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Central Bank Policy Under Strategic Wage Setting

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Section 1:

Introduction

In the last years a growing literature on macroeconomic policy evaluation is concerned with stressing game–theoretic aspects of optimal policy design. Frequently, a government is playing a game against a sophisticated private sector as its counterpart in an economy. Usually the game is assumed to possess a hierarchical structure in the sense that the government, acting as the dominant player, has the power of getting carried out its policies within the private sector. Under such circumstances a government has a credibility problem because of the incentive to seek gains by reneging on a previous announced policy. In the absence of a pre–commitment power of the government such announcements become insignificant and the only sustainable equilibrium in the economy is an inferior Nash–equilibrium in which only a second best solution is realized.

Important recent works analyse the credibility problem by favouring the concept of repeated games or supergames[1]. In this context a government operates under the certainty that an attempt to cheat in any period is met by a loss of credibility or reputation subsequently. Provided a threat of loss of credibility is credible itself, a reputational equilibrium[2] exists which is subgame perfect[3]. This is then superior to the Nash–equilibrium in the single–stage game. In this analytical framework Barro and Gordon[4] consider a repeated macroeconomic policy game with the inability of the government to

[1] See the seminal contribution of
Friedman, J.W. (1971): "A Non–cooperative Equilibrium of Supergames", Review of Economic Studies, 28, pp. 1 – 12.

[2] See
Kreps, D.M. and R. Wilson (1980): "Sequential Equilibria", Econometrica, 50, pp. 863 – 894,
Selten, R. (1975): "Re–examination of the Perfectness Concept for Equilibrium Points in Extensive Games", International Journal of Game Theory, 4, pp. 25 – 55.

[3] Shortly, the perfectness of equilibria rules out equilibria which rely on non–credible threats and ensures that the conditions for the equilibrium are satisfied in every stage of the game. Because of the fact that the incentive to renege on announced policies is referred to as the time inconsistency of optimal policies it can be shown that subgame perfectness is sufficient but not necessary for time consistency.

[4] See
Barro, R. and D. Gordon (1983): " Rules, Discretion and Reputation in a Model of Monetary Policy", Journal of Monetary Economics, 12, pp. 101 – 121.

precommit itself to an announced zero inflation policy because of the anticipated incentive for the government to exploit gains from producing bouts of inflation. Then, an equilibrium with a lower rate of inflation can be sustained when the game is repeated and the government takes into account the effects of its current strategy on future credibility or reputation. Introducing asymmetric information about the preferences of the players is an important extension of this framework.

This paper analyses the effects of stabilization policy in a unionized economy. A government or a central bank[5] announces a policy rule at which in this model the government attempts to minimize unemployment and inflation and where the trade union sets nominal wages in order to reach its objective concerning employment and real wages.

In this paper after when we illustrate the credibility problem in Section 3 itself we analyse in the context of asymmetric information[6] about the characteristics of a rival the decision problem which follows from the observation that it may pay for an opponent of policymakers to attempt to compel the policy instance to reveal its own identity early on in a game.

The scenario can be formalized as an 'announcement' game in which one is looking for an equilibrium which is incentive compatible in the sense that no player can profit by making false statements about its preferences. Two types of equilibria are possible to describe policy choices: First, a separating equilibrium in which the preferences of the types of the government are revealed and neither type of government gives false information in the sense of cheating the other player, secondly, a pooling equilibrium in which the real preferences of the government remain unidentified. The aim is to find out in our labor market model which kind of equilibrium determines policy evaluation.

[5] If we consider the institutional aspect of the interaction of the central bank and the government we assume that the central bank is not independent from the government's political will. This describes the concrete situation in the UK whereas the situation in West–Germany is characterized by a central bank (Bundesbank) which can create the policy of money supply without direct influence of the government. Therefore we suppose a bargaining situation in which the union is trying to enforce its conception of the nominal wage for a period t and is confronted with the government as an opponent.

[6] We assume that only the government knows its own type and therefore the union has to decide under incomplete information.

Section 2:

The Model

Consider an economy in which the labor market is characterized by the demand for labor. Therefore the demand for labor in period t can be represented as a function of the real wage. This function is assumed to be negative exponential.

Formally,

$$N_t = f\left[\frac{w_t}{p_t}\right]$$

$$= \alpha_0 e^{-\frac{w_t}{p_t}}$$

with

$$\frac{dN_t}{d\left[\frac{w_t}{p_t}\right]} < 0 \text{ iff } (w_t, p_t) \in W_t^+ \times P_t^+$$

and

$$\frac{d^2N_t}{d^2\left[\frac{w_t}{p_t}\right]} < 0 \text{ iff } (w_t, p_t) \in W_t^+ \times P_t^+ ,$$

in which w_t denotes the nominal wage and p_t the price level; α_0 represents the slope of the function in question.

The labor market is not perfect competitive and all workers are organized by an encompassing union. In particular we assume that the union is able to set its most preferred nominal wage unilaterally. Moreover we assume that in each period the union sets the nominal wage in a way that maximizes its "expected" real wage bill

$$U_t := N_t \cdot \frac{w_t}{p_t} .$$

Formally this optimization problem has the structure

$$\max_{w_t} \; N_t \cdot \frac{w_t}{p_t}$$

subject to

$$N_t = \alpha_0 e^{-\frac{w_t}{p_t}} .$$

In game–theoretic language, w_t is the action parameter of the union. To capture the governments preferences for the stability of the price level and high employment, we specify a cost function for the government.

The costs for the government in period t increase quadratic in the rate of inflation and decrease linear in employment. Thus we suppose

$$G_t := \frac{1}{2}\gamma P_t^2 - \beta N_t ,$$

where $\gamma > 0$ and $\beta > 0$ must be fulfilled.

The governments objective is to minimize this function with respect to p_t in which p_t is the action parameter of the government. The structure of the last function shows that the cost of a policy increases if the government choose an inflation pattern and vice versa. Reversely the costs of the government decrease if it reaches a high level of employment.

Section 3:

Non-cooperative behavior in a single stage game

This section illustrates the basic problem of credibility in a full information one shot game, i.e. the time horizon of the players is one period only.

In the absence of precommitment the only sustainable equilibrium is when the government treats the setting of w_t as parametric and the union solves its optimization problem on the assumption that p_t is given.

Formally,

$$G_0 = \frac{1}{2}\gamma P_0^2 - \beta \alpha_0 e^{-\frac{w_0}{p_0}} \longrightarrow \min$$

given w_0 ,

and

$$U_0 = \alpha_0 e^{-\frac{w_0}{p_0}} \cdot \frac{w_0}{p_0} \longrightarrow \max$$

given p_0 ,

in which $t = 0$ represents the considered period.

The actors now solve under verification of the first order conditions their optimization problems simultaneously, and for the union we get

$$w_0 = p_0,$$

but for the government we must consider the equation

$$\gamma P_0^3 - \beta \alpha_0 e^{-\frac{w_0}{p_0}} \cdot w_0 = 0 ,$$

which is not explicitly representable as a function of the structure

$$p_0 = f(w_0).$$

A comparison of these two reaction functions gives

$$p_0 = \sqrt{\frac{1}{e} \cdot \frac{\beta a_0}{\gamma}} ,$$

and we have the Nash–equilibrium

$$(p_0^N, w_0^N) \in P_0^t \times W_0^t$$

with the explicit coordinates

$$p_0^N = \sqrt{\frac{1}{e} \cdot \frac{\beta \alpha_0}{\gamma}} \quad \text{and} \quad w_0^N = \sqrt{\frac{1}{e} \cdot \frac{\beta \alpha_0}{\gamma}} .$$

When the Nash–equilibrium determines the policy evaluation of the government the union's realized real wage in the economy is

$$\frac{w_0^N}{p_0^N} = 1$$

with utility of the union of

$$U_0(p_0^N, w_0^N) = \frac{1}{e}\alpha_0$$

and demand for labor of

$$N_0(p_0^N, w_0^N) = \frac{1}{e}\alpha_0$$

whereas the costs for the government are

$$G_0(p_0^N, w_0^N) = -\frac{\beta \alpha_0}{2e} .$$

We define

$$p_0^a := \mu < p_0^N \in P_0^+$$

as the announcement of the government, given that the institutional framework in the economy permits the government to commit itself to a preannounced rule for the price level in this period. When the government is following such a structure of policy design the game starts with the government announcement of its policy rule. Then the union sets the wage rate so as to solve its optimization problem, after which the price level is set according to the (mentioned above) rule.

In this situation the question may be important whether an equilibrium, given this behavior of the government which can be described in game–theoretical terminology, as Stackelberg–equilibrium with the government as leader is enforceable in the economy. The reason for this is that the government can gain from cheating now.

Step one: The government announces

$$p_0^a = \mu .$$

Step two: The union believes this announcement and chooses

$$w_0^a = \mu .$$

Step three: The government deviates from the announcement and takes

$$p_0 = p_0^N .$$

The resulting real wage is then

$$\frac{w_0^a}{p_0^N} = \frac{\mu}{\sqrt{\frac{1}{e}\frac{\beta\,\alpha_0}{\gamma}}}$$

which gives the labor market effect

$$N_0(w_0^a, p_0^N) = \alpha_0\, e^{-\frac{\mu}{\sqrt{\frac{1}{e}\frac{\beta\,\alpha_0}{\gamma}}}} .$$

This labor market effect must be compared with the effects on this market if the government follows its preannounced policy rule.

With

$$N_0(w_0^a, p_0^a) = \frac{1}{e}\alpha_0$$

we have

$$N_0(w_0^a, p_0^N) > N_0(w_0^a, p_0^a),$$

and this strong inequality is fulfilled if

$$\sqrt{\frac{1}{e}\frac{\beta\,\alpha_0}{\gamma}} > \mu .$$

In view of the costs that the government has to occur if one of the two policy design alternatives were implemented, we see

$$G_0(w_0^a, p_0^a) > G_0(w_0^a, p_0^N),$$

given, the union believes the preannounced rule of the government.

Proof: Assume $G_0(w_0^a, p_0^a) \leq G_0(w_0^a, p_0^N)$.

This weak inequality is fulfilled if

$$\sqrt{\frac{1}{e}\frac{\beta\,\alpha_0}{\gamma}}\,(1 + \ln 2 - \ln 3) \leq \mu$$

holds, because the logarithm's monotonicity gives

$$1 + \ln 2 - \ln 3 < 1.$$

If the government chooses $p_0^a = \mu$, it follows that

$$p_0^a = \mu < \sqrt{\frac{1}{e}\frac{\beta\,\alpha_0}{\gamma}}\,(1 + \ln 2 - \ln 3) < \sqrt{\frac{1}{e}\frac{\beta\,\alpha_0}{\gamma}} \in P_0^+ ,$$

however, the last weak inequality cannot be valid, hence

$$G_0(w_0^a, p_0^a) > G_0(w_0^a, p_0^N).$$

q.e.d.

Thus the inequalities

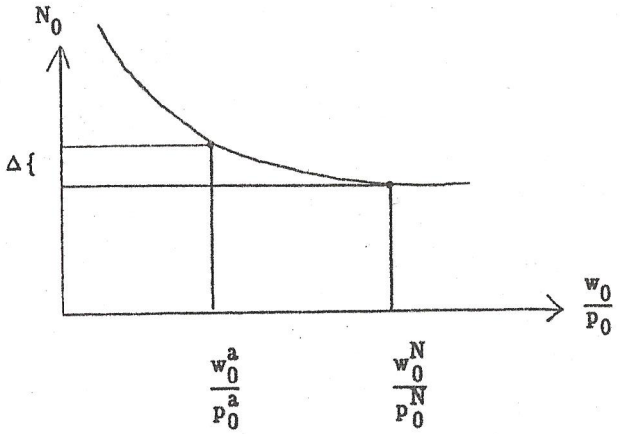
$$N_0(w_0^a, p_0^N) > N_0(w_0^a, p_0^a)$$

and

$$G_0(w_0^a, p_0^a) > G_0(w_0^a, p_0^N)$$

characterize the incentives for the government to differ from the announcement.

Graphically the figure



elucidates the gain of cheating for the government on the labor market Δ.

Given the union can understand this incentive an inferior open loop Nash–equilibrium arises from the lack of credibility of governments announcements. This anticipation of the incentive to cheat nullifies any possibility for the government to stimulate the economy over the creation of surprise inflation and the equilibrium is characterized by a rate of high inflation without positive influence on the labor market.

Section 4:

Preference Revelation as Method to Design Policies

The problem of the credibility of announcements in the full information context which has been discussed in section 3 can be extended in the strategic dimension if we consider a special optimization behavior of the union in the case of asymmetric information. To this end we analyze the question, given this structure of interaction, whether it could be possible that a government is forced to reveal its true preferences. Then an economy in game–theoretic terms is characterized by a separating equilibrium whereas in the other case a pooling equilibrium[7] describes the behavior of the players. The first kind of equilibrium ensures not only incentive compatibility of the implemented policies but also the credibility itself. In our model we use "incentive compatible" in the sense that no player can profit by making false statements about its preferences or that no player can profit from misrepresenting private information. When formalizing uncertainty about the preferences of the policy maker we consider a type concept at which the government can appear as type 1 given by

$$G_t(w_t, p_t) = \tfrac{1}{2}\gamma p_t^2 - \beta N_t$$

or type 2 given by

$$\tilde{G}_t(w_t, p_t) = \tfrac{1}{2}\gamma p_t^2 - \tilde{\beta} N_t$$

with

$$0 < \beta < \tilde{\beta}.$$

Obviously, for a policymaker of type 2, labor market effects are more important than for the other type.

The basic sequence of events begins with a government's announcement about the value of the parameter β or $\tilde{\beta}$, which capture the preference for the developments of the labor

[7] The concepts of a pooling equilibrium and a separating equilibrium were introduced by Milgrom, Roberts in
Milgrom, P. and Robers, J.: Limit Pricing and Entry under incomplete information: an equilibrium analysis, Econometrica, Vol. 50, No. 2, pp. 443 – 459.

market. Before the government can make this, a random equipment chooses its type and the result will be notified to it.

The union then sets the nominal wage conditional on this announcement and the possibility that the announcement is misleading. Denoting this choice of w_t by

$w_t(\beta)$ if the government has announced type 1

and

$w_t(\tilde{\beta})$ if the government has announced type 2.

Now the government specifies for each type a reaction function by

$$p_{\beta t}(w_t|\tilde{\beta}) = \sqrt{\frac{1}{e}\,\frac{\beta\,\alpha_0}{\gamma}}\,,$$

in which

$p_{\beta t}(w_t|\tilde{\beta})$ represents the fact of cheating of the government in the sense that it announces a false type

and

$$p_{\tilde{\beta} t}(w_t|\beta) = \sqrt{\frac{1}{e}\,\frac{\tilde{\beta}\,\alpha_0}{\gamma}}$$

in which

$p_{\tilde{\beta} t}(w_t|\beta)$ has the above interpretation.

Consequently we must consider cost functions for the government

$G_{\beta t}(w_t|\beta)$, $G_{\tilde{\beta} t}(w_t|\tilde{\beta})$ respectively

when it reveals the real type about an announcement, and

$G_{\tilde{\beta} t}(w_t|\beta)$, $G_{\beta t}(w_t|\tilde{\beta})$ respectively

when it sends a false signal.

Formally

$$G_{\beta t}(w_t|\beta) = \tfrac{1}{2}\gamma\,(p_{\beta t}(w_t|\beta))^2 - \beta\,\alpha_0\,e^{-\frac{w_t(\beta)}{p_{\beta t}(w_t|\beta)}}$$

$$G_{\tilde{\beta} t}(w_t|\tilde{\beta}) = \tfrac{1}{2}\gamma\,(p_{\tilde{\beta} t}(w_t|\tilde{\beta}))^2 - \tilde{\beta}\,\alpha_0\,e^{-\frac{w_t(\tilde{\beta})}{p_{\tilde{\beta} t}(w_t|\beta)}}$$

if the government doesn't cheat the union, and

$$G_{\beta t}(w_t|\tilde{\beta}) = \tfrac{1}{2}\gamma\,(p_{\beta t}(w_t|\tilde{\beta}))^2 - \beta\,\alpha_0\,e^{-\frac{w_t(\tilde{\beta})}{p_{\beta t}(w_t|\tilde{\beta})}}$$

$$G_{\tilde{\beta} t}(w_t|\beta) = \tfrac{1}{2}\gamma\,(p_{\tilde{\beta} t}(w_t|\beta))^2 - \tilde{\beta}\,\alpha_0\,e^{-\frac{w_t(\beta)}{p_{\tilde{\beta} t}(w_t|\beta)}}$$

if cheating is the aim of government's policy.

When the context of full information determines the decision situation then a Nash–equilibrium will characterize the policy result. In the case of asymmetric information there is an incentive for a type 2 government to conceal information about preferences in order to induce a strategy $w_t(\beta)$ when in reality $\tilde{\beta}$ represents the characteristic of the government, because it holds

$$\tilde{G}_t(w_t(\beta),p_t) < \tilde{G}_t(w_t(\tilde{\beta}),p_t).$$

We assume that it is possible for the union to understand this incentive structure of the government, i.e. it has the ability to form rational expectations and therefore $w_t(\beta)$ and $w_t(\tilde{\beta})$ will be chosen conditional on this uncertainty.

If we consider the construction of the cost functions as 'reaction functions' for each type of the government explicitly, it could be possible to secure an appropriate choice of the union that ensures that it never pays for a government to misrepresent its preferences. If this is realizable, then the policy choices in the economy could be described by a separating equilibrium.

In order to analyze this question, we must compare the payoff received by a specific type of government when it announces its preferences truthfully and the true preferences are strategically misrepresented. This consideration generates equilibrium strategies in the game in a natural way.

By using the inequalities above one obtains that if

$$G_{\tilde{\beta}t}(w_t|\tilde{\beta}) < G_{\tilde{\beta}t}(w_t|\beta)$$

and

$$G_{\beta t}(w_t|\beta) < G_{\beta t}(w_t|\tilde{\beta})$$

holds, a type β or a type $\tilde{\beta}$ government will reveal its preferences and if

$$G_{\beta t}(w_t|\beta) \geq G_{\beta t}(w_t|\tilde{\beta})$$

and

$$G_{\tilde{\beta}t}(w_t|\tilde{\beta}) \geq G_{\tilde{\beta}t}(w_t|\beta)$$

holds, the government will not reveal its preferences.

When the strong inequality holds that the policy actions in the economy will be made according to a separating equilibrium, whereas the validity of the weak inequality shows that a pooling equilibrium determines the behavior of the actors, where the former conditions may be written as

$$G_{\tilde{\beta}t}(w_t|\tilde{\beta}) - G_{\tilde{\beta}t}(w_t|\beta) < 0$$

and

$$G_{\beta t}(w_t|\beta) - G_{\beta t}(w_t|\tilde{\beta}) < 0.$$

Now the decision problem facing the union is to choose $w_t(\beta)$, $w_t(\tilde{\beta})$ which minimizes first its expected disutility and secondly ensures the revelation of the government's preferences. Considering the structure of the decision problem of the union gives the reason for a reformulation of it as a standard inequality constrained optimization problem in which $w_t(\beta)$, $w_t(\tilde{\beta})$ is chosen to maximize its expected utility, where ξ represents the probability for a type β government.

Formally the optimization problem of the union can be written as

$$\max_{\{w_t(\beta),w_t(\tilde{\beta})\}} E\,U_t(w_t(\beta),w_t(\tilde{\beta}),p_t)$$

$$= \max_{\{w_t(\beta),w_t(\tilde{\beta})\}} E\{\xi\,\alpha_0\, e^{-\frac{w_t(\beta)}{p_t}\frac{w_t(\beta)}{p_t}} + (1-\xi)\,\alpha_0\, e^{-\frac{w_t(\tilde{\beta})}{p_t}\frac{w_t(\tilde{\beta})}{p_t}}\}$$

s.t.

$$G_{\tilde{\beta}t}(w_t|\tilde{\beta}) - G_{\tilde{\beta}t}(w_t|\beta) < 0$$

$$G_{\beta t}(w_t|\beta) - G_{\beta t}(w_t|\tilde{\beta}) < 0$$

where

ξ : =prob (government is type β)

$1-\xi$: =prob (government is type $\tilde{\beta}$).

A technical solution of this problem can be obtained using the Kuhn–Tucker theory and the complementary slackness conditions. Rearrangements of the constraints in the union's optimization problems show that

$$\frac{w_t(\tilde{\beta})}{p_{\tilde{\beta}t}(w_t|\tilde{\beta})} - \frac{w_t(\beta)}{p_{\tilde{\beta}t}(w_t|\beta)} > 0$$

and

$$\frac{w_t(\beta)}{p_{\beta t}(w_t|\beta)} - \frac{w_t(\tilde{\beta})}{p_{\beta t}(w_t|\tilde{\beta})} > 0$$

hold such that

$$1 < \frac{w_t(\tilde{\beta})}{w_t(\beta)} < 1$$

is valid.

The weak inequalities

$$1 \leq \frac{w_t(\tilde{\beta})}{w_t(\beta)} \leq 1$$

show that

$$w_t(\tilde{\beta}) = w_t(\beta) ,$$

which implies that only a pooling equilibrium without preference revelation describes the behavior of the actors in the economy.

Section 5:

Concluding Remarks

A stylized analytical labor market model of two strategically interdependent economic actors is used to examine game–theoretical aspects of macroeconomic policy design. In this connection an 'announcement' game is considered in which an uninformed player (the union) with a special kind of optimization behavior attempts to design strategies which forces its opponent (the government) into revealing its true identity. If this is possible a separating equilibrium describes the economic behavior which is incentive compatible such that no player can gain advantage from misrepresenting private information. In the other case a pooling equilibrium determines the policy choices and the true character of the government is not revealed.

In order to solve this question we must consider the problem of asymmetric information from the perspective of examining correctly the incentive structure which motivates an opponent to conceal information. Moreover we look for mechanisms which are designed under the aspect of preventing this occurrence.

In the current framework the economy is steerable ony by a pooling equilibrium; in other words it is not possible for the union to coerce the government into pursuing a particular course of action such that a government's true identity is revealed.

An extension in our analytical framework gives the observation that a government or a special type of policymaker respectively follows a special optimization behavior. The reason for that is that it could be an advantage for a type to attempt to reveal his own identity early on in a game. If we consider the game structure in our model, it is obvious that the policy design can be considered to be somewhat biased against a type–2 policymaker. Therefore a type–2 player suffers, even if no preference of cheating characterizes its behavior. In the case of uncertainty about the true type of the policymaker with what the union is confronted in our scenario the remarks before describe an incentive of a policymaker of this type to reveal its true preferences early on in a game.

In the terminology of game theory a separating equilibrium should be represented by the policy sequences in the view of a type–2 policymaker.

Analyzing this question is the focal point in a following paper.

Section 6:

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