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Permits or Taxes?
How to Regulate Cournot Duopoly
with Polluting Firms

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Abstract

We consider an asymmetric Cournot Duopoly with firms facing a linear Leontief Technology. A quantity of a pollutant is generated proportional to the quantity of output. The government may regulate the firms by imposing Pigouvian taxes or giving out a number of tradable permits for emitting a certain quantity of pollutants. We characterize the optimal Pigouvian tax as well as the optimal number of permits as a function of a damage parameter. Under imperfect competition, social optimum is in general not enforcable. None of the two policies can be said to be superior to the other in general. For a wide range of parameters, however, the permit policy turns out to be superior.

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1 Introduction

The more seriously societies become involved into environmental pollution problems the louder becomes the public's call to take steps and to design environmental regulatory policies. The policy implemented most frequently in order to reduce the aggregate output of pollutants is to impose uniform standards of pollution levels on the firms. From economic theory we know for almost 30 years that there are more efficient tools than that, like charging Pigouvian taxes on pollutants or to give out a number of marketable permits for pollutants. It is well known that under certain (strong) assumptions both regimes are equivalent and yield the socially efficient outcome if the optimal Pigouvian tax is charged or the optimal number of permits is given out (for an exposition see for example BAUMOL and OATES [2]). SPULBER [12] demonstrates that taxes or permits are also optimal in the long run if the number of firms is determined endogenously.

For this results to hold it is necessary to assume that the government has complete information about the industry's aggregate abatement cost as well as about the social damage which is assumed to depend only on aggregate emissions. Moreover, it has to be assumed that the polluting firms supply their marketable output on a competitive market and *also* behave as price takers on the market for permits if there is any.

Weitzman showed that under incomplete information about aggregate abatement cost and damage, either of both tools can yield a higher welfare, contingent on the ratio of slopes of marginal abatement cost and marginal social damage. Further approaches on incomplete information have been pursued by ROBERTS and SPENCE [9] and also by KWEREL [5], who propose a mixture of measures consisting of permits, taxes and subsidies on abatement. SPULBER also considers incomplete information on the firms' abatement cost in [13]. All these models are partial analyses where firms are assumed to behave as price takers.[1]

Very few has been done, however, on regulating polluting firms which engage in imperfect competition, that is, have market power on either market. HAHN [3] studies a model where one big firm has market power on the market of permits, the remaining firms behave as price takers. He shows that the final allocation of permits depends on the initial allocation and will be inefficient in general. MALUEG [6] considers a Cournot oligopoly on the output market, however, without explicitly considering the "pollution

[1]There is much more literature on taxes and permits under price taking behavior, which we cannot all give credit here. For an excellent overview of different kinds of permit trading see TIETENBERG's book [15].

technology".

According to our knowledge there do not exist oligopoly models where firms engage in imperfect competition, and where the output as well as the pollution sector are treated simultaneously. The reason may be that very few can be said under fairly general assumptions. For example, the analysis of imperfect competition starts to become interesting if firms are assumed to have *different* technologies since otherwise uniform standards work quite all right.

In this paper, we set up a very simple asymmetric duopoly model. Each of two firms owns a Leontief technology, that is, they face constant marginal cost and produce a pollutant proportional to the output of the marketable commodity.

We pursue partial analysis for one marketable commodity and assume that a certain pollutant will be generated by this industry only. (Partial) social welfare is additively separable in consumers' surplus, social damage of pollutant and production cost. The government has complete or almost complete information about the firms' technologies, market demand, and social damage (for a brief discussion of the underlying information structure see section 4). First, we derive the social optimum and then look for the optimal linear tax as well as the optimal number of permits, taking into account the firms' strategic behavior on the output market as well as on the market for permits if there is one. It will turn out that the optimal (linear) tax will be nondecreasing as a function of the s, the slope of marginal damage which is assumed to be a linear function of aggregate pollution. For low s the Pigouvian tax will be negative. In other words, if social damage from pollution is low, pollution will be subsidized in order to increase output, a seemingly perverse phenomenon, and indeed, in the real world we are more concerned with over-pollution (high social damage) rather than with under-pollution.

The optimal number of permits, on the other hand, is nonincreasing in s, maybe discontinuous for some value of s, and may be constant on some interval for s. Both regimes yield the social optimum, allowing only the less polluting firm to produce, if s is sufficiently high. Under the permit regime, however, the social optimum is achieved for a greater range of parameters for which it is socially desirable that only the "cleaner" firm produces. This yields an argument in favor of permits under imperfect competition if social damage is high. For very low social damage from pollution, the permit regime turns out to be undesirable since the lower cost firm exploits the regime by buying all the permits and exercises monopoly power. For intermediate values of the social damage parameter, few can be said in general. Depending on the constellation of parameters, either of both regimes may yield a higher welfare. Recently, the president of the *Institut fuer Weltwirtschaft*, Prof. H. Siebert argued in one of the leading

German political magazines, *Der Spiegel*, in favor of permits by explaining its idea by the example of power plants: the modern power plant, buys all the permits from the odd one compensating it for closing down. This work supports his argument if the odd firm is sufficiently worse or if social damage is sufficiently high (and the corresponding number of permits is low), however, not for *all* values of s for which it is socially desirable that the worse polluter closes down!

The paper is organized as follows: In the following section we set up the model. Section 3 presents the social optimum. In section 4 we briefly discuss the underlying information structure for the tax and the permit regime. In sections 5 and 6 we develop the optimal linear tax, and the optimal number of permits, respectively. In section 7 we compare the two regimes followed by some numerical examples. The last section contains some final remarks. All formal proof are given in the appendix, but will only be sketched whenever standard calculations apply.

2 The Basic Model

Throughout this paper we will consider a Cournot duopoly with firms $i = 1, 2$ setting quantities q_1, q_2. The price is determined by an inverse demand function P, with $P' < 0$, which depends on aggregate output $Q = q_1 + q_2$. Both firms have constant marginal costs c_1 and c_2, with (w.l.o.g.) $c_1 \leq c_2$. Production is not possible without pollution. Producing q_i units of output, firm i produces $e_i = d_i q_i$ units of emissions. Total emissions are written $E = e_1 + e_2$. To evaluate utility and harm of Q and E to the society, we assume to have a partial social welfare function $W(q_1, q_2)$.[2] In the absence of pollution, in the industrial economics literature, a social welfare is simply taken as $W(q_1, q_2) = \int_0^Q P(z)dz - c_1 q_1 - c_2 q_2$, that is, consumers' gross surplus minus aggregate production costs.[3]

We will extend this approach by assuming that benefit of production and damage of pollution are additively separable. This means, in addition to consumers' surplus there is a social damage function S, which depends on aggregate emissions, and which is positive, strictly increasing and strictly convex. The assumption of additivity may be criticized since marginal utility of the marketable output may decrease with the level

[2] We call it *partial* since we neglect income effects and externalities on *other* markets.

[3] This is equivalent to $W(q_1, q_2) = \int_0^Q P(z)dz - P(Q) \cdot Q + (P(Q)q_1 - c_1 q_1) + (P(Q)q_2 - c_2 q_2)$, that is, net surplus plus profits of the firms. Some authors use the latter, and sometimes even multiply surplus and profits with different weights (see for example Baron and Meyerson [1]). Then, however, the two concepts are not equivalent.

of pollution, such that demand for the product goes down if emissions increase. It may also be the other way round if, for example, the marketable product consists of some protective device against the impact of pollution. The last possibility, however, seems to be most unlikely in the light of the following crucial assumption of the model.

Assumption 1 *The pollutant resulting from production of the industry's output, only arises in this industry.*

Under this assumption it is not likely to have an industry which produces a protective device which will only be needed because of the negative externalities of its production. Of course, pollution sometimes *does* influence the utility drawn from consumption of certain commodities. Observing peoples behavior, however, pollution does not seem to have a significant impact on the demand of other consumption goods. In particular, it is unlikely that this impact is significant with respect to demand for the output of the polluting industry under consideration.

Assumption 1 does not hold in *all* industries, of course. For example CO_2, is generated by many different industries. SO_2, on the other hand, is generated basically by power plants. Also in the chemical industry, some poisonous pollutants are generated from production of one certain commodity. Since we want to analyze regulation of firms under imperfect competition, Assumption 1 is crucial to make the analysis interesting.

Assuming separability of social welfare in consumers' surplus, production cost and social damage, the welfare function is given by

$$W(q_1, q_2) := \int_0^Q P(z)dz - S(E) - c_1q_1 - c_2q_2 \tag{2.1}$$

To keep the model mathematically tractable we further assume inverse demand to be linear, that is, $P(Q) = D - dQ$. W.l.o.g. we normalize units such that $D = 1$, $d = 1$, and $c_i < 1$ for $i = 1, 2$, leading to

$$P(Q) = 1 - Q\ , \tag{2.2}$$

The social damage function is assumed to be quadratic:

$$S(E) = \frac{s}{2}E^2\ , \tag{2.3}$$

where $s \geq 0$ is the damage parameter.

Without any kind of regulation, Cournot competition leads to the known Cournot-Nash equilibrium with quantities $q_i = (1 - 2c_i + c_j)/3$ for $i = 1, 2$, $i \neq j$, independently of s.

Before turning to regulatory policies, let us derive the social optimum a fictive social planner would install under complete information. If $c_1 < c_2$, it is clear that for $s = 0$ the higher cost firm 2 should not produce anything. If social damage is very high, one should think that only the firm with the relatively lower pollution level per unit of output should operate, that is, with the smaller d_i. However, it is not quite like this.

3 The social optimum

The social planner has to solve the following program:

$$\max_{q_1,q_2} W(q_1, q_2) := \max_{q_1,q_2} \int_0^{q_1+q_2} P(z)dz - S(d_1q_1 + d_2q_2) - c_1q_1 - c_2q_2 \tag{3.1}$$

s.t. $q_1 \geq 0$, $q_2 \geq 0$.

The following propositions yield the properties of the optimal solution:

Proposition 3.1 *If*

$$\frac{d_1}{1 - c_1} \leq \frac{d_2}{1 - c_2}\ , \tag{3.2}$$

firm 2 will be shut down $\forall s \geq 0$, *that is,* $q_2 = 0$, *whereas firm 1 produces*

$$q_1 = \frac{1 - c_1}{1 + d_1^2 s}\ . \tag{3.3}$$

Thus, we can say that firm one has the better technology if condition (3.2) holds, unless $c_1 = c_2$, $d_1 = d_2$, in this case the output can be arbitrarily distributed among both firms. Inequality (3.2) says that firm 1's emission per unit, d_1, divided by its *competitive* output $1 - c_1$ is not greater than the corresponding ratio of firm 2. Notice that $c_1 < c_2$ and $d_1 = d_2$ as well as $c_1 = c_2$ and $d_1 < d_2$ imply (3.2). But notice also that (3.2) may hold for some $d_1 > d_2$ if c_1 is sufficiently smaller than c_2. In other words, even if firm 2 emits less pollutants per unit of output, it may never produce in social optimum if the cost differential $c_2 - c_1$ is sufficiently high.

Proposition 3.2 *Let*

$$\frac{d_1}{1-c_1} > \frac{d_2}{1-c_2} \tag{3.4}$$

hold. Then, for

$$s \leq \underline{s} := \frac{c_2 - c_1}{d_1[d_1(1-c_2) - d_2(1-c_1)]} \tag{3.5}$$

only firm 1 produces with

$$q_1 = \frac{1-c_1}{1+d_1^2 s}\,.$$

For

$$s \geq \overline{s} := \frac{c_2 - c_1}{d_2[d_1(1-c_2) - d_2(1-c_1)]} \tag{3.6}$$

only firm 2 produces with

$$q_2 = \frac{1-c_2}{1+d_2^2 s}\,. \tag{3.7}$$

Finally, for

$$\underline{s} < s < \overline{s} \tag{3.8}$$

both firms produce with

$$q_1 = \frac{1}{(d_2-d_1)^2}\left[\frac{c_2-c_1}{s} - d_2[d_1(1-c_2) - d_2(1-c_1)]\right] \tag{3.9}$$

$$q_2 = \frac{1}{(d_2-d_1)^2}\left[\frac{c_1-c_2}{s} + d_1[d_1(1-c_2) - d_2(1-c_1)]\right] \tag{3.10}$$

Propositions 3.1 and 3.2 are derived by solving (3.1), taking into account the Kuhn–Tucker conditions with respect to the constraints $q_1 \geq 0$ and $q_2 \geq 0$. Details are relegated to the appendix. Notice that $c_1 \leq c_2$ and (3.4) imply $d_1 > d_2$, that is, firm 2 emits strictly less pollutants per unit of output. Hence, we get $\underline{s} < \overline{s}$. From (3.3) and (3.7) it is obvious that aggregate output is strictly decreasing in s for $s \leq \underline{s}$ and $s \geq \overline{s}$. Interestingly, for $\underline{s} \leq s \leq \overline{s}$ aggregate output is constant in s and equals

$$Q = \frac{d_1(1-c_2) - d_2(1-c_1)}{d_1 - d_2}\,.$$

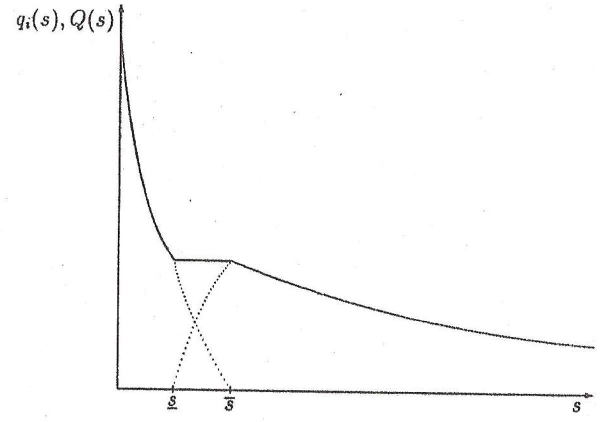

Figure 1: *The quantities in social optimum as a function of s if $d_1(1-c_2)-d_2(1-c_1) > 0$. The solid line depicts aggregate output which equals $q_1(s)$ for $s \leq \underline{s}$ and $q_2(s)$ for $s \geq \overline{s}$. The dotted lines depict q_1 and q_2 for $\underline{s} < s < \overline{s}$.*

This follows simply by adding up (3.9) and (3.10). Moreover, one can show that Q is continuous in s at $\underline{s}$ and $\overline{s}$. Thus, the social planner shifts production continuously from firm 1 to firm 2 as s increases, keeping total output constant, until firm 1, which faces the lower production cost but is the worse polluter, shuts down. These properties are displayed in figure 1. Total emissions E, on the other hand, are continuous and strictly decreasing in s. The same applies to welfare $W^{FB}(s)$ as a function of s when the *first best* production levels are chosen.

4 Regulatory Policies: Some Remarks on the Information Structure

Needless to say that first best solutions are in general not enforceable by prescribing the firms to produce individually different quantities. Not only is there an information problem in the sense that the government does not know the firms' technologies. It is also considered to be unfair to prescribe different policies to the firms. By widespread opinion of the public and their representatives, firms are supposed to make their own decisions about their output in an economy with free enterprise. This paper is not about incomplete information in the sense that the government has prior (probability) beliefs about the firms' technologies. If the government, however, has to choose a "fair" policy that treats all the firms alike, complete information is not necessary anyway. To choose, for instance, an optimal linear tax, it is sufficient to know the existing types of technologies and how many there are of each type, but not exactly, what firm has what technology.[4] In this paper, however, implementation of decentralizing policies is the issue rather than incomplete information. Hence, we will assume for the remainder of this paper that the government knows at least what technologies there are.

We also assume that the emissions generated by each firm can be perfectly monitored by the authorities. So, the firms will pay a tax bill exactly according to the amount of their emitted pollutants (in section 5). In case of holding permits, firms cannot emit more than the number of permits allows them to do. Otherwise, we assume, a high penalty has to be paid. So there is no room for cheating. Needless to say that also this a strong abstraction.

5 Pigouvian Taxes

By a Pigouvian Taxes we mean a linear tax tariff on emissions. Firm i has to pay a bill of $\tau \cdot e_i$ if it emits e_i units of the pollutant, where τ is the tax rate. Producing q_i units, firm i's costs amount to $c_i q_i + \tau e_i = (c_i + \tau d_i) q_i$. We do not impose a condition on the sign of τ. Negative τ's, mean a subsidy. Indeed, we will see that for low social damage it is optimal to subsidize pollution, a seemingly perverse phenomenon at first thought. Since we retain the assumption of Cournot competition, the firms go on choosing Cournot–Nash quantities. Being taxed they produce

$$q_i(\tau) = q_i^N(\tau) := \frac{1 - 2c_i + c_j - (2d_i - d_j)\tau}{3} \qquad \forall i,j = 1,2 \quad i \neq j\,, \tag{5.1}$$

unless one of these expressions is negative. If firm j shuts down due to a high tax rate, firm i will produce the monopoly quantity

$$q_i(\tau) = q_i^M(\tau) := \frac{1 - c_i - \tau d_i}{2}\,. \tag{5.2}$$

[4]This information structure is reminiscent of the second degree price discrimination literature. In MASKIN and RILEY's model [7], the monopolist has to know what kinds of consumer there are, but not which consumers has which utility function. The same structure can be found in ROTHSCHILD and STIGLITZ [10], and STIGLITZ [14] in the analysis of insurance markets. Of course, assuming this information structure is more appealing when there are many agents rather than only two as in our model.

What is the government's program? It wants to find the optimal tax rate under the constraint that the firms set Nash quantities if they both produce, and monopoly quantities if only one of them is active. Hence, it has to solve $\max_\tau W^{PT}(\tau)$

$$:= \max_\tau \int_0^{q_1(\tau)+q_2(\tau)} P(z)dz - S(d_1 q_1(\tau) + d_2 q_2(\tau)) - c_1 q_1(\tau) - c_2 q_2(\tau) \tag{5.3}$$

Observe that the additional costs of size τq_i for the firms and the tax revenue for the government cancel out if we assume that the government redistributes them lump sum back to the firms, or even to consumers. This does not matter. What matters is that the government has no objective to collect tax revenues in this industrial sector. Especially, there is no additional technology the government can buy in order to reduce the aggregate emissions E, once these have been dumped into the environment by the firms. Even if it is not quite realistic that the government does not care about tax revenues, we want to abstract from them in order to highlight the role of taxes as a regulating tool. To solve (5.3), we have to know the behavior of $q_i(\tau)$, $i = 1, 2$. See from (5.1) that $q_i^N(\tau) \geq 0$ if and only if

$$1 - 2c_i + c_j - (2d_i - d_j)\tau \geq 0 \tag{5.4}$$

$$\Leftrightarrow \quad \tau \leq \frac{1 - 2c_i + c_j}{2d_i - d_j} \quad \text{if} \quad 2d_i > d_j\,, \tag{5.5}$$

$$\tau \geq \frac{1 - 2c_i + c_j}{2d_i - d_j} \quad \text{if} \quad 2d_i < d_j\,, \tag{5.6}$$

$$1 - 2c_i + c_j \geq 0 \quad \text{if} \quad 2d_i = d_j\,. \tag{5.7}$$

From this we get $q_i(\tau) = q_i^M(\tau)$ if $\tau \leq (1 - c_i)/d_i$ and

$$\Leftrightarrow \quad \tau \geq \frac{1 - 2c_i + c_j}{2d_i - d_j} \quad \text{if} \quad 2d_i > d_j\,,$$

$$\tau \leq \frac{1 - 2c_i + c_j}{2d_i - d_j} \quad \text{if} \quad 2d_i < d_j\,.$$

Let τ_i^{No} be the "Nash tax rate", at which firm i just closes in competition with firm j, that is,

$$\tau_i^{No} = \frac{1 - 2c_i + c_j}{2d_i - d_j}\,. \tag{5.8}$$

Lemma 5.1 *If*

$$\frac{d_1}{1 - c_1} < \frac{d_2}{1 - c_2}\,, \tag{5.9}$$

firm 2 always closes first as the tax increases or it does not produce at all. In particular:

a] *If* $2d_1 < d_2$ *then* $\tau_1^{NO} < \tau_2^{NO}$ *and*

$$\begin{aligned} q_1^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau > \tau_1^{NO}, \\ q_2^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_2^{NO}. \end{aligned}$$

b] *If* $2d_1 = d_2$, *then*

$$\begin{aligned} q_1^N(\tau) > 0 \quad &\forall \tau, \\ q_2^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_2^{NO}. \end{aligned}$$

c] *If* $2d_1 > d_2 > d_1/2$, *then* $\tau_2^{NO} < \tau_1^{NO}$ *and*

$$\begin{aligned} q_1^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_1^{NO}, \\ q_2^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_2^{NO}. \end{aligned}$$

d] *If* $2d_2 \leq d_1$, *then* $q_2(\tau) = 0 \ \forall \tau$.

Lemma 5.2 *If* $c_1 < c_2$ *and* [5]

$$\frac{d_1}{1-c_1} > \frac{d_2}{1-c_2}, \tag{5.10}$$

firm 1 always closes first as the tax increases or never operates at all, in particular:

a] *If* $d_1/2 < d_2 < 2d_1$ *then* $\tau_1^{NO} < \tau_2^{NO}$ *and*

$$\begin{aligned} q_1^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_1^{NO}, \\ q_2^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_2^{NO}. \end{aligned}$$

b] *If* $d_1 = 2d_2$, *then*

$$\begin{aligned} q_2^N(\tau) > 0 \quad &\forall \tau, \\ q_1^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_1^{NO}. \end{aligned}$$

c] *If* $2d_2 < d_1$, *then* $\tau_2^{NO} < \tau_1^{NO}$ *and*

$$\begin{aligned} q_1^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau < \tau_1^{NO}, \\ q_2^N(\tau) > 0 \quad &\Leftrightarrow \quad \tau > \tau_2^{NO}. \end{aligned}$$

Notice that $d_2 \geq 2d_1$ is not compatible with (5.10) and our assumption that $c_1 \leq c_2$.

[5] If $c_1 = c_2$ interchange the names of the firms and apply Lemma 5.1.

Lemma 5.3 *If*

$$\frac{d_1}{1-c_1} = \frac{d_2}{1-c_2} \tag{5.11}$$

and $2d_2 > d_1$ *both firms close simultaneously as* τ *increases and* $\forall i = 1, 2$:

$$q_i(\tau) = q_i^N(\tau) > 0 \quad \Leftrightarrow \quad \tau < \tau_1^{NO} = \tau_2^{NO}.$$

If $2d_2 \leq d_1$, *firm 2 never produces.*

Lemma 5.4 *Suppose* τ *maximizes* $W^{PT}(\tau)$ *and we have* $q_i(\tau) = q_i^N(\tau) > 0$ *for* $i = 1, 2$, *then the "Nash-tax" is*

$$\tau(s) = \tau^N(s) := \frac{A + Bs}{C + Ds} \tag{5.12}$$

where

$$\begin{aligned} A &= (5c_2 - 4c_1 - 1)d_2 + (5c_1 - 4c_2 - 1)d_1 \\ B &= 2(d_1^2 - d_1 d_2 + d_2^2)[(1 - 2c_2 + c_1)d_2 + (1 - 2c_1 + c_2)d_1] \\ C &= (d_1 + d_2)^2 \\ D &= 4(d_1^2 - d_1 d_2 + d_2^2)^2 \end{aligned}$$

That means, $\tau^N(s)$ is the tax rate at s imposed on the firms if they both are in the market and engage in Cournot competition. Notice that since $C > 0$, $D > 0$, and $s \geq 0$, the denominator is always positive.

Lemma 5.5 *Suppose* τ *maximizes* $W^{PT}(\tau)$ *and for some* i, $j \neq i$, $q_i(\tau) = 0$, *and* $q_j(\tau) = q_j^M(\tau) > 0$, *then the "monopoly-tax" for firm* j *is*

$$\tau(s) = \tau^{M_j}(s) := \frac{1 - c_j}{d_j}\left[1 - \frac{2}{1 + s d_j^2}\right] \tag{5.13}$$

Let $\tau^{N^{-1}}(\cdot)$ be the inverse of $\tau^N(\cdot)$, that is,

$$\tau^{N^{-1}}(t) = \frac{tC - A}{B - tD} \qquad \forall t \neq \frac{B}{D}.$$

We define $\forall \tau_i^{NO} \neq \frac{B}{D}$:

$$s_i^D = \tau^{N^{-1}}(\tau_i^{NO}) \tag{5.14}$$

(The superscript stands for "duopoly".) This means, s_i^D is that damage parameter for which the tax equals τ_i^{No}. By its definition, s_i^D may also take negative values, which, however, are not relevant then.

Similarly, let $\tau^{M_j^{-1}}(\cdot)$ be the inverse of $\tau^{M_j}(\cdot)$. We define

$$s_i^{M_j} := \tau^{M_j^{-1}}(\tau_i^{No}) \,. \tag{5.15}$$

This means, $s_i^{M_j}$ is that damage parameter for which firm i just opens if firm j is taxed as a monopolist.

Lemma 5.6

a) If $d_1/(1-c_1) < d_2/(1-c_2)$, then $s_2^D < s_2^{M_1}$. Moreover, $s_1^{M_2}$, $s_1^D < 0$, hence not relevant.

b) If $d_1/(1-c_1) > d_2/(1-c_2)$, then $s_2^{M_1} < s_2^D$, and $0 < s_1^D < s_1^{M_2}$.

After these preparations we are ready to characterize the optimal linear tax as a function of the damage parameter s.

Proposition 5.1

a) If $d_1/(1-c_1) < d_2/(1-c_2)$, then

$$\tau(s) = \begin{cases} \tau^N(s) & \text{for} \quad 0 \le s \le s_2^D, \quad \text{(both firms produce)} \\ \tau_2^{No} & \text{for} \quad \max\{0, s_2^D\} \le s \le s_2^{M_1}, \quad \text{(only firm 1 produces)} \\ \tau^{M_1}(s) & \text{for} \quad \max\{0, s_2^{M_1}\} \le s \quad \text{(only firm 1 produces)} \end{cases} \tag{5.16}$$

Especially, for $d_1 \ge 2d_2$, $\tau(s) = \tau^{M_1}(s)\ \forall s \ge 0$ and only firm 1 produces.

b) If $d_1/(1-c_1) > d_2/(1-c_2)$, then

$$\tau(s) = \begin{cases} \tau^{M_1}(s) & \text{for} \quad 0 \le s \le s_2^{M_1} \quad \text{(only firm 1 produces)} \\ \tau_2^{No} & \text{for} \quad \max\{0, s_2^{M_1}\} \le s \le s_2^D, \quad \text{(only firm 1 produces)} \\ \tau^N(s) & \text{for} \quad \max\{0, s_2^D\} \le s \le s_1^D, \quad \text{(both firms produce)} \\ \tau_1^{No} & \text{for} \quad s_1^D \le s \le s_1^{M_2}, \quad \text{(only firm 2 produces)} \\ \tau^{M_2}(s) & \text{for} \quad s_1^{M_2} \le s \quad \text{(only firm 2 produces)} \end{cases} \tag{5.17}$$

c) If $d_1/(1-c_1) = d_2/(1-c_2)$, and $2d_2 < d_1$, then

$$\tau(s) = \tau^N(s) \qquad \forall\, s \ge 0 \qquad \text{(both firms produce)} \tag{5.18}$$

If $d_1/(1-c_1) = d_2/(1-c_2)$, and $2d_2 \ge d_1$ then

$$\tau(s) = \tau^{M_1}(s) \qquad \forall\, s \ge 0 \qquad \text{(only firm 1 produces)} \tag{5.19}$$

Proposition 5.1 follows directly from Lemmata 5.1 – 5.6. Lemma 5.6 is most important among all and a bit tricky to prove.

In words, Proposition 5.1 says that if firm 2 has the strictly worse technology, it *may* be the case that for low values of s *both* firms produce. By Lemma 5.1, firm 2 closes first as s increases. For $s \in [\max\{0, s_2^D\}, s_2^{M_1}]$, the tax is constant in s and equals τ_2^{No}. This is due an incentive constraint: Suppose $s_2^D > 0$. If s increases towards s_2^D, $\tau(s)$ goes to τ_2^{No}, that is, firm 2 closes down. For higher taxes than τ_2^{No}, firm 1 is a monopolist. Hence $\tau(s) \ne \tau^N(s)$, and firm 1 has to be taxed as a monopolist. However, if firm 2 could be prohibited to produce for s slightly higher than s_2^D, the optimal tax for the monopoly firm 1 would be lower than τ_2^{No} for $s \le s_2^{M_1}$. This follows immediately from Lemma 5.6. But firm 1 cannot be told to shut down by law. At least this is what we assume. Hence, to prevent firm 1 from producing, the tax must not be lower than τ_2^{No}. For $s \ge s_2^{M_1}$, $\tau(s) = \tau^{M_1}(s) \ge \tau_2^{No}$, and $\tau(s)$ is strictly increasing in s.

Notice that in case a) it can never happen that only firm 2 produces as a monopolist. This follows from the second claim in Lemma 5.6, namely that $s_1^D, s_1^{M_2} < 0$. It is easy to verify that $\tau(0) < 0$, this means, for low damage parameters, the firms' pollution will be subsidized. We know that a monopolist or a (Cournot–) oligopoly produce less than the social optimum (which is equal to the competitive output of the lower cost firm if there are several with different costs). From the theory of regulating monopolies or oligopolies (see BARON and Meyerson [1] or KONISHI et al. [4]) we know that in the absence of externalities and under complete information, the firms' output is to be subsidized in order to increase welfare. A monopolist can even be brought to produce the competitive output. In our model, the subsidies work indirectly via subsidizing emissions, which stand in fixed proportions to the firms' output.

In part b) of the proposition, where firm 1 has the lower cost $c_1 \le c_2$, but firm 2 has the better abatement technology, it may be the case that the lower cost firm 1 produces as a monopolist for low damage parameters. Then, *both* firms produce for intermediate values of s, whereas for high s only the "cleaner" firm produces. Here, there may be two intervals for $\tau(s)$ being constant in s. On the first interval $[\max\{0, s_2^{M_1}\}, s_2^D]$ (which may be empty) we have $\tau(s) = \tau_2^{No} < 0$, that is, we get a subsidy. On the second interval $[s_1^D, s_1^{M_2}]$ we have $\tau(s) = \tau_1^{No} > 0$, that is, τ is a real tax. Depending on the parameters it is also possible that for $s = 0$, both firms produce under the optimal tax. But the case that firm 2 is a monopolist for all s is ruled out.

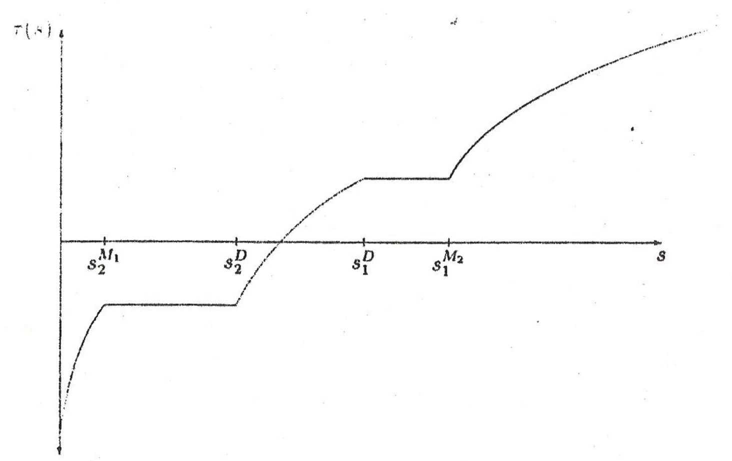

Figure 2: *The optimal linear taxrate as a function of s.*

In figure 2 we have depicted the optimal tax as a function of s for the case b) of the proposition where all the $s_j^{M_i}$, s_j^D are positive.[6]

6 Permits

In this section we assume that the government gives out a number of L permits for pollution which may be traded among the firms. Each permit allows a firm to emit one unit of the pollutant. We need not care about whether the permits will be bought from the government and at what price. We could assume that the government distributes them fairly among the firms such that each firm holds $L/2$ permits at the beginning. Assume that L be arbitrarily divisible.

6.1 The Firms' Behavior

The process going on in the economy may be divided into 3 steps. At first, the firms hold some initial endowment (l_1, l_2) of permits, with $l_1 + l_2 = L$. In the second step they may trade, that is here, one firm sells some or all permits to the other firm. Firms end

[6] By shifting this curve to the left and cutting off at $s = 0$ one gets the shape for the other cases. For case a) interchange the subscripts 1 and 2.

up with a new allocation of permits (e_1, e_2) with $e_1 + e_2 = L$. In the third step, firms engage into (Cournot-)competition and choose quantities q_1^N, q_2^N under the constraint

$$q_i^N \leq e_i/d_i \tag{6.1}$$

which is binding if e_i is sufficiently low.

How will the firms trade the permits? Denote by $\Pi_i^N(e_1, e_2)$ the profit of firm i if the final allocation of permits in the second step has been (e_1, e_2) and both firms choose Nash-quantities under the constraint (6.1). Observe that there is a gain from trade if and only if there is an allocation (e_1, e_2) such that

$$\Pi_1^N(l_1, l_2) + \Pi_2^N(l_1, l_2) < \Pi_1^N(e_1, e_2) + \Pi_2^N(e_1, e_2)$$

In this case there is T which can be interpreted as *transfer-payment* from firm 1 to firm 2 (which may be negative, of course) such that

$$\Pi_1^N(e_1, e_2) + T > \Pi_1^N(l_1, l_2),$$
$$\Pi_2^N(e_1, e_2) - T > \Pi_2^N(l_1, l_2).$$

How the firms figure out T is nothing we have to care about. For example, they could agree on the Nash-bargaining solution. The maximum gain from trading permits is determined by

$$\max_{e_1, e_2} \left[\Pi_1^N(e_1, e_2) + \Pi_2^N(e_1, e_2)\right] \quad \text{s.t. } e_1 + e_2 \leq L,\ e_1 \geq 0,\ e_2 \geq 0. \tag{6.2}$$

Accepting the assumption that firms behave as profit maximizers it is natural to make the following assumption:

Assumption 2 *Firms trade permits in the second phase such that the final allocation* (e_1^*, e_2^*) *solves (6.2).*

Notice that this assumption allows also for the case that one firm buys all the other firm's permits such that the market ends up with monopoly. And indeed, this will happen for some range of values for L as we will.

Before the government can solve the problem how to choose the optimal number of permits contingent on s, we have to analyze how the firms will determine the final allocation by solving (6.2). For this consider the following program:

$$\max_{q_1, q_2} P(q_1 + q_2)[q_1 + q_2] - c_1 q_1 - c_2 q_2 \quad \text{s.t. } d_1 q_1 + d_2 q_2 \leq L. \tag{6.3}$$

After solving (6.3), we will show that the resulting quantities form a Nash equilibrium, under the constraint that $q_i \le e_i/d_i$.

Proposition 6.1 *Let*

$$\tilde{Q} := \frac{d_1(1-c_2) - d_2(1-c_1)}{2(d_1-d_2)}$$

a) *If*

$$\frac{d_1}{1-c_1} \le \frac{d_2}{1-c_2} \tag{6.4}$$

the solution of (6.3) is given by

$$\begin{aligned} q_1(L) &= \min\left\{\frac{1-c_1}{2}, \frac{L}{d_1}\right\}, \quad & q_2(L) = 0 \\ e_1(L) &= \min\left\{d_1\frac{1-c_1}{2}, L\right\}, \quad & e_2(L) = 0 \end{aligned}$$

b) *If* $c_1 < c_2$ *and* [7]

$$\frac{d_1}{1-c_1} > \frac{d_2}{1-c_2} \tag{6.5}$$

the solution of (6.3) is given by

$$q_1(L) = \begin{cases} \min\left\{\frac{1-c_1}{2}, \frac{L}{d_1}\right\} & \textit{if} \quad L \ge d_1\tilde{Q} \\ \frac{1}{d_1-d_2}\left[L - d_2\tilde{Q}\right] & \textit{if} \quad d_1\tilde{Q} \ge L \ge d_2\tilde{Q} \\ 0 & \textit{if} \quad L \le d_2\tilde{Q} \end{cases} \tag{6.6}$$

$$q_2(L) = \begin{cases} 0 & \textit{if} \quad L \ge d_1\tilde{Q} \\ \frac{1}{d_1-d_2}\left[d_1\tilde{Q} - L\right] & \textit{if} \quad d_1\tilde{Q} \ge L \ge d_2\tilde{Q} \\ \frac{L}{d_2} & \textit{if} \quad L \le d_2\tilde{Q} \end{cases} \tag{6.7}$$

We define

$$L_{mon} := d_1\frac{1-c_1}{2}, \tag{6.8}$$

[7] If $c_1 = c_2$ interchange the names of the firms and apply case a).

as the pollution level of the lower cost firm 1 if this firm operates as a monopolist in the absence of regulation.

To interpret the proposition: if (6.4) holds, firm 1 buys all the permits and behaves as a monopolist. If $L > L_{mon}$, firm 1 also buys all the permits but does not use them all. In this case, there is under-production combined with under-pollution. By giving out more permits, however, the government cannot induce the firms to produce more than the monopoly output $\frac{1-c_1}{2}$.

If (6.5) holds, the same thing happens as long as $L \ge d_1\tilde{Q}$. If $d_1\tilde{Q} \ge L \ge d_2\tilde{Q}$, the two firms shift production continuously from firm 1 to firm 2 as L decreases, holding total output constant. This follows from (6.6) and (6.7). Adding up q_1 and q_2 yields $\tilde{Q}$, independently of L.

For $L \le d_2\tilde{Q}$, firm 2, which has the better abatement technology, buys all the permits and produces alone.

Proposition 6.2 *The solution of (6.2) forms a Nash–equilibrium.*

The proof is obvious for $L \ge d_1\tilde{Q}$ and $L \le d_2\tilde{Q}$ since then the firms just produce their monopoly quantities under the constraint that $q_i \le L/d_i$. The other firm does not hold any permits and hence cannot produce. If $d_2\tilde{Q} < L < d_1\tilde{Q}$, for $q_1(L)$ and $q_2(L)$ to form a Nash–equilibrium it is sufficient to show that

$$\frac{\partial \Pi^i}{\partial q_i}(q_1(L), q_2(L)) > 0 \qquad \text{for } i = 1, 2, \tag{6.9}$$

that is, each firm i would like to increase quantities given the other firm produces $q_j(L)$, but cannot since it is constrained by its number of permits. (6.9) will be established in the appendix.

6.2 The Government's program

Given these reactions of the firms when a number of L permits is in the market, the government has to find the optimal size of L given the social damage parameter s. We will denote $q_1(L)$, $q_2(L)$ as the solutions of (6.3) given by Proposition 6.1. Further denote $Q(L) = q_1(L) + q_2(L)$, $e_i(L) = d_iq_i(L)$, $i = 1, 2$. Hence it has to solve the following program:

$$\max_L W^{Per}(L) := \max_L \int_0^{Q(L)} P(z)dz - S(L) - c_1q_1(L) - c_2q_2(L) \tag{6.10}$$

If we want to emphasize the dependence on the damage parameter s we write $W^{Per}(L,s)$. Before we characterize the solution of (6.3), we state some preparatory lemmata. Let $L(s)$ denote the optimal number of permits contingent on s.

Lemma 6.1 *Let s be given. Suppose only firm i is allowed to produce, and $L(s)$ maximizes $W^{Per}(L)$. Then*

$$L(s) = \frac{d_i(1-c_i)}{1+d_i^2 s} =: L^{M_i}(s) . \tag{6.11}$$

(The superscript M_i indicates that firm i is a monopolist.) Welfare is given by

$$W^{Per}(L(s)) = \frac{1}{2}\frac{(1-c_i)^2}{1+d_i^2 s} . \tag{6.12}$$

Lemma 6.2 *Let s be given. Suppose $L(s)$ maximizes $W^{Per}(L)$ and $q_i(L) > 0$ for $i = 1,2$. Then*

$$L(s) = \frac{c_2-c_1}{d_1-d_2}\frac{1}{s} =: L^D(s) . \tag{6.13}$$

(The superscript D indicates that both firms produce in duopoly.) Welfare is given by

$$W^{Per}(L(s)) = \frac{1}{2}\left[3\tilde{Q} + \left(\frac{c_2-c_1}{d_1-d_2}\right)\frac{1}{s}\right] . \tag{6.14}$$

Now let $L^{D^{-1}}$ be the inverse of $L(\cdot)$. Define

$$\sigma_i^D := L^{D^{-1}}(d_i\tilde{Q}) = \frac{c_2-c_1}{d_i(d_1-d_2)\tilde{Q}} , \tag{6.15}$$

that is, the solution of $d_i\tilde{Q} = (c_2-c_1)/(d_1-d_2)s$ in s. In words, σ_1^D is the damage parameter where firm 1 just closes if s *increases* towards σ_1^D and $L(s) = L^D(s)$. Similarly, σ_2^D is the damage parameter where firm 2 just closes if s *decreases* towards σ_2^D and $L(s) = L^D(s)$.

Analogously, let $L^{M_j^{-1}}$ be the inverse to $L^{M_j}(\cdot)$ and define

$$\sigma_i^{M_j} := L^{M_j^{-1}}(d_i\tilde{Q}) = \frac{1-c_j-\tilde{Q}}{d_j^2\tilde{Q}} , \tag{6.16}$$

that is, the solution of $d_j\tilde{Q} = d_j(1-c_j)/(1+d_j^2 s)$ in s. In words, $\sigma_1^{M_2}$ is the damage parameter where firm 1 just closes if s *increases* towards $\sigma_1^{M_2}$ and $L(s) = L^{M_2}(s)$. Similarly, $\sigma_2^{M_1}$ is the damage parameter where firm 2 just closes if s *decreases* towards $\sigma_2^{M_1}$ and $L(s) = L^{M_1}(s)$.

Now we get a claim similar to Lemma 5.5.

Lemma 6.3 *If (6.5) holds, then*

$$\sigma_2^D < \sigma_2^{M_1} \quad \textit{and} \quad \sigma_1^D < \sigma_1^{M_2} . \tag{6.17}$$

Notice that $\sigma_2^{M_1}$ may be smaller, greater or equal to σ_1^D.

Lemma 6.3 says that $L^{M_1}(s)$ is greater than $L^D(s)$ for s close to σ_1^D and $L^{M_2}(s)$ is greater than $L^D(s)$ for s close to σ_2^D. This implies that like the optimal tax, $L(s)$ must be constant on the interval $[\sigma_1^D, \sigma_1^{M_2}]$. For, if $s = \sigma_1^D$, then $L^D(s) = d_2\tilde{Q}$, and by Proposition 6.1, equation (6.7), firm 2 buys all the permits from firm 1. For $s \geq \sigma_1^D$, firm 2 behaves as a monopolist. Forbidding firm 1 to produce, the optimal number of permits equals $L^{M_1}(s)$, which is higher than $d_2\tilde{Q}$ if s is greater but close to σ_1^D. Giving out $L^{M_2}(s) > d_2\tilde{Q}$ many permits, however, firm 2 does *not* buy all the permits. Hence, $L(s)$ has to be constant and equal to $d_2\tilde{Q}$ for $s \in [\sigma_1^D, \sigma_1^{M_2}]$ in order to keep firm 1 out of the market. Notice that this argument is very similar to the optimal linear tax scheme, where the tax rate also has to be constant on certain intervals of damage parameters.

On the other hand, $L(s)$ must be discontinuous somewhere in the interval $(\sigma_2^D, \sigma_2^{M_1})$. To see this, consider first the left hand boundary of this interval, σ_2^D. If we employ the "duopoly–policy" L^D, we get $L^D(\sigma_2^D) = d_1\tilde{Q}$ and $q_2 = 0$. Employing the monopoly policy L^{M_1} w.r.t. firm 1 we get $L^{M_1}(\sigma_2^D) > d_1\tilde{Q}$ by Lemma 6.3. Let us assume that $L^{M_1}(\sigma_2^D) < L_{mon}$. Obviously, $L^{M_1}(\cdot)$ is the better policy than $L^D(\cdot)$ for $s = \sigma_2^D$. Hence,

$$W^{Per}\left(L^{M_1}(\sigma_2^D)\right) > W^{Per}\left(L^D(\sigma_2^D)\right) .$$

By arguing similarly the other way round, we get for $\sigma_2^{M_1}$.

$$W^{Per}\left(L^{M_1}(\sigma_2^{M_1})\right) < W^{Per}\left(L^D(\sigma_2^{M_1})\right) .$$

Since $L^D(\cdot)$, $L^{M_1}(\cdot)$ and $W^{Per}(\cdot)$ are continuous there must be some intersection $\sigma_{int} \in (\sigma_2^D, \sigma_2^{M_1})$ such that

$$W^{Per}\left(L^{M_1}(\sigma_{int})\right) = W^{Per}\left(L^D(\sigma_{int})\right) ,$$

and $L(\cdot)$ jumps down from $L^{M_1}(\cdot)$ to $L^D(\cdot)$, at least if $\sigma_2^{M_1} < \sigma_1^D$. The case $\sigma_2^{M_1} \geq \sigma_1^D$ is similar and will be treated in the proof of the next proposition which characterizes completely the optimal number of permits as a function of the damage parameter s.

Before doing this, we define

$$\sigma_{mon} := L^{M_1^{-1}}(L_{mon}) = \frac{1}{d_1^2} \tag{6.18}$$

as the damage parameter, for which the monopolistic firm 1 faces a real capacity constraint if it is regulated by $L^{M_1}(\cdot)$.

Proposition 6.3 *a) If $d_1/(1-c_1) \leq d_2/(1-c_2)$, the optimal number of permits as a function of s is given by*

$$L(s) = \begin{cases} L_{mon} & \text{for } s \leq \sigma_{mon}\,, \\ L^{M_1}(s) & \text{for } s \geq \sigma_{mon}\,. \end{cases} \tag{6.19}$$

In this case, only firm 1 produces for all $s \geq 0$.

b) If $d_1/(1-c_1) > d_2/(1-c_2)$, the optimal number of permits as a function of s is given by

$$L(s) = \begin{cases} L_{mon} & \text{for } s \leq \min\{\sigma_{mon}, \sigma_{int}\} & \text{(only firm 1 produces)} \\ L^{M_1}(s) & \text{for } \sigma_{mon} \leq s \leq \sigma_{int} & \text{(only firm 1 produces, interval may be empty)} \\ L^D(s) & \text{for } \sigma_{int} < s \leq \sigma_1^D & \text{(both firms produce, interval may be empty)} \\ d_2\tilde{Q} & \text{for } \max\{\sigma_{int}, \sigma_1^D\} < s \leq \sigma_1^{M_2} & \text{(only firm 2 produces)} \\ L^{M_2}(s) & \text{for } s \geq \sigma_1^{M_2} & \text{(only firm 2 produces)} \end{cases} \tag{6.20}$$

where σ_{int} is the solution of

$$\widetilde{W}^{Per}\left(\min\{L_{mon}, L^{M_1}(s)\}, s\right) \stackrel{!}{=} \begin{cases} \widetilde{W}^{Per}\left(L^D(s), s\right) & \text{if } s \leq \sigma_1^D \\ \widetilde{W}^{Per}\left(d_1\tilde{Q}, s\right) & \text{if } s > \sigma_1^D \end{cases}$$

in s.

Proof: a) follows immediately from Proposition 6.1, b) follows also from that result and Lemma 6.3. For $\sigma_2^{M_1} < \sigma_1^D$ and $\sigma_{mon} < \sigma_2^D$ the argument has been elaborated

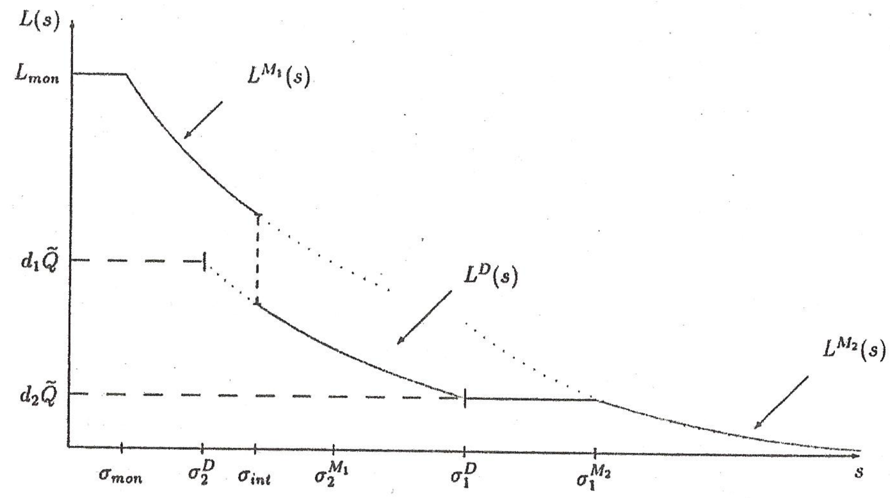

Figure 3: *The solid line depicts the optimal number of permits as a function of s if $d_1(1-c_2) - d_2(1-c_1) > 0$.*

above. For the remaining cases, see the appendix. Plugging (6.20) into $W^{Per}(\cdot)$ we could get the corresponding formulas for the welfare, however, these do not yield further insight.

Notice that for $s \leq \min\{\sigma_{mon}, \sigma_{int}\}$ firm 1 behaves as a monopolist in the absence of regulation. Hence, $L = L_{mon}$ is not unique. Any number of permits $\geq L_{mon}$ would do the job. Notice further that apart from the monopoly effect for large values of L, we get the same structure as in social optimum: If (6.4) (= (5.9)) holds, the worse firm 2 never produces under the first best as well as under the permit solution. If (6.5) (= (5.10)) holds, only firm 1 produces for low values of s, both firms produce for intermediate values of s, and only firm 2 produces for high values of s. In those intervals of s (which, however do not coincide) where both firms produce total output is constant as s increases. Production shifts from the lower production cost but more polluting firm to the higher cost but less polluting firm. $L(s)$ is depicted in figure 3.

7 Comparison and Discussion of the Policies

Recall for the remainder of the paper that s_1^D, s_2^D, $s_1^{M_2}$, $s_2^{M_1}$ denote the border cases for s if we consider *taxes*. For *permits* we use the Greek σ_1^D, σ_2^D, $\sigma_1^{M_2}$, $\sigma_2^{M_1}$, σ_{int} and σ_{mon}.

Throughout this paper we saw that the term $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2}$ played a crucial role in the analysis of the model. If this term is not positive, a social planner will not allow firm 2 to produce for any damage parameter s. Under the permit solution, firm 2 never holds any permits, if we accept Assumption 2. For $L \geq L_{mon}$, firm 1 behaves as a monopolist under "laissez-faire". The government can not induce the monopolist to produce more by giving out more permits. But if $s \geq \sigma_{mon}$, the government can always induce firm 1 to produce the social optimum. Hence, from Proposition 6.3 we get the following

Corollary 7.1 *If firm 2 has the worse technology, that is, if $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} \leq 0$ the permit solution yields the social optimum for all $s \geq \sigma_{mon}$.*

Under the tax solution, firm 2 produces for $s < s_2^D$. For $s \geq s_2^{M_1}$ the optimal tax in order to regulate the monopolistic firm 1 can be imposed. Hence, from Proposition 5.1 we get the following

Corollary 7.2 *If firm 2 has the worse technology, that is, if $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} < 0$, the tax solution yields the social optimum for all $s \geq s_2^{M_1}$.*

Corollary 7.3 *If firm 2 has the much worse technology such that $s_2^{M_1} \leq 0$, which is satisfied if d_2, the emission per unit of firm 2, is sufficiently high, the tax solution yields the social optimum for all $s \geq 0$.*

In this case, the tax is negative for small values of s. So, pollution is subsidized in order to induce the better firm to produce the socially optimal quantity.

On the other hand, it is easy to verify that

$$s_2^{M_1} = \frac{(c_1 - c_2) + \frac{d_1}{2}(1-c_1)}{d_1[d_2(1-c_1) - d_1(1-c_2)]} \,. \tag{7.1}$$

Hence, if the term $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2}$ becomes small, which will be the case if firm 2 is not *much* worse than firm 1, $s_2^{M_1}$ becomes large.

Corollary 7.4 *If firm 2 has the worse technology but is not too bad such that $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2}$ is small and $s_2^{M_1} > \sigma_{mon}$, the permit solution is at least as good as the tax solution, if the better firm pollutes too much as a monopolist, that is, if $s \geq \sigma_{mon}$. The permit solution is strictly better than the tax solution for $s \in (\sigma_{mon}, s_2^{M_1})$.*

The last corollary is striking! For, it says that the permit solution is better than the tax solution if the social damage from pollution is relatively high $s \geq \sigma_{mon}$, and one firm is worse than but not too different from the other firm.

Some special cases:

Let $c_1 < c_2$ and $d_1 = d_2$, that is, firm 1 has the lower production cost but both firms have the same emission per unit. In this case,

$$s_1^{M_2} = 1/d_1^2 \left[\frac{1-c_1}{1-c_2} - 1\right] \,. \tag{7.2}$$

Since $c_1 < c_2$, $s_1^{M_2} > 0$. If $1 - 2c_2 + c_1 > 0$, which is necessary for enabling firm 2 to compete at all with firm 1 without getting subsidies, we even get $s_1^{M_2} > \frac{1}{d_1^2} = \sigma_{mon}$.

If $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} = 0$ and $c_1 < c_2$, $q_i(\tau(s)) > 0$ for all $s \geq 0$. Hence the tax solution never yields the social optimum and is worse than the permit solution for $s \geq \sigma_{mon}$.

If $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} = 0$ and $c_1 = c_2 =: c$, it follows $d_1 = d_2 =: d$, that is, both firms are alike. In this case $\tau(s) = (1-c)\frac{d^2 s - 1}{2d(1+d^2 s)}$ and $q_i(\tau(s)) = \frac{1-c}{2(1+sd^2)}$. Aggregate output is $Q = 2q_1(\tau(s)) = \frac{1-c}{(1+sd^2)}$, which is the same as the socially optimal solution (see Proposition 3.1).

Corollary 7.5 *If both firms are alike, the tax solution yields the socially optimal outcome for all $s \geq 0$. The permit solution is socially optimal only for $s \geq \sigma_{mon}$.*

Let us now turn to the case where $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2}$ is positive. Recall that the social optimum and the permit solution have in common that firm 1 produces alone for low values of s. Then both firms produce, keeping output constant for increasing s. For high values of s only firm 2 produces. However, the intervals where both firms are active may be quite different as the following proposition shows. Recall that firm 1 produces alone for $s \leq \sigma_{int}$ in the permit solution, and for $s \leq \underline{s}$ in social optimum. Firm 2 produces alone for $s \geq \sigma_1^D$ in the permit solution, and for $s \geq \bar{s}$ in social optimum.

Proposition 7.1 $\sigma_{int} > 2\underline{s}, \qquad \sigma_1^D = 2\bar{s}.$

Corollary 7.6 *The permit solution is socially optimal for $s \in [\sigma_{mon}, \underline{s})$ and for $s \geq \sigma_1^{M_1} > 2\bar{s}$.*

Notice that the first interval may be empty.

In other words, under the permit solution the "cleaner" firm 2 opens "much too late" and the worse polluter, firm 1, closes "much too late" under the permit solution compared with social optimum. Notice that the intervals where both firms produce in social optimum and under permits, respectively, may be even disjoint (see also the numerical example below).

Let us consider taxes.

Corollary 7.7 *For $s \in [0, s_2^{M_1}]$ and $s \geq s_1^{M_2}$ the optimal tax induces the social optimum.*

This is easily verified by considering the resulting quantities produced under the tax regime and in social optimum, respectively. It is also clear that it must be like this, for, in those regions of damage parameters, only one firm produces and the optimal tax for regulating a monopolist can be imposed (there is no binding incentive constraint). Notice that the interval $[0, s_2^{M_1}]$ may be empty.

Proposition 7.2 *If $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} > 0$, then $\sigma_1^{M_2} < s_1^{M_2}$.*

In words, the damage parameter, from where on the socially optimal solution is achieved under permits is smaller than the damage parameter, from where on the optimum is achieved under taxes.

Corollary 7.8 *If $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} > 0$, the permit regime achieves the social optimum for a greater range of damage parameters, for which it is desirable that the higher polluting firm shuts down, than the tax regime does.*

In the light of this corollary, the permit solution is not as bad as it seemed to be from Proposition 7.1. From Proposition 7.2 it follows also that the permit solution is better than the tax solution for values slightly lower than $\sigma_1^{M_2}$. If s further decreases welfare under taxes intersects welfare under permits as the following example demonstrates.

Example 7.1 *Let $c_1 = 0.25$, $c_2 = 0.5$, $d_1 = 1$, $d_2 = 0.5$.*

Under this constellation, $d_1(1-c_2) - d_2(1-c_1) > 0$. In this case we get $\underline{s} = 2$, $\bar{s} = 4$, that is, in social optimum both firms are active for $s \in (2, 4)$. Under the optimal Pigouvian tax, both firms are active for $s = 0$. Firm 1 closes for $s_1^D = 16$. For $s \in (s_1^D, s_1^{M_2}) = (16, 20)$, the tax is constant and equals $\tau = \tau_1^{No} = 0.666$. Only when $s > 20$, the social optimum is obtained by the Pigouvian tax. From figure 4 we see that there is over-production for $s \in (0, 2.25)$ and under-production for $s \in (2.25, 20)$, combined with excess pollution for $s \in (2.25, 6)$ and under-pollution for $s \in (0, 0.25) \cup (6, 20)$ (see figure 5). Under permits, social optimum is attained for $s \in [\sigma_{mon}, \sigma_2^D) = (0.125, 2)$ and $s \geq \sigma_1^{M_2} = 12$. For $s \in (2, 12)$ there is under-production combined with excess pollution for $s \in (\underline{s}, \sigma_{int}) = (2, 4.2)$ and under-pollution for $s \in [\sigma_{int}, \sigma_1^{M_2}) = [4.2, 12)$. For the most values of s, welfare is lower under taxes as under permits, however, for $s \in (2, 6.5)$, the optimal Pigouvian tax yields a higher welfare than the optimal number of permits (cf. Figure 6). Compared with "laissez faire", both solutions yield approximately good results as can be seen from figure 7. Other interesting examples could be provided, however, limits on space force us to close here.

Concluding, the permit regime is better if one firm has a better technology for all s and if the lower cost firm would over-pollute as a monopolist. It is also better for a greater range of high damage parameters if $\frac{d_1}{1-c_1} - \frac{d_2}{1-c_2} > 0$ holds. The permit regime is clearly worse if social damage is so low that lower cost firm under produces (and hence under pollutes) as a monopolist such that pollution should be subsidized under the tax regime. In this case, the lower cost firm exploits the permit regime, by buying all the permits and thereby building up its monopoly position. For intermediate values of s nothing can be said in general. Welfare has to be compared under both regimes. But the optimal size of permits or taxes has to be calculated anyway!

8 Final Remarks

Of course, (linear) Pigouvian taxes and permits are not the only tools to regulate pollution. We could look for optimal incentive compatible mechanisms. This would in general mean to offer the firms a menu of quantities of production *and* emission levels combined with taxes or subsidies. Political economists, however, are not too enthusiastic about controlling quantities of the marketable output. In our model, however, quantities of output and emissions are perfectly correlated by the technology. Hence, it is worth looking for optimal nonlinear tax schemes, (which is beyond this paper). Even so, the investigation of *permits* and *linear taxes* is important since those tools are, first, relatively easily enforced and secondly, and more important, they are tools which become more and more known, better understood, and discussed in the public.

Also is there need for investigating the incomplete information case since in most real world situations the government does *not* know the firms' technologies. However, before doing this one should know the outcome under complete information, studied here.

Analyzing a market with more than two firms is not only messy if all the firms have different technologies, it also raises conceptual problems if we consider the permit regime. For more than 2 firms, say 3, Assumption 2 is vulnerable since it may happen that after all the three firms have traded permits, two of them have an incentive to further trade permits exercising a negative externality on the third firm (we have examples!). In other words, the core of permit allocations may be empty.

Finally, further research should go beyond linear technologies, assuming that firms can substitute less pollution by a higher production cost. As far as I can see, however, few can be said under fairly general assumptions.

A Appendix

Proof of Proposition 3.1 and 3.2: This is standard: Setting the partial derivatives of (3.1) with respect to q_1 and q_2 equal to zero (plugging in the functional forms of P and S) we get:

$$1 - (q_1 + q_2) - sd_1(d_1 q_1 + d_2 q_2) - c_1 + \mu_1 = 0$$
$$1 - (q_1 + q_2) - sd_2(d_1 q_1 + d_2 q_2) - c_2 + \mu_1 = 0$$

Here μ_1 and μ_2 are the Lagrange-Multipliers with respect to the constraints $q_1 \geq 0$, $q_2 \geq 0$. Assuming $\mu_1 = 0 = \mu_2$, $\Rightarrow q_1 > 0$, $q_2 > 0$, and solving for q_1, q_2 we get (3.9) and (3.10). It is straight forward to show that these solutions are not negative if and only if (3.4) holds and $0 < \underline{s} \leq s \leq \bar{s}$. $\mu_1 = 0$, $\mu_2 \neq 0$ holds iff (3.4) and $s \leq \underline{s}$, or (3.2) holds in which case $\underline{s} \leq 0$. The latter case refers to Proposition 3.1. $\mu_2 = 0$, $\mu_1 \neq 0$ holds iff (3.4) and $s \geq \underline{s}$, or (3.2) and $s \leq \underline{s} \leq 0$ hold. The latter case is not relevant since $s \geq 0$. If exactly one of the μ_i equals zero we have to solve

$$\max_{q_i} \int_0^{q_i} P(z)dz - S(d_i q_i) - c_i q_i \qquad \text{(A.1)}$$

This yields (3.3) and (3.7), respectively. Now, for $d_1/(1-c_1) \leq d_2/(1-c_1)$ we get $0 \geq \underline{s} \geq \bar{s}$. In this case Proposition 3.1 follows, otherwise we get Proposition 3.2.

Proof of Lemma 5.1 case a) $d_2 > 2d_1$. $\tau_1^{No} < \tau_2^{No}$ is equivalent to $(1-2c_1+c_2)/(2d_1-d_2) < (1-2c_2+c_1)/(2d_2-d_1)$. Since $d_2 > 2d_1$, this is the same as $(1-2c_1+c_2)(2d_2-d_1) > (1-2c_2+c_1)(2d_1-d_2)$. Multiplying, simplifying and rearranging yields $d_1(1-c_2) < d_2(1-c_1)$. $d_2 > 2d_1$ implies $2d_2 > d_1$. Hence from (5.5) we get $q_2^N > 0$ iff $\tau < \tau_2^{No}$. From (5.6) we get $q_1^N > 0$ iff $\tau > \tau_1^{No}$.

case b) $d_2 = 2d_1$. In this case, $q_1^N(\tau) > 0\, \forall \tau$, since $c_1 \leq c_2$. Further, $q_2^N > 0$ iff $\tau < \tau_2^{No}$.

case c) $d_1/2 < d_2 < 2d_1$. To show that $\tau_2^{No} < \tau_1^{No}$ works the same way as in case a), taking into account that both denominators are positive. By (5.5) and (5.6) we get for each $i = 1,2$ $q_i^N(\tau) > 0$ iff $\tau < \tau_i^{No}$.

case d) $2d_2 = d_1$. In this case, (5.9) leads to $d_1(1-c_2) < d_2(1-c_1) = \frac{d_1}{2}(1-c_1)$, yielding $1 - 2c_2 + c_1 < 0$. (5.4) and (5.7) imply that firm 2 never produces.

case e) $2d_2 < d_1$. Again, $\tau_1^{No} < \tau_2^{No}$ like in case a). (5.5) and (5.6) imply $q_2^N > 0$ iff $\tau > \tau_2^{No}$, and $q_1^N > 0$ iff $\tau < \tau_1^{No}$. Hence $q_i^N(\tau_j^{No}) = 0$, therefore, $\forall \tau > \tau_2^{No}$: $q_2(\tau) = \max\{0, q_2^M(\tau)\}$. But $q_2^M(\tau) = 0\, \forall \tau \geq (1-c_2)/d_2$. By simple algebra one shows that $(1-c_2)/d_2 < \tau_1^{No}$. Hence, firm 2 never opens. Q.E.D.

Proof of Lemma 5.2 case a) $d_1/2 < d_2 < 2d_1$. Same as case c) in Lemma 5.1.

case b) $d_1 = 2d_2$. Then, $q_2^N(\tau) = (1-2c_2+c_1)/3\, \forall \tau$. (5.10) implies that this is positive. Since $2d_1 > d_2$, we get $q_1^N(\tau) > 0$ iff $\tau < \tau_1^{No}$. Hence firm 1 closes first.

case c) $d_1 > 2d_2$. (5.10) implies $\tau_2^{No} < \tau_1^{No}$. (5.5) and (5.6) imply $q_1^N(\tau) > 0$ iff $\tau < \tau_1^{No}$ and $q_2^N(\tau) > 0$ iff $\tau > \tau_2^{No}$.

case d) $d_2 > 2d_1$ is not compatible with $c_1 \leq c_2$ and (5.10).

case e) $d_2 = 2d_1$ leads to negative profits of firm 1 if $c_1 < c_2$.

Hence firm 1 always closes first. Firm 2 may open later than firm 1 as τ increases. Q.E.D.

Proof of Lemma 5.3 It is easy to show that $\tau_1^{No} = \tau_2^{No}$. If $c_1 = c_2$ we get $d_1 = d_2$. If $c_1 < c_2$, $d_1 > d_2$ must hold, hence $2d_1 > d_2$. If $2d_2 \leq d_1$, firm 2 never opens by similar arguments as in the cases d) and e) in proof of Lemma 5.1. Q.E.D.

Proof of Lemma 5.4 (Sketched) Differentiating $W^{PT}(\tau)$ yields

$$W^{PT'}(\tau) = P(Q(\tau))[q_1^{N'}(\tau) + q_2^{N'}(\tau)] - S(E)[e_1^{N'}(\tau) + e_2^{N'}(\tau)] - c_1 q_1^{N'}(\tau) - c_2 q_2^{N'}(\tau),$$

where $e_i^{N'}(\tau) = d_i q_i^{N'}(\tau)$. Plugging in (2.2), (2.3), (5.1) and $q_i^{N'}(\tau) = -\frac{2d_i - d_j}{3}$, and solving for τ yields (5.12) after some tedious rearrangements, which we omit. Q.E.D.

Proof of Lemma 5.5 Differentiating $W^{PT}(\tau) = \int_0^{q_i^M(\tau)} - S(d_i q_i^M(\tau)) - c_i q_i^M(\tau)$ yields

$$W^{PT'}(\tau) = [P(q_i^M(\tau)) - S(d_i q_i^M(\tau)) - c_i] q_i^{M'}(\tau)$$

Plugging in (2.2), (2.3), (5.2) and $q_i^{M'}(\tau) = -\frac{d_i}{2}$ and solving for τ yields (5.13) after some manipulations.

Proof of Lemma 5.6 We show that (5.10) implies $s_1^D < s_1^{M_2}$. The remaining inequalities are demonstrated analogously.

Since $\tau(s)$ solves (5.3), we have $\frac{dW^{PT}}{d\tau}(\tau^N(s)) = 0$, if $q_i^N(\tau^N(s)) > 0$, $i = 1,2$. For $s = s_1^D$ the left-sided derivative of W^{PT} equals zero:

$$\frac{dW^{PT}}{d\tau}(\tau_1^{No}) = 0 \tag{A.2}$$

Since also $q_1^N(\tau_1^{No}) = 0$, (A.2) becomes

$$P(q_2^N(\tau))[q_1^{N'}(\tau) + q_2^{N'}(\tau)] - s d_2 q_2^N(\tau)[d_1 q_1^{N'}(\tau) + d_2 q_2^{N'}(\tau)] - c_1 q_1^{N'}(\tau) - c_2 q_2^{N'}(\tau)$$

Plugging in $q_i^{N'}(\tau) = -\frac{2d_i - d_j}{3}$ we get

$$-P(q_2^N(\tau))\frac{d_1 + d_2}{3} + s d_2 q_2^N(\tau)\frac{2}{3}[d_1^2 + d_2^2 - d_1 d_2] + \frac{(2d_1 - d_2)c_1 + (2d_2 - d_1)c_2}{3} = 0$$

solving for s yields

$$s = \frac{P(q_2^N(\tau))(d_1 + d_2) - [(2d_1 - d_2)c_1 + (2d_2 - d_1)c_2]}{2d_2 q_2^N(\tau)[d_1^2 + d_2^2 - d_1 d_2]} \tag{A.3}$$

Consider now W^{PT} when only firm 2 produces and is taxed as a monopolist. First order condition for the optimal monopoly-tax yields

$$\frac{dW^{PT}}{d\tau}(\tau) = [P(q_2^M(\tau)) - S'(d_2 q_2^M(\tau)) - c_2] q_2^{M'}(\tau) = 0 \tag{A.4}$$

Plugging in $q_2^{M'}(\tau) = -\frac{d_2}{2}$ and (A.3) into (A.4) we get $\frac{dW^{PT}}{d\tau}(\tau)$

$$= \frac{d_2}{2}\left[-P(q_2^M(\tau)) + d_2 \frac{P(q_2^N(\tau))(d_1 + d_2) - [(2d_1 - d_2)c_1 + (2d_2 - d_1)c_2]}{2[d_1^2 + d_2^2 - d_1 d_2]} + c_2\right] \tag{A.5}$$

Now we employ that $q_2^M(\tau_1^{No}) = q_2^N(\tau_1^{No}) = [d_1(1 - c_2) - d_2(1 - c_1)]/(2d_1 - d_2)$ and (2.2). Hence the RHS of (A.5) becomes

$$-\frac{d_1 d_2}{4[d_1^2 + d_2^2 - d_1 d_2]}[d_1(1 - c_2) - d_2(1 - c_1)] < 0$$

The last inequality holds since the denominator is positive and the last term in square brackets is positive by (5.10).

Since $\frac{dW^{PT}}{d\tau}(\tau_1^{No}) < 0$ if firm 2 is regulated as a monopolist (suppose firm 1 is not existent for a moment) the *optimal* tax to regulate a monopoly is lower than τ_1^{No}. Since $\tau^M(s)$ is increasing in s by Lemma 5.5, $s_1^{M_2}$ must be greater than s_1^D. Q.E.D.

Proof of Proposition 6.1 Under Assumption 2 the firms maximize $\Pi^1(q_1, q_2) + \Pi^2(q_1, q_2)$ s.t. $d_1 q_1 + d_2 q_2 \leq L$ and $q_i \geq 0$. Lagrange function is: $L(q_1, q_2, \lambda, \mu_1, \mu_2) = (q_1 + q_2)(1 - q_1 - q_2) - c_1 q_1 - c_2 q_2 + \lambda[d_1 q_1 + d_2 q_2 - L] + \mu_1 q_1 + \mu_2 q_2$. Partially deriving yields $\frac{\partial L}{\partial q_i} = 1 - 2[q_1 + q_2] - c_i + \lambda d_i + \mu_i = 0$. For $\lambda \neq 0$, $\mu_i \neq 0$ this yields $q_1 = \frac{1}{d_1 - d_2}\left[L - d_2\tilde{Q}\right]$ and $q_1 = \frac{1}{d_1 - d_2}\left[-L + d_1\tilde{Q}\right]$. These are both positive if $\tilde{Q} > 0$, which holds iff (6.5), and $d_2\tilde{Q} < L < d_1\tilde{Q}$ hold. For $L \leq d_2\tilde{Q}$ we get $q_1 = 0$, and for $L \leq d_2\tilde{Q}$ we get $q_2 = 0$. If $\tilde{Q} \leq 0$ which holds iff (6.4) holds, we get $q_2 = 0\ \forall L$. Q.E.D.

Proof of Proposition 6.2 It remains to establish (6.9): $\frac{\partial \Pi^1}{\partial q_1}(q_1, q_2) = 1 - 2q_1 - q_2 - c_1$. Plugging in $q_1(L)$, $q_2(L)$ from (6.6) and (6.7) yields

$$\begin{aligned}\frac{\partial \Pi^1}{\partial q_1}(q_1, q_2) &= 1 - c_1 - \frac{1}{d_1 - d_2}[L - (2d_2 - d_1)\tilde{Q}] \\ &> 1 - c_1 - 2\tilde{Q} \qquad \text{since } L < d_1\tilde{Q} \\ &= d_1\frac{c_2 - c_1}{d_1 - d_2} > 0\end{aligned}$$

employing the expression for $\tilde{Q}$, $c_1 < c_2$ and the fact $d_1 > d_2$ which follows from (6.5).

Similarly we get $\frac{\partial \Pi^2}{\partial q_2}(q_1, q_2) > d_2\frac{c_2 - c_1}{d_1 - d_2} > 0$. Q.E.D.

Proof of Lemma 6.1 If $q_j = 0$, we have to maximize $W_i(L) := \int_0^{q_i}(1 - z)dz - \frac{s}{2}L^2 - c_i q_i$ where $q_i = L/d_i$. Solving the F.O.C. w.r.t L yields the result. Details are omitted. Q.E.D.

Proof of Lemma 6.2 If $q_i \neq 0$, for $i = 1, 2$ we have to maximize

$$\widetilde{\widetilde{W}}(L) := \int_0^Q (1-z)dz - \frac{s}{2}L^2 - c_1 \frac{1}{d_1 - d_2}\left[L - d_2\tilde{Q}\right] - c_2 \frac{1}{d_1 - d_2}\left[-L + d_2\tilde{Q}\right]$$

Solving the F.O.C. w.r.t L yields the result. Details are omitted. Q.E.D.

Proof of Lemma 6.3 We show $\sigma_2^D < \sigma_2^{M_1}$. This is the same as $(c_2 - c_1)/d_1(d_1 - d_2)\tilde{Q} < (1 - c_1 - \tilde{Q})/d_1^2\tilde{Q}$. By (6.5), $d_1 > d_2$. Hence we get $d_1(c_2 - c_1) < (d_1 - d_2)(1 - c_1) - (d_1 - d_2)\tilde{Q}$. Plugging in $\tilde{Q} = (d_1(1 - c_2) - d_2(1 - c_1))/2(d_1 - d_2)$ and rearranging yields $0 < d_1(1 - c_2) - d_2(1 - c_1)$, which holds by (6.5). The other inequality is shown in the same way.

Proof of Proposition 6.3 **case a):** $\sigma_{mon} \leq \sigma_2^D, \sigma_2^{M_1} < \sigma_1^D$. This case has almost been proven in the text. For $s \leq \sigma_{mon}$, the government can set $L = L_{mon}$ (or any $L \geq L_{mon}$). By the Lemmata 6.1 and 6.2, L^{M_1}, L^{M_2} and L^D are continuous and strictly decreasing. For $L = d_1\tilde{Q}$ we get $q_2(L) = 0$, and $q_2(L) > 0$ for $d_2\tilde{Q} > L > d_1\tilde{Q}$. Now, $\sigma_2^D = L^{D^{-1}}(d_1\tilde{Q})$ and $\sigma_2^{M_1} = L^{M_1^{-1}}(d_1\tilde{Q})$. Since $\sigma_2^D < \sigma_2^{M_1}$ by Lemma 6.3, we get

$$\widetilde{W}(L^{M_1}(s), s) > \widetilde{W}(L^D(s), s)$$

for $\sigma_2^D \leq s < \sigma_2^D + \varepsilon$, if $\varepsilon > 0$ and not too large. Hence $L(s) = L^{M_1(s)}$ for $\sigma_2^D \leq s < \sigma_2^D + \varepsilon$. On the other hand, $q_2\left(L^{M_1}(s)\right) > 0$ if $s > \sigma_2^{M_1}$. Since $\sigma_2^{M_1} < \sigma_1^D$, also $q_1\left(L^{M_1}(s)\right) > 0$ if $s > \sigma_2^{M_1}$. But if both firms produce, $L^D(s)$ is optimal by Lemma 6.2. Hence

$$\widetilde{W}(L^{M_1}(s), s) < \widetilde{W}(L^D(s), s)$$

for $\sigma_2^{M_1} - \varepsilon < s \leq \sigma_2^{M_1}$, if $\varepsilon > 0$ and not too large. Since also $W(L^{M_1}(s))$ and $W(L^D(s))$ are continuous and strictly decreasing in s, there is a unique σ_{int} such that

$$\widetilde{W}(L^{M_1}(\sigma_{int}), \sigma_{int}) = \widetilde{W}(L^D(\sigma_{int}), \sigma_{int}) .$$

Hence, $L(s) = L^{M_1}(s)$ for $\sigma_2^D \leq s \leq \sigma_{int}$ and $L(s) = L^D(s)$ for $\sigma_{int} < s \leq \sigma_1^D$.

For $s = \sigma_1^D$, we have $L^D(\sigma_1^D) = d_2\tilde{Q}$, hence $q_1(d_2\tilde{Q}) = 0$. For $L < d_2\tilde{Q}$ firm 2 is a monopolist. In the absence of firm 1, we had $L(s) = L^{M_2}(s)$ by Lemma 6.1. By Lemma 6.3, however, and since $L^{M_2}(s)$ is decreasing, we get $L^{M_2}(s) > d_2\tilde{Q}$ for $\sigma_1^D \leq s < \sigma_1^D + \varepsilon$ for appropriate ε. Hence firm 2 would operate if $L(s) = L^{M_2}(s)$ and $\sigma_1^D \leq s < \sigma_1^D + \varepsilon$. But then, welfare could be increased by decreasing L. Hence $L(s) = d_2\tilde{Q}$ for $\sigma_1^D \leq s \leq \sigma_1^{M_2}$. For $s > \sigma_1^{M_2}$, we have $L^{M_2(s)} > d_2\tilde{Q}$ by definition of $\sigma_1^{M_2}$. Hence, $L(s) = L^{M_2}(s)$ for $s > \sigma_1^{M_2}$.

case b): $\sigma_{mon} > \sigma_2^D, \sigma_2^{M_1} < \sigma_1^D$. In his case $L^{M_1}(s) > L_{mon}$ for s near σ_2^D. It is easy to show (use Lemma 6.1) that $\frac{\partial \widetilde{W}}{\partial L}(L, \sigma_2^D) > 0$ for $L < L^{M_1}(s)$. Since $L_{mon} > d_1\tilde{Q}$ we get

$$\widetilde{W}(L_{mon}, s) > \widetilde{W}(L^D(s), s) .$$

for $\sigma_1^D \leq s < \sigma_1^D + \varepsilon$. On the other hand, arguing as in case a) we get

$$\widetilde{W}(L^{M_1}(s), s) < \widetilde{W}(L^D(s), s)$$

for $\sigma_2^{M_1} - \varepsilon < s \leq \sigma_2^{M_1}$, if $\varepsilon > 0$ and not too large. Hence, there is $\sigma_{int} \in (\sigma_2^D, \sigma_2^{M_1})$ such that

$$\widetilde{W}\left(\min\{L_{mon}, L^{M_1}(\sigma_{int})\}, \sigma_{int}\right) = \widetilde{W}\left(L^D(\sigma_{int}), \sigma_{int}\right) .$$

The remaining arguments go through as in case a).

case c): $\sigma_{mon} \leq \sigma_2^D, \sigma_2^{M_1} \geq \sigma_1^D$. In order to establish the unique existence of $\sigma_{int} \in (\sigma_2^D, \sigma_2^{M_1})$ such that

$$\widetilde{W}(L^{M_1}(\sigma_{int}), \sigma_{int}) = \widetilde{W}(L^D(\sigma_{int}), \sigma_{int}) .$$

or

$$\widetilde{W}(L^{M_1}(\sigma_{int}), \sigma_{int}) = \widetilde{W}(d_2\tilde{Q}, \sigma_{int}) .$$

it suffices to show that

$$\widetilde{W}(L^{M_1}(\sigma_1^D), \sigma_1^D) < \widetilde{W}(d_2\tilde{Q}, \sigma_1^D) . \tag{A.6}$$

Since $L^{M_1}(\sigma_1^D) = d_1\tilde{Q}$, this leads to $\widetilde{W}(d_1\tilde{Q}, \sigma_1^D) < \widetilde{W}(d_2\tilde{Q}, \sigma_1^D)$. Now, $\widetilde{W}(d_1\tilde{Q}, s) \geq \widetilde{W}(d_2\tilde{Q}, s)$ is equivalent to

$$\tilde{Q} - \frac{\tilde{Q}}{2} - \frac{s}{2}[d_1\tilde{Q}]^2 - c_1\tilde{Q} \;\geq\; \tilde{Q} - \frac{\tilde{Q}}{2} - \frac{s}{2}[d_2\tilde{Q}]^2 - c_2\tilde{Q}$$
$$\Leftrightarrow \frac{s}{2}[d_2^2 - d_1^2]\tilde{Q} \;\geq\; c_1 - c_2$$

Solving for s yields:

$$s \leq \frac{2(c_2 - c_1)}{(d_1 - d_2)(d_1 + d_2)\tilde{Q}} =: s^*$$

Next we show that $s^* < \sigma_1^D = (c_2 - c_1)/d_2(d_1 - d_2)\tilde{Q}$. This is the same as $2/(d_1 + d_2) < 1/d_2 \Leftrightarrow d_2 < d_1$ which follows from (6.5). Since $\sigma_1^D \leq \sigma_2^{M_1}$ we get $s^* < \sigma_2^{M_1}$. This establishes (A.6). The remaining arguments work as in case a) and b).

case d): $\sigma_{mon} > \sigma_2^D, \sigma_2^{M_1} \geq \sigma_1^D$. Combining the arguments of a), b) and c) we get the result. Q.E.D.

Proof of Proposition 7.1 $s_{int} > \sigma_2^D = (c_2 - c_1)/d_1(d_1 - d_2)\tilde{Q} = 2(c_2 - c_1)/d_1[d_1(1 - c_2) - d_2(1 - c_1)]\tilde{Q} = 2\underline{s}$. The last equality holds by the definition of $\underline{s}$. $\sigma_1^D = 2\bar{s}$ is shown by substituting d_1 by d_2.

Proof of Proposition 7.2 $s_1^{M_2} = (1 - c_2 + d_2 t_1^{NO})/d_2^2(1 - c_2 - t_1^{NO})$, plugging in $t_1^{NO} = (1 - 2c_1 + c_2)/(2d_1 - d_2)$ and manipulating we get $s_1^{M_2} = (d_2(c_2 - c_1) + d_1(1 - c_2))/d_2^2[d_1(1 - c_2) - d_2(1 - c_1)]$. On the other hand: $\sigma_1^{M_2} = (1 - c_2 - \tilde{Q})/d_2^2\tilde{Q}$. Plugging in the expression for $\tilde{Q}$ and manipulating we get $\sigma_1^{M_2} = (d_1(1 - c_2) + d_2[(c_2 - c_1) - (1 - c_2)])/d_2^2[d_1(1 - c_2) - d_2(1 - c_1)]$. The denominators of both expressions are alike. The numerators differ in $-(1 - c_2) < 0$, hence $\sigma_1^{M_2} < s_1^{M_2}$. Q.E.D.

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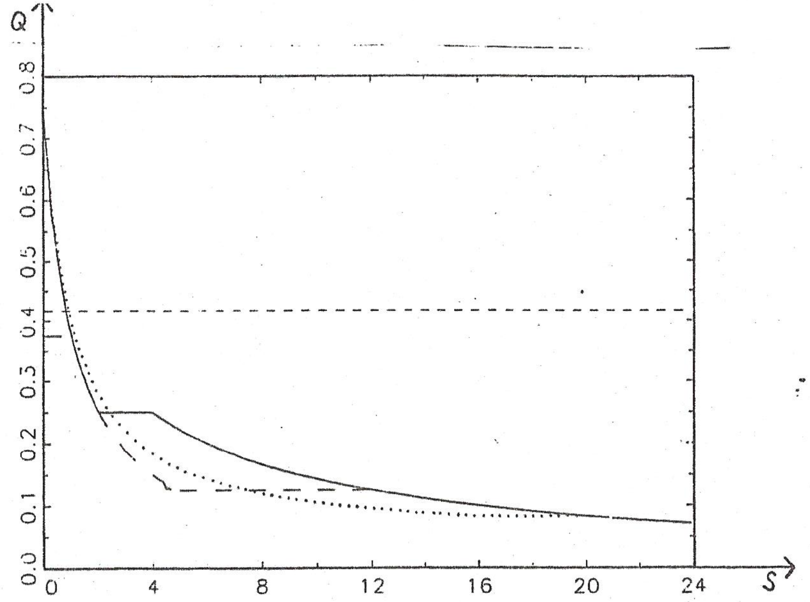

Figure 4: *Aggregate quantities for the marketable output commodity. The solid line depicts the social optimum, the "big dashed" line is for the permit solution, the dotted line for the tax solution, the "small dashed" line denotes "laissez faire".*

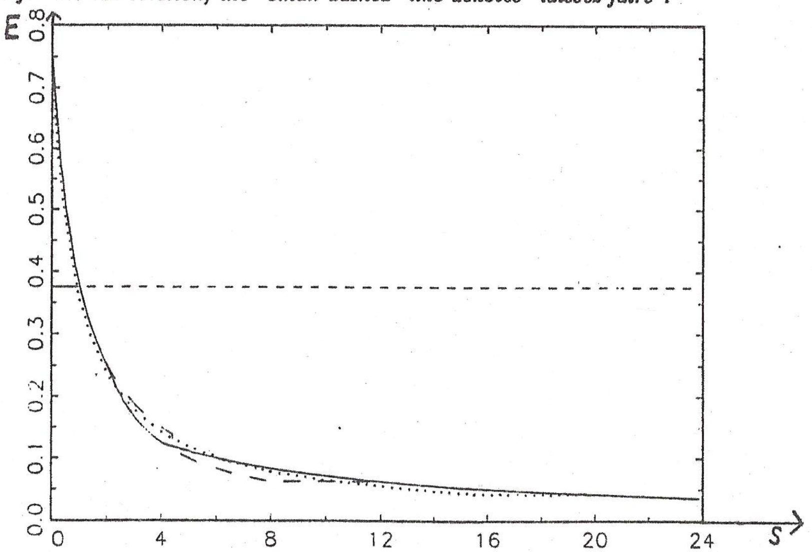

Figure 5: *Aggregate emissions, solid line: social optimum, "big dashed" line: permits, dotted line: taxes, "small dashed" line: "laissez faire".*

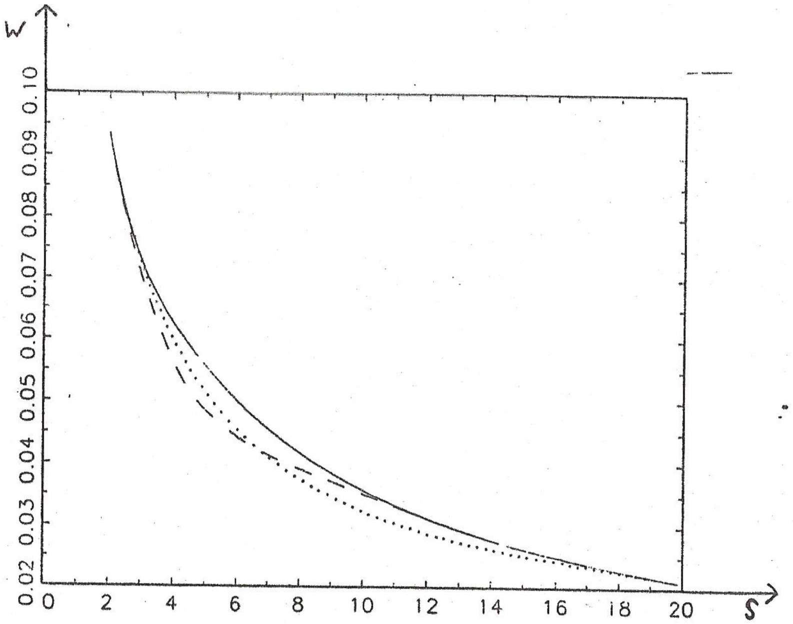

Figure 6: *Welfare without "lassez faire", solid line: social optimum, "big dashed" line: permits, dotted line: taxes.*

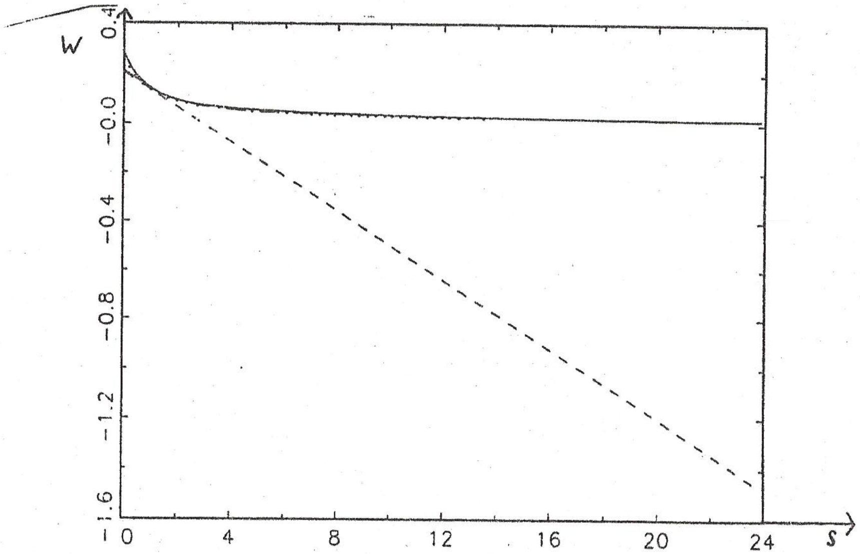

Figure 7: *Welfare including "lassez faire", solid line: social optimum, "big dashed" line: permits, dotted line: taxes, "small dashed" line: "laissez faire".*