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Reinhard Selten and Wilhelm Krischker

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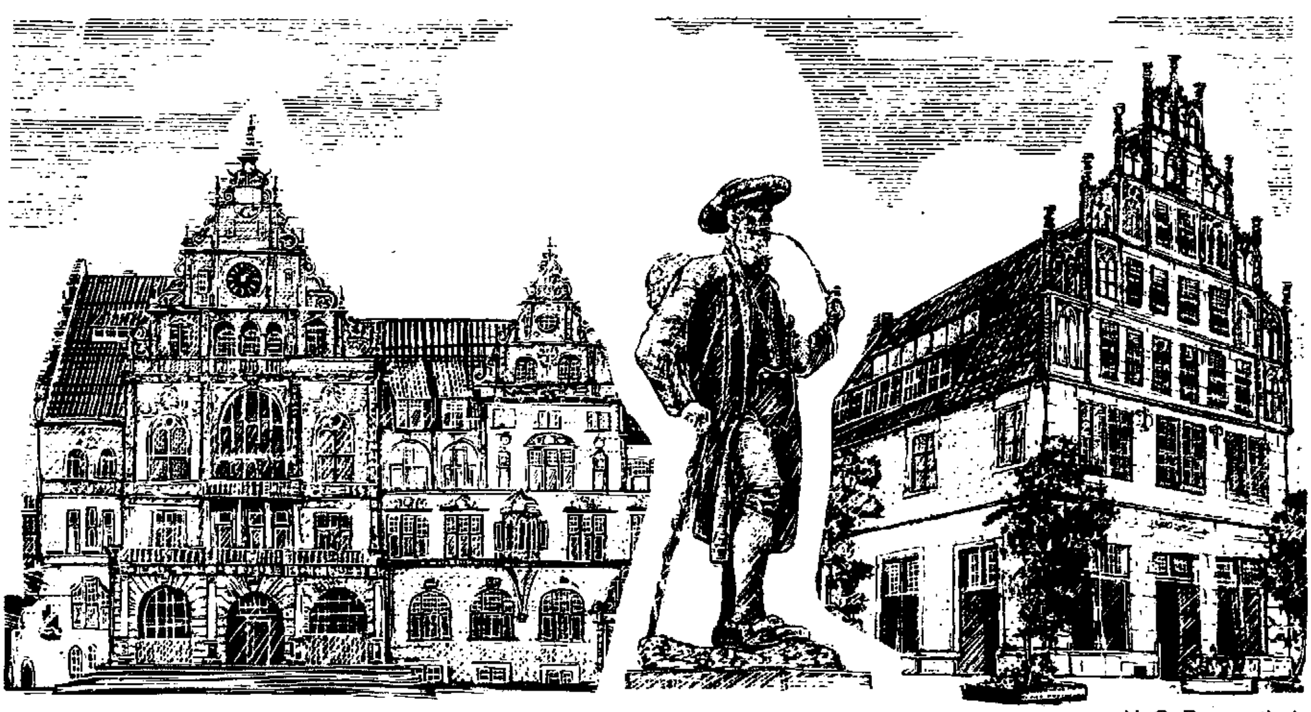

H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

Comparison of Two Theories for Characteristic Function Experiments

by Reinhard Selten and Wilhelm Krischker

There are many theories of characteristic function games. Most of them are normative rather than descriptive and do not seem to have much relevance for the explanation of laboratory experiments. In this paper we shall concentrate on two of the more successful theories: the bargaining set (Aumann and Maschler 1964) and equal share analysis (Selten 1972).

The comparison will be based on 175 plays of 3- and 4-person games reported in the literature (Maschler 1978, Kalish, Milnor, Nash and Nering 1954, Frassine, Fürst and Winter 1967). These are all 3- and 4-person games in the list compiled by Selten in his paper on equal share analysis (Selten 1972). The sample contains 98 plays of 3-person games and 77 plays of 4-person games.

1. Definitions and notations

A characteristic function v assigns a number v(C) to all elements of a set P of non-empty subsets of $N=\{1,...,n\}$ where P contains at least all one element subsets of N and v satisfies the condition.

$$(1) \quad v(C) \geq \sum_{i \in C} v(i) \quad \text{for all } C \in P$$

for the sake of shortness we write v(i) instead of $v(\{i\})$. The set N is interpreted as the set of all players, the subsets of N are called coalitions and those in P are permissible coalitions.

In a play of a characteristic function game a permissible coalition C can be formed by an agreement reached by its members on the division of v(C) among themselves; if several coalitions are formed they must be non-intersecting. The result of a play is a configuration

(2) $\alpha = (C_1,\ldots,C_m;\ x_1,\ldots,x_n)$

which consists of a <u>coalition structure</u> $(C_1,\ldots,C_m)$, a partition of N into permissible coalitions and a <u>payoff vector</u> $(x_1,\ldots,x_n)$ subject to the conditions

(3) $\sum_{i\in C_j} x_i = v(C_j)$ for $j = 1,\ldots,m$

and

(4) $x_i \geq v(i)$ for $i = 1,\ldots,n$

A characteristic function v is called <u>integer valued</u> if v(C) is an integer for all possible coalitions. An <u>integer configuration</u> is a configuration where all payoffs x_i are integers.

The rules of the experimentally played games considered here specify a smallest payoff unit which cannot be further subdivided. We call such games <u>scaled</u> if v(C) is expressed as the number of smallest payoff units available for distribution among the members of C. Obviously, only integer configurations can be realized as results of scaled integer valued characteristic function games. Since no other games will be considered in this paper, a <u>game</u> will always be a scaled integer valued characteristic function game. A <u>configuration</u> will always be an integer configuration.

2. A measure of predictive success

Both theories to be compared here are based on notions of stability. Both of them predict that the result will be in a specified region of stable configurations. Obviously, it is not sufficient to check which theory yields more correct predictions since one theory may have a much larger stability region than the other. The size of the stability region must be taken into account by a reasonable measure of predictive success.

It is instructive to look at the extreme case of that theory which simply predicts the set of all configurations. We shall refer to this theory as the <u>null theory</u>. The null theory always yields a correct prediction, but it is useless since it does not restrict the result in any way.

One needs a measure of the size of a stability region which can be compared with the rate of correct predictions. At first glance, it may seem to be reasonable to construct such a measure by dividing the number of configurations in the stability region by the number of all configurations. (Since configurations are understood to be integer their number is finite). Unfortunately, this simplistic measure of size is inadequate. In order to see this we may look at the following 3-person game

$$v(1) = v(2) = v(3) = 0$$
$$v(12) = 100,\ v(13) = 80,\ v(23) = 60$$
$$v(123) = 100$$

For the sake of shortness we write ij instead of $\{i,j\}$ and 123 instead of $\{1,2,3\}$. There are 101 configurations of the form $(12,3;\ x_1,x_2,x_3)$ and 5151 configurations of the form $(123;\ x_1,x_2,x_3)$. Obviously, the simplistic measure puts too much emphasis on the grand coalition 123. It seems to be preferable to employ a measure which gives equal weight to all possible coalition structures.

For a given characteristic function let K be the number of possible coalition structures. For each coalition structure $(C_1,\ldots,C_m)$ let $N(C_1,\ldots,C_m)$ be the number of configurations with that coalition structure. For every configuration $\alpha = (C_1,\ldots,C_m;\ x_1,\ldots,x_n)$ define the <u>weight</u> $A(\alpha)$ of α.

$$(5)\quad A(\alpha) = \frac{1}{K\cdot N(C_1,\ldots,C_m)}$$

Let Z be a set of configurations. The <u>area</u> A(Z) is defined as follows:

$$(6)\quad A(Z) = \sum_{\alpha\in Z} A(\alpha)$$

The area a(Z) will be our measure of size. Obviously, the set of all configurations belonging to a specific coalition structure has the area 1/K. In this sense equal weight is put on all coalition structures. Within each coalition structure equal weight is put on all configurations.

Suppose that a body of experimental data consists of k plays $1,\ldots,k$ of games $v_1,\ldots,v_k$, respectively (the same game may occur repeatedly). Let s be the number of correct predictions achieved by a theory T and let A_i be the area of the stability set Z_i predicted by T for v_i. Define the <u>gross rate of success</u>:

(7) $R = \frac{s}{k}$

and the <u>average area</u>:

(8) $A = \frac{1}{k} \sum_{i=1}^{k} A_i$

The <u>net rate of success</u> is the difference between the gross rate of success and the average area:

(9) $S = R - A$

Since both A and R are numbers between 0 and 1, the measure S has a range between -1 and +1. The null theory which predicts the set of all configurations always has net rate of success 0, since both R and A are equal to 1.

The net rate of success is the measure which we use in order to compare the predictive success of different theories. The measure has the following interpretation. Consider a bookmaker who offers a prize of 1 for a correct prediction in a one shot experiment; the customer can select any theory T he wants to, but he has to pay A, the area of the predicted region as the fee for the lottery. He is permitted to place just one bet. Obviously, the customer should try to maximize the expected value of S.

It is,of course, true that alternative measures of success could be constructed. The ratio R/A suggests itself as an alternative. However, R/A has very bad properties. Consider two theories T_1 and T_2 with gross rates of success $R_1 = .9$ and $R_2 = .01$ and areas of $A_1 = .3$ and $A_2 = .001$, respectively. We obtain $R_1/A_1 = 3$ and $R_2/A_2 = 10$. The ratio R/A gives preference to T_2 even if this theory is wrong in 99% of the cases. It seems to be clear that one should prefer T_1.

Another argument in favor of S can be seen in the fact that it permits us to combine bodies of data in the obvious way. The measure of success for the combined set is a weighted average of the measures for the sets combined with weights proportional to the number of trials.

3. Theories

On the basis of the sample described in the beginning of the paper gross rates of success average areas and net rates of success have been computed for nine types of configuration sets suggested by bargaining set theory and equal share analysis.

Bargaining set: The version of the bargaining set examined here is $M_1^{(i)}$. The definition of M_1^i can be found in the literature (Davis and Maschler 1967 , Peleg 1979).

Bargaining set with deviations up to 5: The set of all configurations $\alpha = (C_1,\ldots,C_m;\ x_1,\ldots,x_n)$ such that another configuration $\beta = (C_1,\ldots,C_m;\ y_1,\ldots,y_n) \in M_1^{(i)}$ can be found with $|x_i - y_i| \leq 5$ for $i = 1,\ldots,n$. Maschler has expressed the opinion that deviations up to 5 should be tolerated in view of the subjects' tendency to neglect small differences and to prefer round numbers. (Maschler 1978).

Bargaining set without null structure: The bargaining set does not exclude the coalition structure 1, 2,...,n where each player i forms the coalition containing him as the only member. This coalition structure is called the null structure. The bargaining set without null structure contains all configurations in $M_1^{(i)}$ whose coalition structure is different from the null structure.

Bargaining set without null structure and with deviations up to 5: This set is defined analogously to the bargaining set with deviations up to 5. In the definition given above $M_1^{(i)}$ must be replaced by the bargaining set without null structure.

Order of strength: This is one of the three hypotheses of equal share analysis. It asserts that within a coalition a stronger player does not receive a lower payoff than a weaker player. An exact definition can be found in the original literature (Selten 1972).

Exhaustivity: This is another hypothesis of equal share analysis. It excludes coalition structures which permit profitable unions of several coalitions in the structure; a union is profitable if its value is greater than the sum of the values of the coalitions in the union (Selten 1972).

Equal division core: The equal division core is the set of all configurations α with the property that no permissible coalition C can be found such that the equal division of v(C) gives more to each member than his payoff in α. The exact definition can be found in the original literature (Selten 1972).

Order of strength and equal division core: The set of all configurations in the equal division core satisfying the order of strength hypothesis.

Equal share analysis: The set of all configurations in the equal division core which satisfy both the order of strength hypothesis and the exhaustivity hypothesis.

4. Results

The results of the comparison are shown in the table at the end of the paper. It can be seen that the predictive success of the bargaining set is relatively low. The net rate of success is considerably increased by the exclusion of the null structure; the gross rate of success is slightly reduced since the null structure does occur sometimes, but the area is reduced by much more. The inclusion of deviations up to 5 also improves the net rate of success, but not very much.

The bargaining set without null structure and with deviations up to 5 has a net rate of success of .26. Equal share analysis performs much better. There is a considerable difference between .64 and.26. Note that the gross rate of success is greater and the area is smaller for equal share analysis.

Exhaustivity and the order of strength hypothesis do not add much to the predictive power of the equal division core. In fact, the rate of success for "order of strength and equal division core" is slightly greater than that for "equal share analysis".

The sample examined here covers a wide range of 3-person and 4-person characteristic function games. It is unlikely that the composition of the sample is biased in favor of equal share analysis.

In the interpretation of his experimental results Maschler makes use of his theory of the "power of a coalition" (Maschler 1963 and 1978). It is not quite clear how Maschler's power theory should be used in order to construct a modified bargaining set. It is quite possible that a suitable way of doing this will result in a much higher net rate of predictive success. The question must be left open for future investigation.

For the sample examined here, equal share analysis performs remarkably well. It is quite surprising that the equal division core alone achieves a net rate of success of .63. The concept of the equal division core is a very simple one. The bargaining set is much more complicated and its chance to achieve a comparable predictive success rests on the possibility of adding further complications.

Theory	gross rate of success R	area A	net rate of success S
Bargaining set	.28	.24	.04
Bargaining set with deviations up to 5	.43	.36	.07
Bargaining set without null structure	.26	.03	.23
Bargaining set without null structure and with deviations up to 5	.41	.15	.26
Order of strength	.93	.64	.29
Exhaustivity	.95	.60	.35
Equal division core	.79	.16	.63
Order of strength and equal division core	.76	.12	.64
Equal share analysis	.74	.10	.64

Table: Results

References

Aumann, R.J. and M. Maschler, The Bargaining Set for Cooperative Games, in: M. Dresher, L.S. Shapley, A.W. Tucker (eds.), Advances in Game Theory, Princeton University Press, Princeton, New Jersey, 1964, p. 443-476

Davis, M. and M. Maschler, Existence of Stable Payoff Configurations for Cooperative Games, in: Shubik (ed.), Essays in Mathematical Economics in Honor of Oskar Morgenstern, Princeton University Press, Princeton, New Jersey, 1967, p.39-52

Frassine, I., Fürst, E. and Winter, H., Verhandlungsspiele, ein experimenteller Beitrag, Forschungsbericht Nr. 7 des Instituts für Höhere Studien und Wissenschaftliche Forschung Wien, Wien 1967

Kalisch, G.K, Milnor J.W., Nash J. and Nering E.D., Some Experimental n-Person Games, in: Thrall, R.M., Coombs, C.H. and R.L. Davis (eds.), Decision Process, John Wiley and Sons, New York 1959, p. 163-177

Maschler M., The Power of a Coalition, Management Science 10, 1963, p.8-29

Maschler M., Playing an n-Person Game, an Experiment, in:H.Sauermann (ed.) Coalition Forming Behavior, Contributions to Experimental Economics Vol.8, J.C.B. Mohr, Tübingen, 1978

Peleg, B., Verhandlungsbereich, in: Beckmann M.J., G. Menges and R. Selten (eds.), Handwörterbuch der Mathematischen Wirtschaftswissenschaften, Vol. 1, Betriebswirtschaftlicher Verlag Dr. Th. Gabler KG, Wiesbaden 1979, p. 431-436

Selten, R., Equal Share Analysis of Characteristic Function Experiments, in: Sauermann, H. (ed.),Contributions to Experimental Economics, Vol. III, J.C.B. Mohr, Tübingen, 1972, p. 130-165

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