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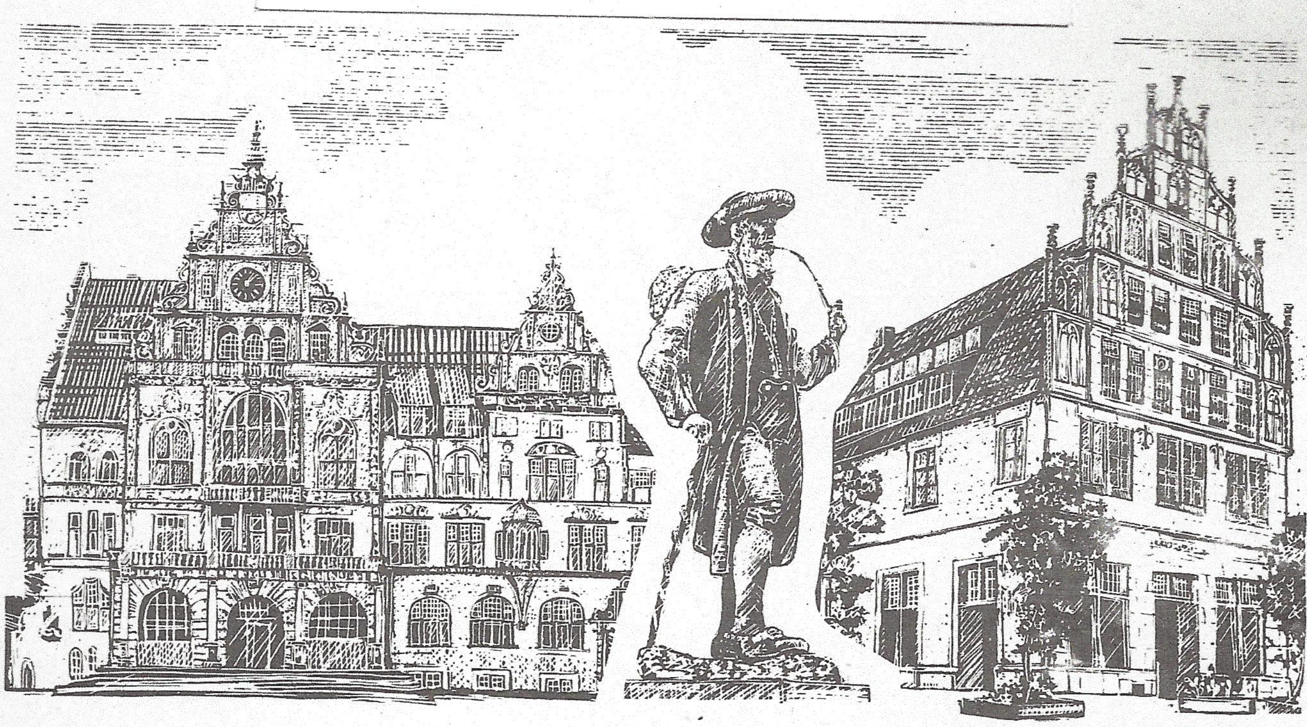

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IMPLEMENTING DEMAND EQUILIBRIA AS STABLE STATES OF A REVEALED DEMAND APPROACH

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Summary: The paper presents the a new approach to sequential bargaining in n-person games. Considered are sequences of dominating states which are restricted by demands revealed by the proposals and agreements reached in the intermediate states of the process. Stable states are defined in a recursive way, starting with states which by the restrictions of the revealed demands do not permit any further dominance. Several theorems concerning the stable states are given. The question is raised, under which conditions demand equilibria (ALBERS, 1974) can be implemented as stable initial states, i.e. as states which players would enter immediately when the bargaining process starts, and which they do not leave (by foresight). Examples show, that the new approach explains experimental results which essentially deviate from the predictions of traditional solution concepts.

1 THE IDEA OF DEMAND EQUILIBRIUM

The concept of demand equilibrium has been introduced by ALBERS (1974), later independently reinvented by TURBAY (1977), WOODERS (1978) and BENNETT (1979,1983). Different Variants of the model and relations to Kernel and other concepts are given in ALBERS (1979) and ALBERS (1981).

Efforts to transfer the concept to spatial games have been made by ALBERS (1979.a,1988.a), BENNETT/WINER (1983).

The basic idea of the concept is, that during the bargaining process the players develop certain demands, in a way that they will only enter a coalition when it fulfills the demands of all of its members. It is then the set of demands which is required to be "in equilibrium".

2 NOTATIONS

A characteristic function game (N,v) is a set N = (1,2,... ,n) (the "players", the subsets of N are called "coalitions"), and a worth-function v which assigns a real number v(S) to any subset S of N. It is assumed that the emty set and the one-player coalitions have worth zero.

A "demand vector" d = (d1,d2,.. ,dn) is a n-vector of reals (the "demands" of the players). The corresponding feasible coalitions are F(d) := (S subset N, d(S) not greater v(S)). The feasible coalitions of a player i in N are Fi(d) := (S in F(d), i in S).

2.1 DEFINITION: The "demand equilibrium" D(v) of a game (N,v) is the set of all demand vectors d = (d1,d2,.. ,dn) which fulfill

(1) (no slack) d(S) not smaller v(S) for all S in N
(2) (feasibility) Fi(d) is not emty for all i in N
(3) (independence) Fi(d) strict subset of Fj(d) for no pair i,j in N

The idea behind this definition tries to capture properties of reasonable endpoints of a dynamic process, which might lead to the demand equilibrium: A player who has no feasible coalition is supposed to reduce his demand (see (1)). If the sum of demans of a coalition are less than the worth of the coalition, then at least some members of the coalition can inrease their demands (see (2)). If Fi(d) is a strict subset of Fj(d) then player i "depends on player j" insofar that i cannot enter a coalition without of j, while j can form a coalition without of i; this means hat j can press (?) i to reduce his demand di by threatening him, otherwise to form a coalition without of i; j can at the same time inrease his demand dj (for instance for the same amount as i reduced his) and has still feasible coalitions. (This is the idea behind (3).) Of course, by the described actions, one condition can be reached to be fulfilled, but another one may become violated. So that a corresponding demand adjustment process will usually need quite a lot of steps to reach equilibrium. In fact additional conditions have to be given to make it sure that the process is finite and leads to a demand equilibrium.

3 EXCURSION: DEMAND CORE AND EXISTENCE OF DEMAND EQUILIBRIA

A solution concept which is mathematically essentially easier to handle than the set of demand equilibria is the demand core:

3.1 DEFINITION: d=(d1,..,dn) is in the "demand core", if

(1) (no slack) d(S) not smaller v(S) for all S in N
(2) (minimal sum) d(N) is minimal subject to (1)

The following theorems are known for the demand core:

3.2 THEOREM: The demand core is notempty for all characteristic function games. It is a cosed facet of the polyhedron of demand profiles fulfilling condition (1).

(In fact the demand core can be obtained by a linear program.)

3.3 REMARK: If a characteristic function game has a nonempty core, then the demand core is the core.

3.4 THEOREM: A demand vector d is in the demand core iff its set of feasible coalitions, F(d), is weakly balanced. It is in the relative interior of the demand core iff F(d) is strictly balanced.

3.5 COROLLARY: For homogeneous weighted majority games the demand core contains one demand profile, namely the vector of weights.

3.6 THEOREM: The demand equilibrium contains the relative interior of the demand core.

From this follows immediately:

3.7 THEOREM: The demand equilibrium is notempty for all characteristic function games.

4 STATES AND DOMINANCE

Approaching a model which is nearer to the explicit dynamics of a demand adjustment process, we consider states and dominance. A traditional approach will define states and dominance as follows:

4.1 DEFINITION (traditional): A "state" (x,S) is a coalition S (subset S of N), and a real-valued n-vector x such that (xi=0 if i not in S) and x(S)=v(S). - A state (y,T) "dominates" a state (x,S) (notation (y,T)<--(x,S)), if yi>xi for all i in T.

The model here is more explicit about the process of domination, i.e. the process of changing from one coalition to another. Here it is assumed that the players can and do agree to a new state in a certain order. Therefore we assume that a state does not only give a coalition S and a payoff distribution (within S), but also an order of S describing the sequence in which the players "are asked to agree to the state". So a state will be given by (x,S1,.. ,Sr), where S = (S1,.. ,Sr), and the indices give the order of S. (In abuse of language we will nevertheless write (x,S) instead, considering S as an ordered set.) In this new model state and dominance are given by

4.2 DEFINITION: A "state" (x,S) is a ordered subset S of N, and a real-valued n-vector x such that (xi=0 if i not in S) and x(S)=v(S) (?). - A state (y,T) "dominates" a state (x,S) (notation (y,T)<--(x,S)) if yi>xi for all players in T who are not after piv(T<--S). (Where piv(T<--S) denotes the first player of T who is also in S.)

Note that in this new approach a dominance from (x,S) to (y,T) is "performed" when the dominance has been agreed to by the first player of T who is also in S.

5 REVEALED DEMANDS

Experimetal results indicate that during a bargaining process the players develop ideas of adequate demands. This will be modelled by assuming that states are not only given by coalition and payoff distribution, but also by assumed or observed demands of the players. It seems that the following model of revealed demands captures the central aspects of the development of demans during a specific negotiation.

The basic idea is that the a player who supports a dominance (y,T)<--(x,S) indicates that he is incontent with his payoff xi and wants to get more than xi. I.e. his (exclusive) lower boundary of demand is from then on given by xi, and it is assumed that he will thereafter enter no state, in which he does not get more than xi. (In fact it is behaviorally unimportant, if he really has this minimum demand or not. It is only necessary that the other players think this, and therefore do not enter a coalition with him, where he gets less than his demand, assuming that he will break coalitions which do not fulfill his demand.) The corresponding model is as follows:

5.1 DEFINITION: A "state" (x,S,d) is an ordered coalition S and an n-vector x (as in the preceding definition), and a real-valued n-vector d (the "revealed demands" of the players). - A "domination" (y,T,e)<--(x,S,d) is given, if

(1)	(y,T)<--(x,S)	
(2)	yi>di	for all i in T
(3)	ei=max(xi,di)	for i = piv(T<--S)
(4)	ei=di	for all i in T, i notequal piv(T<--S) and for all i in N-T

This definition extends the old definition (see (1)). It says that a player can enter a new state (y,T) only when it fulfills his revealed demand (see (2)). And it informs about the way how demands are revealed: A player who "actively" changes from one state to another thereby indicates that he is incontent with his outcome in the preceding state, i.e. that his (exclusive) lower boundary of demand is at least at the preceding outcome (see (3)). A player who is not "actively" involved in the new decision keeps his preceding level of demand (see (4)). The alternative di in the maximum-condition of (3) makes sure that the demand of the active player piv(T<--S) does not decrease.

6 STABLE STATES IN THE REVEALED DEMAND MODEL

There is no doubt that a state should be defined to be stable when it cannot be dominated. Such states can be comparatively easily recognized. - But further conclusions are possible. A player who knows these stable states and has some foresight will probably not enter a new state (i.e. will not agree to perform a dominance (y,T,e)<--(x,S,d)), if thereafter the process moves to a next state which is stable and gives him less than in the first state (x,S,d). - Assuming this foresight behavior additional states can be recognized to be stable. And so forth. - This idea is captured in the following recursive definition:

6.1 DEFINITION:
(1) (start of recursion)
A state (x,S,d) is "stable", if there is no state (y,T,e)<--(x,S,d).
(2) (step of recursion)
Let certain states be recognised as stable.
A state (x,S,d) is "stable", if
for any state (y,T,e)<--(x,S,d) there is
a stable state (z,U,f)<--(y,T,e) such that zi<xi for i=piv(T<--S)

6.2 THEOREM: The set of stable states increases with every step of recursion.

PROOF: Let S(0), S(1), S(2),.. be the sets being recognized as stable in the beginning, in iteration 1,2,... of the procedure. Let (x,S,d) a counterexample with minimal iteration. I.e. (a) (x,S,d) in S(t) but not in S(t+1), and (b) for all s<t ((y,T,e) in S(s)) ==> (y,T,e) in S(s+1).
Since (x,S,d) is not in S(t+1), there is (y,T,e)<--(x,S,d) with no (z,U,f) in S(t) with (z,U,f)<--(y,T,e) which fulfills (2). However, since (x,S,d) in S(t), such an (z,U,f) exists in S(t-1). From the minimality of the counterexample follows that (z,U,f) is in S(t). This is a contradiction. //

6.3 COROLLARY: Let - in addition to the above procedure - within every step (2) a state (x,S,d) be defined to be "definitely unstable", when there is a stable state (y,T,e)<--(x,S,d).
Then the set of definitely unstable states increases with every step of recursion.

PROOF: Let U(1), U(2), .. be the sets of definitely unstable states recognized at iteration 1,2,.. of the procedure. And let (x,S,d) a counterexample, i.e. (x,S,d) in U(t), but not in U(t+1).
Since (x,S,d) is in U(t), there is (y,T,e) in S(t-1) with (y,T,e)<--(x,S,d). From the preceding theorem follows, that (y,T,e) is in S(t). So (x,S,d) is in S(t+1). //

Note that the definition of stability has some similarity with characterization of MC KELVEY, ORDESHOOK's competitive solution (see MC KELVEY ORDESHOOK and WINER, 1978) by FORMAN and LAING (1982). They require a solution to be a set of proposals, which is internally and externally stable. Internal stability means, that no two states of the solution set dominate each other (this is reached here by condition (2)). External stability means, that for any dominance via an alternative outside the solution set there is an alternative inside the solution set which dominates it (this follows immediately from (2)):

6.4 LEMMA: The set of stable states of a given game has the properties
(1) No stable state dominates another stable state.
(2) If a (nonstable) state (y,T,e) dominates a stable state (x,S,d), then there is a stable state (z,U,f) which dominates (y,T,e).

PROOF: 6.4.(1): Let (y,T,e)<--(x,S,d), and both states be stable. Assume that this is a counterexample, for which the repetition, in which the dominator (here (y,T,e)) has been recognized to be stable for the first time, is minimal. Since (x,S,d) is stable, it follows from 6.1.(2) that there is a stable state (z,U,f) <--(y,T,e). This alternative (z,U,f) must have been included in the set of stable states S(t) of repetition t. Since by 6.2 the set S(t) of states that are recognized to be stable increases with t, and (z,U,f) is stable and dominates (y,T,e), (z,U,f) cannot have entered any set S(r) with r>=t. So (y,T,e) must have entered the set of stable states for the first time in some S(r) with r<t. This must have been before (z,U,f) has been for the first time recognized to be stable, which

may have been in S(s), where r<=s<t. At the iteration S(r), when (y,T,e) has been first recognized to be stable there must (by 6.1.(2), since (y,T,e) is not undominated) have been a stable state (z',U',f')<--(z,U,f) which fulfills the conditions of 6.1.(2) as a counter to (z,U,f)<--(y,T,e). I.e. (z',U',f') has been recognized to be stable in S(r-1). So there are two stable states (z',U',f')<--(z,U,f), where the dominating state (z',U',f') was in the set of stable states S(r-1), with r-1<t. This contradicts the selection of (y,T,e)<--(x,S,d) with minimal repetition in which the dominator was recognized to be stable for the first time.
6.4.(2): Let (y,T,e)<--(x,S,d), and (x,S,d) stable. By 6.1.(2) there has been a stable state (z,U,f)<--(y,T,e) at the repetition, when (x,S,d) entered the set of stable states. From 6.2 follows, that (z,U,f) remains stable. //

There is also some similarity to AUMANN's and MASCHLER's bargaining set using arguments and counterarguments. However the criterion of a counterargument to be an adequate candidate for a solution is in that approach only given by the "demands" implicitly "revealed" by the original alternative. - By this shortcut they do restrict their considerations to "bargaining sequences" with not more than 3 states, while in the approach here also longer bargaining sequences are considered. The examples below show, that this approach gives essentially longer bargaining sequences. Moreover example 9.3 shows that bargaining sequences can drift away from initial states, although the bargaining set recognizes stability. (The different approaches to domination seem to be in this context less important, since both models could be easily modified in this respect.)

The following theorem permits to solve the decision, if a state is stable or not by a two-person game. This game is defined in advance:

6.5 DEFINITION: Let (x,S,d) a state of a game (N,v). The two-person "stability game of (x,S,d)" is as follows:
Both players alternatingly select states of (N,v), of which each dominates the preceding one. Player 1 moves first by giving (x,S,d). That player wins the game, who moves last.

6.6 THEOREM: A state (x,S,d) is stable, iff player 1 wins the stability game of (x,S,d).

PROOF: case 1: If (x,S,d) is stable, then player 1 must be able to find for any dominating state (y,T,e)<--(x,S,d) a stable state (z,U,f)<--(y,T,e) (by definition of stability. Moreover it follows from the stability of (x,S,d), that it is reached in finitely many applications of 6.1.(2). So player 1 can reach the set of undominated states in finitely many steps.
case 2: If (x,S,d) is unstable, then player 2 can react on any dominating state (that player 1 selects) with a stable state. The rest of the proof is as in case 1.

(Note that it can happen, that no player wins the stability game. this is the case when the state permits only dominations for which the counterdominations which avoid unstable states improve the outcome of the pivot-player of the first domination. (Compare example 9.3.))

This theorem permits to illustrate the stability of states by showing the paths of corresponding optimal strategies.

7 STABLE INITIAL STATES

We now assume that a bargaining sequence starts with the proposal of a player and follows the rules of domination, including the rules of revealed demands. We are interested in those states, which an initiator could be recommended to make in the initial state, i.e. in a state when not any demands are revealed. Candidates for this selection are those states, which are stable when no demands are revealed:

7.1 DEFINITION: A state (x,S,d) with di=0 for all i in N is called a "stable initial state".

(In the following the demand vector with all zero entries is denoted by 0. Stable initial states are denoted by (x,S,0).)

The question is, under which conditions stable initial states exist, and how these can be characterized. It seems that stable initial states are sometimes related to demand equilibria in the following way:

7.2 DEFINITION: A demand equilibrium d* is called "implementable" for a coalition S* in F(d*), if the initial state (d*/S*,S*,0) is stable.
(Where (d*/S*)i:= d*i (if i in S*) and (d*/S*)i:= 0 (if i in N-S*))

This gives the more specified question: Under which conditions is a demand equilibrium implementable for all/some of its feasible coalitions ?

The next lemma addresses states with "thin" feasible sets:

7.3 DEFINITION: A set F of coalitions is called "thin", if the intersection of any two coalitions S,T of F contains exactly one element.

(The definition could be modified in a way that it permits the intersection to be empty. However, this would permit additional cases with disjoint (feasible) coalitions, which we are not interested in for now.)

7.4 LEMMA: Let d* a demand equilibrium with feasible set F(d*).
If F(d*) is thin, then the proof of implementability or non-implementability can be restricted to sequences of dominations in which the pivotal player piv(T<--S) changes in every step.
(I.e. for any three subseding states (z,U,f)<--(y,T,e)<--(x,S,d) player piv(U<--T) is different from piv(T<--S).)

PROOF: The proof that a given state (x,S,d) (which is not undominated, and therefore obviously stable) is stable can only be done by showing, that every dominating state (y,T,e) is redominated by a state (z,U,f) with disimprovement zi<xi for the pivot piv(T<--S) of the first dominance. This is not possible when piv(T<--S) = piv(U<--S), since then zi>yi>xi.
The other type of error is made, when a state (x,S,d) is recognized as stable under the restriction of dominance, but in fact is not. The situation would then be as follows: (y,T,e)<--(x,S,d)<--(w,R,c) where the dominance to the stable state (y,T,e) is not captured in the analysis, since piv(T<--S)= piv(S<--R)=:i. We distinguish two cases:
case 1: R notequal T: since the revealed demands ej=dj=cj for all j in N-(i), and since yi>xi>wi, we get that (y,T,e)<--(w,R,c), what immediately shows that (w,R,c) is unstable. So the restriction of the analysis does only fail in recognizing a state (here (x,S,d)) wrongly as stable and thereby identifying another state (here (w,R,c)) as unstable, which under complete analysis is unstable too.

So no error has been made in this case.
case 2: R=T: again ej=dj=cj for all j in N-(i), and yi>xi>wi. Assume (w,R,c) is not the first state of the chain, and let (w',R',c') the state before. Then the dominance (x,S,d)<--(w,R,c)<--(w',R',c') with the wrong assumption, that (x,S,d) is stable can only be used, to show, that (w',R',c') is stable. However this is anyway not true, since (y,T,e)<--(w',R',c') and (y,T,e) is stable. This will be recognized in the approach with alternating pivots as well, since piv(T<--R')= piv(T<--R) and the dominance via piv(T<--R) was permitted.
case 3: R=T and (w,R,c) the initial state of the chain: In this case the result of the restricted analysis is that (w,R,c) is not stable since (x,S,d) is wrongly recognized to be stable. But (w,R,c) is also not stable under complete analysis, since the domination (y,T,e)<--(x,S,d), which is missing in the restricted analysis, does not counter the dominance (x,S,d)<--(w,R,c) in the sense of 6.1.(2) since yi>ci. //

(Note that the examples 9.1 and 9.2, below, give thin games, so that the analysis of stability can be restricted to bargaining sequences with alternating pivots.)

8 LOCAL IMPLEMENTABILITY OF DEMAND EQUILIBRIA

The property of a demand profile d* to be a demand equilibrium can be interpreted as "local", since it is completely given by local shape of the polyhedron A(v) := (x in Rn, x(S)>=v(S) for all S in N) in a neighbourhood of d*.

8.1 COROLLARY: Let d* a demand equilibrium of (N,v). Let (N,w) such that A(v) = A(w) in a neighbourhood of d*. Then d* is a demand equilibrium of (N,w).

The PROOF follows immediately from

8.2 LEMMA: Let d* a demand equilibrium of (N,v). Let (N,w) such that cone(d*,v)=cone(d*,w), then d* is a demand equilibrium of (N,w). (Where cone(d*,v):=(a*1S, S in F(d*), a real, a>0.)

Another way of expressing this local property is given by the following definition and remark:

8.3 DEFINITION: Let F a set of subsets of N. The "characteristic game of F" is the game (N,v) with
v(S)=1 if there is a set R in F which is contained in S. (admitting equality)
v(S)=0 otherwise.

8.4 REMARK: Let d* a demand profile, F(d*) the corresponding set of feasible coalitions, and (N,w) the characteristic game of F(d*). Then d* is a demand equilibrium, iff d* is a demand equilibrium of (N,w).

We now come back to the problem of implementability:

8.5 DEFINITION: Let d* a demand equilibrium, S* in F(d*). d* is called "locally implementable for S*", if for any e>0 there is a pair of (perfect) equilibrium-strategies of the corresponding stability game, of which any (selected) state (x,S,d) fulfills

(1) abs(xi-d*i)<e for all i in S
(2) di=0 or abs(di-d*i)<e for all i in N

8.6 THEOREM: Let d* a demand profile, S* in F(d*), and (N,w) the characteristic game of F(d*). Then d* is implementable for S* iff it is locally implementable for S*.

The proof uses the following contraction:

8.7 NOTATION: Let d* a real-valued n-vector.
For any real number e let f<d*,e> the function which maps any state (x,S,d) to f<d*,e>(x,S,d) such that

```
f<d*,e>(xi):=0                 (for all i in N  with  xi=0)
f<d*,e>(xi):=d*i+(xi-d*i)*e    (for all i in N  with  xi>0)
f<d*,e>(di):=0                 (for all i in N  with  di=0)
f<d*,e>(di):=d*i+(di-d*i)*e    (for all i in N  with  di>0)
f<d*,e>(S):=S
```

We call f<d*,e> "the contraction to d* by factor e".

8.8 REMARK: f<d*,e> maps a state (x,S,d) to a state iff d*(S)=v(S).

The PROOF of 8.6 is obtained by applying the contraction above. (Note that the set of demand profiles involved in the equilibrium strategies of the players is bounded.)

Our central question of further investigation is, whether implementability and local implementability are generally equivalent. If this were the case, then the next question would be, whether a demand equilibrium d* can be implemented iff it can be implemented in the characteristic game of F(d*).

9 EXAMPLES

The following example shows, that the revealed demand approach can leed to results implemented by the demand equilibria.

9.1 EXAMPLE: consider a 3-person quota game (N,v) with N=(1,2,3),
v(S)=120 if S contains at least two elements
v(S)=0 otherwise
The bargaining equilibrium of this game is d*=(40,40,40), the corresponding feasible coalöitions are F(d*)=(12,13,23).

For this game the following chains may be given, in which every state dominates the preceding one in the sense of the revealed demands approach:

x1	x2	x3	d1	d2	d3
40	40	--	--	--	--
40+a	--	40-a	40	--	--
--	40	40	40	--	40-a

This sequence illustrates, that ((40,40,0),(1,2),(0,0,0)) is an initial stable state of this game.

The next example shows, that the revealed demand approach is able to select specific feasible coalitions of a demand equilibrium:

9.2 EXAMPLE: consider the 5-person Apex Game, i.e. (N,v) with N=(1,2,3,4,5)
v(S)=100 if 1 in S and S has at least to members
v(S)=100 if S=(2,3,4,5)
v(S)=0 otherwise
The demand equilibrium is d*=(75,25,25,25,25), F(d*)=(12,13,14,15,2345).

For this game the following two chains may be given, in which every state dominates the preceding one in the sense of the revealed demands approach:

x1	x2	x3	x4	x5	d1	d2	d3	d4	d5	
75	25	--	--	--	--	--	--	--	--	
	25+a	25-b	25-b	25-b	--	25	--	--	--	(a=3b)
75		25			--	25	25-b	--	--	
75+c			25-c		75	25	25-b	--	--	
	25+d	25-e	25-f	25-f	75	25	25-b	25-c	--	(d=e+2f,e<b,e<=f,f<c)
75+g				25-g	75	25	25-b	25-c	25-g	(g<f)
75+h			25-h		75+g	25	25-b			(h>b,h>f)

x1	x2	x3	x4	x5	d1	d2	d3	d4	d5
75	25	--	--	--	--	--	--	--	--
75+a	--	25-a	--	--	75	--	--	--	--
	25	25	25	25	75	--	--	--	--

In both chains every uneven state has been selected in (conscious) response to the preceding one. Every even response (except for the respective second) has been selected for the coalition, which might cause the preceding deviator from switching. The result shows, that the state ((75,25,0,0,0),(1,2),(0,0,0,0,0)) is a stable initial state.

x1	x2	x3	x4	x5	d1	d2	d3	d4	d5	
--	25	25	25	25	--	--	--	--	--	
75-a	25+a	--	--	--	--	25	--	--	--	
75	--	25	--	--	75-a	25	--	--	--	
--	25+b	25+c	25-d	25-d	75-a	25	25	--	--	(b+c=2d)
75	--	--	25	--	75-a	25	25	25-d	--	
75+e	--	--	--	25-e	75	25	25	25-d	--	
--	25+f	25+f	25-f	25-f	75	25	25	25-d	25-e	(f<e)
75+g	--	--	25-g	--	75	25	25	25-f	25-e	(g<f)
75+f	--	--	--	25-f	75	25	25	25-f	25-e	
--	25+h	25+h	25-h	25-h	75	25	25	25-f	25-f	(h<g)

This sequence illustrates that ((0,25,25,25,25),(2,3,4,5),(0,0,0,0,0)) is not a stable initial state of this game.

The following example illustrates, that bargaining chains do not necessarily need to end near a quota solution:

9.3 EXAMPLE: Let (N,v) be the 4-person game, where three of four win, i.e.
N=(1,2,3,4), and
v(S)=120 if S has at least three members
v(S)=0 otherwise

x1	x2	x3	x4	d1	d2	d3	d4	
40	40	40	--	--	--	--	--	
40+a	40+a	--	40-b	40	40	--	--	(b=2a)
40+c	40+c	40-d	--	40+a	40+a	--	--	(d=2c)
..								
..								

This chain does not stop and can be completed in a way that it converges to (60,60,0,0). - On the other hand the players 3,4 cannot break this sequence, since none of them can leave the "block" (1,2) without of being excluded "for ever" in the subsequent move:

x1	x2	x3	x4	d1	d2	d3	d4	
..								
40+f	40+f	40-g	--	40+e	40+e	--	--	(d=2c)
40+h	--	40-i	40-i	40+f	40+e	40-g		(h=2i,i<g,h>f)
--	40+f	40-j	40-j	40+f	40+e	40-i	40-i	(j<i since h>f)

This shows that the intuitive prediction, namely that one of the minimal winning three-person coalitions will be formed with an equal split to each member, is not at all obvious. Experimental results uniquely support, that under free communication condition two players "block up" in just the way as players 1 and 2 in the example of the bargaining sequence above. This is a strong evidence which indicates that the revealed demand approach captures essential aspects of bargaining sequences which are not contained in the traditional models.

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