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Robust Implementation Under Alternative Information Structures

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ROBUST IMPLEMENTATION UNDER ALTERNATIVE INFORMATION STRUCTURES[(1)]

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ABSTRACT: *In this paper we consider a model in which agents have complete information about their neighbors and, possibly, incomplete information about the rest of the economy. We consider three different informational frameworks. In the first, agents are ignorant about what is going on in the rest of the economy. In the second, agents are supposed to have beliefs about the unknown characteristics and in the third, there is complete information about the whole economy. We present a mechanism which implements any social choice function satisfying monotonicity and no veto power in all these three informational settings and therefore requires little knowledge from the point of view of the designer of the information in the economy.*

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I: INTRODUCTION

The Theory of Implementation studies the feasibility of achieving social goals when taking into account properly agents' incentives, i.e. the possibility of reconciliating utopia and self-interested behavior. Usually, an agent's behavior is modeled according to some game-theoretical concept which is suited to an informational framework. Social goals are described by a mapping, sometimes called Social Choice Rule (SCR in the sequel), which associates to each economy in a certain class, a feasible allocation which is assumed to be optimal. A Mechanism (sometimes also called a Game Form) is a description of the language in which agents communicate and the consequences of the messages (strategies) they send. A mechanism is said to implement a given SCR if, for any economy in the domain of this Rule, there are equilibrium messages (in some predetermined game-theoretical sense) and the consequences of these messages coincide with the allocation prescribed by the SCR.

A basic assumption in the Theory of Implementation is that the mechanism should be designed before the environment is known, but the designer is allowed to know how agents would behave, i.e. the equilibrium concept. This can be understood assuming that the designer has "knowledge about the knowledge" of agents. The purpose of this paper is to investigate the consequences of assuming that the mechanism should work, at least partially, independently of the structure of information in the economy and therefore, it should implement the SCF for some -and not just for one- equilibrium concepts.

A typical result in the Theory of Implementation says that, given a kind

of rational behavior (implied by the structure of information), some SCR can or can not be implemented in a certain range of environments. Thus, if agents use dominant strategies, implementation of acceptable SCR is essentially impossible (Hurwicz (1972), Gibbard (1973), Satterthwaite (1975), see also Ledyard-Roberts (1974)). However, implementation of acceptable SCR is possible, under certain conditions, if the equilibrium concept is Nash (Maskin (1977)). Moreover these conditions can be adapted to the case in which agents are assumed to be Bayesian maximizers (see Palfrey-Srivastava (1989)) if the designer knows the common prior. All these results suggest a trade-off between the information possessed by an agent about other agents -none in the case of dominant strategies, a common prior in the Bayesian framework and complete information in the case of Nash- and the possibility of implementing a satisfactory SCR.

In this paper we will consider the three cases mentioned above as regards the information in the economy with an additional assumption: throughout the paper we will assume that agents have complete information about a part of the environment. In other words, the economy is composed of islands, with a population of at least three agents each. Every agent knows exactly the preferences of other people on the same island. This is also common knowledge to the agents and the designer. With respect to the information about the rest of the economy, we will assume three different settings.

1. *Uncertainty.* Agents only know the possible types of agents outside their islands, as opposed to knowing their actual ones. Moreover, priors on types are meaningless. In this framework, the equilibrium strategy for an agent should be the best reply to what the rest of the agents in the island play, and to any possible message sent by agents outside her island when they follow their equilibrium strategies. The last requirement resembles dominant

strategies, the difference being that a strategy is dominant if it is a best reply independently of how other players behave, and in our case, the strategy must be a best reply independently of how other players (outside the island) are. This equilibrium concept has been used in the literature (without our assumption about islands) under the name of Uniform Nash Equilibrium (see d'Aspremont-Gerard-Varet (1979) and also Matsushima (1988) for a similar attempt).

2. *Risk.* In this framework, our equilibrium concept is the usual Bayesian Equilibrium introduced by Harsanyi (1967), with the aforementioned condition that players have complete information inside islands. The difference from the usual notion of Bayesian implementation is that we require a mechanism to implement a SCR, a) for any possible prior and b) when the designer does not know the exact prior. This amounts to requiring the mechanism to be designed before the planner has any knowledge of the agent's information. Therefore our implementation results here are, in this sense, stronger than the usual ones.

3. *Complete Information.* Here, our equilibrium notion is the usual Nash equilibrium. Agents have complete information inside, as well as outside, their islands.

We define a mechanism as *Robust* relative to a SCR, if it implements this rule in Uniform Nash Equilibrium, Bayesian Equilibrium (for any possible prior and with the designer ignorant of the actual prior) and Nash Equilibrium. This notion attempts to capture, at least partially, the fact that the mechanism should be flexible enough to cope with different informational settings.

Our main result is that there is a Robust mechanism which implements any SCR satisfying the well-known conditions of Monotonicity and Non-Veto Power introduced by Maskin in the framework of Nash Implementation. Thus, given our

assumption about islands, the fact that information is or is not complete makes no essential difference from the point of view of Implementation. Therefore our results can be understood as a generalization of those of Maskin (for a short proof see Repullo (1987)). The cost of this generalization is the common knowledge assumption on islands. It should be remarked that the paper does not make any progress in other important topics such as the reduction of strategy spaces (even though our strategy spaces are not much larger than those in Williams (1986), Saijo (1988) and McKelvey (1989)), the avoidance of modulo games (see Jackson (1988)) or the consideration of continuous and "realistic" mechanisms. Also, the possibility of coalition formation is not explored. All these points are left for future research.

The rest of the paper goes as follows. The next Section explains the basic model and our main assumptions. Section 3 is devoted to proving our results and, finally, Section 4 gathers our conclusions.

2: THE MODEL

In this Section we present our basic framework. Let N= (1,...,n) be the -finite- set of agents. Let $\mathcal{A}$ stand for the set of social alternatives. Let us denote by $\mathcal{R}_i$ a preorder (i.e. a complete, reflexive and transitive relation) defined on $\mathcal{A}$ for agent i. Let $\mathfrak{R}_i$ be the set of all possible preorders for agent i -which by simplicity will be assumed to be finite- and $\mathfrak{R} \equiv \mathop{X}_{i=1}^{n} \mathfrak{R}_i$. Let $\mathcal{R}$ be an element of $\mathfrak{R}$. Let us denote by $L(a, \mathcal{R}_i)$ the elements of $\mathcal{A}$ which are not preferred to $a \in \mathcal{A}$, according to the preference relation $\mathcal{R}_i$, i.e. the lower contour set of agent i relative to the preference relation $\mathcal{R}_i$. Let us define $a^M_i(\mathcal{R}_i) \subseteq \mathcal{A}$ as the set of maximal elements of $\mathcal{A}$ according to $\mathcal{R}_i$. It is assumed that this set is not empty for all possible preference profiles. This assumption holds if, for instance, preferences are continuous, and the feasible set is compact. We will also assume that for any agent i, the cardinality of $\mathfrak{R}_i$ is greater than or equal to three.

Let $\mathcal{F} : \mathfrak{R} \longrightarrow \mathcal{A}$ be the social choice function (SCF in the sequel). This function is assumed to embed the social objectives. In order to simplify the presentation, we assume that this mapping is single-valued but our results can be extended to the case of a social choice correspondence.

We now describe the informational framework. We will assume throughout the paper that there is a partition of N, $G = (G_1,...,G_s)$ with $\# G_i > 2$, i = 1,...,s such that each agent in a given element of G has complete information about all the characteristics of any agent in this element of the partition, and this is common knowledge for all agents in the partition. We may think of the economy as being composed of islands in the terminology introduced by

Lucas (1972) (but notice that here, any island is composed of at least three agents). Also, each agent i is assumed to have complete information about $\mathcal{A}$, $\mathcal{F}$ and $\mathfrak{R}$. This information is assumed to be common knowledge. Let G_k be a typical element of G and i a typical element of G_k.

With respect to the information possessed by an agent about other islands, we will consider three alternative set-ups which will be fully described later on. In the first one, agents will be assumed to act under uncertainty, i.e. they do not assign probabilities to the occurrence of states of the world (which are the types of agents living on other islands). In the second one, agents are considered to be Bayesian, i.e. they assign probabilities to the states of the world. In the third one we will assume that there is complete information on the whole economy.

We now define the strategic elements. A Mechanism (Game Form) is a pair $(\mathcal{M}, g)$ where $g : \mathcal{M} \longrightarrow \mathcal{A}$ and $\mathcal{M} = \mathop{X}_{i=1}^{n} \mathcal{M}_i$. g is the outcome function and $\mathcal{M}_i$ is the message space of agent i.

In the first informational setting, we assume that agents act in complete ignorance of the characteristics of any agent outside her group. Therefore each agent will play "Nash" against agents in her group and uniform Nash against any agent outside her group. Let $\mathcal{R}^i$ be the information of $i \in G_k$ about the preferences of agents in her group, i.e. $\mathcal{R}^i = (\mathcal{R}_j,...,\mathcal{R}_p)$ where j,...,p are all the agents in G_k. The set of all possible profiles in $\mathcal{R}^i$ is denoted by $\mathfrak{R}^i$. Also let us denote by $\mathcal{R}^{-(i)} = \mathcal{R} \setminus \mathcal{R}^i$ and by $\mathfrak{R}^{-(i)}$ the set of all possible $\mathcal{R}^{-(i)}$. A strategy for i is a function $s_i : \mathfrak{R}^i \longrightarrow \mathcal{M}_i$. Let $s_{(i)} = s_j,...,s_p$ be a tuple of strategies for all agents inside G_k and let $s_{-(i)}$ be a strategy tuple for agents outside G_k.

Definition 1. *$(s^*_1,\dots,s^*_n) = s^*$ is a Uniform Nash Equilibrium with Complete Local Information for a given profile $\mathcal{R}$ if $\forall$ i =1,...,n*

$$g(s^*_{(i)}(\mathcal{R}^{i}),\ s^*_{-(i)}(\mathcal{R}^{-(i)}))\ \mathcal{R}_i\ g(m_i,\ s^*_{(i)\setminus i}(\mathcal{R}^{i}),\ s^*_{-(i)}(\mathcal{R}^{-(i)})),$$
$$\forall\ \mathcal{R}^{-(i)} \in \mathfrak{R}^{-(i)},\ \forall m_i \in \mathcal{M}_i$$

Notice that the equilibrium strategy of any agent must maximize her utility for any possible message recommended by the mapping $s^*_{-(i)}(\mathcal{R}^{-(i)})$ for all agents outside G_k. Let UNE $(\mathcal{R}, \mathcal{M}, g)$ be the set of Uniform Nash Equilibria with Complete Local Information (or Uniform Nash Equilibria for short) for the Game Form $(\mathcal{M}, g)$ when the economy is $\mathcal{R}$.

Definition 2. *$(\mathcal{M}, g)$ implements $\mathcal{F}$ in UNE if $\forall \mathcal{R} \in \mathfrak{R}$*

a) UNE $(\mathcal{R}, \mathcal{M}, g) \neq \emptyset$

b) If $s^* \in$ UNE $(\mathcal{R}, \mathcal{M}, g)$, then $g(s^*) = \mathcal{F}(\mathcal{R})$.

We will say that $\mathcal{F}$ is implementable in *Uniform Nash Equilibrium* if there is a mechanism implementing $\mathcal{F}$ when agents behave according to Definition 1. Our definition of a Uniform Nash equilibrium does not pay attention to the case in which agents have priors. In order to deal with this case let us introduce a new informational setting. Let us denote by t a state of the world. We assume that all relevant information about the economy is embodied in t; i.e. t determines an element of $\mathfrak{R}$ and all the information that agents have. We also assume that the set of all possible states of the world T have a product structure, $T = \underset{i=1}{\overset{n}{X}} T_i$. Since we consider $\mathfrak{R}$ to be a finite set, it is natural to assume that T is finite as well. Given a state of the world $t \in T$ agent i observes her "type" $t_i \in T_i$. We write $t = (t_1,\dots,t_n)$ and $T_{-i} = \underset{j\neq i}{X} T_j$. Each element of T_i can be associated to an element of $\mathfrak{R}_i$ i.e., we assume the

existence of a function $\alpha_i : T_i \longrightarrow \mathfrak{R}_i$. Thus, $\alpha_i(t_i) = \mathcal{R}_i$ denotes agent i's preferences when her type is t_i. Sometimes we write $\alpha_i(t_i) \equiv \mathcal{R}_i(t_i)$, thus, given a state of the world, each agent knows her own preferences on the set $\mathcal{A}$.

We also assume that agent i's preferences, $\mathcal{R}_i$ can be represented by a von Neumann-Morgenstern utility function $U_i: \mathcal{A} \times T_i \longrightarrow R$ where $U_i(a / t_i)$ stands for agent i's utility of the social alternative a, when his type is t_i. We also write $U_i(a/\mathcal{R}_i(t_i))$.

Agents have beliefs about the state of the world $t = (t_1 \dots t_n)$. All of them have a common prior distribution p(t) on T. Let P be the set of all admissible common prior distributions p on T. In accordance with the previous ideas, we assume that there is a partition of N, $G = (G_1,\dots,G_s)$ with $G_i > 2$, $i = 1,\dots,s$ such that each agent in a given element of G, has complete information about the types of all the other agents in that element of G, i.e. agent $i \in G_k$ knows t_i and t_j for all $j \in G_k$, and this is common knowledge to all the agents in G_k. Thus, we can define the conditional probability distribution $q_{(i)}(t_{-(i)} | t_{(i)})$ by

$$q_{(i)}(t_{-(i)} | t_{(i)}) = \frac{p(t_{-(i)}, t_{(i)})}{P_{(i)}(t_{(i)})}$$

where $P_{(i)}(t_{(i)}) = \sum_{t_{-(i)} \in T_{-(i)}} p(t_{-(i)}, t_{(i)})$ and $t_{(i)} = (t_i, t_j, \dots, t_p)$ $t = i, j, \dots, p$ are all the elements of G_k and $t_{-(i)} = t/t_{(i)}$. From now on (i) will stand for the set of agents in the same element of the partition G as agent i, and -(i) for all the agents outside the element of the partition G where i belongs.

The message space for agent i is, again, denoted by $\mathcal{M}_i$. A strategy for i is a function $s_i : T_{(i)} \longrightarrow \mathcal{M}_i$. Let $s_{(i)} = (s_i, s_j, \ldots, s_p)$ and let $s_{-(i)}$ be a strategy profile for agents outside G_k where $i \in G_k$. We assume that N, $\mathcal{A}$, T, $\mathcal{F}$, $\mathcal{M}$, G, p(.) and g are common knowledge to all the agents.

Definition 3. *$s^* \equiv (s^*_1, \ldots, s^*_n)$ is a Bayesian Equilibrium with Complete Local Information if, for a given state of the world t and common prior distribution p(.) and for all $i \in N$, we have*

$$\sum_{t_{-(i)} \in T_{-(i)}} q_{(i)}(t_{-(i)} \mid t_{(i)})\, U_i\left[g\left(s^*_{(i)}(t_{(i)}),\; s^*_{-(i)}(t_{-(i)})\right) \mid R_i(t_i)\right] \geq$$

$$\sum_{t_{-(i)} \in T_{-(i)}} q_{(i)}(t_{-(i)} \mid t_{(i)})\, U_i\left[g\left(m_i, s^*_{(i)\setminus i}(t_{(i)}),\, s^*_{-(i)}(t_{-(i)})\right) \mid R_i(t_i)\right]$$

$$\forall m_i \in \mathcal{M}$$

This is the usual definition of Bayesian equilibrium for the information structure given above. Let $B(\mathcal{M},g,p,t)$ be the set of Bayesian Equilibria with Complete Local Information (Bayesian Equilibrium for short) for the Game form $(\mathcal{M},g)$ when the state of the world is t and the common prior distribution is p.

The standard Bayesian implementation approach assumes that the designer of the mechanism knows the common prior distribution p. In our model, on the on the other hand, the center does not need to know p. However, it must know the partition G which seems to us a less demanding informational requirement, at least in some cases

Definition 4. *The Game Form $(\mathcal{M},g)$ implements the SCF $\mathcal{F}$ in Bayesian Equilibrium with Complete Local Information if for all $t \in T$ we have*

1) $B(\mathcal{M},g,p,t) \neq \emptyset$ *for all* $p \in P$

2) *If* $s^* \in B(\mathcal{M},g,p,t)$, *then* $g\left(s^*(t)\right) = \mathcal{F}\left(R_1(t_1),\ldots,R_n(t_n)\right)$ *for all* $p \in P$.

We now study the third informational framework. In this case we will assume the existence of complete information throughout the economy. The equilibrium concept will be Nash Equilibrium. Therefore there is no distinction between messages and strategies. Formally.

Definition 5. *$(s^*_1,\ldots,s^*_n) = s^*$ is a Nash Equilibrium for a given economy $\mathcal{R}$ if $\forall$ i =1,...,n we have that*

$$g(s^*_i, s^*_{-i})\ \mathcal{R}_i\ g(m_i, s^*_{-i})\ \forall m_i \in \mathcal{M}_i$$

Let NE $(\mathcal{R}, \mathcal{M}, g)$ be the set of Nash Equilibria for the Game Form $(\mathcal{M}, g)$ when the economy is $\mathcal{R}$.

Definition 6. *$(\mathcal{M}, g)$ implements $\mathcal{F}$ in NE if $\forall \mathcal{R} \in \mathfrak{R}$*

a) NE $(\mathcal{R}, \mathcal{M}, g) \neq \emptyset$

b) If $s^* \in$ NE $(\mathcal{R}, \mathcal{M}, g)$ then $g(s^*) = \mathcal{F}(\mathcal{R})$.

Finally, we come to the main notion of the paper.

Definition 7. *Let $\mathcal{F}$ be a SCF. The mechanism $(\mathcal{M}, g)$ is robust relative to $\mathcal{F}$ if it implements $\mathcal{F}$ in Uniform Nash Equilibrium, Bayesian Equilibrium and Nash Equilibrium.*

The idea behind Definition 7 is that a mechanism is robust relative to $\mathcal{F}$ (in short, robust) if it implements $\mathcal{F}$ irrespectively of the information on the economy. Therefore the designer does not need to have much knowledge of the information that agents possess in the economy. Moreover, if agents acquire more information or change their priors, the proposed mechanism implements any $\mathcal{F}$ satisfying Monotonicity and No Veto Power (see Maskin (1977) for a definition of both concepts). Our main result is:

Theorem 1. *Any SCF which satisfies Monotonicity and No Veto Power can be implemented by a robust mechanism relative to $\mathcal{F}$*

We will first present the mechanism and the next Section will then be devoted to proving that this mechanism implements in Uniform Nash, Bayesian and Nash Equilibria.

Let the message space for agent i be given by

$$\mathcal{M}_i = \mathfrak{R}_i \times \mathfrak{R}_j \times \ldots \times \mathfrak{R}_p \times \mathcal{A} \times \mathcal{F}(\mathfrak{R}) \times \mathbb{N} \times \mathcal{A}$$

where i, j, ..., p are all the agents in group G_k, and $\mathcal{F}(\mathfrak{R})$ is the range of the SCF $\mathcal{F}$. We write an element of $\mathcal{M}_i$ in the following way

$$m_i = (\mathcal{R}^i_i, \mathcal{R}^j_i, \ldots\ldots \mathcal{R}^p_i, a_i, f_i, n_i, b_i).$$

Thus, $\mathcal{R}^j_i$ will be the "report" on agent j's preferences given by agent i. Sometimes we omit the superscripts and for example we write

$$m_i = (\mathcal{R}', \mathcal{R}'', \ldots.\mathcal{R}, a_i, f_i, n_i, b_i)$$

where $\mathcal{R}'$ is the "report" on agent i's preferences given by agent i, $\mathcal{R}''$ and $\mathcal{R}$ the report given by agent i on the preferences of agent j and agent p respectively. We now define the outcome function by the following three rules

Rule 1.- If $m = (m_1, \ldots, m_n)$ is such that $\mathcal{R}^j_i = \mathcal{R}^j_j$ for all $i,j \in G_k$, and $k = 1, \ldots, s$. Then, $g(m) = \mathcal{F}(\mathcal{R}^1_1, \ldots, \mathcal{R}^n_n)$

Rule 2.- If for all groups but one, say group G_k, we have the same condition given in Rule 1, and for group G_k we have the following: a unique $i \in G_k$ exists such that for some $l,x \in G_k$ $\mathcal{R}^l_i \neq \mathcal{R}^l_x$, and for all $u,v \in G_k$, $u \neq i$, $v \neq i$, we have $\mathcal{R}^z_u = \mathcal{R}^z_v$ for all $z \in G_k$. Then,

$$g(m)=\begin{cases} a_i, & \text{if } a_i \in L(f_i, \mathcal{R}^i_j) \text{ and } f_i = \mathcal{F}(\mathcal{R}^1_1, \ldots, \mathcal{R}^i_j, \mathcal{R}^j_j, \ldots, \mathcal{R}^n_n) \text{ where } j \in G_k. \\ \mathcal{F}(\mathcal{R}^1_1, \ldots, \mathcal{R}^i_j, \mathcal{R}^j_j, \ldots, \mathcal{R}^n_n), & \text{otherwise} \end{cases}$$

Rule 3.- $g(m) = b_i$ where $i = \max\{j: n_j \geq n_l, \text{ for all } l \in N\}$, if Rules 1 or 2 do not apply.

Rule 1 says that if there is total consistency on the reports on preference profiles, the mechanism selects the allocation given by the SCF $\mathcal{F}$ using the reported profiles.

Rule 2 takes care of the case when all agents but one send consistent reports on preferences. In this case the mechanism will choose the allocation a_i given by the "dissident" whenever 1) f_i coincides with the allocation recommended by the SCF using the preferences reported by all other agents and 2) this allocation belongs to the lower contour set of f_i using the preferences reported by all other agents on i. If a_i does not satisfy 1) and 2), the mechanism chooses the allocation recommended by the SCF when preferences are those reported by all agents but the "dissident".

In all other cases, i.e. when there is more than one "dissident" Rule 3 (the roulette) applies.

These rules allow for some possible interpretation of a_i, f_i and b_i as a choice, a guess and a best alternative in $\mathcal{A}$ respectively.

3: THE PROOF OF THE THEOREM

This Section will be devoted to prove Theorem 1. We will divide the proof in three Propositions. Each Proposition will show that the proposed mechanism implements a SCR in Bayesian, Uniform Nash and Nash Equilibria respectively.

Proposition 1.- *If the Social Choice Function $\mathcal{F}$ satisfies Monotonicity and No Veto Power, then the above Game Form $(\mathcal{M}, g)$ implements $\mathcal{F}$ in Bayesian Equilibrium with Complete Local Information.*

Proof: a) Firstly, we show that part 1 of Definition 4 is satisfied by $(\mathcal{M},g)$, i.e., $\forall t \in T$, $\forall p \in P$ we have that $B(\mathcal{M},g,p,t) \neq \emptyset$. Consider the "truth-telling" strategy $s^*_i: T_{(i)} \longrightarrow \mathcal{M}_i$ given by

$$s^*_i(t_{(i)}) = \left(\mathcal{R}(t_i), \mathcal{R}(t_j), \ldots, \mathcal{R}(t_p), a_i, f_i, n_i, b_i\right)$$

where a_i, f_i, n_i, b_i are arbitrary choices and $\mathcal{R}(t_s)$, $\forall\, s \in G_k$ are the true preferences of agent s. We show now that $s^* \in B(\mathcal{M},g,p,t)$. If all agents follow these strategies, Rule 1 applies and the outcome is $g\left[s^*(t)\right] = \mathcal{F}\left[\mathcal{R}(t_1), \ldots, \mathcal{R}(t_n)\right]$. Suppose that agent i chooses $m_i = (\mathcal{R}^i_i, \mathcal{R}^j_i, \ldots, \mathcal{R}^p_i, \bar{a}_i, \bar{f}_i, \bar{n}_i, \bar{b}_i)$ instead of $s^*_i(t_{(i)})$ with $(\mathcal{R}^i_i, \mathcal{R}^j_i, \ldots, \mathcal{R}^p_i) \neq (\mathcal{R}(t_i), \mathcal{R}(t_j), \ldots, \mathcal{R}(t_p))$. Therefore Rule 2 applies now, and the outcome is either $\bar{a}_i$ or $\mathcal{F}\left[\mathcal{R}(t_1), \ldots, \mathcal{R}(t_n)\right]$. In both cases the outcome cannot be preferred to $g\left[s^*(t)\right]$.

b) Secondly, we show that $(\mathcal{M},g)$ satisfies (2) in Definition 4. We first introduce two definitions and a Lemma.

We say that a vector of strategies s(t) yields a disagreement if, for some group G_k the preference profiles reported by its agents are not all the same, i.e. $\exists i,j \in G_k$ with $s_i(t_{(i)}) = (\mathcal{R}^1_i, \ldots, \mathcal{R}^p_i, \ldots)$ and $s_j(t_{(j)}) = (\mathcal{R}^1_j, \ldots, \mathcal{R}^p_j, \ldots)$ such that $\mathcal{R}^l_i \neq \mathcal{R}^l_j$ for some $l \in G_k$. We say that s(t) yields an agreement, whenever no disagreement is produced. Finally $s_{(i)}(t_{(i)})$ yields a local agreement in G_k if preferences reported on each other are the same. Notice that all these concepts are defined for a given t and $t_{(i)}$.

Let $n_i\left[s_i(t_{(i)})\right]$ be the natural number reported by agent i as part of her message when she follows strategy s_i. We define $\hat{n}(s) = 1 + \arg\max_{i,t_{(i)}} n_i\left[s_i(t_{(i)})\right]$. Thus Since N and T are finite sets $\hat{n}(s)$ is well-defined. Thus $\hat{n}(s)$ is greater than any possible number announced by any agent.

Lemma: *Let $s^* \in B(\mathcal{M},g,p,t)$. Suppose that for some t, $s^*_{(i)}(t_{(i)})$ yields a local agreement and $s^*(t)$ yields a disagreement. If $q_{(i)}(t_{-(i)}|t_{(i)}) > 0$, then $g\left[s^*(t)\right] \in a^M_i\left[R(t_i)\right]$ $\forall i \in G_k$.*

Proof: Suppose not. Then, $\exists t = (t_{(i)}, t_{-(i)})$, $q_{(i)}(t_{-(i)}|t_{(i)}) > 0$ such that $s^*(t)$ yields a disagreement, $s^*_{(i)}(t_{(i)})$ yields a local agreement and $g\left[s^*(t)\right] \notin a^M_i\left[\mathcal{R}(t_i)\right]$ for some $i \in G_k$. Suppose agent i deviates from $s^*_i(t_{(i)}) = (\mathcal{R}^i_i, \mathcal{R}^j_i, \ldots, \mathcal{R}^p_i, a_i, f_i, n_i, b_i)$ to $m_i = (\mathcal{R}^i_i, \mathcal{R}^j_i, \bar{a}_i, \bar{f}_i, \bar{n}_i, \bar{b}_i)$ where $\bar{\mathcal{R}}^i_i \neq \mathcal{R}^i_i$, $\bar{a}_i = \bar{f}_i$, $\bar{b}_i \in a^M_i\left[\mathcal{R}(t_i)\right]$ and $\bar{n}_i = \hat{n}(s^*)$. For the cases where $t_{-(i)}$ is such that $s^*_{(j)}(t_{(j)})$ yields a local agreement for all j, then agent i is indifferent between $s^*_{(i)}(t_{(i)})$ and m_i. In other words, in both cases the outcome will be the same, namely $\mathcal{F}(\mathcal{R}^1_1, \ldots, \mathcal{R}^n_n)$. For the cases where $t_{-(i)}$ is such that $s^*_{(j)}(t_{(j)})$ does not yield a local agreement $\forall j \notin G_k$, reporting m_i assures that Rule 3 applies and, in this case, the outcome is $\bar{b}_i \in a^M_i\left[\mathcal{R}(t_i)\right]$. Thus, by announcing m_l instead of $s^*_l(t_{(l)})$, agent i is strictly better off and this is a contradiction. Notice that this argument does not depend on the choice of any particular p as long as $q_{(l)}(t_{-(l)}|t_{(l)}) > 0$.

Now three cases must be considered.

(i) $g\left[s^*(t)\right]$ is given by Rule 1, i.e. $s^*(t)$ yields an agreement. In this case any agent i could have chosen her strategy

$$m_l = (\bar{\mathcal{R}}^1_l, \mathcal{R}^J_l, \ldots, \mathcal{R}^P_l, \bar{a}_l, \bar{f}_l, \bar{n}_l, \bar{b}_l)$$

instead of

$$s^*_l(t_{(l)}) = (\mathcal{R}^1_l, \mathcal{R}^J_l, \ldots, \mathcal{R}^P_l, a_l, f_l, n_l, b_l)$$

where $\mathcal{R}^1_l \neq \mathcal{R}^1_l$, $\bar{a}_l \in L(\bar{f}_l, \mathcal{R}^1_l)$, $\bar{n}_l = \hat{n}(s^*)$, $\bar{b}_l \in a^M_l\left[\mathcal{R}(t_l)\right]$ and

$$\bar{f}_l = \mathcal{F}\left[\mathcal{R}(t'_1), \mathcal{R}(t'_2), \ldots, \mathcal{R}^1_J, \mathcal{R}^J_J, \ldots, \mathcal{R}^P_J, \ldots, \mathcal{R}(t'_n)\right]$$

for some $t'_{-(l)}$. By the previous Lemma we know that if $\exists \bar{t}_{-(l)}$ and $u \notin G_k$ such that $s^*_{(u)}(\bar{t}_{(u)})$ does not yield a local agreement, then $g\left[s^*_{-(l)}(\bar{t}_{-(l)}), s^*_{(l)}(t_{(l)})\right] \in a^M_l\left[\mathcal{R}(t_l)\right]$. Thus, in this case agent i is indifferent between $s^*_l(t_{(l)})$ and m_l. For the rest of the cases i.e. for all $\hat{t}_{-(l)}$ such that $s^*_{(u)}(\hat{t}_{(u)})$ yields a local agreement for some $u \notin G_k$, announcing m_l yields the outcome $g\left[s^*_{-(l)}(\hat{t}_{-(l)}), s^*_{(l)}(t_{(l)})\right]$ whenever $\bar{f}_l \neq \mathcal{F}(\mathcal{R}^1_1, \mathcal{R}^2_2, \ldots, \mathcal{R}^n_n)$ and $\bar{a}_l$ otherwise. Then, it is clear that the reason agent i did not choose message m_l must be because $\bar{a}_l \in L(\bar{f}_l, \mathcal{R}^1_J) \Rightarrow \bar{a}_l \in L\left[\bar{f}_l, \mathcal{R}(t_l)\right]$. We want to show that $g\left[s^*(t)\right] = \mathcal{F}\left[\mathcal{R}(t_1), \ldots, \mathcal{R}(t_n)\right]$. Since we are in Rule 1, $g\left[s^*(t)\right] = \mathcal{F}\left[\mathcal{R}^1_1, \mathcal{R}^2_2, \ldots, \mathcal{R}^n_n\right]$ and $\mathcal{R}^1_J = \mathcal{R}^1_l$. Letting $\bar{f}_l = \mathcal{F}(\mathcal{R}^1_1, \mathcal{R}^2_2, \ldots, \mathcal{R}^n_n)$ we have $\bar{a}_l \in L\left[\mathcal{F}(\mathcal{R}^1_1, \mathcal{R}^2_2, \ldots, \mathcal{R}^n_n), \mathcal{R}^1_l\right] \Rightarrow \bar{a}_l \in L\left[\mathcal{F}(\mathcal{R}^1_1, \mathcal{R}^2_2, \ldots, \mathcal{R}^n_n), \mathcal{R}(t_l)\right]$, $i = 1,2,\ldots,n$, then by

monotonicity of $\mathcal{F}$ it follows that $\mathcal{F}(\mathcal{R}^1_1, \mathcal{R}^2_2, \ldots, \mathcal{R}^n_n) = \mathcal{F}\left[\mathcal{R}(t_1), \mathcal{R}(t_2), \ldots, \mathcal{R}(t_n)\right]$ and this proves that $g\left[s^*(t)\right] = \mathcal{F}\left[\mathcal{R}(t_1), \mathcal{R}(t_2), \ldots, \mathcal{R}(t_n)\right]$.

ii) $g\left[s^*(t)\right]$ is given by Rule 2. This implies that $\exists i, k \in G_k$ such that $\forall u \notin G_k$, $s^*_{(u)}(t_{(u)})$ yields a local agreement whereas $s^*_{(l)}(t_{(l)})$ does not. Let i denote the "dissident" agent, i.e. $\mathcal{R}^l_J = \mathcal{R}^l_v$, for all l, j, $v \in G_k$ and $j \neq i$, $v \neq i$. By the previous Lemma $\forall u \notin G_k$ we have that $g\left[s^*(t)\right] \in a^M_u\left[\mathcal{R}(t_u)\right]$. Furthermore $\forall j \in G_k$, $j \neq i$, $g\left[s^*(t)\right] \in a^M_J\left[\mathcal{R}(t_J)\right]$. To see this, notice that agent $j \in G_k$ could have chosen message $m_J = \left[\mathcal{R}^1_J, \mathcal{R}^J_J, \ldots, \mathcal{R}^P_J, a_l, f_l, \bar{n}_l, \bar{b}_l\right)$ where $\mathcal{R}^J_J \neq \mathcal{R}^J_n$ for all $n \in G_k$, $\bar{n}_J = \hat{n}(s^*)$ and $\bar{b}_J \in a^M_J\left[(t_J)\right]$. In this way agent j forces the mechanism to go to Rule 3 and since $\bar{n}_J = \hat{n}(s^*)$ the outcome is $\bar{b}_J$ (2). Thus, inevitably $g\left[s^*(t)\right] \in a^M_J\left[\mathcal{R}(t_J)\right]$. Therefore $\forall m \neq i$, $g\left[s^*(t)\right] \in a^M_m\left[\mathcal{R}(t_m)\right]$, and by no veto power of $\mathcal{F}$, $g\left[s^*(t)\right] = \mathcal{F}\left[\mathcal{R}(t_1), \ldots, \mathcal{R}(t_n)\right]$.

iii) $g\left[s^*(t)\right]$ is given by Rule 3. In this case it is clear that all agents obtain the best outcome, i.e. $g\left[s^*(t)\right] \in a^M_l\left[\mathcal{R}(t_l)\right]$ $\forall l$. Then by No Veto Power of $\mathcal{F}$, $g\left[s^*(t)\right] = \mathcal{F}(\mathcal{R}(t_1), \ldots, \mathcal{R}(t_n))$.

q.e.d

Proposition 2.- *If the Social Choice Function $\mathcal{F}$ satisfies Monotonicity and No veto Power, then the above Game form (M,g) implements $\mathcal{F}$ in Uniform Nash Equilibrium with Complete Local Information.*

Proof: This Proposition can be proved in a very similar way to Proposition 1. The first part (proving that *UNE* $(\mathcal{R}, \mathcal{M}, g) \neq \emptyset$) can be done in the same way as

(1) Notice that with more than two agents in each group, since the cardinality of $\mathcal{R}_J$ is greater than three this deviation is possible.

in proposition 1 where it is shown that $B(\mathcal{M},g,p,t) \neq \emptyset$. The second part of the proof -showing that if $s^* \in UNE(\mathcal{R}, \mathcal{M}, g)$ then $g(s^*) = \mathcal{F}(\mathcal{R})$, can be divided in three cases. The first one is when $g(s^*)$ is given by Rule 1. In this case, by using a similar argument to the one given in the proof of Proposition 1 and by Monotonicity of $\mathcal{F}$ it can be shown that $g(s^*) = \mathcal{F}(\mathcal{R})$. The second and third cases are when $g(s^*)$ is given by Rule 2 and Rule 3 respectively. For these cases an argument very similar to the one in the proof of Proposition 1, and the fact that $\mathcal{F}$ satisfies No Veto Power both prove that $g(s^*) = \mathcal{F}(\mathcal{R})$.

Proposition 3.- *If the Social Choice Function $\mathcal{F}$ satisfies Monotonicity and No Veto Power, then the above Game Form $(\mathcal{M}, g)$ implements $\mathcal{F}$ in Nash Equilibrium.*

This Proposition is a straightforward corollary of Proposition 2.

4: CONCLUSIONS

In the literature on Implementation, it is usual to assume a <u>given</u> equilibrium concept. An exception to this, is the work on Double Implementation (see Maskin (1985) and Schmeidler (1980)) which takes care of the problem of coalition formation requiring a mechanism to implement a given SCR in both Nash and Strong equilibria. The idea behind that, is that when designing a mechanism, the planner does not know whether agents will form coalitions or play non-cooperatively. Therefore the mechanism should be robust to the different possibilities that might arise. Our concept of Robust Implementation is in the same spirit, but applied to the problem of information. We require a mechanism to implement a SCR underthree different informational structures: uncertainty, risk and complete information (so it can be said that this mechanism "triple implements" a SCR). We have shown that with some minor assumptions -including that which assumes the SCR to be a function, priors are always strictly positive, and that a most preferred element in the social choice set $\mathcal{A}$ exists for every agent- the existence of complete information inside islands guarantees the existence of a Robust mechanism relative to any SCF satisfying the well-known conditions of monotonicity and no-veto power. Therefore, under our assumptions, the sufficient conditions for implementation in Nash Equilibrium turn out to be sufficient for implementation under Uniform Nash and Bayesian Equilibria as well.

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