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Two-Person Bargaining Between
Threat and Fair Solution

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Summary: The paper gives a model of bargaining involving threats. Two types of solution concepts are introduced, one models bargaining in conflict situations, the other models fair solutions.
Experimental results indicate that the conflict-concept seems to be adequate, if the problems discussed include all issues which have to be ruled between the opponents. It does not make sense to apply the conflict-concept to single issues, since then logrolling is neglected.

The fairness concept only has its place as a helpful tool which can be applied to single issues to obtain overall results corresponding to the conflict-concept. It does, however, not generally lead to these solutions, so that additional arrangements are necessary.

1   Introduction:

Bargaining in two-person or two-party bargaining can be modeled from different points of view. The bargaining strength is the main criterion in the theory of HICKS (1932), who considers the possible costs of a strike as a determinant of the wage level. This approach has been extended by CHAMBERLAIN (1951) and in the bargaining model of CARTTER and MARSHALL (1972). In these concepts the solution of the wage problem is that amount where each player has the same threats, measured as possible lengths of a strike a player would involve.

Other theories, as those of ZEUTHEN (1930), PEN (1952) and SHACKLE (1964) refer more to the risk that a player takes when he starts a strike in order to reach his aims. Here the solution is that point, where both players have the same probability to escalate the conflict. Game theoretic-axiomatic approaches are given by ZEUTHEN (1930), NASH (1953), SHAPLEY (1953), RAIFFA (1953), HARSANYI (1956), HOMANS (1963), CROTT-KALAI-SMORODINSKI (1975) and

(1976), SCHMITZ (1977) and PERLES/MASCHLER (1979). All these theories only refer to the set of possible outcomes. In addition, ADAMS (1965) refers to the stake of the players. In this context the two-step bargaining procedure of NASH (1980) is worthy of note, where in a first round of bargaining the players determine a threat point and then select a cooperative solution which gives equal improvements with respect to the threat point to both players. HARSANYI and SELTEN (1972) generalized this approach to games with incomplete information. The concession-behavior has been modeled by STEVENS (1963) and in the aspiration adjustment theory of SAUERMANN and SELTEN (1962), and its extension by TIETZ (1975) and by OSGOOD (1959). The possible length of a strike is also the main criterion of BISHOP (1964) and FOLDES (1964), they however, model the bargaining process by stepwise concessions of the player with the -in the sense of the theory- weaker position. Dynamic theories of bargaining within time using differential equations have been given by CROSS (1965) and CODDINGTON (1966).

In this approach we model the solution as that point where both players get equal losses if the conflict escalates (fair solution) and as the point that the players reach, if both of them concede such that they just avoid a tremendous escalation of the conflict (threat-solution). Similar to the two step-NASH-procedure we predict that in view of the threat-solution (which gives both players the same utility as if they accept the ideal position of the other in the beginning of the escalation) the players find a cooperative solution, which gives both players equal surplus compared to the threat point.

2. Two-Person Bargaining

Let a bargaining-situation be given by

X space of alternatives

$D = \mathbb{R}_+$ degrees of escalation (which can be used without extending the conflict essentially)

$U_i : X \rightarrow \mathbb{R}, V_i : D \rightarrow$ utility functions of players $i = 1,2$ on X and D respectively.

The utility functions U_1, U_2 are supposed to be strictly quasi concave and to have unique maxima on X. The corresponding points, in which the maxima are attained are denoted as x^1 (for U_1) and x^2 (for U_2). We assume that x_1 and x_2 are the initial demands of the players in the bargaining procedure.

Possible threats are supposed to be orderable in a linear way according to the degree of escalation. At a certain degree of escalation all threats corresponding to that of a lower degree are imposed. $V_i(d)$ gives the utility of all threats belonging to d for player i. V_i is assumed to be monotonically decreasing and continuous.

Example:

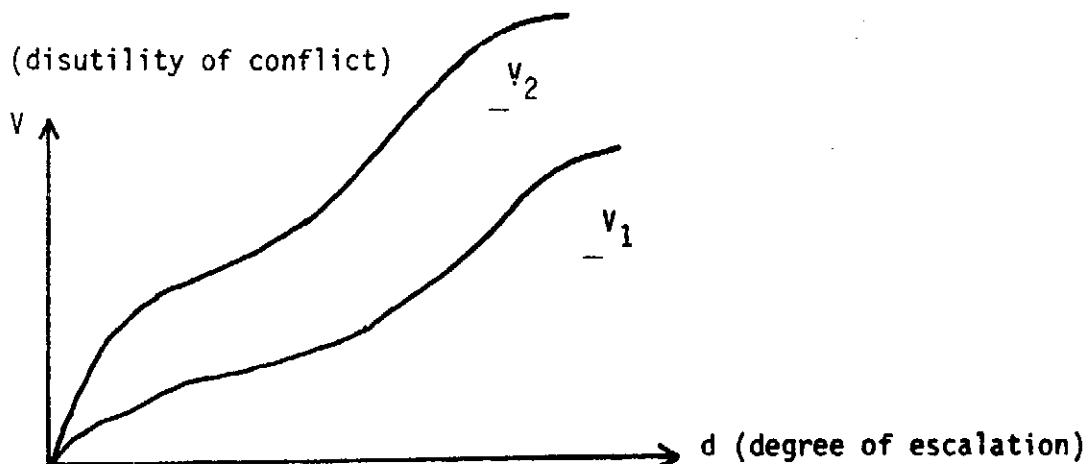(utility of alternative)
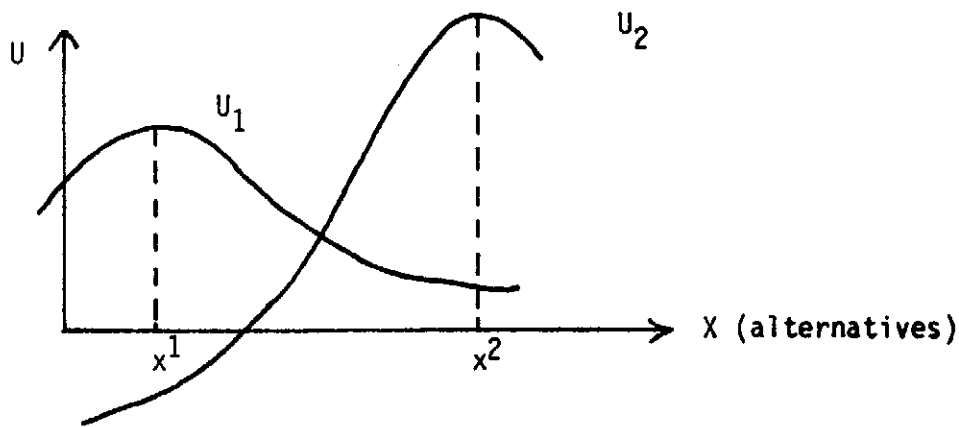
(disutility of conflict)

Figure 1: a) utilities of two players 1,2 on the space of alternatives
b) disutilities of a conflict as a function of the degree of escalation

3   The Fair Solution (for X = IR)

We assume now that $X = \mathbb{R}$ and introduce a "fair solution" by the following two assumptions:

(F1) In view of an extension of the conflict to a degree $d \in D$ the players agree to avoid the conflict by reducing their initial demands x^1 (in favor of the respective other player) to a new demand $x^i(d)$ in such a way that

$$U_i(x^i(t)) = U_i(x^i) - V_i(d)$$

I. e. each player reduces his demand in such a way that his positional loss equals the loss he would suffer, if the threat corresponding to the degree d of escalation would be verified.

(F2) The solution is given by that degree d^F of escalation, for which

$$x_i(d^F) = x_j(d^F) = : x^F.$$

4   The Threat-Solution (for X = IR)

Let still $X = \mathbb{R}$. The second solution-concept models a more conflicting situation. This approach models a bargaining process combined with an escalation of conflict. The assumption is that a player escalates the conflict to an essentially higher degree than modeled in D (which imposes essentially higher costs than the degrees of D), if the net-effect of the offers of the other (utility reduced by the disutility of the imposed threats) gets worse than the utility of the initial offer of the opponent. We assume that the disadvantage of this further escalation is so high that during an escalation following the steps of D both players reduce their demands in such a way that the other (just) does not initiate the further escalation.

Thus, we get the following conditions:

(T1) A player i escalates the conflict to a high degree (which is not modeled in D), if during the escalation of the conflict the other player j does not react on an escalation $d \in D$ by an offer $\tilde{x}^j(d)$ with

$$U_i(\tilde{x}^j(d)) - V_i(d) \geq U_i(\tilde{x}^j)$$

(T2) Each player j tries to give in as slowly as possible, i. e. he se-lects $\tilde{x}^j(d)$ in such a way that

$$U_i(\tilde{x}^j(d)) - V_i(d) = U_i(\tilde{x}^j)$$

for all $d \in D$.

(T3) The threat-solution is given by that degree $\tilde{d} \in D$ of the extension of the conflict, for which

$$\tilde{x}^i(\tilde{d}) = \tilde{x}^j(\tilde{d}).$$

So $\tilde{d}$ is the highest degree of conflict for which it is possible to avoid the further extension of the conflict to alternatives outside D. In $\tilde{d}$ the corre-sponding offers $\tilde{x}^i(\tilde{d})$ and $\tilde{x}^j(\tilde{d})$ of the players coincide, but for each player the utility of the solution is the same as if he had accepted the opening offer of his opponent before the escalation of the conflict.
An example of such an escalated conflict is a strike, where the set D is given by the possible lengths of strike. The essentially higher escalation can be for instance to close or destroy the firm. In this example the conflict is automatically escalated (to a higher length of strike) if the players do not reach an agreement.
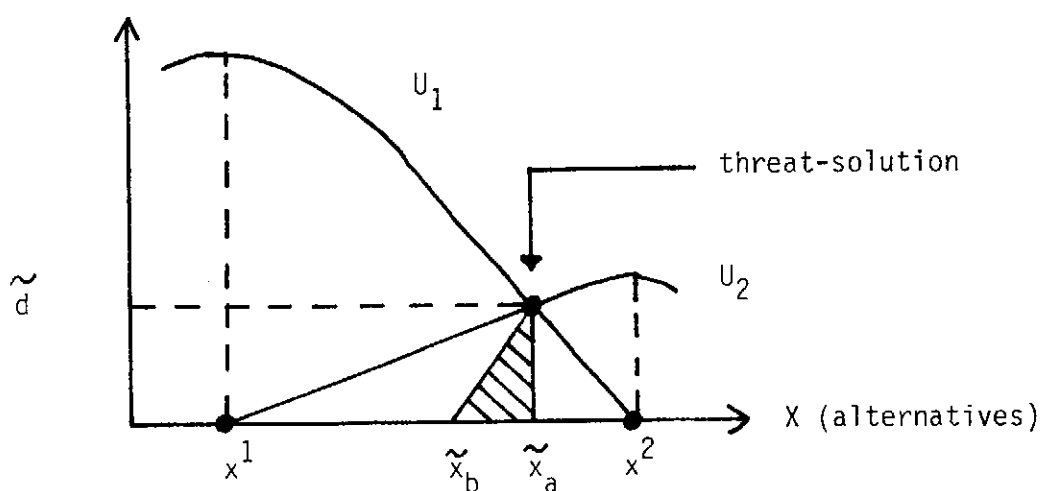Example:

(degree of conflict)
D

Figure 2: Iso-utility lines of the players in the space of alternatives x degree of conflict.
(For each player his level of utility is given his value of the ideal position of the other if the conflict is not escalated (d=0).)

It should be remarked that finding the cooperative solution "below" the threat point with equal improvements of utility for both players involves a comparison of the utility scales. In the situation given here, two extreme alternatives for such a comparison can be suggested:

(a) Utility comparison via degree of threat: Here we assume that the players evaluate their improvements as equal if both of them can avoid the same degree of escalation. The corresponding solution is $\tilde{x}_a$;

(b) Utility comparison via a natural scale of the space of alternatives: If the space of alternatives has a natural scale (as the amount of wages) then the concessions can be measured by the distances on the x - scale. This way of utility comparison gives the mid-point of the ideal positions of the players as a solution, since (for fixed d = 0) the distances to the iso-utility lines U^1 and U^2 through the threat point are equal. The corresponding solution is $\tilde{x}_b$.

Experimental results indicate that condition (T2) is usually met by real behavior. Players try to avoid the maximal possible conflict and then usually find a cooperative solution below the threat point (which improves the utilities of both of them). The problem of this cooperation is, however, how to compare the advantages of the players with respect to the threat point.
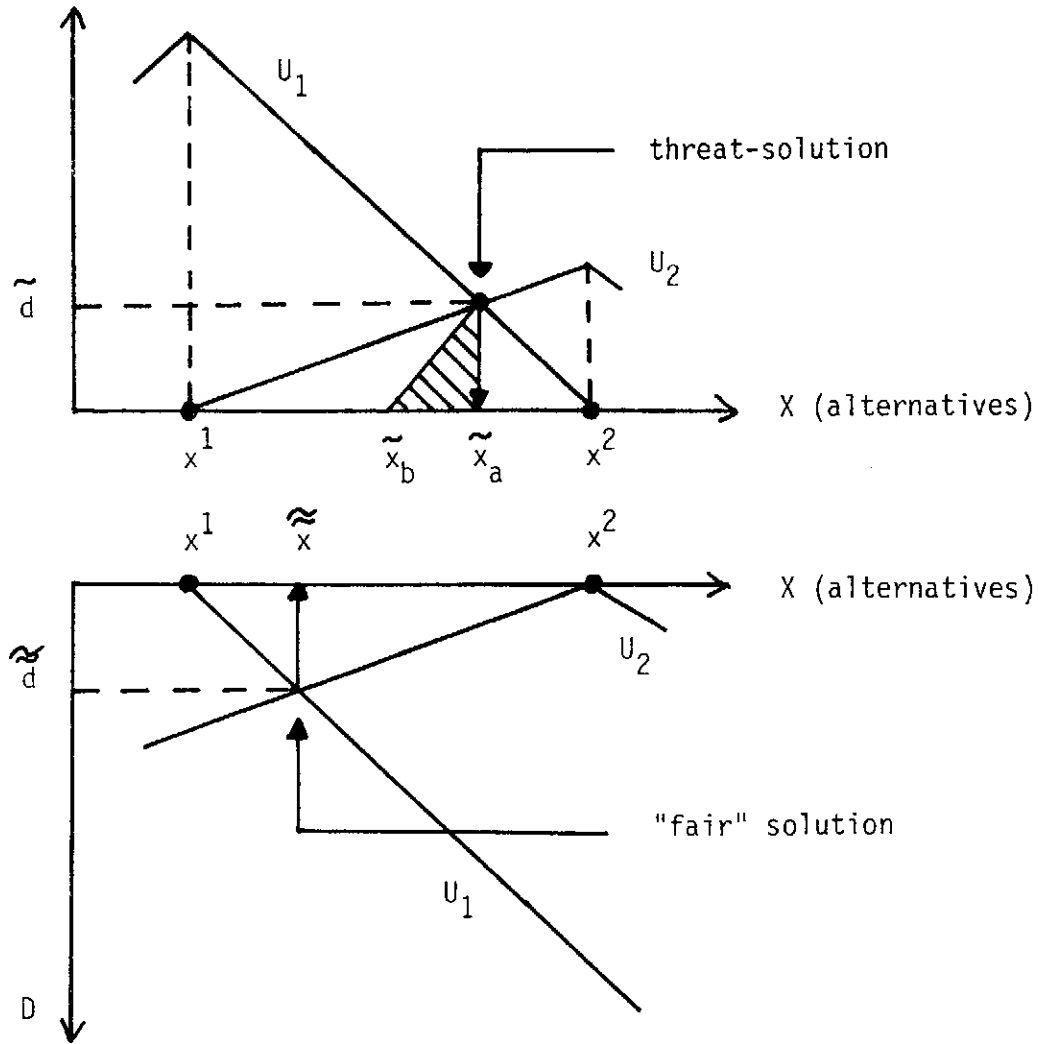
5   Differences of the Two Concepts

We illustrate the differences of the two concepts by an example: (see figure 3).

It can easily be seen that the fair solution and the threat-solution favor different players. The fair solution corresponds to the bargaining theory of HICKS (1963). Experimental results, SCHWIND (1977), support the threat-solution. In this experiment the space of alternatives had a natural measure and the solutions where between $\tilde{x}_a$ and $\tilde{x}_b$.

(degree of escalation)

D

Figure 3: Threat-solution and conflict solution and the corresponding iso-
utility lines of the players in the 1-dimensional case

6 Threat-Solutions for $X = \mathbb{R}$

We now consider the threat-solution for $X = \mathbb{R}^n$. In this case (if for example
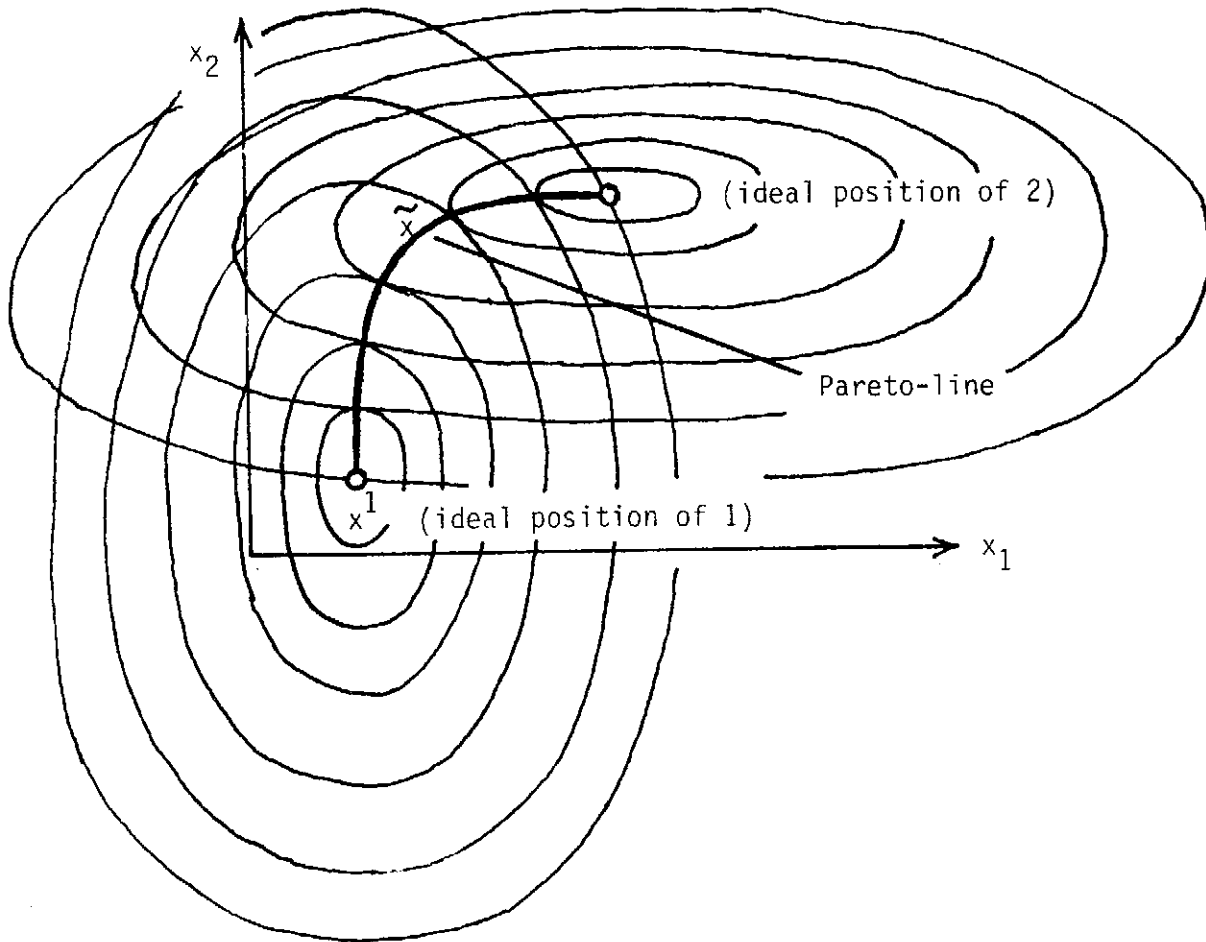no threats are imposed) the situation looks as in the following example:

Figure 4: Iso-utility lines of two players in a two dimensional space if no threats are involved.

To apply the threat-theory we assume that rational players will bargain on a solution point on the Pareto line $x^1 x^2$. We stretch this line and model the solution point as in the 1-dimensional case. [1] Thus, we get the threat-solution above $\tilde{x}$ on the Pareto-line: [2]

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1) This reduction to the 1-dimensional case is generally possible, if the utility functions are strictly quasi concave, since the Pareto-frontier is 1-dimensional in also n-dimensional case.
2) Since there is no natural measure on the Pareto-line, we only predict the cooperative solution $\tilde{x}_a$.
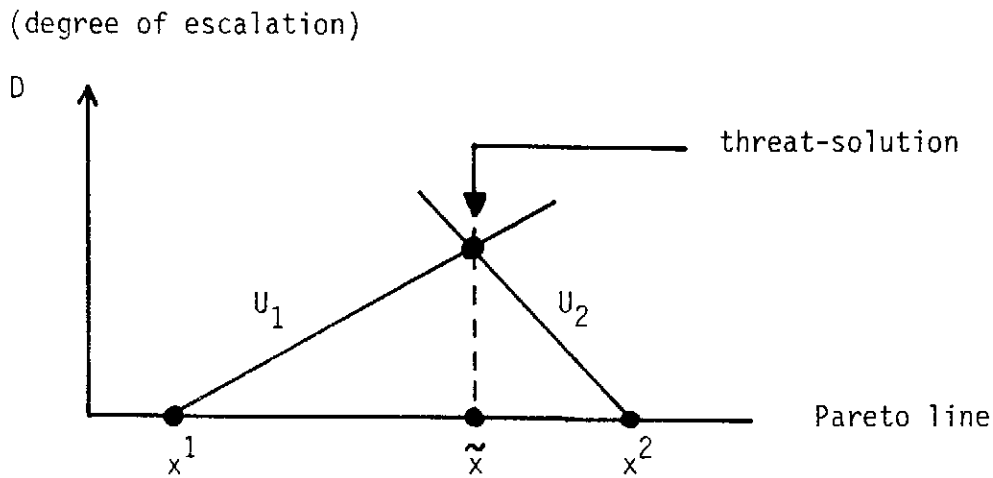
(degree of escalation)

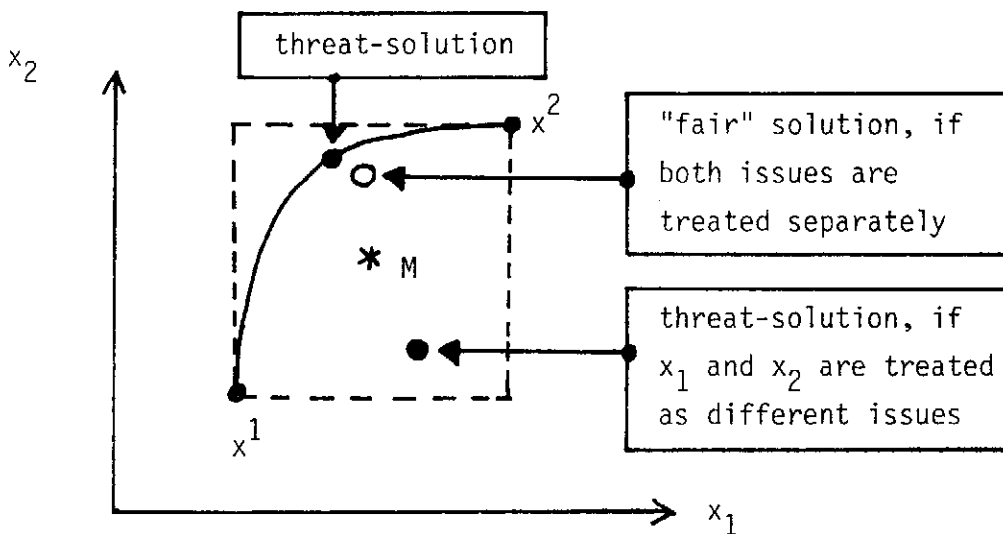Figure 5: Threat-solution on the Pareto-line of the multi-dimensional conflict

Figure 6: Solution points corresponding to different solution concepts in case of a two-dimensional space of alternatives

We remark that the threat-solution selects a point which gives to players 1 and 2 higher utility than the mid-point M of x^1 and x^2. (If the dimensions of the space X would have been bargained separately the solution point would have been worse than M for both players.)

The fair solution, which is obtained, if both issues are treated separately leads to a result, which is "quite near" to the threat-solution of the global

(2-dimensional) problem.

Our opinion is, that generally the threat-solution models <u>possible</u> bargaining results adequately.
However, if problems become too difficult, so that they must be splitted up into different issues (and this is generally the case for the permanent bargaining situation between states) each issue may be treated by the fairness-concept.

This is, however, only a convention and not directly justified by strategic power. To establish a balance of concessions the overall concessions must be counted and eventually corrected by additional arrangements - as for instance the introduction of interest spheres.

Essential new problems will not necessarily be solved on the conventional and unreflected basis of a "fair" solution, but will follow considerations as outlined in the threat-solution concept.

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