartheta \neq \phi$ which fall under the class 11.12) there exist a $\bar{g}(\phi,\vartheta)$ which satisfies 9)i) and 9)ii) and a $\bar{g}(\vartheta,\phi)$ which satisfies 9)iii) and 9)iv).

The implementing mechanism is g which is described in 11.1) – 11.12).

It is obvious that the conditions I), II), III) in proposition 2 are still very restrictive. This can be seen in proposition 3 in which monotonicity of a SCF is used.
A SCF $f : \Theta \rightarrow X$ is called monotone w.r.t a given pair of states $(\vartheta,\phi) \in \Theta \times \Theta$ with $f(\vartheta) \neq f(\phi)$ if there exist an $i \in N$ and a $y \in X$ with : $f(\vartheta)\ R_i(\vartheta) y$ and $y P_i(\phi)\ f(\vartheta)$. The considered $i \in N$ and $y \in X$ are denoted as $i(\vartheta,\phi)$ and $y(\vartheta,\phi)$. A SCF $f : \Theta \rightarrow X$ is called monotone if f is monotone w.r.t. all pairs $(\vartheta,\phi) \in \Theta \times \Theta$ with $f(\vartheta) \neq f(\phi)$.
Monotonicity of a SCF f is a necessary condition for implementability of f in Nash–equilibria (cf. Maskin (1985)).

<u>Proposition 3:</u>
A SCF f which satisfies I), II), III) of proposition 2 is monotone with respect to all pairs of states $(\vartheta,\phi) \in \Theta \times \Theta$, $\vartheta \neq \phi$ which are considered in I), II), III) of proposition 2.

Proof:
The cases 11.6) – 11.12) are analysed separately.
<u>ad 11.6)</u>
It holds $f(\vartheta)\ R_2(\vartheta)\ \bar{g}(\vartheta,\phi)$ and $\bar{g}(\vartheta,\phi)\ R_2(\phi)\ f(\phi)$ because of 9)iii), 9)iv) and it holds $f(\phi)\ P_2(\phi)\ f(\vartheta)$ because of b_ϕ resp. e_ϕ. Therefore it yields:

$$f(\vartheta)\ R_2(\vartheta)\ \bar{g}(\vartheta,\phi) \text{ and } \bar{g}(\vartheta,\phi)\ P_2(\phi)\ f(\vartheta).$$

Thus $i(\vartheta,\phi) = 2$ and $y(\vartheta,\phi) = \bar{g}(\vartheta,\phi)$ can be chosen.
It holds $f(\phi)\ R_1(\phi)\ \bar{g}(\vartheta,\phi)$ and $\bar{g}(\vartheta,\phi)\ R_1(\vartheta)\ f(\vartheta)$ because of 9)iii), 9)iv) and it holds $f(\vartheta)\ P_1(\vartheta)\ f(\phi)$ because of a_ϑ resp. d_ϑ.
Therefore it yields:

$$f(\phi)\ R_1(\phi)\ \bar{g}(\vartheta,\phi) \text{ and } \bar{g}(\vartheta,\phi)\ P_1(\vartheta)\ f(\phi).$$

Thus $i(\phi,\vartheta) = 1$ and $y(\phi,\vartheta) = \bar{g}(\vartheta,\phi)$ can be chosen.

In the cases c_ϑ and b_ϕ, a_ϑ and a_ϕ (11.12)), 11.8), and 11.11) the same result as in 11.6) is shown analogously.

In the cases 11.7), 11.9), 11.10), b_ϑ and c_ϕ, a_ϑ and a_ϕ (11.12)) it can be shown analogously to the case 11.6) that it results:

$f(\vartheta)\ R_1(\vartheta)\ \bar{g}(\phi,\vartheta)$ and $\bar{g}(\phi,\vartheta)\ P_1(\phi)\ f(\vartheta)$ ($\Rightarrow i(\vartheta,\phi) = 1,\ y(\vartheta,\phi) = \bar{g}(\phi,\vartheta)$)

$f(\phi)\ R_2(\phi)\ \bar{g}(\phi,\vartheta)$ and $\bar{g}(\phi,\vartheta)\ P_2(\vartheta)\ f(\phi)$ ($\Rightarrow i(\phi,\vartheta) = 2,\ y(\phi,\vartheta) = \bar{g}(\phi,\vartheta)$)

Thus the proposition is proved.

It shall be mentioned that in the following cases it is not necessary to use $\bar{g}(\vartheta,\phi)$ resp. $\bar{g}(\phi,\vartheta)$ for showing that f is monotone w.r.t. (ϑ,ϕ) and (ϕ,ϑ): (elements of $\{f(\vartheta), f(\phi)\}$ can be chosen as $y(\vartheta,\phi)$ or $y(\phi,\vartheta)$)

a_ϑ and b_ϕ, a_ϑ and e_ϕ (11.6)); a_ϑ and c_ϕ, a_ϑ and d_ϕ (11.7));

c_ϑ and a_ϕ, d_ϑ and a_ϕ (11.8)); e_ϑ and a_ϕ, b_ϑ and a_ϕ (11.9));

a_ϑ and a_ϕ, b_ϑ and c_ϕ, c_ϑ and b_ϕ (11.12)). □

Therefore it is obvious that I), II), III) in proposition 2 are more restrictive than monotonicity of the SCF f w.r.t. the considered pairs of states. Proposition 3 is a first step in analysing the difference between Nash–implementation with renegotiation and Nash–implementation without renegotiation. In special frameworks (for all $i \in N$ all elements of $\mathscr{R}_i$ are also asymmetric and for all $\vartheta \in \Theta$, all $i \in N$ it exists a $b_i(\vartheta)$ with $b_i(\vartheta)\ P_i(\vartheta)\ f(\phi)$ for all $\phi \in \Theta$) a SCF f which satisfies the conditions I), II), III) of proposition 2 can be implemented in Nash–equilibria (without renegotiation). A detailed analysis of such connections is the subject of research in progress.

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