Universität Bielefeld/IMW

Working Papers
Institute of Mathematical Economics

Arbeiten aus dem
Institut für Mathematische Wirtschaftsforschung

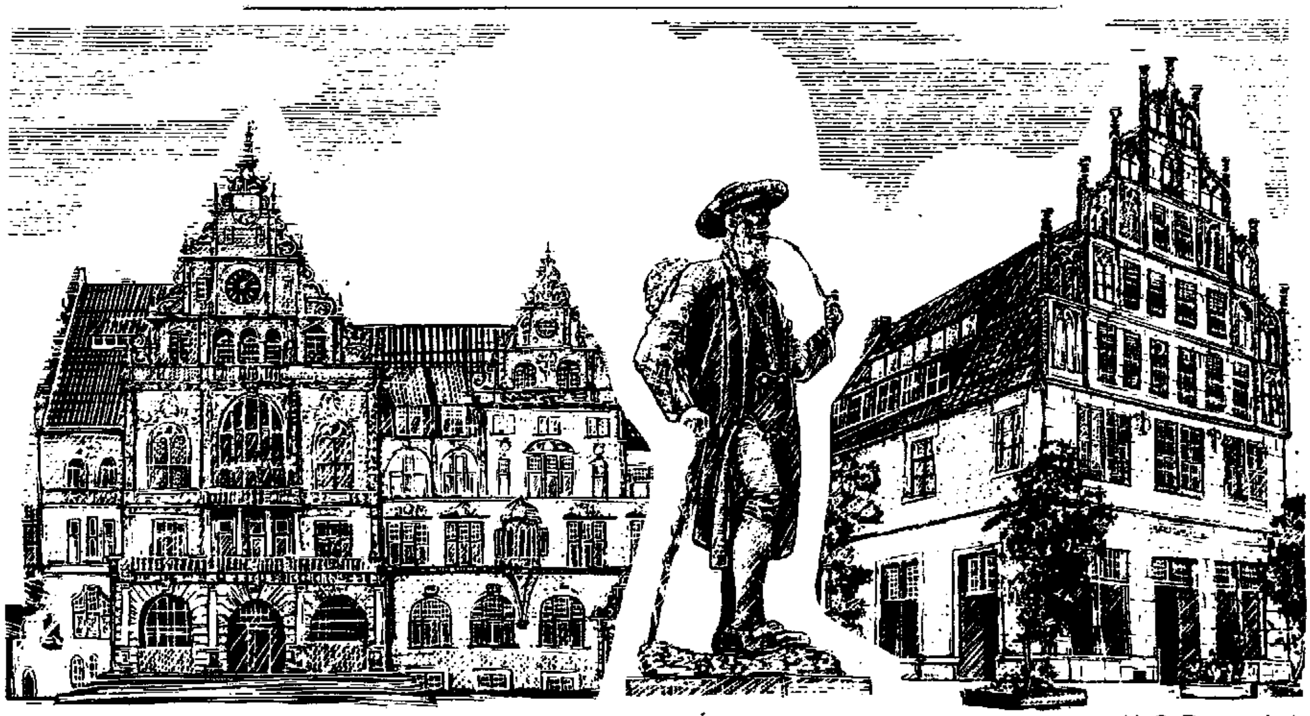
Nr. 183

Revealed Fuzzy Preferences

by

Olga N. Bondareva

January 1990



H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

REVEALED FUZZY PREFERENCES

Bondareva O.N.
Institute of Mathematics and Mechanics
Leningrad State University
I98904 Leningrad, USSR

__Abstract__: It is proved that there is one-to-one mapping between the class of all atireflexive binary unfuzzy relations and a class of choice functions with the special properties. The direct mapping is the rule of choice, the invers mapping is the revealed preference. The fuzzy extention of this notions are given that this result also holds for fuzzy choice on unfuzzy sets i.e. for fuzzy preferences. Becauce the properties of choice functions allow the trivial fuzzy extencion it is convenient to describe the properties of fuzzy relations in term of choice functions i.e. to consider revealed fuzzy preferences. The above principle of fussy extention applies to a fuzzy multiperson choice.

__Keywords__: choice, revealed preference, core, von Neumann-Morgenstern solution.

I. INTRODUCTION

Consider a choice on an arbitrary non-fuzzy set A, finite or infinite. Let $R(x,y)$ be the arbitrary relation on A, $R(x,y)$ may be fuzzy i.e. $0 \leq R(x,y) \leq I$. In this paper it is investigated the fuzzy choice on unfuzzy sets. There are two standart rules of choice with respect to the unfuzzy relation R: I) select the best alternatives, 2) select undominated alternatives. This rules are equvivalent in the following sence:if $C_1(X)$ $(X \subset A)$ is the set selected from X by first rule with the relation R then $C_1(X) = C_2(X)$ where $C_2(X)$ is received by second rule but with the relation $\bar{R}$: $x\bar{R}y$ iff xRy isn't true. In this paper it is prefered the second rule named, as in the game theory, the core.

The rule of choice maps a set of relations in a set of choice functions. If this mapping is one-to-one mapping the relation is named the revealed preference. In other words, the revealed preference has following properties: each choice function C from the given class allows to reconstruct the relation R such that this C is received by this rule of choice with this relation R. Such mapping exists for the set of all antireflexive relations and a set of choice functions fulfiled the well-known axioms of choice. The fuzzy extencion of this axioms is trivial becauce they are expressed in term of operations $\cap$, $\cup$ and $\subset$ only.

The idea of revealed preferences is able to be used both to do the fuzzy extencion of choice with relation and to stand equivalence of some results in term of relations and term of choice function in unfuzzy case.

The von Neumann-Morgenstern solution is a standart notion in the game theory. Inspite of this is the direct extension of the first rule to a wider class of relations it is less known in the theory of choice. The von Neumann- Morgenstern solution is the following set V: all elements of V must be independent and each element x outside of V must be "worse" than some element y V, i.e. yRx. For example if R is a weak order the set V of all maximal elements is the von Neumann-Morgenstern solution.

The result about revealed preferences for this rule is proved. It is given the fussy extension of von Neumann-Morgenstern solution.

The principle of revealed preferences applies to the multiperson choice. A fuzzy multiperson choice is defined.

Becauce the author isn't a specialist in choice theory (may be in game theory) the references aren't full and rather chance. Only the results about the fuzzy choice are pretended to be new, almost all results in unfuzzy choice are known, may be, in other forms. The method of revealed preferences is supposed to be new and useful.

2. NOTATIONS AND DEFINITIONS

Let A be an arbitrary set, finite or infinite, of alternatives for choice. Non-fuzzy subsets of A are denoted by capital letters X, X',...; fuzzy subsets are denoted both special letters and the same letters X.X',.... So X(x) is the membership function of X, if X is non-fuzzy then $X = \chi_X$ i.e. X and χ_X are two representations of the non-fuzzy set X. Similar $R \subset A \times A$ is both a non-fuzzy binary relation and a fuzzy relation $R(x,y) = \chi_R(x,y)$. So a relation, fuzzy or non-fuzzy, is denoted by R(x,y), $R \in [0,I]^{A \times A}$. Only restriction for relations is antireflexivity (irreflexivity), i.e. $R(x,x) = 0$. The relation R isn't need to be transitive one. There are two reasons for transitivity to be unnatural. The first is explained by the next example. Let two persons have preferences R_1 and R_2, linear orders, and a collective relation is $R = R_1 \cup R_2$, i.e. the couple preferes x to y if at least one person preferes x to y. This very natural relation isn't transitive. And second: only the antyreflexivity has evident extension to the fuzzy case.

An unfuzzy choice function is:

$$C: \quad 2^A \longrightarrow 2^A \quad , \qquad C(X) \subset X .$$

A fuzzy choice is considered here on the unfuzzy sets only, so the chioce function is here:

$$C:\ 2^A \to [0,1]^A,\ C(X) \subset X.$$

Note that $C(X) \subset X$ means $C(X)(x) \leq X(x)$. Becauce $X(x)=0$ for $x \notin X$ then $C(X)(x)=0$ for $x \notin X$, this condition will be later omitted but always implied.

Denote 1) $\mathcal{R}$ - a set of antireflexive relations and $\mathcal{R}^f$ - its fuzzy extension, 2) $\mathcal{L}$ - a set of choice functions and $\mathcal{L}^f$ - its fuzzy extension, 3) $\mathcal{R}_1$ - the set of all weak orders, asymetric and negative-transitive relations.

Remind some well-known properties of choice functions. Each of them has many names(and many authors), it is given only one here.

H (heritage condition): $C(X') \supset C(X) \cap X'$, $X' \subset X$.

C (Condorcet condition): $C(X) \supset \bigcup_{y \in X} C(\{x,y\})$, $x \in X$.

S (choice stability): if $X' \subset C(X) \subset X$ then $C(X') = X$.

I (independent of irrelevent alternatives): if $C(X) \subset X' \subset X$ then $C(X') = C(X)$.

K (Chernoff condition): if $X' \subset X$ and $X' \cap C(X) \neq \emptyset$ then $C(X') = C(X) \cap X'$.

Becauce all this conditions are expressed with operations $\cap$, $\cup$ and $\subset$ only, their fuzzy extension are trivial. So all this conditions are considered to be written for fuzzy sets.

3. RULES OF CHOICE AND REVEALED PREFERENCES

The rule of choice F allows to find an "optimal" set (may be fuzzy) for each X $\subset$ A if a preference relation is known. Formally: $F:\ \mathcal{R} \times 2^A \to [0,1]^A,\ F(R,X) \subset X,$

i.e. $F(R,.)$ for fixed R is a choice function. Hence the rule F defines the mapping from the set of relations $\mathcal{R}$ to a set of choice functions

There are two standart rules: first (F_1) - to select all x X each"better"that all other and second (F_2) - to select all x X that there is no $y \in X$ "better" than x.

Thus $F_1(R,X) = \{x \in X: xRy, \forall y \in X\}$,

$F_2(R,X) = \{x \in X: \neg\exists\, y \in X: yRx\}$.

This rules are equivalent in the following sence: put $\bar{R} = A \times A - R$ i.e. $x\bar{R}y$ iff xRy isn't true then $F_1(R,X) = F_2(\bar{R},X)$ (see, for example, Ajserman and Malishevsky (I98I)).

Note that F_1 is correct for reflexive relations and F_2 - for antireflexive ones. F_2 is more preferable becauce $R_2(X)$ is non-empty for more large class of relations.

The rule F_2 is named in game theory the core and denoted C

Therefore $C_R(X) = X - R(X)$, (I)

where $R(X) = \bigcup_{x \in X} R(x)$, $R(x) = \{y: xRy\}$.

$R(x)$ being considered as fuzzy set is represented by the membership function $R(x)(y) = R(x,y)$ so $R(X)(y) = \sup_{x \in X} R(x,y)$ if "$\cup$" is interpreted as "sup". If the negation is defined so that $(X - B)(x) = X(x) - B(x)$ then

$$C_R(X)(y) = I - \sup_{x \in X} R(x,y),\ y \in X, \qquad (2)$$

Definition 3.I If the rule F as a function F: $\mathcal{R} \to \mathcal{L}$ has the inverse function $R(C)$: $F(R(C),X) = C(X)$ then $R(C)$ is named the revealed preference.

Therefore the rule F determines the one-to-one mapping the sets $\mathcal{R}$ and $\mathcal{L}$ in this case.

4. THE THEOREM ABOUT REVEALED PREFERENCES

The following theorem was proved by many authors for unfuzzy case (may be in other form and for finite A), see, for example, Aizerman and Malishevski (I98I). Its fuzzy analogy was proved independent in Aizerman and Litvakov (I988) for special case and in Bondareva (I988). This theorem describes the class $\mathcal{L}$ corresponding to the class $\mathcal{R}$ of all antireflexive relations with respect to the core.

First prove the lemma.

Lemma 4.I.(of binarity). If the choice function C: $2^A \to [0,1]^A$ satisfies the conditions H and C then

$$C(X)(x) = \inf_{y \in X} C(\{x,y\})(x), \; x \in X .$$

Proof. Becauce $\{x,y\} \subset X$, from condition H

$$C(\{x,y\}) \supset C(X) \cap \{x,y\} . \tag{3}$$

The condition (4) being written for x is: $C(\{x,y\})(x) \geqslant C(X)(x)$ for each $y \in X$. Therefore $\inf_{y \in X} C(\{x,y\}) \geqslant C(X)(x)$. (4)

Condition C is rewritten as following:

$$C(X)(x) \geqslant \inf_{y \in X} C(\{x,y\})(x) \tag{5}$$

Hence from (5) and (4): $C(X)(x) = \inf_{y \in X} C(\{x,y\})(x)$ q.e.d.

This result is extended to the case of a choice from fuzzy sets, C : $[0,1]^A \to [0,1]^A$ in Bondareva(I988).

Denote $\mathcal{L}(H,C)$ the set of all choice functions satisfied the conditions H and C.

Propose that $C(\{x,y\})$ is nonempty for all $\{x,y\} \subset A$. Only such classes of choice functions are considered.

Theorem 4.2. The fuzzy core, the rule given by (2), defines the one-to-one correspondence the set $\mathcal{R}^f$ of all antireflexive relations and the set $\mathcal{X}^{f}(H,C)$ the revealed preference is

$$R(x,y) = I - C(\{x,y\})(y). \tag{6}$$

Proof. Show first that each C_R satisfies the conditions H and C. Let $X' \subset X$ then $C_R(X')(y) = I - \sup_{x\in X'} R(x,y) \geq I - \sup_{x\in X} R(x,y) = C_R(X)(y)$, $y \in X'$, because $\sup_{x\in X'} R(x,y) \leq \sup_{x\in X} R(x,y)$. Therefore $C_R(X') \supset X' \cap C_R(X)$ and H is true.

From definition of core: $C(\{x,y\})(y) = I - R(x,y)$ and $C_R(X)(y) = I - \sup_{x\in X} R(x,y) = \inf_{x\in X}(I - R(x,y)) = \inf_{x\in X} C_R(\{x,y\})(y)$ and C is hold.

Prove the inverse. Because $C_R(\{x,y\})(y)$ $I - R(x,y)$, the inverce function R must be $R(x,y) = I - C(\{x,y\})$ for arbitrary $C \in \mathcal{X}(H,C)$. From lemma 4.2: $C(X)(y) = \inf_{x\in X} C(\{x,y\})(y) = \inf_{x\in X}(I - R(x,y)) = I - \sup_{x\in X} R(x,y)$, so $C(X) = C_R(X)$ and proof is over.

Corollary 4.3 The conditions H and C are nessesary conditions for any class of relations to have revealed preferences via the core.

5. PROPERTIES OF REVEALED PREFERENCES

Because of theorem 4.2 it is possible some properties of relations to express with the aid of properties of corresponding choice functions.

Produce the known properties of unfuzzy preferences.

Proposition 5.I. The antireflexivity of a relation is equivivalent to conditions $C(\{x\}) = \{x\}$, $x \in A$.

Proposition 5.2. The nonemptyness of a choice from each finite set is equivivalent to acyclicity of revealed preference.

Proposition 5.3. Each revealed preference R defined by $C \in \mathcal{X}(H,C,I,D_3)$ is transitive where D_3: $C(\{x,y,z\}) \neq \emptyset$ for each triple $\{x,y,z\} \subset A$. If A is finite, the inverse is also true.

Note that the last isn't true for infinite A: there are transitive relations having the choice functions not belonged to $\mathfrak{X}(H,C,I,D)$. Show this by example. Let R be the relation: xRy iff $|x|>|y|$ and $xy>0$, on $A=(-\infty,\infty)$. Put $X=[-5,\infty)$ then for $X'=[-5,5]$ the condition I does not hold because $C(X')=\{-5,5\}$ but $C(X)=\{-5\}$ and $C(X)\subset X'$ but $C(X)\neq C(X')$.

Proposition 5.4. The class $\mathfrak{R}_1$ of wear orders is equivalent to $\mathfrak{X}(K,D_3)$.

From this it follows that each revealed preference $R(C)\in\mathfrak{X}(K,D_3)$ is a weak order.

6. THE POSTULATE OF REVEALED PREFERENCES. STRICT PREFERENCES

It is possible to do some transformations of the relation or the choice function in a problem of choice with a relation R. If the final choice is such that it is possible to reveal the initial relation R then any information isn't lost(if information is received, this is another problem). This condition is here named the principle of revealed preferences. For fuzzy extension this principle implies the following. Let unfuzzy $\mathfrak{R}$ is the set of revealed preferences for $\mathfrak{X}$ with respect to rule F; $\mathfrak{R}^f, \mathfrak{X}^f$ and F^f are their fuzzy extensions then F^f must be a one-to-one mapping from $\mathfrak{R}^f$ to $\mathfrak{X}^f$. So the theorem 4.2 means that the extension of the core (2) satisfies the postulate of revealed preferences.

Consider from this point of view the reduction of a relation R to the strong preference P: xPy iff xRy and not yRx. Only if R is reflexive and symmetric relation it is possible to reconstruct R from P. Hence only in this case it is possible to reveal both R and P from the choice function. The extension of this reduction to fuzzy relations doesn't satisfy the principle of revealed preferences. For example, Orlovski(I978) (see also Zhukovin, Burstein and Korelov(I987)) defines the relation R^S:

$$R^S(x,y) = \begin{cases} R(x,y) - R(y,x), & \text{if } R(x,y) > R(y,x) \\ 0 & \text{otherwise} \end{cases}$$

and the choice function $C(X)(y) = I - \sup_{x \in X} R^S(x,y)$.

As it is following from theorem 4.2, the relation R is a revealed preference but it is impossible to receive R from C (only R^S).

The second example is about the transitivity of a fuzzy relation. Many authors (see for example Batirshin(I979) and Ovchinnikov and Roubens(I989)) define the transitivity as:

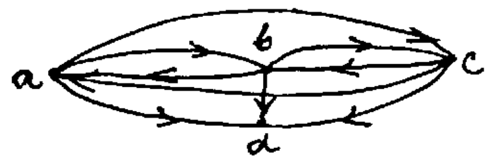
$$R(x.y) \geqslant \inf_{z \in A} (R(x,z), R(z,y))$$

According to the principle of revealed preferences this condition must follow from conditions C, H, I and D_3 (see proposition 5.3). It is difficult to verify this conditions becauce the fuzzy D is undefined. May be notions of transitivity, the strict relation, and so on, are specifically unfuzzy ones. Show that it is true for equivivalence relation. Let R be equivivalence relation on A then $C(X) = X$ or $C(X)(y) = I$ if X is unfuzzy. Becauce C is the core of negation relation (R is reflexive), the condition $C(\{x,y\})(y) = I$ implies $\bar{R}(x,y) = 0$. It means $R(x,y) = I$ for all negation functions (see Ovchinnicov and Roubens(I989)). Hence the equivivalence relation on unfuzzy set must be unfuzzy only.

7. MULTIVALENT CHOICE. FON NEUMANN-MORGENSTERN FUZZY SOLUTIONS

Returne ro rule F_1. Consider the example of two person choice: $A = \{a,b,c,d\}$ and two persons I and 2 have preferences relations

R_1 and R_2, the linear orderings expressed with profiles: R :abcd, R :cbad. Put $R = R_1 \cup R_2$ i.e. an alternative x is "better" than y for the couple if it is "better" at least for one. The graph of the relation R is following



An arrow from x to y means xRy. Note that each of a,b,c is "better" than each other object of choice. Note also that the core is empty. The one-point sets $\{a\}$, $\{b\}$ and $\{c\}$ are von Neumann-Morgenstern solutions in terms og game theory. It is need to define a multivalent choice for this notion to define correctly.

Definition 7.I. The multivalent choice function is

$$\overline{C}: 2^A \to 2^{[0,1]^A},$$

so that $\overline{C} = \{C\}$ where each $C: 2^A \to [0,I]^A$ is the ordinary choice function $C(X) \subset X$.

If there are no $C(X)$ for X,it means $\overline{C}(X) = \emptyset$.

For the above example $\overline{C}(A) = \{C_1(A),\ C_2(A),\ C_3(A)\}$ where $C_1(A) = \{a\}$, $C_2(A) = \{b\}$, $C_3(A) = \{c\}$.

Definition 7.2. Von Neumann-Morgenstern solution (NMS) of unfuzzy relation R on the set $X \subset A$ is

$$V_R(X) = X - R(V_R(X)) \tag{7}$$

A set $V_R(X)$ is the set with the properties:

I) internal stability: it is impossible xRy, $x,y \in V_R(X)$,

2) external stability: for any $y \notin V(X)$ there is $x \in V_R(X)$ such that xRy.

Denote $\overline{V}_R(X) = \{V_R(X)\}$ so $\overline{V}_R(X)$ is a multivalent choice function.

The multivalent choice and NMS as its example was investigated in Kitainik(I988).and in Bondareva(I988).

Each $C_i(A)$, $i=1,2,3$, is NMS in the above example.

Define the revealed preference via NMS as following: xRy iff

$\{x\} \in \bar{C}(\{x,y\})$. (8)

There is obvious extension of properties $\bar{C}$, $\bar{S}$, $\bar{I}$ of choice functions. For example $\bar{C}$ is: if x belongs to each $C(\{x,y\}) \in \bar{C}(\{x,y\})$ for each pair $\{x,y\}$, $y \in X$, then x belongs to each $C(X) \in \bar{C}(X)$. in symbols:

$\bar{C}$: $C(X)(y) \geq \inf_{C(\{x,y\}) \in \bar{C}(\{x,y\})} \inf_{x \in X} C(\{x,y\})(y)$, $y \in X$, $C(X) \in \bar{C}(X)$.

Similarly:

S: if X' is contained in some $C(X) \in \bar{C}(X)$ then each $C(X') = X'$.

I: if some $C(X) \subset X' \subset X$ then $C(X) \in \bar{C}(X')$.

Propose, as above, that $\bar{C}(\{x,y\}) \neq \emptyset$ for each $\{x,y\} \subset A$.

Theorem 7.3. The set $\mathfrak{R}$ of all antireflexive relations corresponds to the set $\mathcal{X}(\bar{C},\bar{S},\bar{I})$ of multivalent choice functions $\bar{C}(X)=\{V_R(X)\}$, and conversely the set $\mathcal{X}(\bar{C},\bar{S},\bar{I})$ defined by (8) the revealed preference $R(\bar{C})$ such that $\bar{C}(X)=\{V_{R(\bar{C})}(X)\}$.

This theorem is proved in Bondareva(1988).

Note that the ordinary choice function is also a multivalent choice function with $\bar{C}(X)=\{C(X)\}$ consisting from unique $C(X)$. So it is possible to find the intersection of the sets

$\mathcal{X}(C,H) \cap \mathcal{X}(\bar{C},\bar{S},\bar{I}) = \mathcal{X}(C,H,I)$ because $H \subset S$.

Corollary 7.4. For $\mathcal{X}(C,H,I)$ both theorem 7.3 and 4.2 are true and conditions (6) and (8) define the same relation.

Indeed, $\{x\}$ is unique NMS iff y doesn't belong to $C(\{x,y\})$.

Extend the notion NMS to the fuzzy choice.

The direct fuzzy extension of condition (7) is rather impossible, but the conditions $\bar{C}$, $\bar{S}$, $\bar{I}$ have trivial extension to the fuzzy choice on arbitrary sets.

Rewrite the conditions $\bar{S}$ and $\bar{I}$ for fuzzy sets in other form.

Condition $\bar{S}$ is: if $\mu' \subset C(\mu) \subset \mu$ then $C(\mu') = \mu'$. Denote $\lambda\mu$ the function $\lambda\mu(x) = \lambda(x)\mu(x)$ then each $\mu' \subset C(\mu)$ is expressed as $\mu' = \lambda C(\mu)$ where $0 \le \lambda(x) \le 1$, $x \in A$. So $\bar{S}$ is equivalent to $C(\lambda C(\mu)) = \lambda C(\mu)$, $\lambda \in [0,I]^A$. Similar $\bar{I}$ is expressed $C(\lambda C(\mu) - (I - \lambda)\mu) = C(\mu)$ where $(I - \lambda)(x) = I - \lambda(x)$

Hence fuzzy NMS satisfies the equations

$$\begin{cases} V(\lambda V(\mu)) = \lambda V(\mu), \\ V(\lambda V(\mu) - (I - \lambda)\mu) = V(\mu) \end{cases}$$

with fixed $\mu \in [0,I]^A$ and arbitrary $\lambda \in [0,I]^A$.

How to solve this equations is unknown.

8. A MULTIPERSON CHOICE AND REVEALED PREFERENCES

Consider a choice on A with n relations $R_1, R_2, \ldots, R_n$. Let the choice be multiperson (it is possible for him to be multicriterial).

There are some forms of the problem of multiperson choice::

I) for given $R_1, \ldots, R_n$ construct the relation $R = \varphi(R_1, \ldots, R_n)$, the preference relation of society, the group preference;

2) for given $R_1, \ldots, R_n$ construct the choice function $C(R_1, \ldots, R_n)$, the group choice function;

3) for given choice functions $C_1, \ldots, C_n$ of n persons construct the group choice function $C(C_1, \ldots, C_n)$.

Consider the example. In Bandyopadhyay(I985) it is investigated the problem in form 2 on the finite A. It is proposed that the revealed preference in the form: xRy iff $x \in C(\{x,y\})$ exists. The nonemptiness of choice is also proposed. In this conditions R satisfies H and C. From condition of Strong Dominance($\mathcal{SD}$): $C(\{x,y\}) \subset C(X) \cap \{x,y\}$ if $C(X) \cap \{x,y\} \neq \emptyset$, it follows that R satisfies K. Therefore R is a weak order. The condition of Weak

Pareto Optimality (PO) is: if x strong prefere to y for each person then xPy or xRy and not yRx. Becauce all R_i is proposed to be weak orders the result: if a social choice function is PO and SD there exists a dictator, is known. Note that the mixture the language of relation(I) and choice function(2) leads to the redundancy of conditions. For example SD with the proposition about above defined R means that if $x \in C(X)$ then xPy for each $y \notin C(X)$, i.e. $C(X)$ is not only a core of P but a NMS.

All further conditions will be given in form I, it isn't difficult to construct their equivalents in forms 2 and 3.

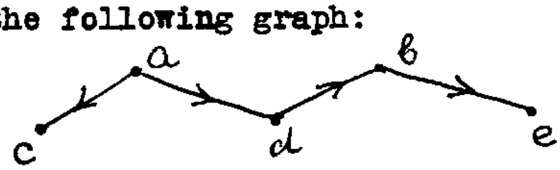
There are two principal conditions of multiperson choice.

PO (Pareto optimality): if $R_1 = \ldots = R_n$ then $\varphi(R_1,\ldots,R_n) = R_1$

AC (Arrow condition): $\varphi(R_1,\ldots,R_n)(x,y) = \varphi(R_1(x,y),\ldots,R_n(x,y))$.

The last is often named the condition of independence of irrelevant alternatives, but it isn't equivalent to I for choice functions with the same name. Show this on an example.

Let $A=\{a,b,c,d,e\}$, $R_1=\{(a,c),(a,d),(b,e)\}$, $R_2 = \{(a,c),(d,b)\}$. Note that both R_1 and R_2 are transitive. Show the preference $R = R_1 \cup R_2$ on the following graph:



Here $C_R(A) = \{a\}$. In the set $X=\{a,c,d,e\}$: e isn't dominated so $C_R(X) = \{a,e\}$ and condition I doesn't hold, although R satisfies AC.

The Arrow problem in our language is to find conditions for a one-to-one mapping to exists from $\mathcal{R}^n$ to $\mathcal{R}$. The Arrow theorem is following: if $\mathcal{R}$ is the set of all weak orders then each PO and AC function $\varphi(R_1,\ldots,R_n)$ equal to some R_i. This result is not seemed to be a paradox becauce the society preference must not be generally transitive if all personal preferences are transitive. Remind the function $R_0=\bigcup_{i=1}^{n} R_i$. This function is democratical in the sence that a is "better" than b for society if a is "better" for at least one of its member. In the choice function language it means $C_{R_0}(X)=\bigcap_{i=1}^{n} C_{R_i}(X)$, $X\subset A$. For weak orders $C_{R_0}(X)$ is the total maximum if it exists. $C_{R_0}(X)$ is often empty.

Another collective preference $R_N=\bigcap_{i=1}^{n} R_i$ leads to $C_{R_N}(X)=\bigcup_{i=1}^{n} C_{R_i}(X)$. $C_{R_N}(X)$ is the strong Pareto-optimum: for $x\in C_{R_N}(X)$ there is no $yR_i x$ for all i. The relation R_N is transitive for tfansitive R_i, $i=1,\dots,n$, but it isn't a weak order if all R are weak orders.

Both R_0 and R_N satisfie AC and PO.

All functions satisfied AC and PO are described as following (see Aizerman and Aleskerov(1983)). Name the coalition $S\subset\{1,\dots,n\}$. Let $\mathfrak{S}_{xy}$ be a system of coalitions for $\{x,y\}\subset A$. Each relation R satisfied AC is expressed in the form:

$$R(x,y)=\bigcup_{S\in\mathfrak{S}_{xy}}\bigcap_{i\in S} R_i(x,y),\quad x,y\in A. \tag{9}$$

This is true for infinite A also.

It isn't difficult to verify the following facts.

Proposition 8.1. The unfuzzy condition (9) describes the class of all unfuzzy relations $R=\varphi(R_1,\dots,R_n)$ satisfied AC and PO iff $\mathfrak{S}_{xy}\neq\emptyset$ for each $(x,y)\in A\times A$.

If all R are antireflexive then R is also antireflexive if $\mathfrak{S}_{xx}\neq\emptyset$.

9. A FUZZY MULTIPERSON CHOICE

The conditions PO and AC have a trivial fuzzy extension. Condition (9) is also read as fuzzy, rewrite him in operations "sup" and "inf":

$$R(x,y) = \max_{S\in \mathfrak{S}_{xy}} \min_{i\in S} R_i(x,y) \tag{I0}$$

Proposition 8.I isn't true in fuzzy case becauce not all relation satisfied PO and AC satisfy (I0). For example, relations

$$R(x,y) = \sum_{i=1}^{n} \alpha_i R_i(x,y)\ ,\ \alpha_i \geq 0\ ,\ \sum_{i=1}^{n} \alpha_i = I\ , \tag{II}$$

satisfy PO and AC but not (I0).

Becauce there is no obvious fuzzy extension of the weak order, it isn't seemed to exist the analogy of Arrow imposibility theorem.

As it has been mentioned the transitive personal relations generate an untransitive group relation. Similar it seems natural for unfuzzy personal relations to generate a fuzzy group relation. For example, the relation $R(x,y) = \frac{1}{n}\sum_{i=1}^{n} R_i(x,y)$ for unfuzzy R_i, $i = I,\dots,n$, is fuzzy. This relation generates all unfuzzy majority relations in a following manner. Put $Q = \{(x,y) \in A\times A : R(x,y) > \frac{k}{n}\}$, $k < n$, then $Q \subset A\times A$ is the relation: xQy iff at least k persons prefere x to y.

There are other espressions majority rules:

$$R(x,y) = \bigcup_{|S|\geq k} \bigcap_{i\in S} R_i(x,y)\ . \tag{I2}$$

This is a special form of (I0) with $\mathfrak{S}_{xy}$ independent of (x,y). Specifying the fuzzy operations (I2) is rewritten:

$$R(x,y) = \max_{|S|\geq k} \min_{i\in S} R_i(x,y)\ . \tag{I3}$$

Define $a_{i_0} = \underset{1\leq i\leq n}{k\text{-max}}\ a_i$ if there are k-I elements $a_i \geq a_{i_0}$. Then (I3) is rewritten $R(x,y) = \underset{1\leq i\leq n}{k\text{-max}}\ R_i(x,y)$, this is the fuzzy k-majority relation.

I0. CONCLUSION REMARKS

There are other aspects of fuzzy extension of choice both one-person and multiperson. The investigation of choice on infinite sets leads to notions of continuous relations and choice functions.

Let A be a compactum in Hausdorf space then sets being considered for choice must be closed. A relation is standart defined as continuous iff R(x) and R (x) y: yRx are open (or closed for reflexive R). This property isn't sufficient the revealed preference to correspondent to continuous choice function.

Name an antireflexive relation R strong continuous if for nets x x and y y : I) x Ry implies xRy if x y and 2) xRy implies x Ry for enough large (see Bondareva(I987)). (I4) This continuity of revealed unfuzzy preference R is equivalent to a continuity of C in exponential topology in 2 (in Hausdorf distance function for metric space). Note that if A E then continuous relations exist only for n I. Therefore unfuzzy choice isn't in general continuous in n-dimension Euclidean space for n I.

The fuzzy extension of (I4) is equivalent to continuity of the function R(x,y) always exept x y. The corresponding continuity of C generates the extension of the Hausdorf distance in the space of fuzzy sets. This notion is enogh complicate and will be investigated in other publications.

REFERENCES

Aizerman M.A., Aleskerov F.T.(I983)Arrow problem in the theory of group choice. Avtomatika i Telemekhanika 9, I27-I5I.

Aizerman M.A., Litvakov B.M.(I988)On some extensionsthe theory of variants choice. Avtomatika i Telemekhanika 3, 92-I04.

Aizerman M.A., Malishevski A.V.(I98I) General theory of best variants choice: some aspects.IEEE Trans. Automat. Control 26, I030-I040.

Bandyopadhyay T. (I985) Pareto optimality and decisive power structure with expansion consistency conditions. J. of Econ. Theory 33, 366-375.

Batirshin I.Z.(I979) On transitivity of fuzzy orders, in: Issledovanie Operazij i Analiticheskoe Proektirovanie v Technike. Kazanski Aviaz. Inst., Kazan, 67-73.

Bondareva O.N.(I987) A finite approximation of choice on infinite set.Izv. Akad. Nauk SSSR. Tekhn. Kibernetika I, I8-23.

Bondareva O.N. (I988) An axiomatization of the core and von Neuman-Morgenstern solution as functions of non-fuzzy anf fuzzy choice. Vestnik LGU 8, 3-7.

Kitainik L.M.(I988) Fuzzy binary relations and decisions procedures. Izv. Akad. Nauk SSSR. Tekhn. Kibernetika 6, I2-30.

Orlovsky S.A. (I978) Decision-making with a fuzzy preference relation. Fuzzy Sets and Systems I(3), I55-I67.

Ovchinnikov S. and Roubens M. (I989) On strict preference relations. Papers to be presented at the workshop on aggregation and best choices of imprecise opinions. Brussels, submitted to Fuzzy Sets and Systems.

Zhukovin V.E., Burstein F.V. and Korelov E.S.(I987) A decision making model with vector fuzzy preference relation. Fuzzy Sets and Systems 22, 7I-79.