INSTITUTE OF MATHEMATICAL ECONOMICS

WORKING PAPERS

No. 216

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December 1992

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June 1992

revised: October 1992

Abstract

The paper investigates a Bertrand duopoly, in which the two firms have different linear technologies. The government can either set a tax per unit of pollution or distribute tradeable licenses to pollute. The optimal emission tax as well as the optimal number of permits are characterized dependently on a critical damage parameter. Although the social optimum cannot be implemented and a comparison between both policies is ambiguous in general, for a wide range of parameters, the tax policy leads to higher welfare. However, the social optimum can be implemented always by combing licenses to pollute with subsidies on output. (JEL: L13, L51, Q28)

forthcoming in: *Journal of Theoretical and Institutional Economics*

*I would like to thank B. Simon, W. Trockel and two anonymous referees for helpful comments and suggestions. Financial support by the state government of Nordrhein–Westfalen (von Bennigsen–Foerder–Preis) is gratefully acknowledged.

Zusammenfassung

Die Arbeit untersucht ein Bertrand–Duopol, in dem die beiden Firmen verschiedene lineare Technologien besitzen. Zunächst sei angenommen, die Regierung könne entweder eine Emissionssteuer erheben oder eine Anzahl handelbarer Emissionslizenzen an die Firmen verteilen. Die optimale Emissionssteuer wird ebenso wie die optimale Anzahl von Lizenzen in Abhängigkeit von einem kritischen Schadensparameter charakterisiert. Obwohl im allgemeinen das soziale Optimum durch keine der beiden Politiken erreicht werden kann, und ein Vergleich beider Instrumente nicht eindeutig ausfällt, führt das steuerliche Instrument für einen großen Bereich von Parametern zu höherer Wohlfahrt. Jedoch kann das soziale Optimum durch Emissionslizenzen in Kombination mit Subventionierung des Outputs erreicht werden.

1 Introduction

The question of how to regulate polluting firms by decentralized mechanisms is not only of theoretical interest but it also is a political issue of increasing importance in many countries. Whereas in the United States markets for licenses to pollute have been created on different regional levels and for different pollutants, in Europe, especially in Germany, the majority of those politicians who care the most about pollution control apparently do not like the idea of tradeable permits, but rather prefer to impose emission taxes. Economic theory teaches that this debate is void as long as markets are perfectly competitive and the environmental jurisdictions have sufficient information about the firms' technologies, consumers' demand, and about the social damage generated by the pollution. Under those assumptions, charging the optimal emission tax or giving out the optimal number of licenses yields the same outcome (see for example BAUMOL and OATES [1988], and SPULBER [1985] for long run considerations). In a series of papers, WEITZMAN [1974], ADAR and GRIFFIN [1976], and FISHELSON [1976] demonstrated that this equivalence result does not hold under imperfect information about the firms' technologies or the social damage. Either of both tools may lead to a higher welfare, contingent on the economies' parameters.

Where firms do *not* behave as price takers, on the other hand, investigation of pollution control has been widely neglected so far. Just recently ULPH [1992] considered a two country model where the governments could impose individually different taxes or quotas on their only firm. The two firms compete a la Cournot or Stackelberg. EBERT [1992] investigated uniform taxation of pollutants when firms engage in Cournot competition. He showed that the optimal tax yields the social optimum if the oligopoly is symmetric. Decentralizing tools like permits or Pigouvian taxes, however, start to display their power if firms are *different*, since otherwise uniform standards work quite well. REQUATE [1992] showed that Pigouvian taxes are in general not efficient under Cournot competition if firms have different linear technologies, moreover, that permits and taxes may yield quite different outcomes even under perfect information. Since

Cournot competition is relatively soft, the inefficiency result should not surprise, for, it is well known that a high cost firm which would drop out under tough price competition may still operate under softer quantity competition. Therefore, it should be clarified whether the equivalence between permits and taxes can be restored under tougher price competition.

For this purpose we consider a duopoly of price setting firms which have different constant marginal costs and also emit different amounts of pollution proportional to output. Welfare is separable into consumers' gross surplus, social damage from the pollution, and the firms' production costs. First, we consider a situation where the government may regulate the firms by imposing an emission tax per unit of pollution or by giving out a number of tradeable emission permits. The pollutant is generated by no other industry, a market for permits will therefore be thin. The social damage function depends on aggregate emissions and an exogenously given damage parameter which determines the steepness of that function. We completely characterize the optimal emission tax as well as the optimal permit policy contingent on the firms' technologies and the damage parameter, and then we investigate and compare the welfare properties of the two policy instruments.

It turns out that in general neither of those two instruments can enforce the social optimum. However, the tax policy can implement the social optimum when it is socially optimal that only the low private cost (but high pollution) firm or only the high private cost (but low pollution) firm produces. The reason is that an emission tax has a direct impact on the firms' total unit cost and almost always induces one of the firms to have a cost advantage after taxes. Hence only the lower total unit cost firm produces under the standard assumptions of the Bertrand model. On the other hand, the firms' technologies may require *both* firms to produce in social optimum for a certain interval of damage parameters. In particular, if the damage parameter increases, production will be shifted continuously from the low private cost to the high private cost (but less polluting) firm. This continuous shift, however, *cannot* be induced by the emission tax. Despite of this shortcoming, the emission tax turns

out to be much more effective under Bertrand competition, investigated here, than under Cournot competition, which was treated in REQUATE (1992). There it has been demonstrated that the socially inefficient firm could not always be held out of the market by the optimal emission tax.

Under *permits*, there may be three critical intervals of damage parameters. The low cost firm produces if social damage is small, production will be shifted continuously from the high to the low pollution firm as the damage parameter increases, and only the high cost (but low pollution) firm produces if social damage is sufficiently high. This structure is very similar to the socially optimal allocation. However, since the firms care about marginal revenues rather than consumers' surplus and marginal damage caused by the pollution, the critical intervals of damage parameters where under permits only firm 1, both firms, or only firm 2 produces, do not coincide with those in social optimum. Typically, the low cost firm produces too much, and the high cost (but less polluting) firm produces too little under permits. In contrast to the tax instrument, the optimal number of permits to be given out is not affected by the kind of competition the firms engage in. This is due to the fact that firms are naturally capacity constrained if they hold a certain number of permits and have linear technologies. Hence, price competition turns into Bertrand–Edgeworth competition under permits. This requires some rationing rule. If we now employ the common efficient or random rationing rules[1], Cournot and Bertrand–Edgeworth competition yield the same allocation under the permit policy.[2]

It follows that the emission tax is not worse and often strictly superior to the permit instrument for a wide range of damage parameters, in particular for those

[1] *Random rationing* is sometimes also referred to as *proportional rationing* in the literature. It is taken for granted that those concepts are well known. Otherwise see for example Tirole [1988], Chapter 5.3.1.

[2] This result is reminiscent of the KREPS–SCHEINKMAN [1983] result which yields equivalence of Cournot and Bertrand–Edgeworth competition if firms choose quantities first, then prices, and the efficient rationing rule applies. By trading permits first and engaging in price competition thereafter, our model has a similar structure.

which require only one of the two firms to produce in social optimum. However, the superiority of taxes does not hold for all constellations of parameters. We will present an example with extremely asymmetric firms where optimal regulation by permits yields a higher welfare than optimal taxation.

Finally we allow the government to give out permits to pollute and at the same time to subsidize output of the marketable commodity. We show that this policy can always implement the social optimum if the optimal subsidy rate is paid and the optimal number of permits is given out.

Altogether, this paper and REQUATE [1992] show that under imperfect competition the optimal choice of decentralizing instruments of pure pollution control, like taxes on emissions and tradeable permits to pollute, depends very sensitively on the degree of social damage caused by the pollution, on the one hand, and the industry structure, that is, the firms' asymmetry and the special way of competition, on the other. Subsidies on output, in many real situations correctly under political attack, turn out to be quite useful if combined with tradeable permits to pollute.

We proceed by setting up the model in the following section. In section 3 we briefly consider the social optimum. Section 4 characterizes the optimal Pigouvian emission tax, section 5 the optimal number of permits, both as a function of the social damage function's steepness. In section 6 we compare both policies with respect to welfare. Section 7 considers subsidies on output combined with permits to pollute. Section 8 offers a summary and conclusions.

2 The Basic Model

Throughout this paper we consider a duopoly with firms $i = 1,2$ setting prices p_1, p_2. We start from a bounded downward sloping inverse demand function P, where $\bar{p} := P(0)$ is the choke off price. Assuming $P' < 0$ on its support we can define demand by $D := P^{-1}$ for all $p \in [0,\bar{p}]$ and $D(p) = 0$ for $p > \bar{p}$. Further we assume that P is not too convex:

Assumption 1 *For all $q > 0$ with $P(q) > 0$:* $P''(q) < 2P'(q)/q$.[3]

The upper bound for P'' is sufficient to guarantee the second order conditions for profit maximization of a monopoly. Both firms have constant marginal costs c_1 and c_2, with (w.l.o.g.) $c_1 \leq c_2 < \bar{p}$. To determine the firm's demand we follow the standard Bertrand model: If firms charge prices p_1 and p_2, firm i's demand is given by

$$D_i(p_i,p_j) := \begin{cases} D(p_i) & \text{if } \; p_i < p_j \,, \\ D(p_i)/2 & \text{if } \; p_i = p_j \,, \\ 0 & \text{if } \; p_i > p_j \,. \end{cases} \tag{2.1}$$

This definition reflects the implicit assumption that consumers are always perfectly informed about the prices, that they always buy at the cheaper firm if prices differ, and split up equally if prices are alike, and the firms are not capacity constrained. The firms' demand functions will be different if we consider regulation by permits, which naturally impose capacity constraints on the firms. This leads to Bertrand–Edgeworth rather than Bertrand competition, and will require a rationing rule. We will turn to that later. It is well known that there is a unique Bertrand–Nash equilibrium, with firms charging a price equal to marginal cost, if those costs are alike. If, say, $c_1 < c_2$ and $c_2 < p_1^m$, where p_1^m is firm 1's monopoly price, we follow the industrial organization literature and take $p_1 = p_2 = c_2$ as the unique Bertrand–Nash equilibrium (actually $p_1 = c_2 - \varepsilon$)[4]. Hence we will call $p = \min\{p_i^m, c_j\}$ *the* Bertrand equilibrium price, if

[3] In terms of elasticity, the derivative of the inverse demand function has elasticity smaller than 2.

[4] Strictly speaking, there is, of course, no Bertrand–Nash equilibrium (in pure strategies) if firms are different. For $(p_1,p_2) = (c_2,c_2)$ both firms would share demand, but firm 1 could improve by undercutting c_2 by ε. But also any price below c_2 is not optimal, since then firm 1 could raise the price a little bit again. This nonexistence, however, is due to the properties of the real numbers rather than a severe economic problem. To overcome the difficulty we could restrict the firms' strategies to a finite set of prices — which raises other inconveniences — or to take ε–equilibrium as a solution concept. Thus the price outcome $(c_2-\varepsilon', c_2)$ is an ε–equilibrium for a suitable ε'. We will do the latter and will then take the limit for $\varepsilon \to 0$, and simply consider the supremum of firm 1's ε–equilibrium prices which is c_2. In this spirit we will interpret the outcome $(p_1,p_2) = (c_2,c_2)$ as the limit of ε–equilibrium outcomes where firm 1 gets the whole demand and makes a profit of $(c_2 - c_1)D(c_2)$,

$c_i < c_j$.

It is assumed that production is not possible without pollution. Producing q_i units of output, firm i generates $e_i = d_i q_i$ units of emissions. Total emissions are written $E := e_1 + e_2$. To evaluate utility and harm of (q_1, q_2) (which determines (e_1, e_2)) to the society, we assume to have social welfare function W. In the absence of pollution, in the industrial economics literature, a social welfare is simply taken as $W(q_1, q_2) = \int_0^Q P(z)dz - c_1 q_1 - c_2 q_2$, that is, consumers' gross surplus minus aggregate production costs.[5] We will extend this approach by assuming that W is separable into benefit from production and damage from pollution. This means, in addition to consumers' surplus there is a social damage function $S : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ with $(E, s) \mapsto S(E, s)$, which depends on aggregate emissions E and a damage parameter s. Employing the usual notation $S_1(E, s) := \frac{\partial S(E,s)}{\partial E}$ and so on, we make the following assumption.

Assumption 2 *o) S is at least twice continuously differentiable with respect to*[6] *E and s; in $(0,0)$ the right sided partial derivatives exist.*
i) $S(0, s) = 0 \ \forall s \geq 0$.
ii) $S(E, 0) = 0 \ \forall E \geq 0$.
iii) $S_1(E, s) \geq 0 \ \forall s > 0$ and strictly greater for $E > 0$.
iv) $S_{11}(E, s) \geq 0 \ \forall s > 0$ and strictly greater for $E > 0$.
v) $S_{12}(E, s) > 0 \ \forall E > 0, s > 0$.

So, S is increasing and convex in E and marginal damage increases in s. Although s is an exogenous parameter of the model, parameterizing S via s allows us to characterize the social optimum as well as regulatory policies as a function of the damage function's

and firm 2 gets nothing.

[5]This is equivalent to $W(q_1, q_2) = \int_0^Q P(z)dz - P(Q) \cdot Q + (P(Q)q_1 - c_1 q_1) + (P(Q)q_2 - c_2 q_2)$, that is, net consumers' surplus plus profits of the firms. Some authors use the latter, and sometimes even multiply surplus and profits with different weights (see for example BARON and MYERSON [1982]). Then, however, the two concepts are not equivalent.

[6]For short: "w.r.t." in the remainder.

steepness. Finally we assume

Assumption 3 *The pollutant resulting from production of the industry's output, only arises in this industry.*[7]

In reality, Assumption 3 does not hold in *all* industries, of course. For example CO_2, is generated by many different industries. SO_2, on the other hand, is generated basically by power plants. Also in the chemical industry, some poisonous pollutants are generated from production of one certain commodity only. Since we want to analyze regulation of firms under imperfect competition, Assumption 3 is crucial to make the analysis interesting.

Assuming separability of social welfare in consumers' surplus, production cost, and social damage, the welfare function W is given by

$$W^s(q_1, q_2) := \int_0^Q P(z)dz - S(E, s) - c_1 q_1 - c_2 q_2 , \tag{2.2}$$

where the superscript s refers to the damage parameter determining the steepness of S. Without any kind of regulation, Bertrand competition leads to the Bertrand–Nash (ε–)equilibrium independently of s.

When considering regulatory policies in the sections 4 and 5, we will assume that the government has sufficient information in the following sense. It knows demand, the social damage function, and the firms' technologies. More precisely, it knows what technologies there are, but not necessarily what firm has what technology. This information structure corresponds to the second degree price discrimination models.

We also assume that the emissions generated by each firm can be perfectly and costlessly monitored by the authorities. So, the firms will pay a tax bill exactly according to the amount of their emitted pollutants. In case of holding permits, firms cannot emit more than the number of permits allows them to do. Otherwise, we assume, a high penalty has to be paid. So there is no room for moral hazard.

[7]By an industry we mean the set of firms which produce a certain commodity, that is here, the two firms which produce the marketable commodity under consideration.

Before turning to regulatory policies, let us characterize the social optimum that a fictive social planner would install under complete information. If $c_1 < c_2$, it is clear that for $s = 0$ the higher cost firm 2 should not produce anything. If social damage is very high, one could think that only the firm with the lower pollution per unit of output should operate, that is, with the smaller d_i. However, it is not quite like this. What will turn out to be crucial is whether the term $(d_1c_2 - d_2c_1)/(d_1 - d_2)$ is greater than the choke-off price or not, or equivalently, what the sign of $\tilde{\Delta} := d_1/(\overline{p} - c_1) - d_2/(\overline{p} - c_2)$ is, which is the difference between the firms' ratio of marginal pollution and maximal marginal consumers' surplus. Instead of $\tilde{\Delta}$ it is more convenient to work with $\Delta := d_1(\overline{p} - c_2) - d_2(\overline{p} - c_1)$ in the remainder of the paper. Notice that $\tilde{\Delta}$ and Δ have the same sign.

3 The social optimum

The social planner has to solve the following program:

$$\max_{q_1,q_2} W^s(q_1, q_2) := \max_{q_1,q_2} \int_0^{q_1+q_2} P(z)dz - S(d_1q_1 + d_2q_2, s) - c_1q_1 - c_2q_2 \qquad (3.1)$$

s.t. $q_1 \geq 0$, $q_2 \geq 0$.

The following proposition yields the properties of the optimal solution (recall that $c_1 \leq c_2$):

Proposition 3.1 *a) If $\Delta \leq 0$, for all $s \geq 0$ firm 2 never produces, and firm 1 produces q_1 which solves*

$$P(q) = c_1 + S_1(d_1q, s)d_1 \ . \qquad (3.2)$$

q_1 is decreasing in s, unless $c_1 = c_2$, $d_1 = d_2$ (if both firms are alike, clearly q may be arbitrarily distributed among both firms).

b) If $\Delta > 0$, there are parameters $\underline{s}, \overline{s}$ with $0 < \underline{s} < \overline{s} \leq \infty$ ($\overline{s} < \infty$ for $d_2 > 0$) such that the solution of (3.1) is characterized by

i) $\forall s \in [0, \underline{s}]$ we get $q_1 > 0$, $q_2 = 0$, and $Q = q_1$ is decreasing in s.

ii) $\forall s \in [\underline{s}, \overline{s}]$ we get $q_1 > 0$, $q_2 > 0$, and q_1 is decreasing, q_2 is increasing, and $Q = q_1 + q_2$ is constant in s.

iii) $\forall s \geq \overline{s}$ we get $q_1 = 0$, $q_2 > 0$, and $Q = q_2$ is decreasing in s.

iv) Moreover, Q, E, and W are continuous, E and W are decreasing in s.

Thus, we can say that firm 1 has the better technology if $\Delta \leq 0$ (unless $c_1 = c_2$, $d_1 = d_2$). Notice that $c_1 < c_2$ and $d_1 = d_2$ as well as $c_1 = c_2$ and $d_1 < d_2$ imply $\Delta \leq 0$. But notice also that $\Delta \leq 0$ may hold for some $d_1 > d_2$ if c_1 is sufficiently smaller than c_2. In other words, even if firm 2 emits less pollutants per unit of output than firm 1, it may never produce in social optimum if the cost differential $c_2 - c_1$ is sufficiently high. Notice on the other hand that $c_1 \leq c_2$ and $\Delta > 0$ imply $d_1 > d_2$, that is, for $\Delta > 0$ firm 2 emits strictly less pollutants per unit of output than firm 1.

The result of Proposition 3.1 is derived by solving (3.1) taking into account the Kuhn–Tucker conditions with respect to the constraints $q_1 \geq 0$ and $q_2 \geq 0$. For later reference we write down the first order conditions[8]. Differentiating the Lagrangian with respect to q_1 and q_2 we get

$$P(q_1 + q_2) - S_1(E, s) \cdot d_1 - c_1 + \mu_1 = 0 \qquad (3.3)$$

$$P(q_1 + q_2) - S_1(E, s) \cdot d_2 - c_2 + \mu_2 = 0 \qquad (3.4)$$

Here μ_1, μ_2 are the Kuhn–Tucker multipliers w.r.t. the constraints $q_1 \geq 0$ and $q_2 \geq 0$.

Assuming $\mu_1 = \mu_2 = 0$, eliminating $S_1(E, s)$, and solving for $P(Q) = P(q_1 + q_2)$ yields:

$$P(Q) = (d_1c_2 + d_2c_1)/(d_1 - d_2) \qquad (3.5)$$

or, equivalently

$$Q = D([d_1c_2 + d_2c_1]/[d_1 - d_2]) \qquad (3.6)$$

on the interval $[\underline{s}, \overline{s}]$, independently of s. Let us denote $\tilde{Q} := Q(s)$ for $s \in [\underline{s}, \overline{s}]$ and $\tilde{p} := P(\tilde{Q})$.

[8] For short: "f.o.c.s", in the remainder.

If now $\mu_i \geq 0$, $\mu_j = 0$ for $j = 3 - i$, that is, $q_i = 0$, $q_j > 0$, the f.o.c. becomes

$$P(q_j) - S_1(d_j q_j, s) \cdot d_j - c_j = 0 \tag{3.7}$$

that is, price equals marginal production cost plus marginal social damage. A more detailed proof can be found in REQUATE [1992].

Interestingly, we find that aggregate output is constant in s for $\underline{s} \leq s \leq \bar{s}$. Thus, the social planner shifts production continuously from firm 1 to firm 2 as s increases, keeping total output constant, until firm 1, which faces the lower production cost but is the worse polluter, shuts down. These properties are displayed in figure 1.

Figure 1 about here.

4 Pigouvian Taxes

Suppose now that a linear tax is imposed on emissions such that the firm's marginal cost amounts to $c_i + \tau d_i$. Let

$$p_i^m(\tau) := \arg\max_p [p - (c_i + \tau d_i)] D(p) \tag{4.1}$$

be firm i's monopoly price under tax τ and let

$$q_i^m(\tau) := D(p_i^m(\tau)) \tag{4.2}$$

be the corresponding monopoly output.

If

$$c_i + \tau d_i \quad < \quad c_j + \tau d_j \ ,$$

under Bertrand-competition, the market price is given by

$$p(\tau) = \min\{c_j + \tau d_j, p_i^m(\tau)\} \tag{4.3}$$

and the quantity by

$$Q(\tau) = \max\{D(c_j + \tau d_j), q_i^m(\tau)\} \ . \tag{4.4}$$

If

$$c_i + \tau d_i \quad = \quad c_j + \tau d_j \ ,$$

then

$$p(\tau) = c_1 + \tau d_1 = c_2 + \tau d_2 \quad \text{and} \quad Q = D(c_1 + \tau d_1) \ ,$$

where $q_1 = q_2 = D(c_1 + \tau d_1)/2$.

Next define τ_i^c by

$$c_j + \tau_i^c d_j = p_i^m(\tau_i^c) \tag{4.5}$$

as the tax rate where firm i switches from monopoly behavior to competition with firm j (or vice versa).

Further define for $d_1 \neq d_2$ the "break even-tax" τ^{be} where marginal costs of the two firms are equal by

$$\tau^{be} := \frac{c_2 - c_1}{d_1 - d_2} \ . \tag{4.6}$$

Lemma 4.1 *If $d_1 > d_2$, then $\tau_1^c < \tau^{be} < \tau_2^c$ and*

$$p(\tau) = \begin{cases} p_1^m(\tau) & \text{for} \quad \tau \leq \tau_1^c \\ c_2 + \tau d_2 & \text{for} \quad \tau_1^c \leq \tau < \tau^{be} \end{cases} \Big\} \ \textit{firm 1 produces}$$
$$\qquad \begin{matrix} \frac{c_2 - c_1}{d_1 - d_2} & \text{for} \quad \tau = \tau^{be} \end{matrix} \quad \textit{both firms produce}$$
$$\qquad \begin{cases} c_1 + \tau d_1 & \text{for} \quad \tau^{be} < \tau \leq \tau_2^c \\ p_2^m(\tau) & \text{for} \quad \tau_2^c \leq \tau \end{cases} \Big\} \ \textit{firm 2 produces}$$

Moreover, $p(\tau)$ is continuous at τ^{be}.

Proof: By definition of τ_1^c we know that $c_2 + \tau_1^c d_2 = P(q_1(\tau_1^c))$. On the other hand, $P(q_1(\tau_1^c))$ satisfies the f.o.c. for monopoly: $P(q_1) + P'(q_1)q_1 - c_1 - \tau_1^c d_1 = 0$. Combining we get $c_2 + \tau_1^c d_2 + P'(q_1)q_1 - c_1 - \tau_1^c d_1 = 0$, thus $\tau_1^c = (c_2 - c_1 + P'(q_1)q_1)/(d_1 - d_2) < (c_2 - c_1)/(d_1 - d_2) = \tau^{be}$. To show $\tau_2^c > \tau^{be}$ works almost the same. The rest can be checked by inspection (see Figure 2). Q.E.D.

Figure 2 about here.

Define for $i = 1, 2$

$$p_i^c(\tau) := c_j + \tau d_j \qquad \text{for } j = 3 - i\,,$$
$$q_i^c(\tau) := D(p_i^c(\tau))\,.$$

The price $p_i^c(\tau)$ is firm j's marginal cost. So firm j acts like a competitive fringe for firm i.

The government's program is now to maximize

$$\int_0^{Q(\tau)} P(z)dz - S(E(\tau), s) - c_1 q_1(\tau) - c_2 q_2(\tau)\,. \tag{4.7}$$

For $\tau \neq \tau^{be}$ only one of the two firms produces and (4.7) amounts to

$$\int_0^{q_i(\tau)} P(z)dz - S(d_i q_i(\tau), s) - c_i q_i(\tau)\,. \tag{4.8}$$

The f.o.c. with respect to τ is

$$[P(q_i(\tau)) - S_1(d_i q_i(\tau), s)d_i - c_i]q_i'(\tau) = 0\,. \tag{4.9}$$

First we claim that $q_i'(\tau) < 0$. To see this we differentiate the f.o.c. of the monopoly firm w.r.t. τ. This yields $[2P' + P''q_i]q_i' - d_i \equiv 0$. By Assumption 1 we get

$$q_i' = \frac{d_i}{2P' + P''q_i} < 0\,. \tag{4.10}$$

Hence, the term in square brackets of (4.9) must be zero. But this is also the f.o.c. for the social optimum if $\Delta \leq 0$ and only firm 1 produces, or if $\Delta > 0$ and $s < \underline{s}$ or $s > \overline{s}$ when only firm 1 or firm 2, respectively, produce. It follows that the social optimum can be achieved by imposing an emission tax as long as $\tau(s) < \tau^{be}$ or $\tau(s) > \tau^{be}$. In the first case, firm 1 produces alone, in the latter firm 2. This holds regardless of whether $p_i(\tau) = p_i^m(\tau)$ or $p_i(\tau) = p_i^c(\tau)$. If $p(\cdot)$ has a kink at τ_i^c, so does $\tau(\cdot)$ at $\tau^{-1}(\tau_{ic})$ for the corresponding value of s. This need not bother us. It remains to investigate for what values of s the taxes $\tau(s)$ are greater, smaller than, or equal to τ^{be}. By Lemma 4.1, $\tau(s) < \tau^{be}$ implies

$$p(\tau) = P(q_1(\tau)) \leq c_2 + \tau d_2 < c_2 + \tau^{be} d_2 = c_2 + \frac{c_2 - c_1}{d_1 - d_2} d_2 = \frac{d_1 c_2 - d_2 c_1}{d_1 - d_2}\,.$$

From section 3 we know that for $\Delta > 0$, in social optimum, $Q(s) = q_1(s)$ is decreasing and $P(Q(s))$ is increasing for $s < \underline{s}$. For $s \in [\underline{s}, \overline{s}]$, $Q(s)$ and $P(Q(s))$ are constant and $P(Q(s)) = [d_1 c_2 - d_2 c_1]/(d_1 - d_2)$. Hence, $\tau(s) < \tau^{be}$ if and only if $s < \underline{s}$. Arguing the same way for firm 2 we get $\tau(s) > \tau^{be}$ if and only if $s > \overline{s}$, and only firm 2 produces.

It remains to figure out the best tax for $s \in [\underline{s}, \overline{s}]$. Now, if $\tau = \tau^{be}$, the unique Nash-equilibrium is given by $p_1 = p_2 = (d_1 c_2 - d_2 c_1)/(d_1 - d_2)$ and q_1 jumps from $D(p_1(\tau))$ to $D(p_1(\tau))/2$ as τ increases from below to τ^{be}. That is, for $s \in [\underline{s}, \overline{s}]$ and $\tau = \tau^{be}$ we have the socially optimal price and the socially optimal quantity, however, we do not have the optimal allocation of production between the two firms. Whereas the social planner would shift production continuously from firm 1 to firm 2 as s increases from $\underline{s}$ to $\overline{s}$, the government cannot induce this continuous shift by taxes. By continuity, however, there is obviously some $s_0 \in (\underline{s}, \overline{s})$ such that for the optimal quantities $q_i(s)$, we get $q_1(s_0) = q_2(s_0)$, and $q_1(s) \approx q_2(s)$ for s in a neighborhood of s_0. On the other hand, $q_1(s) \approx q_1(\underline{s})$ for s close to $\underline{s}$, and $q_2(s) \approx q_2(\overline{s})$ for s close to $\overline{s}$. Hence, the best linear tax on the interval $[\underline{s}, \overline{s}]$ works like this:

Still prevent firm 2 from production for s close to $\underline{s}$ by charging $\tau^{be} - \varepsilon$. Charge τ^{be} for s close to s_0 and charge $\tau^{be} + \varepsilon$ for s close to $\overline{s}$.

With a little abuse of notation let $W(\tau, s) := W^s(q_1(\tau), q_2(\tau))$ and consider welfare as a function of s for fixed tax rates $\tau^{be} - \varepsilon$, τ^{be} and $\tau^{be} + \varepsilon$. Clearly, $W(\tau^{be} - \varepsilon, \cdot)$ and $W(\tau^{be}, \cdot)$ must intersect somewhere between $\underline{s}$ and s_0, whereas $W(\tau^{be}, \cdot)$ and $W(\tau^{be} + \varepsilon, \cdot)$ must intersect somewhere between s_0 and $\overline{s}$. Hence, the switching points where $\tau(s)$ jumps from $\tau^{be} - \varepsilon$ to τ^{be}, and from τ^{be} to $\tau^{be} + \varepsilon$, are given by the intersection of welfare $W(\tau^{be} - \varepsilon, \cdot)$ with $W(\tau^{be}, \cdot)$ in s, and $W(\tau^{be}, \cdot)$ with $W(\tau^{be} + \varepsilon, \cdot)$, respectively. Call these switching points s_1 and s_2. (Strictly speaking, there exists no optimal tax for $s \in (\underline{s}, s_1)$ and $s \in (s_2, \overline{s})$. But it is optimal up to some ε.)

By these arguments we have shown the following proposition.

Proposition 4.1 *If $\Delta > 0$, then*

a) $\tau(s)$ *which solves (4.9) for* $i = 1$ *implements the social optimum for* $s < \underline{s}$, *and only firm 1 produces.*

b) $\tau(s) = \tau^{be} - \varepsilon'$ *for* $s \in [\underline{s}, s_1)$ *(with* $s_1 < s_0$*), and only firm 1 produces.*

c) $\tau(s) = \tau^{be}$ *for* $s \in [s_1, s_2]$ *(with* $s_1 < s_0 < s_2$*), and both firms produce.*

d) $\tau(s) = \tau^{be} + \varepsilon'$ *for* $s \in (s_2, \bar{s}]$ *(with* $s_0 < s_2$*), and only firm 2 produces.*

e) $\tau(s)$ *which solves (4.9) for* $i = 2$ *implements the social optimum for* $s > \bar{s}$, *and only firm 2 produces.*

f) *Moreover,* $Q(s)$ *is decreasing and* $P(Q(s))$ *is increasing for* $s < \underline{s}$ *and* $s > \bar{s}$. $Q(s)$ *and* $P(Q(s))$ *are "almost" constant on* $[\underline{s}, \bar{s}]$. *("Almost" is supposed to mean "up to two small jumps at* s_1 *and* s_2*", which can be held arbitrarily small.)*

Observe moreover that $W(\tau^{be}, s)$ smoothly approximates the optimal social welfare $W^{SO}(s)$ in the interior of $(\underline{s}, \bar{s})$ at the point s_0. It is equal to $W^{SO}(s_0)$ for $s = s_0$ and cannot intersect W^{SO} (otherwise W^{SO} would not be optimal since it is differentiable in s). The same holds for the points $\underline{s}$ and $\bar{s}$ (up to an arbitrarily small ε).

Corollary 4.1 *If* $\Delta > 0$, *the optimal linear tax implements the social optimum for those* s *for which only one firm should produce, that is, for* $s < \underline{s}$ *and* $s > \bar{s}$. *Moreover there is some* $s_0 \in (\underline{s}, \bar{s})$ *where the optimal linear tax implements social optimum and both firms produce.*

Thus the optimal linear tax is not fully efficient for those parameters of s where both firms produce but yields a reasonable approximation.

We turn now to the case $\Delta \leq 0$.

Proposition 4.2 *If* $\Delta \leq 0$, *then only firm 1 produces, and for all* $s \geq 0$, $\tau(s)$ *implements the social optimum.*

Proof: The proof works indirectly. First, let $d_1 > d_2$. Suppose $\tau(s) = \tau^{bc}$ such that both firms produce. Then $p = \tilde{p} = [d_1c_2 - d_2c_1]/[d_1 - d_2]$, or equivalently, $\tilde{p}(d_1-d_2)-d_1c_2+d_2c_1 = 0$. But $\Delta = \bar{p}(d_1-d_2)-d_1c_2+d_2c_1 > \tilde{p}(d_1-d_2)-d_1c_2+d_2c_1 = 0$, a contradiction. Suppose now $\tau(s) > \tau^{be}$ and $p_2(\tau) = c_1 + \tau d_1$. Then $\bar{p} \geq c_1 + \tau d_1 > c_2 + \tau^{be} d_2 = [d_1c_2 - d_2c_1]/[d_1 - d_2]$, implying $\Delta > 0$. Next suppose $p_2(\tau) = p_2^m(\tau)$. Then

$$
\begin{aligned}
0 &= p_2(\tau) + P'(q_2^m(\tau))q_2^m(\tau) - c_2 - \tau d_2 \\
&< p_2(\tau) - c_2 - \tau^{be} d_2 \\
&= p_2(\tau) - \frac{d_1c_2 + d_2c_1}{d_1 - d_2} < \frac{\Delta}{d_1 - d_2}.
\end{aligned}
$$

Hence also $\Delta > 0$, a contradiction again.

Now let $d_1 < d_2$. Firm 2 can only produce if $c_1 + \tau d_1 \geq c_2 + \tau d_2$ which implies $\tau \leq [c_2 - c_1]/[d_1 - d_2] < 0$. But then the market price would not be higher than $c_1 + \tau d_1 < c_1$. Welfare could be improved by setting $\tau = 0$. Q.E.D.

Not surprisingly we get the following

Corollary 4.2 *The optimal emission tax implements the social optimum for symmetric Bertrand duopoly.*

Moreover, it should be mentioned that $\tau(s)$ becomes negative for s close to zero.[9] So, for small s the tax turns into a subsidy. Since a monopolist produces less than socially optimal if the damage from pollution is very small, the emission tax can indirectly subsidize production by subsidizing pollution.

Remark 4.1 *The last statement, however, must be taken with caution. If the subsidy is higher than marginal cost and the firm faces free disposal, it could be tempted to produce pollution, take the subsidy and destroy the produced quantity of the marketable commodity up to the monopoly output. In other words, subsidizing pollution without controlling the output of the marketable good may not work. But subsidizing pollution,*

[9] To see this, notice first that the optimal quantity is $q_1 = D(c_1 + S_1(d_1q_1, s))$. The firm's f.o.c. is $P(q_1) + P'(q_1)q_1 - [c_1 + \tau d_1] = 0$. Solving for τ yields $\tau = \frac{P(q_1)+P'(q_1)q_1-c_1}{d_1} = \frac{S_1(d_1q_1,s)+P'(q_1)q_1}{d_1} < 0$ for s sufficiently small.

or even a non-poisonous by-product, in order to enforce more production seems to be somewhat ridiculous. Direct subsidies on the output should be more effective in such a case. We will return to this in section 7.

The results derived in this section, above all, the corollaries 4.1, 4.2 and Proposition 4.2 suggest that Pigouvian taxes are very powerful instruments in order to control the pollution of firms which engage in Bertrand competition. The optimal tax can indeed implement the social optimum for those cases, where only one of the two firms is supposed to produce in social optimum. The intuition behind this result is straightforward: Under Bertrand competition the lower cost firm always serves the whole market, whereas the higher cost firm produces nothing. Unless the firms are alike, a tax can almost always induce a cost advantage for one of the two firms apart from the case where $\tau = \tau^{be}$. In that case the emission tax cannot induce the optimal allocation of production among the firms. If the interval $(\underline{s}, \overline{s})$ is small, the efficiency distortion can certainly be neglected. If the asymmetry between the firms is considerable, however, especially if d_1 is large and d_2 is relatively small, the interval $(\underline{s}, \overline{s})$ can become rather large and the welfare loss is not to be neglected.

5 Permits

In this section we assume that the government gives out a number of L pollution permits which may be traded among the firms. Each permit allows a firm to emit one unit of the pollutant. Since we assume that the government has no objective to earn money from regulation, we can assume that the permits are given to the firms for free. For example they could be distributed fairly among the firms such that each firm holds $L/2$ permits at the beginning. As we will see, the initial allocation of permits will not effect the outcome. Assume that L be arbitrarily divisible.

5.1 The Firms' Behavior

The process going on in the economy may be divided into 3 steps. At first, the firms hold some initial endowment (l_1, l_2) of permits, with $l_1 + l_2 = L$. In the second step, they may trade, that is here, one firm sells some or all permits to the other firm. Firms end up with a new allocation of permits (e_1, e_2) with $e_1 + e_2 = L$. In the third step, firms engage in Bertrand–Edgeworth competition and choose prices p_1, p_2. Since each firm is capacity constrained by e_i/d_i, that is, the number of permits it owns, divided by pollution per output, its demand $D_i(p_i, p_j)$ involves some rationing rule. We assume that residual demand is determined by either *efficient* or *random rationing* (or any convex combination of both).

To figure out how the firms will trade the permits, denote by $\Pi_i^N(e_1, e_2)$ the profit of firm i if the final allocation of permits in the second step has been (e_1, e_2) and both firms choose prices but are capacity constrained by e_i/d_i. Observe that there is a gain from trade if and only if there is an allocation $(\hat{e}_1, \hat{e}_2)$ such that

$$\Pi_1^N(l_1, l_2) + \Pi_2^N(l_1, l_2) < \Pi_1^N(\hat{e}_1, \hat{e}_2) + \Pi_2^N(\hat{e}_1, \hat{e}_2) .$$

In this case there is a real number T which can be interpreted as a *transfer–payment* from firm 1 to firm 2 (which may be negative, of course) such that

$$\begin{aligned} \Pi_1^N(\hat{e}_1, \hat{e}_2) + T &> \Pi_1^N(l_1, l_2) , \\ \Pi_2^N(\hat{e}_1, \hat{e}_2) - T &> \Pi_2^N(l_1, l_2) . \end{aligned}$$

How the firms figure out T is nothing we have to care about. For example, they could agree on the Nash–bargaining solution. The maximum gain from trading permits is determined by

$$\max_{e_1, e_2} \left[\Pi_1^N(e_1, e_2) + \Pi_2^N(e_1, e_2)\right] \qquad \text{s.t. } e_1 + e_2 \leq L,\ e_1 \geq 0,\ e_2 \geq 0. \tag{5.1}$$

Clearly, this maximum is independent of the initial distribution of permits. Accepting the assumption that firms behave as profit maximizers it is natural to make the following assumption:

Assumption 4 *Firms trade permits in the second phase such that the final allocation (e_1^*, e_2^*) solves (5.1).*

Notice that this assumption allows also for the case that one firm buys all the other firm's permits such that the market ends up with monopoly. And indeed, as we will see, this will happen for some range of values for L.

Before we can solve the government's problem of how to choose the optimal number of permits contingent on s, we have to analyze how the firms will determine the final allocation by solving (5.1). For this consider the following program:

$$\max_{q_1,q_2} P(q_1+q_2)[q_1+q_2] - c_1 q_1 - c_2 q_2 \quad \text{s.t. } d_1 q_1 + d_2 q_2 \le L \,. \tag{5.2}$$

After solving (5.2), we will show that the resulting prices $p_1 = p_2 = P(q_1+q_2)$ form a Nash equilibrium of the *price* game under capacity constraints. Denote by q_{mon} the monopoly output of the lower cost firm 1 in the absence of regulation (which is also the monopoly outcome of the horizontally integrated industry). Denote further by $L_{mon} = d_1 q_{mon}$ the number of permits that are necessary to produce q_{mon}.

Proposition 5.1 a) *If $\Delta \le 0$, $\forall\, L \ge 0$ the solution of (5.2) is given by* [10]

$$q_1(L) = \min\left\{q_{mon}\,,\ \frac{L}{d_1}\right\}\,, \qquad q_2(L) = 0\,.$$

b) *If $c_1 < c_2$ and* [11] *$\Delta > 0$, there are $\underline{L}$, $\overline{L}$ with $0 \le \underline{L} < \overline{L}$ such that the solution of (5.2) is given by*

$$\left.\begin{aligned} q_1(L) &= \min\{q_{mon}\,, \tfrac{L}{d_1}\} \\ q_2(L) &= 0 \end{aligned}\right\} \quad \text{for} \quad L \ge \overline{L}\,,$$

$$\left.\begin{aligned} q_1(L) &> 0 \\ q_2(L) &> 0 \end{aligned}\right\} \quad \text{for} \quad \overline{L} > L > \underline{L}\,,$$

$$\left.\begin{aligned} q_1(L) &= 0 \\ q_2(L) &= \tfrac{L}{d_2} \end{aligned}\right\} \quad \text{for} \quad L \le \underline{L} \quad \text{and } d_2 > 0.$$

[10] If both firms are alike, which implies $\Delta = 0$, the solution is not unique. Either firm could buy all the permits.

[11] If $c_1 = c_2$ interchange the names of the firms and apply case a).

Moreover, $q_i(L)$ are continuous in L and $Q(L) := q_1(L) + q_2(L)$ is continuous and constant for $\overline{L} \ge L \ge \underline{L}$.

To interpret the proposition: if $\Delta \le 0$, firm 1 buys all the permits and behaves as a monopolist. If $L > L_{mon}$, firm 1 also buys all the permits but does not use them all. In this case, there is underproduction combined with underpollution. By giving out more permits, however, the government cannot induce the firms to produce more than the monopoly output q_{mon}.

If $\Delta > 0$, the same thing happens as long as $L \ge \overline{L}$, that is, firm 1 buys all the permits, does not use all of them for $L < L_{mon}$ and exhausts them for $L_{mon} \le L \le \overline{L}$. For $\overline{L} \ge L \ge \underline{L}$, the two firms shift production continuously from firm 1 to firm 2 as L decreases, holding total output constant. For $L \le \underline{L}$, the less polluting firm 2 buys all the permits and produces alone.

The proposition is proven by solving (5.2) taking into account $q_i(L) \ge 0$. A detailed proof can be found in REQUATE [1992].

The next proposition establishes that the price which would come about if the aggregate output resulting from (5.2) were given to an auctioneer, does indeed form a Nash equilibrium if the firms engage in Bertrand competition after trade of permits.

Proposition 5.2 *If residual demand is determined by efficient or proportional rationing (or any convex combination of both), (p_1, p_2) with $p_1 = p_2 = P(q_1(L)+q_2(L)) =: \hat{p}$ is a Nash–equilibrium of the price setting game, where $q_i(L)$ is given by Proposition 5.1.*

Proof: The proof is obvious for $L \ge \overline{L}$ and $L \le \underline{L}$ since then only one firm produces just its monopoly quantity under the constraint $q_i \le L/d_i$. The other firm does not hold any permits and hence cannot produce. For $\underline{L} < L < \overline{L}$, undercutting $\hat{p}$ clearly does not pay since both firms exhaust their capacities at $(p_1, p_2) = (\hat{p}, \hat{p})$. It remains to show that it does also not pay to raise the price. For this we immediately assume that residual demand of the higher price firm is given by a convex combination of efficient and random rationing such that for $p_i > \hat{p}$, firm i's profit is given by

$$\Pi^i(p_i, \hat{p}) = \mu[D(p_i) - q_j(L)](p_i - c_i) + (1-\mu)\left(1 - \frac{q_j(L)}{D(\hat{p})}\right) D(p_i)(p_i - c_i) \quad (5.3)$$

where $j = 3 - i$ and $\mu \in [0, 1]$.

Making a little detour, let $q_1(L) > 0$, $q_2(L) > 0$ be the solution of the program (5.2). Its f.o.c. is given by

$$P'(\hat{Q})\hat{Q} + P(\hat{Q}) - c_i - \lambda d_i = 0 \quad (5.4)$$

where $\hat{Q} = q_1(L) + q_2(L)$ and λ is the Kuhn–Tucker multiplier w.r.t. $d_1 q_1 + d_2 q_2 \leq L$. Further, let $\hat{p} = P(\hat{Q})$.

Back to (5.3), taking the right-sided derivative at $p_i = \hat{p}$ we get:

$$\begin{aligned}
\frac{\partial \Pi^i}{\partial p_i}(\hat{p}, \hat{p}) &= \mu[D'(\hat{p})(\hat{p} - c_i) + D(\hat{p}) - q_j(L)] + \\
&\quad +(1-\mu)\left(1 - \frac{q_j(L)}{D(\hat{p})}\right)[D'(\hat{p})(\hat{p} - c_i) + D(\hat{p})] \\
&< [\mu + (1-\mu)\left(1 - \frac{q_j(L)}{D(\hat{p})}\right)][D'(\hat{p})(\hat{p} - c_i) + D(\hat{p})] \\
&= [\mu + (1-\mu)\left(1 - \frac{q_j(L)}{D(\hat{p})}\right)]\frac{1}{P'(\hat{Q})}[P(\hat{Q}) - c_i + P'(\hat{Q})\hat{Q}] \\
&< [\mu + (1-\mu)\left(1 - \frac{q_j(L)}{D(\hat{p})}\right)]\frac{1}{P'(\hat{Q})}[P(\hat{Q}) - c_i + P'(\hat{Q})\hat{Q} - \lambda d_1] = 0
\end{aligned}$$

where $\lambda > 0$ is the Kuhn–Tucker multiplier from (5.4). The last equality holds by (5.4). Q.E.D.

Notice that this result does not hold for *all* rationing rules. For example, take the rationing rule that maximizes residual demand of the higher price firm. This rule turns the efficient rationing rule upside down: those with the lowest reservation prices are served first rather than those with the highest. In this case the higher price firm clearly has an incentive to raise the price over $\hat{p}$. This rationing rule, of course, is also the most inefficient one and not very realistic.

In REQUATE [1992] it has been demonstrated that the solution of (5.2) in q_1 and q_2 is also a *Cournot*–Nash–equilibrium of the *quantity* setting game. We draw

the interesting conclusion that in order to determine the optimal number of permits it is irrelevant for the regulating authority to know what kind of competition the firms engage in. This allows us to recall from REQUATE [1992] the result of the government's program for the optimal number of permits:

5.2 The Government's program

Given that the reactions of the firms are determined by Proposition 5.1 when a number of L permits is in the market, and given the damage parameter s, the government has to find the optimal size of L. Denoting $Q(L) := q_1(L) + q_2(L)$, and $e_i(L) := d_i q_i(L)$, $i = 1, 2$, the following program has to be solved:

$$\max_L W_s^{Per}(L) := \max_L \int_0^{Q(L)} P(z)dz - S(L, s) - c_1 q_1(L) - c_2 q_2(L) \quad (5.5)$$

Let $L(s)$ denote the optimal number of permits contingent on s, that is, the solution of (5.5), then this has the following properties.

Proposition 5.3 *a) If $\Delta \leq 0$, only firm 1 produces and there is a damage parameter $\sigma_{mon} > 0$ such that*

a1) $L(s) = L_{mon}$ for $s \leq \sigma_{mon}$.

a2) $L(s)$ is decreasing and implementing the social optimum for $s \geq \sigma_{mon}$.

b) If $\Delta > 0$ and $d_2 > 0$, there are parameters $0 < \sigma_{mon} < \underline{\sigma} < \sigma_0 < \overline{\sigma} < \infty$ (for $d_2 = 0$, $\sigma_0 = \overline{\sigma} = \infty$) such that

b1) $L(s) = L_{mon}$ for $s \leq \sigma_{mon}$, and only firm 1 produces.

b2) $L(s)$ is decreasing for $s \in [\sigma_{mon}, \underline{\sigma}]$, and only firm 1 produces.

b3) $L(s)$ is decreasing for $s \in (\underline{\sigma}, \sigma_0]$, and both firms produce.

b4) $L(s) = \underline{L}$ for $s \in [\sigma_0, \overline{\sigma}]$, and only firm 2 produces.

b5) $L(s)$ is decreasing for $s \geq \overline{\sigma}$, and only firm 2 produces.

b6) $L(s)$ is discontinuous and downward jumping at $\underline{\sigma}$, continuous elsewhere.

b7) $\underline{\sigma} > \underline{s}$, $\overline{\sigma} > \overline{s}$.

We will explain the result in the following and give some intuitive hints why it holds. For a formal proof see REQUATE [1992].

The proposition says the following.

In a), where $\Delta \leq 0$, only firm 1 produces which follows immediately from Proposition 5.1 a). Further, there is a damage parameter σ_{mon} ("mon" stands for "monopoly"), where for all $s < \sigma_{mon}$ the pollution level of the monopolistic firm 1 would be less than socially optimum. If we assume that a firm cannot be forced to exhaust its permits, any number of permits $L \geq L_{mon}$ has the same impact: firm 1 behaves like an unregulated monopolist. For $s \geq \sigma_{mon}$, the optimal number of permits does not exceed L_{mon}. Hence, firm 1's constraint $q_1 \leq L/d_1$ becomes binding. Due to the linear technology, and since firm 1 always buys all the permits, the social optimum can be achieved for $s \geq \sigma_{mon}$.

If $\Delta > 0$, the same thing happens for small values of s: for $s < \sigma_{mon}$, a monopolistic firm 1 would underpollute and cannot be induced to producing more and polluting more by giving out more permits [b1)]. For $s \geq \sigma_{mon}$ the constraint $e_1 + e_2 \leq L$ becomes binding. $L(s)$ decreases as s increases, and firm 1 buys all the permits as long as $s \leq \underline{\sigma}$ [b2)]. At the damage parameter $\underline{\sigma}$ the optimal permit policy is discontinuous and downward jumping [b6)] (the reason for this jump will be explained below). For $s > \underline{\sigma}$, the optimal number of permits for regulating *two* active firms is given out. This will last until firm 1 shuts down for $s = \sigma_0$ which is defined by $L(\sigma_0) = \underline{L}$ [b3)]. L(s) is constant on the interval $(\sigma_0, \overline{\sigma}]$. This is so because $L(\sigma_0) = \underline{L}$, and for $L \leq \underline{L}$ firm 2 buys all the permits. In the absence of firm 1, however, the optimal number of permits in order to regulate a monopolistic firm 2 would be greater than $\underline{L}$ if $s \in [\sigma_0, \overline{\sigma}]$. But any $L > \underline{L}$ would induce firm 1 to hold some permits, as we know. Hence, the optimal number of permits has to be held constant and equal to $\underline{L}$ in order to keep firm 1 out of the market [b4)]. For $s \geq \overline{\sigma}$, the *optimal* number of permits to regulate a monopolist can be given out, since this not greater than $\underline{L}$. For these parameters, the social optimum can be implemented [b5)]. $L(s)$ is depicted in figure 3.

Figure 3 about here.

The reason for the discontinuity at $\underline{\sigma}$ can be explained as follows: The f.o.c. of (5.5) takes different forms for the cases in which only one of both firms produces or in which both firms produce. In a neighborhood of $\underline{\sigma}$ we get two local maxima, say $\tilde{L}^1(s)$ and $\tilde{L}^2(s)$, one with $\tilde{L}^1(s) > \overline{L}$, in which case only firm 1 produces, and an other one with $\tilde{L}^2(s) < \overline{L}$, where both firms produce (see Proposition 5.1). To find the global maximum we simply have to compare the two local maxima. For $s < \underline{\sigma}$, the solution $\tilde{L}^1(s) > \overline{L}$ yields the highest welfare [b2)], for $s = \underline{\sigma}$ the welfare values coincide, and for $s > \underline{\sigma}$ the solution $\tilde{L}^2(s) < \overline{L}$, with both firms being active, leads to a higher welfare. Hence, the optimal permit policy obviously is to discontinuously reducing $L(\cdot)$ at $\underline{s}$, switching from one active firm (firm 1) to letting both firms produce.

The last claim [b7)] makes a statement about the welfare properties of the optimal permit policy. It claims that the damage parameter $\underline{s}$, where firm 2 starts to produce under the optimal permit policy, is greater than the damage parameter $\underline{s}$, where firm 2 starts to produce in social optimum. That is, under permits firm 2 starts to produce "later" than socially optimal as s increases. On the other hand, firm 1 also shuts down later (namely for $s = \overline{\sigma}$) than socially optimal, which would be for $s = \overline{s}$. For linear demand and quadratic damage function one can even show that $\underline{\sigma} > 2\underline{s}$, and $\overline{\sigma} = 2\overline{s}$.

Corollary 5.1 *If $\Delta > 0$, the permit solution is socially optimal for $s \in [\sigma_{mon}, \underline{s})$ (interval may be empty) and for $s \geq \overline{\sigma}$.*

Remark 5.1 *We saw that the permit policy yields underpollution going along with underproduction for $s < \sigma_{mon}$ if we assume that firms cannot be forced to use all their permits. If s is equal to or very close to zero, "laissez-faire" is obviously better than any permit policy, since any permit policy $L \geq L_{mon}$ leads to the monopoly price p_1^m, whereas "laissez-faire" induces firm 1 to charge $c_2 - \epsilon$ yielding a higher welfare if*

$p_1^m > c_2$. since more will be produced and there is very little environmental damage from production for s close to zero.

Remark 5.2 *An alternative possibility to restore efficiency for $s < \sigma_{mon}$ would be to modify the licenses in a way that obliges its holder to exactly emit a quantity of the pollutant equal to the number of licenses. In this case, firm 1 would also buy all the permits if $L > \bar{L}$, especially for $L > L_{mon}$. However, the same problem arises as for subsidizing pollution in order to increase output (cf. Remark 4.1). There is no guarantee that the monopolist's excess amount $D(c_1) - D(p_1^m)$ will really be sold to the consumers rather than being destroyed, unless the regulator also controls the quantity of output of the marketable commodity. So, if a minimal output level is supposed to be enforced, this can probably be done more effectively in a direct way, rather than by enforcing a minimal pollution level (see also section 7).*

6 Comparison and Discussion of the Policies:

We saw that if firms engage in tough Bertrand competition, for all $s \geq 0$, the optimal tax induces the socially optimal solution if $\Delta \leq 0$, that is, if firm 2 has the worse technology. Under permits this was only possible for $s \geq \sigma_{mon}$. In REQUATE [1992] it has been demonstrated that this result does not hold under softer Cournot competition. There, for $\Delta \leq 0$ the optimal linear tax does *not* always keep the worse firm 2 out of the market!

Turning to $\Delta > 0$ we get the following result:

Proposition 6.1 *If $\Delta > 0$, the tax policy is superior to the permit policy i) for all $s \in [0, \underline{\sigma}]$, ii) for all $s \geq \bar{s}$, and iii) for all s in a neighborhood of some $s_0 \in (\underline{s}, \bar{s})$.*

Corollary 6.1 *If $\Delta > 0$ and $\underline{\sigma} > \bar{s}$, the tax policy is superior to the permit policy for all $s \geq 0$.*

(Recall that by definition, for $s \leq \underline{\sigma}$ only firm 1 was active under the permit policy,

while both firms were active for $\underline{\sigma} < s < \sigma_0$. At $s = \bar{s}$ firm 1 shuts down in social optimum.)

Proof of Proposition 6.1: For $s \in [0, \underline{s}]$, for $s \geq \bar{s}$, and for s in a neighborhood of s_0, this follows immediately from Corollary 4.1. For $s \in [\underline{s}, \underline{\sigma}]$ observe that both policies induce only firm 1 to produce if s is sufficiently close to $\underline{s}$. Both policies yield the same result, that is, same output, same pollution level and same welfare, since they would regulate firm 1 optimally if firm 2 were not around. Whereas the permit policy induces only firm 1 to produce for all $s < \underline{\sigma}$, the tax policy can eventually do better by switching the tax from $\tau^{be} - \varepsilon$ to τ^{be} or $\tau^{be} + \varepsilon$ as s increases. Q.E.D.

The following example satisfies the conditions of Corollary 4.1, that is, the tax policy dominates the permit policy for all $s \geq 0$.

Example 6.1 Let $P(Q) = 1 - Q$, $S(E, s) = \frac{s}{2}E^2$ and $c_1 = 0.25$, $c_2 = 0.5$, $d_1 = 1$, $d_2 = 0.5$. Under this constellation, $\Delta > 0$, and we get $\underline{s} = 2$, $\bar{s} = 4$, that is, in social optimum both firms are active for $s \in (2, 4)$. The break–even–tax is $\tau^{be} = 0,5$. Under the optimal Pigouvian tax, only firm 1 produces for $0 \leq s < 2.39$, both firms produce for $2.39 \leq s \leq 3.12$, and only firm 1 produces for $s > 3.12$. Under permits, the social optimum is attained for $s \in [\sigma_{mon}, \underline{s}) = (0.125, 2)$ and $s \geq \bar{\sigma} = 12$. Since $\underline{\sigma} > \bar{s}$, a Pigouvian tax yields a higher welfare for all damage parameters $s \geq 0$ (see Figure 4).

Also if $\underline{\sigma} < \bar{s}$, it may be the case that the tax policy is superior to the permit policy for all damage parameters s for some quadruple (c_1, c_2, d_1, d_2), however not for all. In Example 6.2, where firms are extremely different, we find some interval of damage parameters for which the optimal number of permits yields a higher welfare than the optimal emission tax.

Example 6.2 Let $P(\cdot)$ and $S(\cdot, \cdot)$ as in Example 6.1. Let $c_1 = 0.25$, $c_2 = 0.74$, $d_1 = 0.7$, $d_2 = 0.1$. Firm 1 has much lower production costs than firm 2, but also generates much more pollutants than firm 2 does. In this case, $\underline{s} = 6.54$, $\bar{s} = 45.79$ and

$\underline{\sigma} = 14.05 < 45.79 = \bar{s}$. For $16.5 < s < 20.75$, permits yield a higher welfare than the optimal Pigouvian tax does. This is illustrated by Figure 5.

7 Subsidies on Output, Permits for Pollution:

Consider now a situation where the government gives out permits to pollute and simultaneously subsidizes output. Let ζ denote the subsidy. Assumption 4 will be retained. For given s, let further $(L^{\zeta}(s), (\zeta(s))$ be the optimal permit/subsidy policy. Denote further by $q_i^0(s)$, $i = 1,2$ the socially optimal output of firm i and by $E^0(s) := d_1 q_1^0(s) + d_2 q_2^0(s)$ the optimal aggregate pollution level. Then we get the following result:

Proposition 7.1 *For all Δ and for all $s \geq 0$ there is a subsidy $\zeta(s)$ and a number of permits $L^{\zeta}(s)$ which implements the social optimum. The optimal permit/subsidy policy is given by:*

$$L^{\zeta}(s) \;=\; E^0(s) \qquad \forall \Delta \,. \tag{7.6}$$

If $\Delta \leq 0$:

$$\zeta(s) \;=\; \begin{cases} -d_1 S_1(d_1 q_1^0(s), s) - P'(q_1^0(s)) q_1^0(s) & \textit{for } s \leq \sigma_{mon}, \\ 0 & \textit{for } s > \sigma_{mon} \;. \end{cases} \tag{7.7}$$

If $\Delta > 0$:

$$\zeta(s) \;=\; \begin{cases} -d_1 S_1(d_1 q_1^0(s), s) - P'(q_1^0(s)) q_1^0(s) & \textit{for } s \leq \sigma_{mon}, \\ 0 & \textit{for } s \in (\sigma_{mon}, \underline{s}), \\ -P'(\tilde{Q})\tilde{Q} & \textit{for } s \in [\underline{s}, \bar{s}], \\ d_2 \left[\frac{c_2 - c_1}{d_1 - d_2} - S_1(d_2 q_2^0(s), s)\right] - P'(q_2^0(s)) q_2^0(s) & \textit{for } s \in (\bar{s}, \bar{\sigma}), \\ 0 & \textit{for } s \geq \bar{\sigma}. \end{cases} \tag{7.8}$$

where $\tilde{Q}$ is the socially optimal aggregate output for $s \in (\underline{s}, \bar{s})$ given by (3.6).

Before we prove the result, let us give the intuition. For $s < \sigma_{mon}$, the subsidy/permit system leading to first best is very easy: subsidize output such that the

monopolist chooses the optimal level of output. Set $L(s)$ equal to the corresponding amount of pollution (or even higher).

For $\Delta > 0$ we have seen that under the plain permit regime too much production was allocated at the low cost (high pollution) firm for a wide regime of damage parameters. If output is subsidized by ζ, the unit cost of the firms become $c_1 - \zeta$, and $c_2 - \zeta$, respectively. The higher ζ, the lower is firm 1's cost advantage and the less production will be allocated (via permits) at that firm. This suggests that ζ can be chosen such that the interval of damage parameters where *both* firms produce under *permits* can be induced to coincide with the interval of damage parameters where both firms produce in *social optimum*. And indeed, this is the case. Since the social optimum will be achieved, the optimal number of permits $L^{\zeta}(s)$ must coincide with $E^0(s)$, which is continuous and decreasing in s. Thus, the jump and the constant piece of the *pure* permit policy $L(s)$ vanish. However, the subsidy $\zeta(s)$ jumps discontinuously at $s = \underline{s}$ from zero to $-P'(\tilde{Q})\tilde{Q} > 0$. Loosely speaking, for $\Delta > 0$ and $s \geq \underline{s}$ the permit policy $L(s)$ is shifted upwards and compressed to the right position $L^{\zeta}(s) = E^0(s)$. One can show (which we omit) that $\zeta(s)$ is continuous in $\bar{s}$ and $\bar{\sigma}$.

Proof of Proposition 7.1: Given ζ and L, the two firms maximize

$$\max_{q_1, q_2} P(q_1 + q_2) \cdot (q_1 + q_2) - (c_1 - \zeta) q_1 - (c_2 - \zeta) q_2$$

s.t. $d_1 q_1 + d_2 q_2 \leq L$, $q_1 \geq 0$, $q_2 \geq 0$. Denote $Q = q_1 + q_2$. The complementary slackness conditions are

$$q_1 \cdot [P'(Q)Q + P(Q) - c_1 + \zeta - \lambda d_1 + \mu_1] \;=\; 0 \tag{7.9}$$

$$q_2 \cdot [P'(Q)Q + P(Q) - c_2 + \zeta - \lambda d_2 + \mu_2] \;=\; 0 \tag{7.10}$$

First consider $\Delta \leq 0$. If $s \geq \sigma_{mon}$, nothing is to be shown since $\zeta(s) = 0$ and $L(s)$ like in Proposition 5.3 implement the social optimum. If $s < \sigma_{mon}$, the socially optimal choice of q_1^0 satisfies $P(q_1^0) = c_1 + d_1 S_1(d_1 q_1^0, s)$. Choose now $\zeta := -d_1 S_1(d_1 q_1^0, s) - P'(q_1^0) q_1^0$ and $L \geq d_1 q_1^0$ (actually L^{ζ} is not unique for $s \leq \sigma_{mon}$). It is easy to verify that the monopolist's profit maximizing quantity equals q_1^0. Since

$L^\zeta(s) \geq d_1 q_1^0$, $L^\zeta(s)$ is not strictly binding. Hence $\lambda = 0$, and all the production is allocated at the low price firm 1.

Let now $\Delta > 0$. For $s < \sigma_{mon}$, the same thing happens as for $\Delta \leq 0$. For $\sigma_{mon} \leq s < \underline{s}$, choose $\zeta(s) = 0$ and $L^\zeta(s) = L(s)$ from Proposition 5.3. For $s \in [\underline{s}, \overline{s}]$, the social optimum requires $P(\tilde{Q}) = c_1 + d_1 S_1(E^0(s), s) = c_2 + d_2 S_1(E^0(s), s)$. Further it is easy to show that $S_1(E^0(s), s) = (c_2 - c_1)/(d_1 - d_2)$ for $s \in (\underline{s}, \overline{s})$. Set now $\zeta(s) := -P'(\tilde{Q}) \cdot \tilde{Q}$. Then (7.9), (7.10) and $d_1 q_1 + d_2 q_2 = L$ have a unique solution with $\mu_1 = \mu_2 = 0$, $\lambda = (c_2 - c_1)/(d_1 - d_2) = S_1(E^0(s), s)$ and $q_1 = q_1^0(s)$, $q_2 = q_2^0(s)$. (7.9) and (7.10) reduce to

$$P(\tilde{Q}) - c_i - d_i S_1(E(s), s) = 0 \qquad i = 1, 2 ,$$

which is the f.o.c. of the social optimum for $s \in (\underline{s}, \overline{s})$.

Next let $s \in [\overline{s}, \overline{\sigma}]$. Choose $L^\zeta(s) = E(s)$ and

$$\zeta(s) := d_2 \left[\frac{c_2 - c_1}{d_1 - d_2} - S_1(d_2 q_2^0(s), s) \right] - P'(q_2^0(s)) q_2^0(s) .$$

It is easy to verify that (7.9), (7.10), and $d_1 q_1 + d_2 q_2 \leq L$ have a unique solution with $q_1 = 0$, $q_2 = q_2^0(s)$ and $\lambda = (c_2 - c_1)/(d_1 - d_2)$.

For $s > \overline{\sigma}$, we know that the social optimum can be implemented by $\zeta(s) = 0$. Q.E.D.

One may ask now whether other combinations like subsidies on output combined with taxes on pollution or even *pure* subsidies on output will lead to the same result. From Proposition 7.1 we have seen that pure subsidies do the right job if $s < \sigma_{mon}$, that is, for those cases where only firm 1 is active and produces less than socially optimal in the absence of regulation. However, a subsidy on output clearly does not discriminate between the marginal rate of pollution. Hence it will always maintain firm 1's cost advantage with the consequence that firm 2 will never be able to sell anything under Bertrand competition. Therefore, a pure subsidy on output cannot be optimal for $s > \sigma_{mon}$.

Finally, a combination of taxes on pollution and subsidies on output cannot lead to higher welfare than a pure emission tax since it cannot induce the optimal allocation

of production among the two firms for $s \in (\underline{s}, \overline{s})$, either.

8 Concluding Remarks

The paper studied the impact of decentralizing pollution control policies, like emission taxes, licenses to pollute and finally subsidies on output combined with licenses, in a model where price setting firms compete imperfectly. The imperfection results from the firms asymmetry: the lower cost firm does not charge its unit cost but rather will undercut the higher cost firm slightly. A tax on emissions has a direct impact on the firms' total unit costs and turns out to be very effective in cases where, from a social point of view, one of the two firms should serve the whole market and the other one should not produce anything. An emission tax turned out to be not efficient, in general, if *both* firms are supposed to produce in social optimum.

Under a pure permit regime, firms trade the licenses such that the final allocation maximizes the joint profit under the scarcity constraint given by the limited number of permits. If damage from pollution is low, and hence the corresponding number of permits given out by the government is high, the permit policy leads to monopolization of the market: the lower private cost firms buys all the permits since it can compensate the higher cost firm for closing down. A permit policy can therefore lead to unfavorable outcomes if the damage from pollution is low, and if the firms are free to dissipate the number of permits they own. Underpollution, however, is not so much of a problem in reality. If we assume that the damage from pollution is sufficiently high such that even a monopolist would overpollute in the absence of regulation, the permit policy is efficient if one of the firms has a technology bad enough such that it should never produce in social optimum, regardless of how the steep social damage function is. But in this case, also the tax instrument yields the efficient outcome. If, on the other hand, *both* firms are supposed to produce in social optimum for some damage parameters, permit trading does not always induce an efficient allocation. Too much production will be shifted towards the low cost but worse polluting firm. Despite of this, a welfare comparison turned out to be ambiguous, in general. For, there exist parameters where both firms should produce in social optimum, but where the optimal number of permits, though not being efficient, leads to higher welfare than the optimal emission tax.

These results stand in contrast to those in REQUATE [1992] where tax and permit policies were investigated for a different market structure. The firms set quantities rather than prices, leading to softer competition. In that model the permit policy turned out to be superior to the tax policy for a wide range of damage parameters s, since the tax could not always prevent the socially inefficient firm from production. A political recommendation in favor of one of the two pure pollution control instruments has therefore to be handled with care. Not only does the choice of the regulating tool depend on the economies' parameters, that is, the firm's technologic parameters and the steepness of the social damage function, but also on how tough or soft the firms compete.

Subsidies on output, on the other hand, although being abused frequently in order to guarantee the survival of inefficient industries in the real world, can be useful combined with tradeable permits in cases where polluting firms compete imperfectly. In particular, they may cancel out the negative effects of monopolization generated by tradeable permits.

We chose a very simple model in order to get first insights in an area which has not been explored very much so far. Further research should inquire how the results change under more general (non–linear) technologies, or in cases where firms can select among at least two different technologies. For example firms could substitute permits by buying scrubbers to reduce SO_2. It is further important to investigate, whether the results, in particular the efficiency result of the last section, do still hold when more than two firms are involved. Finally, it would be interesting to know how the proposed policy tools will encourage entry of new firms and/or R & D of less polluting technologies.

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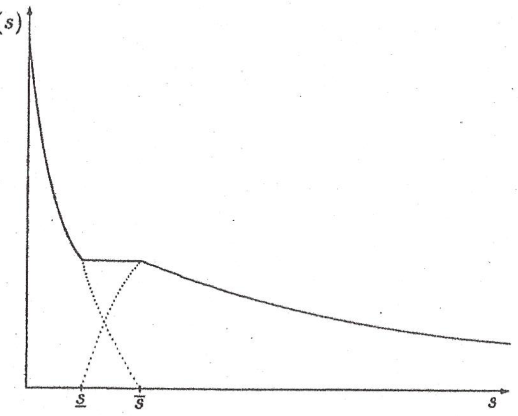

Figure 1: *The quantities in social optimum as a function of s if $\Delta > 0$. The solid line depicts aggregate output which equals $q_1(s)$ for $s \leq \underline{s}$ and $q_2(s)$ for $s \geq \overline{s}$. The dotted lines depict q_1 and q_2 for $\underline{s} < s < \overline{s}$.*

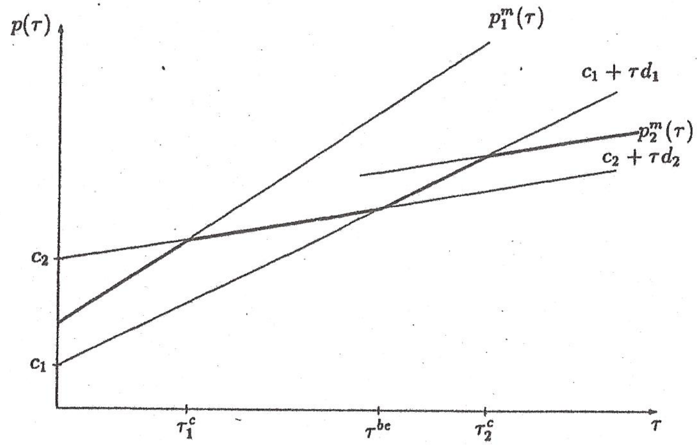

Figure 2: *The bold line denotes the market price as a function of τ.*

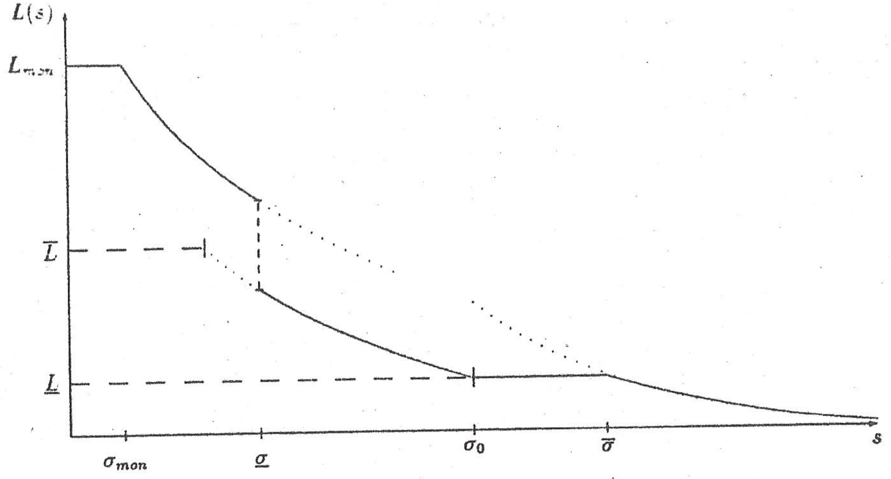

Figure 3: *The solid line depicts the optimal number of permits as a function of s if* $\Delta > 0$.

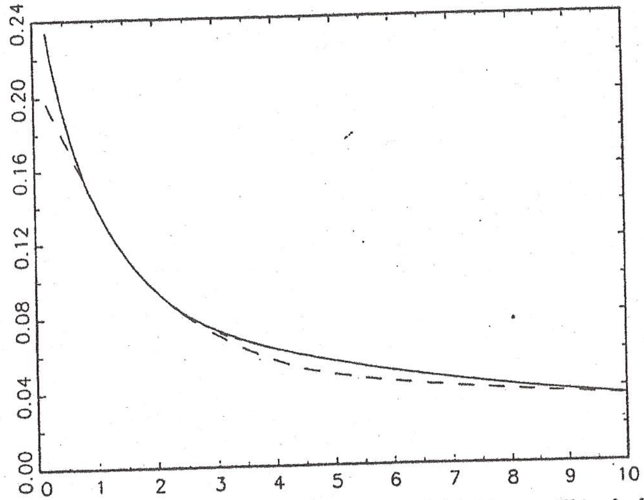

Figure 4: *Welfare as a function of s: solid line: social optimum, "big dashed" line: permits, "small dashed" line: taxes. Welfare under taxes differs from social optimum only for* $s \in (2,4)$. *Parameters such that* $\Delta > 0$ *and* $\underline{\sigma} > \bar{s}$.

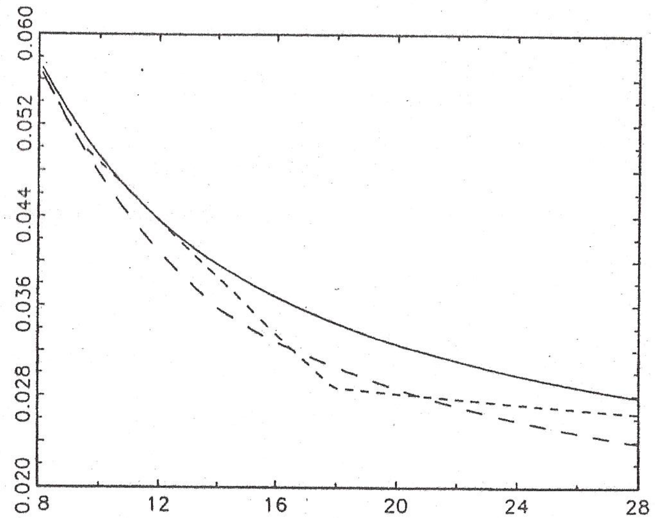

Figure 5: *Welfare as a function of s: solid line: social optimum, "big dashed" line: permits, "small dashed" line: taxes. Parameters such that* $\Delta > 0$ *and* $\underline{\sigma} < \bar{s}$.